27-mavzu. O‘tkinchi jarayonlarni operator usulda hisoblash.

Reja

1. Tasvir va original tushunchalari.
2. Laplas o‘zgartirishi.

1.Tasvir va original tushunchalari.

Oddiy funksiya f(t)=U_0 bo`lganda uning tasviri F(p) ni topish talab etilsin:

$$F(p)=\int_0^{\infty} e^{-pt}\cdot U_0 dt=-\frac{U_0}{p}e^{-pt}\Big|_0^{\infty}=\frac{U_0}{p}.$$

Demak o`zgarmas kattalikning tasviri shu kattalikning operator r ga bo`linganiga teng. Shunigdek eksponensial funksiya f(t)=$e^{\alpha t}$ íèíã òàñâèðè:

$$F(p)=\int_0^{\infty} e^{-pt}\cdot e^{\alpha t}dt=-\frac{1}{p-\alpha}e^{-(p-\alpha)t}\Big|_0^{\infty}=\frac{1}{p-\alpha}$$

Demak: $e^{\alpha t}\ \frac{1}{p-\alpha}$;

1. $$e^{-\alpha t}=\frac{1}{p+\alpha};$$
2. $$1-e^{-\alpha t}=\frac{1}{p}-\frac{1}{p+\alpha}=\frac{\alpha}{p(p+\alpha)};$$
3. $$e^{j(\omega t+\psi)}=e^{j\psi}\cdot e^{j\omega t}=\frac{e^{j\psi}}{p-j\omega};$$
4. $$\sin\omega t=\frac{e^{j\omega t}-e^{-j\omega t}}{j2}=\frac{1}{j2}\left[\frac{1}{p-j\omega}-\frac{1}{p+j\omega}\right]=\frac{\omega}{p^2+j\omega^2};$$
5. $$\cos\omega t=\frac{e^{j\omega t}+e^{-j\omega t}}{2}=\frac{1}{2}\left[\frac{1}{p-j\omega}+\frac{1}{p+j\omega}\right]=\frac{p^2}{p^2+j\omega^2};$$ va q.k.

Vaqt jixatidan X ga siljigan original funksiyaning tasviri $f(t-x)=e^{-px}F(p)$ bo`ladi. Bunda t-x=t’≥0.

Tasvirlar bilan tegishlicha operatsiya bajarilgandan keyin originallarga qaytish mumkin.

2.Laplas o‘zgartirishi.

Berilgan qosila funksiyasi $\frac{d}{dt}[f(t)]=f'(t)$ ning tasvirini topish talab etiladi deylik:

$$f'(t)=\int_0^{\infty} f'(t)e^{-pt}dt$$

Kasirlar bo`yicha integrallash qoidasi:

$$\int_0^t (uv)'dt = \int_0^t uv'dt + \int_0^t vu'dt$$ dan foydalanib quyidagini yozamiz:

$$f'(t) = \left| e^{-pt} \cdot f(t) \right|_0^\infty - \int_0^\infty f(t)(e^{-pt})'dt = p\int_0^\infty f(t)e^{-pt}dt - f(0) = pF(p) - f(0)$$

Àãàð f(0)=0 áo`ëñà :

$$f'(t)=pF(p)$$

Ya'ni f(t) dan qosila olish uning tasvirini operator r ga ko`paytirish bilan baravar. Agar funksiya φ(t), $\varphi(t) = \int_0^T f(t)dt$ uning Laplas bo`yicha tasviri :

$$\psi(p) = \int_0^T \varphi(t)e^{-pt}dt;$$

Qisimlar bo`yicha integrallash qoidasiga binoan:

$$\psi(p) = -\frac{e^{-pt}}{p}\int_0^t f(t)dt\Big|_0^\infty + \int_0^\infty \frac{e^{-pt}f(t)}{p}dt = 0 + \frac{1}{p}F(p)$$

Demak $\varphi(t) = \int_0^T f(t)dt = \frac{F(p)}{p}$ funksiya integralining tasviri funksiya tasvirining operator r ga bo`linganiga teng. Lekin boshlang`ich shartlari nolga teng

$$\psi(p) = \frac{1}{p}F(p) + \frac{f(0)}{p}.$$

Sinov savollari.

1. Laplas almashtirishi formulasini tushuntirib bering?
2. Karson-Xevisayd almashtirishi bilan Laplas almashtirishining farqi nimada ?
3. Funksiyaning qosilasi Laplas almashtirishi bo`yicha tasvirini yozing ?
4. Funksiyaning integralini Laplas almashtirishi bo`yicha tasvirini yozing ?