

26-ma`ruza. Om va Kirxgof qonunlarining operator shakli.

Reja

1. Om va Kirxgof qonunlarining operator shakli.
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1. Om va Kirxgof qonunlarining operator shakli.

1.1 Kirxgof qonunining operator shaklidagi tasviri.

Zanjir tugunidagi toklar qiymatlarining yig`indisi nolga teng:

$$\sum_{k=1}^n i_k = 0.$$

Har qanday tarmoqdagi tok $i_k(t)$

$$\sum_{k=1}^n I_k(p) = 0 \quad (1)$$

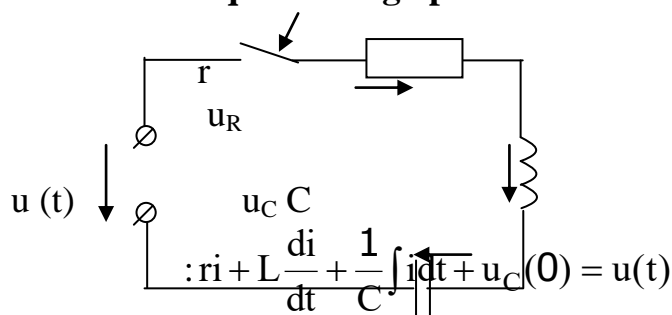
Kirxgofning II-qonuni.

$$\sum_{k=1}^n U_k = \sum_{k=1}^n e_k.$$

Operator ko`rinishda $\sum_{k=1}^n U_k(p) = \sum_{k=1}^n E_k(p) \quad (2)$

(1) va (2) chi Kirxgof qonunlarining operator shaklidagi ifodasi.

1.2. Om qonunining operator shaklidagi tasviri.



$$r \int_0^{\infty} i(t) e^{-pt} dt + L \int_0^{\infty} i'(t) e^{-pt} dt + \frac{1}{C} \int_0^{\infty} \left[\int_0^t i(t) dt \right] e^{-pt} dt + \int_0^{\infty} u_C(0) e^{-pt} dt = \int_0^{\infty} u(t) e^{-pt} dt.$$

$$rI(p) + pLI(p) - Li(0) + \frac{I(p)}{pC} + \frac{u_C}{p} = u(p).$$

$$i(t)=I(p) \hat{=} u(t)=u(p).$$

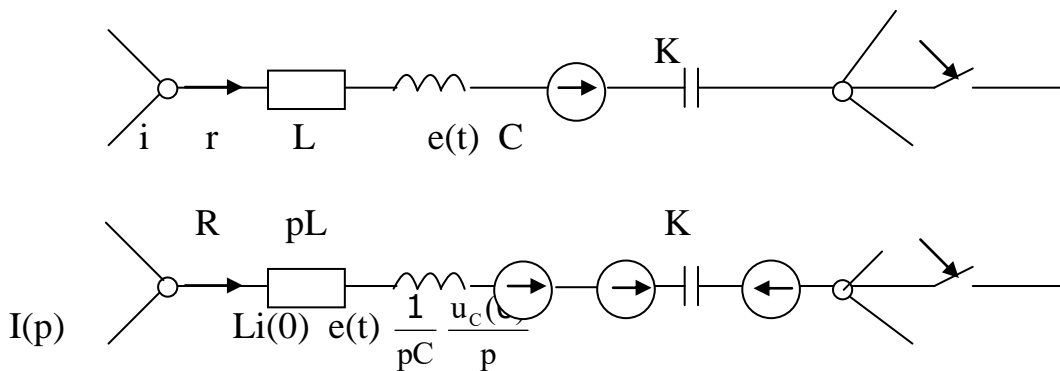
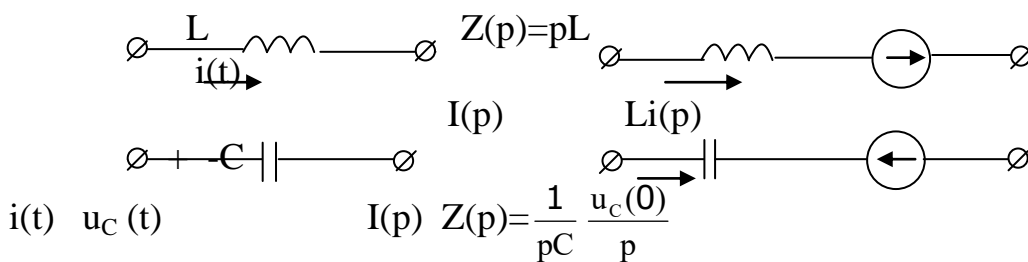
$$I(p) = \frac{u(p) + Li(0) - \frac{u_C(0)}{p}}{r + pL + \frac{1}{pC}} = \frac{u(p) + Li(0) - \frac{u_C(p)}{p}}{Z(p)}$$

$Z(p) = r + pL + \frac{1}{pC}$ - operator qarshilik.

Om qonunining operator shaklidagi ifodasi nolinch boshlang'ich shartlarda:

$$I(p) = \frac{u(p)}{z(p)}$$

Operator sxema oldin keltirilgan sinusoidal turg'un xolatga chizilgan sxemadan farq qiladi. Operator ko'rinishda kompleks kattaliklar $j\omega L \hat{=} pL$ lar $pL \hat{=} pC$ ga almashtiriladi.

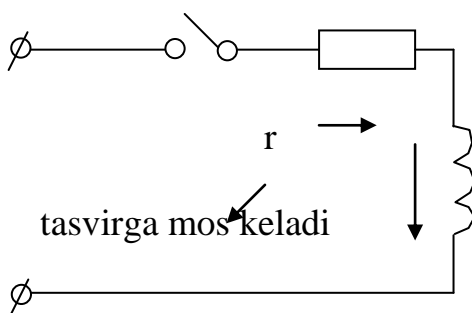


$\sim U$

U_L

L

2.Yoyish Teoremasi



RL zanjirni sinusoidal kuchlanish

$$U(t) = Um \sin(\omega t + \psi)$$

Ulash jadvalga binlan zanjirga berilgan Kuchlanishi $u(t)$ ning funksiyasi quyidagi

$$U(p) = U_m \frac{p \sin \psi + \omega \cos \psi}{p^2 + \omega^2}$$

Operator shaklida o`tkinchi tok

$$I(p) = \frac{U(p)}{Z(p)} = U_m \frac{p \sin \psi + \omega \cos \psi}{(p^2 + \omega^2)(r + pL)} = \frac{\frac{U_m}{L}(p \sin \psi + \omega \cos \psi)}{(p + j\omega)(p - j\omega)(p + \frac{r}{L})} \quad (1)$$

$$\frac{F_1(p)}{F_2(p)} = \frac{A_1}{p - p_1} + \frac{A_2}{p - p_2} + \dots + \frac{A_n}{p - p_n} = \sum_{k=1}^n \frac{A_k}{p - p_k}$$

bu yerda: A_1, A_2, \dots, A_n – yeyish koeffisientlarini ifodalovchi oddiy xaqiqiy sonlar,

r_1, r_2, \dots, r_m – tenglama $F_2(p) = 0$ ning ildizlari
Koeffisient A_k quyidagicha aniqlanadi.

$$A_k = \frac{F_1(p_k)}{F_2'(p_k)}$$

Ko`rib chiqilayotgan misolda tekshirilayotgan operator funksiya quyidagi kasrni ifodalaydi,

$$I(p) = \frac{F_1(p)}{F_2(p)} = \frac{\frac{U_m}{L} \sin \psi p + \omega \frac{U_m}{L} \cos \psi}{(p + j\omega)(p - j\omega)(p + \frac{r}{L})};$$

Moxrae $F_2(p)$ ning ildizlari:

$$R_1 = -j\omega, p_2 = j\omega, p_3 = -\frac{r}{L},$$

Yeyish koeffisientlarini aniqlaymiz:

$$\begin{aligned} A_1 &= \frac{\frac{U_m}{L}(-j\omega) \sin \psi + \frac{U_m \omega}{L} \cos \psi}{(p^3 + \frac{r}{L} p^2 + \omega^2 p + \omega^2 \frac{r}{L})_{[p=-j\omega]}} = \frac{\frac{U_m \omega}{L}(\cos \psi - j \sin \psi)}{(3p^2 + 2\frac{r}{L} p + \omega^2)_{[p=-j\omega]}} = -\frac{\frac{U_m}{L} \omega e^{-j\psi}}{2\omega^2 + j2\omega \frac{r}{L}} \\ &= -\frac{U_m e^{-j\psi}}{2(\omega L + jr)} = -\frac{U_m e^{-j\psi}}{2j(r - j\omega L)} = -\frac{U_m e^{j\psi}}{2jze^{-j\varphi}} = -\frac{U_m e^{-j(\psi - \varphi)}}{2jZ} \end{aligned}$$

$$A_2 = \frac{\frac{Um\omega}{L}(\cos\psi + j\sin\psi)}{(3p^2 + 2\frac{r}{L}p + \omega^2)[p = j\omega]} = \frac{\frac{Um\omega}{L}e^{j\psi}}{-2\omega + 2j\omega\frac{r}{L}} = \frac{Ume^{j\psi}}{2j(r + j\omega L)} = \frac{Ume^{j\psi}}{2jZe^{j\phi}} = \frac{Ume^{j(\psi-\phi)}}{2jZ}$$

$$A_3 =$$

$$= \frac{\frac{Um}{L}(-\frac{r}{L})\sin\psi + \frac{Um\omega}{L}\cos\psi}{(3p^2 + 2\frac{r}{L}p + \omega^2)\left[p = -\frac{r}{L}\right]} = -\frac{Um(r\sin\psi - \omega L\cos\psi)}{L^2(\omega^2 + \frac{r^2}{L^2})} = -\frac{Um^2(\sin\psi\cos\phi - \cos\psi\sin\phi)}{Z^2} =$$

$$= -\frac{Um}{Z}\sin(\psi - \phi).$$

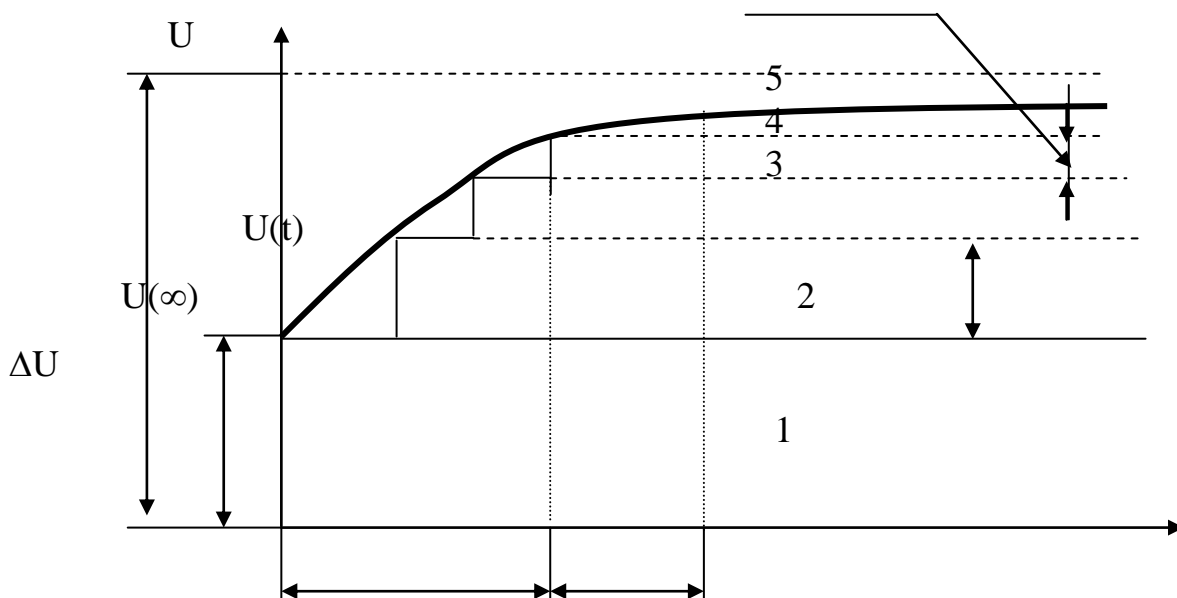
$$I(p) = \frac{F_1(p)}{F_2(p)} = -\frac{Ume^{-j(\psi-\phi)}}{2jZ(p + j\omega)} + \frac{Ume^{j(\psi-\phi)}}{2jZ(p - j\omega)} - \frac{Um\sin(\psi - \phi)}{Z(p + \frac{r}{L})}.$$

Jadval asosida oddiy kasrlar yig'indisining xar bir tashkil etuvchisiga mos originalini tanlaymiz:

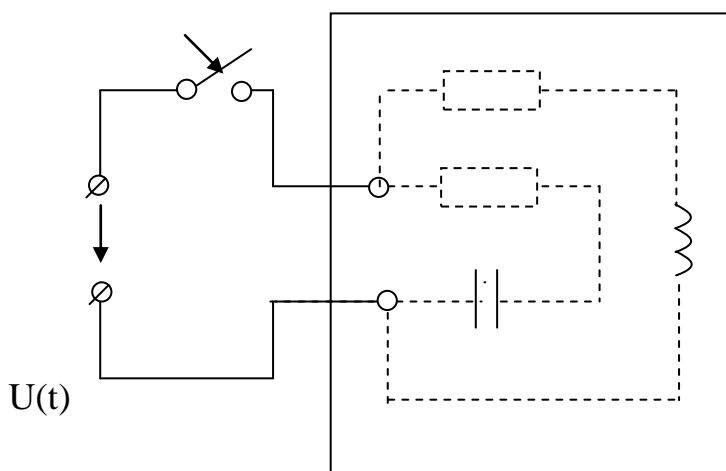
$$\begin{aligned} i(t) &= \frac{Ume^{-j(\psi-\phi)}e^{-j\omega t}}{2jZ} + \frac{Ume^{j(\psi-\phi)}e^{j\omega t}}{2jZ} - \frac{Um}{Z}\sin(\psi - \phi)e^{-\frac{r}{L}t} = \\ &= \frac{Um}{Z} \frac{e^{j(\omega t + \psi - \phi)} - e^{-j(\omega t + \psi - \phi)}}{2j} - \\ &- \frac{Um}{Z}\sin(\psi - \phi)e^{-\frac{r}{L}t} = \frac{Um}{Z} \left[\sin(\omega t + \psi - \phi) - \sin(\psi - \phi)e^{-\frac{r}{L}t} \right] \end{aligned}$$

3.Dyuamel integrali.

$$\Delta U = \frac{\Delta U}{\Delta X} \Delta X$$



Passiv zanjirlarni o'zgarmas yoki sinusoidal kuchlanishga ulaganda bo'ladigan o'tkinchi jarayonlarni qisoblash nisbatan oddiy, chunki kommutatsiyadan keyin turg'unlashgan va erkin tok va kuchlanishlar oddiy qamda yaxshi o'rganilgan qonunlar bo'yicha o'zgaradi. Lekin manbaning kuchlanishi vaqt jixatidan ix tiyoriy qonunga ko'ra o'zgarsa masala birmuncha murakkablashadi.



Zanjir $t=0$ da U_0 . U_0 o'zgarmas kuchlanishdan iste'mol qilinadigan o'tkinchi tok shu kuchlanish U_0 ni o'tkinchi o'tkazuvchanlik deb ataladigan $Y(t)$.

$$i(t) = U_0 Y(t)$$

$Y(t) \hat{=} U_0$. Unda ketma-ket ulangan elementlar uchun:

$$i(t) = U_0 Y(t) = U_0 \frac{1}{r} (1 - e^{-\frac{t}{\tau}})$$

Tokning o'zgarish qonuni o'tkinchi o'tkazuvchanlikning o'rganish qonunida aks etiladi:

$$Y(t) = \frac{1}{r} (1 - e^{-\frac{r}{L}t})$$

Kuchlanish U_0 ni yagona 1V ga teng impuls bilan almashtirsak:

$$i(t) = 1 \cdot Y(t) = Y(t)$$

$Y(t)$ istalgan vaqt paytida o'tkinchi tok $i(t)$ ning oniy qiymati son jiqatidan aniqlaydi. Zanjir ΔU impulsiga ulashdagi o'tkinchi jarayonni vaqt jixatdan bir qancha mikroprotsesslarga yoyamiz. Kuchlanish $U(0)$ ning birinchi sakrashi o'tkinchi tok $i(t)$ ning $U(0) \cdot Y(t)$ tashkil etuvchisini beradi. ΔX vaqtdan so'ng ΔU sakrash bo'lib, o'tkinchi tok $\Delta U \cdot Y(t - \Delta X)$. X vaqtdan so'ng o'tkinchi tokda

$$Y(t - \Delta X) \frac{\Delta U}{\Delta X} \Delta X$$

sakrash bo'ladi va q.k. $t \rightarrow \infty$. Kuchlanish $U(t)$ ning ta'siri quyidagi o'tkinchi tok bilan ifodalanadi:

$$i(t) \cong U(0)Y(t) + \sum_{X=0}^{k=t} Y(t-X) \frac{\Delta U}{\Delta X} \Delta X \quad (1)$$

ΔU va ΔX ortirma qanchalik kichik bo'lsa, o'tkinchi jarayonning xaqiqiy manzarasi shuncha aniq bo'ladi, $\Delta X \rightarrow 0$ bo'lsa:

$$U'(X) = \lim_{\Delta X \rightarrow 0} \frac{\Delta U}{\Delta X} = \left(\frac{dU}{dt} \right)_{t=X}$$

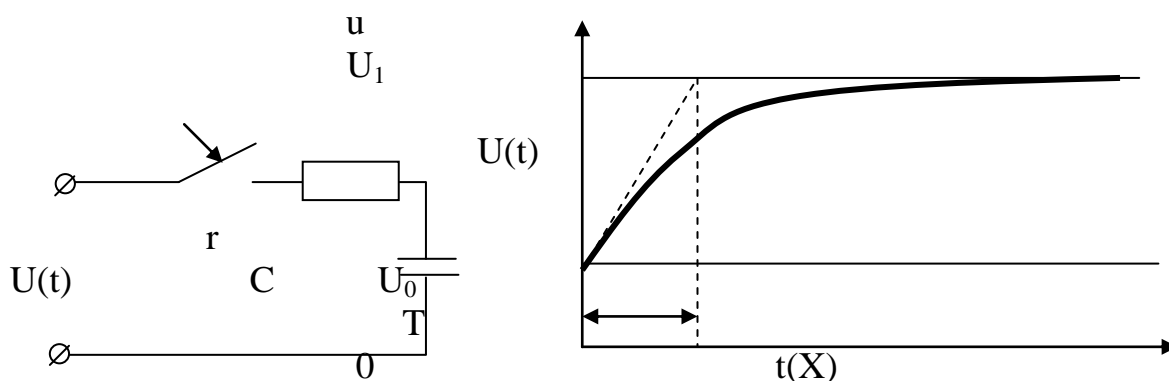
$$i(t) = U(0)Y(t) + \int_0^t Y(t-X) \cdot U'(X) dX \quad (2)$$

Dyuamel integrali yordamida o'tkinchi jarayonlarini hisoblash.

Misol. Berilgan: $U(t) = U_1 - (U_1 - U_0) e^{-\frac{t}{T}}$

$$U_0 = 100 \text{ V}; U_1 = 220 \text{ V}; T = 0,02 \text{ sek}; r = 10^3 \text{ Ohm}; S = 80 \text{ mkf.}$$

$i(t)$ -?



$$Y(t) = \frac{1}{r} e^{-\frac{t}{T}} = 10^{-3} e^{-12,5t} \left[\frac{1}{0,06} \right]; \quad \tau = rC = 0,08.$$

$$U(t) = U_1 - (U_1 - U_0) e^{-\frac{t}{\tau}}$$

$$U(x) = U_1 - (U_1 - U_0) e^{-\frac{x}{T}} = 220 - 120 e^{-50x};$$

$$\begin{aligned} i(t) &= U(0)Y(t) + \int_0^t Y(t-x)U'(x)dx = U_0 \frac{1}{r} e^{-\frac{t}{\tau}} + \int_0^t \frac{1}{r} e^{-\frac{t-x}{\tau}} \times \frac{U_1 - U_0}{T} e^{-\frac{x}{T}} dx = \\ &= \frac{U_0}{r} e^{-\frac{t}{\tau}} + \frac{U_1 - U_0}{rT} e^{\frac{t}{\tau}} \int_0^t e^{-\left(\frac{1}{T} - \frac{1}{\tau}\right)x} dx = \frac{U_0}{r} e^{-\frac{t}{\tau}} - \frac{(U_1 - U_0)\tau}{r(\tau - T)} e^{-\frac{t}{\tau}} \left[e^{-\frac{t}{T}} \cdot e^{\frac{t}{\tau}} \right] = \\ &= \frac{U_0}{r} e^{-\frac{t}{\tau}} + \frac{(U_1 - U_0)}{r(\tau - T)} e^{-\frac{t}{\tau}} - \frac{(U_1 - U_0)\tau}{r(\tau - T)} e^{-\frac{t}{T}} = 0,1 e^{-12,5t} + \frac{120 \cdot 0,08}{10^3 \cdot 0,06} e^{-12,5t} - \\ &= \frac{120 \cdot 0,08}{10^3 \cdot 0,06} e^{-50t} = 0,26 e^{-12,5t} - 0,16 e^{-50t} \quad (A) \end{aligned}$$

zanjirni ulash paytida iste'mol qilanayotgan tok maksimal bo'ladi:

$$i(0) = 0,26 - 0,16 = 0,1 \text{ A}$$

$$i(\infty) = 0$$

Sinov savollari.

1. Yoyish teoremasi qanday xollarda qo'llaniladi?
2. r, C zanjirni qisqa tutashtirish xolati operator usulda qanday qisoblanadi?
3. r, L zanjirga kondensatorning aperiodik zaryadsizlanishi operator usulida xisoblash?

