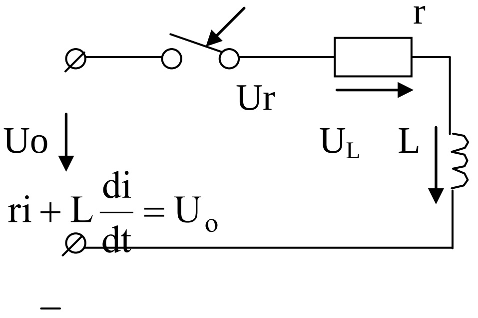
24-ma`ruza. Oddiy elektr zanjirlarida o‘tkinchi jarayonlarni klassik usulda xisoblash.

Reja

1.O`tkinchi jarayonni qisoblashning klassik usuli.
2.Kondensatorning aperiodik, chegaraviy aperiodik va tebranma zaryadsizlanishi.

r,l elementlari ketma-ket ulangan zanjirni o`zgarmas kuchlanish manbaiga ulash.

Zanjir t=0 da o`zgarmas kuchlanish Uo ga ulanadi
Zanjirning differensial tenglamasi:



$$ri + L\frac{di}{dt} = U_o$$

Unga mos bir jinsli tenglama quyidagi ko`rinishga ega:

$$L\frac{di_{эрк}}{dt} + ri_{эрк} = 0$$

$i_{эрк}$ aniqlaymiz, uning xarakteristik tenglamasi quyidagicha bo`ladi:

$$L\alpha + r = 0$$

U birgina xaqiqiy va manfiy $\alpha = -\frac{r}{L}$

$$i_{эрк} = Ae^{\alpha t} = Ae^{-\frac{r}{L}t} = Ae^{-\frac{t}{\tau}}$$

bunda: $\alpha = -\frac{r}{L}$ so`nish koeffisienti

Integrallash doimiysini A boshlang`ich shartlar orqali aniqlaymiz. Kommutatsiyaga qadar zanjirdan tok o`tmagan:

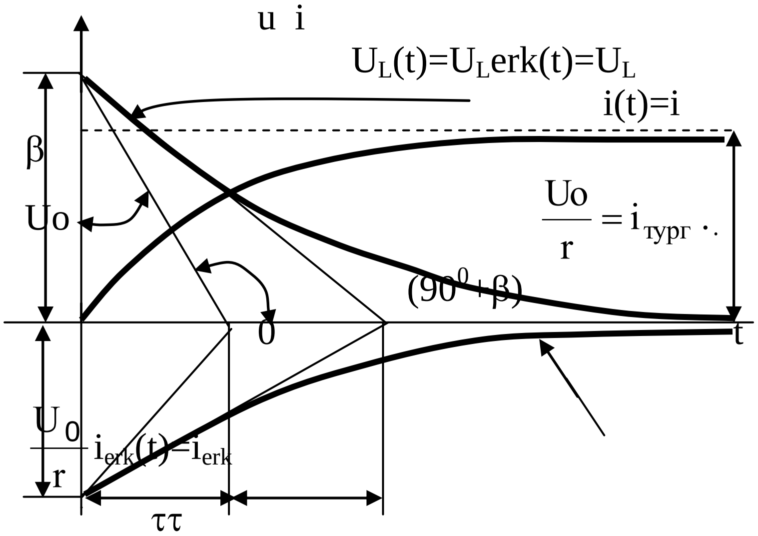
$$i_{-}(0) = 0$$

demak zanjirni ulash paytida xam u nolga teng bo`lgan i(0) =0 Zanjirda o`tkinchi protsess tugagandan so`ng, faqat r ning qarshiligi bilan aniqlanadigan $i_{тург} = \frac{U_0}{r}$ tok bo`ladi, unda:

$$i(0) = i_{тург}(0) + i_{эрк}(0) = \frac{U_0}{r} + A = 0$$

Shunday qilib $A = -\frac{U_0}{r}$ A bo`lib o`tkinchi tok i esa:

$$i = i_{тург} + i_{эрк} = \frac{U_o}{r} - \frac{U_o}{r} e^{-\frac{t}{\tau}} = \frac{U_o}{r}(1 - e^{-\frac{t}{\tau}})$$



τ-vaqt doimiysi; τ=L/r.

Zanjirning ayram elementlaridagi o`tkinchi kuchlanishlarni aniqlash mumkin:

$$U_r = ri = U_o(1 - e^{-\frac{t}{\tau}})$$

$$U_L = L\frac{di}{dt} = \frac{L}{r}U_o\left[0 - (-\frac{1}{\tau})e^{-\frac{t}{\tau}}\right] = U_o e^{-\frac{t}{\tau}}$$

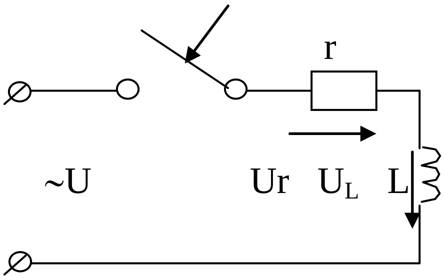
Masala:

Berilgan: r=50 Om, L=0,1 Gn , Uo=200 B.

$\tau = \frac{L}{r} = 0{,}002$сек ; $i_{turg`}$ =Io= $\frac{Uo}{r} = \frac{200}{50} = 4$A; t=5τ =0,01 cek.

$$i = I_0(1 - e^{-\frac{t}{\tau}}) = 4(1 - e^{-5}) = 4 \cdot 0{,}099325 = 3{,}97A$$

R,l elementlari ketma-ket biriktirilgan zanjirni sinusoidal o`zgaruvchan kuchlanish manbaiga ulash.



r va L sinusoidal kuch U=UmSin(ωt+ψè)

$$L\frac{di}{dt}+ri=U_m\sin(\omega t+\psi_u)$$ bo`lib

turg`unlashgan tok:

$$i_{тург}=\frac{Um}{z}\sin(\omega t+\psi_u-\varphi)$$ ni xosil qiladi.

bu yerda: $z=\sqrt{r^2+\omega^2L^2}$ – zanjirning to`la qarshiligi

$\varphi=\mathrm{arctg}\frac{\omega L}{r}$ –kuchlanish u va tok i_{turg} orasidagi siltish burchagi.

Oldingi paragrafdan ma’lumki, tok=erkin tashkil etib:

$$i_{эрк}=Ae^{-\frac{t}{\tau}}$$

Bunda zanjirdagi o`tkinchi tok qo`yidagicha o`zgaradi:

$$i=i_{тург}+i_{эрк}=\frac{U_m}{z}\sin(\omega t+\psi_u-\varphi)+Ae^{-\frac{t}{\tau}}$$

A-ni aniqlash uchun boshlang`ich shartlarga murojat qilamiz, kommutatsiya birinchi qonuniga ko`ra:

$$i_-(0)=(0)=\frac{U_m}{Z}\sin(\psi_u-\varphi)+A=0$$ ga egamiz

bundan: $$A=-\frac{U_m}{Z}\sin(\psi_u-\varphi)$$

$$i=\frac{U_m}{Z}\sin(\omega t+\psi_u-\varphi)-\frac{U_m}{Z}\sin(\psi_u-\varphi)e^{-\frac{t}{\tau}}$$ bo`ladi. (1)

Induktivlik g`altagining qismalaridagi o`tkinchi kuchlanish qo`yidagicha:

$$U_L = L\frac{di}{dt} = \frac{\omega L}{Z} U_m \cos(\omega t + \psi_u - \varphi + \frac{\pi}{2}) + \frac{r}{z} U_m \sin(\psi_u - \varphi) e^{-\frac{t}{\tau}} = \qquad (2)$$

$$= U_m \left[\sin\varphi \cos(\omega t + \psi_u - \varphi + \frac{\pi}{2}) + \cos\varphi \sin(\psi_u - \psi) e^{-\frac{t}{\tau}} \right]$$

Induktivlikdagi o`tkinchi tok va kuchlanish uchun olingan ifodalarning to`g`riligiga vakt t ga 0 dan $\infty$ gacha chegaraviy qiymatlar berib ishonch qosil qilish mumkin. Masalan, vaqt t=0 da (1) ning o`ng qismi nolga aylanadi, bu esa induktivlikdagi tok uchun kommutatsiya qonuni tasdiqlaydi:

$$i_{-}(0) = i(0) = 0$$

O`tkinchi protsess tugagandan so`ng t=∞ da:

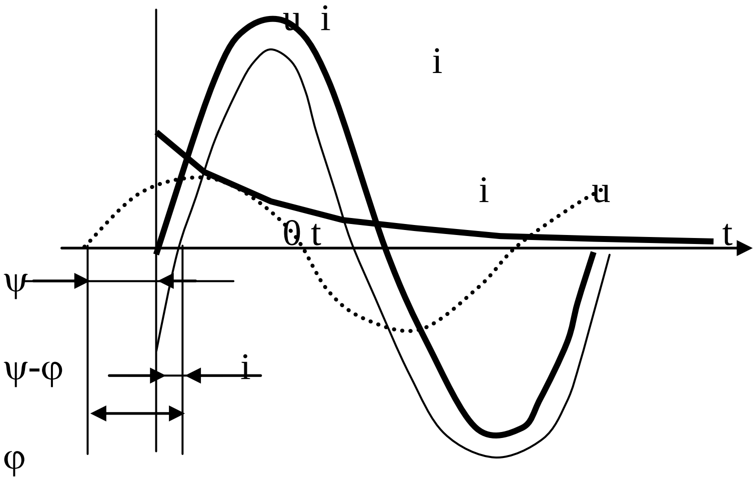
$$i = i_{\text{тург}} = \frac{U_m}{Z} \sin(\omega t + \psi_u - \varphi)$$

chunki i_{erk}=0 2- tenglamaga ko`ra kommutatsiya paytida:

$$U_L(0) = U_m[\sin\varphi\cos(\psi_u - \varphi) + \cos\varphi\sin(\psi_u - \varphi)] = U_m \sin\psi$$

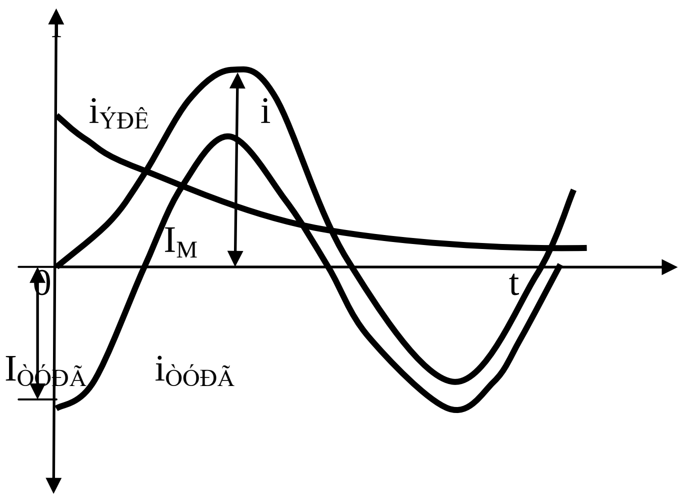
bo`ladi, ya’ni t=0 g`altak qismlardagi kuchlanish zanjirga berilgan kuchlanishning oniy qiymatiga teng.

O`tkinchi tok va o`tkinchi kuchlanish egri chiziqlari



a)

a) rasmda ψ>0, φ>0 va ψ-φ< 0 xollar uchun zanjirga berish kuchlanish u =Umsin(ωt +φ) ning(punktir chiziq) va o`tkinchi tok i ning(yo`g`on chiziq) egri chiziq ko`rsatilgan $i_{\text{эрк}} = -\frac{Um}{z}\sin(\psi_u - \varphi)e^{-\frac{t}{\tau}}$ mikdori va yo`nalish kommutatsiya paytida $i_{turg`}$ ning boshlang`ich fazasiga bog`liq $\psi_i = \psi_u - \varphi$.



á)

Agar kuchlanishíèíã fazasi ψ_U siltish burchagi φ ga teng bo`lib qolsa, u xolda o`tkinchi tok i faqat turg`un tok $i_{TURG}$ ning qiymatidan iborat bo`ladi,chunki 1-tenglamaga asosan i_{ERK} =0 bo`ladi. Shunda zanjir ulanganda tebranishsiz sinusoidal tok $i = \frac{U_m}{Z} Sin\omega t$ turg`unlashadi. Agar zanjirni ulash $\psi = \varphi \pm \frac{\pi}{2}$ da sodir bo`lsa, erkin tokning boshlang`ich chayqalishi o`zining maksimum qiymatiga erishadi $i_{эрк}(0) = \pm \frac{U_m}{z}$ (b-rasm).

Shu tufayli zanjirga berilgan kuchlanish davrining yarimiga teng bo`lgan vaqtdan so`ng, o`tkinchi tok xam o`zining eng katta qiymati i_{max}-ga erishadi. Ammo nazariy jixatdan tokning bu maksimumi xatto vaqt doimiysi τ ning eng katta qiymatlarida xam turg`unlashgan tok amplitudaviy qiymatining ikkilanganidan ortmaydi.

R,l elementlari ketma-ket ulangan zanjirda qisqa tutashuv.

Boshlang`ich toki i-(0)≠0 noldan farq qiladigan r, L elementi ketma-ket ulangan zanjir qisqa tutashganda vujudga keluvchi o`tkinchi protsesslarni ko`ramiz. Zanjir kommutatsiyaga qadar qo`shimcha rezistor r_o orqali o`zgarmas
kuchlanish Uo ga ulangan t=0 da qisqa tutashadi deb faraz qilamiz.
Kommutatsiyaga qadar turg`un tok:

$$I_0 = \frac{U_o}{r_o + r} \text{ bo`lsa,}$$

kommutatsiyadan so`ng nolga teng , chunki kontur manbadan ajratilgan bo`lib, ilgari g`altakka tok I_0 olib kirgan magnit maydonning energiyasi

$$W_m = \frac{LI_0^2}{2}$$

rezistor r da issiqlik energiyasiga aylanib, asta sekin nolgacha kamayadi. O`tkinchi rejimdagi zanjirning differensial tenglamasi:

$$L\frac{di_{эрк}}{dt}+ri_{эрк}=0$$

bo`ladi, o`tkinchi tok esa: $i=i_{эрк}=Ae^{-\frac{t}{\tau}}$

Integrallash doimiysi Ani $i_{-}(0)=i(0)$ shartidan topamiz, ya'ni $\frac{U_o}{r_o+r}=A$

Shunday qilib : $i=\frac{U_o}{r_o+r}e^{-\frac{t}{\tau}}=I_0e^{-\frac{t}{\tau}}$

Induktiv g`altakdagi o`tkinchi kuchlanish:

$$U_L=L\frac{di}{dt}=-\frac{U_0r}{r_0+r}e^{-\frac{t}{\tau}}=-rI_0e^{-\frac{t}{\tau}}$$

Ya'ni rezistordagi kuchlanish tushuvi:

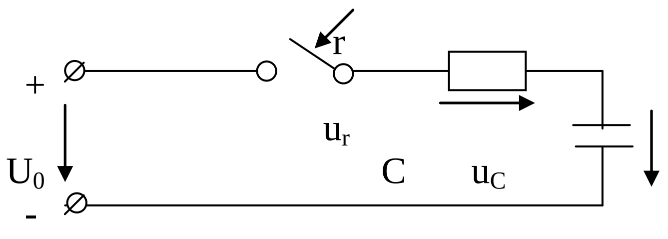
$$U_r=ri=rI_0e^{-\frac{t}{\tau}}$$

ga teng bo`lib ishorasi qarama-qarshi kommutatsiya paytida o`tkinchi tokning avvalgi qiymati:

$$i_{-}(0) = i(0) = I_0$$

ni saqlagan tok i dan farqli induktiv g`altakdagi kuchlanishi $u_{L-}(0)=0$ dan $U_L(0) =- rI_0$ gacha sakrab o`tishi mumkin(b-chizma).

G`altakdagi e.yu.k. $e_L= - L\frac{di}{dt}$ ning qo`yib ishoralari kuchlanishga va tokning vaqt bo`yicha o`zgarishga bog`liq. Agar induktiv tarmoqdagi tok orta borsa e.yu.k. ning ishorasi $e_L=-u_L$ manfiy , kamaya borsa musbat bo`ladi. Shunday qilib induktiv g`altakni o`zgarmas kuchlanishni manbai ulash va ajratish tokning qar qanday o`zgarishi e.yu.k. xosil qilishga va uning zanjirdagi boshlangich muvozanat xolatini saqlab turishiga sabab bo`ladi.



Chizmada vaqtning t=0 paytida r, C elementlari ketma-ket ulangan zanjirni o`zgarmas kuchlanish $U_0$ manbaiga ulash sxemasi ko`rsatilgan. Sig`imdagi kuchlanish $U_C$ ni o`zgaruvchan xisoblab zanjirning differensial tengëàìàñèíè tuzamiz:

$$rC\frac{dU_C}{dt} + U_C = U_0$$

bunda: i=$i = \frac{dq}{dt} = C\frac{dU_C}{dt}$ kondensator S qoplamalaridagi o`tkincha tok.

Sig`imdagi o`tkinchi kuchlanish $Uc=U_{turg`} + U_{erk}$

$$rC\frac{dU_{Cэрк}}{dt} + U_{Cэрк} = 0$$

Bunga mos xarakteristik tenglama:

$$rC\alpha + 1 = 0$$

dan chastota o`lchash $\frac{1}{сек}$, $\alpha = -\frac{1}{rC}$ ni topamiz.

So`nish koeffisienti $\alpha$-ning ishorasiga ko`rà,

$$U_{Cэрк} = Ae^{\alpha t} = Ae^{-\frac{t}{rC}}$$

Integrallash doimiysi A ni boshlangich shartlardan topamiz:

$$Uc_{-}(0) = Uc(0)$$

$$Uc_{-}(0) = 0 = U_{Cтург}(0) + U_{Cэрк}(0) = U_0 + A$$

yoki

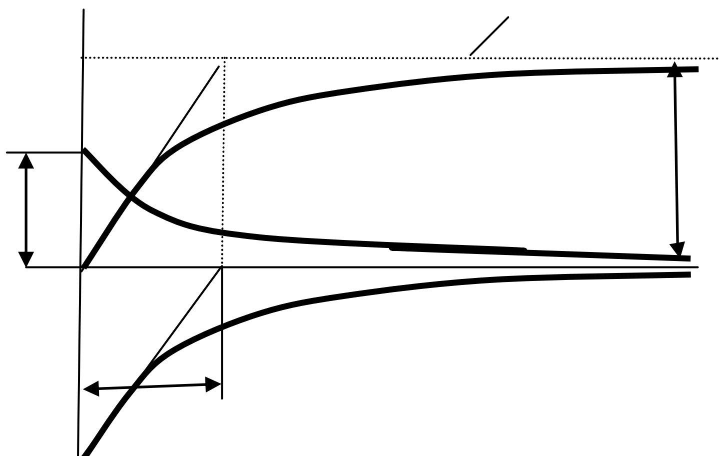
$$A = -U_0$$

Kommutatsiyaga qadar sig`imdagi kuchlanish nolga teng Uc(0)=0. O`tkinchi protsess tugagandagi turg`unlashgan kuchlanish $U_{Sturg`}$=Uo, chunki o`zgarmas tokda kondensatorning qarshiligi $\infty$ (tok o`tmaydi) zanjirdagi tok $i_{turg`}=0$.

Sig`imdagi kuchlanish:

$$U_C = U_{Cтург} + U_{Cэрк} = U_0 - U_0 e^{-\frac{t}{\tau}} = U_0(1 - e^{-\frac{t}{\tau}})$$

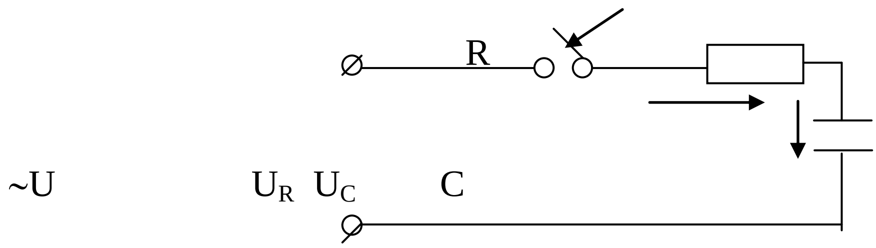
τ=rC –o`tkinchi protsessning intensivligini xarakterlovchi vaqt doimiysi (sek), u qanchalik katta bo`lsa S ning zaryadlanishi shunchalik sekin boradi va aksincha.



O`tkinchi tok $i = C\frac{dU_C}{dt}$ binoan:

$$i = CU_0\left[0 - \left(-\frac{1}{rC}e^{-\frac{t}{\tau}}\right)\right] = \frac{U_0}{r}e^{-\frac{t}{\tau}}$$

R,c elementlari ketma-ket ulangan zanjirni sinusoidal o`zgaruvchan kuchlanish manbaiga ulash.



r C elementlar ketma-ket ulangan zanjir U $=U_m\sin(\omega t+\varphi_u)$ manbaga ulanganda o`tkincha kuchlanish va toklar o`zgarish qoqunlarini aniqlaymiz.
O`tkinchi kuchlanish:

$$U_C = U_{Стур} + U_{Сэрк}$$

Bularning birinchisi i $_{turg`}$ ãà bog`liq ikkinchisi esa $u_{Сэрк} = Ae^{-\frac{t}{\tau}}$ ãà òåíã.

$$i_{тypr} = \frac{U_m}{z}\sin(\omega t + \psi_u - \varphi)$$ bo`ladi

áunda: $Z = \sqrt{r^2 + (\frac{1}{\omega c})^2}$ – zanjiríèíã to`la qarshiligi,

φ=àrctg $(-\frac{1}{r\omega c})$ – o`tkinchi protsess tugagandan keyingi u âà i orasidagi siljish burchagi.

$$U_{C\text{ту рг}} = \frac{U_m}{z\omega C}\sin(\omega t + \psi_u - \varphi - \frac{\pi}{2})$$

chunki u tokdan $\frac{\pi}{2}$ burchakka orqada qoladi, unda:

$$U_C = \frac{U_m}{z\omega C}\sin(\omega t + \psi_u - \varphi - \frac{\pi}{2}) + Ae^{-\frac{t}{\tau}}, \quad \tau = rC$$

Integrallash doimiysi A ni boshlangich shartlardan aniqlanadi:

$$U_C(0) = 0 = \frac{U_m}{z\omega C}\sin(\psi_u - \varphi - \frac{\pi}{2}) + A$$

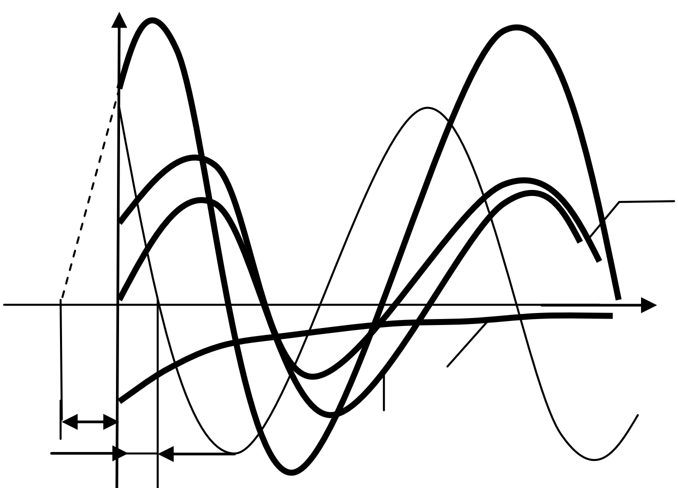
bu yerdan: $A = -\frac{U_m}{z\omega C}\sin(\psi_u - \varphi - \frac{\pi}{2})$

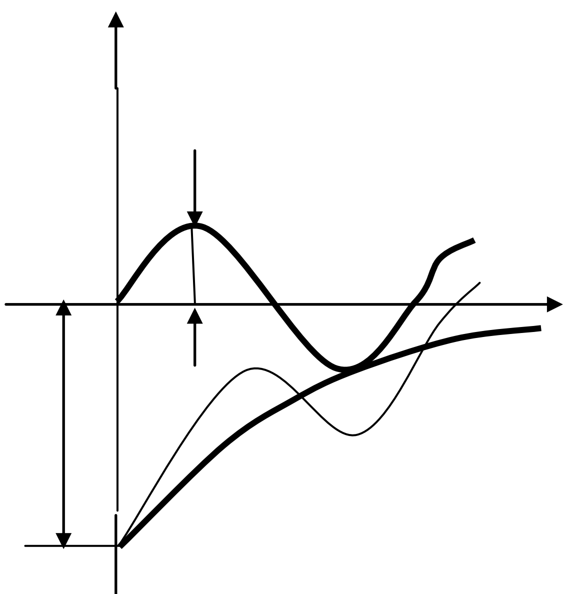
U_S -kuchlanishning o`zgarish qonuni quyidagicha:

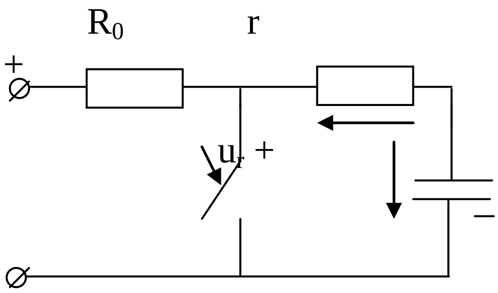
$$U_C = \frac{U_m}{z\omega C}\sin(\omega t + \psi_u - \varphi - \frac{\pi}{2}) - \frac{U_m}{z\omega C}\sin(\psi_u - \varphi - \frac{\pi}{2})e^{-\frac{t}{\tau}}$$

Bundan o`tkinchi tok $i = C\frac{dU_c}{dt}$ ni topamiz:

$$i = \frac{U_m}{z}\sin(\omega t + \psi_u - \varphi) + \frac{U_m}{z}\frac{1}{r\omega C}\sin(\psi_u - \varphi - \frac{\pi}{2})e^{-\frac{t}{rC}}$$







u_0 C

-

$$Uc = Uc\text{эрк} = Ae^{-\frac{t}{\tau}} \quad \tau = rC$$

Integrallash doimiysi boshlang`ich shartlar yordamida aniqlanadi:

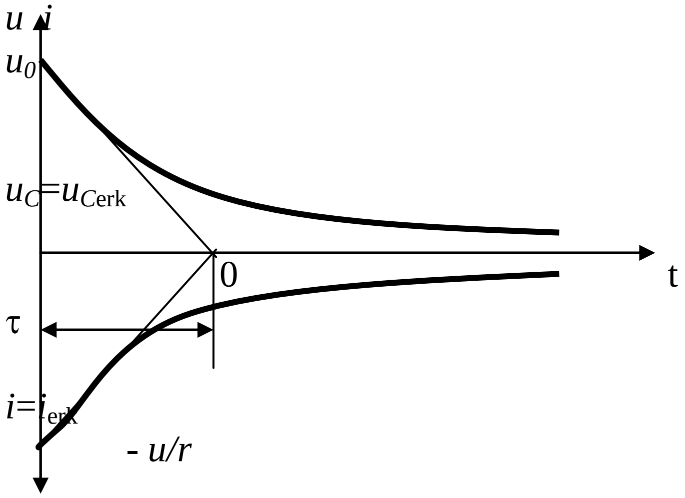
$$u_{C-}(0) = u_C(0) = u_0$$

$u_C(0) = A = u_0$ bo`ladi yoki

$$u_C = u_0 e^{-\frac{t}{\tau}}$$

Zanjirdagi o`tkinchi tok:

$$i = i_{erk} = C\frac{du_c}{dt} = -\frac{u_0}{r}e^{-\frac{t}{\tau}}$$



O`tkinchijarayon (t=0 dant=∞gacha) davomidaqarshilik R

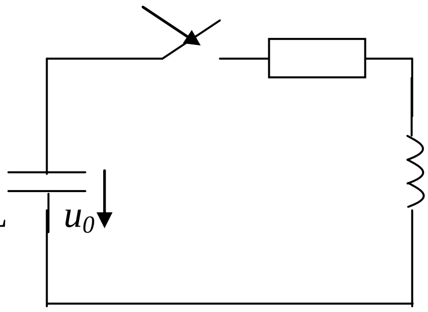
$$w_r = \int_0^{\infty} i^2 r dt = -\frac{u_0^2}{r}\int_0^{\infty} e^{-\frac{2t}{\tau}} dt = -\frac{u_0^2}{r}\cdot\frac{rC}{2}\left| e^{-\frac{2t}{\tau}} \right|_0^{\infty} = \frac{Cu_0^2}{2}$$

Kondensatorning zaryadsizlanishi unda to`plangan elektr energiyasi miqdoriga bog`liq

$$W_э = \frac{Cu_0^2}{2}$$

r

C +   L



Shu. o`tkinchi jarayon tugagandan keyin tok va kuchlanishlarning turg`unlashgan qiymatlari nolga teng.Zanjirning differensial tenglamasi:

$$\frac{d^2i}{dt^2} + \frac{r}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \qquad (1)$$

$$\frac{r}{L} = 2\delta; \qquad \omega_O^2 = \frac{1}{LC}$$

$$\alpha^2 + 2\delta\alpha + \omega_O^2 = 0 \qquad (2)$$

Uning ildizlari:

$$\alpha_1 = -\delta + \sqrt{\delta^2 - \omega_O^2}$$

$$\alpha_2 = -\delta - \sqrt{\delta^2 - \omega_O^2}$$

$$i = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

Zanjir o`tkinchi jarayon r,L va C parametrlarning miqdorlari nisbati bilan aniqlanadi. Unda nisbat uch variantda bo`lishi mumkin:

1) α_1 va α_2 ildizlar qaqiqiy va turlicha, yani δ^2-ω_0^2 >0;
2) α_1 va α_2 ildizlar qaqiqiy va teng, ya'ni δ^2-ω_0^2 = 0;
3) δ^2-ω_0^2 <0 bo`lganda $\alpha_1=-\delta+j\sqrt{\omega_0^2 - \delta^2}$ ildizlar qo`shma kompleslardan $\alpha_2=-\delta-j\sqrt{\omega_0^2 - \delta^2}$ iborat.

Dastlabki ikki variantda kondensatorning zaryadsizlanishi aperiodik xarakterda bo`lib, uchinchi variantda esa bu jarayon davriy yoki tebranma bo`ladi.

Sinov savollari.

1. Qisqa tutashuv qolatida g`altakdagi magnitaviy maydon energiyasi qanday energiya turiga aylanadi?
2. O`tkinchi jarayon intensivligini qanday kattalik xarakterlaydi?

3. Sig`imdagi turg`unlashgan kuchlanishni yozib bering?
4. So`nish koeffisienti α qanday aniqlanadi?