O'ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ISLOM KARIMOV NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

OLIY MATEMATIKA

BIR O‘ZGARUVCHILI FUNKSIYANING

DIFERENSIAL HISOBI. AMALIY MASHG’ULOTDAN

USLUBIY QO’LLANMA

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Ushbu uslubiy qo‘llanma Toshkent davlat texnika universitetining quyi kurs talabalari uchun mo‘ljallangan bo‘lib, oliy matematika fanining asosiy bo‘limlaridan biri bo‘lgan limitlar, differensial tushunchasi hamda hosila yordamida yechiladigan masalalar haqida asosiy tushunchalar beriladi, bundan tashqari talabalarning mustaqil ishlashlari uchun mustaqil ish variantlari keltirilgan.

Islom Karimov nomidagi Toshkent davlat texnika universiteti ilmiy-uslubiy kengashi qaroriga muvofiq chop etildi

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K I R I S H

Mazkur uslubiy qo‘llanma Toshkent davlat texnika universiteti Oliy matematika fanining o‘quv rejasi asosida tayyorlangan bo‘lib, limitlar nazariyasi, bir o‘zgaruvchili funksiyaning differensiali haqida to‘la tushunchalarni o‘z ichiga qamrab oladi, shu bilan bir qatorda differensialga keladigan ayrim fizik va mexanik masalalar ustida mukammal to‘xtalib o‘tiladi. Hosilani hisoblash usullari isbotlari bilan berilgan. Talabalar bilimlarini puxtalash maqsadida uslubiy qo‘llanmada mustaqil ish variantlari ham keltirilgan.

FUNKSIYA. LIMITLAR NAZARIYASI

1. Sonli ketma-ketlik va uning limiti.

Matematik analizning asosiy amallaridan biri limitga o‘tish amalidir. Bu amal analiz kursida turli ko‘rinishlarda uchraydi. Bu bobda o‘zgaruvchi miqdorninglimiti tushunchasiga asoslangan sodda ko‘rinishlar ko‘riladi.

Agar masala shartida berilgan miqdor har xil sonli qiymatlarni qabul qilsa, bu miqdor o‘zgaruvchi miqdor deyiladi.

1-ta’rif. *Agar har bir* $n \in N$ *natural songa biror qonun yoki qoidaga ko‘ra bitta* x_n *haqiqiy son mos qo‘yilgan bo‘lsa,* $x_1, x_2, \ldots, x_n, \ldots$ *sonli ketma-ketlik berilgan deyiladi va u* $\{x_n\}$ *kabi belgilanadi.*

$x_n \quad (n = 1, 2, \ldots)$ miqdorlar $\{x_n\}$ ketma-ketlikning hadlari deyiladi.

$\{x_n\}$ va $\{y_n\}$ ketma-ketliklar berilgan bo‘lsa,

$$\{x_n + y_n\} = \{x_1 + y_1,\ x_2 + y_2, \ldots\},$$

$$\{x_n - y_n\} = \{x_1 - y_1,\ x_2 - y_2, \ldots\},$$

$$\{x_n \cdot y_n\} = \{x_1 \cdot y_1,\ x_2 \cdot y_2, \ldots\},$$

$$\left\{\frac{x_n}{y_n}\right\} = \left\{\frac{x_1}{y_1}, \frac{x_2}{y_2}, \ldots\right\} \quad (y_n \neq 0,\ n = 1, 2, \ldots)$$

ketma-ketliklarga mos ravishda $\{x_n\}$ va $\{y_n\}$ ketma-ketliklarning **yig’indisi, ayirmasi, ko‘paytmasi va nisbati** deyiladi.

2-ta’rif. *Agar* $\exists M \ (\exists m)$ *son mavjud bo‘lsaki,* $\forall n \in N$ *uchun* $x_n \leq M \ (x_n \geq m)$ *tengsizlik o‘rinli bo‘lsa,* $\{x_n\}$ *ketma-ketlik yuqoridan (quyidan) chegaralangan deyiladi. Aks holda esa, ya’ni* $\forall M \ (\forall m)$ *son olinganda ham* $\exists n \in N$ *son mavjud bo‘lsaki,* $x_n > M \ (x_n < m)$ *bo‘lsa,* $\{x_n\}$ *ketma-ketlik **yuqoridan (quyidan) chegaralanmagan** deyiladi.*

3-ta’rif. *Agar* $\exists M > 0$ *son mavjud bo‘lsaki,* $\forall n \in N$ *uchun* $|x_n| \leq M$ *bo‘lsa,* $\{x_n\}$ *ketma-ketlik* ***chegaralangan*** *deyiladi. Aks holda esa, ya’ni* $\forall M > 0$ *son* *olinganda ham* $\exists n \in N$ *son topilsaki* $|x_n| > M$ *bo'lsa,* $\{x_n\}$ ***chegaralanmagan ketma-ketlik*** *deyiladi.*

4-ta'rif. *Berilgan* $\{x_n\}$ *ketma-ketlik uchun shunday* a *son topilib,* $\forall \varepsilon > 0$ *son olinganda ham* $\exists n_0 = n_0(\varepsilon, a) \in N$ *son mavjud bo'lsaki,* $n > n_0$ *tengsizlikni qanoatlantiruvchi barcha natural sonlar uchun* $|x_n - a| < \varepsilon$ *tengsizlik o'rinli bo'lsa,* a *son* $\{x_n\}$ *ketma-ketlikning* ***limiti*** *deyiladi va* $\lim_{n \to \infty} x_n = a$ *ko'rinishda belgilanadi.*

Agar 4-ta'rifdagi shartni qanoatlantiruvchi a son mavjud bo'lmasa, $\{x_n\}$ ketma-ketlik **limitga ega emas** deyiladi.

5-ta'rif (4-ta'rifning inkori). *Agar* $\forall n_0 \in N$ *son olinganda ham* $\exists \varepsilon > 0,\ \exists n > n_0$ *son topilsaki,* $|x_n - a| \geq \varepsilon$ *bo'lsa,* a *son* $\{x_n\}$ *ketma-ketlikning* ***limiti emas*** *deyiladi va* $\lim_{n \to \infty} x_n = a$ *ko'rinishda belgilanadi.*

6-ta'rif. *Agar* $\{x_n\}$ *ketma-ketlik chekli limitga ega bo'lsa, bu ketma-ketlik* ***yaqinlashuvchi*** *deyiladi. Aks holda bu ketma-ketlik* ***uzoqlashuvchi*** *deyiladi.*

2. Cheksiz kichik va cheksiz katta ketma-ketliklar

1-ta'rif. *Agar* $\{x_n\}$ *ketma-ketlikning limiti nolga teng (*$\lim_{n \to \infty} x_n = 0$*) bo'lsa,* $\{x_n\}$ *ketma-ketlik* ***cheksiz kichik*** *ketma-ketlik deyiladi.*

2-Ta'rif. *Agar* $\forall M > 0$ *son olinganda ham* $\exists n_0 \in N$ *son mavjud bo'lsaki,* $\forall n > n_0$ *natural sonlar uchun* $|x_n| > M$ *tengsizlik o'rinli bo'lsa,* $\{x_n\}$ *ketma-ketlik* ***cheksiz katta*** *ketma-ketlik deyiladi.*

Agar $\{x_n\}$ cheksiz katta ketma-ketlik bo'lsa, $\lim_{n \to \infty} x_n = \infty$ ko'rinishda yoziladi. Agar $\{x_n\}$ cheksiz katta ketma-ketlik bo'lib, biror nomerdan boshlab barcha hadlari **musbat (manfiy)** bo'lsa, $\lim_{n \to \infty} x_n = +\infty$ $\left(\lim_{n \to \infty} x_n = -\infty\right)$ ko'rinishda yoziladi.

Har qanday cheksiz katta ketma-ketlik chegaralanmagan bo'ladi, lekin bu tasdiqning teskarisi har doim ham o'rinli bo'lavermaydi.

1-teorema. *Chekli sondagi cheksiz kichik ketma-ketliklar yig'indisi cheksiz kichik ketma-ketlik bo'ladi.*

2-teorema. *Chegaralangan ketma-ketlik bilan cheksiz kichik ketma-ketlik ko'paytmasi cheksiz kichik ketma-ketlik bo'ladi.*

3-teorema. *Agar* $\forall n \in N$ *uchun* $x_n \neq 0$ *bo'lib,* $\{x_n\}$ *- cheksiz katta (cheksiz kichik) ketma-ketlik bo'lsa, u holda* $\left\{\frac{1}{x_n}\right\}$ *cheksiz kichik (cheksiz katta) ketma-ketlik bo'ladi.*

4-teorema. $\lim\limits_{n\to\infty} x_n = a$ *bo'lishi uchun* $\{\alpha_n\} = \{x_n - a\}$ *ketma-ketlikning cheksiz kichik ketma-ketlik bo'lishi zarur va yetarlidir.*

3. Yaqinlashuvchi ketma-ketliklarning xossalari

1-teorema. *Agar* $\{x_n\}$ *ketma-ketlik yaqinlashuvchi bo'lsa, uning limiti yagona bo'ladi.*

2-**teorema**. *Agar* $\{x_n\}$ *ketma-ketlik yaqinlashuvchi bo'lsa, u chegaralangan bo'ladi.*

3-**teorema**. *Agar* $\{x_n\}$ *va* $\{y_n\}$ *ketma-ketliklar yaqinlashuvchi bo'lsa, u holda* $\{x_n \pm y_n\}$, $\{x_n \cdot y_n\}$ *ketma-ketliklar ham yaqinlashuvchi bo'ladi va*

$$\lim_{n\to\infty}(x_n \pm y_n) = \lim_{n\to\infty} x_n \pm \lim_{n\to\infty} y_n,$$

$$\lim_{n\to\infty}(x_n \cdot y_n) = \lim_{n\to\infty} x_n \cdot \lim_{n\to\infty} y_n$$

formulalar o'rinli bo'ladi.

4-teorema. *Agar* $\{x_n\}$ *va* $\{y_n\}$ *ketma-ketliklar yaqinlashuvchi bo'lib,* $\forall n \in N$ *uchun* $y_n \neq 0$ *va* $\lim\limits_{n\to\infty} y_n \neq 0$ *bo'lsa,* $\left\{\frac{x_n}{y_n}\right\}$ *ketma-ketlik ham yaqinlashuvchi bo'ladi va*

$$\lim_{n\to\infty} \frac{x_n}{y_n} = \frac{\lim\limits_{n\to\infty} x_n}{\lim\limits_{n\to\infty} y_n}$$

formula o'rinli bo'ladi.

5-teorema. *Agar* $\lim_{n\to\infty} x_n = a$ *bo'lib, biror nomerdan boshlab* $x_n \geq c$ $(x_n \leq c)$ *bo'lsa, u holda* $a \geq c$ $(a \leq c)$ *bo'ladi.*

6-teorema. ***(«Ikki mirshab haqidagi teorema»).*** *Agar* $\lim_{n\to\infty} x_n = a$, $\lim_{n\to\infty} y_n = a$ *bo'lib, biror nomerdan boshlab* $x_n \leq z_n \leq y_n$ *tengsizlik o'rinli bo'lsa, u holda* $\lim_{n\to\infty} z_n = a$ *bo'ladi.*

Agar $\lim_{n\to\infty} x_n = 0$, $\lim_{n\to\infty} y_n = 0$ bo'lsa, $\lim_{n\to\infty} \frac{x_n}{y_n}$ ga $\frac{0}{0}$ ko'rinishdagi aniqmaslik deyiladi. $\frac{\infty}{\infty}$, $0\cdot\infty$, $\infty-\infty$ va boshqa ko'rinishdagi aniqmasliklar ham shu kabi ta'riflanadi.

Monoton ketma-ketliklar va ularning limiti

1-Ta'rif. *Agar* $\{x_n\}$ *ketma-ketlikning hadlari* $\forall n \in N$ *uchun* $x_n \leq x_{n+1}$ $(x_n \geq x_{n+1})$ *tengsizlikni qanoatlantirsa* $\{x_n\}$ ***o'suvchi (kamayuvchi)*** *ketma-ketlik deyiladi.*

2-Ta'rif. *O'suvchi va kamayuvchi ketma-ketliklar* ***monoton*** *ketma-ketliklar deb ataladi.*

1-teorema. *Agar* $\{x_n\}$ *ketma-ketlik o'suvchi bo'lib, yuqoridan chegaralangan bo'lsa, u holda u yaqinlashuvchi bo'ladi.*

2-teorema. *Agar* $\{x_n\}$ *ketma-ketlik kamayuvchi bo'lib, quyidan chegaralangan bo'lsa, u holda u yaqinlashuvchi bo'ladi.*

Fundamental ketma-ketliklar

1-ta'rif. *Agar* $\forall \varepsilon > 0$ *son olinganda ham* $\exists n_0 = n_0(\varepsilon) \in N$ *son mavjud bo'lsaki,* $\forall n > n_0$ *va* $p \in N$ *sonlar uchun* $|x_{n+p} - x_n| < \varepsilon$ *tengsizlik bajarilsa,* $\{x_n\}$ ***fundamental ketma-ketlik deyiladi.***

2-ta'rif. *(1-ta'rifning inkori).* $\forall n_0 \in N$ *son olinganda ham shunday* $n > n_0$, $p \in N,\ \varepsilon > 0$ *sonlar mavjud bo'lib,* $|x_{n+p} - x_n| \geq \varepsilon$ *tengsizlik o'rinli bo'lsa,* $\{x_n\}$ *ketma-ketlik* ***fundamental emas*** *deyilali.*

Teorema (Koshi). *Ketma-ketlikning yaqinlashuvchi bo'lishi uchun uning fundamental bo'lishi zarur va yetarlidir.*

Qismiy ketma-ketliklar. Ketma-ketlikning yuqori va quyi limitlari

$\{x_n\}$ ketma-ketlik berilgan bo'lib, $k_1, k_2, \ldots, k_n, \ldots\ (k_n \geq n)$ o'suvchi natural sonlar ketma-ketligi bo'lsin. $\{x_n\}$ ketma-ketlikning $k_1, k_2, \ldots, k_n, \ldots$ nomerli hadlaridan $x_{k_1}, x_{k_2}, \ldots, x_{k_n}, \ldots$ ketma-ketlikni tuzamiz. Hosil bo'lgan $\{x_{k_n}\}$ sonli ketma-ketlik $\{x_n\}$ ketma-ketlikning **qismiy ketma-ketligi** deb ataladi.

1-teorema. *Agar* $\lim\limits_{n\to\infty} x_n = a$ *bo'lsa, u holda uning har qanday qismiy ketma-ketligining limiti ham* a *ga teng bo'ladi.*

2-teorema. ***(Bolsano-Veyershtrass).*** *Agar* $\{x_n\}$ *ketma-ketlik chegaralangan bo'lsa, u holda bu ketma-ketlikdan yaqinlashuvchi bo'lgan qismiy ketma-ketlik ajratish mumkin.*

1-ta'rif. $\{x_n\}$ *ketma-ketlikning qismiy ketma-ketligi limiti* $\{x_n\}$ *ketma-ketlikning* ***qismiy limiti*** *deb ataladi.*

2-ta'rif. *Yuqoridan (quyidan) chegaralangan ketma-ketlik qismiy limitlarining eng kattasi (eng kichigi) berilgan ketma-ketlikning* ***yuqori (quyi)*** *limiti deyiladi va* $\overline{\lim\limits_{n\to\infty}} x_n$ $\left(\underline{\lim\limits_{n\to\infty}} x_n\right)$ *ko'rinishda belgilanadi.*

3-teorema. $\lim\limits_{n\to\infty} x_n = a$ *bo'lishi uchun* $\overline{\lim\limits_{n\to\infty}} x_n = \underline{\lim\limits_{n\to\infty}} x_n = a$ *bo'lishi zarur va yetarli.*

4. Funksiya tushunchasi. Funksiya limiti

Bizga biror $X \subset R$ to'plam berilgan bo'lib, x o'zgaruvchi miqdor X to'plamdan olingan bo'lsin. Agar har bir $x \in X$ songa biror qonun yoki qoidaga ko'ra bitta y son mos qo'yilsa, u holda X to'plamda **funksiya** aniqlangan deyiladi va $y = f(x)$ kabi belgilanadi, x o'zgaruvchiga **erkli**

o‘zgaruvchi (yoki funksiyaning argumenti), X to‘plam $f(x)$ funksiyaning **aniqlanish sohasi**, x soniga mos keluvchi y soniga esa funksiyaning x nuqtadagi **xususiy qiymati** deb ataladi. $f(x)$ funksiyaning barcha xususiy qiymatlar to‘plami Y ga $f(x)$ funksiyaning **qiymatlar to‘plami** (yoki **o‘zgarish sohasi**) deyiladi. Shunday qilib,

$$Y = \{y \in R:\ y = f(x),\ x \in X\}$$

Agar a ($a \in X$ ёки $a \notin X$) nuqtaning ixtiyoriy atrofida X to‘plamning a dan farqli kamida bitta nuqtasi bo‘lsa, u holda a nuqta X to‘plamning **limit nuqtasi** deyiladi.

Bundan keyin butun paragraf davomida X - $f(x)$ funksiyaning aniqlanish sohasi, a nuqta X to‘plamning limit nuqtasi deb tushuniladi.

1-ta’rif. (Koshi). *Agar* $\forall \varepsilon > 0$ *uchun* $\exists \delta = \delta(\varepsilon, a) > 0$ *topilsaki,* $0 < |x - a| < \delta$ *tengsizlikni qanoatlantiruvchi* $\forall x \in X$ *uchun* $|f(x) - b| < \varepsilon$ *tengsizlik bajarilsa, u holda* b *soni* $f(x)$ *funksiyaning* a ***nuqtadagi limiti*** *deyiladi va* $\lim\limits_{x \to a} f(x) = b$ *kabi belgilanadi.*

2-ta’rif. (Geyne). *Agar* X *to‘plamning nuqtalaridan tuzilgan,* a *ga intiluvchi* $\forall \{x_n\}$ $(x_n \neq a,\ n = 1, 2, \ldots)$ *ketma-ketlik uchun* $\{f(x_n)\}$ *ketma-ketlik hamma vaqt yagona* b *soniga intilsa, shu* b *soni* $f(x)$ *funksiyaning* a ***nuqtadagi limiti*** *deb ataladi.*

Keltirilgan ta’riflardan ko‘rinib turibdiki, funksiyaning a nuqtadagi limiti mavjud bo‘lishi uchun funksiya a nuqtada aniqlangan bo‘lishi, ya’ni $a \in X$ bo‘lishi, mutlaqo shart emas (a nuqtaning X to‘plam uchun limit nuqta bo‘lishi yetarli, ya’ni, umuman olganda, $a \notin X$).

Endi 1-va 2-ta’riflarga teskari ta’riflarni keltiramiz.

1-ta’rifning inkori. *Agar* $\exists \varepsilon > 0$ *topilsaki,* $\forall \delta > 0$ *uchun* $0 < |x - a| < \delta$ *tengsizlikni qanoatlantiruvchi* $\exists x \in X$ *mavjud bo‘lib,* $|f(x) - b| \geq \varepsilon$ *tengsizlik bajarilsa,* b *soni* $f(x)$ *funksiyaning* a ***nuqtadagi limiti emas*** *deyiladi* $\left(\lim\limits_{x \to a} f(x) \neq b\right)$.

2-ta’rifning inkori. *Agar* a *nuqtaga intiluvchi* $\exists\{x_n\}$ $(x_n \in X,\ x_n \neq a,\ n = 1, 2, \ldots)$ *ketma-ketlik topilsaki, unga mos* $\{f(x_n)\}$ *ketma-ketlik* b *ga intilmasa, u holda* b *son* $f(x)$ *funksiyaning* a ***nuqtadagi limiti emas*** *deyiladi.*

1-teorema. *Funksiya limitining 1- va 2-ta’riflari ekvivalentdir.*

Biz 1-teoremadan quyidagi xulosani chiqaramiz: funksiyaning limitini hisoblayotganda qaysi ta’rif bo‘yicha hisoblash oson va qulay bo‘lsa, shu ta’rifdan foydalanish kerak.

Ba’zi bir hollarda $f(x)$ funksiyaning a nuqtadagi limiti mavjud bo‘lmaydi. Ana shunday hollarda funksiyaning nuqtadagi **bir tomonli (o‘ng va chap)** limitlari to‘g’risida gap yuritiladi.

3-ta’rif *(Koshi).* $\forall\ \varepsilon > 0$ *uchun* $\exists\delta = \delta(a, \varepsilon) > 0$ *topilsaki,* $a < x < a + \delta$ $(a - \delta < x < a)$ *tengsizlikni qanoatlantiruvchi* $\forall x \in X$ *uchun* $|f(x) - b| < \varepsilon$ *tengsizlik bajarilsa,* b *son* $f(x)$ *funksiyaning* a *nuqtadagi* ***o‘ng (chap) limiti*** *deb ataladi va*

$$\lim_{x \to a+0} f(x) = f(a+0) = b \ \left(\lim_{x \to a-0} f(x) = f(a-0) = b\right)$$

kabi belgilanadi.

4-ta’rif *(Geyne).* a *nuqtaga intiluvchi* $\forall\{x_n\}$, $x_n \in X,\ x_n > a\ (x_n < a)$ *ketma-ketlik olinganda ham unga mos* $\{f(x_n)\}$ *ketma-ketlik* b *soniga intilsa,* b *soni* $f(x)$ *funksiyaning* a *nuqtadagi* ***o‘ng (chap) limiti*** *deyiladi.*

2-teorema. $\lim\limits_{x \to a} f(x) = b$ *bo‘lishi uchun* $f(a+0) = f(a-0) = b$ *tenglikning bajarilishi zarur va yetarli.*

Endi funksiyaning $x \to +\infty$ *dagi limiti ta’rifini beramiz.* $f(x)$ *funksiya* $(c, +\infty)$ *cheksiz oraliqda aniqlangan bo‘lsin.*

5-ta’rif. **(Koshi)**. $\forall\varepsilon > 0$ *uchun* $\exists A > 0$ $(A \geq c)$ *topilsaki,* $\forall x > A$ *uchun* $|f(x) - b| < \varepsilon$ *tengsizlik bajarilsa,* b *son* $f(x)$ *funksiyaning* $x \to +\infty$ ***dagi limiti*** *deyiladi va* $\lim\limits_{x \to +\infty} f(x) = b$ *kabi belgilanadi.*

6-ta'rif. (Geyne). $+\infty$ *ga intiluvchi* $\forall\{x_n\}$ $(x_n > c)$ *ketma-ketlik uchun unga mos* $\{f(x_n)\}$ *ketma-ketlik* b *soniga intilsa,* b *soni* $f(x)$ *funksiyaning* $x \to +\infty$ ***dagi limiti*** *deb ataladi.*

3- va 4-ta'riflar hamda 5- va 6-ta'riflar bir-biriga ekvivalent. $\lim\limits_{x\to-\infty} f(x)=b$ ning ta'rifi ham yuqoridagiga o'xshash aniqlanadi. Agar $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to-\infty} f(x)=b$ bo'lsa, u holda $\lim\limits_{x\to\infty} f(x)=b$ deb yoziladi.

7-ta'rif. *Agar* $\lim\limits_{x\to a} f(x)=\infty$ *(*$\lim\limits_{x\to a} f(x)=0$*) bo'lsa,* $f(x)$ *funksiya* a *nuqtada* ***cheksiz katta (cheksiz kichik) funksiya*** *deyiladi.*

Cheksiz katta va cheksiz kichik funksiyalar ham cheksiz katta va cheksiz kichik ketma-ketliklar uchun 2^0-punktda keltirilgan xossalarga ega.

5. Limitga ega bo'lgan funksiyalarning xossalari.

Birinchi va ikkinchi ajoyib limitlar

1-ta'rif. *Ushbu* $\dot{U}_\delta(a)=\{x\in R:\ 0<|x-a|<\delta\}$ *to'plam* a *nuqtaning* ***o'yilgan*** δ ***atrofi*** *deb ataladi.*

1-teorema. $f(x)$ *va* $g(x)$ *funksiyalar* a *nuqtaning biror o'yilgan atrofida aniqlangan bo'lib,* $\lim\limits_{x\to a} f(x)=b$ *va* $\lim\limits_{x\to a} g(x)=c$ *bo'lsin. U holda*

1) $\lim\limits_{x\to a}[f(a)\pm g(x)]=\lim\limits_{x\to a} f(x)\pm\lim\limits_{x\to a} g(x)=b\pm c,$

2) $\lim\limits_{x\to a}[f(a)\cdot g(x)]=\lim\limits_{x\to a} f(x)\cdot\lim\limits_{x\to a} g(x)=b\cdot c,$

3) agar $c\neq 0$ *bo'lsa,* $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}=\dfrac{b}{c}$ *bo'ladi.*

2-teorema. («Ikki mirshab haqidagi teorema»). *Agar* $f(x)$, $g(x)$ *va* $h(x)$ *funksiyalar* a *nuqtaning biror o'yilgan atrofida aniqlangan bo'lib, shu atrofda* $f(x)\le g(x)\le h(x)$ *tengsizlikni qanoatlantirsa va* $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} h(x)=b$ *tenglik bajarilsa, u holda* $\lim\limits_{x\to a} g(x)=b$ *bo'ladi.*

Funksiya limitini hisoblashda quyidagi ajoyib limitlar katta ahamiyatga ega.

Birinchi ajoyib limit:

Teorema. $\frac{\sin x}{x}$ funksiyaning x=0 nuqtadagi limit qiymati mavjud bo‘lib, u birga teng

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

Isboti. Agar 0<x<π/2 uchun 0<sin$x$<x<tg$x$ tengsizlikning o‘rinliligidan, tengsizlikni hadma had sin$x$ ga bo‘lib

$$1<\frac{x}{\sin x}<\frac{1}{\cos x} \quad \text{yoki} \quad \cos x<\frac{\sin x}{x}<1$$

Oxirgi tengsizliklar x ning -π/2<x<0 shartini qanoatlantiruvchi qiymatlari uchun ham o‘rinlidir. Bunga ishonch hosil qilish uchun cos(x)=cos(-x) va $\frac{\sin x}{x}=\frac{\sin(-x)}{(-x)}$ ekanini ko‘zda tutish yetarli. cos$x$ uzluksiz funksiya bo‘lgani uchun $\lim_{x\to 0}\cos x=1$ bo‘ladi. Shunday qilib cos$x$, 1 va $\frac{\sin x}{x}$ funksiyalar uchun x=0 nuqtaning biror δ atrofida bir xil limit qiymatiga ega bo‘ladi.

Demak, bunda

$$\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\cos x=1$$

Teorema isbotlandi.

Ikkinchi ajoyib limit:

Quyidagi o‘zgaruvchi miqdorni ko‘ramiz. $(1+\frac{1}{n})^n$ bunda *n* o‘suvchi o‘zgaruvchi miqdor va *n*=1, 2, 3...

Teorema. O‘zgaruvchi miqdor $(1+\frac{1}{n})^n$ *n*→∞ intilganda limit qiymati mavjud bo‘lib u 2 va 3 son orasida yotadi.

Isbot. Nyuton binomi formulasidan quyidagilarni yozish mumkin.

$$\left(1+\frac{1}{n}\right)^n=1+\frac{n}{1}\cdot\frac{1}{n}+\frac{n(n-1)}{1\cdot 2}\cdot\left(\frac{1}{n}\right)^2+\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\cdot\left(\frac{1}{n}\right)^2+$$

$$...+\frac{n(n-1)(n-2)...[n-(n-1)]^1}{1\cdot 2\cdot,...,n}\cdot\left(\frac{1}{n}\right)^n \qquad (1)$$

(1) ifodada algebraik almashtirishlardan so‘ng

$$\left(1+\frac{1}{n}\right)^2=1+1+\frac{1}{1\cdot 2}\left(1-\frac{1}{n}\right)+\frac{1}{1\cdot 2\cdot 3}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)+...$$

$$...+\frac{1}{1\cdot 2\cdot ...n}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)...\left(1-\frac{n-1}{n}\right) \qquad (2)$$

Oxirgi tenglik n ning o‘sib borishi o‘zgaruvchi miqdor $\left(1+\frac{1}{n}\right)^n$ ni o‘sib borishini ko‘rsatadi. Haqiqatan, agar *n* ni *n*+1 ga almashtirsak $\frac{1}{1\cdot 2}\left(1-\frac{1}{n}\right)<\frac{1}{1\cdot 2}\left(1-\frac{1}{n+1}\right)$ va h.x.

Bundan $\left(1+\frac{1}{n}\right)^n$ ni chegaralanganligini ko‘rsatamiz. Agar shuni e’tiborga olsak $\left(1-\frac{1}{n}\right)<1$; $\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)<1$ va h.k. (2) ifodadan yozish mumkin $\left(1+\frac{1}{n}\right)^n<1+1+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+...\frac{1}{1\cdot 2\cdot 3....n}$

Bundan

$$\frac{1}{1\cdot 2\cdot 3}<\frac{1}{2^2}; \qquad \frac{1}{1\cdot 2\cdot 3\cdot 4}<\frac{1}{2^3}; \qquad \frac{1}{1\cdot 2\cdot 3...n}<\frac{1}{2^{n-1}};$$

Shu tengsizlikni yozish mumkin

$$\left(1+\frac{1}{n}\right)^n<1+1+\frac{1}{2}+\frac{1}{2^2}+....+\frac{1}{2^{n-1}}$$

va o‘ng tarafdagi ifoda geometrik progressiyani tashkil qiladi va q=1/2 a=1 progressiyaning birinchi hadidan iborat.

Shuning uchun

$$\left(1+\frac{1}{n}\right)^n<1+\left[1+\frac{1}{2}+\frac{1}{2^2}+....+\frac{1}{2^{n-1}}\right]=1+\frac{a-aq^n}{1-q}=$$

$$=1+\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=1+\left[2-\left(\frac{1}{2}\right)^{n-1}\right]<3$$

endi hamma n lar uchun

$$\left(1+\frac{1}{n}\right)^n<3$$

hosil qilamiz.

Yuqoridagi (2) dan $\left(1+\frac{1}{n}\right)^n\geq 2$

shunday qilib quyidagi tengsizlikka ega bo‘lamiz

$$2\leq\left(1+\frac{1}{n}\right)^n<3 \qquad (3)$$

Demak, (3) o‘zgaruvchi miqdor $\left(1+\frac{1}{n}\right)^n$ ning chegaralanganligini ko‘rsatadi.

Agar o‘zgaruvchi miqdor $\left(1+\frac{1}{n}\right)^n$ - o‘suvchi va chegaralangan bo‘lsa bu miqdor limit qiymatiga ega bo‘ladi va bu limitni e deb belgilaymiz. (3) tengsizlikni shunday yozish mumkin: 2≤e≤3

Bu esa teoremaning isbotini beradi.

Bunda e irratsional son va uning qiymati

e=2.7182818284... ga teng

Teorema. $f(x)=\left(1+\frac{1}{x}\right)^x$ funksiyaning $x\to\infty$ da limit qiymati mavjud va u e soniga teng.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e \qquad (4)$$

Isboti 1. Aytaylik $x\to+\infty$, va quyidagi shu tengsizlik o‘rinli bo‘lsa,

$$n\leq x<n+1$$

Yozish mumkin $\frac{1}{n}\geq\frac{1}{x}>\frac{1}{n+1}$.

$$1+\frac{1}{n}\geq1+\frac{1}{x}>1+\frac{1}{n+1}$$

$$\left(1+\frac{1}{n}\right)^{n+1}>\left(1+\frac{1}{x}\right)^x>\left(1+\frac{1}{n+1}\right)^n$$

Agar x→∞ da n→∞ intiladi. Endi quyidagi limitlarni topamiz:

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n+1}=\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)=$$

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n\cdot\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)=e\cdot1=$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n=\lim_{n\to+\infty}\frac{\left(1+\frac{1}{n+1}\right)^{n+1}}{1+\frac{1}{n+1}}=\frac{\lim\limits_{n\to+\infty}\left(1+\frac{1}{n+1}\right)^{n+1}}{\lim\limits_{n\to+\infty}\left(1+\frac{1}{n+1}\right)}=\frac{e}{1}=e$$

Demak shularga asosan,

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x=e \qquad (5)$$

2. Aytaylik, endi $x\to-\infty$. Agar yangi o‘zgaruvchi $t=-(x+1)$ olsak yoki $x=-(t+1)$ desak, $t\to+\infty$ da $x\to-\infty$ bo‘ladi.

Yozish mumkin:

$$\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x=\lim_{t\to+\infty}\left(1-\frac{1}{t+1}\right)^{-t-1}=\lim_{t\to+\infty}\left(\frac{t}{t+1}\right)^{-t-1}=\lim_{t\to+\infty}\left(\frac{t+1}{t}\right)^{t+1}=$$

$$\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t+1}=\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t\cdot\left(1+\frac{1}{t}\right)=e\cdot1=e$$

Oxirgi ifoda teoremani isbotini beradi.

Agar (5) da $1/x=\alpha$ bo‘lsa $x\to\infty$ da $\alpha\to0$ ($\alpha\neq0$) va shuni hosil qilamiz

$$\lim_{\alpha\to0}(1+\alpha)^{1/\alpha}=e$$

Funksiya limiti uchun Koshi teoremasi

$f(x)$ funksiya X to‘plamda berilgan bo‘lib, a nuqta X to‘plamning limit nuqtasi bo‘lsin.

Ta’rif. *Agar* $\forall\varepsilon>0$ *uchun* $\exists\delta>0$ *topilsaki, argument* x *ning* $0<|x'-a|<\delta$, $0<|x''-a|<\delta$ *tengsizlikni qanoatlantiruvchi* $\forall x', x''$ $(x'\in X, x''\in X)$ *qiymatlarida* $|f(x'')-f(x')|<\varepsilon$ *tengsizlik o‘rinli bo‘lsa,* $f(x)$ *funksiya uchun* a ***nuqtada Koshi sharti bajariladi*** *deyiladi.*

Ta'rifning inkori. Agar $\exists\varepsilon>0$ son topilsaki, $\forall\delta>0$ son uchun, $0<|x'-a|<\delta,\ \ 0<|x''-a|<\delta$ tengsizlikni qanoatlantiruvchi $\forall x',x''\in X$ lar mavjud bo'lib, $|f(x'')-f(x')|\geq\varepsilon$ tengsizlik bajarilsa, $f(x)$ funksiya uchun a **nuqtada Koshi sharti bajarilmaydi** deyiladi.

Teorema. (Koshi). $f(x)$ *funksiya* a *nuqtada chekli limitga ega bo'lishi uchun bu funksiyaning* a *nuqtada Koshi shartini bajarishi zarur va yetarlidir*.

6. Funksiyaning uzluksizligi va uzilishi

$f(x)$ funksiya a nuqtaning biror to'liq atrofida aniqlangan bo'lsin.

1-ta'rif. *Agar*

$$\lim_{x\to a} f(x)=f(a)$$

bo'lsa, $f(x)$ *funksiya* a ***nuqtada uzluksiz** deyiladi.*

Funksiya uzluksizligi ta'rifini Koshi va Geyne ta'riflari yordamida ham berish mumkin. Biz ularga to'xtalib o'tirmaymiz.

Endi $f(x)$ funksiya a nuqtaning biror o'ng (chap) yarim atrofida, ya'ni $[a,a+\delta)$ (mos ravishda, $(a-\delta,a]$) yarim intervalda aniqlangan bo'lsin.

2-ta'rif. *Agar*

$$\lim_{x\to a+0} f(x)=f(a)\ \left(\lim_{x\to a-0} f(x)=f(a)\right)$$

bo'lsa, $f(x)$ *funksiya* a ***nuqtada o'ngdan (chapdan) uzluksiz** deyiladi.*

Teorema. $f(x)$ *funksiyaning* a *nuqtada uzluksiz bo'lishi uchun uning shu nuqtada o'ngdan va chapdan uzluksiz bo'lishi zarur va yetarlidir*.

Faraz qilaylik, $f(x)$ funksiya a nuqtada uzluksiz bo'lsin. U holda $\lim\limits_{x\to a} f(x)=f(a)$ bo'ladi. $\Rightarrow \lim\limits_{x-a\to 0}[f(x)-f(a)]=0$. Agar $\Delta x:=x-a$ - argument orttirmasi va $\Delta y:=\Delta f(a)=f(x)-f(a)$ - funksiyaning a nuqtadagi orttirmasi belgilashlarini kiritsak, $x=a+\Delta x$ va $\Delta y=\Delta f(a)=f(a+\Delta x)-f(a)$ bo'ladi. Natijada, biz

$$\lim_{x-a\to 0}\left[f(x)-f(a)\right]=\lim_{\Delta x\to 0}\left[f(a+\Delta x)-f(a)\right]=\lim_{\Delta x\to 0}\Delta y=0$$

ekanligini hosil qilamiz. Shunday qilib,

$$\lim_{\Delta x\to 0}\Delta y=0$$

tenglik bajarilsa, $f(x)$ funksiya a nuqtada uzluksiz bo‘ladi.

3-ta’rif. $f(x)$ *funksiya* (c,d) *intervalning har bir nuqtasida uzluksiz bo‘lsa, funksiya* (c,d) *intervalda uzluksiz deyiladi.*

$f(x)$ funksiya (c,d) da uzluksiz bo‘lib, s nuqtada o‘ngdan, d nuqtada chapdan uzluksiz bo‘lsa, unda u $[c,d]$ kesmada uzluksiz deyiladi.

X to‘plamda uzluksiz funksiyalar sinfi $C(X)$ kabi belgilanadi.

4-ta’rif. *Agar*

$$\lim_{x\to a} f(x)=b\neq f(a) \qquad \textit{(1-hol)}$$

$$\lim_{x\to a} f(x)-\nexists \qquad \textit{(2-hol)}$$

$$\lim_{x\to a} f(x)=\infty \qquad \textit{(3-hol)}$$

bo‘lsa, unda $f(x)$ *funksiya* a ***nuqtada uzilishga ega** deyiladi.*

Funksiyaning a nuqtada uzilishga ega bo‘ladigan hollarini alohida-alohida ko‘rib chiqaylik.

a) $\lim_{x\to a} f(x)=b\neq f(a)$ **bo‘lsin.**

Bu holda $\lim_{x\to a+0} f(x)=f(a+0)$ va $\lim_{x\to a-0} f(x)=f(a-0)$ lar mavjud bo‘lib, $f(a+0)=f(a-0)\neq f(a)$ bo‘ladi. Bunday nuqta **bartaraf qilish mumkin bo‘lgan uzilish nuqtasi** deb ataladi.

Misollar.

1. $f(x)=\begin{cases} x^2, \text{agar } x\neq 0 \text{ bo'sa,} \\ 1, \text{agar } x=0 \text{ bo'lsa} \end{cases}$

funksiya uchun $x=0$ nuqta bartaraf qilish mumkin bo‘lgan uzilish nuqtasi bo‘ladi, chunki

$$\lim_{x\to a+0} f(x)=\lim_{x\to -0} f(x)=0 \text{ va } f(0)=1$$

Agar $f(0)=0$ deb qabul qilsak, funksiya uzluksiz bo‘lib qoladi.

2. $f(x)=\begin{cases} 1-x\sin\frac{1}{x}, \text{agar } x\neq 0 \text{ bo'lsa,} \\ 2, \text{agar } x=0 \text{ bo'lsa} \end{cases}$

funksiya uchun ham $x=0$ nuqta bartaraf qilish mumkin bo‘lgan uzilish nuqtasi bo‘ladi, chunki

$$\lim_{x\to a+0} f(x)=\lim_{x\to -0} f(x)=1 \text{ va } f(0)=2.$$

b) $\lim\limits_{x\to a+0} f(x)-\nexists$ bo‘lsin.

Bunda quyidagi uchta hol bo‘lishi mumkin.

1) $\lim\limits_{x\to a-0} f(x)=f(a-0)$ va $\lim\limits_{x\to a+0} f(x)=f(a+0)$ lar $\exists$ va $f(a-0)\neq f(a+0)$.

Funksiyaning bunday nuqtadagi uzilishi birinchi tur uzilish va $|f(a+0)-f(a-0)|$ ayirmaga funksiyaning a nuqtadagi sakrashi deyiladi.

Masalan,

$$f(x)=\begin{cases} \dfrac{1}{1+2^{\frac{1}{x}}}, \text{agar } x\neq 0 \text{ bo'lsa,} \\ 0, \text{agar } x=0 \text{ bo'lsa} \end{cases}$$

funksiya uchun $x=0$ nuqta 1-tur uzilish nuqtasi bo‘ladi va funksiyaning bu nuqtadagi sakrashi 1 ga teng:

$$|f(a+0)-f(a-0)|=|f(+0)-f(-0)|=|0-1|=1$$

2) $x\to a$ da $f(x)$ funksiyaning o‘ng va chap limitlaridan hech bo‘lmaganda biri $\nexists$. Funksiyaning a nuqtadagi bunday uzilishi ikkinchi tur uzilish deyiladi.

Misollar.

1. $f(x)=\begin{cases} \sin\frac{1}{x}, \text{agar } x>0 \text{ bo'lsa,} \\ \text{- x, agar } x\le 0 \text{ bo'lsa} \end{cases}$

funksiya $x=0$ nuqtada ikkinchi tur uzilishga ega, chunki

$\lim\limits_{x\to -0} f(x)=\lim\limits_{x\to -0}(-x)=0=f(0)$, lekin $\lim\limits_{x\to +0} f(x)=\lim\limits_{x\to 0}\sin\frac{1}{x}-\nexists$.

2. $D(x)=\begin{cases} 0, \text{agar } x-\text{irratsional bo'lsa,} \\ 1, \text{agar } x-\text{ratsional bo'lsa} \end{cases}$

funksiya $\forall a\in R$ nuqtada ikkinchi tur uzilishga ega, chunki $x\to a$ da $D(x)$ funksiyaning o'ng limiti ham, chap limiti ham $\nexists$.

3. $x\to a$ da $f(x)$ funksiyaning o'ng va chap limitlaridan biri cheksiz yoki o'ng va chap limitlar turli ishorali cheksiz. Funksiyaning a nuqtadagi bunday uzilishi ham **ikkinchi tur uzilish** deyiladi.

v) $\lim\limits_{x\to a} f(x)=\infty$ bo'lsa, $f(x)$ funksiya $x=a$ nuqtada **ikkinchi tur uzilishga** ega deyiladi.

7. Uzluksiz funksiyalarning xossalari

1-teorema. *Agar* $f(x)$ *va* $g(x)$ *funksiyalar* $X\subset R$ *to'plamda aniqlangan bo'lib, ularning har biri* $a\in X$ *nuqtada uzluksiz bo'lsa, u holda*

1) $f(x)\pm g(x)$,

2) $f(x)\cdot g(x)$,

3) $\frac{f(x)}{g(x)}$ *(*$\forall x\in X$ *uchun* $g(x)\neq 0$*)*

funksiyalar ham shu nuqtada uzluksiz bo'ladi.

Izoh: 1-teoremaning aksi har doim ham o'rinli bo'lavermaydi. Masalan, $f(x)=x$ va

$$g(x)=\begin{cases}\sin\frac{1}{x}, \text{ага } x\neq 0 \text{ булса,}\\ 0, \text{агар } x=0 \text{ булса}\end{cases}$$

funksiyalar ko‘paytmasi $f(x)\cdot g(x)=x\cdot\sin\frac{1}{x}$ funksiya R da uzluksiz, lekin $g(x)$ funksiya $x=0$ nuqtada uzilishga ega.

Aytaylik, $y=f(x)$ funksiya X to‘plamda, $z=\varphi(y)$ funksiya esa $Y=\{y=f(x): x\in X\}$ to‘plamda aniqlangan bo‘lib, ular yordamida X to‘plamda aniqlangan $z=\varphi[f(x)]$ murakkab funksiya tuzilgan bo‘lsin.

2-teorema. *Agar $y=f(x)$ funksiya $a\in X$ nuqtada, $z=\varphi(y)$ funksiya esa, unga mos $y_a=f(a)$ nuqtada uzluksiz bo‘lsa, $z=\varphi[f(x)]$ murakkab funksiya a nuqtada uzluksiz bo‘ladi.*

Bu teorema limit hisoblashda juda muhim rol o‘ynaydi va uning yordamida 1-§ ning 9^0 –punktidagi muhim limitlar keltirib chiqariladi.

3-teorema. *Agar $\lim\limits_{x\to a} f(x)=b$ $(b>0)$ va $\lim\limits_{x\to a} g(x)=c$ bo‘lsa, $\lim\limits_{x\to a}[f(x)]^{g(x)}=c$ bo‘ladi.*

$[f(x)]^{g(x)}$ ko‘rinishdagi funksiyaga **darajali - ko‘rsatkichli funksiya** deb ataladi.

Funksiyaning tekis uzluksizligi

Biror $y=f(x)$ funksiya X to‘plamda berilgan bo‘lsin.

Ta’rif. *Agar $\forall\varepsilon>0$ son uchun $\exists\delta=\delta(\varepsilon)>0$ son topilsaki, X to‘plamning $|x''-x'|<\delta$ tengsizlikni qanoatlantiruvchi $\forall x'$ va x'' $(x', x''\in X)$ nuqtalarida $|f(x'')-f(x')|<\varepsilon$ tengsizlik bajarilsa, $f(x)$ funksiya X to‘plamda* ***tekis uzluksiz*** *deb ataladi.*

Ta’rifning inkori. $\exists\varepsilon>0$ son topilsaki, $\forall\delta>0$ son olinganda ham $|x''-x'|<\delta$ tengsizlikni qanoatlantiruvchi shunday $\forall x'$ $x''\in X$ nuqtalar mavjud bo‘lib $|f(x'') - f(x')| \geq \varepsilon$ tengsizlik bajarilsa, $f(x)$ funksiya X to‘plamda **tekis uzluksiz** emas deyiladi.

Kantor teoremasi. *Agar $f(x)$ funksiya $[a,b]$ kesmada aniqlangan va uzluksiz bo‘lsa, u shu kesmada tekis uzluksiz bo‘ladi.*

8. Namunaviy misollar

1-misol. $\lim\limits_{n\to\infty} x_n = a$ **ekanligi ta’rif yordamida ko‘rsatilsin** $(n_0(\varepsilon)-?)$ **.**

$$x_n = \frac{2n^3}{n^3-2},\ a = 2$$

$$\triangleleft\ (\lim_{n\to\infty} x_n = a) \Leftrightarrow (\forall \varepsilon > 0\ \exists n_0 = n_0(\varepsilon) \in N : \forall n > n_0\ |x_n - a| < \varepsilon)\ .$$

$$|x_n - a| = \left|\frac{2n^3}{n^3-3} - 2\right| = \left|\frac{2n^3 - 2n^3 + 6}{n^3-3}\right| = \frac{6}{|n^3-3|} =$$

$$= \frac{6}{(n-\sqrt[3]{3})(n^2+\sqrt[3]{3}n+\sqrt[3]{3^2})} < \frac{6}{n^2+\sqrt[3]{3}n+\sqrt[3]{9}} < \frac{6}{\sqrt[3]{3}n} <$$

$$< \frac{6}{n} < \varepsilon \Rightarrow n > \frac{6}{\varepsilon} \Rightarrow n_0 = \left[\frac{6}{\varepsilon}\right]$$

Demak, $\forall \varepsilon > 0$ son olinganda ham $n_0 = \max\left\{2, \left[\frac{6}{\varepsilon}\right]\right\}$ deb olsak, $\forall n > n_0$ uchun $|x_n - a| < \varepsilon$ bo‘ladi. $\Rightarrow \lim\limits_{n\to\infty} x_n = a\ \triangleright$

2-misol. Limitni hisoblang: $\lim\limits_{n\to\infty} \frac{3n^2+2n-5}{4n^2-n+6}$.

Yechish: $\lim\limits_{n\to\infty} \frac{3n^2+2n-5}{4n^2-n+6} = \lim\limits_{n\to\infty} \frac{\frac{3n^2}{n^2}+\frac{2n}{n^2}-\frac{5}{n^2}}{\frac{4n^2}{n^2}-\frac{n}{n^2}+\frac{6}{n^2}} = \lim\limits_{n\to\infty} \frac{3+\frac{2}{n}-\frac{5}{n^2}}{4-\frac{1}{n}+\frac{6}{n^2}} = \frac{3}{4}.$

3-misol. Limitni hisoblang: $\lim\limits_{x\to 0} \frac{\sqrt{1+x^2}-1}{x}$.

Yechish: $\lim\limits_{x\to 0}\frac{\sqrt{1+x^2}-1}{x}=\left(\frac{0}{0}\right)=\lim\limits_{x\to 0}\frac{\left(\sqrt{1+x^2}-1\right)\left(\sqrt{1+x^2}+1\right)}{x\left(\sqrt{1+x^2}+1\right)}=\lim\limits_{x\to 0}\frac{\left(\sqrt{1+x^2}\right)^2-1^2}{x\left(\sqrt{1+x^2}+1\right)}=$

$$=\lim_{x\to 0}\frac{1+x^2-1}{x\left(\sqrt{1+x^2}+1\right)}=\lim_{x\to 0}\frac{x^2}{x\left(\sqrt{1+x^2}+1\right)}=\lim_{x\to 0}\frac{x}{\left(\sqrt{1+x^2}+1\right)}=\frac{0}{2}=0$$

4-misol. Limitni hisoblang: $\lim\limits_{x\to 1}\frac{x^3+x-2}{x^3-x^2-x+1}$.

Yechish: $\lim\limits_{x\to 1}\frac{x^3+x-2}{x^3-x^2-x+1}=\left(\frac{0}{0}\right)=\lim\limits_{x\to 1}\frac{x^3+x-1-1}{x^3-x^2-x+1}=\lim\limits_{x\to 1}\frac{\left(x^3-1\right)+(x-1)}{x^2(x-1)-(x-1)}=$

$$=\lim_{x\to 1}\frac{(x-1)\left(x^2+x+1\right)+(x-1)}{\left(x^2-1\right)(x-1)}=\lim_{x\to 1}\frac{(x-1)\left(x^2+x+2\right)}{(x-1)^2(x+1)}=\lim_{x\to 1}\frac{x^2+x+2}{(x-1)(x+1)}=\frac{4}{0}=\infty$$

5-misol. Limitni hisoblang: $\lim\limits_{x\to -2}\frac{x^3+3x^2+2x}{x^2-x-6}$

Yechish: $\lim\limits_{x\to -2}\frac{x^3+3x^2+2x}{x^2-x-6}=\left(\frac{0}{0}\right)=\lim\limits_{x\to -2}\frac{x\left(x^2+3x+2\right)}{x^2-x-6}=$

$$=\lim_{x\to -2}\frac{x(x+1)(x+2)}{(x-3)(x+2)}=\lim_{x\to -2}\frac{x(x+1)}{x-3}=-\frac{2}{5}$$

6-misol. Limitni hisoblang: $\lim\limits_{x\to 0}\frac{\sin 3x}{x}$.

Yechish: Birinchi ajoyib limit formulasidan foydalanamiz.

$$\lim_{x\to 0}\frac{\sin 3x}{x}=\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot 3=1\cdot 3=3$$

7-misol. Limitni hisoblang: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$.

Yechish: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=2\lim\limits_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\cdot\frac{1}{4}=\frac{1}{2}$

8-misol. Limitni hisoblang: $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}$.

Yechish: $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}=\left|\begin{array}{l}\frac{\pi}{2}-x=t\\ x=\frac{\pi}{2}-t,\ x\to\frac{\pi}{2}\ t\to 0\end{array}\right|=\lim\limits_{t\to 0}\frac{1-\sin\left(\frac{\pi}{2}-t\right)}{t^2}=$

$$=\lim_{t\to 0}\frac{1-\cos t}{t^2}=\lim_{t\to 0}\frac{2\sin^2\frac{t}{2}}{t^2}=2\lim_{t\to 0}\left(\frac{\sin\frac{t}{2}}{\frac{t}{2}}\right)^2\cdot\frac{1}{4}=2\cdot 1\cdot\frac{1}{4}=\frac{1}{2}$$

9-misol. Limitni hisoblang: $\lim\limits_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{tgx}\right)$

Yechish: $\lim\limits_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{tgx}\right)=(\infty-\infty)=\lim\limits_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\frac{\sin x}{\cos x}}\right)=\lim\limits_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)=$

$$=\lim_{x\to 0}\frac{1-\cos x}{\sin x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\sin x}=2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\cdot\frac{x^2}{4}\cdot\frac{1}{\frac{\sin x}{x}\cdot x}=\frac{1}{2}\lim_{x\to 0}x=\frac{1}{2}\cdot 0=0$$

10-misol. $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^x$ limitni hisoblang.

Yechish. Quyidagi almashtirishlarni bajaramiz va (5) da yozamiz:

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}\cdot 3}=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}}\right]^3=\left[\lim_{x\to\infty}\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}}\right]^3=e^3$$

11-misol. Limitni hisoblang: $\lim\limits_{x\to\infty}\left(\frac{x}{1+x}\right)^x$.

Yechish: $\lim\limits_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}=\lim\limits_{x\to\infty}\left(\frac{1}{\frac{1+x}{x}}\right)^{x}=\frac{1}{\lim\limits_{x\to\infty}\left(\frac{1+x}{x}\right)^{x}}=\frac{1}{\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}}=\frac{1}{e}$

12-misol. Limitni hisoblang: $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$.

Yechish: $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}=\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x\cdot\frac{x+1}{x^2}}=\lim\limits_{x\to\infty}e^{\frac{x+1}{x^2}}=e^{\lim\limits_{x\to\infty}\frac{x+1}{x^2}}=e^{\lim\limits_{x\to\infty}\left(\frac{1}{x}+\frac{1}{x^2}\right)}=e^{0}=1$

13-misol. Limitni hisoblang: $\lim\limits_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1}$.

Yechish: $\lim\limits_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1}=\lim\limits_{x\to\infty}\left(\frac{x-2+3}{x-2}\right)^{2x-1}=\lim\limits_{x\to\infty}\left(\frac{x-2}{x-2}+\frac{3}{x-2}\right)^{2x-1}=$

$\lim\limits_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{2x-1}=\lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x-2}{3}}\right)^{\frac{x-2}{3}\cdot\frac{3}{x-2}\cdot(2x-1)}=\lim\limits_{x\to\infty}e^{\frac{3(2x-1)}{x-2}}=e^{\lim\limits_{x\to\infty}\frac{3(2x-1)}{x-2}}=e^{\lim\limits_{x\to\infty}\frac{6-\frac{3}{x}}{1-\frac{2}{x}}}=e^{6}$

14-misol. Limitni hisoblang: $\lim\limits_{x\to 0}\frac{\ln(a+x)-\ln a}{x}$.

Yechish: $\lim\limits_{x\to 0}\frac{\ln(a+x)-\ln a}{x}=\lim\limits_{x\to 0}\frac{\ln\frac{(a+x)}{a}}{x}=\lim\limits_{x\to 0}\frac{1}{x}\ln\frac{(a+x)}{a}=$

$=\lim\limits_{x\to 0}\ln\left(1+\frac{x}{a}\right)^{\frac{1}{x}}=\lim\limits_{x\to 0}\ln\left(1+\frac{x}{a}\right)^{\frac{a}{x}\cdot\frac{1}{a}}=\ln e^{\frac{1}{a}}=\frac{1}{a}\ln e=\frac{1}{a}$.

15-misol. $y=f(x)$ funksiya $x=x_0$ nuqtada uzluksiz ekanligi ta’rif yordamida isbotlansin ($\delta(\varepsilon)$ topilsin).

$f(x)=-2x^2-4,\ \ x_0=3$

◁ $f(x)$ funksiyani $x_0=3$ nuqtaning biror atrofida, masalan, (2; 4) intervalda qaraymiz. $\forall\varepsilon>0$ son olamiz va $|f(x)-f(x_0)|=|f(x)-f(3)|$ ayirmani baholaymiz:

$$|f(x)-f(3)|=|-2x^2-4-(-22)|=|-2x^2+18|=2|x^2-9|=$$

$$=2|x+3|\cdot|x-3|<14\cdot|x-3|.$$

Bu tenglikdan ko‘rinib turibdiki, agar $\delta=\frac{\varepsilon}{14}$ deb olsak, $|x-3|<\delta$ tengsizlikni qanoatlantiruvchi $\forall x\in(2;\ 4)$ uchun $|f(x)-f(3)|<14|x-3|<14\delta=14\frac{\varepsilon}{14}=\varepsilon$ bo‘ladi. $\Rightarrow$ $f(x)=-2x^2-4$ funksiya $x_0=3$ nuqtada uzluksiz. ▷

9. Mustaqil yechish uchun vazifalar

$\lim\limits_{n\to\infty}x_n=a$ **(**$n_0(\varepsilon)-$ko‘rsating**)** isbotlang.

1. $x_n=\frac{3n-2}{2n-1},\ a=\frac{3}{2};$	11. $x_n=\frac{6n}{3n+1},\ a=2;$
2. $x_n=\frac{4n-1}{2n+1},\ a=2;$	12. $x_n=\frac{4n-3}{2n+1},\ a=2;$
3. $x_n=\frac{8n+3}{4n-1},\ a=2;$	13. $x_n=\frac{n+1}{1-2n},\ a=-\frac{1}{2};$
4. $x_n=\frac{7n-3}{2n+1},\ a=\frac{7}{2};$	14. $x_n=\frac{6-5n}{10n+7},\ a=-\frac{1}{2};$
5. $x_n=\frac{2n-5}{3n+4},\ a=\frac{2}{3};$	15. $x_n=\frac{4-n^2}{n^2+3},\ a=-1;$
6. $x_n=\frac{5n-4}{n+1},\ a=5;$	16. $x_n=\frac{n}{5n-1},\ a=\frac{1}{5};$
7. $x_n=\frac{3+4n^2}{3-2n^2},\ a=-2;$	17. $x_n=\frac{15n+4}{3-5n},\ a=-3;$
8. $x_n=\frac{5n+6}{3n-4},\ a=\frac{5}{3};$	18. $x_n=\frac{2-3n^2}{5n^2+3},\ a=-\frac{3}{5};$
9. $x_n=\frac{8-n^2}{1+2n^2},\ a=-\frac{1}{2};$	19. $x_n=\frac{1+3n^2}{-2+4n^2},\ a=\frac{3}{4};$
10. $x_n=\frac{n^3+3}{2n^3-4},\ a=\frac{1}{2};$	20. $x_n=\frac{3n-8}{4+2n},\ a=\frac{3}{2};$

a) Sonli ketma-ketlikning limitini toping.

1. $\lim\limits_{n\to\infty}\dfrac{3n^2-2n+4}{2+n+6n^2}$.	16. $\lim\limits_{n\to\infty}\dfrac{(3n-1)^3-(2n+3)^3}{(n+3)^3}$.
2. $\lim\limits_{n\to\infty}\dfrac{4n^3+3n^2-1}{5n^2+4n+2}$.	17. $\lim\limits_{n\to\infty}\dfrac{8n^3-2n}{(n+1)^4+(n-1)^4}$
3. $\lim\limits_{n\to\infty}\dfrac{(3-n)^2+(3+n)^2}{(3+n)^2-(3-n)^2}$.	18. $\lim\limits_{n\to\infty}\dfrac{(n+6)^3-(n+2)^3}{(2n+3)^2}$.
4. $\lim\limits_{n\to\infty}\dfrac{(n+2)^4-(n-2)^4}{(n+2)^4+(n-2)^4}$.	19. $\lim\limits_{n\to\infty}\dfrac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$.
5. $\lim\limits_{n\to\infty}\dfrac{(n+1)!+n!}{(n+2)!}$.	20. $\lim\limits_{n\to\infty}\dfrac{(3n+4)^3-(n+5)^3}{(2n+3)^3+(3n-1)^3}$.
6. $\lim\limits_{n\to\infty}\dfrac{(n+1)!}{(n+2)!-(n+1)!}$.	21. $\lim\limits_{n\to\infty}\dfrac{(n+9)^2+(3n-1)^2}{(2+n)^3+(n-1)^3}$..
7. $\lim\limits_{n\to\infty}\dfrac{(3+n)^4-(2-n)^4}{(2-n)^4+(1+n)^4}$.	22. $\lim\limits_{n\to\infty}\dfrac{(2n-1)^3+(2n+3)^3}{(2n-1)^2+(2n+3)^2}$. .
8. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n+1)^2}{(n-1)^4-(n+1)^3}$.	23. $\lim\limits_{n\to\infty}\dfrac{(n+2)^4-(n-2)^4}{(n+5)^2+(n-5)^2}$.
9. $\lim\limits_{n\to\infty}\dfrac{(6-5n)^2+(6+5n)^2}{(n+3)^2-(1-n)^2}$.	24. $\lim\limits_{n\to\infty}\dfrac{n^3-(n-1)^3}{(n+2)^4+n^4}$.
10. $\lim\limits_{n\to\infty}\dfrac{(3+2n)^3-8n^3}{(2+3n)^2+4n^2}$.	25. $\lim\limits_{n\to\infty}\dfrac{(n+1)^2+(n-1)^2}{(2n+1)^2+(2n-1)^3}$.
11. $\lim\limits_{n\to\infty}\dfrac{(3+n)^3}{(n+2)^2-(n+1)^3}$.	26. $\lim\limits_{n\to\infty}\dfrac{(3n+16)^4}{(2n+1)^3-n^3}$.
12. $\lim\limits_{n\to\infty}\dfrac{(n+1)^2+(n-1)^2-(n+3)^3}{(4+n)^3}$.	27. $\lim\limits_{n\to\infty}\dfrac{(3n+1)^3-(2n-3)^3}{n^3-4n}$.
13. $\lim\limits_{n\to\infty}\dfrac{2(n+1)^3-(n-2)^3}{n^3+3n^2+4}$.	28. $\lim\limits_{n\to\infty}\dfrac{(n+1)^2-(n-1)^2}{(2+n)^2+(n+1)^2}$.
14. $\lim\limits_{n\to\infty}\dfrac{(n+5)^3+4(n-5)^3}{(n+3)^3}$.	29. $\lim\limits_{n\to\infty}\dfrac{(2n+3)^3+(n-3)^3}{n^4+2n^2+3}$.
15. $\lim\limits_{n\to\infty}\dfrac{(2n+1)^2+(2n-1)^2}{(3n+2)^2}$.	30. $\lim\limits_{n\to\infty}\dfrac{(2n+3)^4+(n-1)^4}{(3n+2)^3-(n+1)^3}$.

b) Sonli ketma-ketliklarning limitini toping.

1. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+2n-1}}{n+2}$.	16. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{7n}-\sqrt[4]{81n^8-1}}{\left(n+4\sqrt{n}\right)\sqrt{n^2-5}}$.
2. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+n}}{n+1}$.	17. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^3-7}+3\sqrt[3]{n^2+5}}{\sqrt[4]{n^3+3}+\sqrt{n+1}}$.

3. $\lim\limits_{n\to\infty}\dfrac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}$.

4. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^3-2n^2+1}+\sqrt[3]{n^4+1}}{\sqrt[4]{n^6+6n^5+2}-\sqrt[5]{n^7+3n^3+1}}$.

5. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{5n^2}+\sqrt[4]{n^3+2}}{\left(n+\sqrt{n}\right)\sqrt{7-n+n^2}}$.

6. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n-1}-\sqrt{n^2+1}}{\sqrt[3]{3n^3+3}+\sqrt[4]{n^5+1}}$.

7. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^3+1}-\sqrt{n-1}}{\sqrt[3]{n^3+1}-\sqrt{n-1}}$.

8. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2-1}+7n^3}{\sqrt[4]{n^{12}+n+1}-n}$.

9. $\lim\limits_{n\to\infty}\dfrac{\sqrt{3n-1}+\sqrt[3]{125n^3+n}}{\sqrt[5]{n}-n}$.

10. $\lim\limits_{n\to\infty}\dfrac{\sqrt{5n+2}-\sqrt[3]{8n^3+5}}{\sqrt[4]{n+7}-n}$.

11. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[4]{3n+1}+\sqrt{81n^4-n^2+1}}{\left(n+\sqrt[3]{n}\right)\sqrt{5-n+n^2}}$.

12. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+3}-\sqrt{n^2-3}}{\sqrt[3]{n^5-4}-\sqrt[4]{n^4+1}}$.

13. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n}-9n^2}{3n-\sqrt[4]{9n^8+1}}$.

14. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^5+3}-\sqrt{n-3}}{\sqrt[5]{n^5+3}+\sqrt{n-3}}$.

15. $\lim\limits_{n\to\infty}\dfrac{\sqrt{4n+1}-\sqrt[3]{27n^3+4}}{\sqrt[4]{n}-\sqrt[3]{n^5+n}}$.

18. $\lim\limits_{n\to\infty}\dfrac{3n^2-\sqrt[3]{n^4}}{\sqrt{n^4+n^3+1}-3n}$.

19. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^4+3}+\sqrt{n+3}}{\sqrt{n^3+4n}-\sqrt{n+3}}$.

20. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+2n}+\sqrt{n+3}}{\sqrt{n+5}}$.

21. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{n}+\sqrt{16n^4+3}}{\left(n+3\sqrt{n}\right)\sqrt{n^2-n+3}}$.

22. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+1}-3n^2}{n-\sqrt{n^2+n+2}}$.

23. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+2}-3n\sqrt{n}}{\sqrt{n^4-n+2}}$.

24. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{64n^2+3}-\sqrt[3]{n^6+4}}{\left(n-\sqrt{n}\right)\sqrt{n+n^2}}$.

25. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^2+6}-\sqrt{n^2-6}}{\sqrt[3]{n^3+6}+\sqrt{n^2-6}}$.

26. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+2}-\sqrt[3]{n^3+2}}{\sqrt[6]{n^5+3}-\sqrt[5]{n^5+7}}$.

27. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+1}-\sqrt[3]{n^3+1}}{\sqrt[4]{n+1}-\sqrt[5]{n^5+1}}$.

28. $\lim\limits_{n\to\infty}\dfrac{n!+(n+1)!}{(n+2)!+(n+1)!}$.

29. $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{n^2+1}-\sqrt{n}}{\sqrt{3-n}+n\sqrt{n}}$.

30. $\lim\limits_{n\to\infty}\dfrac{n\sqrt{n^2+1}-\sqrt{n^4+1}}{\left(3n^2+2\right)\left(\sqrt[3]{n^6+1}\right)}$.

d) Sonli ketma-ketliklarning limitini toping.

1. $\lim\limits_{n\to\infty}\left[\dfrac{1+3+5+...+(2n-1)}{n+1}-\dfrac{2n+1}{2}\right]$.

2. $\lim\limits_{n\to\infty}\dfrac{\frac{1}{3}+\frac{1}{3^2}+...+\frac{1}{3^n}}{\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^n}}$.

11. $\lim\limits_{n\to\infty}\dfrac{1-3+5-7+9-11+...+(4n-3)-(4n-1)}{\sqrt{n^2+1}+\sqrt{n^2+n+1}}$.

12. $\lim\limits_{n\to\infty}\dfrac{1+2+3+...+n}{n-n^2-3}$.

13. $\lim\limits_{n\to\infty}\dfrac{2^n-3^n}{2^{n-1}+3^n}$.

3. $\lim\limits_{n\to\infty}\dfrac{2^{n+1}+3^{n+1}}{2^n+3^n}$.	14. $\lim\limits_{n\to\infty}\dfrac{5}{6}+\dfrac{13}{36}+\dfrac{35}{216}+...+\dfrac{2^n+3^n}{6^n}$.
4. $\lim\limits_{n\to\infty}\dfrac{1+2+3+...+n}{\sqrt{9n^4+1}}$.	15. $\lim\limits_{n\to\infty}\dfrac{1-2+3-4+5-6+...+(2n-1)-2n}{n}$.
5. $\lim\limits_{n\to\infty}\dfrac{(2n+1)!+(2n+2)!}{(2n+3)!}$.	16. $\lim\limits_{n\to\infty}\dfrac{2+4+6+...+2n}{1+3+5+...+(2n-1)}$.
6. $\lim\limits_{n\to\infty}\dfrac{1+3+5+...+(2n-1)}{1+2+...+n}$.	17. $\lim\limits_{n\to\infty}\dfrac{7^n+2^n}{2^n-7^n}$.
7. $\lim\limits_{n\to\infty}\left[\dfrac{1+3+5+...+(2n-1)}{n+3}-n\right]$.	18. $\lim\limits_{n\to\infty}\dfrac{1-2+3-4+...-2n}{\sqrt[3]{n^3+3n+4}}$.
8. $\lim\limits_{n\to\infty}\dfrac{2^n-5^{n+1}}{2^{n+1}+5^{n+2}}$.	19. $\lim\limits_{n\to\infty}\dfrac{3+6+9+...+3n}{n^2+4}$.
9. $\lim\limits_{n\to\infty}\dfrac{(3n-1)!+(3n+1)!}{(3n)!(n-1)!}$.	20. $\lim\limits_{n\to\infty}\left(\dfrac{3}{4}+\dfrac{5}{16}+\dfrac{9}{64}+...+\dfrac{1+2^n}{4^n}\right)$.
10. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^3+5}-\sqrt{3n^4+2}}{1+3+5+...+(2n-1)}$.	21. $\lim\limits_{n\to\infty}\dfrac{n!+(n+2)!}{(n-1)!+(n+2)!}$.

e) Quyidagi ketma-ketlikning limitini toping.

1. $\lim\limits_{n\to\infty}\left(1-\dfrac{4}{n}\right)^n$.	16. $\lim\limits_{n\to\infty}\left(\dfrac{n^3+1}{n^3-1}\right)^{2n-n^3}$.
2. $\lim\limits_{n\to\infty}\left(\dfrac{n+2}{n-2}\right)^n$.	17. $\lim\limits_{n\to\infty}\left(\dfrac{12n+5}{12n-3}\right)^n$.
3. $\lim\limits_{n\to\infty}\left(1+\dfrac{3}{4n}\right)^n$.	18. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-6n+5}{n^2-5n+5}\right)^{3n+4}$.
4. $\lim\limits_{n\to\infty}\left(\dfrac{2n+1}{2n-1}\right)^n$.	19. $\lim\limits_{n\to\infty}\left(\dfrac{n^2+3}{n^2-n}\right)^n$.
5. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-1}{n^2}\right)^{n^3}$.	20. $\lim\limits_{n\to\infty}\left(\dfrac{2n+1}{2n-1}\right)^{n^2}$.
6. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+3}{2n^2+1}\right)^{n^2}$.	21. $\lim\limits_{n\to\infty}\left(1+\dfrac{4}{7n^2}\right)^{n^2+1}$.
7. $\lim\limits_{n\to\infty}\left(\dfrac{n-2}{n+3}\right)^{n+1}$.	22. $\lim\limits_{n\to\infty}\left(\dfrac{3n^2+4n-5}{3n^2+2n+1}\right)^{n^2+1}$.
8. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-3n+6}{n^2+5n+2}\right)^{\frac{n}{2}}$.	23. $\lim\limits_{n\to\infty}\left(\dfrac{7n^2+6n+2}{7n^2-2n+1}\right)^n$.
9. $\lim\limits_{n\to\infty}\left(1-\dfrac{4}{n^2+1}\right)^{n^2}$.	24. $\lim\limits_{n\to\infty}\left(\dfrac{4n^2+4n-3}{4n^2+3n+1}\right)^{1-3n}$.

10. $\lim\limits_{n\to\infty}\left(\frac{n-3}{n+2}\right)^{4n+1}$.	25. $\lim\limits_{n\to\infty}\left(\frac{n+4}{n-6}\right)^{\frac{n}{6}+3}$.
11. $\lim\limits_{n\to\infty}\left(\frac{3n^2+4n-1}{3n^2+n+1}\right)^{2n+3}$.	26. $\lim\limits_{n\to\infty}\left(\frac{3n^3+2n^2+4}{3n^3+4n-5}\right)^{n^3+1}$.
12. $\lim\limits_{n\to\infty}\left(\frac{4n+3}{4n+5}\right)^{n}$.	27. $\lim\limits_{n\to\infty}\left(\frac{7n^2-18n+5}{7n^2+11n+5}\right)^{n+3}$.
13. $\lim\limits_{n\to\infty}\left(\frac{n^2+n+1}{n^2-n+1}\right)^{-n^2}$.	28. $\lim\limits_{n\to\infty}\left(1-\frac{5}{6n^2}\right)^{n^2-1}$.
14. $\lim\limits_{n\to\infty}\left(1-\frac{3}{n^2}\right)^{n^2+1}$.	29. $\lim\limits_{n\to\infty}\left(\frac{3n+4}{3n-1}\right)^{2n+1}$.
15. $\lim\limits_{n\to\infty}\left(\frac{n+3}{n+5}\right)^{n+4}$.	30. $\lim\limits_{n\to\infty}\left(\frac{n+5}{n-1}\right)^{n}$.

f) Sonli ketma-ketlikning limitini toping.

1. $\lim\limits_{n\to\infty} n\left[\sqrt{n^2+1}+\sqrt{n^2-1}\right]$.	14. $\lim\limits_{n\to\infty}\left[n-\sqrt{n(n-1)}\right]$.
2. $\lim\limits_{n\to\infty} n\left[\sqrt{n(n-2)}-\sqrt{n^2-3}\right]$	15. $\lim\limits_{n\to\infty}\frac{\sqrt{(n+1)^3}-\sqrt{n(n-1)(n-3)}}{\sqrt{n}}$.
3. $\lim\limits_{n\to\infty}\left(n-\sqrt[3]{n^3-5}\right)n\sqrt{n}$.	16. $\lim\limits_{n\to\infty}\left[\sqrt{n^2+3n-2}-\sqrt{n^2-3}\right]$.
4. $\lim\limits_{n\to\infty}\left[\sqrt{(n^2+1)(n^2-4)}-\sqrt{n^4-9}\right]$.	17. $\lim\limits_{n\to\infty}\sqrt{n}\left(\sqrt{n+2}-\sqrt{n-3}\right)$.
5. $\lim\limits_{n\to\infty}\frac{\left[\sqrt{n^5-8}-n\sqrt{n(n^2+4)}\right]}{\sqrt{n}}$.	18. $\lim\limits_{n\to\infty}\frac{\sqrt{n(n^5+9)}-\sqrt{(n^4-1)(n^2+5)}}{n}$.
6. $\lim\limits_{n\to\infty}\left[\sqrt{n^2-3n+2}-\sqrt{n^2-2n+3}\right]$.	19. $\lim\limits_{n\to\infty}\sqrt{n^3+8}\left(\sqrt{n^3+2}-\sqrt{n^3-1}\right)$.
7. $\lim\limits_{n\to\infty}\left[\sqrt{n^2-2n+3}-n\right]$.	20. $\lim\limits_{n\to\infty}\frac{\sqrt{(n^3+1)(n^2+3)}-\sqrt{n(n^4+2)}}{2\sqrt{n}}$.
8. $\lim\limits_{n\to\infty}\left(n+\sqrt[3]{4-n^3}\right)$.	21. $\lim\limits_{n\to\infty}\left[\sqrt{(n^2+1)(n^2+2)}-\sqrt{(n^2-1)(n^2-2)}\right]$.
9. $\lim\limits_{n\to\infty}\left[\sqrt{(n+2)(n+1)}-\sqrt{(n-1)(n+3)}\right]$.	22. $\lim\limits_{n\to\infty}\frac{\sqrt{(n^5+1)(n^2-1)}-n\sqrt{n(n^4+1)}}{n}$.
10. $\lim\limits_{n\to\infty} n^2\left[\sqrt{n(n^4-1)}-\sqrt{n^5-8}\right]$.	23. $\lim\limits_{n\to\infty}\frac{\sqrt{(n^4+1)(n^2-1)}-\sqrt{n^6-1}}{n}$.
11. $\lim\limits_{n\to\infty}\left[\sqrt{n(n+1)(n+2)}\left(\sqrt{n^3-3}-\sqrt{n^3-2}\right)\right]$.	24. $\lim\limits_{n\to\infty}\sqrt[3]{n}\left[\sqrt[3]{n^2}-\sqrt[3]{n(n-1)}\right]$.
12. $\lim\limits_{n\to\infty}\left[\sqrt{n(n+5)}-n\right]$.	25. $\lim\limits_{n\to\infty}\left[(n+1)\sqrt{n}-\sqrt{(n+1)(n+2)(n+3)}\right]$.
13. $\lim\limits_{n\to\infty}\left[\sqrt[3]{(n+2)^2}-\sqrt[3]{(n-3)^2}\right]$.	

g) Quyidagi tengliklarni isbotlang.

1. $\lim\limits_{x\to -3}\dfrac{2x^2+5x-3}{x+3}=-7.$	16. $\lim\limits_{x\to \frac{5}{2}}\dfrac{2x^2-9x+10}{2x-5}=\dfrac{1}{2}.$
2. $\lim\limits_{x\to 1}\dfrac{5x^2-4x-1}{x-1}=6.$	17. $\lim\limits_{x\to \frac{1}{3}}\dfrac{6x^2+x-1}{x-\frac{1}{3}}=5.$
3. $\lim\limits_{x\to -2}\dfrac{3x^2+5x-2}{x+2}=-7.$	18. $\lim\limits_{x\to -\frac{1}{2}}\dfrac{6x^2-75x-39}{x+\frac{1}{2}}=-81.$
4. $\lim\limits_{x\to 3}\dfrac{4x^2-14x+6}{x-3}=10.$	19. $\lim\limits_{x\to 11}\dfrac{2x^2-21x-11}{x-11}=23.$
5. $\lim\limits_{x\to -\frac{1}{2}}\dfrac{6x^2+x-1}{x+\frac{1}{2}}=-5.$	20. $\lim\limits_{x\to 5}\dfrac{5x^2-24x-5}{x-5}=26.$
6. $\lim\limits_{x\to \frac{1}{2}}\dfrac{6x^2-x-1}{x-\frac{1}{2}}=5.$	21. $\lim\limits_{x\to -7}\dfrac{2x^2+15x+7}{x+7}=-13.$
7. $\lim\limits_{x\to -\frac{1}{3}}\dfrac{9x^2-1}{x+\frac{1}{3}}=-6.$	22. $\lim\limits_{x\to -4}\dfrac{2x^2+6x-8}{x+4}=-10.$
8. $\lim\limits_{x\to 2}\dfrac{3x^2-5x-2}{x-2}=7.$	23. $\lim\limits_{x\to -\frac{1}{3}}\dfrac{6x^2-x-1}{3x+1}=-\dfrac{5}{3}.$
9. $\lim\limits_{x\to -\frac{1}{3}}\dfrac{3x^2-2x-1}{x+\frac{1}{3}}=-4.$	24. $\lim\limits_{x\to -5}\dfrac{x^2+2x-15}{x+5}=-8.$
10. $\lim\limits_{x\to -1}\dfrac{7x^2+8x+1}{x+1}=-6.$	25. $\lim\limits_{x\to 8}\dfrac{3x^2-40x+128}{x-8}=8.$
11. $\lim\limits_{x\to 3}\dfrac{x^2-4x+3}{x-3}=2.$	26. $\lim\limits_{x\to 10}\dfrac{5x^2-51x+10}{x-10}=49.$
12. $\lim\limits_{x\to \frac{1}{2}}\dfrac{2x^2+3x-2}{x-\frac{1}{2}}=5.$	27. $\lim\limits_{x\to \frac{1}{2}}\dfrac{2x^2-5x+2}{x-\frac{1}{2}}=-3.$
13. $\lim\limits_{x\to \frac{1}{3}}\dfrac{6x^2-5x+1}{\frac{3x-1}{3}}=-1.$	28. $\lim\limits_{x\to -6}\dfrac{3x^2+17x-6}{x+6}=-19.$
14. $\lim\limits_{x\to \frac{7}{5}}\dfrac{10x^2+9x-7}{\frac{5x+7}{5}}=-10.$	29. $\lim\limits_{x\to \frac{1}{3}}\dfrac{3x^2+17x-6}{x-\frac{1}{3}}=19.$
15. $\lim\limits_{x\to -\frac{7}{2}}\dfrac{2x^2+13x+21}{2x+7}=-\dfrac{1}{2}.$	30. $\lim\limits_{x\to -\frac{1}{5}}\dfrac{15x^2-2x-1}{x+\frac{1}{5}}=-8.$

h) $y=f(x)$ funksiyani $x=x_0$ nuqtada uzluksiz ekanligini isbotlang ($\delta(\varepsilon)-$ni toping).

1. $f(x)=5x^2-1,\ x_0=6.$	16. $f(x)=5x^2+3,\ x_0=8.$
2. $f(x)=4x^2-2,\ x_0=5.$	17. $f(x)=5x^2+1,\ x_0=7.$
3. $f(x)=3x^2-3,\ x_0=4.$	18. $f(x)=4x^2-1,\ x_0=6.$
4. $f(x)=2x^2-4,\ x_0=3.$	19. $f(x)=3x^2-2,\ x_0=5.$
5. $f(x)=-2x^2-5,\ x_0=2.$	20. $f(x)=2x^2-3,\ x_0=4.$
6. $f(x)=-3x^2-6,\ x_0=1.$	21. $f(x)=-2x^2-4,\ x_0=3.$
7. $f(x)=-4x^2-7,\ x_0=1.$	22. $f(x)=-3x^2-5,\ x_0=2.$
8. $f(x)=-5x^2-8,\ x_0=2.$	23. $f(x)=-4x^2-6,\ x_0=1.$
9. $f(x)=-5x^2-9,\ x_0=3.$	24. $f(x)=-5x^2-7,\ x_0=1.$
10. $f(x)=-4x^2+9,\ x_0=4.$	25. $f(x)=-4x^2-8,\ x_0=2.$
11. $f(x)=-3x^2+8,\ x_0=5.$	26. $f(x)=-3x^2-9,\ x_0=3.$
12. $f(x)=-2x^2+7,\ x_0=6.$	27. $f(x)=-2x^2+9,\ x_0=6.$
13. $f(x)=2x^2+6,\ x_0=7.$	28. $f(x)=2x^2+8,\ x_0=5.$
14. $f(x)=3x^2+5,\ x_0=8.$	29. $f(x)=3x^2+7,\ x_0=6.$
15. $f(x)=4x^2+4,\ x_0=9.$	30. $f(x)=4x^2+6,\ x_0=7.$

i) Funksiyaning limitini hisoblang.

1. $\lim\limits_{x\to 2}\dfrac{x^2+4x-5}{3x+1}.$	17. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^3-x^2-x+1}.$
2. $\lim\limits_{x\to \frac{1}{3}}\dfrac{9x^2+3x-7}{6x+1}.$	18. $\lim\limits_{x\to 1}\dfrac{x+3}{x^2-4x+3}.$
3. $\lim\limits_{x\to 0}\dfrac{x^2+4x-6}{3x+4}.$	19. $\lim\limits_{x\to -3}\dfrac{x^2+3x}{x^3+27}.$
4. $\lim\limits_{x\to -1}\dfrac{x^3-x^2+2x}{3x+4}.$	20. $\lim\limits_{x\to \frac{2}{3}}\dfrac{x^2+\frac{4}{3}x+2}{x^2+3x+1}.$
5. $\lim\limits_{x\to \frac{1}{2}}\dfrac{3x^3+4x^2-2}{4x+3}.$	21. $\lim\limits_{x\to \frac{3}{2}}\dfrac{33x+5}{\frac{4}{3}x^2+4}.$
6. $\lim\limits_{x\to 1}\dfrac{x}{1-x}.$	22. $\lim\limits_{x\to \frac{4}{5}}\dfrac{\frac{5}{4}x^2-3x+5}{4x-3}.$
7. $\lim\limits_{x\to \sqrt{2}}\dfrac{x^2+4}{2x^4-3x^2+5}.$	

8. $\lim\limits_{x\to 4}\dfrac{\frac{1}{4}x^2+3x+\frac{1}{4}}{x+3}$.	23. $\lim\limits_{x\to -3}\dfrac{x^3+3x+4}{x^2-4}$.
9. $\lim\limits_{x\to \sqrt{3}}\dfrac{2x^4+3x^2+6}{x^2+5}$.	24. $\lim\limits_{x\to 0.5}\dfrac{4x^2+31x-5}{17x-1}$.
10. $\lim\limits_{x\to -2}\dfrac{x^2+2x+2}{x^2+3}$.	25. $\lim\limits_{x\to -4}\dfrac{\frac{x^2}{16}-\frac{x}{4}+3}{x^2+5x}$.
11. $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x-1}$.	26. $\lim\limits_{x\to 0.5}\dfrac{x^2+3x+6}{2x-x^2-1}$.
12. $\lim\limits_{x\to -1}\dfrac{x^3+1}{x+1}$.	27. $\lim\limits_{x\to 5}\dfrac{\frac{x^2}{125}+3x-7}{5x+3}$.
13. $\lim\limits_{x\to -2}\dfrac{(x-1)\sqrt{2-x}}{x^2-1}$.	28. $\lim\limits_{x\to -\frac{5}{4}}\dfrac{4x^2-5x+6}{8x+3}$.
14. $\lim\limits_{x\to 1}\left(\dfrac{1}{1-x}-\dfrac{3}{1-x^3}\right)$.	29. $\lim\limits_{x\to \frac{3}{4}}\dfrac{4x^2+7x-3}{5x-31}$.
15. $\lim\limits_{x\to \frac{1}{2}}\dfrac{8x^3-1}{6x^2+5x+1}$.	30. $\lim\limits_{x\to 1}\dfrac{x^3-1}{x^2-1}$
16. $\lim\limits_{x\to \frac{1}{2}}\dfrac{8x^3-1}{6x^2+5x+1}$.	

j) Limitlarni toping

1. $\lim\limits_{x\to +\infty}\dfrac{3x^2-4x+5}{x+4x^2-2}$,	16. $\lim\limits_{x\to \infty}\dfrac{3x^4+2x^5-x-4}{x^4+5x^2+1}$,
2. $\lim\limits_{x\to \infty}\dfrac{4x^3+3x+2}{2x^2+x-4}$,	17. $\lim\limits_{x\to \infty}\dfrac{4x^2-3x^3+4}{x^2+3x^3+5x}$,
3. $\lim\limits_{x\to \infty}\dfrac{4x^3+3x^2+2}{x^2-3x^5+3}$,	18. $\lim\limits_{x\to \infty}\dfrac{5x-4x^2+3}{2x^2+3x-7}$,
4. $\lim\limits_{x\to \infty}\dfrac{x^2+4x-6}{3x^3-2x+1}$,	19. $\lim\limits_{x\to \infty}\dfrac{x^3+4x^2-5x+6}{2x^4+3x^2+6x+2}$,
5. $\lim\limits_{x\to \infty}\dfrac{3x-4x^2+x+6}{3x^2+2x-7}$,	20. $\lim\limits_{x\to \infty}\dfrac{x^3-4x^2+5x+7}{x^2-3x^3+2x-1}$,
6. $\lim\limits_{x\to \infty}\dfrac{6x^4-5x^3+7x^2+3}{3x-4x^2+2}$,	21. $\lim\limits_{x\to \infty}\dfrac{8x^6-1+x}{4x^3-5x^5+2}$,
7. $\lim\limits_{x\to \infty}\dfrac{4x-5x^2+3x^3+3}{2x^3+3x+6x}$,	22. $\lim\limits_{x\to \infty}\dfrac{4x^2-3x-1}{3x^2+2x+3}$,
8. $\lim\limits_{x\to \infty}\dfrac{x^3+4x-6}{3x-x^3+4}$,	23. $\lim\limits_{x\to \infty}\dfrac{x^3-4x^2+3x-5}{3x^3+2x+7}$,

9. $\lim\limits_{x\to\infty}\frac{6x^2+3x+4}{3-6x-7x^2}$,	24. $\lim\limits_{x\to\infty}\frac{x^5-4x^6}{x^3+4x^5-x^6}$,
10. $\lim\limits_{x\to\infty}\frac{x^3+3x^2-4x+5}{x^2+4x-7}$,	25. $\lim\limits_{x\to\infty}\frac{9x^3-4x^2+1}{8x^4+3x^2+4}$,
11. $\lim\limits_{x\to\infty}\frac{x^3+x}{x^4+3x+2}$,	26. $\lim\limits_{x\to\infty}\frac{2x^3-x+5}{4x^3+5}$,
12. $\lim\limits_{x\to\infty}\frac{x^4+6x}{x^2+5x-1}$,	27. $\lim\limits_{x\to\infty}\frac{3x^4-5x^3+2x-1}{3x^2-x^4+1}$,
13. $\lim\limits_{x\to\infty}\left(\frac{x^3}{x+1}-x\right)$,	28. $\lim\limits_{x\to\infty}\frac{3x^2+4x-2}{3x^2+3x+8}$,
14. $\lim\limits_{x\to\infty}\left[\frac{4x^2}{3x+1}-\frac{(2x+1)(3x^2+x+2)}{x^2}\right]$	29. $\lim\limits_{x\to\infty}\frac{10x^3+5x-7}{3-2x^2+5x^3}$,
15. $\lim\limits_{x\to\infty}\frac{\frac{5}{6}x^3-4x^2+5}{x-x^2-x^3}$,	30. $\lim\limits_{x\to\infty}\frac{x^2+4x+3}{6-2x-x^2}$,

k) Quyidagi limitlarni hisoblang.

1. $\lim\limits_{x\to1}\frac{4x^2-6x+2}{2x^2-4x+1}$,	12. $\lim\limits_{x\to-1}\frac{3x^2+4x+1}{x^2-1}$,
2. $\lim\limits_{x\to2}\frac{x^2-3x+2}{x^2-4x+4}$,	13. $\lim\limits_{x\to4}\frac{x^2-2x-8}{x^2-16}$,
3. $\lim\limits_{x\to2}\frac{3x^2-5x+2}{2x-4}$,	14. $\lim\limits_{x\to3}\frac{3x^2-7x-6}{6+x-x^2}$,
4. $\lim\limits_{x\to1}\frac{6x^2-7x+1}{x^3-1}$,	15. $\lim\limits_{x\to2}\frac{x^2-4x+4}{x^3-8}$,
5. $\lim\limits_{x\to-2}\frac{-x^2-3x-2}{x^2+4x+4}$,	16. $\lim\limits_{x\to1}\frac{4x^2-3x-1}{x^2-1}$,
6. $\lim\limits_{x\to2}\frac{7x^2-13x-2}{x^2-3x+2}$,	17. $\lim\limits_{x\to3}\frac{3x^2-8x-3}{x^2-9}$,
7. $\lim\limits_{x\to-2}\frac{4x^2+7x-2}{x^2-4}$,	18. $\lim\limits_{x\to-1}\frac{4x^2+3x-1}{x^3+1}$,
8. $\lim\limits_{x\to-1}\frac{3x^2+x-2}{2x^2+3x+1}$,	19. $\lim\limits_{x\to2}\frac{4x^2-5x-6}{x^3+216}$,
9. $\lim\limits_{x\to-1}\frac{x^2+2x-3}{x^3-1}$,	20. $\lim\limits_{x\to2}\frac{6x^2-10x-4}{x^2-4}$,
10. $\lim\limits_{x\to3}\frac{x^2-5x+7}{x^3-27}$,	21. $\lim\limits_{x\to-3}\frac{x^3+27}{x^2-9}$,
11. $\lim\limits_{x\to5}\frac{2x^2-3x-35}{x^2-3x-10}$,	22. $\lim\limits_{x\to1}\frac{3x^2+x-4}{x^2-3x+2}$.

l) Funksiyaning limitini hisoblang.

1. $\lim\limits_{x\to 4}\dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2}$.	16. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x+2}$.
2. $\lim\limits_{x\to -8}\dfrac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$.	17. $\lim\limits_{x\to 4}\dfrac{\sqrt[3]{16x}-4}{\sqrt{4+x}-\sqrt{2x}}$.
3. $\lim\limits_{x\to 1}\dfrac{\sqrt{x-1}}{\sqrt[3]{x^2-1}}$.	18. $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x^2}-4}$.
4. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}$.	19. $\lim\limits_{x\to \frac{1}{2}}\dfrac{\sqrt[3]{\frac{x}{4}}-\frac{1}{2}}{\sqrt{\frac{1}{2}+x}-\sqrt{2x}}$.
5. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x^3+8}$.	20. $\lim\limits_{x\to \frac{1}{3}}\dfrac{\sqrt[3]{\frac{x}{9}}-\frac{1}{3}}{\sqrt{\frac{1}{3}+x}-\sqrt{2x}}$.
6. $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}$.	21. $\lim\limits_{x\to \frac{1}{4}}\dfrac{\sqrt[3]{\frac{x}{16}}-\frac{1}{4}}{\sqrt{\frac{1}{4}+x}-\sqrt{2x}}$.
7. $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$.	22. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[7]{x}}$.
8. $\lim\limits_{x\to 1}\dfrac{\sqrt{1-2x+x^2}-(1+x)}{x}$.	23. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{\sqrt[3]{x^2}+\sqrt[5]{x}}$.
9. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x+x^2}-2}{x+x^2}$.	24. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x-x^2}-2}{\sqrt[3]{x^2+x^3}}$.
10. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}}$.	25. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-2x+3x^2}-(1+x)}{\sqrt[3]{x}}$.
11. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{1+x}-\sqrt{2x}}$.	26. $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$.
12. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}$.	27. $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt[3]{\left(\sqrt{x}-4\right)^2}}$.
13. $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{4x}-2}{\sqrt{2+x}-\sqrt{2x}}$.	28. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{\sqrt[3]{x^3+8}}$.
14. $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x^2-1}$.	29. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{\sqrt[3]{x^2-16}}$.
15. $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{9x}-3}{\sqrt{x+3}-\sqrt{2x}}$.	30. $\lim\limits_{x\to -8}\dfrac{10-x-6\sqrt{1-x}}{2+\sqrt[3]{x}}$.

m) Funksiyaning limitini toping.

1. $\lim\limits_{x\to0}\frac{\ln(1+\sin x)}{\sin 4x}$.
2. $\lim\limits_{x\to0}\frac{1-\cos 10x}{e^{x^2}-1}$.
3. $\lim\limits_{x\to0}\frac{3x^2-5x}{\sin 3x}$.
4. $\lim\limits_{x\to0}\frac{1-\cos 2x}{\cos 7x-\cos 3x}$.
5. $\lim\limits_{x\to0}\frac{4x}{tg(\pi(x+2))}$.
6. $\lim\limits_{x\to0}\frac{2x}{tg(2\pi(x+\frac{1}{2}))}$.
7. $\lim\limits_{x\to0}\frac{1-\cos^3 x}{4x^2}$.
8. $\lim\limits_{x\to0}\frac{\arcsin 3x}{\sqrt{2+x}-\sqrt{2}}$.
9. $\lim\limits_{x\to0}\frac{2^x-1}{\ln(1+2x)}$.
10. $\lim\limits_{x\to0}\frac{arctg 2x}{\sin(2\pi(x+10))}$.
11. $\lim\limits_{x\to0}\frac{\ln(1-7x)}{\sin(\pi(x+7))}$.
12. $\lim\limits_{x\to0}\frac{\cos(x+\frac{5\pi}{2})tgx}{arc\sin 2x^2}$.
13. $\lim\limits_{x\to0}\frac{9\ln(1-2x)}{4arctg3x}$.
14. $\lim\limits_{x\to0}\frac{1-\sqrt{3x+1}}{\cos\left(\frac{\pi(x+1)}{2}\right)}$.
15. $\lim\limits_{x\to0}\frac{\sin 7x}{x^2+\pi x}$.
16. $\lim\limits_{x\to0}\frac{\sqrt{4+x}-2}{3arctgx}$.
17. $\lim\limits_{x\to0}\frac{2\sin[\pi(x+1)]}{\ln(1+2x)}$.
18. $\lim\limits_{x\to0}\frac{\cos 2x-\cos x}{1-\cos x}$.
19. $\lim\limits_{x\to0}\frac{\sqrt{1+x}-1}{\sin[\pi(x+2)]}$.

41. $\lim\limits_{x\to2\pi}\frac{\sin 7x-\sin 3x}{e^{x^2}-e^{4\pi^2}}$.
42. $\lim\limits_{x\to2}\frac{\sin 7\pi x}{\sin 8\pi x}$.
43. $\lim\limits_{x\to2}\frac{\ln(5-2x)}{\sqrt{10-3x}-2}$.
44. $\lim\limits_{x\to0}\left(\frac{\sin 2x}{x}\right)^{1+x}$.
45. $\lim\limits_{x\to0}\left(\frac{2+x}{3-x}\right)^{x}$.
46. $\lim\limits_{x\to0}\left(\frac{\sin 4x}{x}\right)^{\frac{2}{1+x}}$.
47. $\lim\limits_{x\to0}\left(\frac{x^2+4}{x+2}\right)^{x^2+3x+4}$.
48. $\lim\limits_{x\to0}\left(\frac{tgx}{3x}\right)^{3x+4}$.
49. $\lim\limits_{x\to0}(\cos x)^{\sin x+3}$.
50. $\lim\limits_{x\to0}\left(\frac{\ln(1+x)}{6x}\right)^{\frac{x}{x+3}}$.
51. $\lim\limits_{x\to0}\left(\frac{x+5}{x+6}\right)^{\cos 3x}$.
52. $\lim\limits_{x\to0}\left(\frac{\sin 8x}{5x}\right)^{3+x}$.
53. $\lim\limits_{x\to0}\left(\frac{\sin 2x}{\cos 3x}\right)^{x^2+3x+1}$.
54. $\lim\limits_{x\to0}\left(\frac{x^3+8}{3x^2+10}\right)^{2x+3}$.
55. $\lim\limits_{x\to0}\left(tg\left(x+\frac{\pi}{4}\right)\right)^{x+1}$.
56. $\lim\limits_{x\to0}\left(\frac{11x+8}{12x+1}\right)^{\cos^2 x}$.
57. $\lim\limits_{x\to0}[\sin(x+2)]^{\frac{3}{3+x}}$.
58. $\lim\limits_{x\to0}\left(\frac{x^4+5}{x+10}\right)^{\frac{4}{x+2}}$.

20. $\lim\limits_{x\to 0}\dfrac{\sin[5(x+\pi)]}{e^{3x}-1}$.

21. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{x\cdot\sin x}$.

22. $\lim\limits_{x\to 0}\dfrac{arc\sin 2x}{}$.

23. $\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{\sin[\pi(\frac{x}{2}+1)]}$.

24. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{(e^{3x}-1)^2}$.

25. $\lim\limits_{x\to 0}\dfrac{\sin^2 x-tg^2 x}{x^4}$.

26. $\lim\limits_{x\to 0}\dfrac{arc\sin 2x}{\ln(e-x)-1}$.

27. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{x(1-\cos 2x)}$.

28. $\lim\limits_{x\to 0}\dfrac{\ln(x^2+1)}{1-\sqrt{x^2+1}}$.

29. $\lim\limits_{x\to 0}\dfrac{tg[\pi(1+\frac{x}{2})]}{\ln(x+1)}$.

30. $\lim\limits_{x\to 0}\dfrac{2(e^{\pi x}-1)}{3(\sqrt[3]{1+x}-1)}$.

31. $\lim\limits_{x\to 0}\dfrac{2x\cdot\sin x}{1-\cos x}$.

32. $\lim\limits_{x\to 1}\dfrac{x^2-1}{\ln x}$.

33. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-x+1}-1}{\ln x}$.

34. $\lim\limits_{x\to \pi}\dfrac{1+\cos 3x}{\sin^2 7x}$.

35. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{1-\sin 2x}{(\pi-4x)^2}$.

36. $\lim\limits_{x\to 1}\dfrac{1+\cos \pi x}{tg^2\pi x}$.

37. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\sin 3x}{tgx}$.

38. $\lim\limits_{x\to \pi}\dfrac{\sin^2 x-tg^2 x}{(x-\pi)^4}$.

39. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2-x+1}-1}{tg\pi x}$.

59. $\lim\limits_{x\to 0}\left(\dfrac{arc\sin x}{x}\right)^{\frac{2}{x+3}}$.

60. $\lim\limits_{x\to 0}\left(\cos\dfrac{x}{\pi}\right)^{1+x}$.

61. $\lim\limits_{x\to 0}\left(\dfrac{x^2+6}{x^2+12}\right)^{\frac{3}{x+3}}$.

62. $\lim\limits_{x\to 0}\left(\dfrac{1+8x}{2+11x}\right)^{\frac{3}{x^2+3}}$.

63. $\lim\limits_{x\to \infty}\left(\dfrac{3x+4}{3x-2}\right)^{x}$.

64. $\lim\limits_{x\to -\infty}(3x-2)[\ln(x+3)-\ln(x-1)]$.

65. $\lim\limits_{x\to 2}\left[\dfrac{1}{x(x-2)^2}-\dfrac{1}{x^2-3x+2}\right]$.

66. $\lim\limits_{x\to -1}\left[\dfrac{x+3}{x^2+3x+2}+\dfrac{1}{3(x^2+4x+3)}\right]$.

67. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}$.

68. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{x^2+16}-4}$.

69. $\lim\limits_{x\to 1}\dfrac{x^2-\sqrt{x}}{\sqrt{x}-1}$.

70. $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{1+x^3}-1}{x^3}$.

71. $\lim\limits_{x\to a}\dfrac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}\quad (a>b)$.

72. $\lim\limits_{x\to 0}\dfrac{\sqrt{2+x}-2}{x^2}$.

73. $\lim\limits_{x\to 10}\dfrac{\sqrt{x-1}-3}{x-10}$.

74. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+h}-\sqrt{h}}{x}$.

75. $\lim\limits_{x\to \infty}\sqrt{x+3}-\sqrt{x}$.

76. $\lim\limits_{x\to \infty}(\sqrt{x^2+2}-\sqrt{x^2-2})$.

77. $\lim\limits_{x\to +\infty}(\sqrt{x^2+4}-x)$.

40. $\lim\limits_{x\to\pi}\dfrac{\cos 5x-\cos 3x}{\sin^2 x}$.	78. $\lim\limits_{x\to-\infty} x\cdot(\sqrt{x^4+1}-x^2)$. 79. $\lim\limits_{x\to\infty}\left(\sqrt{(x+3)(x+2)}-x\right)$. 80. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-2x-4}-\sqrt{x^2-7x+5}\right)$.

10. 1-namunaviy hisob

Limitlarni toping.

1

1.1. $\lim\limits_{x\to1}\dfrac{3x^2-2x-1}{x^2+4x+1}$.	1.16. $\lim\limits_{m\to3}\dfrac{3m^2-5m-3}{m^2-5m+6}$.
1.2. $\lim\limits_{x\to2}\dfrac{x^2+x+1}{x^2-x-2}$.	1.17. $\lim\limits_{x\to5}\dfrac{2x^2-11x+5}{x^2-7x+10}$.
1.3. $\lim\limits_{x\to3}\dfrac{x^2-4x+3}{2x^2-5x+1}$.	1.18. $\lim\limits_{x\to6}\dfrac{2x^2-9x-18}{x^2-7x+6}$.
1.4. $\lim\limits_{x\to4}\dfrac{x^2-x-2}{x^2-5x-4}$.	1.19. $\lim\limits_{x\to7}\dfrac{3x^2-17x-28}{x^2-9x+14}$.
1.5. $\lim\limits_{x\to5}\dfrac{x^2-25}{x^2-4x+5}$.	1.20. $\lim\limits_{x\to3}\dfrac{3x^2-8x-3}{x^2-x-6}$.
1.6. $\lim\limits_{x\to-1}\dfrac{x^2+3x+1}{2x^2-3x-5}$.	1.21. $\lim\limits_{x\to-2}\dfrac{x^2-x-6}{2x^2+x-6}$.
1.7. $\lim\limits_{x\to-3}\dfrac{2x^2+5x+1}{x^2+2x-3}$.	1.22. $\lim\limits_{x\to2}\dfrac{x^2-x-2}{x^2+x-6}$.
1.8. $\lim\limits_{x\to-2}\dfrac{x^2+x-2}{2x^2-x+1}$.	1.23. $\lim\limits_{x\to-1}\dfrac{3x^2+x-2}{3x^2+4x+1}$.
1.9. $\lim\limits_{x\to-4}\dfrac{x^2-16}{x^2+5x+2}$.	1.24. $\lim\limits_{t\to-1}\dfrac{2t^2-5t-7}{3t^2+t-2}$.
1.10. $\lim\limits_{x\to3}\dfrac{x^2+x-3}{x^2-4}$.	1.25. $\lim\limits_{x\to-5}\dfrac{x^2+2x-15}{2x^2+7x-15}$.
1.11. $\lim\limits_{x\to1}\dfrac{2x^2+x-3}{x^2+x-2}$.	1.26. $\lim\limits_{x\to-4}\dfrac{2x^2+9x+4}{x^2-x-20}$.
1.12. $\lim\limits_{x\to2}\dfrac{2x^2-3x-2}{x^2-3x+2}$.	1.27. $\lim\limits_{x\to1}\dfrac{3x^2-5x+2}{x^2-4x+3}$.
1.13. $\lim\limits_{x\to3}\dfrac{3x^2-10x+3}{x^2-2x-3}$.	1.28. $\lim\limits_{x\to-2}\dfrac{x^2+3x+2}{2x^2+5x+2}$.
1.14. $\lim\limits_{x\to5}\dfrac{3x^2-14x+5}{x^2-6x+5}$.	1.29. $\lim\limits_{x\to4}\dfrac{x^2-x-12}{x^2-2x-8}$.

1.15. $\lim\limits_{x\to 7}\dfrac{2x^2-13x-7}{x^2-9x+14}$.	1.30. $\lim\limits_{x\to -3}\dfrac{x^2-x-12}{x^2+5x+6}$.

2

2.1. $\lim\limits_{x\to 2}\dfrac{2x^2+3x-2}{3x^2+2x-8}$.	2.16. $\lim\limits_{x\to -2}\dfrac{4x^2+9x+2}{x^2-3x-10}$.
2.2. $\lim\limits_{x\to 1}\dfrac{3x^2-5x+2}{2x^2-x-1}$.	2.17. $\lim\limits_{x\to -1}\dfrac{5x^2+4x+1}{x^2-6x-7}$.
2.3. $\lim\limits_{x\to 2}\dfrac{10x-3x^2-8}{3x^2-8x+4}$.	2.18. $\lim\limits_{x\to -3}\dfrac{x^2+x-6}{2x^2+3x-7}$.
2.4. $\lim\limits_{x\to 1}\dfrac{2x^2-x-3}{x^2-3x-4}$.	2.19. $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{3x^2-4x-3}$.
2.5. $\lim\limits_{x\to 3}\dfrac{7x-x^2-12}{2x^2-11x+15}$.	2.20. $\lim\limits_{x\to 5}\dfrac{x^2-6x+5}{2x^2-7x-18}$.
2.6. $\lim\limits_{x\to -3}\dfrac{3-8x-3x^2}{x^2+x-6}$.	2.21. $\lim\limits_{x\to 4}\dfrac{x^2-x-12}{x^2-4x+3}$.
2.7. $\lim\limits_{x\to 5}\dfrac{2x^2-17x+35}{x^2-x-20}$.	2.22. $\lim\limits_{x\to 2}\dfrac{4x^2-7x-2}{x^2-7x+10}$.
2.8. $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{4-3x^2-x}$.	2.23. $\lim\limits_{x\to -3}\dfrac{x^2-x-12}{x^2+16}$.
2.9. $\lim\limits_{x\to -1}\dfrac{2x^2-16x+1}{3x^2+5x-2}$.	2.24. $\lim\limits_{x\to 1}\dfrac{3x^2+x+4}{2x^2+x-3}$.
2.10. $\lim\limits_{x\to -1}\dfrac{3x^2+2x-1}{x^2-1}$.	2.25. $\lim\limits_{x\to 1}\dfrac{2x^2-3x+2}{4x-3x^2-1}$.
2.11. $\lim\limits_{x\to 1}\dfrac{2x^2+x-3}{3x^2+x+2}$.	2.26. $\lim\limits_{x\to 1}\dfrac{3x^2+x-2}{x^2-2x+3}$.
2.12. $\lim\limits_{x\to 1}\dfrac{4x^2+2x-3}{x^2+x-2}$.	2.27. $\lim\limits_{x\to 2}\dfrac{2x^2+3x-104}{3x^2-7x+2}$.
2.13. $\lim\limits_{x\to 3}\dfrac{3x^2+10x+5}{x^2-2x-3}$.	2.28. $\lim\limits_{x\to 2}\dfrac{3x^2-10x+8}{2x^2-3x-2}$.
2.14. $\lim\limits_{x\to 5}\dfrac{3x^2-14x-5}{2x^2+6x+5}$.	2.29. $\lim\limits_{x\to -1}\dfrac{3x^2-2x-5}{x^2+5x+4}$.
2.15. $\lim\limits_{x\to -2}\dfrac{3x^2-6x+2}{x^2+5x+6}$.	2.30. $\lim\limits_{x\to 3}\dfrac{2x^2-3x-9}{3x^2-5x-10}$.

3

3.1. $\lim\limits_{x\to 2}\dfrac{\sqrt{3x-2}-2}{x^2-4}$.	3.16. $\lim\limits_{b\to 5}\dfrac{\sqrt{b-1}-2}{\sqrt{2b-1}-3}$.
3.2. $\lim\limits_{x\to -3}\dfrac{\sqrt{x+4}-1}{\sqrt{3-2x}-3}$.	3.17. $\lim\limits_{x\to 0}\dfrac{\sqrt{9+x}-3}{x^2+3}$.

3.3. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{9-x^2}-3}$.

3.4. $\lim\limits_{z\to -2}\dfrac{\sqrt{z+6}-2}{z^2-4}$.

3.5. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+13}-4}{x^2-9}$.

3.6. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{\sqrt{2x+1}-3}$.

3.7. $\lim\limits_{x\to 0}\dfrac{\sqrt{9-x}-3}{\sqrt{x+4}-2}$.

3.8. $\lim\limits_{x\to 3}\dfrac{\sqrt{2x+3}-3}{2-\sqrt{x+1}}$.

3.9. $\lim\limits_{n\to 0}\dfrac{\sqrt{n^2+9}-3}{\sqrt{4-n^2}-2}$.

3.10. $\lim\limits_{m\to 4}\dfrac{5-\sqrt{m^2+9}}{\sqrt{2m+1}-3}$.

3.11. $\lim\limits_{x\to 3}\dfrac{\sqrt{5x+1}-4}{x^2-9}$.

3.12. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+3x}-\sqrt{4-3x}}{7x}$.

3.13. $\lim\limits_{x\to 2}\dfrac{\sqrt{5x-1}-3}{x^2-2x}$.

3.14. $\lim\limits_{x\to 1}\dfrac{\sqrt{5x+4}-3}{\sqrt{2x-1}-1}$.

3.15. $\lim\limits_{x\to 2}\dfrac{x^3-8}{\sqrt{4x+1}-3}$.

3.18. $\lim\limits_{a\to 2}\dfrac{\sqrt{3a+10}-4}{a^2-4}$.

3.19. $\lim\limits_{m\to 3}\dfrac{9-m^2}{\sqrt{4m-3}-3}$.

3.20. $\lim\limits_{x\to 7}\dfrac{x^2-49}{\sqrt{2x+11}-5}$.

3.21. $\lim\limits_{z\to 1}\dfrac{\sqrt{1+3z^2}-2}{z^2-z}$.

3.22. $\lim\limits_{x\to 3}\dfrac{\sqrt{2x-2}-2}{\sqrt{x+1}-2}$.

3.23. $\lim\limits_{t\to 5}\dfrac{\sqrt{1+3t}-\sqrt{2t+6}}{t^2-5t}$.

3.24. $\lim\limits_{x\to 2}\dfrac{x^3-8}{\sqrt{2x}-2}$.

3.25. $\lim\limits_{m\to 4}\dfrac{\sqrt{6m+1}-5}{\sqrt{m}-2}$.

3.26. $\lim\limits_{z\to 3}\dfrac{\sqrt{z-1}-\sqrt{2}}{\sqrt{2z+3}-3}$.

3.27. $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{\sqrt{2x-2}-4}$.

3.28. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+9}-3}{\sqrt{x^2+25}-5}$.

3.29. $\lim\limits_{n\to 0}\dfrac{\sqrt{3-n}-\sqrt{3+n}}{5n}$.

3.30. $\lim\limits_{a\to 4}\dfrac{a-4}{\sqrt{5a+5}-5}$.

4

4.1. $\lim\limits_{a\to\infty}\dfrac{2a^3-a+1}{a^2+2a-5}$.

4.2. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{x^2+x-4}$.

4.3. $\lim\limits_{z\to\infty}\dfrac{2z^3+3z-1}{2z^3+z^2-4}$.

4.4. $\lim\limits_{n\to\infty}\dfrac{2n^2-3n+1}{n^2+2n-3}$.

4.5. $\lim\limits_{m\to\infty}\dfrac{3m^3+2m-5}{m^4+5m^2-1}$.

4.6. $\lim\limits_{z\to\infty}\dfrac{2z^2+z-3}{z^2+3z+1}$.

4.16. $\lim\limits_{a\to\infty}\dfrac{a^4-3a^2+2}{5a^4-3a-2}$

4.17. $\lim\limits_{b\to -\infty}\dfrac{9b^5-4b^3+2}{3b^4-2b+3}$

4.18. $\lim\limits_{x\to\infty}\dfrac{3x^2-4x+1}{x^3-2x^2-1}$

4.19. $\lim\limits_{n\to\infty}\dfrac{4n^5-3n^2+1}{2n^5-2n+3}$

4.20. $\lim\limits_{z\to\infty}\dfrac{9z^3-4z^2+1}{6z^3+3z+2}$

4.21. $\lim\limits_{x\to -\infty}\dfrac{3x^5-x^2+x}{x^4+2x+5}$

4.7. $\lim\limits_{a\to\infty}\dfrac{2a^2-3a+1}{a^3+3a-4}$	4.22. $\lim\limits_{n\to-\infty}\dfrac{6n^3-2n+7.}{3n^3-5n+2}$
4.8. $\lim\limits_{n\to\infty}\dfrac{m^3-8m+1}{3m^3-m+4}$	4.23. $\lim\limits_{n\to-\infty}\dfrac{7n^4-2n^3+2}{n^4+2n}$
4.9. $\lim\limits_{n\to\infty}\dfrac{3n^2-4n+1}{2n^2+n-3}$	4.24. $\lim\limits_{a\to\infty}\dfrac{3a^7+6a-5}{4a^7+2a^3-3}$
4.10. $\lim\limits_{z\to\infty}\dfrac{2-3z-z^2}{2z^3+z-1}$	4.25. $\lim\limits_{x\to\infty}\dfrac{8x^5-3x^2+9}{2x^5+2x+5}$
4.11. $\lim\limits_{n\to\infty}\dfrac{3n^2-4n^5+1}{2n^5+3n^3-n}$	4.26. $\lim\limits_{n\to\infty}\dfrac{n^4-5n+2}{2n^4+3n^2-n}$
4.12. $\lim\limits_{a\to\infty}\dfrac{4a^3+3a^2-1}{2a^3-3a+1}$	4.27. $\lim\limits_{x\to-\infty}\dfrac{6x^4-4x^3+8}{2x^3-3x^2+1}$
4.13. $\lim\limits_{y\to-\infty}\dfrac{y^6-3y^2-2}{2y^6+4y+5}$	4.28. $\lim\limits_{a\to-\infty}\dfrac{3a^4-4a^2+5}{6a^4+2a^3-1}$
4.14. $\lim\limits_{z\to\infty}\dfrac{6z^5-3z^2+1}{3z^5-2z+3}$	4.6. $\lim\limits_{n\to\infty}\dfrac{2+3n^2-n^5}{2n+n^2-3n^5}$
4.15. $\lim\limits_{x\to-\infty}\dfrac{x^3-4x^2+5}{3x^4+2x^2-x}$	4.30. $\lim\limits_{z\to\infty}\dfrac{2z^3+7z-4}{6z^3-3z^2+2}$

5

5.1. $\lim\limits_{\alpha\to0}\sin3\alpha\cdot ctg2\alpha$	5.16. $\lim\limits_{x\to0}\dfrac{\cos x-\cos5x}{3x^2}$
5.2. $\lim\limits_{\varphi\to0}\dfrac{1-\cos4\varphi}{\sin^2 3\varphi}$	5.17. $\lim\limits_{x\to0}\dfrac{1-\cos5x}{xtg2x}$
5.3. $\lim\limits_{\beta\to0}\dfrac{\arcsin6\beta}{2\beta}$	5.18. $\lim\limits_{x\to0}\sin5x\cdot ctg3x$
5.4. $\lim\limits_{\varphi\to0}\dfrac{\varphi\sin2\alpha}{tg^2 3\varphi}$	5.19. $\lim\limits_{z\to0}\dfrac{5z^2}{\sin3z\cdot tg2z}$
5.5. $\lim\limits_{\alpha\to0}\dfrac{arctg5\alpha}{3\alpha}$	5.20. $\lim\limits_{\alpha\to0}\sin8\alpha\cdot ctg\alpha$
5.6. $\lim\limits_{x\to0}\dfrac{x\sqrt{1-\cos4x}}{\sin^2 3x}$	5.21. $\lim\limits_{x\to0}\dfrac{5x}{arctg3x}$
5.7. $\lim\limits_{x\to0}\dfrac{\sin^2 6x}{xtg2x}$	5.22. $\lim\limits_{x\to0}tg^2 3x\cdot ctg^2 2x$
5.8. $\lim\limits_{y\to0}\dfrac{\sin5y}{\arcsin2y}$	5.23. $\lim\limits_{\alpha\to0}\dfrac{1-\cos8\alpha}{1-\cos2\alpha}$
5.9. $\lim\limits_{\beta\to0}\dfrac{tg^3 3\beta}{1-\cos4\beta}$	5.24. $\lim\limits_{x\to0}3x\cdot ctg7x$
5.10. $\lim\limits_{\varphi\to0}\dfrac{\arcsin^2\varphi}{3\varphi\sin\varphi}$	5.25. $\lim\limits_{\alpha\to0}\dfrac{\alpha\sin\alpha}{\cos\alpha-\cos^3\alpha}$
5.11. $\lim\limits_{x\to0}\dfrac{tg^2 4x}{\sin^2 3x}$	5.26. $\lim\limits_{x\to0}\dfrac{x\sin2x}{1-\cos4x}$

5.12. $\lim\limits_{\alpha\to 0}\dfrac{1-\cos 4\alpha}{\alpha\sin 3\alpha}$ 5.13. $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{tg^2 5x}$ 5.14. $\lim\limits_{x\to 0}\dfrac{arctg 3x}{4x}$ 5.15. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^3 x}{4x^2}$	5.27. $\lim\limits_{\alpha\to 0}\dfrac{\arcsin^2 3\alpha}{2\alpha\sin 5\alpha}$ 5.28. $\lim\limits_{x\to 0}\dfrac{xtg 2x}{1-\cos 3x}$ 5.29. $\lim\limits_{x\to 0}\sin^2 3x\cdot ctg^2 5x$ 5.30. $\lim\limits_{\varphi\to 0}\dfrac{\sin^2 3\varphi}{arctg^2 2\varphi}$

6

6.1. $\lim\limits_{x\to-\infty}\left(1+\dfrac{3}{2x-1}\right)^{4x}$	6.16. $\lim\limits_{x\to\infty}\ln\left(\dfrac{2x-3}{2x-1}\right)^{x}$
6.2. $\lim\limits_{x\to\infty}\left(1-\dfrac{2}{x+4}\right)^{2x-5}$	6.17. $\lim\limits_{x\to 2}\ln(2x-3)^{\frac{x^2}{(x-2)}}$
6.3. $\lim\limits_{x\to\infty}\left(\dfrac{2x-3}{2x+1}\right)^{3x-4}$	6.18. $\lim\limits_{x\to-\infty}\left(\dfrac{4x+5}{4x-1}\right)^{x+3}$
6.4. $\lim\limits_{x\to-\infty}\left(\dfrac{3x+2}{3x+5}\right)^{4-x}$	6.19. $\lim\limits_{x\to 1}(3x-2)^{\frac{5x}{(x-1)}}$
6.5. $\lim\limits_{y\to\infty}\left(1-\dfrac{4}{3y-1}\right)^{y+2}$	6.20. $\lim\limits_{x\to 3}(3x-8)^{\frac{(x+1)}{(x-3)}}$
6.6. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+5}\right)^{3x-2}$	6.21. $\lim\limits_{x\to\infty}\ln\left(\dfrac{2x+3}{2x-1}\right)^{x}$
6.7. $\lim\limits_{x\to\infty}\left(\dfrac{2x-3}{2x+1}\right)^{5-2x}$	6.22. $\lim\limits_{x\to\infty}\left(\dfrac{3x-2}{3x+4}\right)^{x-1}$
6.8. $\lim\limits_{x\to\infty}\left(\dfrac{3x-1}{3x+2}\right)^{2x-4}$	6.23. $\lim\limits_{n\to\infty}\ln\left(\dfrac{n+1}{n+2}\right)^{n+3}$
6.9. $\lim\limits_{x\to\infty}\left(1-\dfrac{2}{3x-1}\right)^{1-4x}$	6.24. $\lim\limits_{x\to\infty}\left(\dfrac{2x+4}{2x-4}\right)^{x-3}$
6.10. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{3x-4}\right)^{1-6x}$	6.25. $\lim\limits_{x\to\infty}\left(\dfrac{5x+1}{5x-1}\right)^{x-4}$
6.11. $\lim\limits_{x\to-\infty}\left(\dfrac{2+x}{2-x}\right)^{3x+1}$	6.26. $\lim\limits_{x\to 1}(3x-2)^{\frac{x}{(x^2-1)}}$
6.12. $\lim\limits_{x\to\infty}\ln\left(\dfrac{x+2}{x+3}\right)^{2x+3}$	6.27. $\lim\limits_{t\to\infty}\ln\left(\dfrac{4+3t}{1+3t}\right)^{t-2}$
6.13. $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x+5}\right)^{x-1}$	6.28. $\lim\limits_{x\to 2}(5-2x)^{\frac{x^2}{(x-2)}}$
6.14. $\lim\limits_{x\to\infty}\ln\left(\dfrac{2-4x}{1-4x}\right)^{x+3}$	6.29. $\lim\limits_{x\to\infty}\ln\left(\dfrac{x+3}{x-4}\right)^{x}$

6.15. $\lim_{x\to-\infty}\left(\frac{2x+3}{2x-2}\right)^{3x}$	6.30. $\lim_{x\to\infty}(7-6x)^{\frac{x}{(3x-3)}}$

FUNKSIYA HOSILASI

1. Funksiya hosilasi va differensiali.

$y=f(x)$ funksiya (a,b) oraliqda aniqlangan bo‘lib, $x\in(a,b)$ bo‘lsin. Bu x nuqtaga shunday Δx orttirma beraylikki, $x+\Delta x\in(a,b)$ bo‘lsin.

1-ta’rif. $f'(x):=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ – *funksiyaning x nuqtadagi hosilasi.*

Differensiallashning umumiy qoidalari

1. $y=c=const$, $y'=0$.

2. $y=c\cdot u\ (c=const)$, $y'=c\cdot u'$.

3. $y=u\pm v$, $y'=u'\pm v'$.

4. $y=u\cdot v$, $y'=u'\cdot v+u\cdot v'$.

5. $y=\frac{u}{v}\ (v(x)\neq 0)$, $y'=\frac{u'\cdot v-u\cdot v'}{v^2}$.

6. $y=f(u)\ (u=u(x))$, $y'=f'_u\cdot u'_x$.

7. $y=f(x)$, $x=f^{-1}(y)$ $y'_x=\frac{1}{x'_y}$.

8. $y=u^v$, $y'=u^v\cdot v'\cdot\ln u+u^{v-1}\cdot v\cdot u'$.

Asosiy elementar funksiyalarning hosilalari

1. $(x^n)'=nx^{n-1}$.

2. $(x^x)'=x^x\cdot(1+\ln x)$.

3. $(\sin x)'=\cos x$.

4. $(\cos x)'=-\sin x$.

5. $(tgx)'=\frac{1}{\cos^2 x}$.

6. $(ctgx)'=-\frac{1}{\sin^2 x}$.

7. $(\ln x)'=\frac{1}{x}$.

8. $(\log_a x)'=\frac{1}{x\ln a}\ (a>0,\ a\neq 1)$.

9. $(e^x)'=e^x$.

10. $(a^x)'=a^x\ln a\ (a>0)$.

11. $(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$.

12. $(\arccos)'=-\frac{1}{\sqrt{1-x^2}}$.

13. $(arctgx)'=\frac{1}{1+x^2}$.

14. $(arcctgx)'=-\frac{1}{1+x^2}$.

15. $(shx)'=chx$.

16. $(chx)'=shx$.

17. $(thx)'=\frac{1}{ch^2x}$.

18. $(cthx)'=-\frac{1}{sh^2x}$.

19. $(arcshx)'=\frac{1}{\sqrt{x^2+1}}$.

20. $(arcchx)'=\frac{1}{\sqrt{x^2-1}}$.

21. $(arccthx)'=-\frac{1}{1-x^2}$.

2-ta'rif. $f'(x+0):=\lim\limits_{\Delta x\to+0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to+0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ - *o'ng hosila.* $f'(x-0):$ $=\lim\limits_{\Delta x\to-0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to-0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ - *chap hosila.*

1 va 2-ta'riflardan quyidagilar chiqib keladi:

1)Agar $y=f(x)$ funksiya x nuqtada $f'(x)$ hosilaga ega bo'lsa, u holda $f'(x+0)$ va $f'(x-0)$ lar mavjud va $f'(x+0)=f'(x-0)=f'(x)$ bo'ladi.

2)Agar $f'(x+0)$ va $f'(x-0)$ lar mavjud bo'lib, $f'(x+0)=f'(x-0)$ bo'lsa, unda $f'(x)$ ham mavjud va $f'(x)=f'(x+0)=f'(x-0)$ bo'ladi.

1-teorema. *Agar x_0 nuqtada $f'(x_0)$ mavjud bo'lsa, u holda $y=f(x)$ funksiya grafigining $(x_0, f(x_0))$ nuqtasiga urinma o'tkazish mumkin va bu urinmaning burchak koeffitsienti $f'(x_0)$ ga teng bo'ladi.*

$$y=f(x_0)+f'(x_0)\cdot(x-x_0)$$ - ***urinma tenglamasi.***

$$y=f(x_0)-\frac{1}{f'(x_0)}\cdot(x-x_0)$$ - ***normal tenglamasi.***

Agar $S=f(t)$ moddiy nuqtaning sonlar o'qidagi t vaqtga mos keluvchi o'rnini bildirsa, unda $\Delta f=f(t+\Delta t)-f(t)$ - nuqtaning Δt vaqt oralig'idagi ko‘chishi, $\frac{f(t+\Delta t)-f(t)}{\Delta t}$ - o‘rtacha tezlik, $f'(t)$ esa t momentdagi ***oniy tezlik*** bo‘ladi.

3-ta’rif. *Agar* Δy *ni ushbu*

$$\Delta y = f(x+\Delta x) - f(x) = A(x)\cdot \Delta x + \alpha(x,\Delta x)\cdot \Delta x,$$

bu yerda $\Delta x \to 0$ *da* $\alpha(x,\Delta x) \to 0$ *ko‘rinishda ifodalash mumkin bo‘lsa, unda* $y = f(x)$ *funksiya* x *nuqtada* ***differensiallanuvchi*** *deyiladi.*

$A(x)\cdot \Delta x$ ifoda funksiya orttirmasining ***chiziqli bosh qismi*** yoki *funksiya* ***differensiali*** deb ataladi va **dy** kabi belgilanadi.

$\alpha(x,\Delta x)$ ifoda funksiya orttirmasining ***qoldiq hadi*** deb ataladi**.** Agar 0-simvolikadan foydalansak, $\Delta x \to 0$ da $\Delta y = A\cdot \Delta x + o(\Delta x)$ tenglikni xosil qilamiz.

2-teorema. $y = f(x)$ *funksiya* x *nuqtada differensiallanuvchi bo‘lishi uchun shu nuqtada chekli* $f'(x)$ *mavjud bo‘lishi zarur va yetarli.*

3-teorema. *Differensiallanuvchi funksiya uzluksiz bo‘ladi.*

Agar 2-teorema shartlari bajarilsa $df(x) = f'(x)\cdot \Delta x = f'(x)\cdot dx$ bo‘ladi. Differensiallashning asosiy qoidalari va elementar funksiyalar uchun hosilalar jadvali 1-§ ning 13^0 va 14^0 punktlarida keltirilgan.

2. Murakkab, oshkormas va parametrik ko‘rinishda berilgan funksiyalarning hosilalari

Aytaylik, $y = f(u)$ va $u = \varphi(x)$ funksiyalar berilgan bo‘lib, ular yordamida $y = f[\varphi(x)]$ murakkab funksiya tuzilgan bo‘lsin. Agar $u = \varphi(x)$ funksiya x nuqtada va $y = f(u)$ funksiya x nuqtaga mos keluvchi u nuqtada hosilaga ega bo‘lsa, unda

$$y'_x = y'_u \cdot u'_x \tag{5}$$

tenglik o‘rinli bo‘ladi.

Teskari funksiyaning hosilasi

Agar $y = f(x)$ funksiya x nuqtada $f'(x) \neq 0$ hosilaga ega bo‘lsa, bu funksiyaga teskari $x = f^{-1}(y)$ funksiya x nuqtaga mos bo‘lgan y nuqtada hosilaga ega va

$$x'_y = \frac{1}{y'_x}$$

bo‘ladi.

Parametrik ko‘rinishda berilgan funksiyaning hosilasi

Faraz qilaylik, $y = y(x)$ funksiya parametrik ko‘rinishda.

$$\begin{cases} x = \varphi(t) \\ y = \psi(t) \end{cases} \alpha < t < \beta$$

sistema yordamida aniqlangan bo‘lsin. Agar $\varphi(t)$ va$\psi(t)$ funksiyalar differensiallanuvchi bo‘lib, $\varphi'(t) \neq 0$ bo‘lsa, unda sistema differensiallanuvchi $y = \psi\left[\varphi^{-1}(x)\right]$ funksiyani aniqlaydi va

$$y'_x = \frac{y'_t}{x'_t} = \frac{\psi'(t)}{\varphi'(t)}$$

tenglik o‘rinli bo‘ladi.

Oshkormas funksiyaning hosilasi

Agar biror oraliqda differensiallanuvchi bo‘lgan $y = y(x)$ funksiya $F(x, y) = 0$ tenglik yordamida aniqlansa, unda oshkormas ko‘rinishda berilgan funksiyaning $y' = y'(x)$ hosilasini ushbu

$$\frac{d}{dx} F(x, y) = 0$$

tenglikdan topish mumkin.

Masalan, ushbu $y^5 + y^3 + y - x = 0$ tenglik yordamida oshkormas ko‘rinishda berilgan $y = y(x)$ funksiyaning y' hosilasini topaylik.

◁ (5)-tenglikka ko‘ra

$$\left(\ y^5+y^3+y-x\ \right)'_x=0\Rightarrow 5y^4\cdot y'+3y^2\cdot y'+y'-1=0\Rightarrow y'\,\frac{1}{5y^4+3y^2+1}.\ ▷$$

Differensialning taqribiy hisoblashga tatbiqi

Ma’lumki, $y=f(x)$ funksiya x_0 nuqtada differensiallanuvchi bo‘lsa, unda

$$\Delta f(x_0)=df(x_0)+o(\Delta x)$$

tenglik o‘rinli bo‘ladi. Agar $df(x_0)\neq 0$ bo‘lsa, bu tenglikdan yetarlicha kichik Δx lar uchun

$$\Delta f(x_0)\approx df(x_0)$$

yoki

$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\cdot\Delta x$$

taqribiy hisoblash formulasini hosil qilamiz.

3. Yuqori tartibli hosila va differensiallar. Aniqmasliklarni Lopital qoidasi yordamida ochish

$y=f(x)$funksiyaningюqori tartibli hosila va differensiallari ushbu

$$f^{(n)}(x)=\left\{f^{(n-1)}(x)\right\}'\ (n=2,3,\ldots),$$

$$d^n y=d\left(d^{n-1}y\right)(n=2,3,\ldots),$$

tengliklar yordamida aniqlanadi.

Asosiy formulalar

1) $\left(a^x\right)^n=a^x\cdot\ln^n a$ (a>0); $\left(e^x\right)^{(n)}=e^x$

2) $(\sin x)^{(n)}=\sin\left(x+\frac{n\pi}{2}\right)$

3) $(\cos x)^{(n)}=\cos\left(x+\frac{n\pi}{2}\right)$

4) $\left(x^{\alpha}\right)^{(n)}=\alpha(\alpha-1)\ldots(\alpha-n+1)x^{\alpha-n},\ \alpha\in R$

5) $(\ln x)^{(n)}=\dfrac{(-1)^{n-1}\cdot(n-1)!}{x^{n}}$

Leybnis formulasi

Agar $u=u(x)$va$v=v(x)$ funksiyalar n-tartibli hosilalarga ega bo‘lsa, unda $y=u(x)\cdot v(x)$ funksiya ham n-tartibli hosilaga ega bo‘ladi va

$$y^{(n)}=(u\cdot v)^{(n)}=\sum_{k=0}^{n}C_n^k u^{(k)}v^{(n-k)} \tag{6}$$

tenglik o‘rinli bo‘ladi. Bu yerda $u^{(0)}=u,\ \ v^{(0)}=v$ va $C_n^k=\dfrac{n!}{k!\cdot(n-k)!}$.

(6)-formulaga n-tartibli hosilani hisoblash uchun ***Leybnis formulasi*** deyiladi.

$u(x)\cdot v(x)$ funksiyaning n-tartibli differensiali $d^n(u\cdot v)$ uchun ham Leybnis formulasi o‘rinli.

Differensial hisobning asosiy teoremalari

Aytaylik $y=f(x)$ funksiya $[a,b]$ oroliqda aniqlangan bo‘lsin.

1-teorema.(Ferma teoremasi). Agar

1) $f(x)\in C[a,b]$*,*

2) $\forall x\in(a,b)$ *uchun chekli* $f'(x)-\exists$*,*

3)ichki $c\in(a,b)$ *nuqtada* $f(x)$ *funksiya eng katta (yoki eng kichik) qiymatga erishsa,*

unda $f'(c)=0$ *bo‘ladi.*

2-teorema. (Roll teoremasi). *Agar*

1) $f(x)\in C[a,b]$*,*

2) $\forall x\in(a,b)$ *uchun chekli* $f'(x)-\exists$*,*

3) $f(a)=f(b)$

bo‘lsa, $\exists x_0\in(a,b)$ *nuqta topiladiki ,* $f'(x_0)=0$ *bo‘ladi.*

3-teorema. (Lagranj teoremasi). *Agar*

1) $f(x)\in C[a,b]$

2) $\forall x\in(a,b)$ *uchun chekli* $f'(x)-\exists$

bo'lsa $\exists x_0\in(a,b)$ *nuqta topiladiki*

$$f(b)-f(a)=f'(x_0)\cdot(b-a)$$

bo'ladi.

1-natija. *Agar* $\forall x\in(a,b)$ *uchun* $f'(x)=0$ *bo'lsa, unda* (a,b)*da* $f(x)\equiv const$ *bo'ladi.*

2-natija. *Agar* $f(x)$ *funksiya* (a,b) *intervalda chegaralangan* $f'(x)$ *hosilaga ega bo'lsa, u holda* $f(x)$ (a,b) *da tekis uzluksiz bo'ladi.*

Lagranj teoremasini ba'zi bir tengsizliklarni isbotlashda qo'llash mumkin. Masalan, $(1+x)^\alpha\geq 1+\alpha x$ **Bernulli tengsizligi** $\forall x>-1$ va $\alpha>1$ da o'rinli ekanligi isbotlansin.

$\triangleleft$ **1-hol.** ***x>0 bo'lsin***. Unda $f(u)=(1+u)^\alpha$, $u\in[0,x]$ funksiya uchun Lagranj teoremasiga ko'ra $\exists x_0\in(0,x)$ nuqta topiladiki

$f(x)-f(0)=(1+x)^\alpha-1=\alpha\cdot(1+x_0)^{\alpha-1}\cdot x>\alpha x$ bo'ladi $\Rightarrow$ $(1+x)^\alpha>1+\alpha x$

2-hol. ***-1<x<0 bo'lsin***.Unda $f(u)=(1+u)^\alpha$, $u\in[x,0]$ funksiya uchun Lagranj teoremasini qo'llaymiz. $\Rightarrow\exists x_0\in(x;0)$

$f(0)-f(x)=1-(1+x)^\alpha=\alpha\cdot(1+x_0)^{\alpha-1}\cdot(0-x)=((1+x_0<1))<-\alpha x\Rightarrow(1+\alpha x)^\alpha>1+\alpha x$.

3-hol. ***x=0 bo'lsin*.** Unda $(1+x)^\alpha=1+\alpha x=1$ bo'ladi. Endi 3 ta holni umumlashtirsak, isbot qilishimiz kerak bo'lgan Bernulli tengsizligini hosil qilamiz. $\triangleleft$

4-teorema (Koshi teoremasi). *Agar*

1) $f(x),g(x)\in C[a,b]$,

2) $\forall x\in(a,b)$ *uchun chekli* $f'(x)$*va* $g'(x)$-$\exists$ *hamda* $g'(x)\neq 0$ *bo'lsa, unda* $\exists x_0\in(a,b)$ *nuqta topiladiki,*

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(x_0)}{g'(x_0)}$$

tenglik o'rinli bo'ladi.

Aniqmasliklarni ochish. Lopital qoidalari

2-§ da ko'rganimizdek funksiya limitini hisoblashda biz $\frac{0}{0}, \frac{\infty}{\infty}$, $0\cdot\infty,\ \infty-\infty,\ 1^{\infty}$, 0^0 va shu kabi aniqmasliklarga duch keldik. Bu aniqmasliklarni ochishda Lopital qoidalari katta yordam beradi.

T**eorema.** $f(x)$ *va* $g(x)$ *funksiyalar uchun quyidagi shartlar o'rinli bo'lsin.*

1) $f(x)$ *va* $g(x)$ *funksiyalar a nuqtaning biror atrofida aniqlangan va chekli hosilaga ega,*

2) $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} g(x)=0$,

3) a nuqtaning shu atrofida $[f'(x)]^2+[g'(x)]^2\neq 0$,

4) $\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$*-chekli yoki cheksiz.*

U holda

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

tenglik o'rinli bo'ladi.

Izoh: Agar bu teoremaning shartlari a nuqtaning chap (yoki o'ng) yarim atrofida bajarilsa, unda teorema $\frac{f(x)}{g(x)}$ ning *a* nuqtadgi chap (yoki o'ng) limitiga nisbatan o'rinli bo'ladi.

Yuqoridagi $\frac{0}{0}$ ko'rinishidagi aniqmasliklar uchun keltirilgan Lopital teoremasi $\frac{\infty}{\infty}$ ko'rinishidagi aniqmasliklar uchun ham o'rinli bo'ladi. Boshqa ko'rinishdagi aniqmasliklar esa $\frac{0}{0}$ va $\frac{\infty}{\infty}$ ko'rinishidagi aniqmasliklarga keltiriladi.

O-simvolika

Funksiya limitini hisoblashda va funksiyaning asimptotik xarakterini o‘rganishda «o-kichik» va «O-katta» tushunchalari muhim ahamiyatga ega. Biz a nuqta deganda chekli son yoki ∞ni tushunamiz. a chekli bo‘lgan holda nuqtaning atrofi deganda quyidagi to‘plamlardan biri tushuniladi: $(a-\delta;a)$, $(a;a+\delta)$, $(a-\delta;a+\delta)$, bu yerda $\delta>0$. Agar $a=\infty$ bo‘lsa, u holda a nuqtaning atrofi deganda quyidagi to‘plamlardan biri nazarda tutiladi: $(-\infty;-\Delta)$, $(\Delta,+\infty)$ yoki $(-\infty;-\Delta)\cup(\Delta,+\infty)$, bu yerda $\Delta>0$. Aytaylik, berilgan funksiyalar a nuqtaning biror atrofida aniqlangan bo‘lsin.

1-ta’rif. Agar shunday o‘zgarmas K son topilsaki, topilsaki, a nuqtaning biror atrofida

$$|\varphi(x)|\le K\cdot|\psi(x)|$$

tengsizlik bajarilsa, u holda shu atrofda $\varphi(x)$ funksiya $\psi(x)$ ga nisbatan O -katta deyiladi va $\varphi(x)=O(\varphi(x))$ kabi belgilanadi.

2-ta’rif. Agar a nuqtaning biror atrofida $\varphi(x)=\alpha(x)\cdot\psi(x)$ tenglik o‘rinli bo‘lib, $\lim\limits_{x\to a}\alpha(x)=0$ bo‘lsa, unda $x\to a$ da $\varphi(x)$ funksiya $\psi(x)$ ga nisbatan o -kichik deyiladi va $\varphi(x)=o(\psi(x))$ kabi belgilanadi.

1-ta’rifdan ko‘rinadiki, agar $\psi(x)\neq0$ bo‘lsa, unda $\lim\limits_{x\to a}\dfrac{\varphi(x)}{\psi(x)}=0$ bo‘lganda $\varphi(x)=o(\psi(x))$ bo‘ladi.

Izoh. Quyidagi tengliklar o‘rinli:

1) $o(f(x))+o(f(x))=o(f(x)),$

2) $K\cdot o(f(x))=o(f(x)),$

3) $o(f(x))\cdot o(f(x))=o(f(x)),$

4) $o(f(x))\cdot O(f(x))=o(f(x)),$

5) $x\to0$ da $x^m=o(x^n)\Leftrightarrow m>n$

6) $x\to\infty$ da $x^m = o(x^n) \Leftrightarrow m < n$.

3-ta’rif. Agar $x\to a$ da $\varphi(x)-\psi(x)=o(\psi(x))$ bo‘lsa, unda $x\to a$ da $\varphi(x)$ va $\psi(x)$ funksiyalar ekvivalent deyiladi hamda $\varphi(x)\sim\psi(x)$ kabi belgilanadi.

Bu ta’rifdan ko‘rinadiki, agar $\psi(x)\neq 0$ bo‘lsa, unda $\lim\limits_{x\to a}\frac{\varphi(x)}{\psi(x)}=1$ bo‘lganda $\varphi(x)\sim\psi(x)$ bo‘ladi.

1-teorema. Agar ushbu

$$\lim_{x\to a}\frac{\varphi(x)+o(\varphi(x))}{\psi(x)+o(\psi(x))} \text{ yoki } \lim_{x\to a}\frac{\varphi(x)}{\psi(x)}$$

limitlardan birortasi mavjud bo‘lsa, unda

$$\lim_{x\to a}\frac{\varphi(x)+o(\varphi(x))}{\psi(x)+o(\psi(x))}=\lim_{x\to a}\frac{\varphi(x)}{\psi(x)}$$

tenglik o‘rinli bo‘ladi.

1-teoremadan foydalanish samaradorligi Teylor formulasi yordamida yanada oshadi.

2-teorema. Agar $f(x)$ funksiya a nuqtada $f'(a)$, $f''(a),\ldots, f^{(n)}(a)$ hosilalarga ega bo‘lsa, u holda a nuqtaning biror atrofida ushbu

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\ldots+\frac{f^{(n)}(a)}{n!}(x-a)^n+o\left((x-a)^n\right)$$

Peano ko‘rinishidagi qoldiq hadli Teylor formulasi o‘rinli bo‘ladi.

Natija. $x\to 0$ da quyidagi tengliklar o‘rinli bo‘ladi.

1. $(1+x)^m = 1+mx+\frac{m(m-1)}{2!}x^2+\ldots+\frac{m(m-1)\ldots(m-n+1)}{n!}x^n+o(x^n)$

2. $e^x = 1+x+\frac{x^2}{2!}+\ldots+\frac{x^n}{n!}+o(x^n)$

3. $\ln(1+x)=x-\frac{x^2}{2}+\ldots+(-1)^{n-1}\cdot\frac{x^n}{n}+o(x^n)$

4. $\sin x = x - \frac{x^3}{3!} + ... + (-1)^{n-1} \cdot \frac{x^{2n-1}}{(2n-1)!} + o(x^{2n})$

5. $\cos x = 1 - \frac{x^2}{2!} + ... + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$

6. $tgx = x + \frac{1}{3}x^3 + o(x^4)$

7. $arctgx = x - \frac{1}{3}x^3 + o(x^4)$

Misol. $\lim_{x\to 0} \frac{\ln\cos x + x^2}{\sin x \cdot tgx}$ hisoblansin.

$$\lhd \lim_{x\to 0} \frac{\ln\cos x + x^2}{\sin x \cdot tgx} = \lim_{x\to 0} \frac{\ln\left(1 - \frac{1}{2}x^2 + o(x^2)\right) + x^2}{(x + o(x^2))(x + o(x^2))} =$$

$$= \lim_{x\to 0} \frac{\left(-\frac{1}{2}x^2 + o(x^2)\right) + o\left(-\frac{1}{2}x^2 + o(x^2)\right) + x^2}{x^2 + o(x^2)} =$$

$$= \lim_{x\to 0} \frac{-\frac{1}{2}x^2 + o(x^2) + x^2}{x^2 + o(x^2)} = \lim_{x\to 0} \frac{\frac{1}{2}x^2}{x^2} = \frac{1}{2} \rhd$$

Izoh. Limitni hisoblash jarayonida biz natijada keltirilgan 5, 4, 6, 3 tengliklardan va 1-teoremadan foydalandik.

4. Funksiyaning o‘sish va kamayishi. Funksiyaning ekstremumlari

Faraz qilaylik , $y = f(x)$ funksiya (a,b) oroliqda berilgan bo‘lsin.

1-ta’rif. $x_2 > x_1$ *tengsizlikni qanoatlantiruvchi* $\forall x_1, x_2 \in (a,b)$ *uchun* $f(x_2) \geq f(x_1)$ $(f(x_2) \leq f(x_1))$ *bo‘lsa,* $f(x)$ *funksiya* (a,b) *oraliqda* ***o‘suvchi*** ↗ ***(kamayuvchi*** ↘***)*** *deyiladi.*

Agar funksiya o‘suvchi yoki kamayuvchi bo‘lsa, bunday funksiyaga ***monoton*** *funksiya deyiladi.*

1-teorema. $f(x)$ *funksiya* (a,b) *intervalda chekli* $f'(x)$ *hosilaga ega bo'lsin. Bu funksiya shu intervalda o'suvchi (kamayuvchi) bo'lishi uchun* (a,b) *da* $f'(x)\geq 0$ $(f'(x)\leq 0)$ *bo'lishi zarur va yetarli.*

Funksiyaning ekstremumlari

$y=f(x)$ funksiya (a,b) intervalda berilgan bo'lib, $x_0\in(a,b)$ bo'lsin.

2-ta'rif. *Agar* x_0 *nuqtaning* $\exists\bigcup_\delta(x_0)$ *atrofi mavjud bo'lsaki,* $\forall x\in\bigcup_\delta(x_0)$ *uchun*

$$f(x)\leq f(x_0)\;(f(x)\geq f(x_0))$$

tengsizlik o'rinli bo'lsa, $f(x)$ *funksiya* x_0 *nuqtada **maksimum**ga (**minimum**ga) erishadi deyiladi.* $f(x_0)$ *qiymat* $f(x)$ *ning maksimum (minimum) qiymati deyiladi va*

$$f(x_0)=\max_{x\in\bigcup_\delta(x_0)}\{f(x)\}\left(f(x_0)=\min_{x\in\bigcup_\delta(x_0)}\{f(x)\}\right)$$

kabi belgilanadi.

Funksiyani maksimum va minimumi umumiy nom bilan uning ***ekstremumi*** deyiladi.

2-teorema. ***(Ekstremumning zaruriy sharti).*** *Agar* $f(x)$ *funksiya* x_0 *nuqtada (* $x_0\in(a,b)$ *) chekli* $f'(x_0)$ *hosilaga ega bo'lib, bu nuqtada* $f(x)$ *funksiya ekstremumga erishsa, u holda* $f'(x_0)=0$ *bo'ladi.*

Endi funksiya ekstremumga erishishining yetarli shartlarini keltiramiz.

Faraz qilaylik, $y=f(x)$ funksiya x_0 nuqtada uzliksiz bo'lib, $\bigcup_\delta(x_0)$ $\{x_0\}$ da chekli $f'(x)$ hosilaga ega bo'lsin.

3-teorema. *Agar* $f'(x)$ *hosila* x_0 *nuqtadan o'tishda o'z ishorasini musbatdan (manfiydan) manfiydan (musbatdan) o'zgartirsa, unda* $f(x)$ *funksiya* x_0 *nuqtada maksimumga (minimumi) erishadi. Agar* $f'(x)$

ishorasini o'zgartirmasa, u holda $f(x)$ funksiya x_0 nuqtada ekstremumga erishmaydi.

4-Teorema. $f(x)$ *funksiya* x_0 *nuqtada* $f', f'', ... f^{(n)}$ *hosilalarga ega bo'lib,*

$$f'(x_0) = f''(x_0) = ... = f^{(n-1)}(x_0) = 0, \ f^{(n)}(x_0) \neq 0$$

bo'lsin. Unda

1)agar n juft son bo'lib,

$$f^{(n)}(x_0) < 0 \ \left(f^{(n)}(x_0) > 0\right)$$

bo'lsa, $f(x)$ funksiya x_0 nuqtada ***maksimumga (minimumga)*** *erishadi.*

2) agar n toq son bo'lsa, $f(x)$ funksiya x_0 nuqtada ekstremumga erishmaydi.

Funksiyaning hosilasi nolga aylanadigan yoki hosilasi mavjud bo'lmagan nuqtalariga uning ***kritik*** nuqtalari deyiladi.

Izoh: Funksiya hosilasi mavjud bo'lmagan nuqtalarda ham funksiya ekstremumga erishishi mumkin. Masalan, $f(x)=|x|$ fuksiya uchun $f'(0)$– mavjud emas, lekin funksiya $x=0$ nuqtada minimumga erishadi.

$[a,b]$kesmada uzluksiz bo'lgan $f(x)$ funksiya o'zining shu kesmadagi eng katta (eng kichik) qiymatiga kritik nuqtada yoki kesmaning chegaraviy nuqtasida erishadi.

5. Funksiyaning qavariq va botiqligi

3-ta'rif.*Agar (a,b) oraliqda berilgan $y=f(x)$ funksiya grafigi $\forall[x_1, x_2]\subset(a,b)$ kesmaning chetki nuqtalarini tutashtiruvchi vatardan* ***yuqorida (pastda)*** *yotsa, unda $y=f(x)$ funksiya $[a,b]$ oraliqda* ***qavariq(botiq)*** *deb ataladi.*

5-teorema. *$y=f(x)$funksiya(a,b) intervalda aniqlangan va bu intervalda chekli $f'(x)$ hosilaga ega bo'lsin. $f(x)$funksiyaning(a,b) da*

qavariq $\cap$***(botiq***$\cup$***)*** *bo'lishi uchun* $f'(x)$*ning* (a,b)*da* ***kamayuvchi (o'suvchi)*** *bo'lishi zarur va yetarli.*

6-teorema. $y = f(x)$*funksiya* (a,b) *intervalda aniqlangan va bu intervalda ikkinchi tartibli* $f''(x)$ *hosilaga ega bo'lsin.* $f(x)$*ning* (a,b) *intervalda* $\cap(\cup)$ *bo'lishi uchun shu intervalda* $f''(x) \le 0$ $(f''(x) \ge 0)$ *tengsizlikning bajarilishi zarur va yetarli.*

4-ta'rif.*Agar* $x = a$ *nuqtadan o'tishda* $y = f(x)$ *funksiyaning grafigi qovariqligi yoki botiqligini o'zgartirsa, u holda* $x = a$ *nuqta funksiya grafigining* ***egilish nuqtasi*** *deyiladi.*

Funksiya grafigining asimptotalari

5-ta'rif.*Agar* $\lim\limits_{x \to a} f(x) = \infty$*bo'lsa,* $x = a$ *to'g'ri chiziq* $y = f(x)$ *funksiya grafigining* ***vertikal asimptotasi*** *deyiladi.*

6-ta'rif.*Agar* $\lim\limits_{x \to \infty} f(x) = b$ *bo'lsa,* $y = b$ *to'g'ri chiziq* $y = f(x)$ *funksiya grafigining* ***gorizontal asimptotasi*** *deyiladi.*

7-ta'rif.*Agar* $\lim\limits_{x \to \infty}[f(x) - (ax + b)] = 0$ *bo'lsa,* $y = ax + b$ *to'g'ri chiziq* $y = f(x)$ *funksiya grafigining* ***og'ma asimptotasi*** *deyiladi.*

7-teorema. $y = f(x)$*funksiya grafigi* $x \to +\infty$*da* $y = ax + b$ *og'ma asimptotaga ega bo'lishi uchun*

$$\lim_{x \to +\infty} \frac{f(x)}{x} = a, \ \lim_{x \to +\infty}[f(x) - ax] = b$$

bo'lishi zarur va yetarlidir.

Bu teorema $x \to -\infty$ da ham o'rinlidir.

Funksiyalarni to'liq tekshirish va grafiklarini chizish

Funksiyani to'la tekshirish va grafigini yasash quyidagilarni aniqlash yordamida amalga oshiriladi.

1) Funksiyani aniqlanish soxasini topish.
2) Aniqlanish sohasining chegaraviy nuqtalaridagi xarakterini aniqlash.

3) Funksiyaning juft yoki toqligini va, agar imkon bo‘lsa, boshqa markaz va simmetriya o‘qlarini aniqlash.

4) Davriylikka tekshirish.

5) Uzilish nuqtalarini topish va ularning turini aniqlash (2-punktni to‘ldiradi).

6) Koordinata o‘qlari bilan kesishish nuqtalarini topish.

7) Funksiyaning ishorasi o‘zgarmaydigan oraliqlarni aniqlash.

8) Monotonlik va ekstremumga tekshirish.

9) Egilish nuqtalari, qavariqlik va botiqlik oraliqlarini topish.

10) Asimptotalarni aniqlash

11) Tekshirish natijalarini yo‘llari $x, y, f(x), f'(x), f''(x)$ larga mos bo‘lgan jadval ko‘rinishida ifodalash (oxirgi yo‘lda faqat ishora aniqlanadi).

12) Jadvaldagi nuqtalarni tekislikda ifodalash.

13) Asimtotalarni yasash.

14) Yuqoridagi tekshirish natijalarini hisobga olgan holda tekislikdagi nuqtalarni chiziq yordamida tutashtirish.

Izoh: Agar funksiya parametrik ko‘rinishda yoki qutb koordinatalar sistemasida berilgan bo‘lsa ham u yuqoridagi sxema yordamida tekshiriladi.

Nazorat savollari

1. Funksiya hosilasining ta’rifi.

2. Bir tomonli hosilalar.

3. Hosilaning geometrik ma’nosi.

4. Urinma tenglamasi.

5. Normal tenglamasi.

6. Hosilaning mexanik ma’nosi.

7. Funksiya differensialining ta’rifi.

8. Differensiallanuvchi va uzluksiz funksiyalar orasidagi bog’lanish.

9. Murakkab funksiyaning hosilasi.

10. Teskari funksiyaning hosilasi.

11. Parametrik ko‘rinishda berilgan funksiyaning hosilasi.

12. Oshkormas ko‘rinishda berilgan funksiyaning hosilasi.

13. Differensial yordamida taqribiy hisoblash.

14. Yuqori tartibli hosila va differensiallar.

15. Leybnis formulasi.

16. Ferma teoremasi.

17. Roll teoremasi.

18. Lagranj teoremasi.

19. Lagranj teoremasining natijalari.

20. Koshi teoremasi.

21. Lopitalning birinchi qoidasi.

22. Lopitalning ikkinchi qoidasi.

23. O-simvolika.

24. Teylor formulasi.

25. Funksiyaning monotonligi.

26. Birinchi tartibli hosila yordamida funksiyaning ekstremumini topish.

27. Yuqori tartibli hosilalar yordamida funksiyaning ekstremumini topish.

28. Funksiyaning qavariqligi va egilish nuqtalari.

29. Funksiya grafigining asimptotalari.

30. Funksiyani to‘la tekshirish va grafigini yasash.

6. Mustaqil yechish uchun misol va masalalar

1-masala. Hosila ta'rifidan foydalanib $f'(0)$ topilsin (agar u mavjud bo'lsa).

$$f(x)=\begin{cases}\dfrac{e^{x^2}-\cos x}{x}, x\neq 0\\ 0, x=0\end{cases}$$

$$\triangleleft\ f'(0):=\lim_{\Delta x\to 0}\frac{f(0+\Delta x)-f(0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\dfrac{e^{\Delta x^2}-\cos\Delta x}{\Delta x}-0}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{e^{\Delta x^2}-\cos\Delta x}{\Delta x^2}=\lim_{\Delta x\to 0}\frac{\left(e^{\Delta x^2}-1\right)+\left(1-\cos\Delta x\right)}{\Delta x^2}=$$

$$=\lim_{\Delta x\to 0}\frac{e^{\Delta x^2}-1}{\Delta x^2}+\lim_{\Delta x\to 0}\frac{1-\cos\Delta x}{\Delta x^2}=1+\lim_{\Delta x\to 0}\frac{2\sin^2\frac{\Delta x}{2}}{\Delta x^2}=1+\frac{1}{2}\lim_{\Delta x\to 0}\left(\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}\right)^2=1+\frac{1}{2}=\frac{3}{2}.\triangleright$$

1.1 $f(x)=\begin{cases}tg\left(x^3+x^2\sin\frac{1}{x}\right), \ x\neq 0\\ 0, x=0.\end{cases}$	**1.11** $f(x)=\begin{cases}\sin x\cdot\cos\frac{5}{x}, x\neq 0,\\ 0, x=0.\end{cases}$
1.2 $f(x)=\begin{cases}\sin\left(x\sin\frac{3}{x}\right), x\neq 0,\\ 0, x=0.\end{cases}$	**1.12** $f(x)=\begin{cases}2x^2+x^2\cos\frac{1}{x}, x\neq 0,\\ 0, \ x=0.\end{cases}$
1.3 $f(x)=\begin{cases}\arcsin\left(x^2\cos\frac{1}{9x}\right)+\frac{2}{3}x, x\neq 0,\\ 0, x=0.\end{cases}$	**1.13** $f(x)=\begin{cases}x+\arcsin\left(x^2\sin\frac{6}{x}\right), x\neq 0,\\ 0, \ x=0.\end{cases}$
1.4 $f(x)=\begin{cases}0, \ x=0,\\ \sqrt{1+\ln\left(1+x^2\sin\frac{1}{x}\right)^2}-1, x\neq 0.\end{cases}$	**1.14** $f(x)=\begin{cases}\dfrac{\ln\cos x}{x}, x\neq 0,\\ 0, \ x=0.\end{cases}$
1.5 $f(x)=\begin{cases}arctg\left(x\cos\frac{1}{5x}\right), x\neq 0,\\ 0, \ x=0.\end{cases}$	**1.15** $f(x)=\begin{cases}tg\left(2^{x^{2\cdot\cos\frac{1}{8x}}}-1+x\right), x\neq 0,\\ 0, \ x=0.\end{cases}$

1.6 $f(x)=\begin{cases}\sin\left(e^{x^{2\sin\frac{5}{x}}}-1\right)+x, x\neq 0,\\ 0,\ x=0.\end{cases}$	**1.16** $f(x)=\begin{cases}6x+x\sin\frac{1}{x}, x\neq 0\\ 0, x=0\end{cases}$
1.7 $f(x)=\begin{cases}\ln\left[1-\sin\left(x^2\sin\frac{1}{x}\right)\right], x\neq 0,\\ 0,\ x=0.\end{cases}$	**1.17** $f(x)=\begin{cases}arctgx\cdot\sin\frac{7}{x}, x\neq 0,\\ 0,\ x=0.\end{cases}$
1.8 $f(x)=\begin{cases}0,\ \mathrm{x}=0,\\ x^2\cdot\cos\frac{4}{3x}+\frac{x^2}{2}, x\neq 0.\end{cases}$	**1.18** $f(x)=\begin{cases}e^{x\cdot\sin 5x}-1,\ x\neq 0,\\ 0,\ x=0.\end{cases}$
1.9 $f(x)=\begin{cases}arctg\left(x^3-x^{\frac{3}{2}}\sin\frac{1}{3x}\right), x\neq 0,\\ 0, x=0.\end{cases}$	**1.19** $\begin{cases}f(x)=2x^2+x^2\cos\frac{1}{9x},\ \mathrm{x}\neq 0,\\ 0,\ \mathrm{x}=0.\end{cases}$
1.10 $f(x)=\begin{cases}x^2\cdot\cos^2\frac{11}{x}, x\neq 0,\\ 0, x=0.\end{cases}$	**1.20** $f(x)=\begin{cases}3^{x^2\cdot\sin\frac{2}{x}}-1+2x,\ x\neq 0,\\ 0,\ x=0.\end{cases}$

2-masala. Funksiya grafigining abssissasi x_0 bo‘lgan nuqtasiga o‘tkazilgan normal (2.1-2.12 variantlarda) yoki urinma (2.13-2.20 variantlarda) tenglamasi topilsin.

Funksiya grafigining absissasi x_0 bo‘lgan nuqtasiga o‘tkazilgan urinma tenglamasi topilsin.

$$y=\frac{x}{x^2+1}\ ,\ x_0=-2$$

$\triangleleft$ Ma’lumki, urinma tenglamasi

$$y=f(x_0)+f'(x_0)\cdot(x-x_0)$$

ko‘rinishga ega. $f(x_0)=f(-2)=\frac{-2}{(-2)^2+1}=-\frac{2}{5}$

$$f'(x)=\left(\frac{x}{x^2+1}\right)'=\frac{x'\cdot(x^2+1)-x\cdot(x^2+1)'}{(x^2+1)^2}=\frac{x^2+1-2x^2}{(x^2+1)^2}=$$

$$=\frac{1-x^2}{(x^2+1)^2}\Rightarrow f(x_0)=f'(-2)=\frac{1-4}{25}=-\frac{3}{25}\Rightarrow$$

$\Rightarrow y=-\frac{2}{5}-\frac{3}{25}\cdot(x+2)\Rightarrow$ Urinma tenglamasi: $3x+25y+16=0$ ▷

2.1 $y=\frac{4x-x^2}{4}, x_0=2$.	**2.11** $y=\sqrt{x}-3\sqrt[3]{x}, x_0=64$.
2.2 $y=2x^2+3x-1, x_0=-2$.	**2.12** $y=\frac{x^3+3}{x^3-2}, x_0=2$.
2.3 $y=x-x^3, x_0=-1$.	**2.13** $y=2x^2+3, x_0=-1$.
2.4 $y=x^2+8\sqrt{x}-32, x_0=4$.	**2.14** $y=\frac{x^{29}+6}{x^4+1}, x_0=1$.
2.5 $y=x+\sqrt{x^3}, x_0=1$.	**2.15** $y=2x+\frac{1}{x}, x_0=1$.
2.6 $y=\sqrt[3]{x^2}-20, x_0=-8$.	**2.16** $y=-\frac{2(x^8+2)}{3\cdot(x^4+1)}, x_0=1$.
2.7 $y=\frac{1+\sqrt{x}}{1-\sqrt{x}}, x_0=4$.	**2.17** $y=\frac{x^5+1}{x^4+1}, x_0=1$.
2.8 $y=8\sqrt[4]{x}-70, x_0=16$.	**2.18** $y=\frac{x^{16}+9}{1-5x^2}, x_0=1$.
2.9 $y=2x^2-3x+1, x_0=1$.	**2.19** $y=3(\sqrt[3]{x}-2\sqrt{x}), x_0=1$.
2.10 $y=\frac{x^2-3x+6}{x^2}, x_0=3$.	**2.20** $y=\frac{1}{3x+2}, x_0=2$.

3-masala. Differensial yordamida ifodaning taqribiy qiymati hisoblansin.

$$y=\sqrt[3]{x^3+7x},\ x=1{,}012$$

◁ Taqribiy qiymat

$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\cdot\Delta x \qquad (*)$$

formula yordamida hisoblanadi.

Bizda

$$f(x)=\sqrt[3]{x^3+7x},\ x_0=1,\quad \Delta x=0{,}012\Rightarrow f'(x)=\left(\sqrt[3]{x^3+7x}\right)'=\left[\left(x^3+7x\right)^{\frac{1}{3}}\right]'=$$

$$=\frac{1}{3}\cdot\left(x^3+7x\right)^{-\frac{2}{3}}\cdot\left(x^3+7x\right)'=\frac{3x^2+7}{3\cdot\sqrt[3]{\left(x^3+7x\right)^2}}\Rightarrow f(x_0)=\sqrt[3]{1+7}=2,$$

$$f'(x_0)=\frac{10}{3\cdot 4}=\frac{10}{12}=\frac{5}{6}.$$

Topilgan ifodalarni (*) tenglikka olib borib qo‘yamiz:

$$\sqrt[3]{(1{,}012)^3+7\cdot 1{,}012}\approx 2+\frac{5}{6}\cdot 0{,}012=2+5\cdot 0{,}002=2{,}01 \rhd$$

3.1 $y=\sqrt[3]{x}, x=7{,}76$.	**3.11** $y=\sqrt{4x-1}, x=2{,}56$.
3.2 $y=\sqrt[3]{x}, x=27{,}54$.	**3.12** $y=\sqrt[5]{x^2}, x=1{,}03$.
3.3 $y=\dfrac{x+\sqrt{5-x^2}}{2}, x=0{,}98$.	**3.13** $y=\dfrac{1}{\sqrt{2x^2+x+1}}, x=1{,}016$.
3.4 $y=\arcsin x, x=0{,}08$.	**3.14** $y=\sqrt{1+x+\sin x}, x=0{,}01$.
3.5 $y=\sqrt[3]{x^2+2x+5}, x=0{,}97$.	**3.15** $y=\dfrac{1}{\sqrt{x}}, x=4{,}16$.
3.6 $y=\sqrt{x^2+x+3}, x=1{,}97$.	**3.16** $y=\sqrt[3]{3x+\cos x}, x=0{,}01$.
3.7 $y=x^{11}, x=1{,}021$.	**3.17** $y=x^7, x=2{,}002$.
3.8 $y=x^{21}, x=0{,}998$.	**3.18** $y=\sqrt[4]{2x-\sin\dfrac{\pi x}{2}}, x=1{,}02$.
3.9 $y=\sqrt[3]{x^2}, x=1{,}03$.	**3.19** $y=\sqrt{4x-3}, x=1{,}78$.
3.10 $y=x^6, x=2{,}01$.	**3.20** $y=\sqrt{x^2+5}, x=1{,}97$.

4-masala. Hosila hisoblansin.

$$y=\frac{x^6+x^3-2}{\sqrt{1-x^2}}$$

$$\lhd\; y'=\left(\frac{x^6+x^3-2}{\sqrt{1-x^2}}\right)'=\frac{\left(x^6+x^3-2\right)'\cdot\sqrt{1-x^2}-\left(x^6+x^3-2\right)\cdot\left(\sqrt{1-x^2}\right)'}{\left(\sqrt{1-x^2}\right)^2}=$$

$$=\frac{\left(6x^5+3x^2\right)\cdot\sqrt{1-x^2}-\left(x^6+x^3-2\right)\cdot\dfrac{-2x}{2\sqrt{1-x^2}}}{1-x^2}=\frac{\left(6x^5+3x^2\right)\left(1-x^2\right)+x\left(x^6+x^3-2\right)}{\left(1-x^2\right)\cdot\sqrt{1-x^2}}=$$

$$=\frac{x\left(5x^6-6x^4+2x^3-3x+2\right)}{\left(x^2-1\right)\cdot\sqrt{1-x^2}}\;\rhd$$

Funksiyaning hosilasini toping

4.1 $y=\frac{3x+\sqrt{x}}{\sqrt{x^2+2}}$.	**4.11** $y=3\sqrt[3]{\frac{x+1}{(x-1)^2}}$.
4.2 $y=2\cdot\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$.	**4.12** $y=\frac{\sqrt{2x+3}\cdot(x-2)}{x^2}$.
4.3 $y=\frac{(x+3)\sqrt{2x-1}}{2x+7}$.	**4.13** $y=3\cdot\frac{\sqrt[3]{x^2+x+1}}{x+1}$.
4.4 $y=\frac{(2x+1)\sqrt{x^2-x}}{x^2}$.	**4.14** $y=\frac{x^6+8x^3-128}{\sqrt{8-x^3}}$.
4.5 $y=\frac{x^2+2}{2\sqrt{1-x^4}}$.	**4.15** $y=\frac{1}{(x+2)\cdot\sqrt{x^2+4x+5}}$.
4.6 $y=\frac{x-1}{\left(x^2+5\right)\sqrt{x^2+5}}$.	**4.16** $y=\frac{\sqrt{\left(1+x^2\right)^3}}{3x^3}$.
4.7 $y=\frac{x\sqrt{x+1}}{x^2+x+1}$.	**4.17** $y=\frac{\sqrt{x-1}\cdot(3x+2)}{4x^2}$.
4.8 $y=\frac{\left(2x^2+3\right)\cdot\sqrt{x^2-3}}{9x^3}$.	**4.18** $y=\frac{\left(x^2-2\right)\cdot\sqrt{4+x^2}}{24x^3}$.
4.9 $y=\frac{x+7}{6\sqrt{x^2+2x+7}}$.	**4.19** $y=\frac{1+x^2}{2\cdot\sqrt{1+2x^2}}$.
4.10 $y=\left(1-x^2\right)\cdot\sqrt[5]{x^3+\frac{1}{x}}$.	**4.20** $y=\frac{4+3x^3}{x\cdot\sqrt[3]{\left(2+x^3\right)^2}}$.

5-masala. Hosila hisoblansin.

$$y = x^{2x} \cdot 5^x$$

$$\triangleleft\ y' = \left(x^{2x} \cdot 5^x\right)' = \left[e^{\ln\left(x^2 \cdot 5^x\right)}\right]' = e^{\ln\left(x^{2x} \cdot 5^x\right)} \cdot \left[\ln\left(x^{2x} \cdot 5^x\right)\right]' = x^{2x} \cdot 5^x \cdot \left[2x\ln x + x \cdot \ln 5\right]' =$$

$$= x^{2x} \cdot 5^x \cdot \left[2\ln x + 2 + \ln 5\right] = x^{2x} \cdot 5^x \cdot \left(2 + \ln 5x^2\right). \triangleright$$

5.1 $y = (arctgx)^{\frac{1}{2}\ln arctgx}$	**5.11** $y = (x\sin x)^{\sin(x\sin x)}$
5.2 $y = \left(\sin\sqrt{x}\right)^{\ln\sin\sqrt{x}}$	**5.12** $y = \left(x^3 + 4\right)^{tgx}$
5.3 $y = (\sin x)^{5e^x}$	**5.13** $y = x^{\sin x^3}$
5.4 $y = (\arcsin x)^{e^x}$	**5.14** $y = \left(x^4 + 5\right)^{ctgx}$
5.5 $y = (\ln x)^{3^x}$	**5.15** $y = (\sin x)^{\frac{5x}{2}}$
5.6 $y = x^{\arcsin x}$	**5.16** $y = \left(x^2 + 1\right)^{\cos x}$
5.7 $y = (ctg3x)^{2e^x}$	**5.17** $y = 19^{x^{19}} \cdot x^{19}$
5.8 $y = x^{e^{tgx}}$	**5.18** $y = x^{3x} \cdot 2^x$
5.9 $y = (tgx)^{4e^x}$	**5.19** $y = \left(\sin\sqrt{x}\right)^{e^{\frac{1}{x}}}$
5.10 $y = (\cos 5x)^{e^x}$	**5.20** $y = x^{e^{\sin x}}$

6-masala. Funksiya grafigining abssissasi $x_0 = x(t_0)$ bo‘lgan nuqtasiga o‘tkazilgan urinma va normal tenglamalari topilsin.

$$\begin{cases} x = 1 - t^2, \\ y = t - t^3, t_0 = 2 \end{cases}$$

$\triangleleft$ Biz $y = f(x_0) + f'(x_0) \cdot (x - x_0)$ (urinma tenglamasi), $y = f(x_0) - \dfrac{1}{f'(x_0)} \cdot (x - x_0)$ (normal tenglamasi) va $y'_x = \dfrac{y'_t}{x'_t}$ (parametrik ko‘rinishda berilgan funksiyaning hosilasi) formulalardan foydalanamiz:

$$x_0 = 1 - 2^2 = -3;\ f(x_0) = 2 - 2^3 = -6;\ y'_x = \frac{\left(t - t^3\right)'}{\left(1 - t^2\right)'} = \frac{1 - 3t^2}{-2t} \Rightarrow f'(x_0) = \frac{1 - 3 \cdot 4}{-4} = \frac{11}{4}.$$

Topilgan qiymatlarni tenglikka olib borib qo‘yib urinma va normal tenglamalarni topamiz:

$$\begin{cases} y=-6+\dfrac{11}{4}\cdot(x+3) \\ y=-6-\dfrac{4}{11}(x+3) \end{cases} \Rightarrow \begin{cases} 4y-11x-9=0-urinma \\ 4x+11y+78=0-normal \end{cases} \triangleright$$

6.1 $\begin{cases} x=\sqrt{3}\cos t \\ y=\sin t, t_0=\dfrac{\pi}{3} \end{cases}$

6.2 $\begin{cases} x=t(t\cos t-2\sin t) \\ y=t(t\sin t+2\cos t), t_0=\dfrac{\pi}{4} \end{cases}$

6.3 $\begin{cases} x=2t-t^2 \\ y=3t-t^3, t_0=1 \end{cases}$

6.4 $\begin{cases} x=\dfrac{3at}{1+t^2} \\ y=\dfrac{3at^2}{1+t^2}, t_0=2 \end{cases}$

6.5 $\begin{cases} x=2\sin^3 t \\ y=2\cos^3 t, t_0=\dfrac{\pi}{3} \end{cases}$

6.6 $\begin{cases} x=2\ln(ctgt)+1 \\ y=tgt+ctgt, t_0=\dfrac{\pi}{4} \end{cases}$

6.7 $\begin{cases} x=3(t-\sin t) \\ y=3(1-\cos t), t_0=\dfrac{\pi}{3} \end{cases}$

6.8 $\begin{cases} x=at\cos t \\ y=at\sin t, t_0=\dfrac{\pi}{2} \end{cases}$

6.9 $\begin{cases} x=\sin^2 t \\ y=\cos^2 t, t_0=\dfrac{\pi}{6} \end{cases}$

6.10 $\begin{cases} x=\dfrac{t+1}{t} \\ y=\dfrac{t-1}{t}, t_0=-1 \end{cases}$

6.11 $\begin{cases} x=\arcsin\dfrac{t}{\sqrt{1+t^2}} \\ y=\arccos\dfrac{1}{\sqrt{1+t^2}}, t_0=1 \end{cases}$

6.12 $\begin{cases} x=\ln(1+t^2) \\ y=t-arctgt, t_0=1 \end{cases}$

6.13 $\begin{cases} x=\dfrac{1+\ln t}{t^2} \\ y=\dfrac{3+2\ln t}{t}, t_0=1 \end{cases}$

6.14 $\begin{cases} x=t\cdot(1-\sin t) \\ y=t\cdot\cos t, t_0=0 \end{cases}$

6.15 $\begin{cases} x=\dfrac{1+t}{t^2} \\ y=\dfrac{3}{2t^2}+\dfrac{2}{t}, t_0=2 \end{cases}$

6.16 $\begin{cases} x=\dfrac{1+t^3}{t^2-1} \\ y=\dfrac{t}{t^2-1} t_0=2 \end{cases}$

6.17 $\begin{cases} x = a\sin^3 t \\ y = a\cos^3 t, t_0 = \dfrac{\pi}{6} \end{cases}$

6.18 $\begin{cases} x = 3\cos t \\ y = 4\sin t, t_0 = \dfrac{\pi}{4} \end{cases}$

6.19 $\begin{cases} x = a(t\sin t + \cos t) \\ y = a(\sin t - t\cos t), t_0 = \dfrac{\pi}{4} \end{cases}$

6.20 $\begin{cases} x = t - t^4 \\ y = t^2 - t^3, t_0 = 1 \end{cases}$

7-masala. Parametrik ko‘rinishda berilgan funksiyaning ikkinchi tartibli hosilasi hisoblansin.

$$\begin{cases} x = t + \sin t \\ y = 2 + \cos t \end{cases}$$

◁ Bu masalani $y'_x = \dfrac{y'_t}{x'_t}$ - formuladan ikki marta foydalanish yordamida yechamiz.

$$y'_x = \frac{y'_t}{x'_t} = \frac{(2+\cos t)'}{(t+\sin t)'} = \frac{-\cos t}{1+\cos t}$$

$$y''_{x^2} = \frac{(y'_x)'_t}{x'_t} = \frac{\left(\dfrac{-\cos t}{1+\cos t}\right)'}{1+\cos t} = \frac{\sin t\cdot(1+\cos t) - \cos t\sin t}{(1+\cos t)^3} = \frac{\sin t}{(1+\cos t)^3}. \triangleright$$

7.1 $\begin{cases} x = \cos 2t \\ y = 2\sec^2 t \end{cases}$

7.2 $\begin{cases} x = \sqrt{t^3 - 1} \\ y = \ln t \end{cases}$

7.3 $\begin{cases} x = \sqrt{1 - t^2} \\ y = \dfrac{1}{t} \end{cases}$

7.4 $\begin{cases} x = \sqrt{t} - 1 \\ y = \dfrac{1}{\sqrt{t}} \end{cases}$

7.5 $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$

7.6 $\begin{cases} x = \cos^2 t \\ y = tg^2 t \end{cases}$

7.7 $\begin{cases} x = t + \sin t \\ y = 2 - \cos t \end{cases}$

7.8 $\begin{cases} x = \sqrt{t-3} \\ y = \ln(t-2) \end{cases}$

7.9 $\begin{cases} x = \dfrac{1}{t} \\ y = \dfrac{1}{1+t^2} \end{cases}$

7.10 $\begin{cases} x = \sin t \\ y = \ln(\cos t) \end{cases}$

7.11 $\begin{cases} x = \sqrt{t} \\ y = \dfrac{1}{\sqrt{1-t}} \end{cases}$

7.12 $\begin{cases} x = t - \sin t \\ y = 2 - \cos t \end{cases}$

7.13 $\begin{cases} x = \sin t \\ y = \sec t \end{cases}$

7.14 $\begin{cases} x = \cos t \\ y = \ln(\sin t) \end{cases}$

7.15 $\begin{cases} x = tgt \\ y = \dfrac{1}{\sin 2t} \end{cases}$

7.16 $\begin{cases} x = \cos t + t\sin t \\ y = \sin t - t\cos t \end{cases}$

7.17 $\begin{cases} x = \sqrt{t-1} \\ y = \dfrac{t}{\sqrt{t-1}} \end{cases}$

7.18 $\begin{cases} x = e^t \\ y = \arcsin t \end{cases}$

7.19 $\begin{cases} x = \sqrt{t} \\ y = \sqrt[3]{t-1} \end{cases}$

7.20 $\begin{cases} x = 2(t - \sin t) \\ y = 4(2 + \cos t) \end{cases}$

8-masala. n-tartibli hosila hisoblansin.

$$y = \ln(2x+7).$$

$$\lhd\ y = \lg(2x+7) = \frac{\ln(2x+7)}{\ln 10} = \frac{1}{\ln 10}\cdot \ln(2x+7)$$

$$y' = \frac{1}{\ln 10}\cdot\frac{1}{2x+7}\cdot(2x+7)' = \frac{2}{\ln 10}\cdot\frac{1}{2x+7}$$

$$y'' = (y')' = \frac{2}{\ln 10}\cdot\left(-\frac{1}{(2x+7)^2}\right)\cdot(2x+7)' = -\frac{2^2}{\ln 10}\cdot\frac{1}{(2x+7)^2}$$

$$y''' = (y'')' = \frac{2^3}{\ln 10}\cdot\frac{2!}{(2x+7)^3}$$

Bu jarayonni davom ettirish natijasida $\forall n \in N$ uchun

$y^{(n)} = (-1)^{n-1}\cdot\dfrac{2^n}{\ln 10}\cdot\dfrac{(n-1)!}{(2x+7)^n}$ tenglikni hosil qilamiz. $\rhd$

8.1 $y=\sin 2x+\cos(x+1)$.	**8.2** $y=\sqrt[5]{e^{7x-1}}$.
8.3 $y=\frac{4x+7}{2x+3}$.	**8.4** $y=\lg(5x+2)$.
8.5 $y=2^{3x}$.	**8.6** $y=\frac{x}{2(3x+2)}$.
8.7 $y=\frac{2x+5}{13(3x+1)}$.	**8.8** $y=4^{3x+5}$.
8.9 $y=\sin(x+1)+\cos 2x$.	**8.10** $y=\sqrt[3]{e^{2x+1}}$.
8.11 $y=\frac{4x+15}{5x+1}$.	**8.12** $y=\lg(3x+1)$.
8.13 $y=7^{5x}$.	**8.14** $y=\frac{x}{9(4x+9)}$.
8.15 $y=\frac{4}{x}$.	**8.16** $y=\frac{5x+1}{13\cdot(2x+3)}$.
8.17 $y=5^{2x+3}$.	**8.18** $y=\sin(3x+1)+\cos 5x$.
8.19 $y=\sqrt{e^{3x+1}}$.	**8.20** $y=\frac{11+12x}{6x+5}$.

9-masala. Quyidagi tengsizliklar isbotlansin.

Quyidagi

$$b^n-a^n<n\cdot(b-a)\cdot b^{n-1},0<a<b,n\in N$$

tengsizlik isbotlansin.

◁ Bu tengsizlikni Lagranj teoremasidan foydalanib isbotlaymiz. $f(x)=x^n$ funksiya uchun $[a,b]$ kesmada Lagranj teoremasini qo‘llaymiz:

$$f(b)-f(a)=f'(x_0)\cdot(b-a),\ x_0\in(a,b)\Rightarrow b^n-a^n=n\cdot x_0^{\,n-1}\cdot(b-a)<n\cdot(b-a)\cdot b^{n-1}.\ \triangleright$$

9.1 $\ln(1+x)>\frac{x}{1+x},x>0$.	**9.11** $e^x\geq 1+x+\frac{x^2}{2!}+...+\frac{x^n}{n!},x\geq 0,n\in N$.

9.2 $\ln(1+x)<x, x>0$.	**9.12** $arctgx<x-\frac{x^3}{6}, 0<x\le 1$.
9.3 $e^x>1+x, x\in R$.	**9.13** $\ln(1+x)\le x-\frac{x^2}{2}+\frac{x^3}{3}, x\ge 0$.
9.4 $e^x>ex, x>1$.	**9.14** $e^x\ge 1+x+\frac{x^2}{2!}, x\ge 0$.
9.5 $b^n-a^n>n(b-a)a^{n-1}, 0<a<b, n\in N$.	**9.15** $\ln\frac{a}{b}<\frac{a-b}{b}, 0<b<a$.
9.6 $(a+b)^p\le a^p+b^p, 0\le p\le 1$.	**9.16** $e^x\le 1+x+\frac{x^2e^x}{2!}, x\ge 0$.
9.7 $\cos x>1-\frac{x^2}{2}, x>0$.	**9.17** $x^p-y^p\le px^{p-1}\cdot(x-y), 0<y<x,\ p>1$.
9.8 $2\sqrt{x}>3-\frac{1}{x}, x>1$.	**9.18** $\ln(1+x)\ge x-\frac{x^2}{2}, x\ge 0$.
9.9 $\sin x>x-\frac{x^3}{6}, x>0$.	**9.19** $\lvert arctga-arctgb\rvert\le\lvert a-b\rvert$.
9.10 $arctgx>x-\frac{x^3}{3}, 0<x\le 1$.	**9.20** $\ln\frac{a}{b}>\frac{a-b}{a}, 0<b<a$.

10-masala. Limit hisoblansin.

$$\lim_{x\to 0}(\cos x)^{\frac{1}{\sin x}}$$

$\lhd \lim_{x\to 0}(\cos x)^{\frac{1}{\sin x}}=(1^\infty)=e^{\lim_{x\to 0}\frac{\ln(\cos x)}{\sin x}}=e^{\left(\frac{0}{0}\right)}=$((Lopital teoremasidan foydalanamiz)) $=e^{\lim_{x\to 0}\frac{(\ln(\cos x))'}{(\sin x)'}}=e^{\lim_{x\to 0}\frac{\frac{-\sin x}{\cos x}}{\cos x}}=e^{\lim_{x\to 0}\frac{\sin x}{\cos^2 x}}=e^0=1\rhd$

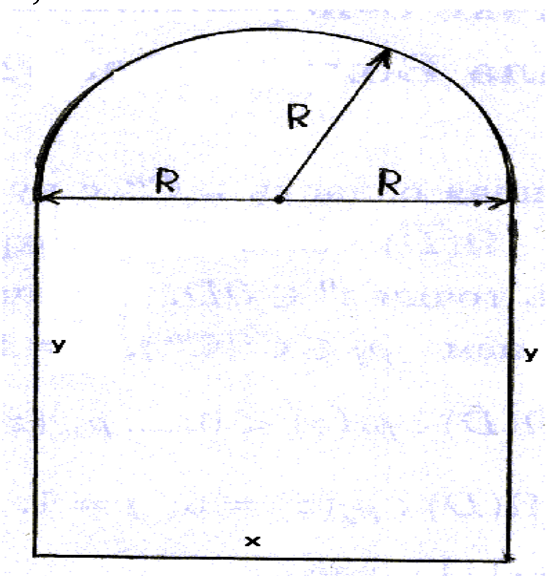
10.1 $\lim_{x\to-\infty}\frac{e^{-x}}{x^{10}}$.	**10.11** $\lim_{x\to+\infty}\frac{x^3+\ln x}{x^3+\cos x}$.
10.2 $\lim_{x\to 1-0}\ln(1-x)\cdot ctg\,\pi x$.	**10.12** $\lim_{x\to 1}\left[(x-1)^{-1}-\ln^{-1}x\right]$.
10.3 $\lim_{x\to 0}\frac{e^{-\frac{1}{x^2}}}{x^{100}}$.	

10.4 $\lim\limits_{x\to\frac{1}{2}}(2-2x)^{tg\pi x}$.	**10.13** $\lim\limits_{x\to+\infty}\dfrac{x^2+e^x}{\sin x+e^{2x}}$.
10.5 $\lim\limits_{x\to+0}\dfrac{e^{-\frac{1}{x}}}{x^{10}}$.	**10.14** $\lim\limits_{x\to 0}\left(x^{-2}-\sin^{-2}x\right)$.
10.6 $\lim\limits_{x\to\frac{\pi}{2}}\left[tgx-(1-\sin x)^{-1}\right]$.	**10.15** $\lim\limits_{x\to+\infty}\dfrac{x^4+\cos x}{e^x+\sin x}$.
10.7 $\lim\limits_{x\to+\infty}x^{100}(0.01)^x$.	**10.16** $\lim\limits_{x\to+0}x^{-2tgx}$.
10.8 $\lim\limits_{x\to+0}\sqrt{x}\ln^2 x$.	**10.17** $\lim\limits_{x\to 0}\left(x^{-2}-ctg^2x\right)$.
10.9 $\lim\limits_{x\to+\infty}\dfrac{x^2+\sin x}{e^x+\cos x}$.	**10.18** $\lim\limits_{x\to 0}\left[(e^x-1)^{-1}-x^{-1}\right]$.
10.10 $\lim\limits_{x\to+0}x^{\sin s}$.	**10.19** $\lim\limits_{x\to 0}\left(\dfrac{tgx}{x}\right)^{\frac{1}{x}}$.
	10.20 $\lim\limits_{x\to\frac{\pi}{4}}(tgx)^{tg2x}$.

11-masala. Quyidagi masalalar yechilsin.

Derazaning perimetri P ga teng, юqori qismi yarim doiradan iborat bo‘lgan to‘g’ri to‘rtburchak shaklga ega. Derazaning o‘lchamlari qanday bo‘lganda undan eng ko‘p yorug’lik o‘tadi?

◃Masala shartiga ko‘ra deraza chizmada ko‘rsatilgan shaklga ega. Chizmadan ko‘rinadiki,



$R=\frac{x}{2}$. Unda $P=x+2y+\pi R=x+2y+\frac{\pi x}{2}\Rightarrow$

$$y=\frac{P}{2}-\frac{x}{2}-\frac{\pi x}{4}.$$

Endi derazaning yuzasini topamiz:

$$S=x\cdot y+\frac{\pi R^2}{2}=\frac{Px}{2}-\frac{x^2}{2}-\frac{\pi x^2}{4}+\frac{\pi x^2}{8}=\frac{Px}{2}-\frac{x^2}{2}-\frac{\pi x^2}{8}$$

Derazadan eng ko‘p yorug’lik o‘tishi uchun derazaning yuzasi eng katta bo‘lishi kerak. Buning uchun (5) - funksiyaga maksimum qiymatni beruvchi x ni topishimiz lozim.

$S'(x)=\frac{P}{2}-x-\frac{\pi x}{4};\ S'(x)=0\Rightarrow x\cdot\left(\frac{\pi}{4}+1\right)=\frac{P}{2}\Rightarrow x_0=\frac{\frac{P}{2}}{\frac{\pi}{4}+1}$ - statsionar nuqta. Bu nuqtada $S''(x_0)=-1-\frac{\pi}{4}<0\Rightarrow\max$. Demak, derazadan yorug’lik eng ko‘p o‘tishi uchun uning asosi $x=\frac{2P}{\pi+4}$ bo‘lishi kerak ekan. Balandligi esa

$y=\frac{P}{2}-\frac{x}{2}-\frac{\pi x}{4}=\frac{P}{2}-\frac{P}{\pi+4}-\frac{\pi P}{2(\pi+4)}=\frac{P(\pi+4-2-\pi)}{2(\pi+4)}=\frac{P}{\pi+4}$ bo‘lar ekan.◁

11.1 Yig’indisi o‘zgarmas a soniga teng bo‘lgan 2 ta musbat sonning m va n darajalari (m>0,n>0) ko‘paytmasining eng katta qiymati topilsin.

11.2 Ko‘paytmasi o‘zgarmas a soniga teng bo‘lgan 2 ta musbat sonning m va n darajalari (m>0,n>0) yig’indisining eng kichik qiymati topilsin.

11.3 Yuzasi Sga teng bo‘lgan barcha to‘g’ri to‘rtburchaklar ichidan perimetri eng kichik bo‘lganini aniqlang.

11.4 Kateti va gipotenuzasi yig’indisi o‘zgarmas bo‘lgan to‘g’ri burchakli uchburchaklar ichida юzasi eng katta bo‘lganini aniqlang.

11.5 V hajmli yopiq tsilindrik bankaning o‘lchamlari qanday bo‘lganda u eng kichik to‘la sirtga ega bo‘ladi?

11.6 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ellipsga tomonlari ellipsning o‘qlariga parallel bo‘lgan shunday ichki to‘g’ri to‘rtburchak chizingki, uning yuzasi eng katta bo‘lsin.

11.7 R radiusli yarim sharga asosi kvadratdan iborat bo‘lgan shunday ichki to‘g’ri parallelepipedni chizingki, uning hajmi eng katta bo‘lsin.

11.8 R radiusli sharga shunday ichki silindr chizingki, uning hajmi eng katta bo‘lsin.

11.9 R radiusli sharga shunday ichki silindr chizingki, uning to‘la sirti eng katta bo‘lsin.

11.10 R radiusli sharga shunday tashqi konus chizingki, uning hajmi eng kichik bo‘lsin.

11.11 Yasovchisi *l* ga teng bo‘lgan eng katta hajmli konusning hajmini toping.

11.12 M(p,p) nuqta va $y^2=2px$ parabola orasidagi eng qisqa masofani toping.

11.13 A(2,0) nuqta va $x^2+y^2=1$ aylana orasidagi eng qisqa va eng uzun masofalar topilsin.

11.14 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (0$<b<a$) ellipsning B($0;-b$) nuqtasidan o‘tuvchi eng katta vatarini toping.

11.15 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ellipsda shunday $M(x,y)$ nuqtani topingki, shu nuqtadan ellipsga o‘tkazilgan urinma va koordinata o‘qlari yordamida hosil bo‘lgan uchburchakning yuzasi eng kichik bo‘lsin.

11.16 R radiusli doiraga shunday ichki to‘g’ri to‘rtburchak chizingki, uning perimetri eng katta bo‘lsin.

11.17 $A(1;2)$nuqtadan shunday to‘g’ri chiziq o‘tkazingki, shu to‘g’ri chiziq va musbat yarim o‘qlar yordamida hosil bo‘lgan uchburchakning yuzasi eng kichik bo‘lsin.

11.18 *a* musbat sonni shunday 2ta musbat qo‘shiluvchiga ajratingki, ular kublarining yig’indisi eng kichik bo‘lsin.

11.19 Uzunligi l ga teng bo‘lgan setka bilan bir tomoni devor bilan to‘silgan shunday to‘g’ri to‘rtburchak shaklidagi yer uchastkasini o‘rash kerakki, uning yuzasi eng katta bo‘lsin.

11.20 Teng yoqli uchburchakni 2 ta teng yuzali uchburchakka ajratuvchi eng kichik kesmaning uzunligi topilsin.

12 - masala. Birinchi tartibli hosiladan foydalanib funksiyaning grafigini chizing.

$$y = \frac{x^2 \cdot (x-4)^2}{16}$$

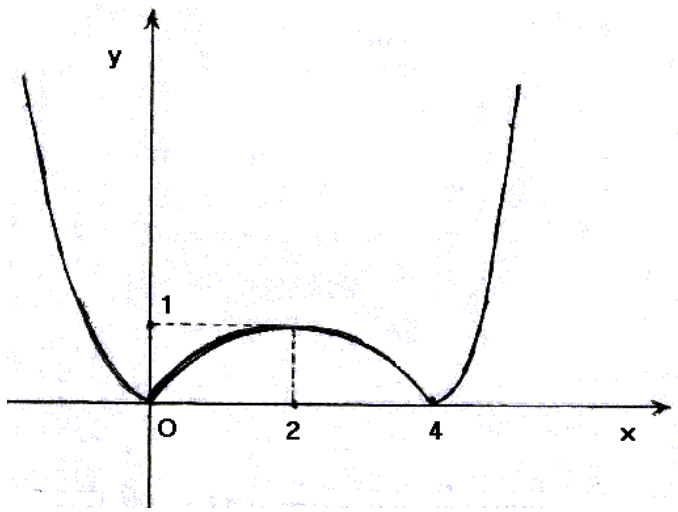
◁ Berilgan funksiyaning hosilasini hisoblaymiz:

$$y' = \frac{1}{16}\left[2x \cdot (x-4)^2 + x^2 \cdot 2(x-4)\right] = \frac{2x \cdot (x-4)}{16} \cdot [x-4+x] = \frac{x(x-4)\cdot(2x-4)}{8} = \frac{x \cdot (x-2)(x-4)}{4}.$$

Intervallar usulidan foydalanib bu ifodaning ishorasi saqlanadigan oroliqlarni topamiz va quyidagi jadvalni tuzamiz.

x	$(-\infty;0)$	0	$(0;2)$	2	$(2;4)$	4	$(4;+\infty)$
y'	-	0	+	0	-	0	+
y	↘	$\min_0$	↗	$\max_1$	↘	$\min_0$	↗

Jadvaldagi ma’lumotlardan foydalanib berilgan funksiyaning grafigini chizamiz: ▷



12.1 $y = x^2(x-2)^2$.	**12.11** $y = 2x^3 + 3x^2 - 5$.
12.2 $y = \frac{x^3 - 9x^2}{4} + 6x - 9$.	**12.12** $y = 2 - 12x^2 - 8x^3$.
12.3 $y = 2 - 3x^2 - x^3$.	**12.13** $y = (2x+1)^2 \cdot (2x-1)^2$.
12.4 $y = (x+1)^2 \cdot (x-1)^2$.	**12.14** $y = 2x^3 + 9x^2 + 12x$.
12.5 $y = 2x^3 - 3x^2 - 4$.	**12.15** $y = 12x^2 - 8x^3 - 2$.
12.6 $y = 3x^2 - 2 - x^3$.	**12.16** $y = (2x-1)^2 \cdot (2x-3)^2$.
12.7 $y = (x-1)^2 \cdot (x-3)^2$.	**12.17** $y = \frac{27(x^3 - x^2)}{4} - x$.
12.8 $y = \frac{x^3 + 3x^2}{4} - 5$.	**12.18** $y = \frac{x \cdot (12 - x^2)}{8}$.
12.9 $y = 6x - 8x^3$.	**12.19** $y = -\frac{(x^2 - 4)^2}{16}$.
12.10 $y = 16x^2 \cdot (x-1)^2$.	**12.20** $y = 16x^3 - 12x^2 - 4$.

13-masala. Funksiyaning asimtotalarini toping va grafigini yasang.

$$y = \frac{4x^3 - 3x}{4x^2 - 1}$$

$$\lhd\ y = \frac{4x^3 - 3x}{4x^2 - 1} = \frac{x \cdot (4x^2 - 3)}{(2x+1)(2x-1)} = \frac{x\left(x + \frac{\sqrt{3}}{2}\right)\left(x - \frac{\sqrt{3}}{2}\right)}{\left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right)}\ \rhd$$

a) Vertikal asimptota: $x = -\frac{1}{2}$ va $x = \frac{1}{2}$ to‘g’ri chiziqlar vertikal asimptota bo‘ladi , chunki $\lim\limits_{x \to -\frac{1}{2}} f(x) = \infty$ va $\lim\limits_{x \to \frac{1}{2}} f(x) = \infty$. Funksiyaning shu nuqtadagi o‘ng va chap limitlarini ham hisoblaymiz:

$$\lim_{x \to -\frac{1}{2}+0} f(x) = -\infty, \quad \lim_{x \to -\frac{1}{2}-0} f(x) = +\infty$$

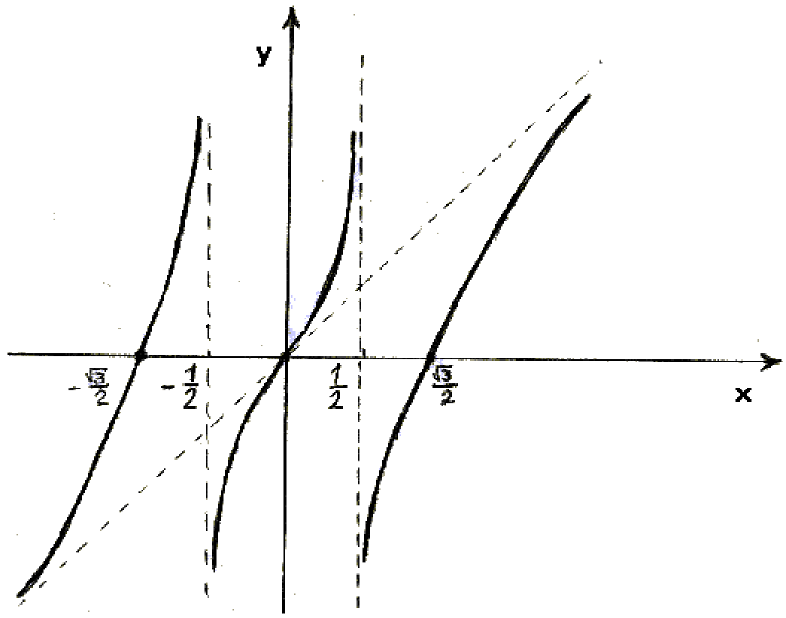
$$\lim_{x \to \frac{1}{2}+0} f(x) = -\infty \quad \lim_{x \to \frac{1}{2}-0} f(x) = +\infty$$

b) Gorizontal asimptota: $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{4x^3-3x}{4x^2-1} = \infty \Rightarrow$ gorizontal asimptota yo‘q.

v) Og’ma asimptota: $a = \lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{4x^3-3x}{x(4x^2-1)} = 1,$

$b = \lim\limits_{x\to\infty}[f(x) - ax\ ] = \lim\limits_{x\to\infty}\left[\dfrac{4x^3-3x}{4x^2-1} - x\right] = \lim\limits_{x\to\infty} \dfrac{4x^3-3x-4x^3+x}{4x^2-1} = \lim\limits_{x\to\infty} \dfrac{-2x}{4x^2-1} = 0 \Rightarrow y = x$ - og’ma asimptota.

Bu asimtotalardan foydalanib funksiya grafigini chizamiz : ▹



13.1 $y = \dfrac{x^3-2x^2-3x+2}{1-x^2}$.	**13.11** $y = \dfrac{2-x^2}{\sqrt{9x^2-4}}$.
13.2 $y = \dfrac{2x^2-9}{\sqrt{x^2-1}}$.	**13.12** $y = \dfrac{x^2+1}{\sqrt{4x^2-3}}$.
13.3 $y = \dfrac{x^2-11}{4x-3}$.	**13.13** $y = \dfrac{17-x^2}{4x-5}$.
13.4 $y = \dfrac{2x^3-3x^2-2x+1}{1-3x^2}$.	**13.14** $y = \dfrac{4x^2+9}{4x+8}$.
13.5 $y = \dfrac{2x^2-1}{\sqrt{x^2-2}}$.	**13.15** $y = \dfrac{x^3-4x}{3x^2-4}$.

13.6 $y=\frac{21-x^2}{7x+9}$.	**13.16** $y=\frac{x^2-3}{\sqrt{3x^2-2}}$.
13.7 $y=\frac{x^3+3x^2-2x-2}{2-3x^2}$.	**13.17** $y=\frac{4x^3+3x^2-8x-2}{2-3x^2}$.
13.8 $y=\frac{x^2+16}{\sqrt{9x^2-8}}$.	**13.18** $y=\frac{2x^3+2x^2-3x-1}{2-4x^2}$.
13.9 $y=\frac{3x^2-7}{2x+1}$.	**13.19** $y=\frac{2x^2-6}{x-2}$.
13.10 $y=\frac{x^2-6x+4}{3x-2}$.	**13.20** $y=\frac{x^3-5x}{5-3x^2}$.

14-masala. Funksiyani to‘liq tekshiring va grafigini yasang.

$$y=\frac{x^2-x+1}{x-1}$$

$\triangleleft$ Funksiyani taklif qilingan sxema asosida to‘liq tekshiramiz.

Funksiyaning aniqlanish sohasi: $Д(y)=\{x\neq 1\}$

Funksiya juft ham, toq ham, davriy ham emas.

$x=1$ nuqta funksiyaning 2-tur uzilish nuqtasi, chunki $\lim\limits_{x\to 1-0} f(x)=-\infty$ va $\lim\limits_{x\to 1+0} f(x)=+\infty$ OY o‘qi bilan kesishish nuqtasi: $y=f(0)=-1$.

OX o‘qi bilan kesishish nuqtasi: $y=0\Rightarrow x^2-x+1=0\Rightarrow x\in\varnothing\Rightarrow$ OX o‘qi bilan kesishishmaydi.

Funksiyaning ishorasi o‘zgarmaydigan oraliqlar:

X	$(-\infty;1)$	$(1;+\infty)$
Y	-	+

Endi funksiyani monotonlik va ekstremumga tekshiramiz:

$$y'=\left(\frac{x^2-x+1}{x-1}\right)'=\frac{(2x-1)\cdot(x-1)-(x^2-x+1)\cdot 1}{(x-1)^2}=\frac{2x^2-3x+1-x^2+x-1}{(x-1)^2}=\frac{x^2-2x}{(x-1)^2}=\frac{x\cdot(x-2)}{(x-1)^2}$$

Intervallar usulidan foydalanib bu ifodaning ishorasi saqlanadigan oraliqlarni topamiz va quyidagi jadvalni tuzamiz:

x	$(-\infty;0)$	0	$(0;1)$	1	$(1;2)$	2	$(2;+\infty)$
y'	+	0	-	$\nexists$	-	0	+
y		↗ max -1		↘ $\nexists$		↘ min 3	↗

Qavariqlikka tekshirish uchun y'' ni hisoblaymiz:

$$y''=(y')'=\left[\frac{x^2-2x}{(x-1^2)}\right]'=\left[1-\frac{1}{(x-1)^2}\right]'=\frac{2}{(x-1)^3}\Rightarrow x<1 \partial a \cap \text{ va } x>1 \partial a \cup .$$

Funksiya asimptotalarini topamiz:

a) Vertikal asimptota: $x=1$-vertikal asimptota.

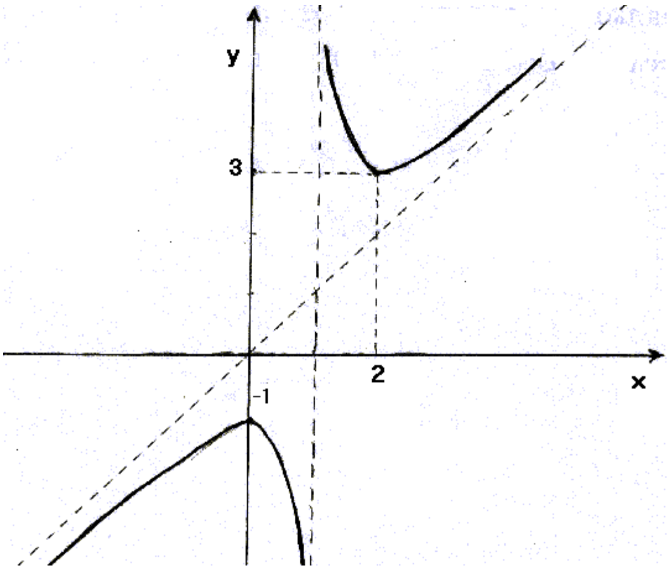
b) Gorizontal asimptota: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\frac{x^2-x+1}{x-1}=\infty\Rightarrow$ gorizontal asimptota yo‘q.

v) Og’ma asimptota: $a=\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{x^2-x+1}{x\cdot(x-1)}=1$

$$b=\lim_{x\to\infty}[f(x)-ax]=\lim_{x\to\infty}\left(\frac{x^2-x+1}{x-1}-x\right)=\lim_{x\to\infty}\frac{x^2-x+1-x^2+x}{x-1}=\lim_{x\to\infty}\frac{1}{x-1}=0\Rightarrow y=x$$

og’ma asimptota.

Endi topilgan ma’lumotlardan foydalanib funksiya grafigini chizamiz:



Quyidagi funksiyalarning hosilalarini toping:

14.1 $y=\frac{x^3+4}{x^2}$.	**14.11** $y=\left(1+\frac{1}{x}\right)^2$.
14.2 $y=\frac{4x^2}{3+x^2}$.	**14.12.** $y=\frac{12-3x^2}{x^2+12}$
14.3 $y=\frac{2}{x^2+2x}$.	**14.13** $\frac{9+6x-3x^2}{x^2-2x+13}$.
14.4 $y=\frac{12x}{9+x^2}$.	**14.14** $y=-\frac{8x}{x^2+4}$.
14.5 $y=\frac{x^2-3x+3}{x-1}$.	**14.15** $y=\left(\frac{x-1}{x+1}\right)^2$.
14.6 $y=\frac{4-x^3}{x^2}$.	**14.16** $y=\frac{3x^4+1}{x^3}$.
14.7 $y=\frac{x^2-4x+1}{x-1}$.	**14.17** $y=\frac{4x}{(x+1)^2}$.
14.8 $y=\frac{2x^3+1}{x^2}$.	**14.18** $y=\frac{8(x-1)}{(x+1)^2}$.
14.9 $y=\frac{(x-1)^2}{x^2}$.	**14.19** $y=\frac{4}{x^2+2x-3}$.
	14.20 $y=\frac{x^2+2x-7}{x^2+2x-3}$.

14.10 $y=\frac{x^2}{(x-1)^2}$.	

a) Hosilasini toping

1. x-ln(2+e^x+2$\sqrt{e^{2x}+e^x+1}$) 2. $\frac{e^{2x}(2-\sin 2x-\cos 2x)}{8}$ 3. $\frac{1}{2}$arctg$\frac{e^x-3}{2}$ 4. $\frac{1}{ln4}\,ln\frac{1+2^x}{1-2^x}$ 5. $2\sqrt{e^x+1}+ln\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$ 6. $\frac{2}{3}\sqrt{(arctg\ e^x)^3}$ 7. $\frac{1}{2}\ln(e^{2x}+1)-2arctge^x$ 8. $\ln(e^x+1)+\frac{18e^{2x}+27e^x+11}{6(e^x+1)^3}$ 9. $\frac{2(\sqrt{2^x-1}-arctg\sqrt{2^x-1})}{ln2}$ 10. $2(x-2)\sqrt{1+e^x}-2ln\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}$ 11. $e^{\alpha x}\frac{\alpha sin\beta x-\beta cos\beta x}{\alpha^2+\beta^2}$ 12. $e^{\alpha x}\frac{\beta sin\beta x+\alpha cos\beta x}{\alpha^2+\beta^2}$	13. $e^{ax}[\frac{1}{2a}+\frac{acos2bx+2bsinbx}{2(a^2+4b^2}]$ 14. $x+\frac{1}{1+e^x}-\ln(1+e^x)$ 15. $x-3\ln\left[1+e^{\frac{x}{6}}\right)\sqrt{1+e^{\frac{x}{3}}}-3arctge^{\frac{x}{6}}]$ 16. $x+\frac{8}{1+e^{\frac{x}{4}}}$ 17. $\ln\left(e^x+\sqrt{e^{2x}-1}\right)+arcsine^{-x}$ 18. $x-e^{-x}arcsine^x-\ln(1+\sqrt{1-e^{2x}})$ 19. $x-\ln(1+e^x)-2e^{-\frac{x}{2}}arctge^{\frac{x}{2}}-(arctge^{\frac{x}{2}})^2$ 20. $\frac{e^{x^3}}{1+x^3}$ 21. $\frac{1}{m\sqrt{ab}}arctg(e^{mx}\sqrt{\frac{a}{b}})$ 22. $3e^{\sqrt[3]{x}}(\sqrt[3]{x^2}-2\sqrt[3]{x}+2)$ 23. $ln\frac{\sqrt{1+e^x+e^{2x}}-e^x-1}{\sqrt{1+e^x+e^{2x}}-e^x+1}$

b) Hosilalarini toping

1. $y=\sqrt{x}\ln\left(\sqrt{x}+\sqrt{x+a}\right)-\sqrt{x+a}$ **2.**$y=\ln(x+\sqrt{a^2+x^2}$ **3.** $y=2-\sqrt{x}-4\ln\left(2+\sqrt{x}\right)$ **4.**$y=ln\frac{x^2}{\sqrt{1-ax^4}}$	**11.** $y=ln\sqrt[4]{\frac{1+2x}{1-2x}}$ **12.**$y=x+\frac{1}{\sqrt{2}}\ln\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+a^{\pi^{\sqrt{2}}}$ **13.**$y=lnsin\frac{2x+4}{x+1}$ **14.**$y=log_{16}log_5tgx$

5. $y = \ln\left(\sqrt{x} + \sqrt{x+1}\right)$ **6.** $y = ln\frac{a^2+x^2}{a^2-x^2}$ **7.** $y = ln^2(x + cosx)$ **8.** $y = ln^3(1 + cosx)$ **9.** $y = ln\frac{x^2}{1-x^2}$ **10.** $y = lntg(\frac{\pi}{4} + \frac{x}{2})$	**15.**$y = log_4 log_2 tgx$ **16.**$y = \frac{x(coslnx+sinlnx)}{2}$ **17.**$y = lncos\frac{2x+3}{2x+1}$ **18.**$y = lglnctgx$ **19.**$y = log_a\frac{1}{\sqrt{1-x^4}}$ **20.**$y = \frac{1}{\sqrt{2}}\ln(\sqrt{2}tgx + \sqrt{1+2tg^2x})$

d) Hosilasini toping

1. $y = sin\sqrt{3} + \frac{1}{3}\frac{sin3x}{cos6x}$ 2. $y = cosln2 - \frac{1}{3}\frac{cos^2 3x}{sin6x}$ 3. $y = tglg\frac{1}{3} + \frac{1}{4}\frac{sin^2 4x}{cos8x}$ 4. $y = ctg\sqrt[3]{5} - \frac{1}{8}\frac{cos^2 4x}{sin8x}$ 5. $y = \frac{cossin5\ sin^2 5x}{2cos4x}$ 6. $y = \frac{sincos3\ cos^2 2x}{4cos4x}$ 7. $y = \frac{cosln7\ sin^2 7x}{7cos14x}$ 8. $y = cosctg2 - \frac{1}{16}\frac{cos^2 8x}{sin16x}$ 9. $y = ctgcos2 + \frac{1}{6}\frac{sin^2 6x}{cos12x}$ 10. $y = \sqrt[3]{ctg2} - \frac{1}{20}\frac{cos^2 10x}{sin20x}$	11. $y = \frac{1}{3}costg\frac{1}{2} + \frac{1}{10}\frac{sin^2 10x}{cos20x}$ 12. $y = lnsin\frac{1}{2} - \frac{1}{24}\frac{cos^2 12x}{sin24x}$ 13. $y = 8sinctg3 + \frac{1}{5}\frac{sin^2 5x}{cos10x}$ 14. $y = \frac{cosctg3\ cos^2 14x}{28sin28x}$ 15. $y = \frac{costg\frac{1}{3}sin^2 15x}{15cos30x}$ 16. $y = \frac{sintg\frac{1}{7}cos^2 16x}{32sin32x}$ 17. $y = \frac{ctgsin\frac{1}{3}sin^2 17x}{17cos34x}$ 18. $y = \frac{\sqrt[5]{ctg2}cos^2 18x}{36sin36x}$ 19. $y = \frac{tgln2\ sin^2 19x}{19cos38x}$ 20. $y = ctgcos5 - \frac{1}{40}\frac{cos^2 20x}{sin40x}$

e) Hosilasini toping

1. $y = arctg\frac{tgx - ctgx}{\sqrt{2}}$ 2. $y = \arcsin\frac{\sqrt{x}-2}{\sqrt{5x}}$ 3. $y = \frac{2x-1}{4}\sqrt{2+x-x^2} + \frac{9}{8}\arcsin\frac{2x-1}{3}$	11. $y = \frac{1}{2\sqrt{x}} + \frac{1+x}{2x}arctg\sqrt{x}$ 12. $y = \frac{3+x}{2}\sqrt{x(2-x)} + 3\arccos\sqrt{\frac{x}{2}}$ 13. $y = \frac{4+x^4}{x^3}arctg\frac{x^2}{2} + \frac{4}{x}$

4. $y = arctg\frac{\sqrt{1+x^2}-1}{x}$ 5. $y = arccos\frac{x^2-4}{\sqrt{x^4+16}}$ 6. $y = \sqrt{\frac{2}{3}}arctg\frac{3x-1}{\sqrt{6x}}$ 7. $y = \frac{1}{4}ln\frac{x-1}{x+1} - \frac{1}{2}arctgx$ 8. $y = \frac{(x-4)\sqrt{8x-x^2-7}}{2-9arccos\sqrt{\frac{x-1}{6}}}$ 9. $y = \frac{(1+x)arctg\sqrt{x}}{x^2} + \frac{1}{3x\sqrt{x}}$ 10. $y = \frac{x^3}{3}\arccos x - \frac{2+x^2}{9}\sqrt{1-x^2}$	14. $y = arcsin\sqrt{\frac{x}{x+1}} + arctg\sqrt{x}$ 15. $y = \frac{1}{2}\sqrt{\frac{1}{x^2} - 1} - \frac{arccosx}{2x^2}$ 16. $y = 6\arcsin\frac{\sqrt{x}}{2} - \frac{6+x}{2}\sqrt{x(4-x)}$ 17. $y = \frac{x-3}{2}\sqrt{6x-x^2-8} + \arcsin\sqrt{\frac{x}{2}-1}$ 18. $y = \frac{(1+x)arctg\sqrt{x}-\sqrt{x}}{x}$ 19. $y = \frac{2\sqrt{1-x}arcsin\sqrt{x}}{x} + \frac{2}{\sqrt{x}}$ 20. $y = \frac{2x-5}{4}\sqrt{5x-4-x^2} + \frac{9}{4}\arcsin\sqrt{\frac{x-1}{3}}$

f) Hosilasini toping

1. $y = \frac{1}{4\sqrt{5}}ln\frac{2+\sqrt{5}thx}{2-\sqrt{5}thx}$ 2. $y = \frac{shx}{4ch^4x} + \frac{3shx}{8ch^2x} + \frac{3}{8}arctg(shx)$ 3. $y = \frac{1}{2}ln\frac{1+\sqrt{thx}}{1-\sqrt{thx}} - arctg\sqrt{thx}$ 4. $y = \frac{1}{2}thx + \frac{1}{4\sqrt{2}}ln\frac{1+\sqrt{2}thx}{1-\sqrt{2}thx}$ 5. $y = \frac{1}{2}thx + \frac{1}{4\sqrt{2}}ln\frac{1-\sqrt{5}thx}{1+\sqrt{5}thx}$ 6. $y = (-\frac{1}{2}lnth\frac{x}{2} - \frac{chx}{2sh^2x})$ 7. $y = \frac{1}{2a\sqrt{1+a^2}}ln\frac{a+\sqrt{1+a^2}thx}{a-\sqrt{1+a^2}thx}$ 8. $y = \frac{1}{18\sqrt{2}}ln\frac{1+\sqrt{2}cthx}{1-\sqrt{2}cthx}$ 9. $y = arctg\frac{\sqrt{sh2x}}{chx-shx}$ 10. $y = \frac{1}{6}ln\frac{1-sh2x}{2+sh2x}$	11. $y = \sqrt[4]{\frac{1+thx}{1-thx}}$ 12. $y = \frac{shx}{1+chx}$ 13. $y = \frac{chx}{\sqrt{sh2x}}$ 14. $y = \frac{sh3x}{\sqrt{ch6x}}$ 15. $y = \frac{1+8ch^2xlnchx}{2ch^2x}$ 16. $y = -\frac{12sh^2x+1}{3ch^3x}$ 17. $y = -\frac{shx}{2ch^2x} + \frac{3}{2}arcsin(thx)$ 18. $y = \frac{1}{\sqrt{8}}arcsin\frac{3+chx}{1+3chx}$ 19. $y = \frac{1}{\sqrt{8}}ln\frac{4+\sqrt{8}th\frac{x}{2}}{4-\sqrt{8}th\frac{x}{2}}$ 20. $y = \left[\frac{1}{4}ln\left\|th\frac{x}{2}\right\| - \frac{1}{4}ln\frac{3+chx}{shx}\right]$

g) Hosilasini toping

1. $y = (arctgx)^{\frac{1}{2}lnarctgx}$ 2. $y = (sin\sqrt{x})^{lnsin\sqrt{x}}$ 3. $y = (sinx)^{5e^x}$ 4. $y = (arcsinx)^{e^x}$ 5. $y = (lnx)^{3x}$ 6. $y = x^{arcsinx}$ 7. $y = (ctg3x)^{2e^x}$ 8. $y = x^{e^{tgx}}$ 9. $y = (tgx)^{4e^x}$ 10. $y = (cos5x)^{e^x}$	11. $y = (xsinx)^{8\ln(xsinx)}$ 12. $y = (x-5)^{chx}$ 13. $y = (x^3+4)^{tgx}$ 14. $y = x^{sinx^3}$ 15. $y = (x^2-1)^{shx}$ 16. $y = (x^4+5)^{ctgx}$ 17. $y = (sinx)^{\frac{5x}{2}}$ 18. $y = (x^2+1)^{cosx}$ 19. $y = 19^{x^{19}}x^{19}$ 20. $y = x^{3^x}2^x$

h) Hosilasi topilsin

1. $y = \frac{1}{24}(x^2+8)\sqrt{x^2-4} + \frac{x^4}{16}arcsin\frac{2}{x}, (x > 0)$ 2. $y = \frac{4x+1}{16x^2+8x+3} + \frac{1}{\sqrt{2}}arctg\frac{4x+1}{\sqrt{2}}$ 3. $y = 2x - \ln\left(1+\sqrt{1-e^{4x}}\right) - e^{-2x}arcsin(e^{2x})$ 4. $y = \sqrt{9x^2-12x+5}arctg(3x-2) - \ln(3x-2+\sqrt{9x^2-12x+5})$ 5. $y = \frac{2}{x-1}\sqrt{2x-x^2} + ln\frac{1+\sqrt{2x-x^2}}{x-1}$ 6. $y = \frac{x^4}{81}arcsin\frac{3}{x} + \frac{1}{81}(x^2+18)\sqrt{x^2-9}, \ (x > 0)$ 7. $y = \frac{1}{\sqrt{2}}arctg\frac{3x-1}{\sqrt{2}} + \frac{1}{3}\frac{3x-1}{3x^2-2x+1}$ 8. $y = 3x - \ln\left(1+\sqrt{1-e^{6x}}\right) - e^{-3x}\arcsin(e^{3x})$	11. $y = (2x+3)^4 arcsin\frac{1}{2x+3} + \frac{2}{3}(4x^2+12x+11)\sqrt{x^2+3x+2}$ 12. $y = \frac{x+2}{x^2+4x+6} + \frac{1}{\sqrt{2}}arctg\frac{x+2}{\sqrt{2}}$ 13. $y = 5x - \ln\left(1+\sqrt{1-e^{10x}}\right) - e^{-5x}\arcsin(e^{5x})$ 14. $y = \sqrt{x^2-8x+17}arctg(x-4) - \ln(x-4+\sqrt{x^2-8x+17})$ 15. $y = ln\frac{1+\sqrt{-3+4x-x^2}}{2-x} + \frac{2}{2-x}\sqrt{-3+4x-x^2}$ 16. $y = \left(3x^2-4x+2\right)\sqrt{9x^2-12x+3} + \arcsin x$ 17. $y = \frac{1}{\sqrt{2}}arctg\frac{x-1}{\sqrt{2}} + \frac{x-1}{x^2-2x+3}$ 18. $y = \ln\left(e^{5x}+\sqrt{e^{10x}-1}\right) + \arcsin(e^{-5x})$

9. $y=\ln(4x-1)+\sqrt{16x^2-8x+2}-\sqrt{16x^2-8x+2}arctg(4x-1)$ 10. $y=ln\frac{1+2\sqrt{-x-x^2}}{2x+1}+\frac{4}{2x+1}\sqrt{-x-x^2}$	19. $y=\ln\left(2x-3+\sqrt{4x^2-12x+10}\right)-\sqrt{4x^2-12x+10}arctg(2x-3)$ 20. $y=ln\frac{1+\sqrt{-3-4x-x^2}}{-x-2}-\frac{2}{x+2}\sqrt{-3-4x-x^2}$

i) Hosilasini toping

1. $y=\frac{xarcsinx}{\sqrt{1-x^2}}+ln\sqrt{1-x^2}$ 2. $y=4ln\frac{x}{1+\sqrt{1-4x^2}}-\frac{\sqrt{1-4x^2}}{x^2}$ 3. $y=x(2x^2+5)\sqrt{x^2+1}+3\ln(x+\sqrt{x^2+1})$ 4. $y=x^3arcsinx+\frac{x^2+2}{3}\sqrt{1-x^2}$ 5. $y=3arcsin\frac{3}{4x+1}+2\sqrt{4x^2+2x-2},\ (4x+1>0)$ 6. $y=\sqrt{1+x^2}arctgx-\ln(x+\sqrt{1+x^2})$ 7. $y=2arcsin\frac{2}{3x+4}+\sqrt{9x^2+24x+12},\ (3x+4>0)$ 8. $y=x(2x^2+1)\sqrt{x^2+1}-\ln(x+\sqrt{x^2+1})$ 9. $y=\ln\left(x+\sqrt{1+x^2}\right)-\frac{\sqrt{1+x^2}}{x}$ 10. $y=\frac{3}{2\sqrt{2}}arcsin\frac{3x+4}{\sqrt{17}}$	11. $y=\sqrt{(4+x)(1+x)}+3\ln\left(\sqrt{4+x}+\sqrt{1+x}\right)$ 12. $y=ln\frac{\sqrt{x^2-x+1}}{x}+\sqrt{3}arctg\frac{2x-1}{\sqrt{3}}$ 13. $y=\frac{1}{12}ln\frac{x^4-x^2+1}{(x^2+1)^2}-\frac{1}{2\sqrt{3}}arctg\frac{\sqrt{3}}{2x^2-1}$ 14. $y=\sqrt{4x^2+12x-7},$ 15. $y=2arcsin\frac{2}{3x+1}+\sqrt{9x^2+6x-3},\ (3x+1>0)$ 16. $y=(2+3x)\sqrt{x-1}+\frac{3}{2}arctg\sqrt{x-1}$ 17. $y=\frac{1}{3}(x-2)\sqrt{x+1}+\ln\left(\sqrt{x+1}+1\right)$ 18. $y=\sqrt{x^2+1}-\frac{1}{2}ln\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+1}$ 19. $y=\ln\sqrt{\frac{x-1}{x+1}}-\frac{1}{2}\left(\frac{1}{2}+\frac{1}{x^2-1}\right)arctgx$ 20. $y=x\ln\left(\sqrt{1-x}+\sqrt{1+x}\right)+\frac{1}{2}(\arcsin x-x)$

j) Hosilasini toping

1. $y=\frac{1}{sin\propto}\ln(tgx+ctg\propto)$ 2. $y=xcos\propto+sinalnsin(x-\propto)$ 3. $y=\frac{1}{2\sqrt{2}}\left[\sin\ln x-\left(\sqrt{2}-1\right)\cos\ln x\right]x^{\sqrt{2}+1}$	11. $y=(1+x^2)e^{arctgx}$ 12. $y=\frac{ctgx+x}{1-xctgx}$

4. $y = arctg(cosx)\sqrt[4]{cos2x}$	13. $y = \frac{1}{2sin\frac{\propto}{2}} arctg \frac{2xsin\frac{\propto}{2}}{1-x^2}$
5. $y = 3\frac{sinx}{cos^2x} + 2\frac{sinx}{cos^4x}$	14. $y = arctg \frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}$,
6. $y = \left(a^2+b^2\right)^{-\frac{1}{2}} \arcsin\left(\frac{\sqrt{a^2+b^2}\sin x}{b}\right)$	15. $y = \frac{6^x(sin4xln6-4cos4x)}{16+ln^2 6}$
7. $y = \frac{7^x(3sin3x+cos3x\, ln7)}{9+ln^2 7}$	16. $y = arctg \frac{\sqrt{2tgx}}{1-tgx}$
8. $y = ln\frac{sinx}{cosx+\sqrt{cos2x}}$	17. $y = arctg \frac{2sinx}{\sqrt{9cos^2x-4}}$
9. $y = \frac{1}{a\left(1+a^2\right)}\left[arctg(a\cos x) + a\ln tg\frac{x}{2}\right]$	18. $y = \frac{5^x(2sin2x+cos2xln5)}{4+ln^2 5}$
10. $y = -\frac{1}{3\sin^3 x} - \frac{1}{\sin x} + \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}$	19. $y = ln\frac{\sqrt{2}+thx}{\sqrt{2}-thx}$
	20. $y = \frac{3^x(4sin4x+ln3cos4x)}{16+ln^2 3}$

$t = t_0$ parametrning qiymatlariga mos keluvchi egri chicziqning urinma va normal tenglamalarini tuzing.

1. $\begin{cases} x = asin^3t \\ y = acos^3t \end{cases}$, $t_0 = \frac{\pi}{3}$	11. $\begin{cases} x = atcost \\ y = atsint \end{cases}$, $t_0 = \frac{\pi}{2}$
2. $\begin{cases} x = \sqrt{3}cost \\ y = sint \end{cases}$, $t_0 = \frac{\pi}{3}$	12. $\begin{cases} x = sin^2t \\ y = cos^2t \end{cases}$, $t_0 = \frac{\pi}{6}$
3. $\begin{cases} x = a(t - sint) \\ y = a(1 - cost) \end{cases}$, $t_0 = \frac{\pi}{3}$	13. $\begin{cases} x = \arcsin(\frac{t}{\sqrt{1+t^2}}) \\ y = \arccos(\frac{1}{\sqrt{1+t^2}}) \end{cases}$, $t_0 = 1$
4. $\begin{cases} x = 2t - t^2 \\ y = 3t - t^3 \end{cases}$, $t_0 = 1$	14. $\begin{cases} x = \frac{1+lnt}{t^2} \\ y = \frac{3+2lnt}{t} \end{cases}$, $t_0 = 1$
5. $\begin{cases} x = \frac{2t+t^2}{1+t^3} \\ y = \frac{2t-t^2}{1+t^3} \end{cases}$, $t_0 = 1$	15. $\begin{cases} x = \frac{1+t}{t^2} \\ y = \frac{3}{2t^2} + \frac{2}{t} \end{cases}$, $t_0 = 2$
6. $\begin{cases} x = \arcsin(\frac{t}{\sqrt{1+t^2}}) \\ y = arccos(\frac{1}{\sqrt{1+t^2}}) \end{cases}$, $t_0 = -1$	16. $\begin{cases} x = asin^3t \\ y = acos^3t \end{cases}$, $t_0 = \frac{\pi}{6}$

7. $\begin{cases} x = t(tcost - 2sint) \\ y = t(tsint + 2cost) \end{cases}, \quad t_0 = \frac{\pi}{4}$	17. $\begin{cases} x = a(tsint + cost) \\ y = a(sint - tcost) \end{cases}, \quad t_0 = \frac{\pi}{4}$
8. $\begin{cases} x = \frac{3at}{1+t^2} \\ y = \frac{3at^2}{1+t^2} \end{cases}, \quad t_0 = 2$	18. $\begin{cases} x = \frac{t+1}{t} \\ y = \frac{t-1}{t} \end{cases}, \quad t_0 = -1$
9. $\begin{cases} x = 2lnctgt + 1 \\ y = tgt + ctgt \end{cases}, \quad t_0 = \frac{\pi}{4}$	19. $\begin{cases} x = 1 - t^2 \\ y = t - t^3 \end{cases}, \quad t_0 = 2$
10. $\begin{cases} x = \left(\frac{1}{2}\right) t^2 - (\frac{1}{4})t^4 \\ y = \left(\frac{1}{2}\right) t^2 + (\frac{1}{3})t^3 \end{cases}, \quad t_0 = 0$	20. $\begin{cases} x = \ln(1 + t^2) \\ y = t - arctgt \end{cases}, \quad t_0 = 1$

n-tartibli hosilasini toping.

1. $y = xe^{ax}$	15. $y = \lg(3x + 1)$
2. $y = sin2x + \cos(x + 1)$	16. $y = 7^{5x}$
3. $y = \sqrt[5]{e^{7x-1}}$	17. $y = \frac{x}{9(4x+9)}$
4. $y = \frac{4x+7}{2x+3}$	18. $y = \lg(1 + x)$
5. $y = \lg(5x + 2)$	19. $y = \frac{4}{x}$
6. $y = a^{3x}$	20. $y = \frac{5x+1}{13(2x+3)}$
7. $y = \frac{x}{2(3x+2)}$	21. $y = a^{2x+3}$
8. $y = \lg(x + 4)$	22. $y = \sin(3x + 1) + cos5x$
9. $y = \sqrt{x}$	23. $y = \sqrt{e^{3x+1}}$
10. $y = \frac{2x+5}{13(3x+1)}$	24. $y = \frac{11+12x}{6x+5}$
11. $y = 2^{3x+5}$	25. $y = \lg(2x + 7)$
12. $y = \sin(x + 1) + cos2x$	26. $y = 2^{kx}$
13. $y = \sqrt{e^{2x+1}}$	27. $y = \frac{x}{x+1}$
14. $y = \frac{4+15x}{5x+1}$	28. $y = log_3(x + 5)$

Ko‘rsatilgan tartibda hosilasini toping

1. $y = (2x^2 - 7)\ln(x - 1), \quad y^V$	11. $y = \frac{lnx}{x^3}, \quad y^{IV}$
2. $y = (3 - x^2)\ln^2 x, \quad y^{III}$	12. $y = (4x + 3)^{2-x}, \quad y^V$
3. $y = xcosx^2, \quad y^{III}$	13. $y = e^{1-2x}\sin(2 + 3x), \quad y^{IV}$
4. $y = \frac{\ln(x-1)}{\sqrt{x-1}}, \quad y^{III}$	14. $y = \frac{\ln(3+x)}{3+x}, \quad y^{III}$

5. $y=\frac{log_2x}{x^3},\ \ y^{III}$ 6. $y=(4x^3+5)e^{2x+1},\ \ y^{V}$ 7. $y=x^2\sin(5x-3),\ \ y^{III}$ 8. $y=\frac{lnx}{x^2},\ \ y^{IV}$ 9. $y=(2x+3)\ln^2 x,\ \ y^{III}$ 10. $y=(1+x^2)arctgx,\ \ y^{III}$	15. $y=(2x^3+1)cosx,\ \ y^{V}$ 16. $y=(x^2+3)\ln(x-3),\ \ y^{IV}$ 17. $y=(1-x-x^2)e^{\frac{x-1}{2}},\ \ y^{IV}$ 18. $y=\frac{1}{x}sin2x,\ \ y^{III}$ 19. $y=(x+7)\ln(x+4),\ \ y^{V}$ 20. $y=(3x-7)^{3-x},\ \ y^{IV}$

Parametrik ko'rinishda berilgan y''_{xx} funksiyaning hosilasini toping

1. $\begin{cases}x=cos2t\\y=2\sec^2 t\end{cases}$	11. $\begin{cases}x=\sqrt{t}\\y=\sqrt[3]{t-1}\end{cases}$
2. $\begin{cases}x=\sqrt{1-t^2}\\y=\frac{1}{t}\end{cases}$	12. $\begin{cases}x=\frac{cost}{1+2cost}\\y=\frac{sint}{1+2cost}\end{cases}$
3. $\begin{cases}x=e^t cost\\y=e^t sint\end{cases}$	13. $\begin{cases}x=\sqrt{t^3-1}\\y=lnt\end{cases}$
4. $\begin{cases}x=sh^2t\\y=\frac{1}{ch^2t}\end{cases}$	14. $\begin{cases}x=sht\\y=th^2t\end{cases}$
5. $\begin{cases}x=t+sint\\y=2-cost\end{cases}$	15. $\begin{cases}x=\sqrt{t-1}\\y=\frac{1}{\sqrt{t}}\end{cases}$
6. $\begin{cases}x=\frac{1}{t}\\y=\frac{1}{1+t^2}\end{cases}$	16. $\begin{cases}x=\cos^2 t\\y=tg^2t\end{cases}$
7. $\begin{cases}x=\sqrt{t}\\y=\frac{1}{\sqrt{1-t}}\end{cases}$	17. $\begin{cases}x=\sqrt{t-3}\\y=\ln(t-2)\end{cases}$
8. $\begin{cases}x=sint\\y=sect\end{cases}$	18. $\begin{cases}x=sint\\y=lncost\end{cases}$
9. $\begin{cases}x=tgt\\y=\frac{1}{sin2t}\end{cases}$	19. $\begin{cases}x=t+sint\\y=2+cost\end{cases}$
10. $\begin{cases}x=\sqrt{t-1}\\y=\frac{t}{\sqrt{t-1}}\end{cases}$	20. $\begin{cases}x=t-sint\\y=2-cost\end{cases}$

y funksiyani tenglamani qanoatlantirishini ko‘rsating

1. $y = xe^{-\frac{x^2}{2}}$ $xy' = (1 - x^2)y.(1)$ 2. $y = \frac{sinx}{x}$ $xy' + y = cosx.(1)$ 3. $y = 5e^{-2x} + \frac{e^x}{3}$ $y' + 2y = e^x.(1)$ 4. $y = 2 + c\sqrt{1 - x^2}$ $(1 - x^2)y' + xy = 2x.(1)$ 5. $y = x\sqrt{1 - x^2}$ $yy' = x - 2x^3.(1)$ 6. $y = \frac{c}{cosx}$ $y' - tgx\, y = 0.(1)$ 7. $y = -\frac{1}{3x+c}$ $y' = 3y^2.(1)$ 8. $y = \ln(c + e^x)$ $y' = e^{x-y}.(1)$ 9. $y = \sqrt{x^2 - cx}$ $(x^2 + y^2)dx - 2xydy = 0.(1)$ 10. $y = x(c - lnx)$ $(x - y)dx + xdy = 0.(1)$	11. $y = e^{tg\left(\frac{x}{2}\right)}$ $y'sinx = ylny.(1)$ 12. $y = \frac{1+x}{1-x}$ $y' = \frac{1 + y^2}{1 + x^2}.(1)$ 13. $y = \frac{b+x}{1+bx}$ $y - xy' = b(1 + x^2y).(1)$ 14. $y = \sqrt[3]{2 + 3x - 3x^2}$ $yy' = \frac{1 - 2x}{y}.(1)$ 15. $y = \sqrt{\ln(\frac{1+e^x}{2})^2 + 1}$ $(1 + e^x)yy' = e^x.(1)$ 16. $y = tgln3x$ $(1 + y^2)dx = xdy.(1)$ 17. $y = -\sqrt{\frac{2}{x^2} - 1}$ $1 + y^2 + xyy' = 0.(1)$ 18. $y = \sqrt[3]{x - lnx - 1}$ $lnx + y^3 - 3xy^2y' = 0.(1)$ 19. $y = a + \frac{7x}{ax+1}$ $y - xy' = a(1 + x^2y').(1)$ 20. $y = atg\sqrt{\frac{a}{x} - 1}$ $a^2 + y^2 + 2x\sqrt{ax - x^2}y' = 0.(1)$

Berilgan funksiyaning birinchi tartibli hosilasi yordamida grafigini chizing

1. $y = 2x^3 - 9x^2 + 2x - 9$	16. $y = 2x^3 + 9x^2 - 12x$
2. $y = 3x - x^2$	17. $y = 12x^2 - 8x^3 - 2$
3. $y = x^2(x-2)^2$	18. $y = (2x-1)^2(2x-3)^2$
4. $y = \frac{x^3-9x^2}{14+6x-9x^2}$	19. $y = \frac{27(x^3-x^2)}{4} - 4$
5. $y = 2 - 3x^2 - x^3$	20. $y = \frac{x(12-x^2)}{8}$
6. $y = (x+1)^2(x-1)^2$	21. $y = \frac{x^2(x-4)^2}{16}$
7. $y = 2x^3 - 3x^2 - 4$	22. $y = \frac{27(x^3+x^2)}{4} - 5$
8. $y = 3x^2 - 2 - x^3$	23. $y = \frac{16-2x^2-x^3}{8}$
9. $y = (x-1)^2(x-3)^2$	24. $y = -\frac{x^2-4}{16}$
10. $y = \frac{x^3+3x^2}{4} - 5$	25. $y = 16x^3 - 36x^2 + 24x - 9$
11. $y = 6x - 8x^2$	26. $y = \frac{16x^2-x^3-16}{8}$
12. $y = 16x^2(x-1)^2$	27. $y = -\frac{(x-2)^2(x-6)^2}{16}$
13. $y = 2x^3 + 3x^2 - 5$	28. $y = 16x^3 - 12x^2 - 4$
14. $y = 2 - 12x^2 - 8x^3$	29. $y = \frac{11+9x-3x^2-x^3}{8}$
15. $y = (2x+1)^2(2x-1)^2$	30. $y = -\frac{(x+1)^{2(x-3)^2}}{16}$

Birinchi tartibli hosilalar yordamida funksiyaning grafigini yasang

1. $y = 1 - \sqrt[3]{x^2 - 2x}$	16. $y = 6\sqrt[3]{(x-2)^2} - 4x + 8$
2. $y = 2x - 3\sqrt[3]{x^2}$	17. $y = \frac{3\sqrt[3]{6(x-5)^2}}{x^2-6x+17}$
3. $y = \frac{2\sqrt[3]{6(x-2)^2}}{x^2+8}$	18. $y = 2 + \sqrt[3]{8x(x+2)}$
4. $y = \frac{2\sqrt[3]{6(x-2)^2}}{x^2+2x+9}$	19. $y = 6x - 6 - 9\sqrt[3]{(x-1)^2}$
5.$y = 1 - \sqrt[3]{x^2 - 2x}$	20. $y = \sqrt[3]{x^2 + 6x + 8}$
6. $y = 2x + 6 - 3\sqrt[3]{(x+1)^2}$	21. $y = \sqrt[3]{4x(x-1)}$
7. $y = \frac{6\sqrt[3]{6(x-1)^2}}{x^2-2x+9}$	22. $y = -\frac{3\sqrt[3]{6(x-2)^2}}{x^2+8x+24}$
	23. $y = \sqrt[3]{x(x-2)}$

8. $y=1-\sqrt[3]{x^2+4x+3}$ 9. $y=\sqrt[3]{(x-3)^2}-2x+6$ 10. $y=-\frac{6\sqrt[3]{(x-1)^2}}{x^2+4x+12}$ 11. $y=4x+8-6\sqrt[3]{(x-2)^2}$ 12. $y=\frac{3\sqrt[3]{(x-1)^2}}{x^2-4x+7}$ 13. $y=\sqrt[3]{x(x+2)}$ 14. $y=\sqrt[3]{x^2+4x+3}$ 15. $y=-\frac{3\sqrt[3]{6(x-1)^2}}{x^2+6x+17}$	24. $y=1-\sqrt[3]{x^2-4x+3}$ 25. $y=9\sqrt[3]{(x+1)^2}-6x-6$ 26. $y=\frac{6\sqrt[3]{6(x-3)^2}}{x^2-8x+24}$ 27. $y=8x-16-12\sqrt[3]{(x-2)^2}$ 28. $y=\frac{6\sqrt[3]{6(x-6)^2}}{x^2+10x+39}$ 29. $y=12\sqrt[3]{(x+2)^2}-8x-16$ 30. $y=\frac{3\sqrt[3]{6(x-1)^2}}{2(x^2+2x+9)}$

Berilgan kesmada funksiyaning eng katta va eng kichik qiymatlarini toping

1. $y=x^2+\frac{16}{x}-16,\ \ [1;4]$ 2. $y=4-x-\frac{4}{x^2},\ \ [1;4]$ 3. $y=\sqrt[3]{2(x-2)^2(8-x)}-1,\ \ [0;6]$ 4. $y=\frac{2(x^2+3)}{x^2-2x+5},\ \ [-3;3]$ 5. $y=2\sqrt{x}-x,\ \ [0;4]$ 6. $y=1+\sqrt[3]{2(x-1)^2(x-7)},\ \ [0;3]$ 7. $y=x-4\sqrt{x}+5,\ \ [1;9]$ 8. $y=\frac{10x}{1+x^2},\ \ [0;3]$ 9. $y=\sqrt[3]{2(x+1)^2(5-x)}-2,\ \ [-3;3]$ 10. $y=2x^2+\frac{108}{x}-19,\ \ [-1;6]$ 11. $y=3-x-\frac{4}{(x+2)^2},\ \ [-1;2]$ 12. $y=\sqrt[3]{2x^2(x-3)},\ \ [-1;6]$ 13. $y=\frac{2(-x^2+7x-7)}{x^2-2x+2}$	16. $y=\frac{4x}{4+x^2},\ \ [-4;2]$ 17. $y=-\frac{x^2}{2}+\frac{8}{x}+8,\ \ [-4;-1]$ 18. $y=\frac{x}{\sqrt[3]{2x^2(x-6)}},\ \ [-2;4]$ 19. $y=-\frac{2x(2x+3)}{x^2+4x+5},\ \ [-2;1]$ 20. $y=-\frac{2(x^2+3)}{x^2+2x+5},\ \ [-5;1]$ 21. $y=\sqrt[3]{2(x-1)^2(x-4)},[0;4]$ 22. $y=\frac{x^2-2x+16}{x-1}-13,\ \ [2;5]$ 23. $y=2\sqrt{x-1}-x+2,\ \ [1;5]$ 24. $y=\sqrt[3]{2(x+2)^2(1-x)},[-3;4]$ 25. $y=-\frac{x^2}{2}+2x+\frac{9}{x-2}+5,[-2;1]$ 26. $y=8x+\frac{4}{x^2}-15,\ \ [\frac{1}{2};2]$ 27. $y=\sqrt[3]{x^2(x+2)^2(x-4)}+3,\ \ [-3;4]$ 28. $y=x^2+4x+\frac{16}{x+2}-9,[-1;2]$ 29. $y=-\frac{4}{x^2}-8x-16,\ \ [-2;4]$

14. $y = x - 4\sqrt{x+2} + 8,\ [-1;7]$ 15. $y=\sqrt[3]{2(x-2)^2(5-x)}, [1;5]$	30. $y=\sqrt[3]{2(x+1)^2(x-1)}, [-2;5]$

Yuqori tartibli hosilalar yordamida berilgan nuqta atrofida funksiyani holatini tekshiring

1. $y = x^2 + x - (x-2)\ln(x-1), x_0 = 2$ 2. $y = 4x - x^2 - 2\cos(x-2),\ x_0 = 2$ 3. $y = 6e^{x-2}x^3 + 3x^2 - 6x,$ $x_0 = 2$ 4. $y = 2\ln(x+1) - 2x + x^2 + 1, x_0 = 0$ 5. $y = 2x - x^2 - 2\cos(x-1), x_0 = 1$ 6. $y = \cos^2(x+1) + x^2 + 2x,\ x_0 = -1$ 7. $y = 2lnx + x^2 - 4x + 3,$ $x_0 = 1$ 8. $y = 1 - x - x^2 - 2\cos(x+1),\ \ x_0 = 1$ 9. $y = x^2 + 6x + 8 - 2e^{x+2},\ x_0 = -2$ 10. $y = 4x + x^2 - 2e^{x+1}, x_0 = -1$ 11. $y = (x+1)\sin(x+1) - x^2,\ x_0 = -1$ 12. $y = 6e^{x-1} - 3x - x^3, x_0 = -1$ 13. $y = 2x + (x+1)\ln(2+x), x_0 = -1$ 14. $y = \sin^2(x+1) - 2x - x^2, x_0 = -1$ 15. $y = x^2 + 4x + \cos^2(x+2), x_0 = -2$	16. $y = x^2 + 2\ln(x+2), x_0 = -1$ 17. $y = 4x - x^2 + (x-2)\sin(x-2),\ x_0 = 2$ 18. $y = 6e^x - x^3 - 3x^2 - 6x - 5,\ x_0 = 0$ 19. $y = x^2 - 2x - 2e^{x-2},\ x_0 = 2$ 20. $y = \sin^2(x+2) - x^2 - 2x,\ x_0 = 1$ 21. $y = \cos^2(x-1) + x^2 - 2x,\ x_0 = 1$ 22. $y = x^2 - 2x - (x-1)sinx,\ x_0 = 1$ 23. $y = (x-2)\sin(x-1) + 2x - x^2,\ x_0 = 1$ 24. $y = x^2 - 4x + \cos^2(x-2),\ x_0 = 2$ 25. $y = x^4 + 4x^3 + 12x^2 - e^x, x_0 = 0$ 26. $y = \sin^2(x-2) - x^2 + 4x - 4,\ x_0 = 2$ 27. $y = 6e^{x+1}x^3 - 6x^2 - 15x - 16,\ x_0 = -1$ 28. $y = sinx + shx - 2x,\ x_0 = -1$ 29. $y = \sin^2(x+1) + 2x - x^2, x_0 = -1$ 30. $y = cosx + chx, x_0 = 0$

Berilgan funksiyaning asimtotasini toping va grafigini toping

1. $y=\frac{17-x^2}{4x-5}$	16. $y=\frac{(21-x^2)}{17x+9}$
2. $y=\frac{x^2+1}{\sqrt{4x^2-3}}$	17. $y=\frac{2x^2-1}{\sqrt{x^2-2}}$
3. $y=\frac{x^2-4x}{3x^2-4}$	18. $y=\frac{2x^3-3x^2-2x+1}{1-3x^2}$
4. $y=\frac{4x^2+9}{(4x+8)}$	19. $y=\frac{x^2-11}{4x-3}$
5. $y=\frac{4x^3+3x^2-8x-2}{2-3x^2}$	20. $y=\frac{2x^2-9}{\sqrt{x^2-1}}$
6. $y=\frac{x^2-3}{\sqrt{3x^2-2}}$	21. $y=\frac{x^3-2x^2-3x-2}{1-x^2}$
7. $y=\frac{2x^2-6}{x-2}$	22. $y=\frac{x^2+6x+9}{x+4}$
8. $y=\frac{2x^3+2x^2-3x-1}{2-4x^2}$	23. $y=\frac{x^3+x^2-3x-1}{2x^2-2}$
9. $y=\frac{x^2-5x}{5-3x^2}$	24. $y=\frac{x^2+2x+1}{2x+1}$
10. $y=\frac{x^2-6x+4}{3x-2}$	25. $y=\frac{3x^2+x^2-3x-1}{2x^2-2}$
11. $y=\frac{2-x^2}{\sqrt{9x^2-4}}$	26. $y=\frac{x^2-2x+2}{x+3}$
12. $y=\frac{4x^2-3x}{4x^2-1}$	27. $y=\frac{2x^3+2x^2-9x-3}{2x^2-3}$
13. $y=\frac{3x^2-7}{2x+1}$	28. $y=\frac{3x^2-10}{3-2x}$
14. $y=\frac{x^2+16}{\sqrt{9x^2-8}}$	29. $y=\frac{-x^2-4x+13}{4x+3}$
15. $y=\frac{x^3+3x^2-2x-2}{2-3x^2}$	30. $y=\frac{-8-x^2}{\sqrt{x^2-4}}$

Funksiyani to‘la tekshiring va grafigini yasang

1. $y=\frac{x^2+4}{x^2}$	16. $y=\left(\frac{x-1}{x+1}\right)^2$
2. $y=\frac{x^2-x+1}{x-1}$	17. $y=\frac{3x^4+1}{x^3}$
3. $y=\frac{2}{x^2+2x}$	18. $y=\frac{4x}{(x+1)^2}$
4. $y=\frac{4x^2}{3+x^2}$	19. $y=\frac{8(x-1)}{(x+1)^2}$
5. $y=\frac{12x}{8+x^2}$	20. $y=\frac{1-2x^3}{x^2}$

6. $y=\frac{x^2-3x+3}{x-1}$	21. $y=\frac{4}{x^2+2x-3}$
7. $y=\frac{4-x^2}{x^2}$	22. $y=\frac{4}{3+2x-x^2}$
8. $y=\frac{x^2-4x+1}{x-1}$	23. $y=\frac{x^2-6x+9}{x^2+2x+4}$
9. $y=\frac{2x^3+1}{x^2}$	24. $y=\frac{1}{x^4-1}$
10. $y=\frac{(x-1)^2}{x^2}$	25. $y=-\left(\frac{x}{x+2}\right)^2$
11. $y=\frac{x^2}{(x-1)^2}$	26. $y=\frac{x^3-32}{x^2}$
12. $y=\frac{x+1}{x^2}$	27. $y=\frac{4(x+1)^2}{x^2+2x+4}$
13. $y=-\frac{8x}{x^2+4}$	28. $y=\frac{3x-2}{x^3}$
14. $y=\frac{9+6x-3x^2}{x^2-2x+13}$	29. $y=\frac{x^2-6x+9}{(x-1)^2}$
15. $y=\frac{12-3x^2}{x^2+12}$	30. $y=\frac{x^3-27x+54}{x^3}$

Egri chiziqning absissasi x_0 nuqtaga o‘tkazilgan (1-2) normal (2-3) urinma tenglamalarini tuzing

1. $y=\frac{4x-x^2}{4},\ \ x_0=2$	13. $y=2x^2+3,\ \ x_0=-1$
2. $y=2x^2+3x-1,\ \ x_0=-2$	14. $y=\frac{(x^{29}+6)}{x^4-1},\ \ x_0=1$
3. $y=x-x^3,\ \ x_0=-1$	15. $y=2x+\frac{1}{x},\ \ x_0=1$
4. $y=x^2+8\sqrt{x}-32,\ \ x_0=4$	16. $y=-\frac{2(x^8+2)}{3(x^4+1)},\ \ x_0=1$
5. $y=x+\sqrt{x^3},\ \ x_0=1$	17. $y=\frac{x^5+1}{x^4+1},\ \ x_0=1$
6. $y=\sqrt[3]{x^2}-20,\ \ x_0=-8$	18. $y=\frac{x^{16}-9}{1-5x^2},\ \ x_0=1$
7. $y=\frac{1+\sqrt{x}}{1-\sqrt{x}},\ \ x_0=4$	19. $y=3(\sqrt[3]{x}-2\sqrt{x}),\ \ x_0=1$
8. $y=8\sqrt[4]{x}-70,\ \ x_0=16$	20. $y=\frac{1}{3x+2},\ \ x_0=2$
9. $y=2x^2-3x+1,\ \ x_0=1$	21. $y=\frac{x}{x^2+1},\ \ x_0=-2$
10. $y=\frac{x^2-3x+6}{x^2},\ \ x_0=3$	22. $y=\frac{x^2-3x+3}{3},\ \ x_0=3$
11. $y=\sqrt{x}-3\sqrt[3]{x},\ \ x_0=64$	23. $y=\frac{2x}{x^2+1},\ \ x_0=1$
12. $y=\frac{x^3+2}{x^3-2},\ \ x_0=2$	24. $y=-2(\sqrt[3]{x}+3\sqrt{x}),\ \ x_0=1$

dy – differensialini toping

1. $y=\sqrt{1+2x}+\ln\left\|x+\sqrt{x^2-1}\right\|,\ x>0$ 2. $y=tg\left(2arccos\sqrt{1-x^2}\right),\ \ x>0$ 3. $y=\arccos\left(\frac{1}{\sqrt{1+2x^2}}\right),\ \ x>0$ 4. $y=\frac{lnx}{1+x^2}-\frac{1}{2}\ln\frac{x^2}{1+x^2}$ 5. $y=2x+\ln\|sinx+2cosx\|$ 6. $y=\sqrt[3]{\frac{x+2}{x-2}}$	7. $y=\ln\|\frac{x+\sqrt{x^2+1}}{2x}$ 8. $y=arctg\frac{x^2-1}{x}$ 9. $y=\sqrt{ctgx}-\frac{\sqrt{tgx^3}}{3}$ 10. $y=e^x(cos2x+2sin2x)$ 11. $y=cosxlntgx-ln\frac{x}{2}$ 12. $y=x\sqrt{x^2-1}+ln\|x+\sqrt{x^2-1}\|$

Differensial yordamida taqribiy hisoblang

1. $y=\sqrt[3]{x},\ \ x=7.76$ 2. $y=\sqrt[3]{x^3+7x},\ \ x=1.012$ 3. $y=\frac{x+\sqrt{5-x^2}}{2},\ \ x=0.98$ 4. $y=\sqrt[3]{x},\ \ x=27.54$ 5. $y=arcsinx,\ \ x=0.08$ 6. $y=\sqrt[3]{x^2+2x+5},\ \ x=0.97$	7. $y=x^{11},\ \ x=1.092$ 8. $y=\sqrt[3]{x^2},\ \ x=1.08$ 9. $y=\sqrt{4x-1},\ \ x=2.56$ 10. $y=\frac{1}{\sqrt{x}},\ \ x=4.16$ 11. $y=\sqrt{x^3},\ \ x=0.98$ 12. $y=x^4,\ \ x=3.908$

Hosilasini toping

1. $y=\frac{2(3x^3+4x^2-x-2)}{15\sqrt{1+x}}$ 2. $y=\frac{(2x^2-1)\sqrt{1+x^2}}{3x^3}$ 3. $y=\frac{(x^4-8x^2)}{2(x^2-4)}$ 4. $y=\frac{2x^2-x-1}{3\sqrt{2+4x}}$ 5. $y=\frac{(4+x^3)\sqrt{1+x^8}}{12x^{12}}$ 6. $y=\frac{x^2}{2\sqrt{1-3x^2}}$ 7. $y=\frac{(x^2-6)\sqrt{(4+x^2)^3}}{120x^5}$	25. $y=\sqrt{x}\ln\left(\sqrt{x}+\sqrt{x+a}\right)-\sqrt{x-a}$ 26. $y=2\sqrt{x}-4\ln(2+\sqrt{x})$ 27. $y=\ln^2(x+cosx)$ 28. $y=ln\frac{a+x^2}{a-x^2}$ 29. $y=ln\frac{x^2}{1-x^2}$ 30. $y=log_{16}log_5tgx$ 31. $y=x(coslnx+sinlnx)$ 32. $y=log_a\frac{1}{\sqrt{1-x^4}}$

8. $y = \sqrt[3]{\frac{\left(1+x^{\frac{3}{4}}\right)^2}{x^{\frac{3}{2}}}}$

9. $y = \frac{x\sqrt{x+1}}{x^2+x+1}$

10. $y = \frac{3x+\sqrt{x}}{\sqrt{x^2-2}}$

11. $y = \frac{3x^6+4x^4-x^2-2}{2x+7}$

12. $y = (1-x^2)\sqrt[5]{x^3+\frac{1}{x}}$

13. $y = x-\ln\left(2+e^x+2\sqrt{e^{2x}+e^x+1}\right)$

14. $y = \frac{1}{2}arctg\frac{e^x-3}{2}$

15. $y = 2\sqrt{e^x+1}+ln\frac{\sqrt{e^x+1}+1}{\sqrt{e^x+1}-1}$

16. $y = \frac{2}{3}\sqrt{(arctge^x)^3}$

17. $y = e^{2x}\frac{(2-sin2x-cos2x)}{8}$

18. $y = 2(x-2)\sqrt{1+e^x}-2\ln\frac{\sqrt{e^x+1}+1}{\sqrt{e^x+1}-1}$

19. $y = 3e^{\sqrt{x}}(\sqrt[3]{x^2}-2\sqrt[3]{2}+2$

20. $y = \frac{e^{x^2}}{1+x^2}$

21. $y = y-e^{2x}\frac{(\beta\sin\beta x+\alpha\cos\beta x)}{\alpha^2+\beta^2}$

22. $y = x^2+\frac{8}{1+e^{\frac{3}{4}}}$

23. $y = \ln\left(e^x\arcsin e^x\right)-\ln\left(1+\sqrt{1-e^{2x}}\right)$

24. $y = \frac{e^{x^3}}{1+x^3}$

33. $y = \ln(arccos\frac{1}{\sqrt{x}}$

34. $y = \ln(e^x+\sqrt{1+e^{2x}})$

35. $y = lnln^3\ln^2 x$

36. $y = \ln(bx+\sqrt{a^2+b^2x^2})$

37. $y = sin\sqrt{3}+\frac{1}{3}\frac{\sin^2 3x}{cos6x}$

38. $y = cosln2-\frac{1}{2}\frac{\cos^2 3x}{sin6x}$

39. $y = tgln\frac{1}{3}+\frac{1}{4}\frac{\sin^2 4x}{cos8x}$

40. $y = ctg\sqrt[3]{5}-\frac{1}{8}\frac{\cos^2 4x}{sin8x}$

41. $y = tgln\,2\frac{\sin^2 9x}{19cos38x}$

42. $y = \sqrt{tg4}+\frac{\sin^2 21x}{23cos46x}$

43. $y = sin\sqrt[3]{tg2}-\frac{\cos^2 28x}{60sin60x}$

44. $y = \frac{ctgsin\frac{1}{8}+\sin^2 15x}{15cos30x}$

45. $y = \frac{1}{3}costg\frac{1}{2}+\frac{1}{10}\frac{\sin^2 10x}{cos10x}$

46. $y = \sqrt[3]{cos\sqrt{2}}-\frac{1}{52}\frac{\cos^2 26x}{sin52x}$

47. $y = 8sinctg3+\frac{1}{5}\frac{\sin^2 5x}{cos10x}$

48. $y = \cos^2 sin3+\frac{\sin^2 29x}{24cos50x}$

7. 2-namunaviy hisob.

Funksiyaning limiti, hosilasi va differensiali.

Funksiyani hosila yordamida tekshirish.

Nazariy savollar

1. Ketma-ketlik limiti. Limit va ketma-ketliklar haqida teoremalar.
2. Funksiyaning limiti. Chekli limitga ega funksiyalarning chegaralanganligi haqidagi teorema.
3. Limitga egafunksiyalar haqida teoremalar.Bir tomonlama limitlar.
4. Birinchi ajoyib limit. Ikkinchi ajoyib limit.
5. Hosilaga ta'rif bering. Funksiyaning nuqtada va sohada differensiallanuvchanligi. Uning geometric va fizik ma'nosi qanday?
6. Asosiy elementlar funksiyalar hosilalari jadvalini keltiring.
7. Funksiyaning nuqtadagi differensiali deb nimaga aytiladi va uning geometric ma'nosi. Differensial shaklining invariantligi.
8. Murakkabfunksiyaning hosilasi. Teskari funksiyaning hosilasi.
9. Logarifmik differensiallash yordamida $y = [u(x)]^{v(x)}$ ko'rinishdagi funksiyaning hosilasi uchun formula keltirib chiqarish.
10. Ikkinchi tartibli hosilaning ta'rifini bering va uning fizik ma'nosini tushuntirng.
11. Parametrik ravishda berilgan funksiyaning birinchi va ikkinchi tartibli hosilalarini topish formulalarini yozing.
12. Oshkormas ravishda berilgan funksiyaning birinchi va ikkinchi tartibli hosilalarini yozing.
13. Funksiya ekstremumi. Ekstremumning zaruriy sharti.
14. Funksiya ekstremumining yetarlishartlari. (1- va 2- tartibli hosilalar yordamida)
15. Funksiyaning asimptotalari. Funksiyaning qavariqligi va botiqligi. Yetarli shartlar. Burilish nuqtalari.

Yechish namunasi

1-misol. $lim_{n\to\infty}\frac{(n+1)(n+2)(n-1)}{3n^3+2n+3}$ limitni hisoblang.

Yechish. Bu limitni hisoblash uchun surati va maxrajida *n* ning eng yuqori darajasini qavsdan tashqariga chiqaramiz.

$$lim_{n\to\infty}\frac{(n+1)(n+2)(n-1)}{3n^3+2n+3} = lim_{n\to\infty}\frac{n^3(1+\frac{1}{n})(1+\frac{2}{n})(1-\frac{1}{n})}{n^3(3+\frac{2}{n^2}+\frac{3}{n^3})} = lim_{n\to\infty}\frac{(1+\frac{1}{n})(1+\frac{2}{n})(1-\frac{1}{n})}{3+\frac{2}{n^2}+\frac{3}{n^3}} = \frac{1}{3}$$

2-misol. $lim_{x\to 0}\frac{\sqrt[3]{x+1}-1}{\sqrt{x+4}-2}$ limitni hisoblang.

Yechish. Bu limitni hisoblash uchun surat va maxrajini $((\sqrt[3]{x+1})^2+\sqrt{x+1}+1)(\sqrt{x+4}-2)$ ga ko‘paytiramiz.

$$lim_{x\to 0}\frac{\sqrt[3]{x+1}-1}{\sqrt{x+4}-2} = lim_{x\to 0}\frac{(\sqrt[3]{x+1}-1)((\sqrt[3]{x+1})^2+\sqrt{x+1}+1)(\sqrt{x+4}-2)}{(\sqrt{x+4}-2)((\sqrt[3]{x+1})^2+\sqrt{x+1}+1)(\sqrt{x+4}-2)} = lim_{x\to 0}\frac{\sqrt{x+4}+2}{(\sqrt[3]{x+1})^2+\sqrt{x+1}+1} = \frac{4}{3}$$

3-misol. $lim_{x\to -1}\frac{x^3+1}{\arcsin(x+1)}$ limitni hisoblang.

Yechish. $lim_{x\to -1}\frac{x^3+1}{\arcsin(x+1)} = lim_{x\to -1}\frac{(x+1)(x^2-x+1)}{\arcsin(x+1)} = lim_{x\to -1}\frac{(x+1)}{\arcsin(x+1)}(x^2-x+1) = 3$

4-misol. $lim_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$ limitni hisoblang.

Yechish.

$$lim_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1} = lim_{x\to\infty}\left(\frac{2x+1+2}{2x+1}\right)^{x+1}$$

$$= lim_{x\to\infty}\left(1+\frac{2}{2x+1}\right)^{x+1} =$$

$$= lim_{x\to\infty}\left[\left(1+\frac{2}{2x+1}\right)^{\frac{2x+1}{2}}\right]^{\frac{2(x+1)}{2x+1}} = lim_{x\to\infty}e^{\frac{2(x+1)}{2x+1}}$$

$$= lim_{x\to\infty}e^{\frac{2x(1+\frac{1}{x})}{x(2+\frac{1}{x})}} = e$$

5-misol.$y = e^{sin2x}$ funksiya grafigining x_0=0 nuqtasiga o‘tkazilgan urinma va normal tenglamalarini tuzing.

Yechish. Berilgan M(x_0,y_0) nuqtasiga o‘tkazilgan $y - y_0 = f'(x_0)(x - x_0)$ ko‘rinishga ega.

M(x_0,y_0) nuqta berilgan, ya’ni M(0,1).

$f'(x_0)$ ni hisoblaymiz.

$$f'(x_0) = e^{sin2x}\cdot cos2x\cdot 2$$

$f'(0)$ =2

$$y - 1 = 2(x - 0) \to y = 2x + 1 \to 2x - y + 1 = 0$$

Endi normalni tenglamasini tuzamiz.

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$

$$y - 1 = -\frac{1}{2}(x - 0) \to 2y - 2 = -x \to x + 2y - 2 = 0$$

6-misol. $y = ln\frac{1-x^2}{1+x^2}$ funksiyaning differensialini toping.

Yechish. Funksiyaning differensialini topish uchun

$$dy = f'(x)dx$$

Formuladan foydalanamiz.

$$dy = \left(ln\frac{1-x^2}{1+x^2}\right)dx = \frac{1-x^2}{1+x^2}\cdot\frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}dx$$

$$= \frac{-2x-2x^3-2x+2x^3}{(1-x^2)(1+x^2)}dx = \frac{-4x}{(1-x^4)}dx$$

7-misol. $y = sinx \cdot lnx$ funksiyaning ikkinchi tartibli hosilasini toping.

Yechish. $y' = cosx \cdot lnx + sinx \cdot \frac{1}{x}$

$$y'' = -sinx \cdot lnx + cosx \cdot \frac{1}{x} + cosx \cdot \frac{1}{x} - sinx \cdot \frac{1}{x^2}$$

$$= 2\frac{cosx}{x} - sinx(lnx + \frac{1}{x^2})$$

8-misol. $arctg\frac{y}{x} = \frac{1}{2}\ln(x^2+y^2)$ oshkormas funksiyaning hosilasini toping.

Yechish. Oshkormas funksiyaning hosilasini topish uchun y ni x ning funksiyasi deb y'_x hosilani topamiz.

$$(arctg\frac{y}{x})'_x = \frac{1}{2}(\ln(x^2+y^2))'_x =$$

$$\frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{y'_x x - y}{x^2} = \frac{1}{2}\cdot\frac{2x+2y\cdot y'_x}{x^2+y^2} \rightarrow \frac{y'_x x - y}{x^2+y^2} = \frac{x+yy'_x}{x^2+y^2}$$

$$\rightarrow y'_x x - y = x + yy'_x \rightarrow y'_x x - yy'_x = x + y$$

$$\rightarrow (x-y)y'_x = x + y \rightarrow y'_x = \frac{x+y}{x-y}$$

9-misol. $\begin{Bmatrix} x = asint \\ y = bcost \end{Bmatrix}$ parametrik ko‘rinishda berilgan funksiyani ikkinchi tartibli hosilasini toping.

Yechish. Bu funksiyaning ikkinchi tartibli hosilasini toppish uchun ushbu formulalardan foydalanamiz:

$$y'_x = \frac{y'_t}{x'_t}, \quad y''_{xx} = \frac{(y'_x)'_t}{x'_t}$$

$$x'_t = acost$$

$$y'_t = -bsint$$

$$y'_x = \frac{y'_t}{x'_t} = -\frac{bsint}{acost} = -\frac{b}{a}tgt$$

$$y''_{xx} = \frac{(y'_x)'_t}{x'_t} = \frac{\left(-\frac{b}{a}tgt\right)'_t}{acost} = -\frac{b \cdot \frac{1}{\cos^2 t}}{a^2 \cdot cost} = -\frac{b}{a^2 \cos^3 t}$$

Parametrik ko‘rinishda berilgan funksiyani ikkinchi tartibli hosilasini quyidagi formula yordamida ham toppish mumkin:

$$y''_{xx} = \frac{y''_{II}x'_I - y'_I x''_{II}}{(x'_I)^3}$$

10-misol. $y = x\sqrt{1-x^2}$ funksiyani o‘sish va kamayish hamda ekstremumlarini toping.

Yechish. Berilgan funksiyani aniqlanish sohasini topamiz.

$D(y): 1 - x^2 \geq 0 \rightarrow |x| \leq 1$. Demak$D(y)$: $[-1,1]$.

Endi o‘sish, kamayish oraliqlari va ekstremumlarini topamiz. Bu funksiyadan hosila olib va nolga tenglab, kritik nuqtalarini topamiz.

$$y' = \sqrt{1-x^2} + \frac{x(-2x)}{2\sqrt{1-x^2}} = \frac{1-x^2-x^2}{\sqrt{1-x^2}} = \frac{1-2x^2}{\sqrt{1-x^2}}$$

$$f'(x) = y' = 0 \rightarrow 1 - 2x^2 = 0 \rightarrow x_{1,2} = \pm\frac{1}{\sqrt{2}}.$$

Bu nuqtalar [-1,1] kesma ichida bo‘lgani uchun, shu kritik nuqtalarda ekstremumlarni izlaymiz. Lekin $x_{3,4} = \pm 1$ nuqtalar ichki nuqta bo‘lmay, chegara nuqtalar bo‘lgani uchun bular kritik nuqtalar bo‘lmaydi. O‘sish, kamayish oraliqlarini hamda ekstremumlarni ushbu jadvalga yozamiz:

	$(-1, -\frac{1}{\sqrt{2}})$	$x_1 = -\frac{1}{\sqrt{2}}$	$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$	$x_2 = \frac{1}{\sqrt{2}}$	$(\frac{1}{\sqrt{2}}, 1)$
y'	-	0	+	0	-
x'	→↓	$Y_{min} = -\frac{1}{2}$	→↑	$Y_{max} = \frac{1}{2}$	→↓

Yechish uchun vazifalar

1-vazifa. Sonli ketma-ketlikning limitini hisoblang.
2,3,4-vazifa. 2,3,4-misolllarda berilgan limitlarni hisoblang.
5-vazifa. Berilgan funksiyaning absissasi x_0 bo‘lgan nuqtaga o‘tkazilgan urinma va normal tenglamalarini tuzing.
6-vazifa. Funksiyaning differensialini toping.
7-vazifa. Berilgan funksiyaning ikkinchi tartibli hosilasini toping.
8-vazifa. Funksiyaning hosilasini toping.
9-vazifa. Oshkormas funksiyaning hosilasini toping.
10-vazifa. Parametrik ko‘rinishda berilgan funksiyaning oraliqlari va ekstremumlarini toping.
11-vazifa. Berilgan funksiyaning o‘sish va kamayish oraliqlari va ekstremumlarini toping.

Variantlar

1-variant	2-variant
1. $\lim\limits_{x\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n-1}{n^2}\right.$	1. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^3+3}-\sqrt[4]{n^4-3n+5}}{\sqrt{2n^2+4n+2}+n}$
2. $\lim\limits_{x\to\infty}\frac{(x+1)(2x+1)(3x+1)(4x+1)}{(2x-1)^4-(x+3)^4}$	2. $\lim\limits_{x\to\infty}(\sqrt{x+b}-\sqrt{x-b})$
3. $\lim\limits_{x\to\infty}\frac{1-\cos^2 2x}{x sin\frac{x}{2}}$	3. $\lim\limits_{x\to\infty}\frac{1-cos3x}{1-cosx}$
4. $\lim\limits_{x\to\infty}\left(\frac{x+4}{x+8}\right)^{-3x}$	4. $\lim\limits_{x\to\infty}\left(\frac{2x}{1+2x}\right)^{-4x}$
5. $y=xtgx,\ \ x_0=\frac{\pi}{4}$	5. $y=\frac{5sinx}{cos2x},\ \ x_0=\frac{\pi}{2}$
6. $y=lnarccos\sqrt{1-e^x}$	6. $y=e^{sin2x}-\log_3(\sqrt{x+1})$
7. $y=5^{-x^2}$	7. $y=\sin\sqrt[3]{tg2x}-\cos^2\frac{6x}{sin30^\circ}$
8. $y=\left(sin\sqrt{x}\right)^{ctgx}$	8. $y=(x^3+2)e^{4x+3}$
9. $xy-lny=1$	9. $x+y=2^{x+y}$
10. $\begin{cases} x=3^{t^2} \\ y=sin\sqrt{t^2-1} \end{cases}$	10. $\begin{cases} x=\mathrm{a}\cos^2 t \\ y=bsin^2 t \end{cases}$
11. $y=\frac{x^2-2x+2}{x-1}$	11. $y=e^{\frac{1}{5+x}}$

3-variant 1. $\lim_{n\to\infty}(\sqrt[3]{2n-3}-\sqrt[3]{2n+3})$ 2. $\lim_{x\to\infty}x(\sqrt{x^2+1}-\sqrt{x^2-1})$ 3. $\lim_{x\to\infty}\frac{1+cos2x}{1+cos6x}$ 4. $\lim_{x\to\infty}\left(\frac{2x+5}{2x+1}\right)^{5x}$ 5. $y=xctgx,\ \ x_0=\frac{\pi}{2}$ 6. $y=tg^3 2x$ 7. $y=(1-x^2)\sqrt[5]{x^3}$ 8. $y=(cos2x)^{e^x}$ 9. $x-y=arctg\sqrt{y}$ 10. $\begin{cases} x=arctgt \\ y=ln\frac{\sqrt{1+t^2}}{1+2} \end{cases}$ 11. $y=\frac{4x-x-4}{x}$	4-variant 1. $\lim_{n\to\infty}\frac{2^n-6^n+3^n}{3^n+4^n}$ 2. $\lim_{x\to\infty}\frac{\sqrt[3]{8x+5}-\sqrt[6]{x^2+3x}}{\sqrt[3]{27x+3}-\sqrt[3]{2-64x}}$ 3. $\lim_{x\to\infty}\frac{1+sin3x}{\left(\frac{\pi}{2}-x\right)^2}$ 4. $\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{1+2x}$ 5. $y=x^2e^x,\ \ x_0=0$ 6. $y=arccosx+\ln(tgx)$ 7. $y=x^2\cdot\log_3 x$ 8. $y=(cos5x)^{2x}$ 9. $y^3+2xy+x^2=0$ 10. $\begin{cases} x=1+\cos^2 t \\ y=\frac{cost}{sint} \end{cases}$ 11. $y=\frac{x+1}{(x-1)^2}$
5-variant 1. $\lim_{n\to\infty}\frac{(n+1)!+n!}{(n+2)!}$ 2. $\lim_{x\to\infty}\frac{\sqrt{4x^2-1}-\sqrt{7+9x^2}}{\sqrt{16x^2+7x+8}}$ 3. $\lim_{x\to0}\frac{1-cos3x}{xsin5x}$ 4. $\lim_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{3x}$ 5. $y=xsinx,\ \ x_0=\frac{\pi}{2}$ 6. $y=\frac{2}{3}\cdot\sqrt{(arctge^x)^3}$ 7. $y=ln\frac{x+1}{x^3}$ 8. $y=(x^4+5)^{ctgx}$ 9. $x-y=arcsinx-arcsiny$ 10. $\begin{cases} x=arcsin\sqrt{t} \\ y=\sqrt{1+\sqrt{t}} \end{cases}$	6-variant 1. $\lim_{n\to\infty}\frac{(2n+1)^4-(n-1)^4}{(2n+1)^4+(n-1)^4}$ 2. $\lim_{x\to-2}\frac{\sqrt{12-2x}-\sqrt{2x+20}}{x^2+6x+8}$ 3. $\lim_{x\to\frac{1}{2}}\frac{\arcsin(2x-1)}{4x-2}$ 4. $\lim_{x\to\infty}\left(\frac{x-1}{x+4}\right)^{3x+2}$ 5. $y=\sqrt[3]{x^3+4},\ \ x_0=0$ 6. $y=\frac{(2x^2-1)\sqrt{1+x^2}}{3x^2}$ 7. $y=\left(arcsin\sqrt{1-x}\right)^3+5^{ctgx}$ 8. $y=x^{cosx^2}$ 9. $x+1=e^{xy}$ 10. $\begin{cases} x=\ln(1-t^2) \\ y=arcsin\sqrt{1-t^2} \end{cases}$

11. $y=\frac{x}{9-x}$	11. $y=\frac{lnx}{\sqrt{x}}$
7-variant 1. $\lim\limits_{n\to\infty}\frac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{1-5n^2+27n^6}}$ 2. $\lim\limits_{x\to 2}\frac{\sqrt[3]{x+6}-2}{\sqrt{6-2x}-\sqrt{2}}$ 3. $\lim\limits_{x\to\frac{5}{6}}\frac{5x-6}{\arcsin(6-5x)}$ 4. $\lim\limits_{x\to\infty}\left(\frac{x-2}{x+1}\right)^{2x-3}$ 5. $y=xcos2x,\ \ x_0=\frac{\pi}{2}$ 6. $y=arctg\sqrt{\sin\frac{5}{x}}$ 7. $y=\ln\frac{x^2}{\sqrt{1-ax^4}}$ 8. $y=(tgx-1)^{cosx}$ 9. $\sqrt{x+y}=e^{x+y}$ 10. $\begin{cases}x=t^3+1\\ y=t^2\end{cases}$ 11. $y=x^3-9x^2+24x$	8-variant 1. $\lim\limits_{n\to\infty}\frac{(n+1)!+n!}{n!-(n-2)!}$ 2. $\lim\limits_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$ 3. $\lim\limits_{x\to 2}\frac{(x-2)^2}{\arcsin^2(x-2)}$ 4. $\lim\limits_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{2x}$ 5. $y=x\cdot 3^x,\ \ x_0=0$ 6. $y=arccos\sqrt{1-x^2}$ 7. $y=\ln(x+\sqrt{1+x^2})$ 8. $y=(1-cosx)^{tgx}$ 9. $x^3+y^3=3^{xy}$ 10. $\begin{cases}x=\ln tgt\\ y=\frac{1}{\sin^2 t}\end{cases}$ 11. $y=x+\frac{\ln x}{x}$
9-variant 1. $\lim\limits_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}}$ 2. $\lim\limits_{x\to e}\frac{\sqrt{e}-\sqrt{2e-x}}{x-e}$ 3. $\lim\limits_{x\to\infty}\frac{arctg^2 3x}{9x^2}$ 4. $\lim\limits_{x\to\infty}\left(\frac{2x-4}{2x}\right)^{-3x}$ 5. $y=\frac{x}{sinx},\ \ x_0=\frac{\pi}{2}$ 6. $y=arccos\frac{x^2-4}{\sqrt{x^2+16}}$ 7. $y=\sqrt[3]{x^3+7x}$ 8. $y=(x^3+4)^{tgx}$ 9. $e^{x+y}=arctgx\cdot y$	10-variant 1. $\lim\limits_{n\to\infty}\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{(2n-1)(2n+1)}$ 2. $\lim\limits_{x\to\frac{1}{2}}\frac{\sqrt{6x+1}-\sqrt{2x+3}}{\sqrt[3]{x+7.5}-2}$ 3. $\lim\limits_{x\to -1}\frac{4x+4}{arctg2(x+1)}$ 4. $\lim\limits_{x\to\infty}\left(\frac{x-7}{x+1}\right)^{4x-2}$ 5. $y=\ln\frac{x+1}{x+2},\ \ x_0=1$ 6. $y=\frac{2}{3}\sqrt{(arctge^x)^3}$ 7. $y=\frac{2x^2-x-1}{\sqrt[3]{2+4x}}$ 8. $y=(x^2-1)^{sinx}$

10. $\begin{cases} x=\sqrt[3]{1-t^2} \\ y=tg(1+t) \end{cases}$ 11. $y=x-\ln(1+x^2)$	9. $\sqrt{x^2+y^2}-tgxy=0$ 10. $\begin{cases} x=\frac{3t^2+1}{3t^2} \\ y=\sin t^3 \end{cases}$ 11. $y=\frac{x^2}{x^2-x+1}$
11-variant 1. $\lim\limits_{n\to\infty}\frac{(n+1)^5-(n-1)^5}{(2n+1)^5-(2n-1)^5}$ 2. $\lim\limits_{x\to 0.25}\frac{\sqrt{4x}-\sqrt{2-4x}}{4x^2+2x-0.75}$ 3. $\lim\limits_{x\to\infty}\frac{\cos\frac{\pi x}{12}-\sin\frac{\pi x}{12}}{\cos\frac{\pi x}{6}}$ 4. $\lim\limits_{x\to\infty}\left(\frac{2-3x}{5-3x}\right)'$ 5. $y=\frac{x}{x^2+1},\quad x_0=0$ 6. $y=\left(ln\sqrt[3]{e^{2x}+1}\right)^4$ 7. $y=x^2\cdot lnx$ 8. $y=sinx^{5e^4}$ 9. $ysinx=\cos(x-y)$ 10. $\begin{cases} x=\ln(t+\sqrt{t^2+1}) \\ y=\sqrt{t^2+1} \end{cases}$ 11. $y=3x^4-4x^3+1$	12-variant 1. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{2-3n+n^3}+(\sqrt{n}-1)^2}{\sqrt{2n^2+n-1}}$ 2. $\lim\limits_{x\to 0.(3)}\frac{\sqrt{5-3x}-\sqrt{3,(3)+2x}}{x^2-2x+\frac{5}{9}}$ 3. $\lim\limits_{x\to a}\frac{\cos^2 x-\cos^2 a}{x-a}$ 4. $\lim\limits_{x\to\infty}\left(\frac{4x-1}{4x+1}\right)^{2x}$ 5. $y=\frac{x}{cosx},\quad x_0=0$ 6. $y=\ln(\cos^2 x+\sqrt{1+\cos^2 x})$ 7. $y=x\cdot\sqrt{x^2-8}$ 8. $y=(lnx)^{3^x}$ 9. $xy=\sin(x+y)$ 10. $\begin{cases} x=\frac{3t^2+1}{3t^2} \\ y=\sin t^3 \end{cases}$ 11. $y=x^2-2lnx$
13-variant 1. $\lim\limits_{n\to\infty}\frac{\sqrt[4]{n^5+2}-\sqrt[3]{n^2+1}}{\sqrt[5]{n^4+2}-\sqrt{n^3+1}}$ 2. $\lim\limits_{x\to -3}\frac{x^3+27}{\sqrt{12+x}-\sqrt{3-2x}}$ 3. $\lim\limits_{x\to\infty}\frac{\sin^2 x-\sin^2 a}{x-a}$ 4. $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^{2x+3}$ 5. $y=\frac{sinx}{x},\quad x_0=\frac{\pi}{2}$ 6. $y=\ln^3(\arccos\frac{1}{\sqrt{x}})$ 7. $y=arctg\,\frac{1-2x}{\sqrt{1-x-x^2}}$	14-variant 1. $\lim\limits_{n\to\infty}\frac{(n+2)!}{(n_1)!+n!}$ 2. $\lim\limits_{x\to\infty}\frac{2x^2+7x+5}{\sqrt{8-x}-\sqrt{5-4x}}$ 3. $\lim\limits_{x\to 0}\frac{cos\alpha x-cos\beta x}{x^2}$ 4. $\lim\limits_{x\to\infty}\left(\frac{x-1}{x}\right)^{2-3x}$ 5. $y=sin2\sqrt{x},\quad x_0=\frac{\pi^2}{4}$ 6. $y=\ln(arcsin\sqrt{1-e^{2x}})$ 7. $y=\frac{1}{\sqrt{2}}arctg\,\frac{x}{\sqrt{1-x^3}}$ 8. $y=x^{e^{cosx}}$

8. $y = (x^3 + 1)^{cosx}$

9. $xy = \sin(x + y)$

10. $\begin{cases} x = ln\sqrt{1+t} \\ y = \cos\frac{1}{2} \end{cases}$

11. $y = x^3 e^{-\frac{x^2}{2}}$

9. $x + y = sinxy$

10. $\begin{cases} x = \frac{3at}{1+t^2} \\ y = \frac{3at^2}{1+t^2} \end{cases}$

11. $y = \frac{x^3}{1-x^2}$

15-variant

1. $\lim\limits_{n\to\infty} \frac{n!-(n+1)!}{n!+(n+1)!}$

2. $\lim\limits_{x\to 2.5} \frac{\sqrt{6.5+x}-\sqrt{11.5-x}}{\sqrt[3]{6-2x}-1}$

3. $\lim\limits_{x\to a} \frac{cosx-cosa}{sinx-sina}$

4. $\lim\limits_{x\to 0} \left(\frac{x+3}{x}\right)^{-5x}$

5. $y = \ln(x^2 + 1), \;\; x_0 = 1$

6. $y = \log_a \frac{1}{\sqrt{1-x^4}}$

7. $y = \arcsin\sqrt{\frac{x}{x+1}}$

8. $y = (lnx + 1)^{\sin 2x}$

9. $xcosx - \sin(y^2) = 0$

10. $\begin{cases} x = \sqrt{2t - t^2} \\ y = \arcsin(t-1) \end{cases}$

11. $y = \frac{x^2-x-1}{x^2-2x}$

16-variant

1. $\lim\limits_{n\to\infty} \frac{(2n-1)^4-(2n+1)^4}{(3n-1)^4+(3n+1)^4}$

2. $\lim\limits_{x\to 2} \frac{\sqrt[3]{21+3x}-\sqrt[3]{31-2x}}{\sqrt{x+2}-\sqrt{6-x}}$

3. $\lim\limits_{x\to\frac{\pi}{2}} \left(\frac{\pi}{2} + x\right)^2 tgx$

4. $\lim\limits_{x\to\infty} \left(\frac{x+3}{x-1}\right)^{x-4}$

5. $y = \ln^2 x + x^2, \;\; x_0 = 1$

6. $y = \frac{1}{2} arctg \frac{e^x-3}{2}$

7. $y = arctg \frac{x-1}{2}$

8. $y = x^{3^x} \cdot 2^x$

9. $xy = 3^x + x \cdot e^y$

10. $\begin{cases} x = ctg(2e^x) \\ y = lntg(e^x) \end{cases}$

11. $y = (x-1)x^2$

17-variant

1. $\lim\limits_{n\to\infty} \frac{3^{2n+1}-3^{2n-1}}{9^{n+1}-9^{n-1}}$

2. $\lim\limits_{x\to -2} \frac{3x^2+5x+8}{\sqrt[3]{11x-5}+\sqrt[3]{19-4x}}$

3. $\lim\limits_{x\to\infty} (x+2)^2 tg\frac{\pi x}{4}$

4. $\lim\limits_{x\to\infty} \left(\frac{x-7}{x}\right)^{2x+1}$

5. $y = xlnx, \;\; x_0 = 1$

6. $y = \frac{4+3x^2}{\sqrt[5]{2+x^4}}$

7. $y = 3^{\sqrt{x+1}}$

18-variant

1. $\lim\limits_{n\to\infty} \frac{\sqrt[5]{2-3n^4+n^5}-\sqrt[3]{2n^3-1}}{\sqrt{n-2+4n^2}-\sqrt[3]{1-n^3}}$

2. $\lim\limits_{x\to 1} \frac{\sqrt{19-3x}-\sqrt{9+7x}}{\sqrt[3]{7x+1}-2}$

3. $\lim\limits_{x\to 1} (x-1)^2 ctg\pi x$

4. $\lim\limits_{x\to\infty} \left(\frac{2x+1}{2x-1}\right)^{x+2}$

5. $y = \frac{x}{lnx}, \;\; x_0 = e$

6. $y = arctg^3(e^x - e^{-x})$

7. $y = x\sqrt{4-x^2} + 4\arcsin\frac{x}{2}$

8. $y = (cosx)^{lnx}$

9. $lnx + e^{\frac{y}{x}} = a^2$

10. $\begin{cases} x = arccos\frac{1}{\sqrt{1+t^2}} \\ y = arcsin\frac{t}{\sqrt{1+t^2}} \end{cases}$

11. $y = \frac{(x-2)^2}{x+1}$

8. $y = (lnx)^{x^2-1}$

9. $\sin(xy) + \cos(xy) = tg(x + y)$

10. $\begin{cases} x = arctge^{\frac{1}{2}} \\ y = \sqrt{e^x + 1} \end{cases}$

11. $y = \ln(x^2 + 1)$

19-variant

1. $\lim\limits_{n\to\infty}\left[\frac{1+3+5+7+\cdots+(2n-1)}{n+1} - \right.$ $\left. - \frac{2n+1}{2}\right]$

2. $\lim\limits_{x\to-5}\frac{x^2+125}{\sqrt{18+3x}-\sqrt{8+x}}$

3. $\lim\limits_{x\to\infty}(\pi - x)ctgx$

4. $\lim\limits_{x\to\infty}\left(\frac{x}{x-3}\right)^{x-5}$

5. $y = \sqrt{x^2 + 3}, \;\; x_0 = 1$

6. $y = arccos\sqrt{1 + 2x^3}$

7. $y = x^4 \cdot ln3x$

8. $y = (sinx)^{arctgx}$

9. $sinxy = y^2$

10. $\begin{cases} x = \arccos\frac{1}{t} \\ y = \sqrt{t^2 - 1} \end{cases}$

11. $y = \frac{x^2+6}{x^2+1}$

20-variant

1. $\lim\limits_{n\to\infty}\frac{1+5+9+\cdots+(4n-3)}{n^2-3}$

2. $\lim\limits_{x\to2}\frac{\sqrt[3]{11-5x}+\sqrt[3]{7-4x}}{0.5x^2+x-4}$

3. $\lim\limits_{x\to3}(x^2 - 9)tg\frac{\pi}{6}x$

4. $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+4}\right)^{3x-1}$

5. $y = \sqrt{x} + \frac{1}{x}, \;\; x_0 = 1$

6. $y = x^2 \cdot sin2x$

7. $y = 3^x \cdot cos2x$

8. $y = (sinx)^{\sqrt{x}}$

9. $arctg\frac{y}{x} = x$

10. $\begin{cases} x = \ln^3 cost \\ y = e^{sint} \end{cases}$

11. $y = (x - 1)e^{3x-1}$

21-variant

1. $\lim\limits_{n\to\infty}\left(\frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \cdots + \frac{1}{(3n+1)(3n+4)}\right)$

2. $\lim\limits_{x\to2}\left(\frac{1}{\sqrt{x}-\sqrt{2}} - \frac{2\sqrt{2}}{x-2}\right)$

3. $\lim\limits_{x\to-2}(x^2 - 4)ctg\frac{\pi}{2}x$

4. $\lim\limits_{x\to\infty}\left(\frac{x+5}{x}\right)^{3x+4}$

22-variant

1. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^2-1}-\sqrt{(n-1)}+1}{\sqrt[3]{n^2-1}+\sqrt{n-1}+1}$

2. $\lim\limits_{x\to\frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}$

3. $\lim\limits_{x\to1}(x^3 - 1)tg\frac{\pi}{2}x$

4. $\lim\limits_{x\to0}\left(\frac{x+2}{x}\right)^{3-2x}$

5. $y = arcsinx - x, \;\; x_0 = 0$

5. $y = arctg^2 x,\ \ x_0 = 1$ 6. $y = lnarctg\sqrt{e^x + sin3x}$ 7. $y = \frac{2(3x^3+4x^2-x-2)}{15\sqrt{1-x}}$ 8. $y = (tgx)^{4e^x}$ 9. $cosx = x + y$ 10. $\begin{cases} x = \sqrt{2t - t^2} \\ y = \sqrt[3]{(1-t)^2} \end{cases}$ 11. $y = \frac{x^3}{x^4-1}$	6. $y = arctg(x^2 \cdot \cos\frac{1}{x})$ 7. $y = \cos(ctg2x) - \frac{1}{16}\frac{\cos^2 8x}{sin16x}$ 8. $y = (x^7 + 1)^{tgx}$ 9. $\sqrt{x^2 + y^2} = \arccos\frac{y}{x}$ 10. $\begin{cases} x = \arccos(sint) \\ y = \arcsin(cost) \end{cases}$ 11. $y = \frac{x^5}{x^4-1}$
23-variant 1. $\lim\limits_{n\to\infty}\frac{2\cdot 2^n+4^n+8^n}{3\cdot 8^n-4^n+2^n}$ 2. $\lim\limits_{x\to\frac{1}{2}}\frac{4x^2-1}{\sqrt{2x+5}-\sqrt{1-6x}}$ 3. $\lim\limits_{x\to 0}(\frac{1}{sin3x} - \frac{1}{tg3x})$ 4. $\lim\limits_{x\to\infty}\left(\frac{1-x}{2-x}\right)^{3x}$ 5. $y = \frac{x}{2} + \frac{2}{x},\ \ x_0 = 1$ 6. $y = \ln^2(x^3 + cos3x)$ 7. $y = \frac{2}{3}arcsin\sqrt{\frac{x^2-1}{3}}$ 8. $y = \left(\sqrt{x+5}\right)^{tg\frac{x^3}{2}}$ 9. $3^y + 2x = 2^{x+y}$ 10. $\begin{cases} x = arctge^{\frac{1}{2}} \\ y = \sqrt{e^x + 1} \end{cases}$ 11. $y = \ln(x^2 - 2x + 6)$	24-variant 1. $\lim\limits_{n\to\infty}(\sqrt{2n^2-1} - \sqrt{2n^2+1})$ 2. $\lim\limits_{x\to 2}\frac{\sqrt[3]{x+6}-\sqrt[3]{19-5.5x}}{x^2-3x+2}$ 3. $\lim\limits_{x\to\frac{\pi}{2}}\frac{1}{cosx} - \frac{1}{ctgx}$ 4. $\lim\limits_{x\to\infty}\left(\frac{3x+4}{3x}\right)^{-2x}$ 5. $y = x^2 + arcctgx,\ \ x_0 = 0$ 6. $y = \ln^3(e^{2x} + cos\sqrt{x})$ 7. $y = tg(\ln\frac{1}{3}) + \frac{1}{4}\frac{sin4x}{4cos8x}$ 8. $y = (x^3 - 4)^{sin2x}$ 9. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ 10. $\begin{cases} x = tcost - 2sint \\ y = tsint + 2cost \end{cases}$ 11. $y = \frac{e^{2x+1}}{e^x}$
25-variant 1. $\lim\limits_{n\to\infty}\frac{(3+2n+n^2)^5-(2n^2-1)^5}{(2n-1)^{10}}$ 2. $\lim\limits_{x\to 4}\frac{\sqrt{x+1}-\sqrt{8-x}}{\sqrt{2x+1}-\sqrt{17+8x}}$ 3. $\lim\limits_{x\to a}\frac{\cos^2 x-\cos^2 a}{x^2-a^2}$	26-variant 1. $\lim\limits_{n\to\infty}(\frac{1}{2\cdot 5} + \frac{1}{3\cdot 5} + \cdots + \frac{1}{(n+1)(2n+3)})$ 2. $\lim\limits_{x\to -2}\frac{8+x^3}{\sqrt{4x+9}-\sqrt{17+8x}}$

4. $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+4}\right)^{-x}$

5. $y=\frac{x^2}{x^2+1},\quad x_0=1$

6. $y=\frac{1}{4}\ln\frac{\sqrt[4]{1+x^4}+x}{\sqrt[4]{1+x^4}-x}$

7. $y=\frac{1}{18}ln\frac{4+\sqrt{8}tg\frac{x}{2}}{4-\sqrt{8}tg\frac{x}{2}}$

8. $y=a^{2x+3}$

9. $e^x\cdot sinx=e^{-x}\cdot cosy$

10. $\begin{cases}x=ctg(2e^x)\\ y=\ln tg(e^x)\end{cases}$

11. $y=\frac{5x^4+3}{x}$

3. $\lim\limits_{x\to\beta}\frac{\sin^2x-\sin^2\beta}{x^2-\beta^2}$

4. $\lim\limits_{x\to0}\left(\frac{3x+4}{2x+5}\right)^{x+1}$

5. $y=\sqrt{x^2+4x+1},\quad x_0=0$

6. $y=\ln(tg\left(\frac{x}{2}+\frac{\pi}{4}\right))$

7. $y=arccos\frac{x^2-4}{\sqrt{x^4+16}}$

8. $y=(\sin^2x)e^x$

9. $x^4+y^4=x^2y^3$

10. $\begin{cases}x=lnctgx\\ y=\frac{4}{\cos^2t}\end{cases}$

11. $y=\frac{5x}{4-x^2}$

27-variant

1. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^2+2n-1}}{\sqrt{n-10}}$

2. $\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

3. $\lim\limits_{x\to h}\frac{\sqrt[3]{x+h}-2\sin(x-h)-sin2h}{x^2-h^2}$

4. $\lim\limits_{x\to\infty}\left(\frac{1+2x}{3+2x}\right)^{-x}$

5. $y=\sqrt{sin2x},\quad x_0=\frac{\pi}{4}$

6. $y=arctg\left(tg\frac{x}{2}+1\right)-lncos\sqrt{x}$

7. $y=ln\sqrt[3]{1-sin\frac{5}{x}}$

8. $y=(x^3+4)^{3x}$

9. $tg\frac{2}{y}=ctgxy$

10. $\begin{cases}x=\sqrt{1-t^2}\\ y=\frac{t}{\sqrt{1-t^2}}\end{cases}$

11. $y=\frac{4-2x}{1-x^2}$

28-variant

1. $\lim\limits_{n\to\infty}\frac{\sqrt[4]{n^3+5}+\sqrt[3]{n^2-5}}{\sqrt[3]{n^2+1}+\sqrt[4]{16n^3+1}}$

2. $\lim\limits_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2},\quad (0<b<a)$

3. $\lim\limits_{x\to0}\frac{\cos(2x+a)-\cos(2x-a)}{x^2}$

4. $\lim\limits_{x\to0}\left(\frac{x}{x-1}\right)^{3-2x}$

5. $y=\sqrt[3]{x^2+1},\quad x_0=0$

6. $y=\log_a\frac{1}{\sqrt{3+x^2}}-\ln arcsin\sqrt{1+e^{2x}}$

7. $y=\log_a cos\frac{2x+3}{2x-1}$

8. $y=(2x^3+1)^{cos2x}$

9. $xarcsiny=yarcsinx$

10. $\begin{cases}x=ln\sqrt{\frac{1-t}{1+t}}\\ y=\sqrt{1-t^2}\end{cases}$

11. $y=x^2+\frac{1}{x^2}$

29-variant 1. $\lim\limits_{n\to\infty}\frac{(n+1)^5+(n-1)^4+(2n-3)^3}{(2n-1)^5+(2n+1)^4+(2n+3)^3}$ 2. $\lim\limits_{x\to-1}\frac{\sqrt{8-x}-\sqrt{15+6x}}{2x^2+5x+3}$ 3. $\lim\limits_{x\to0}\frac{\sqrt{2}-\sqrt{1+cosx}}{\sin^2 x}$ 4. $\lim\limits_{x\to\infty}\left(\frac{4-2x}{3x+2}\right)^{x-2}$ 5. $y=\sqrt[3]{\frac{1}{x}+7},\ \ x_0=1$ 6. $y=x-e^{-x}\cdot arcsine^{x}+x^2$ 7. $y=arcctg\,\frac{1-3x}{\sqrt{1+x+x^2}}$ 8. $y=(e^x+1)^{x^2-1}$ 9. $xy-\sin(x+y)=\cos(x-y)$ 10. $\begin{cases}x=arcsin\sqrt{1-t^2}\\ y=(\arccos t)^{-2}\end{cases}$ 11. $y=xlnx$	30-variant 1. $\lim\limits_{n\to\infty}\frac{(n+1)^5+(n-1)^4+(2n-3)^3}{(2n-1)^5+(2n+1)^4+(2n+3)^3}$ 2. $\lim\limits_{x\to-1}\frac{\sqrt{8-x}-\sqrt{15+6x}}{2x^2+5x+3}$ 3. $\lim\limits_{x\to0}\frac{\sqrt{2}-\sqrt{1+cosx}}{\sin^2 x}$ 4. $\lim\limits_{x\to\infty}\left(\frac{4-2x}{3x+2}\right)^{x-2}$ 5. $y=\sqrt[3]{\frac{1}{x}+7},\ \ x_0=1$ 6. $y=x-e^{-x}\cdot arcsine^{x}+x^2$ 7. $y=arcctg\,\frac{1-3x}{\sqrt{1+x+x^2}}$ 8. $y=(e^x+1)^{x^2-1}$ 9. $xy-\sin(x+y)=\cos(x-y)$ 10. $\begin{cases}x=arcsin\sqrt{1-t^2}\\ y=(\arccos t)^{-2}\end{cases}$ 11. $y=xlnx$

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MUNDARIJA

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