Branislav Hučko; Roland Jančo

Introduction to Mechanics of Materials: Part I





Roland Jančo & Branislav Hucko

Introduction to Mechanics of Materials Part I

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A	Area
b	width
B.C.	buckling coefficient
D	diameter
E	modulus of elasticity, Young's modulus
f.	shearing factor
F	external force
F.S.	factor of safety
G	modulus of rigidity
h	height
24	second moment, or moment of inertia, of the area A respect to the z or y axis
J.,	polar moment of inertia of the area A
L	length
DL	elongation of bar
м	bending moment, couple
N	normal or axial force
Q, Q.	first moment of area with respect to the z or y axis
r.	radius of gyration of area A with respect to the z axis
R	radius
R	reaction at point i
5	length of centreline
Т	torque
t	thickness
ΔΤ	change of temperature
ы	strain energy density
U	strain energy
V	volume
V	transversal force
W	uniform load
y(x)	deflection
A	area bounded by the centerline of wall cross-section area
a	coefficient of thermal expansion (in chapter 2)
۵	parameter of rectangular cross-section in torsion
Y	shearing strain
8	stmin
•	angle of twist
0,	slope at point i
τ	thearing stress

List Of Symbol

Introduction to Mechanics of Materials: Part I

т о о о о о allowable shearing stress stress or normal stress allowable normal stress maximum normal stress von Misses stress normal or axial stress

Preface

This book presents a basic introductory course to the mechanics of materials for students of mechanical engineering. It gives students a good background for developing their ability to analyse given problems using fundamental approaches. The necessary prerequisites are the knowledge of mathematical analysis, physics of materials and statics since the subject is the synthesis of the above mentioned courses.

The book consists of six chapters and an appendix. Each chapter contains the fundamental theory and illustrative examples. At the end of each chapter the reader can find unsolved problems to practice their understanding of the discussed subject. The results of these problems are presented behind the unsolved problems.

Chapter 1 discusses the most important concepts of the mechanics of materials, the concept of stress. This concept is derived from the physics of materials. The nature and the properties of basic stresses, i.e. normal, shearing and bearing stresses; are presented too.

Chapter 2 deals with the stress and strain analyses of axially loaded members. The results are generalised into Hookes law. Saint-Venant's principle explains the limits of applying this theory.

In chapter 3 we present the basic theory for members subjected to torsion. Firstly we discuss the torsion of ciscular members and subsequently, the torsion of non-circular members is analysed.

In chapter 4, the largest chapter, presents the theory of beams. The theory is limited to a member with at least one plane of symmetry and the applied loads are acting in this plane. We analyse stresses and strains in these types of beams.

Chapter 5 continues the theory of beams, focusing mainly on the deflection analysis. There are two principal methods presented in this chapter: the integration method and Castiglianos theorem.

Chapter 6 deals with the buckling of columns. In this chapter we introduce students to Euler's theory in order to be able to solve problems of stability in columns.

In closing, we greatly appreciate the fruitful discussions between our colleagues, namely prof. Pavel Elesztős, Dr. Michal Čekan. And also we would like to thank our reviewers' comments and suggestions.

> Roland Jančo Branislav Hučko

1 Introduction – Concept of stress

1.1 Introduction

The main objective of the mechanics of materials is to provide engineers with the tools, methods and technologies for

- analysing existing load-bearing structures;
- designing new structures.

Both of the above mentioned tasks require the analyses of stresses and *deformations*. In this chapter we will firstly discuss the stress.

1.2 A Short Review of the Methods of Statics



Fig. 1.1

Let us consider a simple trues structure, see Fig. 1.1. This structure was originally designed to carry a load of 15kN. It consists of two rods; BC and CD. The rod CD has a circular cross-section with a 30-mm diameter and the rod BC has a rectangular cross-section with the dimensions 20×80 mm. Both rods are connected by a pin at point C and are supported by pins and brackets at points B and D. Our task is to analyse the rod CD to obtain the answer to the question: is rod CD sufficient to carry the load? To find the answer we are going to apply the methods of statics. Firstly, we determine the corresponding load acting on the rod CD. For this purpose we apply the joint method for calculating axial forces n each rod at joint C, see Fig. 1.2.

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Introduction - Concept of stress

(1.1)



$$\sum F_x = 0$$
 $F_{BC} - F_{CD} = 0$

$$\sum F_{v} = 0 \qquad F_{CD} \frac{3}{r} - 15 \, kN = 0$$

Solving the equations (1.1) we obtain the forces in each member: $F_{ac} = 20 \ kN$, $F_{co} = 25 \ kN$. The force F_{ac} is compressive and the force F_{co} is tensile. At this moment we are not able to make the decision about the safety design of rod CD.

Secondly, the safety of the rod BC depends mainly on the material used and its geometry. Therefore we need to make observations of processes inside of the material during loading.



Fig. 1.3

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Introduction - Concept of stres





Pp. 1.5

Let us consider a crystalline mesh of rod material. By detaching two neighbour atoms from the crystalline mesh, we can make the following observation. The atoms are in an equilibrium state, see Fig. 1.3(a). Now we can pull out the right atom from its equilibrium position by applying external force, see Fig. 1.3(b). The applied force is the action force. Due to Newton's first law a reaction force is pulling back on the atom to the original equilibrium. During loading, the atoms find a new equilibrium state. The action and the reaction are in equilibrium too. If we remove the applied force, the atom will go back to its initial position, see Fig. 1.3(a). If we push the right atom towards the left atom, we will observe a similar situation: see Fig. 1.3(c). Now we can build the well-known diagram from the physics of materials internal force versus interatomic distance, see Fig. 1.4. From this diagram we can find the magnitudes of forces in corresponding cases. Now we can extend our observation to our rod *CD*. For simplicity let us draw two parallel layers of atoms inside the rod considered, see Fig. 1.5. After applying the force of the external load on *CD* we will observe the elongation of the rod. In other words, the internal reaction forces will result between two neighbouring atoms will increase. Then due to Newton's first law the internal reaction forces will result between two neighbouring atoms will increase.

$$\sum_{i=1}^{N} F_{i} = F_{cp}$$
 or \sum internal forces = external applied force (1.2)

The next task is to determine the internal forces. Considering the continuum approach we can replace equation (1.2) with the following one:

Resultant of internal forces = external applied force



Pip. 1.6

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(1.3)

Introduction - Concept of stress

The resultant can be determined by applying the method of acction. Passing the section at some arbitrary point Q we get two portions of the red: CQ and DQ, see Fig. 1.6. Since force $F_{c0} = 25$ kN must be applied at point Q for both portions to keep them in equilibrium, we can conclude that the resultant of internal forces of 100 kN is produced in the red CD, when a load of 15kN is applied at C.



The above mentioned method of section is a very helpful tool for determining all internal forces. Let us now consider the arbitrary body subjected to a load. Dividing the body into two portions at an arbitrary point Q, see Fig. 1.7, we can define the positive outgoing normal n^+ , the normal force $N_{(X)}$ is the force component in the direction of positive normal. The force component derived by turning the positive normal clockwise about $\frac{\pi}{2}$ at Q is known as the shear force $V_{(X)}$, the moment $M_{(X)}$ about the z-axis defines the bending moment (the positive orientation will be explain in Chapter 4). The moment $T_{(X)}$ defines the torque with a positive orientation according to the right-hand rule.



Ro. 1.8

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For assessing the safety of rod *CD* we need to ask material scientists for the experimental data about the materials response. When our rod is subjected to tension, we can obtain the experimental data from a simple tensile test. Let us arrange the following experiments for the rod made of the same material. The output variables are the applied force and the elongation of the rod, i.e. the force vs. elongation diagram the first test is done for the rod of length *L*, and cross-sectional area **A**, see Fig 1.8 (a). The output can be plotted in Fig 1.8 (d), seen as curve number 1. For the second test we now define the rod to have a length of 2L while all other parameters remain, see Fig. 1.8 (b). The result is represented by curve number 2, see Fig. 1.8 (d). It is only natural that the total elongation is doubled for the same load level. For the third test we keep the length parameter L but increase the cross-sectional area to 2A. The result are represented by curve number 3, see Fig. 1.8 (d). The conclusion of these three experiments is that the load vs. elongation diagram is not as useful for designers as one would initially expect. The secults are very sensitive to geometrical parameters of the samples. Therefore we need to exclude the geometrical sensitivity from experimental data.

1.3 Definition of the Stresses in the Member of a Structure

The results of the proceeding section represent the first necessary step in the design or analysing of structures. They do not tell us whether the structure can support the load safely or not. We can determine the distribution functions of internal forces along each member. Applying the method of section we can determine the resultant of all elementary internal forces acting on this section, see Fig. 1.9. The average intensity of the elementary force ΔN over the elementary area ΔA is defined as $\Delta N/\Delta A$. This ratio represents the internal force per unit area. Thus the intensity of internal force at any arbitrary point can be derived as



Fig. 1.9

intensity =
$$\lim_{\Delta A \to 0} \frac{\Delta N}{\Delta A} = \frac{dN}{dA}$$

(1.4)

Introduction Concept of stress

(16)

Whether or not the rod will break under the given load clearly depends upon the ability of the insterial to withstand the corresponding value, see the above mentioned definition, of the distributed internal forces. It is clear that this depends on the applied load F_{CD} , the cross-section area A and on the material of the rod considered.

The internal force per unit area, or the intensity of internal forces distributed over a given cross-sectional area, is called sizes. The stress is denoted by the Greek letter sigma σ . The unit of stress is called the Fascal which has the value N/m^2 . Then we can rewrite equation (1.4) into

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta W}{\Delta A} = \frac{dW}{dA}$$
(1.5)

The positive sign indicates tendle stress in a member or that the member is in tendion. The asystem sign of stress indicates compressive stress in a member or that the member is subjected to compression

The equation (1.5) is not so convenient to use in engineering design so solving for this equation we get

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 $N = \int \sigma dA$

.7)

If we apply Saint Venant's principle, see Section 2.6 for more details, we can assume the uniform stress distribution function over the cross-section, except in the immediate vicinity of the loads points of application, thus we have

$$A = \sigma \int dA = \sigma A \qquad \text{or} \qquad \sigma = \frac{w}{A} \qquad (1)$$

Fig. 1.18

A graphical representation is presented in Fig. 1.10. If an internal force N was obtained by the section passed perpendicular to the member axis, and the direction of the internal force N coincides with the member axis, then we are talking about axially loaded members. The direction of the internal force N also determines the direction of stress σ . Therefore we define this stress σ as the normal stress. Thus formula (1.7) determines the mormal stress in the axially loaded member.

From elementary statics we get the resultant N of the internal forces, which then must be applied to the centre of the cross-section under the condition of uniformly distributed stress. This means that a distribution of stress is possible only if the action line of the applied loads passes through the centre of the section considered, see Fig. 1.11. Sometimes we this type of loading is known as centric loading. In the case of an eccentrically loaded member, see Fig. 1.12, this condition is not satisfied, therefore the stress distribution function is not uniform. The explanation will be done in Chapter 4. The normal force $N_C = F$ and the moment $M_C = Fd$ are the internal forces obtained through the method of section.

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introduction - Concept of stress

Basic Stresses (Avdal, Normal, Shearing and Bearing stress) 1.4



In the previous Section we discussed the case when the resultant of internal forces and the resulting str normal to the cross-section are considered. Now let us consider the cutting process of material in aciasors, are Fig. 1.13. The applied load F is transversal to the axis of the member. Therefore the loss is called the transversal load. Thus we have a physically different stress. Let us pass a section the point C between the application points of two forces, see Fig. 1.14 (a). Detaching portion DC form member we will get the diagram of the portion DC shown in Fig. 1.14(b). The zero valued inte forces are excluded. The resultant of internal forces is only the shear force. It is placed perpendicula forces are excluded. The resultant of internal forces is only the state more an applied force. The corresponding stress is calk the normal or consider the axially loaded member CD, see Fig. 1.16. If we pass the section at any arbitrary the member axis is the section and is equal to the applied lotte. The contraption and in equal to the applied lotte. The contraption and in equal to the applied lotte. The contraption and in equal to the applied lotter and any arbitrary section, we will get the to the normal stress, we cannot assume that the shearing stress is uniform over the cross-sectionto the normal stress, we cannot assume that the shearing stress is uniform over the cross section, publicities with the axial force R is P = P. This axial force P represents the resultant of internal forces shearing stress:

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The presented case of cutting is known as the shear.

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The cutaway effect can be commonly found in bolts, screws, pins and rivets used to connect various structural components, see Fig. 1.15(a). Two plates are subjected to the tansile force F. The corresponding cutting stress will develop in plane CD. Considering the method of section in plane CD, for the top portion of the rivet, see Fig. 1.15(b), we obtain the shearing stress according to formula (1.9)



intal now we have discussed the application of section in a perpendicular direction to the member axis. te body diagram shown in Fig. 1.17. From the free body diagram we see that the applied force F is in ting in this section. The components of axial force are

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 $N = P \cos \theta$ and $V = P \sin \theta$

(1.11



$$\sigma = \frac{N}{A_0} = \frac{P\cos\theta}{\frac{K_0}{\cos\theta}} = \frac{F}{A_0}\cos^2\theta \tag{112}$$

$$\tau_{ave} = \frac{V}{A} = \frac{P \sin \theta}{A_{a}} = \frac{1}{A_{a}} \sin \theta \cos \theta$$
(1.13)

For the perpendicular section, when $\theta = 0$, we get $\sigma = \sigma_{max} = \frac{1}{\tau_{ave}}$ and $\tau_{ave} = 0$. These results correspond to the ones we found earlier. In the point of view of mathematics, the magnitudes of stresses depend upon the orientation of the section.





The resultant stress from the normal and shearing stress components is called the axial stress (the stress in the direction of the axia) and it is denoted as p; see Fig. 1.18. Then using elementary mathematics we get

$$p = \sqrt{\sigma^2 + \tau_{ave}^2} \tag{114}$$

The exact mathematical definition of the axial stress is the same as previously defined stress types, i.e.

$$p = \lim_{d,A \to 0} \frac{\Delta P}{\Delta A} = \frac{dP}{dA}$$
(1.15)

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Introduction - Concept of stress



Fig. 5.19



Pig. 1.30

Failage, bolts, or screws have a lateral contact within the connected member, see Fig. 1.19. They create the stress in the connected member along the bearing surface of the contact surface. For example let in consider the bolt JK connecting two plates 3 and C, which are subjected to shear. see Fig. 1.19. The holi shank exerts a force P on the plate B which is equal to the applied force F. The force P represents the resultant of all elementary forces distributed over the half of the cylindrical hole in plate B, see Fig. 1.20. The diameter of the cylindrical hole is D and the height is t. The distribution function of the atorementioned staesses is very complicated and therefore we usually use the average value of contact ar bearing stress. In this case the average engineering bearing stress is defined as

$$\sigma_b = \frac{F}{A} = \frac{F}{Dt}$$
(1.16)

Application to the Analysis and Design of Simple Structures 1.5

Let us recall the simple truss structure that we discussed in Section 1.2. see Fig. 1.1. Let us now detach rod CD for a more detailed analysis, see Fig. 1.21. The detailed pla connection at point D is presented in Fig 1.22. The following stresses acting in the rod CD can be calculated



Fig. 1.21



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The normal stress in the shank of the rod CD:

The normal force acting in the circular shank is $F_{CD} = 25 \text{ kN}$, the corresponding cross sectional area is $A_{\text{shank}} = \pi \left(\frac{1}{2}\right)^2 = 706.9 \text{ mm}^2$. Then we have

$$\sigma_{shank} = \frac{F_{CD}}{A_{shank}} = \frac{25000 N}{706 N mm^2} = 35.4 MPa.$$

• The normal stresses in the flat end of D:

The normal force acting in the flat end is $P_{CD} = 25 \text{ kN}$ again, the corresponding cross sectional areas are at the section *a-a* $A_{aa} = (50 - 20).30 = 900 \text{ mm}^2$ and at the section *b-b* $A_{bb} = 50.30 = 1500 \text{ mm}^2$. Thus we get

$$\sigma_{\text{max}} = \frac{F_{CP}}{A_{max}} = \frac{25000 N}{100 \text{ mm}^3} = 27.8 MPa \text{ and } \sigma_{bb} = \frac{F_{CP}}{A_{bb}} = \frac{25000 N}{1500 \text{ mm}^3} = 16.7 MPa$$

The shearing stress in the pin connection D:

The shear force acting in the pin is $F_{CD} = 25 \text{ kN}$, the corresponding cross-sectional area $A_{pm} = \pi \left(\frac{2b}{2}\right)^2 = 214.2 \text{ mm}^2$. Then we have

$$\tau_{pln} = \frac{P_{Pll}}{n_{min}} = \frac{25600 N}{314,2805^2} = 79,6 MPa$$

• The bearing stress at D:

The contact force acting in the cylindrical hole is $P_{hearing} = 25 kN_{ext}$, the corresponding cross sectional area in $A_{hearing} = 30.30 = 900 mm^2$. Using formula (1.16) we get

$$\sigma_{lataring} = \frac{\overline{r}_{lataring}}{A_{latering}} = \frac{25000 N}{500 \text{ mm}^3} = 27.8 MPa$$

1.6 Method of Problem Solution and Numerical Accuracy

Every formula previously mentioned and derived has its own validity. This validity predicts the applicatiarea, i.e. the limitations on the applicability. Our solution must be based on the fundamental principlesstatics and mechanics of materials. Every step, which we apply in our approach, must be justified on the basis. After obtaining the results, they must be checked. If there is any doubt in the senalts obtained, we should check the problem formulation, the validity of applied methods, input data (material parameter) boundary conditions) and the accuracy of computations.

The method of problem solution is the step by step solution. This approach consists of the following step

- i. Clear and precise problem formulation. This formulation should contain the given data and indicate what information is required.
- E. Simplified drawing of a given problem, which indicates all essential quantities, which should be included.
- iii Free body diagram to obtaining reactions at the supports.
- in Applying method of section in order to obtain the internal forces and moments.
- g Solution of problem oriented equations in order to determine stresses, strains, and deformations.

Subsequently we have to check the results obtained with respect to some simplifications, for example boundary conditions, the neglect of some structural details, etc.

The numerical accuracy depends upon the following items:

- the accuracy of input data;
- the accuracy of the computation performed.

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For example it is possible that we can get inaccurate material parameters. Let us consider an error of 5% in Young's modulus. Then the calculation of stress contains at least the same error, the explanation can be found in Section 2.5. The accuracy of computation is tightly connected with the computation, method applied. We can apply either the analytical solution or the iterative solution.

1.7 Components of Stress under General Loading Conditions





Until now we have limited the discussion to axially loaded members. Let us generalise the results obtained in the previous sections. Thus we can consider a body subjected to several forces, see Fig. 1.23. To analys the stress conditions created by the loads inside the body, we must apply the method of sections. Let us analyse stresses at an arbitrary point Q. The Euclidian space is defined by three perpendicular plane therefore we will pass three parallel sections to the Euclidian ones through point Q.



27 Download free ellipoits at boolsboon.com Instity we pass a section parallel to the principal plane yz, see Fig. 1.24 and take into account the left to a of the body. This portion is subjected to the applied forces and the resultants of all internal forces these forces replace the effect of the removed part). In our case we have the normal force N_x and the hear force The lower subscript means the direction of the positive outgoing normal. The general hear force V_x has two components in the directions of y and z, i.e. V^y and V_x^z . The superscript indicates the direction of the shear component. For determining the stress distributions over the section we need to define a small area ΔA surrounding point Q, see Fig. 1.24. Then the corresponding internal forces are ΔV_x^- , ΔV_x^z . Recalling the mathematical definition of stress in equations (1.5) and (1.5), we get

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$$\sigma_{x} = \lim_{\Delta A \to 0} \frac{\Delta w_{x}}{\Delta A} \quad \tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta w_{x}^{2}}{\Delta A} \quad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta w_{z}^{2}}{\Delta A} \quad (1.17)$$

These results are presented in Fig.1.25 Remember that the first subscript in σ_x , τ_{xy} and τ_{xz} is used to indicate that the stresses under consideration are exerted on a surface perpendicular to the x axis. The second subscript in the shearing stresses identifies the direction of the component. The same results will be obtained if we apply the same approach for the right side of the body considered, set Fig. 1.26.







Fig.1.38

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Secondly we now pass a section parallel to the principal plane of xz, where we will get the structure components σ_y , τ_{yx} and τ_{yx} in a similar way. Thirdly, passing a section parallel to the principal plane of xy, we can also get the stress components σ_x , τ_{xx} and τ_{yy} by the same way. Thus the stress τ_{xx} at point Q is defined by nine stress components. With respect to statics, it is attaically indetermined problem, since we only have six equilibrium equations.



The relation obtained shows that the y component of the shearing stress exerted on a face perpendicular to the x axis is equal to the x component of the shearing exerted on a face perpendicular to the y axis to the x axis will be obtained for the rest of the moment equilibrium equations, i.e.

$$\Sigma_{ref} = T_{ref}$$
 and $T_{ref} = T_{ref}$ (1.20)

The equations (1.19) and (1.20) represent the shear law The explanation of the shear law is if the shearing stress exerts on any plane, then the shearing stress will also exert on the perpendicular plane to that one. Thus the stress state at any arbitrary point is determined by six stress components: $\sigma_{x}, \sigma_{y}, \sigma_{x}, \tau_{xy}, \tau_{xz}, \tau_{yz}$.

1.8 Design Considerations and Factor of Safety

In the previous sections we discussed the stress analysis of existing structures. In engineering applications we must design with safety as well as economical acceptability in mind. To reach this compromise stress analyses assists us in fulfilling this task. The design procedure consists of the following steps:



To visualise the stress conditions at point Q, we can represent point Q as a small cube, see Fig. 1.7 There are only three faces of the cube visible in Fig. 1.27. The stresses on the hidden parallel faces π equal and opposite of the visible ones. Such a cube must satisfy the condition of equilibrium. Therefor we can multiply the stresses by the face area ΔA to obtain the forces acting on the cube faces. Focusin on the moment equation about the local axis, see Fig. 1.28 and assuming the positive moment in ψ counter-clockwise direction, we have

$$\sum M_x = 0 \qquad \tau_{yx} \Delta A - \tau_{yx} \Delta A \frac{\sigma}{2} + \tau_{xy} \Delta A \frac{\sigma}{2} - \tau_{yx} \Delta A - \frac{\sigma}{2} = 0 \qquad (1.1)$$

we then conclude

 $\tau_{xy} = \tau_{yx}$

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- Determination of the ultimate stress of a material. A cartified laboratory will make material tents in respect to the defined load. For example they can determine the ultimate tensile stress, the ultimate compressive stress and the ultimate shearing stress for a given material, see Fig. 1.29.
- Allowable load and allowable stress, Factor of Safety. Due to any uniorusen loading during the structures operation, the maximum stress in the designed structure can not be equal to the ultimate stress. Usually the maximum stress is less than this ultimate stress. Low stress corresponds to the smaller loads. This smaller loading we call the allowable load or design load. The ratio of the ultimate load to the allowable load is used to define the Factor of Safeth which is:

An alternative of this definition can be applied to stresses

- Selecting the appropriate Factor of Safety. The appropriate Factor of Safety (ES.) for a given design application requires good engineering judgment based on many considerations, such as the following:
 - Type of loading, i.e. static or dynamic or random loading.
 - Variation of material properties, i.e. composite structure of different materials.
 - Type of failure that is expected, i.e. brittle or ductile failure, etc.
 - Importance of a given member, i.e. less important members can be designed with allowed ES.
 - Uncertainty due to the analysis method. Usually we use some simplifications in our analysis.
 - The nature of operation, i.e. taking into account the properties of our surrounding, in example: corrosion properties.

For the majority of structures, the recommended E.S. is specified by structural Standards and othe documents written by engineering authorities.

Stress and Strain – Axial Loading

2.1 Introduction

induction to Mechani s of Materials; Part I

In the previous chapter we discussed the stresses produced in the structures under various conditions, le loading, boundary conditions. We have analyzed the stresses in simply loaded members and we learned how to design some characteristic dimensions of these members due to allowable stress. Another important aspect in the design and analysis of structures are their deformations, and the reasons are very simple. For example, large deformations in the structure as a result of the stress conditions under the applied load should be avoided. The design of a bridge can fulfil the condition for allowable stress but the deformation (in our case deflection) at mid-span may not be acceptable. The deformation analysis is very helpful in the stress determination too, mainly for statically indeterminated problems. Statically it is animed that is structure is a composition of rigid bodies. But now we would like to analyse the structure as a deformable body.

(1.22 2.2 Normal Stress and Strain under Axial Loading



(2.1)

force and elongation can be measured. And we can plot the load va. elongation, nor Fig. 2.2.

As we mentioned in the previous chapter, we would like to avoid plotting geometrical charactering, i.e. cross-sectional area and length. We cannot use such a graph directly to predict the rod elongate of the same material with different dimensions. Let us consider the following examples:



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Let us assume that the rod BC, of length L with constant cross-sectional area A, is hanging on a fig. The state of the rol of the ro point B, see Fig. 2.1. If we apply the load F we can observe an elongation of the rod BC. Both the apple section perpendicular to the axis of the rod (method of sections) applying this method we obtain F/A, see Fig. 2.1. If we apply the same load to the rod of length 21, and the same crossused area A, we will observe an elongation of $2\Delta L$ with the same normal stress $\sigma_x = F/A$, see Fig 2.3. Thus means the deformation is twice as large as the previous case. But the ratio of deformation mer the rod length is the same, i.e. is equal to ALIL. This result brings us to the concept of strain.

> We can now define the normal strain a caused by axial loading as the deformation per unit length of the rod Since length and elongation have the same units, the normal strain is a dimensionless quantity Mathematical, we can express the normal strain by:









This equation is valid only for a rod with constant cross-sectional area. In the case of variable cross sectional area, the normal stress varies over the axis of the rod by $\sigma_x = F/A_{(x)}$ Then we must define the normal strain at an arbitrary point Q by considering a small element of undeformed length Dz. The corresponding Mongation of this element is D(DL), see Fig 2.4. Thus we can define the normal strain at point Q as:

$$I_{\pm} = \lim_{\Delta x \to 0} \frac{\Delta(\Delta L)}{\Delta x} = \frac{\Delta L}{\Delta x}$$
(2.2)

which spin remuts in a dimensionless quantity.

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2.3 Stress-Strain Diagram, Hooke's Law, and Modulus of Elasticity



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As discussed before, plotting load vs. elongation is not useful for engineers and designers due to its strong sensitivity on the sample geometry. Therefore we explained the concepts of stress and strain 13 and Sec. 2.2 in detail. The result is a stress-strain diagram that represents the relationship mean stress and strain. This diagram is an important characteristic of material and can be obtained to conducting a tensile test. The typical specimen can be shown in Fig. 2.5. The cross-sectional area is the cylindrical central portion of the specimen has been accurately determined and two gage marks have been made in this portion at a distance L_{g} from each other. The distance L_{g} is known as the gage angule (or referential length) of the specimen. The specimen is then placed into the test machine seen in Fig. 2.6, which is used for centric load application. As the load F increases, the distance L between gage marks also increases. The distance can be measured by several mechanical gages and both quantities (load and distance) are recorded continuously as the load increases. As a result we obtain the total elongation of the cylindrical portion $DL=L-L_{g}$ for each corresponding load step. From the measured quantities we can recalculate the values of stress and strain using equations (1.5) and (2.1). For different materials we obtain different stress-strain diagrams. In Fig. 2.7 one can see the typical diagrams for ductile and brittle materials

Stress and Strain - Axial

to Mechani s of Materials: Part I

(2.3)



For a more detailed discussion about the diagrams we recommend any book which is concerned will material sciences for engineers.

Many engineering applications undergo small deformations and small strains. Thus the response material can be expected in an elastic region. For many engineering materials the elastic response linear, i.e. the straight line portion in a stress-strain diagram. Therefore we can write:

$$\sigma_x = E e_x$$

This equation is the well-known Hooke's law, found by Robert Hooke (1635–1703), the English pion of applied mechanics. The coefficient E is called the modulus of elasticity for a given material. Young's modulus, named after the English scientist Thomas Young (1773–1829). Since the strain t^{\pm} dimensionless quantity, then the modulus of elasticity E has the same units as the stress o, in Pascals 1 physical meaning of the modulus of elasticity is the stress occurring in a material undergoing a striequal to one, i.e. the measured specimen is elongated from its initial length L_{μ} .

if the response of the material is independent from the direction of loading, it is known as Materials whose properties depend upon the direction of loading are *anisotropic*. Typical example anisotropic materials are laminates, composites etc.

As we can see in the previous sections (2.2 and 2.3) the normal stress and strain have the same direction as the applied load. Let us assume that the homogenous and isotropic rod is axially loaded by a force P as in Fig. 2.8. Then the corresponding normal stress is $\sigma_x = \frac{\pi(y)}{4} = F/A$ and applying Hooke's law we obtain:



It is natural to assume that normal streams on the faces of a unit cube which represents the arbitrary point Q are zero. $a_p = a_i = 0$. This could convince one to assume that the corresponding strains e_i are zero too. But is not our case. In many engineering materials the elongation in the direction of applied load is impanied in a contraction in any transversal direction, see Fig. 2.9. We are assuming homogeneous and isotropic materials, i.e. mechanical properties are independent of position and direction. Therefore have $e_p = e_i$. This common value is called the *lateral strain*. Now we can define the important material e_i and e_i there is a strain the common value is called the lateral strain.

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stroduction to Mechanics of Materials: Part I

Stress and Strain - Axial Loading

 $y = -\frac{\text{interval strain}}{\text{asial strain}}$

$$y = -\frac{t_{k}}{t_{k}} = -\frac{t_{k}}{t_{k}}$$

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Note that the contraction in the lateral direction means that the reduction of lateral dimension $m_{\rm Hurs}$ negative value of strain and a positive value of Poisson's ratio. Usually Poisson's ratio has a value with the interval of (0, -) for common engineering materials like steel, iron, brass, aluminium, etc. If we applied to be an engineering materials like steel, iron, brass, aluminium, etc. If we applied to be an engineering materials like steel, iron, brass, aluminium, etc. If we applied to be an engineering materials like steel, iron, brass, aluminium, etc. If we applied to be an engineering material steel and engineering materials and engineering materials and engineering materials are steel and the steel and engineering materials are steel and engineering materials and engineering materials are steel and engineering materials are steel

$$z_x = \frac{\sigma_x}{\varepsilon} = \frac{\varepsilon}{\varepsilon s}$$
 and $\bar{z}_y = \bar{z}_z = -\frac{w_z}{\varepsilon} = -\frac{w_z}{\varepsilon s}$

(24

120

(23



Fig. 2.16 Open foam

28

there exist some materials with a negative value of Poisson's ratio. These materials are known as cellular, i.e. foams and honeycombs instead of contraction, they elongate in the lateral direction. The structure of these materials is presented in Fig. 2.10. For more information see any book written by L.J. Gibson M.F. Ashby

2.5 Generalised Hooke's Law for Multiaxial Loading



Until now we have discussed slender members (rods, bars) under axial loading alone. This resulted in a stress state at any arbitrary point of Q: $\sigma_x = -$, $\sigma_y = \sigma_z = 0$. Now let us consider multiaxial loading acting in the direction of all three coordinate axes and producing non-zero normal stresses: $\sigma_x \neq \sigma_y \neq \sigma_x \neq 0$, see Fig. 2.11.



Stress and Strain - Axial

(2)

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Stress and Strain - Axial Loading

Let us consider that our material is isotropic and homogeneous. Our arbitrary point Q is supresented a unit cube (where the dimensions of each side are a unit of the length), see Fig. 2.12. Under the multiaxial loading the unit cube is deformed into a rectangular parallelepiped with the following tide $(1 + e_x), (1 + e_y), (1 + e_y)$, where e_x are strains in the directions of the coordinate axes to e_x Fig. 2.12(b). It is necessary to emphasis that the unit cube is undergoing the deformation motion e_y with no rigid motion (translation). Then we can express the strain components e_y, e_y, e_y in terms of the stress components e_x, e_y . For this purpose, we will first consider the effect of each stress componenseparately. Secondly we will combine the effects of all contributing stress components by applying the principle of superposition. This principle states that the final effect of combined loading can be obtaine by determining the effects for individual loads separately and subsequently these separate effects an combined into the final result.

In our case the strain components are caused by the stress component σ : in the x direction $\varepsilon'_x = \sigma_x/E$ as in the y and z directions $\varepsilon'_y = \varepsilon'_x = -v\sigma_x/E$ recalling eq. (2.6). Similarly, the stress component s cause the strain components in the y direction $\varepsilon'_y = \sigma_y/E$ and in x and z directions $z = \varepsilon'_x = -v\sigma_y/i$. And finally the stress component s, causes the strain components in z direction $z = \sigma_x/E$ and is: and y directions $\varepsilon_x = z = -v\sigma_x/E$. These are separate effects of individual stress components. In final strain components are then the sums of individual contributions, i.e.



The equation (2.7) are known as a part of the generalised Hooke's law or a part of the elasticity equation for homogeneous and isotropic materials.

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Until now, shearing stresses have not been involved in our discussion. Therefore consider the more generalized stress state defines with six stress components σ_x , σ_y , σ_x , τ_{xy} , τ_{xx} , τ_{yz} , see Fig. 2.13. The shearing stresses τ_{xy} , τ_{xx} , τ_{yx} , τ_{xx} , τ_{yx} , see Fig. 2.13. The shearing stresses τ_{xy} , τ_{xx} , τ_{yx} , τ_{xx} , τ_{yx} , see Fig. 2.13. The shearing stresses τ_{xy} , τ_{xx} , τ_{yx} , τ_{xx} , τ_{yx} , see Fig. 2.13. The shearing stresses τ_{xy} , τ_{xx} , τ_{yx} , τ_{yx} , τ_{xx}



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 $\varepsilon_{x} = \frac{\sigma_{x}}{E} - \frac{v\sigma_{y}}{E} - \frac{v\sigma_{z}}{E}$ $\varepsilon_{y} = -\frac{v\sigma_{z}}{E} + \frac{\sigma_{y}}{E} - \frac{v\sigma_{z}}{E}$ $\varepsilon_{y} = -\frac{v\sigma_{z}}{E} - \frac{v\sigma_{z}}{E} + \frac{\sigma_{y}}{E}$

 $\gamma_{xy} = \frac{\tau_{xy}}{c}$ $\gamma_{xz} = \frac{\tau_{yz}}{c}$ $\gamma_{yz} = \frac{\tau_{yz}}{c}$

(2.11)

(2.10)

The validity of these equations is limited to isotropic materials, the proportionality limit stress that can not be exceeded by none of the stresses, and the superposition principle. Equation (2.10) contains three material constants E, G, v that must be determined experimentally. In reality we need only two of them, because the following relationship can be derived

For simplicity, let us consider a unit cube of material which undergoes a simple shear in the $xy pl_m$ material see Fig.2.14. The unit cube is deformed into the rhombold with sides equal to one, see Fig.2.15. In othe because on the angular changes. The four angles undergo a change in their values. Two of them reduces their values from $\frac{n}{2}$ to $\frac{n}{2} - \gamma_{xy}$, while the other two increase from $\frac{n}{2}$ to $\frac{n}{2} - \gamma_{xy}$. This angular change γ_{xy} while the other two increase from $\frac{n}{2}$ to $\frac{n}{2} - \gamma_{xy}$. This angular change γ_{xy} (measured in radians) defines the *shearing strain* is both directions x and y. The shearing strain γ_{xy} formed by two faces with the same direction as the positive x and y are see Fig. 2.15. Otherwise it is negative.

In a similar way as the normal stress-strain diagram for tensile test we can obtain the shear stress-strup plot for simple shear or simple torsion, discussed in Chapter 3. From a mathematical point of views can write Hooke's law for the straight part of the diagram by:

$$G\gamma_{xy} = G\gamma_{xy}$$

The material constant G is the shear modulus for any given material and has the similar physical meaning as Young's modulus.

If we consider shear in the xz and yz planes we will get similar solutions to Eq. (2.8) for stresses in the planes, i.e.

$$u = G \gamma_{10}, \qquad \tau_{yy} = G \gamma_{yy} \qquad (1)$$

Finally we can conclude that the generalised Hooke's law or elasticity equations for the generalise d set state are written by:

Saint Venant's Principle

(21



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Pig. 2.17

Until now we have discussed axially loaded members (bars, rods) with uniformly distributed size over the cross-section perpendicular to the axis of the member. This assumption can cause errors in (vicinity of load application. For simplicity let us consider a homogeneous rubber-like member that axially loaded by a compressive force F, see Fig. 2.16. Let us make the following two experiments 1 int we draw a squared mesh over the momber, see Fig. 2.17(a). Then we apply the compressive load through two rigid plates, see Fig. 2.17(b). The member is deformed in such a manner that it remains straight h the original square element change into a rectangular elements, see Fig. 2.17(b). The deformed no is uniform; therefore the strain distribution over a perpendicular cross-section is also uniform If 0 strain is uniform, then we can conclude that the stress distribution is also similarly uniform describby Hooken law. Secondly we apply the compressive force to the same meshed member throughout it sharp points, see Fig. 2.18. This is the effect of a concentrated load. We can observe strong deformation in the vicinity of the load application point. At certain distances from the end of a member the part is again uniform and rectangular. Therefore we can say that there are large deformations and street around the load application point while uniform deformations and stresses occur farther from this pair In other words, except for the vicinity of load application point, the stress distribution function mat assumed independently to the load application mode. This statement which can be applicable to # type of loading is known as Seint-Venant's principle, after Adhémar Barré de Saint-Venant (1797-188



While Saint-Venant's principle makes it possible to replace actual loading with a simpler one for computational purposes, we need to keep in mind the following:

- . The actual loading and loading used to compute stresses must be statically equivalent.
- Stremes cannot be computed in the vicinity of load application point. In these cases
 advanced theoretical and experimental method must be applied for stress determination.

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Let us consider a homogeneous isotropic member BC of length L, cross-sectional area A, a Young's modulus E subjected to the centric axial force F, see Fig. 2.18. If the resulting normal m $\sigma_x = N_{(n)}/A = F/A$ does not exceed the proportional limit stress and applying Saint-Venant's pring Until now we could solve problems starting with the free body diagram, and subsequently determine the we can then apply Hooks's law

$$\epsilon = \mathbf{E} \mathbf{e}_x$$
 or $\mathbf{e}_y = \frac{\mathbf{e}_x}{\mathbf{e}_y}$

And substituting for the normal stress $\sigma_x = N_{(x)}/A = F/A$ we have

$$T_{\rm m} = \frac{N_{\rm (s)}}{E\,A} = \frac{F}{E\,A}$$

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Recalling the definition of normal strain, equation (2.1) we get

$$\Delta L = z_{1}L$$

and substituting equation(2.13) into equation (2.14) we have

$$\Delta L = \frac{N_{(n)}L}{EA} = \frac{FL}{EA}$$

Now we can conclude that the application of this equation: Equation (2.15) may be used only if the rod homogeneous (canstant E), has a uniform cross-sectional area A, and is loaded at both ends. If the number is loaded at any other point or is composed from several different homogeneous parts having different cross-sectional areas we must apply the division into parts satisfying the previous conclusion Denoted $N_{i(x)}$, E_i , A_i , L_i the internal normal force, Young's modulus, cross-sectional area and length comesponding to the part i respectively. Then the total elongation is the sum of individual elongations (principle of superposition):

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$$\Delta L = \sum_{i=1}^{n} \Delta L_{i} = \sum_{i=1}^{n} \frac{m_{i+1} L_{i}}{n_{i}}$$
(2.16)

In the case of variable cross-sectional area, as in Fig. 2.4, the strain depends on the position of the arbitrary point Q, therefore we must apply equation (2.2) for the strain computation. After some mathematical manipulation we have the total elongation of the member

$$\Delta L = \int_{(d_c)} \frac{N_{(c)}}{EA} dx \tag{2.17}$$

reactions from equilibrium equations. Recalling the method of sections in (chapter 2.2) we can compute internal forces at any arbitrary section, allowing us to then proceed with computing stresses, strains (2) and deformations. But many engineering problems can not be solved by the approach of statics alone.



For simplicity, let us consider a simple problem, see Fig. 2.19. Using statics we cannot solve the problem through equilibrium equations. The main difficulty in this problem is that the number of unknown reactions is greater than the number of equilibrium equations. From a mathematical point of view the problem is ill-conditioning. For our case we obtain one equilibrium equation as

$$\Sigma F_x = 0 + R_c - F + R_m = 0$$

(2.18)

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Fig. 2.20

There are two unknown reactions in equation (2.18). Problems of this type are called statical indeterminate problems.

**





To overcome the static indeterminacy we need to complete the system of equilibrium equations with relations involving deformations by considering the geometry of the problem. These additional relations are called *deformation conditions*. For practical solution let us consider the following transformation in Fig. 2.20. The problem presented is exactly the same as the problem in Fig. 2.19. This problem is statically indeterminate to the first degree. Removing the redundant support at point C and replacing it with the unknown reaction **R**, we obtain the so-called statically indeterminate problem with unknown reaction, see Fig. 2.20(b). Now our task is to receive the same response for the statically indeterminate problem as in the original statically indeterminate problem. To get the same response of the structure we need to impose the deformation condition for point C, that the displacement for this point is equal to zero, see Fig. 2.21, or mathematically

$$u_c = 0$$
 (2.19)

This condition (2.19) coincides with the total elongation of the member also equal to zero. We then have:

$$\mathbf{u}_{\mathsf{C}} = \Delta \mathbf{L} = \mathbf{0} \tag{2.20}$$

The member presented in Fig. 2.21 can be divided into two homogeneous parts. Therefore the total congation is a sum of individual elongation, equation (2.16), i.e. $\Delta L = \Delta L_1 + \Delta L_2$. Then we have

$$\Delta L = \frac{n_{12g}L_3}{\mu_0} + \frac{n_{2(g)}L_2}{\mu_0} = 0 \qquad (2.21)$$

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Stress and Strain - Axial I

Both normal forces $N_{1(x)} = -R_c$, $N_{2(x)} = F - R_c$ are functions of unknown reaction R_c . Solving equation (2.21) we obtain the value of reaction R_c . We can then continue by solving in the usual for statically determinate problems.

2.8 Problems Involving Temperature Changes





In this case there is no stress in a rod. We can prove this very easily by applying the method of sections and writing equilibrium equations.

In the previous discussions we assumed constant temperature as the member was being loaded Lett now consider a homogeneous rod *BC* with the constant cross-sectional area A and the initial lengt L_r see Fig. 2.22. If the temperature of the rod grows by ΔT then we will observe the elongation of th rod by ΔL_{γ} , see Fig. 2.22. This elongation is proportional to the temperature increase ΔT and the initial length L. Using basic physics we have

$$\Delta L_T = \alpha(\Delta T) L$$

where a is the coefficient of thermal expansion. The thermal strain ε_r is associated with the aforemention elongation ΔI_r , i.e. $\Gamma_r = \Delta L_T/L$. Then we have

$$\varepsilon_T = \alpha(\Delta T)$$

(2.2

(2.2

Stress and Strain - Axial (

2.9

By modify the previous rod by placing it between two rigid plates and subjecting it to a temperaturchange of ΔT we will observe no elongation because of the fixed supports at its ends. We know that the problem is statically indeterminate due to the supports at each end. Let us then transform the problem into the so-called statically determinate problem. Removing the support at point C and replace its unknown reaction R_c . Now we can apply the principle of superposition in the following way limit we heat the rod by ΔT , see Fig. 2.24(a), then we can observe the elongation $\Delta L_T = \alpha(\Delta T)L$. so Fig. 2.24(b). Secondly, we push the rod by the reaction R_c back to its initial length, see Fig. 2.24(c). The effect of pushing in the opposite of elongation ΔL_{R_c} . Applying the formulas (2.22) and (2.15) we have

$$\Delta L_{T} = \alpha(\Delta T)L$$
 and $\Delta L_{R_{C}} = \frac{R_{C}L}{EA}$

Expressing the condition that the total elongation must be zero, we get

$$\Delta \mathbf{L} = \Delta \mathbf{L}_{\mathrm{T}} + \Delta \mathbf{L}_{\mathrm{R}_{\mathrm{C}}} = \alpha (\Delta T) \mathbf{L} + \frac{\mathbf{E}_{\mathrm{C}} \mathbf{L}}{\mathbf{E}_{\mathrm{A}}} = \mathbf{0} \tag{C}$$

This equation represents the deformation condition. And we can compute the reaction as

 $R_{c} = -EA\alpha(\Delta T)$

and corresponding stress

$$\sigma_x = \frac{n_{(x)}}{A} = \frac{n_x}{A} = -E\alpha(\Delta T)$$

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The trues is a structure consisting of several slender members (rods, bars) that are subjected to axial loading only. The mple trues structure is presented in Fig. 2.25. This trues consists of two bars of the same cross-sectional area A and Young's modulus E. The trues is loaded by a vertical force F. Our task is to compute the vertical and horizontal displacements of joint C. Applying the methods of statics we can determine axial fonces in each bar: $N_1 = F/\sin \theta$, $N_2 = F/\tan \theta$. Consequently, we can determine elongations for individual bars using equation (2.15)

$$\Delta L_1 = \frac{N_1 L_1}{\epsilon A} = \frac{F L_1}{\epsilon A \sin \theta} \qquad \text{and} \qquad \Delta L_2 = \frac{N_2 L_2}{\epsilon A} = \frac{F L_2}{\epsilon A \sin \theta}$$
(2.28)

The deformed configuration can be founded by drawing two circles with centres at joints B and D with (2)1 the following radii, see Fig. 2.26

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$$r_1 = L_1 + \Delta L_1 = L_1 \left(1 + \frac{F}{RA \sin \theta} \right)$$
$$r_2 = L_2 - \Delta L_2 = L_2 \left(1 - \frac{F}{RA \sin \theta} \right)$$

The deformations are relatively small, therefore we can replace the circles with tangents perpendicu, to the undeformed bars, see Fig. 2.27. One can then compute the horizontal and vertical displacement as follows:

$$u_{L} = \Delta L_{2} = \frac{rL_{2}}{\epsilon A \tan \theta}$$
$$v_{L} = \Delta L_{1} \sin \theta + \frac{AL_{2} + AL_{1} \cos \theta}{\tan \theta} = \frac{rL_{1}}{r_{A} \sin \theta} + \frac{rL_{2}}{r_{A} \cos \theta}$$



Figs 2.37 Vertical and horizontal displacements

Examples, Solved and Unsolved Problems 2.10

Problem 2.1



55 Download tree eBooks at bootboon.com A used has the following dimensions: a = 100 mm, b = 50 mm, L = 1500 mm, shown in Fig. 2.28. If an axial force of F = 80 kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross-section after the load is applied. Assume that the material behaves elastically. where the Young's modulus for steel is E = 200 GPa and Poisson's ratio ν = 0.32.

The normal stress in the bar is

$$a_{1} = \frac{F}{A} = \frac{F}{ab} = \frac{80(10^{2}) N}{(0.1 m)(0.05 m)} = 16.0 \times 10^{4} Pa = 16.0 MPa$$

(2.3

The strain in the x direction is

$$r_{\rm e} = \frac{r_{\rm e}}{E} = \frac{16 \times 10^6 {\rm Pa}}{200 \times 10^6 {\rm Pa}} = 80 \times 10^6$$

The axial elongation of the bar then becomes

$$\Delta L_{x} = r_{1}L - \frac{m_{1}}{E}L - \frac{H_{1}}{abE} = (80 \times 10^{-6}) \times 1.5m = 120 \mu m.$$

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Using Eq. (2.6) for the determination of Poisson's ratio, where v = 0.32 as given for steel, the contract Stress and Equilibrium for part BC strain in the y and z direction are

$$a_{\rm c} = -0.32(10 \times 10^{-6}) = -25.6 \,\mu{\rm m}{\rm m}$$

Thus the changes in the dimensions of cross-section are given by

$$\Delta L_{\gamma} = c_{\gamma} L_{\gamma} \sim -\pi c_{\gamma} a = -i\pi \frac{\sigma_{\chi}}{E} = -i\pi \frac{F}{abE}$$
$$\Delta L_{\gamma} = -\frac{p_{\Psi}}{bE} = -2.56 \ \mu m$$

$$L_1 = c_1 L_1 = -v b_1 \frac{\sigma_1}{E} = -v b_1 \frac{\sigma_2}{E}$$

Problem 2.2

A



A composite steel har shown in Fig. 2.29 is made from two segments, BC and CH, having circular crof section with a diameter of D = - D and D = - 2D. Determine the diameter D, if we have an allowed stress of σ_{al} = 147 MPa and the applied load is F = 20 kN.

Solution

We can divide the bar into three parts (BC, CG and GH) which have constant cross-section area constant loading.

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Solution of normal (axial) load N

$$\Sigma F_{r} = 0$$
: $F - N_{r} = 0 \implies N_{r} = F = 20 \text{ kN}$

Stress in the part BC

$$a = \frac{N_1}{A_1} = \frac{F}{aD^2} = \frac{4F}{aD^2} = \frac{4 \times 20000N}{aD^2} = 25404.8 \frac{1}{13^2}$$

Equilibrium and stress in part CG





Solution of normal (axial) load N.

$$\sum F_{n} = 0$$
 $F - \frac{F}{2} - \frac{F}{2} - N_{n} = 0$ $\implies N_{n} = 0$

Stress in part BC

$$\pi_1 = \frac{N_1}{A_1} = \frac{1}{\frac{RD^2}{A}} = \frac{4F}{RD^2} = \frac{4 \times 20000N}{RD^2} = 25464.8 \frac{1}{D^4}$$

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Equilibrium of part and stress in part GH



Solution of normal (axial) load N

$$\sum F_{n_{m}} = 0: \quad F - \frac{F}{2} - \frac{3}{2}F - \frac{3}{2}F - N_{m} = 0 \qquad \qquad N_{m} = -3F$$

 $N_m = -3P = -3 \times 20000 \text{ N} = -90000 \text{ N}$



For all parts, draw the diagram of normal force and stress. The maximum stress is in the first part (BC), which we can compare with the allowable stress and obtain the parameter D

$$\sigma_{MAX} = \sigma_{I} = \frac{4F}{m^{2}} \le \sigma_{AB} \implies D \ge \sqrt{\frac{4F}{m\sigma_{AB}}}$$
$$D \ge \sqrt{\frac{4}{147 MPa}} \implies D \ge 13.2 \text{ mm}$$

Problem 2.3



Des mane the clongation of a conical bar shown in Fig. 2.30 at point B without constituering its weight

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Transforment the second part

2,=(1.2.2)

Given by maximum cone diameter of D, length L, modulus of elasticity E and applied force P. $Deter_{D_{ij}}$ the maximum stress in the conical bar.

Solution

The problem is divided into two parts.

Equilibrium of the first part

 $x_{i}\in\left\langle 0,L/2\right\rangle$



We determine the normal force N, and normal stress o,

Normal force N_p

$$\sum F_{n_1} = 0$$
: $N_1(x_1) = 0 \implies N_1(x_1) = 0$

Calculate angle β from the geometry of the cone given by diameter D, at position x,

$$\mathbf{m} \ \boldsymbol{\beta} = \frac{\mathbf{D}}{\mathbf{L}} \underbrace{\mathbf{D}_{1}(\mathbf{x}_{1})}{\mathbf{2}} \Rightarrow \mathbf{D}_{1}(\mathbf{x}_{1}) = \frac{\mathbf{x}_{1}}{\mathbf{L}}$$

Cross-sectional area (function of position) in the first part is

$$A_{1}(x_{1}) = \frac{\pi D_{1}^{2}}{4} = \frac{\pi}{4} \left(\frac{x_{1}}{L} D \right)^{2} = \frac{\pi D^{2}}{4} \frac{x^{2}}{L^{2}}$$

Normal stress o, is as follows

$$c_1(x_1) = \frac{N_1(x_1)}{A_2(x_1)} = \frac{0}{\frac{3D^2x_1^2}{4L^2}} \approx 0$$



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We determine the normal force $N_{\rm g}$ and normal stren $\sigma_{\rm g}^{-1}$

Normal force Nat

$$\sum F_{n_1} = 0 \quad N_n(\mathbf{x}_n) - \mathbf{F} = 0 \quad \Rightarrow \quad N_n(\mathbf{y}) = \mathbf{F}$$

Stress and Strain - Axial Line

Calculation of angle b from geometry and diameter D_a at position x_a

$$= \beta - \frac{D}{L} - \frac{D_h(x_h)}{x_h} \implies D_h(x_h) - \frac{x_h}{L} D$$

Cross-section area (function of position) in second part is

$$A_{n}(x_{n}) = \frac{nD_{n}^{2}}{4} = \frac{n}{4} \left(\frac{x_{n}}{L}D\right)^{2} = \frac{nD^{2}}{4} \frac{x_{n}^{2}}{L^{2}}$$

Normal stress of is then

$$a_{n}(x_{n}) = \frac{N_{n}(x_{n})}{A_{n}(x_{n})} = \frac{F}{\frac{nD^{2}x_{n}^{2}}{4t^{2}}} = \frac{4FL^{2}}{nD^{2}x_{n}^{2}}$$



The graphical result of the normal force and stress is shown in the Fig. 2.31.

Elongation is found by summing the elongation of each part using integration, because cross-andiarea is a function of position in all parts, which is given by

$$\begin{split} \Delta L_{n} &= \Delta L_{n} + \Delta L_{n} = \int_{0}^{10} \frac{N_{1}(\mathbf{x}_{0})}{EA_{1}(\mathbf{x}_{1})} d\mathbf{x}_{1} + \int_{10}^{10} \frac{N_{n}(\mathbf{x}_{n})}{EA_{n}(\mathbf{x}_{n})} d\mathbf{x}_{n} \\ \Delta L_{n} &= \int_{0}^{10} \frac{0}{EA_{n}(\mathbf{x}_{n})} d\mathbf{x}_{1} + \int_{10}^{10} \frac{F}{E\frac{dD^{2}\mathbf{x}_{n}^{2}}{4L^{2}}} d\mathbf{x}_{n} \\ \Delta L_{n} &= \frac{4FL^{2}}{EnD^{2}} \int_{10}^{10} \frac{1}{\mathbf{x}_{n}^{2}} d\mathbf{x}_{n} = \frac{4EL^{2}}{EnD^{2}} \left[-\frac{1}{L} \right]_{01}^{1} = \frac{4FL^{2}}{EnD^{2}} \frac{1}{L} \\ \Delta L_{n} &= \frac{4FL}{EnD^{2}} \end{split}$$

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our sion to Materials: Part I

Problem 2.4



A har BC and CG of length L is attached to rigid supports at B and G. Part BC have a square cross-section and between point C and G the cross section is circular. What are the stresses in portions BC and OG due to the application of load F at point C in Fig. 2.32. The weight of the bar is neglected. Design the parameter D to accommodule for the given allowable stras of length L, modulus of elasticity E and applied force F are known Problem is statically indeterminate.

Solution



Fig. 2.33

At less, we detach the bar at point B and define a reaction at its location, which will be solved from the delormation condition (See Fig. 2.33).

The solution is divided into two solutions part BC and CG.

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Free-body diagram on portion I (part BC):

 $\mathbf{x}_i \in (0, L)$



From the equilibrium equation in the first part, we obtain

$$\sum \mathbf{F}_{\mathbf{n}_{1}} = 0: \quad \mathbf{N}_{1}(\mathbf{x}_{1}) - \mathbf{R} = 0 \quad \Longrightarrow \quad \mathbf{N}_{2}(\mathbf{x}_{1}) = \mathbf{R}$$

Solution of cross-section area is given From Pythagoras theorem where we determine the side log of the square:

-

$$(2D)^{3} = a^{3} + a^{3}$$

$$A_{1} = a^{3} \implies 4D^{3} = 2a^{3} \implies A_{1} = 2D^{3}$$

$$2D^{3} = a^{3}$$

Stress and Strain - Axial Loading

Stress in portion BC is

$$\mathbf{x}_{\mathrm{f}}(\mathbf{x}_{\mathrm{f}}) = \frac{\mathbf{N}_{\mathrm{f}}(\mathbf{x}_{\mathrm{f}})}{\mathbf{A}_{\mathrm{f}}(\mathbf{x}_{\mathrm{f}})} = \frac{\mathbf{R}}{2\mathbf{D}^{2}}$$

Free body diagram an portion II (part CG):

x, e(L.2L)



From the equilibrium equation in the second part, we obtain

$$\sum F_n = 0$$
 $N_n(\mathbf{x}_n) + F - R = 0 \implies N_n(\mathbf{x}_n) = R - F$

Stress in portion OG in

$$h_{\mu}(X_{R}) = \frac{N_{R}(X_{R})}{A_{H}(X_{R})} = \frac{R-F}{\frac{\pi D^{2}}{4}} = \frac{4(R-F)}{\pi D^{2}}$$

We determine the unknown reaction from the deformation condition, total elongation (movement of Point B) is equal to zero:

 $\Delta \mathbf{L}_{\mathbf{D}} = \mathbf{0} \quad \Longrightarrow \quad \Delta \mathbf{L}_{\mathbf{D}} = \Delta \mathbf{L}_{\mathbf{1}} + \Delta \mathbf{L}_{\mathbf{H}} = \mathbf{0} \quad \Longrightarrow \quad \Delta \mathbf{L}_{\mathbf{1}} + \Delta \mathbf{L}_{\mathbf{H}} = \mathbf{0}$

from which we have

$$\frac{RL}{E_{A}} + \frac{P_{A}L_{a}}{E_{B}A_{a}} = 0 \implies \frac{RL}{E2D^{3}} + \frac{4(R-F)L}{ERD^{3}} = 0$$

$$\frac{RR}{R} + 8(R-F) = 0 \implies R = \frac{8F}{R+8}$$

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Problem 2.5

Stress and Strain - Axial Loading



We insert the solved reaction into the result of parts BC and OG.

$$N_{1}(x_{t}) = R = \frac{16}{R^{+}8} = 0.72F$$

$$\sigma_{1}(x_{t}) = \frac{14_{1}(x_{t})}{A_{1}(x_{t})} = \frac{8F}{(R^{+}8)2D^{1}} = 0.36\frac{F}{D^{1}}$$

$$N_{R}(x_{t}) = R - F = \frac{8F}{R^{+}8} - F = -\frac{8F}{R^{+}8} = -0$$

and draw the diagram of normal forces and stresses for both portions, which is shown in the Fig.25

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Design of parameter D

The maximum (absolute value) of stresses is the same for both portions, we compare them with " allowable stress and we get the designed parameter D:

$$\sigma_{MAX} = 0.36 \frac{F}{D^3} \le \sigma_{AI} = D \ge \sqrt{\frac{D.36F}{\sigma_{AI}}}$$



Fig. 2.35

is Fig. 2.35, a bar of length 21, with uniform circular cross-section area and made of the same material with a modulus of elasticity E, is subjected to an applied force F. determine the stress in the bar. Consider the weight of bar (density p and gravity g are known).

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Stress and Strain - Axial Local

Solution



Problem is statically indeterminate and for the solution we use the deformation condition at point 1

First step of solution is to substitute an unknown reaction at point B (see Fig. 2.36).

Because the problem is in pure tension, the reaction R and moment M are zero, reaction R is non-ze-

Solution of this problem is divided into two parts.

x, c (0.L)



Equilibrium of first part

 $\sum F_{h_1} = 0$: $N_1(\mathbf{x}_1) + \mathbf{R} - \mathbf{G}_1 = 0 \implies N_1(\mathbf{x}_1) = \mathbf{G}_1 - \mathbf{R}$

where G₁ is gravitational load of first part, defined by

 $\mathbf{O}_1 = \mathbf{mg} = \rho \mathbf{Vg} = \rho \mathbf{gA}_1 \mathbf{x}_1$

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ternal time and stress is gathered by

$$p_1(\mathbf{x}_1) = \frac{N_1(\mathbf{x}_1)}{A_1(\mathbf{x}_1)} = \frac{\rho_{\mathbf{x}}A_1\mathbf{x}_1 - \mathbf{R}}{A_1} = \rho_{\mathbf{x}}\mathbf{x}_1 - \frac{\mathbf{R}}{A}$$

= (L. 2L)



I quilibrium at the second part, is given by

$$\Sigma F_n = 0$$
 $N_n(x_n) + F - R - G_n = 0 \Rightarrow N_n(x_n) = F + G_n - R$

Normal force and stress is as follows

$$\mathbf{N}_{\mathbf{x}}(\mathbf{x}_{\mathbf{x}}) = \mathbf{F} + \rho \mathbf{g} \mathbf{A}_{\mathbf{x}} \mathbf{x}_{\mathbf{x}} - \mathbf{R} = \mathbf{F} + \rho \mathbf{g} \mathbf{A} \mathbf{x}_{\mathbf{x}} - \mathbf{R}$$

$$\pi_{1} \bar{\kappa} \pi_{0} \bar{\rho} = \frac{N_{0}(\pi_{0})}{A_{0}(\pi_{0})} = \frac{F + \rho g A \pi_{g} - R}{A} = \frac{F}{A} + \rho g \pi_{0} - \frac{R}{A}$$

Deformation condition at point A

Total elongation at point A is equal to zero, which is consisting of the first part of the bar ΔL_1 and second part ΔL_2 . For solution of each part we used the integral form because normal force is a function of position. Unknown reaction R after calculation becomes

$$\Delta L_{A} = \Delta L_{A} + \Delta L_{B} = 0 \implies \frac{P_{1}L_{A}}{E_{T}A_{1}} + \frac{P_{B}L_{B}}{E_{B}A_{B}} = 0$$

$$\frac{1}{2}\frac{(P = A_{T} - R)L}{EA} dx_{1} + \int_{T}^{T} \frac{(F + \rho = A_{T} - R)L}{EA} dx_{2} = 0$$

$$2\rho = AL + F = 2R \implies R = \rho = AL + \frac{F}{2}$$

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Stress and Strain - Axiat



We insert the result of reaction R into the function of normal force and stress for both parts and a diagram for force and stress is shown in Fig. 2.37.

$$\begin{split} N_{1}(x_{t}) &= \rho_{II}Ax_{t} - \left(\rho_{III}AL + \frac{F}{2}\right) = \rho_{III}A(x_{t} - L) - \frac{F}{2} \\ \sigma_{1}(x_{t}) &= \frac{N_{1}(x_{t})}{A_{1}(x_{t})} + \frac{\rho_{III}A_{t} - R}{A_{t}} = \rho_{IIII}x_{t} - L) - \frac{F}{2A} \\ N_{II}(x_{t}) &= F + \rho_{III}Ax_{t} - R = F + \rho_{III}Ax_{t} - \left(\rho_{IIII}AL + \frac{F}{2}\right) = \frac{F}{2} + \rho_{III}A(x_{t} - L) \\ \sigma_{III}(x_{t}) &= \frac{N_{II}(x_{t})}{A_{II}(x_{t})} = \frac{F}{2A} + \rho_{IIII}(x_{t} - L) \end{split}$$

Problem 2.6



A rod of length L. cross-sectional area A,, and modulus of elasticity E, has been place inside a sector the same length 1, but of differing cross section area A, and modulus of elasticity E, (Fig. 2.38). is use detormination of the rod and tube when F is applied to the end of the plate as shown?

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solution to Machain s of Materials: Part I

Stress and Strain - Axial Londing

(a)

(b)

Solution

The areal force in the rod and in the tube is denoting by N_{hop} and N_{runs}, respectively, we draw a freebody disgram for the rigid plate in Fig. 2 39:



$$\Sigma F_{n} = 0$$
: $N_{rom} + N_{non} - F = 0 \implies N_{rom} + N_{hom} = F$

The problem is statically indeterminate. However, the geometry of the problem shows that the deformation Dires and Dires of the rod and tube must be equal:

$$\Delta L_{\text{ress}} = \Delta L_{\text{hop}} = \frac{N_{\text{ress}}}{E_{\text{ress}}A_{\text{ress}}} = \frac{N_{\text{hop}}L_{\text{hop}}}{E_{\text{hop}}A_{\text{hop}}}$$

Equation (a) and (b) can be solved simultaneously for Naop and Nrus by:

$$N_{max} \frac{E_{max}A_{Tim}}{E_{max}A_{aax}} + N_{max} = F$$
$$N_{max} \left(\frac{E_{max}A_{Tim}}{E_{axx}A_{axx}} + 1\right) = F$$

$$= \frac{F}{\left(\frac{E_{max}A_{TVM}}{E_{max}A_{LOD}} + 1\right)}$$

N_{ne}

$$\frac{N_{max}}{E_{max}\Lambda_{max}} \frac{F}{\left(\frac{E_{max}\Lambda_{max}}{E_{max}\Lambda_{max}}+1\right)}$$

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Stress and Strain - Astanta

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Stress and Strain - Axial Loading

Problem 2.7



Determine the value of stress in the steel bar shown on Fig. 2.40 when the temperature change of bar is $\Delta T = 30$ °C. Assume a value of E = 200 GPa and a = 12 x 10⁴ 1/°C for stoel.

Solution

We first determine the reaction at the support. Since the problem is statically indeterminate, we deter the bas from its support at B.



Pip. 2.41

The corresponding deformation from temperature exchange (Fig. 2.41) in

$\Delta L_{2} = \alpha \Delta T L$

Applying the unknown force N______ at the end of the bar at B (Fig. 2.41). We use eq. (2.15) to end the corresponding deformation AL____

$$M_{a} = \frac{N_{a}}{EA}$$

73 Download free efforts at bootboon com This deformation of the bar must be zero at point B, from which we have the following deformation

ondition

 ΔL_{T} = ΔL_{T} ,

treat this we obtain N

Stress in the bar is then given by

$$a = \frac{N_{max}}{\Lambda} = a \Delta T E = 12 \times 10^{-6} 1/{}^{\circ}C \times 30 {}^{\circ}C \times 200 \times 10^{9} Pa = 72 MPa$$

Stress and Strain - Aug

Biochem s of Materials: Part I

Problem 2.8





by $\Delta T = 50$ °C. Use the value E = 70 GPs and $\alpha = 22.2 \times 10^{4}$ 1/°C for aluminum. The intege in Fig. 2.44 is made of three 304 stainless members connected together by pins, each member

Solution

We determine the elongation of the bar from temperature exchange from the following equation

Determine the stress of the aluminum bar L = 500 mm shown in Fig. 2.42, when its temperature characteristics

x & (0, L)





We divide the bar into one component part shown in Fig. 2.43. From equilibrium equation in this p we find the unknown normal force

 $\sum F_{x} = 0$: N(x) = 0

Stress in the aluminum bar we describe by

$$a = \frac{N}{A} = \frac{0}{A} = 0 Pa$$

he a cross-sectional area of A = 1000 mm². If a vertical force P = 250 kN is applied to the end of the member at D. Determine the stresses of all members and the maximum stress output



First an disconnected the member CD and draw a free-body diagram (shown in Fig. 2.45) We then " for the force N co by the following equilibrium equation

$$\sum F_n = 0$$
: $N_{co} = F = 0 \implies N_{co} = F = 250 \text{ kN}$

The normal forces Ha and Nar we determined from equilibrium at point C (shown in Fig. 2.46), treas by:

$$\alpha = \frac{L}{L_s} = \frac{1.0 \text{ m}}{1.5 \text{ m}} = 0.666 \qquad \alpha = 33.69^\circ$$

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Problem 2.9



Stress and Strain - Axial L

s of Materials: Part I

Stress and Strain - Axial Loading

In the z direction

$$\sum I_n = 0$$
: $-N_{nr} = nn + N_{nr} = 0 \implies -N_{nr} + N_{nr} = 0$





The assembly consists of two titanium rods and a rigid beam AC in Fig. 2.47. The cross section area is $= 60 \text{ mm}^3$ and $A_{co} = 45 \text{ mm}^3$. The force is applied at a = 0.5 m. Determine the stress at rod GB and CD; if a the vertical force is equal to F = 30 kN.

Hg. 2.47

F = 30 kN 2a L = 2m

 $N_{\rm He}=N_{\rm Ac}$

In the y direction

 $\sum F_{e} = 0$: $N_{ex} \cos a + N_{ex} \cos a - N_{cb} = 0$

$$N_{mc} \cos \alpha = N_{mc} = F \implies N_{mc} = \frac{P}{2 \cos \alpha} = \frac{230 \text{ kN}}{2 \cos \alpha} = 150.23 \text{ kN}$$

 $N_{mc} = N_{mc} = 150.23 \text{ kN}$

Stresses in the members are

$$\sigma_{cc} = \frac{N_{cc}}{A} = \frac{150.23 \text{ } 10^3 \text{ N}}{1000 \text{ } \text{ mm}^3} = 150.23 \text{ MPa}$$

$$\sigma_{cc} = \frac{N_{cc}}{A} = \frac{150.23 \text{ } 10^3 \text{ N}}{1000 \text{ } \text{ cm}^3} = 150.23 \text{ MPa}$$

$$\sigma_{cc} = \frac{N_{cc}}{A} = \frac{250 \text{ } 10^3 \text{ N}}{1000 \text{ } \text{ cm}^3} = 250 \text{ MPa}$$

Maximum value of stress is at Jink CD.

0_{MAX} = 0_{cb} = 250 MPa

Stress and Strain - Axial L

Solution



The unknown normal forces in the titanium rod are found from the equilibrium equation of rigid and GC in Fig. 2.48, given by

$$\sum F_{e} = 0$$
: $N_{em} + N_{em} - F = 0$

$$\sum M_{co} = 0$$
: $N_{co} 3a - Pa = 0 \implies N_{co} = \frac{F}{3} = \frac{30 \text{ kN}}{3} = 10 \text{ kN}$

 $N_{cm} = F - N_{cp} = 30 \text{ kN} - 10 \text{ kN} = 20 \text{ kN}$

N. - 20 EN

Stress in rod AB and CD is given by the following

$$a_{cm} = \frac{N_{cm}}{A_{cm}} = \frac{20000 \text{ N}}{60 \text{ mm}^3} = 333.3 \text{ MPa}$$

 $a_{cm} = \frac{N_{cm}}{A} = \frac{10000 \text{ N}}{45 \text{ mm}^3} = 222.2 \text{ MPa}$

Problem 2.11



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Stress and Strain - Axial Loading

(a)

rigid bar BD is supported by two links AC and CD in Fig. 2.49. Link CH is made of aluminum ($E_{DO} = 68.9$ GPa) and has a cross-section area $A_{CH} = 14$ mm²; link DG is made of aluminum ($E_{DO} = 68.9$ GPa) has a cross-section of $A_{DC} = 2$. 280 mm². For the uniform load w = 9 kN/m, determine of a point D and stremes in the link CH and DG.

solution

Fine heady diagrams of rigid bar BD



Equilibrium equation of moment at point B in the bar BC (Fig. 2.50), is expressed as

 $\sum M_{es} = 0$: $N_{cs}Lana + N_{so} 2L - w2LL = 0$

$$= N_{cl} \frac{\sqrt{2}}{2} + 2N_{ac} = 2wL$$

sheet

$$\tan \alpha = \frac{L}{1} = i \qquad \alpha = 45^{\circ}$$

in equation (a) we have two unknowns. We need a second equation for the solution of normal forces in the links from the deformation condition in Fig. 2.51, from the similar triangles



Pip.2.51

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Stress and Strain - Axial L

 $\Delta BDD' \approx \Delta BCC'$ $\tan \beta = \frac{DD'}{BD} = \frac{CC'}{BC} \implies \frac{\Delta L_{max}}{L \ max} = \frac{\Delta L_{max}}{2L}$

In these triangles the angle β are the same from which we have the following equation.

$$\sin u = \frac{M_{max}}{CC} \Rightarrow CC = \frac{M_{max}}{u = a}$$

$$\frac{M_{max}}{E_{ca}A_{ca}} = \frac{N_{ca}\sqrt{2t}}{EA}$$

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$$\Delta L_{ac} = \frac{N_{pe}L_{pe}}{E_{be}A_{ac}} = \frac{N_{pe}L}{ZEA}$$

$$\frac{\Delta L_{OL}}{L \text{ and }} = \frac{\Delta L_{DO}}{2L} \implies \frac{N_{m}\sqrt{2}L^{2}}{EA\sqrt{2}} = \frac{N_{m}L}{2EA} \implies N_{m} = \frac{N_{DO}}{4}$$

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solving for the system of equations (a) and (b), we get

$$\frac{N_{max}}{4} \frac{\sqrt{2}}{2} + 2N_{max} = 2wL \implies N_{max} = \frac{2wL}{\left(\frac{\sqrt{2}}{8} + 2\right)} = 0.92wL$$

N_{ma} = 0.92wL = 0.92 300N m im = 276 N

$$N_{m0} = \frac{N_{m0}}{4} = \frac{wL}{2\left(\frac{d2}{5}+2\right)} = \frac{0.92wL}{4} = 0.23wL.$$

Nm = 0.23 wL = 0.23 300Nm 1m = 69 N

stress in link CH is

$$\sigma_{cu} = \frac{N_{cu}}{A_{cu}} = \frac{69 \text{ N}}{14 \text{ mm}^3} = 4.93 \text{ MPa}$$

Stress in link DG is

$$\frac{N_{DC}}{A_{DC}} = \frac{276 \text{ N}}{28 \text{ cm}^2} = 9.86 \text{ MPa}$$

Deflection of point D is given by the following

$$\frac{N_{mo}L_{per}}{L} = \frac{0.92 \text{wL L}}{2\text{EA}} = \frac{0.92 \text{ 300N m (1m)}^3}{2.68.9 \text{ 10}^3\text{Pa 14 10}^4\text{ mm}^3}$$

ΔL_{ng} = 1 43 10⁴m = 0.143 mm

Unsolved problems



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Problem 2.12

Both portions of red GBC in Fig. 2.52 are made of aluminum for which E = 70 GPa. Knowing the magnitude of P is 4 kN, determine (a) the value of F, so that the deflection at point A is zero, or a corresponding deflection of point B, (c) the value of stress for each portion.

$$|P_1 = 32.8 \text{ kN}; \Delta L_0 = 0.073 \text{ mm}, \sigma_{c0} = 12.73 \text{ MPz}; \sigma_{c1} = 10.19 \text{ m}.$$

Problem 2.13

Link DB in Fig. 2.53 is made of aluminum (E = 72 GPa) and has a cross sectional area of 300 m Link CG is made of brass (E = 105 GPa) and has a cross-sectional area of 240 mm². Knowing that he support rigid member HBC, determine the maximum force F that can be applied vertically at points if the deflection of H cannot exceed 0.35 mm.

F - 16.4 km

Problem 2.14

In Fig 2.54 a vertical load F is applied at the center B of the upper section of a homogeneous conic frustum with height h. minimum radius a, and maximum radius 2a. Youngs modulus for the materia is denoted by E and we can neglect the weight of the structure, determine the deflection of point B.



Problem 2.15

Determine the reaction at D and B for a steel bar loaded according to Fig. 2.55, assume that a 4.5 mm clearance exists between the bar and the ground before the load is applied. The bar # ## (E = 200 GPa).

(R. - 430.8 kN. R. - 769.2 kM

Problem 2.16

Comparison centric force of N - 1000 N is applied at both ends of the assembly shown in Fig 2.56 by means of mail and plates. Knowing that Emm. = 200 GPa and BAUMONUM = 70 GPa. determine (a) normal stresses in the steel core and the aluminum shell. (b) the deflection of the assembly.

[σ_{serences} = 3.32 MPa σ_{seren} 9.55 MPa; ΔL = 4.74×10³ mm]

Problem 2.17

Two cylindrical rods in Fig. 2.57, one made of steel (E_{strate} = 200 GPa) and the other of brass (E_{strate} = 105 GPs), are joined at B and restrained by supports at G and C. For the given load, determine (a) the reaction at G and C. (b) the deflection of point B.

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[R. - 134 kN; R. - 266 kN; DL. = - 0.3 mm]

stress and Strain - Asta

Problem 2.12

Both portions of red GBC in Fig. 2.52 are made of aluminum for which E = 70 GPa. Knowing that the magnitude of F is 4 kN, determine (a) the value of F₁ so that the deflection at point A is zero. (b) the corresponding deflection of point B, (c) the value of stress for each partice.

 $[F_1 = 32.8 \text{ kN}; \Delta L_0 = 0.073 \text{ mm}; \sigma_{c0} = 12.73 \text{ MPa; } \sigma_{pc} = 10.19 \text{ Mpc}$

Problem 2.13

Link DB in Fig. 2.53 is made of aluminum (E = 72 GPa) and has a cross-sectional area of 300 may Link CG is made of brass (E = 105 GPa) and has a cross-sectional area of 240 mm³. Knowing that they support rigid member HBC, determine the maximum force F that can be applied vertically at point $\frac{1}{10}$ if the deflection of H cannot exceed 0.35 mm.

[F - 164 km

Problem 2.14

In Fig 2.54 a vertical load F is applied at the center B of the upper section of a homogeneous contain frustum with height h. minimum radius a, and maximum radius 2a. Youngs modulus for the material is denoted by E and we can neglect the weight of the structure, determine the deflection of point B.



Problem 2.15

Determine the reaction at D and B for a steel bar loaded according to Fig. 2.55, assume that a 10^{-1} mm clearance exists between the bar and the ground before the load is applied. The bar is 10^{-1} (E = 200 GPa),

IR. = 430.8 kN. R. = 769.2 kN

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Problem 2.16

Compressive centric force of N = 1000 N is applied at both ends of the assembly shown in Fig 2.56 by means of real and place. Knowing that $E_{max} = 200$ GPs and $E_{AUDENDA} = 70$ GPs, determine (a) normal areas in the steel core and the aluminum shell. (b) the deflection of the assembly.

 $[\sigma_{arrange}=3.32~\text{MPa};\sigma_{arrag}=9.55~\text{MPa};~\text{SL}=4.74\times10^{9}~\text{mm}]$

Problem 2.17

Two cylinderical rods in Fig. 2.57, one made of steel (E_{max} = 200 GPa) and the other of brass (E_{max} = 105 GPa), are joined at B and restrained by supports at G and C. For the given load, determine (a) the reaction at G and C, (b) the deflection of point B.

 $[R_{a} = 134 \text{ kN}; R_{c} = 266 \text{ kN}; DL_{a} = -0.3 \text{ mm}]$

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Machanics of Materia

Problem 2.18



The rigid bar HBC is supported by a pin connection at the end of rol CB which has a cross-sectional area of 20 mm² and is made of aluminum (E = 68.9 GPs). Determine the vertical deflection of the $b_{\rm H}$ at point D in Fig. 2.58 when the following distributed load w=300N/s is applied.



 $\left[\Delta L = \frac{\gamma L^2}{2E} + \frac{FL}{EA}\right]$

Problem 2.19



The bar has length L and cross-sectional area A. (see Fig. 2.59) Determine its elegation due to the limit F and its own weight. The material has a specific weight y (weight / volume) and amodulus of elasticity E

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3 Torsion

31



-

In the previous chapter we discussed axially loaded members and we analyzed the strenges and strains in these members, but we only considered the internal force directed along the axis of each members without observing any other internal force. Now we are going to analyze strenges and strains in members subjected to *twisting couples* or *torques T* and *T*, see Fig. 3.1. Torques have a common magnitude as opposite sense and can be represented either by curved arrows or by couple vectors, see Fig. 3.2.



Fig. 3.2 Alternative representations of torques

Members in torsion are encountered in many engineering applications and are primarily used to transmit power from one point to another. These shafts play important roles in the automotive and power industry. Some applications are presented in Fig. 3.3.



Fig. 2.3 Transmitting else is http://www.deschodustor.com/

There is a parallelism between an axially loaded member and a member in torsion. Both vectors d applied force F and applied torque \tilde{T} act in the direction of the member area, see Fig. 3.4. Further ds will see the results of a deformation analysis speak more about this parallelism.

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This chapter contains two different approaches in solving tersion problems. Firstly we will present the theory for members with circular cross-sectional areas (circular members in short) and secondly we will extend our knowledge of this theory for application on non-circular members.

3.2 Deformation in a Circular Shaft



88 Download five eBooks as bookboon.com Let us consider a circular shaft fixed to a support at point B while the other end is free, see Fig. 3.5 The shaft is of length L with constant circular cross-sectional area A. If the torque T is applied at point (free end of shaft), then the shaft will twist, i.e. the free end will rotate about the shaft axis through angle of twist φ and the shaft axis remains straight after applying the load.

Before applying the load, we can draw a square mesh over the cylindrical surface of the shaft as well warying diameters on the front circular surface of the shaft, see Fig. 3.6(a). After applying the load an under the assumption of a small angle of twist (less than 5°) we can observe the distortion in Fig. 3.6(b).

- 1. All surface lines on the cylindrical part rotate through the same angle y
- 2. The frontal cross-sections remain in the original plane and the shape of every circle remains undistorted as well.
- 3. Diameters on the front face remain straight.
- 4. The distances between concentric circles remain unchanged.

Torsio



Fig. 3.4

These experimental observations allow us to conclude the following hypotheses:

- 1. All cross-sectional areas remain in the original plane after deformation.
- 2. Dismeters in all cross-sections remain straight.
- 3. The distances between any arbitrary cross-sections remain unchanged

The acceptability of these hypotheses is proven by experimental results. The aforementioned hypotheses is no strain along the member axis. Applying equation (2.5) for isotropic material, we get

 $\varepsilon_z = 0 \implies \varepsilon_y = \varepsilon_z = 0$

(3.1)

90

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Tor sin



Pig. 3.7

Using equations of elasticity (2.10) we have $\sigma_x = 0$. Equation (3.1) means that the edge dimensions of the unit cube are unchanged, but the shape of unit cube is changing. This can be proven with a small experiment. Let us imagine a circular member composed of two wooden plates which represent the faces on the front of the member. Now consider several wooden slats that are nailed to these plates and make up the cylindrical marface of the member, see Fig. 3.7. Let us make two markers on each meighbouring slat, see Fig. 3.7(a). These markers represent the top surface of the unit cube. After applying a load, the markers will slide relative to each other, see Fig. 3.7(b). The square configuration will then be deformed into a rhombus which proves the existence of a shearing strain.



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Torsion





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We can now determine the shearing strain distribution in a circular shaft, see Fig. 3.5, and which has been twisted through the angle φ , see Fig. 3.8(a). Let us detach the inner cylinder of radius ρ , $\rho \in \{0, z\}$ from the shaft. Now lets consider a small square element on its surface formed by two adjacent circles and two adjacent straight lines traced on the surface of the cylinder before any load is applied, see Fig. 3.8(b). Now subjecting the shaft to the torque T, the square element becomes deformed into a rhombus see Fig. 3.8(c). Recalling that, in section 2.5, the angular change of element represents the shearing strain. This angular change must be measured in radians.

From Fig. 3.8(c) one can determine the length of arc *EE* using basic geometry: *EE* = Ly or *EE* = $_{Om}$

Then we can derive

 $\gamma = \frac{\mu \mu}{L}$

(31)

where y, φ are both considered to be in radians. From equation (3.2) it is clear for a given point on the shaft that the shearing strain varies linearly with the distance ρ from the shaft axis.

Due to the definition of inner radius ρ the shearing strain reaches its maximum on the outer surface of the shaft, where $\rho = R$. Then we get

 $\dot{\gamma}_{max} = \frac{\rho_W}{L}$ (3.3)

Using equations (3.2) and (3.3) we can eliminate the angle of twist. Then we can express the shearing strain y at an arbitrary distance form the shaft axis by the following:

 $\gamma = \frac{p}{g} \gamma_{max}$



tion to Mechani > of Meterials: Part I

Stress in the Elastic Region



Fig. 3.8





Pip. 3.10

Torsion

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Let us consider a section BC of the circular shaft with constant diameter D along its length L, subjected to torques T and T' at its ends, see Fig. 3.9. Applying the method of sections, we can divide the shaft two arbitrary portions BQ and QC at any arbitrary point Q. In order to satisfy conditions of equilibrium for each part separately, we need to represent the removed part with internal forces. In our case, i_{Prink} the equilibrium equations, we get non-zero values only for the torque T(x), see Fig. 3.10(a) torque represents the resultant of all elementary abearing forces dP exerted on a section at point Q, see Fig. 3.10 (b). If the portion BQ is twisted, we can write

$$\int \rho dF = T_{(\pi)} \tag{3c}$$

where ρ is the perpendicular distance from the force dF to the shaft axis. The shearing force dF can be expressed as follows $dF = \tau dA$, then substituting into equation (3.5) we get

$$\rho \tau dA = T_{(x)} \tag{3.6}$$

(3.7)

Recalling Hookes law from Section 2.5 we can write

 $T = G\gamma$

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to Mechanics of Materials: Part I

ad applying equation (3.4) we get

$$G\gamma = \frac{p}{R}G\gamma_{max}$$



This equation shows that the shearing stress also varies linearly with the distance ρ from the shaft axis, as long as the yield stress is not exceeded. The distribution functions of shearing stress are presented in Fig. 3.11(a), for a solid circle, and in Fig. 3.11(b) for a hollow circle ($\rho \in (R_1, R_2)$). For the latter case we can write

$$\tau_{max} = \frac{\mu_1}{\mu_2} \tau_{max} \qquad (3.10)$$

The integral equation (3.6) determines the relationship between the resultant of internal forces T(x) and the elements areas r. Substituting r from equation (3.9) into (3.6) we get

$$T_{\rm (x)} = \frac{m_{\rm HL}}{2} \int \rho^2 dA \tag{3.12}$$

The integral in the last member represents the polar moment of inertia J with respect to its centre O, for more detail see Appendix A. Then we have

$$T_{(\mu)} = \frac{\tau_{max}}{\mu} f$$
 or $\tau_{max} = \frac{T_{(\mu)}}{l} R$ (3.13)

Substituting equation (3.9) into (3.13) we get

$$\hat{\tau} = \frac{\hat{\tau}_{(x)}}{2}$$
(3.14)

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Torslos

(3.8)

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3.4 Angle of Twist in the Elastic Region



When observing the deformation of a circular shaft subjected to a torque *T*, are Fig. 3.12, we can us the rotation of the free end *C*, about the shafts axis or angle of twist φ . The entire shaft remains in the elastic region after applying the load. The considered shaft has a constant, circular cross-section with a maximum radius *R*, and a length of *L*. Now we can recall equation (3.3) where the maximum shears strain *Ymax* and the angle of twist are related by the following

$$\eta_{max} = \frac{\mu_F}{L}$$
(3.9)

We are assuming that there is elastic response, therefore we can apply Hookes law for simple sher $\gamma_{max} = \tau_{max}/G$. After substituting equation (3.13) into Hookes law, and knowing that T(x) = 7. T(x) = T along the whole axis of the shaft, we get

$$m_{RR} = \frac{T_{(k)}}{c_l}R = \frac{T}{c_l}R$$
(3.19)

Equating the right-hand members of equations (3.3) and (3.15), and solving for φ we have

$$p = \frac{T_{GZ}}{G} = \frac{TL}{G}$$
(3.16)

The obtained formula shows that the angle of twist is proportional to the applied torque within the elastic region. If we compare the results of equation (2.15) from chapter 2, one can conclude the following parallelism: $\Delta L \triangleq \varphi$. $N_{(x)} \triangleq T_{(x)} \models E \equiv G$, $A \triangleq J$. This equation is valid only if the sheft is homogenous material (constant G), has a uniform cross-sectional area (constant J), and is loaded at its

If the shaft is composed from several different parts, each individually satisfying the validity of equation (3.16), we can extend formula (3.16) using the principles of superposition as follows:

$$\varphi = \sum_{i=1}^n \varphi_i = \sum_{i=1}^n rac{\mathbb{T}_{1:i=1}}{\mathbb{T}_{i=1}}$$

(3.17)

Machanics of Materials: Part I

duste Ten), Gi, Ji, Li is the internal torque, shear modulus, polar moment of inertia and length monding to the part i respectively.

the case of variable cross-sectional area along the shaft, as in Fig.3.12, the strain depends on the position of the arbitrary point Q, therefore we must apply a similar equation to (2.2) for the computation the destring strain. After some mathematical manipulation the total angle of twist of the member is

$$= \int_{(L)} \frac{T_{(n)}}{q} dx \qquad (3.18)$$

35 Statically indeterminate Shafts



Torsion

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Until now, we have discussed statically determinate problems. But there are some situations, where the internal torques can not be determinated using statics alone. For simplicity, let us consider a single problem, see Fig. 3.13. In this case we cannot solve the problem through equilibrium equations from statics alone. The main difficulty in this problem is that the number of usknown reactions is greater than the number of equilibrium equations. From a mathematical point of view, the problem is ill-conditional. For our case we obtain one equilibrium equation to be

$$\sum T_x = 0 : T_c - T + T_n = 0 \tag{3.10}$$

This problem is statically indeterminate. To overcome this difficulty we must use the same approach as in Chapter 2, Section 2.7, i.e. to add deformation conditions. In our case the angle of twist at point C is equal to zero, and corresponds to the total angle of twist

$$\varphi = \varphi_c = \sum_{i=1}^2 \varphi_i = 0 \tag{3.19}$$

Using equation (3.17) we obtain

$$\varphi = \sum_{i=1}^{2} \varphi_i = \varphi_1 + \varphi_2 = \frac{T_{1(j_1)}L_1}{u_{f_1}} + \frac{T_{2(j_2)}L_2}{u_{f_2}} = 0$$
 (5.20)

Both internal torques $T_{1(\pi)} = T - T_c$, $T_{2(\pi)} = T_c$ are functions of unknown reaction T_c . Solving equation (3.20) we obtain the value of reaction $T_c = \frac{f_2 t_1}{(f_2 t_1 - f_1 t_2)}T$. We can then continue by solving in the usual way (for statically determinate problems).

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to Mechanics of M. . . Part I

Design of Transmission Shafts

transmission shafts the principal specifications that must be satisfied are the power to be the velocity of rotation. Our task now is to select the material and the type and the size tion to satisfy the strength condition, i.e. the maximum shearing stress will not exceed the double shearing stress $T_{max} \leq T_{dit}$, when the shaft is transmitting the required power at the specified Recalling elementary physics we have

$$P = T\omega = 2\pi f T \tag{3.21}$$

where P is the transmitted power, ω is the angular velocity, and f is the frequency of rotation. Solving (3.21) for T obtains the torque exerted on our shaft which is transmitting the required power P is a frequency of rotation f.

$$T = \frac{P}{2\pi f}$$
(3.22)

Now we can apply the strength condition using equation (3.13) as follows

$$\tau_{\text{max}} = \frac{\tau}{l} R \le \tau_{all} \tag{3.23}$$

Substituting equation (3.22) into (3.23) we get

$$\frac{r}{1 \pi t/r} R \le \tau_{\alpha R} \qquad \text{or} \qquad \frac{1}{R} \ge \frac{r}{2 \pi f \tau_{\alpha R}}$$
(3.24)

The value *UR* represents the allowable minimum. This variable is known as *the section modulus* and can be found in any common section standards.

37 Torsion of Non-Circular Members



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Applying the methods of mathematical theory of elasticity for the bar with rectangular cross technicity, we will get the stress distribution functions presented in Fig. 3.17. The corner stresses are training zero. We can find the two local stresses which are largest at point I and II (Roman numerals). Denote L as the length of the bar, b and h as the narrow and wide side of bar cross-section respectively as the applied torque, see Fig. 3.18, we have

$$\tau_I = \tau_{max} = \frac{\tau}{ahb^2}$$
 and $\tau_H = \beta \tau_H$ (3.27)

The coefficient α, β depend only upon the ratio h/b. The angle of twist can be expressed as

$$\varphi = \frac{\pi}{\gamma Ghs^3}$$
(3.2b)

The coefficient yalao depends only upon the ratio h/b. All coefficients $\alpha_{i}\beta_{i}$ y are presented in the following Tab. 3.1.

h/b	1.00	0.50	1,75	2.00	2.50	3,00	4,00	6,00	8.00	1000	00
α	0,208	0,231	0.239	0,246	0.258	0,267	0,282	0.299	0,307	0,313	0.333
β	1,000	0,899	0.820	0,795	Q776	Q.753	0,745	0,743	0,742	Q742	0.742
7	0,141	0,196	0.214	0,229	0,249	0,263	0.261	0.299	0,307	Q313	0.333

Teh. 3.1

and the Machanics of Materials Part (

(3.29)



the stress distribution function over the non-circular cross-section can be visualised by the membrane energy. Firstly, what does this analogy mean? Two processes are analogous if both can be describe by the same type of equations. In our case we have the twisting of a non-circular bar and the deformation of a this membrane subjected to internal pressures, see Fig. 3.19. Both processes are determined by the same type of differential equations. Secondly, we need to determine the analogous variables. In our case we have

T ▲ volume bouded by the deformed membrane and horizontal plane value of shearing strain ▲ tangent of maximum slope direction of shearing strain ▲ horizontal tangent



implies representation of these equations is presented in Fig. 3.20.

Its manifitance analogy can be efficiently applied for members whose cross-section can be unrolled into the basic rectangle *lack*, see Fig. 3.21. Another application of the membrane analogy is for members with cross-sections composed from several rectangles, see Fig. 3.22. These cross-sections cannot be unrolled one simple rectangle *basis*. For this case we can assume that the total volume of deformed membrane the sum of individually deformed membranes, see Fig. 3.23. If the torque is analogous to the rectangle, and then we can write

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 $i=\varphi_1=\varphi_2=\cdots=\varphi_n$

ter simple asthematical manipulations of these equations we determine that the total torsional stiffness quals to thesum of individual torsional stiffness' of each rectangle, i.e.

$$\gamma h b^{3} = \sum_{i=1}^{n} \gamma_{i} h_{i} b_{i}^{3}$$
(33)

sequently the largest stress corresponding to each rectangle can be found by

 $T_I = \frac{T_I}{x_I h_I h_I^2}$



Fig. 3.23

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Thin-Walled hollow Members



is the previous section we discussed members with open non-circular crom-sections subjected to remional loading. The results obtained in the previous section required advanced theory of elasticity. For thin-walled hollow members we can apply some simple computations to obtain results.

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Let us consider the thin-walled hollow member of non-circular cross-section, see Eig. 3.24 The wall thickness varies within the transverse section and remains very small in comparison to other dimensions. Let us detach a small coloured portion DE. This portion is bounded by two parallel transverse sections by the distance Δx and two parallel longitudinal planes. Focusing on the equilibrium of part DE is the longitudinal direction x, the shear law says that the shear forces F_D , F_E are exerted on faces D and E see Fig. 3.25. We then get the corresponding equation

$$\sum F_{x} = 0 : F_{D} - F_{E} = 0 \tag{330}$$

The longitudinal shear forces F_D , F_E are acting on the small faces of areas $\Delta x t_D$ and $\Delta x t_E$. Thus we can express the force as a product of shearing stress and area, i.e.

$$F_D = \tau_B A_D = \tau_D \Delta x t_0 \quad F_E = \tau_E A_E = \tau_E \Delta x t_E \tag{335}$$

Substituting equation (3.35) into (3.34) we get

$$\tau_D \Delta x t_D - \tau_E \Delta x t_F = 0$$

or

$$\mathbf{t}_D \mathbf{t}_D = \mathbf{t}_E \mathbf{t}_E$$

Since the selection of portion DE is arbitrary, and then the product πi is constant throughout the member. Denoting this product by q we get

$$q = \tau t = constant$$



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(3.37)

(3.36)

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Torsior



Pip. 3.27

This new variable describes the shear flow in the member. The direction of shearing stress is determined by the direction of shear forces and the application of the shear haw as one can see in Fig. 3.26 and Fig. 3.27.



Fig. 3.28

let us consider a small element ds which is a portion of the wall section, see Fig. 3.28. The componding area is dA - tds. The resultant of shearing stresses exerted within this area is denoting by dF or

$$dF = \tau dA = \tau t ds = q ds \tag{3.38}$$

The moment dM_c of this force about the arbitrary point C is

 $dM_c = pdF = pqds = qpds$

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(3.39)

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Where p is the distance of C to the action line of dF. The action line passes through the centre of the element and the product pds represents the doubled area dA, see Fig. 3.29. We then have

$$dM_c = g2dA$$

In a mathematical point of view, the integral of moments around the wall section represents the resulting moment that is in equilibrium with the applied torque *T*. Thus we have

$$T = \oint dM_c = \oint g 2 dA$$

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(3.41)

(3.60)

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sace the shear flow is constant, we get

$$T = q \neq 2dA = q2A$$



Where A is the area bounded by the centreline of the section, see Fig. 3.30. From the previous equation we can easily derive the formula for calculating the shearing stress

$$r = \frac{T}{2LA}$$
 (3.43)

The corresponding angle of twist can be derived by using the method of strain energy, see Appendix A.4.2. We then get

$$\varphi = \frac{7L}{4m^2 G} \oint \frac{dx}{L}$$
(3.44)

If the antiion can be built from several parts of constant thicknesses it is known to be piecewise constant, equation (3.44) can then be simplified

$$p = \frac{TL}{4\pi d^2 G} \sum_{l=1}^{n} \frac{\Delta a_l}{t_c}$$
(3.45)

39 Bramples, Solved and Unsolved Problems

Problem 3.1



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(3.42)

Introduction to Mechany s of Materials: Part I

For the steel shaft with applied torque T = 2400 Nm shown in Fig. 3.31 (G = 77 GPa), determine (a) the maximum and minimum shearing stress in the shaft, (b) the angle of twist at the free end. The shafthen the following dimensions: L = 500 mm, $D_s = 40 \text{ mm}$, $D_s = 50 \text{ mm}$.

Solution



Fig. 3.33



Pig. 3.32

The shaft in Fig. 3.32 consists of one portion, which has uniform cross-section area and constant internal torque. From the free body diagram in Fig. 3.33 we find that:

$$\sum M_{x} = 0: T(x) + T = 0$$

T(x) = -T = -2400 Nm

The polar moment of inertia (see Appendix A.2) is

$$J = J_{mul} - J_{mul} = \frac{\pi D_{mul}^{i}}{32} - \frac{\pi D_{mul}^{i}}{32}$$
$$J = \frac{\pi (50 \text{ mm})^{4}}{32} - \frac{\pi (40 \text{ mm})^{4}}{32} = 362265 \text{ mm}^{4}$$

Machanics of Materia Part

Maximum shearing stress. On the outer surface, we have

$$r_{m} = \frac{T}{J}\rho_{m} = \frac{T}{J}\frac{D_{max}}{2} = \frac{2400 \times 10^3 \text{ N mm}}{362265 \text{ mm}^4} \times \frac{50 \text{ mm}}{2}$$

Tent = 165.5 MPa

Mannum shearing stress. The stress is proportional to its distance from the axis of the shaft







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lorsion

ntroduction to Mechanics of Materials: Part I

$$\frac{\mathbf{D}_1}{\mathbf{r}_m} = \frac{\mathbf{D}_1}{\frac{\mathbf{D}_2}{2}} = \frac{\mathbf{D}_1}{\mathbf{D}_2} \implies \mathbf{r}_m = \mathbf{r}_m = \frac{\mathbf{D}_1}{\mathbf{D}_2}$$
$$\mathbf{r}_m = 165.6 \text{ MPa} = \frac{40 \text{ mm}}{50 \text{ mm}} = 132.5 \text{ MPa}$$

Another way th determine this is by:

$$f = -\frac{T}{J} = -\frac{T}{J} \frac{D_{HGR}}{2} = \frac{2400 \times 10^3 \text{ N mm}}{362265 \text{ mm}^4} + \frac{40 \text{ mm}}{3}$$

r_ = 132.5 MPa



Fip. 3.25

Graphically we can show shearing stress in Fig. 3.34 and the diagram of torque along the length of the shaft is shown in Fig. 3.35.

Angle of twist.

Using Eq. (3.16) and recalling that G = 77 GPa for the shaft we obtain

$$= \frac{TL}{GJ} = \frac{2400 \times 10^3 \text{ N mm} \times 500 \text{ mm}}{77 \times 10^3 \text{ N/mm}^2 \times 362265 \text{ mm}^4}$$

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Torsion



The vertical shaft AC is attached to a fixed base at C and subjected to a torque T shown in Fig. 3.36. Determine the maximum shearing atress for each portion of the shaft and the angle of twist at A. Portion AB is made of steel for which G = 77 GPa with a diameter of $D_{\text{symat}} = 30$ mm. Portion BC is made of brass for which G = 37 GPa with a diameter of $D_{\text{symat}} = 50$ mm. Parameter L is equal to 100 mm

Solution



The complete shaft consists of two portions, AB and BC (see Fig. 3.37), each with uniform cross-section and constant Internal torque.

 $x_1 \in (0, L)$



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Solution of portion AB

Passing a section though the shaft between A and B and using the free body diagram shown $Fi_{B, 3,34}$ we find

$$\sum M_{-t} = 0$$
: $T_t(x) + T = 0$ $T_t(x) = -T$



Fig. 3.30

The maximum shearing stress is on the outer surface, we have

$$t_{\max,1} = \frac{[T]}{J} \rho_{\max,1} = \frac{[T]}{T} \frac{D_{\max,1}}{2} = \frac{[T]}{\frac{\pi D_{\max,1}^2}{32}} \frac{D_{\max,2}}{2}$$

$$r_{max} = \frac{101}{m^2} = \frac{101}{m^2} = 1.886 \times 10^{-4} \text{T}$$

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Torsion

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Disgram of the shearing stress across the cross-section area is shown in Fig. 3.39.

Selution of portion BC



Now passing a section between B and C (see Fig. 3.40) we obtain

$$\sum M_{ust} = 0: T_{u}(x) + T - 2T = 0 T_{u}(x) = T$$

Again, the maximum shearing stress is on the outer surface, found by the following

$$r_{max x} = \frac{|T_{a}|}{y} \rho_{max x} = \frac{|T|}{y} \frac{D_{a}}{2} = \frac{|T|}{y} \frac{D_{max m}}{32} \frac{D_{max m}}{2}$$

$$r_{max} = \frac{16 T}{y} \frac{16 T}{y} = \frac{16 T}{y} = 4.074 \times 10^{-5} T$$

$$T_{11} x_{11}$$



Graphically, the shearing stress is shown in Fig. 3.41.

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When we compare the results from both portions the maximum shearing stress is in portion AB. which compares with the allowable stress. From this inequality, we have the unknown torque T.

$$\tau_{max} = \tau_{max1} \leq \tau_{ax}$$

$$\tau_{max1} = \frac{16 \text{ T}}{\text{mD}_{max1}^2} \leq \tau_{ax} \implies T \leq \frac{\tau_{AB} \pi D_{STRM}^2}{16}$$

$$T \leq \frac{\tau_{AB} \pi D_{STRM}^2}{16} = \frac{150 \text{ MPa} \times \pi \times (30 \text{ mm})^2}{16} = 795215.6 \text{ Nmm}$$

T ≤ 795215,6 Nm



Choosing the torque T = 795 kNm. We can graphically represent the torque along the length of shaft in Fig. 3.42.

Angle of twist

Using Eq. (3.17), we have

$$\varphi = \sum_{r} \frac{T_{L}}{J_{r}G_{1}}$$

$$\varphi_{n} = \frac{T_{nn} L_{nn}}{J_{AB} G_{AB}} + \frac{T_{nc} L_{nc}}{J_{Bc} G_{Bc}}$$

$$\varphi_{n} = \frac{T_{nn} L_{nn}}{\frac{\pi D_{max}^{h}}{32} G_{AB}} + \frac{T_{nc} L_{nc}}{\frac{\pi D_{maxB}}{32} G_{Bc}}$$

$$\varphi_{n} = \frac{32 T_{AB} L_{AB}}{\pi D_{max}} + \frac{32 T_{nc} L_{Bc}}{\pi D_{maxB}^{h}} = -9.48 \text{ mm}$$

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tion in Mechanics of Materials: Part I

Torsion





A torque T is applied as shown in Fig. 3.43 to a solid tapered shaft Armine the maximum shearing, stress and show, by integration, that the angle of twist at A is

$$\varphi_{\Lambda} = \frac{7 \,\mathrm{T} \,\mathrm{L}}{12 \,\mathrm{x} \,\mathrm{G} \,\mathrm{c}^4}.$$

The radius c, length L, modulus of rigidity G and applied torque 1 ren

ntroduction to Mechanics of Materials: Part 1

Torsion

Solution



Weonly have one part so from free body diagram (see Fig. 3.44), we find

$$\sum M_{n} = 0$$
: $T(x) - T = 0 \implies T(x) = T$







The maximum of shearing is on the outer sum face. The radius c(x) at location x is found from similarity of triangles, Fig. 3.45.

$$m \beta = \frac{c}{L} = \frac{c(x) - c}{x} \implies cr(x) = c \left(1 + \frac{x}{L}\right),$$

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The diameter IX(x) at location x in

$$D(x) = 2 c(x) \implies D(x) = 2 c \left(1 + \frac{x}{L}\right)$$

Moment of inertia at location x is

ю

$$x_{1} = \frac{x D(x)^{4}}{32} = \frac{x \left[2x \left(1 + \frac{x}{L}\right)\right]^{4}}{32} = \frac{1}{32}$$

The maximum shearing stress at position x on the outer surface is

$$r_{max}(x) = \frac{|T|}{J(x)}\rho_{max} = \frac{|T|}{J(x)}\frac{D(x)}{2} = \frac{16 \text{ T}}{\pi \left[2c\left(1+\frac{x}{L}\right)\right]^2}$$

Angle of twist is determined from the definition of the angle of twist Eq. (3.18), and we have

$$\varphi = \int_{0}^{L} \frac{T(x)}{G J(x)} dx = \int_{0}^{L} \frac{32 T}{G = \left[2e \left(1 + \frac{x}{L}\right)\right]^{4}} dx = \frac{32 T}{G = 16 e^{4}} \int_{0}^{L} \frac{1}{\left(1 + \frac{x}{L}\right)} \varphi = \frac{7 T L}{12 G = e^{4}}$$

in the fig. 3.46 is a graph of the torque along length L.

Problem 3.4

103	Ta	6335
B 8	C	H
100	14	1000

Fig. 3.47

A circular shaft BH is attached to fixed supports at both ends with a torque T applied at the midsection (1) 3.47). Determine the torque exerted on the shaft by each of the supports and determine the maximum shearing stress.

Is length L, modulus of rigidity G and applied torque T, are given.

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Solution



The problem is statically indeterminate. The support at point H is replaced by an unknown support reaction T_{μ} (horizontal and vertical reactions are equal to zero, because this is a problem of pure torsion). The solution is divided into two part (see Fig. 3.48).

Free-body diagram on portion 1 (part HC):

x, e (0, L)



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non the equilibrium equation of the first part, we obtain

$$\sum M_{m} = 0 \qquad T_{1}(x_{1}) + T_{m} = 0 \qquad \Rightarrow \qquad T_{1}(x_{1}) = -T_{m}$$

net-body diagramon portion II (part CB):

x, (L. 2L)



From the equilibrium equation of the second part, we obtain

$$\sum M_{u_{u}} = 0: \quad T_{u}(x_{u}) - T + T_{H} = 0 \quad \Rightarrow \quad T_{u}(x_{u}) = -T_{H} - T$$

The unknown runction is determined from the deformation condition, that the total angle of twist of shaft BH must be zero, since both of its ends are restrained. J, and J, denote the angle of twist for portions AC and CB respectively, we write

 $\varphi_{\rm H} = 0 \implies \varphi_{\rm H} = \varphi_{\rm I} + \varphi_{\rm H} = 0 \implies \varphi_{\rm I} + \varphi_{\rm H} = 0,$

from which we have

$$\frac{\mathbf{T}_{\mathbf{L}_{1}}}{\mathbf{G}_{\mathbf{J}_{1}}} + \frac{\mathbf{T}_{\mathbf{D}}\mathbf{L}_{\mathbf{L}}}{\mathbf{G}_{\mathbf{D}}\mathbf{J}_{\mathbf{D}}} = 0$$

where $G_i = G_n = G$, $J_i = J_n = J$ and $L_i = L$ because both parts of shaft are made from same material, have the same cross-section area, and the same length. Then solving for T_n , we have

$$T_{t}(x_{1}) + T_{t}(x_{t}) = 0 \implies -T_{t} - T_{t} + T = 0 \implies T_{t} = \frac{T}{2}$$

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Substituting the results for each part, we obtain

$$T_{I}(x_{I}) = -T_{H} = \frac{T}{2}$$
 $T_{H}(x_{H}) = -T_{H} - T = -\frac{T}{2} - T = -\frac{T}{2}$

The diagram of torque is shown in Fig. 3.49.



Fip.3.49



Fig. 3.50



Fip. 1.51

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Fig. 3.52

Reaction at point B.

Drawing a free-body diagram of the shaft and denoting the torques exerted by supports $T_{\rm p}$ and $T_{\rm pr}$ (see Fig. 3.50) we obtain the equilibrium equation

$$\sum M_{n_1} = 0; \quad T_n + T_n - T = 0 \quad \Rightarrow \quad T_n = T - T_n = \frac{T}{2}$$

The maximum shearing stress at part HC (outer surface) is

$$I_{1} = -\frac{|T_1|}{J_1} \rho_{max} = \frac{\left|-\frac{1}{2}\right|}{\frac{\pi D^4}{32}} \frac{D}{2} = \frac{16}{2} \frac{T}{\pi D^3} = \frac{8}{\pi D^3}$$

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The maximum shearing stress at part BC (outer surface) is



The diagram of shearing stresses for each part is shown in the Fig. 3.51and Fig. 3.52.

Problem 3.5



The bars in Fig. 3.53 have a square and rectangular cross-section area. Knowing that the magnitude of torque T is 800 Nm determine the maximum shearing stress for each bar.

The dimensions are given by L = 400 mm, a = 50 mm and b = 35 mm

Solution

For a bar with square cross-section area (see Fig. 3.53a) and bar with rectangular cross-section area (see Fig. 3.53b), the maximum shearing stress is defined by Eq. (3.27)

$$r_{m} = \frac{T}{\alpha a b^2}$$

where the coefficient als obtained from tab. 3.1 in section 3.7. We have

 $\frac{a}{b} = \frac{50 \text{ mm}}{50 \text{ mm}} = 1 \implies \alpha = 0.20B \text{ for square cross section}$

and

$$\frac{a}{b} = \frac{50 \text{ mm}}{30 \text{ mm}} = 1.43 \implies \alpha = 0.231 \text{ for rectangular cross section.}$$

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starimum shearing stress for square cross-section in Fig. 3.53a is

$$r_{\rm me} = \frac{T}{\alpha \ {\rm a} \ {\rm b}^2} = \frac{800 \ {\rm Nm}}{0.208 \times 0.050 \ {\rm m} \times (0.050 \ {\rm m})} = 30.77 \ {\rm MPa}$$

Maximum shearing stress for rectangular cross-section in Fig. 3.53b is

$$T_{\rm m} = \frac{T}{\alpha + b^2} = \frac{800 \text{ Nm}}{0.208 \times 0.050 \text{ m} \times (0.035 \text{ m})^2} = 1.98 \text{ MPa}.$$

Problem 3.6





Two shafts of the name length and made by the same materials is connected by a welded rigid beam. On the ends of the rigid beam amoment couple given by force P is applied. Cross-section area of the shaft is in Fig. 3.54. Design parameter D if wearegiven an allowable stress of $a_{\rm e}$ = 150 MPa.

Given F = 1000 N, c = 200 mm, a = 2D, t = 0.1D, L = 400 mm.

Solution

from the given force, we find the total magnitude of the torque T applied to both shafts.

 $T = Fc = 1000N \times 0.2 m = 200 Nm$

This tarque will then be dived on both shafts and from the equilibrium of the rigid beam, we have

$$T = T_{c} + T_{c}$$

We have two unknowns torques T₃ and T₂ so we need a second equation, which is found from the deformation condition

$$\varphi_1 = \varphi_2 \qquad \Longrightarrow \qquad \frac{T_{\mu}L}{GJ_{\mu}} = \frac{T_{\mu}L}{GJ_{\mu}}, \qquad (b)$$

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where angle of twist for the first cross-section area is

$$J_{1} = \frac{4A^{3}}{\int_{A}^{A} \frac{d_{1}}{t}} = \frac{4(1 \text{ 9D} \times 1 \text{ 9D})^{2}}{2\left(\frac{1.9\text{ D}}{0.1\text{ D}} + \frac{1.9\text{ D}}{0.1\text{ D}}\right)} = \frac{52.1284\text{ D}^{4}}{76} = 0.686\text{ D}^{4}$$

1000

(1)

16)

05

and for the second cross-section is

$$J_2 = \frac{\pi D^4}{32}$$

inserting (c) and (d) into (b), we get

$$T_1 = 6.998 T_2$$

Solving the system of equations (a) and (f), we give

$$T_1 = 0.875T = 0.875 F c = 0.875 \times 200 Nm = 175 Nm$$

$$T_1 = 0.125T = 0.125 F c = 0.125 \times 200Nm = 25 Nm$$

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Torsion

eterimum shearing stress in the first cross-section is

$$r_{max} = \frac{T_1}{2 \text{ A t}_m} = \frac{0.875 \text{ F c}}{2 \times (1.9 \text{ D})^2 0.1 \text{ D}} = \frac{175 \text{ Nm}}{0.722 \text{ D}^2} = \frac{242.4}{\text{ D}^3} \text{ Nm}$$

Maximum shearing stress in the second cross-section is

$$e_{\max B} = \frac{T_2}{\frac{\pi D^3}{16}} = \frac{16 T_2}{\pi D^3} = \frac{16 \times 25 \text{Nm}}{\pi D^3} = \frac{127.3}{D^3} \text{Nm}$$

To design parameter D, we get the maximum shearing stress (from all parts), which compare with the allowable stress, we then get

$$r_{max1} = \frac{242.4}{D^3} \text{ Nm} \le r_{AB} \implies \overline{D} \ge \sqrt{\frac{242.4 \text{ Nm}}{r_{AB}}} = \sqrt{\frac{242.4 \text{ Nm}}{150 \times 10^8 \text{ Nm}^3}}$$

D≥0.012 m

Problem 3.7



A torque T = 850 Nm is applied to a hollow shaft with uniform wall thickness t = 6 mm shown in Fig. 3.55. Neglecting the effect of stress concentration, determine the shearing stress at points a and b. Determine the angle of twist at the end of shaft when L is 200 mm and the modulus of rigidity is G = 77 GPa.

Given: R = 30 mm, t = 6 mm, L₁ = 60 mm, L = 200 mm.

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Solution



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From the definition of maximum shearing stress for thin-walled hollow shafts, we have

$$r = \frac{T}{2At}$$

where A is the area bounded by the centerlins of wall cross-section area (Fig. 3.56 - hatching area), we have

$$\mathbf{A} = \mathbf{\pi}\mathbf{R}_{1}^{2} + \mathbf{J}\mathbf{R}_{1}\mathbf{L}_{1} = \mathbf{\pi}\left(\mathbf{R} + \frac{\mathbf{t}}{2}\right)^{2} + 2\left(\mathbf{R} + \frac{\mathbf{t}}{2}\right)\mathbf{L}_{1}$$

The shearing stress at point a and b is

$$\tau_{1} = \tau_{1} = \frac{T}{2 \,\text{A} \, \tau_{1}} = \frac{850000 \,\text{Nmm}}{2 \times 6 \,\text{mm} \times 7381, 19 \,\text{mm}^{4}} = 9.6 \,\text{MPa}$$

The angle of twist of a thin-walled shaft of length L and modulus of rigidity G is defined

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 $J = \frac{4A^2}{\int \frac{1}{A^2}} = \frac{4 \times (7381.19 \text{ mm}^2)^2}{54.5575} = 3994460.65 \text{ mm}^3$

forsion.

Angle of twist at the end of the shaft is given by the following

integral $\iint \frac{dt}{dt}$ is computed along the contactine of the wall section and we get

$$\varphi = \frac{T L}{G J} = \frac{850000 \text{ Nmm} \times 200 \text{ mm}}{77 \times 10^3 \text{ MPa} \times 3994460.65 \text{ mm}^4} = 5.527 \times 10^4 \text{ rad} = 0.032^\circ$$

 $f \frac{dz}{dt} = \frac{s_1}{t} + \frac{s_2}{t} + \frac{s_3}{t} + \frac{s_4}{t} = \frac{\pi 33 \text{ mm}}{6 \text{ mm}} + \frac{60 \text{ mm}}{6 \text{ mm}} + \frac{\pi 33 \text{ mm}}{6 \text{ mm}} + \frac{60 \text{ mm}}{6 \text{ mm}} = 54 5575$

where the assument of inertia is $J = \frac{4A^3}{\int \frac{ds}{dt}}$

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Unsolved problems



Problem 3.8

A torque T = 750 Nm is applied to the hollow shaft shown in the Fig. 3.58 that has a uniform wall thickness of t = 8 mm. Neglecting the effect of stress concentration, determine the shearing stress at points a and b.

 $[\tau_{a} = \tau_{b} = 16.1 \text{MPa}]$

10 Carl

Problem 3.9

The composite shaft in the Fig. 3.59 is twisted by applying a torque T at its end. Knowing that the maximum shearing stress in steel is 150 MPa, determine the corresponding maximum shearing stress in the aluminum core. Use G = 77 GPa for steel and G = 27 GPa for aluminum.

[1 = 39.44 MPa, T = 10.31kNm]

Problem 3.10

A statically indeterminate circular shaft BH consists of length L and diameter D (portion CH) and length L with diameter 2D (portion BC). The shaft is attached by fixed supports at both ends, and a torque T is applied at point C (see Fig. 3.60). Determine the maximum shearing stress in portion BC and CH, and reaction at the support in point H.

$$T_{m} = \frac{T}{17}, \ r_{max BC} = \frac{32 \text{ T}}{17 \text{ m D}^{2}}, \ r_{max BC} = \frac{16 \text{ T}}{17 \text{ m D}^{2}}$$



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Problem 3.11

Using τ_{a} = 150 MPa, determine the largest torque T that may by applied to each of the steel bars and to the steel tube shown in Fig. 3.61. Given is a = 50 mm, b = 24 mm, t = 8 mm and L = 200 mm.

[(a) T - 531.2 Nm. (b) T - 4233.6 Nm]

Problem 3.12

A 1.25 m long angle iron with L cross-section (shown in Fig. 3.62). Knowing that the allowable shearing meas $t_{ab} = 60$ MPa and modulus of rigidity G = 77 GPa and ignoring the effects of stress concentration. (a) determine the largest magnitude of torque T that may by applied. (b) the corresponding angle of twist at the free ends. The dimensions are h = 50 mm, h = 25 mm, t = 5 mm and L = 200 mm.

[(a) T = 35kNm. (b) j = 31.2 rad]

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Appendix

A.1 Centrold and first moment of areas



Consider an area A located in the zy plane (Fig. A.1). The first moment of area with respect to the z axis is defined by the integral

$$Q_x = \int_A y \, dA \tag{A.1}$$

Similarly, the first moment of area A with respect to the y axis is

$$Q_y = \int_A z \, \mathrm{d}A \tag{A.2}$$

If we use SI units are used, the first moment of Q_a and Q_y are expressed in m³ or mm³.



The centroid of the area A is defined at point C of coordinates y and Z (Fig. A.2), which satisfies the relation

$$\overline{y} = \frac{\int y \, dA}{A}$$

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Appendib



Rs.A.I

When an area possesses an axis of symmetry, the first moment of the area with respect to that axis is zero.

Considering an area A, such as the trapezoidal area shown in Fig. A.3, we may divide the area into simple geometric shapes. The solution of the first moment Q of the area with respect to the z area can be divided into components A_1, A_2 , and we can write

$$Q_s = \int_{A} y \, \mathrm{d} t = \int_{A_1} y \, \mathrm{d} A + \int_{A_2} y \, \mathrm{d} A = \sum_i \overline{y}_i A_i \tag{A.4}$$

Solving the centroid for composite area, we write



Example A.01



For the triangular area in Fig. A.4, determine (a) the first moment Q_{z} of the area with respect to the z task. (b) the \overline{y} ordinate of the centroid of the area.

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(A.3)

(A.5)

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Solution

(a) First moment Q.



We selected an element area in Fig. A.5 with a horizontal length u and thickness dy. From thesimilarity in triangles, we have

$$\frac{u}{b} = \frac{h - y}{h} \qquad u = b \frac{h - y}{h}$$

and

$$dA = u dy = b \frac{h - y}{h} dy$$

using Eq. (A.1) the first moment is

$$Q_s = \int y \, dt = \int y \frac{h-y}{h} \, dy = \frac{b}{h} \int (hy - y^2) \, dy$$
$$Q_s = \frac{b}{h} \left[h \frac{y^2}{2} - \frac{y^2}{3} \right] = \frac{1}{6} b h^2$$

(b) Ordinate of the castroid

Recalling the first Eq. (A.4) and observing that $A = -\frac{1}{2}bh$, we get

$$Q_i = A\overline{y} \implies = \frac{1}{6}bh^2 = \frac{1}{2}bh^2\overline{y} \implies \overline{y} = \frac{1}{3}h$$

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Appendb

A.2 Second moment, moment of areas

Consider spain an area A located in the zy plane (Fig. A.1) and the element of area dA of coordinate y and a. The second moment, or moment of inertia, of area Awith respect to the z-axis is defined as

$$I_{q} = \int_{A} y^{2} \, \mathrm{d}A \tag{A.6}$$

Example A.82

Locate the centroid C of the area A shown in Fig. A.6



Fig. A.4

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Solution

Selecting the coordinate system shown in Fig. A.7, we note that centroid C must be located on the y axis, since this axis is the axis of symmetry than z = 0.



Dividing A into its component parts A, and A,, determine the yordinate of the centroid, using Eq. (A.5)



Similarly, the second moment, or moment of inertia, of area A with respect to the y axis in

$$I_s = \int z^2 dA . \qquad (A.7)$$

We now define the polar moment of inertia of area A with respect to point O (Fig. A.8) as the integral

$$J_a = \int \rho^2 \, \mathrm{d}A \,. \tag{A.8}$$

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Appendi



where μ is the distance from O to the element dA. If we use SI units, the moments of inertia are expressed in m⁴ or mm⁴.

An important relation may be established between the polar moment of inertia J_a of a given area and the moment of inertia J_a and J_a of the same area. Noting that $\rho^2 = y^2 + z^2$, we write

$$J_{\phi} = \int_{A} \rho^{2} dA = \int_{A} (y^{2} + z^{2}) dA = \int_{A} y^{2} dA + \int_{A} z^{2} dA$$

10

$$J_{g} = I_{i} + I_{g}$$
 (A.9)

The radius of gyration of area A with respect to the z axis is defined as the quantity r_a, that satisfies the relation

$$I_s = r_s^2 A \implies r_s = \sqrt{\frac{I_s}{A}} \tag{A.10}$$

In a similar way, we defined the radius of gyration with respect to the y axis and origin O. We then have

$$I_s = r_s^2 A \implies r_s = \sqrt{\frac{I_s}{A}}$$
(A.11)

$$J_a = r_a^3 A \implies r_a = \sqrt{\frac{J_a}{A}}$$
 (A.12)

Industinating for [], and I, in terms of its corresponding radi of gyration in Eg. (A.9), we observe that

$$r_{0}^{2} = r_{1}^{2} + r_{1}^{2}$$
(A.13)

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Example A.03

For the rectangular area in Fig. A.9, determine (a) the moment of inertia *I* of the area with respect to the centroidal axis. (b) the corresponding radius of gyration r.



Fig. A.S

Solution

(a) Moment of inertia I. We select, as an element area, a horizontal strip with length b and thickness dy (see Fig. A.10). For the solution we use Eq. (A.6), where dA = b dy, we have

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$$I_{a} = \int_{A} y^{2} dA = \int_{-b/2}^{-b/2} y^{3} (b dy) = b \int_{-b/2}^{-b/2} y^{2} dy = \frac{b}{3} \left[y^{3} \right]_{-b/2}^{-b/2}$$
$$I_{a} = \frac{b}{3} \left(\frac{h^{3}}{8} + \frac{h^{3}}{8} \right) \implies I_{a} = \frac{1}{12} b h^{3}$$



(b) Radius of gyration r. From Eq. (A.10), we have

$$r_r = \sqrt{\frac{I_r}{A}} = \sqrt{\frac{\frac{1}{12}bh^3}{bh}} = \sqrt{\frac{h^2}{12}} \implies r_s = \frac{h}{\sqrt{12}}$$

Example A.04

But the circular cross-section in Fig. A.11. Determine (a) the polar moment of inertia I_{ij} (b) the moment of inertia I_{ij} and I_{ij}



Rp. A.11

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Solution

(a) Polar moment of inertia. We select, as an element of area, a ring of radius ρ and thickness $d\rho$ (Fig. A.12). Using Eq. (A.8), where $dA = 2 \pi \rho d\rho$, we have





Fig. A.12

(b) Moment of Inertia. Because of the symmetry of a circular area I, = 1, Recalling Eg. (A.9), we can write

$$J_o = l_a + l_g = 2l_a \implies l_a = \frac{J_a}{2} = \frac{\frac{\pi}{32}}{2}$$

$$I_s = I_p = \frac{\pi D^4}{64}$$

A.3 Parallel axis theorem



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Appendix

Considering the moment of inertia I_{x} of an area A with respect to an arbitrary x axis (Fig. A.13). Let us now draw the control of a size, i.e., the axis parallel to the x axis which passes though the areas control C. Denoting the distance between the element dA and axis passes though the centrold Cby y', we write y = y' + d. Substituting for y in Eq. (A.6), we write

$$I_{a} = \int_{A} y^{2} dA = \int_{A} (y'+d)^{2} dA ,$$

$$I_{a} = \int_{A} y^{d} dA + 2d \int_{A} y' dA + d^{2} \int_{A} dA ,$$

$$I_{a} = \overline{I}_{a'} + Q_{a'} + Ad^{2}$$
(A.14)

where $\overline{I}_{a'}$ is the area is moment of inertia with respect to the centroidal z axis and Q_a is the first moment of the area with respect to the z axis, which is equal to zero nince the centroid C of the area is located on that axis. Finally, from Eq. (A.14)we have

$$I_{c} = \overline{I}_{c} + Ad^{2}$$

(A.15)

A similar formula may be derived, which relates the polar moment of inertia J_a of an area to an arbitrary point O and polar moment of inertia J_c of the same area with respect to its centroid C. Denoting the distance between O and Cby d, we write

$$J_{c} = J_{c} + Ad^{1}$$

Example A.05

Determine the moment of inertia l, of the area shown in Fig. A.14 with respect to the centroidal z axis



Solution

The first step of the solution is to locate the centroid C of the area. However, this has already been done in Example A.02 for a given area A.

We divide the area A into two rectangular areas A_1 and A_2 (Fig. A.15) and compute the moment of inertia of each area with respect to the *z* axis. Moment of inertia of the areas are

$$I_i = I_{ii} + I_{i3}$$

where I_{a} is the moment of inertia of A₁ with respect to the z axis. For the solution, we use the parallelaxis theorem (Eq. A.15), and write

$$I_{s1} = \overline{I}_{s} + A_{1}d_{1}^{2} = \frac{1}{12}b_{1}h_{1}^{2} + b_{1}h_{1}d_{1}^{2}$$
$$I_{s1} = \frac{1}{12} \times 8t \times (2t)^{3} + 8t \times 2t \times (7t - 4.6t)^{2}$$
$$I_{s1} = 97.5 t^{4}$$

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(A.16)

Appendit



In a similarly way, we find the moment of inertia I_{a} of A_{1} with respect to the z axis and write

$$I_{s2} = \overline{I}_{s^2} + A_3 d_2^2 = \frac{1}{12} b_1 b_2^3 + b_2 b_2 d_2^2$$
$$I_{s2} = \frac{1}{12} \times 4t \times (6t)^3 + 4t \times 6t \times (4.6t - 3t)^3$$
$$I_{s2} = 133.4 t^4$$

The moment of inertia i, of the area shown in Fig. A.14 with respect to the centroidal z axis is

$$I_{1} = I_{11} + I_{12} = 97.5t^4 + 133.4t^4 = 230.9t^4$$
.

Example A.06



Fig. A.16

Determine the moment of inertia l_{μ} of the area shown in Fig. A.14 with respect to the centroidal z axis and the moment of inertia l_{μ} of the area with respect to the centroidal y axis.

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Rp. A.17

Solution

The first step of the solution is to locate the centroid C of the area. This area has two axis of symmetry, the location of the centroid C is in the intersection of the axes of symmetry.

Appensendb



We divide the area A into three rectangular areas A_1 , A_1 and A_2 . The first wan divide area A of A can be seen in Fig. A.17, a second way can be seen in Fig. A.18.

Solution the division of area A by Fig. A.17 (the first way) themoment days

$$I_{4} = I_{44} + I_{42} + I_{43}$$

where.

$$I_{ab} = \overline{I}_{a} + A_{3}d_{1}^{2} = \frac{1}{12}b_{3}h_{1}^{4} + b_{1}h_{1}d_{1}^{2} = \dots = 196t^{4},$$

$$I_{a3} = \overline{I}_{a} + A_{2}d_{2}^{2} = \frac{1}{12}b_{3}h_{2}^{3} + b_{3}h_{2}d_{2}^{2} = \dots = 36t^{4},$$

$$I_{a9} = \overline{I}_{a} + A_{3}d_{3}^{2} = \frac{1}{12}b_{3}h_{3}^{3} + b_{3}h_{3}d_{3}^{2} = \dots = 196t^{4}.$$

Resulting In

$$I_{1} = I_{2} + I_{1} + I_{2} = 196t^{4} + 36t^{4} + 196t^{4} = 428t^{4}$$

For the moment of inertia I, we have

$$I_{u} = I_{u1} + I_{u2} + I_{u3}$$

where

$$I_{y1} = \overline{I}_{y} = \frac{1}{12}\mathbf{h}_{1}\mathbf{b}_{1}^{3} = \frac{1}{12} \times 2t \times (6t)^{3} = 36t^{4},$$

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$$I_{st} = \overline{I}_s = \frac{1}{12}h_3h_2^3 = \frac{1}{12}\times 6t \times (2t)^3 = 4t^4,$$

$$I_{st} = \overline{I}_s = \frac{1}{12}h_3h_3^3 = \frac{1}{12}\times 2t \times (6t)^3 = 36t^4.$$

Resulting in

$$I_{y} = I_{y1} + I_{y2} + I_{y3} = 36t^{4} + 4t^{4} + 36t^{4} = 76t^{4}$$

The solution for the division of area A according to Fig. A. 18 (by the second way) the moment of inertial in

$$I_{a} = I_{a1} - I_{a2} - I_{a3}$$

where

$$I_{s1} = \overline{I}_s = \frac{1}{12} \mathbf{b}_1 \mathbf{h}_1^3 = \frac{1}{12} \times 6\mathbf{t} \times (10\mathbf{t})^3 = 500\mathbf{t}^4,$$

$$I_{s2} = \overline{I}_s = \frac{1}{12} \mathbf{b}_2 \mathbf{h}_2^3 = \frac{1}{12} \times 2\mathbf{t} \times (6\mathbf{t})^3 = 36\mathbf{t}^4,$$

$$I_{s3} = \overline{I}_s = \frac{1}{12} \mathbf{b}_3 \mathbf{h}_3^3 = \frac{1}{12} \times 2\mathbf{t} \times (6\mathbf{t})^3 = 36\mathbf{t}^4.$$

Resulting in

$$I_{s} = I_{s1} - I_{s3} - I_{s3} = 500t^4 - 36t^4 - 36t^4 = 428t^4.$$

For the moment of inertial, we have

$$I_{y} = I_{y1} - I_{y2} - I_{y3},$$

where

$$I_{p1} = \overline{I}_{1} = \frac{1}{12}\mathbf{h}_{1}\mathbf{b}_{1}^{3} = \frac{1}{12} \times 10t \times (6t)^{3} = 180t^{4},$$

$$I_{p2} = \overline{I}_{1} = \frac{1}{12}\mathbf{h}_{2}\mathbf{b}_{2}^{3} + \mathbf{h}_{2}\mathbf{b}_{2}\mathbf{d}_{2}^{2} = \frac{1}{12} \times 6t \times (2t)^{3} + 6t \times 2t \times (2t)^{2} = 52t^{4},$$

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$$I_{y3} = \overline{I}_y = \frac{1}{12}h_y b_3^2 + h_y b_y d_3^2 = \frac{1}{12} \times 6t \times (2t)^2 + 6t \times 2t \times (2t)^2 = 52t^4$$

Beaulting in

$$I_{y} = I_{y1} - I_{y2} - I_{y3} = 180t^{4} - 52t^{4} - 52t^{4} = 76t^{4}.$$

Example A.07



Pp.A.19

In order to solve the torsion of a rectangular cross-section in Fig. A.19, we defined (See S.P. Thimoshenk), and J.N. Goodier, Theory of Hasticity, 3d ed. McGraw-Hill, New York, 1969, sec. 109) the following parameters for b>h:

$$J = \gamma \mathbf{b}^{1} \mathbf{h}$$
(A.17)

$$S_{\rm c} = a b^2 h, \tag{A.18}$$

$$S_2 = \beta \mathbf{b} \mathbf{h}^2, \tag{A.19}$$

where parameters α , β and γ are in Tab.A.J.

The shearing stresses at point 1 and 2 are defined as

$$r_1 = r_{max} = \frac{T}{S_1}, \qquad r_2 = \frac{T}{S_2}, \qquad (A.20)$$

where T is the applied torque.

Tab.A.I

b /b	1	1.2	1.5	2	3	5	10	>10
α	0.208	0.219	0.231	0.246	0.267	0.291	0.313	1/3
β	0 208	0 1 96	0180	0.155	0118	0.078	0 042	0
۲	0.1404	0 166	0196	0.229	0.263	0.291	0.313	1/3

A.4 Product of Inertia, Principal Axes

Definition of product of inertia is

$$I_{\mu} = \int_{A} y \, x \, \mathrm{d}A \tag{A 20a}$$

in which each element of area dA is multiplied by the product of its coordinates and integration is extended over the entire area A of a plane figure. If a cross-section area has an axis of symmetry which is taken for the y or z axis (Fig. A.19), the product of inertia is equal to zero. In the general case, for any point of any cross-section area, we can always find two perpendicular area such that the product of inertia for these vanishes. If this quantity becomes zero, the area in these directions are called the principal axes. Usually the centroid is taken as the origin of coordinates and the corresponding principal axes are then called the centroidal principal axes.

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internal tion to Mechanics of Materials: Part I



If the product of inertia of a cross-section area is known for area y and z (Fig. A.19a) thought the centroid, the product of inertia for parallel area y' and z' can be found from the equation

$$I_{yy} = I_{\mu} + Ama$$
 (A.20b)

The coordinates of an element dA for the new axes are

Hence.

$$I_{yy} = \int y' z' dA = \int (y + n)(z + m) dA = \int yz dA + \int mn dA + \int ym dA + \int nz dA.$$

The last two integrals vanish because C is the centroid so that the equation reduces to (A.20b).

A.5 Strain energy for simple loads



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Apparentic

Consider a rod BC of length L and uniform cross-section area A, attached at B to a fixed support. The rod is subjected to a slowly increasing axial load F at C (Fig. A.20). The work done by the load F as it is slowly applied to the rod must result in the increase of some energy associated with the deformation of the rod. This energy is referred to as the *stram energy* of the rod. Which is defined by

Strain energy
$$= U = \int_0^x \mathbf{F} \, \mathrm{d}\mathbf{x}$$
 (A.21)

Dividing the strain energy U by the volume V = A L of the rod (Fig. A.20) and using Eq. (A.21), we have

$$\frac{U}{V} = \int_{0}^{x} \frac{F}{AL} dr$$
(A.22)

Recalling that F/A represents the normal stress σ_i in the rod, and x/L represents the normal strain ε_i , we write

$$\frac{U}{V} = \int_0^a \sigma_n \, d\sigma_n \tag{A.23}$$

The strain energy per unit volume, U/V, is referred to as the strain-energy density and will be denoted by the letter u. We therefore have

Aspendix

$$u = \int_{a}^{a} \sigma_{x} \, \mathrm{d}s_{x} \tag{A.24}$$

A.5.1 Electic strain energy for normal stresses

In a machine part with non-uniform stress distribution, the strain energy density u can be defined by considering the strain energy of a small element of the material with the volume ΔV , writing

$$u = \lim_{dT\to 0} \frac{\Delta U}{\Delta V}$$
 or $u = \frac{dU}{dV}$. (A.25)

for the value of σ_i within the proportional limit, we may set $\sigma_i = E \epsilon_i$ in Eq. (A.24) and write

$$u = \frac{1}{2} \sum z_{x}^{2} = \frac{1}{2} \sigma_{x} z_{y} = \frac{\sigma_{x}^{2}}{2E}.$$
 (A.26)

The value of strain energy U of the body subject to uniaxial normal stresses can by obtain by substituting Eq. (A.26) into Eq. (A.25), to get

$$U = \int \frac{\dot{\sigma}_{*}}{2E} dV. \qquad (A.27)$$

Electic strain energy under axial loading

When a rod is acted on by centric axial loading, the normal streams are $\sigma_1 = N/A$ from Sec. 2.2. Substituting for σ_1 into Eq. (A.27), we have

$$U = \int \frac{N^2}{2EA^2} \, \mathrm{d}F' \text{ or, setting } \, \mathrm{d}V = A \, \mathrm{d}V \,, \qquad U = \int \frac{1}{2EA} \frac{N^2}{2EA} \, \mathrm{d}V \tag{A.20}$$

If the rod hass uniform cross-section and is acted on by a constant axial force R we then have

$$U = \frac{N^2 L}{2E t}$$
(A.29)

Elestic strain energy in Bending

The normal stresses for pure bending (neglecting the effects of shear) is $\sigma_1 = My/l$ from Sec. 4. Substituting for σ_1 into Eq. (A.27), we have

$$U = \int \frac{\sigma_s}{2E} dV = \int \frac{M^3 y^2}{2EJ^2} dV \qquad (A.30)$$

Setting dV = dA dx, where dA sepresents an element of cross-sectional area, we have

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$$U = \int_{0}^{1} \frac{M^2}{2Ef^2} \left(\int y^2 dA \right) dx = \int_{0}^{1} \frac{M^2}{2Ef} dx$$

Example A.08



Pis. A.21

Determine the strain energy of the prismatic castilever beam in Fig. A.21. taking into account the effects of normal stressesonly.

Solution

The bending moment at a distance x from the free end is M = -F x. Substituting this expression into Eq. (A.31), we can write

$$U = \int_{0}^{1} \frac{M^2}{2EI} dx = \int_{0}^{1} \frac{(F x)^2}{2EJ} dx = \frac{F^2 L^3}{6EI}$$

A.5.2 Elastic strain energy for shearing stresses

When a material is acted on by plane shearing stresses to the strain-energy density at a given point can be expressed as

$$u = \int_{0}^{\infty} \tau_{\mu\nu} dr_{\mu\nu}, \qquad (A.32)$$

where with the shearing strain corresponding to the For the value of the within the proportional limit. we have T = G y and write

$$u = \frac{1}{2}G\gamma_W^2 = \frac{1}{2}r_W\gamma_{trr} = \frac{r_W^2}{2G}$$
 (A.33)

Substituting Eq. (A.33) into Eq. (A.25), we have

$$U = \int \frac{\tau_{w}^{2}}{2G} dV'.$$
 (A.34)

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(A.31)

Electic strain energy in Terrice.

The shearing stremes for pure torsion are: $T\rho / J$ from Sec. 3. Substituting for r into Eq. (A.27), we have

$$U = \int \frac{\tau_{\rm T}^2}{2G} \, \mathrm{d}V = \int \frac{T^2 n}{2GE J^2} \, \mathrm{d}V \tag{A.35}$$

Setting dV = dA dx, where dA represents an element of the cross-sectional area, we have

$$U = \int_{0}^{1} \frac{T^{2}}{2GJ^{2}} \left(\int \rho^{2} dA \right) dx = \int_{0}^{1} \frac{T^{2}}{2GJ} dx$$
(A.36)

In the case of a shaft of uniform cross-sectionacted on by a constant torque T, we have

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Electic stream energy in transversel loading

If the internal abear at section x is V, then the shear stress acting on the volume element, having a length of dx and an area of dA, is $\tau = V Q / I t$ from Sec. 4. Substituting for τ into Eq. (A.37), we have

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(A.37)

$$U = \int_{0}^{\infty} \frac{\pi^{2}}{2\Omega} dV = \int_{0}^{\infty} \frac{1}{2\Omega} \left(\frac{VQ}{Tt} \right)^{2} d4 dx = \int_{0}^{0} \frac{V^{2}}{2\Omega t^{2}} \left(\int_{0}^{0} \frac{Q^{2}}{t^{2}} d4 \right) dx \qquad (A.38)$$

The integral in parentheses is evaluated over the beam's cross-sectional area. To simplify this expression we define the form factor for shear

$$J_x = \frac{A}{I^2} \int \frac{Q^2}{I^2} dA \qquad (A.39)$$

Substituting Eq. (A.39) into Eq. (A.38), we have

$$U = \int f_{\mathcal{B}} \frac{V^2}{2GA} \, \mathrm{d} \mathbf{x} \tag{A.40}$$

N.A.



The form factor defined by Eq. (A.39) is a dimensionless number that is unique for each specific crosssection area. For example, if the beam has a rectangular cross-section with a width b and height h, as in Fig. A.22, then

$$t = \mathbf{b}, \qquad \mathbf{A} = \mathbf{b} \mathbf{h}, \qquad \mathbf{f} = \frac{1}{12} \mathbf{b} \mathbf{h}^{3}$$
$$Q = \overline{\mathbf{y}}^{2} \mathbf{A}^{\prime} = \left(\mathbf{y} + \frac{\mathbf{h}}{2} - \mathbf{y}\right) \mathbf{b} \left(\frac{\mathbf{h}}{2} - \mathbf{y}\right) = \frac{\mathbf{b}}{2} \left(\frac{\mathbf{h}^{2}}{4} - \mathbf{y}^{2}\right)$$

h/2

h/2

Substituting these terms into Eq. (A.39), we get

$$f_{g} = \frac{bh}{\left(\frac{1}{12}bh^{4}\right)^{2}} \int_{-k_{2}}^{-k_{2}} \frac{b^{2}}{4b^{2}} \left(\frac{h^{4}}{4} - y^{2}\right) b \, dy = \frac{6}{5}$$
(A.41)

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Example A.00



Determine the strain energy in the cantilever beam due to shear if the beam has a rectangular crosssection and is subject to a load R Fig. A.23, assume that El and G are constant.

Solution

From the free body diagram of the arbitrary section, we have

$$V(x) = F$$
.

Since the cross-section is rectangular, the form factor $f_g = \frac{6}{5}$ from Eq. (A.41) and therefore Eq. (A.40)

becames

$$U_{above} = \int_{0}^{1} \frac{6}{5} \frac{F^{2}}{2GA} dx = \frac{3}{5} \frac{F^{2}L}{GA}$$

Using the results of Example A.08, with A = b h. $I = \frac{1}{12}b$ h¹, the ratio of the shear to the bending strain energy is

$$\frac{U_{++}}{U_{++}} = \frac{\frac{3 \text{ F}^2 \text{L}}{5 \text{ GA}}}{\frac{F^2 \text{L}^3}{6FI}} = \frac{3 \text{ h}^2 \text{ E}}{10 \text{ L}^3 \text{ G}}$$

Since G = E / 2(1+n) and n = 0.5, then E = 3G, so

$$\frac{U_{\text{share}}}{U_{\text{share}}} = \frac{3}{10} \frac{h^2}{L^2} \frac{3G}{G} = \frac{9}{10} \frac{h^2}{L^2}$$

In can be seen that the result of this ratio will increasing as L decreases. However, even for short beams, where, say L = 5 h, the contribution due to shear strain energy is only 3.6% of the bending strain energy. For this reason, the shear strain energy stored in beams is usually neglected in engineering analysis.

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