**O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI**

**ABU RAYHON BERUNY NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI**

**OLIY O‘QUV YURTLARIGA KIRUVCHILAR UCHUN MATEMATIKADAN QO‘LLANMA ( 1996-2007)**

# TRIGOHOMETRIK IFODALARNI SODDALASHTIRISN VA HISOBLASH

Toshkent 2016

Muallif: A. Abdukarimov . Oliy o‘quv yurtlariga kiruvchilar uchun matematikadan qollanma.(1996-2007).Trigonometrik ifodalarni soddalashtirish va hisoblash.-T.:ToshDTU,2016.78 b.

Mazkur o‘quv qo‘llanma Oliy oquv yurtlariga kiruvchilar, Akademik litsey va kasb-hunar kollejlarining o‘quvchilari uchun mo‘ljallangan bo‘lib, u elementar matematikaning trigonometriya bo‘limining “Trigonometrik ifodalarni soddalashtirish va hisoblash” mavzusiga bag‘ishlangan.O‘quv qollanmada matematikadan abituriyentlar uchun mavzulashtirilgan testlar toplamidagi (1996 – 2007) trigonometriya bo’limining trigonometrik ifodalarni soddalashtirish va hisoblash mavzusiga tegishli barcha misollar yechimlari bilan berilgan.

ToshDTU ilmiy-uslubiy kengasi qaroriga asosan chop etishga tavsiya etilgan.

Taqrizchilar:

| | |
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## So‘z boshi

Ma’lumki hozirgi kunda Respublikamizning barcha oliy o‘quv yurtlarida qabul imtixonlari test usulida o‘tkaziladi. Test sinovlarini muvaffaqiyatli topshirishlari uchun abituriyentlar test topshiriladigan fanlarning barcha mavzulariga puxta tayyorgarlik ko‘rishlari kerak.

Buning uchun birinchi navbatda Davlat test markazi tomonidan chop etilgan “Axborotnoma”larda berilgan test savollari asosida tuzilgan “Abituriyentlar uchun mavzulashtirilgan testlar to‘plami (1996-2007) “ dagi barcha test savollarining yechimlarini o‘rganib chiqishlari kerak bo‘ladi.

Qo‘lingizdagi ushbu uslubiy qo‘llanma aynan shu maqsadda tayyorlangan bo‘lib, u elementar matematikaning trigonometriya bo‘limining тrigonometrik ifodalarni soddalashtirish va hisoblash mavzusiga bag‘ishlangan va “Abituriyentlar uchun mavzulashtirilgan testlar to‘plami (1996-2007) “ dagi barcha тrigonometrik ifodalarni soddalashtirish va hisoblashga doir misollar to‘liq yechimlari bilan berilgan.

Uslubiy qo‘llanmada turli ko‘rinishdagi trigonometrik ifodalar soddalashtirilgan va yechilgan bo‘lib, har bir turdagi trigonometrik ifodalarni soddalashtirish yoki yechish uchun kerak bo‘ladigan barcha formulalar isbotsiz keltirilgan.

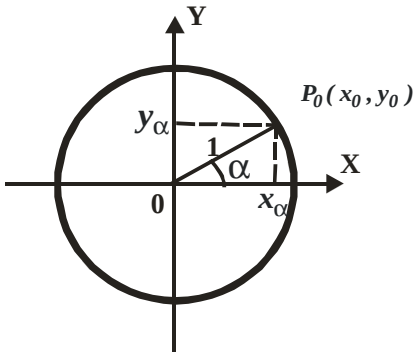
Ushbu uslubiy qo‘llanma Akademik litsey o‘quvchilari, kasb-hunar kollejlari talabalari va abiturientlar uchun тrigonometrik ifodalarni soddalashtirish va hisoblashni mustaqil o‘rganishda katta yordam beradi deb o‘ylayman.

Qo‘lyozmani sinchiklab o‘qib chiqib, o‘zlarining foydali maslahatlarini va ko‘rsatmalarini berganliklari uchun “Matematika va mexanika” kafedrasi dotsentlari G.Shodmonov va E.Esonovga muallif o‘z minnatdorchiligini bildiradi.

Muallif

# Trigonometriya.
# Trigonometrik funksiyalarning chorаklardagi ishoralariga, burchaklarning radian va gradus o‘lchovlariga doir misollar

## Burchaklarning radian va gradus o‘lchovlari




$$a^\circ = \frac{180^\circ}{p} \cdot a_{рад}, \quad a_{рад} = \frac{p}{180^\circ} \cdot a^\circ, \quad \sin a = y_a, \quad \cos a = x_a,$$

$$tga = \frac{y_a}{x_a}, \quad ctga = \frac{x_a}{y_a}, \quad \sec a = \frac{1}{x_a}, \quad \operatorname{cosec} a = \frac{1}{y_a}.$$

$$1\ рад \approx 57^\circ 47' 15''; \quad p = 3{,}141592\ldots$$

## Trigonometrik funksiyalarning ayrim burchaklardagi qiymatlari

1-jadval

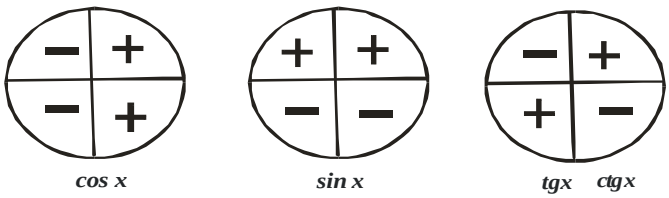
| Gradus o‘lchovi | Radian o‘lchovi | $\sin x$ | $\cos x$ | $tgx$ | $ctgx$ | $\sec x$ | $\csc x$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | - | 1 | - |
| $30^\circ$ | $\frac{p}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | $\sqrt{3}$ | $\frac{2}{\sqrt{3}}$ | 2 |
| $45^\circ$ | $\frac{p}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 | 1 | $\sqrt{2}$ | $\sqrt{2}$ |
| $60^\circ$ | $\frac{p}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{\sqrt{3}}{3}$ | 2 | $\frac{2}{\sqrt{3}}$ |
| $90^\circ$ | $\frac{p}{2}$ | 1 | 0 | - | 0 | - | 1 |
| $120^\circ$ | $\frac{2p}{3}$ | $\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $-\sqrt{3}$ | $-\frac{\sqrt{3}}{3}$ | $-2$ | $\frac{2}{\sqrt{3}}$ |
| $135^\circ$ | $\frac{3p}{4}$ | $\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-1$ | $-1$ | $-\sqrt{2}$ | $\sqrt{2}$ |
| $150^\circ$ | $\frac{5p}{6}$ | $\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{3}$ | $-\sqrt{3}$ | $-\frac{2}{\sqrt{3}}$ | 2 |
| $180^\circ$ | $p$ | 0 | $-1$ | 0 | - | $-1$ | - |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $210^0$ | $\frac{7p}{6}$ | $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | $\sqrt{3}$ | $-\frac{2}{\sqrt{3}}$ | $-2$ |
| $225^0$ | $\frac{5p}{4}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $1$ | $1$ | $-\sqrt{2}$ | $-\sqrt{2}$ |
| $240^0$ | $\frac{4p}{3}$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $\sqrt{3}$ | $\frac{\sqrt{3}}{3}$ | $-2$ | $-\frac{2}{\sqrt{3}}$ |
| $270^0$ | $\frac{3p}{2}$ | $-1$ | $0$ | - | $0$ | - | $-1$ |
| $360^0$ | $2p$ | $0$ | $1$ | $0$ | - | $1$ | - |

2 - jadval

| Gradus o‘lchovi | Radian o‘lchovi | $\sin x$ | $\cos x$ | $tgx$ | $ctgx$ |
|---|---|---|---|---|---|
| $15^0$ | $\frac{p}{12}$ | $\frac{\sqrt{3}-1}{2\sqrt{2}}$ | $\frac{\sqrt{3}+1}{2\sqrt{2}}$ | $2-\sqrt{3}$ | $2+\sqrt{3}$ |
| $18^0$ | $\frac{p}{10}$ | $\frac{\sqrt{5}-1}{4}$ | $\frac{\sqrt{5+\sqrt{5}}}{2\sqrt{2}}$ | $\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$ | $\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$ |
| $36^0$ | $\frac{p}{5}$ | $\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$ | $\frac{\sqrt{5}+1}{4}$ | $\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$ | $\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$ |
| $54^0$ | $\frac{3p}{10}$ | $\frac{\sqrt{5}+1}{4}$ | $\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$ | $\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$ | $\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$ |
| $75^0$ | $\frac{5p}{12}$ | $\frac{\sqrt{3}+1}{2\sqrt{2}}$ | $\frac{\sqrt{3}-1}{2\sqrt{2}}$ | $2+\sqrt{3}$ | $2-\sqrt{3}$ |

**Trigonometrik funksiyalarning chorаklardagi ishoralari**




**1- misol**. (97-8-30) $216^0$ ning radian o‘lchovini toping.

**Yechilishi**: $n^0 \cdot \frac{\pi}{180^0}$ formulaga asosan, $216^0 \cdot \frac{\pi}{180^0} = \frac{36^0}{30^0}\pi = \frac{6}{5}\pi$.
Javobi : E

**2 – misol**. (97 – 12 – 32 ) Quyidagi sonlarning qaysi biri manfiy ?
A ) $\sin 122^0 \cdot \cos 322^0$ , B ) $\cos 148^0 \cdot \cos 289^0$ , S ) $tg 196^0 \cdot ctg\ 189^0$
D) $tg\ 220^0 \cdot \sin 100^0$, E ) $ctg 320^0 \cdot \cos 186^0$

**Yechish:** A ) $\sin 122^0 = \sin(90^0+32^0) = (+)$ . $\cos 322 = \cos(270 + 52) = (+)$. $\sin 122^0 \cdot \cos 322^0 = (+)$ .
B ) $\cos 148^0 = \cos(90^0 + 58^0) = (-)$. $\cos 289^0 = \cos(270^0 + 19^0) = (+)$ . $\cos 148^0 \cdot \cos 289^0 = (-)$. Javobi : B

**3 – misol**. ( 96 – 3 – 54 ). Agar $\cos\alpha \cdot \sin\alpha > 0$ bo'lsa, $\alpha$ burchak qaysi chorakka tegishli ?
**Yechilishi**: 1) $\cos\alpha$ II , IV choraklarda musbat , $\sin\alpha$ I , II choraklarda musbat , bundan, $\alpha$ burchak II chorakda bo'lsa , $\sin\alpha \cdot \cos\alpha > 0$ .
2 ) $\cos\alpha$ II , III choraklarda manfiy. $\sin\alpha$ III , IV choraklarda manfiy , bundan, $\alpha$ burchak III chorakda bo'lsa , $\sin\alpha \cdot \cos\alpha > 0$, demak , $\alpha$ burchak II , III choraklarga tegishli .Javobi : ( D )

**4 – misol** . ( 96 – 12 – 58 ). Agar $\sin\alpha \cdot \cos\alpha < 0$ bo'lsa , $\alpha$ burchak qaysi chorakka tegishli ?
**Yechilishi**: $\sin\alpha$ II chorakda ( + ), $\cos\alpha$ II chorakda ( - ) , demak II chorakda $\sin\alpha \cdot \cos\alpha < 0$. IV chorakda $\sin\alpha < 0$, $\cos\alpha > 0$, bundan $\sin\alpha \cos\alpha < 0$. Javobi : D

**5 – misol.** ( 97– 8 – 32 ). Quyidagi sonlardan qaysi biri musbat ?
A) $\frac{ctg\ 187^0}{\sin 316^0}$ ; B) $\frac{\cos 340^0}{\sin 185^0}$ ; C) $\frac{\sin 148^0}{\cos 317^0}$ ; D) $\frac{ctg\ 105^0}{tg\ 165^0}$ ; E) $\frac{tg\ 215^0}{\cos 125^0}$
**Yechilishi**: A ) $\operatorname{ctg} 187^0 = (+)$ , $\sin 316^0 = -$ . $\frac{ctg\ 187^0}{\sin 316^0} = -$ .
B ) $\cos 340^0 = +$, $\sin 185^0 = -$ . $\frac{\cos 340^0}{\sin 185^0} = -$ . D ) $\operatorname{ctg} 105^0 = -$ . $\operatorname{tg} 165^0 = -$ . $\frac{ctg\ 105^0}{tg\ 165^0} = +$. Javobi: D

**6 – misol**. ( 96 – 3 – 42 ). R ( 3 ; 0 ) nuqtani koordinata boshi atrofida $90^0$ ga burganda u qaysi nuqtaga o'tadi ?
A ) (-3 ; 0 ) V) ( 0 ; 3 ) S) ( 3 ; 3 ) D) ( 0;3 ) E ) ( 3 ; -3 )
**Yechilishi**: $90^0$ ga burganda $x=0, y=3$ bo'ladi. Javobi : ( D )

**7 – misol.** ( 98 – 11 – 101 ). Agar $\sin x - \frac{1}{\sin x} = -3$ bo'lsa, $\sin^2 x + \frac{1}{\sin^2 x}$ ning qiymati qanchaga teng bo'ladi?
**Yechilishi**: Har ikkala tomonni kvadratga ko'taramiz.
$(\sin x - \frac{1}{\sin x})^2 = (-3)^2$. $\sin^2 x - 2\sin x \cdot \frac{1}{\sin x} + \frac{1}{\sin^2 x} = 9$, bundan,
$\sin^2 x + \frac{1}{\sin^2 x} = 9+2 = 11$. Javobi : D

**8 – misol.** ( 00–10 – 78 ). To‘g‘ri tengsizlikni aniqlang.
A) cos( sin $\alpha$ ) > 0, B) cos 2 > 0 , C) - $\frac{\pi}{2}$ + 2 ≤ 0, D) /cos $\alpha$/ + /sin $\alpha$/ < 1, E) sin $5^0$ – tg $4^0$ > 0
**Yechilishi**:A) -1 ≤ sin $x$ ≤ 1, cos (sin $x$ ) > 0. B) cos$x$ ning grafigidan, cos 2 < 0. C) - $\frac{\pi}{2}$ < −2 => −1,57 < −2, noto‘g‘ri. D) / cos $\alpha$/ + /sin $\alpha$/ < 1 noto‘g‘ri. E) sin$x$ ning grafigidan $\sin 5^0 < 0$, $tg4^0 = \frac{\sin 4^0}{\cos 4^0}$. $\sin 4^0 < 0$, $cos4^0 < 0$, bundan $tg4^0 > 0$. berilganga ko‘ra $\sin 5^0 - tg4^0 > 0$ tenglik noto‘g‘ri. Javobi : A

**9 – misol**. (01 – 2 – 58 ). Hisoblang. $cos\frac{12\pi}{5}$ ( $log_2 0{,}25 + log_{0{,}25} 2$ )
**Yechilishi**: $log_2 0{,}25 = log_2 \frac{1}{4} = log_2 2^{-2} = -2$, $log_{0{,}25} 2 = log_{\frac{1}{4}} 2 = = log_{2^{-2}} 2 = -\frac{1}{2}$, $\cos(\frac{12\pi}{5} \cdot \left(-2 - \frac{1}{2}\right) = \cos\left(\frac{12\pi}{5} \cdot \left(-\frac{5}{2}\right)\right) = \cos(-6\pi) = 1$. Javobi : B

**10 – misol**. ( 01 – 7 – 3 ). Ushbu a = 2,7 (2), b = $\sqrt{2 + \sqrt[3]{8}}$ , c = $\pi - 3{,}14$ va d= $\sin\frac{\pi}{3}$ sonlardan qaysilari irratsional sonlar ?
**Yechilishi:** a = $2\frac{72-7}{90} = 2\frac{65}{90} = 2\frac{13}{18}$ davriy o‘nli kasr. b= $\sqrt{2 + 2}$ = 2. c = 3,1417 … - 3,14 = 0,0017 … irratsional son. d = $sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$ irratsional son. Javobi : C va D

**11 – misol.** ( 01 – 9 – 29 ). Qaysi ko‘paytma musbat ?
1) sin4,11 · tg3,52, 2) cos2,53 · $log_{\frac{1}{2}} \frac{\pi}{3}$ , 3) ctg5,73 · cos1,19
**Yechilishi: 1)** sin4,11= (−) ; $tg3{,}52 = \frac{sin3{,}52}{cos3{,}52} = \frac{(-)}{(-)} = (+)$, $sin4{,}11 \cdot tg3{,}52 = = (-) \cdot (+) = (-)$. 2)cos2,53=(−), $log_{\frac{1}{2}} \frac{\pi}{3} = log_2 1{,}04 = (-)$, cos2,53 · $log_{\frac{1}{2}} \frac{\pi}{3} = = (-) \cdot (-) = (+)$ . 3) cos1,19 = (+), $ctg5{,}73 = \frac{cos5{,}73}{sin5{,}73} = \frac{(+)}{(-)} = (-)$, cos1,19 · ctg 5,73 = (+) · (−) = (−). Javobi : C

**12 – misol**. ( 01–9 – 33 ). Ushbu k = $2^{\frac{\pi}{4}}$, $p = log_2\frac{\pi}{5}$ va $q = sin\frac{5\pi}{6}$ sonlarni o‘sish tartibida yozing.

**Yechilishi** : $k = 2^{\frac{3,14}{4}} = 2^{0,8} = 2^{\frac{4}{5}} = \sqrt[5]{16} = 1,\ldots$ , $p = log_2\frac{3,14}{5} =$ $= log_2 0,62 = (-)$. $q = sin\frac{5\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right) = sin\pi \cdot cos\frac{\pi}{6} - cos\pi$ $sin\frac{\pi}{6} = = sin\frac{\pi}{6} = \frac{1}{2} = 0,5$. $p < q < k$. Javobi : A

**13-misol**. ( 02 – 2 – 48 ). Agar $\frac{1}{2}x = (sin30^0 + tg60^0 \cdot cos30^0)^2$ bo‘lsa, x = ?

**Yechilishi**: $\frac{1}{2}x = (\frac{1}{2} + \sqrt{3} \cdot \frac{\sqrt{3}}{2})^2 \Rightarrow \frac{1}{2}x = (\frac{1}{2} + \frac{3}{2})^2 \Rightarrow \frac{1}{2}x =$ 4. x = 8.Javobi: A

## Asosiy trigonometrik ayniyatlar yordamida soddalashtirishlar va hisoblashlar.

### Asosiy trigonometrik ayniyatlar.

1. $sin^2 x + cos^2 x = 1.$
2. $tgx \cdot ctgx = 1.$
3. $1 + tg^2 x = \frac{1}{cos^2 x}.$
4. $tgx = \frac{sin\, x}{cos\, x}.$
5. $ctgx = \frac{cos\, x}{sin\, x}.$
6. $1 + ctg^2 x = \frac{1}{sin^2 x}.$

### Trigonometrik funksiyalarning birini ikkinchisi orqali ifodalash

3- jadval

| | $sin\, x$ | $\cos\ x$ | $tgx$ | $ctgx$ |
|---|---|---|---|---|
| $sin\, x$ | $sin\, x$ | $\pm\sqrt{1-cos^2 x}$ | $\frac{tgx}{\pm\sqrt{1+tg^2 x}}$ | $\frac{1}{\pm\sqrt{1+ctg^2 x}}$ |
| $\cos\ x$ | $\pm\sqrt{1-sin^2 x}$ | $\cos\ x$ | $\frac{1}{\pm\sqrt{1+tg^2 x}}$ | $\frac{ctgx}{\pm\sqrt{1+ctg^2 x}}$ |
| $tgx$ | $\frac{\sin x}{\pm\sqrt{1-\sin^2 x}}$ | $\frac{\pm\sqrt{1-cos^2 x}}{cos\, x}$ | $tgx$ | $\frac{1}{ctgx}$ |
| $ctgx$ | $\frac{\pm\sqrt{1-\sin^2 x}}{\sin x}$ | $\frac{\cos x}{\pm\sqrt{1-\cos^2 x}}$ | $\frac{1}{tgx}$ | $ctgx$ |
| $\sec\ x$ | $\frac{1}{\pm\sqrt{1-\sin^2 x}}$ | $\frac{1}{cos\, x}$ | $\pm\sqrt{1+tg^2 x}$ | $\frac{\pm\sqrt{1+ctg^2 x}}{ctgx}$ |

| | | | | |
|---|---|---|---|---|
| $\csc x$ | $\frac{1}{\sin x}$ | $\frac{1}{\pm\sqrt{1-\cos^2 x}}$ | $\frac{\pm\sqrt{1+tg^2 x}}{tgx}$ | $\pm\sqrt{1+ctg^2 x}$ |

**14 – misol**. ( 98 – 5 – 48 ). Agar $sin\alpha = \frac{3}{5}$ va $\frac{\pi}{2} < \alpha < \pi$ bo‘lsa, $tg\alpha$ ni toping.

**Yechilishi:** $cos\alpha = \mp\sqrt{1-sin^2\alpha} = \pm\sqrt{1-\frac{9}{25}} = \pm\frac{4}{5}$ .II chorakda cos $\alpha$ manfiy bo‘lgani uchun, $cos\alpha = -\frac{4}{5}$, $tg\alpha = \frac{sin\alpha}{cos\alpha} = \frac{\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4} =$ 0,75. Javobi: V

**15 – misol**. ( 00 – 8 – 61 ). Agar $0 < \alpha < \frac{\pi}{2}$ va tg $\alpha$=2 bo‘lsa, $cos\alpha$ ni toping.

**Yechilishi:** $1 + tg^2\alpha = \frac{1}{cos^2\alpha}$ formuladan $cos^2\alpha = \frac{1}{1+tg^2\alpha}$ yoki $cos\alpha = \pm\frac{1}{\sqrt{1+tg^2\alpha}} = \pm\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$. Javobi : S

**16 – misol**. ( 98 – 4 – 17 ). Agar $tg\alpha = 3$ bo’lsa, $\frac{3sin\alpha}{5sin^3\alpha+10cos^3\alpha}$ ning qiymati qanchaga teng bo‘ladi ?

**Yechilishi** : Kasrning surati va maxrajini $cos^3\alpha$ ga bulamiz.

$$\frac{3\frac{sin\alpha}{cos^3\alpha}}{5tg^3\alpha + 10} = \frac{3tg\alpha\cdot\frac{1}{cos^2\alpha}}{5tg^3\alpha + 10} = \frac{3tg\alpha\cdot(1 + tg^2\alpha)}{5tg^3\alpha + 10}$$

$tg\alpha = 3$ ni e’tiborga olsak , $\frac{3sin\alpha}{5sin^3\alpha+10cos^3\alpha} = \frac{3\cdot3\cdot(1+9)}{5\cdot3^3+10} => \frac{90}{135+10} = \frac{90}{145} = \frac{18}{29}$. Javobi : A.

**17 – misol.** ( 96 – 6 – 52 ). Agar $\sqrt{1-cos^2 x} - \sqrt{1+sin^2 x} = k$ bo‘lsa, $\sqrt{1-cos^2\alpha} + \sqrt{1+sin^2\alpha}$ ni toping.

**Yechilishi:**

$$(\sqrt{1-cos^2\alpha} - \sqrt{1+sin^2\alpha})(\sqrt{1-cos^2\alpha} + \sqrt{1+sin^2\alpha} =$$

$$= k\cdot(\sqrt{1-cos^2\alpha} + \sqrt{1+sin^2\alpha}$$

$$1 - cos^2\alpha - (1 + sin^2\alpha) = = k\cdot\left(\sqrt{1-cos^2\alpha} + \sqrt{1+sin^2\alpha}\right)$$

$$-(cos^2\alpha + sin^2\alpha) = k(\sqrt{1-cos^2\alpha} + \sqrt{1+sin^2\alpha}$$

$\sqrt{1-cos^2\alpha}+\sqrt{1+sin^2\alpha}=-\frac{1}{k}$. Javobi : E

**18 – misol.** ( 98 – 8 – 62). Agar tg $\alpha$+ ctg$\alpha$ = p bo‘lsa, $tg^2\alpha + ctg^2\alpha$ ni r orqali ifodalang.

**Yechilishi**: Berilgan ifodaning har ikkala tomonini kubga ko‘taramiz.

$$(tg\alpha + ctg\alpha)^3 = p^3$$

$tg^3\alpha + 3tg^2\alpha \cdot ctg\alpha + 3tg\alpha \cdot ctg^2\alpha + ctg^3\alpha = p^3$ bundan,

$tg^3 + ctg^3\alpha = p^3 - 3p$. Javobi : V

**19 – misol**. ( 98 – 12 – 54 ). Agar sin$\alpha$+cos$\alpha$ = a bo‘lsa, /sin$\alpha$ – cos$\alpha$/ ni a orqali ifodalang.

**Yechilishi**:

$|sin\alpha - cos\alpha| = \sqrt{(sin\alpha - cos\alpha)^2} = \sqrt{sin^2\alpha - 2sin\alpha cos\alpha + cos^2\alpha}$

$= = \sqrt{1-2sin\alpha cos\alpha}$.

$sin\alpha + cos\alpha = a$ dan $(sin\alpha + cos\alpha)^2 = a^2$. $sin^2\alpha + 2sin\alpha cos\alpha + cos^2\alpha = = a^2$. $2sin\alpha cos\alpha = a^2 - 1$. Demak, $|sin\alpha - cos\alpha| = \sqrt{1-(a^2-1)} = = \sqrt{2-a^2}$ . Javobi: A

**20 – misol**. ( 99 – 6 – 51 ). Soddalashtiring.

$$sin^6\alpha + cos^6\alpha + 3sin^2\alpha \cdot cos^2\alpha$$

**Yechilishi**:$(sin^2\alpha)^3 + (cos^2\alpha)^3 = (sin^2\alpha + cos^2\alpha)(sin^4\alpha - -sin^2\alpha cos^2\alpha + + cos^4\alpha) = (cos^2\alpha + sin^2\alpha)^2 - 3sin^2\alpha cos^2\alpha = 1 - 3sin^2\alpha cos^2\alpha$.

$sin^6\alpha + cos^6\alpha + 3sin^2\alpha \cdot cos^2\alpha = 1 - 3sin^2\alpha cos^2\alpha + 3sin^2\alpha \cdot cos^2\alpha = 1$. Javobi : S

**21 – misol**. (00-10–01). Agar ctg$\alpha = \sqrt{3}$ bo‘lsa, $\frac{9}{sin^4\alpha + cos^4\alpha}$ ni hisoblang.

**Yechilishi**: $sin^2\alpha + cos^2\alpha = 1$, $1 + ctg^2\alpha = \frac{1}{sin^2\alpha}$, $\frac{9}{sin^4\alpha(1+ctg^4\alpha)} = \frac{9}{1+(\sqrt{3})^4} \cdot \cdot \frac{1}{sin^4\alpha} = \frac{9}{1+9} \cdot 16 = \frac{9 \cdot 16}{10} = \frac{9 \cdot 8}{5} = \frac{72}{5} = 14{,}4$

$\frac{1}{(sin^2\alpha)^2} = (1 + ctg^2\alpha)^2 = 16$. javobi: E

**22 – misol**.(97–1–47). Soddalashtiring. $(\text{ctg}\alpha - \cos\alpha)\left(\frac{\sin^2\alpha}{\cos\alpha} + \text{tg}\alpha\right)$.

**Yechilishi :** $(\text{ctg}\alpha - \cos\alpha)\left(\frac{\sin^2\alpha}{\cos\alpha} + \text{tg}\alpha\right) = \left(\frac{\cos\alpha}{\sin\alpha} - \cos\alpha\right)\left(\frac{\sin^2\alpha}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}\right) = = \frac{\cos\alpha(1-\sin\alpha)}{\sin\alpha} \cdot \frac{\sin\alpha(1+\sin\alpha)}{\cos\alpha} = 1 - \sin^2\alpha = \cos^2\alpha$. Javobi : A

**23 – misol**. ( 97 – 6 – 46 ). Soddalashtiring. $\frac{1-\sin^4\alpha-\cos^4\alpha}{\cos^4\alpha}$

**Yechilishi:** $\frac{1-\sin^4\alpha-\cos^4\alpha}{\cos^4\alpha} = \frac{(1-\sin^2\alpha)(1+\sin^2\alpha)-\cos^4\alpha}{\cos^4\alpha} =$

$= \frac{\cos^2\alpha[1+\sin^2\alpha-\cos^2\alpha]}{\cos^4\alpha} = \frac{2\sin^2\alpha}{\cos^2\alpha} = 2\text{tg}^2\alpha$. Javobi : A

**24 – misol**. ( 97 – 11 – 46 ). Soddalashtiring. $\frac{\sin^2\alpha+\cos^2\alpha+\cos^4\alpha}{\cos^2\alpha-\sin^2\alpha+\sin^4\alpha}$ .

$\frac{\sin^2\alpha-\cos^2\alpha+\cos^4\alpha}{\cos^2\alpha-\sin^2\alpha+\sin^4\alpha} = \frac{\sin^2\alpha+\cos^2\alpha(1-\cos^2\alpha)}{\cos^2\alpha-\sin^2\alpha(1-\sin^2\alpha)} = \frac{\sin^2\alpha(1+\cos^2\alpha)}{\cos^2\alpha(1-\sin^2\alpha)} = \frac{\sin^4\alpha}{\cos^4\alpha} =$

$= \text{tg}^4\alpha$. Javobi : A

**25 – misol** ( 98 – 1 – 55 ). Soddalashtiring. $\frac{3\sin^2\alpha+\cos^4\alpha}{1+\sin^2\alpha+\sin^4\alpha}$

**Yechilishi:**

$\frac{3\sin^2\alpha+\cos^4\alpha}{1+\sin^2\alpha+\sin^4\alpha} = \frac{3\sin^2\alpha+(1-\sin^2\alpha)^2}{1+\sin^2\alpha+\sin^4\alpha} = \frac{3\sin^2\alpha+1-2\sin^2\alpha+\sin^4\alpha}{1+\sin^2\alpha+\sin^4\alpha} = 1$. Javobi: D

**26–misol.** ( 99 – 1 – 8). Agar $|-1+\cos\alpha| + 2\cos\alpha$ va $\sin\alpha = \frac{\sqrt{3}}{2}, \frac{\pi}{2} < \alpha < < \pi$ bo‘lsa, $|\frac{\text{tg}\alpha}{\sqrt{3}} - 0{,}5|$ ni hisoblang.

**Yechilishi :**

$|\cos\alpha - 1| = \sqrt{(\cos\alpha-1)^2} = \sqrt{\cos^2\alpha - 2\cos\alpha + 1} = \sqrt{\frac{1}{4}+2} = \frac{3}{2}$

$\cos\alpha = \pm\sqrt{1-\frac{3}{4}} = \pm\frac{1}{2} => \cos\alpha = -\frac{1}{2}, \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

$$\left|\frac{\text{tg}\alpha}{\sqrt{3}} - 0{,}5\right| = \left|\frac{-\sqrt{3}}{\sqrt{3}} - 0{,}5\right| = |-1{,}5| = \frac{3}{2}$$

$(|-1+\cos\alpha| + 2\cos\alpha)/|\text{tg}\alpha/\sqrt{3} - 0{,}5| = (1/2)/(3/2) = 1/3.$
Javobi : A

**27 – misol**. ( 91 – 1 – 46 ). Ifodani soddalashtiring. $\frac{cos^2\alpha - ctg^2\alpha}{tg^2\alpha \cdot sin^2\alpha}$

**Yechilishi:** $\frac{cos^2\alpha - ctg^2\alpha}{tg^2\alpha - sin^2\alpha} = \frac{cos^2\alpha - \frac{cos^2\alpha}{sin^2\alpha}}{\frac{sin^2\alpha}{cos^2\alpha} - sin^2\alpha} = \frac{(cos^2\alpha \cdot sin^2\alpha - cos^2\alpha)cos^2\alpha}{(sin^2\alpha - sin^2\alpha cos^2\alpha)sin^2\alpha} =$

$= \frac{cos^4\alpha(sin^2\alpha - 1)}{sin^4\alpha(1 - cos^2\alpha)} = -\frac{cos^6\alpha}{sin^6\alpha} = -ctg^6\alpha$. Javobi: A

**28 – misol**. ( 01 – 1 – 69 ). Agar sinx + cosx = 0,5 bo‘lsa, $16(sin^3x + cos^3x)$ ni toping.

**Yechilishi :** $sin^3x + cos^3x = (sin x + cos x)(sin^2x - sinxcosx + cos^2x) =$ $= \frac{1}{2}(1 - sinxcos x)$. $(sinx + cos x)^2 = (\frac{1}{2})^2$, $sin^2x + cos^2x + 2sinxcosx =$ $= \frac{1}{4}$, $2sinxcosx = \frac{1}{4} - 1$, $sinxcosx = -\frac{3}{8}$, $16(sin^3x + cos^3x) = \frac{16}{2}\left(1 + \frac{3}{8}\right) =$ $= \frac{16 \cdot 11}{16} = 11$. Javobi: C

**29 – misol.** ( 01 – 7 – 37 ). Agar $tg\alpha = -\frac{3}{4}$ va $\frac{\pi}{2} < \alpha < \pi$ bo‘lsa, $sin\alpha - cos\alpha$ ning qiymatini toping.

**Yechilishi:** $1 + tg^2\alpha = \frac{1}{ctg^2\alpha}$, $cos\alpha = \pm\frac{1}{\sqrt{1+tg^2\alpha}} =$

$\pm\frac{1}{\sqrt{1+\frac{9}{16}}} = = -\frac{4}{5}$; $sin\alpha = \pm\sqrt{1 - cos^2\alpha} = \pm\sqrt{1 - \frac{16}{25}} = \frac{3}{4}$ ; $sin\alpha - cos\alpha = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$;

Javobi : C

**30 – misol.** ( 02 – 4 – 30 ). ( tg x + ctg x )$^2$ – ( tg x – ctg x )$^2$ ni soddalashtiring.

**Yechilishi** : $tg^2 x + 2tgx \cdot ctg x + ctg^2x - tg^2x + 2tg x \cdot ctg x - ctg^2x = 4.$

Javobi : E

**31 – misol**. ( 02 – 6 – 31 ). $\frac{5\,sin^2\alpha + 4cos^2\alpha}{4cos^2\beta + 5sin^2\beta}$ Ifodaning eng katta qiymatini toping.

**Yechilishi:** $\frac{5 \cdot \frac{1-cos2\alpha}{2} + 4 \cdot \frac{1+cos2\alpha}{2}}{4 \cdot \frac{1+cos2\beta}{2} + 5 \cdot \frac{1-cos2\beta}{2}} = \frac{5 - 5cos2\alpha + 4 + 4cos2\alpha}{4 + 4cos2\beta + 5 - 5cos2\beta} = \frac{9 - cos2\alpha}{9 - cos2\beta} = \frac{10}{8} =$

$= \frac{5}{4} = 1{,}25$. Javobi : A

**32- misol.** (03-1-32). Agar tg x = 0,5 bo‘lsa, $\cos^8\alpha - \sin^8\alpha$ ning qiymatini toping.

**Yechilishi:** $(cos^4\alpha)^2 - (sin^4\alpha) = (cos^4\alpha - sin^4\alpha)(cos^4\alpha + sin^4\alpha) = = ((cos^2\alpha)^2 - (sin^2\alpha)^2\,[(sin^2\alpha + cos^2\alpha)^2 - 2sin^2\alpha cos^2\alpha] = (cos^2\alpha - - sin^2\alpha)(cos^2\alpha + sin^2\alpha)$

$sin^2\alpha + cos^2\alpha = 1;\ cos^2\alpha = \frac{1}{1+tg^2\alpha};\ tg^2\alpha + 1 = \frac{1}{cos^2\alpha};\ cos^2\alpha = = \frac{1}{\pm\sqrt{1+tg^2\alpha}};\ cos\alpha = \frac{1}{\sqrt{1+\frac{1}{4}}} = \frac{2}{\sqrt{5}};\ sin\alpha = \sqrt{1-\frac{4}{5}} = \frac{1}{\sqrt{5}};\ cos^8\alpha = = (\frac{2}{\sqrt{5}})^8 = \frac{2}{5^4};\ sin^8\alpha = (\frac{1}{\sqrt{5}})^8 = \frac{1}{5^4};\ cos^8\alpha - sin^8\alpha = \frac{2^8-1}{5^4} = = \frac{255}{25\cdot 25} = \frac{51}{125} = 0{,}408$. Javobi: V

**33 – misol.** ( 03-08-54 ). cosα-sinα = 0,2 bo’lsa, $\cos^3\alpha$- $\sin^3\alpha$ ni hisoblang.

**Yechilishi:** $cos^3\alpha - sin^3\alpha = (cos\alpha - sin\alpha)(cos^2\alpha + sin\alpha cos\alpha + sin^2\alpha) = = 0{,}2(1 + sin\alpha\cdot cos\alpha);\ (cos\alpha - sin\alpha)^2 = (0{,}2)^2;\ cos^2\alpha - 2sin\alpha cos\alpha + sin^2\alpha = 0{,}04;\ -2sin\alpha cos\alpha = 0{,}04 - 1 = -0{,}96;\ sin\alpha\cdot cos\alpha = 0{,}48;$

$cos^3\alpha - sin^3\alpha = 0{,}2\cdot(1 + 0{,}48) = 0{,}2\cdot 1{,}48 = 0{,}296;$ Javobi: A

**34 – misol.** ( 03-8-55 ). Agar $\cos x = \frac{1}{\sqrt{10}}$ bo‘lsa, $(1 + tg^2x)(1 - sin^2x) - -sin^2x$ ifodaning qiymatini toping.

**Yechilishi :** $(1 + tg^2x)(1 - sin^2x) - sin^2x = \frac{1}{cos^2x}\cdot cos^2x - sin^2x = 1 - -sin^2x = cos^2x = (\frac{1}{\sqrt{10}})^2 = \frac{1}{10} = 0{,}1$ ; Javobi : A

**35 – misol.** ( 03-12-25 ). $1 + \frac{sin^2\alpha + sin^2\alpha\cdot cos^2\alpha}{cos^2\alpha}$ ni soddalashtiring.

**Yechilishi :** $1 + \frac{sin^2\alpha + sin^2\alpha\cdot cos^2\alpha}{cos^2\alpha} = 1 + \frac{sin^2\alpha(sin^2\alpha + cos^2\alpha)}{cos^2\alpha} = 1 + tg^2\alpha = \frac{1}{cos^2\alpha}$

Javobi : B

**36 – misol**. ( 99-7-47 ). Agar $2 \in \left(\frac{\pi}{2};\pi\right)$ va $\sin\alpha = \frac{1}{4}$ bo‘lsa, ctg α ni hisoblang.

**Yechilishi :**

$$ctg\alpha = \frac{cos\alpha}{sin\alpha}; cos\alpha = \pm\sqrt{1 - sin^2\alpha} = \pm\sqrt{1 - \frac{1}{16}} = \pm\sqrt{\frac{15}{16}}$$

$$= \pm\frac{\sqrt{15}}{4};$$

cosα III chorakda manfiy bo‘lgani uchun, $cos\alpha = -\frac{\sqrt{15}}{4}$. Demak,

$ctg\alpha = = -\frac{\frac{\sqrt{15}}{4}}{\frac{1}{4}} = -\sqrt{15}$. Javobi : E

**37 – misol**. ( 98-12-55 ). Agar $tg\alpha + ctg\alpha = p$ bo‘lsa, $tg^2\alpha + ctg^2\alpha$ ni r orqali ifodalang.

**Yechilishi** : $(tg\alpha + ctg\alpha)^2 = p^2$; $tg^2\alpha + 2tg\alpha \cdot ctg\alpha + ctg^2\alpha = p^2$. Bundan , $tg^2\alpha + ctg^2\alpha = p^2$; Javobi : A

**38 – misol.** ( 99-6-21 ). Soddalashtiring.

$$sin^2\alpha + sin^2\beta - sin^2\alpha \cdot sin^2\beta + cos^2\alpha \cdot cos^2\beta$$

**Yechilishi** : $sin^2\alpha = 1 - cos^2\alpha$ buni berilgan ifodaga qo‘yamiz va quyidagicha guruhlaymiz. $1 - (cos^2\alpha - cos^2a \cdot cos^2\beta) + sin^2\beta - sin^2\alpha \cdot \cdot sin^2\beta = 1 - cos^2a(1 - cos^2\beta) + sin^2\beta(1 - sin^2a) = 1 - cos^2a \cdot sin^2\beta + sin^2\beta \cdot cos^2a = 1$. Javobi : A

**39 – misol.** ( 99-6-33 ) tgx ni hisoblang.

$$\frac{2sinx - cosx}{2\cos x + \sin x} = 3$$

**Yechilishi** : Kasrning surat va maxrajini cosx ga bo‘lib, quyidagini hosil qilamiz.

$$\frac{2tgx - 1}{2 + tgx} = 3$$

Bundan, 2 tg x -1 = 6 + 3 tg x ; tg x = - 7. Javobi : D

**40 – misol**. ( 98-8-55 ). Soddalashtiring.

$$\frac{1 + cos^2a + cos^4a}{3cos^2a + sin^4a}$$

**Yechilishi** : $\frac{1+cos^2a+(1-sin^2a)^2}{3cos^2a+sin^4a} = \frac{1+cos^2a+1-2sin^2a+sin^4a}{3cos^2a+sin^4a} =$

$= \frac{cos^2a+2(1-sin^2a)+sin^4a}{3cos^2a+sin^4a} = \frac{3cos^2a+sin^4a}{3cos^2a+sin^4a} = 1$. Javobi : E

**41 – misol**. ( 97-4-36 ). Hisoblang. $\left[\frac{log_\pi 2\pi}{tg\frac{\pi}{12}+1}\right]^{sin^2\frac{\pi}{5}+cos^2\frac{\pi}{5}-1}$

**Yechilishi**: $sin^2\frac{\pi}{5} + cos^2\frac{\pi}{5} - 1 = 1 - 1 = 0$. Har qanday sonning nolinchi darajasi 1 ga teng. Javobi : D

**42–misol**. ( 01-9-23 ). Agar $\cos a = \frac{\sqrt{3}}{2}$ bo‘lsa, $\frac{1-sina+cos^2a\cdot sina}{1+sina}$ ifodaning qiymatini toping.

**Yechilishi :** $1 - sin^2a + cos^2a \cdot sina = cos^2a + cos^2a \cdot sina = cos^2a(1 + sina)$, buni berilgan ifodaga qo‘ysak, $\frac{cos^2a(1+sina)}{1+sina} = cos^2a = \frac{3}{4}$.

Javobi : A

**43 – misol** ( 02-5-32 ). Agar $\frac{2sina+3cosa}{5sina-cosa}$ bo‘lsa, ctg a= - 2 ning qiymatini hisoblang.

**Yechilishi :** $\frac{2\frac{sina}{sina}+3\frac{cosa}{sina}}{5\frac{sina}{sina}-\frac{cosa}{sina}} = \frac{2+3\cdot(-2)}{5-(-2)} = -\frac{4}{7}$. Javobi : E

## Keltirish formulalari yordamida trigonometrik ifodalarni soddalashtirish va hisoblash.

### Keltirish formulalari.

4-jadval

| x | Sinx | cosx | tgx | ctgx |
|---|---|---|---|---|
| $\frac{\pi}{2} - \alpha$ | cos$\alpha$ | sin$\alpha$ | ctg$\alpha$ | tg$\alpha$ |
| $\frac{\pi}{2} + a$ | cos$\alpha$ | - sin$\alpha$ | - ctg$\alpha$ | - tg a |
| $\pi$ - a | sin$\alpha$ | - cos$\alpha$ | - tga | - ctg$\alpha$ |
| $\pi$ + a | - sin$\alpha$ | - cos$\alpha$ | tga | ctg$\alpha$ |
| $\frac{3\pi}{2} - a$ | - cos$\alpha$ | - sin$\alpha$ | ctg$\alpha$ | tg$\alpha$ |
| $\frac{3\pi}{2} + a$ | - cos$\alpha$ | sin$\alpha$ | -ctga | - tg$\alpha$ |

| | | | | |
|---|---|---|---|---|
| $2\pi$-a | - sin$\alpha$ | cos$\alpha$ | - tg$\alpha$ | - ctg$\alpha$ |
| $2\pi$+a | sin$\alpha$ | cos$\alpha$ | tg$\alpha$ | ctg$\alpha$ |

**44 – misol.** ( 96-1-54 ). Ushbu $2\text{tg}\,(-765^0)$ ifodaning qiymatini toping.

**Yechilishi**: $2tg\,(-765^0) = 2tg\,765^0 = -2tg\,(720^0 + 45^0) = -2tg\,45^0 = = -2$. Javobi: S

**45 – misol** ( 00-5-31 ). $\text{Sin}2010^0$ ni hisoblang.

**Yechilishi** : $\sin 2010^0 = (5 \cdot 360^0 + 210^0) = sin\,210^0 = \sin(180^0 + 30^0) = = sin\,30^0 = -\frac{1}{2}$. Javobi: A

**46 – misol.** (97-6-43). Hisoblang. $\sin(-45^0) + \cos(405^0) + +tg(-945^0)$

**Yechilishi:** $\sin(-45^0) + \cos(405^0) + tg(-945^0) = -sin\,45^0 + +\cos(360^0 + 45^0) - tg\,(2 \cdot 360^0 + 225^0) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} - tg(180^0 + 45^0) = = -1$. Javobi : B

**47 – misol.** ( 00-1-25 ). Keltirilgan sonlardan eng kattasini toping.

A) $\sin 1$ ; V) $\cos\left(\frac{\pi}{2} - \frac{1}{2}\right)$; S) $\sin 4$ ; D) $\cos\left(\frac{3\pi}{2} + \frac{1}{4}\right)$; E) $\text{tg}\frac{\pi}{4}$

**Yechilishi:** $\cos\left(\frac{\pi}{2} - \frac{1}{2}\right) = \sin\frac{1}{2}$; $\sin 4 < 0$ ; $\cos\left(\frac{3\pi}{2} + \frac{1}{4}\right) = \sin\frac{1}{4}$; $\text{tg}\frac{\pi}{4} = 1$;

Javobi : E

**48 – misol**. ( 97-8-33) ). Ifodani soddalashtiring.

$$\sin\left(\frac{3\pi}{2} + a\right) c\,tg(\pi + \beta)$$

**Yechilishi :** $\sin\left(\frac{3\pi}{2} + a\right) ct\,g(\pi + \beta) = -cos\,\alpha \cdot c\,t\,g\beta$ Javobi : B

**49 – misol**. ( 98-11-105 ). Ifodani soddalashtiring. $cos^2(\pi + x) + +cos^2(\frac{\pi}{2} + x)$.

**Yechilishi :** $\cos(\pi + x) = -\cos x$; $\cos\left(\frac{\pi}{2} + x\right) = -sin\ x$ ; $cos^2(\pi + x) + +cos^2\left(\frac{\pi}{2} + x\right) = cos^2x + sin^2x = 1$. Javobi : E

**50 – misol**. ( 98-5-49). $tg1^0 \cdot tg2^0 \cdots tg88^0 \cdot tg89^0$ ni hisoblang.

**Yechilishi** : $tg46^0 = tg(90^0 - 44^0) = ctg44^0;$

$$tg\ 47^0 = tg\left(\frac{\pi}{2} - 43^0\right) = ctg43^0$$

……

$$tg89^0 = tg\left(\frac{\pi}{2} - 1^0\right) = ctg1^0;$$

$tg1^0 \cdot tg\ 2^0 \cdots tg44^0 \cdot tg45^0 \cdot ctg\ 44^0 \cdots ctg1^0 = tg45^0 = 1;$

Javobi : S

**51 – misol**. (00-8-42). Hisoblang. $\log_5 tg36^0 + \log_5 tg54^0$

**Yechilishi** : $\log_5(tg36^0 \cdot tg54^0) = \log_5 tg(90^0 - 54^0) \cdot tg54^0 =$

$= \log_5 ctg54^0 \cdot tg54^0 = \log_5 1 = 0.$ Javobi : A

**52 – misol**. ( 00-9-56 ). Hisoblang.

$$((3 \cdot 128^{3/7} \cdot e^{-\ln 8})^{-1/2} - (tg\frac{7\pi}{6})^{-1})^2 + \frac{12}{\sqrt{6}}$$

**Yechilishi** : $(3 \cdot (2^7)^{3/7} \cdot e^{\ln 48^{-1}})^{-1/2} = (3 \cdot 8 \cdot \frac{1}{48})^{-\frac{1}{2}} = (24 \cdot \frac{1}{48})^{-\frac{1}{2}} =$

$(\frac{1}{2})^{-\frac{1}{2}} = 2^{\frac{1}{2}} = \sqrt{2}$ ; $(tg\frac{7\pi}{6})^{-1} = (tg\left(\pi + \frac{\pi}{6}\right))^{-1} = (tg\frac{\pi}{6})^{-1} =$

$(\frac{1}{\sqrt{3}})^{-1} = \sqrt{3};$ $(\sqrt{2} - -\sqrt{3})^2 + \frac{12}{\sqrt{6}} = 2 - 2\sqrt{2 \cdot 3} + 3 + 2\sqrt{6} = 5 -$

$2\sqrt{6} + 2\sqrt{6} = 5;$ Javobi : E

**53 – misol.** ( 01-9-28 ). Qaysi ifoda ma'noga ega emas ?

1) $\log_3 \sin\frac{6\pi}{5}$; 2) $\log_2 \cos\frac{23\pi}{12}$; 3) $\sqrt{tg\frac{7\pi}{12}}$;

**Yechilishi:** $\sin\frac{6\pi}{5} = \sin\left(\pi + \frac{\pi}{5}\right) = -\sin\frac{\pi}{5}$; 1) $\log_3(-\sin\frac{\pi}{5})$ ma'noga ega emas. 2) $\cos\left(2\pi + \frac{\pi}{12}\right) = \cos\frac{\pi}{12}$; $\log_2 \cos\frac{\pi}{12}$ ma'noga ega .

3) $\sin\frac{7\pi}{12} = \sin\left(\pi - \frac{5\pi}{12}\right) = \sin\frac{5\pi}{12}$; $\cos\frac{7\pi}{12} = \cos(\pi - \frac{5\pi}{12}) - \cos\frac{5\pi}{12}$;

$tg\frac{7\pi}{12} = -$ ; $\sqrt{tg\frac{7\pi}{12}}$ - ma'noga ega emas. Javobi : C

**54 – misol**. ( 01-12-31 ). Qaysi ifoda ma'noga ega ?

Ye**chilishi:** 1) $\sqrt{\lg\frac{11\pi}{8}} = \sqrt{\lg\frac{34.54}{8}}$ -ma'noga ega; 2) $\sqrt{\sin\frac{19\pi}{12}} = = \sqrt{\sin(\pi + \frac{7\pi}{12})} = \sqrt{-\cos\frac{7\pi}{12}}$ - $\emptyset$; 3) $\log_{\sqrt{\frac{\pi}{3}}}\sqrt[3]{\frac{3\pi}{8}} = \log_{1.0} 1{,}01$ ma'noga ega. Javobi : C

**55 – misol** (01-12-42). Tenglamani yeching. $\frac{3}{2}x - \frac{4}{5} = \sqrt{\frac{1}{2} + (-\frac{\sqrt{2}}{2})}$ .

**Yechilishi** : $\frac{3}{2}x - \frac{4}{5} = \sqrt{\frac{1}{2} + (-\frac{\sqrt{2}}{2})}$ ; $\sin\frac{7\pi}{4} = \sin\left(2\pi - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = = -\frac{1}{\sqrt{2}}; \frac{3}{2}x - \frac{4}{5} = \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{2}}; \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{2}} < 0$. $\emptyset$. Javobi: B

**56 – misol**. ( 99-10-37 ). Hisoblang.

$$\lg tg22^0 + \lg tg68^0 + \lg\sin 90^0$$

**Yechilishi :** $\sin 90^0 = 1$; $demak$, $\lg\sin 90^0 = \lg 1 = 0$.
$\lg tg22^0 \cdot tg68^0 = \lg\cdot tg22^0 \cdot tg(90^0 - 22^0) = \lg tg\ 22^0 \cdot ctg\ 22^0 = = \lg 1 = 0$; Javobi: C

**57 – misol**. ( 02-3-76 ). $\sin\frac{\pi}{9} - \cos\frac{7\pi}{8}$ ni hisoblang.

**Yechilishi:** $\sin\frac{\pi}{9} - \cos\frac{7\pi}{18} = \sin\frac{\pi}{9} - \cos\left(\frac{\pi}{2} - \frac{\pi}{9}\right) = \sin\frac{\pi}{9} - \sin\frac{\pi}{9} = 0$. Javobi: A

**58 – misol**. ( 03-2-43 ). tg $1395^0$ ni hisoblang.

**Yechilishi** : $tg(3\cdot 360^0 + 315^0) = tg\ 315^0 = tg(270^0 + 45^0) = = -ctg45^0 = -1$. Javobi : C

**59– misol**. ( 03-2-43 ) $ctg37^0 \cdot ctg38^0 \cdot ctg39^0 \cdots\cdots ctg52^0 \cdot \cdot ctg53^0$ ni hisoblang.

**Yechilishi** : $ctg37^0 \cdot ctg38^0 \cdots ctg44^0 \cdot ctg45^0 \cdot ctg46^0 \cdots ctg53^0$
$ctg46^0 = ctg(90^0 - 44^0) = tg\ 44^0$; $ctg\ 52^0 = ctg(90^0 - 38^0) = tg38^0$. $ctg53^0 = ctg(90^0 - 37^0) = tg37^0$; Bundan, $tg37^0 \cdot tg\ 37^0 \cdot ctg38^0 \cdot tg38^0 \cdots ctg\ 44^0 \cdot tg44^0 \cdot tg45^0 = tg45^0 = 1$.
Javobi : V

**60 – misol.** ( 03-4-22 ). $tg240^0$, $\sin 120^0$, $\cos 150^0$ va $\operatorname{ctg} 225^0$ sonlardan eng kattasining eng kichigiga ko‘paytmasini toping.

**Yechilishi :** $tg240^0 = tg(180^0 + 60^0) = tg60^0 = \sqrt{3};$

$$\sin 120^0 = \sin(90^0 + 30^0) = \cos 30^0 = \frac{\sqrt{3}}{2};$$

$$\cos 150^0 = \cos(90^0 + 60^0) = -\sin 60^0 = -\frac{\sqrt{3}}{2};$$

Misolning shartiga asosan. $tg240^0 \cdot \cos 150^0 = \sqrt{3} \cdot \left(-\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} = = -1{,}5$ ; Javobi: V

**61 – misol**. ( 03-5-44 ) ni soddalashtiring.

$y = \cos(-7{,}9\pi)tg(-1{,}1\pi) - \sin 5{,}6\pi \cdot ctg4{,}4\pi.$

**Yechilishi**: Keltirish formulasiga asosan,

$$\cos(-7{,}9\pi) = \cos\frac{79}{10}\pi = \cos(8\pi - \frac{\pi}{10}) = -\cos\frac{\pi}{10};$$

$$tg\,(-1{,}1\pi) = -tg\,\frac{11}{10}\pi = -tg\left(\pi + \frac{\pi}{10}\right) = -tg\frac{\pi}{10};$$

$$\sin 5{,}6\pi = \sin\frac{56\pi}{10} = \sin\left(6\pi - \frac{4\pi}{10}\right) = -\sin\frac{2\pi}{5};$$

$$ctg\frac{44\pi}{10} = ctg\frac{22\pi}{2} = ctg\left(4\pi + \frac{2\pi}{5}\right) = ctg\frac{2\pi}{5};$$

Bularni berilgan ifodaga qo‘ysak, $y = -\cos\frac{\pi}{10} \cdot \frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} +$

$+\sin\frac{2\pi}{5} \cdot \frac{\cos\frac{2\pi}{5}}{\sin\frac{2\pi}{5}} = -\sin\frac{\pi}{10} + \cos\frac{2\pi}{5} = \cos\frac{4\pi}{5} - \sin\frac{\pi}{10} = = \cos\frac{2\pi}{5} -$

$\sin\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) = \cos\frac{2\pi}{5} - \cos\frac{2\pi}{5} = 0.$ Javobi : A

**62 – misol**. ( 03-9-29 ). Agar $\alpha - \beta = \frac{\pi}{2}$ bo‘lsa, $\frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta}$ ning qiymatini toping.

**Yechilishi** : $\alpha = \frac{\pi}{2} + \beta;$ $\sin\left(\frac{\pi}{2} + \beta\right) = \cos\beta; \cos\left(\frac{\pi}{2} + \beta\right) = -\sin\beta$

Bularni berilgan ifodaga qo‘ysak,

$\frac{\cos\beta - \sin\beta}{-\sin\beta + \cos\beta} = \frac{\cos\beta - \sin\beta}{\cos\beta - \sin\beta} = 1;$ Javobi : D

## Qo‘shish formulalari yordamida trigonometric ifodalarni soddalashtirish va hisoblash.

### Qo‘shish formulalari.

1. $\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$;
2. $\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$;
3. $\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$;
4. $\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$;
5. $\operatorname{tg}(x+y) = \frac{\operatorname{tg}x+\operatorname{tg}y}{1-\operatorname{tg}x\cdot\operatorname{tg}y}$; 6. $\operatorname{tg}(x-y) = \frac{\operatorname{tg}x-\operatorname{tg}y}{1+\operatorname{tg}x\cdot\operatorname{tg}y}$;
7. $\operatorname{ctg}(x+y) = \frac{\operatorname{ctg}x\cdot\operatorname{ctg}y-1}{\operatorname{ctg}y+\operatorname{ctg}x}$; 8. $\operatorname{ctg}(x+y) = \frac{\operatorname{ctg}x\cdot\operatorname{ctg}y+1}{\operatorname{ctg}y-\operatorname{ctg}x}$.

**63 – misol**. (01-1-42). Agar $tg\left(\frac{\pi}{4}-a\right) = 2$ bo‘lsa, tga ning qiymatini toping.

**Yechilishi** : $tg\left(\frac{\pi}{4}-a\right) = \frac{tg\frac{\pi}{4}-tga}{1+tg\frac{\pi}{4}\cdot tga} = \frac{1-tga}{1+tga}$; $\frac{1-tga}{1+tga} = 2 \Rightarrow$

$\Rightarrow 1 - tga = 2 + 2tga$; $3tga = -1$; $tga = -\frac{1}{3}$; Javobi : D

**64 – misol**. ( 96-3-111). Agar $a = \frac{1}{2}, tg\beta = \frac{1}{3}$ va $\pi < a + \beta < 2\pi$ bo‘lsa, α+β ning qiymatini toping.

**Yechilishi**: $tg(a+\beta) = \frac{tga+tg\beta}{1-tga\cdot tg\beta}$ ; $tg(a+\beta) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}$; $tg(a+\beta) = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$. $tg(a+\beta) = 1$; $a + \beta = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$; Javobi : C

**65 – misol**. ( 97 – 1-60 ). Agar $tg(x+y) = 3, tg(x-y) = 2$ bo‘lsa, tg 2 x ni hisoblang.

**Yechilishi** : $\begin{cases} x + y = a \\ x - y = \beta \end{cases} = 2x = a + \beta$; $tg(a+\beta) = \frac{tga+tg\beta}{1-tga\cdot tg\beta}$;

$tg2x = \frac{tg(x+y)+tg(x-y)}{1-tg(x+y)tg(x-y)} = \frac{3+2}{1-3\cdot 2} = \frac{5}{-5} = -1$; Javobi : D

**66 – misol**. ( 97-1-66). Agar $tga = \frac{5+\sqrt{x}}{2}; tg\beta = \frac{5-\sqrt{x}}{2}$ va $a + \beta = 45^0$ bo‘lsa, x ni toping.

**Yechilishi** : $tg\ (a+\beta) = \frac{t\,g\,a + t\,g\,\beta}{1 - t\,ga \cdot t\,g\beta}$;

$$1 = \frac{\frac{5+\sqrt{x}}{2} + \frac{5-\sqrt{x}}{2}}{1 - \frac{5+\sqrt{x}}{2} \cdot \frac{5-\sqrt{x}}{2}} => 1 = \frac{5}{1 - \frac{25-x}{4}} =>$$

$=> 1 = \frac{20}{x-21} => x - 21 = 20 => x = 41$. Javobi : A

**67-misol.** ( 97-6-60). Agar $\begin{cases} t\,g(a+\beta) = 5 \\ t\,g(a-\beta) = 3 \end{cases}$ bo‘lsa, $t\,g2\beta$ ni hisoblang.

**Yechilishi** : Quyidagicha belgilash kiritamiz.

$\begin{cases} a + \beta = x \\ a - \beta = y \end{cases}$ bundan, $2\beta = x - y$ ; $t\,g(x-y) = \frac{tg\ x - tg\ \ y}{1 + t\,g\ x\ t\,gy}$ formuladan

$t\,g2\beta = \frac{t\,g(a+\beta) - t\,g(a-\beta)}{1 + t\,g\ (a-\beta) \cdot t\,g(a+\beta)} => \frac{5-3}{1+15} = \frac{1}{8}$ ; Javobi : C

**68-misol**. (98-6-48). Agar tg ( x+y ) = 5 va tg x = 3 bo‘lsa, tgy ni toping

**Yechilishi** : $t\,g(x+y) = \frac{t\,g\,x + t\,g\,y}{1 - t\,g\,x \cdot t\,g\,y}$; $5 = \frac{3 + t\,gy}{1 - 3t\,gy} => 5 - 15t\,gy =$

$= 3 + t\,gy => 16t\,gy = 2 => tgy = \frac{1}{8}$; Javobi : B

**69-misol**. (98-8-61). Agar $b = \sin(40^0 + \alpha)$ va $0^0 < a < 45^0$ bo‘lsa, $\cos(70^0 + \alpha)$ ni b orqali ifodalang.

**Yechilishi** : $\cos(70^0 + \alpha) = \cos\left(30^0 + (40^0 + \alpha)\right) = \cos 30^0 \cdot \cos(40^0 + \ +\alpha) - \sin 30^0 \cdot \sin(40^0 + \alpha) = \frac{\sqrt{3}}{2}\cos(40^0 + \alpha) - \frac{1}{2} \cdot b =$

$\frac{\sqrt{3}}{2}\sqrt{1 - sin^2(40^0 + \alpha)} - \frac{b}{2} = \frac{\sqrt{3}}{2}\sqrt{1 - b^2} - \frac{b}{2} = \frac{1}{2}\left(\sqrt{3(1-b^2)} - b\right)$. Javobi : C

**70-misol**. (98-10-33). Agar $a = 46^0$ va $\beta = 16^0$ bo‘lsa, $\sin(a+\beta) - 2\sin\beta \cdot \cos a$ son 21,5 dan qancha kam bo’ladi ?

**Yechilishi** : $\sin(a+\beta) - 2\sin\beta \cdot \cos a = \sin a \cdot \cos\beta + \cos a \cdot \sin\beta - 2\sin\beta \cdot \cos a = \sin a \cdot \cos\beta - \sin\beta \cdot \cos a =$

$\sin(a-\beta) = \sin(46^0 - 16^0) = \sin 30^0 = \frac{1}{2}$; 21,5-0,5=21; Javobi : E

**71 – misol**. ( 98 – 11 – 73 ). Agar $5x^2 - 3x - 1 = 0$ tenglamaning ildizlari $tg\alpha$ va $tg\beta$ bo‘lsa, $tg\,(\alpha + \beta)$ qanchaga teng bo‘ladi ?

**Yechilishi** : $5x^2 - 3x - 1 = 0$; D = 9 + 20 = 29; $x_1 = \frac{3+\sqrt{29}}{10}$; $x_2 = \frac{3-\sqrt{29}}{10}$. $tg\alpha = \frac{3+\sqrt{29}}{10}$; $tg\,\beta = \frac{3-\sqrt{29}}{10}$; $tg\,(\alpha + \beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha \cdot tg\beta} = \frac{\frac{3+\sqrt{29}}{10} + \frac{3-\sqrt{29}}{10}}{1 - \frac{3+\sqrt{29}}{10} \cdot \frac{3-\sqrt{29}}{10}} = \frac{\frac{6}{10}}{1 - \frac{9-29}{100}} = \frac{\frac{6}{10}}{1 + \frac{2}{10}} = \frac{6}{12} = \frac{1}{2}$; Javobi : D

**72 – misol.** ( 98 – 11 – 104 ). Agar $\sin\alpha = \frac{3}{5}$ va $\sin\beta = \frac{5}{13}$, $\frac{\pi}{2} < \alpha < \pi$ va $\frac{\pi}{2} < \beta < \pi$ bo‘lsa, $\sin(\alpha - \beta)$ ning qiymati qanchaga teng ?

**Yechilishi:** $\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$; $\cos\alpha = \pm\sqrt{1 - sin^2\alpha} = \pm\sqrt{1 - \frac{9}{24}} = \pm\frac{4}{5}$; $\frac{\pi}{2} < \alpha < \pi$ shartdan $\cos\alpha = -\frac{4}{5}$; $\cos\beta = \pm\sqrt{1 - \sin\beta} = \pm\sqrt{1 - \frac{25}{169}} = \pm\frac{12}{13}$; $\frac{\pi}{2} < \beta < \pi$ shartdan $\cos\beta = -\frac{12}{13}$; Demak, $\sin(\alpha - \beta) = \frac{3}{5} \cdot \left(-\frac{12}{13}\right) - \left(-\frac{4}{5}\right) \cdot \frac{5}{13} = \frac{20}{65} - \frac{36}{65} = = -\frac{16}{65}$. Javobi : A

**73 – misol**. ( 99 – 5 – 25 ). Agar $\begin{cases} \alpha, \beta \in (0; \frac{\pi}{2}) \\ (tg(\alpha + 1)(tg\beta + 1) = 2 \end{cases}$ bo‘lsa, $3{,}2(\frac{\alpha+\beta}{\pi})^2$ ning qiymati nechaga teng ?

**Yechilishi:** ( tg a +1 ) ( tg$\beta$ + 1 )= 2; tg a · tg β + tg a + tg β + 1 = 2; tg a + tg β = 1 – tg a · tg β; $\frac{tg\alpha + tg\beta}{1 + tg\alpha \cdot tg\beta} = 1$, tg (a – β) = 1; $\alpha + \beta = \frac{\pi}{4}$; $3{,}2 \cdot (\frac{\pi}{4 \cdot \pi})^2 = 3{,}2 \cdot \frac{1}{16} = 0{,}2$. Javobi : B

**74 – misol**. ( 00 – 1 – 32 _. Agar $tg\alpha + tg\beta = \frac{5}{6}$; $tg\alpha \cdot tg\beta = \frac{1}{6}$ bo‘lsa, α+β nimaga teng bo‘ladi ?

**Yechilishi** : $tg(\alpha + \beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha \cdot tg\beta}$ formuladan $tg(\alpha + \beta) = \frac{5/6}{1 - 1/6} = = 1$; $\alpha + \beta = \frac{\pi}{4} + k\pi$ $k \in Z$; Javobi : D

**75 – misol**. ( 00 – 1 – 31 ). Soddalashtiring. $ctg2\alpha - ctg\alpha$

**Yechilishi** : $\frac{\cos 2a}{\sin 2a} - \frac{\cos a}{\sin a} = \frac{\cos^2 a - \sin^2 a - 2\cos^2 a}{2\sin a \cos a} = \frac{-\sin^2 a - \cos^2 a}{2\sin a \cdot \cos a} =$

$= -\frac{1}{\sin 2a}$; Javobi : C

**76 – misol**. ( 98 – 6 – 56 ). Agar $\sin^2 x = \cos x \cdot \cos y$ ; $\cos^2 x = \sin x \cdot \sin y$ bulsa, $\cos(x-y)$ ni toping.

**Yechilishi :** $\sin^2 x + \cos^2 x = \cos x \cdot \cos y + \sin x \cdot \sin y$; $\cos(x-y) = 1$; Javobi : B

**77 – misol.** ( 01 – 10 – 40 ). cos (x-y) ni toping. $\begin{cases} \sin x \cdot \sin y = \frac{1}{4} \\ ctgx \cdot ctgy = 3 \end{cases}$

**Yechilishi :** $\frac{\cos x}{\sin x} \cdot \frac{\cos y}{\sin y} = 3$; cos x· cos y = 3 sin x ·sin y; $\cos x \cdot \cos y = \frac{3}{4}$,

$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y = \frac{3}{4} + \frac{1}{4} = 1$; Javobi : C

**78 – misol**. ( 97 – 3 – 54). Soddalashtiring.

$$\frac{\sin 56^0 \cdot \sin 124^0 - \sin 34^0 \cdot \cos 236^0}{\cos 28^0 \cdot \cos 88^0 + \cos 178^0 \cdot \sin 208^0}$$

**Yechilishi :**

$\sin 124^0 = \sin(90^0 + 34^0) = \cos 34^0$;

$\cos 236^0 = \cos(180^0 + 56^0) = \cos 56^0$;

$\sin 56^0 \sin 124^0 - \sin 34^0 \cos 236^0 = \sin 56^0 \cos 34^0 - \sin 34^0 (-\cos 56^0) = \sin 56^0 \cos 34^0 + \sin 34^0 \cos 56^0 = \sin(56^0 + 34^0) = \sin 90^0 = 1$;

$\cos 178^0 = \cos(90^0 + 88^0) = -\sin 88^0$;

$\sin 178^0 = \sin(180^0 + 28^0) = -\sin 28^0$.

$\cos 28^0 \cdot \cos 88^0 + \sin 88^0 \cdot \sin 28^0 = \cos(88^0 - 28^0) =$

$\cos 60^0 = \frac{1}{2}$;

Javobi : C

**79 – misol.** ( 97 – 10 – 54 ). Soddalashtiring.

$$\frac{\cos 18^0 \cdot \cos 28^0 + \cos 108^0 \cdot \sin 208^0}{\sin 18^0 \cdot \sin 78^0 + \sin 108^0 \cdot \sin 168^0}$$

**Yechilishi** :

*Cos 108 $^0$ = cos ( 90 $^0$ + 18 $^0$ ) = - sin 18 $^0$ ;*
*sin 208 $^0$ = sin ( 180 $^0$ + 28 $^0$ ) = - sin 28 $^0$ ;*
*cos 18 $^0$ cos 28 $^0$ + sin 18 $^0$ sin 28 $^0$ = cos ( 28 $^0$ – 18 $^0$ ) = cos 10 $^0$ ;*
*sin 108 $^0$ = sin ( 90 $^0$ + 18 $^0$ ) = cos 18 $^0$;*
*sin 168 $^0$ = sin ( 90 $^0$ + 78 $^0$ ) = cos 78 $^0$;*
*sin 18 $^0$ sin 78 $^0$ + cos 18 $^0$ cos 78 $^0$ = cos ( 78 $^0$ – 18 $^0$ ) = cos 60 $^0$ = ½ ;*
$\frac{\cos 18^0\cdot\cos 28^0+\cos 108^0\cdot\sin 208^0}{\sin 28^0\cdot\sin 88^0+\sin 108^0\cdot\sin 168^0} = \frac{\cos 10^0}{\frac{1}{2}} = 2\cos 10^0$ ; Javobi : A

**80 – misol**. ( 01 – 1 – 49 ). Agar $\sin a = -\frac{1}{3}$ va $\cos\beta = -\frac{1}{2}$ bo‘lsa,
$\sin(a+\beta)\cdot\sin(a-\beta)$ ning qiymatini toping.
**Yechilishi** : $\sin(a+\beta) = \sin a\cdot\cos\beta + \cos a\cdot\sin\beta$ ; $\cos a = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$;
$\sin\beta = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$; $\sin(a+\beta) = (-\frac{1}{3})\cdot\left(-\frac{1}{2}\right) + \frac{2\sqrt{2}}{3}\cdot\frac{\sqrt{3}}{2} = =$
$\frac{1}{6} + \frac{\sqrt{6}}{3} = = \frac{1+2\sqrt{6}}{6}$; $\sin(a-\beta) = \left(-\frac{1}{3}\right)\left(-\frac{1}{2}\right) - \frac{2\sqrt{2}}{3}\cdot\frac{\sqrt{3}}{2} = \frac{1}{6} - -\frac{\sqrt{6}}{3} =$
$\frac{1-2\sqrt{6}}{6} = \frac{1-24}{36} = -\frac{23}{36}$; Javobi : A

**81 – misol**. ( 01 – 6 – 27 ). Hisoblang : $\cos 15^0 + \sqrt{3}\sin 15^0$
**Yechilishi** : $2\cdot\left[\frac{1}{2}\cos 15^0 + \frac{\sqrt{3}}{2}\sin 15^0\right] = 2[\sin 30^0\cdot\cos 15^0 + \cos 30^0\cdot\sin 15^0] = 2[\sin(30^0+15^0)] = 2\sin 45^0 = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$ ; Javobi : B

**82 – misol**. ( 01 – 11 – 24 ). Soddalashtiring. $\frac{\sin a+\cos a}{\sqrt{2}\cos(\frac{\pi}{4}-a)}$
**Yechilishi** : $\frac{\sin a+\cos a}{\sqrt{2}(\cos 45^0\cdot\cos a+\sin 45^0\cdot\sin a)} = \frac{\sin a+\cos a}{\sqrt{2}(\cos a+\sin a)} = 1$. Javobi : D

**83 – misol.** ( 02 – 3 – 71 ). $a,\beta,\gamma$ utkir burchaklar bo’lsa, $tga = = \frac{1}{2}, tg\beta = \frac{1}{5}$ va $tg\gamma = \frac{7}{9}$ bo’lsa , γ ni α va β lar orqali ifodalang.

**Yechilishi** : $tg(a+\beta)=\frac{tga+tg\beta}{1-tga\cdot tg\beta}=\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\cdot\frac{1}{5}}=\frac{\frac{7}{10}}{\frac{9}{10}}=\frac{7}{9}$; $tg\gamma=\frac{7}{9}$

bundan, $tg\gamma=tg(a+\beta)$ yoki $\gamma=a+\beta$. Javobi : A

**84 – misol**. ( 02 – 5 – 35 ). Agar $tg\,a=3$, $tg\beta=-\frac{1}{2}$, $0<a<\pi$ va $-\frac{\pi}{2}<\beta<0$ bo‘lsa, $a+\beta$ ni toping.

**Yechilishi :** $tg(a+\beta)=\frac{tga+tg\beta}{1-tga\cdot tg\beta}=\frac{3-\frac{1}{2}}{1-3\cdot\left(-\frac{1}{2}\right)}=\frac{\frac{5}{2}}{1+\frac{3}{2}}=\frac{\frac{5}{2}}{\frac{5}{2}}=1.$

$a+\beta=\frac{\pi}{4}$; Javobi : D

**85 – misol**. ( 03–1–25 ). $\sin x\cdot\cos y=-\frac{1}{2}$; $\cos x\cdot\sin y=\frac{2}{3}$; $ctg(x-y)$ =?

**Yechilishi** : Birinchi tenglikdan ikkinchisini ayiramiz.

$\sin x\cdot\cos y+\cos x\cdot\sin y=-\frac{1}{3}-\frac{3}{2}$; sin (x-y)= -1; $\cos(x-y)=\sqrt{1-1}=0.$

$ctg(x-y)=\frac{\cos(x-y)}{\sin(x-y)}=0$; Javobi : A

**86 – misol.** ( 03 – 5 – 46 ). $4\cos 20^0-\sqrt{3}ctg20^0$ ni hisoblang

**Yechilishi :** $4\cos 20^0-\sqrt{3}\frac{\cos 20^0}{\sin 20^0}=4\cos 20^0+1-\frac{\sqrt{3}\cos 20^0}{\sin 20^0}-1=$

$4\cos 20^0+\frac{\sin 20^0-\sqrt{3}\cos 20^0}{\sin 20^0}-1=4\cos 20^0+\frac{2(\frac{1}{2}\sin 20^0-\frac{\sqrt{3}}{2}\cos 20^0}{\sin 20^0}-$

$1=4\cos 20^0+\frac{2\sin(-40^0)}{\sin 20^0}-1=4\cos 20^0-\frac{4\sin 20^0\cdot\cos 20^0}{\sin 20^0}-1=$

$4\cos 20^0-4\cos 20^0-1=-1$. Javobi : A

**87 – misol** . 03 – 12 – 77 ). $\left(\left(tg^2\frac{7\pi}{24}-tg\frac{2\pi}{24}\right):\left(1-tg^2\frac{7\pi}{24}\cdot tg^2\frac{\pi}{24}\right)\right)^2$

ni hisoblang.

**Yechilishi :** $\left(\left(\frac{tg\frac{7\pi}{24}-tg\frac{\pi}{24}}{1+tg\frac{7\pi}{24}\cdot tg\frac{\pi}{24}}\right)\left(\frac{tg\frac{7\pi}{24}+tg\frac{\pi}{24}}{1-tg\frac{7\pi}{24}\cdot tg\frac{\pi}{24}}\right)\right)^2=\left(tg\left(\frac{7\pi}{24}-\frac{\pi}{24}\right)\cdot tg\left(\frac{7\pi}{24}++\frac{\pi}{24}\right)\right)^2=tg^2\frac{6\pi}{24}\cdot tg^2\frac{8\pi}{24}=tg^2\frac{\pi}{24}\cdot tg^2\frac{\pi}{3}=3.$ Javobi : E

## Ikkilangan burchak formulalari yrdamida trigonometric ifodalarni soddalashtirish va hisoblash.

### Ikkilangan burchak formulalari

1. $\sin 2\alpha = 2\sin\alpha\cos\alpha$;
2. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$;
3. $\cos 2\alpha = 1\text{-}2\sin^2\alpha$;
4. $\cos 2\alpha = 2\cos^2\alpha - 1$;
5. $\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$; 6. $\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}$;

**88 – misol.** ( 97 – 6 – 68 ). Agar $tga = \frac{3+\sqrt{x}}{2}$, $tg\beta = \frac{3-\sqrt{x}}{2}$, $a + \beta = = \frac{\pi}{4}$ bo‘lsa, x ni toping.

**Yechilishi** : $tg(a+\beta) = \frac{\frac{3+\sqrt{x}}{2}+\frac{3-\sqrt{x}}{2}}{1-\frac{(3+\sqrt{x})(3-\sqrt{x}}{4}}$, $1 = \frac{3-4}{4-9+x}$, $x - 5 = 12$, $x = 17$. Javobi : D

**89 – misol**. ( 99 – 10 – 30 ) Agar $tg(a-\beta) = 5$ va $a = 45^0$ bo‘lsa, $tg\,\beta$ ning qiymatini toping.

**Yechilishi :** $tg(a-\beta) = \frac{tga - tg\beta}{1+tga\cdot tg\beta}$, bundan, $5 = \frac{1-tg\ \beta}{1+tg\ \beta}$ yoki $5 + 5tg\beta = 1 - tg\beta$, $ctg\beta = -4$, $tg\beta = -\frac{2}{3}$. Javobi : E

**90 – misol**. ( 00 – 1 – 29 ). Agar $a = -45^0$ va $\beta = 15^0$ bo‘lsa, $\cos(a+\beta) + +2\sin a\cdot\sin\beta$ ning qiymatini toping

**Yechilishi:** $\cos a\cdot\cos\beta - \sin a\cdot\sin\beta + 2sina\cdot\sin\beta = \cos a\cdot\cos\beta + +\sin a\cdot\sin\beta = \cos(a-\beta) = \cos(-45^0 - 15^0) = \cos(-60^0) = \frac{1}{2}$. Javobi: E

**91 – misol**. ( 00 – 9 – 65 ). Agar $a,\beta \in (0;\frac{\pi}{2})$ va $(tg\ a + \sqrt{3})(tg\,\beta + +\sqrt{3}) = 4$ bo‘lsa, $9\cdot(\frac{a+\beta}{\pi})^2$ ning qiymatini hisoblang.

**Yechilishi :** $(tga + \sqrt{3})(tg\beta + \sqrt{3}) = tga\cdot tg\beta + \sqrt{3}tga + \sqrt{3}tg\beta + 3 == 4$ ; $tga\cdot tg\beta + \sqrt{3}(tga + tg\beta) = 1$; $\sqrt{3}(tga + tg\beta) = 1 - tga\cdot tg\beta$;

$\frac{tg\,a+tg\,\beta}{1-tg\,a\cdot tg\beta} = \frac{1}{\sqrt{3}}$; Bundan $tg(a+\beta) = 1/\sqrt{3}$ yoki $a+\beta = \frac{\pi}{6}$;
$9\cdot(\frac{\pi}{6}\cdot\frac{1}{\pi})^2 = 9\cdot\frac{1}{36} = \frac{1}{4} = 0{,}25$; Javobi : A

**92 – misol**. ( 98 – 2 – 25 ). Quyidagi sonlardan qaysi biri qolgan uchtasiga teng emas ?

$p = \frac{1}{sin^2 x} - ctg^2 x$, $q = tg\,x\cdot tg(270^0 - x)$, $x \neq \frac{k\pi}{2}$, $k \in z$),

$r = cos^2(270^0 - x) + cos^2 x$, $l = sin\,42^0\cdot cos\,48^0 + sin\,48^0\cdot cos\,42^0$

**Yechilishi** : p = 1 + ctg $^2$ x – ctg $^2$ x = 1; q = tg x · ctg x = 1;
l = sin ( 42 $^0$ + 48 $^0$ ) = 1; javobi : C

**93 – misol**. ( 00 – 7 – 30 ). Agar $\sin a\cdot\cos\beta = 1$ va $\sin\beta\cdot\cos a = \frac{1}{2}$ bo‘lsa, $a-\beta$ ning qiymatini toping.

**Yechilishi** : $-\begin{cases}\sin a\cdot\cos\beta = 1\\ \sin a\cdot\cos a = \frac{1}{2},\end{cases}$ bundan $\sin(a-\beta) = \frac{1}{2}, a-\beta = (-1)^n\frac{\pi}{6} + +n\,\pi$Javobi : E

**94 – misol**. ( 02 – 6 – 46 ). $\begin{cases}\cos x\cdot\cos y = \frac{1}{6}\\ tgx\cdot tgy = 2\end{cases}$, $\cos(x+y) = ?$

**Yechilishi:** $\frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y} = 2$ bundan, $\sin x\cdot\sin y = 2\cdot\frac{1}{6} = \frac{1}{3}$, $\cos(x++y) = = \cos x\cdot\cos y - \sin x\cdot\sin y = \frac{1}{6} - \frac{1}{3} = -\frac{1}{6}$.

Javobi : E

**95 – misol** ( 96 – 7 – 54 ) . Soddalashtiring.

$$\frac{\cos 18^0\cdot\cos 28^0 + \cos 108^0\cdot\sin 208^0}{\sin 34^0\cdot\sin 146^0 + \sin 236^0\cdot\sin 304^0}$$

**Yechilishi** : Keltirish formulasidan foydalanamiz.
cos 108 $^0$ = cos ( 90 $^0$ + 18 $^0$ ) = - sin 18 $^0$ ;
sin 208 $^0$ = sin ( 180 $^0$ + 28 $^0$ ) = - sin 28 $^0$ ;
sin146 $^0$ = sin ( 180 $^0$ - 34 $^0$ ) = sin 34 $^0$ ;
sin 236 $^0$ = sin ( 270 $^0$ - 34 $^0$ ) = - cos 34 $^0$;
sin 304 $^0$ = sin ( 270 $^0$ +34 $^0$ ) = - cos 34 $^0$ ;
Bularni berilgan ifodaga kuysak,

$$\frac{\cos 18^0 \cdot \cos 28^0 + (-\sin 18^0) \cdot (-\sin 28^0)}{\sin 34^0 \cdot \sin 34^0 + (-\cos 34^0) \cdot (-\cos 34^0)} = \frac{\cos(28^0 - 18^0)}{cos^2 34^0 + sin^2 34^0}$$
$$= \cos 10^0$$

Javobi : B

**96 – misol**. ( 02 – 3 – 72 ). $\frac{2\cos\left(\frac{\pi}{4}-\alpha\right)+\sqrt{2}\sin\left(\frac{3\pi}{2}-\alpha\right)}{2\sin\left(\frac{2\pi}{2}+\alpha\right)-\sqrt{3}\cos(2\pi-\alpha)}$ ni soddalashtiring.

**Yechilishi:**

$$\frac{2\cdot\left[\cos\frac{\pi}{4}\cdot\cos\alpha+\sin\frac{\pi}{4}\cdot\sin\alpha\right]+\sqrt{2}(-\cos\alpha)}{2[\sin 120^0\cos\alpha+\cos 120^0\cdot\sin\alpha]-\sqrt{3}\cos\alpha} =$$

$$\left|\begin{array}{l}\sin(90^0 + 30^0) = \cos 30^0 = \frac{\sqrt{3}}{2} \\ \cos(90^0 + 30^0 = -30^0 = -\frac{1}{2}\end{array}\right| = \frac{2\cdot\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)-\sqrt{2}\cos\alpha}{2(\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}\sin\alpha)-\sqrt{3}\cos\alpha} =$$

$$\frac{\sqrt{2}\cos\alpha+\sqrt{2}\sin\alpha-\sqrt{2}\cos\alpha}{\sqrt{3}\cos\alpha+\sin\alpha-\sqrt{3}\cos\alpha} = \frac{\sqrt{2}\sin\alpha}{-\sin\alpha} = -\sqrt{2}.$$ Javobi : A

**97 – misol**. ( 02 – 10 – 22 ). Agar $\sin\alpha = -\frac{1}{3}, \cos\beta = -\frac{1}{2}$, bo‘lsa, $\sin(\alpha + \beta) \cdot \sin(\alpha - \beta)$ ning qiymatini hisoblang.

**Yechilishi:** $\cos\alpha = \sqrt{1 - sin^2\alpha} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$; $\sin\beta\sqrt{1 - \frac{1}{4}} =$

$= \frac{\sqrt{3}}{2}$; $\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta = \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{2}\right) + \frac{2\sqrt{2}}{3} \cdot \frac{\sqrt{3}}{2} = = \frac{1}{6} + \frac{\sqrt{6}}{3} = \frac{1+2\sqrt{6}}{3}$; $\sin(\alpha - \beta) = \frac{1}{6} - \frac{\sqrt{6}}{3} = \frac{1-2\sqrt{6}}{6}$;

$\sin(\alpha + \beta) \cdot \sin(\alpha - \beta) = \frac{1+2\sqrt{6}}{6} \cdot \frac{1-2\sqrt{6}}{6} = \frac{1-24}{36} = -\frac{23}{36}$; Javobi : A

**98 – misol**: ( 03 – 4 – 23 ). $(tg60^0 \cdot \cos 15^0 - \sin 15^0) \cdot 7\sqrt{2}$ ning qiymatini toping.

**Yechilishi:** $\sqrt{3}\cos 15^0 - \sin 15^0 = 2(\frac{\sqrt{3}}{2}\cos 15^0 - \frac{1}{2}\sin 15^0) =$

$2(\sin 60^0 \cdot \cos 15^0 - \cos 60^0 \cdot \sin 15^0) = 2\sin(60^0 - 15^0) =$

$2\sin 45^0 = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

Demak , $\sqrt{2} \cdot 7\sqrt{2} = 14$; Javobi : D

**99 – misol**. ( 96 -3 – 112 ). Soddalashtiring. $\frac{\sin 3\alpha}{\sin\alpha} - \frac{\cos 3\alpha}{\cos\alpha}$

**Yechilishi:** $\frac{\sin 3\alpha}{\sin\alpha} - \frac{\cos 3\alpha}{\cos\alpha} = \frac{\sin 3\alpha\cdot\cos\alpha - \cos 3\alpha\cdot\sin\alpha}{\sin\alpha\cdot\cos\alpha}; \frac{2\sin(3\alpha-\alpha)}{2\sin\alpha\cos\alpha} =$

$= \frac{2\sin 2\alpha}{\sin 2\alpha} = 2$. Javobi : B

**100 – misol**. ( 97 – 1 – 52 ). Hisoblang.

$$\sin\frac{\pi}{16}\cdot\cos^3\frac{\pi}{16} - \sin^3\frac{\pi}{16}\cdot\cos\frac{\pi}{16}$$

**Yechilishi** : $\sin\frac{\pi}{16}\cdot\cos^3\frac{\pi}{16} - \sin^3\frac{\pi}{16}\cdot\cos\frac{\pi}{16} = \sin\frac{\pi}{16}\cos\frac{\pi}{16}\left(\cos^2\frac{\pi}{16} - \right.$

$\left. -\sin^2\frac{\pi}{16}\right) = \frac{1}{2}\sin 2\frac{\pi}{16}\cdot\cos 2\frac{\pi}{16} = \frac{1}{2}\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \frac{1}{4}\sin\frac{\pi}{4} = \frac{1}{4}\cdot\frac{\sqrt{2}}{2} =$

$\frac{\sqrt{2}}{8}$. Javobi:D

**101 – misol**. ( 96 – 10 – 35 ). Agar $\cos a = \frac{1}{2}$ bo‘lsa, $\frac{2\sin a+\sin 2a}{2\sin a-\sin 2a}$ ni hisoblang.

**Yechilishi :** $\frac{2\sin a+\sin 2a\cos a}{2\sin a-\sin 2a\cos a} = \frac{2\sin(1+\cos a)}{2\sin(1-\cos a)} = \frac{1+0,5}{1-0,5} = \frac{1,5}{0,5} = 3$. Javobi : C

**102 – misol**. ( 98 – 3 – 53 ). Hisoblang. $\frac{\sin 6^0}{\sin 12^0} - \frac{\cos 36^0}{\cos 12^0}$

**Yechilishi** : Umumiy maxraj beramiz. $\frac{2(\sin 36^0\cdot\cos 12^0 - \cos 36^0\cdot\sin 12^0)}{2\sin 12^0\cdot\cos 12^0} =$

$= \frac{2\cdot\sin(36^0-12^0)}{\sin 24^0} = 2$. Javobi : A

**103 – misol**. ( 98 – 10 – 37 ). Agar $a = 15^0$ bo‘lsa, $(1 + \cos 2a)tga$ ning qiymatini $\frac{1}{8}$ bilan solishtiring.

**Yechilishi :** $\frac{2\cdot(1+\cos 2a)}{2}\cdot tga = 2\cos^2 a\cdot\frac{\sin a}{\cos a} = 2\cos a\cdot$ $\sin a = = \sin 2a = \sin 30^0 = \frac{1}{2}; \frac{1}{2}\div\frac{1}{8} = 4.$ $\frac{1}{8}$ *dan* 4 marta katta. Javobi : D

**104 – misol**. ( 98 – 10 – 101 ). Agar $tga = \frac{1}{2}$ bo‘lsa, tg 2α ni toping.

**Yechilishi :** $tg2a = \frac{tg2a}{1-tg^2 a}$ *dan* $tg\,2a = \frac{2\cdot\frac{1}{2}}{1-(\frac{1}{2})^2} = \frac{4}{3}$. Javobi : B

**105 – misol**. ( 98 – 11 – 17 ). Hisoblang. $\text{tg } 22{,}5^0 + \text{tg}^{-1} 22{,}5^0$

**Yechilishi:** :

$tg22,5^0 + \frac{1}{tg22,5} = \frac{sin22,5^0}{cos22,5^0} + \frac{cos22,5^0}{sin22,5^0} = \frac{sin^2 22,5^0 + cos^2 22,5^0}{sin22,5^0 \cdot cos22,5^0} = \frac{2}{2sin22,5^0 \cdot cos22,5^0} = \frac{2}{sin45^0} = 2 \cdot \frac{1}{\frac{1}{\sqrt{2}}} = 2\sqrt{2}$. Javobi : E

**106 – misol**. ( 98 – 12 – 78 ). Agar $\sin\frac{a}{2} + \cos\frac{a}{2} = -\frac{1}{2}$ va $\frac{3\pi}{2} < a < 2\pi$ bo‘lsa, $\sin 2\alpha$ ning qiymatini toping.

**Yechilishi** : $(\sin\frac{a}{2} + \cos\frac{a}{2})^2 = (-\frac{1}{2})^2$; $sin^2\frac{a}{2} + 2\sin\frac{a}{2} \cdot \cos\frac{a}{2} + cos^2\frac{a}{2} = \frac{1}{4}$; $1 + \sin a = \frac{1}{4}$; $\sin a = -\frac{3}{4}$; $\cos a = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$; $\sin 2a = 2\sin a \cdot \cos a = 2 \cdot \left(-\frac{3}{4}\right) \cdot \frac{\sqrt{7}}{4} = -\frac{3\sqrt{7}}{8}$. Javobi : A

**107 – misol**. ( 98–4 – 29 ). Hisoblang. $\cos92^0\cos2^0 + 0{,}5\sin 4^0 + 1$

**Yechilishi :** cos ( $90^0 + 2^0$ ) = - $\sin2^0$ ; $-\sin 2^0 \cdot \cos 2^0 + \frac{1}{2} \cdot 2\sin 2^0 \cdot \cos 2^0 + 1 = 1$; Javobi : B

**108 – misol.**(96–11- 59). Hisoblang.$Sin10^0 \cdot sin\,30^0 \cdot sin50^0 \cdot sin70^0$

**Yechilishi :** $\frac{1}{2}\sin 10^0 \cdot \sin 50^0 \cdot \sin 70^0 = \frac{1}{2}\sin 10^0 \cdot \sin(90^0 - 20^0) \cdot \sin(90^0 - 40^0) =$

$\frac{1}{2}\sin 10^0 \cdot \cos 20^0 \cdot \cos 40^0 = \frac{2\sin10^0 \cdot \cos10^0 \cdot \cos20^0 \cdot \cos40^0}{2 \cdot 2\cos10^0} = \frac{2\sin20^0 \cdot \cos20^0 \cdot \cos40^0}{2 \cdot 4 \cdot \cos10^0} = \frac{2 \cdot \sin40^0 \cos40^0}{2 \cdot 8\cos10^0} = \frac{\sin80^0}{16 \cdot \cos10^0} = \frac{\sin(90^0 - 10^0)}{16\cos10^0} = \frac{1}{16}$.

Javobi : E

**109 – misol**. (96–12– 12 ). Hisoblang. Cos $20^0 \cdot$ cos $40^0 \cdot$ cos $80^0$

**Yechilishi** $\frac{2\sin20^0 \cdot \cos20^0 \cdot \cos40^0 \cdot \cos80^0}{2\sin20^0} = \frac{2\sin40^0 \cdot \cos40^0 \cos80^0}{4\sin20^0} =$

$-\frac{2\sin80^0 \cdot \cos80^0}{8 \cdot \sin20^0} = \frac{\sin160^0}{8\sin20^0} = \frac{\sin(180^0 - 20^0)}{8 \cdot \sin20^0} = \frac{\sin20^0}{8\sin20^0} = \frac{1}{8}$

Javobi : C

**110 – misol**. ( 00 – 10 – 13 ) . Hisoblang. $\cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}$

**Yechilishi :** $\frac{2\sin\frac{\pi}{5} \cdot \cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}}{2\sin\frac{\pi}{5}} = \frac{2\sin\frac{2\pi}{5} \cdot \cos\frac{2\pi}{5}}{2 \cdot 2 \cdot \sin\frac{\pi}{5}} = \frac{\sin\frac{4\pi}{5}}{4 \cdot \sin\frac{\pi}{5}} = \frac{\sin(\pi - \frac{\pi}{5})}{4 \cdot \sin\frac{\pi}{5}} = \frac{1}{4}$.

Javobi : C

**111 – misol**. ( 96 – 13 – 38 ). Soddalashtiring . $\frac{2}{ctga-tga}$

**Yechilishi :** $ctga-tga=\frac{\cos a}{\sin a}-\frac{\sin a}{\cos a}=\frac{\cos^2 a-\sin^2 a}{\sin a\cdot\cos a}$ ; $\frac{2}{ctga-tga}=-\frac{2\sin a\cos a}{\cos 2a}=\frac{\sin 2a}{\cos 2a}=tg2a$; Javobi : C

**112 – misol**. ( 98 – 8 – 57 ). Hisoblang. $\sin^4\left(\frac{23\pi}{12}\right)-\cos^4\left(\frac{13\pi}{12}\right)$

**Yechilishi :** $\sin^4\left(\frac{23\pi}{12}\right)-\cos^4\left(\frac{13\pi}{12}\right)=\left(\sin^2\frac{23\pi}{12}-\cos^2\frac{13\pi}{12}\right)\cdot\left(\sin^2\frac{23\pi}{12}++\cos^2\frac{13\pi}{12}\right)$ ; $\sin\frac{23\pi}{12}=\sin\left(2\pi-\frac{\pi}{12}\right)=-\sin\frac{\pi}{12}$ ; $\cos\frac{13\pi}{12}=\cos\left(\pi+\frac{\pi}{12}\right)==-\cos\frac{\pi}{12}$ ; $\left(\sin^2\frac{\pi}{2}-\cos^2\frac{\pi}{2}\right)\left(\sin^2\frac{\pi}{12}+\cos^2\frac{\pi}{12}\right)=-\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$; Javobi : C

**113 – misol**. ( 98 – 12 – 90 ). Hisoblang . $\frac{\sqrt{3}}{\sin 100^0}+\frac{1}{\cos 260^0}$

**Yechilishi :** $\sin 100^0=\sin(90^0+10^0)=\cos 10^0$ ; $\cos 260^0=\cos(270^0-10^0)==-\sin 10^0$; $\frac{\sqrt{3}}{\sin 100^0}+\frac{1}{\cos 260^0}=\frac{\sqrt{3}}{\sin 10^0}-\frac{1}{\sin 10^0}=\frac{\sqrt{3}\sin 10^0-\cos 10^0}{\sin 10^0\cdot\cos 10^0}==\frac{2\cdot 2(\frac{\sqrt{3}}{2}\sin 10^0-\frac{1}{2}\cos 10^0}{2\cdot\sin 10^0\cdot\cos 10^0}=\frac{4(\cos 30^0\cdot\sin 10^0-\sin 30^0\cdot\cos 10^0)}{\sin 20^0}==\frac{4\sin(10^0-30^0)}{\sin 20^0}=-\frac{4\sin 20^0}{\sin 20^0}=-4$. Javobi : B

**114 – misol**. ( 99 – 3 – 32 ). Soddalashtiring.

$$\sin^6 a+\cos^6 a+\frac{3}{4}\sin^2 a$$

**Yechilishi:** $(\sin^2 a)^3+(\cos^2 a)^3=(\sin^2 a+\cos^2 a)(\sin^4 a-\sin^2 a\cdot\cdot\cos^2 a+\cos^4 a)=(\sin^2 a+\cos^2 a)^2-3\sin^2 a\cos^2 a=1-3\sin^2 a\cos^2 a$;

$$\sin^6 a+\cos^6 a+\frac{3}{4}\sin^2 2a$$

$$=1-3\sin^2 a\cos^2 a+\frac{3}{4}\cdot(2\sin a\cdot\cos a)^2=$$

$=1-3\sin^2 a\cdot\cos^2 a+3\sin^2 a\cdot\cos^2 a=1$. Javobi : A

**115 – misol.** ( 99 – 9 – 30 ). $\sin^4 a+\cos^4 a$ ning eng kichik qiymatini toping.

**Yechilishi :** $\sin^4 a+\cos^4 a=(\sin^2 a+\cos^2 a)-2\sin^2 a\cdot\cos^2 a=$

$= 1 - 2 \cdot (\frac{1}{2}$

$\cdot 2 \sin a \cos a)^2 = 1 - 2 \cdot \frac{1}{4} sin^2 2a = 1 - \frac{1}{2} \cdot \frac{1 - \cos 2a}{2} =$

$= 1 - \frac{1 - \cos 2a}{4} = 1 - \frac{1}{4} + \frac{1}{4} \cos 2a = \frac{3}{4} + \frac{1}{4} \cos 2a; \Longrightarrow$

Cos 2α = - 1 desak, $\Longrightarrow \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ . Javobi : C

**116 – misol**. ( 00 – 8 – 14 ). Hisoblang. $\log_2 \cos 20^0 = \log_2 \cos 40^0 + + \log_2 \cos 60^0 + \log_2 \cos 80^0$.

**Yechilishi :** $\log_2 \cos 20^0 = \log_2 \cos 40^0 + \log_2 \cos 60^0 + \log_2 \cos 80^0 =$

$= \log_2 ( \cos 20^0 \cdot \cos 40^0 \cdot \cos 60^0 \cdot \cos 80^0 ) =$

$= \log_2 \left( \frac{1}{2} \cdot \frac{2 \sin 20^0 \cdot \cos 20^0 \cdot \cos 40^0 \cdot \cos 80^0}{2 \sin 20^0} \right) =$

$= \log_2 \left( \frac{1}{2} \cdot \frac{2 \sin 40^0 \cdot \cos 40^0 \cdot \cos 80^0}{4 \sin 20^0} \right) = \log_2 ( \frac{2 \sin 80^0 \cdot \cos 80^0}{16 \sin 20^0} ) =$

$\log_2 \frac{\sin 160^0}{16 \cdot \sin 20^0} = \log_2 \frac{\sin (180^0 - 20^0)}{16 \sin 20^0} = \log_2 \frac{\sin 20^0}{16 \cdot \sin 20^0} = \log_2 \cdot 2^{-4} = -4.$

Javobi : A

**117 – misol**. ( 99 – 9 – 31 ). $tga + ctga = 4$ bo‘lsa, sin2α ni xisoblang.

**Yechilishi** : $\frac{\sin a}{\cos a} + \frac{\cos a}{sin a} = 4; \frac{1}{\sin a \cdot \cos a} = 4 \Longrightarrow \sin 2a = \frac{1}{2}$. Javobi : A

**118 – misol**. ( 99-10-29 ). Ifodaning qiymatini toping.

$$\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0}$$

**Yechilishi :** $\frac{\cos 10^0 - \sqrt{3} \sin 10^0}{\sin 10^0 \cdot \cos 10^0} = \frac{2 \cdot 2 (\frac{1}{2} \cos 10^0 - \frac{\sqrt{3}}{2} \sin 10^0)}{2 \sin 10^0 \cdot \cos 10^0} =$

$= \frac{4 (\sin 30^0 \cdot \cos 10^0 - \cos 30^0 \cdot \sin 10^0)}{\sin 20^0} = \frac{4 \sin 20^0}{\sin 20^0} = 4.$ Javobi : A

**119 – misol.** ( 00-2-48). Soddalashtiring.

$$(\cos 3x + \cos x)^2 + (\sin 3x + \sin x)^2$$

**Yechilishi** : ( cos 3 x + cos x )$^2$ + ( sin 3 x + sin x )$^2$= cos$^2$ 3x + 2 cos 3x · cos x + +cos$^2$ x +sin $^2$ 3x + 2 sin 3x sin x + sin $^2$ x = ( cos $^2$ 3x +sin$^2$ 3x ) + ( cos $^2$ x + +sin $^2$ x ) + 2 ( cos 3x cos x + sin 3x sin x ) = 2 + 2 cos 2x = 2 ( 1 + cos 2x ) =

$= 4\cos^2 x$. Javobi : A

**120 – misol**. (00-4-45 ). Agar tg x = 2 , bo‘lsa, $\frac{2-5\cos 2a}{6+10\sin 2a} - \frac{13+3tg2a}{10\cos 2a-15\sin 2a}$ ning qiymatini toping

**Yechilishi**: $\sin a = \frac{2tg\frac{a}{2}}{1+tg^2\frac{a}{2}}$ ; $\cos a = \frac{1-tg^2\frac{a}{2}}{1+tg^2\frac{a}{2}}$ ; $tg2a = \frac{2tga}{1-tg^2 a}$; $\sin 2a = \frac{2tga}{1+tg^2 a}$ ;

$\cos 2a = \frac{1-tg^2 a}{1+tg^2 a}$ formulalardan foydalansak va tg α=2 ekanligini e’tiborga olsak, $\sin 2a = \frac{2\cdot 2}{1+4} = \frac{4}{5}$; $\cos 2a = \frac{1-4}{1+4} = -\frac{3}{5}$; $tg2a = \frac{2\cdot 2}{1-4} = = -\frac{4}{3}$; U holda $tg2a = -\frac{4}{3}$; $\cos 2a = -\frac{3}{5}$; $\sin 2a = \frac{4}{5}$;

$$\frac{2-5\cdot(-\frac{3}{5})}{6+10\cdot(\frac{4}{5})} - \frac{13+3\cdot(-\frac{4}{3})}{10\cdot\left(-\frac{3}{5}\right)-15\cdot(\frac{4}{5})} = \frac{5}{6+8} - \frac{13-4}{-6-12} =$$

$= \frac{5}{14} + \frac{9}{18} = \frac{5}{14} + \frac{1}{2} = \frac{12}{14} = \frac{6}{7}$; Javobi : C

**121 – misol**. ( 00-8-46 ). Hisoblang. $\cos 50^0 \cdot \cos 40^0 - 2\cos 20^0 \cdot \sin 50^0 \cdot \sin 20^0$

**Yechilishi :** $\cos 50^0 = \cos(90^0 - 40^0) = \sin 40^0$; $\cos 50^0 \cdot \cos 40^0 = = \sin 40^0 \cdot \cos 40^0 = \frac{1}{2}\sin 80^0$; $2\sin 20^0 \cdot \cos 20^0 \cdot \sin 50^0 = \sin 40^0 \cdot \sin 50^0 = \sin 40^0 \cdot \sin(90^0 - 40^0) = \sin 40^0 \cdot \cos 40^0 = \frac{1}{2}\sin 80^0$. Bularni berilgan ifodaga qo‘yamiz. $\frac{1}{2}\sin 80^0 - -\frac{1}{2}\sin 80^0 = 0$. Javobi : A

**122 – misol**. ( 97-3-56 ). Soddalashtiring. $\frac{\cos 2a+\cos(\frac{\pi}{2}-a)\cdot\sin a}{\sin(\frac{\pi}{2}+a)}$

**Yechilishi** : $\frac{\cos 2a+\sin^2 a}{\cos a} = \frac{\cos^2 a-\sin^2 a+\sin^2 a}{\cos a} = \cos a$. Javobi : A

**123 – misol**. ( 01-1-50 ). Ifodaning qiymatini toping .

$$1 - \sqrt{3}ctg40^0 + \frac{1}{\cos 20^0}$$

**Yechilishi :** $1-\sqrt{3}\frac{\cos 40^0}{\sin 40^0}=\frac{\sin 40^0-\sqrt{3}\cos 40^0}{\sin 40^0}=\frac{2(\frac{1}{2}\sin 40^0-\frac{\sqrt{3}}{2}\cos 40^0}{\sin 40^0}=$

$=\frac{2(\cos 60^0\cdot\sin 40^0-\sin 60^0\cdot\cos 40^0)}{\sin 40^0}=\frac{-2\sin(60^0-40^0)}{2\sin 20^0\cdot\cos 20^0}=-\frac{\sin 20^0}{\sin 20^0\cdot\cos 20^0}==$

$-\frac{1}{\cos 20^0};\quad -\frac{1}{\cos 20^0}+\frac{1}{\cos 20^0}=0.$ Javobi : C

**124 – misol**. ( 01-2-86 ). Agar $\sin 2x=\frac{2}{5}$ bo'lsa, $\sin^8 x+\cos^8 x$ ning qiymatini aniqlang

**Yechilishi** : $\sin^8 x+\cos^8 x=(\sin^4 x+\cos^4 x)^2-2\sin^4 x\cdot\cos^4 x=[(\sin^2 x++\cos^2 x)^2-2\sin^2 x\cos^2 x]^2-2\sin^4 x\cos^4 x=(1-2(\frac{1}{2}\cdot 2\sin x\cos x)^2)^2--2(\frac{1}{2}\cdot 2\sin x\cos x)^4=(1-2\cdot\frac{1}{4}\sin^2 2x)^2-2\cdot\frac{1}{2^4}\cdot\sin^4 2x=$

$=(1-\frac{1}{2}\cdot(\frac{2}{5})^2)^2-\frac{1}{2^3}\cdot(\frac{2}{5})^4=(1-\frac{1}{2}\cdot\frac{4}{25})^2-\frac{1}{2^3}\cdot\frac{2^4}{5^4}=(1-\frac{2}{25})^2-\frac{2}{625}==\frac{23^2}{625}-\frac{2}{625}=\frac{529-2}{625}=\frac{527}{625}.$ Javobi : C

**125 – misol**. ( 01-3-1 ). Ifodaning qiymatini aniqlang.

$\sin 50^0+\sin 40^0\cdot\operatorname{tg}20^0$

**Yechilishi :** $\sin 50^0+2\sin 20^0\cdot\cos 20^0\cdot\frac{\sin 20^0}{\cos 20^0}=\sin 50^0+2\sin^2 50$ ;

$\sin 50^0=\sin(90^0-40^0)=\cos 40^0$ ; $\cos 40^0=\cos 20^0-\sin 20^0$ ;

$\cos 20^0-\sin 20^0+2\sin 20^0=\cos 20^0+\sin 20^0=1$. Javobi : C

**126 – misol**. ( 01-3-15 ). Ushbu $\sin\frac{x}{2}\cdot\cos^3\frac{x}{2}-\sin^3\frac{x}{2}\cdot\cos\frac{x}{2}$ ning eng katta qiymatini toping.

**Yechilishi :** $\sin\frac{x}{2}\cdot\cos^3\frac{x}{2}-\sin^3\frac{x}{2}\cdot\cos\frac{x}{2}=\sin\frac{x}{2}\cdot\cos\frac{x}{2}(\cos^2\frac{x}{2}-\sin^2\frac{x}{2})=\frac{1}{2}\cdot 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}\cos x=\frac{1}{2}\sin x\cdot\cos x=\frac{1}{4}\cdot\sin 2x$ ; Eng katta qiymati $\frac{1}{4}$. Javobi : D

**127 – misol**. ( 01-6-28 ). Soddalashtiring .

$\cos^6 x+\sin^6 x-\sin^2 x\cdot\cos^2 x$

**Yechilishi :** $\cos^6 x+\sin^6 x-\sin^2 x\cdot\cos^2 x=(\cos^2 x)^3+(\sin^2 x)^2-$

$-\sin^2 x\cdot\cos^2 x=(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cdot\cos^2 x+\cos^4 x)-$

$-\sin^2 x\cdot\cos^2 x=(\cos^2 x-\sin^2 x)^2=\cos^2 2x.$ Javobi : E

**128 – misol**. ( 01-7-38 ). Agar tg α=2- $\sqrt{3}$ bo‘lsa, α o‘tkir burchakning qiymatini toping .

**Yechilishi** : $sin^2\alpha + cos^2\alpha = 1;\ tg\alpha = 2-\sqrt{3};\ 1 + tg^2\alpha = \frac{1}{cos^2\alpha};$

$$tg^2\alpha = 4 - 4\sqrt{3} + 3 = 7 - 4\sqrt{3};\ cos^2\alpha = \frac{1}{1 + tg^2\alpha} \Rightarrow$$

$$cos^2\alpha = \frac{1}{8-4\sqrt{3}} = \frac{1}{4(2-\sqrt{3})};\ cos^2\alpha = \frac{1\cdot(2+\sqrt{3})}{4(2-\sqrt{3})(2+\sqrt{3}} = \frac{2+\sqrt{3}}{4};$$

$$cos^2\alpha = \frac{1\cdot(2+\sqrt{3})}{4(2-\sqrt{3})(2+\sqrt{3}} = \frac{2+\sqrt{3}}{4};\ \frac{1+\cos 2\alpha}{2} = \frac{2+\sqrt{3}}{4};\ 1+\cos 2\alpha = \frac{2}{}$$

$$\frac{1+\cos 2\alpha}{2} = \frac{2+\sqrt{3}}{4};\quad 1+\cos 2\alpha = \frac{2+\sqrt{3}}{2};$$

$\cos 2\alpha = \frac{2+\sqrt{3}}{2} - 1 \Rightarrow \cos 2\alpha = \frac{\sqrt{3}}{2} \Rightarrow 2\alpha = \frac{\pi}{6} \Rightarrow \alpha = \frac{\pi}{12}$ ; Javobi : B

**129 – misol** ( 01-8-54 ). Agar tgα=0,2 bo‘lsa , $\frac{1}{3+4\cos 2\alpha}$ ning qiymatini toping.

**Yechilishi** : $tg\alpha = \frac{1}{5};\ tg^2\alpha = \frac{1}{25};\ cos^2\alpha = \frac{1}{1+tg^2\alpha} \Rightarrow cos^2\alpha = \frac{1}{1+\frac{1}{25}} = \frac{25}{26};$

$$\frac{1+\cos 2\alpha}{2} = \frac{25}{26};\ 1+\cos 2\alpha = \frac{25}{13} \Rightarrow 1+\cos 2\alpha = 1 + \frac{12}{13};$$

$\cos 2\alpha = \frac{12}{13};\ \frac{2}{3+4\cdot\frac{12}{13}} = \frac{13\cdot 2}{39+48} = \frac{26}{87}$. Javobi : C

**130 – misol** ( 01-9-21 ). Soddalashtiring : $sin^2\alpha \cdot tg\alpha = cos^2\alpha \cdot ctg\alpha + sin2\alpha$

**Yechilishi** : $sin^2\alpha \cdot tg\alpha = cos^2\alpha \cdot ctg\alpha$ ifodani quyidagicha yozib olamiz.

$$sin^2\alpha \cdot \frac{\sin\alpha}{\cos\alpha} + cos^2\alpha \cdot \frac{\cos\alpha}{\sin\alpha} + \sin 2\alpha = \frac{sin^4\alpha + cos^4\alpha}{\sin\alpha\cos\alpha} \Rightarrow$$

$$\Rightarrow \frac{(sin^2\alpha + cos^2\alpha) - 2sin^2\alpha cos^2\alpha}{\sin\alpha\cdot\cos\alpha} \Rightarrow \frac{1-2sin^2\alpha cos^2\alpha}{\sin\alpha\cdot\cos\alpha}$$

Hosil qilingan ifodani berilgan ifodaga qo‘yamiz .

$\frac{1-2\sin^2\alpha\cdot\cos^2\alpha}{\sin\alpha\cos\alpha}+$

$2\sin\alpha\cdot\cos\alpha \Rightarrow \frac{1-2\sin^2\alpha\cdot\cos^2\alpha+2\sin^2\alpha\cos^2\alpha}{\sin\alpha\cos\alpha}=\frac{1}{\sin\alpha\cos\alpha}=\frac{2}{\sin 2\alpha}$ ;

Javobi: A

**131 – misol** ( 02-2-51 ). Agar *2sin6x (cos$^4$ 3x – sin$^4$ 3x ) = sin k x* tenglik hamma vaqt urinli bo‘lsa, k ni toping.

**Yechilishi :** Berilgan ifodaning chap tomonini soddalashtiramiz.

*2sin 6x ( cos$^2$3x – sin$^2$3x ) ( cos$^2$3x + sin$^2$3x ) = 2sin 6x · cos 6x = sin 12x*

u holda, *sin 12x = sinkx*, bundan *k* = 12. Javobi : A

**132 – misol** ( 02-3-75 ). $\sqrt{\sin^4\alpha+\cos^2\alpha}+\sqrt{\cos^4\alpha+\cos 2\alpha}$ ni soddalashtiring.

**Yechilishi** : Birinchi ildiz ostidagi ifodani soddalashtiramiz.

$(\sin^2\alpha)^2+\cos 2\alpha=(\frac{1-\cos 2\alpha}{2})^2+\cos 2\alpha=\frac{1-2\cos 2\alpha+\cos^2 2\alpha}{4}+$

$+\cos 2\alpha=$

$=\frac{1-2\cos 2\alpha+\cos^2 2\alpha+4\cos 2\alpha}{4}=\frac{1+2\cos 2\alpha+\cos^2 2\alpha}{4}=\frac{(1+\cos 2\alpha}{4}$

$$\sqrt{\sin^4\alpha+\cos 2\alpha}=\sqrt{\frac{(1+\cos 2a)^2}{4}}=\frac{1+\cos 2a}{2}.$$

Endi ikkinchi ildiz ostidagi ifodani soddalashtiramiz.

$\cos^4 a-\cos 2a=(\frac{1+\cos 2a}{2})^2-\cos 2a=\frac{1-2\cos 2a+\cos^2 2a}{4}==$

$\frac{(1-\cos 2a)^2}{4}$; $\sqrt{\cos^4 a-\cos 2a}=\sqrt{(\frac{1-\cos 2a}{2})^2}=\frac{1-\cos 2a}{2}$;

Bundan , $\sqrt{\sin^4 a+\cos 2a}+\sqrt{\cos^4 a-\cos 2a}=\frac{1+\cos 2a}{2}+\frac{1-\cos 2a}{2}=$

$=\frac{1+\cos 2a+1-\cos 2a}{2}=1$. Javobi : A

**133 – misol** ( 02-7-11 ). $\sin^4 105^0\cdot\cos^4 75^0$ ni hisoblang.

**Yechilishi** : $\sin 105^0=\sin(90^0+15^0)=\cos 15^0$; $\cos^4 15^0=(\cos^2 15^0)^2=$

$$=(\frac{1+\cos 30^0}{2})^2=(\frac{1+\frac{\sqrt{3}}{2}}{2})^2=(\frac{2+\sqrt{3}}{4})^2=\frac{4+4\sqrt{3}+3}{16}=\frac{7+4\sqrt{3}}{16};$$

$\text{Cos}^4 75^0 = (\text{Cos}^2 75^0)^2 = (\frac{1+\cos 150^0)^2}{2} = (\frac{1+\cos(90^0+60^0)}{2})^2 = (\frac{1-\sin 60^0}{2})^2 = = \frac{(1-\frac{\sqrt{3}}{2})^2}{4} = \frac{4-4\sqrt{3}+3}{16} = \frac{7-4\sqrt{3}}{16}$; $\sin^4 105^0 \cdot \cos^4 75^0 = \frac{(7-4\sqrt{3})(7+4\sqrt{3}}{16\cdot 16} = \frac{49-48}{256} = = \frac{1}{256}$. Javobi : A

**134 – misol** ( 02-8-40 ). $\cos(\pi + 2\alpha) + \sin(\pi + 2\alpha) \cdot tg(\frac{\pi}{2} + \alpha)$ ni soddalashtiring.

**Yechilishi** : Keltirish formulalariga asosan, $\cos(\pi + 2\alpha) = -\cos 2\alpha$, $\sin(\pi+2\alpha)=-\sin 2\alpha$, $tg\left(\frac{\pi}{2}+\alpha\right)=-ctg\ \alpha$, *bularni berilgan ifodaga qo'yamiz* .

$-\cos 2a + \sin 2a \cdot ctga$, bundan, - $\cos 2\alpha + 2\sin a \cos a \cdot \frac{\cos a}{\sin a} \Rightarrow$ $-\cos 2a + 2cos^2 a = -cos^2 a + sin^2 a + 2cos^2 a = 1$. Javobi : A

**135 – misol** ( 02-10-59 ). $\sin a = \frac{1}{3}$ Va $90^0 < a < 180^0$ bo‘lsa, tg 2α ni hisoblang.

**Yechilishi :**

$$tg2a = \frac{2tga}{1-tg^2 a};\ \cos a = \sqrt{1-sin^2 a} = \sqrt{1-\frac{1}{9}} = \frac{-2\sqrt{2}}{3};$$

$$tga\frac{\sin a}{\cos a} = \frac{\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} = -\frac{\sqrt{2}\cdot 8}{2\cdot 7} = -\frac{4\sqrt{2}}{7}.$$

Javobi : A

**136 – misol** ( 02-11-41 ). $\frac{1+\cos\frac{a}{2}-\sin\frac{a}{2}}{1-\cos\frac{a}{2}-\sin\frac{a}{2}}$ ni soddalashtiring.

**Yechilishi :** Suratini soddalashtiramiz. $\sin\frac{a}{2} = 2\sin\frac{a}{4}\cos\frac{a}{4}$;1+cos $\frac{a}{2} = 2cos^2\frac{a}{4}$;

$1+\cos\frac{a}{2} - \sin\frac{a}{2} = 2cos^2\frac{a}{4} - 2\sin\frac{a}{4}\cdot\cos\frac{a}{4} = 2\sin\frac{a}{4}(\cos\frac{a}{4} - \sin\frac{a}{4})$

Maxrajini soddalashtiramiz.

$1-\cos\frac{a}{2} = 2sin^2\frac{a}{4}$; $2sin^2\frac{a}{2} - 2\sin\frac{a}{4}\cdot\cos\frac{a}{4} = 2sin\frac{a}{4}(\sin\frac{a}{4} - \cos\frac{a}{4})$;

U holda berilgan ifoda

$\frac{1+\cos\frac{a}{2}-\sin\frac{a}{2}}{1-\cos\frac{a}{2}-\sin\frac{a}{2}} = \frac{2\cos\frac{a}{4}(\cos\frac{a}{4}-\sin\frac{a}{4})}{2\sin\frac{a}{4}(\sin\frac{a}{4}-\cos\frac{a}{4})} = -ctg\frac{a}{4}$. Javobi : C

**137 – misol.** ( 03-1-33 ) $1 - \sin^6 22{,}5^0 + \cos^6 22{,}5^0$ ni hisoblang.

**Yechilishi :** $1 - (\sin^6 22{,}5^0 - \cos^6 22{,}5^0) = 1 - [(\sin^2 22{,}5^0)^3 - (\cos^2 22{,}5^0)^3] = 1 - [(\sin^2 22{,}5^0 - \cos^2 22{,}5^0)(\sin^4 22{,}5^0 + \sin^2 22{,}5^0 \cdot \cos^2 22{,}5^0 + \cos^4 22{,}5^0)] = 1 + \frac{\sqrt{2}}{2}((sin^2 22{,}5^0 + cos^2 22{,}5^0)^2 - sin^2 22{,}5^0 \cdot cos^2 22{,}5^0) =$

$1 + \frac{\sqrt{2}}{2}(1 - (\frac{1}{2} \cdot 2\sin 22{,}5^0 \cos 22{,}5^0)^2) = 1 + \frac{\sqrt{2}}{2}\left(1 - \frac{1}{4} \cdot \frac{1}{2}\right) = 1 + \frac{\sqrt{2}}{2} \cdot \frac{7}{8} = \frac{16+7\sqrt{2}}{16}$; Javobi: D

**138 – misol** ( 03-1-37 ). $\cos a \cdot \cos\frac{a}{2} \cdot \cos\frac{a}{4} \cdot \cos\frac{a}{8} \cdots \cos\frac{a}{128}$ $n$i soddalashtiring.

**Yechilishi :** $\frac{2\sin\frac{a}{128} \cdot \cos\frac{a}{128} \cdot \cos\frac{a}{64} \cdots \cos\frac{a}{8} \cdot \cos\frac{a}{4} \cdot \cos\frac{a}{2} \cdot \cos a}{2\sin\frac{a}{128}} =$

$$-\frac{2\sin\frac{a}{64} \cdot \cos\frac{a}{64} \cdots \cos\frac{a}{8} \cdot \cos\frac{a}{4} \cdot \cos\frac{a}{2} \cdot \cos a}{2^2 \sin\frac{a}{128}} =$$

$= \frac{2\sin\frac{a}{32} \cdots \cos\frac{1}{8} \cdot \cos\frac{a}{4} \cdot \cos\frac{a}{2} \cdot \cos a}{2 \cdot 2^2 \cdot \sin\frac{a}{128}} = \frac{\sin 2a}{2^8 \sin\frac{a}{128}} = \frac{\sin 2a}{256 \cdot \sin\frac{a}{128}}$. Javobi : B

**139 – misol** ( 03-2-25 ). Agar $ctg\alpha = \sqrt{2} - 1$ bo‘lsa, $\cos 2\alpha$ ning qiymatini toping.

**Yechilishi**: $\cos 2a = \frac{1 - tg^2 a}{1 + tg^2 a}$; $tga = \frac{1}{ctg\, a} = \frac{1}{\sqrt{2}-1} =$

$= \sqrt{2} + 1$; $\cos 2a = = \frac{1-(\sqrt{2}+1)^2}{1+(\sqrt{2}+1)^2} = \frac{1-(2+2\sqrt{2}+1)}{1+2+2\sqrt{2}+1} = =$

$\frac{1-3-2\sqrt{2}}{4+2\sqrt{2}} = = \frac{-2-2\sqrt{2}}{2(2+\sqrt{2})} = -\frac{2(1+\sqrt{2})}{2\sqrt{2}(\sqrt{2}+1)} = -\frac{1}{\sqrt{2}}$; Javobi : C

**140 – misol**. ( 03-3-39 ). $\text{ctg}\, 35^0 - \text{tg}\, 35^0 - 2\text{tg}\, 20^0$ ni hisoblang.

**Yechilishi :** $\text{ctg}\, 35^0 - \text{tg}\, 35^0 = \frac{\cos 35^0}{\sin 35^0} - \frac{\sin 35^0}{\cos 35^0} = \frac{2(cos^2 35^0 - sin^2 35)}{2\sin 35^0 \cdot cos 35^0} =$

$$= \frac{2\cos 70^0}{\sin 70^0} = \frac{2\cos(90^0 - 20^0)}{\sin(90^0 - 20^0)} = \frac{2\sin 20^0}{\cos 20^0} = 2tg\, 20^0$$

$2\text{tg}\, 20^0 - 2\text{tg}\, 20^0 = 0$. Javobi : B

**141 – misol**. ( 03-3-42 ). Agar $\sin\left(\alpha-\frac{\pi}{2}\right)=\frac{2\sqrt{6}}{5}$ va $\alpha\in(\frac{\pi}{2};\pi)$ bo‘lsa, $tg2\alpha$ ning qiymatini toping.

**Yechilishi** : $\sin\left(\alpha-\frac{\pi}{2}\right)=-\cos\alpha=\frac{2\sqrt{6}}{5}$; $\cos\alpha=-\frac{2\sqrt{6}}{5}$; $tg2\alpha=\frac{2tg\alpha}{1-tg^2\alpha}$;

$$1+tg^2\alpha=\frac{1}{cos^2\alpha}\Rightarrow tg^2\alpha=\frac{1}{cos^2\alpha}-1=\frac{25}{24}-1=\frac{1}{24};$$

$tg\alpha=-\frac{1}{\sqrt{24}}$; $tg2\alpha=-\frac{2\cdot\frac{1}{\sqrt{24}}}{1-\frac{1}{24}}=-\frac{\frac{1}{\sqrt{6}}}{\frac{23}{24}}=-\frac{24}{23\sqrt{6}}=-\frac{4\cdot(\sqrt{6})^2}{23\sqrt{6}}=$

$-\frac{4\sqrt{6}}{23}$; Javobi : C

**142 – misol**. ( 03-4-24 ). $\frac{1-\cos 2a}{1+tg^2a}$ ni soddalashtiring.

**Yechilishi :** $\frac{1-cos^2a+sin^2a}{1+\frac{sin^2a}{cos^2a}}=\frac{cos^2a+sin^2a-cos^2a+sin^2a}{\frac{cos^2a+sin^2a}{cos^2a}}=\frac{2sin^2a\cdot cos^2a}{1}=$

$=2\cdot(\frac{1}{2}\cdot 2\sin a\cdot\cos a)^2=2\cdot\frac{1}{4}sin^22a=\frac{1}{2}sin^22a$ ; Javobi : B

**143 – misol**. ( 03-6-06 ). Agar $\sin 37^0=a$ bo‘lsa, $\sin 16^0$ ni **a** orqali ifodalang.

**Yechilishi :** $sin^237^0=a^2\Rightarrow\frac{1-\cos 2\cdot 37^0}{2}=a^2\Rightarrow 1-\cos 74^0=2a^2\Rightarrow$ $\cos 74^0=1-2a^2$, $\cos 74^0=\cos(90^0-16^0)=\sin 16^0$; $\sin 16^0=1-2a^2$ ;

Javobi : D

**144 – misol**. ( 03-7-34 ). $\cos 37^0=a$ bo‘lsa, $\sin 16^0$ ni **a** orqali ifodalang.

**Yechilishi** : $\cos^2 37^0=a^2$; $\frac{1+\cos 74}{2}=a^2\Rightarrow\cos 74^0=2a^2-1$ ,

$\cos 74^0=\cos(90^0-16^0)=\sin 16^0$; $\sin 16^0=2a^2-1$. Javobi : C

**145 – misol**. ( 03-8-53 ) $\sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8}\cdot tg\frac{\pi}{8}\cdot ctg\frac{9\pi}{8}$ ni hisoblang.

**Yechilishi** : $ctg\frac{9\pi}{8}=ctg\left(\pi+\frac{\pi}{8}\right)=ctg\frac{\pi}{8}$; $\frac{1}{2}\cdot 2\sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8}\cdot tg\frac{\pi}{8}\cdot ctg\frac{\pi}{8}==\frac{1}{2}\sin 2\cdot\frac{\pi}{8}=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}=\frac{1}{2\sqrt{2}}$ . Javobi : A

**146 – misol**. ( 03-8-57 ). Agar $\cos a + \sin a = m$ bo‘lsa, $\frac{1+\cos 2a}{ctg\frac{a}{2}-tg\frac{a}{2}}$ ni **m** orqali ifodalang.

**Yechilishi** : $1 + \cos 2a = 2cos^2 a$; ctg $\frac{a}{2} - tg\frac{a}{2} = \frac{\cos\frac{a}{2}}{\sin\frac{a}{2}} - \frac{\sin\frac{a}{2}}{\cos\frac{a}{2}} = \frac{2\left(\cos^2\frac{a}{2}-\sin^2\frac{a}{2}\right)}{2\sin\frac{a}{2}\cdot\cos\frac{a}{2}} = = \frac{2\cos a}{\sin a}; \frac{1+\cos 2a}{ctg\frac{a}{2}-tg\frac{a}{2}} = \frac{2cos^2 a}{2\frac{\cos a}{\sin a}} = \cos a \cdot \sin a = \frac{1}{2}\sin 2a.$

$(\cos a + \sin a)^2 = m^2$,

$1 + 2\sin a \cdot \cos a = m^2 \Rightarrow \sin 2a = m^2 - 1, \frac{1}{2}sin2a = \frac{1}{2}(m^2 - 1)$; Javobi :A

**147 – misol ( 03-9-27 ).** $\frac{\sin 106^0 - \sin 14^0}{1-2cos^2 22^0}$ ni hisoblang.

**Yechilishi** : sin $106^0$ – sin $14^0$ $= 2\cos\frac{106^0+14^0}{2}\cdot\sin\frac{106^0-14^0}{2} = 2\cos\frac{120^0}{2}\cdot\sin\frac{92^0}{2} = 2\cdot\frac{1}{2}\cdot\sin 46^0 = \sin 46^0 = \sin(90^0 - 44^0) = \cos 44^0$. $1 - 2\cos^2 22^0 = = \cos^2 22^0 + \sin^2 22^0 - 2\cos^2 22^0 = -(\cos^2 22^0 - \sin^2 22^0) = -\cos 44^0$ $\frac{\sin 106^0 - \sin 14^0}{1-2cos^2 22^0} = \frac{\cos 44^0}{-\cos 44^0} = -1$. Javobi : E

**148 – misol**. ( 03 – 9 – 28 ) . $\frac{1-\sin^2\frac{a}{8}-\cos^2 a-\sin^2 a}{4\sin^4\frac{a}{16}}$ ni hisoblang.

**Yechilishi** : $1 - \sin^2\frac{a}{8} - (\cos^2 a + \sin^2 a) = -\sin^2\frac{a}{8}$,

$\sin\frac{a}{8} = 2\sin\frac{a}{16}\cdot\cos\frac{a}{16}; \sin^2\frac{a}{8} = 4\sin^2\frac{a}{16}\cdot\cos^2\frac{a}{16}$;

$\frac{-4\sin^2\frac{a}{16}\cdot\cos^2\frac{a}{16}}{4\sin^4\frac{a}{16}} = -\frac{\cos^2\frac{a}{16}}{\sin^2\frac{a}{16}} = -ctg^2\frac{a}{16}$ . Javobi : E

**149 – misol**. ( 03-9-31 ). Agar $tg\frac{a}{2} = -2$ bo‘lsa, $\sin a + 2\cos a$ ning qiymatini hisoblang.

**Yechilishi** : $\sin a = \frac{2tg\frac{a}{2}}{1+tg^2\frac{a}{2}}$ formulaga asosan, $\sin a = \frac{2\cdot(-2)}{1+4} = \frac{-4}{5}$;

$\cos a = \frac{1-tg^2 a}{1+tg^2 a}$; U holda $-\frac{4}{5} + 2\cdot\left(-\frac{3}{5}\right) = -\frac{10}{5} = -2$. Javobi : C

**150 – misol**. ( 03-10-40 ). $tga = \frac{1}{2}$ , $\sin\left(2a + \frac{\pi}{4}\right) = ?$

**Yechilishi**: $\sin\left(2a+\frac{\pi}{4}\right)=\sin 2a\cdot\cos\frac{\pi}{4}+\cos 2a\cdot\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}(\sin 2a++\cos 2a)$; $\sin 2a=\frac{2tg\ a}{1+tg^2a}$; $\cos 2a=\frac{1-tg^2a}{1+tg^2a}$; formuladan foydalansak, $\sin 2a=\frac{2\cdot\frac{1}{2}}{1+\frac{1}{4}}=\frac{4}{5},\cos 2a=\frac{1-\frac{1}{4}}{1+\frac{1}{4}}=\frac{3}{5}$; $\sin\left(2a+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\left(\frac{4}{5}+\frac{3}{5}\right)=\frac{7\sqrt{2}}{10}$. Javobi : E

**151 – misol**. ( 03-11-22 ) α o‘tkir burchak $sin^2\alpha\cdot cos^4\alpha=\frac{1}{64}$ bo‘lsa , α quyidagilarning qaysi biriga teng ?

**Yechilishi**: $sin^4\alpha\cdot cos^4\alpha=(\frac{1}{2}\cdot 2\cos\alpha\cdot\sin\alpha)^4=\frac{1}{16}sin^4 2\alpha$;

$\frac{1}{16}sin^4 2\alpha=\frac{1}{64}\Rightarrow sin^4 2\alpha=\frac{1}{4}$; $\sin 2\alpha=\pm\frac{1}{\sqrt{2}}\Rightarrow 2\alpha=\pm\frac{\pi}{4}+n\pi$;

$\alpha=\pm\frac{\pi}{8}+\frac{n\pi}{2}; n=0;\ \alpha_1=\frac{\pi}{8};\ \alpha_2=\frac{\pi}{2}-\frac{\pi}{8}=\frac{3\pi}{8}$; Javobi : A

**152 – misol** (03-11-25). Agar $\cos 29^0$ = a bo‘lsa, $\sin 32^0$ ni α orqali ifodalang.

**Yechilishi**: $\cos^2 29^0=a^2$ ; $\frac{1+\cos 58^0}{2}=a^2$;$1+\cos 58^0=2a^2$ ; $\cos 58^0=2a^2-1$ ; $\cos(90^0-32^0)=2a^2-1$; $\sin 32^0=2a^2-1$; Javobi : A

**153 – misol**. (03-11-26). Agar $tg\alpha=-2$ bo‘lsa, $1+5\sin 2\alpha-3\cos^{-1}2\alpha$ ning qiymatini toping.

**Yechilishi**: $\sin 2\alpha=\frac{2tg\alpha}{1+tg^2\alpha}=-\frac{4}{5}$; $\cos 2\alpha=\frac{1-tg^2\alpha}{1+tg^2\alpha}=-\frac{3}{5}$; $1+5\cdot\left(-\frac{4}{5}\right)++3\cdot\frac{1}{\frac{3}{5}}=1-4+5=2$; Javobi : A

**154 – misol**. (98-1-54). Agar $tga=-\frac{1}{4}$ bo‘lsa, $\frac{2cos^2a-\sin 2a}{2sin^2a-\sin 2a}$ ni hisoblang.

**Yechilishi:** Berilgan ifodani soddalashtiramiz. $\frac{2cos^2a-\sin 2a}{2sin^2a-\sin 2a}=\frac{2cos^2a-2\sin a\cos a}{2sin^2a-2\sin a\cdot\cos a}=\frac{2\cos a(\cos a-\sin a)}{2\sin a\ (\sin a-\cos a)}=-ctg=-\frac{1}{tga}=4.$
Javobi : D

**155 – misol.** ( 98-3-53 ). Hisoblang. $\frac{\sin 36^0}{\sin 12^0}-\frac{\cos 36^0}{\cos 12^0}=?$

**Yechilishi** : $\frac{\sin 36^0}{\sin 12^0}-\frac{\cos 36^0}{\cos 12^0}=\frac{\sin 36^0\cdot\cos 12^0-\cos 36^0\cdot\sin 12^0}{\sin 12^0\cdot\cos 12^0}=\frac{2\cdot\sin(36^0-12^0)}{\sin 12^0\cdot 2}=$

$= \frac{2\sin 24^0}{\sin 24^0} = 2$; Javobi : A

**156 – misol**. ( 98-10-32 ). Hisoblang. tg $15^0$ – ctg $15^0$

**Yechilishi**: $\frac{\sin 15^0}{\cos 15^0} - \frac{\cos 15^0}{\sin 15^0} = \frac{\sin^2 15 - \cos^2 15}{\sin 15^0 \cdot \cos 15^0} = \frac{-2(\cos^2 15^0 - \sin^2 15^0)}{2 \cdot \sin 15^0 \cdot \cos 15^0} = -\frac{2\cos 30^0}{\sin 30^0} == -ctg\ 30^0 = -2\sqrt{3}$ ; Javobi : B

**157 – misol**. ( 99-6-12 ). Hisoblang. $\frac{2tg(240^0)}{1 - tg^2 240^0}$

**Yechilishi**: $\frac{2tga}{1-\tan 2a} = tg2a, bundan \frac{2tg(240^0)}{1-tg^2 240^0} = tg\ (2 \cdot 240^0) = tg(480^0) = tg(360^0 + +120^0) = tg120^0 = tg(90^0 + 30^0) = -ctg30^0 = -\sqrt{3}$. Javobi : A

**158 – misol**. ( 96-9-47 ). Soddalashtiring. $\frac{1+\sin 2a}{\sin a + \cos a} = -\sin a$.

**Yechilishi:** $\frac{\sin^2 \alpha + \cos^2 \alpha + 2\sin \alpha \cos \alpha}{\sin a + \cos a} - \sin \alpha = \frac{(\sin \alpha + \cos \alpha)^2}{\sin \alpha + \cos \alpha} - \sin \alpha = \sin \alpha + +\cos \alpha - \sin \alpha = \cos \alpha$. Javobi : A

` **159 – misol**. ( 99-9-29 ) . sin $150^0$ ning qiymati $\cos 20^0 \cdot \cos 40^0 \cdot \cos 80^0$ ning qiymatidan qanchaga katta ?

**Yechilishi :** $\sin 150^0 = \sin(90^0 + 60^0) = \cos 60^0 = \frac{1}{2}$;

$$\frac{2\sin 20^0 \cdot \cos 20^0 \cdot \cos 40^0 \cdot \cos 80^0}{2 \cdot \sin 20^0} = \frac{2 \cdot \sin 40^0 \cdot \cos 40^0 \cdot \cos 80^0}{2 \cdot 2 \cdot \sin 20^0}$$

$$== \frac{2 \cdot \sin 80^0 \cdot \cos 80^0}{2 \cdot 4 \cdot \sin 20^0} = \frac{\sin 160^0}{8 \cdot \sin 20^0} = \frac{\sin(180^0 - 20^0)}{8 \cdot \sin 20^0}$$

$$= \frac{\sin 20^0}{8 \sin 20^0} = \frac{1}{8};$$

$\frac{1}{2} - \frac{1}{8} = \frac{7}{8}$; ga katta. Javobi : D

**160 – misol**. ( 00-1-27 ). Soddalashtiring. $\frac{1-\cos 2a}{1+\cos 2a}$

**Yechilishi:** $1 - \cos 2a = 2\sin^2 a,\ 1 + \cos 2a = 2\cos^2 a,$ shularni e'tiborga olsak, $\frac{2\sin^2 a}{2\cos^2 a} = 1 = \frac{\sin^2 a + \cos^2 a}{\cos^2 a} = \frac{1}{\cos^2 a} = \cos^{-2} a$. Javobi : A

**161 – misol**. ( 00-7-29 ). Soddalashtiring. $\frac{1+\cos 2a+\cos^2 a}{\sin^2 a}$

**Yechilishi** : $\frac{1+\cos 2a+\cos^2 a}{\sin^2 a} = \frac{3\cos^2 a}{\sin^2 a} = 3ctg^2 a;$ Javobi : A

**162 – misol.** ( 99-6-23 ). Soddalashtiring $1 + \frac{tg^2(-a)-1}{\sin(0{,}5\pi+2a)}$

**Yechilishi** : $\sin(0{,}5\pi + 2a) = \cos 2a;\ tg^2 a - 1 = \frac{\sin^2 a}{\cos^2 a} - 1 =$ $\frac{\sin^2 a-\cos^2 a}{\cos^2 a} = = -\frac{\cos 2a}{\cos^2 a}$; Bularni berilgan ifodaga qoysak,

$1 + \frac{-\cos 2a}{\cos^2 a \cdot \cos 2a} = 1 - \frac{1}{\cos^2 a} = 1 - 1 - tg^2 a = -tg^2 a;$ Javobi : A

**163 – misol**. ( 99-9-32 ). Soddalashtiring. $\frac{\sqrt{3}\cos 2a+\sin 2a}{\cos a+\sqrt{3}\sin a}$

**Yechilishi :** $\frac{2(\frac{\sqrt{3}}{2}\cos 2a+\frac{1}{2}\sin 2a)}{2(\frac{1}{2}\cos a+\frac{\sqrt{3}}{2}\sin a)} = \frac{\sin(2a+60^0)}{\sin(a+30^0)} = \frac{\sin 2(a+30^0)}{\sin(a+30^0)} =$

$= \frac{2\sin(a+30^0)\cos(a+30^0)}{\sin(a+30^0)} = 2\cos(a + \frac{\pi}{6});$ Javobi : E

**164 – misol**. ( 99-10-31 ). Soddalashtiring. $\frac{2\cos^2 a}{ctg\frac{a}{2}-tg\frac{a}{2}}$

**Yechilishi :** $ctg\frac{a}{2} - tg\frac{a}{2} = \frac{\cos\frac{a}{2}}{\sin\frac{a}{2}} - \frac{\sin\frac{a}{2}}{\cos\frac{a}{2}} = \frac{(\cos^2\frac{a}{2}-\sin^2\frac{a}{2})^2}{2\sin\frac{a}{2}\cdot\cos\frac{a}{2}} = \frac{2\cos a}{\sin a};$

Demak,

$2\cos^2 a : \frac{2\cos a}{\sin a} = \frac{2\cos^2 a\cdot\sin a}{2\cos a} = \sin a \cdot \cos a = \frac{1}{2}\sin 2a;$ Javobi : D

**165 – misol.** ( 01-1-43 ) . Agar $tga = -\frac{4}{3}$ bo‘lsa, $\sin 2\alpha$ ning qiymatini toping.

**Yechilishi** : $\sin 2a = \frac{2tg\ \alpha}{1+tg^2\alpha}$ ekanligini e’tiborga olsak, $\sin 2\alpha =$ $\frac{-\frac{4}{3}\cdot 2}{1+\frac{16}{9}} = \frac{-8\cdot 9}{3\cdot 25} = -\frac{24}{25} = -0{,}96;$ Javobi : B

**166 – misol**. ( 03-4-24 ). Soddalashtiring $\frac{1-\cos 2a}{1+tg^2\alpha}$

**Yechilishi :** $1-\cos 2\alpha = 2\sin^2\alpha,\ 1+\operatorname{tg}^2\alpha = \frac{1}{\cos^2\alpha}$ ekanligini e'tiborga olsak, $sin^2 : \frac{1}{cos^2\alpha} = 2sin^2\alpha \cdot \cos^2\alpha = \frac{1}{2}(2\sin\alpha \cdot \cos\alpha)^2 = \frac{1}{2}\sin^2 2\alpha$; Javobi: B

**167 – misol**. ( 0-9-28 ). $\frac{1-sin^2\frac{\alpha}{8}-cos^2\alpha-sin^2\alpha}{4sin^4\frac{\alpha}{16}}$ ni soddalashtiring.

**Yechilishi :** $\frac{1-cos^2\alpha-sin^2\alpha-sin^2\frac{\alpha}{2}}{4sin^4\frac{\alpha}{16}} = -\frac{sin^2\frac{\alpha}{8}}{4sin^4\frac{\alpha}{16}} = -\frac{(sin^2\frac{\alpha}{16})^2}{4sin^4\frac{\alpha}{16}} = -\frac{4sin^2\frac{\alpha}{16}\cdot cos^2\frac{\alpha}{16}}{4sin^4\frac{\alpha}{16}} = = -ctg^2\frac{\alpha}{16}$; Javobi : E

**168 – misol.** ( 03-12-75 ). Agar tg α = 3 bo'lsa, $\sin 2\alpha - \cos 2\alpha$ ning qiymatini toping.

**Yechilishi** : $\sin 2\alpha = \frac{2tg\ \alpha}{1+tg^2\alpha} = \frac{2\cdot 3}{1+9} = \frac{3}{5}; \cos 2\alpha = \frac{1-tg^2\alpha}{1+tg^2\alpha} = \frac{1-9}{1+9} = -\frac{4}{5};$ $\sin 2\alpha - cos\ 2\alpha = \frac{3}{5} - \left(-\frac{4}{5}\right) = \frac{7}{5} = 1{,}4$; Javobi : D

## Trigonometrik funksiylarning yig'indisini va ayirmasini ko'paytmaga keltirish formulalardan foyidalanib trigonometrik ifodalarni soddalashtirish va hisoblash.

1. $\sin x + \sin y = 2\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$;
2. $\sin x - \sin y = 2\cos\frac{x+y}{2}\cdot\sin\frac{x-y}{2}$;
3. $\cos x + \cos y = 2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$;
4. $\cos x - \cos y = -2\sin\frac{x+y}{2}\cdot\sin\frac{x-y}{2}$;
5. $\cos x+\sin x = \sqrt{2}\sin(\frac{\pi}{4}+x) = \sqrt{2}\cos(\frac{\pi}{4}+x)$;
6. $\cos x - \sin x = \sqrt{2}\cos(\frac{\pi}{4}+x) = \sqrt{2}\sin(\frac{\pi}{4}+x)$;

**169 – misol.** ( 00-8-59 ). Hisoblang $\sin 10^0 + \sin 50^0 - \cos 20^0$

**Yechilishi** : (1) formulaga asosan, $\sin 10^0 + \sin 50^0 = 2 \sin 30^0 \cdot \cos 20^0 =$

$= \cos 20^0$ ; Cos $20^0 - \cos 20^0 = 0$; Javobi : A

**170 – misol**. ( 96-6-35 ). Soddalashtiring $\frac{\cos\alpha+\cos 3\alpha}{\sin\alpha}$

**Yechilishi** : ( 4 ) formuladan $\cos\alpha - \cos 3\alpha = -2 \sin\frac{4\alpha}{2}\sin\frac{2\alpha}{2}$;

$\frac{\cos\alpha-\cos 3\alpha}{\sin\alpha} = \frac{2\sin 2\alpha\cdot\sin\alpha}{\sin\alpha} = 2\sin 2\alpha$ ; Javobi : E

**171 – misol.** ( 97-2-35 ). Quyidagilardan qaysi biriga teng $\frac{\sin\alpha}{\cos\alpha-\cos 3\alpha}$

**Yechilishi** : (2) formulaga asosan, $\cos\alpha - \cos 3\alpha = -2 \sin 2\alpha \cdot \sin(-\alpha)$;

$\frac{\sin\alpha}{\cos\alpha-\cos 3\alpha} = \frac{\sin\alpha}{2\sin 2\alpha\sin\alpha} = \frac{1}{2\sin 2\alpha}$; Javobi : E

**172 – misol**. ( 98-11-103 ). Hisoblang $\sin 70^0 - \sin 15^0$

**Yechilishi** : (2) formulaga asosan $\sin 75^0 - \sin 15^0 = 2\cos\frac{75^0+15^0}{2} \cdot$

$\cdot \sin\frac{75^0-15^0}{2} = 2\cos 45^0 \cdot \sin 30^0 = 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}$ ; Javobi: A

**173 – misol**. ( 98-1-58 ) . Hisoblang $\frac{\sin 2\alpha+2\sin\alpha\cdot\cos 2\alpha}{1+\cos\alpha+\cos 2\alpha+\cos 3\alpha}$

**Yechilishi**: Suratini soddalashtiramiz. $\sin 2\alpha + 2\sin\alpha \cdot \cos 2\alpha =$ $= 2\sin\alpha \cdot \cos\alpha + 2\sin\alpha \cdot \cos 2\alpha =$ $2\sin\alpha(\cos\alpha + \cos 2\alpha)$;

Maxrajini soddalashtiramiz. $1 + \cos\alpha + \cos 2\alpha + \cos 3\alpha = (1 +$ $+\cos 2\alpha) + (\cos\alpha + \cos 3\alpha) = 2\cos^2\alpha + 2\cos 2\alpha \cdot \cos\alpha =$ $2\cos\alpha(\cos\alpha + \cos 2\alpha)$; Demak, $\frac{\sin 2\alpha+2\sin\alpha\cdot\cos 2\alpha}{1+\cos\alpha+\cos 2\alpha\cdot\cos 3\alpha} = \frac{2\sin\alpha(\cos\alpha+\cos 2\alpha)}{2\cos\alpha(\cos\alpha+\cos 2\alpha)} = tg\,\alpha$; Javobi : B

**174-misol**. (98-8-58). Soddalashtiring $\frac{1-\sin\alpha-\cos 2\alpha+\sin 3\alpha}{\sin 2\alpha+2\cos\alpha\cdot\cos 2\alpha}$

**Yechilishi** : Suratini soddalashtiramiz. $1 - \sin\alpha - \cos 2\alpha +$ $\sin 3\alpha =$ $= (1 - \cos 2\alpha) + (\sin 3\alpha - \sin\alpha) = 2\sin^2\alpha +$ $2\cos 2\alpha \cdot \sin\alpha = 2\sin\alpha(\sin\alpha + \cos 2\alpha)$;

Mahrajini soddalashtiramiz. $\sin 2a + 2\cos a \cdot \cos 2a = 2\sin a \cdot \cos a + 2\cos a \cdot \cos 2a = 2\cos a(\sin a + \cos 2a)$; Demak , $\frac{1-\sin a-\cos 2a+\sin 3a}{\sin 2a+2\cos a\cdot\cos 2a} = \frac{2\sin a(\sin a+\cos 2a)}{2\cos a(\sin a+\cos 2a)} = tga$; Javobi : B

**175–misol**.(99-5-54).Hisoblang .

$$\sqrt[3]{8 + (\cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{3\pi}{5} + \cos\frac{4\pi}{5})^3}$$

**Yechilishi**: Ildiz ostidagi ifodani soddalashtiramiz.

$(\cos\frac{\pi}{5} + \cos\frac{3\pi}{5}) + (\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}) = = 2\cos\frac{\frac{\pi}{5}+\frac{3\pi}{5}}{2}\cdot\cos\frac{\frac{\pi}{5}-\frac{3\pi}{5}}{2} +$

$2\cos\frac{\frac{2\pi+4\pi}{5}}{2}\cdot\cos\frac{\frac{2\pi-4\pi}{5}}{2} =$

$= 2\cos\frac{4\pi}{10}\cdot\cos\frac{2\pi}{10} + 2\cos\frac{6\pi}{10}\cdot\cos\frac{2\pi}{10} = 2\cos\frac{\pi}{5}(\cos\frac{2\pi}{5} + \cos\frac{3\pi}{5}) =$

$= 2cos\frac{\pi}{5}\cdot 2\cos\frac{\frac{2\pi+3\pi}{5}}{2}\cdot\cos\frac{\frac{2\pi-3\pi}{5}}{2} = 4\cos\frac{\pi}{5}\cdot\cos\frac{\pi}{2}\cdot\cos\frac{\pi}{10} = 0;$

Demak,

$$\sqrt[3]{8 + (\cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{3\pi}{5} + \cos\frac{4\pi}{5})^3} = \sqrt[3]{8} = 2;$$ Javobi : B

**176 – misol**. ( 01-7-40 ). Soddalashtiring. $\frac{\sin a+\sin 2a-\sin(\pi+3a)}{2\cos a+1}$

**Yechilishi** : $\sin(\pi + 3a) = -\sin 3a$ ; $\sin a + \sin 2a + \sin 3a = (\sin a + \sin 3a) + \sin 2a = 2\sin\frac{a+3a}{2}\cdot\cos\frac{a-3a}{2} + \sin 2a =$ $2\sin 2a\cdot\cos a + +\sin 2a = \sin 2a(2\cos a + 1)$; Demak , $\frac{\sin 2a(2\cos a+1)}{2\cos a+1} = \sin 2a$. Javobi : C

**177 – misol**. ( 02-2-47 ). $(\frac{\sin 100^0+\sin 20^0}{\sin 50^0})$ ni hisoblang

**Yechilishi :** $\sin 100^0 + \sin 20^0 = 2\sin\frac{100^0+20^0}{2}\cdot\cos\frac{100^0-20^0}{2} =$

$= 2\sin 60^0\cdot\cos 40^0 = 2\ \frac{\sqrt{3}}{2}\cos(90^0 - 50^0) =$

$\sqrt{3}\sin 50^0$; $\frac{\sin 100^0+\sin 20^0}{\sin 50^0} = (\frac{\sqrt{3}\sin 50^0}{\sin 50^0})^2 = 3$; Javobi : A

**178 – misol**. ( 03-7-55 ). Sin $87^0$ – sin $59^0$ – sin $93^0$ + sin $61^0$ ni soddalashtiring.

**Yechilishi :** $(\sin 87^0 - \sin 93^0) + (\sin 61^0 - \sin 59^0) =$
$= 2\cos\frac{87^0+93^0}{2}\sin\frac{87^0-93^0}{2} + 2\cos\frac{61^0+59^0}{2}\cdot\sin\frac{61^0-59^0}{2} = 2\cos 90^0 \cdot$
$\cdot \sin(-3^0) + 2\cos 60^0 \cdot \sin 1^0 = \sin 1^0$; Javobi : B

**179 – misol**. ( 96-6-35 ). Soddalashtiring. $\frac{\cos a - \cos 3a}{\sin a}$

**Yechilishi:** $\frac{\cos a - \cos 3a}{\sin a} = \frac{-2\sin\frac{a+3a}{2}\cdot\sin\frac{a-3a}{2}}{\sin a} = \frac{2\sin 2a\sin a}{\sin a} =$
$-\sin 2a$. Javobi: E

**180– misol**. ( 97-8-34 ). Ushbu $\frac{\cos 4a}{\sin 5a - \sin 3a}$ ifoda quyidagalardan kaysi biriga teng ?

A) $\frac{1}{2\cos a}$, V) $\frac{1}{\sin a}$, S) $\frac{1}{\cos a}$, D) $\frac{\cos 4a}{\sin 2a}$, E) $\frac{1}{2\sin a}$

**Yechilishi :** $\frac{\cos 4a}{2\cos 4a \cdot \sin a} = \frac{1}{2\sin a}$; Javobi : E

**181 – misol**. ( 00-1-28 ). Hisoblang. $\frac{\sin 35^0 + \cos 65^0}{2\cos 5^0}$

**Yechilishi**: $\frac{\sin(90^0 - 55^0) + \cos 65^0}{2\cos 5^0} = \frac{\cos 55^0 + \cos 65^0}{2\cos 5^0} = \frac{2\cos 60^0 \cdot \cos 5^0}{2\cos 5^0} =$
$\frac{1}{2}$; Javobi : C

**182 – misol**. ( 02-5-33 ) $\frac{\sin a + \sin 2a - \sin(\pi + 3a)}{1 + 2\cos a}$ ni soddalashtiring.

**Yechilishi :** $\frac{\sin a + \sin 2a + \sin 3a}{1 + 2\cos a} = \frac{2\sin 2a\cos a + \sin 2a}{1 + 2\cos a} = \frac{\sin 2a(2\cos a + 1)}{2\cos a + 1} =$
$\sin 2a$;
Javobi : E

**183 – misol**. ( 03-7-55 ). $\sin 87^0 - \sin 59^0 - \sin 93^0 + \sin 61^0$ ni soddalashtiring.

**Yechilishi** : $(\sin 87^0 - \sin 93^0) + (\sin 61^0 - \sin 59^0) =$
$= 2\sin\frac{(87^0 - 93^0)}{2}\cos\frac{(87^0 + 93^0)}{2} + 2\sin\frac{61^0 - 59^0}{2}\cdot\cos\frac{61^0 + 59^0}{2} =$
$-2\sin 3^0 \cdot \cos 90^0 + 2\sin 1^0 \cdot \cos 60^0 = \sin 1^0$; Javobi : B

## Trigonometrik funksiylarda ko‘paytmani yig‘indiga keltirish formulalardan foydalanib trigonometrik ifodalarni soddalashtirish va hisoblash.

## Ko‘paytmani yig‘indiga keltirish formulalari

$$1.\ \sin x \cdot \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)];$$

$$2.\ \cos x \cdot \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)];$$

$$3.\ \sin x \cdot \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)];$$

**184 – misol**. ( 00-8-48 ). Hisoblang. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

**Yechilishi** : Bu ifodani $\sin\frac{2\pi}{7}$ ga bo‘lamiz va ko‘paytiramiz

$$\sin\frac{2\pi}{7}\cdot\cos\frac{2\pi}{7} + \sin\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7} + \sin\frac{2\pi}{7}\cdot\cos\frac{6\pi}{7}$$

Suratidagi har bir qo‘shiluvchiga (3) formulani qo‘llaymiz.

$$\frac{1}{2}[\sin\left(\frac{2\pi}{7}+\frac{2\pi}{7}\right) + \sin(\frac{2\pi}{7}-\frac{2\pi}{7})] + \frac{1}{2}[\sin\frac{2\pi}{7}+\frac{4\pi}{7}) + \sin(\frac{2\pi}{7}-\frac{4\pi}{7})] +$$

$$+\frac{1}{2}[\sin\left(\frac{2\pi}{7}+\frac{6\pi}{7}\right) + \sin(\frac{2\pi}{7}-\frac{6\pi}{7})] = \frac{1}{2}[\sin\frac{4\pi}{7} + \sin\frac{6\pi}{7} - \sin\frac{2\pi}{7} + \sin\frac{8\pi}{7} -$$

$$-\sin\frac{4\pi}{7}] = \frac{1}{2}[\sin\left(\pi-\frac{\pi}{7}\right) - \sin\frac{2\pi}{7} + \sin(\pi+\frac{\pi}{7})] = -\frac{1}{2}\sin\frac{2\pi}{7};$$

Demak, $\frac{-\frac{1}{2}\sin\frac{2\pi}{7}}{\sin\frac{2\pi}{7}} = -\frac{1}{2}$; Javobi : A

**185 – misol**. ( 96-3-57 ). Hisoblang. $\sin 20^0 \cdot \sin 40^0 \cdot \sin 80^0$

**Yechilishi**: $\sin 20^0 \cdot \sin 40^0 = \frac{1}{2}[\cos 20^0 - \cos 60^0] = \frac{1}{2}[\cos 20^0 - \frac{1}{2}] =$ $= \frac{1}{2}\cos 20^0 - \frac{1}{4}$; $[\frac{1}{2}\cos 20^0 - \frac{1}{4}]\cdot\sin(90^0 - 10^0) =$ $(\frac{1}{2}\cos 20^0 - \frac{1}{4})\cos 10^0 = \frac{1}{2}\cos 20^0\cdot\cos 10^0 - \frac{1}{4}\cos 10^0 =$ $=$ $\frac{1}{4}(\cos 10^0 + \cos 30^0) - \frac{1}{4}\cos 10^0 = \frac{1}{4}\cos 10^0 + \frac{1}{4}\cos 30^0 -$ $\frac{1}{4}\cos 10^0 = \frac{1}{4}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$; Javobi : D

**186 – misol.** ( 00-10-79 ). Hisoblang. $\cos 5^0 \cdot \cos 55^0 \cdot \cos 65^0$

**Yechilishi :** $\cos 5^0 \cdot \cos 55^0 = \frac{1}{2}[\cos 50^0 + \cos 60^0] = \frac{1}{2}\cos 50^0 + \frac{1}{4}$;

$$\cos 5^0 \cdot \cos 55^0 \cdot \cos 65^0 = \frac{1}{2}\cos 50^0 \cdot \cos 65^0 + \frac{1}{4}\cos 65^0 =$$

$$= \frac{1}{4}[\cos 115^0 + \cos 15^0] + \frac{1}{4}\cos 65^0 = \frac{1}{4}\cos(180^0 - 65^0)$$

$$+ \frac{1}{4}\cos 15^0 + \frac{1}{4}\cos 65^0 =$$

$$= -\frac{1}{4}\cos 65^0 + \frac{1}{4}\cos 15^0 + \frac{1}{4}\cos 65^0 = \frac{1}{4}\cos 15^0 =$$

$$= \frac{1}{4}\cdot\sqrt{\frac{1+\cos 30^0}{2}} = \frac{1}{4}\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{1}{4}\cdot\sqrt{\frac{2+\sqrt{3}}{4}} = \frac{1}{8}\sqrt{2+\sqrt{3}}\ .$$

$$\sqrt{2+\sqrt{3}} = \sqrt{\frac{2+1}{2}} + \sqrt{\frac{2-1}{2}} = \frac{\sqrt{3}+1}{\sqrt{2}}\ ;\ A = \sqrt{4-3} = 1;$$

Demak , $\cos 5^0 \cdot \cos 55^0 \cdot \cos 65^0 = \frac{1}{8}\cdot\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{16}$; Javobi : A

**187 – misol**. ( 96-3-57 ). Hisoblang. $\text{Sin } 20^0 \cdot \sin 40^0 \cdot \sin 80^0$

**Yechilishi** : $\sin 20^0 \cdot \sin 40^0 = \frac{1}{2}[\cos 20^0 - \cos 80^0] = \frac{1}{2}(\cos 20^0 - \frac{1}{2})$;

$$\sin 80^0 = \sin(90^0 - 10^0) = \cos 10^0;$$

$$\sin 20^0 \cdot \sin 40^0 \cdot \sin 80^0 = \frac{1}{2}(\cos 20^0 - \frac{1}{2}) \cdot \cos 10^0 =$$

$$= \frac{1}{2}\cos 20^0 \cdot \cos 10^0 - \frac{1}{4}\cos 10^0 =$$

$= \frac{1}{4}[\cos 10^0 + cos30^0] - + \cos 30^0] - \frac{1}{4}\cos 10^0 = \frac{1}{4}\cos 30^0 = \frac{1}{4}\cdot$ $\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$; Javobi : D

**188 – misol**. ( 98-3-54 ). Hisoblang. $\frac{4\cdot\sin 40^0\cdot\sin 50^0}{\cos 10^0}$

**Yechilishi :** $\sin 40^0 \cdot \sin 50^0 = \frac{1}{2}(\cos 10^0 - \cos 90^0) = \frac{1}{2}\cos 10^0$;

Demak, $\frac{4\cdot\frac{1}{2}\cos 10^0}{\cos 10^0} = 2$; Javobi : B

**189 – misol**. ( 01-1-45 ). $5^0$, $10^0$, $15^0$, … burchaklarning qiymatlari arifmetik progressiyani tashkil qiladi. Shu progressiyaning birinchi qadidan boshlab eng kamida nechtasini olganda ularning konuslari yig‘indisi nolga teng buladi ?

**Yechilishi**: $\cos 5^0 + \cos 10^0 + \cos 15^0 + \ldots = 0$ yoki $\cos 5^0 + \cos(5^0 + 1\cdot 5^0) + \cos(5^0 + 2\cdot 5^0) + \ldots + \cos(5^0 + n\cdot 5^0) = 0$ ; $\cos 5^0 + \cos 175^0 =$ $2\cos 90^0 \cdot \cos 85^0 = 0$ ekanligini e’tiborga olsak,

$5^0 + n \cdot 5^0 = 175^0$ dan $n = \frac{170^0}{5^0} = 34^0$. Birinchi hadini hisobga olsak, $n = 35^0$ . Javobi : D

**190 – misol**. ( 03-9-30 ). $\cos 55^0 \cdot \cos 65^0 \cdot \cos 175^0$ ni hisoblang.
**Yechilishi** : (2) formulaga asosan $\cos 55^0 \cdot \cos 65^0 = \frac{1}{2}(\cos 10^0 + \cos 120^0)$;
$\cos 120^0 = \cos(90^0 + 30^0) = -\sin 30^0 = -\frac{1}{2}$; ekanligini e'tiborga olsak, $\cos 55^0 \cdot \cos 65^0 = \frac{1}{2}(\cos 10^0 - \frac{1}{2}) = \frac{1}{2}\cos 10^0 - \frac{1}{4}$ ;

$$(\frac{1}{2}\cos 10^0 - \frac{1}{4})\cos 175^0 = \frac{1}{2}\cos 10^0 \cdot \cos 175^0 - \frac{1}{4}\cos 175^0 =$$

$$= \frac{1}{4}(\cos 165^0 + \cos 185^0) - \frac{1}{4}\cos 175^0 = \frac{1}{4}\cos 165^0 + \frac{1}{4}\cos 185^0 - \frac{1}{4}\cos 175^0 =$$

$$= \frac{1}{4}\cos 165^0 + \frac{1}{4}\cos(180^0 + 5^0) - \frac{1}{4}\cos(180^0 - 5^0) = \frac{1}{4}\cos 165^0 - \frac{1}{4}\cos 5^0 + \frac{1}{4}\cos 5^0 =$$

$$= \frac{1}{4}\cos 165^0 = \frac{1}{4}\cos(180^0 - 15^0) = -\frac{1}{4}\cos 15^0 = -\frac{1}{4}\sqrt{\frac{1+\cos 30^0}{2}} =$$

$$= -\frac{1}{4}\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = -\frac{1}{8}\sqrt{2+\sqrt{3}}\ ;$$ Javobi : E

**191 – misol**. ( 01-5-15 ). Hisoblang. $\operatorname{tg} 10^0 \cdot \operatorname{tg} 50^0 \cdot \operatorname{tg} 70^0$
**Yechilishi** : $\frac{\sin 10^0 \cdot \sin 50^0 \cdot \sin 70^0}{\cos 10^0 \cdot \cos 50^0 \cdot \cos 70^0}$

$$\sin 10^0 \cdot \sin 50^0 = \frac{1}{2}(\cos 40^0 - \cos 60^0) = \frac{1}{2}(\cos 40^0 - \frac{1}{2}) = \frac{1}{2}\cos 40^0 - \frac{1}{4};$$

$$\sin 70^0 = \sin(90^0 - 20^0) = \cos 20^0;$$

$$(\frac{1}{2}\cos 40^0 - \frac{1}{4}) \cdot \cos 20^0 = \frac{1}{2}\cos 40^0 \cdot \cos 20^0 - \frac{1}{4} \cdot \cos 20^0 =$$

$$\frac{1}{4}(\cos 60^0 + \cos 20^0) - \frac{1}{4}\cos 20^0 = \frac{1}{4}\cos 60^0 + \frac{1}{4}\cos 20^0 - \frac{1}{4}\cos 20^0 = \frac{1}{4}\cos 60^0 =$$

$$= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}\ ;\ \cos 10^0 \cdot \cos 50^0 = \frac{1}{2}(\cos 60^0 + \cos 40^0) = \frac{1}{4} + \frac{1}{2}\cos 40^0;$$

$$(\frac{1}{4} + \frac{1}{2}\cos 40^0) \cdot \cos 70^0 = \frac{1}{4}\cos 70^0 + \frac{1}{2}\cos 40^0 \cdot \cos 70^0 = \frac{1}{4}\cos 70^0 + \frac{1}{2}\cos 110^0 +$$

$$+ \frac{1}{4}\cos 30^0 = \frac{1}{4}\cos(90^0 - 20^0) + \frac{1}{4}\cos(90^0 + 20^0) + \frac{1}{4}\cos 30^0 = \frac{1}{4}\sin 20^0 - \frac{1}{4}\sin 20^0 +$$

$+ \frac{1}{4} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$. Demak, $tg10^0 \cdot tg50^0 \cdot tg70^0 = \frac{1}{8} \div \frac{\sqrt{3}}{8} = \frac{1}{\sqrt{3}}$; Javobi : A

## Trigonometrik funksiylarning yarim burchaklari uchun formulalardan foydalanib trigonometrik ifodalarni soddalashtirish va hisoblash.

### Yarim burchak formulalari.

1. $sin^2x = \frac{1-cos\ 2x}{2}$; 2. $cos^2x = \frac{1+cos\ 2x}{2}$;

3. $sin^2\frac{x}{2} = \frac{1-cos x}{2}$; 4. $cos^2\frac{x}{2} = \frac{1+cos\ x}{2}$;

5. $tg\frac{x}{2} = \frac{sin x}{1+cos\ x} = \frac{1-cos\ x}{sin x}$; 6. $ctg\frac{x}{2} = \frac{sin x}{1-cos\ x} = \frac{1+cos\ x}{sin\ x}$

7. $tg^2\frac{x}{2} = \frac{1-cos\ x}{1+cos\ x}$; 8. $ctg^2\frac{x}{2} = \frac{1+cos x}{1-cos\ x}$;

9. $tg\frac{x}{2} = \frac{sin\ x}{1+cos\ x} = \frac{1-cos\ x}{sin\ x}$ 10. $ctg\frac{x}{2} = \frac{sin\ x}{1-cos\ x} = \frac{1+cos\ x}{sin\ x}$

**192 – misol**. ( 98-10-100 ). Hisoblang. $\sin 105^0 + \sin 75^0$

**Yechilishi:**

$sin\ 105^0+sin\ 75^0=2\ sin(\frac{105^0+75^0}{2})\cdot cos\frac{105^0-75^0}{2}=2\ sin90^0\cdot cos\ 15^0=2\ cos\ 15^0=$

$= 2\sqrt{\frac{1+cos\ 30^0}{2}} = 2\cdot\frac{\sqrt{2+\sqrt{3}}}{2} = \sqrt{2+\sqrt{3}}$; Javobi: D

**193 – misol**. ( 96-1-55 ). Agar $\cos 2a = \frac{1}{2}$ bo‘lsa, $\cos^2 a$ ni hisoblang.

**Yechilishi** : $\frac{1+cos\ 2a}{2} = cos^2a$; $cos^2a = \frac{1+\frac{1}{2}}{2} = \frac{3}{4}$; Javobi : C

**194 – misol**. ( 96-7-55 ). $\sin\frac{\pi}{12}$ ni hisoblang.

**Yechilishi:** $\sin\frac{\pi}{12} = \sin 15^0 = \sqrt{\frac{1-cos\ 30^0}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$; Javobi : C

**195 - misol** ( 97-1-45 ). Agar $\cos\alpha = -\frac{1}{2}$ va $\pi < \alpha < \frac{3\pi}{2}$ bo‘lsa, $\sin(\frac{\pi}{2} + \frac{\alpha}{2})$ ni toping.

**Yechilishi** : $\sin\left(\frac{\pi}{2} + \frac{\alpha}{2}\right) = \cos\frac{\alpha}{2}$; $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1-cos\ \alpha}{2}}$, $\pi < \alpha < \frac{3\pi}{2}$ $dan$, $\cos\frac{\alpha}{2} = -\sqrt{\frac{1+cos\alpha}{2}}$ $ydki$, $\cos\frac{\alpha}{2} = -\sqrt{\frac{1-\frac{1}{2}}{2}} = -\frac{1}{2}$; Javobi : D

**196 – misol**. ( 00-3-53 ). Agar $0 < \alpha < \frac{\pi}{2}$ va $\cos\alpha = \frac{1}{2}\frac{\sqrt{2+\sqrt{3}}}{2}$ b o‘lsa, $\alpha$ ni toping.

**Yechilishi** :

$cos^2\alpha = \frac{1}{4}(2+\sqrt{3})$ $\frac{1+\cos 2\alpha}{2} = \frac{2+\sqrt{3}}{4} \Rightarrow 1+\cos 2\alpha = \frac{2+\sqrt{3}}{2}$;

$1+\cos 2\alpha = 1+\frac{\sqrt{3}}{2} \Rightarrow \cos 2\alpha = \frac{\sqrt{3}}{2} \Rightarrow 2\alpha = \frac{\pi}{6} \Rightarrow \alpha = \frac{\pi}{12} = 15^0$;

Javobi : E

**197 – misol**. ( 99-3-34 ). Agar $0 < a < \frac{\pi}{2}$ va $\cos a = \frac{1}{2}\sqrt{2+\sqrt{2}}$ bo‘lsa, $a$ ni toping.

**Yechilishi** : $cos^2 a = \frac{1}{4}(2+\sqrt{2}) \Rightarrow \frac{1+\cos 2a}{2} = \frac{2+\sqrt{2}}{4}$; $1+\cos 2a = \frac{2+\sqrt{2}}{2} \Rightarrow$ $1+\cos 2a = 1+\frac{\sqrt{2}}{2} \Rightarrow \cos 2a = \frac{\sqrt{2}}{2} \Rightarrow 2a = \frac{\pi}{4} \Rightarrow a = \frac{\pi}{8}$; Javobi : D

**198 – misol.** ( 99-8-69 ). Hisoblang. $\sin(202^0 30')$

**Yechilishi** : $\sin(202^0 30') = \pm\sqrt{\frac{1-\cos 405^0}{2}}$; $\cos 405^0 = \cos(360^0+45^0) = \cos 45^0$; $\sin(202^0 30') = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2} = \frac{-2-\sqrt{2}}{2}}$;

Javobi : A

**199 – misol**. ( 97-5-28 ). Hisoblang. $8\cdot\cos 30^0 + \mathrm{tg}^2\ 15^0$

**Yechilishi :** $8\cdot\cos 30^0 + tg^2 15^0 = 8\cdot\frac{\sqrt{3}}{2} + \frac{sin^2 15^0}{cos^2 15^0}$;

$sin^2 15^0 = \frac{1-\cos 30^0}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4}$; $cos^2 15 = \frac{2+\sqrt{3}}{4}$;

$tg^2 15 = \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{(2-\sqrt{3})^2}{4-3} = 4-4\sqrt{3}+3 = 7-4\sqrt{3}$;

$8\cdot\cos 30^0 + tg^2 15^0 = 4\sqrt{3}+7-4\sqrt{3} = 7$ ;

Javobi : C

**200 – misol**. ( 97-7-55 ). Hisoblang. $\cos\frac{5\pi}{12}$

**Yechilishi :**

$$\cos\frac{5\pi}{12} = \sqrt{\frac{1+\cos 2\cdot\frac{5\pi}{12}}{2}} = \sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1+\cos 150^0}{2}}$$

$$= \sqrt{\frac{1+\cos(90^0+60^0)}{2}} =$$

$$= \sqrt{\frac{1-\sin 60^0}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2};$$ Javobi : E

**201 – misol**. ( 97-11-44 ). Agar cos α = $-\frac{1}{2}$ va $\pi < \alpha < \frac{3\pi}{2}$ bo‘lsa, $\cos(\frac{\pi}{2}+\frac{\alpha}{2})$ ni toping.

**Yechilishi :** $\cos\left(\frac{\pi}{2}+\frac{\alpha}{2}\right) = -\sin\frac{\alpha}{2}$; $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos a}{2}} = \sqrt{\frac{1-\cos a}{2}}$;

$-\sin\frac{a}{2} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$; $\sin\frac{a}{2} = -\frac{\sqrt{3}}{2}$; Javobi : E

**202 – misol**. ( 99-3-33). Agar $450^0 < a < 540^0$ va $ctga = -\frac{7}{24}$ bo‘lsa, $\cos\frac{a}{2}$ ni qisoblang.

**Yechilishi**: $\cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}}$; $1 + ctg^2 a = \frac{1}{\sin^2 a} \Rightarrow \sin a = \pm\frac{1}{\sqrt{1+ctg^2 a}}$;

$a$ II chorakda bo‘lgani uchun, $\sin a = \frac{1}{\sqrt{1+ctg^2 a}} = \frac{1}{\sqrt{1-\frac{49}{176}}} = \frac{1}{\sqrt{\frac{625}{576}}} = \frac{24}{25}$;

$\cos a = -\sqrt{\frac{576}{625}} = -\sqrt{\frac{49}{625}} = -\frac{7}{25}$; α II chorakda bo‘lgani uchun

$\cos\frac{\alpha}{2} = -\sqrt{\frac{1-\frac{7}{25}}{2}} = -\sqrt{\frac{18}{50}} = -\frac{3}{5} = -0{,}6$; Javobi : D

**203 – misol.** ( 99-2-27 ). Soddalashtiring. $\frac{\cos^2 x+\cos x}{2\cos^2\frac{x}{2}} + 1$

**Yechilishi** : $cos^2\frac{x}{2} = \frac{1+\cos x}{2} \Rightarrow 1 + \cos x = 2cos^2\frac{x}{2}$; $\frac{cos^2 x+\cos x}{1+\cos x} + 1 = \frac{\cos x\ (1+\cos x)}{1+\cos x} + 1 = \cos x + 1 = 2cos^2\frac{x}{2}$; Javobi : E

**204–misol.** ( 99-9-28 ). Soddalashtiring. $\sqrt{\frac{1+\sin(\frac{3\pi}{2}+a)}{1+\sin(\frac{\pi}{2}+a)}} = \sqrt{\frac{1-\cos a}{1+\cos a}}$

**Yechilishi :** $\begin{cases} 1-\cos a = 2sin^2\frac{a}{2} \\ 1+\cos = 2cos^2\frac{a}{2} \end{cases}$ formulalardan foydalansak,

$\sqrt{\frac{1+\sin(\frac{3\pi}{2}+a)}{1+\sin(\frac{\pi}{2}+a)}} = \sqrt{\frac{2sin^2\frac{a}{2}}{2cos^2\frac{a}{2}}} = \left|tg\ \frac{a}{2}\right|$; Javobi : A

**205 – misol**. ( 99-6-25 ). Soddalashtiring. $\frac{sin^4 a+2\sin a\cos a-cos^4 a}{2cos^2 a-1}$

**Yechilishi** : Suratini soddalashtiramiz.

$sin^4 a + 2\sin a \cdot \cos a - cos^4 a = (sin^4 a - cos^4 a) + 2\sin a \cdot \cos a =$

$= (sin^2 a - cos^2 a)(sin^2 a + cos^2 a) + 2\sin a\cos a =$

$= sin^2 a - cos^2 a + 2\sin a\cos a = \sin 2a - \cos 2a$

Maxrajini soddalashtiramiz. $2cos^2 a - 1 = \cos 2a$

Demak, $\frac{sin^4 a+2\sin a\cos a-cos^4 a}{2cos^2 a-1} = \frac{\sin 2a-\cos 2a}{\cos 2a} = tg2a - 1$; Javobi : A

**206 – misol**. ( 01-1-68 ). Agar $\sin a = -0{,}8$ va $\alpha \in (\pi; \frac{3\pi}{2})$ bo‘lsa, $tg\ \frac{a}{2}$ ni aniqlang.

**Yechilishi** : $\cos a = \pm\sqrt{1-0{,}64} = \pm\sqrt{0{,}36} = \pm 0{,}6$; $\cos a = -0{,}6$:

$tg\ \frac{a}{2} = \frac{\sin a}{1+\cos a} = \frac{-0{,}8}{1-0{,}6} = \frac{-0{,}8}{0{,}4} = -2$; Javobi : D

**207 – misol.** (01-3-3) Hisoblang. $sin^4 15^0 + cos^4 15^0$

**Yechilishi :** $(sin^2 15^0)^2 + (cos^2 15^0)^2 = (\frac{1-\cos 30^0}{2})^2 + (\frac{1+\cos 30^0}{2})^2 =$

$= \frac{(2-\sqrt{3})^2}{16} + \frac{(2+\sqrt{3})^2}{16} = \frac{4-4\sqrt{3}+3+4+4\sqrt{3}+3}{16} = \frac{14}{16} = \frac{7}{8}$; Javobi : C

**208 – misol.** (01-7-36). Hisoblang. $cos^2 5^0 + cos^2 1^0 - \cos 6^0 \cdot \cos 4^0$

**Yechilishi :** $\frac{1+\cos 10^0}{2} + \frac{1+\cos 2^0}{2} - \frac{1}{2}(\cos 10^0 + \cos 2^0) =$

$= \frac{2+\cos 10^0+\cos 2^0-\cos 10^0-\cos 2^0}{2} = 1$; Javobi : E

**209 – misol.** ( 01-10-35 ). Ifodaning qiymatini toping.

$$cos^8 22^0 30' - sin^8 22^0 30'$$

**Yechilishi:** $(cos^2 22^0 30^0)^4 - (sin^2 22^0 30') = (\frac{1+\cos 45^0}{2})^4 -$
$(\frac{1-\cos 45^0}{2})^4 = (\frac{1+\frac{\sqrt{2}}{2}}{2})^4 - (\frac{1-\frac{\sqrt{2}}{2}}{2})^4 = \frac{(2+\sqrt{2})^4}{4^4} - \frac{(2-\sqrt{2})^4}{4^4} = \frac{1}{256}[((2 +$
$\sqrt{2})^2)^2 - ((2-\sqrt{2})^2)^2] = = \frac{1}{256}[((2+\sqrt{2})^2 - (2-\sqrt{2})^2] \cdot$
$((2+\sqrt{2})^2 + (2-\sqrt{2})^2)\ ]=$
$= \frac{1}{256}[\left(4+\sqrt{2}+2-\left(4-4\sqrt{2}+2\right)\right)\left(4+4\sqrt{2}+2+4-2\sqrt{2}+2\right) =$
$= \frac{1}{256}\left(8\sqrt{2}\cdot 12\right) = \frac{1}{256}\cdot 96\sqrt{2} = \frac{3}{8}\sqrt{2}$; Javobi : C

**210 – misol.** ( 02-1-24) Soddalashtiring.

$$\cos^2 73^0 + \cos^2 47^0)^2 + \cos 73^0 \cdot \cos 47^0$$

**Yechilishi :** $(\cos 73^0 + \cos 47^0)^2 - \cos 73^0 \cdot \cos 47^0 =$

$$(2\cos\frac{73^0+47^0}{2}\cdot\cos\frac{73^0-47^0}{2})^2 - \frac{1}{2}(\cos 120^0 + \cos 26^0) =$$

$= 4cos^2 60^0 \cdot cos^2 13^0 - \frac{1}{2}\cdot\left(-\frac{1}{2}\right) - \frac{1}{2}\cos 26^0 = \frac{1}{2} + \frac{1}{2}\cos 26^0 + \frac{1}{4} -$
$\frac{1}{2}\cos 26^0 = \frac{3}{8}$; Javobi : A

**211 – misol**. (02-3-73 ) $8sin^2\frac{7\pi}{8}\cdot cos^2\frac{9\pi}{8}$ ni hisoblang.

**Yechilishi :** $\sin\frac{7\pi}{8} = \sin\left(\pi-\frac{\pi}{8}\right) = +\sin\frac{\pi}{8}$; $\cos\frac{9\pi}{8} = \cos\left(\pi+\frac{\pi}{8}\right) =$
$-\cos\frac{\pi}{8}$;

$8sin^2\frac{7\pi}{8}\cdot cos^2\frac{9\pi}{8} = 8\cdot sin^2\frac{\pi}{8}\cdot cos^2\frac{\pi}{8} =$

$= 8\cdot(\frac{1}{2}\cdot 2\sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8})^2 = 8\cdot\frac{1}{4}sin^2 2\cdot\frac{\pi}{8} = = 2sin^2\frac{\pi}{4} = 2\cdot(\frac{\sqrt{2}}{2})^2 =$
$= 2\cdot\frac{2}{4} = 1$; Javobi : C

**212–misol**. (02-3-74) $Agar$ $\cos(\pi - 4a) = -\frac{1}{3}$ bo‘lsa $cos^4(\frac{3\pi}{2} -$
$-2a)$ ni hisoblang.

**Yechilishi :** $\cos\left(\frac{3\pi}{2}-2a\right)=-\sin 2a;\ \cos^4\left(\frac{3\pi}{2}-2a\right)=sin^4 2a=$

$=(sin^2 2a)^2=(\frac{1-\cos 4a}{2})^2;\qquad \cos(\pi-4a)=-\cos 4a=$

$-\frac{1}{3}\Rightarrow\cos 4a=\frac{1}{3};\qquad cos^4\left(\frac{3\pi}{2}-2a\right)=(\frac{1-cos4a}{2})^2=(\frac{1-\frac{1}{3}}{2})^2=(\frac{2}{6})^2=\frac{1}{9};$

Javobi : A

**213 – misol**. (02-5-31 ) Agar $\sin(a+\beta)=\frac{4}{5},\sin(a-\beta)=\frac{5}{13}\ va\ 0<\beta<a<\frac{\pi}{4}$; bo‘lsa $\sin a+\sin\beta$ ning qiymatini hisobang.

**Yechilishi :**

$$\sin a+\sin\beta=2\sin\frac{a+\beta}{2}\cdot\cos\frac{a-\beta}{2}\ ,$$

$$\cos(a+\beta)=\sqrt{1-sin^2(a+\beta)}=\sqrt{1-\frac{11}{25}}=\frac{3}{5};$$

$$\cos(a+\beta)=1-2sin^2\frac{a+\beta}{2}=\frac{3}{5}\Rightarrow 2sin^2\frac{a+\beta}{2}=1-\frac{3}{5};$$

$$sin^2\frac{a+\beta}{2}=\frac{1}{5};\ \sin\frac{a+\beta}{2}=\frac{1}{\sqrt{5}};$$

$$\cos(a-\beta)=\sqrt{1-sin^2(a-\beta)}=\sqrt{1-\frac{25}{169}}=\frac{12}{13};$$

$$\cos(a-\beta)=2cos^2\frac{a-\beta}{2}-1\Rightarrow 2cos^2\frac{a-\beta}{2}=\frac{12}{13}+1;$$

$$cos^2(\frac{a+\beta)}{2}=\frac{25}{26}\Rightarrow\cos\frac{a+\beta}{2}=\frac{5}{\sqrt{26}};\quad 0<\frac{a+\beta}{2}<\frac{\pi}{4}$$

$$\sin a+\sin\beta=2\cdot\sin\frac{a+\beta}{2}\cdot\cos\frac{a-\beta}{2}=2\cdot\frac{1}{\sqrt{5}}\cdot\frac{5}{\sqrt{26}}=\frac{10}{\sqrt{130}};$$

Javobi : A

**214 – misol**. (02-5-34 ) $\frac{\cos^2 68-\cos^2 28}{\sin 106^0}$ ni hisoblang.

**Yechilishi:** $\cos^2 68^0-\cos^2 38^0=(\cos 68^0-\cos 38^0)(\cos 68^0+$

$+\cos 38^0)=-2\sin\frac{68^0+38^0}{2}\cdot\sin\frac{68^0-38^0}{2}\cdot\ 2\cos\frac{68^0+38^0}{2}\cdot\cos\frac{68^0-38^0}{2}=$

$=-2\sin 53\cdot\sin 15^0\cdot 2\cos 53^0\cdot\cos 15^0=-\sin 106^0\cdot\sin 30^0$

Demak, $\frac{\cos^2 68^0-\cos^2 28^0}{\sin 106^0}=\frac{-\sin 106^0\cdot\sin 30^0}{\sin 106^0}=-\sin 30^0=-\frac{1}{2}$; Javobi: B

**215 – misol**. (02-6-40 ) $\cos^8 165^0-\sin^8 165^0$ ni hisoblang.

**Yechilishi :** $(cos^4 165^0)^2-(sin^4 105^0)^2=(cos^4 165^0-sin^4 165^0)\cdot$

$\cdot(cos^4 165^0+sin^4 165^0)^2=((cos^2 165^0)^2-(sin^2 165^0)^2)\cdot$

$[(cos^2 165^0+sin^2 165^0)^2-2cos^2 165^0 sin^2 165^0]=$

$$= (cos^2 165^0 - sin^2 165^0)\left[1 - 2\cdot(\frac{1}{2}\cdot 2\sin 165^0\cdot\cos 165^0)^2\right] =$$

$$= \cos 2\cdot 165^0\cdot\left(1 - 2\cdot\frac{1}{4}sin^2 2\cdot 165^0\right) =$$

$$= \cos 330^0\left(1 - \frac{1}{2}sin^2 330^0\right) =$$

$$= \left|\begin{matrix}\cos 330^0 = \cos(360^0 - 30^0) = \cos 30^0\\ \sin 330^0 = \sin(360^0 - 30^0) = -\sin 30^0\end{matrix}\right| =$$

$= \cos 30^0(1 - \frac{1}{2}(-\sin 30^0)^2) = \frac{\sqrt{3}}{2}\left(1 - \frac{1}{2}\cdot\frac{1}{4}\right) = \frac{\sqrt{3}}{2}\cdot\frac{7}{8} = \frac{7\sqrt{3}}{16}$; *Javobi B*

**216 – misol**. (02-7-16 ) $\frac{2\cos^2(45^0 - \frac{a}{2})}{\cos a}$ ni soddalashtiring.

**Yechilishi** : $\cos\left(45^0 - \frac{a}{2}\right) = \cos 45^0\cdot\cos\frac{a}{2} + \sin 45^0\cdot\sin\frac{a}{2} = \frac{\sqrt{2}}{2}(\cos\frac{a}{2} + +\sin\frac{a}{2})$;

$$\cos a = cos^2\frac{a}{2} - sin^2\frac{a}{2} = (\cos\frac{a}{2} - \sin\frac{a}{2})(\cos\frac{a}{2} + \sin\frac{a}{2});$$

$$\frac{2cos^2(45^0 - \frac{a}{2})}{\cos a} = \frac{2[\frac{\sqrt{2}}{2}(\cos\frac{a}{2} + \sin\frac{a}{2})]^2}{(\cos\frac{a}{2} + \sin\frac{a}{2})(\cos\frac{a}{2} - \sin\frac{a}{2})} =$$

$= \frac{(\cos\frac{a}{2} + \sin\frac{a}{2})}{(\cos\frac{a}{2} - \sin\frac{a}{2})} = \frac{\sqrt{2}(\frac{1}{\sqrt{2}}\cos\frac{a}{2} + \frac{1}{\sqrt{2}}\sin\frac{a}{2})}{\sqrt{2}(\frac{1}{\sqrt{2}}\cos\frac{a}{2} - \frac{1}{\sqrt{2}}\sin\frac{a}{2})} = = \frac{\cos(45^0 - \frac{a}{2})}{\sin(45^0 - \frac{a}{2})} = ctg(45^0 - \frac{a}{2})$.

Javobi : B

**217 – misol**. (02-09-39 ) $\frac{2sin^2 70^0 - 1}{2ctg115^0\cdot cos^2 155^0}$ ni soddalashtiring.

**Yechilishi:** :

$2sin^2 a - 1 = -(1 - 2sin^2 a) = -\cos 2a$ *formuladan* $2sin^2 70^0 - 1 = = -\cos 140^0$; $2\cdot\frac{\cos 115^0}{\sin 115^0}\cdot[\cos(90^0 + 65^0)]^2 = -2\cos 65^0\cdot\sin 65^0 = -\sin 130^0$;

$\frac{2sin^2 70^0 - 1}{2ctg115^0\cdot cos^2 115^0} = \frac{-\cos 140^0}{-\sin 130^0} = \frac{\cos(180^0 - 40^0)}{\sin(90^0 + 40^0)} = \frac{-\cos 40^0}{\cos 40^0} = -1$;Javobi: A

**218 – misol.** (02-11-40 ) $2\sin 44^0 \cdot \cos 16^0 + 2\sin^2 31^0 - 1$ ni hisoblang.

**Yechilishi:**

$$2\sin 44^0 \cdot \cos 16^0 = 2 \cdot \frac{1}{2}[\sin(44^0 - 16^0) + \sin(44^0 + 16^0)] =$$

$= \sin 28^0 + \sin 60^0 = \sin 28^0 + \frac{\sqrt{3}}{2}$; $2\sin^2 31^0 - 1 = -\cos 2 \cdot 31^0 =$ $-\cos 62^0 = -\cos(90^0 - 28^0) = -\sin 28^0$; Demak, $2\sin 44^0 \cos 16^0 + +2\sin^2 31^0 - 1 = \sin 28^0 + \frac{\sqrt{3}}{2} - \sin 28^0 = \frac{\sqrt{3}}{2}$;

Javobi : C

**219 – misol**. (02-12-38 ) $\frac{tga+\sin a}{2\cos^2 \frac{a}{2}}$ ni soddalashtiring.

**Yechilishi:**

$$\frac{\sin a}{\cos a} + \sin a = \frac{\sin a(1 + \cos a)}{\cos a} = \left|1 + \cos a = 2\cos^2 \frac{a}{2}\right| = \frac{\sin a \cdot 2\cos^2 \frac{a}{2}}{\cos a};$$

$$\frac{tga + \sin a}{2\cos^2 \frac{a}{2}} = \frac{2\sin a \cdot \cos^2 \frac{a}{2}}{2\cos a \cdot \cos^2 \frac{a}{2}} = tga; \; Javobi B$$

**220–misol**. (02-11-42 ) Agar $tga = \frac{5}{12}$ $va$ $a \in (540^0; 630^0)$ $bo'lsa$, $\sin\frac{a}{2}$ ning qiymatini hisoblang.

**Yechilishi:**

$$1 + ctg^2 a = \frac{1}{\sin^2 a} = \sin a = \pm\frac{1}{\sqrt{1 + ctg^2 a}} = \pm\frac{1}{\sqrt{1 + \frac{25}{144}}} = \pm\frac{12}{13};$$

$$540^0 < a < 630^0 \; bo'lganiuchun \; \sin a = -\frac{12}{13}$$

$$\cos a = \pm\sqrt{1 - \sin^2 a} = \pm\sqrt{1 - \frac{144}{169}} = \pm\frac{5}{13}; \; \cos a = -\frac{5}{13};$$

$$\sin\frac{a}{2} = \pm\sqrt{\frac{1 - \cos a}{2}} = \pm\sqrt{\frac{1 + \frac{5}{13}}{2}} = \pm\sqrt{\frac{18}{26}} = \pm\sqrt{\frac{9}{13}};$$

$$540^0 < a < 630^0$$

$$\frac{540^0}{2} < \frac{a}{2} < \frac{630^0}{2}; \quad 270^0 < \frac{a}{2} < 315^0 \; \sin a = -\frac{3}{\sqrt{13}}; \; Javobi E$$

**221 – misol.** (02-12-37) $tg105^0$ ning qiymatini hisoblang.

**Yechilishi:** $tg(90^0 + 15^0) = -ctg15^0;\ ctg15^0 = \pm\sqrt{\frac{1+\cos 2\cdot 15^0}{1-\cos 2\cdot 15^0}} =$

$= \pm\sqrt{\frac{1+\cos 30^0}{1-\cos 30^0}} = \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}} = \pm\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \pm\sqrt{\frac{(2+\sqrt{3})^2}{4-3}} =$

$= \pm(2+\sqrt{3}) = 2+\sqrt{3};\ -ctg15^0 = 2+\sqrt{3} \Rightarrow ctg15^0 = -\sqrt{3}-2;$
Javobi : D

**222 – misol**. (01-2-85 ) $\cos 2227^0 30'$ ni hisoblang.

**Yechilishi:**

$\cos a = \sqrt{\frac{1+\cos 2a}{2}}$ formuladan $\cos a = \sqrt{\frac{1+\cos 4455^0}{2}};$

$\cos 4455^0 = \cos(12\cdot 360^0 + 135^0) = = \cos 135^0 = \cos(90^0 +$

$45^0) = -\sin 45^0;\ \cos a = \sqrt{\frac{1-\sin 45^0}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2};$ Javobi : E

**223 – misol.** (01-11-19 ) $\sin 195^0$ ning qiymatini hisoblang.

**Yechilishi:**

$\sin a = \sqrt{\frac{1-\cos 2a}{2}}$ formuladan $\sin 195^0 = \pm\sqrt{\frac{1-\cos 390}{2}} =$

$= \pm\sqrt{\frac{1-\cos(360^0+30)}{2}} = \pm\sqrt{\frac{1-\cos 30}{2}} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{2}};\quad \sin 195^0 =$

$= -\sqrt{\frac{2-\sqrt{3}}{2}}$ ; Javobi : E

**224 – misol** (98 – 1 – 57 ) $8\sin^2\frac{15\pi}{16}\cdot\cos^2\frac{17\pi}{16} - 1$ ni hisoblang.

**Yechilishi:** $\sin\frac{15\pi}{16} = \sin\left(\pi - \frac{\pi}{16}\right) = \sin\frac{\pi}{16};$

$\cos\frac{17\pi}{16} = \cos\left(\pi + \frac{\pi}{16}\right) = -\cos\frac{\pi}{16};$ bundan,

$8\sin^2\frac{15\pi}{16}\cdot\cos^2\frac{17\pi}{16} = 8\cdot\sin^2\frac{\pi}{16}\cos^2\frac{\pi}{16} = 2\cdot(2\sin\frac{\pi}{16}\cdot\cos\frac{\pi}{16})^2 =$

$= 2\cdot\sin^2\frac{\pi}{8};\ \sin^2\frac{15\pi}{16}\cdot\cos^2\frac{17\pi}{16} - 1 = 2\sin^2\frac{\pi}{8} - 1;$

yoki $2 \cdot \frac{1-\cos\frac{\pi}{4}}{2} - 1 = 1 - \cos\frac{\pi}{4} - 1 = -\frac{\sqrt{2}}{2}$; Javobi : A

**225– misol**. (99-8-76 ) $\frac{\sin^2 2{,}5a - \sin^2 1{,}5a}{\sin 4a \sin a + \cos 3a \cos 2a}$ ni soddalashtiring.

**Yechilishi:** Suratini soddalashtiramiz.

$$\sin^2 2{,}5a = \frac{1-\cos 5a}{2};\ \sin^2 1{,}5a = \frac{1-\cos 3a}{2};$$

$$\sin^2 2{,}5a - \sin^2 1{,}5a = \frac{1-\cos 5a}{2} - \frac{1-\cos 3a}{2} =$$

$$-\frac{1-\cos 5a - 1 + \cos 3a}{2} = \frac{1}{2}(\cos 3a - \cos 5a);$$

$$\cos 3a - \cos 5a = -2\sin\frac{3a+5a}{2} \cdot \sin\frac{3a-5a}{2} = 2\sin 4a \cdot \sin a$$

$$Surati: \frac{1}{2} \cdot 2\sin 4a \cdot \cos a = \sin 4a \cdot \cos a$$

Maxrajini soddalashtiramiz. $\sin 4a \cdot \sin a = \frac{1}{2}(\cos 3a - \cos 5a)$,

$$\cos 3a \cdot \cos 2a = \frac{1}{2}(\cos a + \cos 5a), bundan,$$

$$\sin 4a \cdot \sin a + \cos 3a \cdot \cos 2a =$$

$$= \frac{1}{2}\cos 3a - \frac{1}{2}\cos 5a + \frac{1}{2}\cos a + \frac{1}{2}\cos 5a = \frac{1}{2}(\cos 3a + \cos a);$$

$\cos 3a + \cos a = 2\cos 2a \cdot \cos a$; $Maxraji: \frac{1}{2} \cdot 2\cos 2a \cdot \cos a = = \cos 2a \cdot \cos a$; $Demak: \frac{\sin^2 2{,}5a - \sin^2 1{,}5a}{\sin 4a \sin a + \cos 3a \cos a} = \frac{\sin 4a \cdot \sin a}{\cos 2a \cdot \cos a} =$

$= \frac{2\sin 2a \cdot \cos 2a \sin a}{\cos 2a \cdot \cos a} = = \frac{4\sin a \cdot \cos a \sin a}{\cos a} = 4\sin^2 a$; Javobi : E

**226 – misol**. (00-6-53 ) Soddalashtiring. $\frac{4\cos^2 a - 4\cos^2 a + 3\sin^2 a}{4\cos^2\left(\frac{5\pi}{2} - a\right) - \sin^2 2(a-\pi)}$.

**Yechilishi:** Maxraji: $\cos(450^0 - a) = \cos\left(360^0 + (90^0 - a)\right) = \cos(90^0 - -a) = \sin a$; $\sin 2(\alpha - \pi) = \sin(2\alpha - 2\pi) = \sin 2\alpha$; $4\sin^2 a - \sin^2 2a = 4\sin^2 a - -(2\sin a \cos a)^2 = 4\sin^2 a - 4\sin^2 a \cos^2 a = 4\sin^2 a(1 - \cos^2 a) = 4\sin^4 a$; $Surati\ 4\cos^2 2a - \cos^2 a - 3\cos^2 a + +3\sin^2 a = = 4\cos^2 2a - \cos^2 a - 3(\cos^2 a - \sin^2 a) =$

$$= 4\cos^2 a - \cos^2 a - 3\cos 2a = 4\cos^2 2a - \frac{1+\cos 2a}{2} - 3\cos 2a =$$

$$= \frac{8\cos^2 2a - 1 - \cos 2a - 6\cos 2a}{2} =$$

$$= \frac{1}{2}(8\cos^2 2a - 7\cos 2a - 1);\ \cos 2a = x;$$

$8x^2 - 7x - 1 = 0; D = 49 + 32 = 9^2; x_1 = \frac{7+9}{16} = 1; x_2 = \frac{7-9}{16} = -\frac{1}{8};$

$\frac{8\cdot\frac{1}{2}(\cos 2a - 1)(\cos 2a + \frac{1}{8})}{4\sin^4 a} = \frac{(\cos 2a - 1)(8\cos 2a + 1)}{8\sin^4 a} = |\cos 2a - 1 = -2\sin^2 a| = \frac{-2\sin^2 a(8\cos 2a + 1)}{8\sin^4 a} = -\frac{8\cos 2a + 1}{4\cdot\frac{1-\cos 2a}{2}} = = \frac{8\cos 2a + 1}{2\cos 2a - 2}$; Javobi : B

**227 – misol**. (03-1-48 ) $tg555^0$ ni hisolang.

**Yechilishi:** $\sin 555^0 = \sqrt{\frac{1-\cos 1110^0}{2}} = \sqrt{\frac{1-\cos 30^0}{2}}; \cos 555^0 = \sqrt{\frac{1+\cos 1110^0}{2}} = \sqrt{\frac{1+\cos 30^0}{2}}; \cos 1110^0 = \cos(3\cdot 360^0 + 30^0) =$

$= \cos 30^0;$ $tg555^0 = \frac{\sin 555^0}{\cos 555^0} = \sqrt{\frac{1-\cos 30^0}{1+\cos 30^0}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} =$

$|2-\sqrt{3}| = 2-\sqrt{3}$; Javobi : C

**228 – misol.** (03-2-27 ) $\sin^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$ ni hisolang.

**Yechilishi:** $\cos\frac{3\pi}{8} = \cos\left(\pi - \frac{5\pi}{8}\right) = -\cos\frac{5\pi}{8}; \cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right) = -\cos\frac{\pi}{8};$ $\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^4\frac{5\pi}{8} + \cos^4\frac{5\pi}{8} = \left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right) - 2\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8} + (\sin^2\frac{5\pi}{8} + \cos^2\frac{5\pi}{8})^2 - 2\sin^2\frac{5\pi}{8}\cdot\cos^2\frac{5\pi}{8} =$

$1 - 2\cdot(\frac{1}{2}\cdot 2\sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8})^2 + 1 - 2(\frac{1}{2}\cdot 2\sin\frac{5\pi}{8}\cdot\cos\frac{5\pi}{8})^2 = 1 - -2\cdot\frac{1}{4}\sin^2\frac{\pi}{4} + 1 - 2\cdot\frac{1}{4}\sin^2\frac{5\pi}{4} = 2 - \frac{1}{2}\cdot(\frac{\sqrt{2}}{2})^2 - \frac{1}{2}(\sin\left(\pi + \frac{\pi}{4}\right))^2 =$

$2 - \frac{1}{4} - \frac{1}{2}\cdot \cdot (\frac{1}{\sqrt{2}})^2 = = 2 - \frac{1}{4} - \frac{1}{4} = 2 - \frac{1}{2} = \frac{3}{2} = 1{,}5;$ Javobi : D

**229 – misol.** (03-2-40 ) $\cos 92^0\cos 73^0 - \sin 92^0\sin 73^0$ ni hisolang.

**Yechilishi:** $\cos(92^0 + 73^0) = \cos 165^0 = \cos(180^0 - 15^0) = -\cos 15^0;$

$\cos 15^0 = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2};$ $Demak: \cos 165^0 = -\frac{\sqrt{2+\sqrt{3}}}{2};$

Javobi : C

**230 – misol**. (03-5-40 ) Agar $\sin a\left(1 - 2sin^2\frac{a}{2}\right) = \frac{1}{3}$ $bo'lsa, \cos(\frac{\pi}{4} - -a) \cdot \sin(\frac{3\pi}{4} - a)$ ni hisoblang.

**Yechilishi:** $1 - 2sin^2 a = \cos 2a$ $formulaga, 1 - 2sin^2\frac{a}{2} = \cos a;$

$sina\left(1 - 2sin^2\frac{a}{2}\right) = sina\cos a = \frac{1}{2}sin 2a = \frac{1}{3};$ $sin 2a = \frac{2}{3}$

$\sin\left(\frac{3\pi}{4} - a\right) = \sin\left[\pi - \left(\frac{\pi}{4} + a\right)\right] = \sin\left(\frac{\pi}{4} + a\right);$

$$\cos(\frac{\pi}{4} - a)\sin\left(\frac{3\pi}{4} - a\right) =$$

$$= \cos\left(\frac{\pi}{4} - a\right)\sin\left(\frac{\pi}{4} + a\right) =$$

$$= \frac{1}{2}\sin 2\left(\frac{\pi}{4} - a\right) = \frac{1}{2}\sin\left(\frac{\pi}{2} - 2a\right) = \frac{1}{2}\cos 2a;$$

$\cos 2a = \sqrt{1 - sin^2 2a} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3};$ $\frac{1}{2}\cos 2a = \frac{\sqrt{5}}{6};$ Javobi : A

**231 – misol**. (03-3-40 ) $2\sin 32^0 \cdot \cos 2^0 + 2sin^2 28^0 + \frac{1}{2}$ ni hisoblang.

**Yechilishi:** $2\sin 32^0 \cdot \cos 2^0 = 2 \cdot \frac{1}{2}[\sin 30^0 + \sin 34^0] = \frac{1}{2} + \sin 34^0;$

$$2sin^2 28 = 2 \cdot \frac{1 - \cos 56^0}{2} = 1 - \cos(90^0 - 34^0) = 1 - \sin 34^0;$$

$\frac{1}{2} + \sin 34^0 + 1 - \sin 34^0 + \frac{1}{2} = 2;$ Javobi : D

**232 – misol**. (03-7-1) $\cos 15^0 - \sin 15^0 = \frac{a}{4\cdot\cos 15^0}$ $bo'lsa, a =?$

**Yechilishi:** $a = 4cos^2 15^0 - 4\sin 15^0 \cdot \cos 15^0 = 4 \cdot \frac{1+\cos 30^0}{2} -$

$-2\sin 30^0 = 2\left(1 + \frac{\sqrt{3}}{2}\right) - 2 \cdot \frac{1}{2} = 2 + \sqrt{3} - 1 = 1 + \sqrt{3};$ Javobi : B

**233– misol**.(03-7-35) $\cos 15^0 + \sin 15^0 = \frac{a}{4\cos 15^0};$ $bo'lsa, a =?$

**Yechilishi:** $4cos^2 15^0 + a\sin 15^0 \cdot \cos 15^0 = a;$

$a = 4 \cdot \frac{1+\cos 30^0}{2} + 2\sin 30^0 = 2\left(1 + \frac{\sqrt{3}}{2}\right) + 1 = 3 + \sqrt{3};$ Javobi : B

**234–misol.**(03-7-47)

$\sin\frac{a}{2} = \frac{1}{2}\sqrt{2-\sqrt{3}}$ bo'lsa, $\cos a$ ning qiymatini hisoblang.

**Yechilishi:** $(\sqrt{\frac{1-\cos a}{2}})^2 = (\frac{1}{2}\sqrt{2-\sqrt{3}})^2;$ $\frac{1-\cos a}{2} = \frac{2-\sqrt{3}}{4};$

$$1 - \cos a = \frac{2-\sqrt{3}}{2} \Rightarrow \cos a = 1 - \frac{2-\sqrt{3}}{2}; \cos a =$$

$$= \frac{2-2+\sqrt{3}}{2}; \quad \cos a = \frac{\sqrt{3}}{2};$$ Javobi : E

**235– misol.** (97-9-28) Soddalashtiring. $4ctg30^0 + tg^2 15^0$

**Yechilishi:**

$$tg^2 15 = \frac{1-\cos 30^0}{1+\cos 30^0} = \frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{(2-\sqrt{3})^2}{4-3} =$$

$$= 4 + 4\sqrt{3} + 3 = 7 - 4\sqrt{3};$$

$$4ctg30^0 + tg^2 15^0 = 4\sqrt{3} + 7 - 4\sqrt{3} = 7;$$ Javobi B

**236 – misol**. (97-10-55 ) Hisoblang $\sin\frac{5\pi}{12}$

**Yechilishi:** $\sin\frac{5\pi}{12} = \sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}; \cos\frac{5\pi}{6} = \cos\left(\pi - \right.$

$\left. -\frac{\pi}{6}\right) = -\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2};$ Javobi D

**237 – misol**. (98-11-20) Agar $\cos a = \frac{7}{18}$, $0 < a < \frac{\pi}{2}$ bo'lsa, $6\cos\frac{a}{2}$ ni hisoblang.

Yechish: $\cos a = 2cos^2\frac{a}{2} - 1,$ $2cos^2\frac{a}{2} = 1 + \cos a;$ $2cos^2\frac{a}{2} = 1 + \frac{7}{18} = \frac{25}{18}; cos^2\frac{a}{2} = \frac{25}{36};$ $\cos\frac{a}{2} = \frac{5}{6};$ $6\cos\frac{a}{2} = 6 \cdot \frac{5}{6} = 5.$ Javobi B

## Arcsinus, arccosinus, arctangens va arccotangens funksiylarning ba'zi bir qiymatlarining jadvali.

5-jadval

| X | -1 | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
|---|---|---|---|---|---|---|---|---|---|
| Arcsinx | $-\frac{\pi}{2}$ | $-\frac{\pi}{3}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
| Arccosx | $\pi$ | $\frac{5\pi}{6}$ | $\frac{3\pi}{4}$ | $\frac{2\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{\pi}{3}$ | $\frac{\pi}{4}$ | $\frac{\pi}{6}$ | 0 |

6-jadval

| X | -∞ | $-\sqrt{3}$ | $-1$ | $-\frac{1}{\sqrt{3}}$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ |
|---|---|---|---|---|---|---|---|---|---|
| Arctg x | $-\frac{\pi}{2}$ + 0 | $-\frac{\pi}{3}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ − 0 |
| Arcctgx | -π-0 | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{\pi}{2}$ | $\frac{\pi}{3}$ | $\frac{\pi}{4}$ | $\frac{\pi}{6}$ | -0+0 |

**Teskari trigonometrik funksiyalarning asosiy hossalaridan quyidagi tengliklarga ega bo'lamiz.**

1. $\sin(\arcsin x) = x,\ -1 \le x \le 1;$ 2. $\arcsin(\sin x) = x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2};$
3. $\cos(\arccos x) = x,\ -1 \le x \le 1;$ 4. $\arccos(\cos x) = x,\ 0 \le x \le \pi;$
5. $tg(arctgx) = x,\ \infty \le x \le \infty;$ 6. $arctg(tgx) = x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2};$
7. $arcctg(ctgx) = x, 0 < x < \pi;$ 8. $ctg(arcctgx) = x, 0 < x < \infty;$

**Teskari trigonometrik funksiyalar orasidagi munosabatlar**

1. $\sin(\arccos x) = \pm\sqrt{1-x^2}, |x| \le 1;$
2. $\cos(arcsinx) = \pm\sqrt{1-x^2}, |x| \le 1;$
3. $tg(\arcsin x) = \frac{x}{\sqrt{1-x^2}};\ |x| < 1;$
4. $ctg(\arcsin x) = \frac{\sqrt{1-x^2}}{x}, |x| \le 1, x \ne 0;$
5. $tg(\arccos x) = \frac{\sqrt{1-x^2}}{x}, |x| \le 1, x \ne 0;$
6. $ctg(\arccos x) = \frac{x}{\sqrt{1-x^2}},\ |x| < 1;$
7. $\sin(arcctgx) = \frac{x}{\sqrt{1+x^2}};$
8. $\cos(arctgx) = \frac{1}{\sqrt{1+x^2}};$ 9. $ctg(arctgx) = \frac{1}{x},\ x \ne 0;$
10. $ctg(arcctgx) = x;$ 11. $\sin(arcctgx) = \frac{1}{\sqrt{1+x^2}};$

12. $\cos(arctg\ x) = \frac{x}{\sqrt{1+x^2}}$; 13. $tg(arcctg\ x) = \frac{1}{x}$, $x \neq 0$;

14. $\arcsin x = -\arcsin(-x) = \frac{\pi}{2} - \arccos x = arctg\ \frac{x}{\sqrt{1-x^2}}$;

15. $\arccos x = \pi - \arccos(-x) = \frac{\pi}{2} - \arcsin x = arcctg\ \frac{x}{\sqrt{1-x^2}}$;

16. $arctg\ x = -arctg(-x) = \frac{\pi}{2} - arcctg\ x = \arcsin\frac{x}{\sqrt{1+x^2}}$;

175. $arcctg\ x = \pi - arcctg\ (-x) = \frac{\pi}{2} - arctg\ x = arccos\frac{x}{\sqrt{1+x^2}}$;

**238 – misol**. (97-4-37 ) $m = \arcsin\frac{\sqrt{3}}{2}$, $n = \arccos\left(-\frac{1}{2}\right)$, $p = arctg\ 1$ sonlarni kamaish tartibida joylashtiring.

**Yechilishi:** $m = \arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3} = 60^0; n = \pi - \arccos\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3} = 120^0$, $p = arctg\ 1 = \frac{\pi}{3} = 45^0$; $n > m > p$. Javobi: C

**239–misol**. (97-4-37 ) ma'noga ega ifodalarni ko'rsating.

1) $\arcsin(\log_2 5)$; 2) $\arccos\frac{\pi}{\sqrt{17}}$;

3) $\arccos\frac{a^2+b^2}{a^2+b^2+c^2}$; 4) $\arcsin\frac{a^2+b^2+\sqrt{2}}{a^2+b^2+1}$;

**Yechilishi:**

$\log_2 5 > 2$, bundan $\arcsin(\log_2 5)$ ma'noga ega emas; $\frac{\pi}{\sqrt{17}} \approx \frac{3,17}{4,12} < 1$, $\arccos\frac{\pi}{17}$ ma'noga ega; $\frac{a^2+b^2}{a^2+b^2+c^2} < 1$, $\arccos\frac{a^2+b^2}{a^2+b^2+c^2}$ – ma'noga ega; $\frac{a^2+b^2+\sqrt{2}}{a^2+b^2+1} > 1$, $\arcsin\frac{a^2+b^2+\sqrt{2}}{a^2+b^2+1}$ – ma'noga ega emas.

Javobi:C

**240– misol**. (97-9-97) ma'noga ega ifodalarni ko'rsating.

1) $\lg(\arccos 1)$, 2) $\arcsin(\lg\frac{1}{2})$, 3) $\arccos\left(\frac{a^4+1}{(a^2+1)^2}\right)$, 4) $\arcsin(\sqrt[10]{2})$

**Yechilishi:** 1) $\arccos - = 0, \lg(\arccos 1) -$ ma'noga ega emas.

2) $\lg\frac{1}{2} - \lg 2 < 1, \arcsin(\lg\frac{1}{2}) -$ ma'noga ega;

3) $\frac{a^4+1}{a^4+2a^2+1} < 1, \arccos\left(\frac{a4^{+1}}{(a^2+1)^2}\right) -$ ma'noga ega;

4) $\sqrt[10]{2} > 1, \arcsin\sqrt[10]{2} -$ ma'noga ega emas; Javobi: E

**241–misol.** (97-9-96) Agar $A = \frac{97,6^2 - 2\cdot 97,6\cdot 96,6 + 96,6^2 + 5}{\sin^2 5 + \cos^2 5 + 5}$ bo'lsa, $(\arccos A)^{sin^2 5} + \cos^2 6 + 2\sin 5\cdot\cos 6$ ni hisoblang.

**Yechilishi:** $A = \frac{(97,6-96,6)+5}{6} = 1$; $\arccos = 0$; Demak, javobi: C

**242– misol.** (98-2-22) Hisoblang. $\arccos\left(-\frac{\pi}{2}\right) - arctg\ \frac{1}{\sqrt{3}}$

**Yechilishi:** $arccos\ \left(-\frac{\sqrt{2}}{2}\right) - arctg\frac{1}{\sqrt{3}} = \pi - \frac{\pi}{4} - \frac{\pi}{6} = \frac{12\pi - 5\pi}{12} = \frac{7\pi}{12} =$

$= 105^0$; Javobi: E

**243– misol**. (98-5-47) Hisoblang. $\sin(\arcsin\frac{\sqrt{2}}{2} - \arccos\frac{\sqrt{2}}{2})$

**Yechilishi:** $\arcsin\frac{\sqrt{2}}{2} = \frac{\pi}{4}$; $\arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}$; Javobi: A

**244– misol**. (98-9-20) Hisoblang. $\arccos\left(-\frac{1}{2}\right) - \arcsin(-\frac{\sqrt{2}}{2})$

**Yechilishi:**

$\arccos\left(-\frac{1}{2}\right) = \pi - \arccos\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$; $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$;

$\frac{2\pi}{3} - \left(-\frac{\pi}{4}\right) = \frac{2\pi}{3} + \frac{\pi}{4} = \frac{8\pi+3\pi}{12} = \frac{11\pi}{12}$; Javobi: A

**245– misol**. (99-7-46) Hisoblang. $tg\ (\arcsin\frac{\sqrt{3}}{2} + arctg\sqrt{3})$

**Yechilishi:** $\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$; $arctg\sqrt{3} = \frac{\pi}{3}$; $tg\left(\frac{\pi}{3} + \frac{\pi}{3}\right) = tg\ \frac{2\pi}{3} = -\sqrt{3}$;

Javobi:D

**246– misol**. (01-6-31) Hisoblang. $\sin(2\arccos\frac{\sqrt{3}}{2})$

**Yechilishi:** $\arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$; $\sin\left(2\cdot\frac{\pi}{6}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$; Javobi: C

**247– misol**. (01-9-19) Hisoblang. $\frac{\sin(\pi + \arcsin\frac{\sqrt{3}}{2})}{\cos(0{,}5\pi + \arcsin\frac{1}{2})}$

**Yechilishi:** $\sin(\pi + a) = -\sin a$;

$\sin(\pi + \arcsin\frac{\sqrt{3}}{2}) = -\sin(\arcsin\frac{\sqrt{3}}{2}) = -\frac{\sqrt{3}}{2}$;

$\cos(\frac{\pi}{2} + \arcsin\frac{1}{2}) = -\sin(\arcsin\frac{1}{2}) = -\frac{1}{2}$;

$\frac{\sin(\pi+\arcsin\frac{\sqrt{3}}{2})}{\cos(0{,}5\pi+\arcsin\frac{1}{2})} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$; Javobi: A

**248– misol**. (97-9-30) Soddalashtiring. $arctg(ctg(-3))$

**Yechilishi:** $arctg(ctgx) = x, agar 0 < x < \pi bo'lsa,$
$arcctg(-ctg3) = arcctg(ctg(\pi - 3) = \pi - 3;$ Javobi: E

**249– misol.** (98-4-16) Hisoblabg. $\arccos(\sin\frac{\pi}{8})$

**Yechilishi:** $\arccos(\cos x) = x,\ 0 < x < \pi.$ $\arccos(\cos(\frac{\pi}{2} - \frac{\pi}{8})) = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8};$ $Javobi: E$

**250– misol**. (98-10-104) Hisoblang.

$$arctg\left(tg\left(-\frac{3\pi}{5}\right)\right) + arcctg(ctg\left(-\frac{3\pi}{5}\right))$$

**Yechilishi:** $arcctg(ctgx) = x,\ agar\ 0 < x < \pi.$ arctg(tgx) = x, $-\frac{\pi}{2} \le x \le \frac{\pi}{2};$

$arctg\left(-tg\frac{3\pi}{5}\right) = arctg(tg\left(\pi - \frac{3\pi}{3}\right) = \pi - \frac{3\pi}{5} = \frac{2\pi}{5};$

$arcctg\left(-ctg\frac{3\pi}{5}\right) = arcctg(ctg\left(\pi - \frac{3\pi}{5}\right) = \pi - \frac{3\pi}{5} = \frac{2\pi}{5}; \frac{2\pi}{5} + \frac{2\pi}{5} = \frac{4\pi}{5};$

Javobi: C

**251– misol**. (00-3-54) $arcctg(tg - 37^0)$ necha gradus bo'ladi ?.

**Yechilishi:** $arcctg(tg(-37^0)) = arcctg(-tg37^0),$

$$-tg37^0 = ctg\left(\frac{\pi}{2} + 37^0\right) va\ \ arcctg\ (ctgx) = x,$$

$$-tg37^0 = ctg\left(\frac{\pi}{2} + 37^0\right)\ va\ arcctg\ (ctgx) = x,$$

$$agar 0 < x < \pi$$

$$arcctg(-tg37^0) = arcctg(ctg\frac{\pi}{2} + 37^0)) = 90^0 + 37^0 = 127^0$$

**252– misol**. (99-3-36 ) Hisoblang. $\cos(\arcsin\frac{40}{41} - \arcsin\frac{4}{5})$

**Yechilishi:** $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha \cdot \sin\beta$ formuladan

$\cos(\arcsin\frac{40}{41} - \arcsin\frac{4}{5}) = \cos(\arcsin\frac{40}{41}) \cdot \cos(\arcsin\frac{4}{5}) +$

$\sin(\arcsin\frac{40}{41}) \cdot \sin(\arcsin\frac{4}{5});$

$\cos(\arcsin x) = \sqrt{1-x^2}, 0 < x < 1.$ $\sin(\arcsin x) = x, -1 \le x \le 1$ formulalardan foydalansak, $\cos(\arcsin\frac{40}{41} - \arcsin\frac{4}{5}) = \sqrt{1-(\frac{40}{41})^2} \cdot$

$$\cdot \sqrt{1-(\frac{4}{5})^2} + \frac{40}{41}\cdot\frac{4}{5} = \frac{9}{41}\cdot\frac{3}{5} + \frac{32}{41} = \frac{187}{205};$$ Javobi: E

**253– misol**. (99-3-39) Hisoblang. $arctg\frac{1}{2} + arctg\frac{1}{3}$

**Yechilishi:** $arctg\frac{1}{2} + arctg\frac{1}{3} = x$; belgilash kiritamiz $tg\left(arctg\frac{1}{2} + \right.$

$\left. + arctg\frac{1}{3}\right) = tg\ x$ ;

$tg\ (a+\beta) = \frac{tga+tg\beta}{1-tga\cdot tg\beta}$ formulalardan foydalansak,

$\frac{tg\left(arctg\frac{1}{2}\right)+tg\ (arctg\frac{1}{3})}{1-tg\left(arctg\frac{1}{2}\right)\cdot tg(arctg\frac{1}{3})} = = tgx;$ $tgx = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1;$ $x = \frac{\pi}{4};$

$arctg\ \frac{1}{2} + arctg\frac{1}{3} = \frac{\pi}{4}$; Javobi: E

**254– misol**. (97- 4-63) Hisoblang. $\sin(2\arccos\frac{1}{3})$

**Yechilishi:** $\sin 2a = 2\sin a\cos a$ formulada $a = \arccos\frac{1}{3}$;

$\sin(2\arccos\frac{1}{3}) = 2\sin(\arccos\frac{1}{3})\cdot\cos(\arccos\frac{1}{3}) = \frac{2}{3}\sin(\arccos\frac{1}{3})$;

$\sin(\arccos a) = \sqrt{1-cos^2(\arccos a)}$ formula dan $\sin(\arccos\frac{1}{3}) =$

$= \sqrt{1-\frac{1}{9}} = \frac{\sqrt{8}}{3}$; Demak $\sin 2a = \frac{2}{3}\cdot\frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{9}$; Javobi: C

**255– misol**. (96-7-60) Hisoblang. $\sin(2\arcsin\frac{1}{3})$

**Yechilishi:** $\sin(2\arcsin\frac{1}{3}) = 2\sin(\arcsin\frac{1}{3})\cdot\cos(\arcsin\frac{1}{3}) = 2\cdot\frac{1}{3}\cdot$

$\sqrt{1-\frac{1}{9}} = = \frac{2}{3}\cdot\frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{9}$; Javobi: C

**256– misol**. (00-3-56) Hisoblang. $\cos(2\arcsin\frac{1}{3})$

**Yechilishi:** $\cos 2a = cos^2a - sin^2a$ formulada $a = \arcsin\frac{1}{3}$

$\cos 2a = cos^2(\arcsin\frac{1}{3}) - sin^2(\arcsin\frac{1}{3}) = (\sqrt{1 - sin^2(\arcsin\frac{1}{3})})^2 - \frac{1}{9} = 1 - \frac{1}{9} - \frac{1}{9} = \frac{7}{9}$; Javobi: C

**257– misol**. ( 00-5-44). Hisoblang. $tg(2\arcsin\frac{3}{4})$

**Yechilishi:** $tg2a = \frac{2tga}{1-tg^2a}$ formulada $a = \arcsin\frac{3}{4}$ deb olsak,

$tg(2\arcsin\frac{3}{4}) = \frac{2tg(\arcsin\frac{3}{4})}{1-tg^2(\arcsin\frac{3}{4})}$; $tg(\arcsin\frac{3}{4}) = \frac{\frac{3}{4}}{\sqrt{1-\frac{9}{16}}} = \frac{3}{\sqrt{7}}$,

$tg(\arcsin\frac{3}{4}) = = \frac{2\cdot\frac{3}{\sqrt{7}}}{1-\frac{9}{7}} = \frac{2\cdot 3\cdot\sqrt{7}}{-2} = -3\sqrt{7}$; Javobi: E

**258– misol**. ( 00-6-54). Ifodaning qiymatini toping. $\cos(2\arcsin\frac{2}{5})$

Yechilishi:

$\arcsin\frac{2}{5} =$

deb olsak, $\cos 2a = cos^2a - sin^2a$ formulaga asosan,

$\cos(2\arcsin\frac{2}{5}) = cos^2(\arcsin\frac{2}{5}) - sin^2(\arcsin\frac{2}{5})$;

$\cos(\arcsin\frac{2}{5}) = \sqrt{1-\frac{4}{25}} = \frac{\sqrt{21}}{5}$; $sin^2(\arcsin\frac{2}{5}) = (\frac{2}{5})^2 = \frac{4}{25}$;

Demak, $\cos(\arcsin\frac{2}{5}) = \frac{21}{25} - \frac{4}{25} = \frac{17}{25}$; Javobi: D

**259– misol**. ( 98-9-23). Hisoblang. $\arccos\left(-\frac{\sqrt{2}}{2}\right) - \arcsin(-\frac{\sqrt{3}}{2})$

**Yechilishi:** $\arccos(-a) = \pi - \arccos a$; $\arcsin(-a) = -\arcsin a$;

$\arccos\left(-\frac{\sqrt{2}}{2}\right) - \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \pi - \arccos\frac{\sqrt{2}}{2} + \arcsin\frac{\sqrt{3}}{2} =$

$= \pi - \frac{\pi}{4} + \frac{\pi}{3} = \frac{13\pi}{12}$; Javobi: C

**260– misol.** ( 99-8-68). Hisoblang. $2\arcsin\left(-\frac{1}{2}\right) + \frac{1}{2}\arccos\frac{\sqrt{3}}{2}$

**Yechilishi:** $2\cdot\left(-\frac{\pi}{6}\right) + \frac{1}{2}\cdot\frac{\pi}{6} = -\frac{\pi}{3} + \frac{\pi}{12} = -\frac{\pi}{4}$; Javobi : A

**261– misol**. (00-7-27 ). Hisoblang. $\sin(\frac{\pi}{2} - \arccos\frac{3}{5})$

**Yechilishi:** Ma'lumki, $\sin\left(\frac{\pi}{2} - a\right) = \cos a$ *u holda*,

$\sin(\frac{\pi}{2} - \arccos\frac{3}{5}) = \cos(\arccos\frac{3}{5}) = \frac{5}{3} = 0{,}6$; Javobi : E

**262– misol**. ( 97-12-66). Hisoblang. $\cos(2\arccos\frac{1}{3})$

**Yechilishi**: $\arccos\frac{1}{3} = a$ *deb olamiz U holda*

$\cos(2\arccos\frac{1}{3}) = cos^2(\arccos\frac{1}{3}) - sin^2(\arccos\frac{1}{3})$; $\cos(\arccos\frac{1}{3}) = \frac{1}{3}$,

$\sin(\arccos\frac{1}{3}) = \sqrt{1 - cos^2(\arccos\frac{1}{3}} = \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}$, $sin^2(\arccos\frac{1}{3}) = \frac{8}{9}$

*Demak*: $\cos(2\arccos\frac{1}{3}) = \frac{1}{9} - \frac{8}{9} = -\frac{7}{9}$; Javobi : D

**263– misol**. ( 00-10-37). Hisoblang. $\sin(2arctg0{,}75)$

**Yechilishi:** $arctg0{,}75 = a$ *belgilab olamiz . U holda*,

$\sin(2arctg0{,}75) = 2\sin(arctg0{,}75)\cos(arctg0{,}75)$;

$arctg\ a = \arcsin\frac{a}{\sqrt{1+a^2}} = \arccos\frac{1}{\sqrt{1+a^2}}$ *formuladan foydalansak*,

$$\sin(2arctg0{,}75) = 2\sin(\arcsin\frac{0{,}75}{\sqrt{1+(0{,}75)^2}}) \cdot \cos(\arccos\frac{1}{\sqrt{1+(0{,}75)^2}} =$$

$$= 2 \cdot \frac{0{,}75}{\sqrt{1+\frac{9}{16}}} \cdot \frac{1}{\sqrt{1+\frac{9}{16}}} = \frac{1{,}5 \cdot 16}{25} = \frac{24}{25};\ Javobi B$$

**264– misol.** (98-11-42 ). Hisoblang. $tg(\frac{1}{2}\arcsin\frac{5}{13})$

**Yechilishi:** $\arcsin\frac{5}{13} = a$ *deb belgilaymiz u holda* $\sin a = \frac{5}{13}$;

$tg\frac{a}{2} = \frac{\sin a}{1+\cos a} = \frac{\sin(\arcsin\frac{5}{13})}{1+\cos(\arcsin\frac{5}{13})} = \frac{\frac{5}{\beta}}{1+\frac{12}{13}} = \frac{5}{25} = \frac{1}{5}$; Javobi: D

**265– misol**. ( 98-12-76). Hisoblang. $\sin(\frac{1}{2}\arccos\frac{1}{9})$

**Yechilishi**: $\arccos\frac{1}{9} = a$ *deb belgilasak* , $\sin\frac{a}{2} = \sqrt{\frac{1-\cos a}{2}} =$

$= \sqrt{\frac{1-\cos(\arccos\frac{1}{9})}{2}} = \sqrt{\frac{1-\frac{1}{9}}{2}} = \frac{2}{3}$; Javobi: A

**266 – misol.** ( 01-1-47). Ifodaning qiymatini toping. $arctg3 - \arcsin\frac{\sqrt{5}}{5}$

**Yechilishi:** $arctg3 - \arcsin\frac{\sqrt{5}}{5} = a$; deb belgilaymiz va har ikkala tarafining tangensini topamiz. $tg(arctg3 - \arcsin\frac{\sqrt{5}}{5}) = tga$
Chap tomonidagi ifodani $tg(a-\beta)$ yoyilmadan foydalanib quyidagicha yozamiz.

$tg(arctg3 - \arcsin\frac{\sqrt{5}}{5}) = \frac{tg(arctg3) - tg(\arcsin\frac{\sqrt{5}}{5})}{1 + tg(arctg3)\cdot tg(\arcsin\frac{\sqrt{5}}{5})}$;

$tg(arctg3) = 3$; tg(arcsinx) = $\frac{x}{\sqrt{1-x^2}}$ formuladan $tg(\arcsin\frac{1}{\sqrt{5}}) = \frac{\frac{1}{\sqrt{5}}}{\sqrt{1-\frac{1}{5}}} = \frac{\frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}} = \frac{1}{2}$; $Demak: tg(arctg3 - \arcsin\frac{\sqrt{5}}{5}) = \frac{3-\frac{1}{2}}{1+3\cdot\frac{1}{2}} = \frac{\frac{5}{2}}{\frac{5}{2}} = 1$, $ya'ni$

$tga = 1$; $a = \frac{\pi}{4}$; $shunday$ $qilib$ $arctg3 - \arcsin\frac{\sqrt{5}}{5} = 45^0 = \frac{\pi}{4}$;
Jovobi: E

**267 – misol**. ( 01-5-14). Hisoblang. $arctg\frac{1}{3} + arctg\frac{1}{9} + arctg\frac{7}{19}$

**Yechilishi:** $arctg\frac{1}{3} + arctg\frac{1}{9} = a$; $arctg\frac{7}{19} = \beta$ $deb$ $olib$ $tg(a+\beta)$ $yoyilmadan$ $foydalanamiz$; $tga = tg\left(arctg\frac{1}{3} + arctg\frac{1}{9}\right) = \frac{\frac{1}{3}+\frac{1}{9}}{1-\frac{1}{3}\cdot\frac{1}{9}} = \frac{\frac{4}{9}}{\frac{26}{27}} = \frac{4\cdot 27}{9\cdot 26} = \frac{12}{26} = \frac{6}{13}$; $tg\beta = tg\left(arctg\frac{7}{19}\right) = \frac{7}{19}$;

$U$ $holda$ $tg(a+\beta) = \frac{tga + tg\beta}{1 - tg\,a\cdot tg\beta} = \frac{\frac{6}{13}+\frac{7}{19}}{1-\frac{6}{13}\cdot\frac{7}{19}} = \frac{\frac{205}{247}}{\frac{205}{247}} = 1$; $a+\beta = \frac{\pi}{4}$; $Demak$: $arctg\frac{1}{3} + arctg\frac{1}{9} = \frac{\pi}{4}$; $Javob$: $A$;

**268 – misol**. ( 0-7-41). Ifodaning qiymatini toping. $cos\left(arcctg\left(-\frac{1}{5}\right)\right)$

**Yechilishi**: $\cos(arcctg\ x) = \frac{x}{\sqrt{1+x^2}}$ $formulada$ $foydalanamiz$,

$\cos\left(arcctg\left(-\frac{1}{5}\right)\right) = \frac{-\frac{1}{5}}{\sqrt{1+\frac{1}{25}}} = \frac{-\frac{1}{5}}{\frac{\sqrt{26}}{5}} = -\frac{1}{\sqrt{26}} = -\frac{\sqrt{26}}{26}$; Javobi: C

**269 – misol**. (01-11-20). Hisoblang. $\sin(2\arcsin\frac{4}{5})$

**Yechilishi:** $\arcsin\frac{4}{5} = a$ *desak*, $\sin 2a = 2\sin a \cdot \cos a$,

*ya'ni* $\sin(2\arcsin\frac{4}{5}) = 2\sin(\arcsin\frac{4}{5})\cos(\arcsin\frac{4}{5}) = 2\cdot\frac{4}{5}\cdot$

$\sqrt{1-\frac{16}{25}} = \frac{8}{5}\cdot\ \cdot\frac{3}{5} = \frac{24}{25} = 0{,}96$; Javobi: C

**270–misol**.(01-12-34).Ushbu

$q = \log_2\sqrt{5}; p = tg\left(arcctg\frac{1}{5}\right)$ *va* $k = 2$

sonlarni kamayishi tartibida yozing.

**Yechilishi:** $q = \log_2\sqrt{5} = \log_2 2{,}24 \approx 1{,}1$; $tg(arcctgx) = \frac{1}{x}$ *formulidan foydalanib* $p = tg\left(arcctg\frac{1}{5}\right) = 5; k = 2$; Javobi : C

**271 – misol**. (02-1-54) $\cos(arctg\sqrt{3} + \arccos\frac{\sqrt{3}}{2})$ *ni hisoblang*

**Yechilishi:**

$\cos(\alpha+\beta)$ *ning yoyilmasi dan foydalanib*, $\cos(arctg\sqrt{3} +$

$+\arccos\frac{\sqrt{3}}{2}) = \cos(arctg\sqrt{3})\cdot\cos(\arccos\frac{\sqrt{3}}{2}) - \sin(arctg\sqrt{3}\cdot$

$\cdot\sin(\arccos\frac{\sqrt{3}}{2}) = = \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{1+3}} - \frac{\sqrt{3}}{\sqrt{1+3}}\cdot\sqrt{1-\frac{3}{4}} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2}\cdot\frac{1}{2} = 0;$

Javobi: E

**272–misol**. ((02-2-50). $\frac{\pi}{24}(8x+1) = 2\arccos\left(-\frac{1}{2}\right) +$

$\arcsin\frac{1}{2} - \frac{1}{2}arctg1$ *tenglamani yeching.*

**Yechilishi** : $\frac{\pi}{24}(8x+1) = 2\arccos\left(-\frac{1}{2}\right) + \arcsin\frac{1}{2} - \frac{1}{2}arctg1$,

$\frac{\pi}{24}(8x+1) = 2\left(\pi - \frac{\pi}{3}\right) + \frac{\pi}{6} - \frac{1}{2}\cdot\frac{\pi}{4}$;

$\frac{\pi}{24}(8x+1) = \frac{33\pi}{24}$, *bunda* $8x+1 = = 33$, $8x = 32$; $x = 4$;

Javobi : A

**273 – misol.** (02-4-31) $12\arcsin\frac{-\frac{1}{2}}{\pi}$ *ni hisoblang*

**Yechilishi**: $12\arcsin\frac{-\frac{1}{2}}{\pi} = 12\cdot(-\sin\frac{1}{2})\cdot\frac{1}{\pi} = \frac{12}{\pi}\cdot\left(-\frac{\pi}{6}\right) - 2$; Javobi : B

**274 – misol**. (02-4-32) $tg(arctg\ 3 + arctg\ 7)$ ni hisoblang.
**Yechilishi :** $tg(\alpha+\beta)$ning yoyilmasidan foydalansak,
$tg(arctg3 + arctg7) = \frac{tg(arctg3)+tg(arctg7)}{1-tg(arctg3)tg(arctg7)} = \frac{3+7}{1-3\cdot7} = \frac{10}{-20} =$
$-\frac{1}{2} = -0{,}5$; Javobi: C

**275 – misol**. (02-6-42) $\cos(2\arcsin\frac{3}{5})$ ni hisoblang .
**Yechilishi:** $\arcsin\frac{3}{4} = a$ desak, $\cos 2a = \cos^2(2\arcsin\frac{3}{5}) =$
$\cos^2(\arcsin\frac{3}{5}) - \sin^2(\arcsin\frac{3}{5}) = 1 - \frac{9}{25} - \frac{9}{25} = \frac{7}{25} = 0{,}28$; Javobi : C

**276 – misol**. (02-7-17) $\arcsin\frac{4}{5} + \arccos\frac{1}{\sqrt{50}}$ ni hisoblang
**Yechilishi:** $\arcsin\frac{4}{5} + \arccos\frac{1}{\sqrt{50}} = a$
debbel gilab, har ikkita omoni dan sinus olsak ,
$\sin(arcsin\frac{4}{5} + \arccos\frac{1}{\sqrt{50}}) = \sin a$ ;
$\sin\left(arcsin\frac{4}{5}\right)\cos(\arccos\frac{1}{\sqrt{50}}) + \cos(\arcsin\frac{4}{5})\cdot\sin(\arccos\frac{1}{\sqrt{50}}) =$
$= \frac{4}{5}\cdot\frac{1}{\sqrt{50}} + \sqrt{1-\frac{16}{25}}\cdot\sqrt{1-\frac{1}{50}} = \frac{4}{5\sqrt{50}} + \frac{3}{5}\cdot\frac{7}{\sqrt{50}} = \frac{25}{5\sqrt{50}} = \frac{5}{\sqrt{25}\cdot2} = \frac{1}{2}$;
$\sin a = \frac{1}{2}$ yoki $a = \frac{\pi}{6}$; Javobi : D

**277 – misol**. (02-7-19) $arcctg3 - arctg2$ ni hisoblang
Yechilishi : $arcctg\ 3 - arctg2 = a$;
$tg(arcctgx) = \frac{1}{x}$ ekanligini e'tiborga olib,
$\operatorname{tg}(\alpha-\beta)$ ni ayirmasidan foydalanamiz
$tg(arcctg\ 3 - arctg\ 2) = \frac{tg(arcctg3)-tg(arctg2)}{1+tg(arcctg3)\cdot tg(arctg\ 2)} = \frac{\frac{1}{3}-2}{1+\frac{1}{3}\cdot2} = \frac{-\frac{5}{3}}{\frac{5}{3}} = -1$;
$tga = -1$, $a = -\frac{\pi}{4}$ yoki $arcctg3 - arctg2 = -\frac{\pi}{4}$; Javobi : A

**278 – misol**. (02-7-34) $tg(\pi - \arcsin\frac{3}{5})$ ni hisoblang
**Yechilishi :**

$tg(\pi - \arcsin\frac{3}{5}) = -tg(\arcsin\frac{3}{5})$, $tg(\arcsin x) = \frac{x}{\sqrt{1-x^2}}$, $|x| < 1$

$$-tg(\arcsin\frac{3}{5}) - \frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}} = -\frac{\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}; \quad Javob: A$$

**279 – misol.** (02-7-35) $tg(arctg2 - \arccos\frac{12}{13})$ nihi sobl ang

**Yechilishi :** $tg(\arccos x) = \frac{\sqrt{1-x^2}}{x}$; $|x| < 1$, $tg(arctg2 - \arccos\frac{12}{13}) = \frac{tg(arctg2) - tg(\arccos\frac{12}{13})}{1+tg(arctg2)\cdot tg(\arccos\frac{12}{13})} = \frac{2-\frac{13\cdot\sqrt{144}}{12}}{1+\frac{2\cdot 13\cdot 5}{12}} = \frac{2-\frac{5}{12}}{1+\frac{10}{12}} = \frac{24-5}{22} = \frac{19}{22}$;

Javobi : A

**280 – misol**. (02-11-45) $ctg(\arccos\left(-\frac{1}{3}\right) - \pi)$ nihi sobl ang

**Yechilishi** : $ctg(\pi - \arccos\frac{1}{3} - \pi) = -ctg(\arccos\frac{1}{3}) =$

$= -\frac{\frac{1}{3}}{\sqrt{1-\frac{1}{9}}} = -\frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$; Javobi : E

**281 – misol.** (02-5-36) $arctg\sqrt{2} - arctg\frac{1}{\sqrt{2}}$ nihi sobl ang

**Yechilishi** : $arctg\sqrt{2} - arctg\frac{1}{\sqrt{2}} = a$;

$tg\left(arctg\sqrt{2} - arctg\frac{1}{\sqrt{2}}\right) = tga = \frac{tg(arctg\sqrt{2}) - tg(arctg\frac{1}{\sqrt{2}})}{1+tg(arctg\sqrt{2})\cdot tg(arctg\frac{1}{\sqrt{2}})} =$

$\frac{\sqrt{2}-\frac{1}{\sqrt{2}}}{1+2\cdot\frac{1}{\sqrt{2}}} = \frac{\frac{1}{\sqrt{2}}}{2}$; $tga = \frac{1}{2\sqrt{2}}$; $a = arctg\frac{1}{2\sqrt{2}} = arctg\frac{\sqrt{2}}{4}$; $arctg\sqrt{2} - arctg\frac{1}{\sqrt{2}} = arctg\frac{\sqrt{2}}{4}$; Javobi : A

**282 – misol.** (02-12-39) $\cos(2\arcsin\frac{4}{5})$ nihi sobl ang

**Yechilishi** : $\arcsin\frac{4}{5} = a$ deb olsak ,

$\cos 2a = \cos^2(\arcsin\frac{4}{5}) - \sin^2(\arcsin\frac{4}{5}) = (\sqrt{1-\frac{16}{25}})^2 - \frac{16}{25} = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25} = -0{,}28$; Javobi : A

**283 – misol**. (03-3-44) $\sin\left(arctg\left(-\frac{2}{3}\right)\right)$ ni hisoblang

**Yechilishi :** $\sin\left(arctg\left(-\frac{2}{3}\right)\right) = -\sin\left(arctg\frac{2}{3}\right) =$

$= -\frac{\frac{2}{3}}{\sqrt{1+\frac{4}{9}}} = -\frac{\frac{2}{3}}{\frac{\sqrt{13}}{3}} = -\frac{2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$; Javobi : A

**284 – misol.** (03-4-26) $tg\left(arctg\frac{1}{3} + arctg\frac{1}{9}\right)$ ni hisoblang.

**Yechilishi :**

$tg(\alpha+\beta)$ ning yoyilmasidan foydalansak va $tg(arctgx) = x$, $-\infty < x < \infty$; Ekanligini e'tiborga olsak,

$tg\left(arctg\frac{1}{3} + arctg\frac{1}{9}\right) = \frac{\frac{1}{3}+\frac{1}{9}}{1-\frac{1}{3}\cdot\frac{1}{9}} = \frac{\frac{4}{9}}{\frac{26}{27}} = \frac{4\cdot 27}{9\cdot 26} = \frac{6}{13}$; Javobi : E

**285 – misol**. (03-5-45) $\sin(2arctg3) - \cos(2arctg2)$ ni hisoblang

Yechilishi : $arctg\alpha = a$ debbelgilasak, $\sin 2a = \frac{2tga}{1+tg^2a}$;

$\sin(2arctg3) = \frac{2tg(arctg3)}{1+tg^2(arctg3)} = \frac{2\cdot 3}{1+9} = \frac{6}{10} = \frac{3}{5}$; $arctg2 = x$ desak,

$\cos 2x = \frac{1-tg^2x}{1+tg^2x}$; $\cos(2arctg2) = \frac{1-4}{1+4} = -\frac{3}{4}$. Demak,

$\sin(2arctg3) - \cos(2arctg2) = \frac{3}{5} - \left(-\frac{3}{5}\right) = \frac{3}{5} + \frac{3}{5} = \frac{6}{5} = 1{,}2$; Javobi : A

**286 – misol.** (03-6-66) $\sin(\arcsin\frac{1}{2} + \arccos\frac{1}{2})$ ni hisoblang.

**Yechilishi** : $\sin(\alpha+\beta)$ ning yoyilmasidan foydalansak,

$$\sin(\arcsin\frac{1}{2} + \arccos\frac{1}{2}) = \sin(\arcsin\frac{1}{2})\cdot\cos(\arccos\frac{1}{2}) +$$
$$+ \cos(\arcsin\frac{1}{2})\sin(\arccos\frac{1}{2}) =$$

$= \frac{1}{2}\cdot\frac{1}{2} + \sqrt{1-\frac{1}{4}}\cdot\sqrt{1-\frac{1}{4}} = \frac{1}{4} + \left(1-\frac{1}{4}\right) = 1$; Javobi : C

**287 – misol.** (03-9-35)

$tg(\arcsin\left(-\frac{1}{3}\right) + \frac{\pi}{2}$ ni nggiymatini topiing

**Yechilishi :**

$tg(\arcsin\left(-\frac{1}{3}\right) + \frac{\pi}{2}) = tg(\frac{\pi}{2} - \arcsin\frac{1}{3} = ctg(\arcsin\frac{1}{3}) = \frac{\sqrt{1-\frac{1}{9}}}{\frac{1}{3}} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$;

Javobi : C

**288 – misol**. (03-11-24) $ctg(2\pi - 3\arcsin\frac{\sqrt{2}}{2})$ ni hisoblang

**Yechilishi :** $ctg\left(2\pi - 3\cdot\frac{\pi}{4}\right) = ctg\left(\frac{8\pi - 3\pi}{4}\right) =$

$= ctg\frac{5\pi}{4} = ctg\left(\pi + \frac{\pi}{4}\right) = ctg\frac{\pi}{4} = 1$; Javobi :A

**289 – misol**. (03-11-24) $\sin(300\cdot\arccos(-\frac{\sqrt{2}}{2}))$ ni hisoblang

**Yechilisi :** $\sin(300\cdot(\pi - \arccos\frac{\sqrt{2}}{2})) = \sin\left(300\cdot\left(\pi - \frac{\pi}{4}\right)\right) =$

$\sin\left(300\cdot\frac{3\pi}{4}\right) = \sin 225\pi = \sin(225\cdot 180^0) = \sin(112^0\cdot 360 +$

$+180^0) = \sin 180^0 = 0$; Javobi : E

**290 – misol.** (2009-8-103). Agar $tg(\frac{\pi}{4} + a) = -\frac{41}{19}$ bo'sa, ctga ning qimatini toping.

**Yechilishi** : $tg\left(\frac{\pi}{4} + a\right) = \frac{1+tga}{1-tga} = -\frac{41}{19}$; $19 + 19tga = -41 +$

$41tga$; $(41 - 19)tga = 19 + 41$;

$22tga = 60$; $tga = \frac{60}{22} = \frac{30}{11}$; $ctga = \frac{1}{tga} = \frac{1}{\frac{30}{11}} = \frac{11}{30}$; Javobi : B

Foydalanilgan adabiyotlar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Mundarija

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