O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA
MAXSUS TA'LIM VAZIRLIGI

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TEXNIKA OLIY O'QUV YURTLARIDA OLIY MATEMATIKA KURSIDAN MUSTAQIL SHUG'ULLANISH UCHUN TOPSHIRIQLAR TO'PLAMI

(O'QUV QO'LLANMA)

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Техниka oliy o'quv yurtlarida oliy matematiкa кursidan mustaqil shug'ullanish uchun topshiriqlar to'plami: Oquv qollanma. Sodiqov M., Shamsiyev E.A.-Toshkent, ToshDTU, 2006,-202 bet.

Ushbu o'quv qo'llanma oliy texnika o'quv yurtlarida o'qitiladigan «Oliy matematika» kursi dasturining asosiy qismini o'z ichiga oladi. Har bir bo'lim boshida zarur nazariy ma'lumot batafsil yechimi bilan namunaviy misol va masalalar keltirilgan, so'ng mustaqil yechish uchun 20 ta variant beriladi.

«Oliy matematika» kafedrasi

Abu Rayhon Beruniy nomli Toshkent davlat texnika universitetining ilmiy-uslubiy kengashi qarori asosida chop etildi.

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“OLIY MATEMATIKA” FANINI O’RGANISH BO’YICHA USLUBIY KO’RSATMALAR

Oliy matematika fanini o’rganishga o’tishdan avval talabalarga /3/ kitobning kirish qismini o’qish tavsiya qilinadi, u yerda matematika fanining predmeti va ahamiyati qisqacha bayon etilgan.

1- MAVZU. CHIZIQLI ALGEBRA VA ANALITIK GEOMETRIYA ELEMENTLARI

1. Ikkinchi va uchinchi tartibli determinantlar, ularning asosiy xossalari /I, I bob, 3-, 8- bo’limlar/.
2. n- chi tartibli determinantlar. Determinantlarni hisoblash /I, V bob, 2-bo’lim/.
3. Koordinatalar sistemasi. R^2 va R^3 fazolar. Vektorlar, ular ustida amallar. /I, I bob, 1,2-bo’limlar.
4. Vektorlarning skalyar ko’paytmasi va uning xossalari. Skalyar ko’paytmaning mexanik ma’nosi /I, I bob, 1, 2, 3- bo’limlar/.
5. Vektorlarning vektorli, aralash ko’paytmalari, ularning xossalari va geometrik ma’nolari /I, I bob, 3-, 3-5 bo’limlar, 7-8 bo’limlar/.
6. Matritsalar va ular ustida amallar. Matritsaning rangi. Teskari matritsa. Chiziqli almashtirishlar /I, V bob, 1, 2- bo’limlar XXI bob, 1-7- bo’limlar/.
7. Kvadrat matritsaning xos sonlari va xos vektorlari, uning xarakteristik tenglamasi. Kvadratik formalar va ularni kanonik holga keltirish /2, XXI bob, 2-4- bo’limlar/.
8. Chiziqli tenglamalar sistemasi, ularni matritsalar yordami bilan yechish. Kroneker- Kapeli teoremasi. Kramer qoidasi. Gauss usuli. /I, V bob, 3-5- bo’limlar, 2, XXI bob, 8,9,15- bo’limlar/.
9. To’g’ri chiziq va tekislikning tenglamalari, asosiy masalalar. Fazoviy chiziq tenglamalari /I, II bob, 1-3- bo’limlar/.

10. Ikkinchi tartibli sirtlar. Sirt tenglamasi, aylana, silindrik va konus sirtlar tenglamasi. /I, III bob, 4-bo'lim/.

Asosiy nazariy ma'lumotlar

1. Ushbu

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

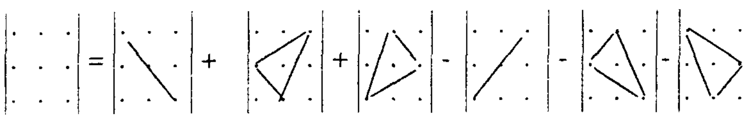
ikkinchi tartibli determinant deyiladi.

Quyidagini

$$\begin{vmatrix} a_{11} a_{12} a_{13} \\ a_{21} a_{22} a_{23} \\ a_{31} a_{32} a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$

$$- a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

uchinchi tartibli determinant deyiladi. Uchburchaklar bo'yicha quyidagi ko'rinishda hisoblanadi:



a_{ij} elementlardan tashkil topgan $n!$ hadlarning yig'indisidan iborat – bo'lgan D songa n-chi tartibli determinant deyiladi:

$$D = \det(a_{ij}) = \begin{vmatrix} a_{11}a_{12}.......a_{1n} \\ a_{21}a_{22}.......a_{2n} \\ .... \\ a_{n1}a_{n2}.......a_{nn} \end{vmatrix}$$

ko'rinishda belgilanadi, a_{ij}- elementda i-yo'l, j- ustunni ko'rsatadi.

Determinantning a_{ij} elementi turgan yo'l va ustun o'chirilganda hosil bo'ladigan (n-1)- tartibli determinantga

minor deyiladi va M_{ij} bilan belgilanadi. Minorni $(-1)^{i+j}$ ga ko'paytirishdan hosil bo'lgan ifoda a_{ij} elementning algebraik to'ldiruvchisi deyiladi va $A_{ij}=(-1)^{i+j}M_{ij}$ ko'rinishda yoziladi.

2. Ikki $\bar{a}$ va $\bar{b}$ vektorlarning skalyar ko'paytmasi deb $\bar{a}\,\bar{b}=\bar{a}\,\bar{b}\cos a\hat{\ }b$ songa aytiladi.

Agar $\bar{a}=\{a_x,a_y,a_z\}$ va $\bar{b}=\{b_x,b_y,b_z\}$ ko'rinishda belgilangan bo'lsa, u holda $\bar{a}\,\bar{b}=a_xb_x+a_yb_y+a_zb_z$ bo'ladi.

$\bar{a}$ va $\bar{b}$ vektorlarning vektorli ko'paytmasi deb, quyidagi shartlarni qanoatlantiruvchi $\bar{c}$ vektorga aytiladi.

1) $\bar{c}$ vektor $\bar{a}$ va $\bar{b}$ vektorlarga perpendikulyar;

2) $|\bar{c}|=|\bar{a}||\bar{b}|\sin(\bar{a}\hat{\ }\bar{b})$;

3) $\bar{a},\bar{b},\bar{c}$ vektorlarning tartiblangan uchligi o'ng uchlikni tashkil etadi.

Vektorli ko'paytma $\bar{a}\,\bar{b}=\bar{c}$ yoki $[\bar{a},\bar{b}]=\bar{c}$ ko'rinishda yoziladi.

Agar, $\bar{a}$ va $\bar{b}$ vektorlar kollinear bo'lsa, u holda $\bar{c}=|\overline{ab}|$ son $\bar{a}$ va $\bar{b}$ vektorlarga yasalgan parallelogrammning yuziga teng.

Agar $\bar{a}=\{a_x,a_y,a_z\}$, $\bar{b}=\{b_x,b_y,b_z\}$ ko'rinishida berilgan bo'lsa, u holda

$$\bar{c}=\bar{a}\,\bar{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$

bo'ladi. $\bar{a},\bar{b},\bar{c}$ vektorlar tartiblangan uchligining aralash ko'paytmasi deb, $\bar{a}\,\bar{b}$ vektor bilan $\bar{c}$ vektorning skalyar ko'paytmasiga teng songa aytiladi va $(\bar{a}\,\bar{b})\bar{c}$ yoki $[\bar{a},\bar{b}]\bar{c}$ deb belgilanadi.

Agar $\bar{a}=\{a_x,a_y,a_z\}$, $\bar{b}=\{b_x,b_y,b_z\}$, $\bar{c}=\{c_x,c_y,c_z\}$ berilgan bo'lsa, u holda

$$[\bar{a}\,\bar{b}]\bar{c}=\begin{vmatrix} a_x a_y a_z \\ b_x b_y b_z \\ c_x c_y c_z \end{vmatrix}$$

bo'ladi.

Agar $\bar{a},\bar{b},\bar{c}$ vektorlar komplanar bo'lmasa, aralash ko'paytmaning mutlaq qiymati shu $\bar{a},\bar{b},\bar{c}$ vektorlarga yasalgan parallelepipedning hajmiga teng, ya'ni

$$V=\left|\left(\overline{a}\overline{b}\right)\overline{c}\right|$$

3. n ta ustun va m ta yo'ldan iborat bo'lgan to'g'ri burchakli jadvalda joylashgan nm ta sonlar to'plami **matritsa** deyiladi va

$$A=\begin{pmatrix} a_{11}a_{12}.....a_{1n} \\ a_{21}a_{22}.....a_{2n} \\ .... \\ a_{m1}a_{m2}.....a_{mn} \end{pmatrix}$$

ko'rinishda bo'ladi.

Agar A- matritsa uchun n=m bo'lsa, n-chi tartibli **кvadrat matritsa** deyiladi.

$$E=\begin{pmatrix}10.....00\\01.....00\\............\\00.....10\\00.....01\end{pmatrix}$$

ko'rinishdagi matritsaga birlik matritsa deyiladi.

A matritsa uchun $AB=E$ tenglikni qanoatlantiruvchi B matritsa A ga teskari matritsa deyiladi. $B=A^{-1}$ deb belgilanadi, $A^{-1}A=AA^{-1}=E$ bo'lishi kerak. Teskari matritsa A^{-1} hisoblanadi,

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix}A_{11} & A_{21} & ... & A_{n1}\\ ... & ... & ... & ...\\ A_{1n} & A_{2n} & ... & A_{nn}\end{pmatrix}$$

bu yerda: $A_{ij}-A$ matritsaning a_{ij} elementining algebraik to'ldiruvchisi, $|A|$- uning determinanti.

Matritsaning rangi deb, matritsaning noldan farqli minorlarining eng yuqori tartibiga aytiladi va $r(A)$ deb yoziladi.

Agar,

$$x_1=a_{11}y_1+a_{12}y_2+....+a_{1n}y_n$$
$$x_2=a_{21}y_1+a_{22}y_2+....+a_{2n}y_n$$
$$...$$
$$x_n=a_{n1}y_1+a_{n2}y_2+....+a_{nn}y_n$$

$x_1,x_2,....,x_n$ o'zgaruvchilarni $y_1,y_2,.....,y_n$ ko'rinishda o'zgaruvchilar orqali ifodalasak, x o'zgaruvchilar y o'zgaruvchilar orqali **chiziqli almashtirilgan** deyiladi. Bu chiziqli almashtirish koeffitsientlardan tuzilgan n- tartibli

$$A = \begin{pmatrix} a_{11}a_{12}.....a_{1n} \\ a_{21}a_{22}.....a_{2n} \\ .... \\ a_{n1}a_{n2}.....a_{nn} \end{pmatrix}$$

kvadrat matritsa bilan to'la xarakterlanadi.

$(A-\lambda E)X = 0$ matritsa ko'rinishidagi tenglamani qanoatlantiruvchi noldan farqli $\bar{x} \neq 0$ bo'lgan shartda λ xos soni topish uchun $\det|A - \lambda E|$ xarakteristik tenglamani hosil qilamiz.

λ_i xos sonlarga mos x_i xos vektorning koordinatalari berilgan tenglamalar sistemasining yechimlari bo'ladi:

$$\begin{cases} (a_{11}-\lambda_i)x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0 \\ a_{21}x_1 + (a_{22}-\lambda_i)x_2 + ... + a_{2n}x_n = 0 \\ .... \\ a_{n1}x_1 + a_{n2}x_2 + ... + (a_{nn}-\lambda_i)x_n = 0 \end{cases}$$

4. Chiziqli tenglamalar sistemasining yechimini 3 noma'lumli 3ta tenglamalar sistemasida ko'raylik. 3 noma'lumli 3ta chiziqli tenglamalar sistemasi quyidagi ko'rinishda bo'ladi:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

bu yerda: a_{ij}-sistema koeffitsientlari, b_i- ozod hadlar. Noma'lumlar oldidagi koeffitsientlardan tuzilgan determinant sistema determinanti va ∇ bilan belgilanadi:

$$\Delta = \begin{vmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{vmatrix}$$

Berilgan sistemani $AX = B$ matritsa ko'rinishida yozish mumkin. Bu yerda:

$$A = \begin{pmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Kengaytirilgan matritsa deb, A matritsaga tenglamalar sistemasining ozod hadlarini qo'shib yozishdan hosil bo'lgan matritsaga aytiladi.

Kroneker – Kapelli teoremasi: Chiziqli tenglamalar sistemasining birgalikda bo'lishi uchun asosiy matritsaning rangi shu sistemaning kengaytirilgan matritsasining rangiga teng bo'lishi uchun zarur va yetarli.

Agar $\Delta \neq 0$ bo'lsa, berilgan sistemaning yagona yechimi Kramer formulalari bilan topiladi.

$x_1 = \frac{\Delta_1}{\Delta}, x_2 = \frac{\Delta_2}{\Delta}, x_3 = \frac{\Delta_3}{\Delta}$, bu yerda:

$$\Delta_1 = \begin{vmatrix} b_1a_{12}a_{13} \\ b_2a_{22}a_{23} \\ b_3a_{32}a_{33} \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_{11}b_1a_{13} \\ a_{21}b_2a_{23} \\ a_{31}b_3a_{33} \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_{11}a_{12}b_1 \\ a_{21}a_{22}b_2 \\ a_{31}a_{32}b_3 \end{vmatrix}$$

Gauss usulini misolda ko'ramiz.

5. To'g'ri chiziq tenglamalari: $Ax + By + C = 0$ umumiy tenglama, bu yerda: A, B, C - o'zgarmas koeffitsientlar.

$y = kx + b$ burchak koeffitsienti tenglama, bu yerda: k - to'g'ri chiziqning burchak koeffitsienti, b - to'g'ri chiziqning boshlang'ich ordinatasi.

$y - y_1 = k(x - x_1)$ -bir nuqtadan o'tuvchi to'g'ri chiziq tenglamasi.

$tg\varphi = \frac{k_2 - k_1}{1 + k_1k_2}$ - ikki to'g'ri chiziq orasidagi burchakni topish formulasi.

$k_1 = k_2$ - to'g'ri chiziqning parallellik sharti.

$k_1 k_2 = -1$ to'g'ri chiziqning perpendikulyarlik sharti.

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$, berilgan ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi.

$x\cos\alpha + y\sin\alpha - p = 0$ to'g'ri chiziqning normal tenglamasi. Bu tenglamada $\sin^2\alpha + \cos^2\alpha = 1$ o'rinli bo'ladi, chunki x va y koordinatalarining koefffitsientlari bitta burchakning sinus va kosinuslari va P perpendikulyarning kesmasi doim musbat bo'ladi:

$-p \leq 0$. Bu ikkala shart to'g'ri chiziqning normal tenglamasini xarakterlovchi belgidir. (P) tekislikning umumiy tenglamasi:

$Ax + By + Cz + D = 0$. Bu tekislikning normal vektori $\vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$, $M_0(x_0, y_0, z_0)$, $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ uch nuqtadan o'tgan tekislikning tenglamasi:

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} = 0$$

Agar (P_1) tekislikning umumiy tenglamasi $A_1x + B_1y + C_1z + D_1 = 0$ $\left(\vec{n}_1 = \{A_1, B_1, C_1\}\right)$, (P_2) tekislikning umumiy tenglamasi $A_2 + B_2y + C_2z + D_2 = 0$ $\left(\vec{n}_2 = \{A_2, B_2, C_2\}\right)$, ko'rinishlarida bo'lsa, u holda (P_1) va (P_2) tekisliklarning orasidagi burchak

$$\cos\varphi = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\cdot|\vec{n_2}|}$$ formula bilan topiladi.

Fazodagi to'g'ri chiziqning vektorli, parametrik va kanonik tenglamalari mos ravishda quyidagicha bo'ladi:

$\vec{r} = \vec{r_0} + \vec{S}t$ bu yerda $\vec{r}\{x,y,z\}$, $\vec{r}_0\{x_0,y_0,z_0\}$, .

$\vec{S} = \{m,n,p,\}$, t - o'zgaruvchi parametr.

$$\left.\begin{aligned} x &= x_0 + mt \\ y &= y_0 + nt \\ z &= z_0 + pt \end{aligned}\right\}, \qquad \frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}.$$

$M_0(x_0,y_0,z_0)$, $M_1(x_1,y_1,z_1)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x-x_0}{x_1-x_0} = \frac{y-y_0}{y_1-y_0} = \frac{z-z_0}{z_1-z_0}$$ ko'rinishida bo'ladi.

6. Aylana, ellips, giperbola va parabolalar ikkinchi tartibli egri chiziqlarga kiradi. Ikkinchi tartibli chiziqning umumiy tenglamasi $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Markazi $C(a,b)$ nuqtada, radiusi R bo'lgan aylananing kanonik tenglamasi:

$(x-a)^2 + (y-b)^2 = R^2$. Ellipsning kanonik tenglamasi:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad e = \frac{c}{a}, \quad c < a, \quad 0 < l < 1$$

ellipsning ekssentrisiteti deyiladi, $c^2 = a^2 - b^2$ bo'ladi. Giperbolaning kanonik tenglamasi: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $y = \pm\frac{b}{a}x$ giperbolaning asimptotalari, giperbolada $c > 0$ va $c^2 - a^2 = b^2$ bo'ladi. Parbolaning kanonik tenglamasi $y^2 = 2px$.

7. $$A_{11}x^2 + A_{22}y^2 + A_{33}z^2 + 2A_{12}xy + 2A_{23}yz +$$
$$+ 2A_{14}x + 2A_{14}y + 2A_{34}z + A_{44} = 0$$

tenglamani qanoatlantiruvchi nuqtalar to'plami ikkinchi tartibli sirt deyiladi. $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ tenglama markazi $C(a,b,c)$ nuqtada bo'lgan, R radiusli sfera tenglamasi deyiladi.

Yuqoridagi tenglamadan aylana, silindrik va konus sirtlarining tenglamalarini keltirib chiqarish mumkin.

O'z–o'zini tekshirish uchun savollar

1. Determinant deb nimaga aytiladi? Determinantlarning asosiy xossalari nimalardan iborat?
2. Determinantlarni hisoblash usulini tushuntiring. Misollar keltiring. Minor va algebraik to'ldiruvchilarni ta'riflang.
3. R^2 va R^3 fazolarni tushuntiring. Vektor deb nimaga aytiladi? Kollinear, komplanar va teng vektorlar deb qanday vektorlarga aytiladi?
4. Ikkita vektorning skalyar ko'paytmasi deb nimaga aytiladi? Ular qanday xossalarga ega?
5. Vektorlarning vektorli ko'paytmasi nima? Ularning xossalari qanday? Aralash ko'paytmani ta'riflang.
6. Matritsa deb nimaga aytiladi? Matritsalar ustida qanday amallar bajariladi?
7. Matritsaning rangi nima? Teskari matritsa qanday topiladi?
8. Kvadrat matritsaning xos son va xos vektorlari, uning xarakteristik tenglamalari qanday topiladi?
9. Kvadratik formalar va ularni kanonik holga keltirishni tushuntiring. Misol keltiring.
10. Chiziqli tenglamalar sistemasi deb nimaga aytiladi?

11. Chiziqli tenglamalar sistemasini matritsalar yordamida yechishni tushuntiring.
12. Kramer formulasi va Gauss usulini tushuntiring.
13. To'g'ri chiziq va tekislikning tenglamalarini yozing.
14. Ikkinchi tartibli egri chiziq tenglamalarini keltiring. Asosiy xossalari nimadan iborat?
15. Fazodagi to'g'ri chiziq tenglamalarini yozing. Sirt tenglamalarini yozing.

2- MAVZU. MATEMATIK TAHLILGA KIRISH

11. Matematik mantiq elementlari. To'g'ri va teskari teoremalar. Nyuton binomi. /IX, 2-bo'lim, I, II boblar; I bob, 13-§/.
12. Haqiqiy sonlar to'plami. Funksiya. Asosiy elementar funksiyalar, ularning xossalari va grafiklari /1-tom, I bob, 12-§, 6-§, 7, 8, 9-§ /, /3. I bob, 3.1-§, 3.8-§/.
13. Sonli ketma – ketliklar, ularning limiti /3, II bob, II, II bob, 1, 2, 3, 7-§ /.
14. Funksiyaning nuqtada va intervalda uzluksizligi, uzluksiz funksiyalar ustida amallar. Uzluksiz funksiyaning kesmadagi asosiy xossalari /2, II bob, 9-11-§lar/.

Asosiy nazariy ma'lumotlar

1. Nyuton binomi deb,

$$(a+b)^n = a^n + c_n^1 a^{n-1} b + c_n^2 a^{n-2} b^2 + c_n^3 a^{n-3} b^3 + \ldots + c_n^{n-1} a b^{n-1} + b^n$$

ifodaga aytiladi. n - butun musbat son. C_n^m - koeffitsientlar binominal koeffitsientlar deyiladi.

2. Funksiyaning nuqtadagi chekli limitining ta'rifi: Agar ixtiyoriy kichik $\varepsilon > 0$ uchun shunday $\delta(\varepsilon) > 0$ topilsaki, unda $|x-a| < \delta$ bo'lganda $|f(x) - A| < \varepsilon$ tengsizlik o'rinli bo'lsa, $x \to a$ da A soni $f(x)$ funksiyaning limiti deyiladi, $\lim\limits_{x\to a} f(x) = A$ yoki $x \to a$ da $f(x) \to A$ deb yoziladi.

Agar $\lim\limits_{x\to 0} f(x) = 0$ (yoki $\lim\limits_{x\to 0} F(x) = \infty$) bo'lsa. $f(x)$ funksiya (yoki $F(x)$) cheksiz kichik (cheksiz katta) deyiladi. $x \to a$ bo'lganda, bir vaqtda 0 ga yoki ∞ ga intiluvchi 2 ta $f(x)$ va $\varphi(x)$ funksiyalar uchun

$$\lim_{x\to a}\frac{f(x)}{\varphi(x)}=1$$

bo'lsa, bu funksiyalar ekvivalent deyiladi, $f(x)\approx\varphi(x)$ ko'rinishida belgilanadi.

3. 1) $y=x^n$ darajali funksiya; 2) $y=a^x$ ko'rsatkichli funksiya; 3) $y=\log_a x$ logarifmik funksiya; 4) $y=\sin x$, $y=\cos x$, $y=tgx$, $y=ctgx$ trigonometrik funksiyalar; 5) $y=\arcsin x$, $y=\arccos x$, $y=arctgx$, $y=arcctgx$ teskari trigonometrik funksiyalar asosiy elementar funksiyalarga kiradi.

Elementar funksiyaning biror nuqtadagi limiti funksiyaning shu nuqtadagi xususiy qiymatiga teng: $\lim_{x\to a} f(x)=f(a)$. $x\to a$ $a=0$ yoki $a\to\infty$ bo'lganda funksiyaning limitini hisoblashda quyidagi aniqmasliklar bo'lishi mumkin: $\infty-\infty$, $\infty\cdot 0$, $0/0$, 1^∞, 0^0, ∞^0, $\frac{\infty}{\infty}$ bu aniqmasliklarni yechishning oddiy usullari:

1) Aniqmaslikka olib keladigan ko'paytuvchilarni qisqartirish;

2) ($x\to\infty$ da ko'phadning nisbati berilsa) surat, maxrajini argumentining eng yuqori darajasiga bo'lish;

3) cheksiz kichik va cheksiz katta ekvivalentlarni qo'llash;

4) ikkita ajoyib limitlardan foydalanish:

$$\lim_{\alpha(x)\to 0}\frac{\sin\alpha(x)}{\alpha(x)}=1 \qquad \lim_{\alpha(x)\to 0}(1+\alpha(x))^{\frac{1}{\alpha(x)}}=e$$

4. Agar

1) $f(x)$ funksiya a nuqtada aniqlangan

2) $\lim_{x\to a} f(x)=f(a)$

3) funksiyaning α nuqtadagi qiymati bilan uning shu nuqtadagi limiti teng bo'lsa, u holda $f(x)$ funksiya D to'plamning nuqtasida uzluksiz deyiladi.

Agar

$$\lim_{x\to a-0} f(x) = f(a-0) \qquad \lim_{x\to a+0} f(x) = f(a+0)$$

bo'lsa, u holda $f(x)$ funksiya. α nuqtada chapdan (o'ngdan) uzluksiz deyiladi.

$f(\alpha-0) = f(\alpha+0) \neq f(\alpha)$ va $f(\alpha-0) \neq f(\alpha+0)$ bo'lib, chekli sondagi sakrashga ega bo'lsa, $f(x)$ funksiya 1- tur uzilishga, $\lim_{x\to\infty} f(x)$ mavjud bo'lmasa, yoki $\lim_{x\to\infty} f(x) = \pm\infty$ bo'lsa, $f(x)$ funksiya 2- tur uzilishga ega deyiladi.

O'z-o'zini tekshirish uchun savollar

1. Matematik mantiq elementlari nima?
2. To'g'ri va teskari teoremalami tushuntiring.
3. Nyuton binomining yoyilmasini yozing.
4. Funksiyaning ta'rifini ayting. Funksiyaning aniqlanish sohasi deb nimaga aytiladi?
5. Funksiya qanday usullarda beriladi? Misollar keltiring
6. Murakkab funksiya deb nimaga aytiladi?
7. Elementar funksiya deb nimaga aytiladi?
8. Funksiyaning limiti tushunchasi bilan o'ng va chap limitlar tushunchasi orasidagi bog'lanishni ayting.
9. Birinchi ajoyib limitni ko'rsating. Ikkinchi ajoyib limitni tushuntiring.
10. Funksiyaning nuqtadagi va kesmadagi uzluksizligini ta'riflang. Funksiyaning uzilish nuqtalari deb nimaga aytiladi?

3- MAVZU. BIR O'ZGARUVCHILI FUNKSIYANING DIFFERENSIAL HISOBI

15. Hosila tushunchasiga olib keluvchi masalalar, hosilaning ta'rifi. Hosilanng fizik va geometrik ma'nolari. Funksiya differensiali
16. Hosilalar topishning asosiy qoidalari.
17. Yuqori tartibli hosila va differensiallar.
18. Roll, Lagranj va Koshi teoremalari. Teylor formulasi.
19. Lopital qoidasi.
20. Funksiyani tekshirish.

Asosiy nazariy ma'lumotlar

1. Harakatning berilgan momentdagi tezligi quyidagicha topiladi;

$$\upsilon = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$$

bu yerda: $s = f(t) - (t$-vaqt) harakat tenglamasi. $y = f(x)$ funksiyaning x bo'yicha hosilasi deb, funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining argument orttirmasi $\Delta x \to 0$ dagi limitiga aytiladi va $y', y'_x, \frac{dy}{dx}$ ko'rinishlarda belgilanadi.

Ta'rifga asosan

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

yoziladi.

2. Lopital qoidasi: 2 ta cheksiz kichik miqdor yoki cheksiz katta miqdor ($\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarni ochish) nisbatining limiti ularning hosilalari nisbatlarining limitiga teng, ya'ni agar

1) $\lim\limits_{x\to a} f(x)=0(\infty)$, $\lim\limits_{x\to a} \vartheta(x)=0(\infty)$

2) $\lim\limits_{x\to a} \frac{f'(x)}{\vartheta'(x)}=A$ bo'lsa, u holda

$\lim\limits_{x\to a} \frac{f(x)}{\vartheta(x)}=\lim\limits_{x\to a} \frac{f'(x)}{\vartheta'(x)}=A$ bo'ladi.

3. Agar x_0 nuqtaning yetarli kichik atrofida $f(x)<f(x_0)$ yoki $f(x)>f(x_0)$ tengsizliklar bajarilsa, $f(x)$ funksiya x_0 nuqtada maksimum yoki minimumga ega bo'ladi. (max yoki min)

Funksiyaning maksimum va minimumlari funksiyaning ekstremumlari deyiladi. Ekstremumning zaruriy sharti agar x_0 nuqta $f(x)$ funksiyaning ekstremum nuqtasi bo'lsa, u holda $f'(x_0)=0$, yoki bu nuqtada hosila mavjud bo'lmaydi. Ekstremumning yetarli sharti: agar $f'(x)x_0$ nuqtaning chap tomonidan o'ng tomoniga o'tishda o'z ishorasini «+» dan «-» ga o'zgartirsa, shu nuqtada funksiya maksimumga ega bo'ladi, agar chapdan o'ngga o'tishda $f'(x)$ ning ishorasi «-» dan «+» ga o'zgarsa, funksiya shu nuqtada minimumga ega bo'ladi.

4. Agar (a,b)ning hamma nuqtalarida $f''(x)<0$ bo'lsa, shu intervalda tenglamasi $y=f(x)$ bo'lgan egri chiziq qavariq bo'ladi, agar (d,c) intervalda $f''(x)>0$ bo'lsa, shu intervalda tenglamasi $y=f(x)$ bo'lgan egri chiziq botiq bo'ladi. Uzluksiz egri chiziqning qavariq qismini botiq qismidan ajratgan nuqta burilish nuqtasi bo'ladi, bu nuqtada $f''(x)=0$ yoki $f''(x_0-\delta)\cdot f''(x_0+\delta)<0$ bo'ladi.

5. $y=kx+b$ to'g'ri chiziq $y=f(x)$ egri chiziqning og'ma asimptotasi deyiladi, bu yerda:

$$k=\lim_{x\to\infty}\frac{f(x)}{x} \qquad b=\lim_{x\to\infty}(f(x)-kx)$$

formulalar bilan hisoblanadi.

$k = 0$ bo'lganda, $y = b$ bo'ladi, bu gorizontal asimptota deyiladi. Agar $\lim_{x \to a-0} f(x) = \infty$ yoki $\lim_{x \to a+0} f(x) = \infty$ bo'lsa, $x = a$ to'g'ri chiziq vertikal asimptota deyiladi.

6. Funksiyalarni tekshirishning umumiy plani va grafiklar yasash.

1) Funksiyaning aniqlanish sohasi topiladi;
2) Funksiyaning uzilish nuqtalari topiladi, juft-toq, davriyligi aniqlanadi;
3) Funksiyaning o'sish va kamayish intervallari topiladi;
4) Kritik nuqtalar va ekstremum qiymatlar topiladi;
5) Qavariqlar, botiqlik sohalari, burilish nuqtasi topiladi;
6) Asimptotalar topiladi ;
7) Topilgan natijalardan foydalanib, funksiyaning grafigi chiziladi.

O'z-o'zini tekshirish uchun savollar

1. Hosila tushunchasiga olib keluvchi qanday masalalarni bilasiz? Hosila deb nimaga aytiladi? Uning geometrik va fizik ma'nolarini tushuntiring
2. Yig'indining, ko'paytmaning hosilalarini topish formulasini yozing, misollar keltiring.
3. Murakkab funksiyani differensiallash formulasini yozing. Misollar keltiring.
4. Trigonometrik va logarifmik funksiyalarning hosilalari qanday topiladi?
5. Yuqori tartibli hosila va differensiallarni ta'riflang.
6. Ikkinchi tartibli hosilaning mexanik ma'nosi nimadan iborat?
7. Roll, Lagranj va Koshi teoremalarini ta'riflang.
8. Funksiya differensiali tushunchasini keltiring.
9. Qanday nuqtalarda funksiyaning differensiali orttirmasiga aynan teng bo'ladi? Funksiya differensialining invariantlik formasi nimadan iborat?

10. Teylor formulasini yozing. Lopital qoidasini tushuntiring.
11. Funksiya ekstremumining zaruriy va yetarli shartlarini ta'riflang.
12. Funksiyaning burilish nuqtasi qanday topiladi?
13. Asimptota deb nimaga aytiladi?

4- MAVZU. KOMPLEKS SONLAR

21. Kompleks sonlarni tekislikda tasvirlash, uning moduli va argumenti, kompleks sonning trigonometrik shakli. Eyler formulasi. Kompleks sonning ko'rsatkichli shakli.
22. Ko'phadlar. Bezu teoremasi. Algebraning asosiy teoremasi.
23. Interpolyatsiya va approksimatsiya haqida tushuncha.

Asosiy nazariy ma'lumotlar

1. $z = x + yi = \rho(\cos\varphi + i\sin\varphi)$ ko'rinishdagi ifoda kompleks son deyiladi (mos ravishda algebraik va trigonometrik ko'rinishlari), bu yerda: $i^2 = -1$, $x = R \in z - z$ kompleks sonning haqiqiy qismi mavhum qismi, $y = Lmz$ - mavhum qismi, ρ va $\varphi - z$ sonning moduli va argumenti: $\rho = |z|, \varphi = \arg z(tg\varphi = \frac{y}{x})$ kompleks sonlar kompleks tekislikdagi nuqtalar bilan tasvirlanadi.

$z = \rho(\cos\varphi + i\sin\varphi)(z \neq 0)$ kompleks sondan n- darajali ildiz chiqarish:

$$\sqrt[n]{z} = \sqrt[n]{\rho}(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n})$$
$$k = 0, 1, 2, ..., n-1$$

Eyler formulasi: $e^{i\varphi} = \cos\varphi + i\sin\varphi$ bu formulaga ko'ra kompleks sonning ko'rsatkichli shakli $z = \rho e^{i\varphi}$ bo'ladi.

2. $f(x) = A_0x^n + A_1x^{n-1} + ... + A_n$ (n- butun musbat son) ko'rinishdagi ifoda ko'phad deyiladi.

n-son ko'phadning darajasi deyiladi. $A_0, A_1, \ldots\ldots, A_n$. koeffitsientlar - haqiqiy va kompleks sonlar: o'zgaruvchi x haqiqiy va kompleks qiymatlar olishi mumkin. O'zgaruvchi x ning ko'phadni 0 ga aylantiradigan qiymati ko'phadning ildizi deyiladi.

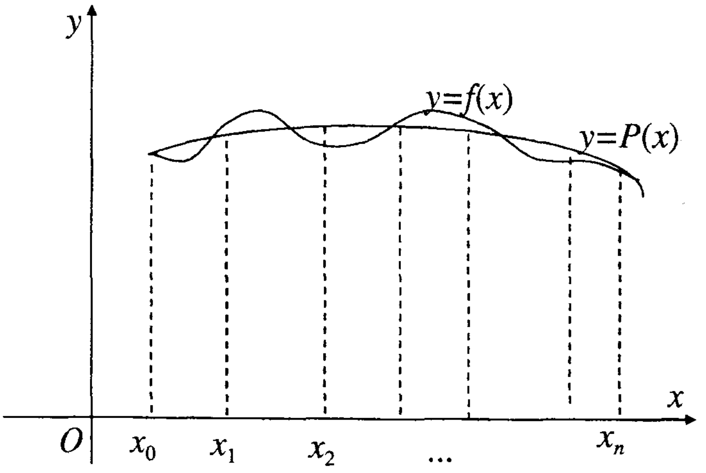
Bezu teoremasi: $f(x)$ ko'phadni $x-a$ ayirmaga bo'lganda $f(a)$ ga teng qoldiq hosil bo'ladi.

3. Biror hodisani o'rganishda x va y miqdorlar orasida shu hodisaning miqdor tomonini ta'riflovchi funksional bog'lanish borligi aniqlangan bo'lsin; bunda $y = f(x)$ funksiya noma'lum bo'lib, lekin tajriba asosida argumentning $[a,b]$ kesmadagi $x_0, x_1, \ldots\ldots, x_n$ qiymatlarida funksiyaning $y_0, y_1, \ldots\ldots, y_n$ qiymatlari aniqlangan bo'lsin.

Masala quyidagicha qo'yiladi: $[a,b]$ kesmada noma'lum $y = f(x)$ funksiyaning $n+1$ ta har xil $x_0, x_1, \ldots\ldots, x_n$ nuqtalardagi $y = f(x), y_1 = f(x_1), \ldots., y_n = f(x_n)$ qiymatlari berilgan, funksiyani taqribiy ifodalovchi, darajasi n dan katta bo'lmagan $P(x)$ ko'phadni topish talab etiladi.

Bunday ko'phad sifatida $x_0, x_1, \ldots\ldots, x_n$ nuqtalaridagi qiymatlari $f(x)$ funksiyaning $f(x), f(x_1), \ldots., f(x_n)$ qiymatlari bilan mos ravishda teng bo'lgan $P(x)$ ko'phad olinadi:

Topilgan $P(x)$ funksiya interpolyatsion formula deyilib, $x_0, x_1, \ldots\ldots, x_n$ lar interpolyatsiya tugunlari deyiladi. Tugunlar orasidagi masofa $h = x_i - x_{i-1}$ interpolyatsiya qadami deyiladi.



3. Approksimatsiya tushunchasi bitta matematik obyektni ma'no jihatidan berilgan obyektga yaqin bo'lgan boshqasi bilan almashtirishdan iborat.

Berilgan masalani oddiyroq yoki xossalari avvaldan ma'lum bo'lgan, hisoblash uchun qulay obyektlarni o'rganishga keltirib, ularning sonli xarakteristikalarini va sifatining xususiyatlarini tekshiradi.

Matematikaning ba'zi bo'limlari approksimatsiya masalasiga bag'ishlangan. Masalan, differensial tenglamalar ayirmali tenglamalarga, sonlar nazariyasida irratsional sonlarni ratsional sonlarga keltirilib o'rganiladi.

O'z-o'zini tekshirish uchun savollar

1. Kompleks sonlarning ta'rifini bering. Kompleks sonlar ustida qanday amallar bajariladi?
2. Kompleks sonlarning trigonometrik shakli, ko'rsatkichli shaklini yozing.
3. Kompleks sondan ildiz chiqarishni ko'rsating.
4. Eyler formulasi deb qanday formulaga aytiladi?

5. Bezu teoremasini ta'riflang.
6. Algebraning asosiy teoremasini ta'riflang.
7. Lagranj interpolyatsion formulasini yozing. Berilgan funksiyani interpolyatsiyalash jarayonining ma'nosi nimadan iborat?
8. Funksiyani interpolyatsiyalash masalasi qanday geometrik ma'noga ega?
9. Approksimatsiya nima?

5- MAVZU. ANIQMAS INTEGRALLAR

24. Boshlang'ich funksiya. Aniqmas integral va uning xossalari /2, X bob, 1-3-§lar/
25. Integrallash usullari. Aniqmas integral jadvallaridan foydalanish/2, X bob, 4-15-§lar/

6- MAVZU. ANIQ INTEGRALLAR

26. Aniq integral tushunchasiga olib keluvchi masalalar. Aniq integral va uning xossalari. O'rta qiymat haqida teorema /2, XI bob, 1-3-§lar/
27. Nyuton - Leybnis formulasi /2, XI bob, 4-6-§lar/
28. Aniq integralni taqribiy hisoblashda to'g'ri to'rtburchak, trapetsiya va Simpson formulalari. Xosmas integrallar, ularning asosiy xossalari /2, XI bob, 7-8-§lar/

Asosiy nazariy ma'lumotlar

I. Agar $F'(x) = f(x)$ bo'lsa, $\int f(x)dx = F(x) + C$ ko'rinishdagi ifodaga $f(x)$ funksiyadan olingan aniqmas integral deyiladi. $F(x)$ funksiya berilgan $f(x)$ funksiyaning boshlang'ich funksiyasi deyiladi. Integrallashda quyidagi usullardan foydalaniladi:

1. Agar $\int f(x)dx = F(x) + C$ bo'lsa, u holda $\int f(ax)dx = \frac{1}{a}F(ax) + C; \int f(x+b)dx = F(x+b) + C$ bo'ladi, bu yerda: a va b ixtiyoriy o'zgarmaslar.
2. Differensial ostiga kirish:

$$\int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d(\varphi(x)) = \int f(u)du,$$ chunki $\varphi'(x)dx = d\varphi(x), [u = \varphi(x)]$

3. Bo'laklab integrallash formulasi $\int udv = uv - \int vdu$

Odatda dv ifoda integrallashda qiyinchilik tug'dirmaydigan qilib tanlab olinadi. Bo'laklab integrallanadigan funksiyalar sinfiga, xususan

$$P(x)\cdot e^{dx}, P(x)\sin ax, P(x)\cos ax, P(x)\ln x,$$

$$P(x)\arcsin x, P(x)arctgx$$

kiradi, bu yerda: $P(x)$- ko'phad.

4. Ratsional kasrlarni integrallash.

$$R(x) = \frac{P_k(x)}{Q_k(x)}$$

Ko'phadlar nisbati berilgan bo'lsa, $R(x)$ elementar kasrlarga ajratiladi.

$$\frac{A}{(x-a)^e}, \qquad \frac{MX+N}{(x^2+px+q)^m}$$

Ko'rinishda bo'lib, bu yerda: e va m - butun musbat sonlar.

5. O'zgaruvchilarni almashtirish usuli bilan integrallash (o'rniga qo'yish usuli), bu usul integrallashning eng samarali usullaridan hisoblanadi. Bu usulning ma'nosi x o'zgaruvchidan t o'zgaruvchiga o'tish: $x = \varphi(t)$ ko'p uchraydigan funksiyalarga standart almashtirishlar ko'rsatish mumkin.

$$\int R(x, \sqrt[n]{\frac{ax+b}{cx+d}})dx, \sqrt[n]{\frac{ax+b}{cx+d}} = t;$$

$$\int R(x, \sqrt{a^2 - x^2})dx, x = a\sin t;$$

$$\int R(x, \sqrt{a^2 + x^2})dx, x = atgt;$$

bu yerda: R - ratsional funksiyaning ramzi.

1. Agar $F'(x) = f(x)$ va $F(x)$ $[a,b]$ kesmada uzluksiz bo'lsa, aniq integralni hisoblashda Nyuton-Leybnis

formulasi $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$ ko'rinishda bo'ladi.

Aniq integral son jihatdan $x = a, x = b, y = 0$ to'g'ri chiziqlar va $y = f(x)$ funksiya grafigining bir qismi bilan chegaralangan trapetsiyaning yuziga teng, agar $f(x) \geq 0$ bo'lsa "+" ishora bilan $f(x) \leq 0$ bo'lsa "–" ishora bilan olinadi.

3. Agar $\lim\limits_{b \to +\infty} \int_a^b f(x)dx$ chekli limit mavjud bo'lsa, bu limit $f(x)$ funksiyaning $[a, \infty]$ dagi xosmas integrali deyiladi va

$$\int_a^{+\infty} f(x)dx = \lim_{b \to +\infty} \int_a^b f(x)dx$$ ko'rinishda yoziladi.

$f(x)$ funksiya $a \leq x < c$ oraliqda aniqlangan va uzluksiz bo'lsin, $x = c$ bo'lganda esa yoki aniqlanmagan yoki uzilishga ega bo'lsin. Bunday holda $\int_a^c f(x)dx$ integralni integral yig'indining limiti deb qarash mumkin bo'lmaydi, chunki $f(x)$ funksiya $[a, c]$ kesmada uzlukli va shuning uchun bu limit mavjud bo'lmasligi ham mumkin.

c nuqtada uzilishga ega bo'lgan $f(x)$ funksiyaning $\int_a^c f(x)dx$ integrali:

$$\int_a^c f(x)dx = \lim_{b \to c-0} \int_a^b f(x)dx$$ ko'rinishda yoziladi.

Agar o'ng tomonda turgan limit mavjud bo'lsa, u holda integral yaqinlashuvchi xosmas integral deyiladi, aks holda integral uzoqlashuvchi deyiladi.

Agar $f(x)$ funksiya $[a,c]$ ning chap uchida uzlukli bo'lsa, u holda ta'rifga ko'ra

$$\int_a^{+\infty} f(x)dx = \lim_{b\to+\infty} \int_a^b f(x)dx .$$

Agar $f(x)$ funksiya $[a,c]$ kesma ichidagi biror $x = x_0$ nuqtada uzlukli bo'lsa, u holda

$$\int_a^c f(x)dx = \int_a^{x_0} f(x)dx + \int_{x_0}^c f(x)dx$$

yoziladi, bunda o'ng tomondagi xosms integrallarning ikkalasi ham mavjud bo'lishi shart.

O'z-o'zini tekshirish uchun savollar

1. Boshlang'ich funksiya deb nimaga aytiladi?
2. Aniqmas integralni tushuntiring, uning asosiy xossalari nimalardan iborat?
3. $\int(2x-1)^2 dx$ integralni ikki xil usul bilan :

a) murakkab argumentli ko'rsatkichli funksiyadan olingan integral ko'rinishda;

b) qavslarni ochib, oddiy integrallash ko'rinishida yeching.

4. Ratsional kasrlarni oddiy kasrlarga yoyib, integrallashni ko'rsating.
5. Irratsional kasrlar qanday integrallanadi?
6. Aniq integralni ta'riflang. Aniq integral tushunchasiga olib keluvchi qanday masalalarni bilasiz?
7. $f(x) \neq 0$ bo'lganda $\int_a^b f(x)dx = 0$ integralning geometrik ma'nosini tushuntiring.

8. $\int_{-a}^{a} f(x^2)dx = 2\int_{0}^{a} f(x^2)dx$ ekanini ko'rsating.

9. Aniq integralning asosiy xossalari nimalardan iborat? O'rta qiymat haqidagi teoremani tushuntiring.

10. Nyuton–Leybnis formulasini yozing va misollar keltiring.

11. Aniq integralni taqribiy hisoblash formulalarini yozing va tushuntiring. Bir-biridan farqi nimada?

12. Xosmas integral deb qanday integralga aytiladi? Uning asosiy xossalari nimalardan iborat?

7- MAVZU. KO'P O'ZGARUVCHI FUNKSIYALAR

29. Ko'p o'zgaruvchi funksiyalar. Ko'p o'zgaruvchi funksiyalarning uzluksizligi /2, VIII bob, 1-4-§lar/
30. Xususiy hosilalar. To'liq differensial. /2, VIII bob, 5-10-§lar/
31. Yuqori tartibli hosilalar va to'liq differensiallar. Teylor formulasi.
32. Ko'p o'zgaruvchili funksiyalarning ekstremumi: ekstremumning zaruriy va yetarli shartlari. /2, VIII bob, 17-§lar/
33. Oshkormas funksiyalar, mavjudlik teoremasi. Oshkormas funksiyalarni differensiallash /2, VIII bob, 11-§lar/
34. Shartli ekstremumlar. Lagranj ko'paytuvchilari usuli, optimal yechimlarni topishda uning qo'llanilishi /2, VIII bob, 18-§lar/

Asosiy nazariy ma'lumotlar

1. $M(x,y)$ nuqta funksiyaning aniqlanish sohasidan chiqmagan holda, $M_0(x_0,y_0)$ nuqta ixtiyoriy usulda intilganda $\lim\limits_{M\to M_0} f(M)=f(M_0)$ yoki $\lim\limits_{\substack{x\to x_0\\ y\to y_0}} f(x,y)=f(x_0,y_0)$ tenglik o'rinli bo'lsa, $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada uzluksiz deyiladi.

2. $\dfrac{\partial z}{\partial x}=\lim\limits_{\Delta x\to 0}\dfrac{\Delta_x z}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x,y)-f(x,y)}{\Delta x}, z=f(x,y)$ funksiyaning x bo'yicha xususiy hosilasi deyiladi va $z'_x, f'_x(x,y), \dfrac{\partial f}{\partial y}$ ko'rinishda belgilanadi. Agar $z=f(x,y)$ funksiya uzluksiz xususiy hosilalarga ega bo'lsa, bu funksiya

(x,y) da nuqta differensiallanuvchi va uning to'liq differensiali:

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy \text{ bo'ladi.}$$

3. $\frac{\partial^n z}{\partial x^p \partial y^{n-p}}$ ifoda n-tartibli xususiy hosilaning umumiy formulasi, bu yerda $f(x,y)=z$ funksiyadan x bo'yicha p marta hosila olingandan keyin, y bo'yicha $(n-p)$ marta hosila olinadi.

Agar x ning uzluksiz funksiyasi $y: F(x,y)=0$ oshkormas ko'rinishda berilgan bo'lsa, uning hosilasi:

$y'_x = -\frac{\partial F/\partial x}{\partial F/\partial y}$ bo'ladi, bu yerda, $\frac{\partial F}{\partial y} \neq 0$ bo'lishi kerak.

O'z-o'zini tekshirish uchun savollar

1. Ikki o'zgaruvchining funksiyasi va uning aniqlanish sohasi deb nimaga aytiladi?
2. Murakkab funksiyaning hosilasini yozing.
3. Xususiy hosila qanday topiladi?
4. $z=f(u,v)$ murakkab funksiya $(u=u(x), v=v(x)$ bo'lganda) dan to'la hosilani hisoblash formulasini yozing.
5. Berilgan $F(x,y)=0$ oshkormas funksiyadan olingan hosila formulasini chiqaring.
6. Yuqori tartibli xususiy hosilani ta'riflang. Ikki o'zgaruvchili funksiyaning aralash xususiy hosilalarining tengligi to'g'risidagi teoremani ta'riflang.
7. Lagranj ko'paytiruvchilari usulini tushuntiring. Bu usul optimal yechimlarni topishda qanday qo'llanadi?

8- MAVZU. ODDIY DIFFERENSIAL TENGLAMALAR.

35. Differensial tenglamaga olib keluvchi masalalar. Oddiy differensial tenglamalar. Koshi masalasi. Birinchi tartibli differensial tenglama yechimining mavjudligi va yagonaligi haqidagi teorema /2, XIII bob, 1-3-§lar, 3-§ning I bo'limi/

36. Birinchi tartibli differensial tenglamalarni yechish. /2, XIII bob, 3-§ning 2-5 bo'limlari/.

37. Yuqori tartibli differensial tenglamalar. /2, XIII bob, 16-18-§lar/.

38. Chiziqli differensial tenglamalar. Bir jinsli tenglamalar. /2, XIII bob, 20,21-§lar/. 42. O'zgarmas koeffitsientli chiziqli differensial tenglamalar, o'ng tomoni maxsus ko'rinishda berilgan differensial tenglamalar. /2, XIII bob, 22-25-§lar/.

Asosiy nazariy ma'lumotlar

1. Erkli o'zgaruvchi x, noma'lum funksiya y va uning hosilalari $y', y'', ..., y^{(n)}$ orasidagi bog'lanishga differensial tenglama deyiladi. Ramziy ko'rinishda $F(y', y'', ..., y^{(n)}) = 0$ deb yoziladi. Birinchi tartibli differensial tenglama $F(y, y, y') = 0$ ko'rinishda bo'ladi. Bu tenglamaning $x = x_0$ bo'lganda $y = y_0$ (yoki $y / x = x_0 = y_0$) boshlang'ich shartni qanoatlantiruvchi xususiy yechimini topish masalasi Koshi masalasi deyiladi.

Berilgan differensial tenglamaning xoy tekislikda chizilgan har qanday $y = \varphi(x)$ yechimining grafigi bu tenglamaning integral egri chizig'i deyiladi.

2. $y' + P(x)y = Q(x)$ ko'rinishdagi tenglama birinchi tartibli chiziqli tenglama deyiladi. Agar $Q(x) = 0$ bo'lsa, bir jinsli, agar $Q(x) \neq 0$ bo'lsa, bir jinslimas tenglama deyiladi.

Bir jinsli tenglamaning umumiy yechimi

o'zagaruvchilarni ajratish yo'li bilan topiladi. Bir jinslimas tenglamaning umumiy yechimi esa bir jinsli tenglamaning umumiy yechimidan ixtiyoriy o'zgarmas c ni variatsiyalash yordamida topiladi yoki $y=uv$, (uv-izlanayotgan funksiyalar) ko'rinishida almashtirish yordamida integrallash mumkin.

3. $y^{(n)}=f(x,y,y',...,y^{(n-1)})$ – n-tartibli tenglama bo'lib, uning $y\big|_{x=x_0}=y_0; y'\big|_{x=x_0}=y'_0;....,y^{(n-1)}\big|_{x=x_0}=y_0^{(n-1)}$ boshlang'ich shartlarni qanoatlantiruvchi $y=\vartheta(x)$ yechimini topish masalasi Koshi masalasi deyiladi. n – tartibli differensial tenglamaning umumiy yechimi $y=\vartheta(x,C_1,C_2,...,C_n)$ ko'rinishida bo'ladi.

$$y^{(n)}+a_1y^{(n-1)}+a_2y^{(n-2)}+....+a_ny=f(x)$$

n – chi tartibli bir jinslimas chiziqli tenglama berilgan bo'lsa, bu tenglamaga mos bir jinsli $y^{(n)}+a_1y^{(n-1)}+....+a_ny=0$ tenglamaning umumiy yechimi $\overline{y}=c_1y_1+c_2y_2+....+c_ny_n$ bo'ladi. U holda bir jinslimas tenglamaning yechimi $y=\overline{y}+y^*$ ko'rinishda olinadi, bu yerda: y^* – bir jinslimas tenglamaning xususiy yechimi.

4. O'zgarmas koeffitsientli ikkinchi tartibli chiziqli differensial tenglama $a_0y''+a_1y'+a_2y=f(x)$ ko'rinishda bo'ladi, bu yerda: a_0,a_1,a_2 – o'zgarmas sonlar, $a_0\neq 0$. Agar $f(x)=0$ bo'lsa, tenglama bir jinsli, $f(x)\neq 0$ bo'lsa, bir jinslimas deyiladi.

5. $a_0k^2+a_1k+a_2=0$ kvadrat tenglama $a_0y''+a_1y''+a_2y=0$ differensial tenglamaning xarakteristik tenglamasi deyiladi. $D=a_1^2-4a_0\cdot a_2$ - kvadrat tenglamaning diskriminanti, D ning ishorasiga qarab, umumiy yechimlar topiladi.

6. Bir jinslimas tenglamaning umumiy yechimi $y=\bar{y}+y^*$ ko'rinishda olinadi, bu yerda; $\bar{y}$- bir jinsli tenglamaning umumiy yechimi, y^*- bir jinslimas tenglamaning xususiy yechimi, $f(x)$- o'ng tomon, har xil ko'rinishda bo'lishi mumkin:

1) Agar $f(x)=b_0x^2+b_1x+b_2$ ko'rinishda bo'lsa, xususiy yechim:

a) $y^*=Ax^2+Bx+C$ (agar 0 xarakteristik tenglamaning yechimi bo'lmasa)

b) $y^*=Ax^3+Bx^2+Cx$ (agar 0 xarakteristik tenglamaning oddiy yechimi bo'lsa);

2) Agar $f(x)=be^{\alpha x}$ bo'lsa, y^* lar tanlab olinadi.

3) Agar $f(x)=e^{\alpha x}(M\cos bx+N\sin bx)$ bo'lsa. y^* yechimlar tanlab olinadi. Bu xususiy yechimlarda A,B,C o'zgarmas aniqmas koeffitsientlar usuli bilan topiladi.

O'z-o'zini tekshirish uchun savollar

1. Birinchi tartibli differensial tenglama va uning umumiy, xususiy yechimlari tushunchasini keltiring. Birinchi tartibli differensial tenglama uchun Koshi masalasini ta'riflang va uning geometrik ma'nosini tushuntiring.
2. Birinchi tartibli differensial tenglama yechimining mavjudligi va yagonaligi to'g'risidagi teoremani tushuntiring.
3. O'zgaruvchilari ajraladigan differensial tenglama deb nimaga aytiladi? Uning umumiy yechimini topish usulini ko'rsating. Misol keltiring.
4. Birinchi tartibli chiziqli differensial tenglama deb qanday tenglamaga aytiladi?
5. Qanday tenglamaga Bernulli tenglamasi deyiladi? Uning umumiy yechimi qanday topiladi?

6. To'liq differensialli tenglamani tushuntiring, umumiy yechimini topishni ko'rsating.
7. Yuqori tartibli differensial tenglama deb qanday tenglamaga aytiladi? Chiziqli, bir jinsli, bir jinslimas differensial tenglamalarni tushuntiring.
8. O'zgarmas koeffitsientli bir jinslimas differensial tenglamaning o'ng tomoniga qarab xususiy yechimlari qanday topiladi?

9- MAVZU. ODDIY DIFFERENSIAL TENGLAMALAR SISTEMASI

39. Differensial tenglamalar normal sistemasi. Avtonom sistema, differensial tenglamalarning normal sistemasi uchun Koshi masalasi. Yechimning mavjudligi va yagonaligi haqida teorema./2, XIII bob, 29-§/
40. Differensial tenglamalar normal sistemasini yechish uchun noma'lumlarni yo'qotish usuli./2, XIII bob, 34-§/
41. Chiziqli differensial tenglamalar sistemasi. O'zgarmas koeffitsientli chiziqli differensial tenglamalar sistemasi. /2, XIII bob, 30-§/

Asosiy nazariy ma'lumotlar

1)

$$\begin{cases} \frac{dy_1}{dx} = f_1(x, y_1, y_2, ..., y_n) \\ \frac{dy_2}{dx} = f_2(x, y_1, y_2, ..., y_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{dy_n}{dx} = f_n(x, y_1, y_2, ..., y_n) \end{cases}$$

ko'rinishdagi sistemaga differensial tenglamalar normal sistemasi deyiladi, bu erda, $y_1, y_2, ..., y_n$ - noma'lum funksiyalar.

O'zgarmas koeffitsientli n-chiziqli differensial tenglamalar sistemasi quyidagi ko'rinishga ega:

$$\begin{cases} \dfrac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ \dfrac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n \end{cases}$$

bu yerda, $a_{ij} - const$.

Bu sistemani matritsa ko'rinishda

$$\frac{dX}{dt} = AX$$

deb yozish mumkin, bu yerda,

$$A = \begin{pmatrix} a_{11}a_{12}\ldots a_{1n} \\ a_{21}a_{22}\ldots a_{2n} \\ \ldots\ldots\ldots\ldots \\ a_{n1}a_{n2}\ldots a_{nn} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}, \frac{\partial X}{\partial t} = \begin{pmatrix} \dfrac{\partial x_2}{\partial t} \\ \ldots \\ \ldots \\ \dfrac{\partial x_n}{\partial t} \end{pmatrix}$$

Sistemaning yechimini $x_1 = P_1e^{\lambda t}; x_2 = P_2e^{\lambda t}, \ldots, x_n = P_ne^{\lambda t}$ ko'rinishda qidiramiz. $x_1, x_2, \ldots, x_n$ qiymatlarni tenglamalar sistemasiga qo'yib, $P_1, P_2, \ldots, P_n$ larda nisbatan chiziqli algebraik tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} (a_{11} - \lambda)P_1 + a_{12}P_2 + \ldots + a_{1n}P_n = 0 \\ a_{21}P_1 + (a_{22} - \lambda)P_2 + \ldots + a_{2n}P_n = 0 \\ a_{n1}P_1 + a_{n2}P_2 + \ldots + (a_{nn} - \lambda)P_n = 0 \end{cases}$$

Bu sistema noldan farqli yechimlarga ega bo'lishi uchun uning determinanti nolga teng bo'lishi kerak:

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{1n} \\ a_{21} & a_{22}-\lambda & a_{2n} \\ \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{nn}-\lambda \end{vmatrix} = 0$$

Bu esa λ ga nisbatan $n-$chi darajali tenglama, uning ildizlari haqiqiy, har xil, kompleks, karrali bo'lishi mumkin.

Xarakteristik tenglamaning ildizlari haqiqiy va har xil bo'lgan hol. $k_1, k_2, ..., k_n$ bilan xarakteristik tenglamaning ildizlari belgilanadi. Har bir k_i ildiz uchun sistemani yozib, $\alpha_1^{(i)}, \alpha_2^{(i)}, ..., \alpha_n^{(i)}$ koeffitsientlar aniqlanadi.

k_i ildiz uchun sistemaning yechimi

$$x_1^{(1)} = \alpha_1^{(1)} e^{k_1 t}, x_2^{(1)} = \alpha_2^{(1)} e^{k_1 t}, ..., x_n^{(1)} = \alpha_n^{(1)} e^{k_1 t},$$

k_2 ildiz uchun

$$x_1^{(2)} = \alpha_1^{(2)} e^{k_1 t}, x_2^{(2)} = \alpha_2^{(2)} e^{k_1 t}, ..., x_n^{(2)} = \alpha_n^{(2)} e^{k_1 t},$$

....

k_n ildiz uchun

$$x_1^{(n)} = \alpha_1^{(n)} e^{k_1 t}, x_2^{(n)} = \alpha_2^{(n)} e^{k_1 t}, ..., x_n^{(n)} = \alpha_n^{(n)} e^{k_1 t}.$$

Bevosita tenglamaga qo'yib:

$$\left.\begin{array}{l} x_1 = c_1 \alpha_1^{(1)} e^{k_1 t} + c_2 \alpha_1^{(2)} e^{k_2 t} + ... + c_n \alpha_1^{(n)} e^{k_n t} \\ x_2 = c_1 \alpha_2^{(1)} e^{k_1 t} + c_2 \alpha_2^{(2)} e^{k_2 t} + ... + c_n \alpha_2^{(n)} e^{k_n t} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_n = c_1 \alpha_n^{(1)} e^{k_1 t} + c_2 \alpha_n^{(2)} e^{k_2 t} + ... + c_n \alpha_n^{(n)} e^{k_n t} \end{array}\right\}$$

sistema hosil qilinadi. Bunda $C_1, C_2, ..., C_n$ ixtiyoriy o'zgarmas miqdorlar bo'lib, bu miqdorlarning shunday qiymatlari topiladiki, bu qiymatlar yechimning berilgan boshlang'ich shartlarini qanoatlantiradi.

Bu berilgan tenglamalar sistemasining umumiy yechimi bo'ladi.

O'z-o'zini tekshirish uchun savollar

1. Birinchi tartibli differensial tenglamalar normal sistemasining umumiy yechimini topish usullarini tushuntiring. Misol keltiring.
2. O'zgarmas koeffitsientli chiziqli differensial tenglamalar sistemasini matritsalar ko'rinishida yozing va yechimlarini topishni misolda tushuntiring.
3. O'zgarmas koeffitsientli chiziqli bir jinsli tenglamalar sistemasini xarakteristik tenglamaning oddiy ildizlarini topish usuli bilan yeching.

10- MAVZU. SONLI VA FUNKSIONAL QATORLAR

42. Sonli qatorlar, qatorlarning yaqinlashishi va uzoqlashishi. Yaqinlashishning zaruriy va yetarli sharti, qator yaqinlashishining asosiy alomatlari. / II-XVI bob, 1-8§lar/
43. Funksional qatorlar Yaqinlashish sohasi, uni aniqlash usullari / II XVI bob, 9-12§lar/
44. Darajali qatorlar. Funksiyalarni darajali qatorga yoyish. Taqribiy hisoblarda darajali qatorni qo'llanilishi. / II XVI bob, 13-28§lar/
45. Trigonometrik qator. Fure qatorlari. Fure funksiyalarini Fure qatoriga yoyish / II XVII bob, 1-11§lar/
46. Kompleks hadli qatorlar. Kompleks shakldagi Fure qatorlar. Spektr. Fure integrali. Fure almashtirishlari / II-XVII bob, 12-16§lar/

Asosiy nazariy ma'lumotlar

1. $U_1+U_2+...+U_n+...=\sum_{n=1}^{\infty}U_n$ sonli qator deyiladi. Agar $S_n=\sum_{k=1}^{n}U_k$ qismiy yig'indining limiti $S=\lim_{n\to\infty}S_n$ mavjud bo'lsa, sonli qator yaqinlashuvchi deyiladi, S-sonli qatorning yig'indisi deyiladi.

Agar limit mavjud bo'lmasa, berilgan sonli qator uzoqlashuvchi deyiladi.

$\lim_{n\to\infty}u_n=0$ sonli qator yaqinlashuvchi bo'lishining zaruriy shartidir. Musbat hadli sonli qatorlar yaqinlashishning yetarli shartlari $(U_{n\geq0})$:

a) Agar $\lim_{n\to\infty}\frac{U_n}{V_n}=k(k\neq0,\infty)$ bo'lsa, u holda $\sum_{n=1}^{\infty}U_n$ va

$\sum_{n=1}^{\infty} V_n$ qatorlar bir vaqtda yaqinlashuvchi va uzoqlashuvchi bo'ladi.

Musbat hadli sonli qatorlar odatda quyidagi qatorlar bilan taqqoslanadi:

$$\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$$

$\alpha > 1$ bo'lganda yaqinlashuvchi va $\alpha \leq 1$ bo'lganda uzoqlashuvchi bo'ladi;

$$\sum_{n=1}^{\infty} q^{n-1}$$

$0 \leq q < 1$ bo'lganda yaqinlashuvchi va $q \geq 1$ uzoqlashuvchi qator bilan;

b) Dalamber alomati:

$\lim\limits_{n\to\infty} \frac{U_{n+1}}{U_n} = q$ bo'lsa, $\sum_{n=1}^{\infty} U_n$ qator $0 \leq q < 1$ bo'lganda yaqinlashuvchi bo'ladi, $q > 1$ uzoqlashuvchi bo'ladi. Agar $q = 1$ bo'lsa, qatorning yaqinlashishini bu alomat bilan aniqlab bo'lmaydi.

Huddi shunga o'xshash, Koshi alomati mavjud. Bundan tashqari integral alomati bilan ham qatorning yaqinlashuvchiligini tekshiriladi.

2. $U_1(x) + U_2(x) + \ldots + U_n(x)$ ko'rinishidagi qatorga funksional qator deyiladi. $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ qator darajali qator deyiladi. a_i-lar qatorning koeffitsientlari. Darajali qatorning yaqinlashish sohasi markazi koordinatalar boshiga bo'lgan intervaldan iborat.

$-R$ dan $+R$ gacha bo'lgan intervalga darajali qatorning yaqinlashish qatori deyiladi, bu interval ichida yotgan har qanday x nuqtada qator yaqinlashadi, shu bilan birga absolyut yaqinlashadi, uning tashqarisidagi x nuqtalarda qator uzoqlashadi, R- darajali qatorning yaqinlashish radiusi deyiladi.

$$R=\lim_{n\to\infty}\left|\frac{U_n}{U_{n+1}}\right|$$

formula bilan hisoblanadi. Darajali qatorlarning yaqinlashish intervalida hadlab differensiallash va integrallash mumkin.

3. 2ℓ davrli $f(x)$ funksiya uchun Fure qatori:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{e}+b_n\sin\frac{n\pi x}{e}$$

bu erda

$$a_n=\frac{1}{e}\int_{-e}^{e}f(x)\cos\frac{n\pi x}{e}dx, n=0,1,2....$$

$$b_n=\frac{1}{e}\int_{-e}^{e}f(x)\sin\frac{n\pi x}{e}dx, n=0,1,2....$$

Juft funksiya uchun Fure qatori:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{e}; b_n=0, a=\frac{2}{e}\int_{0}^{e}f(x)\cos\frac{n\pi x}{e}dx$$

faqat kosinuslarga bog'liq.

Toq funksiyalar uchun Fure qatori:

$$f(x)=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{e}; a_n=0, b=\frac{2}{e}\int_{0}^{e}f(x)\sin\frac{n\pi x}{e}dx$$

faqat sinuslarga bog'liq bo'lib qoladi.

4. $z_1+z_2+......+z_n+....$ qatorga kompleks hadli qator deyiladi, bu yerda: $z_n=x_n+iy_n (n=1,2,3)$. Qator yaqinlashsa, $\lim_{n\to\infty}z_n=0$ bo'ladi.

$$f(x)=c_0+\sum_{n=1}^{\infty}(c_ne^{inx}+c_ne^{-inx})$$

yoki qisqaroq

$$f(x)=\sum_{n=-\infty}^{\infty} c_n e^{inx}$$

ko'rinishdagi qator kompleks shakldagi Fure qatori deyiladi. Bu ifoda

$$\cos nx=\frac{c_n e^{inx}+c_n e^{-inx}}{2} \qquad \sin nx=\frac{c_n e^{inx}+c_n e^{-inx}}{2i}$$

larni Fure qatoriga qo'yib, ixchamlashtirishdan hosil bo'ladi. Koeffitsientlar

$$C_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx;\ C_{-n}=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$$

formulalar orqali hisoblanadi va $f(x)$ funksiya Furening kompleks koeffitsientlari deyiladi.

Agar $f(x)$ 2ℓ davrli davriy funksiya bo'lsa, kompleks shakldagi Fure qatori

$$f(x)=\sum_{n=-\infty}^{\infty} C_n e^{i\frac{-n\pi}{\ell}x}$$

ko'rinishda bo'ladi, koeffitsientlar esa

$$C_n=\frac{1}{2e}\int_{-e}^{e} f(x)e^{i\frac{-n\pi x}{e}}dx (n=0,\pm1,\pm2...)$$

bo'ladi. Bu yerda: $e^{i\frac{-n\pi x}{e}}$ ifodalar garmoniklar deyilib,

$\alpha_n=\frac{n\pi}{e}(n=0\pm,\pm1,\pm2...)$ sonlar $f(x)=\sum_{n=-\infty}^{\infty} C_n e^{i\alpha_n k}$

Funksiyalarning to'lqin sonlari deyiladi. To'lqin sonlar to'plami **spektr** deyiladi.

$$f(x)=\frac{1}{\pi}\int_{0}^{\infty}\left(\int_{-\infty}^{\infty} f(t)\cos\alpha(t-x)dx\right)dx$$

ko'rinishdagi integralga **Fure integrali** deyiladi.

O'z-o'zini tekshirish uchun savollar

1) Qatorlarning yaqinlashuvchi va uzoqlashuvchi bo'lishini ta'riflang. Geometrik progressiyaning hadlaridan tuzilgan qatorning yaqinlashuvchi bo'lishini tekshiring.
2) Qatorlarning yaqinlashuvchi bo'lishi uchun zaruriy shartni ko'rsating.
3) Qatorning oxirgi hadlarini tashlab yuborish qatorning yaqinlashuvchiligiga (uzoqlashuvchiligiga) ta'sir qilmasligini ko'rsating.
4) Dalamber, Koshi, integral alomatlari bilan qatorning yaqinlashuvchi bo'lishini ko'rsating. Misollar keltiring.
5) Qanday qatorlar mutlaq va shartli yaqinlashuvchi qatorlar deyiladi? Absolyut va shartli yaqinlashuvchi qatorlarga misollar keltiring.
6) Funksional qatorlarning yaqinlashish sohasini ta'riflang. Tekis yaqinlashuvchi qator deb qanday qatorga aytiladi?
7) Darajali qatorlarni yaqinlashishida Abel teoremasini ko'rsating. Darajali qatorning yaqinlashish radiusi qanday topiladi?
8) Darajali qatorlarni taqribiy hisoblarda qo'llanilishini ko'rsating.
9) Trigonometrik Fure qatorning koeffitsientlari qanday formulalar bilan hisoblanadi?
10) Juft va toq funksiyalar uchun Fure koeffitsientlarini topish formulalarini yozing.
11) Kompleks formadagi Fure qatorini tushuntiring.
12) Spektr deb nimaga aytiladi?
13) Fure integralini yozing.
14) Fure almashtirishini ta'riflang.

11- MAVZU. OPERATSION HISOB VA UNING TATBIQI

47. Laplas tasviri, uning xossalari. Operatsion hisobning asosiy teoremalari. Tasvirlar jadvali.
48. Tasvir orqali originallarni tiklash usullari. Laplas tasvirining o'ramasi.
49. Differensial tenglamalar va sistemalarni operatsion hisob usullari bilan yechish.

Asosiy nazariy ma'lumotlar

I. Operatsion hisobning asosiy tushunchasi Laplas tasviri hisoblanadi. Bunda $t \geq 0$ da aniqlangan haqiqiy o'zgaruvchining $f(t)$ funksiyasiga

$$F(p) = \int_0^{\infty} e^{-pt} f(t) dt$$

(bu yerda: p- kompleks o'zgaruvchi) $F(p)$ funksiya mos qo'yiladi.

Bu holda $f(t)$ funksiya **original**, $F(p)$ esa tasvir deyiladi, qisqacha $F(p) \to f(t)$ tasvir original $f(t) \leftarrow F(p)$ original- tasvir bilan belgilandi.

Tasvirning mavjudligi va xosmas integrallarning hammasi yaqinlashishi uchun $f(t)$ original quyidagi shartlarni qanoatlantirishi yetarli bo'ladi: 1) istalgan chekli intervalda $f(t)$ va $f'(t)$ chekli sondan ko'p bo'lmagan 1-tur uzilish nuqtalariga ega;

2) $T < 0$ uchun $f(t) \equiv 0$

3) $M > 0$ va $S > 0$ sonlar mavjudki, $|f(t)| \leq Me^{st}$ bo'ladi.

Chiziqlilik xossasi:

$$\sum_{i=1}^{n} c_i f_i(t) \Rightarrow \sum_{i=1}^{n} c_i F_i(P)$$

ya'ni originallarning chiziqli kombinatsiyasiga tasvirlarning chiziqli kombinatsiyasi mos keladi va aksincha.

O'xshashlik teoremasi: agar $a > 0$ va $f(t) \to F(p)$ bo'lsa, u holda

$$f(at) \to \frac{1}{a} F(\frac{P}{a})$$

bo'ladi. Tasvirning siljish teoremasi: $f(t) \to F(p)$ bo'lsa, u holda istalgan P_0 uchun $e^{-p_0 t} f(t) \to F(p + p_0)$ bo'ladi. Originalning kechikish teoremasi: Agar $t_0 > 0$ bo'lsa, u holda $f(t) \to F(p)$ dan

$$f(t - t_0) \to e^{-t_0 p} F(p)$$

kelib chiqadi.

Originalning o'zib ketish teoremasi: Agar $t_0 > 0$ bo'lsa, u holda

$$f(t) \to F(P) \text{ dan } f(t + t_0) \to e^{t_0 p} \left[F(P) - \int_0^{t_0} e^{-pt} f(t) dt \right]$$

kelib chiqadi.

2. Tasvir orqali originallarni tiklash uchun ajratish teoremasini qo'llash kerak. Tasviri to'g'ri ratsional kasr bo'lgan originalni topish uchun ratsional kasrlarni har xil elementar kasrlarning yig'indisi ko'rinishida yozib olinadi va original topiladi.

O'rama haqida teorema (tasvirlarni ko'paytirish teoremasi):

Agar $f_1(t), f_2(t)$ (1-3) shartlarni qanoatlantirsa, ular o'ramasining tasviri ko'paytuvchilar tasvirining ko'paytmasidan iborat bc'ladi, ya'ni

$$f_1(t) * f_2(t) \to F_1(P) F_2(P)$$

bo'ladi.

Teorema. Agar $f_1(t), f_2(t)$ funksiyalar $[0, \infty]$ da uzluksiz hosilalarga ega bo'lsa va $f_1(t) \to F_1(P)$, $f_2(t) \to F_2(P)$ bo'lsa, u holda

$$\frac{d}{dt}[f_1(t) * f_2(t)] \to pF_1(p)F_2(p) \qquad \text{bo'ladi.}$$

3. Operatsion hisob usuli bilan o'zgarmas koeffitsientli chiziqli differensial tenglamalarni va sistemalarni yechish mumkin. Bu usulda originalni o'z ichiga olgan tenglama (yoki sistema)ni tasvirga mos tenglama bilan almashtiriladi, so'ngra tasvirni topish uchun chiziqli algebraik tenglama (yoki sistema)ni yechish kifoya. Keyin esa topilgan tasvirdan foydalanib, originalni tiklash mumkin.

Operatsion usulni qo'llash uchun avvalo tasvirlar jadvalini yaxshi bilib olish kerak va operatsion hisobning asosiy teoremalarini qo'llashni bilish kerak.

O'z-o'zini tekshirish uchun savollar

1. Laplas tasviri qanday ta'riflanadi? Tasvir va original deb nimaga aytiladi?
2. 2 ta har xil uzluksiz funksiyalar bitta tasvirga ega bo'lishi mumkinmi?
3. Agar $F_1(p) \to f_1(t)$ va $F_2(p) \to f_2(t)$, bo'lsa, qaysi tasvir $af_1(t) + bf_2(t)(a, b - const)$ ifodani tasviri bo'ladi.
4. Agar $F(p) \to f(t)$ bo'lsa $e^{-\partial t} f(t)$ ifodaning tasviri qanday bo'ladi? Siljish teoremasini tasvirlang.

5. $\sin at$ va $\cos at$ funksiyalarining tasvirlarini toping.

6. Originallarni differensiallash va integrallash teoremalarini tushuntiring.

7. $\int_0^t f_1(\tau) f_2(t-\tau) d\tau$ ifodaning tasviri qanday bo'ladi? O'rama teoremasini tushuntiring.

8. O'zgarmas koeffitsientli differensial tenglamalar va sistemalarni operatsion usul bilan yechishni ko'rsating. Misol keltiring.

12- MAVZU. KARRALI VA SIRT INTEGRALLAR

50. Karrali, egri va sirt bo'yicha integrallar tushunchasiga olib keluvchi masalalar /2, XIV bob 1, 2-§lar , XV bob, 1-§/
51. Ikki va uch karrali integrallar va ularning xossalari. Karrali integrallarni takroriy integrallash yo'li bilan hisoblash. /2, XIV bob 1-4 §lar , 11-15-§/
52. Sirt yuzasi. Sirt integralining eng sodda xossalari va ularni ikki karrali integralga keltirish yo'li bilan hisoblash. /2, XV bob, 5-7-§lar./

Asosiy nazariy ma'lumotlar

XOY tekislikda *L* yopiq chiziq bilan chegaralangan *D* yopiq sohada $z = f(x,y)$ uzluksiz funksiya berilgan bo'lsa,

$$\lim_{\max dlam\Delta S_i \to 0} \sum_{i=1}^{n} f(p_i)\Delta S_i = \iint_D f(x,y)\alpha x dy$$

ifodaga ikki o'lchovli integral deyiladi.

Agar *D* soha $a \le x \le b,\ f_1(x) \le f_2(x)$ da o'zgarsa, ikki o'lchovli integralni hisoblash:

$$\iint_D f(x,y)dxdy = \int_a^b dx \int_{f_1(x)}^{f_2(x)} f(x,y)dy \qquad (1)$$

ko'rinishidagi ikki karrali integralga keltiriladi.

Agar *D* soha $c \le y \le d,\ \varphi_2(y) \le x \le \varphi_2(y)$ da o'zgarsa,

$$\iint_D f(x,y)dxdy = \int_c^d dy \int_{\varphi_1(y)}^{\varphi_2(y)} f(x,y)dx \qquad (2)$$

ko'rinishiga keltiriladi.

(1) dan (2) ga yoki aksincha o'tish integrallash tartibini

o'zgartirish deyiladi, ikki o'lchovli integralning qiymati integrallash tartibiga bog'liq.

2. D sohada aniqlangan $f(x,y,z)$ funksiyadan olingan uch o'lchovli integralni hisoblash

$$\iiint_V f(x,y,z)dxdydz = \iint_{D_{xy}} dxdy \int_{\varphi_1(x,y)}^{\varphi_2(x,y)} f(x,y,z)dz$$

ko'rinishdagi integralni hisoblashga keltiriladi, bu yerda, D_{xy} xOy tekislikning V sohaga proek-siyasi, $z=\varphi_1(x,y)$ va $z=\varphi_2(x,y)$ lar pastdan, yuqoridan chegaralangan V sohadagi sirtning tenglamasi. Ikki o'lchovli integralga o'xshash uch o'lchovli integralda ham integrallash tartibi o'zgartirilishi mumkin.

3. Birinchi tur egri chiziqli integral $\int_{(e)} f(x,y)dS$, ikkinchi to'g'ri egri chiziqli integral

$\int_{(r)} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz$ berilgan

bo'lib, agar r- egri chiziq $x=x(t)$, $y=y(t)$, $z=z(t)$ $(\alpha \le t \le \beta)$ parametrik holda berilgan, u holda

$$\int_{(r)} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz =$$

$$\int_\alpha^\beta (P(x(t),\ y(t), z(t))\, x`(t) + Q(x(t), y(t), z(t))\, y`(t) + R(x(t),$$
$$y(t), z(t))\, z`(t))dt$$

bo'ladi.

4. Birinchi tur sirt integrali $\iint_{(S)} f(x,y,z)dS$ ikkinchi tur sirt integrali

$$\iiint_{(S)} (P(x,y,z)dxdy + Q(x,y,z)dydz + R(x,y,z)dzdx$$

ko'rinishida bo'lib, sirt integralini hisoblash ikki o'lchovli integralni hisoblashga keltiriladi.

5. Uch o'lchovli integral yordamida:

a) jismning V hajmi va uning massasi M:

$$M = \iiint\limits_V \mu(x,y,z)dxdydz, \quad V = \iiint\limits_V dxdydz$$

bu yerda: μ- massa hajmiy zichligining tarqalishi;

b) bir jinsli jism inersiya momenti (masalan, oz o'qi bo'yicha) $I_z = \iiint\limits_V (x^2 + y^2)dxdydz$ hisoblanadi.

O'z-o'zini tekshirish uchun savollar

1. D soha bo'yicha $f(x,y)$ funksiyadan olingan 2 o'lchovli integral deb nimaga aytiladi? Uning geometrik ma'nosi nimadan iborat?
2. D soha bo'yicha $f(x,y)$ funksiyadan olingan ikki karrali integral deb nimaga aytiladi, qanday hisoblanadi?
3. Ikki o'lchovli integral uchun o'rta qiymat haqidagi teoremani ta'riflang va geometrik ma'nosini ayting.
4. $\iint\limits_D \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2} dxdy$ integralning geometrik ma'nosini tushuntiring, bu yerda, D sohada uzluksiz xususiy hosilalarga ega bo'lgan $z = z(x,y)$ funksiya.
5. Uch o'lchovli integralni uch karrali integral yordamida hisoblash formulasini yozing.
6. Hajmiy zichlik $\mu = \mu(x,y,z)$ bo'lgan V jismning og'irlik markazining koordinatalarini hisoblash formulasini yozing.
7. Koordinatalar bo'yicha egri chiziqli integral deb nimaga aytiladi?
8. Sirt integralini ikki karrali integralga keltirish formulasidan foydalanib hisoblang.

13- MAVZU. EHTIMOLLAR NAZARIYASI VA MATEMATIK STATISTIKA ELEMENTLARI

53. Hodisalar va ularning klassifikatsiyasi. Tasodifiy hodisalar haqida tushuncha. Nisbiy chastota va ehtimol. Ehtimollarning asosiy xossalari. Ehtimollarni hisoblash usullari /2, XX bob, 1-5-§lar; IV bob, 1-4-§lar/.
54. Bernulli sxemasi /2, XX bob, 6-§, IV, V bob, 1-4-§lar/.
55. Diskret va uzluksiz tasodifiy miqdorlar. Taqsimot qonuni. Taqsimot funksiyasi va uning xossalari /2, XX bob, 7-§lar; IV, VI bob, 1-6-§lar/.
56. Katta sonlar qonuni. Limit teoremalar haqida tushuncha /2, IX bob, 1-6-§lar; 5, V bob, VII bob/.
57. Tasodifiy miqdorning sonli xarakteristikalari. Matematik kutilma, dispersiya /2, 20-bob, 9, 10-§lar, IV, VII-VIII boblar/.
58. Korrelyatsiya va regressiya haqida tushuncha. Chiziqli korrelyatsiya, korrelyatsiya koeffitsienti. Tajribada olingan natijalarga ko'ra chiziqli korrelyatsiyani tekshirish. /4, XIV bob, 17, 18-§ lar; V, IV bob, 4-§/.
59. Matematik statistika elementlari. /IV, XV bob, 1-8-§ lar/.

Asosiy nazariy ma'lumotlar

1. Hodisalar uch turga ajratiladi: muqarrar, ro'y bermaydigan va tasodifiy hodisalar.

Hodisalarning klassik ta'rifi: A hodisaning ehtimoli deb, tajribaning bu hodisani ro'y berishiga imkon tug'diruvchi natijalari sonining tajribaning yagona mumkin bo'lgan va teng imkoniyatli elementar natijalari jami soni nisbatiga aytiladi:

$$P(A)=\frac{m}{n}$$

bilan belgilanadi, bu yerda; $m-A$ hodisaning ro'y berishiga qulaylik tug'diruvchi elementar natijalar soni; n- tajribaning mumkin bo'lgan barcha elementar natijalari soni.

2. Hodisaning nisbiy chastotasi; hodisa ro'y bergan natijalar sonining aslida o'tkazilgan jami tajribalar soni nisbatiga aytiladi:

$$W(A)=\frac{M}{N}$$

M - hodisalarning ro'y berish soni;
N - tajribalarning jami soni.

3. $P_n(k)=\frac{n!}{k!(n-k)!}P^k q^{n-k}$ Bernulli formulasi.

4. Diskret tasodifiy miqdor deb, ayrim qiymatlarni ma'lum ehtimollar bilan qabul qiladigan miqdorga aytiladi. Uzluksiz tasodifiy miqdor deb, chekli yoki cheksiz oraliqdagi barcha qiymatlarni qabul qilish mumkin bo'lgan miqdorga aytiladi. Tasodifiy miqdorning qabul qiladigan qiymatlari bilan ularning ehtimollar orasidagi munosabatiga tasodifiy miqdorning taqsimot qonuni deyiladi.

5. $M[x]=x_1p_1+x_2p_2+........+x_np_n$ ifodaga diskret tasodifiy miqdorning matematik kutilishi deyiladi.

$x_1,x_2,......x_n$ tasodifiy miqdorning mumkin bo'lgan qiymatlari.

$P_1,P_2,....P_n$ mos ehtimollari.

$$D(x)=M(x^2)-[M(x)]^2$$

ifodaga dispersiya deyiladi.

6. X tasoddifiy o'z matematik kutilishidan chetlanishi mutlaq qiymati E musbat sondan kichik bo'lish ehtimoli.

$$1-\frac{D[x]}{E^2}$$

dan kichik emas.

$$P(|x-M[x]<E|)\geq 1-\frac{D[x]}{E^2}$$

Bu Chebishev tengsizligi deyiladi.

7. X tasodifiy miqdor ustida tajriba o'tkazish natijasida hosil qilingan $x_1,x_2,....x_n$ miqdorlarga boshqa Y tasodifiy

miqdorning ta'sirini o'rganishga to'g'ri keladi.

Agar X tasodifiy miqdorning har bir qiymatiga biror qonun asosida Y tasodifiy miqdorning aniq qiymati mos kelsa, u holda X va Y orasidagi munosabat statistik yoki korellyatsion munosabat deyiladi. Bu munosabatni jadval ko'rinishida ifodalash mumkin. Korrelyatsion munosabatlar to'g'ri, teskari, to'g'ri chiziqli, egri chiziqli va hokazo bog'lanishlar ko'rinishida bo'lishi mumkin.

Bog'liqlik miqdori – r_{xy} korrelyatsiya koeffitsientini

$$r_{xy} = \frac{\sum x_i y_i - n\overline{x}\overline{y}}{n\ell_x \ell_y}$$

formula yordamida topish mumkin, bu yerda: σ_x, σ_y - X va Y ning o'rtacha kvadratik chetlanishi. X, Y - tenglamaning o'rtacha arifmetigi. Korrelyatsiya koeffitsientlari $r_{xy} r_{xz} r_{yz}$ ko'rinishda belgilanishi mumkin.

8. Statistika tabiatda va jamiyatda bo'ladigan ommaviy hodisalarni o'rganadi. Matematik statistikaning asosiy masalalari quyidagilardan iborat: noma'lum taqsimot funksiyani baholash; noma'lum parametrlarni baholash; statistik gipotezalarni tekshirish.

Tanlanma deb ajratiladigan obyektlar soniga aytiladi.

To'plam hajmi deb to'plamdagi obyektlar soniga aytiladi. Olingan obyekt kuzatish o'tkaziladigandan keyin bosh to'plamga qaytarilsa, takror tanlama qaytarilmasa, notakror tanlanma deyiladi.

Yuqorida keltirilgan masalalarni yechish uchun avvalo dastlabki statistik tekshirishlarni bajarish kerak.

Biror ξ tasodifiy miqdor ustida n marta kuzatish o'tkazib,

$$x_1, x_2, \ldots, x_n \qquad (1)$$

natijalar olingan bo'lsin. Agar $x_1, x_2, \ldots, x_n$ larni $x_1^* \le x_2^* \le \ldots \le x_n^*$ (yoki $x_1^* \ge x_{n-1}^* \ge \cdots x_2^* \ge x_1^*$) kabi joylashtiilrsa, $x_1^*, x_2^*, \dots, x_n^*$ variatsion qator deyiladi.

(1)tanlanmadagi x_i, $i=1,2,...,n$ larni esa variantalar deb yuritiladi.

Variantalarning har biri bir necha marta takrorlanishi mumkin: masalan, x_1^* varianta n_1 marta, x_2^* varianta n_2 marta, $...,x_n^*$ varianta esa n_k marta takrorlansin va $n=n_1+n_2+\dots+n_k$ bo'lsin, $n_1,n_2,\dots,n_k$ sonlar chastotasi deyiladi.

Har bir chastotaning tanlanma hajmi n ga nisbati shu variantaning nisbiy chastotasi deyiladi:

$W_i=\frac{n_i}{n}$, $\quad i=\overline{1,k}$ kabi belgilanadi.

$$x_i: \quad x_1, x_2, \dots, x_k;$$
$$W_i: \quad W_1, W_2, \dots, W_k;$$

jadval ξ tasodifiy miqdorning statistik yoki empirik taqsimoti deyiladi. Nisbiy chastotalar yigindisi

$$W_1+W_2+\dots+W_k=\frac{n_1}{n}+\frac{n_2}{n}+\dots+\frac{n_k}{n}=\frac{n_1+n_2+\dots+n_k}{n}=\frac{n}{n}=1$$

bo'ladi.

Variantalarning x sondan kichik bo'lgan qiymatlarining chastotasi

$$F_n(x)=\frac{m(x)}{n}$$

empirik taqsimot funksiya deyiladi, $m(x)$ x dan kichik bo'lgan variantalar soni, n - tanlanma to'plamining hajmi.

Empirik taqsimotni grafik usulda tasvirlashda poligon va gistogramma muhim rol o'ynaydi.

Chastotalar poligoni deb, (n_1,n_1); (n_2,n_2); ..., (n_k,n_k) nuqtalarni tutashtiruvchi siniq chiziqqa aytiladi. Nisbiy chastotalar poligoni deb esa $(X_1,W_1),(X_2,W_2),\dots,(X_k,W_k)$ nuqtalarni tutashtiruvchi siniq chiziqlarga aytiladi.

Chastotalar gistogrammasi deb, asoslari h uzunlikdagi intervallar, balandliklari esa, n_i dan iborat bo'lgan to'g'ri to'rtburchaklardan iborat pog'onasimon shaklga aytiladi, bu yerda; h- bosh to'plamning kuzatiladigan qiymatlarini o'z ichiga olgan interval to'plamining kuzatiladigan qiymatlarni o'z ichiga olgan interval uzunligi, n_i- intervalga tushgan variantalar soni.

Matematik statistika o'rganadigan masalalardan biri taqsimotning turli sonli xarakteristikalarini baholashdan iborat.

(L) tanlanmaning o'rta arifmetik qiymati

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

bo'ladi, tanlanma dispersiyasi deb,

$$\sigma^2 = \frac{\sum_{i=1}^{n} n_i (x_i - \bar{x})^2}{n}$$

ifodaga aytiladi.

Tanlanma dispersiyasidan musbat ishora bilan olingan kvadrat ildiz

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} n_i (x_i - \bar{x})^2}{n_i}}$$

taqsimotining o'rtacha kvadratik chetlanishi deyiladi.

O'z-o'zini tekshirish uchun savollar

1) Ehtimolning klassik ta'sirini bering. Nisbiy chastota bilan ehtimollik orsidagi farq nimadan iborat?
2) Hodisalar yig'indisi deb nimaga aytiladi? Qo'shish teoremasini isbotlang.

3) Bernulli formulasini ta'riflang. Laplasning lokal va integral teoremasi qanday?
4) To'liq ehtimol formulasini yozing.
5) Beyes formulasini keltirib chiqaring. Puasson taqsimoti qanday?
6) Tasodifiy miqdorning taqsimot finktsiyasi deb nimaga aytiladi? Tekis taqsimlangan tasodifiy miqdor deb nimaga aytiladi?
7. Matematik kutilish deb nimaga aytiladi? Uning xossalarini tushuntiring.
8. Dispersiya va uning xossalarini tushuntiring. O'rta kvadratik chetlanish deb nimaga aytiladi?
9. Katta sonlar qonunini ta'riflang. Katta sonlar qonunining bajarilishi uchun zaruriy va yetarli shartlarni yozing.
10. Limit teoremalar haqida tushuncha bering. Korrelyatsiya koeffitsienti deb nimaga aytiladi?
11. Korrelyatsiya va regressiya tushunchasini bering. Korrelyatsiya koeffitsienti deb nimaga aytiladi?
12. Tanlanma va variatsion qator deb nimaga aytiladi?
13. Poligon va gistogramma nima?

1- NAZORAT ISHI

1- masala.

Quyidagi tenglamalar sistemasini birgalikda ekanligini ko'rsating va uch xil usulda:

1. Gauss usuli;
2. Kramer qoidasi;
3. Matritsa hisobi bilan yeching.

$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ 3x_1 - x_2 + x_3 = 1 \\ 2x_1 - x_2 + 3x_3 = 1 \end{cases}$$

Yechish:

Sistemaning asosiy determinantini hisoblaymiz:

$$|A| = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = -3 + 32 + -2 - 9 + 1 = -8 \neq 0$$

demak sistema birgalikda ekan, chunki $r(A) = r(B) = 3$.

Bu yerda: B sistemaning kengaytirilgan matritsasi. Endi sistemaning yechimini topamiz.

1. Gauss usuli. Yetakchi tenglama sifatida birinchi tenglamani olamiz va uni avval (-3)ga , so'ngra (-2)ga ko'paytirib, mos ravishda ikkinchi va uchinchi tenglamalarga qo'shamiz.

$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ -4x_2 + 4x_3 = -5 \\ -3x_2 + 5x_3 = -3 \end{cases}$$

Endi yetakchi tenglama sifatida ikkinchi tenglamani olamiz. Uni (-4) ga bo'lib, so'ngra 3 ga ko'paytiramiz va uchinchi tenglamaga qo'shamiz:

$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 - x_3 = \dfrac{5}{4} \\ 2x_3 = \dfrac{3}{4} \end{cases}$$

Endi teskari yo'l bilan noma'lumlarni topamiz:

$$x_3 = \frac{3}{8}$$

$$x_2 = x_3 + \frac{5}{4} = \frac{3}{8} + \frac{5}{4} = \frac{13}{8}$$

$$x_1 = 2 + x_3 - x_2 = 2 + \frac{3}{8} - \frac{13}{8} = \frac{3}{4}$$

Biror tenglamaga qo'yib tekshirish mumkin. Masalan: ikkinchi tenglamada noma'lumlar o'rniga topilganlarni qo'yaylik:

$$3 \cdot \frac{3}{4} - \frac{13}{8} + \frac{3}{8} = 1$$

$$\frac{9}{4} - \frac{13}{8} + \frac{3}{8} = 1, \qquad 1 = 1 \qquad \text{demak to'g'ri.}$$

2. Kramer qoidasi. Biz asosiy determinantni hisobladik. Endi yordamchi determinantni hisoblaylik.

$$\Delta_1 = \begin{vmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = -6 + 1 + 1 - 1 - 3 + 2 = -6$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 3 - 3 + 4 + 2 - 18 - 1 = -13$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = -1-6+2+4-3+1 = -3$$

$$x_1 = \frac{\Delta_1}{|A|} = \frac{-6}{-8} = \frac{3}{4}, \qquad x_2 = \frac{\Delta_2}{|A|} = \frac{-13}{-8} = \frac{13}{8},$$

$$x_3 = \frac{\Delta_3}{|A|} = \frac{-3}{-8} = \frac{3}{8}$$

3. Matritsa hisobi. Sistemani $Ax = b$ matritsa ko'rinishida yozaylik. Bundan $x = A^{-1} \cdot b$. Bu yerda sistema matritsasi

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 2 & -1 & 3 \end{pmatrix}$$

A^{-1} esa A ga teskari matritsa; x, b lar ustun matritsalar:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad x = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

A^{-1} teskari matritsani hisoblaymiz.

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

bu yerda A_{ij} lar algebraik to'ldiruvchilar.

$$A_{11} = \begin{vmatrix} -1 & 1 \\ -1 & 3 \end{vmatrix} = -3+1 = -2,$$

$$A_{21} = -\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$A_{12}=\begin{vmatrix}3 & 1\\ 2 & 3\end{vmatrix}=-(9-2)=-7,\qquad A_{22}=\begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix}=3+2=5$$

$$A_{13}=\begin{vmatrix}3 & -1\\ 2 & -1\end{vmatrix}=-3+2=-1,\qquad A_{23}=\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}=-(-1-2)=3$$

$$A_{31}=\begin{vmatrix}1 & -1\\ -1 & 1\end{vmatrix}=1-1=0$$

$$A_{32}=\begin{vmatrix}1 & -1\\ 3 & 1\end{vmatrix}=-(1+3)=-4$$

$$A_{33}=\begin{vmatrix}1 & 1\\ 3 & -1\end{vmatrix}=-1-3=-4$$

$$A^{-1}=\frac{1}{-8}=\begin{pmatrix}-2 & -2 & 0\\ -7 & 5 & -4\\ -1 & 3 & -4\end{pmatrix}=\begin{pmatrix}\frac{1}{4} & \frac{1}{4} & 0\\ \frac{7}{8} & -\frac{5}{8} & \frac{1}{2}\\ \frac{1}{8} & -\frac{3}{8} & \frac{1}{2}\end{pmatrix}$$

$$X=A^{-1}\cdot b=\begin{pmatrix}\frac{1}{4} & \frac{1}{4} & 0\\ \frac{7}{8} & -\frac{5}{8} & \frac{1}{2}\\ \frac{1}{8} & -\frac{3}{8} & \frac{1}{2}\end{pmatrix}\cdot\begin{pmatrix}2\\ 1\\ 1\end{pmatrix}=\begin{pmatrix}\frac{1}{4}\cdot 2+\frac{1}{4}\cdot 1+0\cdot 1\\ \frac{7}{8}\cdot 2-\frac{5}{8}\cdot 1+\frac{1}{2}\cdot 1\\ \frac{1}{8}\cdot 2-\frac{3}{8}\cdot 1+\frac{1}{2}\cdot 1\end{pmatrix}=\begin{pmatrix}\frac{3}{4}\\ \frac{13}{8}\\ \frac{3}{8}\end{pmatrix}$$

demak, $x_1=\frac{3}{4},\quad x_2=\frac{13}{8},\quad x_3=\frac{3}{8}$

Biz uch xil usulda ham bir xil javob oldik.

Javob: $x_1=\frac{3}{4},\quad x_2=\frac{13}{8},\quad x_3=\frac{3}{8}$

2– masala.

Ikki chiziqli almashtirishlar berilgan. Matritsa hisobi bilan x_1'', x_2'' , x_3'' larni x_1, x_2, x_3 lar orqali ifodalovchi almashtirishlarni toping.

$$\begin{cases} x_1' = 2x_1 - x_2 + 3x_3 \\ x_2' = x_1 - x_2 \\ x_3' = x_2 + 4x_3 \end{cases} \qquad \begin{cases} x_1'' = 3x_1' + x_2' - x_3' \\ x_2'' = x_1' - 2x_3' \\ x_3'' = x_1' + 5x_2' \end{cases}$$

Yechish:

Almashtirishlarni matritsa ko'rinishida yozaylik: $x` = Ax$, $x`` = Bx`$ bundan $x`` = BAx$ kelib chiqadi.

Demak,

$$B\cdot A = \begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & -2 \\ 1 & 5 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{pmatrix} =$$

$$\begin{pmatrix} 3\cdot 2+1\cdot 1+(-1)\cdot 0 & 3\cdot(-1)+1\cdot(-1)+(-1)\cdot 1 & 3\cdot 3+1\cdot 0+(-1)\cdot 4 \\ 0\cdot 2+1\cdot 1+(-2)\cdot 0 & 0\cdot(-1)+1\cdot(-1)+(-2)\cdot 1 & 0\cdot 3+1\cdot 0+(-2)\cdot 4 \\ 1\cdot 2+5\cdot 1+0\cdot 0 & 1\cdot(-1)+5\cdot(-1)+0\cdot 1 & 1\cdot 3+5\cdot 0+0\cdot 4 \end{pmatrix} =$$

$$= \begin{pmatrix} 7 & -5 & 5 \\ 1 & -3 & -8 \\ 7 & -6 & 3 \end{pmatrix}$$

Demak, izlanayotgan almashtirishlar quyidagicha bo'ladi:

$$\begin{cases} x_1^{``} = 7x_1 - 5x_2 + 5x_3 \\ x_2^{``} = x_1 - 3x_2 - 8x_3 \\ x_3^{``} = 7x_1 - 6x_2 + 3x_3 \end{cases}$$

3- masala.

Biror bazisda matritsa holida berilgan chiziqli almashtirishlarning xos son va xos vektorlarini toping.

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

Yechish:

Xarakterestik tenglamani tuzamiz.

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 3 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2(5-\lambda)+3+3-9(5-\lambda)-(1-\lambda)-(1-\lambda)=0$$

elementar shakl almashtirishdan so'ng $(\lambda+2)(-\lambda^2+9\lambda-18)=0$ ni hosil qilamiz. Bundan $\lambda_1=-2$, $\lambda_2=3$, $\lambda_3=6$ xos sonlarni topamiz.

$\lambda_1=-2$ xos songa mos keluvchi vektorni topamiz. Buning uchun

$$\begin{cases} (1-(-1))U_1+U_2+3U_3=0 \\ U_1+(5-(-2))U_2+U_3=0^0 \\ 3U_1+U_2+(1-(-2))U_3=0 \end{cases}$$

tenglamalar sistemasini yechamiz.

$$\begin{cases} 3U_1+U_2+3U_3=0 \\ U_1+7U_2+U_3=0 \\ 3U_1+U_2+3U_3=0 \end{cases}$$

Bitta tenglamani tashlab yuboramiz va $U_2=0$ ni topamiz.

$U_3=-U_1$ (U_1 - erkli o'zgaruvchi, U_3 - bazis o'zgaruvchi). Bundan birinchi xos vektorni topamiz.

$\vec{U}(U_1, U_2, U_3)=U_3(1, 0, -1)$.

$\lambda_2=3$ xos son uchun

$$\begin{cases}(1-3))V_1+V_2+3V_3=0\\ V_1+(5-3)V_2+V_3=0\\ 3V_1+V_2+(1-3)V_3=0\end{cases}$$ sistemani tuzamiz.

$$\begin{cases}-2V_1+V_2+3V_3=0\\ V_1+2V_2+V_3=0\\ 3V_1+V_2-2V_3=0\end{cases}$$ Ikkkinchi tenglamani (-3)ga

ko'paytirib uchinchi tenglamaga qo'shsak, $V_2=-V_3$ kelib chiqadi. Bundan $V_1=V_3$

Ikkinchi xos vektor $\vec{V}(V_1, V_2, V_3)=V_3(1,-1,1)$

$$\lambda_3=6 \qquad \begin{cases}-5W_1+W_2+3W_3=0\\ W_1-W_2+W_3=0\\ 3W_1+W_2-5W_3=0\end{cases}$$

Birinchi va ikkinchi tenglamalarni qo'shsak, $W_1=W_3$ kelib chiqadi.

Bundan $W_2=W_1+W_3=2W_3$

Uchinchi xos vektor $\vec{W}(W_1, W_2, W_3)=W_3(1,2,1)$

JAVOB: $\lambda_1=-2,\ \lambda_2=3,\ \lambda_3=6$

$$\vec{U}=U_1(1,0,-1)$$

$$\vec{V}=U_3(1,-1,1)$$

$$\vec{W}=W_3(1,2,1)$$

4 - masala.

Quyidagi egri chiziq tenglamasini kanonik holga keltiring.

$$4x^2+4\sqrt{3}xy+5y^2=10$$

Yechish:

Uning matritsasi

$$A=\begin{pmatrix}4 & 2\sqrt{3}\\ 2\sqrt{3} & 5\end{pmatrix}$$ bo'ladi.

$$\begin{vmatrix}4-\lambda & 2\sqrt{3}\\ 2\sqrt{3} & 5-\lambda\end{vmatrix}=0$$

xarakteristik tenglamani yechamiz:

$$(4-\lambda)(5-\lambda)-12=0$$

$$\lambda^2-9\lambda+8=0$$

$\lambda_1=1$, $\lambda_2=8$ xos sonlarni topamiz. Demak,

$4x^2+4\sqrt{3}xy+5y^2$ kvadratik forma

$1(x`)^2+8(y`)^2$ ko'rinishida berilgan, tenglama esa

$(x`)^2+8(y`)^2=10$ ko'rinishiga keladi. Yoki

$\frac{(x`)^2}{10}+\frac{(y`)^2}{5/4}=1$ bu ellips tenglamasi.

5- masala

A_1, A_2, A_3, A_4 - piramida uchlarining koordinatalari berilgan. Quyidagilarni toping:

1. $A_1\ A_2$ qirra uzunligini;
2. $A_1\ A_2$ va $A_1\ A_4$ qirralar orasidagi burchakni;
3. $A_1\ A_4$ va $A_1\ A_2\ A_3$ yoq orasidagi burchakni;
4. $A_1\ A_2\ A_3$ yoq yuzasini;
5. piramida hajmini.

$A_1(8;\ 6;\ 4)$, $A_2(10;\ 5;\ 5)$, $A_3(5;\ 6;\ 8)$, $A_4(8;\ 10;\ 7)$

Yechish:

1. Ikki nuqta orasidagi masofani hisoblash formulasidan qirraning uzunligini hisoblaymiz.

$$|A_1\ A_2|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}=$$

$$=\sqrt{(10-8)^2+(5-6)^2+(5-4)^2}=\sqrt{6}$$ uz. b

2. $A_1 \ A_2$ va $A_1 \ A_4$ qirralar orasidagi burchakni

$$\cos\varphi = \frac{\overline{A_1A_2}\cdot\overline{A_1A_4}}{\left|\overline{A_1A_2}\right|\cdot\left|\overline{A_1A_4}\right|}$$ formula bilan hisoblaymiz.

Bizda $\overline{A_1A_2} = (10-8)\bar{i} + (5-6)\bar{j} + (5-4)\bar{k} = 2\bar{i} - \bar{j} + \bar{k}$

$$\overline{A_1A_2} = (8-8)\bar{i} + (10-6)\bar{j} + (7-4)\bar{k} = 4\bar{j} + 3\bar{k}$$

$\left|\overline{A_1A_2}\right| = \sqrt{6}$, $\quad \left|\overline{A_1A_4}\right| = 5$

$$\overline{A_1A_2}\cdot\overline{A_1A_4} = 2\cdot 0 + (-1)4 + 1\cdot 3 = -1$$

$$\cos\varphi = -\frac{1}{5\sqrt{6}} \approx -\frac{1}{12.25} \approx -0.08, \qquad \varphi \approx 95^0$$

(Bradis jadvalidan).

3. Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasidan $A_1 \ A_4$ tenglamasini tuzamiz:

$$\frac{x-8}{8-8} = \frac{y-6}{10-6} = \frac{z-4}{7-4}$$

$$\frac{x-8}{0} = \frac{y-6}{4} = \frac{z-4}{3}$$

Endi $A_1 \ A_2 \ A_3$ yon tenglamasini tuzish uchun uch nuqtadan o'tuvchi tekislik tenglamasidan foydalanamiz:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-8 & y-6 & z-4 \\ 10-8 & 5-6 & 5-4 \\ 5-8 & 6-6 & 8-4 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-8 & y-6 & z-4 \\ 2 & -1 & 1 \\ -3 & 0 & 4 \end{vmatrix} = 0$$

$$-4(x-8)-11(y-6)-3(z-4)=0$$

$$4x+11y+3z-110=0$$

$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

to'g'ri chiziq va $Ax+By+Cz+D=0$ tekislik orasidagi burchagi

$$\sin\varphi=\frac{Al+Bm+Cn}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{l^2+m^2+n^2}}$$

hisoblash formulasidan A_1 A_4 qirra va A_1 A_2 A_3 yoq orasidagi burchagini topamiz:

$$\sin\varphi=\frac{4\cdot 0+11\cdot 4+3\cdot 3}{\sqrt{4^2+4^2+3^2}\cdot\sqrt{0^2+4^2+3^2}}=\frac{53}{5\sqrt{146}}\approx 0.88$$

4. A_1 A_2 A_3 yoq yuzasi $\overline{A_1A_2}\cdot\overline{A_1A_3}$ vektor ko'paytma modulining yarmiga teng.

$$\overline{A_1A_2}\cdot\overline{A_1A_3}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & -1 & 1\\ -3 & 0 & 4\end{vmatrix}=-4\bar{i}-11\bar{j}-3\bar{k}$$

$$\overline{A_1A_2}\cdot\overline{A_1A_3}=\sqrt{4^2+11^2+3^2}=\sqrt{146}$$

$$S_{\Delta A_1B_1C_1}=\frac{1}{2}\cdot\sqrt{146}\approx 6.0 \text{ kv. b.}$$

5. A_2 A_3 A_4 piramida hajmi $\overline{A_1A_2}\cdot\overline{A_1A_3}\cdot\overline{A_1A_4}$ aralash ko'paytmaning $\frac{1}{6}$ qismiga teng:

$$\begin{vmatrix}2 & -1 & 1\\ -3 & 0 & 4\\ 0 & 4 & 3\end{vmatrix}=-12-9-32=-53$$

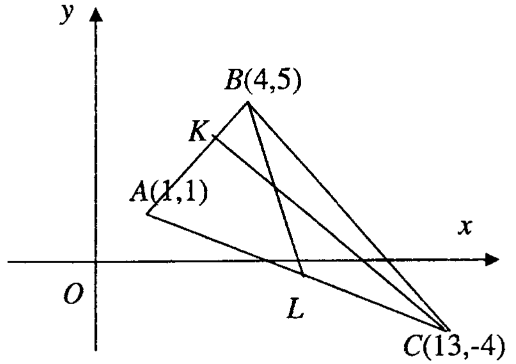
$$V_{\ddot{\imath}\grave{e}\eth}=\frac{1}{6}|-53|=\frac{53}{6}\approx 8{,}83 \text{ hajm birligi.}$$

6- masala

Uchburchak uchlarining koordinatalari berilgan: A(1; 1), V(4; 5) va S(13; -4) V uchidan tushirilgan mediana tenglamasi va S uchidan tushirilgan balandlik tenglamasi tuzilsin.

Uchburchakning yuzini hisoblang.

Yechish:



BL medriananing tenglamasini tuzish uchun L nuqtaning koordinatalarini topamiz. Shartga ko'ra $AL = LC$

$$X_L = \frac{X_A + X_C}{2} = \frac{13+1}{2} = 7$$

$$Y_L = \frac{Y_A + Y_C}{2} = \frac{1-4}{2} = -\frac{3}{2}$$

$$BL: \quad \frac{y-5}{-\frac{3}{2}-5} = \frac{x-4}{7-4}, \qquad \frac{y-5}{-\frac{13}{2}} = \frac{x-4}{3}$$

bundan $13x + 6y - 82 = 0$, CK balandlik tenglamasini tuzish uchun C nuqtadan to'g'ri chiziq tenglamasini topamiz. $y + 4 = k(x-13)$. Endi k ni topish uchun AB tenglamasini tuzib, uning burchak koeffitsientini topish lozim.

$$AB: \frac{y-1}{5-1}=\frac{x-1}{4-1} \quad \text{yoki} \quad \frac{y-1}{4}=\frac{x-1}{3}$$

bundan $y=\frac{4x}{3}-\frac{1}{3}$ yoki $4x-3y-1=0$

shartga ko'ra $CK \perp AB$ $\quad k=-\frac{1}{\frac{3}{4}}=-\frac{3}{4}$

Demak, $CK: \quad y+4=-\frac{3}{4}(x-13)$

yoki $3x+4y-23=0$

Endi ΔABC ning yuzini hisoblash uchun CK balandlik va AB tomon uzunligini topish kerak:

$$S_\Delta=\frac{CK\cdot AB}{2}$$

Berilgan nuqtadan to'g'ri chiziqqacha masofa formulasidan

$$|CK|=L=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}=\frac{|4\cdot 13-3\cdot(-4)-1|}{\sqrt{4^2+3^2}}=\frac{|52+12-1|}{5}=\frac{63}{5}$$

$$|AB|=\sqrt{(4-1)^2+(5-1)^2}=\sqrt{3^2+4^2}=5$$

Demak, $S_\Delta=\frac{\frac{63}{5}\cdot 5}{2}=31.5$ kv.bir.

JAVOB: $13x+16y-82=0$

$3x+4y-23=0$

$S=31.5$ kv.b.

7- masala

a - kompleks son berilgan. Quyidagilarni toping.

1. a sonini algebraik va trigonometrik shaklda yozing.
2. $z^3+a=0$ tenglamaning hamma ildizlarini toping.

$$a=\frac{1}{1+i\sqrt{3}}$$

Yechish:

1. $a=x+yi$ - algebraik ko'rinishga keltiramiz.

$$a=\frac{1}{1+i\sqrt{3}}=\frac{1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}=\frac{1-i\sqrt{3}}{1^2+(\sqrt{3})^2}=\frac{1}{4}-\frac{\sqrt{3}}{4}i$$

$a=r(\cos\varphi+i\sin\varphi)$ ko'rinishiga keltiramiz

$$\varphi=arctg\frac{y}{x}=arctg\frac{-\frac{\sqrt{3}}{4}}{\frac{1}{4}}=arctg\left(-\sqrt{3}\right)=\frac{5\pi}{3},$$

chunki $x>0$, $y<0$ IV chorak. $a=\frac{1}{2}\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

2. $z^3=-a$

$$-a=-\frac{1}{1+i\sqrt{3}}=-\frac{1}{4}+\frac{\sqrt{3}}{4}i$$

$r=\frac{1}{2}$, $\varphi=\frac{2\pi}{3}$, ya'ni $x<0$, $y>0$ II chorak.

$$-a=\frac{1}{2}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$z=\sqrt[3]{-a}=\sqrt[3]{\frac{1}{2}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)}$$

$$\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}\left(\cos\frac{\varphi+k\pi}{n}+i\sin\frac{\varphi+2k\pi}{n}\right)$$

Bu yerda $k=/0,1,2,\ldots,n-1/$ formuladan foydalanamiz.

$$z=\sqrt[3]{\frac{1}{2}}\cdot\left(\cos\frac{\frac{2\pi}{3}+2k\pi}{3}+i\sin\frac{\frac{2\pi}{3}+2k\pi}{3}\right)$$

$$k=0, \quad z=\sqrt[3]{\frac{1}{2}}\cdot\left(\cos\frac{2\pi}{9}+i\sin\frac{2\pi}{9}\right)$$

$$k=1, \quad z=\sqrt[3]{\frac{1}{2}}\cdot\left(\cos\frac{8\pi}{9}+i\sin\frac{8\pi}{9}\right)$$

$$k=2, \quad z=\sqrt[3]{\frac{1}{2}}\cdot\left(\cos\frac{14\pi}{9}+i\sin\frac{14\pi}{9}\right)$$

1- NAZORAT ISHI BO'YICHA MASALALAR

1–20 masalalarda chiziqli tenglamalar sistemasini birgalikda ekanini ko'rsating va uch xil usulda:

1. Gauss usuli;
2. Kramer qoidasi;
3. Matritsa hisobi bilan yeching.

1. $\begin{cases} 3x_1+4x_2+2x_3=8 \\ 2x_1-4x_2-3x_3=-1 \\ x_1+5x_2+x_3=0 \end{cases}$

2. $\begin{cases} 7x_1-5x_2=31 \\ 4x_1+11x_3=-43 \\ 2x_1+3x_2+4x_3=-20 \end{cases}$

3. $\begin{cases} 5x_1+8x_2-x_3=7 \\ 2x_1-3x_2+2x_3=1 \\ x_1+2x_2+3x_3=1 \end{cases}$

4. $\begin{cases} x_1+2x_2+x_3=4 \\ 3x_1-5x_2+3x_3=1 \\ 2x_1+7x_2-x_3=8 \end{cases}$

5. $\begin{cases} 3x_1+x_2+x_3=5 \\ x_1-4x_2-2x_3=-3 \\ -3x_1+5x_2+6x_3=7 \end{cases}$

6. $\begin{cases} x_1+x_2-x_3=1 \\ 8x_1+3x_2-6x_3=2 \\ -4x_1-x_2+3x_3=6 \end{cases}$

7. $\begin{cases} 2x_1-x_2+5x_3=4 \\ 5x_1+2x_2+13x_3=-23 \\ 3x_1+x_2+5x_3=0 \end{cases}$

8. $\begin{cases} x_1-2x_2+3x_3=6 \\ 2x_1+3x_2-4x_3=20 \\ 3x_1-2x_2-5x_3=6 \end{cases}$

9. $\begin{cases} x_1 + x_2 - x_3 = -2 \\ 4x_1 - 3x_2 + x_3 = 1 \\ 2x_1 + x_2 - 5x_3 = 1 \end{cases}$
10. $\begin{cases} 4x_1 - 3x_2 + 2x_3 = 9 \\ 2x_1 + 5x_2 - 3x_3 = 14 \\ 5x_1 + 6x_2 - 2x_3 = 18 \end{cases}$

11. $\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ x_1 - 12x_2 + 4x_3 = -7 \\ 3x_1 - 5x_2 + 3x_3 = 1 \end{cases}$
12. $\begin{cases} x_1 - 5x_2 - x_3 = -14 \\ x_1 - 2x_2 + 3x_3 = 6 \\ 2x_1 + 3x_2 - 4x_3 = 20 \end{cases}$

13. $\begin{cases} x_1 + 2x_2 - x_3 = 7 \\ 2x_1 - x_2 + x_3 = 2 \\ 3x_1 - 5x_2 + 2x_3 = -7 \end{cases}$
14. $\begin{cases} x_1 - x_2 + 7x_3 = 6 \\ 2x_1 + 3x_2 - 3x_3 = 16 \\ 3x_1 + 2x_2 + 5x_3 = 17 \end{cases}$

15. $\begin{cases} 8x_1 + 4x_2 + 3x_3 = 7 \\ 2x_1 + 6x_2 - 2x_3 = 4 \\ 3x_1 + 10x_2 + x_3 = 11 \end{cases}$
16. $\begin{cases} -2x_1 + x_2 + 7x_3 = 1 \\ 3x_1 - 3x_2 + 8x_3 = 20 \\ 5x_1 + 4x_2 - x_3 = 1 \end{cases}$

17. $\begin{cases} x_1 + 5x_3 = 0 \\ 3x_1 + 2x_2 + 7x_3 = 4 \\ 5x_1 + 9x_3 = 16 \end{cases}$
18. $\begin{cases} x_1 + 4x_2 + 6x_3 = 14 \\ -2x_1 + 7x_2 + 4x_3 = 18 \\ 3x_1 + 2x_2 + 2x_3 = 6 \end{cases}$

19. $\begin{cases} -2x_1 + 3x_2 + 5x_3 = 18 \\ x_1 - 3x_2 + 4x_3 = 25 \\ 7x_1 + 8x_2 - x_3 = 1 \end{cases}$
20. $\begin{cases} 3x_1 - 4x_2 + 2x_3 = 1 \\ 4x_1 - 2x_2 + x_3 = 3 \\ 5x_1 - x_2 + 3x_3 = 2 \end{cases}$

21–40 ikkita chiziqli almashtirishlar berilgan. Matritsa hisobi bilan x_1'', x_2'', x_3'' larni x_1, x_2, x_3 lar orqali ifodalovchi almashtirishlarni toping.

21. $\begin{cases} x_1' = 5x_1 - x_2 + 3x_3 \\ x_2' = x_1 - 2x_2 \\ x_3' = 7x_2 - x_3 \end{cases}$ $\begin{cases} x_1'' = 2x_1' + x_3' \\ x_2'' = x_1' - 5x_3' \\ x_3'' = 2x_1' \end{cases}$

22. $\begin{cases} x_1' = x_1 + 2x_2 + 2x_3 \\ x_2' = -3x_2 + x_3 \\ x_3' = 2x_1 + 3x_3 \end{cases}$ $\begin{cases} x_1'' = 3x_1' + x_2' \\ x_2'' = x_1' - 2x_2' - x_3' \\ x_3'' = 3x_1' + 2x_3' \end{cases}$

23. $\begin{cases} x_1' = x_1 - 3x_2 + 4x_3 \\ x_2' = 2x_1 + x_2 - 5x_3 \\ x_3' = 3x_1 + 5x_2 + x_3 \end{cases}$ $\begin{cases} x_1'' = 4x_1' + 5x_2' - 3x_3' \\ x_2'' = x_1' - x_2' - x_3' \\ x_3'' = 7x_1' + 4x_3' \end{cases}$

24. $\begin{cases} x_1' = 4x_1 + 3x_2 + 5x_3 \\ x_2' = 6x_1 + 7x_2 + x_3 \\ x_3' = 9x_1 + x_2 + 8x_3 \end{cases}$ $\begin{cases} x_1'' = -x_1' + 5x_2' - 3x_3' \\ x_2'' = x_1' - x_2' - x_3' \\ x_3'' = 7x_1' + 4x_3' \end{cases}$

25. $\begin{cases} x_1' = -x_1 - x_2 - x_3 \\ x_2' = -x_1 + 4x_2 + 7x_3 \\ x_3' = 8x_1 + x_2 - x_3 \end{cases}$ $\begin{cases} x_1'' = 9x_1' + 3x_2' + 5x_3' \\ x_2'' = 2x_1' + 3x_3' \\ x_3'' = x_2' - x_3' \end{cases}$

26. $\begin{cases} x_1' = 4x_1 + 3x_2 + 2x_3 \\ x_2' = -2x_1 + x_2 - x_3 \\ x_3' = 3x_1 + x_2 + x_3 \end{cases}$ $\begin{cases} x_1'' = x_1' - 2x_2' - x_3' \\ x_2'' = 3x_1' + x_2' + 2x_3' \\ x_3'' = x_1' + 2x_2' + 2x_3' \end{cases}$

27. $\begin{cases} x_1' = 3x_1 + 5x_3 \\ x_2' = x_1 + x_2 + x_3 \\ x_3' = 3x_2 - 6x_3 \end{cases}$ $\begin{cases} x_1'' = 2x_1' - x_2' - 5x_3' \\ x_2'' = 7x_1' + x_2' + 4x_3' \\ x_3'' = 6x_1' + 4x_2' - 7x_3' \end{cases}$

28. $\begin{cases} x_1' = 2x_2 \\ x_2' = -2x_1 + 3x_2 + 2x_3 \\ x_3' = 4x_1 - x_2 + 5x_3 \end{cases}$ $\begin{cases} x_1'' = -3x_1' + x_3' \\ x_2'' = 2x_2' + x_3' \\ x_3'' = -x_2' + 3x_3' \end{cases}$

29. $\begin{cases} x_1' = 7x_1 + 4x_3 \\ x_2' = 4x_2 - 9x_3 \\ x_3' = 3x_1 + x_2 \end{cases}$ $\begin{cases} x_1'' = x_2' - 6x_3' \\ x_2'' = 3x_1' + 7x_3' \\ x_3'' = x_1' + x_2' - x_3' \end{cases}$

30. $\begin{cases} x_1' = 7x_1 + 3x_2 + 4x_3 \\ x_2' = 4x_2 - 9x_3 \\ x_3' = 3x_1 + x_2 + x_3 \end{cases}$ $\begin{cases} x_1'' = x_1' + x_2' - 6x_3' \\ x_2'' = 3x_2' + 7x_3' \\ x_3'' = x_1' + x_2' - x_3' \end{cases}$

31. $\begin{cases} x_1' = 4x_1 + 3x_2 + 2x_3 \\ x_2' = -2x_1 + x_2 - x_3 \\ x_3' = x_1 - x_2 + 3x_3 \end{cases}$ $\begin{cases} x_1'' = x_1' - 2x_2' + x_3' \\ x_2'' = 2x_1' + x_2' - 5x_3' \\ x_3'' = x_1' + 2x_2' + 2x_3' \end{cases}$

32. $\begin{cases} x_1' = 3x_1 - x_2 + 5x_3 \\ x_2' = x_1 + 2x_2 + 4x_3 \\ x_3' = 3x_1 + 2x_2 - x_3 \end{cases}$ $\begin{cases} x_1'' = 4x_1' + 3x_2' + x_3' \\ x_2'' = 3x_1' + x_2' + 2x_3' \\ x_3'' = x_1' - 2x_2' + x_3' \end{cases}$

33. $\begin{cases} x_1' = 3x_1 + x_2 \\ x_2' = -x_1 + 2x_3 \\ x_3' = x_1 - x_2 + 2x_3 \end{cases}$ $\begin{cases} x_1'' = -x_1' + 2x_2' - x_3' \\ x_2'' = x_1' - x_2' - x_3' \\ x_3'' = x_2' + 6x_3' \end{cases}$

34. $\begin{cases} x_1' = -x_1 + 2x_2 + 3x_3 \\ x_2' = 3x_1 - x_2 + x_3 \\ x_3' = 3x_1 - x_3 \end{cases}$ $\begin{cases} x_1'' = 5x_1' - x_2' \\ x_2'' = 3x_1' + x_2' - x_3' \\ x_3'' = x_1' - x_2' + 2x_3' \end{cases}$

35. $\begin{cases} x_1' = 4x_1 + 5x_2 - 3x_3 \\ x_2' = 3x_1 - x_2 + 5x_3 \\ x_3' = -x_1 + x_2 \end{cases}$ $\begin{cases} x_1'' = -x_2' + 3x_3' \\ x_2'' = x_1' - x_2' - 9x_3' \\ x_3'' = -4x_1' + 5x_3' \end{cases}$

36. $\begin{cases} x_1' = -x_1 + 4x_2 - x_3 \\ x_2' = x_1 - x_2 + 5x_3 \\ x_3' = 5x_1 - x_2 + 6x_3 \end{cases}$ $\begin{cases} x_1'' = 2x_2' - 5x_3' \\ x_2'' = x_1' + x_2' - x_3' \\ x_3'' = 5x_1' - x_2' + 8x_3' \end{cases}$

37. $\begin{cases} x_1' = 4x_1 - x_2 + 5x_3 \\ x_2' = -x_1 + 6x_2 - x_3 \\ x_3' = x_1 - x_2 - 9x_3 \end{cases}$ $\begin{cases} x_1'' = x_1' - 5x_2' + x_3' \\ x_2'' = 5x_1' - x_3' + 7x_3' \\ x_3'' = x_1' + x_2' \end{cases}$

38. $\begin{cases} x_1' = 3x_1 + 9x_2 - x_3 \\ x_2' = 4x_1 - x_2 \\ x_3' = 7x_2 - x_3 \end{cases}$ $\begin{cases} x_1'' = 4x_1' - 5x_2' + x_3' \\ x_2'' = 6x_1' + x_2' \\ x_3'' = 5x_1' - x_2' + 7x_3' \end{cases}$

39. $\begin{cases} x_1' = 5x_1 + 3x_2 - x_3 \\ x_2' = 7x_1 - 6x_2 + x_3 \\ x_3' = 7x_1 - 6x_2 + x_3 \end{cases}$ $\begin{cases} x_1'' = 7x_1' + x_2' - x_3' \\ x_2'' = 6x_1' - x_2' + x_3' \\ x_3'' = 9x_1' + 5x_2' - x_3' \end{cases}$

40. $\begin{cases} x_1' = 9x_1 + x_2 + 6x_3 \\ x_2' = 8x_1 - x_2 + x_3 \\ x_3' = 9x_1 - 8x_2 + 7x_3 \end{cases}$ $\begin{cases} x_1'' = 4x_1' - 5x_2' \\ x_2'' = 6x_1' + x_2' - x_3' \\ x_3'' = 7x_1' - x_2' + x_3' \end{cases}$

41-60. Biror bazisda matritsa holida chiziqli almashtirishning xos son va xos vektorlarini toping.

41. $\begin{pmatrix} 2 & 19 & 30 \\ 0 & -5 & -12 \\ 0 & 2 & 5 \end{pmatrix}$ 42. $\begin{pmatrix} -3 & 2 & 0 \\ -2 & 1 & 0 \\ 15 & -7 & 4 \end{pmatrix}$

43. $\begin{pmatrix} -1 & -2 & 12 \\ 0 & 4 & 3 \\ 0 & 5 & 6 \end{pmatrix}$ 44. $\begin{pmatrix} 5 & -7 & 0 \\ -3 & 1 & 0 \\ 12 & 6 & -3 \end{pmatrix}$

45. $\begin{pmatrix} 1 & 8 & 23 \\ 0 & 5 & 7 \\ 0 & 3 & 1 \end{pmatrix}$ 46. $\begin{pmatrix} 4 & 0 & 5 \\ 7 & -2 & 9 \\ 3 & 0 & 6 \end{pmatrix}$

47. $\begin{pmatrix} 1 & -1 & 16 \\ 0 & 1 & -1 \\ 0 & 1 & 3 \end{pmatrix}$ 48. $\begin{pmatrix} -3 & 11 & 7 \\ 0 & 5 & -4 \\ 0 & 1 & 1 \end{pmatrix}$

49. $\begin{pmatrix} 5 & 9 & 7 \\ 0 & 3 & -2 \\ 0 & 2 & -1 \end{pmatrix}$ 50. $\begin{pmatrix} 5 & 0 & 21 \\ 21 & 2 & 16 \\ 1 & 0 & -1 \end{pmatrix}$

51. $\begin{pmatrix} 0 & 1 & 0 \\ -3 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$ 52. $\begin{pmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{pmatrix}$

53. $\begin{pmatrix} 4 & -5 & 5 \\ 1 & -4 & 9 \\ -4 & 0 & 5 \end{pmatrix}$ 54. $\begin{pmatrix} 5 & 6 & 3 \\ -1 & 0 & 1 \\ 1 & 2 & -1 \end{pmatrix}$

55. $\begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}$ 56. $\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}$

57. $\begin{pmatrix} 7 & 0 & 0 \\ 10 & -19 & 10 \\ 12 & -24 & 13 \end{pmatrix}$ 58. $\begin{pmatrix} 3 & 1 & 0 \\ -4 & -1 & 0 \\ 4 & -8 & -2 \end{pmatrix}$

59. $\begin{pmatrix} 1 & -3 & 4 \\ 4 & -7 & 8 \\ 6 & -7 & 7 \end{pmatrix}$ 60. $\begin{pmatrix} 0 & 7 & 4 \\ 0 & 1 & 0 \\ 1 & 13 & 0 \end{pmatrix}$

61–80 masalalarda ikkinchi tartibli egri chiziq tenglamasini (kvadratik formalar nazariyasidan foydalanib) kanonik ko'rinishga keltiring.

61. $5x^2+4xy+2y^2=18$

62. $4x^2+2\sqrt{6}xy+3y^2=24$

63. $6x^2+2\sqrt{5}xy+2y^2=21$

64. $5x^2+4\sqrt{2}xy+3y^2=14$

65. $7x^2+2\sqrt{18}xy+4y^2=15$

66. $3x^2+2\sqrt{14}xy+y^2=10$

67. $7x^2+2\sqrt{6}xy+2y^2=24$

68. $9x^2+4\sqrt{2}xy+2y^2=20$

69. $6x^2+2\sqrt{10}xy+3y^2=16$

70. $17x^2+xy+8y^2=20$

71. $15x^2-2\sqrt{55}xy+9y^2=20$

72. $5x^2+4\sqrt{6}xy+3y^2=12$

73. $5x^2+4\sqrt{6}xy+7y^2=22$

74. $xy+3y^2=36$

75. $5x^2+8xy+5y^2=9$

76. $13x^2-48xy+37y^2=45$

77. $4x^2+24xy+11y^2=20$

78. $3x^2-2\sqrt{5}xy-y^2=8$

79. $6x^2-4\sqrt{14}xy+5y^2=26$

80. $x^2-2\sqrt{21}xy+5y^2=24$

81–100 $A_1\,A_2\,A_3\,A_4$ piramida uchlarining koordinatalari berilgan. Quyidagilarni toping:

1. $A_1\,A_2$ qirra uzunligi;
2. $A_1\,A_2$ va $A_3\,A_4$ qirralar orasidagi burchakni;

3. $A_1 A_4$ qirra va $A_1 A_2 A_3$ yoq orasidagi burchakni;
4. $A_1 A_2 A_3$ yoq yuzasini;
5. piramida hajmini.

81. $A_1(4;2;5)$, $A_2(0;7;2)$, $A_3(0;2;7)$, $A_4(1;5;0)$
82. $A_1(4;4;10)$, $A_2(4;10;2)$, $A_3(2;8;4)$, $A_4(9;5;9)$
83. $A_1(4;6;5)$, $A_2(6;9;4)$, $A_3(2;10;10)$, $A_4(7;5;9)$
84. $A_1(3;5;4)$, $A_2(8;7;4)$, $A_3(5;10;4)$, $A_4(4;7;8)$
85. $A_1(10;6;6)$, $A_2(-2;8;2)$, $A_3(5;10;4)$, $A_4(4;7;8)$
86. $A_1(1;8;2)$, $A_2(5;2;6)$, $A_3(5;7;4)$, $A_4(4;10;9)$
87. $A_1(6;6;5)$, $A_2(4;9;5)$, $A_3(4;6;11)$, $A_4(6;9;3)$
88. $A_1(7;2;2)$, $A_2(5;7;7)$, $A_3(5;3;1)$, $A_4(2;3;7)$
89. $A_1(7;6;4)$, $A_2(9;5;5)$, $A_3(5;6;8)$, $A_4(7;10;7)$
90. $A_1(7;7;3)$, $A_2(6;5;8)$, $A_3(3;5;8)$, $A_4(8;4;1)$
91. $A_1(3;1;4)$, $A_2(-1;6;1)$, $A_3(-1;1;6)$, $A_4(0;4;-1)$
92. $A_1(3;3;9)$, $A_2(6;9;1)$, $A_3(1;7;3)$, $A_4(8;5;8)$
93. $A_1(3;5;4)$, $A_2(5;8;3)$, $A_3(1;9;9)$, $A_4(6;4;8)$
94. $A_1(2;4;3)$, $A_2(7;6;3)$, $A_3(4;9;3)$, $A_4(3;6;7)$
95. $A_1(9;5;5)$, $A_2(-3;7;1)$, $A_3(5;7;8)$, $A_4(6;9;2)$
96. $A_1(0;7;1)$, $A_2(4;1;5)$, $A_3(4;6;3)$, $A_4(3;9;8)$
97. $A_1(5;5;4)$, $A_2(3;8;4)$, $A_3(3;5;10)$, $A_4(5;8;2)$
98. $A_1(6;1;1)$, $A_2(4;6;6)$, $A_3(4;2;0)$, $A_4(1;2;6)$
99. $A_1(7;5;3)$, $A_2(9;4;4)$, $A_3(4;5;7)$, $A_4(7;9;6)$
100. $A_1(6;6;2)$, $A_2(5;4;7)$, $A_3(2;4;7)$, $A_4(7;3;0)$

101. Agar parallelogrammning diagonallari (-1;0) nuqtada kesisishi ma'lum bo'lsa, uning $x+y-1=0$ va $y+1=0$ tomonlarining kesishish nuqtasi o'tmaydigan diagonallarining tenglamasini toping.

102. $2x+y+11=0$ to'g'ri chiziqda berilgan ikki A(1;1) va V(3;0) nuqtadan baravar uzoqlikdagi nuqtani toping.

103. (2;-4) nuqtaga $4x+3y+1=0$ to'g'ri chiziqqa nisbatan simmetrik bo'lgan nuqtaning koordinatalarini toping.

104. Uchlari A(-1;1), V(2;-1), S(4;0) bo'lgan uchburchakka tashqi chizilgan aylana markazining koordinatalarini toping.

105. (2;6) nuqtadan o'tuvchi va koordinata o'qlari bilan ikkinchi chorakda joylashib 3 kv. birlik yuzga ega bo'lgan uchburchak tashkil etuvchi to'g'ri chiziqning tenglamasini tuzing.

106. A(2;2) nuqtadan o'tuvchi to'g'ri chiziq tenglamasini shunday tuzingki, uning $x+2y+1=0$ va $x+2y-3=0$ parallel to'g'ri chiziqlar orasida joylashgan $x-y-6=0$ to'g'ri chiziqda yotsin.

107. Uchburchakning ikki tomonini tenglamalari berilgan: $4x-5y+9=0$ va $x+4y-3=0$. Agar bu uchburchakning medianalari (3;1) nuqtada kesishishi ma'lum bo'lsa, uchburchak uchinchi tomonining tenglamasini toping.

108. Romb ikkita tomonining tenglamasi: $2x-y+4=0$ va $2x-y+10=0$ hamda diagonallaridan birining tenglamasi $x+y+2=0$ ma'lum bo'lsa, romb uchlarining koordinatalarini hisoblang.

109. Agar $A(-5;5)$ va $B(3;1)$ uchburchakning ikkita uchi, $M(2;5)$ esa uning balandliklarining kesishgan nuqtasi bo'lsa, uchburchak tomonlarining tenglamasini tuzing.

110. $x+3y-7=0$ kvadrat tomonlaridan birining tenglamasi va $C(0;1)$ bu kvadratning diagonallarining kesishgan nuqtasi berilgan. Qolgan tomonlarining tenglamalarini tuzing.

111. $x+3y-5=0$ kvadrat tomonlaridan birining tenglamasi va S(-1;0) bu kvadrat diagonallarining kesishgan nuqtasi berilgan. Qolgan tomonlarining tenglamalarini tuzing.

112. Rombning bitta tomonining tenglamasi $x-3y+10=0$ va diagonallaridan birining tenglamasi $x+4y-4=0$ berilgan. Rombning diagonallari (0;1) nuqtada kesishadi. Rombning qolgan tomonlarining tenglamalarini tuzing.

113. Parallelogrammning ikkita tomoni tenglamalari berilgan $x+2y+2=0$ va $x+y-4=0$, diagonallarining biri tenglamasi $x-2=0$. Parallelogramm uchlarining koordinatalarini toping.

114. Uchburchakning ikki uchini A(-3;3), B(5;-1) va D(4;3) balandliklarining kesishgan nuqtasi. Uning tomonlari tenglamalarini tuzing.

115. ABCD (AD ‖ BC) trapetsiyaning uchlari A(-3;2), V(4;-1), S(1;3) berilgan. Trapetsiyaning diagonallari o'zaro perpendikulyar bo'lsa, D uchining koordinatalarini toping.

116. Uchburchakning ikkita tomonining tenglamalari berilgan: $5x-4y+15=0$ va $4x+y-9=0$. Medianalari (0;2) nuqtada kesishadi. Uchburchakning uchinchi tomoni tenglamasini tuzing.

117. Uchburchakning ikki uchi A(2;-2), V(3;-1) va medianalarining kesishish nuqtasi R(1;0) berilgan. S uchidan tushirilgan balandlik tenglamasini tuzing.

118. Uchburchakning ikki balandligi tenglamalari $x+y=4$ va $y=2x$ va uchlaridan biri A(0;2) berilgan. Uchburchak tomonlarining tenglamalarini tuzing.

119. Uchburchakning ikki medianasi $x-2y+1=0$ va $y-1=0$ va uchlaridan biri (1;3) berilgan. Uning tomonlari tenglamalarini tuzing.

120. Uchburchakning ikkita tomoni tenglamalari $5x-2y-8=0$ va $3x-2y-8=0$ berilgan. Uchinchi tomonining o'rtasi esa koordinata boshiga mos keladi. Shu tomon tenglamasini tuzing.

121–140. a kompleks soni berilgan. Quyidagilarni toping.

1) a sonini algebraik va trigonometrik shaklga keltiring;

2) $z^2+a=0$ tenglamaning barcha ildizlarini toping.

121. $a=\dfrac{2\sqrt{2}}{1+i}$

122. $a=\dfrac{4}{1+i\sqrt{3}}$

123. $a=\dfrac{-2\sqrt{2}}{1-i}$

124. $a=\dfrac{4}{1-i\sqrt{3}}$

125. $a=\dfrac{4}{1-i\sqrt{3}}$

126. $a=\dfrac{2\sqrt{2}}{1-i}$

127. $a=\dfrac{4}{1-i\sqrt{3}}$

128. $a=\dfrac{2\sqrt{2}}{\sqrt{3}-i}$

129. $a=\dfrac{4}{\sqrt{3}+i}$

130. $a=\dfrac{1}{\sqrt{3}-i}$

131. $a=-\dfrac{2}{\sqrt{3}+i}$

132. $a=\dfrac{2}{\sqrt{3}-i}$

133. $a=\dfrac{2}{\sqrt{3}+i}$

134. $a=-\dfrac{1}{1-i\sqrt{3}}$

135. $a=\dfrac{2}{1+i}$

136. $a=-\dfrac{2}{\sqrt{3}-i}$

137. $a=-\dfrac{1}{1+i\sqrt{3}}$

138. $a=-\dfrac{2}{1-i}$

139. $a=-\dfrac{2}{1+i\sqrt{3}}$

140. $a=-\dfrac{1}{\sqrt{3}-i}$

2- NAZORAT ISHI

1- masala.

Funksiya limitlarini Lopital qoidasidan foydalanmasdan turib hisoblang.

a) $\lim\limits_{x\to\infty}\dfrac{7x+5}{3x+4}$

Yechish: $\dfrac{\infty}{\infty}$ ko'rinishidagi aniqmaslik. Surat va maxrajini x – ga bo'lib limitga o'tamiz.

$$\lim_{x\to\infty}\frac{7x+5}{3x+4}=\lim_{x\to\infty}\frac{7+\frac{5}{x}}{3+\frac{4}{x}}=\frac{7}{3}$$

b) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+7}-\sqrt{7}}{x}$

Yechish: $\dfrac{0}{0}$ ko'rinishidagi aniqmaslik. Surat va maxrajini qo'shma son $\sqrt{x+7}+\sqrt{7}$ ga ko'paytiramiz:

$$\lim_{x\to 0}\frac{\sqrt{x+7}-\sqrt{7}}{x}=\lim_{x\to 0}\frac{\left(\sqrt{x+7}-\sqrt{7}\right)\left(\sqrt{x+7}+\sqrt{7}\right)}{x\left(\sqrt{x+7}+\sqrt{7}\right)}=$$

$$=\lim_{x\to 0}\frac{x+7-7}{x\left(\sqrt{x+7}+\sqrt{7}\right)}=\lim_{x\to 0}\frac{1}{\sqrt{x+7}+\sqrt{7}}=\frac{1}{2\sqrt{7}}$$

v) $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{x^2}$

Yechish:

$$\lim_{x\to 0}\frac{1-\cos 5x}{x^2}=\lim_{x\to 0}\frac{2\sin^2\frac{5x}{2}}{x^2}=2\lim_{x\to 0}\left(\frac{\sin\frac{5x}{2}}{\frac{5x}{2}}\cdot\frac{5}{2}\right)^2=2\cdot 1\cdot\frac{25}{4}=\frac{25}{2}$$

bu erda $\lim\limits_{x\to 0}\dfrac{\sin\alpha}{\alpha}=1$ ajoyib limitdan foydalandik.

g) $\lim\limits_{x\to 0}\left(\dfrac{x+4}{x-2}\right)^x$

Yechish:

1^∞ ko'rinishidagi aniqmaslik, 2-ajoyib limitdan foydalanamiz.

$$\lim_{x\to 0}\left(\frac{x+4}{x-2}\right)^x=\lim_{x\to 0}\left(1+\frac{x+4}{x-2}-1\right)^x=\lim_{x\to 0}\left(1+\frac{6}{x-2}\right)^x=$$

$$=\left[\lim_{x\to 0}\left(1+\frac{6}{x-2}\right)^{\frac{x-2}{6}}\right]^{\frac{6x}{x-2}}=e^6,$$

ko'rsatkichli funksiyaning uzluksizligidan limitni daraja – ko'rsatkichga olib chiqildi.

2- masala

$y=f(x)$ funksiya va argumentning x_1 va x_2 qiymati berilgan. Quyidagilar talab qilinadi: 1) x ning berilgan qiymatlarining har qaysisi uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishini aniqlang; 2) funksiya uzilishga ega bo'lgan x ning berilgan qiymatlaridan har biriga chapdan va o'ngdan yaqinlashganda funksiyaning limitini toping; 3) sxematik chizma yasang.

$f(x)=4^{\frac{1}{x-3}}$ $\qquad x_1=3 \qquad x_2=5$

Yechish:

1) $x_1=3$ da daraja – ko'rsatkichining maxraji 0 ga aylanadi. Demak $x_1=3$ da funksiya uzilishga ega.

$f(5)=4^{\frac{1}{2}}=2$, ya'ni $x_2=5$ da funksiya uzluksiz.

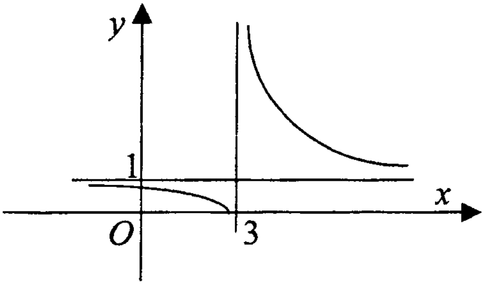
2) $x_1=3$ uchun chap va o'ng limitlarni hisoblaymiz.

$$\lim_{x\to 3-0} 4^{\frac{1}{x-3}} = 4^{-\frac{1}{0}} = \frac{1}{\infty} = 0$$

$$\lim_{x\to 3+0} 4^{\frac{1}{x-3}} = 4^{\infty} = \infty$$

$x = 3$ da funksiya II tur uzilishiga ega.

3) Sxematik chizma chizamiz



3- masala

$$f(x) = \begin{cases} 1-x & x \le 0 \\ x^2+1 & 0 < x \le 3 \\ x+2 & x > 3 \end{cases}$$

funksiya berilgan. Agar uzilish nuqtalari bo'lsa topib, xarakterlang. Chizma chizing.

Yechish:

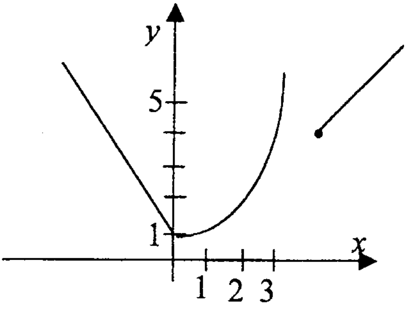
$x = 1$ va $x = 3$ nuqtalardagi bir tomonlama limitlarni ko'ramiz.

$$\lim_{x\to 0-0} f(x) = \lim_{x\to 0-0} (1-x) = 1$$

$$\lim_{x\to 0+0} f(x) = \lim_{x\to 0+0} (x^2+1) = 1$$

$$\lim_{x\to 3-0} f(x) = \lim_{x\to 3-0} (x^2+1) = 10$$

Demak, funksiya x=3 da I tur uzilishga ega. Funksiyaning sakrashi $\Delta=|5-10|=5$ bo'ladi. Chizma chizamiz.



4- masala

Berilgan funksiyaning $\frac{dy}{dx}$ hosilalarini toping.

a) $y=3\sqrt{2x+3}-\frac{2}{\sqrt{x^2+1}}$ b) $y=(e^{tgx}-2)^3$

v) $y=\ln\cos(3x-1)$ g) $y=x^{\sin x}$

d) $3x-y+ctgy=0$ e) $x=t+2t^3, y=3t^4$

Yechish:

a) Murakkab funksiyadan hosila olinganda «tashqi» funksiyaga e'tibor qilish kerak. Bu yerda ildizlar «tashqi» funksiya bo'ladi.

$$\frac{dy}{dx}=(3\sqrt{2x+3}-\frac{2}{\sqrt{x^2+1}})'=3(\sqrt{2x+3})'-2(\frac{1}{\sqrt{x^2+1}})'=$$

$$=3(\frac{(2x+3)'}{2\sqrt{2x+3}})-2(\frac{-(\sqrt{x^2+1})'}{x^2+1})=\frac{3}{\sqrt{2x+3}}+$$

$$+\frac{2*2x}{2(x^2+1)\sqrt{x^2+1}}=\frac{3}{2x+3}+\frac{2x}{(\sqrt{x^2+1})^3}$$

b) $y=(e^{tgx}-2)^3$ «tashqi» funksiya darajali funksiyadir

$$\frac{dy}{dx}=3(e^{tgx}-2)^2(e^{tgx}-2)^1=3(e^{tgx}-2)^2e^{tgx}(tgx)'=$$

$$=3(e^{tgx}-2)^2e^{tgx}\frac{1}{\cos^2 x}=\frac{3(e^{tgx}-2)^2e^{tgx}}{\cos^2 x}$$

v) $y=\ln\cos(3x-1)$ logarifmik funksiya «tashqi» bo'ladi.

$$\frac{dy}{dx}=(\ln\cos(3x-1))'=\frac{1}{\cos(3x-1)}(\cos(3x-1))'=$$

$$=\frac{-\sin(3x-1)}{\cos(3x-1)}(3x-1)'=-3tg(3x-1)$$

g) $y=x^{\sin x}$ tenglikni logarifmlash qulayroqdir:

$$\ln y=\ln x^{\sin x}=\sin x\ln x,\quad (\ln y)'=\frac{y'}{y}$$

$$(\sin x\ln x)'=(\sin x)'\ln x+\sin x(\ln x)'=\cos x\ln x+\frac{\sin x}{x}$$

$$\frac{y'}{y}=\cos x\ln x+\frac{\sin x}{x}$$

$$y'=y\left(\cos x\ln x+\frac{\sin x}{x}\right)$$

yoki $y'=x^{\sin x}\left(\cos x\ln x+\frac{\sin x}{x}\right)$

d) $3x-y+ctgy=0$ oshkormas funksiyadan to'g'ridan-to'g'ri hosila olamiz.

$$3-y'-\frac{y'}{\sin^2 y}=0$$

y' ga nisbatan chiziqli tenglama hosil bo'ldi. Undan y' ni topamiz:

$$-y' - \frac{y'}{\sin^2 y} = -3$$

$$y' \cdot \frac{1+\sin^2 y}{\sin^2 y} = 3$$

$$y' = \frac{3\sin^2 y}{1+\sin^2 y}$$

e) $\begin{cases} x = t + 2t^2 \\ y = 3t^4 \end{cases}$

Bu parametrik funksiya bo'lib, t parametr bo'yicha hosila olamiz:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}; \qquad \frac{dx}{dt} = \left(t + 2t^3\right)'_t = 1 + 6t^2$$

$$\frac{dy}{dt} = \left(3t^4\right)'_t = 12t^3, \qquad \frac{dy}{dx} = \frac{12t^3}{1+6t^2}$$

5- masala

Differensial hisob usullari bilan $y = \frac{x^3+4}{x^2}$ funksiyani tekshiring va grafigini chizing.

Yechish:

1. Funksiyaning aniqlanish sohasi $x = 0$ dan tashqari barcha haqiqiy sonlarni o'z ichiga oladi;
2. $x = 0$ uzilish nuqtasi $\lim\limits_{x \to 0} y = \infty$ bundan $x = 0$ (OY o'qi) vertikal asimptotaligi kelib chiqadi. Og'ma asimptotani $y = kx + b$ ko'rinishida izlaymiz. Bunda

$$b = \lim_{x\to\infty}\left[f(x) - kx\right]$$

$$k = \lim_{x\to\infty} \frac{x^3+4}{x^3} = \lim_{x\to\infty}\left(1 + \frac{4}{x^3}\right) = 1$$

$$b = \lim_{x\to\infty}\left(\frac{x^3+4}{x^2} - x\right) = \lim_{x\to\infty}\frac{4}{x^2} = 0$$

demak, $y = x$ og'ma asimptota.

3. Funksiya toq ham, juft ham, davriy ham emas.
4. Grafikning koordinata o'qlari bilan kesishish nuqtalarini topamiz.

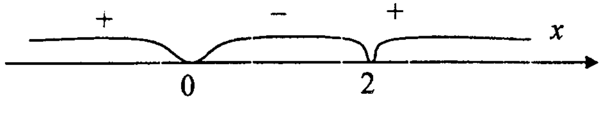
$$\frac{x^3+4}{x^2} = 0 \quad \text{bundan} \quad x = -\sqrt[3]{4}$$

5. O'sish, kamayish oraliqlari va ekstremumlarini topamiz. Buning uchun hosila olamiz:

$$y` = \left(\frac{x^3+4}{x^2}\right)` = \frac{3x^2x^2 - (x^3+4)2x}{x^4} = \frac{x^3-8}{x^3}$$

$$\frac{x^3-8}{x^3} = 0$$

x=2 kritik nuqta x=0 va x=2 nuqtalar sonlar o'qini oraliqlarga bo'ladi



-∞ <x<0 va x>2 da $f'(x) > 0$ demak, bu oraliqlarda funksiya o'suvchi,

$0 < x < 2$ da $f`(x) < 0$ kamayuvchi bo'ladi.

$x = 2$ nuqtaning atrofida funksiyaning ishorasi (-) dan (+) ga o'zgargani uchun bu nuqtada min ga erishadi.

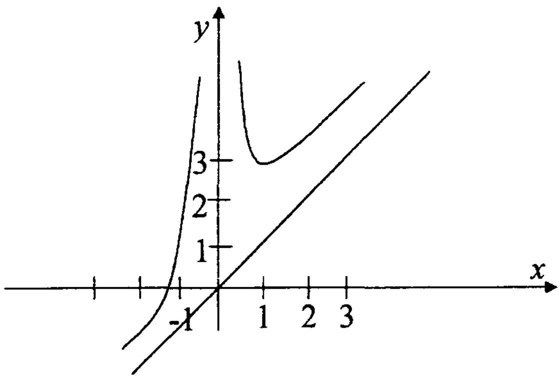
$$y_{\min} = y(2) = \frac{2^2+4}{2^2} = 3$$

6. Botiqlik, qavariqlik oraliqlari va burilish nuqtalarini aniqlaymiz. Buning uchun ikkinchi tartibli hosilani olamiz

$$y'' = \left(\frac{x^3-8}{x^3}\right)' = \frac{3x^2 \cdot x^3 - (x^3-8)3x^2}{x^6} = \frac{24}{x^4}$$

$x \neq 0$ va x ning hamma qiymatida $y'' > 0$. Demak, funksiya doimo botiq. Burilish nuqtalari yo'q.

Hosil qilingan ma'lumotlar asosida funksiyaning grafigini chizamiz.



6– masala

To'la kvadrat shakldа bo'lgan 32 m³ hajmli ochiq basseynning devorlari va tubiga ishlov berishda eng kam miqdorda material ketishi uchun uning o'lchovlari qanday bo'lishi kerak?

Yechish:

Kvadratning tomonini x, balandligini y deylik. U holda basseyn hajmi $V = x^2 y = 32$ bo'ladi. Basseynga ishlov beradigan sirtni S desak, $S = x^2 + 4xy$ bo'ladi.

y ni x orqali ifodalaymiz.

$S = x^2 + \frac{128}{x}$ funksiyani hosil qilamiz.

Hosila olamiz:

$$S = 2x - \frac{128}{x^2} = 0$$

$$x = 4$$

Bu yagona nuqta S ning eng kichik qiymatini beradi, chunki ikkinchi tartibli hosila bu nuqtada musbat. Demak, bu basseyn tubining tomoni 4 m va balandligi $y = 2$.

2- NAZORAT ISHI BO'YICHA MASALALAR

141–160 misollarda Lopital qoidasidan foydalanmasdan funksiya limitlarini hisoblang.

141. a) $\lim\limits_{x\to\infty}\frac{1-2x}{3x-2}$ b) $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{3x}$

d) $\lim\limits_{x\to 0}\frac{1-\cos x}{5x^2}$ e) $\lim\limits_{x\to\infty}\left(\frac{x+3}{x-2}\right)^x$

142. a) $\lim\limits_{x\to\infty}\frac{x^3+1}{2x^3+1}$ b) $\lim\limits_{x\to 7}\frac{\sqrt{2+x}-3}{x-7}$

d) $\lim\limits_{x\to 0}\frac{arctg 3x}{x}$ e) $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^x$

143. a) $\lim\limits_{x\to\infty}\frac{2x^3+x^2-5}{x^3+x-2}$ b) $\lim\limits_{x\to 1}\frac{x-\sqrt{x}}{x^2-x}$

d) $\lim\limits_{x\to 0}\frac{\sqrt{1-\cos^2 x}}{x}$ e) $\lim\limits_{x\to\infty}\left(\frac{4x+1}{4x}\right)^{2x}$

144. a) $\lim\limits_{x\to\infty}\frac{3x^4+x^2-6}{2x^4-x+2}$ b) $\lim\limits_{x\to 0}\frac{x}{\sqrt{1+3x}-1}$

d) $\lim\limits_{x\to 0}\frac{\cos x-\cos^3 x}{x^2}$ e) $\lim\limits_{x\to\infty} x[\ln(x+1)-\ln x]$

145. a) $\lim\limits_{x\to\infty}\frac{3+x+\sqrt{x}}{\sin 3x}$ b) $\lim\limits_{x\to 0}\frac{\sqrt{1+3x}\sqrt{1-2x}}{x-x^2}$

d) $\lim\limits_{x\to 0}\frac{x^2 ctg2x}{\sin 3x}$ e) $\lim\limits_{x\to\infty}(2x+1)[\ln(x+3)-x]$

146. a) $\lim\limits_{x\to\infty}\frac{2x^2+6x-5}{5x^2-x-1}$ b) $\lim\limits_{x\to 0}\frac{1-\sqrt{x-x^2}}{x^2}$

d) $\lim\limits_{x\to 0}\frac{5x}{arctgx}$ e) $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}}$

147. a) $\lim\limits_{x\to\infty}\frac{x-2x^2+5x^4}{2+3x^2+x^4}$ b) $\lim\limits_{x\to 0}\frac{\sqrt{1+3x^2}-1}{x^{2+x^3}}$

d) $\lim\limits_{x\to 0}\frac{1-\cos 6x}{1-\cos^2 x}$ e) $\lim\limits_{x\to 3}(x-3)[\ln(x-3)-\ln x]$

148. a) $\lim\limits_{x\to\infty}\frac{5x^2-3x+1}{3x^2+x-5}$ b) $\lim\limits_{x\to 3}\frac{\sqrt{2x-1}-\sqrt{5}}{x-3}$

d) $\lim\limits_{x\to 0}\frac{1-\cos 4x}{2xtg2x}$ e) $\lim\limits_{x\to 2}(3x-5)^{\frac{2x}{x^2-4}}$

149. a) $\lim\limits_{x\to\infty}\frac{7x^4-2x^3+2}{x^4+3}$ b) $\lim\limits_{x\to 5}\frac{\sqrt{1+3x}-\sqrt{2x+6}}{x^2-5x}$

d) $\lim\limits_{x\to 0}\frac{tg^2\frac{x}{2}}{x^2}$ e) $\lim\limits_{x\to 1}(7-6x)^{\frac{x}{3x-3}}$

150. a) $\lim\limits_{x\to\infty}\frac{8x^3-3x^2+9}{2x^3+2x^2+5}$ b) $\lim\limits_{x\to 2}\frac{x-2}{\sqrt{2x}-2}$

d) $\lim\limits_{x\to 0}5xctg3x$ e) $\lim\limits_{x\to 3}(3x-8)^{\frac{2}{x-3}}$

151. a) $\lim\limits_{x\to\infty}\frac{2x^3+7x^2-2}{6x^3-4x+3}$ b) $\lim\limits_{x\to 3}\frac{x^2+x-12}{\sqrt{x-2}-\sqrt{4-x}}$

d) $\lim\limits_{x\to 0}\dfrac{\sin^2\dfrac{x}{4}}{x^2}$ e) $\lim\limits_{x\to 1}(3-2x)^{\frac{x}{1-x}}$

152. a) $\lim\limits_{x\to\infty}\dfrac{1+4x-x^4}{x+3x^2+2x^4}$ b) $\lim\limits_{x\to -4}\dfrac{\sqrt{x+12}-\sqrt{4-x}}{x^2+2x-4}$

d) $\lim\limits_{x\to 0}\dfrac{1-\cos 3x}{x^2}$ e) $\lim\limits_{x\to 1}(2x+3)[\ln(x+2)-\ln x]$

153. a) $\lim\limits_{x\to\infty}\dfrac{14x^2+3x}{7x^2+2x-8}$ b) $\lim\limits_{x\to -5}\dfrac{\sqrt{3x+17}-\sqrt{2x+12}}{x^2+8x+15}$

d) $\lim\limits_{x\to 0}\dfrac{xtg3x}{\cos x-\cos^2 x}$ e) $\lim\limits_{x\to\infty}(3x+5)[\ln(x+5)-\ln x]$

154. a) $\lim\limits_{x\to\infty}\dfrac{3x^5-4x^2+1}{2x^5-3x^3+x}$ b) $\lim\limits_{x\to 3}\dfrac{\sqrt{5x+1}-4}{x-3}$

d) $\lim\limits_{x\to 0}\dfrac{tg^2 4x}{x\sin 3x}$ e) $\lim\limits_{x\to 2}(7-3x)^{\frac{x}{2x-4}}$

155. a) $\lim\limits_{x\to\infty}\dfrac{8x^2+4x+5}{4x^2-3x+2}$ b) $\lim\limits_{x\to 2}\dfrac{x^2-3x+2}{\sqrt{5-x}-\sqrt{x+1}}$

d) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x\sin x}$ e) $\lim\limits_{x\to 1}(2-x)^{\frac{2x}{1-x}}$

156. a) $\lim\limits_{x\to\infty}\dfrac{5x^4+x^2+1}{x^3-3x^4}$ b) $\lim\limits_{x\to 4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

d) $\lim\limits_{x\to 0}\dfrac{arctg\alpha\, x}{\sin 3x}$ e) $\lim\limits_{x\to 8}(2x-3)^{\frac{x}{x-2}}$

157. a) $\lim\limits_{x\to\infty}\dfrac{6x^5+4x^3+8}{2x^5-3x^2-1}$ b) $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{\sqrt{x-2}-4}$

d) $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{5x}$ e) $\lim\limits_{x\to 3}(2x-5)^{\frac{2x}{x-3}}$

158. a) $\lim\limits_{x\to\infty}\frac{5x^4+x^3+1}{3x^4-5x^2+7}$ b) $\lim\limits_{x\to 0}\frac{\sqrt{x^2+9}-3}{x^2}$

d) $\lim\limits_{x\to 0}\frac{3x}{arctg8x}$ e) $\lim\limits_{x\to 2}(3x-7)^{\frac{4x+5}{x-2}}$

159. a) $\lim\limits_{x\to\infty}\frac{2x^5+7x^3-4}{8x^5+3x^3+2}$ b) $\lim\limits_{x\to 3}\frac{\sqrt{2x+3}-3}{\sqrt{x-1}-\sqrt{2}}$

d) $\lim\limits_{x\to 0}xctg3x$ e) $\lim\limits_{x\to\infty}(\frac{3x-1}{3x+5})^x$

160. a) $\lim\limits_{x\to\infty}\frac{1+3x^2}{1-3x+4x^3}$ b) $\lim\limits_{x\to 0}\frac{\sqrt{x^2+16}-4}{x^2}$

d) $\lim\limits_{x\to 0}\frac{3x}{5tg2x}$ e) $\lim\limits_{x\to\infty}(\frac{x-5}{x+5})^{2x}$

161-180 masalalarda $y=f(x)$ funksiya va argumentning x_1 va x_2 qiymati berilgan. Quyidagilar talab qilinadi:

1) x ning berilgan qiymatlarining har qaysisi uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishini aniqlang;

2) Funksiya uzilishga ega bo'lgan x ning berilgan qiymatlaridan har biriga chapdan va o'ngdan yaqinlashganda funksiyaning limitini topish;

3) Sxematik chizma chizing.

161. $f(x)=9^{\frac{1}{x-2}}$ $x_1=2, x_2=4$

162. $f(x)=25^{\frac{1}{x-8}}$ $x_1=8, x_2=10$

163. $f(x)=9^{\frac{1}{x-7}}$ $x_1=7, x_2=9$

164. $f(x)=16^{\frac{1}{x-4}}$ $x_1=4, x_2=6$

165. $f(x)=4^{\frac{1}{x-5}}$ $x_1=5, x_2=7$

166. $f(x)=8^{\frac{1}{5-x}}$ $x_1=3, x_2=5$

167. $f(x)=9^{\frac{1}{x-2}}$ $x_1=2, x_2=4$

168. $f(x)=11^{\frac{1}{4+x}}$ $x_1=-4, x_2=-2$

169. $f(x)=17^{\frac{1}{x}}$ $x_1=0, x_2=2$

170. $f(x)=3^{\frac{1}{4-x}}$ $x_1=2, x_2=4$

171. $f(x)=13^{\frac{1}{5-x}}$ $x_1=-5, x_2=-3$

172. $f(x)=7^{\frac{1}{x-1}}$ $x_1=4, x_2=4$

173. $f(x)=5^{\frac{1}{x-4}}$ $x_1=4, x_2=6$

174. $f(x)=3^{\frac{1}{2+x}}$ $x_1=-4, x_2=-2$

175. $f(x)=9^{\frac{1}{x}}$ $x_1=-2, x_2=0$

176. $f(x)=6^{\frac{1}{4+x}}$ $x_1=-4, x_2=-2$

177. $f(x)=7^{\frac{1}{3+x}}$ $x_1=-5, x_2=-3$

178. $f(x)=2^{\frac{1}{x+5}}$ $x_1=-5, x_2=0$

179. $f(x)=5^{\frac{1}{x-1}}$ $x_1=1, x_2=2$

180. $f(x)=4^{\frac{1}{x-2}}$ $x_1=2, x_2=3$

181-200 misollarda $y=f(x)$ funksiya berilgan. Agar funksiyaning uzilish nuqtalari bo'lsa topib xarakterlang. Chizma chizing.

181. $f(x)=\begin{cases} x+4 & x<-1 \\ x^2+2 & -1\le x<1 \\ 2x & x\ge 1 \end{cases}$

182. $f(x)=\begin{cases} x+2 & x\le -1 \\ x^2+1 & -1<x\le 1 \\ -x+3 & x>1 \end{cases}$

183. $f(x)=\begin{cases} -x & x\le -1 \\ -(x-1)^2 & 0<x<2 \\ x-3 & x\ge 2 \end{cases}$

184. $f(x)=\begin{cases} \cos x & x\le 0 \\ x^2+1 & 0<x<1 \\ x & x\ge 1 \end{cases}$

185. $f(x)=\begin{cases} -x & x\le 0 \\ x^2 & 0<x\le 2 \\ x+1 & x>2 \end{cases}$

186. $f(x)=\begin{cases} -x & x\le 0 \\ \sin x & 0<x<\pi \\ x-2 & x\ge \pi \end{cases}$

187. $f(x)=\begin{cases} -x^2 & x\le 0 \\ tgx & 0<x\le \frac{\pi}{4} \\ 2 & x>\frac{\pi}{4} \end{cases}$

188. $f(x)=\begin{cases}-(x+1) & x\le -1\\ 1+x^2 & -1<x\le 0\\ x & x>0\end{cases}$

189. $f(x)=\begin{cases}-2x & x\le 0\\ x^2+1 & 0<x\le 1\\ 2 & x>1\end{cases}$

190. $f(x)=\begin{cases}-2x & x\le 0\\ \sqrt{x} & 0<x<4\\ 1 & x\ge 4\end{cases}$

191. $f(x)=\begin{cases}x^2+1 & x\le 1\\ 2x & 2<x\le 3\\ x+2 & x>3\end{cases}$

192. $f(x)=\begin{cases}x-3 & x<0\\ x+1 & 0\le x\le 4\\ 3-\sqrt{x} & x>4\end{cases}$

193. $f(x)=\begin{cases}2x^2 & x\le 0\\ x & 0<x\le 1\\ 2 & x>1\end{cases}$

194. $f(x)=\begin{cases}x-1 & x\le 0\\ x^2 & 0<x<2\\ 2x & x\ge 2\end{cases}$

195. $f(x)=\begin{cases}\sqrt{1-x^2} & x\le 0\\ 1 & 0<x\le 2\\ x-2 & x>2\end{cases}$

196. $f(x)=\begin{cases}\cos x & x\le 0\\ 1-x & 0<x\le 2\\ x^2 & x>2\end{cases}$

197. $f(x)=\begin{cases}\sin x & x\le 0\\ x & 0\le x\le 2\\ 0 & x>2\end{cases}$

198. $f(x)=\begin{cases}\sin x & x<0\\ x^2 & 0\le x<1\\ x-1 & x\ge 1\end{cases}$

199. $f(x)=\begin{cases}3x+1 & x\le 0\\ x^2+1 & 0<x\le 1\\ 1 & x>1\end{cases}$

200. $f(x)=\begin{cases}x+4 & x<-1\\ x^2+2 & -1\le x<1\\ x & x\ge 1\end{cases}$

201-220 misollarda funksiyalarning $\frac{dy}{dx}$ hosilalarini toping.

a) $y=2\sqrt{4x+3}-\frac{3}{\sqrt{x^3+x+1}}$ b) $y=\frac{1}{tg^2 2x}$

d) $y=\ln\sin(2x+5)$ e) $x^{2x-1}=y$

f) $tg\frac{y}{x}=5x$ g) $x=\cos\frac{t}{2}, y=t-\sin t$

202. a) $y=x^2\sqrt{1-x^2}$ b) $\frac{4\sin x}{\cos^2 x}$

d) $y=arctge^{2x}$ e) $y=x^{\frac{1}{x}}$

f) $x-y+arctgy=0$ g) $x1=t^3+8t, y=t^5+2t$

203. a) $y=x\sqrt{\frac{1+x^2}{1-x}}$ b) $y=(e^{\cos x}+3)^2$

d) $y=\arcsin\sqrt{1-3x}$ e) $y=x^{\ln x}$

f) $y=\sin x=cs(x-y)$ g) $x=t-\sin t, y=1-\cos t$

204. a) $y=\frac{3+6x}{\sqrt{3-4x+5x^2}}$ b) $y=\sin x-x\cos x$

d) $y=x^m\ln x$ e) $y=x^{-tgx}$

f) $\frac{y}{x}=arctg\frac{x}{y}$ g) $x=e^{2t}, y=\cos t$

205. a) $y=\frac{x}{\sqrt{a^2-x^2}}$ b) $y=\frac{-\sin^2 x}{2+3\cos^2 x}$

d) $y=\frac{x}{x-1}\ln x$ e) $y=(arctgx)^{\ln x}$

f) $(e^x-1)(e^y-1)=1$ g) $x=3\cos^2 t, y=\sin^2 x$

206. a) $y=\frac{1}{\sqrt{x^2+1}}+5\sqrt[5]{x^3+1}$ b) $y=2tg^3(x^2+1)$

d) $y=3^{arctgx^3}$ e) $y=(arctgx)^x$

f) $y^2x=e^{\frac{z}{x}}$ g) $x=3\cos t, y=4\sin^2 t$

207. a) $y=\sqrt[3]{\frac{1+x^2}{1-x^2}}$ b) $y=\frac{1}{2}tg^2x+\ln\cos x$

d) $y=arctg\frac{x}{1+\sqrt{1-x^2}}$ e) $y=(x+x^2)^x$

f) $x^3+y^3-3axy=0$ g) $x=3t-t^3, y=3t^2$

208. a) $y=3\sqrt[3]{x^5+5x^4-\frac{5}{x}}$ b) $y=\ln\sqrt{\frac{1-\sin x}{1+\sin x}}$

d) $y=arctgy(tg^2x)$ e) $y=(\sin x)^{\ln x}$

f) $x-y+a\sin y=0$ g) $x=2^t-t^3, y=2t^2$

209. a) $y=5\sqrt[5]{x^2+x+\frac{1}{x}}$ b) $y=2^x-e^{-x}$

d) $y=\frac{\arcsin x}{\sqrt{1-x^2}}$ e) $(\cos x)^x=y$

f) $\ln e=arctg\frac{x}{y}$ g) $x=t+\ln\cos t,$ $y=t-\ln\sin t$

210. a) $y=\sqrt{x^2+1}+\sqrt[3]{x^3+1}$ b) $y=\frac{1}{3}tg^3x+tgx+x$

d) $y=arctg\sqrt{\frac{3-x}{x-2}}$ e) $y=(\cos x)^{x^2}$

f) $x-y+e^y arctgx=0$ g) $x=\ln t, y=\frac{1}{2}(t+\frac{1}{2})$

211. a) $y=\sqrt[3]{x^4+5x}-\sqrt[4]{(5x-1)^3}$ b) $y=\frac{1+tgx}{1-tgx}$

d) $y=arctg\sqrt{x}-\sqrt{x}$ e) $y=x^{\frac{2}{x}}$

f) $x\sin y-y\cos x=0$ g) $x=t+\ln t, y=t-\ln t$

212. a) $y=\frac{3x}{\sqrt[3]{2+x}}-6\sqrt[3]{2+x}$ b) $y=\sin^3 2x$

d) $y=x\arcsin x+\sqrt{1-x^2}$ e) $g=x^{e^x}$

f) $e^{xy}-x^2+y^2=0$ g) $x=2t-\sin 2t, y=\sin^3 t$

213. a) $y=\sqrt{\frac{1+x^2}{1-x^2}}$ b) $y=e^{1+\ln^2 x}$

d) $y=arctg\frac{1}{x}$ e) $y=x^{\arcsin x}$

f) $y\sin x+\cos(x-y)=\cos y$ g) $x=t\frac{1}{2}\sin 2t$

214. a) $y=x\sqrt{\frac{1+x^2}{1-x^2}}$ b) $y=tg\ln\sqrt{x}$

d) $y=3^{\cos^2 x}$ e) $y=\frac{x}{e^x}$

f) $\cos(x-y)-2x+4y=0$ g) $x=t^5+2t, y=t^3+8t-1$

215. a) $y=x+\frac{1}{x+\sqrt{x^2+1}}$ b) $y=\sin\sqrt{1+x^2}$

d) $y=\ln ctg\sqrt[3]{x}$ e) $y=x^{\frac{1}{x^2}}$

f) $xe^y+ye^x=xy$ g) $x=\frac{1}{3}t^3+\frac{1}{2}t^2+t, y=\frac{1}{2}t^2+\frac{1}{3}$

216. a) $y=\sqrt[3]{2x-1}+\sqrt[4]{(x^3+2)^3}$ b) $y=\cos\ln^2 x$

d) $y=(e^{\sin x}-1)^2$ e) $y=2x^{\sqrt{x}}$

f) $\cos(xy)=\frac{y}{x}$ g) $x=\arcsin(t^3-1), y=\arccos 2t$

217. a) $y = x\sqrt[3]{\frac{2}{1+x}}$ b) $y = \frac{1+\sin 3x}{1-\sin 3x}$

d) $y = 2^{\frac{1-x}{1+x}}$ e) $y = (\ln x)^{2x}$

f) $xy + \ln y - 2\ln x = 0$ g) $x = t^2 + t + 1, y = t^3 - t$

218. a) $y = \sqrt[3]{1 + x\sqrt{x+3}}$ b) $y = \sqrt{1 + \ln^2 x}$

d) $y = e^{\frac{1}{x^2}}$ e) $y = (\sin x)^{\cos x}$

f) $x = e^{x+y} = \sin\frac{y}{x}$ g) $x = ctgt, y = \frac{1}{\cos^2 t}$

219. a) $y = \sqrt{\frac{x+\sqrt{x}}{x-\sqrt{x}}}$ b) $y = x\arcsin\frac{2x+1}{3}$

d) $y = (arctg2x^{\sin 3x})$ e) $(x+y)^2 = (x-2y)^3$

f) $x = \frac{2t}{2+t^2}, y = \frac{t^2}{2+t^2}$

220. a) $y = \frac{\sqrt{1+3x^2}}{2+3x^2}$ b) $y = e^{-x^2}\cos^3(2x+3)$

d) $y = xarctg^3 5x + \ln tgx$ e) $y = (\sin 3x)^{\sqrt{x}}$

f) $y\ln x - x\ln y = x + y$ g) $x = 2\cos^3 2t, y = \sin^3 2t$

221-240. Differensial hisob usullari bilan funksiyani tekshiring va grafigini chizing.

221. $y = \frac{x}{x^2+1}$ 222. $y = (\frac{x+1}{x-1})^2$

223. $y = \frac{x}{(x-1)^2}$ 224. $y = \frac{2x+1}{(x+1)^2}$

225. $y=\frac{x^2}{x^2-1}$

226. $y=\frac{x^3}{2(x+1)^2}$

227. $y=\frac{x^3+16}{x}$

228. $y=(\frac{x+2}{x-1})^2$

229. $y=\frac{x^3-1}{4x^2}$

230. $y=\frac{2}{x^2+x+1}$

231. $y=\frac{e^x}{x}$

232. $y=\ln(2x^2+3)$

233. $y=x^3e^{-x}$

234. $y=\frac{1}{e^x-1}$

235. $y=x-\ln(x+1)$

236. $y=e^{\frac{1}{x+2}}$

237. $y=\frac{1}{e^{2x}-1}$

238. $y=x^2\ln x$

239. $y=\ln\frac{x+1}{x+2}$

240. $y=x-\ln x$

241. Berilgan V hajmli silindrning to'la sirti eng kichik bo'lganda, uning radiusining balandligiga nisbatini toping.

242. V hajmli palatka to'g'ri doiraviy konus shakliga ega. Palatkaga eng kam miqdorda material ketishi uchun, konus balandligining asosiy radiusga nisbati qanday bo'lishi kerak.

243. Silindrik idishning tagi yarim sfera shaklida bo'lib, unga 18 l suv ketadi. Idishning tayyorlash uchun eng kam miqdorda material ketishi uchun uning o'lchovlari qanday bo'lishi kerak?

244. Doiraviy sektorning perimetri l ga teng. Sektorning yuzasi eng katta bo'lishi uchun uning radiusi qanday bo'lishi kerak?

245. 8 sonini shunday ikki qo'shiluvchiga ajratingki, ularning kublari yig'indisi eng kichik bo'lsin.

246. 36 sonini shunday ikki qo'shiluvchiga ajratingki, ularning kvadratlari yig'indisi eng kichik bo'lsin.

247. R radiusli sharga ichki joylashgan eng katta yon sirtli silindrning asosiy radiusi va balandligini toping.
248. (3;5) nuqtadan manfiy burchak koeffitsientli shunday to'g'ri chiziq o'tkazingki, uning koordinata o'qlari bilan hosil qilgan yuzasi eng kichik bo'lsin.
249. r radiusli doiraga to'g'ri to'rtburchak ichki chizilgan. To'g'ri to'rtburchakning yuzasi eng katta bo'lishi uchun uning o'lchamlari qanday bo'lishi kerak?
250. Yasovchi l bo'lgan konusning hajmi eng katta bo'lishi uchun, uning balandligi va asos radiusi qanday bo'lishi kerak?
251. r radiusli sharga tashqi chizilgan eng kichik hajmli konusning balandligini toping.
252. Tunukadan V sig'imli silindr shaklidagi qopqoqsiz idish yasalgan. Bunda material eng kam sarflash uchun idishning o'lchamlari qanday bo'lishi kerak?
253. R radiusli doiraning qolgan qismidan eng ko'p sig'imli voronka yasash uchun undan qanday sektor kesib olish mumkin?
254. To'g'ri burchakli uchburchakning gipotenuzasi $9\sqrt{2}$ ga teng. Uchburchakning perimetri eng kichik bo'lishi uchun uning katetlari qanday bo'lishi kerak?
255. $A(2;1)$ nuqtadan manfiy burchak koeffitsientli to'g'ri chiziq o'tkazingki, uning koordinata o'qlari bilan kesishganda hosil bo'lgan kesimlar uzunligining yig'indisi eng kichik bo'lsin.
256. Uchburchakli muntazam prizmaning hajmi V ga teng. Prizmaning to'la sirti eng kichik bo'lishi uchun asosining tomoni qanday bo'lishi kerak?
257. Aylanada A nuqta berilgan. A nuqtadan o'tkazilgan urinmaga parallel qilib BC vatar o'tkazingki, ABC uchburchakning yuzasi eng katta bo'lsin.
258. Teng yonli uchburchakning perimetri $2p$. Uning asosini atrofida aylantirishdan hosil bo'lgan jismning eng katta hajmga ega bo'lishi uchun uchburchak tomonlari qanday bo'lishi kerak?

259. Doiradan α markaziy burchakli sektor qirqib olingan. Sektordan kanonik sirt o'ralgan. α ning qanday qiymatida hosil qilingan konus eng katta hajmga ega bo'ladi?

260. Teng yonli uchburchak perimetri $2p$ ga teng. Uchburchakning asosiga tushirilgan balandligi atrofida aylantirishdan hosil bo'lgan konus eng katta hajmga ega bo'lishi uchun uchburchak tomonlari qanday bo'lishi kerak?

3- NAZORAT ISHI

1- masala

Quyidagi aniqmas integrallarni toping. Oxirgi uchtasida natijani differensiallab tekshiring.

A) $\int e^{\cos^2 x}\sin 2x dx$ b) $\int (x-1)e^{2x}dx$

V) $\int \frac{dx}{x^2-5x+6}$ g) $\int \frac{dx}{1+\sqrt{x+1}}$

d) $\int \cos^2 x dx$

Yechish:

A) $\int e^{\cos^2 x}\sin 2x dx$; $e^{\cos^2 x}=t$ almashtirish kiritamiz va buni differensiallaymiz

$$e^{\cos^2 x}(-2\cos x\sin x)dx = dt$$

$$-e^{\cos^2 x}\sin 2x dx = dt$$

$$-\int dt = -t + c = -e^{\cos^2 x} + c$$

b) $\int U dv = Uv - \int v dU$

bo'laklab integrallash formulasidan foydalanib yechamiz:

$$\int (x-1)e^{2x}dx = \left| \begin{array}{l} U = x-1, dU = dx \\ e^{2x}dx = dv \\ v = \frac{e^{2x}}{2} \end{array} \right| = (x-1)\frac{e^{2x}}{2} -$$

$$-\int \frac{e^{2x}}{2}dx = (x-1)\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

v) $x^2-5x+6=(x-2)(x-3)$

ko'paytuvchilarga ajratamiz. Integral ostidagi kasrni elementar kasrlarga ajratamiz

$$\frac{1}{(x-2)(x-3)}=\frac{a}{x-2}+\frac{b}{x-3}$$

$$a(x-3)+b(x-2)=1$$
$$x=3\partial a, b=1$$
$$x=2\partial a, a=-1$$

$$\int\frac{dx}{x^2-5x+6}=\int\frac{dx}{(x-2)(x-3)}=-\int\frac{dx}{(x-2)}+\int\frac{dx}{(x-3)}=$$
$$=-\ln|x-2|+\ln|x-3|+c=\ln\left|\frac{x-3}{x-2}\right|+c$$

Tekshirish:

$$\left(\ln\left|\frac{x-3}{x-2}\right|+c\right)'=\frac{x-2}{x-3}\cdot\frac{1}{(x-2)^2}=\frac{1}{x^2-5x+6}$$

g) $1+\sqrt{x+1}=t$

$x+1=(t-1)^2$

$dx=2(t-1)dt$

$$\int\frac{dx}{1+\sqrt{x+1}}=\int\frac{2(t-1)dt}{t}=2\int(1-\frac{1}{t})dt=2(t-\ln t)+c=$$
$$=2(1+\sqrt{x+1}-\ln(1+\sqrt{x+1}))+c$$

Tekshirish:

$$2((1+\sqrt{x+1}-\ln(1+\sqrt{x+1}))+c)'=2\left(\frac{1}{2\sqrt{x+1}}-\frac{\frac{1}{2\sqrt{x+1}}}{1+\sqrt{x+1}}\right)=$$
$$=\frac{1}{\sqrt{x+1}}\left(1-\frac{1}{1+\sqrt{x+1}}\right)=\frac{\sqrt{x+1}}{\sqrt{x+1}(1+\sqrt{x+1})}=\frac{1}{1+\sqrt{x+1}}$$

d) $\int\cos^2 x dx=\int\frac{1+\cos 2x}{2}dx=\frac{x}{2}+\frac{\sin 2x}{4}+c$

Tekshirish:

$$\left(\frac{x}{2}+\frac{\sin 2x}{4}+c\right)'=\frac{1}{2}+\frac{\cos 2x}{2}=\frac{1+\cos 2x}{2}=\cos^2 x$$

2- masala.

Xosmas integralni hisoblang yoki uzoqlashuvchi ekanligini ko'rsating

A) $\int\limits_0^{\infty}\frac{arctgx}{1+x^2}dx$

Yechish:

$$\int\limits_0^{\infty}\frac{arctgx}{1+x^2}dx=\lim_{a\to\infty}\int\limits_0^{a}\frac{arctgx}{1+x^2}dx=\lim_{a\to\infty}\int\limits_0^{a}arctgx\,d(arctgx)=$$

$$=\lim_{a\to\infty}\frac{(arctgx)^2}{2}\Bigg|_0^a=\lim_{a\to\infty}\frac{arctg^2 a}{2}=\frac{\left(\frac{\pi}{2}\right)^2}{2}=\frac{\pi^2}{8}$$

integral yaqinlashuvchi

b) $\int\limits_0^{1}\frac{dx}{x}$

chegaralari chekli bo'lsa ham $x=0$ da integral ostidagi funksiya cheksiz bo'ladi.

$$\int\limits_0^{1}\frac{dx}{x}=\lim_{a\to 0}\int\limits_0^{1}\frac{dx}{x}=\lim_{a\to 0}\ln x\Big|_a^1=\lim_{a\to 0}(\ln 1-\ln a)=+\infty$$

demak, integral uzoqlashuvchi.

3- masala

$y=-x^2, x+y+2=0$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

Yechish:

$$\begin{cases} y = -x^2 \\ x + y + 2 = 0 \end{cases}$$

sistema yechimi

$$-x^2 + x + 2 = 0$$

$$x_1 = -1, x_2 = 2$$

$$y_1 = -1, y_2 = -4$$

$$S = \int_1^2 (-x^2 + x + 2)dx = (-\frac{x^3}{3} + \frac{x^2}{2} + 2x)\Big|_{-1}^{2} =$$

$$= (-\frac{2^3}{3} + \frac{2^2}{3} + 2 \cdot 2) - (\frac{(-1)^3}{3} + \frac{(-1)^2}{3} + 2 \cdot (-1)) =$$

$$= -\frac{8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2 = 4{,}5 \text{ kv.}$$

4- masala

$y^2 = (x-2)^3$ egri chiziq va $x = 2$ to'g'ri chiziq bilan chegaralangan hamda *OX* o'qi atrofida aylanishdan hosil bo'lgan jism hajmini hisoblang.

Yechish:

$$V_x = \pi \int_1^2 y^2 dx = \pi \int_1^2 (x-1)^3 dx = \pi \frac{(x-1)^4}{4}\Bigg|_1^2 = \frac{\pi}{4} \text{ hajm bir.}$$

5- masala

$t = 0$ dan $t = 3$ gacha bo'lgan $x = \frac{1}{3}.\ t^3 - t, y = t^2 + 2$ egri chiziq yoyining uzunligini hisoblang.

Yechish:

$$L=\int_{t_1}^{t_2}\sqrt{(x_t')^2+(y_t')^2}dt$$ formuladan foydalanamiz.

$$x'_t=(\frac{1}{3}t^3-t)=t^2-1, y'_t=(t^2+2)'=2t$$

$$L=\int_0^3\sqrt{(t^2-1)^2+(4t^2)}dt=\int_0^3\sqrt{t^4+2t^2+1}dt=$$

$$=\int_0^3(t^2+1)dt=\left(\frac{t^3}{3}+t\right)\Bigg|_0^3=\frac{3^3}{3}+3=12$$

6- masala

$y=\sin 2x$ sinusoidaning $x=0$ dan $x=\frac{\pi}{2}$ gacha bo'lgan yoyning OX o'qi atrofidan hosil bo'lgan sirt yuzasini hisoblang.

Yechish:

$$S_x=2\pi\int_a^b y\sqrt{1+y'^2}dx$$ formuladan

$$y'=2\cos 2x$$

$$S_x=2\pi\int_0^{\frac{\pi}{2}}\sin 2x\sqrt{1+4\cos^2 2x}dx$$

$$2\cos 2x=t, -4\sin 2xdx=dt$$

$$\sin 2x=-\frac{dt}{4},\ x=0\ \partial a\ t=2,\ x=\frac{\pi}{2}\ \partial a\ t=-2$$

$$S=2\pi\int_{2}^{-2}\sqrt{1+t^2}\left(-\frac{dt}{4}\right)=\frac{\pi}{2}\int_{2}^{-2}\sqrt{1+t^2}\,dt=$$

$$=\frac{\pi}{2}\left[\frac{t}{2}\sqrt{1+t^2}+\frac{1}{2}\ln\left(t+\sqrt{1+t^2}\right)\right]_{-2}^{2}=$$

$$=\frac{\pi}{2}\left[2\sqrt{5}+\ln(\sqrt{5}+2)\right]$$

7- masala

$x=a\cos t$, $y=b\cos t$ ellips yoyi (1 chorakda) va koordinata o'qlari bilan chegaralangan figuraning og'irlik markazi koordinatalarini toping.

Yechish:

1 chorakda x dan a gacha oshib borganda $t\,\frac{\pi}{2}$ dan 0 gacha kamayadi, shuning uchun

$$\bar{x}=\frac{1}{S}\int_0^a xy\,dx=\frac{1}{S}\int_0^a a\cos t\cdot b\sin t(-a\sin t)dt=$$

$$=\frac{a^2b}{S}\int_0^{\frac{\pi}{2}}\sin^2 t\cos t\,dt=\frac{a^2b}{S}\cdot\frac{1}{3}\sin^3 t\Big|_0^{\frac{\pi}{2}}=\frac{a^2b}{3S}$$

$S=\pi\,ab$ ellips yuzasidan $\bar{x}=\dfrac{4a}{3\pi}$, xuddi shunday

$$\bar{y}=\frac{1}{2S}\int_0^a y^2dx=\frac{1}{2S}\int_{\frac{\pi}{2}}^{0}b^2\sin^2 t\cdot(-a\sin t)dt=$$

$$=\frac{2ab^2}{\pi ab}\int_{\frac{\pi}{2}}^{0}(-1-\cos^2 t)d(\cos t)=\frac{2b}{\pi}\left[\cos t-\frac{1}{3}\cos^3 t\right]_{\frac{\pi}{2}}^{0}=\frac{4b}{3\pi}$$

3- NAZORAT ISHI BO'YICHA MASALALAR

261-280 misollarda aniqmas integrallarni toping. Oxirgi uchtasida natijani tekshiring.

261. a) $\int e^{\sin^2 x}\sin 2x dx$ b) $\int arctg\sqrt{x}dx$

d) $\int \frac{dx}{x^3+8}$ e) $\int \frac{dx}{1+\sqrt[3]{x+1}}$

f) $\int \frac{(\sin x+\sin^3 x)}{\cos 2x}dx$

262. a) $\int \frac{xdx}{(x^2+4)^6}$ b) $\int e^x \ln x(1+3e^x)dx$

d) $\int \frac{2x^2-3x+1}{x^3+1}dx$ e) $\int \frac{dx}{\sqrt{x^2-x-1}}$

f) $\int \frac{dx}{\sin x+tgx}$

263. a) $\int \frac{(3x-7)dx}{x^3+4x^2+4x+16}$ b) $\int \frac{dx}{\sin x+tgx}$

d) $\int \frac{(3x-7)dx}{x^3+4x^2+4x+16}$ e) $\int \frac{dx}{\sqrt{x+3}+\sqrt[3]{(x+3)^2}}$

f) $\int \sin^4 x\cos^5 xdx$

264. a) $\int \frac{dx}{\cos^2 x(3tgx+1)}$ b) $\int \frac{x\arcsin x}{\sqrt{1-x^2}}dx$

d) $\int \frac{dx}{x^3+x^2+2x+2}$ e) $\int \frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$

f) $\int \sin^3 x dx$

265. a) $\int \frac{\cos 3x dx}{4+\sin 3x}$ b) $\int x^2 e^{3x} dx$

d) $\int \frac{x^2 dx}{x^3+5x^2+8x+4}$ e) $\int \frac{\cos x dx}{1+\cos x}$

f) $\int \sin^4 x - \cos^4 x dx$

266. a) $\int \frac{\sin dx}{\sqrt[3]{\cos^2 x}}$ b) $\int x \arcsin \frac{1}{x} dx$

d) $\int \frac{(x+3)dx}{x^3-x^2-2x}$ e) $\int \frac{\sqrt[4]{x}+1}{(\sqrt{x}+4)\sqrt[4]{x^3}} dx$

f) $\int \cos^4 x dx$

267. a) $\int \frac{x+arctgx}{1+x^2} dx$ b) $\int x \ln(x^2+1) dx$

d) $\int \frac{(x^2-3)}{x^4+5x^2+6} dx$ e) $\int \frac{\sqrt{x+5}dx}{1+\sqrt[3]{x+5}}$

f) $\int ctg^3 3x dx$

268. a) $\int \frac{arctg\sqrt{x}dx}{\sqrt{x}(1+x)}$ b) $\int \sin x \cos x dx$

d) $\int \frac{x^2 dx}{x^4-81}$ e) $\int \frac{dx}{3\cos x+4\sin x}$

f) $\int \frac{\cos^2 x}{\sin 4x} dx$

269. a) $\int \frac{\sin x dx}{\sqrt[3]{3+2\cos x}}$ b) $\int x^2 \sin 4x dx$

d) $\int \frac{(x^2-x+1)}{x^4+2x^2-3} dx$ e) $\int \frac{(\sqrt{x}-1)(\sqrt[3]{x}+1)}{\sqrt[3]{x^2}} dx$

f) $\int \frac{dx}{\cos^3 x} dx$

270. a) $\int \frac{(\sqrt[3]{4+\ln x})dx}{x}$ b) $\int x \ln^2 x dx$

d) $\int \frac{(x^3-6)dx}{x^4+6x^2+8}$ e) $\int \frac{dx}{2\sin x + \cos x + 2}$

f) $\int \sin 3x - \sin x dx$

271. a) $\int \frac{dx}{(\arcsin x)^3 \sqrt{1-x^2}}$ b) $\int x^3 e^{2x} dx$

d) $\int \frac{dx}{x^3 - x^2}$ e) $\int \frac{dx}{(1+\sqrt[4]{z})\sqrt{z}}$

f) $\int \frac{\sin^3 x}{\sqrt[4]{\cos x}} dx$

272 a) $\int \frac{\sin 2x}{\sqrt{1+\cos^2 x}} dx$ b) $\int x^2 \cos x dx$

d) $\int \frac{dx}{x^4 - 1}$ e) $\int \frac{x dx}{\sqrt{x+1}+\sqrt{x+3}}$

f) $\int \frac{\sin^4 x dx}{\cos^2 x}$

273. a) $\int \frac{x^2 dx}{\sqrt{2-x}}$ b) $\int x^3 \ln x dx$

d) $\int \frac{dx}{x^3+x^2+4x+4}$ e) $\int \frac{x^2dx}{\sqrt{4-x^2}}$

f) $\int \frac{dx}{\sin x}$

274. a) $\int \frac{dx}{2\sin^2 x+3\cos^2 x}$ b) $\int \ln(x+\sqrt{1+x^2})dx$

d) $\int \frac{dx}{x^3+1}$ e) $\int \frac{dx}{x^2\sqrt{x^2+1}}$

f) $\int \frac{\sin xdx}{1-\sin x}$

275. a) $\int \frac{1-2x}{\sqrt{1-4x^2}}dx$ b) $\int \frac{x\cos x}{\sin^3 x}dx$

d) $\int \frac{x^2dx}{x^4-16}$ e) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

f) $\int \frac{dx}{5+3\cos x}$

276. a) $\int \frac{\cos xdx}{\sqrt[5]{\sin^2 x}}$ b) $\int (x+1)\ln^2(x+1)dx$

d) $\int \frac{xdx}{x^3-1}$ e) $\int \frac{dx}{x^3\sqrt{x^2-1}}$

f) $\int (3+\cos 2x)\sin^2 xdx$

277. a) $\int \frac{x^2dx}{\sqrt{5+x^6}}$ b) $\int \frac{xdx}{\cos^2 x}$

d) $\int \frac{(x-1)dx}{x^3+x}$ e) $\int \frac{x^3dx}{\sqrt{(1+x^2)^5}}$

f) $\int ctg^4 x dx$

278. a) $\int \frac{\cos x \sin 2x}{3\cos^2 x+2}dx$ b) $\int \cos x^2 dx$

d) $\int \frac{x^2dx}{x^4+x^2-2}$ e) $\int \frac{(\sqrt[6]{x}+1)dx}{\sqrt[6]{x^7}+\sqrt[6]{x^5}}$

f) $\int \frac{\cos 2x}{\sin^4 x}dx$

279. a) $\int \frac{e^{2x}dx}{\sqrt[4]{e^x+1}}$ b) $\int x^3 arctgx dx$

d) $\int \frac{x^4dx}{x^4+5x^2+4}$ e) $\int \frac{dx}{\sqrt[3]{(2x+1)^2}+\sqrt{2x+1}}$

f) $\int \frac{dx}{4\sin x+3\cos x+5}$

280. a) $\int \frac{\sqrt{1+\ln x}}{x}dx$ b) $\int x^3 e^{x^3} dx$

d) $\int \frac{(x^3-2xd)x}{x^4+2x^2+1}$ e) $\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx$

f) $\int tg^4 x dx$

281-300 misollarda xosmas integrallarni hisoblang yoki uzoqlashuvchi ekanligini ko'rsating.

281. a) $\int\limits_{-\infty}^{+\infty} \frac{dx}{x^2+2x+2}$ b) $\int\limits_{1}^{2} \frac{dx}{x\ln x}$

282. a) $\int_{1}^{+\infty}\frac{\ln(x^3+1)}{x^2}dx$ b) $\int_{0}^{1}\frac{dx}{x^3-5x^2}$

283. a) $\int_{4}^{+\infty}\frac{dx}{x\ln^3 x}$ b) $\int_{0}^{2}\frac{x^5dx}{\sqrt{4-x^2}}$

284. a) $\int_{1}^{+\infty}\frac{dx}{(1+x)\sqrt{x}}$ b) $\int_{2}^{3}\frac{xdx}{\sqrt{(x^2-4)^3}}$

285. a) $\int_{2}^{+\infty}\frac{dx}{x\ln x}$ b) $\int_{0}^{1}\frac{x^4dx}{\sqrt{1-x^5}}$

286. a) $\int_{1}^{+\infty}\frac{x^2dx}{1+x^6}$ b) $\int_{0}^{1}\frac{x^2dx}{\sqrt{1-x^3}}$

287. a) $\int_{1}^{+\infty}\frac{dx}{x^2+x+1}$ b) $\int\frac{dx}{(x-1)^2}$

288. a) $\int_{0}^{+\infty}xe^{-x^2}dx$ b) $\int_{-3}^{2}\frac{dx}{(x+3)^2}$

289. a) $\int_{-\infty}^{-3}\frac{xdx}{(x^2+1)^2}$ b) $\int_{0}^{3}\frac{dx}{(x-2)^2}$

290. a) $\int_{-\infty}^{+\infty}\frac{dx}{x^2+4x+5}$ b) $\int_{0}^{4}\frac{dx}{\sqrt[3]{(x-3)^2}}$

291. a) $\int_{4}^{+\infty}\frac{dx}{x^2-5x+6}$ b) $\int_{1}^{2}\frac{dx}{(x-1)^2\ln x}$

292. a) $\int_{1}^{+\infty}\frac{xdx}{1+x^2}$ b) $\int_{-1}^{0}\frac{dx}{(x+1)^2}$

293. a) $\int_{1}^{+\infty}\frac{xdx}{1+x^4}$ b) $\int_{2}^{3}\frac{dx}{x^2-3x+2}$

294. a) $\int_{-\infty}^{0}\frac{dx}{4-x^2}$ b) $\int_{0}^{1}\frac{dx}{\sqrt{x(1-x)}}$

295. a) $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$; b) $\int_{-1}^{1}\frac{dx}{x^2}$;

296. a) $\int_{1}^{\infty}\frac{\ln a}{x^2}$; b) $\int_{0}^{1}x\ln^2 x dx$;

297. a) $\int_{2}^{+\infty}\frac{ldx}{x\ln^2 x}$; b) $\int_{0}^{1}\frac{dx}{\sqrt{x}+x\sqrt{x}}$;

298. a) $\int_{-\infty}^{0}\frac{dx}{1+x+x^2}$; b) $\int_{0}^{0.25}\frac{dx}{\sqrt{x+x^2}}$;

299. a) $\int_{0}^{\infty}\frac{dx}{q+x^2}$; b) $\int_{0}^{1}\ln x dx$;

300. a) $\int_{-\infty}^{+\infty}\frac{dx}{(x+1)^2}$; b) $\int_{-1}^{0}\frac{dx}{1+x}$;

301. $y^2=x+1$ va $y^2=9-x$ parabolalar bilan chegaralangan figuraning yuzini hisoblang.
302. $x=3\cos^3 t$, $y=3\sin^3 t$ astroida bilan chegaralangan figuraning yuzini hisoblang.
303. $x-y+1=0$ to'g'ri chiziq, $y=\cos x$ kosinusoidda yoyi va OX o'q bilan chegaralangan figuraning OX o'q atrofida aylanishidan hosil bo'lgan jismning hajmini hisoblang.
304. $y=\frac{8}{x^2+4}$ Anezi chizig'i va $x^2=4y$ parabola bilan chegaralangan figuraning OY o'q atrofida aylanishidan hosil bo'lgan jismning hajmini hisoblang.

305. $y=\frac{x^2}{2}$ parabola yoyining koordinatalari boshidan abssissasi $x=6$ bo'lgan nuqtagacha uzunligini hisoblang.

306. $x=s(t-\sin t)$, $y=3(1-\cos t)$, $0\le t\le 2\pi$ sikloida bir arkining uzunligini hisoblang.

307. $y^2=x(0\le x\le 4)$ parabolaning OX o'qi atrofida aylanishidan hosil bo'lgan bir jinsli jismning og'irlik markazining koordinatalarini toping.

308. Agar $x=\cos^3 t$, $y=\sin^3 t$ astroida har bir nuqtasi chiziqli zichligi shu nuqtaning abssissasiga teng bo'lsa, birinchi kvadratda joylashgan astroida yoyi og'irlik markazining koordinatalarini hisoblang.

309. $r=3(1-\cos y)$ kardioida va $r=3\cos\varphi$ aylana bilan chegaralangan figuraning yuzini hisoblang.

310. $x=0$ dan $x=\frac{1}{2}$ bo'lgan $y=x^3$ yoy bo'lagining o'q atrofida aylanishidan hosil bo'lgan sirt yuzasi hisoblansin.

311. $x=t-\sin t$, $y=1-\cos t$ bir markazining aylanishi yuzasi egri chiziq yoyining OX o'q atrofida hosil bo'lgan sirt yuzasini hisoblang.

312. $y=\sqrt{x}\,e^x$, $x=1$, $y=0$ chiziqlar bilan chegaralangan figuraning OX o'q atrofida hosil bo'lgan jismning hajmini toping.

313. $x=\frac{\pi}{3}$ dan $y=\frac{\pi}{2}$ gacha bo'lgan $y=\ln\sin x$ yoy uzunligini hisoblang.

314. $0=0$ va $0=\pi$ gacha bo'lgan $y=\ln\sin x$ yoy uzunligini hisoblang.

315. $y=\frac{16}{x^2}$, $y=17-x^2$ I chorak chiziqlari bilan chegaralangan figuraning yuzini hisoblang.

316. $xy=20$, $x^2+y^2=41$ I chorak chiziqlari bilan chegaralangan figuraning yuzini hisoblang.

317. $y=\frac{1}{4}x^2$, $y=3x-\frac{1}{2}x^2$ chiziqlari bilan chegaralangan figuraning yuzini hisoblang.

318. $y=\sqrt{r^2-x^2}$ yarim aylananing og'irlik markazining koordinatalarini toping.

319. $y=4-x^2$, $y=0$ chiziqlar bilan chegaralangan parabola segmentining og'irlik markazi koordinatalarini hisoblang.

320. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning OX o'qi atrofida aylanishidan hosil bo'lgan sirtni hisoblang.

4– NAZORAT ISHI

1– masala

$z=\frac{x}{y}$ funksiya berilgan.

$x\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial z}{\partial y}=0$ ekanligini isbotlang.

Isbot:

Xususiy hosilalarini olamiz:

$\frac{\partial z}{\partial x}=\frac{1}{y}$, $\frac{\partial z}{\partial y}=-\frac{x}{y^2}$, $\frac{\partial^2 z}{\partial x\partial y}=-\frac{1}{y^2}$ berilgan tenglikka qo'ysak,

$x\left(-\frac{1}{y^2}\right)-\left(-\frac{x}{y^2}\right)=-\frac{x}{y^2}+\frac{x}{y^2}=0$ kelib chiqadi.

2– masala

$z=x^2+3y^2+x-y$ funksiyaning $x=1$, $y=1$, $x+y=1$ chiziqlar bilan chegaralangan uchburchakdagi eng katta va eng kichik qiymatlarini toping.

Yechish:

ΔABC ning hamma tomonlarini ko'rib chiqamiz: AB: $y=I$ buni funksiyaga qo'ysak $z=x^2+x+2$ ni hosil qilamiz.

$z`=2x+1=0$, $x=-1$

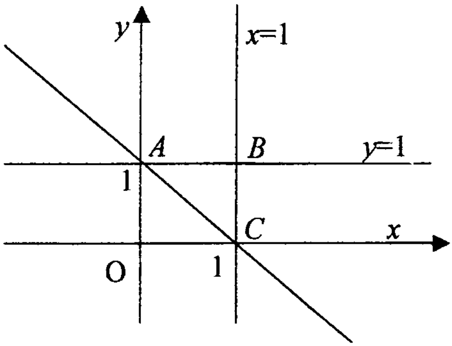
$x=-\frac{1}{2}$ berilgan sohaga kamaydi. Bu yerda $z(A)$, $z(B)$ larni hisoblash kifoya.

$z(A)=0^2+3\cdot 1^2+0-1=2$,

$z(B)=1^2+3\cdot 1^2+1-1=4$

BC: $x=1$, $z=3y^2-y+2$, $z`=6y-1$, $y=\frac{1}{6}$

$$z\left(1;\frac{1}{6}\right)=1^2+3\cdot\frac{1}{36}+1-\frac{1}{6}=2+\frac{1}{12}-\frac{1}{6}=\frac{23}{12};$$



AC: $y=1-x$ buni funksiyaga qo'yamiz:

$$z=x^2+3(1-x)^2+x-(1-x)=4x^2-4x+2=0$$

$$z`=8x-4,\quad x=\frac{1}{2},\quad z\left(\frac{1}{2};\frac{1}{2}\right)=\frac{1}{4}+3\frac{1}{4}+\frac{1}{2}+\frac{1}{2}=1$$

$$z=1,\quad z=2,\quad z=\frac{23}{6},\quad z=4$$

larni hosil qildik. Bundan $z_{\text{eng kichik}}=1$, $z_{\text{eng katta}}=4$.

3– masala.

$z=x^2-xy+y^2$ funksiya $A(1;1)$ nuqta va $\bar{a}=6\bar{i}+8\bar{j}$ vektor berilgan bo'lsa, A nuqtadagi *grad* z ni, $\bar{a}$ vektor yo'nalish bo'yicha z ning hosilasini toping.

Yechish:

$$grad\ z=\frac{\partial z}{\partial x}\bar{i}+\frac{\partial z}{\partial y}\bar{j}$$

formula bo'yicha gradiyentini hisoblaymiz:

$$grad\ z=(2x-y)_A\bar{i}+(-x-2y)_A\bar{j}=\bar{i}+\bar{j}$$

Yo'nalish bo'yicha hosila

$$\frac{\partial z}{\partial \bar{a}} = \frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha$$

formula bilan hisoblanadi.

$$\cos\alpha = \frac{6}{\sqrt{6^2+8^2}} = \frac{6}{10} = 0.6, \qquad \sin\alpha = \frac{8}{\sqrt{6^2+8^2}} = 0.8$$

$$\left(\frac{\partial z}{\partial \bar{a}}\right)_A = 1\cdot 0.6 + 1\cdot 0.8 = 1.4$$

4- masala

$xyy` = y^2 + 2x^2$ differensial tenglamani yeching.

Yechish:

Bu bir jinsli tenglama bo'lib, $y = tx$ almashtirish kiritamiz.

$y` = t`x + t$ tenglamaga qo'ysak

$xtx(t`x + t) = x^2 + 2x^2$, $x^2 \neq 0$ ga bo'lsak, $t(t`x + t) = t^2 + 2$

yoki $tt`x = 2$, $\frac{tdt}{dx}\cdot x = 2$

o'zgaruvchilarni ajratib integrallaymiz

$$\int tdt = \int 2\frac{dx}{x} + C, \qquad \frac{t^2}{2} = 2\ln Cx$$

C ni $\ln C$ deb oldik. $t^2 = 4\ln Cx$ avvalgi o'zgaruvchiga qaytib umumiy yechimni hosil qilamiz:

$$y^2 = 4x^2 \ln Cx.$$

5- masala

$$y` - \frac{y}{x\ln x} = x\ln x.$$

Differensial tenglamani yeching.

Yechish:

Bu chiziqli tenglama. $y = UV$ almashtirish kiritamiz.

$y` = U`V + UV`$ bularni tenglamaga qo'yamiz.

$$U`V+UV`-\frac{UV}{x\ln x}=x\ln x$$

$$U`V+U\left(V`-\frac{V}{x\ln x}\right)=x\ln x$$

qavsdagini «0» ga tenglaymiz.

$$V`=\frac{V}{x\ln x}, \qquad \int\frac{dV}{V}=\int\frac{dx}{x\ln x}$$

$$\ln V=\ln\ln x, \qquad V=\ln x$$

$U`V=x\ln x$ yoki $U`\ln x=x\ln x$, $\ln x\neq 0$ $U`=x$,

$$U=\int x+c=\frac{x^2}{2}+c$$

demak, $y=UV=\left(\frac{x^2}{2}+C\right)\ln x$.

6– masala

$(1-x^2)y``-xy`=2$ tenglamani yeching.

Yechish:

Bu ikkinchi tartibli differensial tenglama. $y`=P(x)$ almashtirish kiritamiz. $y``=p`$ bo'ladi. Bularni tenglamaga qo'yamiz: $(1-x^2)P`-xP=2$. Birinchi tartibli chiziqli tenglama hosil bo'ldi. $P=UV$ desak, $P`=U`V+UV`$

$$(1-x^2)(U`V+UV`)-UVx=2$$

$$U`V+UV`-x^2U`V-x^2UV`-UVx=2$$

$$U`V-x^2U`V-U(V`x^2V`-Vx)=2$$

$$V`-x^2V`=Vx, \qquad V`(1-x^2)=Vx$$

$$\int\frac{dV}{V}=\int\frac{xdx}{1-x^2}, \qquad V=\frac{1}{\sqrt{1-x^2}}$$

$$U`V(1-x^2)=2, \qquad U`(1-x^2)\cdot\frac{1}{\sqrt{1-x^2}}=2$$

$$U' = \frac{2}{\sqrt{1-x^2}};$$

$$U = \int \frac{2}{\sqrt{1-x^2}} dx + C_1 = 2\arcsin x + C_1$$

$$P = UV = (\arcsin x + C_1)\frac{1}{\sqrt{1-x^2}};$$

yoki

$$y = \int \frac{2\arcsin x + C_1}{\sqrt{1-x^2}} dx + C_1 =$$

$$= \int \frac{2\arcsin x}{\sqrt{1-x^2}} dx + C_1 \int \frac{dx}{\sqrt{1-x^2}} + C_2 = \arcsin^2 x + C_1 \arcsin x + C_2$$

Umumiy echim: $y = \arcsin^2 x + C_1 \arcsin x + C_2$

7– masala

$y'' + y = \cos 3x$ tenglamani $y\left(\frac{\pi}{2}\right) = 4$, $y'\left(\frac{\pi}{2}\right) = 1$ boshlang'ich shartlar asosida xususiy yechimini toping.

Yechish:

$y'' + y = 0$ bir jinsli tenglamani yechamiz. $y = e^{kx}$ deylik,

$$y' = ke^{kx}, \qquad y'' = k^2 e^{kx}$$

Bundan $k^2 + 1 = 0$, $k_{1,2} = \pm i$

Bir jinsli tenglamaning umumiy yechimini hosil qilamiz.

$$\tilde{y} = C_1 \cos x + C_2 \sin x.$$

Endi bir jinsli bo'lmagan tenglamaning xususiy yechimini

$$y^* = A\cos 3x + B\sin 3x$$

ko'rinishida qidiramiz.

$$y^{*'} = -3A\sin 3x + 3B\cos 3x$$

$$y^{*''} = -9A\cos 3x - 9B\sin 3x$$

Tenglamaga qo'yamiz.

$$-9A\cos 3x - 9B\sin 3x + A\cos 3x + B\sin 3x = \cos 3x$$

koeffitsientlarini solishtirib quyidagilarni topamiz:

$$-8A=1, \qquad A=-\frac{1}{8};$$

$$-8B=0, \qquad B=0$$

Bundan $\quad y^*=-\frac{1}{8}\cos 3x.$

Tenglamaning umumiy yechimi bu yechimlar yig'indisidan iborat.

$$y=\widetilde{y}+y^*=C_1\cos x+C_2\sin x-\frac{1}{8}\cos 3x$$

Tenglamaning xususiy yechimini topamiz, ya'ni C_1 va C_2ning konkret qiymatlarini boshlang'ich shartlar yordamida topamiz, buning uchun hosilani olamiz:

$$y' = -C_1\sin x+C_2\cos x+\frac{3}{8}\sin 3x$$

$$\begin{cases} C_1\cos\frac{\pi}{2}+C_2\sin\frac{\pi}{2}-\frac{1}{8}\cos 3\frac{3\pi}{2}=4 \\ -C_1\sin\frac{\pi}{2}+C_2\cos\frac{\pi}{2}+\frac{3}{8}\sin\frac{3\pi}{2} \end{cases}$$

$$C_2=4$$

$$C_1-\frac{3}{8}=1 \quad C_1=-\frac{11}{8}$$

xususiy yechim: $y=-\frac{11}{8}\cos x+4\sin x-\frac{1}{8}\cos 3x.$

8– masala.

$$\begin{cases} \frac{dx_1}{dt}=-x_1+8x_2 \\ \frac{dx_2}{dt}=x_1+x_2 \end{cases}$$

sistemaning umumiy yechimini toping.

Yechish:

Xarakteristik tenglamani yechamiz:

$$\begin{vmatrix} -1-\lambda & 8 \\ -1 & 1-\lambda \end{vmatrix}=0\,;\; -1+\lambda^2-8=0\,.\; \lambda^2=9\,,\; \lambda_{1,2}=\sqrt{9}\quad \lambda_{1,2}=\pm 3$$

$\lambda_{1,2}=\pm 3$ - xos sonlardir

$\lambda_1=3$ da $\quad 4P_1+8P_2=0$, $\qquad P_1=2P_2$

(2; 1)vektorni hosil qilamiz.

$\lambda_2=-3$ da $\quad -2P_1+8P_2=0$, $\qquad P_1=4P_2$

(-4; 1)vektorni hosil qilamiz.

$\lambda_1=3$ da $\qquad x_{11}=2e^{3t}$, $\qquad x_{21}=e^{3t}$

$\lambda_2=-3$ da $\qquad x_{12}=-4e^{-3t}$, $\qquad x_{22}=e^{-3t}$

Fundamental yechimlar sistemasini hosil qilamiz.
Umumiy yechim quyidagicha yoziladi:

$$\begin{cases} x_1=2C_1e^{3t}-4C_2e^{-3t} \\ x_2=C_1e^{3t}+C_2e^{-3t} \end{cases}$$

4- NAZORAT ISHI BO'YICHA MASALALAR.

321. $z=e^{xy}$ funksiya berilgan

$$x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}=0$$ ekanini ko'rsating.

322. $z=e^{-\cos(ax+y)}$ funksiya berilgan

$$x^2\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial x^2}$$ ekanini ko'rsating.

323. $z=\ln(x^2+y^2+2y+1)$ funksiya berilgan

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$$ ekanini ko'rsating.

324. $z=\sin^2(y-ax)$ funksiya berilgan

$$x^2\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial x^2}$$ ekanini ko'rsating.

325. $z=\frac{y}{x}$ funksiya berilgan

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0$$ ekanini ko'rsating.

326. $z=\sqrt{\frac{y}{x}}$ funksiya berilgan

$$x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}=0$$ ekanini ko'rsating.

327. $z=\sqrt{\frac{x}{y}}$ funksiya berilgan

$$x^2\frac{\partial^2 z}{\partial x^2}-\frac{\partial}{\partial y}(y^2\frac{\partial z}{\partial y})=0$$ ekanini ko'rsating.

328. $z=arctg\frac{x}{y}$ funksiya berilgan

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$$ ekanini ko'rsating.

321. $z=\frac{\sin(x-y)}{x}$ funksiya berilgan

$$\frac{\partial}{\partial x}(x^2\frac{\partial z}{\partial x})-x^2\frac{\partial^2 z}{\partial y^2}=0$$ ekanini ko'rsating.

321. $z=e^{y/x}$ funksiya berilgan

$$\frac{\partial}{\partial x}(x^2\frac{\partial z}{\partial x})-y^2\frac{\partial^2 z}{\partial y^2}=0$$ ekanini ko'rsating.

331. $z=\sin^2(3x-4y)$ funksiya berilgan

$$4\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=0$$ ekanini ko'rsating.

332. $z=\sqrt{x^2-3y^2}$ funksiya berilgan

$$3y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=0$$ ekanini ko'rsating.

333. $z=tg^2(2x-3y)$ funksiya berilgan

$$3\frac{\partial z}{\partial x}+2\frac{\partial z}{\partial y}=0$$ ekanini ko'rsating.

334. $z=\frac{xy}{x+y}$ funksiya berilgan

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z$$ ekanini ko'rsating.

335. $z=\ln(x^2-y^2)$ funksiya berilgan

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=\frac{2}{x+y}$$ ekanini ko'rsating.

336. $z=\sqrt[3]{2y^2-x^2}$ funksiya berilgan

$$2y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=0$$ ekanini ko'rsating.

337. $z=e^{xy}$ funksiya berilgan.

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}2\frac{\partial^2 z}{\partial x\partial y}=-2z$$ ekanini ko'rsating.

338. $z=\ln(x^2+xy+y^2)$ funksiya berilgan

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2$$ ekanini ko'rsating.

339. $z = x^y$ funksiya berilgan

$$y\frac{\partial^2 z}{\partial x \partial y} = (1 + y\ln x)\frac{\partial z}{\partial x}$$ ekanini ko'rsating.

340. $z = e^{x/y^2}$ funksiya berilgan.

$$2x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$ ekanini ko'rsating.

341- 360 masalalarida funksiyaning yopiq sohadagi eng kichik va eng katta qiymatlarini toping.

341. $z = x^2 - xy + y^2 - 4x$, $x = 0, y = 0$ $2x + 3y - 12 = 0$ chiziqlar bilan chegaralangan uchburchakda.

342. $z = x^3 - y^3 - 3xy$, $0 \le x \le 2, 0 \le y \le 1$ to'g'ri to'rtburchakda.

343. $z = x^2 - 2y^2 - 2y^2 + 4xy - 6x - 1$, $x = 0, y = 0,$ $x + y = 3$ chiziqlar bilan chegaralangan uchburchakda.

344. $z = xy - 2x - y$, $0 \le x \le 3, 0 \le y \le 4$ to'g'ri uchburchakda.

345. $z = \frac{1}{2}x^2 - xy$ $y = \frac{1}{3}x^2$ parabola va $y = 3$ chiziq bilan chegaralangan sohada.

346. $z = 2x + y - xy$, $0 \le x \le 4, 0 \le y \le 4$ kvadratda.

347. $z = x^2 + 2xy - 4x + 8y$, $x = 0, y = 0, x = 1, y = 2$ to'g'ri to'rtburchakda.

348. $z = x^2 + y^2 - xy + x + y$, $x = 0, y = 0, x + y = -3$ uchburchakda.

349. $z = x^3 + 8y^3 - 6xy + 1$, $y = 1, y = -1, x = 0, x = 2$ to'g'ri to'rtburchakda.

350. $z = 6xy - 9x^2 + 4x + 4y$, $0 \le x \le 1, 0 \le y \le 2$ to'g'ri to'rtburchakda.

351. $z = x^2 + xy - 2$ $y = 4x^2 - 4$ parabola va OX

o'q bilan chegaralangan soha.

352. $2z=4x^2+4xy-y^2-8y$ $y=2x, y=2, x=0$ to'g'ri chiziqlar bilan chegaralangan uchburchakda.

353. $z=x^2+2xy+4x-y^2$ $x=0, y=0, x+y+2=0$ to'g'ri chiziqlar bilan chegaralangan uchburchakda.

354. $z=5x^2-3xy+y^2$, $-1\le x\le 1, -1\le y\le 1$ kvadratda.

355. $2z=x^2-2xy, y=2x^2$ parabola va $y=8$ to'g'ri chiziq bilan chegaralangan sohada.

356. $z=3x+y-xy, y=x, y=4$ va $x=0$ to'g'ri chiziqlar bilan chegaralangan uchburchakda.

357. $z=xy-3x-2y$, $0\le x\le 4, 0\le y\le 4$ kvadratda.

359. $z=xy-x-2y$, $y=x, x=3, y=3$ to'g'ri chiziqlar bilan chegaralangan uchburchakda.

360. $z=xy$, $x^2+y^2\le 1$ doirada.

361-380 masalalarda $z=z(x,y)$ funksiya, A nuqta va $\vec{a}$ vektor berilgan. Quyidagilarni toping:

1) A nuqtadagi *grad* z ni;

2) $\vec{a}$ vektorning yo'nalishi bo'yicha A nuqtadagi hosilani.

361. $z=5x^2+6xy$ $A(2;1)\vec{a}=\vec{i}+2\vec{j}$

362. $z=\ln(5x+3y)$ $A(2;2)\vec{a}=2\vec{i}-3\vec{j}$

363. $z=arctg(xy^2)$ $A(2;1)\vec{a}=3\vec{i}+4\vec{j}$

364. $z=2x^3y+3x^2y^2$ $A(1;-2)\vec{a}=6\vec{i}-8\vec{j}$

365. $z=\arcsin(\frac{x^2}{y})$ $A(1;2)\vec{a}=-12\vec{i}+5\vec{j}$

366. $z=\ln(3x^2+5y^2)$ $A(2;3)\vec{a}=-4\vec{i}+3\vec{j}$

367. $z=2x^2+3xy+4y^2$ $A(2;-2)\vec{a}=\vec{i}-3\vec{j}$

368. $z = arctg(\frac{x}{y^2})$ $A(3;1)\vec{a} = \vec{i} - \vec{j}$

369. $z = \ln(2x^2 + 3y^3)$ $A(3;-1)\vec{a} = \vec{i} - 2\vec{j}$

370. $z = 2x^4 + 8x^2y^3$ $A(2;-1)\vec{a} = -3\vec{i} + 4\vec{j}$

371. $z = 2x^2 + 3xy$ $A(-1;2)\vec{a} = 3\vec{i} + 4\vec{j}$

372. $z = arctg\frac{y}{x}$ $A(-1;1)\vec{a} = \vec{i} - \vec{j}$

373. $z = x^3y + xy^2$ $A(1;3)\vec{a} = -5\vec{i} + 12\vec{j}$

374. $z = \ln(2x + 3y)$ $A(2;2)\vec{a} = 2\vec{i} - 3\vec{j}$

375. $z = 5x^2y + 3xy^2$ $A(1;1)\vec{a} = 6\vec{i} - 8\vec{j}$

376. $z = \frac{3x}{y^2}$ $A(3;4)\vec{a} = -3\vec{i} - 4\vec{j}$

377. $z = arctg(xy)$ $A(2;3)\vec{a} = 4\vec{i} + 3\vec{j}$

378. $z = \ln(3x^2 + 2xy^2$ $A(1;2)\vec{a} = 3\vec{i} - 4\vec{j}$

379. $z = \frac{x+y}{x^2+y^2})$ $A(1;-2)\vec{a} = \vec{i} + 2\vec{j}$

380. $z = 5x^2 - 2xy + y^2$ $A(1;1)\vec{a} = 2\vec{i} - \vec{j}$

381–400. Differensial tenglamalarning umumiy yechimlarini toping.

381. $xy' - y = x^2\cos x$ 382. $y' + 2xy = xe^{-x^2}$

383. $xy' = xe^{\frac{y}{x}} + y$ 384. $xy' = -(y - \sqrt{xy})$

385. $(1+x^2)y' + y = arctgx$ 386. $y'(x + y^2) = y$

387. $(x^2 - y^2)y' = 2xy$ 388. $xy' + y - 3 = 0$

389. $x^2y' = 2xy + 3$ 390. $xy' = y\ln\frac{y}{x}$

391. $y'''\sin^4 x = \sin 2x$

392. $xy'' = y'\ln\frac{y}{x}$

393. $(1-x^2)y'' = xy'$

394. $2yy''+(y')^2+(y^0)^4=0$

395. $y''tgy = 2(y')^2$

396. $xy''+2y' = x^3$

397. $1+(y')^2+yy''=0$

398. $y''(1+y)-5(y')^2=0$

399. $y''\div\frac{y}{x}=x^2$

400. $3yy''+(y')^2=0$

401–420. Differensial tenglamaning ko'rsatilgan boshlang'ich shartlarini qanoatlantiruvchi xususiy yechimlarini toping.

401. $y''-8y'+16=e^{4x}$ $\quad y(0)=0,\quad y'(0)=1$

402. $y''-4y'+3y=e^{5x}$ $\quad y(0)=3,\quad y'(0)=9$

403. $y''-5y'+6y=x^2-x$ $\quad y(0)=0,\quad y'(0)=\frac{1}{9}$

404. $y''-7y'+12y=e^{2x}$ $\quad y(0)=0,\quad y'(0)=1$

405. $y''-3y'-4y=17\sin x$ $\quad y(0)=5,\quad y'(0)=6$

406. $y''-3y'-4y=5\cos x$ $\quad y(0)=0,\quad y'(0)=0$

407. $y''-2y'+y=x+\cos x$ $\quad y(0)=0,\quad y'(0)=0$

408. $y''-y'+3y=xe^{2x}$ $\quad y(0)=0,\quad y'(0)=0$

409. $y''-y'+5y=2x^2e^x$ $\quad y(0)=0,\quad y'(0)=0$

410. $y''-2y'+y=x+\sin x$ $\quad y(0)=0,\quad y'(0)=0$

411. $y''-4y'-12y=8\sin 2x$ $\quad y(0)=0,\quad y'(0)=0$

412. $y''+4y=e^{-2x}$ $\quad y(0)=0,\quad y'(0)=0$

413. $y''-2y'+5y=xe^{2x}$ $\quad y(0)=1,\quad y'(0)=0$

414. $y''-5y'+6y=(12x)e^{-x}$ $\quad y(0)=0,\quad y'(0)=0$

415. $y''-y'+5y=2x^2e^x$ $\quad y(0)=0,\quad y'(0)=0$

416. $y''-4y'=6x^2+1,$ $\quad y(0)=2,\quad y'(0)=3$

417. $y''-6y'+9y=x^2-x+3$ $\quad y(0)=\frac{4}{3},\quad y'(0)=\frac{1}{27}$

418. $y''+6y'+9y=10e^{-3x}$ $\quad y(0)=3, \quad y'(0)=2$

419. $y''-4y'=6x^2+1$ $\quad y(0)=2, \quad y'(0)=3$

420. $y''-2y'+y=16e^{x}$ $\quad y(0)=1, \quad y'(0)=2$

421-440. O'zgarmas koeffitsientli chiziqli differensial tenglamalar sistemasini yeching.

421. $\begin{cases} \dfrac{dx_1}{dt}=7x_1+3x_2 \\ \dfrac{dx_2}{dt}=6x_1+4x_2 \end{cases}$

422. $\begin{cases} \dfrac{dx_1}{dt}=4x_1-3x_2 \\ \dfrac{dx_2}{dt}=3x_1+4x_2 \end{cases}$

423. $\begin{cases} \dfrac{dx_1}{dt}=5x_1-x_2 \\ \dfrac{dx_2}{dt}=x_1+3x_2 \end{cases}$

424. $\begin{cases} \dfrac{dx_1}{dt}=12x_1-5x_2 \\ \dfrac{dx_2}{dt}=5x_1+12x_2 \end{cases}$

425. $\begin{cases} \dfrac{dx_1}{dt}=4x_1+6x_2 \\ \dfrac{dx_2}{dt}=4x_1+2x_2 \end{cases}$

426. $\begin{cases} \dfrac{dx_1}{dt}=-5x_1-4x_2 \\ \dfrac{dx_2}{dt}=-2x_1-3x_2 \end{cases}$

427. $\begin{cases} \dfrac{dx_1}{dt}=-x_1+5x_2 \\ \dfrac{dx_2}{dt}=x_1+3x_2 \end{cases}$

428. $\begin{cases} \dfrac{dx_1}{dt}=3x_1+x_2 \\ \dfrac{dx_2}{dt}=8x_1+x_2 \end{cases}$

429. $\begin{cases} \dfrac{dx_1}{dt}=6x_1+3x_2 \\ \dfrac{dx_2}{dt}=-8x_1-5x_2 \end{cases}$

430. $\begin{cases} \dfrac{dx_1}{dt}=3x_1-2x_2 \\ \dfrac{dx_2}{dt}=2x_1+8x_2 \end{cases}$

431. $\begin{cases} \dfrac{dx_1}{dt} = -4x_1 - 6x_2 \\ \dfrac{dx_2}{dt} = -4x_1 - 2x_2 \end{cases}$

432. $\begin{cases} \dfrac{dx_1}{dt} = -5x_1 - 8x_2 \\ \dfrac{dx_2}{dt} = -3x_1 - 3x_2 \end{cases}$

433. $\begin{cases} \dfrac{dx_1}{dt} = -x_1 - 5x_2 \\ \dfrac{dx_2}{dt} = -7x_1 - 3x_2 \end{cases}$

434. $\begin{cases} \dfrac{dx_1}{dt} = -7x_1 + 5x_2 \\ \dfrac{dx_2}{dt} = 4x_1 - 8x_2 \end{cases}$

435. $\begin{cases} \dfrac{dx_1}{dt} = x_1 + 4x_2 \\ \dfrac{dx_2}{dt} = 2x_1 + 3x_2 \end{cases}$

436. $\begin{cases} \dfrac{dx_1}{dt} = 5x_1 + 4x_2 \\ \dfrac{dx_2}{dt} = -2x_1 + 11x_2 \end{cases}$

437. $\begin{cases} \dfrac{dx_1}{dt} = -3x_1 + 2x_2 \\ \dfrac{dx_2}{dt} = -2x_1 + x_2 \end{cases}$

438. $\begin{cases} \dfrac{dx_1}{dt} = x_1 + 4x_2 \\ \dfrac{dx_2}{dt} = x_1 + x_2 \end{cases}$

439. $\begin{cases} \dfrac{dx_1}{dt} = 3x_1 + x_2 \\ \dfrac{dx_2}{dt} = x_1 + 3x_2 \end{cases}$

440. $\begin{cases} \dfrac{dx_1}{dt} = x_1 + 6x_2 \\ \dfrac{dx_2}{dt} = -2x_1 + 9x_2 \end{cases}$

5- NAZORAT ISHI

1- masala

Operatsion hisob usuli bilan differensial tenglamaning xususiy yechimini toping.

$$y`` + y` - 2y = e^t, \qquad y(0) = -1, \qquad y`(0) = 0$$

Yechish:

Original differensiallash tenglamasidan

$$f`(t) \leftarrow p \cdot \vec{f}(p) - f(0)$$

$$f``(t) \leftarrow p^2 \cdot \vec{f}(p) - p \cdot \vec{f}(0) - f`(0)$$

tenglamada tasvirlarga o'tsak:

$$\left[p^2 \overline{y} - py(0) - y`(0)\right] + \left[p(\overline{y}) - y(0)\right] - 2y = \frac{1}{p-1}$$

e^t ni asosiy elementar funksiyalarning tasvirida oldik, ya'ni

$$e^{\alpha t} = \frac{1}{p-\alpha}$$

№	$f(t) \quad t>0$	$f(p)$
1.	2.	3.
1.	$\frac{1}{p}$	1
2.	$\frac{t^n}{n!}$	$\frac{1}{p^n+1}$
3.	$e^{\alpha t}$	$\frac{1}{p-\alpha}$
4.	$\cos\beta t$	$\frac{p}{p^2+\beta^2}$
5.	$\sin\beta t$	$\frac{\beta}{p^2+\beta^2}$
6.	$e^{\alpha t}\cos\beta t$	$\frac{p-\alpha}{(\lambda-\alpha)^2+\beta^2}$

1.	2.	3.
7.	$e^{\alpha t}\sin\beta t$	$\dfrac{\beta}{(p-\alpha)^2+\beta^2}$
8.	$\dfrac{t^n}{n!}e^{\alpha t}$	$\dfrac{1}{(p-\alpha)^{n+1}}$
9.	$t\cdot\cos\beta t$	$\dfrac{p^2-\beta^2}{(p^2+\beta^2)^2}$
10.	$t\cdot\sin\beta t$	$\dfrac{2p\beta}{(p^2+\beta^2)^2}$

Boshlang'ich shartlardan

$$p^2\bar{y}+p+p\bar{y}+1=\frac{1}{p-1}$$

$$\bar{y}(p^2+p-2)=\frac{1}{p-1}-p-1$$

$$\bar{y}(p^2+p-2)=\frac{2-p^2}{p-1}$$

$$\tilde{y}(p-1)(p+2)=\frac{2-p^2}{p-1}$$

$$\tilde{y}=\frac{2-p^2}{(p-1)^2(p+2)}$$

kasrni elementar kasrlarga ajratamiz:

$$\frac{2-p^2}{(p-1)^2(p+2)}=\frac{a}{(p-1)^2}+\frac{b}{p-1}+\frac{c}{p+2}$$

$$a(p+2)+b(p-1)(p+2)+c(p-1)^2=2-p^2$$

$p=1$ da $3a=1$ $a=\dfrac{1}{3}$

$p=-2$ da $9c=-2$ $c=-\dfrac{2}{9}$

$$b+c=-1, \qquad b=-1-c=-1+\frac{2}{9}=-\frac{7}{9}$$

$$y=\frac{1}{3(p-1)^2}-\frac{7}{2(p-1)}-\frac{2}{9(p+2)}$$

jadvaldan foydalanib,

$$y=\frac{1}{3}te^t-\frac{7}{9}e^t-\frac{2}{9}e^{-2t}$$ ni topamiz.

2- masala

$\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$ sonli qatorning yaqinlashishini tekshiring.

Yechish:

$$U_n=\left(\frac{n}{3n+1}\right)^n$$

Demak, Koshi alomati qulay

$$C=\lim_{n\to\infty}\sqrt[n]{U_n}=\lim_{n\to\infty}\frac{n}{3n+1}=\lim_{n\to\infty}\frac{1}{3+\frac{1}{n}}=\frac{1}{3}$$

$$C=\frac{1}{3}<1$$

qator yaqinlashuvchi.

3- masala

$\sum_{n=1}^{\infty}\frac{x^n}{n}$ darajali qatorning yaqinlashish oralig'ini toping.

Yechish:

$$a_n=\frac{1}{n}, \qquad a_{n+1}=\frac{1}{n+1}$$

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{n+1}{n}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=1.$$

Demak, qator x ning $-1<x<1$ tengsizlikni qanoatlantiruvchi hamma qiymatlarida yaqinlashuvchi. Oraliqning chekkalarida tekshiramiz. Agar $x=1$ bo'lsa, $1+\frac{1}{2}+\frac{1}{3}\cdots$ garmonik qatorni hosil qilamiz. Bu esa uzoqlashuvchi. Agar $x=-1$ bo'lsa, $-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\cdots$ qator Leybnis alomatiga ko'ra yaqinlashuvchi.

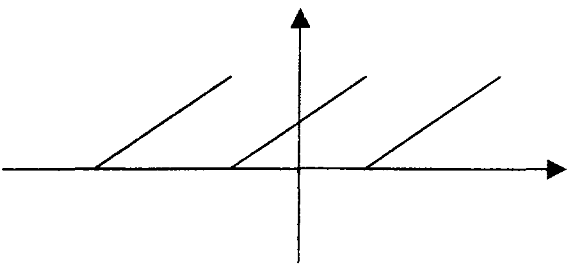
Demak, darajali qatorning yaqinlashish oralig'i $-1\le x<1$ bo'ladi.

4- masala

$f(x)=\pi+x$ funksiyani $[-\pi;\pi]$da Fure qatoriga yoying.

Yechish:

Bu funksiyaning grafigi $(-\pi;0)$ va $(-\pi;2\pi)$ nuqtalarni birlashtiruvchi kesma bo'ladi.



Chizmada $y=f(x)$ funksiyaning grafigi ko'rsatilgan. $S(x)$ - funksiyaning Fure qatori yig'indisi. Bu yig'indi 2π davriy funksiya.

Fure qatorining koeffitsentlarini aniqlaymiz.

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}\int_{-\pi}^{\pi}f(\pi+x)dx=2\pi$$

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos mx dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\pi + x)\cos mx dx =$$

$$= \int_{-\pi}^{\pi} \cos mx dx + \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos mx dx$$

oxirgi integral «0» ga teng. bm ni aniqlaymiz:

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin mx dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\pi + x)\sin mx dx =$$

$$\int_{-\pi}^{\pi} \sin mx dx + \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin mx dx = \frac{2}{\pi}\int_{0}^{\pi} x\sin mx dx =$$

$$= \left| \begin{matrix} u = x, \ \ du = dx \\ \alpha V = \sin mx dx, \ \ V = -\frac{1}{m}\cos mx \end{matrix} \right| = \frac{2}{\pi}\left[-\frac{x}{m}\cos mx \Big|_0^{\pi} + \right.$$

$$\left. + \frac{1}{m}\int_0^{\pi} \cos mx dx \right] = \frac{2}{\pi}\left[-\frac{x}{m}\cos m\pi + \frac{1}{m^2}\sin mx dx \Big|_0^{\pi} \right] =$$

$$= -\frac{2}{m}\cos m\pi = -\frac{2}{m}(-1)^m = \frac{2}{m}(-1)^{m+1}$$

Endi yoyilmani yozamiz:

$$f(x) = \pi + 2\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m}\sin mx = \pi +$$

$$+ 2\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \cdots \right)$$

5- masala

Quyidagi funksiyaning Fure almashtirishlarini toping.

$$f(x)=\begin{cases} x+1, & agar \quad -1\le x\le -\frac{1}{2} \\ 1\ , & agar \quad |x|<\frac{1}{2} \\ -x+1, & agar \quad \frac{1}{2}\le x\le 1 \\ 0\ , & agar \quad |x|>1 \end{cases}$$

Yechish:

Fure almashtirish formulasidan

$$F(Z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)e^{izu}\alpha u$$

$f(x)$ ning ko'rinishi bo'yicha

$$\sqrt{2\pi}F(Z)=\int_{-\infty}^{-1}0\cdot e^{izu}du+\int_{-1}^{1/2}(u+1)e^{izu}du+\int_{-1/2}^{1/2}1\cdot e^{izu}du+$$

$$+\int_{1/2}^{1}(-u+1)e^{izu}du+\int_{1}^{\infty}0\cdot e^{izu}du$$

Birinchi va oxirgisi nolga teng.

qolganlarni I_1, I_2, I_3, deylik

$$I_1=\int_{-1}^{-1/2}(u+1)e^{izu}du=\left[\frac{1}{zi}(u+1)e^{izu}-\frac{1}{i^2z^2}e^{izu}\right]_{-1}^{1/-}=$$

$$=\frac{1}{zi}\cdot\frac{1}{2}e^{-\frac{iz}{2}}-\frac{1}{i^2z^2}e^{-\frac{iz}{2}}+\frac{1}{i^2z^2}e^{-iz}=$$

$$=\frac{1}{zi}\cdot\frac{1}{2}e^{-\frac{iz}{2}}-\frac{1}{i^2z^2}e^{-\frac{iz}{2}}+\frac{1}{i^2z^2}e^{-iz}=\frac{1}{2iz}e^{-\frac{zi}{2}}+\frac{1}{z^2}e^{-\frac{zi}{2}}-\frac{1}{z^2}e^{-zi}$$

$$I_2=\int_{-1/2}^{1/2}e^{izu}du=\frac{1}{z_i}e^{izu}\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}}=\frac{1}{z_i}\left(e^{\frac{zi}{2}}-e^{-\frac{zi}{2}}\right)=\frac{2\sin\frac{z}{2}}{z}$$

$$I_3 = \int_{1/2}^{1}(-U+1)e^{izu}du = \left[\frac{1}{z_i}(-U+1)e^{izu} + \frac{1}{i^2 z^2}e^{izu}\right]_{\frac{1}{2}}^{1} =$$

$$= \frac{1}{z^2}e^{z_i} - \frac{1}{2z_i}e^{\frac{z_i}{2}} + \frac{1}{z^2}e^{\frac{z_i}{2}}$$

$$F(z) = \frac{1}{\sqrt{2\pi}}$$

$$\left(\frac{1}{2z_i}e^{-\frac{z_i}{2}} + \frac{1}{z^2}e^{-\frac{z_i}{2}} - \frac{1}{z^2}e^{-\frac{z_i}{2}} + \frac{2\sin z/2}{z} - \frac{1}{z^2}e^{zi} - \frac{1}{2\pi i}e^{\frac{z_i}{2}} + \frac{1}{z^2}e^{\frac{z_i}{2}}\right) =$$

$$= \frac{1}{\sqrt{2\pi}}\left(-\frac{2\cos z}{z^3} + \frac{\sin z/2}{z} + \frac{2\cos z/2}{z^2}\right)$$

5- NAZORAT ISHI BO'YICHA MASALALAR:

441-460. Operatsion hisob usuli bilan differensial tenglamaning boshlang'ich shartlarini qanoatlantiruvchi hususiy yechimini toping.

441. $y''-2y-3y = e^{3t}; y(0) = o, y'(0) = 0$
442. $y''-y-2y = e^{t}; y(0) = o, y'(0) = 1$
443. $y'''+y'' = \sin t; y(0) = 1, y'(0) = 1, y''(0) = 0$
444. $y''-y' = te^{t}; y(0) = o, y'(0) = 0$
445. $y'''-2y''+y' = 4; y(0) = 1, y'(0) = 2, y''(0) = 2$
446. $y''-9y = e^{-2t}; y(0) = o, y'(0) = 0$
447. $y''+y' = t; y(0) = o, y'(0) = 1$
448. $y''+8y = \cos 3t; y(0) = 1, y'(0) = 0$
449. $y'''+y = 1; y(0) = o, y'(0) = 0, y''(0) = 0$
450. $y''-4y = t; y(0) = o, y'(0) = 0$
451. $y''+2y'+y = \cos t; y(0) = o, y'(0) = 0$

452. $y''+3y'+2y=\sin t; y(0)=o, y'(0)=0$

453. $y'''-6y''+11y'-6y=0; y(0)=o, y'(0)=1; y''(0)=0$

454. $y''+5y'+6y=e^{2t}; y(0)=o, y'(0)=0$

455. $y''+3y'+2y=\cos t; y(0)=o, y'(0)=0$

456. $y''-7y'+6y=e^{t}\sin t; y(0)=o, y'(0)=0$

457. $y''-7y'+6y=e^{t}\sin t; y(0)=o, y'(0)=0$

458. $y''+y'-2y=\cos 2t; y(0)=o, y'(0)=0$

459. $y''+3y'-4y=\sin t; y(0)=o, y'(0)=0$

460. $y''-5y'-6y=e^{2t}\sin t; y(0)=o, y'(0)=0$

461-480. Sonli qatorning yaqinlashishini tekshiring.

461. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{4n+3}\right)^{n}$

462. $\sum_{n=1}^{\infty}\frac{1}{(10n-1)\ln(10n-1)}$

463. $\sum_{n=1}^{\infty}\frac{5^{n}(n+1)^{2}}{3^{n}}$

464. $\sum_{n=1}^{\infty}\frac{e^{-\sqrt{n}}}{\sqrt{n}}$

465. $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

466. $\sum_{n=1}^{\infty}\frac{2n+1}{\sqrt{n2^{n}}}$

467. $\sum_{n=1}^{\infty}\frac{n^{2}}{2+n^{3}}$

468. $\sum_{n=1}^{\infty}\frac{n^{2}}{(3n)!}$

469. $\sum_{n=1}^{\infty}\frac{n+3}{n^{3}-2}$

470. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2}-1}$

471. $\sum_{n=1}^{\infty}\frac{n^{3}}{e^{n}}$

472. $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^{2}}$

473. $\sum_{n=1}^{\infty}\frac{n^{n+1}}{(n+1)!}$

474. $\sum_{n=1}^{\infty}\frac{1}{n^{2}+3n+2}$

475. $\sum_{n=1}^{\infty}\frac{8^{n}n^{2}}{5^{n}}$

476. $\sum_{n=1}^{\infty}\left(\frac{11}{10}\right)^{n}\cdot\frac{1}{n^{5}}$

477. $\sum_{n=1}^{\infty}\left(\frac{2n^2+2n+1}{5n^2+2n+1}\right)^n$

478. $\sum_{n=1}^{\infty}\frac{2^n}{5^n+1}$

479. $\sum_{n=1}^{\infty}\frac{1}{\ln^n}$

480. $\sum_{n=1}^{\infty}\frac{2n-1}{n^2+6n+5}$

481-500. Darajali qatorning yaqinlashish oralig'ini toping.

481. $\sum_{n=1}^{\infty}\frac{(x+1)^n}{n!}$

482. $\sum_{n=1}^{\infty}\frac{(x-4)^4}{\sqrt{n}}$

483. $\sum_{n=1}^{\infty}\frac{(x-1)^n}{2^n}$

484. $\sum_{n=1}^{\infty}\frac{x^n}{2^n+3^n}$

485. $\sum_{n=1}^{\infty}\frac{n^{n/3}}{n!}x^n$

486. $\sum_{n=1}^{\infty}\frac{5^n}{\sqrt{n}}x^n$

487. $\sum_{n=1}^{\infty}\frac{2^n}{n(n+1)}x^n$

488. $\sum_{n=1}^{\infty}\frac{n^n}{(n+1)^n}x^n$

489. $\sum_{n=1}^{\infty}\frac{3^n n!}{(n+1)^n}x^n$

490. $\sum_{n=1}^{\infty}\frac{(x+2)^n}{3^n(n+1)(n+2)}$

491. $\sum_{n=1}^{\infty}\frac{3^n n!}{n^n}x^n$

492. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n x^n$

493. $\sum_{n=1}^{\infty}\frac{(n+1)^{n/3}}{n!}x^n$

494. $\sum_{n=1}^{\infty}\frac{(2n)!}{n^n}x^n$

495. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{(n+1)^n}}{n!}x^n$

496. $\sum_{n=1}^{\infty}\frac{n^n}{(n+1)^n}x^n$

497. $\sum_{n=1}^{\infty}\frac{n}{3^n(n+1)}x^n$

498. $\sum_{n=1}^{\infty}\frac{n+1}{3^n(n+2)}x^n$

499. $\sum_{n=1}^{\infty}\frac{3^n}{\sqrt{2^n(3n-1)}}x^n$

500. $\sum_{n=1}^{\infty}\frac{n+2}{n(n+1)}x^n$

501-520. $f(x)$ ni ko'rsatilgan kesmada Fure qatoriga yoying.

501. $f(x)=x$ $[-\pi,\pi]$

502. $f(x)=|x|$ $[-1,1]$

503. $f(x)=e^x$ $[-\pi,\pi]$

504. $f(x)=x^3$ $[-\pi,\pi]$

505. $f(x)=\begin{cases}-h, & agar \quad -\pi\le x\le 0\\ h, & agar \quad 0<x\le\pi\end{cases}$

506. $f(x)=\begin{cases}-2x, & agar \quad -\pi\le x\le 0\\ 3x, & agar \quad 0<x\le\pi\end{cases}$

507. $f(x)=\cos x$

508. $f(x)=x-\frac{1}{2}x^2$ $[0,2]$

509. $f(x)=x^2$ $[-1,1]$

510. $f(x)=\pi-x$ $[-\pi,\pi]$

511. $f(x)=x+1$ $[-\pi,\pi]$

512. $f(x)=x^2+1$ $[-2,2]$

513. $f(x)=\frac{\pi-x}{2}$ $[-\pi,\pi]$

514. $f(x)=1+|x|$ $[-1,1]$

515. $f(x)=|1-x|$ $[-2,2]$

516. $f(x)=x-1$ $[-1,1]$

517. $f(x)=x^3+1$ $[-\pi,\pi]$

518. $f(x)=\begin{cases}1, & agar \quad -\pi\le x\le 0\\ 2, & agar \quad 0<x\le\pi\end{cases}$

519. $f(x)=2\cos x$ $[-\pi,\pi]$

520. $f(x)=3\sin x$ $\qquad [-\pi,\pi]$

521-540. $f(x)$ funksiyaning Fure almashtirishlarini toping.

521. $f(x)=\begin{cases}\cos\dfrac{x}{2}, & agar\ |x|\le\pi\\ 0, & agar\ |x|>\pi\end{cases}$

522. $f(x)=\begin{cases}-1, & agar\ \ -1\le x\le -\dfrac{1}{2}\\ 0, & agar\ \ -\dfrac{1}{2}\le x\le\dfrac{1}{2}\\ 1, & agar\ \ \dfrac{1}{2}\le x\le 1\end{cases}$

523. $f(x)=\begin{cases}h\left(1-\dfrac{|x|}{a}\right) & agar\quad |x|<a\\ 0 & agar\quad |x|>a\end{cases}$

524. $f(x)=\begin{cases}x^2 & |x|<1\\ 0 & |x|>1\end{cases}$

525. $f(x)=\begin{cases}\cos\alpha x & |x|<\dfrac{\pi}{2}\\ 0 & |x|>\dfrac{\pi}{2}\end{cases}\quad \alpha>0$

526. $f(x)=\begin{cases}\sin t & |t|<1\\ 0 & |t|>1\end{cases}$

527. $f(x)=sign(t-a)-sign(t-b), b>a$

528. $f(x)=e^{-a|x|}, a>0$

529. $f(x)=e^{-ax^2}, a>0$

530. $f(x)=\begin{cases} -x^2+2x+1, & 0\le x\le 2 \\ \frac{1}{3}x+1, & -3\le x\le 0 \\ -x+3, & 2\le x\le 3 \\ 0, & |x|>3 \end{cases}$

531. $f(x)=\begin{cases} \sin|x| & |x|<\frac{\pi}{2} \\ 0 & |x|\ge\frac{\pi}{2} \end{cases}$

532. $\begin{cases} x+1 & -1\le x\le -\frac{1}{2} \\ 1 & |x|<\frac{1}{2} \\ -x+1 & \frac{1}{2}\le x\le 1 \\ 0 & |x|>1 \end{cases}$

533. $f(x)=\begin{cases} -x+1 & 0\le x\le 1 \\ x+1 & -1\le x<0 \\ 0 & |x|\ge 1 \end{cases}$

534. $f(x)=\begin{cases} \sin x & |x|\le\pi \\ 0 & |x|>\pi \end{cases}$

535. $f(x)=\begin{cases} \cos x & |x|\le\frac{\pi}{2} \\ 0 & |x|>\frac{\pi}{2} \end{cases}$

536. $f(x)=\begin{cases} -x+1 & 0\le x\le 1 \\ -x-1 & -1\le x<0 \\ 0 & |x|>1 \end{cases}$

537. $f(x)=\begin{cases} e^{-x} & 0<x\le 1 \\ e^{x} & -1\le x\le 0 \\ 0 & |x|>1 \end{cases}$

538. $f(x)=\begin{cases} -x^2+2x+1, & 0\le x\le 2 \\ \frac{1}{3}x+1, & -3\le x\le 0 \\ -x+3, & 2\le x\le 3 \\ 0, & |x|>3 \end{cases}$

539. $f(x)=\begin{cases} \sin|x| & |x|<\frac{\pi}{2} \\ 0 & |x|\ge\frac{\pi}{2} \end{cases}$

540. $\begin{cases} x+1 & -1\le x\le -\frac{1}{2} \\ 1 & |x|<\frac{1}{2} \\ -x+1 & \frac{1}{2}\le x\le 1 \\ 0 & |x|>1 \end{cases}$

6– NAZORAT ISHI

1– masala

$$\int_{-6}^{2} dx \int_{\frac{x^2}{4}-1}^{2-x} f(x,y)dy$$

integralning integrallash tartibini o'zgartirish.

Yechish:

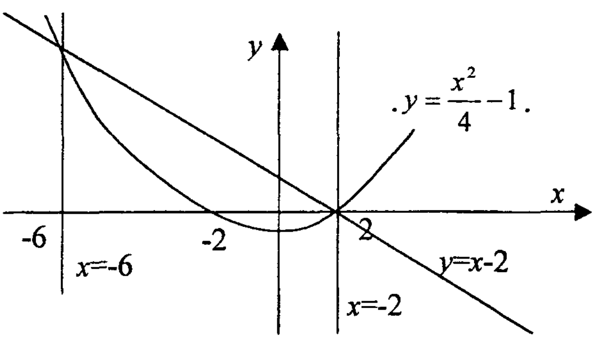
Integrallash sohasi D quyidagicha: $x=-6$, $x=2$, $y=\frac{x^2}{4}-1$, $y=2-x$ oxirgi chiziqlarning kesishish nuqtasini aniqlaymiz.

$\frac{x^2}{4}-1=2-x$ $\qquad x^2+4x-12=0$ $\qquad x=y-2$

$x_1=-6$, $\qquad x_2=2$

$x=t2\sqrt{1+y}$

$y_1=8$, $y_2=0$



Sohani ikkiga bo'lamiz, D_2 soha chapdan $x=-2\sqrt{1+y}$ va o'ngdan $x=2-y(0\le y\le 2)$ bilan chegaralansa, D_2 soha chapdan $x=-2\sqrt{1+y}$ va o'ngdan $x=-2\sqrt{1+y}$ $(-1\le y\le 0)$

chiziqlar bilan chegaralangan:

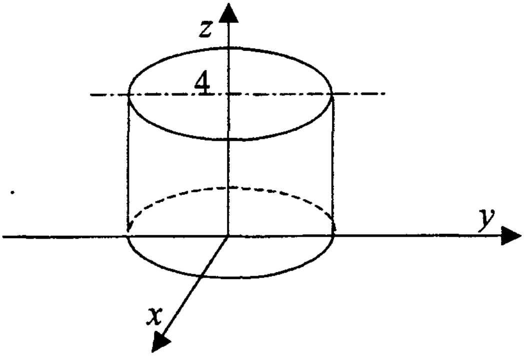
$$\int_{-6}^{2} dx \int_{\frac{x^2}{4}-1}^{2-x} f(x,y)dy = \int_{1}^{0} dy \int_{-2\sqrt{1+y}}^{2\sqrt{1+y}} f(x,y)dx + \int_{0}^{2} dy \int_{-2\sqrt{1+y}}^{2-y} f(x,y)dx$$

2– masala.

$4z = x^2 + y^2$, $z = 4$ sirtlar bilan chegaralangan jism hajmini uch o'lchovli integral yordamida hisoblang.

Yechish:

Bu jism quyidan $z = \dfrac{x^2 + y^2}{4}$ paraboloid va yuqoridan $z = 4$ tekislik bilan chegaralangan. Bu tekislik tekisligiga $x^2 + y^2 \le 4^2$ aylana sifatida proeksiyalanadi:



$$x = \rho\cos\varphi,\ \ y = \rho\sin\varphi,\ \ z = z$$
$$(0 \le \rho < +\infty,\ \ 0 \le \varphi < 2\pi,\ \ -\infty \le z < +\infty)$$

silindrik koordinatalarga o'tamiz. Yakobian $I = \rho$ bo'ladi.

$z = \dfrac{\rho^4}{4}$ ni hosil qilamiz.

$$V=\iiint\limits_T dxdydz=\iiint\limits_T \rho d\rho d\varphi dz=\int_0^{2\pi}d\varphi\int_0^4\rho d\rho\int_{\frac{\rho^2}{4}}^4 dz=$$

$$=\int_0^{2\pi}d\varphi\left(4-\frac{\rho^2}{4}\right)\rho d\rho=\int_0^{2\pi}\left[\frac{4\rho^2}{2}-\frac{\rho^4}{4\cdot 4}\right]=16\int_0^{2\pi}d\varphi=32\pi$$

3– masala

$$\int\limits_{AB}(x^2-y^2)dx+xydy$$

egri chiziqli integralni $A(1;1)$ nuqtadan $B(3;4)$ nuqtagacha bo'lgan yo'l bo'yicha hisoblang.

Yechish:

AB to'g'ri chiziq tenglamasini tuzamiz:

$$\frac{y-1}{4-1}=\frac{x-1}{3-1},\qquad \frac{y-1}{3}=\frac{x-1}{2},\qquad y=\frac{3x}{2}-\frac{1}{2}1\quad dy=3\frac{dx}{2},$$

$$\int\limits_{AB}(x^2-y^2)dx+xydy=\int_1^3\left[a^2-\left(\frac{3x}{2}-\frac{1}{2}\right)^2+x\left(\frac{3x}{2}-\frac{1}{2}\right)\cdot\frac{3}{2}\right]dx=$$

$$=\int_1^3\left(x^2+\frac{3x}{4}-\frac{1}{4}\right)dx=\left(\frac{x^2}{3}+\frac{3x^2}{8}-\frac{x}{4}\right)\Bigg|_1^3=$$

$$=\frac{27}{3}+\frac{27}{8}-\frac{3}{4}-\frac{1}{3}-\frac{3}{8}+\frac{1}{4}=11\frac{1}{6}.$$

4– masala.

$\int\limits_K\sqrt{2y}ds$ egri chiziqli integralni hisoblang. Bu yerda

$$K: x=t,\quad y=\frac{1}{2}t^2,\qquad z=\frac{1}{3}t^3(0\le t\le 1)$$

egri chiziq yoyi bo'lagi.

Yechish:

Bu yerda $ds=\sqrt{x'^2+y'^2+z'^2}=\sqrt{1^2+t^2+t^4}$

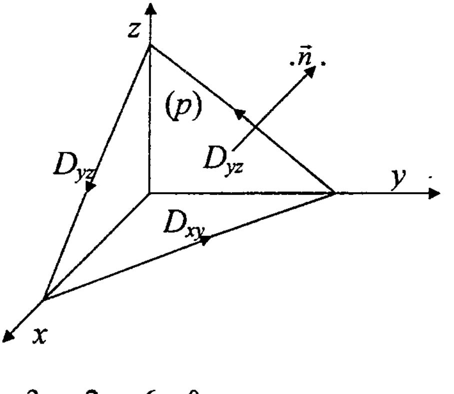
$$\int_K \sqrt{2y}ds=\int_0^1 t\sqrt{1+t^2+t^4}dt=\frac{1}{2}\sqrt{\left(t^2+\frac{1}{2}\right)^2+\frac{3}{4}}\cdot d\left(t^2+\frac{1}{2}\right)=$$

$$=\frac{1}{2}\left[\frac{t^2+\frac{1}{2}}{2}\cdot\sqrt{t^4+t^2+1}+\frac{3}{8}\ln\left(t^2+\frac{1}{2}+\sqrt{t^4+t^2+1}\right)\Bigg|_0^1=\right.$$

$$=\frac{1}{8}\left(3\sqrt{3}-1+\frac{3}{2}\ln\frac{3+2\sqrt{3}}{3}\right)$$

5– masala.

$\vec{F}_0=(3x+z)\vec{i}+(x-2y+z)\vec{j}+(3x+y)\vec{k}$ vektor maydonining (R) tekislikdagi olingan V uchburchak bo'yicha sirkulyatsiyasini toping.



$(P): 2x+3y+2z-6=0$

Yechish:

Stoks formulasi bo'yicha $U=\int_\lambda \vec{F}d\vec{r}=\iint_\varsigma \vec{n}\,rt\,\vec{F}\,d\varsigma$

$$r0t\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ 3x+z & x-2y+z & 3x+y\end{vmatrix}=$$

$$=\left[\frac{\partial}{\partial y}(3x+y)-\frac{\partial}{\partial z}(x-2y+z)\right]\vec{i}-\left[\frac{\partial}{\partial x}(3x+y)-\frac{\partial}{\partial z}(3x+z)\right]\vec{j}+$$

$$+\left[\frac{\partial}{\partial x}(x-2y+z)-\frac{\partial}{\partial y}(3x+z)\right]\vec{k}=$$

$$=(1-1)\vec{i}-(3-1)\vec{j}+(1-0)\vec{k}=-2\vec{y}+\vec{k}$$

$$U=\int_{\lambda}\vec{F}\,d\vec{r}=\iint_{\varsigma}\vec{n}\,rd\,\vec{F}\,d\varsigma=$$

$$\iint_{\varsigma}(r0\vec{F})_x dydz+(r0t\vec{F})_y dzdx+(r0t\vec{F})_z dxdy=\iint_{\varsigma}-2dzdx+dxdy=$$

$$=-2\iint_{Dzx}dzdx+\iint_{Dxy}dxdy=-2\int_0^3 dx\int_0^{3-x}dz+\int_0^3 dx\int_0^{2-\frac{2}{3}x}dy=$$

$$=-2\int_0^3(3-x)dx+\int_0^3\left(2-\frac{2}{3}x\right)dx=$$

$$=-2\left(3x-\frac{x^2}{2}\right)\Bigg|_0^3+\left(2x-\frac{x^2}{3}\right)\Bigg|_0^3=-2\left(9-\frac{9}{2}\right)+(6-3)=-9+3=-6$$

6– masala

Gauss – Ostrogradskiy formulasidan foydalanib,

$\vec{F}=x^3\vec{i}+y^3\vec{j}+R^3z\vec{k}$ vektorning

$\frac{H}{R^2}(x^2+y^2)\le z\le H$

jismni barcha G sirti bo'yicha tashqi normal yo'nalishdagi oqimini toping.

Yechish:

Gauss – Ostrogradskiy formulasi bo'yicha

$$\Pi = \int_{\varsigma} \vec{F} \cdot \vec{n}\, d\varsigma = \iiint_V div\, \vec{F}\, dv$$

$$div\, \vec{F} = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 3x^2 + 3y^2 + R^2 = 3(x^2 + y^2) + R^2$$

demak,

$$\Pi = \iiint_V \left[3(x^2 + y^2) + R^2\right] dv$$

bu uch o'lchovli integralni hisoblash uchun silindrik koordinatalariga o'tsak,

$$Z = \frac{Hr^2}{R^2}$$ ni hosil qilamiz.

$$\Pi = \iiint_V \left[3(x^2 + y^2) + R^2\right] dv = \int_0^{2\pi} d\varphi \int_0^R (3r^2 + R^2) r dr \cdot \int_{\frac{Hr^2}{R^2}}^{H} dz =$$

$$= 2\pi \int_0^R (3r^2 + R)\left(H - \frac{Hr^2}{R^2}\right) r dr =$$

$$= \frac{2\pi H}{R^2} \int_0^R \left(R^2 + 2R^2 r^2 - \frac{Hr^2}{R^2} - 3z\right) r dr =$$

$$= \frac{2\pi H}{R^2} \left(R^4 \frac{r^2}{2} + 2R^2 \frac{r^4}{4} - \frac{r^6}{2}\right)\Bigg|_0^2 = \pi H R^4$$

7– masala

$\vec{F}$ maydonning potensial va solenoidalligini tekshiring. Agar potensial bo'lsa, $U(x, y, z)$ potensialini toping.

$$\vec{F} = (2x + 3yz)\vec{i} + (2y + 3xz)\vec{j} + (2z + 3xy)\vec{k}$$

Yechish:

$rot\vec{F}$ ni hisoblaymiz:

$$rot\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 2x+3yz & 2y+3xz & 2z+3xy\end{vmatrix}=[\frac{\partial}{\partial y}(2z+3xy)-\frac{\partial}{\partial z}(2y+3xz)]\vec{i}-$$

$$-[\frac{\partial}{\partial x}(2z+3xy)-\frac{\partial}{\partial z}(2x+3yz)]\vec{j}+[\frac{\partial}{\partial x}(2y+3xz)-\frac{\partial}{\partial y}(2x+3yz)]\vec{k}=$$

$=(3x-3x)\vec{i}-(3y-3y)\vec{j}+(3z-3z)\vec{k}=0$ demak, maydon potensial ekan.

$U(x,y,z)$ potensialni quyidagi formula bilan hisoblaymiz:

$$U(x,y,z)=\int_{(x_0,y_0,z_0)}^{(x,y,z)}x_1dx+ydy+zdz=$$

$$=\int_{x_0}^{x}X(x\ ,y_0,z_0)dx+\int_{y_0}^{y}Y(x\ ,y\ ,z_0)dy+\int_{z_0}^{z}X(x\ ,y\ ,z\ )dz$$

bu yerda: $M_0(x_0,y_0,z_0)$ qurilayotgan sohadagi fiksirlangan nuqta.

$$U(x,y,z)=\int_{x_0}^{x}(2x+3y_0z_0)dx+\int_{y_0}^{y}(2y+3xz_0)dy+\int_{z_0}^{z}(2z+3xy)dx=$$

$$=(x^2+3y_0z_0x)\Big|_{x_0}^{x}+(y^2+3xz_0y)\Big|_{y_0}^{y}+(z^2+3xyz)\Big|_{z_0}^{z}=$$

$$=x^2+3y_0z_0x-x_0^2-3y_0z_0x_0+y^3+3xz_0y-y_0^2-3xz_0y_0+z^2+$$

$$+3xyz - z^2 - 3xyz_0 = x^2 + y^2 + z^2 + 3xyz + C_1,$$

bu yerda $C_1 = -x_0^2 - y_0^2 - z_0^2 - 3x_0y_0z_0$. Agar maydonning har bir nuqtasida $div\vec{F} = 0$ bo'lsa, $\vec{F}$ vektor maydon solenoidal deyiladi.

$$div\vec{F} = \frac{\partial}{\partial x}(2x + 3yz) + \frac{\partial}{\partial y}(2y + 3xz) + \frac{\partial}{\partial z}(2z + xy) =$$

$$= 2 + 2 + 2 = 6 \neq 0$$

Demak maydon solenoidal emas.

8- masala

Berilgan $w = f(z)$, funksiyani $w = u(x, y) + iV(x, y)$ ko'rinishida yozing. Bu yerda $z = x + iy$; funksiyaning analitik ekanligini tekshiring. Agar shunday bo'lsa, uning z_0 nuqtadagi hosilasining qiymatini toping.

$w = x^2 - z + i$, $\quad z_0 = 1$

Yechish: $z = x + iy$ dan

$$w = (x + iy)^2 - (x + iy) + i = x^2 + 2xyi - y^2 - x - iy + i =$$

$$= x^2 - y^2 - x + i(2xy - y + i)$$

bu yerda $u(x, y) = x^2 - y^2 - x$, $\quad v(x, y) = 2xy - y + 1$

Koshi – Riman shartlaridan

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = 2x - 1, \qquad \frac{\partial v}{\partial y} = 2x - 1$$

$$\frac{\partial u}{\partial y} = -2y, \qquad \frac{\partial v}{\partial x} = 2y$$

Ya'ni shartlar bajariladi, ya'ni funksiya hosilaga ega va berilgan sohada analitik. Endi hosilani topamiz:

$$f'(z) = 2z - 1 \quad \text{va} \quad f'(1) = 2 \cdot 1 - 1 = 1$$

9- masala

$f(z)$ funksiyani z_0 nuqta atrofida Loran qatoriga yoying va qatorning yaqinlashish sohasini toping.

$$f(z)=\frac{1}{5z-8}, \qquad z_0=0$$

Yechish: $f(z)$ ni quyidagicha yozamiz.

$$f(z)=-\frac{\frac{1}{8}}{1-\frac{5z}{8}}$$

$z_0=0$ nuqta atrofida $\left|\frac{5z}{8}\right|<1$ tengsizlik bajariladi, shuning uchun $-\frac{\frac{1}{8}}{1-\frac{5z}{8}}$ kasrning birinchi hadi $a=-\frac{1}{8}$ va $q=\frac{5z}{8}$ bo'lgan cheksiz kamayuvchi geometrik progressiya yigindisi desak bo'ladi. Bundan

$$f(z)=-\frac{1}{8}-\frac{5z}{8^2}-\frac{5^2z^2}{8^3}-\frac{5^3z^3}{8^4}-\dots$$

ya'ni $f(z)=-\sum_{n=1}^{\infty}\frac{5^{n-1}z^{n-1}}{8^n}$ deb yozamiz. Ravshanki, qator faqat to'g'ri qismdan iborat. $\left|\frac{5z}{8}\right|<1$ dan $|z|<\frac{8}{5}$ qatorning yaqinlashish sohasi kelib chiqadi.

10– masala.

Dalamber usuli bilan – to'lqin tenglamasi bir jinsli cheksiz tor shakli tenglamasi $u = u(x,t)$ ni aniqlang. Bu yerda, $t_0 = 0$ boshlang'ich momentda tor shakli x abssissasi bilan mos ravishda $u\big|_{t_0=0} = f(x)$ va $\frac{\partial u}{\partial t}\big|_{t_0=0} = F(x)$ funksiyalar bilan berilgan.

$f(x) = 0$, $F(x) = \cos x$

Yechish.

Dalamber formulasidan

$$u = \frac{f(x-at)+f(x+at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} F(z)dz \quad \text{demak,}$$

$$u = \frac{1}{2a}\int_{x-at}^{x+at} \cos z dz = \frac{1}{2a}\sin z\Big|_{x-at}^{x+at} =$$

$$= \frac{1}{2a}\left[\sin(x+at) - \sin(x-at)\right] = \frac{\cos x \sin at}{a}$$

6– NAZORAT ISHI BO'YICHA MASALALAR

541-560 masalalarda integraldagi integrallash tartibini o'zgartiring. Integrallash sohasini chizmada tasvirlang.

541. $\int_0^4 dx \int_0^{\sqrt{25-x^2}} f(x,y)dy$ 542. $\int_0^1 dy \int_0^{2-x} f(x,y)dx$

543. $\int_0^4 dx \int_0^{\sqrt{x}} f(x,y)dy$ 544. $\int_0^2 dx \int_0^{3-x} f(x,y)dy$

545. $\int_0^{3/4} dx \int_{x^2}^{x} f(x,y)dy$ 546. $\int_0^{1/2} dy \int_y^{\sqrt{y}} f(x,y)dy$

547. $\int^{2} dy \int_{0}^{y^2} f(x,y)dx$

548. $\int_{0}^{1} dy \int_{0}^{x^2+1} f(x,y)dy$

549. $\int^{2} dy \int_{0}^{2y} f(x,y)dx$

550. $\int_{0}^{1} dx \int_{0}^{4-x^2} f(x,y)dy$

551. $\int_{0}^{4} dx \int_{3x^2}^{12x} f(x,y)dy$

552. $\int_{0}^{1} dx \int_{2x}^{3x} f(x,y)dy$

553. $\int_{0}^{a} dx \int_{\frac{a^2-x^2}{2a}}^{\sqrt{a^2-x^2}} f(x,y)dy$

554. $\int_{a/2}^{a} dx \int_{0}^{\sqrt{2ax-x^2}} f(x,y)dy$

555. $\int_{0}^{2a} dx \int_{\sqrt{2ax-x^2}}^{\sqrt{4ax}} f(x,y)dy$

556. $\int_{0}^{1} dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx$

557. $\int_{0}^{1} dy \int_{y^2/2}^{\sqrt{3-y^2}} f(x,y)dx$

558. $\int_{0}^{\frac{R-\sqrt{2}}{2}} dx \int_{2}^{x} f(x,y)dy \int_{\frac{R\sqrt{2}}{2}}^{R} dx$

559. $\int_{0}^{\pi} dx \int_{0}^{\sin x} f(x,y)dy$

560. $\int_{-1}^{1} dx \int_{0}^{\sqrt{1-x^2}} f(x,y)dy$

561-580 masalalarda ko'rsatilgan sirtlar bilan chegaralangan jismning hajmini uch o'lchovli integral yordamida hisoblang. Berilgan jismni va uning *XOY* tekislikdagi proeksiyasini chizmalarda tasvirlang.

561. $z=0$ $\quad z=x^2$, $\quad x+y=2, y=0$

562. $z=0$ $\quad z=x$, $\quad x=\sqrt{4-y^2}$

563. $z=0$ $\quad x^2+y^2$, $\quad x=0, y=0, x+y=2$

564. $z=0$ $\quad z=x^2+y^2$, $\quad x=0, y=0, x+y=2$

565. $z=0$ $\quad z=2y$, $\quad y=\sqrt{9-x^2}$

566. $z=0$ $\quad z=3x$, $\quad y^2=2-x$

567. $z=0$ $\quad z=\sqrt{y}$, $\quad y=3x, x=2$

568. $z=0$ $\quad z=y^2$, $\quad y=2x, x=0$

569. $z=0$ $\quad z=\sqrt{y}$, $\quad y=2x, y=3, x=0$

570. $z=0$ $\quad z=x^2$, $\quad y=2x, y=2x, x=4, y=0$

571. $z=0$ $\quad z=2x$, $\quad x+y=3, x=\sqrt{\frac{y}{2}}$

572. $z=0$ $\quad z=\sqrt{1-y}$, $\quad y=x^2$

573. $z=0$ $\quad x=0, y=0$, $\quad x+y=1, z=x^2+3y^2$

574. $z=0$ $\quad z=x^2$, $\quad 2x-y=0, x+y=0$

575. $z=0$ $\quad z=2-x$ $\quad y=2\sqrt{x}, y=\frac{1}{4}x^2$

576. $z^2=4-y$ $\quad x^2+y^2=4y$,

577. $z=0$ $\quad x=0$, $\quad z=y^2$, $\quad 2x+3y=6$

578. $z=0$ $\quad z=(x-1)^2$, $\quad y^2=x$

579. $z=0$ $\quad z=4-x^3$ $\quad x^2+y^2=4$

580. $z=0$ $\quad x=0$, $\quad y=0$, $\quad z=y^2+1$, $\quad x+y=1$

581. $\int\limits_c (x+y)dx+(2x-y)dy$ egri chiziqli integralni $x=5\cos\varphi$, $y=5\sin\varphi$ aylana bo'ylab soat strelkasi harakatiga teskari yurib hisoblang.

582. $\int\limits_{AB} (x^2-2xy)dx+(2xy+y^2)dy$ egri chiziqli integral $y=x^2$ parabola bo'yicha $A(1;1)$ nuqtadan $B(2;4)$ nuqtagacha joylashgan.

583. $\int\limits_c (2x-3y)dx+xdy$ egri chiziqli integralni

$x=4\cos\varphi, y=3\sin\varphi$ ellips bo'ylab soat strelkasi harakatiga teskari yurib hisoblang.

584. $\int\limits_{AB} 2ydx+(3x-y)dy$ egri chiziqli integralni $y=\sqrt{x}$ parabola bo'yicha $A(1;1)$ nuqtadan $B(4;2)$ nuqtagacha hisoblang.

585. $\int\limits_{AB}(x^2-y)dx+(y^2-x)dy$ egri chiziqli integralni $A(0;0)$ nuqtadan $B(3;4)$ nuqtagacha shu nuqtalar orqali o'tuvchi to'g'ri chiziq bo'ylab hisoblang.

586. $\int\limits_{AB}\frac{(y^2+1)}{y}dx-\frac{x-2y^2}{y^2}dy$ egri chiziqli integralni $A(1;2)$ nuqtadan $B(2;4)$ nuqtagacha shu nuqtalar orqali o'tuvchi to'g'ri chiziq bo'ylab hisoblang.

587. $\int\limits_{AB}(3x^2y+1)dx+(x^2+2)dy$ egri chiziqli integralni $A(0;0)$ nuqtadan $B(2;4)$ nuqtagacha shu nuqtalar orqali o'tuvchi to'g'ri chiziq bo'ylab hisoblang.

588. $\int\limits_{AB}(3x^2y+1)dx+(x^2+2)dy$ egri chiziqli integralni $A(0;0)$ nuqtadan $B(1;2)$ nuqtagacha $y=2\sqrt{x}$ porabola bo'ylab hisoblang.

589. $\int\limits_{AB}(y^2+x)dx+\frac{2x}{y}dy$ egri chiziqli integralni $A(0;1)$ nuqtadan $B(1;2)$ nuqtagacha $y=e^x$ egri chiziq bo'ylab hisoblang.

590. $\int\limits_{AB}\frac{y}{x}dx+xdy$ egri chiziqli integralni $A(1;0)$ nuqtadan $B(0;1)$ nuqtagacha $y=\ln x$ egri chiziq bo'ylab hisoblang.

591. $\int\limits_{AB}(x^2+y^2)dx+(x^2-y^2)dy$ egri chiziqli integralni

$A(-1;1)$ nuqtadan $B(2;2)$ nuqtagacha $y=|x|$ chiziq bo'ylab hisoblang.

592. $\int\limits_{AB} xdy+ydx$ egri chiziqli integralni $A(2\pi a;0)$ nuqtadan $B(0;0)$ sikloida yoyi bo'yicha hisoblang.

593. $\int\limits_{c} (x^2+y)dx+(y^2+x)dy$ egri chiziqli integralni $x=4\cos t$, $y=4\sin t$ aylana bo'ylab soat strelkasi harakatiga teskari yo'nalishda hisoblang.

594. $\int\limits_{c} (x^2+y)dx+(y^2+x)dy$ egri chiziqli integralni $A(1;2)$ nuqtadan $B(3;5)$ nuqtagacha $x=1, y=5$ to'g'ri chiziq kesmalaridan iborat siniq chiziq bo'ylab hisoblang.

595. $\int\limits_{c} xdy+ydy$ egri chiziqli integralni $A(-2,0)$, $B(2;0)$, $D(0;2)$, uchburchak uchlari bo'ylab soat strelkasiga karshi yo'nalishda hisoblang.

596. $\int\limits_{AB} xe^{x^3}dy+ydx$ egri chiziqli integralni $A(0;0)$ nuqtadan $B(1;1)$ nuqtagacha $y=x^2$ parabola yoyi bo'yicha hisoblang.

597. $\int\limits_{AB} 2xydy+x^2dx$ egri chiziqli integralni $A(0;0)$ nuqtadan $B(2;1)$ nuqtagacha $x=2y^2$ parabola yoyi bo'yicha hisoblang.

598. $\int\limits_{AB} \frac{x^2dy-y^2dx}{x^{\frac{5}{3}}+y^{\frac{5}{3}}}$ egri chiziqli integralni $A(8;0)$ nuqtadan $B(0;8)$ nuqtagacha $x=8\cos^3 t, y=8\sin^3 t$ astroida yoyi bo'yicha hisoblang.

599. $\int\limits_{AB} (x^2-y^2)dx+xydy$ egri chiziqli integralni $A(1;1)$ nuqtadan $B(3;4)$ o'tgan to'g'ri chiziq bo'ylab hisoblang.

600. $\int\limits_{AB}(x-y)^2dx+(x+y)^2dy$ egri chiziqli integralni OAB siniq chiziq bo'ylab hisoblang.

Bu yerda: $O(0;0), A(2;0), B(4;2)$.

601–620 masalalarda $\vec{F}=x_{\vec{ir}}\,y_{\vec{j}}+z\vec{k}$ vektor maydon va (P) $Ax+By+CzfD=0$ tekislik berilgan. Ular koordinata tekisliklari bilan V piramidani hosil qiladi. (P) tekislikka tegishli piramida asosini δ bilan belgilaymiz, konturning chegarasi λ, δ-ga o'tkazilgan normal $\vec{n}$ bo'lsa, quyidagilarni toping.

1) $\vec{F}$ maydonning yopiq λ kontur bo'yicha Stoks teoremasidan sirkulyatsiyasini hisoblang;

2) $\vec{F}$ vektor maydonning V piramidani to'la sirti bo'yicha tashqi normal bo'ylab Gauss–Ostrogradskiy teoremasini qo'llab oqimni hisoblang, chizma chizing.

601. $\vec{F}=(x+z)\vec{i}$, $(P): x+y+2-2=0$

602. $\vec{F}=(y-x+z)\vec{j}$, $(P): 2x-y+2z-2=0$

603. $\vec{F}=(x+3z)\vec{л}$, $(P): 2x+y+z-4=0$

604. $\vec{F}=(x+2y-z)\vec{i}$, $(P): -x+2y+2z-4=0$

605. $\vec{F}=(2x+3y-3z)\vec{j}$, $(P): 2x-3y+2z-6=0$

606. $\vec{F}=(2x+4y+3z)\vec{k}$, $(P): 3x+2y+3z-6=0$

607. $\vec{F}=(x-y+z)\vec{i}$, $(P): -x+2y+z-4=0$

608. $\vec{F}=(3x+4y+2z)\vec{j}$, $(P): x+y+2z-4=0$

609. $\vec{F}=(5x+2y+3z)\vec{k}$, $(P): x+y+3z-3=0$

610. $\vec{F}=(x-3y+6z)\vec{i}$, $\quad (P): -x+y+2z-4=0$

611. $\vec{F}=(2z-x)\vec{i}+(x-y)\vec{j}+(3x+z)\vec{k}$,
$(P): x+y+2z-2=0$

612. $\vec{F}=4z\vec{i}+(x-y-z)\vec{j}+(3y+z)\vec{k}$,
$(P): x-2y+2z-2=0$

613. $\vec{F}=(x-z)\vec{i}+(2x+y)\vec{j}+(x+y+z)\vec{k}$,
$(P): 2x+y+z-2=0$

614. $\vec{F}=(x-2z)\vec{i}+(2x+y)\vec{j}+(x+y+z)\vec{k}$
$(P): -x+2y+2z-2=0$

615. $\vec{F}=(2x-z)\vec{i}+(2x+y)\vec{j}+(x+y+z)\vec{k}$,
$(P): x-y+z-2=0$

616. $\vec{F}=(2z+z)\vec{i}+(x-3z)\vec{j}+(y+z)\vec{k}$,
$(P): -3x+2y+4z-6=0$

617. $\vec{F}=(x+y)\vec{i}+(y+z)\vec{j}+(2x+2z)\vec{k}$,
$(P): 3x-2y+2z-6=0$

618. $\vec{F}=(x+y+z)\vec{i}+2z\vec{j}+(y-7z)\vec{k}$,
$(P): 2x+3y+z-6=0$

619. $\vec{F}=4z\vec{i}+(x-y-z)\vec{j}+(3y+2z)\vec{k}$,
$(P): -2x+y+z-4=0$

620. $\vec{F}=(2z-x)\vec{i}+(x+2z)\vec{j}+3z\vec{k}$,
$(P): x+4y+z-4=0$

621 – 640. masalalarda $\vec{F}$ maydonning potensial va solenoidalligini tekshiring. Agar maydon potensial bo'lsa, uning $U(x,y,z)$ potensialini toping.

621. $\vec{F} = (-2x - yz)\,\vec{i} + (-2x + xz)\,\vec{j} + (-2z - xy)\,\vec{k}$

622. $\vec{F} = (2x - yz)\,\vec{i} + (2y + xz)\,\vec{j} + (2z - xy)\,\vec{k}$

623. $\vec{F} = (2x + yz)\,\vec{i} + (2y + xz)\,\vec{j} + (2z - xy)\,\vec{k}$

624. $\vec{F} = (2x - 4yz)\,\vec{i} + (2y - 4xz)\,\vec{j} + (2z + 4xy)\,\vec{k}$

625. $\vec{F} = (2x - 3yz)i + (2y - 8xz)j + (2z - 3xy)\,\vec{k}$

626. $\vec{F} = (-3x + yz)\,\vec{i} + (-3x + xz)\,\vec{j} + (-3z + xy)\,\vec{k}$

627. $\vec{F} = (2x + 2yz)\,\vec{i} + (2y + 2xz)\,\vec{j} + (2z + 2xy)\,\vec{k}$

628. $\vec{F} = (4x + yz)\,\vec{i} + (4y + xz)\,\vec{j} + (4z + xy)k$

629. $\vec{F} = (2x + 5yz)\,\vec{i} + (2y + 5xz)\,\vec{j} + (2z + 5xy)\,\vec{k}$

630. $\vec{F} = (2x + 3yz)\,\vec{i} + (2y + 3xz)\,\vec{j} + (2z + 2xy)\,\vec{k}$

631. $\vec{F} = (6x + 7yz)\,\vec{i} + (6y + 7xz)\,\vec{j} + (6z + 7xy)\,\vec{k}$

632. $\vec{F} = (8x - 5yz)\,\vec{i} + (6y - 5xz)\,\vec{j} + (8z - 5xy)\,\vec{k}$

633. $\vec{F} = (10x - 8yz)\,\vec{i} + (10y - 3xz)\,\vec{j} + (10z - 3xy)\,\vec{k}$

634. $\vec{F} = (12x + yz)\,\vec{i} + (12y + xz)\,\vec{j} + (12z + xy)\,\vec{k}$

635. $\vec{F} = (4x - 7yz)\,\vec{i} + (4y - 7xz)\,\vec{j} + (4z - 4xy)\,\vec{k}$

636. $\vec{F} = (x + 2yz)\,\vec{i} + (y + 2xz)\,\vec{j} + (z + 2xy)\,\vec{k}$

637. $\vec{F} = (5x + 4yz)\,\vec{i} + (5y + 4xz)\,\vec{j} + (5z + 4xy)\,\vec{k}$

638. $\vec{F} = (7x - 2yz)\,\vec{i} + (7y - 2xz)\,\vec{j} + (7z - 2xy)\,\vec{k}$

639. $\vec{F} = (3x - yz)\,\vec{i} + (3y - xz)\,\vec{j} + (3z - xy)\,\vec{k}$

640. $\vec{F} = (9x + 5yz)\,\vec{i} + (2y + 5xz)\,\vec{j} + (9z + 5xy)\,\vec{k}$

641-660-masalalarda berilgan $w = f(z)$, funksiyani $w = u(x, y) + iV(x, y)$ ko'rinishida yozing. Bu yerda $z = x + iy$; funksiyaning analitik ekanligini tekshiring. Agar shunday bo'lsa, uning z_0 nuqtadagi hosilasining qiymatini toping.

641. $w = z^2 - 2z + 1$ $\quad z_0 = i$

642. $w = e^{-2z}$ $\quad z_0 = 1 + i$

643. $w = z - z^2$ $\quad z_0 = -i$

644. $w = \ln(1 + z)$ $\quad z_0 = 1 - i$

645. $w = z^3 - 2z^2$ $\quad z_0 = 2i$

646. $w = 1 - z^2$ $\quad z_0 = -2i$

647. $w = 8e^{-z}$ $\quad z_0 = 1 - i\pi$

648. $w = e^{z^2}$ $\quad z_0 = -i$

649. $w = e^z - z$ $\quad z_0 = 0$

650. $w = z - e^{-z}$ $\quad z = i$

651. $w = (iz)^3$ $\quad z_0 = -1 + i$

652. $w = e^{-z^2}$ $\quad z_0 = i$

653. $w = i(1 - z^2) - 2z$ $\quad z_0 = 1$

654. $w = e^{1-2z}$ $\quad z_0 = \frac{\pi i}{3}$

655. $w = z^3 + 3z - i$ $\quad z_0 = -i$

656. $w = e^{1-2iz}$ $\quad z_0 = 1 - i$

657. $w = 2z^2 - iz$ $\quad z_0 = 1 - i$

658. $w = e^{iz^2}$ $\quad z_0 = \frac{\sqrt{\pi}\, i}{2}$

659. $w = z^3 + z^2 + i$ $\qquad z_0 = \dfrac{2i}{3}$

660. $w = ze^z$ $\qquad z_0 = -1 + i\pi$

661-680-masalalarda $f(z)$ funksiyani z_0 nuqta atrofida Loran qatoriga yoying va qatorning yaqinlashish sohasini toping.

661. $f(z) = \dfrac{1}{1-z}$ $\qquad z_0 = 0$

662. $f(z) = \sin\dfrac{1}{1-z}$ $\qquad z_0 = 0$

663. $f(z) = \dfrac{1}{z(z-2)}$ $\qquad z_0 = 0$

664. $f(z) = \dfrac{1}{(1-z)^n}$ $\qquad z_0 = 1$

665. $f(z) = e^{\frac{1}{z^2}}$ $\qquad z_0 = 0$

666. $f(z) = \dfrac{1}{1-z^2}$ $\qquad z_0 = 0$

667. $f(z) = \ln\dfrac{z-2}{z+3}$ $\qquad z_0 = 1$

668. $f(z) = \dfrac{1}{4z-5}$ $\qquad z_0 = \dfrac{5}{4}$

669. $f(z) = \cos\dfrac{1}{1-z}$ $\qquad z_0 = 0$

670. $f(z) = e^{\frac{1}{1+z}}$ $\qquad z_0 = -1$

671. $f(z) = \dfrac{1}{3z-5}$ $\qquad z_0 = \dfrac{5}{3}$

672. $f(z) = \sin\dfrac{z}{1-z}$ $\qquad z_0 = 1$

673. $f(z)=e^{\frac{1}{z}}$ $\qquad z_0=0$

674. $f(z)=\frac{1}{(z^2+1)^2}$ $\qquad z_0=i$

675. $f(z)=\ln\frac{z-1}{z-2}$ $\qquad z_0=1$

676. $f(z)=\cos\frac{z}{z-1}$ $\qquad z_0=1$

677. $f(z)=\frac{1}{z(z-1)}$ $\qquad z_0=0$

678. $f(z)=e^{\frac{1}{(1-z)}}$ $\qquad z_0=1$

679. $f(z)=\frac{z}{1+z^2}$ $\qquad z_0=i$

680. $f(z)=\frac{1}{(z-3)^2}$ $\qquad z_0=3$

681-700-masalalarda Dalamber usuli bilan – to'lqin tenglamasi bir jinsli cheksiz tor shakli tenglamasi $u=u(x,t)$ ni aniqlang. Bu yerda $t_0=0$ boshlang'ich momentda tor shakli x abssissasi bilan mos ravishda $u|_{t_0=0}=f(x)$ va $\frac{\partial u}{\partial t}|_{t_0=0}=F(x)$ funksiyalar bilan berilgan.

681. $f(x)=x(2-x)$, $\quad F(x)=e^{-x}$

682. $f(x)=x^2$, $\quad F(x)=\sin x$

683. $f(x)=e^x$, $\quad F(x)=wx$

684. $f(x)=\sin x$, $\quad F(x)=v_0$

685. $f(x) = \sin x$, $F(x) = \cos x$

686. $f(x) = \cos x$, $F(x) = wx$

687. $f(x) = x$, $F(x) = \cos x$

688. $f(x) = x(x-2)$, $F(x) = e^x$

689. $f(x) = \cos x$, $F(x) = \sin x$

690. $f(x) = e^{-x}$, $F(x) = v_0$

691. $f(x) = x^2 - 2$ $F(x) = e^{-2x}$

692. $f(x) = e^{3x}$ $F(x) = \sin 2x$

693. $f(x) = x + 4$ $F(x) = 3x - 2$

694. $f(x) = 5 - x$ $F(x) = tgx$

695. $f(x) = x(x-3)$ $F(x) = \cos 2x$

696. $f(x) = e^{-2x}$ $F(x) = x$

697. $f(x) = \cos 3x$ $F(x) = 5 - x$

698. $f(x) = e^{1-x}$ $F(x) = x^2 - x$

699. $f(x) = 3 + x$ $F(x) = 5 - x$

700. $f(x) = x - x^2$ $F(x) = \sin 3x$

7-NAZORAT ISHI

1- masala

Qurilma o'zaro erkli ishlaydigan ikkita elementni o'z ichiga oladi. Qurilmaning buzilishi uchun kamida bitta elementning buzilishi yetarli bo'lsa, qurilmaning ishlamay qolish ehtimolini toping.

Yechish:

Berilgan ehtimolliklar $P_1 = 0.05$ va $P_2 = 0.08$ bo'lsin. qarama-qarshi hodisalar ehtimollari, ya'ni elementlarning buzilmaslik ehtimollari $q_1 = 1 - P_1 = 0.95$, $q_2 = 1 - P_2 = 0.92$ bo'ladi. Izlanayotgan ehtimol:

$$P(A) = 1 - q_1 q_2 = 1 - 0.95 \cdot 0.92 = 0.126$$

2- masala

Uchta stanokda bir xil va bog'liqsiz sharoitda bir turdagi detallar ishlab chiqariladi. Barcha detallarning $q_1 \%u$ birinchi stanokda, $q_2 \%u$ ikkinchi stanokda, $q_3 \%u$ uchinchi stanokda ishlab chiqariladi. Birinchi stanokda ishlab chiqarilgan detalning kamchiliksiz bo'lish ehtimoli P_1, P_2 esa ikkinchi stanok va P_3 uchinchi stanokda ishlangan detalning kamchiliksiz bo'lish ehtimoli bo'lsin.

Tasodifan olingan detalning kamchiliksiz bo'lish ehtimolini toping.

Yechish:

B_i hodisa tasodifan olingan detalning i-nchi stanokda ishlangan hodisasi bo'lsin (i=1,2,3). A hodisa olingan detalning kamchiliksiz bo'lish hodisasi bo'lsin. Shartga ko'ra

$$P(B_i) = \frac{q_i}{100}, \qquad P(A/B_i) = P_i$$

To'la ehtimol formulasidan

$$P(A)=\sum_{i=1}^{3} P(A/B_i)P(B_i)=\frac{1}{100}\sum_{i=1}^{3} q_i \cdot p`_i$$

3- masala

Mahsulot ikki zavod tomonidan ishlab chiqariladi. Bunda ikkinchi zavodning mahsulot hajmi birinchi zavodga qaraganda K marta ko'p. Birinchi zavod mahsuloti yaroqsiz bo'lish ehtimoli P_1 va ikkinchisi P_2. Ikkinchi zavoddan tasodifan olingan mahsulotning yaroqsiz bo'lish ehtimolini toping.

Yechish:

B_1- olingan mahsulotning birinchi zavoddan olinish hodisasi va B_2 ikkinchi zavoddan olinish hodisasi bo'lsin.

Unda $P(B_1)=\frac{1}{1+k}$, $P(B_2)=\frac{1}{1+k}$

A- yaroqsiz mahsulot bo'lish hodisasi bo'lsin. Shartga ko'ra $P(A/B_i)=P_1$, $P(A/B_i)=P_2$. Beyes formulasidan foydalansak,

$$P(B_2/A)=\frac{\frac{k}{1+k}\cdot P_2}{\frac{1}{1+k}\cdot P+\frac{1}{1+k}\cdot P_2}=\frac{kP_2}{P_1+kP_2}$$

Xuddi shunday

$$P\left(B_1/A\right)=\frac{P_1}{P_1+kP_2}$$

4- masala

Bir xil va bog'liqsiz sinovlarning har birida xossaning sodir bo'lishi ehtimolligi 0,9 ga teng. 900 ta sinovda hodisaning 800 marta sodir bo'lish ehtimolini toping.

Yechish:

Shartga ko'ra $n=900$, $M=800$, $P=0,9$, $1-p=1-0,9=0,1, n=900$ katta son bo'lgani uchun Muavr-

Laplasning lokal teoremasidan foydalanamiz.

$$x = \frac{m - np}{\sqrt{np(1-p)}} = \frac{800 - 900 \cdot 0,9}{\sqrt{900 \cdot 0,9 \cdot 0,1}} = -\frac{10}{9} \approx -1,11$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

funksiya jadvalidan $\frac{1}{\sqrt{2\pi}} e^{-\frac{(1,11)^2}{2}} = 0,2155$ ehtimollikni topamiz:

$$P = \frac{1}{\sqrt{np(1-p)}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \frac{1}{\sqrt{900 \cdot 0,9 \cdot 0,1}} \cdot 0,2155 =$$

$$= \frac{0,2155}{9} \approx 0,024$$

5- masala

X tasodifiy miqdor x_1 va x_2 qiymatlar qabul qiladi, lekin $x_1 < x_2; x_1$ qiymatning ehtimolligi $P_1 = 0,5;$ matematik kutilma $M(x) = 2,5$ va dispersiya $D(x) = 0,25$ berilgan. Bu tasodifiy miqdorning taqsimot qonunini toping.

Yechish:

$P_1 + P_2 = 1$ dan $p_2 = 1 - P_1 = 0,5$

Shartga ko'ra $\begin{cases} 0,5x_1 + 0,5x_2 = 2,5 \\ 0,5x_1^2 + 2x_2^2 - 2,5^2 = 0,25 \end{cases}$

Yoki $\begin{cases} x_1 + x_2 = 5 \\ 2x_1^2 + 0,5x_2^2 - 25 = 1 \end{cases}$

$$2(5 - x_2)^2 + 2x_2^2 - 26 = 0$$

$$x_2^2 - 5x_2 + 6 = 0$$

Bundan $x_2^{(1)} = 3, x_2^{(2)} = 2$

$$x_1^{(1)} = 2, x_1^{(2)} = 3$$

Shartga ko'ra $x_1 < x_2$ demak,

X	2	3
P	0,5	0,5

6- masala

X tasodifiy miqdor quyidagi taqsimot funksiyasi bilan berilgan :

$$F(x)=\begin{cases}0, & x\le 0\\ \dfrac{x^2}{100}, & 0<x\le 10\\ 1, & x>10\end{cases}$$

differensial funksiyani, matematik kutilma va dispersiyani toping. Grafigini chizing.

Yechish:

Taqsimot zichligini (differensial funksiyani) topamiz:

$$f(x)=F'(x)=\begin{cases}0, & x\le 0\\ \dfrac{x}{50} & 0<x\le 10\\ 1 & x>10\end{cases}$$

$$M(x)=\int_0^{10} x\cdot\frac{x}{50}dx=\left.\frac{x^3}{150}\right|_0^{10}=\frac{10^3}{150}=6\frac{2}{3}$$

$$D(x)=\int_0^{10}(x-M(x))^2 f(x)dx=\int_0^{10}\left(x-6\frac{2}{3}\right)^2\frac{x}{50}dx=$$

$$=\int_0^{10}\left(\frac{x^3}{50}-\frac{40}{150}x^2+\frac{400}{500}x\right)dx=$$

$$=\left.\left(\frac{x^4}{200}-\frac{4x^3}{45}+\frac{4}{9}x^2\right)\right|_0^{10}=\frac{10000}{200}-\frac{4000}{45}+\frac{400}{9}\approx 5,5555$$

7- masala

$\gamma = 0,95$ ishonchlilik bilan normal taqsimot matematik kutilmasini baholash uchun ishonchlilik oraliqlarini toping. Tanlanma o'rta qiymat $\overline{x_T} = 24$, o'rtacha kvadratik chetlanish $\sigma = 5$ va tanlanma hajmi $n = 25$.

Yechish:

$\overline{x_T} - t\dfrac{\sigma}{\sqrt{n}} < a < \overline{x_T} + t\dfrac{\sigma}{\sqrt{n}}$ oralig'ini topish talab qilinadi.

Bu yerda t noma'lum.

$2\phi(t) = 0,95$ dan $\phi(t) = 0,475$. Laplas formulasi jadvalidan $t = 1,96$ ni topamiz.

$$24 - 1,96 \cdot \frac{5}{\sqrt{25}} < a < 24 + 1,96 \cdot \frac{5}{\sqrt{25}}$$

22,04<a<25,96 oraliqni topamiz.

8- masala

Y ning X ga regressiya tenglama to'g'ri chiziq tenglamasi $\overline{Y}_X - \overline{Y} = r_T \dfrac{\sigma_y}{\sigma_x}(x - \overline{x})$ quyidagi korrelyatsion jadval bo'yicha tuzilsin.

1- jadval

Y\X	20	25	30	35	40	n_y
16	4	6	-	-	-	10
26	-	88	10	-	-	18
36	-	-	32	3	9	44
46	-	-	4	12	6	22
56	-	-	-	1	5	6
nx	14	14	46	16	20	$n = 100$

Yechish:

Soxta nollar sifatida $c_1 = 30$ va $c_2 = 36$ larni tanlab (bular variatsion qatorning o'rtalarida joylashgan) 2- jadvalni tuzamiz, bu yerda

$$U_i = \frac{x_i - 30}{5}, V_i = \frac{y_i - 36}{10}$$

2-jadval

U\V	-2	-1	0	1	2	nv
-2	4	6	-	-	-	10
-1	-	8	10	-	-	18
0	-	-	32	3	9	44
1	-	-	4	12	6	22
2	-	-	-	1	5	6
nx	4	14	46	16	20	$n=100$

Yordamchi kattaliklar $\overline{U}$, $\overline{V}$, $\overline{U}^2$, $\overline{V}^2$, σ_U va σ_V larni

$$\overline{U} = \frac{\sum n_U \cdot U}{n} = \frac{4(-2)+14(-1)+46\cdot 0+16\cdot 1+20\cdot 2}{100} = 0.34$$

$$\overline{V} = \frac{\sum n_V \cdot \mathrm{V}}{n} = \frac{10(-2)+18(-1)+44\cdot 0+22\cdot 1+6\cdot 2}{100} = 0.04$$

$$\overline{U}^2 = \frac{\sum n_U \cdot U^2}{n} = \frac{4\cdot 4+14\cdot 1+46\cdot 0+16\cdot 1+20\cdot 4}{100} = 1.26$$

$$\overline{V}^2 = \frac{\sum n_V \cdot V^2}{n} = \frac{10\cdot 4+16\cdot 1+44\cdot 0+12\cdot 1+6\cdot 4}{100} = 1.04$$

$$\sigma_U = \sqrt{\overline{U}^2 - (\overline{U})^2} = \sqrt{1.26 - 0.34^2} \approx 1.07$$

$$\sigma_U = \sqrt{\overline{V}^2 - (\overline{V})^2} = \sqrt{1.04 - 0.04^2} \approx 1.02$$

r_T ni topamiz, buning uchun 3 – jadvalni tuzamiz:

3-jadval.

v \ m	-2	-1	0	1	2	$V=\sum n_{n2}\cdot U$	$V\cdot U$
2	-	-	-	1	5	11	22
1	-	-	4	12	6	24	24
0	-	-	32	3	9	21	0
-1	-	8	10	-	-	-8	8
-2	4	6	-	-	-	-14	28
$V=\sum U_{UV}\cdot V$	-8	-20	-6	14	16	-	$\sum_U UV=82$
$U\cdot V$	16	20	0	14	32	$\sum_U UV=82$	Teksh.

3 jadvalni tuzishga doir tushuntirishlar:

1. n_{UV} chastotaning $\overline{U}$ variantaga ko'paytmasi $n_{UV}\cdot U$ ni bu chastotani o'z ichiga olgan katakning yuqori o'ng burchagiga yoziladi.

Masalan, $U(-2)=-8$.

2. Bir satr kataklarning yuqori o'ng burchaklarida joylashgan barcha sonlar qo'shiladi va ular yig'indisini V ustunning shu satrdagi katagiga yoziladi. Masalan, $V=-8+(-6)=-14$

3. V variantani U ga ko'paytiriladi va hosil bo'lgan ko'paytmani $V\cdot U$ ustunning tegishli katagiga yoziladi. Masalan, 1 – satrda $V=-2$, $U=-14$.

4. $V\cdot U$ ustunning barcha sonlarini qo'shib, $\sum V\cdot U$ yig'indisini hosil qilinadi, u izlanayotgan $\sum n_{VU}U\cdot V$ yig'indiga teng bo'ladi. Masalan, $\sum_U UV=82$, demak, yig'indi $\sum n_{U\cdot V}U\cdot V=82$. Tanlanma korrelyatsiya koeffitsientini topamiz:

$$r_T = \frac{\sum n_{UV} U \cdot V - nUV}{n\sigma_U \cdot \sigma_V} = \frac{82 \cdot 100 \cdot 0.34 \cdot (-0.04)}{100 \cdot 1.07 \cdot 1.02} \approx 0.76$$

$h_1 = 25 - 20 = 5$; $\qquad h_2 = 26 - 16 = 10$

$\bar{x} = \bar{U} h_1 + C_1 = 0.34 \cdot 5 + 30 = 31.7$

$\bar{y} = \bar{U} h_2 + C_2 = -0.04 \cdot 10 + 36 = 35.6$

$\sigma_z = h_1 \cdot \sigma_U = 5 \cdot 1.07 = 5.35$

$\sigma_y = h_2 \cdot \sigma_V = 10 \cdot 1.02 = 10.2$

topilganlarni (1) ga qo'yamiz:

$$\bar{y}_x - 35.6 = 0.76 \cdot \frac{10.2}{5.35}(x - 31.7)$$

7–NAZORAT ISHI BO'YICHA MASALALAR

701-720-masalalarda qurilma o'zaro erkli ishlaydigan ikkita elementni o'z ichiga oladi. Qurilmaning buzilishi uchun kamida bitta elementning buzilishi yetarli bo'lsa, qurilmaning ishlamay qolish ehtimolini toping.

701. Uch tadqiqotchi bir-biridan erkli ravishda biror kattalikni o'lchashmoqda. Birinchi tadqiqotchining asbob ko'rsatishini o'qishda xatoga yo'l qo'yish ehtimoli 0,1 ga teng. Ikkinchi va uchinchi tadqiqotchi uchun bu ehtimol mos ravishda 0,5 va 0,2 ga teng. Bir martadan o'lchashda tadqiqotchilardan birining xatoga yo'l qo'yish ehtimolini toping.

702. Elektron raqamli mashinani ishlash vaqtida arifmetik qurilmalar, operativ xotira qurilmasida, qolgan qurilmalarda buzilish yuz berish ehtimollari 3:2:5 kabi nisbatda. Arifmetik qurilmada, operativ xotira qurilmasida va boshqa qurilmalardagi buzilishni topish ehtimoli mos ravishda 0.8, 0.9, 0.9 ga teng. Mashinadagi yuz bergan buzilishning topilish ehtimolini toping.

703. Uch mergan bir yo'la o'q uzishdi, bunda ikki o'q nishonga tegdi. Agar birinchi, ikkinchi va uchinchi

merganlarning nishonga tekkizish ehtimollari mos ravishda 0,6; 0,5; 0,4 ga teng bo'lsa, uchinchi merganning nishonga tekkizganligining ehtimolini toping.

704. Talaba fonogrammadagi 50 savoldan 35 tasini biladi. Talaba imtihon biletidagi ikkita savolni bilish ehtimolini toping.

705. Ikki qutichaning har birida uchta qora va bitta oq shar bor. Ikkinchi qutichadan tavakkal qilib bitta shar olingan va birinchisiga solingan, shundan so'ng birinchi qutichadan tavakkaliga bitta shar olindi. Birinchi qutichadan olingan shar oq bo'lish ehtimolini toping.

706. Uchta avtomat: detal yasaydi, ular umumiy konveyerga tushadi. Birinchi, ikkinchi, uchinchi avtomatlarning mehnat unumi 2:3:5 nisbat kabi. Birinchi avtomat yasagan detalning a'lo sifatli ekanligini ehtimolini 0,9, ikkinchi va uchinchi avtomatlar uchun bu ehtimollik mos ravishda 0,8 va 0,7; konveyerdan tavakkal qilib olingan detallar a'lo sifatli bo'lish ehtimolini toping.

707. Avariya haqida xabar berish uchun uchta bir-biriga bog'liq bo'lmagan holda ishlaydigan qurilma o'rnatilgan. Avariya yuz berganda birinchi qurilmaning ishlash ehtimoli 0,8, ikkinchi va uchinchi qurilmalar uchun bu ehtimol mos ravishda 0,9 va 0,8 ga teng. Avariya yuz berganda: a) faqat bitta qurilma; b) faqat ikkinchi qurilma; v) uchala qurilma ishlash ehtimolini toping.

708. Ikkita o'q otganda nishonga hech bo'lmaganda bittasining tegishi ehtimoli 0,96 ga teng. Uchta o'q otilganda ikkitasining nishonga tegish ehtimolini toping.

709. Otilgan uchta o'qdan hech bo'lmaganda bittasining nishonga tegishi ehtimoli 0,992 ga teng. Beshta o'qdan to'rttasini nishonga tegish ehtimolini toping.

710. Bog'liq bo'lmagan sinovlarning har birida hodisaning sodir bo'lish ehtimoli 0,8 ga teng. 144 ta sinovda hodisaning 100 marta ro'y berishi ehtimolini toping.

711. Bog'liq bo'lmagan sinovlarning har birida hodisaning sodir bo'lish ehtimoli 0,2ga teng. 100 sinovda hodisaning 100 marta ro'y berishi ehtimolini toping.

712. Bog'liq bo'lmagan sinovlarning har birida hodisaning sodir bo'lish ehtimoli 0,8 ga teng. 100 ta sinovda hodisaning eng kamida 70 va ko'pi bilan 80 marta ro'y berishi ehtimolini toping.

713. *A* hodisa kamida 4 marta ro'y bergan holda V hodisa ro'y beradi. Agar har birida A hodisaning ro'y berish ehtimoli 0,8 ga teng bo'lgan beshta erkli sinov o'tkaziladigan bo'lsa, V hodisaning ro'y berish ehtimolini toping.

714. Hodisaning 2100 erkli sinovining har biridan ro'y berish ehtimoli 0,7 ga teng. Hodisaning:

a) kamida 1470 marta va ko'pi bilan 1500 marta;

b) kamida 1470 marta;

v) ko'pi bilan 1469 marta ro'y berish ehtimolini toping.

715. Hodisaning 10000 marta erkli sinovining ro'y berish ehtimoli 0,75 ga teng. Hodisa ro'y berishi nisbiy chastotasining ehtimolidan chetlanishi mutlaq kattaligi bo'yicha 0,01 dan ortiq bo'lmaslik ehtimolini toping.

716. Oilada 5 farzand bor. Bu bolalar orasida; a) ikki o'g'il bola, b) ko'pi bilan ikkita o'g'il bola, d) ikkitadan ortiq o'g'il bolalar, e) kamida ikkita ko'pi bilan uchta o'g'il bolalar bo'lish ehtimolini toping. O'g'il bolalar ehtimolini 0,51 ga teng deb olingan.

717. Bitta o'q uzilganda nishonga tegish ehtimoli 0,8 ga teng. 100 ta o'q uzilganda rosa 75 ta o'qning nishonga tegish ehtimolini toping.

718. Hodisaning 900 erkli sinovining har birida ro'y berish ehtimoli 0,5 ga teng. Shunday $\exists$ musbat sonini topingki, hodisa ro'y berishi nisbiy chastotaning uning ehtimoli 0,5 dan chetlanishining mutlaq kattaligi $\exists$ dan katta bo'lmasligini 0,7698 ehtimol bilan kutish mumkin bo'lsin.

719. Fakultet talabalarining imtihon komisiyasidan «4» va «5» baholar bilan o'tish ehtimoli 0,9 ga teng. Tavakkaliga olingan 400 ta talabadan 34 dan 55 tachasi hech bo'lmaganda 1 ta fandan «4» dan past baho olish ehtimolini toping.

720. Bir soat davomida istagan abonentning kommutatorga

telefon qilish ehtimoli 0,01 ga teng. Telefon stansiyasi 300 ta abonentga xizmat qiladi. Bir soat davomida to'rtta abonentning telefon qilish ehtimolini toping.

721-740-masalalarda faqat 2 ta mumkin bo'lgan x_1 va x_2 qiymatlarga ega (bunda $x_1 < x_2$) bo'lgan diskret tasodifiy miqdor X ning taqsimot qonunini toping. $M(x)$ matematik kutish, $D(x)$ dispersiya va mumkin bo'lgan x_1 qiymatning P_1 ehtimoli ma'lum.

721. $P_1 = 0{,}9$	$M(x) = 3{,}1$	$D(x) = 0{,}09$
722. $P_1 = 0{,}8$	$M(x) = 3{,}2$	$D(x) = 0{,}16$
723. $P_1 = 0{,}7$	$M(x) = 3{,}3$	$D(x) = 0{,}21$
724. $P_1 = 0{,}6$	$M(x) = 3{,}4$	$D(x) = 0{,}24$
725. $P_1 = 0{,}5$	$M(x) = 3{,}5$	$D(x) = 0{,}25$
726. $P_1 = 0{,}4$	$M(x) = 3{,}6$	$D(x) = 0{,}24$
727. $P_1 = 0{,}3$	$M(x) = 3{,}7$	$D(x) = 0{,}26$
728. $P_1 = 0{,}2$	$M(x) = 3{,}8$	$D(x) = 0{,}16$
729. $P_1 = 0{,}1$	$M(x) = 3{,}9$	$D(x) = 0{,}09$
730. $P_1 = 0{,}9$	$M(x) = 2.2$	$D(x) = 0{,}36$
731. $P_1 = 0{,}6$	$M(x) = 1.4$	$D(x) = 0{,}24$
732. $P_1 = 0{,}2$	$M(x) = 2.6$	$D(x) = 0{,}8$
733. $P_1 = 0{,}2$	$M(x) = 7.8$	$D(x) = 0{,}16$
734. $P_1 = 0{,}2$	$M(x) = 4.9$	$D(x) = 9.79$
735. $P_1 = 0{,}3$	$M(x) = 4.7$	$D(x) = 0{,}21$
736. $P_1 = 0{,}3$	$M(x) = 1.7$	$D(x) = 0{,}21$
737. $P_1 = 0{,}4$	$M(x) = 1.6$	$D(x) = 0{,}24$
738. $P_1 = 0{,}2$	$M(x) = 2.8$	$D(x) = 0{,}16$
739. $P_1 = 0{,}1$	$M(x) = 9.9$	$D(x) = 0.99$
740. $P_1 = 0{,}2$	$M(x) = 8.8$	$D(x) = 18.76$

741–760 masalalarda X tasodifiy miqdor $F(x)$ integral funksiya bilan berilgan. Quyidagi talab qilinadi: a) differensial funksiyani topish; b) X miqdorning matematik kutilmasini topish; d) X ning dispersiyasini topish; e) integral va differensial funksiyalarning grafigini yasash.

741. $$F(x)=\begin{cases}0, & x\le 0\\ x^3, & 0<x\le 1\\ 1, & x>1\end{cases}$$

742. $$F(x)=\begin{cases}0, & x\le 0\\ 3x^3+2x, & 0<x\le 1\\ 1, & x>\frac{1}{3}\end{cases}$$

743. $$F(x)=\begin{cases}0 & x\le 2\\ \frac{1}{2}x-1 & 2<x\le 4\\ 1 & x>4\end{cases}$$

744. $$F(x)=\begin{cases}0 & x\le 0\\ \frac{x^2}{9} & 0<x\le 3\\ 1 & x>3\end{cases}$$

745. $$F(x)=\begin{cases}0 & x\le 0\\ \frac{x^2}{4} & 0<x\le 2\\ 1 & x>2\end{cases}$$

746. $$F(x)=\begin{cases}0 & x\le -\frac{\Pi}{2}\\ \cos x & -\frac{\Pi}{2}<x\le 0\\ 1 & x>0\end{cases}$$

747. $$F(x)=\begin{cases}0 & x\le 0\\ 2\sin x & 0<x\le \frac{\Pi}{6}\\ 1 & x>\frac{\Pi}{6}\end{cases}$$

748. $$F(x)=\begin{cases}0 & x\le \frac{3\Pi}{4}\\ \cos 2x & \frac{3}{4}<x\le \Pi\\ 1 & x>\Pi\end{cases}$$

749. $$F(x)=\begin{cases}0 & x\le 1\\ \frac{x^2}{2}-\frac{x}{2} & 1<x\le 2\\ 1 & x>2\end{cases}$$

750. $$F(x)=\begin{cases}0 & x\le 0\\ x^2 & 0<x\le 1\\ 1 & x>1\end{cases}$$

751. $$F(x)=\begin{cases}0 & x\le 0\\ \frac{x^2}{50}+\frac{x}{50} & 0<x\le 5\\ 1 & x>5\end{cases}$$

752. $F(x)=\begin{cases}0 & x\le 0\\ \dfrac{x^2}{81} & 0<x\le 9\\ 1 & x>9\end{cases}$

753. $F(x)=\begin{cases}0 & x\le 0\\ \dfrac{x^1}{64} & 0<x\le 8\\ 1 & x>8\end{cases}$

754. $F(x)=\begin{cases}0 & x\le 0\\ \dfrac{x^2}{49} & 0<x\le 7\\ 1 & x>7\end{cases}$

755. $F(x)=\begin{cases}0 & x\le 0\\ \dfrac{x^2}{36} & 0<x\le 6\\ 1 & x>6\end{cases}$

756. $F(x)=\begin{cases}0 & x\le 0\\ \dfrac{x^2}{25} & 0<x\le 5\\ 1 & x>5\end{cases}$

757. $F(x)=\begin{cases}0 & x\le \\ \dfrac{x^2}{16} & 0<x\le 4\\ 1 & x>4\end{cases}$

758. $F(x)=\begin{cases}0 & x\le 0\\ \sin x & 0<x\le \frac{\Pi}{2}\\ 1 & x>\frac{\Pi}{2}\end{cases}$

759. $F(x)=\begin{cases}0 & x\le 0\\ 1-\cos x & 0<x\le \frac{\Pi}{2}\\ 1 & x>\frac{\Pi}{2}\end{cases}$

760. $F(x)=\begin{cases}0 & x\le \frac{\Pi}{6}\\ -\cos 3x & \frac{\Pi}{6}<x\le \frac{\Pi}{3}\\ 1 & x>\frac{\Pi}{3}\end{cases}$

761-780 masalalarda $\bar{x}$ tenglama o'rtacha qiymatini n tanlanma hajmini va b o'rtacha chetlanishini bilgan holda normal taqsimotning a matematik kutilmasini, $\gamma=0{,}95$ ishonchlilik bilan baholash uchun ishonchli oraliqlarni topish talab qilinadi.

761. $\bar{x}=84{,}21$	$b=15$	$n=225$
762. $\bar{x}=84{,}22$	$b=14$	$n=196$
763. $\bar{x}=84{,}23$	$b=13$	$n=169$
764. $\bar{x}=84{,}24$	$b=12$	$n=144$
765. $\bar{x}=84{,}25$	$b=11$	$n=121$
766. $\bar{x}=84{,}26$	$b=10$	$n=100$
767. $\bar{x}=84{,}27$	$b=9$	$n=81$

768. $\bar{x}=84{,}28$ $b=8$ $n=64$

769. $\bar{x}=84{,}29$ $b=7$ $n=49$

770. $\bar{x}=84{,}30$ $b=6$ $n=36$

771. $\bar{x}=75{,}17$ $b=6$ $n=36$

772. $\bar{x}=75{,}16$ $b=7$ $n=49$

773. $\bar{x}=75{,}15$ $b=8$ $n=64$

774. $\bar{x}=75{,}14$ $b=9$ $n=81$

775. $\bar{x}=75{,}13$ $b=10$ $n=100$

776. $\bar{x}=75{,}12$ $b=11$ $n=121$

777. $\bar{x}=75{,}11$ $b=12$ $n=144$

778. $\bar{x}=75{,}10$ $b=13$ $n=169$

779. $\bar{x}=75{,}09$ $b=14$ $n=196$

780. $\bar{x}=75{,}08$ $b=15$ $n=225$

781-800 $\overline{y_x}-\bar{y}=z_T\frac{b_y}{b_x}(x-\bar{x})$ regressiya to'g'ri chiziq tenglamasini korrelyatsiya jadval bo'yicha tuzing.

781.

y \ x	5	10	15	20	25	30	n_y
45	2	4	—	—	—	—	6
55	—	3	5	—	—	—	8
65	—	—	5	35	5	—	45
75	—	—	2	8	17	—	27
85	—	—	—	4	7	3	14
n_x	2	7	12	47	29	3	n=100

782.

y \ x	10	15	20	25	30	35	n_y
40	2	4	—	—	—	—	6
50	—	3	7	—	—	—	10
60	—	—	5	30	10	—	45
70	—	—	7	10	8	—	27
80	—	—	—	5	6	3	14
n_x	2	7	19	45	24	3	n=100

783.

y \ x	15	20	25	30	35	40	n_y
15	4	1	—	—	—	—	5
25	—	6	4	—	—	—	10
35	—	—	5	50	2	—	54
45	—	—	1	9	7	—	17
55	—	—	—	4	3	7	14
n_x	4	7	7	63	12	7	n=100

784.

y \ x	2	7	12	17	22	27	n_y
110	1	5	—	—	—	—	6
120	—	5	3	—	—	—	8
130	—	—	3	40	12	—	55
140	—	—	2	10	5	—	17
150	—	—	—	3	4	7	14
n_x	1	10	8	53	21	7	n=100

785.

x / y	5	10	15	20	25	30	n_y
10	3	5	—	—	—	—	8
20	—	4	4	—	—	—	8
30	—	—	7	35	8	—	50
40	—	—	2	10	8	—	20
50	—	—	—	5	6	3	14
n_x	3	9	13	50	22	3	n=100

786.

x / y	12	17	22	27	32	37	n_y
25	2	4	—	—	—	—	6
35	—	6	3	—	—	—	9
45	—	—	6	35	4	—	45
55	—	—	2	8	6	—	16
65	—	—	—	14	7	3	24
n_x	2	10	11	57	17	3	n=100

787.

x / y	15	20	25	30	35	40	n_y
25	3	4	—	—	—	—	7
35	—	6	3	—	—	—	9
45	—	—	6	35	2	—	43
55	—	—	12	8	6	—	26
65	—	—	--	4	7	4	15
n_x	3	10	21	47	15	4	n=100

788.

y \ x	4	9	14	19	24	29	n_y
30	3	3	—	—	—	—	6
40	—	5	4	—	—	—	9
50	—	—	40	2	8	—	50
60	—	—	5	10	6	—	21
70	—	—	—	4	7	3	14
n_x	3	8	49	16	21	3	n=100

789.

y \ x	5	10	15	20	25	30	n_y
30	2	6	—	—	—	—	8
40	—	5	3	—	—	—	8
50	—	—	7	40	2	—	49
60	—	—	4	9	6	—	19
70	—	—	—	4	7	5	16
n_x	2	11	14	53	15	5	n=100

790.

y \ x	10	15	20	25	30	35	n_y
20	5	1	—	—	—	—	6
30	—	6	2	—	—	—	8
40	—	—	5	40	5	—	50
50	—	—	2	8	7	—	17
60	—	—	—	4	7	8	19
n_x	5	7	9	52	19	8	n=100

791.

y \ x	5	10	15	20	25	30	35	40	n_y
100	2	1	—	—	—	—	—	—	3
120	3	4	3	—	—	—	—	—	10
140	—	—	5	10	8	—	—	—	23
160	—	—	—	1	—	6	1	1	9
180	—	—	—	—	—	—	4	1	5
n_x	5	5	8	11	8	6	5	2	n=50

792.

y \ x	18	23	28	33	38	43	48	n_y
125	—	1	—	—	—	—	—	1
150	1	2	5	—	—	—	—	8
175	—	3	2	12	—	—	—	17
200	—	—	1	8	7	2	2	16
225	—	—	—	—	3	3	—	6
250	—	—	—	—	—	1	1	2
n_x	1	6	8	20	10	4	1	n=50

793.

y \ x	5	10	15	20	25	30	35	n_y
100	—	—	—	—	—	6	1	7
120	—	—	—	—	—	4	2	6
140	—	—	8	10	5	—	—	23
160	3	4	3	—	—	—	—	10
180	2	1	—	1	—	—	—	4
n_x	5	5	11	11	5	10	3	n=100

794.

y \ x	13	18	23	28	33	n_y
25	3	2	—	—	—	7
35	—	6	4	—	—	6
45	—	1	9	5	—	23
55	—	1	2	4	8	10
65	—	—	1	—	4	4
n_x	3	10	16	9	12	n=100

795.

y \ x	30	35	40	45	50	n_y
46	2	6	—	—	—	8
56	2	8	10	—	—	20
66	—	—	32	3	9	44
76	—	—	4	11	6	21
86	—	—	—	2	5	7
n_x	4	14	46	16	20	n=100

796.

y \ x	33	38	43	48	53	58	n_y
65	4	8	1	—	—	—	13
75	—	4	4	2	—	—	10
85	—	1	6	6	1	—	14
95	—	—	—	1	5	—	6
105	—	—	—	1	4	1	6
115	—	—	—	—	2	4	6
n_x	4	13	11	10	12	5	n=100

797.

y \ x	3	7	11	15	19	23	27	n_y
6	5	3	—	2	—	—	—	10
16	7	10	1	2	—	—	—	20
26	2	18	15	20	—	—	—	55
36	—	—	30	26	—	—	—	56
46	—	—	—	19	12	—	—	31
56	—	—	—	—	14	1	—	15
66	—	—	—	—	7	4	2	13
n_x	14	31	46	69	33	5	2	n=100

798.

y \ x	20	30	40	50	60	70	n_y
10	2	5	—	—	—	—	7
20	—	7	3	—	—	—	10
30	—	—	7	30	5	—	42
40	—	—	1	9	6	—	16
50	—	—	—	10	8	7	25
n_x	2	7	11	49	19	7	n=100

799.

y \ x	25	35	45	55	65	75	n_y
20	1	4	—	—	—	—	5
30	—	4	3	—	—	—	7
40	—	—	5	41	4	—	50
50	—	—	2	10	15	—	7
60	—	—	—	2	5	4	11
n_x	1	8	10	53	24	4	n=100

800.

y \ x	30	40	50	60	70	80	n_y
25	1	7	—	—	—	—	8
35	—	5	4	—	—	—	9
45	—	—	8	40	1	—	49
55	—	—	3	9	6	—	18
65	—	—	—	4	8	4	16
n_x	1	12	15	53	15	4	n=100

«OLIY MATEMATIKA» FANI DASTURI

1. Chiziqli algebra va analitik geometriya elementlari

1. Ikkinchi va uchinchi tartibli determinantlar, ularning asosiy xossalari.
2. n-tartibli determinantlar. Determinantlarni hisoblash.
3. Koordinatalar sistemasi. R^2 va R^3 fazolar. Vektorlar, ular ustida amallar.
4. Vektorlarning skalyar ko'paytmasi va uning xossalari. Skalyar ko'paytmaning mexanik ma'nosi.
5. Vektorlarning vektorli aralash ko'paytmalari, ularning xossalari va geometrik ma'nolari.
6. Matritsalar va ular ustida amallar. Matritsaning rangi. Teskari matritsa. Chiziqli almashtirishlar.
7. Kvadrat matritsaning xos sonlari va xos vektorlari, uning xarakteristik tenglamasi. Kvadratik formalar va ularni kanonik holga keltirish.
8. Chiziqli tenglamalar sistemasi, ularni matritsalar yordami bilan yechish. Kroneker-Kapelli teoremasi. Kramer formulasi. Gauss usuli.
9. To'g'ri chiziq va tekislikning tenglamalari, asosiy masalalar. Fazoviy chiziq tenglamalari.
10. Ikkinchi tartibli egri chiziqlar, ularning kanonik tenglamalari, asosiy xossalari.
11. Ikkinchi tartibli sirtlar. Sirt tenglamasi. Aylanma, silindrik va konus sirtlar tenglamasi.

2. Matematik tahlilga kirish

12. Matematik mantiq elementlari. To'g'ri va teskari teoremalar. Nyuton binomi.

13. Haqiqiy sonlar to'plami. Funksiya. Asosiy elementar funksiyalar, ularning xossalari va grafiklari.
14. Sonli ketma-ketliklar, ularning limiti.
15. Funksiyaning nuqtada va intervalda uzluksizligi, uzluksiz funksiyalar ustida amallar. Uzluksiz funksiyaning kesmadagi asosiy xossalari.

3. Bir o'zgaruvchili funksiyaning differensial hisobi

16. Hosila tushunchasiga olib keluvchi masalalar. Hosilaning ta'rifi. Hosilaning geometrik va fizik ma'nolari. Funksiya differensiali.
17. Hosilalarni topishning qoidalari.
18. Yuqori tartibli hosila va differensiallar.
19. Roll, Lagranj va Koshi teoremalari.
20. Teylor formulasi. Lopital qoidasi.
21. Funksiyalarning o'sishi va kamayishi.
22. Funksiyalarning ekstremumlari, ekstremumning zaruriy va yetarli shartlari. Kesmada funksiyaning eng katta qiymatlari.
23. Funksiya grafigining qavariqligi va botiqligi, burilish nuqtasi. Asimptotalar.

4. Kompleks sonlar

24. Kompleks sonlar va ular ustida amallar. Kompleks sonlarni tekislikda tasvirlash, uning moduli va argumenti, kompleks sonning trigonometrik shakli. Eyler formulalari, kompleks sonning ko'rsatkichli shakli.
25. Ko'phadlar. Bezu teoremasi. Algebraning asosiy teoremasi.
26. Interpolyatsiya va approksimatsiya haqida tushuncha.

5. Aniqmas integrallar

27.Boshlang'ich funksiya. Aniqmas integral va uning xossalari.
28.Integrallash usullari. Aniqmas integral jadvallaridan foydalanish.

6. Aniq integrallar

29.Aniq integral tushunchasiga olib keluvchi masalalar. Aniq integral va uning xossalari, o'rta qiymat haqida teorema.
30.Nyuton-Leybnis formulasi.
31.Aniq integralni taqribiy hisoblashda to'g'ri to'rtburchak, trapetsiya va Simpson formulalari. Xosmas integrallar, ularning asosiy xossalari.

7. Ko'p o'zgaruvchili funksiyalar

32.Ko'p o'zgaruvchili funksiyalar. Ko'p o'zgaruvchili funksiyalarning uzluksizligi.
33.Xususiy xossalar. To'liq differensial.
34.Yuqori tartibli xususiy hosilalar va to'liq differensiallar. Teylor formulasi.
35.Oshkormas funksiya, mavjudlik teomerasi. Oshkormas funksiyalarni differensiallash.
36.Ko'p o'zgaruvchili funksiyaning ekstremumi, ekstremumning zaruriy va yetarli shartlari.
37.Shartli ekstremumlar. Lagranj ko'paytuvchilari usuli, optimal yechimlarni topishda uning qo'llanilishi.

8. Oddiy differensial tenglamalar

38.Differensial tenglamaga olib keluvchi masalalar. Oddiy differensial tenglamalar. Koshi masalasi. Birinchi tartibli differensial tenglama yechimining mavjudligi va yagonaligi haqidagi teorema.

39.Birinchi tartibli differensial tenglamalarni yechish.

40.Yuqori tartibli differensial tenglamalar. Koshi masalasi.

41.Chiziqli differensial tenglamalar, bir jinsli va bir jinssiz tenglamalar. Umumiy yechim.

42.O'zgarmas koeffitsentli chiziqli differensial tenglamalar. O'ng tomoni maxsus ko'rinishda berilgan differensial tenglamalar.

9. Oddiy differensial tenglamalar sistmemasi

43.Differensial tenglamalar normal sistemasi. Avtonom sistemalar. Koshi masalasi. Yechimning mavjudligi va yagonaligi haqida teorema.

44.Differensial tenglamalar normal sistemasini yechish uchun noma'lumlarni yo'qotish usuli.

45.Chiziqli differensial tenglamalar sistemasi. O'zgarmas koeffitsentli chiziqli differensial tenglamalar sistemasi.

10. Sonli va funksional qatorlar

46.Sonli qatorlar, qatorlarning yaqinlashish v uzoqlashishi. Yaqinlashishning zaruriy sharti, qator yaqinlashishining asosiy alomatlari.

47.Funksional qətorlar, yaqinlashish sohasi, uni aniqlash usullari.

48.Darajali qatorlar. Funksiyalarni darajali qatorlarga yoyish. Taqribiy hisoblarda darajali qatorlarning qo'llanilishi.

49.Trigonometrik Fure qatorlari. Funksiyalarni Fure qatorlariga yoyish.
50.Kompleks hadli qatorlar. Kompleks shakldagi Fure qatori. Spektr. Fure integrali. Fure almashtirish.

11. Operatsion hisob va uning tatbiqi

51.Laplas tasviri, uning xossalari. Operatsion hisobning asosiy teoremalari. Tasvirlar jadvali.
52.Tasvir orqali originalllarni tiklash usullari. Laplas tasvirining o'ramasi. Dyuamel teoremasi.
53.Differensial tenglamalar va sistemasilarni operatsion usullar bilan yechish.

12. Karrali va sirt integrallar

54.Karrali, egri chiziqli va sirt bo'yicha integrallar tushunchasiga olib keluvchi masalalar.
55.Ikki va uch karrali integrallar va ularning xossalari. Karrali integrallarni takroriy integrallash yo'li bilan hisoblash.
56.Sirt yuzasi. Sirt integralining eng sodda xossalari va ularni ikki karrali integralga keltirish yo'li bilan hisoblash.

13. Ehtimollar nazariyasi va matematik statistika elementlari

57.Hodisalar va ularning klassifikatsiyasi. Tasodifiy hodisalar haqida tushuncha. Nisbiy chastota va ehtimol. Ehtimollarning asosiy xossalari. Ehtimollarni hisoblash usullari.
58.Bernulli sxemasi. Bernulli formulasi. Laplasning normal teoremasi. Laplasning integral teoremasi.

59.Diskret va uzluksiz tasodifiy miqdorlar. Taqsimot qonuni. Taqsimot funksiyasi, uning xossalari.
60.Tasodifiy miqdorning sonli xarakteristikalari. Matematik kutilish, dispersiya.
61.Katta sonlar qonuni. Limit teoremalar haqida tushuncha.
62.Korrelyatsiya va regressiya haqida tushuncha. Chiziqli korrelyatsiya, korrelyatsiya koeffitsienti. Tajribalardan olingan natijalarga ko'ra chiziqli korrelyatsiyani tekshirish.
63.Matematik statistika elementlari.

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