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OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

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OLIY MATEMATIKADAN QO'LLANMA

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Ushbu o'quv qo'llanmada tabiiy yo'nalishdagi fakultetlar talabalari uchun Oliy matematikadan amaliy mashg'ulotlar, masala yechish namunalari, joriy nazorat uchun uy vazifa variantlari keltirilgan.

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Mirzo Ulug'bek nomidagi O'zbekiston Milliy Universiteti Ilmiy Kengashining 2008 yil 5 fevral majlis qarori bilan nashrga tavsiya etilgan.

Mundarija

I–qism. Analitik geometriya va oliy algebra

1–Bob. Tekislikda analitik geometriya.

§1. Analitik geometriyaning sodda masalalari.

Abtsissalar o'qidagi $A(x_1)$ va $B(x_2)$ nuqtalar orasidagi masofa

$$|AB| = |x_2 - x_1|$$

formula yordamida topiladi. AB kesmadagi $|AC|:|CB| = \lambda$ shartni qanoatlantiruvchi C nuqtaning koordinatasi

$$x = \frac{x_1 + \lambda \cdot x_2}{1+\lambda}$$

formula yordamida topiladi. Xususan, $\lambda = 1$ bo'lganda, AB kesma markazi koordinatasi

$$x = \frac{x_1 + x_2}{2}$$

ko'rinishda topiladi.

Tekislikdagi ikki $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar orasidagi masofa

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

koordinatalar boshidan $A(x_1;y_1)$ nuqtagacha bo'lgan masofa

$$|OA| = \sqrt{x_1^2 + y_1^2}$$

formuladan topiladi. AB kesmani $|AC|:|CB| = \lambda$ nisbatda bo'luvchi $C(x,y)$ nuqta koordinatalari

$$x = \frac{x_1 + \lambda \cdot x_2}{1+\lambda}, \quad y = \frac{y_1 + \lambda \cdot y_2}{1+\lambda}$$

formulalardan, o'rtasining koordinatalari esa

$$x = \frac{x_1 + x_2}{2}, \qquad y = \frac{y_1 + y_2}{2}$$

formulalardan topiladi.

Uchlari $A(x_1;y_1)$, $B(x_2;y_2)$, $C(x_3;y_3)$ nuqtalarda bo'lgan uchburchak yuzi

$$S=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

formula yordamida topiladi, og'irlik markazi, ya'ni medianalar kesishish nuqtasi koordinatalari

$$x=\frac{1}{3}(x_1+x_2+x_3),\quad y=\frac{1}{3}(y_1+y_2+y_3)$$

dan topiladi.

1.1. $A(3)$ va $B(-5)$ orasidagi masofani toping.

1.2. $C(2)$ nuqtaga nisbatan $A(-3)$ nuqtaga simmetrik bo'lgan nuqtani toping.

1.3. AB kesma ikki nuqta yordamida teng uch qismga bo'lingan. $A(-1)$, $B(5)$ bo'lsa, bo'linish nuqtalari koordinatalarini toping.

1.4. $A(3;\ 8)$ va $B(-5;\ 14)$ nuqtalar orasidagi masofani toping.

1.5. Uchlari $A(-3;\ -2)$, $B(0;\ -1)$, $C(-2;\ 5)$ nuqtalarda bo'lgan uchburchak to'g'ri burchakli ekanligini isbotlang.

1.6. Ordinatalar o'qida $A(4;\ -1)$ nuqtadan 5 birlik uzoqlikdagi nuqtani toping.

1.7. Uchlari $A(2;\ 0)$, $B(5;\ 3)$, $C(2;\ 6)$ nuqtalarda bo'lgan uchburchak yuzini toping.

1.8. Uchlari $A(3;\ 1)$, $B(4;\ 6)$, $C(6;3)$, $D(5;\ -2)$ nuqtalarda bo'lgan to'rtburchak yuzini toping.

1.9. $A(1;\ 2)$ va $B(4;\ 4)$ nuqtalar berilgan. Abtsissalar o'qida shunday S nuqta topingki, ABC uchburchak yuzi 5 kv.b. ga teng bo'lsin.

1.10. Kvadratning ikki yonma–yon uchlari $A(3;\ -7)$, $B(-1;\ 4)$ nuqtalarda bo'lsa, uning yuzini toping.

1.11. $E(3;\ 5)$ va $F(1;\ -3)$ nuqtalar kvadrat qarama–qarshi uchlari bo'lsa, uning yuzini toping.

1.12. $A(-3;\ 2)$ va $B(1;\ 6)$ nuqtalar muntazam uchburchak uchlari bo'lsa, bu uchburchak perimetri va yuzini toping.

1.13. ABCD parallelogramm uchta uchi $A(3;-7), B(5;-7), C(-2;\ 5)$ nuqtalarda. D nuqta B ga qarama-qarshi uchi bo'lsa, bu parallelogramm dioganallari uzunligini toping.

1.14. Agar $A(3;\ 0), C(-4;\ 1)$ nuqtalar kvadrat qarama-qarshi uchlari bo'lsa, qolgan ikki uchini toping.

1.15. Uchta uchi $A(-2;\ 3), B(4;-5), C(-3;\ 1)$ nuqtalarda bo'lgan parallelogramm yuzini hisoblang.

1.16. Yuzi 3 ga teng, ikki uchi $A(3;\ 1), B(1;\ -3)$ nuqtalarda, uchinchi uchi Oy o'qida yotuvchi uchburchak berilgan. Uchinchi uchi koordinatalarini toping.

1.17. Parallelogramm ikki uchi $A(-1;\ 3), B(-2;\ 4)$ nuqtalarda, yuzi 12 kv.b. bo'lsa, va dioganallari abtsissalar o'qida kesishsa, qolgan ikki uchini toping.

§2. To'g'ri chiziq tenglamalari.

a, b, c – o'zgarmas sonlar, $a^2 + в^2 \neq 0$ shart bajarilganda $ax + вy + c = 0$ tenglama to'g'ri chiziqning umumiy tenglamasi deyiladi.

1) s=0, $a \neq 0$, $в \neq 0$ bo'lsa, $ax + вy = 0$ ko'rinishga kelib, koordinatalar boshidan o'tuvchi to'g'ri chiziqlar hosil bo'ladi.

2) $a = 0$, $в \neq 0$, $c \neq 0$ bo'lsa, tenglama $y = -\frac{c}{в}$ ko'rinish olib, OX o'qiga parallel to'g'ri chiziqni ifodalaydi.

3) $в = 0$, $a \neq 0$, $c \neq 0$ bo'lsa,

tenglama $x = -\frac{c}{a}$ ko'rinish oladi va OY o'qiga parallel to'g'ri chiziqni ifodalaydi.

4) $в = c = 0$, $a \neq 0$ bo'lsa, $ax = 0$ yoki $x = 0$ ko'rinish oladi va Ou o'qini ifodalaydi.

5) $a = c = 0$, $в \neq 0$ bo'lsa, $вy = 0$ yoki $y = 0$ ko'rinish oladi va Ox o'qini anglatadi.

To'g'ri chiziq Ox o'qi musbat yo'nalishi bilan α burchak hosil qilsa va $tg\alpha = k$ deyilsa,

$y = kx + b$ tenglama to'g'ri chiziqning burchak koeffitsientli tenglamasi deyiladi. Bunda b – soni to'g'ri chiziqning Oy o'qi bilan kesishish nuqtasi ordinatasi. Umumiy tenglama tomonlari b ga bo'linib, y topilsa, burchak koeffitsientli tenglama hosil bo'ladi. Agar to'g'ri chiziq Ox o'qini a – abtsissali, Oy o'qini b – ordinatali nuqtalarda kesib o'tsa, to'g'ri chiziq tenglamasi

$\frac{x}{a} + \frac{y}{в} = 1$ ko'rinishda yoziladi va to'g'ri chiziqning kesmalar bo'yicha tenglamasi deyiladi.

To'g'ri chiziqqa koordinatalar boshidan tushirilgan perpendikulyar uzunligi p bo'lib, bu perpendikulyar Ox o'qining musbat yo'nalishi bilan α burchak hosil qilsa, to'g'ri chiziq tenglamasi

$$x\cos\alpha + y\sin\alpha - p = 0$$

ko'rinishda yoziladi va to'g'ri chiziqning normal tenglamasi deyiladi. Bu tenglamani umumiy tenglama tomonlarini $\mu = \pm\frac{1}{\sqrt{a^2 + b^2}}$ ga ko'paytirib hosil qilish mumkin.

2.1. Quyidagi to'g'ri chiziqlarni yasang: 1) $2y+7=0$
2) $5x-2=0$ 3) $4x-3y=0$ 4) $x-3y-3=0$

2.2. Koordinata boshidan o'tuvchi va Ox o'qi musbat yo'nalishi bilan 1) 45^0 2) 60^0 3) 90^0 4) 120^0 burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

2.3. To'g'ri chiziq $y=kx+b$ A(2; 3) nuqtadan o'tadi va Ox o'qi musbat yo'nalishi bilan 45^0 burchak hosil qiladi. k va b ni aniqlang.

2.4. To'g'ri chiziq umumiy tenglamasi $12x-5y-65=0$. Bu to'g'ri chiziqning 1) burchak koeffitsientli; 2)kesmalar bo'yicha; 3)normal tenglamalarini yozing.

2.5. $2x-5y=0$ to'g'ri chiziqning kesmalar bo'yicha tenglamasini yozish mumkinmi?

2.6. Agar to'g'ri chiziq $A(2;5)$ nuqtadan o'tsa va ordinatalar o'qidan $b=7$ kesma ajratsa, uning tenglamasini yozing.

2.7. To'g'ri chiziq son o'qlaridan bir xil kesma ajratadi, son o'qlari orasidagi kesmasining uzunligi $5\sqrt{2}$ bo'lsa, tenglamasini yozing.

2.8. B(−4; 6) nuqtadan o'tib, son o'qlari bilan yuzasi 6 kv.b. uchburchak hosil qiluvchi to'g'ri chiziq tenglamasini toping.

2.9. Rombning dioganallari 10 va 6 sm bo'lib, mos ravishda Ox va Oy o'qlarida joylashsa, tomonlari tenglamalarini yozing.

§3. To'g'ri chiziqqa doir masalalar.

$y=k_1x+b_1$ va $y=k_2x+b_2$ to'g'ri chiziqlar orasidagi o'tkir burchak $tg\varphi=\left|\dfrac{k_2-k_1}{1+k_1k_2}\right|$

formuladan topiladi. Ikki to'g'ri chiziqning parallellik sharti $k_2 = k_1$, perpendikulyarlik sharti esa $k_1 = -\frac{1}{k_2}$ ko'rinishda bo'ladi.

Agar to'g'ri chiziqlar $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ umumiy tenglamalar bilan berilsa $tg\varphi = \left|\frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}\right|$ ko'rinish oladi. Parallellik, perpendikulyarlik shartlari mos ravishda $\frac{a_1}{a_2} = \frac{в_1}{в_2}$ va $a_1a_2 + в_1в_2 = 0$ bo'ladi

k – burchak koeffitsientli, $A(x_0, y_0)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi $y - y_0 = k(x - x_0)$ ko'rinishida bo'ladi.

$A(x_0, y_0), B(x_1, y_1)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0}$$

ko'rinishda bo'ladi.

$A(x_0, y_0)$ nuqtadan $x\cos\alpha + y\sin\alpha - p = 0$ to'g'ri chiziqgacha bo'lgan masofa $d = |x_0\cos\alpha + y_0\sin\alpha - p|$ formuladan, $ax + by + c = 0$ to'g'ri chiziqgacha masofa esa $d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$ formula yordamida topiladi.

3.1. $y = -3x + 4$ va $y = 2x - 1$ to'g'ri chiziqlar orasidagi o'tkir burchakni aniqlang.

3.2. $3x-2y+7=0$, $6x-4y-9=0$, $6x+4y-5=0$, $2x+3y-6=0$ to'g'ri chiziqlardan o'zaro parallellari va perpendikulyarlarini ko'rsating.

3.3. $A(2;-1), B(4;3), C(12;-3)$ nuqtalar uchburchak uchlari bo'lsa, uning tomonlari tenglamalarini yozing.

3.4. $A(0;7), B(6;-6), O(0;0)$ uchlarga ega uchburchak tomonlari tenglamalarini va ichki burchaklarini toping.

3.5. $A(-2;1)$ nuqtadan o'tuvchi, $3x+4y-1=0$ to'g'ri chiziqda parallel va perpendikulyar to'g'ri chiziqlar tenglamalarini yozing.

3.6. Uchburchak tomonlari tenglamalari berilgan: $x+2y=0$, $x+4y-6=0$, $x-4y-6=0$. Ichki burchaklarini aniqlang.

3.7. $A(3;1)$ nuqtadan o'tib, $x-3y+1=0$ to'g'ri chiziq bilan 45^0 li burchak hosil qiluvchi to'g'ri chiziq tenglamasini toping.

3.8. Uchlari $A(-4;2), B(2;-5), C(5;0)$ nuqtalarda bo'lgan uchburchak medinalari va balandliklari kesishadigan nuqtalarni toping.

3.9. Normalining uzunligi 5 bo'lib, Ox o'qi musbat yo'nalishi bilan 45^0 burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

3.10. Koordinata boshidan $12x-5y+39=0$ to'g'ri chiziqqacha bo'lgan masofani toping.

3.11. $A(5;2), B(1;2)$ nuqtalardan $x-2y-1=0$ to'g'ri chiziqqacha masofani hisoblang.

3.12. O'zaro parallel $2x+y-7=0$, $2x+y+1=0$ to'g'ri chiziqlar orasidagi masofani toping.

3.13. Agar $y = kx + 5$ to'g'ri chiziqdan koordinata boshigacha masofa $\sqrt{5}$ bo'lsa k ni aniqlang.

3.14. $x + y - 5 = 0$, $7x - y - 19 = 0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalari tenglamalarini yozing.

§4. Ikkinchi tartibli chiziqlar

Ikkinchi tartibli chiziqlar $Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$ umumiy tenglama bilan beriladi. Bu paragrafda aylana, ellips, giperbola, parabolalar, ularning xossalariga doir masalalar o'rganiladi.

1. Markazi $C(a,b)$ nuqtada, radiusi R bo'lgan aylana tenglamasi

$$(x-a)^2 + (y-b)^2 = R^2.$$

Koordinata boshi $O(0;0)$ nuqtada bo'lsa, aylana tenglamasi

$$x^2 + y^2 = R^2$$

ko'rinish oladi

2. Tekislikda fokuslar deb ataluvchi F_1 va F_2 nuqtalargacha bo'lgan masofalar yig'indisi $2 \cdot a$ songa teng bo'lgan nuqtalarning geometrik o'rni ellips deyiladi.

Fokuslar orasidagi masofani $2c$, $a^2 - c^2 = b^2$ desak, ellips

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

kanonik tenglamaga ega bo'ladi $e = \frac{c}{a} < 1$ miqdor ellips ekstsentrisiteti deyiladi.

Ellips $A(x,y)$ nuqtasidan fokuslargacha masofa (fokal radiuslari) $r=a-ex,\ r=a+ex$ formulalardan topiladi.

3. Tekislikda fokuslar deb ataluvchi F_1va F_2 nuqtalargacha bo'lgan masofalar ayirmasi $2a$ songa teng bo'lgan nuqtalar geometrik o'rni giperbola deyiladi.

Fokuslar orasidagi masofa $2c$, $c^2-a^2=b^2$ bo'lsa, giperbola

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

kanonik ko'rinishiga keladi.

$e=\frac{c}{a}>1$ nisbat giperbola ekstsentrisitetetidir.

Giperbola $A(x,y)$ nuqtasidan fokuslargacha masofa (fokal radiuslari)

$$r_1=|ex-a|\ ,\ r_2=|ex+a|$$

formulalardan topiladi.

4. Tekislikda fokus deb ataluvchi $F(\frac{p}{2};0)$ nuqtadan va direktrisa deb ataluvchi $x=-\frac{p}{2}$ to'g'ri chiziqdan bir xil uzoqlashgan nuqtalar geometrik o'rni parabola deyib,

$$y^2=2px$$

kanonik tenglamaga ega bo'ladi.

Parabola $A(x,y)$ nuqtasidan fokusgacha masofa $r=x+\frac{p}{2}$ formuladan topiladi

4.1. $A(1;2), B(0;1), C(-3;0)$ nuqtalardan o'tuvchi aylana tenglamasini yozing.

4.2. $A(-4;8)$ nuqta berilgan Diametri OA bo'lgan aylana tenglamasini yozing.

4.3. Aylanalar markazlari va radiuslarini toping.

1) $x^2 + y^2 - 6x + 4y - 23 = 0$

2) $x^2 + y^2 + 5x - 7y + 2,5 = 0$

3) $x^2 + y^2 + 7y = 0$

4.4. $x^2 + y^2 - 8x - 4y + 16 = 0$ aylanaga koordinata boshidan o'tkazilgan urinmalar tenglamalarini yozing.

4.5. $x^2 + 4y^2 = 16$ ellips fokusi va ekstsentrisitetini toping

4.6. Er shari biror fokusda Quyosh joylashgan ellips bo'yicha harakatlanadi. Yerdan quyoshgacha eng qisqa masofa 147,5 mln.km, eng uzun masofa 152,5 mln.km bo'lsa, Yer orbitasi katta yarim o'qini va ekstsentrisitetini toping.

4.7. $x^2 + y^2 = 36$ aylana ordinatalari ikki marta qisqartirilsa qanday chiziq hosil bo'ladi?

4.8. A($\sqrt{3};\sqrt{2}$) nuqtadan o'tuvchi, ekstsentrisiteti $\sqrt{2}$ ga teng bo'lgan giperbola tenglamasini yozing.

4.9. $\frac{x^2}{64} - \frac{y^2}{36} = 1$ giperbolaning chap shoxida shunday nuqta topingki, o'ng fokal radiusi 18 ga teng bo'lsin.

4.10. $x^2 + 4x + y^2 = 0$ aylanadan va $M(2;0)$ nuqtadan bir xil masofada yotuvchi nuqtalar tenglamasini yozing.

4.11. Fokus $4x-3y-4=0$ to'g'ri chiziq va OX o'qi kesishgan nuqtada bo'lgan parabola tenglamasini yozing.

4.12. $y^2=2x$ parabola koordinatalar boshidan o'tuvchi to'g'ri chiziqdan $\frac{3}{4}$ uzunlikdagi vatar ajratadi. To'g'ri chiziq tenglamasini toping.

§5. Koordinatalarni almashtirish.

Qutb koordinatalar boshidan qutb o'qi OX o'qi musbat yo'nalishi bilan ustma-ust qo'yilsa, tekislikdagi nuqta qutb va dekart koordinatalari quyidagiga bog'lanadi:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2} \\ tg\varphi = \dfrac{y}{x} \end{cases}$$

XOY koordinatalar sistemasidan koordinata boshi $O_1(a,b)$ bo'lgan $x'Oy'$ koordinatalariga o'tish-parallel ko'chirishda tekislikning biror A nuqtasi eski va yangi koordinatalari quyidagicha bog'lanadi.

$$\begin{cases} x = x'+a \\ y = y'+в \end{cases} \qquad \begin{cases} x = x'-a \\ y' = y-в \end{cases}$$

Agar koordinata o'qlari musbat α burchakka burilsa nuqtaning eski x, y koordinatalari va yangi x', y' koordinatalari qo'yidagicha bog'lanadi:

$$\begin{cases} x = x'\cos\alpha - y'\sin\alpha \\ y = x'\sin\alpha - y'\cos\alpha \end{cases}$$

$$\begin{cases} x' = x\cos\alpha + y\sin\alpha \\ y' = -x\sin\alpha + y\cos\alpha \end{cases}$$

Agar ikkinchi tartibli chiziq (ITCh) $Ax^2 + Cy^2 + 2Dx + 2Ey + F = 0$ ko'rinishida berilsa, parallel ko'chirish yordamida kanonik ko'rinish oladi.

Agar ITCh

$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$ tenglama bilan aniqlansa, koordinata o'qlarini

$$\mathrm{Ctg}2\alpha = \frac{A-C}{2B}$$

shart bajarilganda α – burchakka burish yordamida oldingi holga keltirish mumkin, bunda xy had yo'qoladi.

5.1. $A(2\sqrt{3};2)$, $B(0;-3)$, $C(-4;4)$, $D(\sqrt{2};-\sqrt{2})$ nuqtalar qutb koordinatalarini toping.

5.2. $A\left(10;\frac{\pi}{2}\right)$, $B\left(2;\frac{5\pi}{4}\right)$, $C\left(0;\frac{\pi}{10}\right)$, $D\left(1;-\frac{\pi}{4}\right)$ nuqtalar dekart koordinatalarini toping.

5.3. $A\left(6;-\frac{\pi}{4}\right)$ va $B\left(8;\frac{\pi}{4}\right)$, $C\left(4;\frac{\pi}{6}\right)$ va $D\left(4;\frac{\pi}{2}\right)$, $E\left(5;\frac{\pi}{4}\right)$ va $F(12;\pi)$ nuqtalar orasidagi masofalarni toping.

5.4. Agar parallel ko'chirishda $O(0;0)$ nuqta $O_1(3;-4)$ nuqtaga o'tsa, $A(5;6)$ nuqta qanday nuqtaga o'tadi?

5.5. Agar parallel ko'chirishda $A(4;3)$ nuqta $A_1(-3;-4)$ nuqtaga o'tsa, koordinata boshi qanday nuqtaga o'tadi?

5.6. Koordinatalar sistemasi $\alpha = \frac{\pi}{4}$ burchakka burilsa, A $(\sqrt{3};3)$ nuqtaning koordinatalari qanday o'zgaradi?

5.7. Dastlab koordinatalar boshi $O_1(3;4)$ nuqtaga ko'chirildi, so'ngra bu sistema son o'qlari $\alpha = \frac{\pi}{6}$ ga burildi, bunda $A(2;1)$ nuqta koordinatalari qanday o'zgaradi?

5.8. Parallel ko'chirish yordamida $y = Ax^2 + Bx + C$ ko'rinishidagi parabolani $y' = Ax'^2$ ko'rinishga keltiring:

a) $y = 9x^2 - 6x + 2$

b) $y = 4x - 2x^2$

5.9. Parallel ko'chirish yordamida $y = \frac{kx + l}{px + q}$ ko'rinishidagi giperbolani $y' = \frac{m}{x'}$ ko'rinishda yozing.

a) $y = \frac{4x + 5}{2x - 1}$

b) $y = \frac{2x}{4x - 1}$

5.10. $Ax^2 + Cy^2 + 2Дx + 2Ey + F = 0$ ko'rinishdagi ITCh kanonik ko'rinishga keltirilsin.

a) $4x^2 + 9y^2 - 8x - 36y + 4 = 0$

b) $x^2 - 9y^2 + 2x + 36y - 44 = 0$

v) $16x^2+25y^2-32x+50y-359-0$

g) $x^2+4y^2-4x-8y+8=0$

d) $x^2-y^2-6x+10=0$

e) $x^2+2x+5=0$

5.11. Kanonik ko'rinishga keltiring.

a) $5x^2+4xy+8y^2+8x+14y+5=0$

b) $x^2-2xy+y^2-10x-6y+25=0$

v) $4xy+4\sqrt{3}y^2+16x+12y-36=0$

5.12. Quyidagi chiziqlarni yasang:

a) $r=a\varphi$ (arximed spirali)

b) $r=a(1-\cos\varphi)$ (kardoida)

v) $r^2=a^2\cos 2\varphi$ (lemniskata)

g) $r=\dfrac{a}{\varphi}$ (Giperbolik spiral)

d) $r=a\sin 3\varphi$ (uch yaproqli gul)

e) $r=a\sin 2\varphi$ (to'rt yaproqli gul

5.13. Qutb koordinatalar sistemasiga o'tkazing:

a) $x^2-y^2=a^2$

b) $x^2+y^2=a^2$

v) $\left(x^2+y^2\right)^2=a^2\left(x^2-y^2\right)$

g) $y=x$

5.14. Dekart koordinatalar sistemasiga o'tkazing.

a) $r\cos\varphi=a$

b) $r^2\sin 2\varphi=2a^2$

v) $r\sin(\varphi+\frac{\pi}{4})=a\sqrt{2}$

g) $r = a(1+\cos\varphi)$

5.15. Kanonik tenglamasini yozing.

а) $r = \dfrac{9}{5-4\cos\varphi}$ б) $r = \dfrac{3}{1-\cos\varphi}$

в) $r = \dfrac{1}{2-\sqrt{3}\sin\varphi}$

Bobga doir misollar echish namunalari

1.Kesma bir uchi $A(-7)$, o'rtasi $C(2)$ bo'lsa ikkinchi uchi koordinatasini toping.

Ikkinchi uchi $B(x_2)$ nuqtada bo'lsa, $2 = \dfrac{-7+x_2}{2}$ kelib chiqadi. Bundan $B(11)$ ekanligini topamiz.

2. Uchlari $A(-4;2)$, $B(0;-1)$, $C(3;3)$ nuqtalarda bo'lgan uchburchak yuzi, perimetri va ichki burchaklarini toping.

$$S = \frac{1}{2}\left|-4(-1-3)+0(3-2)+3(2+1)\right| = \frac{1}{2}\cdot\left|16+9\right| = 12{,}5$$

$$|AB| = \sqrt{(0+4)^2+(-1-2)^2} = \sqrt{4^2+3^2} = 5,$$

$$|BC| = \sqrt{(3-0)^2+(3+1)^2} = \sqrt{3^2+4^2} = 5,$$

$$|AC| = \sqrt{(3+4)^2+(3-2)^2} = \sqrt{7^2+1^2} = \sqrt{50} = 5\sqrt{2}$$

Demak, $P = 5+5+5\sqrt{2} = 5(2+\sqrt{2})$.

Kosinuslar teoremasidan

$$\cos < A = \frac{|AC|^2+|AB|^2-|BC|^2}{2\cdot|AC|\cdot|AB|} = \frac{50+25-25}{2\cdot 2\sqrt{5\cdot 5}} = \frac{1}{\sqrt{2}},$$

ya'ni $< A = 45^0$ ekanligi kelib chiqadi.

Uchburchakning teng yonli ekanligidan $< C = 45^0$, $< B = 90^0$.

Bu uchburchak to'g'ri burchakli ekanligi $|AB|^2 + |BC|^2 = |AC|^2$ tenglikdan kelib chiqadi.

3. Uchburchakning ikkita uchi $A(3; 8), B(10; 2)$ nuqtalarda bo'lib, medianalari $O(1; 1)$ nuqtada kesishsa, uchi $C(x, y)$ koordinatalarini toping.

A nuqtadan chiqqan mediana AD bo'lsa, medianalar xossasidan $|AO| : |OD| = 2 : 1 = \lambda$ bo'lib, D nuqta koordinatalari x_1, y_1 uchun quyidagi tengliklar o'rinli:

$$1 = \frac{3 + 2 \cdot x_1}{1 + 2}; \quad 1 = \frac{8 + 2 \cdot y_1}{1 + 2}$$

Bundan $D(0; -2{,}5)$. O'z navbatida, bu nuqta CB kesma markazi ekanligidan

$$\frac{x + 10}{2} = 0; \quad \frac{y + 2}{2} = -2{,}5$$

ya'ni $C(-10; -7)$ kelib chiqadi.

4. Agar $O(0; 0)$, $E(3; 0)$, $F(0; 4)$ uchburchak tomonlari o'rtalari bo'lsa, bu uchburchak yuzini toping.

$|OE| = 3$; $|OF| = 4$; $|EF| = 5$ kesmalar berilgan uchburchak o'rta chiziqlari ekanligidan, uning tomonlari 6, 8, 10 uzunlikka egaligi kelib chiqadi. Pifagor teoremasi o'rinliligidan bu uchburchak to'g'ri burchakli va

$$S = \frac{6 \cdot 8}{2} = 24 \quad \text{(kv.b)}$$

5. Uchlari $O(0; 0)$, $A(8; 0)$, $B(0; 6)$ nuqtalarda bo'lgan uchburchakda OS – mediana, OD – bissektrisa, OE – balandlik uzunliklarini hisoblang.

$a = |OA| = 8$, $b = |OB| = 6$, $c = |AB| = 10$ ekanligidan, uchburchak to'g'ri burchakli.

$$|OC| = \frac{|AB|}{2} = 5;$$

$$L_c = |OD| = \frac{1}{a+b}\sqrt{ab(a+b+c)(a+b-c)}$$

formuladan $|OD| = \frac{24\sqrt{2}}{7}$; $S = \frac{ab}{2} = \frac{c \cdot h}{2}$

formuladan $h = |OE| = 4{,}8$.

6. $2x - 3y - 12 = 0$ to'g'ri chiziqning son o'qlari bilan kesishish nuqtalarini aniqlang.

To'g'ri chiziq Ox o'qi bilan kesishish nuqtasida $y = 0$ bo'ladi. Demak, $2x - 12 = 0$. Bundan $x = 6$, ya'ni to'g'ri chiziq Ox o'qini (6; 0) nuqtada kesib o'tadi.

Aksincha, $x = 0$ bo'lsa, $y = -4$ kelib chiqadi. To'g'ri chiziq Ou o'qini $(0; -4)$ nuqtada kesib o'tadi.

7. $A(0;4)$ nuqtadan o'tuvchi va Ox o'qi musbat yo'nalishi bilan $\alpha = \frac{2\pi}{3}$ burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

$tg\frac{2\pi}{3} = -\sqrt{3}$ ekanligidan $k = -\sqrt{3}$. Demak, to'g'ri chiziq burchak koeffitsientli tenglamasi $y = -\sqrt{3}x + 4$ bo'ladi.

8. To'g'ri chiziq koordinata o'qlarida teng musbat kesmalar ajratadi. Bu to'g'ri chiziq va son o'qlari bilan chegaralangan uchburchak yuzi 8 kv.b. bo'lsa, to'g'ri chiziq tenglamasini yozing.

Bu uchburchak to'g'ri burchaklidir. Agar uning katetlarini a, b deb belgilasak, ularning tengligi va $a^2 = 16$, $a = 4$ ekanligi kelib chiqadi. Demak, to'g'ri chiziqning kesmalar bo'yicha tenglamasi

$$\frac{x}{4}+\frac{y}{4}=1$$ ko'rinishda bo'ladi.

9. To'g'ri chiziq $\frac{(x+2\sqrt{5})}{4}+\frac{y-2\sqrt{5}}{2}=0$ tenglama bilan berilgan. Bu to'g'ri chiziqning umumiy, burchak koeffitsientli, kesmalar bo'yicha va normal tenglamalarini yozing.

1) Berilgan tenglamani umumiy maxrajga keltiramiz:

$$x+2\sqrt{5}+2(y-2\sqrt{5})=0$$

Bundan $x+2y-2\sqrt{5}=0$ umumiy tenglamasi kelib chiqadi.

2) Umumiy tenglamani y ga nisbatan echamiz:

$$2y=-x+2\sqrt{5} \qquad \text{ya'ni} \qquad y=-\frac{1}{2}x+\sqrt{5}$$

burchak koeffitsientli tenglamasidir.

3) Umumiy tenglama tomonlarini $2\sqrt{5}$ ga bo'lamiz:

$$\frac{x}{2\sqrt{5}}+\frac{y}{\sqrt{5}}=1$$

Kesmalar bo'yicha tenglama hosil bo'ldi.

4) Umumiy tenglama tomonlarini $\mu=\pm\frac{1}{\sqrt{a^2+b^2}}=\pm\frac{1}{\sqrt{1^2+2^2}}$ ya'ni $\mu=\frac{1}{\sqrt{5}}$ ga ko'paytiramiz.

$\frac{1}{\sqrt{5}}x+\frac{2}{\sqrt{5}}y-2=0$ bu to'g'ri chiziqning normal tenglamasi bo'lib, $\sin\alpha=\frac{2}{\sqrt{5}}$, $\cos\alpha=\frac{1}{\sqrt{5}}$, $p=2$.

10. Ixtiyoriy nuqtasidan $x=-3$ to'g'ri chiziqqacha masofa Ox o'qigacha bo'lgan masofadan ikki marta kichik bo'ladigan chiziq tenglamasini toping.

Bu chiziqning ixtiyoriy $M(x,y)$ nuqtasini olamiz. Bu nuqtadan $x=-3$ to'g'ri chiziqqacha masofa $|x+3|$ ga, Ox o'qigacha masofa esa u ga teng. Shartga ko'ra, $y=2\cdot|x+3|$ yoki $y=\pm 2(x+3)$.

11. $y=-2x$ va $y=3x+4$ to'g'ri chiziqlar orasidagi o'tkir burchakni toping. $k_1=-2$; $k_2=3$ ekanligidan

$$tg\varphi=\left|\frac{3+2}{1-3\cdot 2}\right|=\left|\frac{5}{-5}\right|=1. \quad \text{Demak: } \varphi=45^0.$$

12. $A(-2;0), B(2;6)$ va $C(1;2)$ nuqtalar uchburchak uchlari bo'lsa, uchburchak AS tomoni, BE medinasi, BD – balandligi tenglamalarini yozing.

A va S nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi $\frac{x-(-2)}{4-(-2)}=\frac{y-0}{2-0}$ yoki $y=\frac{x}{3}+\frac{2}{3}$ dir.

E nuqta A va S nuqtalarni tutashtiruvchi kesma o'rtasi, demak uning koordinatalari

$$x=\frac{-2+4}{2}=1,\ y=\frac{0+2}{2}=1$$

ekanligi kelib chiqadi. Mediana B va E nuqtalardan o'tganligi uchun uning tenglamasi $\frac{x-1}{2-1}=\frac{y-1}{6-1}$, ya'ni $y=5x-4$ ko'rinishda bo'ladi.

BD balandlik AC tomonga perpendikulyarligidan, tenglamasi $y=-3x+b$ ko'rinishda bo'ladi. Uning $B(2;6)$ nuqtadan o'tishidan foydalanib $6=-3\cdot 2+b$, ya'ni $b=12$ ekanligini topamiz.

Demak, BD balandlik $y=-3x+12$ tenglama bilan aniqlanadi.

13. m va n ning qanday qiymatlarida $mx+8y+n=0$ va $2x+my-1=0$ to'g'ri chiziqlar 1) parallel 2) ustma-ust 3) perpendikulyar bo'ladi?

1) bu to'g'ri chiziqlar $\frac{m}{2}=\frac{8}{m}$ shart bajarilsa, ya'ni $m=\pm 4$ bo'lsa parallel bo'ladi.

2) Ular ustma-ust tushishi uchun $\frac{m}{2}=\frac{8}{m}=\frac{n}{-1}$ shartlar bajarilishi zarur. Bundan $m=\pm 4$; $n=\mp 2$ kelib chiqadi.

3) Perpendikulyarlik shartidan $2m+8m=0$, ya'ni $m=0$ kelib chiqadi.

14. Ikki to'g'ri chiziq $x+3y=0$ va $x-2y+3=0$ larning kesishish nuqtasini toping.

Noparallel bu to'g'ri chiziqlar kesishish nuqtasini topish uchun, ularning tenglamasini birgalikda echish kerak.

$$\Rightarrow \begin{cases} y=-\frac{x}{3} \\ y=\frac{x+3}{2} \end{cases} \Rightarrow -\frac{x}{3}=\frac{x+3}{2} \Rightarrow x=-\frac{9}{5}; y=-\frac{3}{5}$$

Demak, bu to'g'ri chiziqlar $A(-\frac{9}{5};-\frac{3}{5})$ nuqtada kesishadi.

15. $A(-1;1)$ nuqtadan o'tib $y=\frac{6-2x}{3}$ to'g'ri chiziq bilan 45^0 burchak hosil qiluvchi to'g'ri chiziqlar tenglamasini yozing.

Berilgan to'g'ri chiziq burchak koeffitsienti $k_1 = -\frac{2}{3}$, qidirilayotgan to'g'ri chiziqlardan birinikini k_2 deb olamiz.

$tg45^0 = \left|\dfrac{k_2 + \frac{2}{3}}{1 - \frac{2}{3} \cdot k_2}\right| = 1$. dan $|3k_2 + 2| = |3 - 2k_2|$, ya'ni

$3k_2 + 2 = \pm(3 - 2k_2)$. Bundan $k_2 = \frac{1}{5}$ yoki $k_2 = -5$ kelib chiqadi. Qidiralayotgan to'g'ri chiziqlar $y - 1 = -5(x + 1)$ ko'rinishdadir. Soddalashtirib $y = \frac{x}{5} + \frac{6}{5}$ va $y = -5x - 4$ tenglamalarga ega bo'lamiz.

16. $A(3;0), B(5;-3)$ nuqtalardan $2x - 3y - 6 = 0$ to'g'ri chiziqqacha bo'lgan masofani toping.

A nuqtadan to'g'ri chiziqqacha masofa

$$d_1 = \frac{|2 \cdot 3 - 3 \cdot 0 - 6|}{\sqrt{2^2 + (-3)^2}} = 0$$

Demak, bu nuqta chiziqda yotadi. B nuqtadan bu to'g'ri chiziqqacha bo'lgan masofa

$$d_2 = \frac{|2 \cdot 5 - 3 \cdot 5 - 6|}{\sqrt{2^2 + (-3)^2}} = \frac{13}{13} = 1$$

17. Ikki parallel $2x - 3y - 6 = 0$ va $4x - 6y - 25 = 0$ to'g'ri chiziqlar orasidagi masofani toping.

Birinchi to'g'ri chiziqdan biror nuqta tanlab olamiz, buning uchun $y = 0$ desak, $x = 3$ chiqadi. $A(3;0)$ nuqta birinchi chiziqda yotadi. Shu nuqtadan ikkinchi to'g'ri chiziqqacha masofa

$$d=\frac{|4\cdot3-6\cdot0-25|}{\sqrt{4^2+(-6)^2}}=\frac{13}{\sqrt{52}}=\frac{13}{2\sqrt{13}}=\frac{\sqrt{13}}{2}$$

bo'lib, bu izlanayotgan masofadir.

18. $2x+3y=10$ va $3x+2y=10$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalari tenglamalarini yozing.

$ax+by+c=0$ va $a_1x+b_1y+c_1=0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalari

$$\frac{ax+by+c}{\sqrt{a^2+b^2}}=\pm\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}$$

formula yordamida topiladi. Bundan

$$\frac{2x+3y-10}{\sqrt{2^2+3^2}}=\pm\frac{3x+2y-10}{\sqrt{3^2+2^2}}$$ ya'ni

$y=x$ va $y=4-x$.

19. Parallelogramm ikki tomoni tenglamalari $y=x-2$ va $5y=x+6$ bo'lib, dioganallari koordinata boshida kesishsa, qolgan ikki tomon va dioganallar tenglamalarini yozing.

Berilgan ikki tomon kesishish nuqtasini topamiz:

$$x-2=\frac{x+6}{5},\ \text{ya'ni}\ x=4,\ y=2.$$

Parallelogramm A(4;2) uchiga qarama-qarshi uchini $C(x,y)$ desak, $\frac{4+x}{2}=0,$ $\frac{2+y}{2}=0$ ekanligidan $C(-4;-2)$ bo'ladi.

$C(-4;-2)$ dan o'tib, berilgan tomonlarga parallel bo'lgan to'g'ri chiziqlar izlanayotgan tomonlar bo'ladi:

$y+2=1\cdot(x+4)$ va $y+2=\frac{1}{5}\cdot(x+4)$ dan

$y=x+2,\ y=\frac{x}{5}-\frac{6}{5}$ kelib chiqadi.

Bu tomonlarning berilgan tomonlar bilan kesishadigan nuqtalari

$\begin{cases} y=x+2 \\ y=\frac{x}{5}+\frac{6}{5} \end{cases}$ va $\begin{cases} y=x-2 \\ y=\frac{x}{5}-\frac{6}{5} \end{cases}$ sistemalar

echimlari, ya'ni $B(-1;1), D(1;-1)$ dir.

AC dioganal $\frac{x-4}{-4-4}=\frac{y-2}{-2-2}$ dan $y=\frac{x}{2}$

va

BD dioganal $\frac{x+1}{1+1}=\frac{y-1}{-1-1}$ dan $y=-x$

ekanligi kelib chiqadi.

20. $x^2+6x+y^2-8y+21=0$ aylana markazi koordinatalari va radiusini toping.

$(x+3)^2-9+(y-4)^2-16+21=0$ ko'rinishda to'la kvadratlar ajratsak,

$(x+3)^2+(y-4)^2=2^2$

Aylana markazi A$(-3;4)$ nuqtada va radius $R=2$

21. $A(5;0)$, $B(1;4)$ nuqtalardan o'tuvchi, markazi $y=-x+3$ to'g'ri chiziqda yotuvchi aylana tenglamasini tuzing.

AB kesma o'rtasining koordinatalari

$x=\frac{5+1}{2}=3$; $y=\frac{0+4}{2}=2$ C(3;2)

AB chiziq tenglamasi

$$\frac{x-5}{1-5}=\frac{y-0}{4-0}, \qquad \text{ya'ni } y=x+5.$$

Aylana markazi $C(3;2)$ dan o'tuvchi, $y=-x+5$ ga perpendikulyar to'g'ri chiziqda yotadi, ya'ni

$$y-2=1(x-3), \quad y=x-1$$

Demak, aylana markazi $y=-x+3$ va $y=x-1$ to'g'ri chiziqlar kesishish nuqtasi $O_1(2;1)$ dir

Aylana radiusi esa.

$$R=|OA|=\sqrt{(5-2)^2+(0-1)^2}=\sqrt{10}$$

Demak,

$$(x-2)^2+(y-1)^2=10$$

22. $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsda fokal radiuslari ayirmasi 6,4 ga teng bo'lgan nuqtani aniqlang.

$a=5,\ b=3$ ekanligidan $\quad s=\sqrt{5^2-3^2}=4$.

Demak, $\varepsilon=\frac{4}{5}$

$6{,}4=|r_2-r_1|=2\varepsilon x$ dan $x=\pm 4$.

Topilgan x ni ellips tenglamasiga qo'yib $y=\pm 1{,}8$.

Demak: $(4;1{,}8)$; $(4;-1{,}8)$; $(-4;1{,}8)$, $(-4;-1{,}8)$ nuqtalar shartni qanoatlantiradi.

23. $\frac{x^2}{16}-\frac{y^2}{9}=1$ giperbola o'ng shoxida shunday nuqta topingki, undan o'ng fokusgacha masofa chap fokusgacha bo'lgan masofadan ikki marta qisqa bo'lsin.

$\varepsilon x + a = 2(\varepsilon x - a)$ shartdan $X = \dfrac{3a}{\varepsilon}$ kelib chiqadi.

$a = 4$; $\varepsilon = \dfrac{c}{a} = \dfrac{\sqrt{a^2 + в^2}}{a} = \dfrac{\sqrt{16+9}}{4} = \dfrac{5}{4}$

ekanligidan $x = 9,6$ demak,

$$y = \pm\frac{3}{4}\sqrt{x^2 - 16} = \pm\frac{3}{4}\sqrt{(\frac{48}{5})^2 - 16} = \pm\frac{3}{4}\sqrt{119}.$$

Shartni $A_1(9,6;\frac{3}{4}\sqrt{119})$ va $A_2(9,6;-\frac{3}{4}\sqrt{119})$ nuqtalari qanoatlantirar ekan.

24. $y^2 = 8x$ parabolada direktrisadan 4 birlik uzoqlikdagi nuqtani toping.

Bunday nuqtadan direktrisagacha masofa $d = \dfrac{p}{2} + x$ ga teng demak, $\dfrac{p}{2} + x = 4$

$p = 4$ ekanligini hisobga olsak, $x = 2$ kelib chiqadi. Uni tenglamaga qo'yib $y = \pm 4$ ekanligini topamiz. Demak, $A_1(2;4), A_2(2;-4)$ izlanayotgan nuqtalardir.

25. $\dfrac{x^2}{100} + \dfrac{y^2}{225} = 1$ ellips va $y^2 = 24x$ parabola kesishish nuqtalarini toping.

$$\frac{x^2}{100} + \frac{24x}{25 \cdot 9} = 1 \quad \text{dan} \quad 3x^2 + 32x - 300 = 0$$

tenglamaga ega bo'lamiz. Bundan $x_1 = -\dfrac{50}{3}$, $x_2 = 6$ kelib chiqib, $x = 6$ shartni qanoatlantiradi, $y^2 = 144$.

Demak, $A_1(6;12), A_2(6;-12)$.

26. $M(1;\sqrt{3}))$ nuqtaning qutb koordinatalarini $A(2;\frac{5П}{4})$ nuqta dekart koordinatalarini toping.

$r=\sqrt{1^2+(\sqrt{3})^2}=2$, $tg\varphi=\frac{\sqrt{3}}{1}$ dan $\varphi=\frac{\pi}{3}$

Demak, $M(2;\frac{\pi}{3}))$

$$x=2\cos\frac{5\pi}{4}=2\cos(\pi+\frac{\pi}{4})=-2\cos\frac{\pi}{4}=-\sqrt{2}$$

$$y=2\sin\frac{5\pi}{4}=2\sin(\pi+\frac{\pi}{4})=-2\sin\frac{\pi}{4}=-\sqrt{2}$$

Demak, $A\left(-\sqrt{2};-\sqrt{2}\right))$

27. $E\left(3;\frac{\pi}{4}\right))$ va $F\left(4;\frac{3\pi}{4}\right)$ nuqtalar orasidagi masofani toping

$$\angle EOF=\frac{3\pi}{4}-\frac{\pi}{4}=\frac{\pi}{2}.$$

Kosinuslar (xususan, Pifagor) teoremasidan foydalanib:

$|EF|^2=3^2+4^2-2\cdot3\cdot4\cdot\cos\frac{\pi}{2}$, ya'ni $|EF|=5$

5.3. $x^2+y^2=ax$ ni qutb koordinatalarida, $r=2a\sin\varphi$ ni dekart koordinatalarda yozing.

$a)x^2+y^2=r^2, x=r\cos\varphi$, $y=r\sin\varphi$ ekanligidan $r^2=ar\cos\varphi$, ya'ni $r=a\cos\varphi$;

b) $\sqrt{x^2+y^2}=2a\cdot\frac{y}{\sqrt{x^2+y^2}}$ dan

$x^2+y^2=2ay$ yoki $x^2+(y-a)^2=a^2$.

28. a) $2x^2+5y^2-12x+10y+13=0$ tenglamani kanonik ko'rinishga keltiring.

$2x^2-12x+5y^2+10y+13=0$ tenglamada to'la kvadratlar ajratamiz:

$2\left[x^2-6x+9-9\right]+5\left[y^2+2y+1\right]+8=0$. Bundan,

$2(x-3)^2-18+5(y-1)^2-5+13=0$, ya'ni

$2(x-3)^2+5(y-1)^2=10$

yoki $x'=x-3,\quad y'=y+1$ parallel ko'chirish natijasida

$2x'^2+5y'^2=10$ yoki $\dfrac{x'^2}{5}+\dfrac{y'^2}{2}=1$

Ellips tenglamasiga ega bo'lamiz.

b) $5x^2-4xy+2y^2-24=0$

29. a) $2x^2+6\sqrt{3}xy-4y^2+20x+10y+\dfrac{480\sqrt{3}-85}{7}=0$

kanonik tenglamasini yozing.

$ctg2\alpha=\dfrac{2+4}{6\sqrt{3}}=\dfrac{1}{\sqrt{3}}$ dan $2\alpha=60^0$, ya'ni

$\alpha=30^0$

$$x=x'\cos30^0-y'\sin30^0=\frac{\sqrt{3}x'-x'}{2}$$

$$y=x'\sin30^0+y'\cos30^0=\frac{x'+\sqrt{3}y'}{2}$$

almashtirish yordamida tenglama quyidagi ko'rinishga keladi:

$$5\left\{\left[x'+2(\sqrt{3}+1)\right]^2-4(\sqrt{3}+1)^2\right\}-7\left\{\left[y'-\frac{10}{7}(\sqrt{3}-1)\right]^2-\frac{100}{49}(\sqrt{3}-1)^2\right\}+$$

$$+\frac{480\sqrt{3}-85}{7}=0$$

$$\begin{cases} x'' = x' + (\sqrt{3}+1) \\ y'' = y' - \frac{10}{7}(\sqrt{3}-1) \end{cases}$$

almashtirishdan so'ng $5x''^2 - 7y''^2 = 35$ ko'rinishga keladi, ya'ni

$$\frac{x''^2}{7} - \frac{y''^2}{5} = 1$$

giperbola tenglamasiga ega bo'lamiz.

b) $r = \frac{1}{2-\sqrt{3}\cos\varphi}$ dekart kordinatalari sistemasiga o'tkazing va kanonik ko'rinishga keltiring.

$\sqrt{x^2+y^2} = \frac{1}{2-\sqrt{3}\cdot\frac{x}{\sqrt{x^2+y^2}}}$ dan

$2\sqrt{x^2+y^2} - \sqrt{3}x = 1$ yoki $4(x^2+y^2) = 1 + 2\sqrt{3}x + 3x^2$ kelib chiqadi. $x^2 - 2\sqrt{3}x + 4y^2 = 1$ da $(x-\sqrt{3})^2 - 3 + 4y^2 = 1$ uchun $x' = x - \sqrt{3}$; $y' = y$ deb almashtirsak:

$x'^2 + 4y'^2 = 4$ ellips tenglamasiga ega bo'lamiz.

I–bob bo'yicha uy vazifalari.

1.Tekislikda uchta $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ nuqtalar berilgan. Quyidagilar topilsin.

1^0. ABC uchburchak perimetri;

2^0. ABC uchburchak medinalari kesishishi nuqtasi;

3^0. ABC uchburchak yuzi;

4^0. C nuqtadan o'tuvchi to'g'ri chiziqlar dastasi;

5^0. A va B nuqtalardan o'tuvchi to'g'ri chiziq 4 xil tenglamasi;

6^0. C nuqtadan o'tib, AB chiziqqa parallel va perpendikulyar bo'lgan to'g'ri chiziqlar;

7^0. C nuqtadan AB to'g'ri chiziqqacha bo'lgan masofa;

8^0. ABC uchburchak ichki burchaklari.

9^0. C nuqtadan o'tkazilgan mediana va bessektrisa tenglamalari.

1). $A(-4;2), B(-1;4), C(1;2)$

2). $A(-5;0), B(1;4), C(1;-4)$

3). $A(1;4), B(3;-1), C(4;3)$

4). $A(-4;1), B(1;4), C(4;-1)$

5). $A(-2;-2), B(1;1), C(+2;6)$

6). $A(-4;-3), B(-1;-2), C(1;0)$

7). $A(-1;-1), B(1;4), C(4;-2)$

8). $A(-3;2), B(0;-4), C(2;4)$

9). $A(-2;3), B(-1;4), C(1;-3)$

10). $A(-1;-4), B(1;2), C(4;-2)$

11). $A(5;1), B(-2;1), C(0;5)$

12). $A(3;4), B(0;-3), C(-2;4)$

13). $A(0;3), B(1;1), C(3;1)$

14). $A(-2;4), B(1;-4), C(3;5)$

15). $A(1;1), B(5;3), C(2;-3)$

2.Quyida beriladigan shartni qanoatlantiruvchi $M(x,y)$ nuqtalar tenglamasini tuzing va grafigini chizing.

1) Koordinatalar boshigacha va $A(5;0)$ nuqtagacha masofalar $2:1$ nisbatda;

2) $A(-1;0)$ gacha masofa $x=-4$ gacha masofadan ikki marta kichik;

3) $A(2;0)$ gacha va $5x+8=0$ gacha masofalar 5:4 nisbatda

4) $B(1;0)$ gacha masofa $A(4;0)$ gacha masofadan 2 marta kichik;

5) $A(2;0)$ gacha va $2x+5=0$ gacha masofalar 4:5 nisbatda;

6) $A(3;0)$ gacha masofa $B(26;0)$ gacha masofadan 2 marta kichik;

7) $A(0;2)$ nuqtadan va $y-4=0$ to'g'ri chiziqdan bir xil uzoqlikda;

8) Ordinatalar o'qidan va $x^2+y^2=4x$ aylanadan bir xil uzoqlikda;

9) $A(2;6)$ nuqtadan va $y+2=0$ to'g'ri chiziqdan bir xil uzoqlikda;

10) $A(-4;0)$ gacha masofa $O(0;0)$ gacha masofadan 3 marta katta;

11) $A(-4;2)$ nuqtadan va $x=1$ to'g'ri chiziqdan bir xil uzoqlikda;

12) $B(2;0)$ gacha masofa $A(6;0)$ gacha masofadan 3 marta kichik;

13) $A(4;0)$ gacha va $x+3=0$ gacha masofalar 2:3 nisbatda;

14) $A(2;2)$ gacha masofa $B(16;0)$ gacha masofadan 3 marta katta;

15) $A(1;1)$ dan va $B(6;4)$ dan bir xil uzoqlikda;

3.Qutb koordinatalar sistemasida $(r = r(\varphi)$ **tenglama bilan chiziq berilgan.**

a) φ ga [0,2π] oraliqdagi qiymatlarni berib, nuqtalar bo'yicha chiziqni yasang.

b) Dekart koordinatalariga o'tkazing va qanday chiziqligini aniqlang

v)Kanonik ko'rinishiga keltiring.

1) $r = \dfrac{1}{(1+\cos\varphi)}$ 2) $r = \dfrac{1}{(2+\cos\varphi)}$

3) $r = \dfrac{4}{(2-3c\,os\varphi)}$ 4) $r = \dfrac{8}{(3-\cos\varphi)}$

5) $r = \dfrac{1}{(2+2\cos\varphi)}$ 6) $r = \dfrac{5}{(3-4\cos\varphi)}$

7) $r = \dfrac{10}{(2+\cos\varphi)}$ 8) $r = \dfrac{3}{(1-2\cos\varphi)}$

9) $r = \dfrac{1}{(3-3\cos\varphi)}$ 10) $r = \dfrac{5}{(6+3\cos\varphi)}$

11) $r = \dfrac{9}{(5-4\cos\varphi)}$ 12) $r = \dfrac{9}{(4-5\cos\varphi)}$

13) $r = \dfrac{3}{(1-\cos\varphi)}$ 14) $r = \dfrac{1}{\left(4-\sqrt{3}\cos\varphi\right)}$

15) $r = \dfrac{1}{\left(2-\sqrt{5}\cos\varphi\right)}$

4.Ikkinchi tartibli chiziq $Ax^2+2Bxy^2+Cy^2+2Dx+2Ey+F=0$ tenglama bilan berilgan. Kanonik ko'rinishga keltiring va dastlabki koordinatalar sistemasida chizing.

1) $x^2-2xy+y^2+6x-14y+29=0$

2) $x^2-2xy+y^2-12x+12y-14=0$

3) $3x^2-2xy+3y^2-4x-4y-12=0$

4) $x^2-6xy+y^2-4x-4y+12=0$

5) $x^2-xy+y^2-2x-2y-2=0$

6) $3x^2+10xy+3y^2-12x-12y+4=0$

7) $x^2-2xy+y^2-10x-6y+25=0$

8) $x^2+2xy+y^2-4y+3=0$

9) $2x^2+6\sqrt{3}xy-4y^2-9=0$

10) $x^2-3y^2+16x+12y-36=0$

11) $4xy+3y^2+16x+12y-36=0$

12) $25x^2+10xy+y^2-1=0$

13) $8x^2-18xy+9y^2+2x-1=0$

14) $14x^2+24xy+21y^2-4x+18y-139=0$

15) $9x^2-24xy+16x^2-20x+110y-50=0$

§ 6.Determinantlar, xossalari. Kramer qoidasi.

nxn ta elementdan tuzilgan, quyidagicha

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2n} \\ . & . & . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & . & . & . & a_{nn} \end{vmatrix}$$

yoziluvchi son n–tartibli determinant deyiladi. Bunda a_{ij} –son I–yo'l (satr), j– ustunda turadi.

I–yo'l, j–ustun o'chirilishidan hosil bo'ladigan (p–1)–tartibli determinant a_{ij} –element minori deyiladi va M_{ij} ko'rinishda belgilanadi.

$A_{ij} = (-1)^{i+j} \cdot M_{ij}$ esa a_{ij} element algebraik to'ldiruvchisi deyiladi.

Ikkinchi tartibli determinant.

$$\begin{vmatrix} a_{11}a_{12} \\ a_{21}a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

tenglik yordamida aniqlanadi.

Agar n–tartibli determinant Δ–songa teng bo'lsa,

$$\Delta = a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$$

$$\Delta = a_{1j}A_{1j} + a_{2j}A_{2j} + \ldots . + a_{nj}A_{nj}$$

tengliklar o'rinli, ya'ni determinant istalgan satri (yoki ustuni) elementlari bilan shu elementlar algebraik to'ldiruvchilari ko'paytmalarining yig'indisiga teng (bu determinant hisoblashning asosiy teoremasidir)

Determinantlar quyidagi xossalarga ega:

1^0. Mos yo'l va ustunlar o'rinlarini almashtirilsa, ya'ni transponirlansa, determinant qiymati o'zgarmaydi.

2^0. Ikki yo'l (ustun) o'rinlari almashtirilsa, determinant ishorasi o'zgaradi.

3^0. Ikki yo'li (ustuni) bir xil bo'lgan determinant nolga teng.

4^0. Ixtiyoriy yo'l (ustun) umumiy elementini determinant belgisidan tashqariga chiqarish mumkin.

5^0. Biror yo'l (ustun) elementlariga boshqa yo'l (ustun) ning bir xil ko'paytuvchiga ko'paytirilgan mos elementlarini qo'shishdan determinant o'zgarmaydi.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = в_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = в_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = в_n \end{cases}$$

chiziqli tenglamalar sistemasi berilgan bo'lsin. Noma'lumlar oldidagi koeffitsientlardan tuzilgan determinant Δ asosiy determinant deyiladi. Bu determinantda I–ustundagi a_{ij} elementlar o'rniga b_j –elementlar qo'yilishidan hosil bo'lgan determinant j–yordamchi determinant deyiladi va Δ_j tarzida belgilanadi.

Bunday sistema echimlari quyidagi Kramer qoidasi yordamida topiladi:

$$X_j = \frac{\Delta_j}{\Delta}.$$

6.1.Ikkinchi tartibli determinantlarni hisoblang:

1) $\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix}$ 2) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ 3) $\begin{vmatrix} a^2 & ae \\ ae & e^2 \end{vmatrix}$ 4) $\begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix}$

5) $\begin{vmatrix} a^2+ae+e^2 & a^2-ae+e^2 \\ a+e & a-e \end{vmatrix}$

6) $\begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$ 7) $\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$

8) $\begin{vmatrix} \sin\alpha+\sin\beta & \cos\beta+\cos\alpha \\ \cos\beta-\cos\alpha & \sin\alpha-\sin\beta \end{vmatrix}$

6.2.Uchburchak qoidasi yordamida quyidagi uchinchi tartibli determinantlarni hisoblang:

1) $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$ 2) $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}$

3) $\begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$ 4) $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}$

6.3.Qulay qator bo'yicha yoyib hisoblang:

1) $\begin{vmatrix} 5 & 6 & 3 \\ 0 & 1 & 0 \\ 7 & 4 & 5 \end{vmatrix}$ 2) $\begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}$

3) $\begin{vmatrix} 1 & e & 1 \\ 0 & e & 0 \\ e & 0 & -e \end{vmatrix}$ 4) $\begin{vmatrix} 0 & a & -e \\ -a & 0 & c \\ e & -c & 0 \end{vmatrix}$

6.4.Determinant xossalaridan foydalanib hisoblang:

1) $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$ 2) $\begin{vmatrix} a^2 & a & 1 \\ в^2 & в & 1 \\ c^2 & c & 1 \end{vmatrix}$

3) $\begin{vmatrix} a & в & c \\ в & c & a \\ c & a & в \end{vmatrix}$ 4) $\begin{vmatrix} a & x & x \\ x & в & x \\ x & x & c \end{vmatrix}$

5) $\begin{vmatrix} a+x & x & x \\ x & в+x & x \\ x & x & c+x \end{vmatrix}$ 6) $\begin{vmatrix} 1+\cos\alpha & 1+\sin\alpha & 1 \\ 1-\sin\alpha & 1+\cos\alpha & 1 \\ 1 & 1 & 1 \end{vmatrix}$

7) $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$ 8) $\begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}$

9) $\begin{vmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \end{vmatrix}$

10) $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$

6.5.Tenglamalar sistemasini a)Kramer formulasi yordamida, b)Gauss usulida eching.

$$1)\begin{cases}2x-y-z=4\\3x+4e-2z=11\\3x-2y+4z=11,\end{cases}\qquad 2)\begin{cases}x+y+2z=-1\\2x-e+2z=-4\\4x+e+4z=-2\end{cases},$$

$$3)\begin{cases}3x+2y+z=5\\2x+3y+z=1\\2x+y+3z=11\end{cases}\qquad 4)\begin{cases}x_1+y_2+2z+3t=1\\3x-y-z-2t=-4\\2x+3y-z-t=-6\\x+2y+3z-t=-4\end{cases}$$

$$5)\begin{cases}x+2y+3z+4t=5\\2x+y+2z+3t=1\\3x+2y+z+2t=1\\4x+3y+2z+t=-5\end{cases}\qquad 6)\begin{cases}y-3z+4t=-5\\x-2z+3t=-4\\3x+2y-5t=12\\4x+3y-5z=5\end{cases}$$

$$7)\begin{cases}x-3y+5z-7t=12\\3x-5y+7z-t=0\\5x-7y+z-3t=4\\7x-y+3z-5t=16\end{cases}\qquad 8)\begin{cases}x+2y=5\\3y+4z=18\\7n+8v=68\\9v+10x=55\end{cases}$$

6.6. $A(x_1;y_1)$, $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini 3—tartibli determinant yordamida yozing.

6.7.Uchlari $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ nuqtalarda bo'lgan uchburchak юzi formulasini 3—tartibli determinant yordamida yozing.

6.8. Tenglamalarni eching:

$$1)\begin{vmatrix}x^2 & 4 & 9\\x & 2 & 3\\1 & 1 & 1\end{vmatrix}=0\qquad 2)\begin{vmatrix}x^2 & 3 & 2\\x & -1 & 1\\0 & 1 & 4\end{vmatrix}=0$$

Yuqori tartibli determinantlarni hisoblash

Yuqori tartibli determinantlar, asosan, xossalar yordamida dioganalning bir tomonidagi elementlarni nolga aylantirish yordamida hisoblanadi.

$$\begin{vmatrix} d_1 & & & & \\ & d_2 & & * & \\ & & \cdot & & \\ & 0 & & \cdot & \\ & & & & d_n \end{vmatrix} = d_1 d_2 \ldots d_n = \prod_{k=1}^{n} d_k$$

Misol.

$$1)\ \Delta = \begin{vmatrix} 1 & a_1 & a_2 & \cdot & \cdot & \cdot & a_n \\ 1 & a+b_1 & a_2 & \cdot & \cdot & \cdot & a_n \\ 1 & a_1 & a_2+b_2 & \cdot & \cdot & \cdot & a_n \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & a_1 & a_2 & \cdot & \cdot & \cdot & a_n+b_n \end{vmatrix} =$$

$$q = \begin{vmatrix} 1 & a_1 & a_2 & \cdot & \cdot & a_n \\ 1 & a_1 & a_2 & \cdot & \cdot & a_n \\ 1 & a_1 & a_2 & \cdot & \cdot & a_n \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & a_1 & a_2 & \cdot & \cdot & a_n \end{vmatrix} + \begin{vmatrix} 1 & a_1 & a_2 & \cdot & \cdot & a_n \\ 1 & b_1 & a_2 & \cdot & \cdot & a_n \\ 1 & a_1 & b_2 & \cdot & \cdot & a_n \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & a_1 & a_2 & \cdot & \cdot & b_n \end{vmatrix}$$

1 – determinant nolga teng, chunki $\overline{2,n+1}$ ustunlardan $a_i \left(i=\overline{1,n}\right)$ ko'paytuvchilar determinant belgisidan tashqariga chiqarilsa, hosil bo'lgan determinat barcha elementlari bir xil bo'lib qoladi.

2—determinat 1—yo'lini (−1) ga ko'paytirib qolgan barcha yo'llariga qo'shsak, u quyidagi ko'rinishga keladi.

$$\begin{vmatrix} 1 & a_1 & a_2 & . & a_n \\ 0 & b_1-a_1 & 0 & . & 0 \\ 0 & 0 & b_2-a_2 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & b_n-a_n \end{vmatrix}$$

Demak, $\prod_{k=1}^{n}(b_k-a_k)$.

Ba'zi hollarda determinat bir echim bir necha o'zgaruvchi ko'phadi deb qaralib, bu ko'phad chiziqli bo'luvchilarini topish mumkin bo'ladi. Chiziqli bo'luvchilar ko'paytmasi tartibi ko'phad tartibiga teng bo'lsa, ular ishorasi farqli bo'lishi mumkin, xolos.

Misol. 1) $f(x)=\begin{vmatrix} 1 & 2 & 3 & . & . & n \\ 1 & x+1 & 3 & . & . & n \\ 1 & 2 & x+1 & . & . & n \\ . & . & . & . & . & . \\ 1 & 2 & 3 & . & . & x+1 \end{vmatrix}$

Agar x o'rniga mos ravishda $1,2,...,n-1$ qo'yilsa determinant nolga teng bo'ladi, ya'ni $f(x)=\pm(x-1)(x-2)...(x-n+1)$ bo'lishi mumkin. Determinat yoyilmasida x^{n-1} had $+1$ koeffitsient bilan qatnashishini hisobga olsak, $f(x)=(x-1)(x-2)...(x-n+1)=\prod_{k=1}^{n-1}(x-k)$ bo'lishi kelib chiqadi.

2) $\begin{vmatrix} 1 & x_1 & x_2 & . & . & x_n \\ 1 & x & x_2 & . & . & x_n \\ 1 & x_1 & x_2 & . & . & x_n \\ . & . & . & . & . & . \\ 1 & x_1 & x_2 & . & . & x_n \\ 1 & x_1 & x_2 & . & . & x \end{vmatrix} = 0$ tenglamani eching.

Echish.

$$\begin{vmatrix} 1 & x_1 & x_2 & . & . & x_n \\ 0 & x-x_1 & 0 & . & . & 0 \\ 0 & 0 & x-x_2 & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & x-x_n \end{vmatrix} = (x-x_1)(x-x_2)...(x-x_n) =$$

$$= \prod_{k=1}^{n}(x-x_k)$$

$\prod_{k=1}^{n}(x-x_k)=0$ tenglama echimlari $x_1, x_2, ... x_n$ bo'ladi.

1. Determinantlar xossalaridan foydalanib hisoblang:

a) $\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$ b) $\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix}$

v) $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & a & b \\ 1 & a & 0 & c \\ 1 & b & c & 0 \end{vmatrix}$ g) $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$

v) $\begin{vmatrix} 0 & a & b & c & d \\ -a & 0 & e & f & g \\ -b & -e & 0 & h & k \\ -c & -f & -h & 0 & e \\ -d & -g & -k & -l & 0 \end{vmatrix}$

e) $\begin{vmatrix} 1 & 2 & 3 & . & . & n \\ -1 & 0 & 3 & . & . & n \\ -1 & -2 & 0 & . & . & n \\ . & . & . & . & . & . \\ -1 & -2 & -3 & . & . & 0 \end{vmatrix}$ f) $\begin{vmatrix} 1 & 2 & 2 & . & . & 2 \\ 2 & 2 & 2 & . & . & 2 \\ 2 & 2 & 3 & . & . & 2 \\ . & . & . & . & . & . \\ 2 & 2 & 2 & . & . & n \end{vmatrix}$

g) $\begin{vmatrix} 0 & 1 & 1 & . & . & 1 \\ 1 & 0 & 1 & . & . & 1 \\ 1 & 1 & 0 & . & . & 1 \\ . & . & . & . & . & . \\ 1 & 1 & 1 & . & . & 0 \end{vmatrix}$ h) $\begin{vmatrix} n & 1 & 1 & . & . & 1 \\ 1 & n & 1 & . & . & 1 \\ 1 & 1 & n & . & . & 1 \\ . & . & . & . & . & . \\ 1 & 1 & 1 & . & . & n \end{vmatrix}$

z) $\begin{vmatrix} 1 & 2 & 0 & 0 & . & . & 0 \\ 1 & 3 & 2 & 0 & . & . & 0 \\ 0 & 1 & 3 & 2 & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{vmatrix}$

2. Chiziqli ko'paytuvchilarni ajratish yordamida hisoblang.

a) $\begin{vmatrix} 1 & x_1 & x_2 & . & . & x_{n-1} & x_n \\ 1 & x & x_2 & . & . & x_{n-1} & x_n \\ 1 & x_1 & x & . & . & x_{n-1} & x_n \\ . & . & . & . & . & . & . \\ 1 & x_1 & x_2 & . & . & x_{n-1} & x \end{vmatrix}$ b) $\begin{vmatrix} -x & a & b & c \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix}$

i) $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{vmatrix}$

3. $f(x,\xi,...)=0$ tenglamani eching.

a) $f(x)=\begin{vmatrix} x & a_1 & a_2 & . & . & a_{n-1} & 1 \\ a_1 & x & a_2 & . & . & a_{n-1} & 1 \\ a_1 & a_2 & x & . & . & a_{n-1} & 1 \\ . & . & . & . & . & . & . \\ a_1 & a_2 & a_3 & . & . & x & 1 \\ a_1 & a_2 & a_3 & . & . & a_n & 1 \end{vmatrix}$

b) $f(x)=\begin{vmatrix} 1 & x_1 & x_1^2 & . & . & x_2^{n-1} \\ 1 & x_2 & x_2^2 & . & . & x_2^{n-1} \\ 1 & x_3 & x_3^2 & . & . & x_3^{n-1} \\ . & . & . & . & . & . \\ 1 & x_n & x_n^2 & . & . & x_n^{n-1} \end{vmatrix}$

§7.Matritsalar. Chiziqli tenglamalar sistemasini teskari matritsa yordamida echish.

Turli tabiatli a_{ij} $(i=\overline{1,m},\ \ j=\overline{1,\ n})$ sonlardan tuzilgan

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2n} \\ . & . & . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & . & . & . & a_{nn} \end{pmatrix}$$

jadval n x m o'lchamli matritsa deyiladi.
n=m bo'lgan holda matritsa kvadrat matritsa deyiladi.

Kvadrat matritsa elementlari nollardan iborat bo'lsa, nol matritsa, $i=j$ da 1, $j=1$ da 0 bo'lsa, birlik matritsa deyiladi. Agar kvadrat matritsa elementlari $a_{ij} = +a_{ji}$ shartni qanoatlantirsa, matritsa simmetrik, $a_{ij} = -a_{ji}$ shart o'rinli bo'lsa, kososimmetrik deyiladi.

$A = \left(a_{ij}\right), B = \left(b_{ij}\right)$ kvadrat matritsalar ustida quyidagicha amallar kiritish mumkin:

$A \pm B = \left(a_{ij} \pm b_{ij}\right)$, $\lambda \cdot A = \left(\lambda a_{ij}\right)$

$$A \cdot B = \begin{pmatrix} \sum_{j=1}^{n} a_{1j}b_{j1}, \sum_{j=1}^{n} a_{1j}b_{j2} \dots \sum_{j=1}^{n} a_{1j}b_{jn} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \sum_{j=1}^{n} a_{nj}b_{j1} \sum a_{nj}b_{j2} \dots \sum_{j=1}^{n} a_{3j}b_{jn} \end{pmatrix}$$

Nol matritsa 0, birlik matritsa E harflari bilan belgilanadi, ya'ni $A + O = A$; $A \cdot E = E \cdot A = A$.

$A \cdot B = B \cdot A = E$ shartni qanoatlantiruvchi V matritsa A matritsaga teskari deyiladi, A^{-1} tarzida belgilanadi va quyidagicha topiladi:

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & . & . & . & A_n \\ A_{12} & A_{22} & . & . & . & A_{n2} \\ . & . & . & . & . & . \\ A_{1n} & A_{2n} & . & . & . & A_{nn} \end{pmatrix},$$

Bunda |A| – matritsa determinanti.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots + a_{nn}x_n = bn \end{cases}$$ tenglamalar

sistemasini matritsalar yordamida.

$$\begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{pmatrix},$$

ya'ni $A \cdot X = B$

ko'rinishda yozish mumkin. Demak, $X = A^{-1} \cdot B$.

7.1. $A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ bo'lsa, $A^2 + 2B - 5C$ ni hisoblang.

7.2. $A = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ bo'lsa, $A^4 - B^4$ ni hisoblang.

7.3. $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^n$ ni hisoblang.

7.4. $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix}$ bo'lsa, $A \cdot B - B \cdot A$ ni hisoblang.

7.5. Berilgan matritsalarga teskari matritsalarni toping.

1) $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$, 2) $A = \begin{pmatrix} a & в \\ c & d \end{pmatrix}$,

3) $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ 4) $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 3 \end{pmatrix}$.

5) $A=\begin{pmatrix}1 & 3 & -5 & 7\\ 0 & 1 & 2 & -3\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 1\end{pmatrix}$, 6) $A=\begin{pmatrix}0 & 1 & . & 1\\ 1 & 0 & 1 & 1\\ . & . & . & .\\ 1 & 1 & 1 & 0\end{pmatrix}$

7.6. Noma'lum matritsani toping

1) $\begin{pmatrix}2 & 5\\ 1 & 3\end{pmatrix}\cdot X=\begin{pmatrix}4 & -6\\ 2 & 1\end{pmatrix}$

2) $\begin{pmatrix}1 & 2\\ 3 & 3\end{pmatrix}\cdot X=\begin{pmatrix}4 & -6\\ 2 & 1\end{pmatrix}$

3) $\begin{pmatrix}3 & -1\\ 5 & -2\end{pmatrix}\cdot X\cdot\begin{pmatrix}5 & 6\\ 7 & 8\end{pmatrix}=\begin{pmatrix}14 & 16\\ 9 & 10\end{pmatrix}$,

4) $\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -4\\ 2 & -1 & 0\end{pmatrix}\cdot X=\begin{pmatrix}1 & -3 & 0\\ 10 & 2 & 7\\ 10 & 7 & 8\end{pmatrix}$.

7.7. Quyidagi sistemalarni matritsaviy usulda eching.

1) $\begin{cases}3x+4y = 11\\ 5y+6z=28,\\ x+2z=7\end{cases}$ 2) $\begin{cases}x+3y+5z+7z=12\\ 3x+5y+7z+t=0\\ 5x+7y+z+3t=4\\ 7x+y+3z+5t=16\end{cases}$

$$3)\begin{cases} x+2z=0 \\ y+2n=0 \\ x+y+v=0 \\ z-n=2 \\ n+v=-1 \end{cases}$$

§ 8.Kompleks sonlar, formalari. Muavr formulalari.

Haqiqiy x, y sonlar yordamida tuzilgan $z=x+iy$ son kompleks son $i=\sqrt{-1}$ esa mavhum birlik deyiladi. Bunda x kompleks sonning haqiqiy qismi, y esa mavhum qismi deyiladi.

$z=x+iy$ yozuv kompleks sonning algebraik formasi deyiladi, amallar bu formada quyidagicha kiritiladi:

$$z_1 \pm z_2 = (x_1+iy_1)\pm(x_2+iy_2)=(x_1 \pm x_2)+(y_1 \pm y_2)i$$

$$z_1 \cdot z_2 = (x_1+iy_1)\cdot(x_2+iy_2)=(x_1x_2-y_1y_2)+i(x_1y_2+y_1x_2)$$

$$\frac{z_1}{z_2}=\frac{x_1+iy_1}{x_2+iy_2}=\frac{x_1x_2+y_1y_2}{x^2{}_2+iy^2{}_2}+i\frac{x_2y_1-x_1y_2}{x^2{}_2+y^2{}_2}.$$

$z=x+iy$ uchun $x-iy$ son qo'shma kompleks son deyiladi va $\bar{z}$ tarzida belgilanadi. $z=x+iy$ songa tekislikdagi $A(x,y)$ nuqtani mos qo'yish mumkin, $|OA|=r=\sqrt{x^2+y^2}$ kompleks son moduli,

OX o'qi bilan hosil qilgan φ burchagi kompleks son argumenti deyiladi va $tgy = \frac{y}{x}$ ko'rinishda yoziladi.

$x = r\cos\varphi, \quad y = r\sin\varphi$ ekanligidan

$z = x + yi = r(\cos\varphi + i\sin\varphi)$ kelib chiqadi. Oxirgi yozuv kompleks son trigonometrik formasi deyiladi, amallar quyidagicha kiritiladi:

$$Z_1 \cdot Z_2 = r_1(\cos\varphi_1 + \sin\varphi_1) \cdot r_2(\cos\varphi_1 + i\sin\varphi_2) =$$

$$= r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)],$$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)],$$

$$Z^n = [r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi),$$

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}), \quad k = \overline{0,1,..(n-1)}$$

Oxirgi ikkita formula Muavr formulalari deyiladi.

$e^{i\varphi} = \cos\varphi + i\sin\varphi$ tenglik o'rinli bo'lib Eyler formulasi deyiladi.

8.1. Amallarni bajaring.

1) $(2+3i)(3-2i)$, 2) $(a+bi)(a-bi)$,

3) $(3-2i)^2$ 4) $(1+i)^3$,

5) $\frac{1+i}{1-i}$, 6) $\frac{2i}{1+i}$

8.2. Tenglamalarni eching.

1) $x^2 + 4 = 0$, 2) $x^2 - 2x + 5 = 0$,

3) $x^2 + 4x + 13 = 0$ 4) $x^8 - 1 = 0$,

5) $x^6 + 1 = 0$, 6) $x^4 + 1 = 0$

8.3. Berilgan kompleks sonni trigonometrik formada tasvirlang.

1) $z = 3$, 2) $z = 2i$,

3) $z=2-2i$, 4) $z=\sqrt{3}+i$

5) $z=-\sqrt{3}-i$, 6) $z=\sqrt{2}-\sqrt{2}i$,

7) $z=\sin\alpha+i(1-\cos\alpha)$

8.4. Muavr formulasidan foydalanmay, hisoblang:

1) $\frac{(1-i)^5-1}{(1+i)^5+1}$, 2) $\frac{(1+i)^5}{(1-i)^7}$, 3) $\sqrt{2i}$, 4) $\sqrt{1-i\sqrt{3}}$.

8.5. Muavr formulalari bo'yicha hisoblang.

1) $(1+i)^{25}$, 2) $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$, 3) $\sqrt[3]{i}$,

4) $\sqrt[3]{2-2i}$, 5) $\sqrt[6]{-27}$, 6) $\sqrt[6]{1}$

7) $\sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}$

§9. Юqori darajali tenglamalar. Algebraning asosiy teoremasi.

$a_nx^n+a_{n-1}x^{n-1}+....+a_1x+a_0=0$ tenglama n−darajali tenglama deyiladi.

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+....+a_1x+a_0$$

ko'phadni $(x-a)$ga bo'lganda hosil bo'ladigan qoldiq berilgan ko'phadning $x=a$ dagi qiymati $f(a)$ ga teng bo'ladi.

Teorema (Bezu): a son $f(x)$ ko'phadning ildizi bo'lishi uchun $f(x)$ ning $(x-a)$ ga bo'linishi zarur va etarli.

Teorema (algebraning asosiy teoremasi): Darajasi birdan kichik bo'lmagan ixtiyoriy ko'phad kamida bitta ildizga ega.

Natija: ixtiyoriy n−darajali $(n\geq 1)$ ko'phad n ta ildizga ega va

$$(x-\alpha_1)^{l_1}\cdot(x-\alpha_2)^{l_2}....(x-\alpha_m)^{l_m}\cdot(x^2+p_1x+q_1)^{r_1}\cdot$$
$$(x^2+p_2x+q_2)^{r_2}...(x^2+p_kx+q_k)^{r_k}$$

ko'rinishda yoziladi, bunda

$$l_1+l_2+....+l_m+2(r_1+r_2+...+r_k)=k$$

$x^3+a_1x^2+a_2x+a_3=0$ tenglama $x=z-\dfrac{a_1}{3}$ almashtirishda

$z^3+pz+q=0$ ko'rinish oladi, ildizlari esa Kardano formulasidan topiladi:

$$z=u+v=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{3}}}$$

$$\Delta=\frac{q^2}{4}+\frac{p^3}{27}$$

diskriminant deyiladi. $u_1=\text{Re}\,u, v_1=\text{Re}\,v.$

1) $\Delta>0$ bo'lsa,

$$z_1=u_1+v_1, z_{213}=-\frac{u_1+v_1}{2}\pm\frac{u_1-v_1}{2}\cdot i\sqrt{3}.$$

2) $\Delta=0$ bo'lsa, $z_1=\dfrac{3q}{p}, z_2=z_3=-\dfrac{z_1}{2}.$

3) $\Delta<0$ bo'lsa, $z_1=2\sqrt{-\dfrac{p}{3}}\cos\dfrac{\varphi}{3},$

$z_{2,3}=2\sqrt{\dfrac{-p}{3}}\cos(\dfrac{\varphi}{3}\pm120^0),$ bunda

$$\cos\varphi=-\frac{q}{2}\div\sqrt{\frac{-p^3}{27}}.$$

Agar $f(a)$ va $f(b)$ turli ishorali bo'lsa (a,b) intervalda $f(x)=0$

Tenglamaning kamida bitta ildizi bor.

9.1.Biror ildizni tanlab tenglamani eching:

a) $x^3-4x^2+x+6=0$

b) $x^3-4x^2-4x-5=0$

v) $x^4+x^3+2x-4=0$

g) $4x^3-4x^2+x-1=0$

9.2.Kardano formulasi bo'yicha eching:

a) $z^3-6z-9=0$

b) $z^3-12z-8=0$

v) $z^3-6z-7=0$

g) $x^3+9x^2+18x+9=0$

9.3. Ildizlari berilgan sonlar bo'lgan tenglamani tuzing:

a) 2; 1; -2; 3

b) -1 uch karrali, I va $2+i$

9.4. Ko'paytuvchilarga ajrating:

a) x^8-1

b) x^5+1

v) x^5-1

g) x^4+4

Yuqori tartibli tenglamalar ratsional ildizlari.

1. Agar qisqarmas $\frac{p}{q}$ $-$ratsioanl son $(p\in Z, q\in N)$

$$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n=0$$

tenglama ildizi bo'lsa, $q\in N$ soni a_0 ning, $p\in Z$ soni esa a_n ning bo'luvchilari bo'ladi. Chunki

$$a_0p^n + a_1p^{n-1}q + a_2p^{n-2}q^2 + ... + a_{n-1}pq^{n-1} + a_nq^n = 0$$

dan

$$\begin{cases} a_0p^n = -q\left(a_1p^{n-1} + a_2p^{n-2}q + ... + a_{n-1}pq^{n-2} + a_nq^{n-1}\right) \\ a_nq^n = -p\left(a_0p^{n-1} + a_1p^{n-2}q + a_2p^{n-3}q^2 + ... + a_{n-1}q^{n-1}\right) \end{cases}$$

tengliklar kelib chiqib, yuqoridagilarni tasdiqlaydi.

Misollar: 1) $x^4 + 2x^2 - 13x^2 - 38x - 24 = 0$ tenglamada $a_0 = 1, a_n = -24$. Demak, $q = 1$: $p = \pm1; \pm2; \pm3; \pm4; \ \pm6; \pm8; \pm12; \pm24$ bo'lishi mumkin. O'rniga qo'yib tekshirishlar $x_1 = -1; x_2 = -2; x_3 = -3; x_4 = 4$ ekanligini, ya'ni tenglama $(x+1)(x+2)(x+3)(x-4) = 0$ Ko'rinishda yozilishi mumkinligini ko'rsatadi.

2) $24x^5 + 10x^4 - x^3 - 19x^2 - 5x + 6 = 0$ tenglama uchun $p = \pm1; \pm2; \pm3; \pm6$, $q = 1;2;3;4;6;8;12;24$ bo'lishi mumkin.

$\left\{\frac{p}{q}\right\} = \left\{\frac{1}{2}; -\frac{2}{3}; \frac{3}{4}\right\}$ ekanligini topish mumkin, xolos.

Qolgan ikki ildiz irratsional yoki kompleks ekanligi kelib chiqdi. Berilgan tenglama chap tomonidagi ko'phadni $\left(x - \frac{1}{2}\right)\left(x + \frac{2}{3}\right)\left(x - \frac{3}{4}\right)$ ga qisqartirib (bo'lib), bo'linma ko'phadni nolga tenglab qolgan ikki ildizi topiladi.

1. Yuqori darajali tenglamalar uchun Viet teoremasi.

Keltirilgan $x^2 + px + q = 0$ tenglama uchun x_1, x_2 ildizlar bo'lsa, $x_1 + x_2 = -p$, $x_1 \cdot x_2 = q$ (Viet teoremasi) kelib chiqishi elementar matematikadan ma'lum. $x^3 + a_0x^2 + a_1x + a_2 = 0$ tenglama ildizlari x_1, x_2, x_3

bo'lsa, tenglama $(x-x_1)(x-x_2)(x-x_3)=0$ ga ekvivalent bo'ladi va

$$x^3-(x_1+x_2+x_3)x^2+(x_1x_2+x_2x_3+x_1x_3)x-x_1x_2x_3=0$$

kubik tenglama uchun Viet tengliklari

$$\begin{cases} x_1+x_2+x_3=-a_0 \\ x_1x_2+x_1x_3+x_2x_3=a_1 \\ x_1x_2x_3=-a_2 \end{cases}$$

ko'rinishda bo'lishi kelib chiqadi.

$$x^4+a_0x^3+a_1x^2+a_2x+a_3=0$$

Tenglama uchun Viet tengliklari

$$\begin{cases} a_0=-(x_1+x_2+x_3+x_4) \\ a_1=x_3x_4+x_1x_3+x_1x_4+x_2x_3+x_2x_4+x_1x_2 \\ a_2=-(x_1x_3x_4+x_2x_3x_4+x_1x_2x_3+x_1x_2x_4) \\ a_3=x_1x_2x_3x_4 \end{cases}$$

ko'rinishda bo'ladi.

Misollar:

1. Agar $x^3+px+q=0$ tenglama ildizlari x_1,x_2,x_3 bo'lsa, $\frac{x_1}{x_2}+\frac{x_2}{x_3}+\frac{x_3}{x_1}+\frac{x_2}{x_1}+\frac{x_3}{x_2}+\frac{x_1}{x_3}$ ni hisoblang.

$x_1+x_2+x_3=0$ ekanligi ma'lum, undan $x_1+x_2=-x_3$ deyish mumkin. U holda

$$\frac{x_1}{x_2}+\frac{x_2}{x_3}+\frac{x_3}{x_1}+\frac{x_2}{x_1}+\frac{x_3}{x_2}+\frac{x_1}{x_3}=$$

$$=\frac{1}{x_1x_2x_3}\cdot\left[x_1^2x_3+x_2^2x_1+x_2x_3^2+x_2^2x_3+x_1x_3^2+x_1^2x_2\right]=$$

$$=\frac{1}{x_1x_2x_3}\cdot\left[x_1^2(x_2+x_3)+x_2x_3(x_3+x_2)+x_1(x_2^2+x_3^2)\right]=$$

$$= \frac{1}{x_1x_2x_3} \cdot \left[-x_1^3 - x_1x_2x_3 + x_1(x_2^2 + x_3^2)\right] = \frac{1}{x_2x_3}\left[-x_1^2 - x_2x_3 + x_2^2 + x_3^2\right] =$$

$$= \frac{1}{x_2}\left[x_1 - x_2 - x_2 + x_3\right] = \frac{1}{x_2}\left[x_1 + x_2x_3 - 3x_2\right] = \frac{1}{x_2}\left(-3x_2\right) = -3$$

.

2. $x^3 + a_0x^2 + a_1x + a_2 = 0$ tenglama ildizlari x_1, x_2, x_3 bo'lsa, ildizlari $x_1 + x_2,\ x_2 + x_3,\ x_3 + 1$ bo'lgan tenglama tuzing.

3. $x^3 + 2x - 3 = 0$ uchun $x_1 + x_2 + x_3,\ x_1 \cdot x_2 \cdot x_3$ va $x_1^2 + x_2^2 + x_3^2$ larni toping.

Bob bo'yicha misollar echish namunalari

1. $\begin{vmatrix} a+в & a-в \\ a-в & a+в \end{vmatrix}$ va $\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$ larni hisoblang

a) $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix} = (a+b)(a+b) - (a-b)(a-b) =$

$$= (a+b)^2 - (a-b)^2 = (a+b+a-b)(a+b-a+b) =$$

$$= 2a2b = 4ab$$

b)

$$\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha$$

2. $\Delta = \begin{vmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{vmatrix}$ determinantni hisoblang.

Teoremaga ko'ra $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} =$

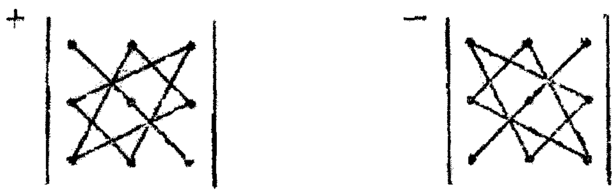
$= a_{11}(-1)^{1+1}M_{11} + a_{12}(-1)^{1+2}M_{12} + a_{13}(-1)^{1+3}M_{13} =$

$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21}a_{23} \\ a_{31}a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21}a_{22} \\ a_{31}a_{32} \end{vmatrix} =$

$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$

$-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$

Oxirgi oltita qo'shiluvchilar uchburchak qoidasi deb ataluvchi quyidagi chiziqlardan topiladi:



3. Determinant xossalaridan foydalanib hisoblang.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 4 & 9 & 16 & 25 \\ 8 & 27 & 64 & 125 \end{vmatrix}$$

i−yo'lni 2 ga ko'paytirib $(i+1)$ −yo'ldan ayiramiz:

$$\Delta=\begin{vmatrix}1&1&1&1\\2&3&4&5\\4&9&16&25\\8&27&64&125\end{vmatrix}=\begin{vmatrix}1&1&1&1\\0&1&2&3\\0&3&8&15\\0&9&32&75\end{vmatrix}=$$

$$\begin{vmatrix}1&2&3\\3&8&15\\9&32&75\end{vmatrix}$$

i- yo'lni 3 ga ko'paytirib $(i+1)$ – yo'ldan ayiramiz:

$$\begin{vmatrix}1&2&3\\0&2&6\\0&8&30\end{vmatrix}=\begin{vmatrix}2&6\\8&30\end{vmatrix}=60-48=12$$

4. $$\begin{cases}x+2y+3Z=14\\y+2z+3t=20\\3x+Z+2t=14\\2x+3y+t=12\end{cases}$$

tenglamalar sistemasini Kramer qoidasi va Gaussning noma'lumlarni ketma – ket yo'qotish usuli yordamida eching.

a) Asosiy determinant va yordamchi determinantlar quyidagicha ko'rinishda bo'ladi:

$$\Delta=\begin{vmatrix}1&2&3&0\\0&1&2&3\\3&0&1&2\\2&3&0&1\end{vmatrix},\quad \Delta_x=\begin{vmatrix}14&2&3&0\\20&1&2&3\\14&0&1&2\\12&3&0&1\end{vmatrix},\quad \Delta_y=\begin{vmatrix}1&14&3&0\\0&20&2&3\\3&14&1&2\\2&12&0&1\end{vmatrix},$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 14 & 0 \\ 0 & 1 & 20 & 3 \\ 3 & 0 & 14 & 2 \\ 2 & 3 & 12 & 1 \end{vmatrix}, \qquad \Delta_t = \begin{vmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 2 & 20 \\ 3 & 0 & 1 & 14 \\ 2 & 3 & 0 & 12 \end{vmatrix};$$

Determinant xossalaridan foydalanib $\Delta=96$, $\Delta x=96$

$\Delta_y = 192, \Delta_z = 288, \Delta_t = 384$ ekanligini topamiz,

Demak, $x = 1; y = 2; t = 4$

b) tenglamalar sistemasi koeffitsientlaridan tuzilgan quyidagi jadvalni tuzamiz:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 3 & 0 & 1 & 2 & 14 \\ 2 & 3 & 0 & 1 & 12 \end{array}\right)$$

Tenglamalar sistemasidagi biror tenglamani songa ko'paytirib, boshqa tenglamaga qo'shsak, ekvivalent tenglama hosil bo'ladi, Demak, 1–tenglamani mos ravishda (–3) va (–2) ga ko'paytirib, 3– va 4– tenglamalarga qo'shamiz.

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 0 & -6 & -8 & 2 & -28 \\ 0 & -1 & -6 & 1 & -16 \end{array}\right)$$

Endi 3–va 4–tenglamalarni mos ravishda 6 va 1 ga ko'paytirilgan 2–tenglamaga qo'shamiz:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 0 & 0 & 4 & 20 & 92 \\ 0 & 0 & -4 & 4 & 4 \end{array}\right)$$

Oxirgi tenglamalarni qo'shib quyidagi natijaga kelamiz:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 0 & 0 & 4 & 20 & 92 \\ 0 & 0 & 0 & 24 & 96 \end{array}\right)$$

Oxirgi jadvalga mos sistema quyidagi ko'rinishda bo'ladi:

$$\left.\begin{array}{r} x+2y+3z=14 \\ y+2z+3t=20 \\ 4z+20t=92 \\ 24t=96 \end{array}\right\}$$

Bundan $t=4, z=3, y=2, x=1$ ekanligini ketma–ket topishimiz qiyin emas.

Qarab chiqilgan usul Gaussning noma'lumlarni ketma–ket yo'qotish usuli bo'lib, talabalarga o'rta maktab kursidan tanish.

5. $A=\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ bo'lsa, A^2-2B ni hisoblang.

$A^2=\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}\cdot\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}=\begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix}$ va

$2\cdot B=\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$ ekanligidan,

$A^2-2B=\begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix}-\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}=\begin{pmatrix} 1 & 1 \\ -11 & 0 \end{pmatrix}$ kelib chiqadi.

6. $A=\begin{pmatrix}2 & 3 & 2\\1 & 2 & -3\\3 & 4 & 1\end{pmatrix}$ ga teskari A^{-1} matritsani toping.

$$|A|=\begin{vmatrix}2 & 3 & 2\\1 & 2 & -3\\3 & 4 & 1\end{vmatrix}=-6.$$

$$A_{11}=\begin{vmatrix}2 & -3\\4 & 1\end{vmatrix}=14,\qquad A_{21}=-\begin{vmatrix}3 & 2\\4 & 1\end{vmatrix}=5,$$

$$A_{31}=\begin{vmatrix}3 & 2\\2 & -3\end{vmatrix}=13,\qquad A_{12}=-\begin{vmatrix}1 & -3\\3 & 1\end{vmatrix}=-10,$$

$$A_{22}=\begin{vmatrix}2 & 2\\3 & 1\end{vmatrix}=-4,\qquad A_{32}=-\begin{vmatrix}2 & 2\\1 & -3\end{vmatrix}=8,$$

$$A_{13}=\begin{vmatrix}1 & 2\\3 & 4\end{vmatrix}=-2,\qquad A_{23}=-\begin{vmatrix}2 & 3\\3 & 4\end{vmatrix}=1,$$

$$A_{33}=\begin{vmatrix}2 & 3\\1 & 2\end{vmatrix}=1$$

Demak:

$$A^{-1}=\frac{-1}{6}\cdot\begin{pmatrix}14 & 5 & -13\\-10 & -4 & 8\\-2 & 1 & 1\end{pmatrix}$$

7. $\begin{cases}3x+2y+2z=13\\x+3y+z=10\\5x+3y+4z=23\end{cases}$

tenglamalar sistemasini matritsaviy usulda eching.

Sistemani $A \cdot X = B$ ko'rinishda yozib olamiz, bunda

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}, \qquad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad B = \begin{pmatrix} 13 \\ 10 \\ 23 \end{pmatrix}.$$

A matritsa determinanti va elementlar to'ldiruvchilarini hisoblaymiz:

$$|A| = \begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} = 27 + 2 - 24 = 5.$$

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 3 & 4 \end{vmatrix} = 9, \qquad A_{21} = -\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2,$$

$$A_{31} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -4,$$

$$A_{12} = -\begin{vmatrix} 1 & 1 \\ 5 & 4 \end{vmatrix} = 1, \qquad A_{22} = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 2,$$

$$A_{32} = -\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -1, \qquad A_{13} = \begin{vmatrix} 1 & 3 \\ 5 & 3 \end{vmatrix} = -12,$$

$$A_{23} = -\begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} = 1, \qquad A_{33} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7.$$

Demak, $$A^{-1} = \frac{1}{5} \cdot \begin{pmatrix} 9 & -2 & -4 \\ 1 & 2 & -1 \\ -12 & 1 & 7 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \frac{1}{5}\begin{pmatrix} 9 & -2 & -4 \\ 1 & 2 & -1 \\ -12 & 1 & 7 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 10 \\ 23 \end{pmatrix} =$$

$$= \frac{1}{5} \cdot \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Ya'ni $x = 1,\ y = 2,\ z = 3$.

8..Amallarni bajaring:

a) $(3 - i) + (4 + 2i) = (4 + 3) + (2 - 1)i = 7 + i$

b) $(3 - i) \cdot (4 + 2i) = (12 + 2) + i(6 - 4) = 14 + 2i$

v) $\frac{3 - i}{2(2 + i)} = \frac{3 - i}{2(2 + i)} \cdot \frac{2 - i}{(2 - i)} = \frac{7 - 5i}{2 \cdot 5} = 0{,}7 - 0{,}5i$

r) $i^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = -i.$

9. $\sqrt{3 - 4i}$ ni hisoblang.

$\sqrt{3 - 4i} = \alpha + \beta i$ chiqadi deb faraz qilamiz, demak,

$(\alpha + \beta i)^2 = \alpha^2 - \beta^2 + 2\alpha\beta i = 3 - 4i.$

Bundan: $\begin{cases} \alpha^2 - \beta^2 = 3 \\ 2\alpha\beta = -4 \end{cases}$ sistema hosil bo'lib,

$(2; -1)$, $(-2; 1)$ echimlar ekanligini topish mumkin.

Demak: $\sqrt{3 - 4i} = \begin{cases} 2 - 2i \\ -2 + 2i \end{cases}$

10. $(1 + i)^{10}$ ni hisoblang.

$r = \sqrt{2},\qquad tg\varphi = \frac{y}{x} = \frac{1}{1} = 1,\qquad$ ya'ni $\qquad \varphi = \frac{\pi}{4}$

ekanligidan foydalanib:

$$(1+i)^{10}=\left(\sqrt{2}\right)^{10}\cdot\left(\cos 10\cdot\frac{\pi}{4}+i\sin 10\cdot\frac{\pi}{4}\right)=$$

$$=2^5\left(\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}\right)=$$

$$=32\left[\cos\left(2\pi+\frac{\pi}{2}\right)+i\sin\left(2\pi+\frac{\pi}{2}\right)\right]=$$

$$=32(0+i)=32i$$

11. $\sqrt[3]{-2+2i}$ ni hisoblang.

$r=\sqrt{4+4}=2\sqrt{2}$; $\quad tg\varphi=\frac{2}{-2}=-1, \quad \varphi=\frac{3\pi}{4}$

bo'lgani uchun

$$\sqrt[3]{-2+2i}=\sqrt[3]{2\sqrt{2}}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{3}+i\sin\frac{\frac{3\pi}{4}+2k\pi}{3}\right),$$

$$z_0=\sqrt[3]{\sqrt{2^3}}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=\sqrt[6]{2^3}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=1+i;$$

$$z_1=\sqrt{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right)=\sqrt{2}\left(-\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$$

$$z_2=\sqrt{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)=\sqrt{2}\left(+\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$$

12. Biror $x=\alpha_1$ ildizini tanlab topib, so'ngra qolgan ildizlarini toping.

$a) x^5 - x^4 - 16x + 16 = 0$ tenglama ildizi, $x_0 = 1$

shuning uchun $(x-1)$ ni ajratamiz:

$x^4(x-1) - 16(x-1) = 0$ yoki

$(x-1)(x^2+4)(x-2)(x+2) = 0.$

bundan ildizlar $1, \pm 2; \pm 2i$ ekanligi kelib chiqadi.

b) $x^4 + x^3 + 2x - 28 = 0; \quad x_1 = 2$

$$x^4 - 2x^3 + 3x^3 - 6x^2 + 6x^2 - 12x + 14x - 28 = 0$$

$$x^3(x-2) + 3x^2(x-2) + 6x(x-2) + 14(x-2) = 0$$

$$(x-2)(x^3 + 3x^2 + 6x + 14) = 0$$

Ikkinchi qavsda $x = z - 1$ almashtirish bajaramiz va

$z^3 + 3z + 10 = 0$ ko'rinishga kelamiz;

Demak, $\Delta > 0$ ekanligidan, Kardano formulasiga ko'ra:

$$x_2 = \sqrt[3]{-5 + \sqrt{25+1}} + \sqrt[3]{-5 - \sqrt{26}}$$

va x_3, x_4 ni ham topish mumkin.

13. Ko'phad ildizlari berilgan: ikki karrali 1, 2, 3 va $1+i$.

Bu ko'phadni yozing:

$(1-i)$ ham ko'phad ildizi bo'ladi, chunki $1 \pm i$

$x^2 - 2x + 2 = 0$ ning ildizlardir. Demak, izlangan ko'phad quyidagicha bo'ladi;

$$(x-1)^2(x-2)(x-3)(x^2 - 2x + 2)$$

II–Bob bo'yicha uy vazifalari.

1. $\begin{cases} ax + by + cz = d \\ bx - cy + az = e \\ cx + ay - bz = f \end{cases}$ tenglamalar sistemasi berilgan. Uch usulda eching: a) Kramer qoidasi, b) Gauss usuli, v)Matritsaviy.

1) $a = 1; b = 2; c = 3; d = 14; e = -1; f = -1$.
2) $a = 2; b = 1; c = 4; d = 5; e = -2; f = 10$.
3) $a = 3; b = 2; c = -2; d = 1; e = 7; f = -2$
4) $a = 2; b = -1; c = 1; d = -1; e = -5; f = -4$
5) $a = 1; b = 3; c = 0; d = 17; e = 18; f = -5$
6) $a = 1;\ b = 0;\ c = 2;\ d = 8;\ e = -3;\ f = 5$
7) $a = 2;\ b = -1;\ c = 3;\ d = 15;\ e = 9; f = 8$
8) $a = 3; b = -2; c = -1; d = 14; t = 2; f = -11$
9) $a = 2; b = -2;\ c = 3; d = 1; e = -3; f = 9$
10) $a = 4; b = 1; c = 1; d = 4; e = -6; f = 4$
11) $a = 5; b = 1; c = -1; d = 12; e = -2; f = 4$
12) $a = 5; b = -1; c = 12; d = 12; e = 26; f = 10$
13) $a = 2;\ b = -1;\ c = 1;\ d = 5;\ e = -17; f = 24$
14) $a = 4; b = 1;\ c = 2;\ d = 5; e = -7;\ f = 24$
15) $a = 8;\ b = 2;\ c = 3;\ d = -1;\ e = -22;\ f = 9$.

2). $z = \dfrac{a}{b + ci}$ berilgan.

a)Algebraik formada yozing va z^2 ni hisoblang.

b)Trigonometrik formada yozing va z^{20}, $\sqrt[3]{z}$ larni hisoblang.

1) $a = 1; b = \sqrt{2};\ c = -\sqrt{2}$.
2) $a = 2;\ b = \sqrt{2}; c = \sqrt{2}$.
3) $a = 3;\ b = 1;\ c = \sqrt{3}$.

4) $a=4;\ b=1; c=-\sqrt{3}$.

5) $a=5;\ b=\sqrt{3}; c=1$.

6) $a=-1; b=-\sqrt{3};\ c=1$.

7) $a=-2;\ b=-1;\ c=1$.

8) $a=-3;\ b=3; c=\sqrt{3}$.

9) $a=-4;\ b=1;\ c=-1$.

10) $a=-5; b=-3;\ c=\sqrt{3}$.

11) $a=1;\ b=\sqrt{3};\ c=-3$.

12) $a=2;\ b=1;\ c=-\sqrt{3}$.

13) $a=3;\ b=1;\ c=\sqrt{3}$.

14) $a=4;\ b=-\sqrt{3};\ c=1$.

15) $a=5;\ b=2;\ c=2\sqrt{3}$.

3. $x^3+ax^2+bx+c=0$ **tenglamani eching.**

1) $a=2;\ b=-3;\ c=0$.

2) $a=-4; b=2; c=1$.

3) $a=3; b=-5;\ c=2$.

4) $a=1;\ b=-2;\ c=-5$.

5) $a=2;\ b=3;\ c=-1$.

6) $a=4;\ b=1;\ c=2$.

7) $a=3;\ b=2;\ c=3$.

8) $a=2;\ b=4;\ c=1$.

9) $a=-2;\ b=2;\ c=-4$.

10) $a=10;\ b=-10;\ c=1$.

11) $a=-3;\ b=2;\ c=-6$.

12) $a=1;\ b=-1;\ c=2$.

13) $a=-4;\ b=3;\ c=2$.

14) $a=4; b=-7; c=2$.

15) $a=3; b=2; c=6$.

3–bob. FAZODA ANALITIK GEOMETRIYA.
§10. Vektorlar nazariyasi va tatbiqlari.

Fazoda M nuqta berilib, to'g'ri burchakli dekart koordinatalar sistemasi $OXYZ$ aniqlansa, nuqta uchta koordinata x (abtsissa) y (ordinata), z (applikata) ga ega bo'ladi va $M(x,y,z)$ tarzida yoziladi.

$O(0;0;0)$ dan $M(x,y,z)$ gacha masofa $|OM|=\sqrt{x^2+y^2+z^2}$, $A(x_1,y_1,z_1)$ va $B(x_2;y_2;z_2)$ nuqtalar orasidagi masofa esa

$$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

formula yordamida topiladi.

AB kesmani $|AB|:|CB|=\lambda$ nisbatda bo'luvchi $C(x,y,z)$ nuqta koordinatalari tekislikdagiga o'xshash topiladi, ya'ni

$$x=\frac{x_1+\lambda x_2}{1+\lambda}; y=\frac{y_1+\lambda y_2}{1+\lambda}; z=\frac{z_1+\lambda z_2}{1+\lambda}.$$

Yo'naltirilgan kesma vektor bo'lib, $OXYZ$ koordinatalari fazosida o'qlardagi birlik $\vec{i},\vec{j},\vec{k}$ vektorlar – ortlar orqali quyidagicha yoyiladi:

$$\vec{a}=x\vec{i}+y\vec{j}+z\vec{k}.$$

Bu holda $\vec{a}$ vektor x, y, z – koordinatalarga ega bo'ladi va

$\vec{a}(x, y, z)$ tarzida yoziladi. Bu vektor uzunligi

$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$ formuladan topiladi.

$\vec{a}$ vektorning ox, oy, oz o'qlari bilan hosil qilgan burchaklari mos ravishda α, β, γ bo'lsa, bu burchaklar kosinuslari

$$\cos\alpha = \frac{x}{|\vec{a}|}, \quad \cos\beta = \frac{y}{|\vec{a}|}, \quad \cos\gamma = \frac{z}{|\vec{a}|}$$

yo'naltiruvchi kosinuslar deyiladi. Ular o'zaro quyidagicha bog'langan: $cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

koordinatalari bilan berilgan $\vec{b}(x_1, y_1, z_1), \vec{c}(x_2, y_2, z_2)$ vektorlar ustida amallar quyidagicha aniqlanadi:

$\vec{b} \pm \vec{c} = (x_1 \pm x_2; y_1 \pm y_2; z_1 \pm z_2)$

$\lambda\vec{b} = (\lambda x_1; \lambda y_1; \lambda z_1)$

Boshi $A(x_0, y_0, z_0)$ oxiri $B(x_1, y_1, z_1)$ nuqtada bo'lgan $\overrightarrow{AB}$ vektor koordinatalari

$\overrightarrow{AB}(x_1 - x_0, y_1 - y_0, z_1 - z_0)$ tarzida aniqlanadi.

1.Skalyar ko'paytma xossalari.

Ikki $\vec{a}$ va $\vec{в}$ vektorlar skalyar ko'paytmasi deb ular uzunliklari va ular orasidagi burchak kosinusi ko'paytmasiga aytiladi, $\vec{a} \bullet \vec{в}$ yoki $(\vec{a}, \vec{в})$ tarzida belgilanadi.

$$\vec{a} \bullet \vec{в} = |\vec{a}| \cdot |\vec{в}| \cdot \cos\varphi$$

$cos\varphi = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ formuladan $\vec{a}$, $\vec{b}$ vektorlar

parallellik sharti $\dfrac{x_1}{x_2} = \dfrac{y_1}{y_2} = \dfrac{z_1}{z_2}$,

perpendikulyarlik sharti $x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2 = 0$.
quyidagi xossalar o'rinli:

1) $\vec{a} \cdot \vec{a} = |a|^2$

2) $\vec{a} \cdot \vec{в} = 0 \Leftrightarrow \vec{a} \perp \vec{в}$ yoki $\vec{a}$, $\vec{в}$ lardan kamida bittasi nol vektor.

3) $\vec{a} \cdot \vec{в} = \vec{в} \cdot \vec{a}$ 4) $\vec{a}(\vec{в} + \vec{c}) = \vec{a}\vec{в} + \vec{a}\vec{c}$

5) $\vec{i}^2 = \vec{j}^2 = \vec{k}^2 = 1$, $\vec{i}\vec{j} = \vec{i}\vec{k} = \vec{j}\vec{k} = 0$

Koordinatalari bilan berilgan $\vec{a}(x_1, y_1, z_1)$, $\vec{b}(x_2, y_2, z_2)$ vektorlar skalyar ko'paytmasi quyidagicha topiladi:

$$\vec{a} \cdot \vec{в} = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2$$

Ikki vektor orasidagi burchak esa

$$\cos\varphi = \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_2^2}}$$

formuladan topiladi.

2. Vektor ko'paytma.

Ikki $\vec{a}$ va $\vec{в}$ vektorlar vektor ko'paytmasi deb shunday $\vec{c}$ vektorga aytiladiki:

1) $\vec{c}$ vektori $\vec{a}$ va $\vec{в}$ vektorlarga perpendikulyar;

2) $\vec{c}$ vektor uzunligi $\vec{a}$ va $\vec{в}$ vektorlarga qurilgan parallelogramm yuziga teng, ya'ni

$$|\vec{c}| = |\vec{a}| \cdot |\vec{в}| \cdot \sin\varphi ,$$

3) $\vec{c}$ vektor uchidan qaralganda, $\vec{a}$ dan $\vec{в}$ ga yo'nalish soat mili yo'nalishiga teskari bo'lishi kerak.

Vektor ko'paytma $\vec{a}x\vec{b}$ yoki $[\vec{a},\vec{в}]$ tarzida belgilanadi,

Quyidagi xossalar o'rinli:

1) $\vec{a}x\vec{b} = -\vec{b}x\vec{a}$

2) $\vec{a}=0$ yoki $\vec{в}=0$ dan tashqari $\vec{a} \parallel \vec{в}$ bo'lsa ham $\vec{a}x\vec{b} = 0$

3) $(k\vec{a})x\vec{b} = \vec{a}x(k\vec{b}) = k(\vec{a}x\vec{b})$

4) $\vec{a}x(\vec{b}+\vec{c}) = \vec{a}x\vec{b} + \vec{a}x\vec{c}$

5) $\vec{i}x\vec{i} = \vec{j}x\vec{j} = \vec{k}x\vec{k} = 0, \qquad \vec{i}x\vec{j} = -\vec{j}x\vec{i} = \vec{k},$

$\vec{j}x\vec{k} = -\vec{k}x\vec{j} = \vec{i}, \quad \vec{k}x\vec{i} = -\vec{i}x\vec{k} = \vec{j}$

Koordinatalari bilan berilgan $\vec{a}(x_1, y_1, z_1)$, $\vec{в}(x_2, y_2, z_2)$ vektorlar vektor ko'paytamasi quyidagicha topiladi:

$$\vec{a}x\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Demak, $\vec{a}$ va $\vec{в}$ vektorlarga qurilgan parallelogramm va uchburchak yuzi uchun quyidagi formulalar o'rinli:

$$S = |\vec{a}x\vec{b}|, S_{\Delta} = \frac{1}{2}|\vec{a}x\vec{b}|$$

3.Aralash ko'paytma.

Uchta $\vec{a}, \vec{b}, \vec{c}$ vektorlar aralash ko'paytmasi deb, $\vec{a} x \vec{b}$ vektor ko'paytmaning $\vec{c}$ vektor bilan skalyar ko'paytmasiga teng songa aytiladi va $\vec{a}\,\vec{b}\,\vec{c}$ ko'rinishda belgilanadi.

Bu ko'paytma moduli berilgan vektorlarga qurilgan parallelopiped hajmiga teng, ya'ni

$$V_{пар} = \left|\vec{a}\vec{b}\vec{c}\right|$$

Yasovchilari $\vec{a}, \vec{в}, \vec{c}$ bo'lgan piramida hajmi esa

$$V_{пир} = \frac{1}{6}\left|\vec{a}\vec{b}\vec{c}\right|$$

Aralash ko'paytma quyidagi xossalarga ega:

1)Ko'paytuvchilardan kamida bittasi nol vektor, kamida ikkitasi parallel, uchalasi bir tekislikda yotadigan hollarda aralash ko'paytma nolga teng.

2) $(\vec{a} x \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \cdot \vec{c})$

Koordinatalari bilan berilgan $\vec{a}(x_1, y_1, z_1)$, $\vec{b}(x_2, y_2, z_2)$, $\vec{c}(x_3, y_3, z_3)$ vektorlar aralash ko'paytmasi

$$\left(\vec{a} x \vec{b}\right)\vec{c} = \vec{a}\vec{b}\vec{c} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

formula yordamida topiladi.

10.1. Ox o'qida $A(2;-4;5)$ va $B(-3;2;7)$ nuqtalardan bir xil uzoqlikda joylashgan nuqtani toping.

10.2. Uchlari $A(2;3;4), B(3;1;2)$, $C(4;-1;3)$ nuqtalarda bo'lgan uchburchak og'irlik markazi koordinatalarini toping.

10.3. XOY tekislikda $A(1;-1;5)$, $B(3;4;4)$, $C(4;6;1)$ nuqtalardan bir xil uzoqlikda joylashgan nuqtani toping.

10.4. $\vec{a}(4;-12;z)$ vektor berilgan. $|\vec{a}|=13$ bo'lsa, Z ni toping.

10.5. $A(3;-1;2)$, $B(-1;2;1)$ bo'lsa, $\overrightarrow{AB}$ va $\overrightarrow{BA}$ vektorlar koordinatalarini toping.

10.6. Vektor OX va OZ o'qlari bilan $\alpha=120^0$, $\beta=45^0$ burchaklar hosil qilsa, OY o'qi bilan qanday burchak hosil qiladi?

10.7. $|\vec{a}|=13$, $|\vec{b}|=19$ va $|\vec{a}+\vec{b}|=24$ bo'lsa, $|\vec{a}-\vec{b}|$ ni hisoblang.

10.8. $|\vec{a}|=11$, $|\vec{b}|=23$ va $|\vec{a}-\vec{b}|=30$ bo'lsa, $|\vec{a}+\vec{b}|$ ni hisoblang.

10.9. $\vec{a}$ va $\vec{b}$ vektorlar $\varphi=120^0$ burchak hosil qiladi. $|\vec{a}|=3$, $|\vec{b}|=5$ bo'lsa, $|\vec{a}+\vec{b}|$, $|\vec{a}-\vec{b}|$ larni hisoblang.

10.10. $\vec{a}(-2;3;\beta)$, $\vec{b}(\alpha;-6;2)$ vektorlar α,β larning qanday qiymatlarida kollinear bo'ladi?

10.11. $\vec{a}(\alpha;2;\beta)$, $\vec{b}(\alpha;-2;3)$ vektorlar α ning qanday qiymatlarida perpendikulyar bo'ladi?

10.12. ABC uchburchak uchlari $A(2;-1;3)$, $B(1;1;1)$, $C(0;0;5)$ nuqtalarda bo'lsa, bu uchburchak ichki burchaklarini aniqlang.

10.13. Qavslarni oching: $(2\vec{i}-\vec{j})\cdot\vec{j}+(\vec{j}-2\vec{k})\cdot\vec{k}+(\vec{i}-2\vec{k})^2$

10.14. $(\vec{a}+\vec{b})^2+(\vec{a}-\vec{b})^2=2(a^2+b^2)$ tenglikni isbotlang, geometrik ma'nosini tushuntiring.

10.15. Oldingi masala yordamida medianalar xossasini isbotlang.

10.16. $|\vec{a}|=3,\ |\vec{b}|=26,\ |\vec{a}x\vec{b}|=72$ bo'lsa, $\vec{a}\cdot\vec{b}$ ni hisoblang.

10.17. Uchlari $A(1;2;0),\ B(3;0;-3),\ C(5;2;6)$ nuqtalarda bo'lgan uchburchak yuzini hisoblang.

10.18. $\vec{a}(2;-3;1),\ \vec{b}(-3;1;2),\ \vec{c}(1;2;3)$ bo'lsa, $(\vec{a}x\vec{b})x\vec{c}$ va $\vec{a}x(\vec{b}x\vec{c})$ larni hisoblang.

10.19. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak 30^0, $|\vec{a}|=6,\ |\vec{b}|=3$. Bu vektorlarga perpendikulyar $\vec{c}$ vektor berilgan va $|\vec{c}|=3$. $(\vec{a}x\vec{b})\cdot\vec{c}$ ni hisoblang.

10.20. $\vec{a}(1;-1;3),\ \vec{b}(-2;2;1),\ \vec{c}(3;-2;5)$ bo'lsa $(\vec{a}x\vec{b})\cdot\vec{c}$ ni hisoblang.

10.21. $A(1;2;-1),\ B(0;1;5),\ C(-1;2;1),\ D(2;1;3)$ nuqtalarning bitta tekislikda yotishini isbotlang.

10.22. Uchlari $A(2;-1;1), B(5;5;4)$, $C(3;2;-1)$, $D(4;1;3)$ nuqtalarda bo'lgan tetraedr hajmini hisoblang.

§11 Fazoda tekislik tenglamalari.

$\vec{N}\{A,B,C\}$ vektorga perpendikulyar, $F(x_0,y_0,z_0)$ nuqtadan o'tuvchi tekislik tenglamasi $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ ko'rinishda bo'lib,undan $Ax+By+Cz+D=0$ umumiy tenglamasini keltirib chiqarish mumkin.

I. $D=0$ bo'lsa, $Ax+By+Cz=0$ tekislik koordinatalar boshidan o'tadi.

II. $C=0$ bo'lsa, $Ax+By+D=0$ tenglama Oz o'qiga parallel bo'ladi.

III. $C=D=0$ bo'lsa $Ax+By=0$ tekislik Oz o'qidan o'tadi.

IV. $B=C=0$ bo'lsa $Ax+D=0$ tekislik YOZ tekisligiga parallel.

V. Koordinata tekisliklari tenglamalari: $x=0,\ y=0,\ z=0$

Tekislikning o'qlardagi kesmalar bo'yicha tenglamasi: $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ko'rinishda bo'ladi

$A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tekisliklar orasidagi burchak

$$\cos\varphi=\pm\frac{\vec{N}_1\vec{N}_2}{|\vec{N}_1||\vec{N}_2|}=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}*\sqrt{A_2^2+B_2^2+C_2^2}}$$

formuladan topiladi.

Bu tekisliklar parallel bo'lsa $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$, perpendikulyar bo'lsa $A_1A_2+B_1B_2+C_1C_2=0$ shartlar bajariladi. $M_0(x_0,y_0,z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha masofa

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

formuladan topiladi.

Agar koordinata boshidan tekislikka uzunligi P bo'lgan perpendikulyar o'tkazilib, u son o'qlari bilan α,β,γ burchaklar hosil qilsa,tekislik tenglamasi

$x\cos\alpha+y\cos\beta+z\cos\gamma-p=0$ ko'rinishda bo'ladi va normal tenglama deyiladi. Bu holda

$$d = |x_0 \cos\alpha + y_0 \cos\beta + z_0 \cos\gamma - p|$$

$A(x_0, y_0, z_0)$ $B(x_1, y_1, z_1)$ $C(x_2, y_2, z_2)$ nuqtalardan o'tuvchi tekislik tenglamasi

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} = 0$$ ko'rinishda bo'ladi.

11.1 $E(2;-1;1)$ nuqta koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosi bo'lsa, tekislik tenglamasini yozing.

11.2 $F(3;4;-5)$ nuqtadan o'tib $\vec{a}_1(3;1;-1), \vec{a}_2(1;-2;1)$ vektorlarga parallel tekislik tenglamasini tuzing

11.3 Koordinata boshidan o'tib, $5x - 3y + 2z - 3 = 0$ tekislikka parallel bo'lgan tekkislik tenglamasini toping

11.4 $x - 2y + 3z - 5 = 0$ tekislikka perpendikulyar va $E(1;-1;-2)$, $F(3;1;1)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing

11.5 $5x - 6y + 3z + 120 = 0$ tekislik va koordinata tekisliklari bilan chegaralangan piramida hajmini toping

11.6 $x - 2y - 2z - 12 = 0$ *ва* $x - 2y - 2z - 6 = 0$ tekisliklar orasidagi masofani toping

11.7 $x + 2z - 6 = 0$ *ва* $x + 2y - 4 = 0$ tekisliklar orasidagi burchakni toping.

§12.Fazoda to'g'ri chiziq.

$A(a, b, c)$ nuqtadan o'tib, $\vec{P}(m, n, p)$ vektorga parallel to'g'ri chiziq tenglamasi

$$\frac{x - a}{m} = \frac{y - b}{n} = \frac{z - c}{p}$$

ko'rinishda bo'lib, uni kanonik tenglama deyiladi.

Kanonik tenglamani t ga tenglab to'g'ri chiziqning parametrik tenglamalarini olish mumkin:

$$\begin{cases} x = mt + a \\ y = nt + b \\ z = pt + c \end{cases}$$

$A(x_0, y_0, z_0)$, $B(x_1, y_1, z_1)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$$

ko'rinishda bo'ladi.

Yo'naltirilgan vektorlari $\vec{P}(m_1, n_1, p_1)$, $\vec{q}(m_2, n_2, p_2)$ bo'lgan ikki to'g'ri chiziq orasidagi burchak

$$\cos\varphi = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| \cdot \vec{q}} = \frac{m_1 m_2 + n_1 n_2 + p_1 p_2}{\sqrt{m_1^2 + n_1^2 + p_1^2} \cdot \sqrt{m_2^2 + n_2^2 + p_2^2}}$$

formuladan topilib, parallellik sharti $\frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2}$, peripendikulyarlik sharti esa $m_1 m_2 + n_1 n_2 + p_1 p_2 = 0$ dir.

$A(a, b, c)$ va $A_1(a_1, b_1, c_1)$ nuqtalardan o'tuvchi to'g'ri chiziqlarning komplanarlik sharti

$$\begin{vmatrix} a - a_1 & b - b_1 & c - c_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix} = 0$$

bo'lib, $\overrightarrow{p_1}$ va $\overrightarrow{p_2}$ parallel bo'lmasa, bu to'g'ri chiziqlar kesishuvchiligi sharti ham bo'ladi.

Normal vektori $\overrightarrow{N}(A,B,C)$ bo'lgan tekislik va yo'naltiruvchi vektori $\overrightarrow{P}(m,n,p)$ bo'lgan to'g'ri chiziq orasidagi burchak

$$\sin\varphi = \frac{\overrightarrow{N}\cdot\overrightarrow{p}}{|\overrightarrow{N}|\cdot|\overrightarrow{p}|} = \frac{m\cdot A + n\cdot B + p\cdot C}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{m^2+n^2+p^2}}$$

formuladan topiladi.

Bunda parallellik sharti $mA+nB+pC=0$, perpendikulyarlik sharti esa

$$\frac{m}{A}=\frac{n}{B}=\frac{p}{C}$$ ko'rinish oladi.

12.1. $\begin{cases} x-2y+3z-4=0 \\ 3x+2y-5z-4=0 \end{cases}$ to'g'ri chiziq kanonik tenglamasini yozing.

12.2. $\begin{cases} 2x+3y-z-4=0 \\ 3x-5y+2z+1=0 \end{cases}$ to'g'ri chiziq parametrik tenglamasini yozing.

12.3. $\frac{x+2}{3}=\frac{y-1}{-2}=\frac{z}{1}$ va $\begin{cases} x+y-z=0 \\ x-y-5z-8=0 \end{cases}$ to'g'ri chiziqlar parallelligini ko'rsating.

12.4. $x=2t+1,\ y=3t-2,\ z=-6t+1$ va $\begin{cases} 2x+y-4z+2=0 \\ 4x-y-5z+4=0 \end{cases}$ to'g'ri chiziqlarning perpendikulyarligini isbotlang.

12.5. $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z}{6}$ va $2x+3y+z-1=0$ kesishish nuqtasi topilsin.

12.6. S ning qanday qiymatida $\begin{cases} 3x-2y+z+3=0 \\ 4x-3y+4z+1=0 \end{cases}$

to'g'ri chiziq va $2x-y+cz-2=0$ tekislik o'zaro parallel bo'ladi?

12.7. $R(1;-1;-2)$ nuqtadan $\frac{x+3}{3}=\frac{y+2}{2}=\frac{z-8}{-2}$

to'g'ri chiziqqacha masofani toping.

12.8. $\begin{cases} 2x+2y-z-10=0 \\ x-y-z-22=0 \end{cases}$ va $\frac{x+7}{3}=\frac{y-5}{-1}=\frac{z-9}{4}$

to'g'ri chiziqlar parallelligini isbotlang va ular orasidagi masofani toping.

§13. Ikkinchi tartibli sirtlar.

Fazoda

$$Ax^2+By^2+Cz^2+2Dyz+2Exz+2Fxy+2Gx+2My+2Kz+L=0$$

(1) tenglamani qanoatlantiruvchi nuqtalar to'plami ikkinchi tartibli sirt (ITS) deyiladi.

ITS lar koordinata o'qlarini burish, parallel ko'chirish yordamida quyidagi 15 holga keltiriladi:

1) Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $a=b=c$ bo'lsa sfera tenglamasi hosil bo'ladi.

2) Bir pallali giperboloid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

3) Ikki pallali giperboloid:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

4) Konus: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

5) Elliptik paraboloid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$

6) Giperbolik paraboloid: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$

7) Elliptik silindr: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

8) Giperbolik silindr: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

9) Parabolik silindr: $y^2=2px$.

10) Ikki kesishuvchi tekislik: $y^2-k^2x^2=0$.

11) Ikki parallel tekislik: $y^2-k^2=0$.

12) Tekislik: $y^2=0$.

13) To'g'ri chiziq: $x^2+y^2=0$.

14) Nuqta: $x^2+y^2+z^2=0$.

15) Bo'sh to'plam: $x^2=-1$

Fazodagi $P(x,y,z)$ nuqtaning o'rnini uning XOY tekislikka proektsiyasi $P'(x,y,o)$ qutb koordinatalari va P nuqta applikatasi z yordamida aniqlash mumkin:

$\varphi=\angle XOP',\quad r=OP',\quad z=P'P$.

r,φ,z kattaliklar silindrik koordinatalar deyiladi. R nuqta dekart va silindrik koordinatalari quyidagicha bog'langan: $x=r\cos\varphi,\ y=r\sin\varphi$. Applikata o'zgarmaydi: $P(r,\varphi,z)$. Aksincha, $r=\sqrt{x^2+y^2},\ tg\varphi=\frac{y}{x}$.

Misol. $P(2;-2;-3)$ ning silindrik koordinatalarini toping:

$$r=\sqrt{2^2+(-2)^2}=2\sqrt{2},\quad \varphi=arctg\frac{-2}{2}=\frac{3\pi}{4}$$

ekanligidan $P\left(2\sqrt{2};\frac{3\pi}{4};-3\right)$.

2) $Q(4\cos 25^0,-4\sin 15^0,1)$ ning silindrik koordinatalarini toping.

$$r=\sqrt{16\cos^2 15^0+16\sin^2 15^0}=4,$$

$\varphi=arctg\frac{-4\sin 15^0}{4\cos 15^0}=-\frac{\pi}{12}$. Demak, $Q\left(4;-\frac{\pi}{12};1\right)$.

3) $x^2+y^2+z^2=1$ tenglamani silindrik koordinatalarda yozing.

$r^2=x^2+y^2$ ekanligidan $r^2+z^2=1$ kelib chiqadi.

Fazodagi $P(x,y,z)$ nuqta va uning OXY tekislikka proektsiyasi $P'(x,y,0)$ berilgan bo'lsin. $OP=\rho$, $<ZOP=\theta$, $<XOP'=\varphi$ kattaliklar nuqtaning sferik koordinatlari deyiladi va $P(\rho,\theta,\varphi)$ tarzida yoziladi. Dekart va sferik koordinatalar sistemalari o'zaro quyidagicha bog'langan:

$$x=\rho\cos\varphi\sin\theta,\ y=\rho\sin\varphi\sin\theta,\ z=\rho\cos\theta$$

. Aksincha,

$$\rho^2=x^2+y^2+z^2,\ \cos\varphi=\frac{x}{\sqrt{x^2+y^2}},\ \sin\varphi=\frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta=\frac{z}{\rho},\ \sin\theta=\frac{\sqrt{x^2+y^2}}{\rho}.$$

Misollar: 1) $P(1;1;1)$ ning sferik koordinatalarini yozing.

$$\rho=\sqrt{1^2+1^2+1^2}=\sqrt{3},$$

$$\cos\varphi=\frac{1}{\sqrt{2}};\ \sin\varphi=\frac{1}{\sqrt{2}}\ \text{lardan}\ \varphi=\frac{\pi}{4}.$$

$$\cos\theta=\frac{1}{\sqrt{3}}\ \text{dan}\ \theta=\arccos\frac{1}{\sqrt{3}}.$$

Demak, $P\left(\sqrt{3};\arccos\frac{1}{\sqrt{3}};\frac{\pi}{4}\right)$.

2) $Q(\cos 77^0, \sin 77^0, 0)$

$$\rho=\sqrt{\cos^2 77^0+\sin^2 77^0+0^2}=1,$$

$$\cos\varphi=\frac{\cos 77^0}{1}\ \text{dan}\ \varphi=77^0;$$

$$\cos\theta=\frac{0}{1}=0\ \text{dan}\ \theta=90^0.$$

Demak, $Q\left(1;\frac{\pi}{2};\frac{77\pi}{180}\right)$.

3) $x^2+(y-1)^2+z^2=1$ tenglamani sferik koordinatalarda yozing.

$x^2+y^2+z^2-2y+1=1$ tenglikdan $\rho^2-2y=0$ yoki $\rho^2=2y$ ga egamiz.

$$\rho^2=2\cdot\sin\varphi\sin\theta,\ \rho=2\sin\varphi\sin\theta.$$

3. Silindrik va sferik koordinatalarini toping.

Buning uchun dastlab, son o'qlarini ko'paytmalarni yo'qotadigan Eyler burchaklari deb ataluvchi φ,θ,ψ burchaklarga quyidagi tartibda buriladi. (Ular ITCh dagiday $ctg 2\alpha=\frac{A-C}{2B}$ shart yordamida aniqlanadi).

$$\begin{cases} x_1 = x\cos\varphi - y\sin\varphi \\ y_1 = x\sin\varphi + y\cos\varphi \\ z_1 = z \end{cases}$$

Bu almashtirishda $x \cdot y$ had yo'qoladi. So'ngra

$$\begin{cases} x_2 = x_1 \\ y_2 = y_1\cos\theta - z_1\sin\theta \\ z_2 = y_1\sin\theta + z_1\cos\theta \end{cases}$$

almashtirishlar $y \cdot z$ hadni yo'qotadi.

$$\begin{cases} x' = x_2\cos\psi - z_2\sin\psi \\ y' = y_2 \\ z' = x_2\sin\psi + z_2\cos\psi \end{cases}$$

almashtirishlarda $x \cdot z$ had yo'qoladi.

Natijada (1) – tenglama

$$A'x'^2 + B'y'^2 + C'z'^2 + 2ax' + 2by' + 2cz' + d = 0$$

(2)

ko'rinishga keladi. Bu tenglama parallel ko'chirish yordamida kanonik ko'rinish oladi.

13.1 $M(4;-1;-3)$ va $N(0;3;-1)$ sferaning biror diametri uchlari bo'lsa, bu sfera tenglamasini yozing.

13.2 $x^2 + y^2 - 2z^2 = 0$ konus va y=2 tekislik qanday chiziq bo'yicha kesishadi?

13.3 $x^2 = yz$ tenglama qanday sirtni aniqlaydi?

13.4 Kanonik ko'rinishga keltiring:

1) $x^2 - xy - xz + yz = 0$

2) $x^2 + z^2 - 4x - 4z + 4 = 0$

3) $x^2 + 2y^2 + z^2 - 2xy - 2yz = 0$

4) $x^2 + y^2 - z^2 - 2y + 2z = 0$

5) $x^2+2y^2+2z^2-4y+4z+4=0$

6) $4x^2+y^2-z^2-2yx-4y+2z+35=0$

7) $x^2+y^2-6x+6y-4z+18=0$

8) $9x^2-z^2-18x-18y-6z=0$

Bobga doir misollar echish namunalari

1. $A(2;-4;5)$ va $B(-3;2;7)$ nuqtalar orasidagi masofani toping.

$$|AB|=\sqrt{(-3-2)^2+(2+4)^2+(7-5)^2}=\sqrt{5^2+6^2+2^2}=$$

$$=\sqrt{25+36+4}=\sqrt{65}$$

2. $A(-1;5;7)$, $B(3;3;3)$ nuqtalar berilgan,

$$|AC|:|CB|=\frac{1}{3}$$

shartni qanoatlantiruvchi $C(x,y,z)$ nuqtani toping.

$\lambda=\frac{1}{3}$ ekanligidan foydalanib,

$$x=\frac{-1+\frac{1}{3}\cdot 3}{1+\frac{1}{3}}=0,\quad y=\frac{5+\frac{1}{3}\cdot 3}{1+\frac{1}{3}}=\frac{9}{2},\quad z=\frac{7+\frac{1}{3}\cdot 3}{1+\frac{1}{3}}=6$$

ekanligini topamiz, ya'ni $C\left(0;\frac{9}{2};6\right)$.

3. ABC uchburchak AB tomoni EF nuqtalari yordamida teng uch qismga ajratiladi. Agar $\overrightarrow{CA}=\vec{a}$, $\overrightarrow{CB}=\vec{в}$ bo'lsa, $\overrightarrow{CE}$ vektorni toping.

$\overrightarrow{AB} = \vec{в} - \vec{a}$ ekanligidan $\overrightarrow{AE} = \frac{1}{3}(\vec{в} - \vec{a})$. Demak,

$$\overrightarrow{CE} = \overrightarrow{CA} + \overrightarrow{AE} = \vec{a} + \frac{1}{3} + (\vec{в} - \vec{a}) = \frac{1}{3}(2\vec{a} + \vec{в}).$$

4 $A(1;2;3), B(4;5;6)$ bo'lsa, $\vec{a} = \overrightarrow{AB}$ vektor koordinatalarini toping.

$x = 4 - 1 = 3;\ y = 5 - 2 = 3;\ z = 6 - 3 = 3$ ekanligidan $\vec{a}('3;3;3)$.

5. $\vec{a}(2;1;0), \vec{b}(-1;2;1), \vec{c}(0;1;1)$ bo'lsa,

$\vec{d} = 2\vec{a} - 3\vec{b} + 4\vec{c}$ ni toping.

$$2\,2\vec{a} = (4;2;0),\ 3\vec{b} = (-3;6;3),\ 4\vec{c} = (0;4;4)$$

ekanligidan

$\vec{d} = (4;2;0) - (-3;6;3) + (0;4;4) = (7;0;1)$, ya'ni $\vec{d}(7;0;1)$

6. $\vec{a} = \vec{i} - 2\vec{j} - 2\vec{k}$ vektor uzunligini, yo'naltiruvchi kosinuslarini toping.

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$cos\,\alpha = \frac{1}{3};\quad \cos\beta = \frac{-2}{3};\quad \cos\gamma = -\frac{2}{3.}$$

7. $\vec{a}(1;2;1), \vec{b}(0;4;2)$ bo'lsa, $\vec{a}\,\vec{в}$ va $\vec{a}\,x\,\vec{b}$ ni toping.

$$\vec{a} \cdot \vec{b} = 1 \cdot 0 + 2 \cdot 4 + (-1) \cdot 2 = 8 - 2 = 6$$

$$\vec{c}=\vec{a}x\vec{в}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 2 & -1\\ 0 & 4 & 2\end{vmatrix}=\vec{i}\begin{vmatrix}2 & -1\\ 4 & 2\end{vmatrix}-\vec{j}\begin{vmatrix}1 & -1\\ 0 & 2\end{vmatrix}$$

$$+\vec{k}=\begin{vmatrix}1 & 2\\ 0 & 4\end{vmatrix}=8\vec{i}-2\vec{j}+4\vec{k},$$ ya'ni

$\vec{c}(8;-2;4)$.

8. $\vec{a}(1;2;-2)$, $\vec{b}(0;6;8)$ vektorlar orasidagi burchakni, ularga qurilgan parallelogramm yuzini toping.

$$\cos\varphi=\frac{1\cdot0+2\cdot6+(-2)\cdot8}{\sqrt{1^2+2^2+(-2)^2}\cdot\sqrt{0^2+6^2+8^2}}=\frac{12-16}{3\cdot10}=$$

$$=\frac{-4}{30}=-\frac{2}{15}$$

$$\varphi=\arccos\left(-\frac{2}{15}\right)$$

$$\vec{a}x\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 2 & -2\\ 0 & 6 & 8\end{vmatrix}=28\vec{i}-8\vec{j}+6\vec{k}$$ dan

$$S=\left|\vec{a}x\vec{b}\right|=\sqrt{28^2+(-8)^2+6^2}=\sqrt{2^2\left(14^2+4^2+3^2\right)}=$$

$=2\sqrt{221}$ (kv.b)

9. $\vec{a}(2;3;-1)$, $\vec{b}(1;-2;3)$, $\vec{c}(2;-1;1)$ vektorlar berilgan. Shunday

$\vec{d}$ vektor topingki, u $\vec{a}$ va $\vec{в}$ vektorlarga perpendikulyar, $\vec{d}\cdot\vec{c}=-6$ bo'lsin.

$\vec{d}(x,y,z)$ desak, shartlardan:

$$\vec{a}\cdot\vec{d}=2x+3y-z=0$$

$$\vec{b}\cdot\vec{d}=x-2y+3z=0$$

$\vec{c}\cdot\vec{d}=2x-y+z=-6$ kelib chiqadi. Hosil bo'lgan sistemani echib $x=-3, y=3,\ z=3$ ekanligini topamiz, ya'ni

$$\vec{d}(-3;3;3).$$

10. $|\vec{a}|=10,\ |\vec{в}|=2,\ \vec{a}\vec{в}=12$ bo'lsa, $|\vec{a}x\vec{в}|$ ni hisoblang.

$\cos\varphi=\frac{12}{10\cdot 2}=\frac{3}{5}$ ekanligidan φ – o'tkir burchak va $\sin\varphi=\frac{4}{5}$

$$|\vec{a}x\vec{b}|=|\vec{a}|\cdot|\vec{b}|\cdot\sin\varphi=10\cdot 2\cdot\frac{4}{5}=16.$$

11. Uchburchakli piramida uchlari $A(2;2;2)$, $B(4;3;3)$, $C(4;5;4)$ va $D(4;4;7)$ nuqtalarda bo'lsa, uning hajmini toping.

$\vec{a}=\overrightarrow{AB}=(2;1;1), \vec{в}=\overrightarrow{AC}=(2;3;2), \vec{c}=\overrightarrow{AD}=(2;2;5)$ vektorlar piramida yasovchilaridir.

$$(\vec{a}x\vec{b})\vec{c}=\begin{vmatrix}2 & 1 & 1\\ 2 & 3 & 2\\ 2 & 2 & 5\end{vmatrix}=22-6+(-2)=14$$

12. $F(1;2;3)$ nuqtadan o'tuvchi va $\vec{N}=2\vec{i}+3\vec{j}+2\vec{k}$ vektorga perpendikulyar tekislik tenglamasini yozing.

$2(x-1)+3(y-2)+2(z-3)=0$ *ya'ni* $2x+3y+2z-14=0$

13. $E(2;-1;2)$ nuqtadan o'tib $x+y-3z+4=0$ tekislikka parallel bo'lgan tekislik tenglamasini yozing. Izlanayotgan tekislik normal vektori

$\vec{N}(1;1;-3)$ bo'lishini hisobga olib $1(x-2)+1(y+1)-3(z-2)=0$ ya'ni $x+y-3z+5=0$ ekanligini topamiz.

14. $2x+3y+6z-12=0$ tekislikni chizing.

Buning uchun tekislikni kesmalar bo'yicha tenglamasini yozamiz:

$$\frac{x}{6}+\frac{y}{4}+\frac{z}{2}=1$$

Endi $\vec{N}$ nuqtalardan o'tuvchi tekislikni chizish mumkin

15. $A(2;3;-5)$ nuqtadan $4x-2y+5z-12=0$ tekislikka tushirilgan perpendikulyar uzunligini toping.

Nuqtadan tekislikkacha bo'lgan eng qisqa masofa izlanayotgan perpendikulyar uzunligi bo'ladi. Demak,

$$|p|=d=\frac{|4\cdot 2-2\cdot 3+5\cdot(-5)-12|}{\sqrt{4^2+(-2)^2+5^2}}=\frac{|8-6-25-12|}{\sqrt{45}}=\frac{35}{\sqrt{45}}$$

16. $x-2y+2z-8=0$ *va* $x+z-6=0$ tekisliklar orasidagi burchakni toping.

$\vec{N}_1(1;-2;2)$, $\vec{N}_2(1;0;1)$ ekanidan:

$$\cos\varphi=\frac{1\cdot 1+(-2)\cdot 0+2\cdot 1}{\sqrt{1^2+(-2)^2+2^2}\cdot\sqrt{1^2+1^2}}=\frac{1}{\sqrt{2}}$$

ya'ni $\varphi=45^0$

17. $x+3y+5z-4=0$ *ва* $x-y-2z+7=0$ tekisliklar kesishish chizig'i va $A(1;0;1)$ nuqtadan o'tuvchi tekislik tenglamasini toping.

Berilgan tekisliklar kesishish chizig'idan o'tuvchi tekisliklar bog'lami tenglamasi

$x+3y+5z-4+\lambda(x-y-2z+7)=0$ *ёки*

$(1+\lambda)x+(3-\lambda)y+(5-2\lambda)z+7\lambda-4=0$

ko'rinishda bo'ladi uning $A(1;0;1)$ nuqtadan o'tishidan foydalanib $1+5\cdot 1-4+\lambda(1-2+7)=0,$

ya'ni $\lambda=-\frac{2}{6}=-\frac{1}{3}$ ekanligini topamiz.

Demak:

$$\frac{2}{3}x+\frac{10}{3}y+\frac{17}{3}z-\frac{19}{3}=0, \text{ яъни } 2x+10y+17z-19=0$$

Demak, $V_{пир}=\frac{1}{6}\cdot 14=\frac{7}{3}$ (kub.b)

18. Ikki tekislik

$2x-y+3z-1=0, 5x+4y-z-7=0$ kesishishidan hosil bo'lgan to'g'ri chiziq kanonik tenglamasini yozing.

Dastlab 1–tenglamani 4 ga ko'paytirib, 2–tenglamaga qo'yamiz:

$$13x+11z-11=0$$

2–tenglamani 3 ga ko'paytirib 1–tenglikka qo'yamiz:

$$17x+11y-22=0$$

Bulardan x ni topamiz:

$$x=\frac{11(y-2)}{-17}=\frac{11(z-1)}{-13}, \text{ ya'ni}$$

$$\frac{x}{-11}=\frac{y-2}{17}=\frac{z-1}{13}$$

19. $A(1;-2;1), B(3;1;-1)$ nuqtalardan o'tuvchi to'g'ri chiziq kanonik tenglamasini yozing.

$\vec{p} = \overrightarrow{AB} = (2;3;0)$ desak, $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-1}{0}$ kelib chiqadi.

20. $B(2;-1;3)$ nuqtadan $\frac{x+1}{3} = \frac{y+2}{4} = \frac{z-1}{5}$ to'g'ri chiziqqacha bo'lgan masofani toping.

Yo'naltiruvchi vektor $\vec{p}$ (3;4;5) va $A(-1;-2;1)$ to'g'ri chiziqdagi nuqta ekanligidan:

$$|AB| = \sqrt{(2+1)^2 + (-1+2)^2 + (3-1)^2} = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$$

$\overrightarrow{AB} = (3;1;2)$ va $\vec{p}(3;4;5)$ orasidagi burchak

$$\cos\varphi = \frac{9+4+10}{\sqrt{9+1+4} \cdot \sqrt{9+16+25}} = \frac{23}{\sqrt{14} \cdot 5\sqrt{2}};$$

$$\sin\varphi = \frac{\sqrt{117 \cdot 163}}{140}$$

$$d = |AB| \sin\varphi = \sqrt{14} \cdot \frac{\sqrt{117 \cdot 163}}{140} = 0{,}3 \cdot \sqrt{38}$$

21. $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z-3}{3}$ to'g'ri chiziq va $2x + y - z = 0$ tekislik orasidagi burchakni toping.

$\vec{p}$ (2; − 1;3), $\vec{N}$ (2;1; − 1) ekanligidan

$$\sin\varphi = \frac{2 \cdot 2 + 1 \cdot (-1) + (-1) \cdot 3}{\sqrt{2^2 + 1^2 + 3^2} \cdot \sqrt{2^2 + 1^2 + 1^2}} = 0, \text{ ya'ni } \varphi=0$$

Berilgan to'g'ri chiziq va tekislik o'zaro parallel ekan.

22. $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$ sfera markazi va radiusini toping.

$$(x - 4x) + (y^2 + 2y) + z^2 - 4 = 0$$

$$(x-2)^2-4+(y+1)^2-1+z^2-4=0$$
$$(x-1)^2+(y+1)^2+z^2=3^2$$

Demak, sfera markazi $O(1;-1;0)$ nuqtada, radiusi 3 ga teng.

23. $x^2-y^2-4x+8y-2z=0$ sirt kanonik ko'rinishga keltirilsin.

$$\left(x^2-4x\right)-\left(y^2-8y\right)=2z$$

$(x-2)^2-(y-4)^2=2(z-6)$ dan

$x'=x-2,\ y'=y-4,\ z'=z-6$ parallel ko'chirish yordamida $x'^2-y'^2=2z'$ ga ega bo'lamiz. Bu giperbolik paraboladir.

24.

a) $x^2+4y^2+9z^2+12yz+6xz+4xy-4x-8y-12z+23=0$ tenglama qanday sirtni ifodalaydi?

Bu tenglamani

$(x+2y+3z)^2-4(x+2y+3z)+3=0$ ko'rinishda yozib $x+2y+3z=1$ va $x+2y+3z=3$ parallel tekisliklarni olamiz.

b). $xy+yz+xz=1$ tenglama kanonik ko'rinishga keltirilsin.

$ctg2\alpha=\dfrac{A-B}{2D}=0$, bundan $\varphi=\dfrac{\pi}{4}$.

Parallel ko'chirish yordamida quyidagi ko'rinishga ega bo'lamiz::

$$\begin{cases} x=\dfrac{\sqrt{2}}{2}(x'-y') \\ y=\dfrac{\sqrt{2}}{2}(x'+y') \\ z=z' \end{cases}$$

ITS ko'rinishi: $x'^2-y'^2+2\sqrt{2}x'z'=2$

$$\mathrm{ctg}2\theta = \frac{A-C}{2F} = \frac{1}{2\sqrt{2}}$$

2D burchakni 0^0 va 90^0 orasida, yoki 2θ to'g'ri burchakli uchburchakning o'tkir burchagi deb hisoblaymiz. 2θ burchak qarshisida yotgan katet $2\sqrt{2}$ teng bo'lsa, u holda boshqa katet 1 ga teng, gipotenuza esa 3.

Demak, $\sin 2\theta = \frac{2\sqrt{2}}{3}$, $\cos 2\theta = \frac{1}{3}$.

Bundan, $\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \frac{1}{\sqrt{3}}$, $\cos\theta = \sqrt{\frac{2}{3}}$

va
$$\begin{cases} x' = x''\sqrt{\frac{2}{3}} - z''\frac{1}{\sqrt{3}} \\ y' = y'' \\ z' = z''\frac{1}{\sqrt{3}} + \sqrt{\frac{2}{3}}z'' \end{cases}$$

ITS quyidagi ko'rinishda bo'ladi:

$$\frac{2}{3}x''^2 - \frac{2\sqrt{2}}{3}x''z'' + \frac{1}{3}z''^2 - y''^2 +$$

$$+2\sqrt{2}\left(\frac{x''^2\sqrt{2}}{3} + \frac{2}{3}x''z'' - \frac{1}{3}x''z'' - \frac{\sqrt{2}}{3}z''^2\right) = 2$$

Demak: $2x''^2\ y''^2 - z''^2 = 2$ yoki

$$-x''^2 + \frac{y''^2}{2} + \frac{z''^2}{2} = -1$$

3–bobga doir uy vazifalari

1. ***ABCD*** **piramida uchlari koordinatalari berilgan. Quyidagilarni toping:**

1) ***AB*** **qirra uzunligini;**

2) AB va AD qirra orasidagi burchak;

3) ABC tekislik tenglamasi;

4) AD qirra va ABC yoq orasidagi burchak;

5) ABC yoq yuzi;

6) Piramida hajmi;

7) AB chiziq tenglamasi;

8) D uchidan tushirilgan balandlik tenglamasi va uzunligi.

9) D uchidan tushirilgan balandlik va asosining kesishish nuqtasini toping.

10) AD va BC to'g'ri chiziqlar orasidagi masofani toping.

Koordinatalar sistemasida tasvirlang.

1) $A(4; 2; 5)$, $B(0; 7; 2)$, $C(0; 2; 7)$, $D(1; 5; 0)$.
2) $A(4; 4;10)$, $B(4; 10; 2)$, $C(2; 8; 4)$, $D(9; 6; 4)$.
3) $A(4; 6; 5)$, $B(6; 9; 4)$, $C(2; 10; 10)$, $D(7; 5; 9)$.
4) $A(3; 5; 4)$, $B(8; 7; 4)$, $C(5; 10; 4)$, $D(4; 7; 8)$.
5) $A(10; 6; 6)$, $B(-2; 8; 2)$, $C(6; 8; 9)$, $D(7; 10; 3)$.
6) $A(1; 8; 2)$, $B(5; 2; 6)$, $C(5; 7; 4)$, $D(4; 10; 9)$.
7) $A(6; 6; 5)$, $B(4; 9; 5)$, $C(4; 6; 11)$, $D(6; 9; 3)$.
8) $A(7; 2; 2)$, $B(5; 7; 7)$, $C(5; 3; 1)$, $D(2; 3; 7)$.
9) $A(8; 6; 4)$, $B(10; 5; 5)$, $C(5; 6; 8)$, $D(8; 10; 7)$.
10) $A(7; 7; 3)$, $B(6; 5; 8)$, $C(3; 5; 8)$, $D(8; 4; 1)$.
11) $A(1; 1; 1)$, $B(5; 3; 4)$, $C(2; 0; 2)$, $D(6; 8; 10)$.
12) $A(0; 1; 1)$, $B(1; 0; 1)$, $C(2; 3; 0)$, $D(6; 4; 3)$.
13) $A(1; 2;3)$, $B(-1; 3; 2)$, $C(7; -3; 5)$, $D(6; 10; 17)$.
14) $A(1; 4; 3)$, $B(6; 8; 5)$, $C(3; 1; 4)$, $D(21;18; 33)$.
15) $A(7; 2; 1)$, $B(4; 3; 5)$, $C(3; 4; -2)$, $D(2; -5;-13)$.

2. Berilgan tenglamani kanonik ko'rinishga keltiring va ikkinchi tartibli sirt turini aniqlang.

1) $x^2 - y^2 + z^2 - 2y - 2z = 0$

2) $xy + yz + xz = 0$

3) $z^2 - xy + 4x - 4y = 12$

4) $x^2 + y^2 + z^2 - 4x + 6y - 1 = 0$

5) $x^2 - 4xy + y^2 - z^2 = 0$

6) $x^2 + 4y^2 + 4z^2 - -2x + 4y - 8z = 0$

7) $x^2 - xy + xz + yz = 0$

8) $z^2 - xy + y^2 = 10$

9) $y^2 - x^2 + z^2 - 2xy = 0$

10) $x^2 + y^2 - 4xz + 4yz = 0$

11) $x^2 - 4xy - 4xz - 4yz = 0$

12) $2x^2 + y^2 - 18z - 3z + y + 14 = 0$

13) $3x^2 - 4y^2 - 24z - x + 2y - 2 = 0$

14) $9x^2 + 4y^2 - z^2 - 9x + 6y - 2z - 27 = 0$

15) $x^2 + 3y^2 + 4z^2 + 2x - 3y + 4z - 10 = 0$

II–qism. MATEMATIK ANALIZ
4–bob. To'plam. Funktsiya. Limit va uzluksizlik.
§14. To'plam. Amallar. To'plam turlari.

Biror xususiyatiga ko'ra jamlangan predmetlar majmuasi to'plam deb qaralishi mumkin. To'plamlar A, B,....X,Y,... harflar bilan, ularni tashkil qiluvchi predmetlar – elementlari a,b,....x,y,... harflar yordamida belgilanadi. a element A to'plamga tegishliligi $a \in A$, tegishli emasligi $a \notin A$ tarzida belgilanadi.

Agar A to'plam elementlari B to'plamga ham element hisoblansa, A to'plam V to'plamning qismi deyiladi va $A \subset B$ tarzida yoziladi. $A \subset B$, $B \subset A$ bir paytda bajarilsa, $A = B$ kelib chiqadi.

A va B to'plamlar barcha elementlaridan tuzilgan to'plam ularning yig'indisi deyiladi va $A \cup B$ ko'rinishda belgilanadi. Ularning faqatgina umumiy elementlaridan tuzilgan to'plam kesishma deyilib, $A \cap B$ tarzida belgilanadi. Faqatgina A ga tegishli elementga (A ga xos elementlar) to'plami A dan B ni ayrilgani deyiladi, $A \setminus B$ tarzida belgilanadi. A va B xos elementlaridan tuzilgan to'plam ularning to'g'ri ko'paytmasi deyilib, $A \Delta B$ tarzida yoziladi. Demak, $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

$a \in A$, $b \in B$ elementlar olinib hosil qilingan barcha (a,b) ko'rinishidagi juftliklar dekart ko'paytma deyiladi va AxB kabi belgilanadi.

Birorta ham elementi bo'lmagan to'plam bo'sh to'plam deyilib, $\varnothing$ ko'rinishda yoziladi.

Qaralayotgan A, B to'plamlar biror E to'plam qismlari bo'lsa, $E \setminus A$ to'plam A ning E ga qadar to'ldiruvchisi deyiladi va $C_E A$ yoki SA ko'rinishda yoziladi.

Quyidagi ikkilanganlik printsipi deb ataluvchi tengliklar o'rinli:

$$E\setminus\bigcup_n A_n=\bigcap_n(E\setminus A_n),\quad E\setminus\bigcap_n A_n=\bigcup_n(E\setminus A_n).$$

Agar A to'plamning har bir elementiga B to'plamning bitta elementini, B ning har bir elementiga A ning bitta elementi mos qo'yilsa, A va B to'plamlar o'zaro bir qiymatli moslikda deyiladi. Bunday to'plamlar o'zaro ekvivalent deyilib, $A\tilde{\ }B$ tarzida yoziladi.

To'plamlar elementlari soniga qarab solishtiriladi:

1) elementlari soni chekligi bo'lgan to'plam chekli to'plam deyiladi;
2) elementlarini sanash mumkin bo'lgan, ya'ni natural sonlar to'plamiga ekvivalent to'plamlar sanoqli to'plamlar deyiladi.
3) Elementlarini sanash mumkin bo'lmagan cheksiz elementli to'plamlar sanoqsiz to'plamlar (yoki S—kontinuum quvvatli) deyiladi.

14.1. Isbotlang.

1) $(A\cup B)\cup C=A\cup(B\cup C)$;

2) $(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$;

3) $A\cap(B\Delta C)=(A\cap B)\Delta(A\cap C)$;

4) $(A\setminus B)\cup(B\setminus A)=(A\cup B)\setminus(A\cap B)$;

5) $C(A\setminus B)=CA\cup B$;

6) $(A\cup B)xC=(AxC)\cup(BxC)$;

7) $Ax(B\cap C)=(AxB)\cap(AxC)$;

8)
$(A\cup B)\setminus(A\cap B)=(A\cap CB)\cup(B\cap CA)$.

14.2. Soddalashtiring: $C[C(CA\cup B)\cup(A\cup CB)]$.

14.3. Haqiqiy sonlar to'plami R va irratsional sonlar to'plami $(R\setminus Q)$ orasida o'zaro bir qiymatli moslik o'rnating.

14.4. $[0;1]$ kesmadagi sonlarni o'nli kasrga yoyganda 9 raqami qatnashmaydigan sonlar to'plami bilan $[0;1]$ kesma orasida o'zaro bir qiymatli moslik o'rnating.
14.5. Tekislikda uchlari koordinatalari ratsional bo'lgan uchburchaklar to'plami quvvatini toping.
14.6. Tekislikda o'zaro kesishmaydigan T harflari to'plami quvvatini toping.

§15. Funktsiya tushunchasi. Elementar funktsiyalar. Ketma–ketliklar.

Agar X to'plamdan olingan har bir songa biror f qoidaga yoki qonunga ko'ra Y to'plamning bitta y soni mos qo'yilgan bo'lsa, u holda X to'plamda funktsiya aniqlangan deyiladi.

Bu moslik $y=f(x)$ tarzida yoziladi. X to'plam funktsiya aniqlanish sohasi, Y – o'zgarish sohasi deyiladi, x – argument, y – funktsiya deyiladi.

Aniqlanish sohasi natural sonlardan iborat funktsiya ketma–ketlik deyiladi, $y=f(n)$ o'rniga y_n yoki a_n kabi belgilanadi.

Funktsiya juft (toq) deyiladi, agar $f(-x)=f(x)$ $[f(-x)=-f(x)]$ bajarilsa.

Ixtiyoriy $x\in X$ da $m\le f(x)\le M$ shart bajarilsa, funktsiya quyidan m soni, yuqoridan M soni bilan chegaralangan deyiladi.

Agar x ning X to'plamdagi x_1 va x_2 qiymatlari uchun $x_1<x_2$ shartdan $f(x_1)<f(x_2)$ $[f(x_1)>f(x_2)]$ kelib chiqsa, $f(x)$ bu to'plamda o'suvchi (kamayuvchi) deyiladi.

Agar shunday T son mavjud bo'lib, ixtiyoriy $x\in X$ da

1) $x\pm T\in X$, 2) $f(x+T)=f(x)$

bo'lsa, bu funktsiya davriy, bu shartga bo'ysunuvchi eng kichik T soni davr deyiladi.

$y = f(x)$ funktsiyada x va y larning o'rinlarini almashtirishdan hosil bo'ladigan funktsiya teskari funktsiya deyiladi va $x = f^{-1}(y)$ tarzida belgilanadi.

Ketma–ketlik uchun chegaralanganlik, o'suvchilik yoki kamayuvchilikni aniqlash mumkin.

15.1. Berilgan funktsiyalar aniqlanish sohasini toping.

1) $y = \dfrac{x}{1-x}$ 2) $y = \sqrt{3x - x^2}$

3) $y = \log(x^2 - 4)$ 4) $y = \sqrt{\sin\sqrt{x}}$

5) $y = \log_2 \log_3 x$ 6) $y = \sqrt{\lg tgx}$

7) $y = \sqrt{\sin 2x} + \sqrt{\sin 3x}$, $x \in [0;\, 2\pi]$.

15.2. 1) $f(x+1) = x^2 - 3x + 2$ bo'lsa, $f(x)$ ni toping.

2) $f(\dfrac{1}{x}) = x + \sqrt{1 + x^2}$ bo'lsa, $f(x)$ ni toping $(x > 0)$.

15.3. Berilgan funktsiyalarga teskari funktsiyalarni toping.

1) $y = 2x + 3$

2) $y = x^2$, $0 \le x < +\infty$

3) $y = \dfrac{1-x}{1+x}$, $x \ne -1$

4) $y = \sqrt{1 - x^2}$, $0 \le x \le 1$.

5) $y = \begin{cases} x, & -\infty < x < 1 \\ x^2, & 1 \le x \le 4 \\ 2^x, & 4 < x < +\infty \end{cases}$

15.4. Juft – toqligini tekshiring.

1) $y = 3x - x^3$;

2) $y = \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2}$;

3) $y = a^x + a^{-x}$

4) $y = \ln(x + \sqrt{1+x^2})$.

15.5. 1) $y = kx$ grafigini $k = 0,\ 1,\ -1$ hollarda chizing.

2) $y = x + b$ grafigini $b = 0,\ 1,\ -1$ bo'lganda chizing.

3) $y = ax^2$, $y = (x-a)^2$, $y = x^2 + a$ grafiklarini $a = 0$, $a = \pm 1$, $a = \pm 2$ bo'lgan hollarda chizing.

15.6. Quyidagi funktsiyalar grafiklarini chizing.

1) $y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

2) $y = \operatorname{sgn} x = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

3) $y = [x] = \{x$ ning butun qismi$\}$

4) $y = \{x\} = \{x$ ning kasr qismi$\}$

5) $y = \sin^2 x$

6) $y = \sin x \cdot \sin 3x$

7) $y = cthx$

8) $y = [x] \cdot |\sin x|$

9) $y = 1 + x + e^x$

10) $y = \sin^4 x + \cos^4 x$

11) $y = |1 - x| + |1 + x|.$

12) $y = x \sin x$

13) $y = \ln(\sin x)$

14) $y = \ln \cos x$

15) $y = shx = \dfrac{e^x - e^{-x}}{2}$

16) $y = chx = \dfrac{e^x + e^{-x}}{2}$

17) $y = tgx;$

18) $y = x \cdot \mathrm{sgn}(\sin x)$

19) $y = \cos x \cdot \mathrm{sgn}(\sin x)$

20) $y = x + \sin x$

21) $y = |1 - x| - |1 + x|$

§16. Ketma–ketlik va funktsiya limiti. Uzluksizlik. Ajoyib limitlar.

Agar a nuqtaning ixtiyoriy $(a-\varepsilon, a+\varepsilon)$ atrofi $(\varepsilon > 0)$ olinganda ham $\{x_n\}$ ketma–ketlikning biror hadidan boshlab, keyingi barcha hadlari shu atrofga tegishli bo'lsa, a son $\{x_n\}$ ketma–ketlikning limiti deyiladi va

$$\lim_{n\to\infty} x_n = a$$

tarzda belgilanadi.

Ketma–ketlik chekli songa intilsa yaqinlashuvchi, aks holda uzoqlashuvchi deyiladi.

Agar X to'plamning nuqtalaridan tuzilgan, a ga yaqinlashuvchi har qanday $\{x_n\}$ ketma–ketlik olinganda ham, funktsiya qiymatlaridan iborat $\{f(x_n)\}$ ketma–ketlik yagona b limitga intilsa, shu

b ga $f(x)$ ning a nuqtadagi (x ning a intilgandagi) limiti deyiladi va $\lim\limits_{n\to\infty} x_n = b$ kabi yoziladi. Bunda, agar $\{x_n\}$ ketma–ketlik a dan faqat katta (kichik) bo'lib a ga intilsa, o'ng (chap) limit deyiladi va

$$\lim_{n\to a+0} f(x) = F(a+0) = b \qquad (\lim_{n\to a-0} f(x) = f(a-0) = b)$$

tarzida yoziladi.

Limitlar quyidagi xossalarga ega:

1) O'zgarmas son limiti o'ziga teng.
2) $\lim(u+v) = \lim u + \lim v.$
3) $\lim(u \cdot v) = \lim u \cdot \lim v$
4) $\lim \dfrac{u}{v} = \dfrac{\lim u}{\lim v}$

Agar $\lim\limits_{x\to x_0} f(x) = f(x_0)$ o'rinli bo'lsa, $f(x)$ $x = x_0$ nuqtada uzluksiz deyiladi. Funktsiya x_0 nuqtada uzluksiz bo'lishi uchun

$$\lim_{x\to x_0-0} f(x) = f(x_0) = \lim_{x\to x_0+0} f(x)$$

tengliklar bir paytda bajarilishi zarur va etarli. Birorta tenglik bajarilmasa, $f(x)$ funktsiya x_0 nuqtada uzilishga ega deyiladi.

$\lim\limits_{x\to x_0+0} f(x) - \lim\limits_{x\to x_0-0} f(x) = \lambda \neq 0$ bo'lsa, I–tur uzilishga ega, λ esa funktsiyaning x_0 nuqtadagi sakrashi deyiladi. Boshqa turdagi uzilishlar barchasi II–tur uzilishlar deyiladi.

Limitlarni hisoblashda quyidagi ko'p uchraydigan «ajoyib limitlar»dan foydalaniladi:

1) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$.

2) $\lim\limits_{n\to\infty}(1+\dfrac{1}{n})^n=\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e.$

3) $\lim\limits_{x\to 0}=\dfrac{(1+x)^\mu-1}{x}=\mu$

4) $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}=1$

5) $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\ln a$

16.1. Limitlar hisoblansin.

1) $\lim\limits_{n\to\infty}\dfrac{5n-1}{n+3}$

2) $\lim\limits_{n\to\infty}\dfrac{4n^2+n}{1-n^2}$

3) $\lim\limits_{n\to\infty}\dfrac{(-2)^n+3^n}{(-2)^{n+1}+3^{n+1}}$

4) $\lim\limits_{n\to\infty}(\dfrac{1}{n^2}+\dfrac{2}{n^2}+\ldots+\dfrac{n-1}{n^2})$

5) $\lim\limits_{n\to\infty}(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}\cdots\sqrt[2^n]{2}$

6) $\lim\limits_{n\to\infty}\dfrac{\sqrt{2n^2+1}}{2n-1}$

7) $\lim\limits_{n\to\infty}\dfrac{1+2+3+\ldots.+n}{\sqrt{9n^4+1}}$

8) $\lim\limits_{n\to\infty}\dfrac{1+3+5+\ldots.+(2n-1)}{1+2+3+\ldots.+n}$

9) $\lim\limits_{n\to+\infty}\dfrac{1-10^n}{1+10^{n+1}}$

10) $\lim\limits_{n\to-\infty}\dfrac{3-10^n}{2+10^{n+1}}$

11) $\lim\limits_{n\to\infty}\left(\dfrac{1+2+\ldots.+n}{n+2}-\dfrac{n}{2}\right)$

12) $\lim\limits_{n\to\infty}(1+\dfrac{2}{n})^{3n}$

13) $\lim\limits_{n\to\infty}\left(1-\dfrac{5}{n}\right)^n$

14) $\lim\limits_{n\to\infty}\left(\dfrac{3n-2}{3n+1}\right)^{2n}$

16.2 Limitlar hisoblansin.

1) $\lim\limits_{n\to 2}\frac{x^2-4}{x-2}$ 2) $\lim\limits_{n\to 3}\frac{x^2-9}{x^2-2x-3}$

3) $\lim\limits_{n\to 3}\frac{x^2-5x+6}{x^2-8x+15}$ 4) $\lim\limits_{n\to 2}\frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}}$

5) $\lim\limits_{n\to 1}\frac{x^m-1}{x^n-1}$ 6) $\lim\limits_{n\to 1}\left(\frac{m}{1-x^m}-\frac{n}{1-x^n}\right)$

7) $\lim\limits_{n\to 0}\frac{x}{\sqrt{1+3x}-1}$ 8) $\lim\limits_{n\to 0}\frac{\sqrt[3]{1+mx}-1}{x}$

9) $\lim\limits_{n\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$ 10) $\lim\limits_{n\to\infty}\left(\sqrt{x^2+3x}-x\right)$

11) $\lim\limits_{n\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x}\right)$ 12) $\lim\limits_{n\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$

13) $\lim\limits_{x\to 0}\frac{\sin 5x}{x}$ 14) $\lim\limits_{x\to 0}\frac{tgx}{x}$

15) $\lim\limits_{x\to 0}\frac{tgx-\sin x}{\sin^3 x}$ 16) $\lim\limits_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}$

17) $\lim\limits_{x\to a}\frac{\sin x-\sin a}{x-a}$ 18) $\lim\limits_{x\to a}\frac{tgx-tga}{x-a}$

19) $\lim\limits_{x\to 0}\frac{\sqrt{1+tgx}-\sqrt{1+\sin x}}{x^3}$ 20) $\lim\limits_{x\to 0}\frac{\cos x-\sqrt[3]{\cos x}}{\sin^2 x}$

21) $\lim\limits_{x\to\infty}\left(\frac{x+2}{2x-1}\right)^{x^2}$ 22) $\lim\limits_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{\frac{x-1}{x+1}}$

23) $\lim\limits_{x\to 0}\sqrt[x]{1-2x}$ 24) $\lim\limits_{x\to 0}\left(\frac{1+tgx}{1+\sin x}\right)^{\frac{1}{\sin^3 x}}$

25) $\lim_{x\to a}\frac{a^x - x^a}{x-a}$

26) $\lim_{x\to 0}\left(\frac{1+x\cdot 2^x}{1+x\cdot 3^x}\right)^{\frac{1}{x^2}}$

27) $\lim_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{\sin\alpha x-\sin\beta x}$

28) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$

16.3 Uzluksizlikka tekshiring va uzilish xarakterini aniqlang.

1) $y=\frac{x}{(1+x)^2}$

2) $y=\frac{\frac{1}{x}-\frac{1}{x+1}}{\frac{1}{x-1}-\frac{1}{x}}$

3) $y=\operatorname{sgn}(\sin x)$

4) $y=x-[x]$

5) $y=x\cdot[x]$

6) $y=e^{-\frac{1}{x}}$

7) $f(x)=\begin{cases}x, & |x|\le 1\\ 1, & |x|>1\end{cases}$

8) $f(x)=\begin{cases}0{,}5x^2, & |x|<2\\ 2{,}5\ , & |x|=2\\ 3\ , & |x|>2\end{cases}$

9) $f(x)=\begin{cases}2, & x=0\ \text{ва}\ x=\pm 2\\ 4-x^2\ , & 0<|x|<2\\ 4\ , & |x|>2\end{cases}$

Bobga doir misollar echish namunalari

1. $(A \cap B) \cup (A \cap CB) \cup (CA \cap B)$ ifodani soddalashtiring.

$(A \cap B) \cup (A \cap CB) \cup CA \cap B) = (A \cap B) \cup (a \setminus B) \cup (B \setminus A) =$

$= (A \cap B) \cup (A \Delta B) = A \cup B.$

2. Ixtiyoriy A, B, S to'plamlar uchun $Ax(B \cup C) = (AxB) \cup (AxC)$ tenglik o'rinli ekanligini isbotlang.

a) $(x, y) \in Ax(B \cup C)$ bo'lsin. $x \in A,\ y \in B \cup C$, ya'ni $x \in A,\ y \in B$ yoki $x \in A,\ y \in C$. Demak, $(x, y) \in AxB$ yoki $(x, y) \in AxC$. Bulardan, $(x, y) \in (AxB) \cup (AxC)$.

b) $(x, y) \in (AxB) \cup (AxC)$. Demak, $(x, y) \in AxB$ yoki $(x, y) \in AxC$. Undan $x \in A,\ y \in B$ yoki $x \in A,\ y \in C$ kelib chiqib, $x \in A,\ y \in B \cup C$. Nihoyat, $(x, y) \in Ax(B \cup C)$.

a) va b) munosabatlar tenglik o'rinliligini bildiradi.

3. $A = (0;1)$ va $B = (0;1]$ to'plamlar orasida o'zaro bir qiymatli moslik o'rnating.

A va V to'plamlarda $\{x_1 = \frac{1}{2},\ x_2 = \frac{1}{3},\ \ldots, x_n = \frac{1}{n+1}, \ldots\}$ nuqtalar to'plamini ajratamiz.

B dagi 1 ga A dagi x_1 ni, x_n ga x_{n+1} ni mos qo'yamiz. B dagi qolgan $x \in (0;1)$ nuqtalarga A dagi mos $x \in (0;1)$ nuqtalarni qo'yamiz.

4. Ratsional sonlar to'plami Qning sanoqliligini isbotlang.

Har bir ratsional son qisqarmas $\frac{p}{q}$ ($p \in Z,\ q \in N$) kasr ko'rinishda yoziladi. $|p|+q$ yig'indi $\frac{p}{q}$ ratsional son balandligi deyiladi.

Ratsional sonlarni balandligi o'sish tartibida joylashtiramiz:

$$\frac{0}{1}, \frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{2}, \frac{2}{1}, \frac{-2}{1}, \frac{1}{3}, \frac{-1}{3}, \frac{3}{1}, \frac{-3}{1}, \ldots$$

Bunda har bir ratsional son biror nomer oladi va $Q \sim N$ ekanligi kelib chiqadi.

5. $[0;1]$ segment nuqtalari to'plami sanoqsizligini isbotlang.

Ular sanab chiqilgan: $\alpha_1, \alpha_2, \ldots \alpha_n, \ldots$ va cheksiz o'nli kasrga yoyilgan deylik:

$\alpha_1 = 0, a_{11}\, a_{12}\, a_{13} \ldots$

$\alpha_{21} = 0, a_{21}\, a_{22}\, a_{23} \ldots$

. . . .

$\alpha_n = 0, a_{n1}\, a_{n2}\, a_{n3} \ldots$

. . . .

Lekin, sanalmay qolib ketgan elementlarni topish mumkin. Masalan,

$$\alpha_0 = b_1 b_2\, b_3 \ldots \qquad (b_1 \neq a_{11},\ b_2 \neq a_{22}, \ldots)$$

sanalmagan.

Demak, $[a,b]$, $[a,b)$, $(a,b]$, (a,b) nuqtalar to'plamlari sanoqsiz ekan.

6. $a)$ $y = (x-2)\sqrt{\frac{1+x}{1-x}}$ aniqlanish sohasini toping.

$\frac{1+x}{1-x} \geq 0$ o'rinli bo'lishi shart, demak, intervallar metodidan

$D(y) = [-1;1)$ kelib chiqadi.

b) $y = \lg \sin x$ aniqlanish sohasini toping.

Logarifmik funktsiya $\sin x > 0$ qiymatlarda aniqlangan, xolos, ya'ni

$$\sin x > 0 \Rightarrow 2k\pi < x < \pi + 2k\pi, \ k \in N.$$

7. *a*) $f = x^2$, $g = 2^x$ bo'lsa, $f(g(x))$ – murakkab funktsiyani tuzing.

$$f(g(x)) = (2^x)^2 = 2^{2x} = 4^x.$$

b) $f(x) = \frac{x}{\sqrt{1+x^2}}$ bo'lsa, $f_n(x) = \underbrace{f(f(\ldots(f(x)\ldots)}_{n}$ ni toping.

$$f(f(x)) = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^{2}}} = \frac{x}{\sqrt{1+x^2+x^2}} = \frac{x}{\sqrt{1+2x^2}}$$

$$f(f(f(x))) = \frac{x}{\sqrt{1+3x^2}}, \ldots, f_n(x) = \frac{x}{\sqrt{1+nx^2}}.$$

8. $f(x) = \ln\frac{1-x}{1+x}$ funktsiyani juft – toqligini tekshiring.

$$f(-x) = \ln\frac{1+x}{1-x} = \ln\left(\frac{1-x}{1+x}\right)^{-1} = -\ln\frac{1-x}{1+x} = -f(x).$$

9. Quyidagi ifodalar qiymatini toping:

1) $$\lim_{x\to\infty}\frac{2n^2-1}{n^2+n}=\lim_{x\to\infty}\frac{\frac{2n^2}{n^2}-\frac{1}{n^2}}{\frac{n^2}{n^2}+\frac{n}{n^2}}=\lim_{x\to\infty}\frac{2-\frac{1}{n^2}}{1+\frac{1}{n}}=\frac{2}{1}=2.$$

2) $$\lim_{x\to\infty}[\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+....+\frac{1}{n(n+1)}]=\lim_{x\to\infty}[1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+....$$

$$+\frac{1}{n}-\frac{1}{n+1}]=\lim_{x\to\infty}[1-\frac{1}{n+1}]=1.$$

3) $$\lim_{x\to\infty}\left(\frac{n}{n+1}\right)^n=\lim_{x\to\infty}\left(\frac{n+1-1}{n+1}\right)^n=\lim_{x\to\infty}\left(1-\frac{1}{n+1}\right)^n=$$

$$=\lim_{x\to\infty}\left(1+(-\frac{1}{n+1})\right)^{-(n+1)\cdot\frac{n}{-(n+1)}}=e^{\lim\limits_{x\to\infty}\frac{-n}{n+1}}=e^{-1}.$$

10. Ifodalar qiymatini toping.

1) $$\lim_{x\to 2}\frac{x^2-5x+6}{x-2}=\lim_{x\to 2}\frac{(x-2)(x-3)}{x-2}=\lim_{x\to 2}(x-3)=-1.$$

2) $$\lim_{x\to 1}\frac{x+x^2+...+x^n-n}{x-1}=$$

$$=\lim_{x\to 1}\frac{(x-1)+(x^2-1)+...+(x^n-1)}{x-1}=$$

$$=\lim_{x\to 1}\frac{(x-1)[1+(x+1)+(x^2+x+1)+...+(x^{n-1}+x^{n-2}+...+1)]}{(x-1)}=$$

$$=1+2+3+...+n=\frac{n+1}{2}\cdot n$$

3)

$$\lim_{x\to -8}\frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}=\lim_{x\to -8}\frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}\cdot\frac{\sqrt{1-x}+3}{\sqrt{1-x}+3}\cdot\frac{(4-2\sqrt[3]{x}+\sqrt[3]{x^2}}{(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}=$$

$$= \lim_{x\to -8} \frac{(1-x-9)(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{(8+x)(\sqrt{1-x}+3)} = -\lim_{x\to -8} \frac{(4-2\sqrt[3]{x}+\sqrt[3]{x^2}}{\sqrt{1-x}+3} =$$

$$= \frac{-12}{6} = -2.$$

4) $\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx} = \lim\limits_{x\to 0} \frac{\sin mx}{mx} \cdot \frac{nx}{\sin nx} \cdot \frac{m}{n} = \frac{m}{n}.$

5)

$$\lim_{x\to 0}\left(\frac{1+tgx}{1+\sin x}\right)^{\frac{1}{\sin x}} = \lim_{x\to 0}\left(\frac{1+\sin x+tgx-\sin x}{1+\sin x}\right)^{\frac{1}{\sin x}} =$$

$$= \lim_{x\to 0}\left(1+\frac{tgx-\sin x}{1+\sin x}\right)^{\frac{1}{\sin x}} = \lim_{x\to 0}\left(1+\frac{tgx-\sin x}{1+\sin x}\right)^{\frac{+\sin x 1}{tgx-\sin x}\cdot\frac{tgx-\sin x}{1+\sin x}\cdot\frac{1}{\sin x}} =$$

$$e^{\lim\limits_{x\to 0}\frac{\sin x(\frac{1}{\cos x}-1)}{1+\sin x}\cdot\frac{1}{\sin x}} = e^{\lim\limits_{x\to 0}\frac{1-\cos x}{\cos x(1+\sin x)}} = e^0 = 1.$$

6) $\lim\limits_{x\to b} \frac{a^x - a^b}{x-b} = \lim\limits_{x\to b} \frac{a^b(a^{x-b}-1)}{x-b} = a^b \cdot \ln a.$

7) $\lim\limits_{x\to +\infty} [\sin\ln(x+1) - \sin\ln x] =$

$$= \lim_{x\to +a} 2\sin\frac{\ln(x+1)-\ln x}{2}\cos\frac{\ln(x+1)+\ln x}{2} =$$

$$= \lim_{x\to +\infty} \frac{2\sin\frac{1}{2}\ln(1+\frac{1}{x})\cdot\cos\ln\sqrt{x^2+x}}{\frac{1}{x}\cdot x} = 0$$

8) $y = 2^{\frac{1}{x-2}}$ funktsiyani $x = 2$ da uzluksizlikka tekshiring.

$x = 2$ da funktsiya aniqlanmagan.

$$\lim_{x\to 2+0} f(x) = \lim_{x\to 2+0} 2^{\frac{1}{x-2}} = +\infty; \quad \lim_{x\to 2-0} 2^{\frac{1}{x-2}} = 0$$

II – tur uzulishga ega.

9) $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$ funktsiyani

uzluksizlikka tekshiring.

$$\lim_{x\to 1-0} f(x) = \lim_{x\to 1-0} x^2 = 1; \quad \lim_{x\to 1+0} f(x) = \lim_{x\to 1+0} (2-x) = 1;$$

$f(1) = 1^2 = 1$

Funktsiya $x = 1$ nuqtada uzluksiz ekan.

4–bobga doir uy vazifalari

1. Funktsiya aniqlanish sohasini toping.

1) $y = \sqrt{\dfrac{x^2 - 4x + 3}{x^2 - 4}}$ 2) $y = \sqrt{\dfrac{x^2 - 9}{x^2 - 7x + 6}}$

3) $y = \sqrt{\dfrac{x^2 - 4x - 5}{x^2 + x - 2}}$ 4) $y = \sqrt{\dfrac{(x-1)(x^2 - 16)}{x^2 - 10x + 16}}$

5) $y = \sqrt{\dfrac{x^2 - 16}{x^2 - 7x + 6}}$ 6) $y = \sqrt{\dfrac{x^2 - 36}{x^2 - 5x}}$

7) $y = \log_x \dfrac{x-1}{x+3}$ 8) $y = \ln \dfrac{x^2 - 4}{9 - x^2}$

9) $y = \lg \dfrac{x^2 - 4x}{x^2 + 5x - 6}$ 10) $y = \lg \sin 2x$

11) $y = \lg \cos 3x$ 12) $y = \sqrt{tg 2x}$

13) $y = \lg(1 - 2\cos x)$ 14) $y = \lg(\sqrt{3} - ctgx)$

15) $y = \sqrt[4]{\lg tgx}$

2. Berilgan funktsiyaga teskari funktsiyani toping.

1) $y = 4x - 1$ 2) $y = 2x + 3$

3) $y = 1 - 4x$ 4) $y = 3 - 2x$

5) $y = 5x + 1$ 6) $y = 2 - 5x$

7) $y = 1 - x$ 8) $y = 4 + 3x$

9) $y = 4 - 5x$ 10) $y = 5x - 4$

11) $y = x - 10$ 12) $y = \dfrac{1+x}{1-x}$

13) $y = \dfrac{2x-1}{1+2x}$ 14) $y = \dfrac{3x-1}{1+3x}$

15) $y = \dfrac{1}{4x-1}$.

3. Quyidagi funktsiyalar grafigini chizing:

1) $y = \sin^2 x$ 2) $y = \sin^3 x$

3) $y = [\sin x]$ 4) $y = \{\sin x\}$

5) $y = \operatorname{sgn}(\cos x)$ 6) $y = [x^2]$

7) $y = x + e^x$ 8) $y = x + \sin x$

9) $y = \sin^4 x + \cos^4 x$ 10) $y = |x+3| + |x-3|$

11) $y = x \cdot \cos x$ 12) $y = 1 - e^{-x}$

13) $y = [x] \cdot |\cos x|$ 14) $y = \cos x \operatorname{sgn}(\sin x)$

15) $y = x + arctgx$

4. Limitlarni hisoblang

1) *a*) $\lim\limits_{n\to\infty} \dfrac{n^4 - 3n}{1 - 2n^4}$ *б*) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x} - \sqrt{1-x}}{5x}$

в) $\lim\limits_{x\to 0} \dfrac{1 - \cos 10x}{5x^2}$ *г*) $\lim\limits_{n\to\infty ч-2} \left(\dfrac{x+3}{x-2}\right)^x$

2) *а*) $\lim\limits_{x\to\infty}\dfrac{x^3+1}{2x^3+1}$; *б*) $\lim\limits_{x\to 7}\dfrac{\sqrt{2+x}-3}{x-7}$;

в) $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{5x}$; *г*) $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+1}\right)^x$.

3) *а*) $\lim\limits_{x\to\infty}\dfrac{2x^3+x^2-5}{x^3+x-2}$; *б*) $\lim\limits_{x\to 1}\dfrac{x-\sqrt{x}}{x^2-x}$;

в) $\lim\limits_{x\to 0}\dfrac{\sqrt{1-\cos 2x}}{x}$; *г*) $\lim\limits_{x\to\infty}\left(\dfrac{4x+1}{4x}\right)^{2x}$.

4) *а*) $\lim\limits_{x\to\infty}\dfrac{3x^4+x^2-6}{2x^4-x+2}$; *б*) $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{1+3x}-1}$;

в) $\lim\limits_{x\to 0}\dfrac{5x}{arctgx}$; *г*) $\lim\limits_{x\to 0}(1+2x)^{1/x}$.

5) *а*) $\lim\limits_{x\to\infty}\dfrac{2x^2+6x-5}{5x^2-x-1}$; *б*) $\lim\limits_{x\to 1}\dfrac{1-\sqrt{1-x^2}}{x^2}$;

в) $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^3 x}{x^2}$; *г*) $\lim\limits_{x\to\infty}x[\ln(x+1)-\ln x]$.

6) *а*) $\lim\limits_{x\to\infty}\dfrac{3+x+5x^4}{x^4-12x+1}$; *б*) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x}-\sqrt{1-2x}}{x+x^2}$;

в) $\lim\limits_{x\to 0}\dfrac{x^2 ctgx}{\sin 3x}$; *г*) $\lim\limits_{x\to\infty}(2+1)[\ln(x+3)-\ln x]$.

7) *а*) $\lim\limits_{x\to\infty}\dfrac{x-2x^2+5x^4}{2+3x^2+x^4}$; *б*) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x^2}-1}{x^2+x^3}$;

в) $\lim\limits_{x\to 0}\dfrac{1-\cos 6x}{1-\cos 2x}$;

г) $\lim\limits_{x\to\infty}(x-5)[\ln(x-3)-\ln x]$.

8) а) $\lim\limits_{x\to\infty}\frac{5x^2-3x+1}{3x^2+x-5}$; б) $\lim\limits_{x\to 0}\frac{\sqrt{2x-1}-\sqrt{5}}{x-3}$;

в) $\lim\limits_{x\to 0}\frac{tg^2\frac{x}{2}}{x^2}$; г) $\lim\limits_{x\to 1}(7-6x)^{x/(3x-3)}$.

9) а) $\lim\limits_{x\to\infty}\frac{7x^4-2x^3+2}{x^4+3}$; б) $\lim\limits_{x\to 0}\frac{\sqrt{1+3x}-\sqrt{2x+6}}{x^2-5x}$;

в) $\lim\limits_{x\to 0}\frac{1-\cos 4x}{2xtg2x}$ г) $\lim\limits_{x\to 2}(3x-5)^{\frac{2x}{x^2-4}}$.

10) а) $\lim\limits_{x\to\infty}\frac{8x^5-3x^2+9}{x^5+2x^2+5}$; б) $\lim\limits_{x\to 2}\frac{x-2}{\sqrt{2x}-2}$;

в) $\lim\limits_{x\to 0}5xctg3x$; г) $\lim\limits_{x\to 3}(3x-8)^{\frac{2}{x-3}}$.

11) а) $\lim\limits_{x\to\infty}\frac{2x^2-4x}{5x^2-6}$; б) $\lim\limits_{x\to 3}\frac{\sqrt{x+1}-3}{x^2-9}$;

в) $\lim\limits_{x\to 0}\frac{1-\cos 8x}{\sin^2 x}$; г) $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

12) а) $\lim\limits_{x\to\infty}\frac{1-2x+3x^2}{x^2+1}$; б) $\lim\limits_{x\to -1}\frac{\sqrt{5+x}-2}{x+1}$;

в) $\lim\limits_{x\to 0}\frac{\sqrt{1-\cos 4x}}{\sin x}$; г) $\lim\limits_{x\to 1}(4-3x)^{\frac{1}{x-1}}$.

13) а) $\lim\limits_{x\to\infty}\frac{4x^2-4x+5}{x^2-3x+10}$; б) $\lim\limits_{x\to 3}\frac{\sqrt{x+6}-3}{x^2-9}$;

в) $\lim\limits_{x\to 0}\frac{\cos x-\cos^3 x}{\sin^2 4x}$; г) $\lim\limits_{x\to 2}(5-2x)^{\frac{1}{2-x}}$.

14) $a)\ \lim_{x\to\infty}\frac{x^3-4x+1}{1-2x+2x^3}$ $б)\ \lim_{x\to 1}\frac{x^2-1}{\sqrt{x+3}-2}$;

$в)\ \lim_{x\to 0}\frac{tg^2\frac{x}{2}}{\sin^2 2x}$; $г)\ \lim_{x\to 0}(1+6x)^{\frac{2}{x}}$.

15) $a)\ \lim_{x\to\infty}\frac{4x^3-2x}{2+4x+5x^3}$; $б)\ \lim_{x\to 2}\frac{\sqrt{x+2}-2}{x^2-4}$;

$в)\ \lim_{x\to 0}\frac{\arcsin 5x}{2x}$; $г)\ \lim_{x\to\infty}\left(\frac{x+1}{x-4}\right)^x$.

5. Berilgan funktsiyalarni uzluksizlikka tekshiring, grafigini chizing.

1) $f(x)=\begin{cases} x+4, & x<-1 \\ x^2+2, & -1\le x<1 \\ 2x, & x\ge 1 \end{cases}$

2) $f(x)=\begin{cases} x+2, & x\le -1 \\ x^2+1, & -1<x\le 1 \\ -x+3, & x>1 \end{cases}$

3) $f(x)=\begin{cases} -x & x\le 0 \\ -(x-1)^2, & 0<x<2 \\ x-3, & x\ge 2 \end{cases}$

4) $f(x)=\begin{cases} \cos x, & x\le 0 \\ x^2+1, & 0<x<1 \\ x, & x\ge 1 \end{cases}$

5) $f(x)=\begin{cases} -x & x\le 0 \\ x^2 , & 0<x\le 2 \\ x+1 , & x>2 \end{cases}$

6) $f(x)=\begin{cases} -x & x\le 0 \\ \sin x , & 0<x\le \pi \\ x-2 , & x>\pi \end{cases}$

7) $f(x)=\begin{cases} -(x+1) & x\le -1 \\ (x+1)^2 , & -1<x\le 0 \\ x , & x>0 \end{cases}$

8) $f(x)=\begin{cases} -x^2 & x\le 0 \\ tgx , & 0<x\le \dfrac{\pi}{4} \\ 2 , & x>\dfrac{\pi}{4} \end{cases}$

9) $f(x)=\begin{cases} -2x & x\le 0 \\ x^2+1 , & 0<x\le 1 \\ 2 , & x>1 \end{cases}$

10) $f(x)=\begin{cases} -2x & x\le 0 \\ \sqrt{x} , & 0<x<4 \\ 1 , & x\ge 4 \end{cases}$

11) $f(x)=\begin{cases} -1 & x<-2 \\ |x| , & |x|\le 2 \\ 2, & x>2 \end{cases}$

12) $f(x)=\begin{cases} x^2 & x<0 \\ x-1, & 0<x<1 \\ 4, & x\geq 1 \end{cases}$

13) $f(x)=\begin{cases} x & x<0 \\ x^2, & |x|\leq 1 \\ x+4, & x>1 \end{cases}$

14) $f(x)=\begin{cases} -x^2, & x<0 \\ 1, & 0\leq x<1 \\ (x-1)^2+1, & x\geq 1 \end{cases}$

15) $f(x)=\begin{cases} -x & x\leq 0 \\ \sin x, & 0<x\leq \pi \\ x+\pi, & x>\pi \end{cases}$

5–bob. Hosila va differentsial. Differentsial hisob teoremalari.

§17. Hosila. Geometrik va fizik ma'nolari. Hosila hisoblash qoidalari.

$y = f(x)$ funktsiyaning x nuqtadagi hosilasi deb

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

limitga aytiladi va $y'(x)$, $f'(x)$, $\frac{dy}{dx}$, $\frac{df(x)}{dx}$ tarzida belgilanadi. Agar hosila chekli bo'lsa, $f(x)$ funktsiya x nuqtada differentsiallanuvchi deyiladi. Agar $x = x_1$ $x + \Delta x = x_2$ bo'lsa, $f(x)$ funktsiyaning x_1 nuqtadagi hosilasi

$\lim_{x_1 \to x_2} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$ ko'rinishda bo'ladi. Limitlardagiga o'xshash, chap va o'ng hosila tushunchasini kiritish mumkin. Hosila $y = f(x)$ funktsiyaga x_1 nuqtada o'tkazilgan urinmaning burchak koeffitsientidir:

$$y'(x_1) = tg\alpha$$

Urunma to'g'ri chiziqning tenglamasi $y - y_1 = y'(x_1) * (x - x_1)$, unga perpendikulyar $(x_1; y_1)$ nuqtadan o'tuvchi normal to'g'ri chiziq tenglamasi:

$$y - y_1 = -\frac{1}{y'(x_1)}(x - x_1)$$ ko'rinishida bo'ladi.

Moddiy nuqta S=S (t) qonun bo'yicha harakat qilsa, uning tezligi $V=\lim\limits_{\Delta t\to 0}\frac{\Delta S}{\Delta t}$, tezlanishi esa $a=\lim\limits_{\Delta t\to 0}\frac{\Delta V}{\Delta t}$ formulalar yordamida topiladi.

Hosilasi mavjud bo'lgan $u(x), v(x)$ funktsiyalar uchun quyidagi qoidalar o'rinli:

1. $(C\cdot u)'=C\cdot u'$;
2. $(C_1u\pm C_2v)'=Cu'\pm C_2v'$;
3. $(u\cdot v)'=u'v+uv'$;
4. $(\frac{u}{v})'=\frac{u'v-uv'}{v^2}$;
5. $y=f(x)$ va $x=g(y)$ o'zaro teskari funktsiyalar bo'lsa, $y'(x)=\frac{1}{x'(y)}$;
6. $y=f(u), u=g(x)$, bo'lsa, u holda $y=f(g(x))$ murakkab funktsiya hosilasi $y'=f'(u)\cdot g'(x)$ formula yordamida olinadi.
7. $y=u^V$ ko'rinishdagi funktsiya daraja–ko'rsatkichli deyilib, hosilasi quyidagi Bernulli formulasidan topiladi: $y'=u^v\cdot\left[v'\cdot\ln u+\frac{u'v}{u}\right]$
8. y ga nisbatan echilmagan tenglama bilan berilgan funktsiya hosilasi, murakkab funktsiya kabi olinadi, hosil bo'lgan tenglamadan y' topiladi.

9. $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ sistema yordamida parametrik berilgan funktsiya hosilasi $y'_t = \dfrac{y'_t}{x'_t}$ formula yordamida topiladi.

Hosilalar jadvali:

1. $(C)' = 0;$
2. $(x^p)' = p \cdot x^{p-1};$
3. $(Sinx)' = Cosx;$
4. $(Cosx)' = -Sinx;$
5. $(tgx)' = \dfrac{1}{Cos^2 x};$
6. $(ctgx)' = -\dfrac{1}{Sin^2 x};$
7. $(a^x) = a^x \cdot \ln a;$
8. $(e^x)' = e^x;$
9. $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}};$
10. $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}};$
11. $(arctgx)' = \dfrac{1}{1+x^2};$
12. $(arcctgx)' = -\dfrac{1}{1+x^2};$
13. $(\log_a x)' = \dfrac{1}{x \cdot Lna};$
14. $(\ln x)' = \dfrac{1}{x};$
15. $(shx)' = chx;$
16. $(chx)' = chx;$
17. $(thx)' = \dfrac{1}{ch^2 x};$
18. $(Cthx)' = -\dfrac{1}{sh^2 x};$

17.1. Ta'rif yordamida hosilasini toping.

1) $y = x^3;$ 2) $y = x^{100};$ 3) $y = \sqrt{x};$

4) $y=\frac{1}{x}$; 5) $y=\frac{1}{\sqrt{x}}$; 6) $y=\frac{1}{x^3}$;

7) $y=\sqrt{1+x^2}$; 8) $y=tgx$

9) $f(x)=x(x-1)(x-2)......(x-100)$ bo'lsa, $f'(0)$ ni hisoblang.

17.2. Hosila hisoblash qoidalari va jadvalidan foydalanib toping.

1) $y=\frac{x^4}{4}-\frac{x^2}{2}+4x$; 2) $y=\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}$;

3) $y=4\sqrt[3]{x}-3\sqrt[4]{x}$; 4) $y=x-\sin x$

5) $y=x^2\cdot ctgx$ 6) $y=x^2\cdot ctgx$

7) $y=\frac{2x}{1-x^2}$ 8) $y=\frac{\cos x}{x^2}$

9) $y=\sqrt{x}\cdot\cos x$ 10) $y=\frac{x^2-1}{x^2+1}$

11) $f(x)=\sqrt[3]{x^2}$ bo'lsa, $f'(-8)$ topilsin.

12) $f(x)=\frac{x}{2x-1}$ bo'lsa, $f'(0)$, $f'(2)$, $f'(-2)$ topilsin.

13) $y=sh^2x$ 14) $y=thx+cthx$

15) $y=x-cthx$

17.3. Murakkab funktsiyalar hosilasini oling:

1) $y=\sin 5x$; 2) $y=(1-2x)^3$;

3) $y=\sqrt{1-x^2}$; 4) $y=\sqrt{\cos 4x}$;

5) $y=\sqrt{2x-\sin 2x}$; 6) $y=\sin^2 x$;

7) $y=\sin^3 x+\cos^3 x$; 8) $y=\sin\sqrt{x}$;

9) $y=3^{4x}$; 10) $y=\ln 5x$;

11) $y = e^{x^2}$; 12) $y = \ln tg\frac{x}{2}$;

13) $y = \frac{\cos x}{2\sin^2 x}$; 14) $y = \frac{\sin x - x\cos x}{\cos x + x\sin x}$

15) $y = tgx - \frac{1}{3}tg^3 x + \frac{1}{5}tg^5 x$; 16) $y = \lg^3 x$;

17) $y = \frac{1}{2}ctg^2 x + \ln\sin x$; 18) $y = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}$

19) $y = \arccos\frac{1-x}{\sqrt{2}}$; 20) $y = arctg\frac{x^2}{a}$

21) $y = \arcsin(\sin x)$;

22) $y = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\arcsin\frac{x}{a}$;

23) $y = \arccos(\sin x^2 - \cos x^2)$; 24) $y = \sqrt[x]{x}$;

25) $y = x^x$; 26) $y = x^{tgx}$;

27) $y = x^{x^x}$; 28) $x^2 + y^2 = a^2$;

29) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 30) $y^2 = 2px$;

31) $\sqrt{x} + \sqrt{y} = \sqrt{a}$; 32) $\sqrt[3]{x^2} + \sqrt[3]{y^2} = \sqrt[3]{a^2}$;

33) $x = y + arcctgy$; 34) $e^y - e^{-x} + xy = 0$;

35) $y = \ln[chx]$; 36) $y = \arcsin[thx]$;

37) $y = \sqrt{1 + sh^2 4x}$. 38) $\begin{cases} x = \sin^2 t \\ y = \cos^2 t \end{cases}$

39) $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$ 40) $\begin{cases} x = e^{2t}\cdot\cos^2 t \\ y = e^{2t}\cdot\sin^2 t \end{cases}$

17.4. 1) $y=(x+1)\sqrt[3]{3-x}$ funktsiyaga $A(-1;0)$, $B(2;3)$, $C(3;0)$ nuqtalarda o'tkazilgan urinma va normal tenglamalarini yozing.

2) Qanday nuqtalarda $y=2+x-x^2$ chiziqqa o'tkazilgan urinma

a) Ox o'qiga parallel b) $y=x$ ga parallel bo'ladi?

3) $y=\sin x$ va $y=\cos x$ chiziqlar qanday burchak ostida kesishadi?

17.5. ITCh (aylana, ellips, giperbola, parabola) larga urinma tenglamasini chiqaring.

17.6. Berilgan funktsiyalar uchun chap va o'ng hosilalarni hisoblang.

1) $y=\sqrt[3]{x^2}$; $x_0=0$ 2) $y=|\ln x|$, $x_0=1$

3) $y=\sqrt{\sin^2 x}$; $x_0=0$ 4) $y=|x-2|$, $x_0=2$

17.7. 1) Jism harakat qonuni $S(t)=\frac{1}{2}t^2+3t+2$ formula bilan berilgan 4s da jism qanday yo'l bosib o'tgan? Shu vaqt momentida harakat tezligi qanday?

2) $S(t)=6t^2-t^3$ qonun bilan harakatlanayotgan jismning eng katta tezligi qancha?

§18. Funktsiya differentsiali.
Yuqori tartibli hosila va differentsial.

Funktsiyaning $(n-1)$ tartibli hosilasidan olingan hosila $n-$ tartibli hosila deyiladi va $y^{(n)}$ orqali belgilanadi:

$$y^{(n)}=(y^{(n-1)})'$$

$$\left(a^x\right)^{(n)} = a^x \cdot \ln^n a; \quad (e^x)^{(n)} = e^x;$$

$$(\sin x)^{(n)} = \sin(x + \frac{n\pi}{2});$$

$$(\cos x)^{(n)} = \cos(x + \frac{n\pi}{2}); \quad (x^m)^{(n)} =$$

$$= m(m-1)\cdots(m-n+1)x^{m-n};$$

$$(\ln x)^{(n)} = \frac{(-1)^{n-1}\cdot(n-1)!}{x^n}$$

formulalarni isbotlash mumkin.

Funktsiya orttirmasini $\Delta y = y'\cdot\Delta x + 0(\Delta x)$ ko'rinishda yozish mumkin. Δy ning bosh qismi $y'\cdot\Delta x$ funktsiya differentsiali deyiladi va dy tarzida belgilanadi.

$dy = y'dx$, chunki $dx = 1\cdot\Delta x$.

Hosila yordamida differentsiallash qoidalari quyidagicha ko'rinish oladi:

$$d(u \pm v) = du + dv,$$

$$d(u \cdot v) = vdu + udv,$$

$$d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}$$

Yuqori tartibli differentsial $d^n y = y^{(n)}dx^n$ formuladan topiladi.

$dy \approx \Delta y$ dan $f(x+\Delta x) \approx f(x) + f'(x)\cdot\Delta x$ taqribiy hisoblash formulasini keltirib chiqarish mumkin.

18.1. 2 – tartibli hosilalarini toping.

1) $y = \sin^2 x$ 2) $y = tgx$

3) $y = \sqrt{1+x^2}$ 4) $y = x\cdot\sin x$

18.2. n – tartibli hosilalarini toping.

1) $y = e^{-ax}$ 2) $y = \ln x$

3) $y=\sqrt{x}$ 4) $y=x^n$

5) $y=\dfrac{1+x}{\sqrt{1-x}}$ 6) $y=xshx$ 7) $y=x\cdot\ln x$

18.3. Leybnits formulasidan foydalanib, funktsiyalarning 2,3 – tartibli hosilalarini yozing.

1) $y=e^x\cdot\cos x$ 2) $y=x\cos x$

3) $y=x^3\cdot e^x$ 4) $y=x^2\cdot\ln x$

18.4. 1) $f(x)=\dfrac{x}{\sqrt{1+x}}$ uchun $f^{(n)}(0)$ ni hisoblang.

2) $f(x)=x^n$ uchun

$f(1)+\dfrac{f'(1)}{1!}+\dfrac{f''(1)}{2!}+....+\dfrac{f^{(n)}(1)}{n!}=2^n$ ekanligini ko'rsating.

18.5. Taqribiy qiymatini toping.

1) $\sqrt{24}$ 2) $\sqrt[3]{65}$ 3) $\sin 31^0$ 4) $\lg 11$

§19. Differentsial hisob asosiy teoremalari.

Teorema (Ferma). Agar $f(x)$ funktsiya $c\in(a,b)$ nuqtada o'zining eng katta (kichik) qiymatiga erishsa, bu nuqtada chekli hosilaga ega bo'lsa, $f'(c)=0$ bo'ladi.

Teorema (Roll'). Agar $f(x)$ funktsiya $[a,b]$ segmentda uzluksiz, $f(a)=f(b)$ va (a,b) da chekli hosilaga ega bo'lsa, u holda kamida bitta $c\in(a,b)$ nuqta topiladiki, $f'(c)=0$ bo'ladi.

Teorema (Koshi): f, g funtsiyalar $[a,b]$ da uzluksiz, (a,b) da chekli hosilalarga ega, $g'(x) \neq 0$ bo'lsa, shunday $c \in (a,b)$ topiladiki:

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)} \text{ bo'ladi.}$$

Teorema (Lagranj): $f(x)$ funktsiya Koshi teoremasi shartlarini qanoatlantirsa, u holda shunday $c \in (a,b)$ topiladiki:

$$\frac{f(b)-f(a)}{b-a} = f'(c) \text{ o'rinli.}$$

19.1. $f(x)=(x-1)(x-2)(x-3)$ funktsiya uchun Roll' teoremasi o'rinliligini tekshiring.

19.2. $f(x)=1-\sqrt[3]{x^2}$ funktsiyaga $[-1;\ 1]$ oralig'ida Roll' teoremasini tatbiq etib bo'ladimi? Chizmada tushuntiring.

19.3. Qaysi nuqtada $y=x^2$ ga o'tkazilgan urinma $A(-1;1)$, $B(3;\ 9)$ nuqtalarni tutashtiruvchi vektorga parallel bo'ladi?

19.4. $f(x)=x^2$ uchun $[a,b]$ da Lagranj formulasini yozing va c ni toping.

19.5. $f(x)=arctgx$ uchun $[0;1]$ da Lagranj formulasini yozing va c ni toping.

19.6. $f(x)=\ln x$ uchun $[1;2]$ da Lagranj formulasini yozing va c ni toping.

19.7. $\sin x$ va $\cos x$ uchun $[0;\frac{\pi}{2}]$ da Koshi formulasini yozing va c ni toping.

19.8. x^2 va $\sqrt{x}$ uchun $[1;4]$ da Koshi formulasini yozing va c ni toping.

19.9. x^2 va x^3 uchun $[-1;1]$ kesmada nega Koshi formulasi o'rinli emasligini tushuntiring.

19.10. Isbotlang:

1) $|\sin x - \sin y| \le |x - y|$

2) $\dfrac{a-b}{a} < \ln\dfrac{a}{b} < \dfrac{a-b}{b}, \quad 0 < b < a.$

Bobga doir misollar echish namunalari

1. Ta'rif yordamida hosilasini oling.

1) $y = \dfrac{1}{\sqrt{x}}$

$$y' = \lim_{\Delta x \to 0} \frac{\dfrac{1}{\sqrt{x+\Delta x}} - \dfrac{1}{\sqrt{x}}}{\Delta x} = \lim_{\Delta x \to 0} \frac{-(\sqrt{x+\Delta x} - \sqrt{x})}{\sqrt{x+\Delta x}\cdot\sqrt{x}\cdot\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-\sqrt{x}\left(\sqrt{1+\dfrac{\Delta x}{x}} - 1\right)}{\sqrt{x+\Delta x}\cdot\sqrt{x}\cdot\dfrac{\Delta x}{x}\cdot x} =$$

$$= \lim_{\Delta x \to 0} \frac{1}{x\cdot\sqrt{x+\Delta x}} \cdot \lim_{\Delta x \to 0} \frac{(1+\frac{\Delta x}{x})^{\frac{1}{2}} - 1}{\dfrac{\Delta x}{x}} = -\frac{1}{2}\cdot\frac{1}{x\sqrt{x}}$$

2). $y = (x-1)(x-2)^2(x-3)^3$ uchun $y'(1)$, $y'(2)$, $y'(3)$ qiymatlarini hisoblang.

$$y' = \lim_{\Delta x \to 0} \frac{(x+\Delta x-1)(x+\Delta x-2)^2(x+\Delta x-3)^3 - (x-1)(x-2)^2(x-3)^3}{\Delta x}$$

uchun

$y'(1) = \lim\limits_{\Delta x \to 0}(1+\Delta x-2)^2(1+\Delta x-3)^3$, ya'ni

$y'(1) = (-1)^2\cdot(-2)^3 = -8$

Shunga o'xshash, $y'(2) = y'(3) = 0$ topiladi.

2. Hosila hisoblash qoidasi va jadvalidan foydalanib toping:

1). $y = \sin(\cos^2 x)\cdot\cos(\sin^2 x)$

$$y' = \left[\sin(\cos^2 x)\right]'\cdot\cos(\sin^2 x) + \sin(\cos^2 x)\cdot\left[\cos(\sin^2 x)\right]' =$$

$$= \cos(\cos^2 x)\cdot 2\cos x\cdot(-\sin x)\cdot\cos(\sin^2 x) +$$

$$+\sin(\cos^2 x)\cdot\left[-\sin(\sin^2 x)\cdot 2\sin x\cdot\cos x\right] =$$

$$= -\sin 2x\left[\cos(\cos^2 x)\cdot\cos(\sin^2 x) + \sin(\sin^2 x)\cdot\sin(\cos^2 x)\right] =$$

$$= -\sin 2x\cdot\cos(\cos^2 x - \sin^2 x) = -\sin 2x\cdot\cos(\cos 2x);$$

2). $y = \ln(x + \sqrt{x^2+1});$

$$y' = \frac{1}{x+\sqrt{x^2+1}}\cdot\left(1 + \frac{1}{2\sqrt{x^2+1}}\cdot 2x\right) = \frac{1}{x+\sqrt{x^2+1}}\cdot\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}};$$

3). $\mathrm{y} = \sin \mathrm{x}^{\cos \mathrm{x}}$

$$\mathrm{y}' = \sin \mathrm{x}^{\cos \mathrm{x}}\cdot\left[-\sin \mathrm{x}\cdot\ln\sin \mathrm{x} + \frac{\cos \mathrm{x}\cdot\cos \mathrm{x}}{\sin \mathrm{x}}\right];$$

4). $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1;$

$\frac{2\mathrm{x}}{\mathrm{a}^2} - \frac{2\mathrm{y}\cdot\mathrm{y}'}{\mathrm{b}^2} = 0$ dan $\mathrm{y}' = \frac{\mathrm{b}^2\cdot\mathrm{x}}{\mathrm{a}^2\cdot\mathrm{y}};$

3. 1). $y = \frac{x^3}{3}$ funktsiyaga $x = -1$ nuqtada o'tkazilgan urinma va normal tenglamasini yozing.

$y(-1) = -\frac{1}{3};$ $y' = x^2;$

$y'(-1) = (-1)^2 = 1$ ekanligidan urinma $y + \frac{1}{3} = x + 1;$

normal esa $y+\frac{1}{3}=-x+1$ tenglama bilan aniqlanadi.

2). $y=x^2$ va $x=y^2$ funktsiyalar qandan burchak ostida kesishadilar?

Bu funktsiyalar kesishish nuqtasini topamiz: $y=y^4$ dan $y_1=0;\ y_2=1$. Demak, $(0;0),\ (1;1)$ nuqtalarda kesishar ekan. Masalan, $(0;0)$ nuqtada berilgan funktsiyalar urinmalari orasidagi burchakni topamiz.

$y=x^2$ uchun $y'=2\cdot x;\ y'(0)=0;\ y(0)=0,$ ya'ni $y=0$

$y=\pm\sqrt{x}$ uchun $y'=\frac{1}{2\sqrt{x}};\ y'(0)$ da aniqlanmagan, lekin urinmasi $x=0$ desak, ular 90^0 burchak ostida kesishishi kelib chiqadi.

(1; 1) nuqtada $y-1=2(x-1)$;

$y-1=\pm\frac{1}{2}(x-1)$ urinmalarga ega bo'lamiz:

$k_1=2,\ k_2=-\frac{1}{2}$ uchun $\varphi=90^0$; $k_1=2;\ k_2=\frac{1}{2}$

uchun esa $tg\varphi=\frac{2-\frac{1}{2}}{1+2\cdot\frac{1}{2}}=\frac{\frac{3}{2}}{2}=\frac{3}{4}$ dan $\varphi=arctg\frac{3}{4}$

kelib chiqadi.

4. $y=|x|$ ning $x=0$ nuqtada hosilasi mavjud emasligini isbotlang.

$y=|x|=\begin{cases}-x, & x<0\\ x, & x\geq 0\end{cases}$ ekanligidan

$$y'_{-}=\lim_{\Delta x\to -0}\frac{|x+\Delta x|-|x|}{\Delta x}=\lim_{\Delta x\to -0}\frac{|x|+|\Delta x|-|x|}{\Delta x}=-1;\ \ y'_{+}=1$$

ekanligidan $y'(0)$ mavjud emas.

5. Moddiy nuqta Ox o'qi bo'ylab $x=\frac{t^3}{3}-2t^2+3t$ qonun bo'yicha harakatlanayapti. Uning tezligi va tezlanishini aniqlang. Nuqta qaysi koordinatalarda yo'nalishini o'zgartiradi?

$x'=t^2-4t+3=V(t)$

Bundan $t_1=1,\ t_2=3$ da yo'nalishini o'zgartirishi kelib chiqadi.

$a(t)=V'(t)=2t-4$ dir.

6. $(u\cdot v)^{(n)}$ uchun Leybnits formulasini yozing.

$$(u\cdot v)'=u'\cdot v+u\cdot v'$$

$$(u\cdot v)''=u''v+u'v'+u'v'+u\cdot v''=u''+2u'\cdot v'+v''$$

$$(u\cdot v)'''=u'''+3u''v'+3u'\cdot v''+v'''$$

Demak:

$$(u\cdot v)^{(n)}=u^{(n)}+n\cdot u^{(n-1)}\cdot v'+\frac{n(n-1)}{2!}u^{(n-2)}\cdot u^{(n-2)}+.....+v^{(n)}.$$

7. $y=\frac{1}{1-x^2}$ uchun $y^{(n)}(0)$ ni hisoblang.

$$y=\frac{1}{1-x^2}=\frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)=\frac{1}{2}\left[(1+x)^{-1}+(1-x)^{-1}\right]$$

ekanligidan:

$$y'=\frac{1}{2}[-1(1+x)^{-2}+(1-x)^{-2}]$$

$$y''=\frac{1}{2}[(-1)(-2)\cdot(1+x)^{-3}-2(1-x)^{-3}]$$

$$y'''=\frac{1}{2}[(-1)(-2)(-3)(1+x)^{-4}-2\cdot(-3)(1-x)^{-4}]$$

. . . .

$$y^{(n)}=\frac{1}{2}[(-1)^n(1+x)^{-(n+1)}\cdot n!+(-1)^{n+1}\cdot n!(1-x)^{-(n+1)}]=$$

$$=\frac{(-1)^n\cdot n!}{2}[(1+x)^{-(n+1)}-(1-x)^{-n-1}]$$

Demak: $y_{(0)}^{(n)}=\begin{cases} n!, & agar\ \ n=2m \\ 0, & agar\ \ n=2m+1 \end{cases}$

8. Taqribiy qiymatini toping.

1) $\sqrt{17}=\sqrt{16+1}\approx\frac{1}{2\sqrt{16}}\cdot 1+\sqrt{16}=\frac{1}{8}+4=4\frac{1}{8}=4{,}125.$

2) $e^{2,1}=e^{2+0,1}\approx e^2+e^2\cdot 0{,}1=1{,}1\cdot e^2$

9. $f(x)=x^2-4x+3$ ildizlari orasida hosilaning ildizi mavjud. Sababini tushuntiring.

$$f(x)=x^2-4x+3=(x-1)(x-3)$$

ekanligidan $f(1)=f(3)=0$. Funktsiya $[1,3]$da uzluksiz, chekli hosilaga ega. Demak, Roll' teoremasi shartlari bajarilayapti. Shunday $c\in(1;\ 3)$ nuqta mavjudki $f'(c)=0$ bo'ladi.

Rostdan ham $f'(x)=2x-4=0,\ x=2$ ya'ni $c=2$ ekanligini ko'rish mumkin.

10. $f(x)=\sqrt{x}$ uchun $[1;4]$ kesmada Lagranj formulasini yozing va c ni toping.

Funktsiya Lagranj teoremasi shartlarini qanoatlantiradi, demak,

$$\frac{f(4)-f(1)}{4-1}=f'(c) \text{ o'rinli.}$$

$\frac{\sqrt{4}-\sqrt{1}}{4-1}=\frac{1}{2\sqrt{c}}$ ekanligidan $\sqrt{c}=\frac{3}{2}$ ya'ni $c=\frac{9}{4}$.

11. $f(x)=x^3$ va $g(x)=x^2$ funktsiyalar uchun Koshi formulasini yozing va c ni toping.

Koshi teoremasi shartlari bajariladi, shuning uchun

$$\frac{b^3-a^3}{b^2-a^2}=\frac{3c^2}{2c}.$$

Bundan $c=\frac{2(b^3-a^3)}{3(b^2-a^2)}=\frac{2(b^2+ab+a^2)}{3(b+a)}$.

5–bobga doir uy vazifalari.

I. Berilgan funktsiyaning 1) ta'rif bo'yicha hosilasi; 2)–6) jadval bo'yicha hosilasi; 7) n–tartibli hosilasi topilsin.

1. 1). $y=x^2-3x$ 2). $y=2\sqrt{4x+3}-\frac{3}{\sqrt{x^2+x}}$

3). $y=(c^{\cos x}+1)^2$ 4). $y=\ln\sin 2x$

5). $y=x^{\ln x}$ 6). $tg(y/x)=5x$ 7). $y=\sin 10x$

2. 1). $y=x^3-1$ 2). $y=x^2\sqrt{1-x^2}$

3). $y=\sin x/\cos^2 x$ 4). $y=arctge^{2x}$ 5). $y=x^{1/x}$

6). $x-y+arctgy=0$ 7). $y=\cos 11x$

3.

1). $y=\frac{1}{1-x}$ 2). $y=\frac{1-x}{1+x^2}$ 3). $y=\frac{1}{tg^2 2x}$

4). $y=\arcsin\sqrt{1-3x}$ 5). $y=(\sin x)^x$

6). $y\sin x=\cos(x-y)$ 7). $y=\ln 2x$

4.

1). $y=\frac{1}{x^2}$ 2). $y=x^2\cos x$ 3). $y=\sin x-x\cos$

4). $y=x^m\ln x$ 5). $y=x^{tgx}$ 6). $\frac{y}{x}=arctg\frac{x}{y}$

7). $y=\frac{1}{1-x^2}$

5.

1). $y=\sin 3x$ 2). $y=\frac{x}{\sqrt{a^2-x^2}}$ 3). $y=\frac{\sin^{2x}}{2+3\cos^2 x}$

4). $y=\frac{x\ln x}{x-1}$ 5). $y=(arctgx)^{\ln x}$

6). $(e^x-1)(e^y-1)-1=0$ 7). $y=\frac{1}{x(1+x)}$

6. 1) $y=\sqrt[4]{x}$ 2) $y=\frac{1}{\sqrt{x^2+1}}+5\sqrt[5]{x^3+1}$

3) $y=tg^3(x^2+1)$ 4) $y=3^{arctgx^3}$

5) $y=(arctgx)^x$ 6) $y^2x=e^{y/x}$

7) $y=\frac{x}{\sqrt{1-x}}$

7.1) $y=\frac{1}{\sqrt[3]{x}}$ 2) $y=\sqrt[3]{\frac{1+x^2}{1-x^2}}$

3) $y=\frac{1}{2}tg^2x+\ln\cos x$

4) $y=arctg\frac{x}{1+\sqrt{1-x^2}}$

5) $y=\left(x+x^2\right)^x$ 6) $x^3+y^3-3axy=0$

7) $y=\dfrac{1}{x(x+3)}$

8. 1) $y=\log_2(1+x)$ 2) $y=3\sqrt[3]{x^5+5x^4-\dfrac{5}{x}}$

3) $y=\ln\sqrt{\dfrac{1-\sin x}{1+\sin x}}$ 4) $y=arctg(tg^2x)$

5) $y=(\sin x)^{\ln x}$ 6) $x-y+a\sin y=0$

7) $y=\ln(1+x^2)$.

9.

1) $y=\sqrt[5]{x+1}$ 2) $y=5\sqrt[5]{x^2+x+\dfrac{1}{x}}$ 3) $y=2^xe^{-x}$

4) $y=\dfrac{\arcsin x}{\sqrt{1-x^2}}$ 5) $y=(\cos x)^x$ 6) $\ln y=arctg\dfrac{x}{y}$

7) $y=\dfrac{x}{x-1}$

10.

1) $y=x^3-x$ 2) $y=\sqrt{x^2+1}+\sqrt{x^3+1}$

3) $y=\dfrac{1}{3}tg^3x-tgx+x$ 4) $y=arctg\sqrt{\dfrac{3-x}{x-2}}$

5) $y=(\cos x)^{x^2}$ 6) $x-y+e^yarctgx=0$

7) $y=\sin 10x$.

11.

1) $y = 3 - x^2$; 2) $y = \frac{\sqrt{x^2 - 1}}{x} + \arcsin\frac{1}{x}$;

3) $y = \ln\frac{1 - \cos x}{1 + \cos x}$; 4) $y = \frac{tg^2 x}{2} + \ln\cos x$;

5) $y = (1 + x)^{\frac{1}{x}}$; 6) $x^2 + 2xy + y^2 - 5x = 0$; 7) $y = e^{2x} + \frac{1}{x}$.

12.

1) $y = \frac{1}{x}$; 2) $y = tg\sqrt{\frac{\sin x + \cos x}{\sin x - \cos x}}$;

3) $y = e^{2x} \cdot \sin x$ 4) $y = \frac{\arccos x}{\sqrt{1 - x^2}}$;

5) $y = (\ln x)^{\sin x}$ 6) $\ln(x + y) = 2xy - y^2$ 7) $y = x\sin x$

13.

1) $y = \frac{2}{1 + 4x}$ 2) $y = \ln\sqrt{\frac{1 - x}{1 + x^2}}$

3) $y = tg^2(4x + 1)$ 4) $y = \arccos\sqrt{1 + 4x}$

5) $y = x^{\ln x}$ 6) $y + yx = \ln\frac{x}{y}$ 7) $y = \frac{1}{(x - 1)(x + 3)}$

14.

1) $y = chx$ 2) $y = x\sqrt{1+x^2}$

3) $y = \lg^3 x^2$ 4) $y = \dfrac{\ln 3 \sin x + cox}{3^x}$

5) $y = x^{x^x}$ 6) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ 7) $y = \sin^2 x$

15. 1) $y = \dfrac{1}{x^2+1}$ 2) $y = 4\sqrt[3]{ctg^2 x} + \sqrt[3]{ctg^8 x}$

3) $y = \sqrt{x+1} - \ln\left(1+\sqrt{x+1}\right)$

4) $y = \arccos\left(\sin^2 x - \cos^2 x\right)$

5) $y = \dfrac{(\ln x)^x}{x^{\ln x}}$ 6) $(x+y)^2 = x - y$

7) $y = \cos^2 x$

II. Berilgan funktsiyalar uchun [a,v] oralig'da Koshi formulasini yozing va c ni toping.

1) x^3 *va* $\sqrt{x}$, $[1;4]$

2) x^3 *va* $3 - x^2$, $[1;\sqrt{3}]$

4) $\sin x$ *va* $\cos x$, $[0;\dfrac{\pi}{2}]$

5) tgx *va* $\cos x$, $[0;\dfrac{\pi}{4}]$

6) $1 + x^3$ *va* $\sqrt[3]{x}$, $[0;4]$

7) $\ln x$ *va* x^2, $[1;3]$

8) $\cos x$ *va* $x + 4$, $[0;\dfrac{\pi}{4}]$

9) x^2 *va* $1 - x^3$, $[0;2]$

10) x^3 *va* $1 - x^2$, $[0;3]$

11) $x + 3$ *va* $\sqrt{x}$, $[1;4]$

12) $ctgx$ *va* x, $[\frac{\pi}{4};\frac{\pi}{2}]$

13) x^4 *va* $\sqrt[4]{x}$, $[1;4]$

14) x^2-1 *va* $1-x$, $[0;4]$

15) $\sin x$ *va* $ctgx$, $[\frac{\pi}{6};\frac{\pi}{3}]$

§20 Teylor formulasi.

f(x) funktsiya $x_0 \in R$ nuqtaning biror atrofida $f', f'', ... f^{(n)}, f^{(n+1)}$ hosilalarga ega va $f^{(n+1)}$ hosila x_0 nuqtada uzluksiz bo'lsin.

U holda

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + ...$$

$$.. + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^{(n)} + 0(x - x_0)^n$$

tenglik o'rinli va bu yoyilma yagona bo'lib Teylor formulasi deyiladi.

Xususan $x = 0$ da Makloren formulasini olamiz.

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(n)}(0)}{n!}x^n + 0(x^n)$$

Makloren formulasidan quyidagi yoyilmalarni olish mumkin.

I. $e^x = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + 0(x^n)$

II. $\sin x = x - \frac{x^3}{3!} + ... + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + 0(x^{2n})$

III. $\cos x = x - \frac{x^2}{2!} + ... + (-1)^n\frac{x^{2n}}{(2n)!} + 0(x^{2n+1})$

VI.

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + ...$$

$$+ \frac{m(m-1)...(m-n+1)}{n!}x^n + 0(x^n)$$

V. $\ln(1+x) = x - \frac{x^2}{2!} + ... + (-1)^{n-1}\frac{x^n}{n!} + 0(x^n)$

20.1 Quyidagi funktsiyalarni x ning darajalari bo'yicha yoying

1) $\sin^2 x$ 2) e^{2x-x^2} *ni* x^5 *gacha*

3) $\ln\cos x$ *ni* x^6 *gacha*

20.2 1) $f(x)=x^3-3x$ *ni* $(x-1)$ darajalari bo'yicha yoying.

2) $f(x)=\sin 3x$ *ni* $(x+\frac{\pi}{3})$ darajalari bo'yicha yoying.

3) $f(x)=\sqrt[3]{x}$ *ni* $(x+1)$ darajalari bo'yicha yoying.

20.3 Teylor formulasi yordamida taqribiy hisoblang

1) $\sqrt[3]{30}$ 2) $\sqrt[5]{250}$ 3) $\sqrt{e}$ 4) $\ln 1,2$ 5) $\sin 18^0$

20.4 Yoyilmalardan foydalanib limitni toping

1) $\lim\limits_{x\to 0}\frac{e^x\sin x - x(1+x)}{x^3}$ 2) $\lim\limits_{x\to 0} x^{\frac{2}{3}}(\sqrt{x+1}+\sqrt{x-1}-2\sqrt{x}]$

3) $\lim\limits_{x\to 0}\frac{a^x - a^{-x}-2}{x^2}$ $(a>0)$ 4) $\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

5) $\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{1}{x}-ctgx\right)$

§21 Lopital qoidalari.

Agar $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} g(x)=0$ $(yoki\ \infty)$ bo'lib,

$\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$ mavjud bo'lsa, u holda

$\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$ o'rinlidir. Bu qoida $\frac{0}{0},\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarni echishda Lopitalning I (II) qoidasi deyiladi.

$0\cdot\infty, \infty-\infty, 1^{\infty}, 0^{0}$ tipidagi aniqmasliklar algebraik almashtirishlar yordamida yuqoridagi ikki holga keltiriladi.

21.1 Limitlarni hisoblang

1) $\lim\limits_{x\to 0}\frac{\sin 5x}{x}$ 2) $\lim\limits_{x\to 0}\frac{\sin ax}{\sin bx}$ 3) $\lim\limits_{x\to 0}\frac{tgx-x}{x-\sin x}$

4) $\lim\limits_{x\to 0}\frac{3tg4x-12tgx}{3\sin 4x-12\sin x}$ 5) $\lim\limits_{x\to 0}\frac{tg3x}{tgx}$

6) $\lim\limits_{x\to 0}\frac{\ln(\cos ax)}{\ln(\cos bx)}$ 7) $\lim\limits_{x\to +0}x^{x}$

8) $\lim\limits_{x\to 1}x^{\frac{1}{1-x}}$ 9) $\lim\limits_{x\to 0}(ctgx)^{\sin x}$

10) $\lim\limits_{x\to 0}\left(\frac{a^{x}-x\ln a}{b^{x}-x\ln b}\right)^{\frac{1}{x^{2}}}$ 11) $\lim\limits_{x\to a}\frac{a^{x}-x^{a}}{x-a}$

12) $\lim\limits_{x\to 0}\frac{(a+x)^{x}-a^{x}}{x^{2}}$

§ 22 Funktsiyani to'liq tekshirish.

1) Agar (a,b) intervalda $f'(x)\geq 0$ $(f'(x)\leq 0)$ bo'lsa funktsiya bu oraliqda o'suvchi (kamayuvchi) bo'ladi.

2) Hosilasi nol'ga teng bo'ladigan nuqtalar kritik nuqtalar deyiladi.

Agar kritik nuqtada funktsiya hosilasi o'z ishorasini + dan – ga (– dan + ga) o'zgartirsa, bu kritik nuqta maksimum (minimum) nuqtadir.

Funktsiya kritik nuqtada ishorasini o'zgartirmasa, bu nuqtada ekstremum mavjud emas.

Bu qoida ekstremum topishning I – qoidasidir.

3) Agar (a,b) intervalda $f''(x) \geq 0$ $(f''(x)) \leq 0$ bo'lsa, funktsiya grafigi botiq (qavariq) bo'ladi.

Demak kritik nuqtada $f'(x_0) > 0$ $(f'(x_0)) < 0$ bo'lsa, $x = x_0$ nuqta minimum (maksimum) nuqtasidir.

Bu qoida ekstremum topishning II – qoidasidir.

Ikkinchi tartibli hosila ishora o'zgartiradigan nuqtalar egilish nuqtalari deyiladi.

Agar $f(x)$ funktsiya $a \in R$ nuqtaning biror atrofida aniqlanib,

$$\lim_{x \to a+0} f(x), \ \lim_{x \to a-0} f(x)$$

lardan biri yoki ikkalasi cheksiz bo'lsa, $x = a$ to'g'ri chiziq $f(x)$ funktsiya grafigiga vertikal asimptota deyiladi.

$\lim\limits_{x \to \infty}[f(x) - (kx + b)] = 0$ bo'lsa, $y = kx + b$ to'g'ri chiziq $f(x)$ funktsiya grafigiga og'ma ($k = 0$ da gorizontal) asimptota deyiladi.

Bunda $k = \lim\limits_{x \to \infty} \dfrac{f(x)}{x}$, $b = \lim\limits_{x \to \infty}[f(x) - kx)]$

Funktsiya son o'qlarini kesib o'tadigan nuqtalar funktsiyaning nollari deyiladi.

Funktsiyani to'liq tekshirish uchun navbatma – navbat quyidagi ishlar amalga oshiriladi.

1) Funktsiyani aniqlanish va o'zgarish sohalarini topish;
2) Funktsiyani uzluksizlikka tekshirish; uzilish nuqtalarini topish;
3) Funktsiya juft – toqligi, davriyligini tekshirish;
4) Funktsiyani monotonlikka tekshirish;
5) Funktsiyani ekstremumga tekshirish;

6) Funktsiyani botiqlik, qavariqlikka tekshirish;
7) Funktsiya grafigi asimptotalarini topish;
8) Funktsiya nollarini topish;
9) Funktsiya grafigini chizish.

Funktsiya ekstremumlarini topishni masalalar echishga ham tatbiq etish mumkin.

22.1 Quyidagi funktsiyalar grafigini chizing.

1) $y = 3x - x^3$ 2) $y = \frac{x^3}{3} + x^2$

3) $y = \frac{x^2}{x-2}$ 4) $y = \frac{(x-1)^2}{x^2+1}$

5) $y = x^3 + 6x^2 + 9x$ 6) $y = 2x - 3\sqrt[3]{x^2}$

7) $y = xe^{-\frac{x^2}{2}}$ 8) $y = x - 2\ln x$

9) $y = 2x + ctgx,\ x \in (0;\pi)$

10) $y = \sin x + \frac{1}{3}\sin 3x$; 11) $y = \cos x \cos 3x$

12) $y = e^{2x-x^2}$ 13) $x + e^{-x}$

14) $y = \frac{e^x}{1+x}$ 15) $y = (x+2)e^{\frac{1}{x}}$

16) $y = x^x$ 17) $y = x^{\frac{1}{x}}$

18) $y = x + 2\sqrt{-x}$ 19) $y = \arcsin\frac{2x}{1+x^2}$

20) $y = \arccos\frac{1-x}{1-2x}$

22.2 1) 120 sm li simni yuzasi eng katta bo'ladigan to'g'ri to'rtburchak shaklida bukilgan. Bu to'g'ri to'rtburchak o'lchamlarini toping.

2) 10 sonini shunday ikki qo'shiluvchiga ajratingki, bu qo'shiluvchilar ko'paytmasi eng katta bo'lsin.

3) Berilgan R radiusli doiraga ichki chizilgan uchburchaklardan yuzi eng katta bo'lganini toping.

4) Trapetsiya kichik asosi va yon tomonlari 10 sm dan. Katta asosi qancha bo'lganda trapetsiya yuzi eng katta bo'ladi?

5) Shar hajmi unga ichki chizilgan eng katta hajmli tsilindr hajmidan necha marta katta?

6) R radiusli sharga ichki chizilgan, to'la sirti eng katta bo'lgan tsilindr o'lchamlarini toping.

7) M(p,p) nuqtadan $y^2 = 2px$ gacha eng qisqa masofani toping.

8) A(2,0) nuqtadan $x^2 + y^2 = 1$ aylanagacha bo'lgan eng qisqa va eng katta masofalarni toping.

9) Kengligi a metr daryoga perpendikulyar ravishda kengligi b metrli kanal qazildi. Maksimal uzunligi qanday bo'lgan kemalar bu kanalga o'ta oladi?

10) Yasovchisi R bo'lgan konuslar ichidan eng katta hajmlisini toping.

Bobga doir misollar echish namunalari

1. $f(x) = xe^x$ ni x darajalari bo'yicha qatorga yoying.

$$f(x) = xe^x = x[1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + 0(x^n)] = x + x^2 + \frac{x^3}{2!} + ..$$

$$.. + \frac{x^{n+1}}{n!} + 0(x^{n+1})$$

2. $f(x) = \ln\frac{1+x}{1-x}$ ni x darajalari bo'yicha qatorga yoying:

$$f(x)=\ln\frac{1+x}{1-x}=\ln(1+x)-\ln(1-x)=x-\frac{x^2}{2}+...+(-1)^{n-1}\frac{x^n}{n}-$$

$$-(-x-\frac{x^2}{2}-...-\frac{x^n}{n})=2(x+\frac{x^3}{3}+\frac{x^5}{5}+.....)$$

3. x^4 ni $(x+1)$ darajalari bo'yicha qatorga yoying.

$$f(-1)=(-1)^4=1;\quad f'(-1)=-4\quad f''(-1)=12$$

$$f'''(-1)=-24;\quad f^{IV}(-1)=24$$

$$f'(x)=4x^3,\quad f''(x)=12x^2,\ f'''(x)=24x,\ f^{IV}=24$$

Demak, Teylor formulasidan:

$$x^4=1-4(x+1)+\frac{12}{2!}(x+1)^2-\frac{24}{3!}(x+1)^2+\frac{24}{4!}(x+1)^3$$

4. ln2 ni hisoblang.

$$\ln\frac{1+x}{1-x}=\ln 2 \quad \text{dan}$$

$$\frac{1+x}{1-x}=2\Rightarrow 1+x=2-2x\Rightarrow x=\frac{1}{3}$$

$$\ln 2=\ln\frac{1+\frac{1}{3}}{1-\frac{1}{3}}=2[\frac{1}{3}+\frac{1}{3}(\frac{1}{3})^3+\frac{1}{5}(\frac{1}{3})^5+.....]$$

5. Yoyilmalardan foydalanib hisoblang.

$$\lim_{x\to 0}\frac{\cos x-e^{-\frac{x^2}{2}}}{x^4}=\lim_{x\to 0}\frac{1}{x^4}[1-\frac{x^2}{2!}+\frac{x^4}{4!}-(1-\frac{x^2}{2}+\frac{x^4}{8})]=$$

$$=\lim_{x\to 0}\frac{1}{x^4}[\frac{x^4}{24}-\frac{x^4}{8}]=\lim_{x\to 0}\frac{-2}{24}=-\frac{1}{12}$$

6. $$\lim_{x\to 0}\frac{a^x-a^{\sin x}}{x^3}=\lim_{x\to 0}\frac{a^x\ln a-a^{\ln x}\ln a\cos x}{3x^2}=$$

$$=\lim_{x\to 0}\frac{\ln a}{3}\cdot\frac{a^x-a^{\sin x}\cos x}{x^2}=$$

$$=\lim_{x\to 0}\frac{\ln a}{3}\cdot\frac{a^x\ln a-a^{\sin x}\ln a\cos^2 x+a^{\sin x}\sin x}{2x}=$$

$$\lim_{x\to 0}\frac{\ln a}{3}\cdot\frac{[a^x\ln^2 a-a^{\sin x}\ln^2 a\cos^3 x+a^{\sin x}\ln a 2\sin x\cos x+ +a^{\sin x}\ln a\sin x\cos x+a\sin x\cos x]}{2}=$$

$$=\frac{\ln a}{2\cdot 3}(\ln^2 a-\ln^2 a+1)=\frac{1}{6}\ln a$$

7.

$$\lim_{x\to 0}\frac{\ln(\sin ax)}{\ln(\sin bx)}=\lim_{x\to 0}\frac{\frac{1}{\sin ax}\cdot\cos ax\cdot a}{\frac{1}{\sin bx}\cdot\cos bx\cdot b}=\lim_{x\to 0}\frac{a\sin bx\cos ax}{b\sin ax\cos bx}=$$

$$=\frac{a}{b}\lim_{x\to 0}\frac{b\cos bx\cos ax-a\sin bx\sin ax}{a\cos ax\cos bx-b\sin ax\sin bx}=1.$$

8. $\lim\limits_{x\to\frac{\pi}{4}}(tgx)^{tg2x}$; $A=(tgx)^{tg2x}$ bo'lsa,

$\ln A=\ln(tgx)tg2x=\dfrac{\ln(tgx)}{\frac{1}{tg2x}}$ deb,

$$\lim_{x\to\frac{\pi}{4}}\ln A=\lim_{x\to\frac{\pi}{4}}\frac{\ln(tgx)}{\frac{1}{tg2x}}=\lim_{x\to\frac{\pi}{4}}\frac{\frac{1}{tgx}\cdot\frac{1}{\cos^2 x}}{-tg^{-2}2x2\frac{1}{\cos^2 2x}}=$$

$$=-\frac{1}{2}\lim_{x\to\frac{\pi}{4}}\frac{\frac{1}{\sin x\cos x}}{\frac{1}{\sin^2 2x}}=$$

$$=-\lim_{x\to\frac{\pi}{4}}\frac{\sin^2 2x}{\sin 2x}=-\lim_{x\to\frac{\pi}{4}}\sin 2x=-1;\ LnA\to -1;$$

Demak $A\to e^{-1}$;

9. $y=\dfrac{x^2}{x-1}$ funktsiyani to'liq tekshiring.

1) $D(y)=(-\infty;1)\cup(1;+\infty),\quad E(y)=(-\infty;+\infty)$

2) $x=1$ uzilish nuqtasi bo'lib,

$\lim\limits_{x\to 1-0} f(x)=-\infty,\ \lim\limits_{x\to 1+0} f(x)=+\infty$ ekanligidan, funktsiya bu nuqtada II − tur uzilishga egaligi kelib chiqadi.

3) $y(-x)\neq \pm y(x)$ ekanligidan, funktsiya juft ham, toq ham emas.

Funktsiya davriy emas.

4,5) $y'=\dfrac{x(x-2)}{(x-1)^2}$ dan kritik nuqtalari $x_1=0; x_2=2$ ekanligini topamiz.

Monotonlik oraliqlari, ekstremumlarni topish uchun quyidagi jadvalni to'ldiramiz.

x	$(-\infty;0)$	0	(0;1)	(1;2)	2	$(2;+\infty)$
y'	+	0	—	—	0	+
y	↗	0	↘	↘	4	↗

6) $y''=\dfrac{2}{(x-1)^3}$ ekanligidan, egilish nuqtalari yo'q, lekin uzilish nuqtasi bu hosila ishorasini o'zgartiradi.

'x	$(-\infty;1)$	$(1;+\infty)$
y''	—	+
y	$\cap$	$\cup$

7) $x=1$ vertikal asimptota ekanligini bilamiz, og'ma asimptotani qidiramiz:

$$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2}{x(x-1)}=1$$

$$b=\lim_{x\to\infty}\left[\frac{x^2}{x-1}-x\right]=\lim_{x\to\infty}\frac{x}{x-1}=1$$

demak $y=x+1$ og'ma asimptota ekan.

8) $x=0$ dan $y=0$; $y=0$ dan $x=0$ kelib chiqadi. Demak, grafik koordinata boshida son o'qlarini kesadi, holos.

Funktsiya grafigini chizamiz

10. R radiusli sharga ichki chizilgan eng katta hajmli tsilindr o'lchamlarini toping.

Silindr asosi radiusini r, balandligini H desak, Pifagor teoremasiga ko'ra:

$$(2r)^2=(2R)^2-H^2,\ \text{yaъni}\ r=\frac{1}{2}\sqrt{4R^2-H^2}$$

$$V_ц(H)=\frac{\pi}{4}(4R^2-H^2)\cdot H=\frac{\pi}{4}(4R^2H-H^3)$$

$$V'(H)=\frac{\pi}{4}(4R^2-3H^2)\quad дан\ H=\frac{2}{\sqrt{3}}R$$

kritik nuqta ekanligi kelib chiqadi.

H	$(0;\frac{2}{\sqrt{3}}R)$	$\frac{2}{\sqrt{3}}R$	$(\frac{2}{\sqrt{3}}R;2R)$
V'	+	0	–

V	↗	$\frac{4\pi}{3\sqrt{3}}R^3$	↘

max

$V_{\max}(\frac{2}{\sqrt{3}}R) = \frac{4}{3\sqrt{3}}R^2$. O'lchamlari $r = \sqrt{\frac{2}{3}}R$, $H = \frac{2}{\sqrt{3}}R$ bo'lgan silindr eng katta hajmga ega bo'ladi.

6 bobga doir uy vazifalari

I. $f(x) = e^x$ **funktsiyaga Teylor formulasini qo'llab** e^a **qiymatni, a ning bor qiymatida taqribiy hisoblang.**

1) a=0.49 2) 0.33 3) 0.75 4) 0.63 5) 0.21 6) 0.55 7) 0.37 8) 0.83 9) 0.13 10) 0.59 11) 0.95 12) 0.27 13) 0.47 14) 0.18 15) 0.72

II. Berilgan $f(x)$ **funktsiyaning** $[a,b]$ **oraliqdagi eng katta va eng kichik qiymatlarini toping.**

1) $f(x) = x^3 - 12x;\ [0;3]$ 2) $f(x) = x^5 - \frac{5}{3}x^3;\ [0;2]$

2) $f(x) = \frac{\sqrt{3}}{2}x + \cos x;\ [0;\frac{\pi}{2}]$ 4) $f(x) = 3x^4 - 16x^2 + 2;\ [-3;1]$

5) $f(x) = x^3 - 3x + 1;\ [0.5;2]$ 6) $f(x) = x^4 + 4x\ \ [-2;2]$

7) $f(x) = \frac{\sqrt{3}}{2}x - \sin x;\ [0;\frac{\pi}{2}]$ 8) $f(x) = 81x - x^4;\ [-1;4]$

9) $f(x) = 3 - 2x^2;\ [-1;3]$ 10) $f(x) = x - \sin x;\ [-\pi;\pi]$

11) $f(x) = 2^x;\ [-1;5]$ 12) $f(x) = x^2 - 4x + 6;\ [-3;10]$

13) $f(x)=\left|x^2-3x\right|$ [1;4] 14) $f(x)=\left|1-x^2\right|$ [0;2]

15) $f(x)=\ln\left|(x-4)\right|$ [0;5]

III. Funktsiyani to'la tekshiring, grafigini chizing.

1) $y=\frac{4x}{4+x^2}$ 2) $y=\frac{x^2-1}{x^2+1}$

3) $y=\frac{x^2+1}{x^2-1}$ 4) $y=\frac{x^2}{x-1}$

5) $y=\frac{x^3}{x^2+1}$ 6) $y=\frac{x^3}{x^2-1}$

7) $y=\frac{x^2-5}{x-3}$ 8) $y=\frac{x^4}{x^3-1}$

9) $y=\frac{4x^3}{x^3-1}$ 10) $y=\frac{2-4x^2}{1-x^2}$

11) $y=\frac{\ln x}{\sqrt{x}}$ 12) $y=xe^{-x^2}$

13) $y=e^{2x-x^2}$ 14) $y=xe^{-x^2}$

15) $y=x^2-2\ln x$

IV. Funktsiya ekstremumlarini topish qoidalari yordamida masalani eching.

1) Tomoni a ga teng kvadrat shaklidagi tunuka burchaklaridan teng kvadratlar qirqilib, ochiq quticha yasaldi. Qanday qilib eng katta sig'imli quti yasash mumkin?

2) Diametri d ga teng doiraviy kesimli xoda, kesimi to'g'ri to'rtburchak bo'lgan to'singa tilindi. To'g'ri to'rtburchak asosi b, balandligi h desak, to'sin eng katta mahkamlikka erishishi uchun b va h qanday o'lchamlarga ega bo'lishi kerak?

Ko'rsatma: Mahkamlik bh^2 ko'paytmaga proportsional.

3) R radiusli sharga eng katta sirtli tsilindr (ichki chizilgan) toping.

4) R radiusli doiradan eng katta hajmli konus yasash uchun qanday sektorni qirqib olish kerak?

5) V hajmli silindrik banka qanday o'lchamlarda eng kichik to'la sirtga ega bo'ladi?

6) R radiusli sharga tashqi chizilgan eng kichik hajmli konus o'lchamlarini toping.

7) R radiusli sferaga ichki chizilgan barcha uchburchakli muntazam prizmalar orasidan hajmi eng kattasini toping.

8) Asosining radiusi R, balandligi H bo'lgan konusga ichki chizilgan tsilindrlar orasidan hajmi eng kattasini toping.

9) O'q kesimi perimetri r ga teng bo'lgan barcha tsilindrlar orasidan hajmi eng kattasini toping.

10) To'g'ri prizmaning asosi teng yonli to'g'ri burchakli uchburchakdan iborat, uning katta yon yog'ining perimetri 24 sm. Prizma hajmi eng katta bo'lishi uchun, uning asosi tomonlari qanday uzunliklarga ega bo'lishi kerak?

11) k ning qanday qiymatida $y = x^2 + 2x$ va $y = kx + 1$ lar bilan chegaralangan figura yuzi eng kichik bo'ladi?

12) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning $M(x, y)$ nuqtasidan shunday urinma o'tkazingki, son o'qlari va urinma bilan chegaralangan uchburchak yuzi eng kichik bo'lsin.

13) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(0 < b < a)$ ellipsda $B(0; -b)$ nuqtasidan o'tuvchi eng katta vatarni toping.

14) Yuk avtomobili ochiq kuzovi sirtining yuzi 2S bo'lgan to'g'ri burchakli paralleliped shaklida. Kuzovning hajmi eng katta, bo'yining eniga nisbati esa $\frac{5}{2}$ bo'lishi uchun uning bo'yi va eni qanday bo'lishi kerak?

15) Eni bir xil uchta taxtadan nov yasalmoqda. Nov yon devorlarining asosga og'ish burchaklari qanday bo'lganda nov ko'ndalang kesimining yuzi eng katta bo'ladi?

7–bob. Aniqmas integral

§ 23. Aniqmas integral. Jadval yordamida integrallash.

Biror (a,b) intervalda $F'(x)=f(x)$ bo'lsa, $F(x)$ funktsiya $f(x)$ ning boshlang'ich funktsiyasi deyiladi.

$F(x)$ boshlang'ich funktsiya bo'lsa, $F(x)+C$ ham boshlang'ich funktsiya bo'ladi. Ixtiyoriy boshlang'ich funktsiya $f(x)$ ning (a,b) intervaldagi aniqmas integrali deyiladi va $\int f(x)dx=F(x)+C$ tarzida yoziladi.

Aniqmas integral quyidagi xossalarga ega

I. $d\int udx=udx$ II. $\int du=u+C$

III. $\int (Au+Bv)dx=A\int udx+B\int vdx$

Aniqmas integral jadvali

1) $\int x^p dx=\dfrac{x^{p+1}}{p+1}+C$ 2) $\int \dfrac{dx}{x}=\ln|x|+C$

3) $\int a^x dx=\dfrac{a^x}{\ln a}+C$ 4) $\int e^x dx=e^x+C$

5) $\int \cos xdx=\sin x+C$

6) $\int \sin xdx=-\cos x+c$

7) $\int \dfrac{1}{\cos^2 x}=tgx+C$

8) $\int \frac{1}{\sin^2 x} = -ctgx + C$

9) $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = -\arccos x + C$

10) $\int \frac{dx}{1+x^2} = arctgx + C = -arcctgx + C$

11) $\int shxdx = chx + C$ 12) $\int chxdx = shx + C$

13) $\int \frac{dx}{sh^2 x} = -cthx + C$ 14) $\int \frac{dx}{ch^2 x} = thx + C$

23.1 Integrallarni toping.

1) $\int (x^2 + 4x + \frac{1}{x})dx$ 2) $\int (\frac{(x^2+1)^2 - \sqrt{x}}{x^3})dx$

3) $\int (\frac{\sqrt{x} = 2\sqrt[3]{x^2} + 1}{\sqrt[4]{x}})dx$ 4) $\int (2 - x^3)^2 dx$

5) $\int (\frac{a}{x} + \frac{a^2}{x^2} + \frac{a^3}{x^3})dx$ 6) $\int (1 - \frac{1}{x^2})\sqrt{x\sqrt{x}}dx$

7) $\int e^x (1 - \frac{e^{-x}}{x^2})dx$ 8) $\int a^x (1 + \frac{a^{-x}}{\sqrt{x^3}})dx$

9) $\int \frac{dx}{\sin^2 x \cos^2 x}$ 10) $\frac{3 - 2ctg^2 x}{\cos^2 x} dx$

11) $\int \left(\sin^2 \frac{x}{2} + \cos^2 x \right) dx$

12) $\int (tg^2 x + ctg^2 x + 2)dx$ 13) $\int \frac{x^4}{1+x^2} dx$

§24. Bevosita va yangi o'zgaruvchi kiritib integrallash

$x=\varphi(u),\ dx=\varphi'(u)du$ bo'lsa,

$\int f(x)dx=\int f[\varphi(u)]\varphi'(u)du$ ko'rinish olib, bunday integrallash yangi o'zgaruvchi kiritib integrallash deyiladi.

$\int f(u(x))d(u(x))=F(u(x))+C$ ko'rinishdan foydalanish bevosita integrallash hisoblanadi, masalan,

$$\int f(ax+b)dx=\frac{1}{a}\int f(ax+b)d(ax+b)=\frac{1}{a}F(ax+b)+C$$

24.1 Bevosita integrallang:

1) $\int\frac{2x-5}{x^2-5x+7}dx$ 2) $\int\frac{2x}{x^2+1}dx$

3) $\int tgxdx$ 4) $\int\sin^2 x\cos xdx$

5) $\int\cos^3 x\sin xdx$ 6) $\int\frac{\sin xdx}{1+3\cos x}$

7) $\int\frac{\sin xdx}{\cos^3 x}$ 8) $\int\frac{\cos xdx}{\sin^4 x}$

9) $\int\sin x\cos xdx$ 10) $\int e^{\cos x}\sin xdx$

11) $\int e^{x^3}x^2dx$ 12) $\int e^{-x^2}xdx$

13) $\int\sqrt{x^2+1}xdx$ 14) $\int\sqrt[3]{x^3-8}x^2dx$

15) $\int\frac{dx}{x\ln x\ln(\ln(x))}$ 16) $\int\frac{dx}{(1+x)\sqrt{x}}$

17) $\int\sin\frac{1}{x}\frac{dx}{x^2}$

24.2 Yangi o'zgaruvchi kiritib integrallang:

1) $\int\cos 3xdx$ 2) $\int\sin\frac{x}{2}dx$

3) $\int e^{-3x}dx$ 4) $\int \frac{dx}{\cos^2 5x}$

5) $\int \sqrt{4x-1}dx$ 6) $\int (3-2x)^4 dx$

7) $\int \sin(a-bx)dx$

24.3.Integrallang:

1) $\int \frac{\sin x+\cos x}{\sqrt[3]{\sin x-\cos x}}dx$ 2) $\int \frac{dx}{(\arcsin x)^2\sqrt{1-x^2}}$

3) $\int \frac{x^2+1}{x^4+1}dx$ 4) $\int \frac{dx}{\sin^2 x\sqrt[4]{ctgx}}$

5) $\int \frac{dx}{\sin^2 x+2\cos^2 x}$ 6) $\int \frac{dx}{\sin x}$

7) $\int \sin^2 xdx$ 8) $\int \cos^2 xdx$

9) $\int \frac{dx}{\cos^4 x}$

Yangi o'zgaruvchi kiritish, bevosita integrallash yordamida quyidagilarni isbotlash mumkin

I. $\int \frac{dx}{a^2+x^2}=\frac{1}{a}arctg\frac{x}{a}+C$

II. $\int \frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left|\frac{a+x}{a-a}\right|+C$

III. $\int \frac{xdx}{a^2\pm x^2}=\pm\frac{1}{2}\ln\left|a^2\pm x^2\right|+C$

IV. $\int \frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$

V. $\int \frac{dx}{\sqrt{a^2\pm x^2}}=\ln\left|x+\sqrt{x^2\pm a^2}\right|+C$

VI. $\int \frac{xdx}{\sqrt{a^2 \pm x^2}} = \pm\sqrt{a^2 \pm x^2} + C$

VII. $\int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

VIII. $\int \sqrt{x^2 \pm a^2} = \frac{x}{2}\sqrt{a^2 \pm x^2} \pm \frac{a^2}{2}\arcsin\frac{x}{a} + C$

24.4 Integrallarni toping

1) $\int \frac{dx}{x^2+9}$ 2) $\int \frac{dx}{x^2-25}$

3) $\int \frac{dx}{\sqrt{4-x^2}}$ 4) $\int \frac{dx}{\sqrt{x^2+5}}$

5) $\int \frac{dx}{2x^2+5}$ 6) $\int \frac{dx}{4x^2-3}$

7) $\int \frac{dx}{\sqrt{5-x^2}}$ 8) $\int \frac{xdx}{\sqrt{3-x^2}}$

9) $\int \frac{5x-2}{x^2+4}dx$ 10) $\int \frac{3x-4}{x^2-4}dx$

11) $\int \frac{x+1}{\sqrt{x^2+2}}dx$ 12) $\int \frac{x+2}{\sqrt{4-x^2}}dx$

13) $\int \frac{dx}{x^2+4x+5}$ 14) $\int \frac{dx}{x^2+4x+4}$

15) $\int \frac{dx}{x^2+4x+3}$ 16) $\int \frac{dx}{\sqrt{x^2+2x+3}}$

17) $\int \frac{dx}{\sqrt{1-2x-x^2}}$ 18) $\int \frac{dx}{\sqrt{4x-x^2}}$

19) $\int \frac{x^4dx}{x^2+2}$ 20) $\int \frac{e^x dx}{\sqrt{1-e^{2x}}}$

21) $\int \frac{dx}{3\sin^2 x - 8\sin x\cos x + 5\cos^2 x}$

§25. Bo'laklab integrallash

Ko'paytmaning differentsiali d(u,v)=udv+vdu formulasidan quyidagi bo'laklab integrallash formulasi kelib chiqadi:

$$\int udv = uv - \int vdu$$

Bu formula yordamida

$\int x^k \ln^m xdx;\ \int x^k \sin bxdx;\ \int x^k \cos bxdx;\ \int x^k e^{ax} dx$

ва $\int e^{ax} \sin bxdx$ ko'rinishdagi integrallarni topish mumkin.

25.1 Bo'laklab integrallang

1) $\int x\ln xdx$ 2) $\int x^2 \ln xdx$

3) $\int arctgxdx$ 4) $\int x^2 \cos xdx$

5) $\int \arcsin xdx$ 6) $\int \frac{xdx}{\sin^2 x}$

7) $\int \frac{\ln xdx}{x^2}$ 8) $\int \cos(\ln x)dx$

9) $\int \sin x\ln(tgx)dx$ 10) $\int x^3 e^{-x} dx$

11) $\int \frac{x\cos xdx}{\sin^3 x}$ 12) $\int e^{ax} \sin bxdx$

§26 Ratsional algebraik funktsiyalarni integrallash

$P_n(x), Q_m(x)$ – mos darajali ko'phadlar bo'lganda $\int \frac{P_n(x)}{Q_m(x)}dx$ ifoda quyidagicha integrallanadi:

1) $n \geq m$ bo'lsa, $\int \frac{P_n(x)}{Q_m(x)}dx$ noto'g'ri kasr deyilib, suratni mahrajga bo'lish yordamida butun qismi ajratiladi va integrallanadi:

$\int \frac{P_n(x)}{Q_m(x)}dx = \int \left(R(x) + \frac{P_1(x)}{Q_m(x)} \right) dx$ bunda R – butun qism, $\frac{P_1}{Q}$ – to'g'ri kasrdir.

2) $n < m$ bo'lsa, $Q_m(x)$ ko'phad algebra asosiy teoremasi natijasiga ko'ra

$$(x-\alpha_1)^{l_1} \cdot (x-\alpha_2)^{l_2} \cdots (x-\alpha_{k_1})^{l_{k_1}} \cdot (x^2 + p_1 x + q_1)^{r_1} \cdot \ldots$$

$$\ldots (x^2 + p_{k_2} x + q_{k_2})^{r_{k_2}}$$

ko'rinishda yoziladi, bunda

$$\sum_{i=1}^{k_1} l_i + 2\sum_{j=1}^{k_2} r_j = m.$$

Kasrni quyidagicha sodda kasrlar yig'indisi sifatida yozish mumkin:

$$\frac{P_n(x)}{Q_m(x)} = \frac{A_1}{x-\alpha_1} + \frac{A_2}{(x-\alpha_1)^2} + \ldots + \frac{C_1 x + D_1}{x^2 + px + q} +$$

$$+ \frac{C_2 x + D_2}{(x^2 + px + q)^2} + \ldots + \frac{C_{r_k} x + D_{r_k}}{(x^2 + px + q)^{r_k}} + \ldots$$

Koeffitsientlar esa aniqmas koeffitsientlar metodi yordamida topiladi.

26.1 Integrallarni toping.

1) $\int \frac{x^3}{x-2}dx$ 2) $\int \frac{x^4}{x^2 + a^2}dx$

3) $\int \frac{x^5}{x^3 - a^3} dx$ 4) $\int \frac{x-4}{(x-2)(x-3)} dx$

5) $\int \frac{2x+7}{x^2+x-2} dx$ 6) $\int \frac{3x^2+2x-3}{x^3-x} dx$

7) $\int \frac{(x+1)^3}{x^2-x} dx$ 8) $\int \frac{x^{10}}{x^2+x-2} dx$

9) $\int \frac{x^3+1}{x^3-5x^2+6x} dx$ 10) $\int \frac{x^2+1}{(x+1)^2(x-1)} dx$

11) $\int \frac{dx}{(x+1)(x^2+1)}$ 12) $\int \frac{dx}{x^8-1}$

13) $\int \frac{xdx}{x^3-1}$ 14) $\int \frac{xdx}{x^4-1}$

15) $\int \frac{dx}{x^4+4}$ 16) $\int \frac{dx}{x^6+1}$

17) $\int \frac{dx}{(1+x)(1+x^2)(1+x^3)}$

18) $\int \frac{dx}{x^5-x^4+x^3-x^2+x-1}$

26.2 Aniqmas koiffitsientlar metodini qo'llamay toping:

1) $\int \frac{dx}{x(x+a)}$ 2) $\int \frac{dx}{(x+a)(x+6)}$

3) $\int \frac{dx}{x^2-2}$ 4) $\int \frac{dx}{(x^3-3)(x^2+2)}$

5) $\int \frac{dx}{x^4-x^2}$ 6) $\int \frac{dx}{x^3+4x}$

§ 27 Irratsional ifodalarni integrallash

$$1^0.\ \int R\left(x, \sqrt[\lambda]{\frac{ax+b}{cx+d}}, \ldots, \sqrt[\mu]{\frac{ax+b}{cx+d}}\right)dx$$

ko'rinishdagi ifoda

EKUK $(\lambda, \ldots, \mu) = m$ bo'lsa,

$$t^m = \frac{ax+b}{cx+d}$$

almashtirish yordamida ratsionallashadi.

$2^0.\ \int R(x, \sqrt{a+b+cx^2})dx$ ko'rinishdagi ifoda quyidagi Eyler almashtirishlari yordamida ratsionallashadi.

a) $D = b^2 - 4ac \geq 0$ bo'lsa,

$a + bx + cx^2 = C(x-\alpha)(x-\beta)$ dan $t = \sqrt{\frac{C(x-\beta)}{x-\alpha}}$

almashtirish o'tkaziladi.

b) $D < 0$ bo'lsa, $\sqrt{a+bx+cx^2} = t \mp x\sqrt{c}$
almashtirish o'tkaziladi.

3^0. $\int x^m (a+bx^n)^p dx$ differentsial binom quyidagi uch holda elementar funktsiyalarda integrallanadi.

1) R – butun bo'lsa, yoyish yordamida.

2) $\frac{m+1}{n}$ butun son bo'lsa, $a + bx^n = t^s$ bunda S soni P ning maxraji.

3) $\frac{m+1}{n} + p$ butun son bo'lsa,

$ax^{-n} + b = t^s$ yordamida ifodalanadi.

4^0. $\int R(x,\sqrt{a^2-x^2})dx$ ko'rinishdagi integral x=asint almashtirish yordamida, $\int R(x,\sqrt{a^2-x^2})dx$ esa $x=\dfrac{a}{\cos t}$ almashtirishda, $\int R(x,\sqrt{a^2-x^2})dx$ esa $x=atgt$ yordamida integrallanadi.

27.1 Integrallarni toping.

1) $\displaystyle\int\frac{dx}{1+\sqrt{x}}$ 2) $\displaystyle\int\frac{dx}{\sqrt[3]{x}+\sqrt{x}}$

3) $\displaystyle\int\frac{x\sqrt[3]{2+x}}{x+\sqrt[3]{2+x}}dx$ 4) $\displaystyle\int\frac{dx}{(1+\sqrt[4]{x})\sqrt[3]{x}}$

5) $\displaystyle\int\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}dx$ 6) $\int x\sqrt{a-x}dx$

27.2 Integrallarni toping.

1) $\displaystyle\int\frac{dx}{x\sqrt{2x^2+2x+1}}$ 2) $\displaystyle\int\frac{dx}{x+\sqrt{x^2+x+1}}$

3) $\displaystyle\int\frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}}dx$ 4) $\displaystyle\int\frac{dx}{1+\sqrt{1-2x-x^2}}$

5) $\int x^2\sqrt{x^2-2x+2}\,dx$

27.3 Integrallarni toping.

1) $\displaystyle\int\frac{dx}{\sqrt[4]{1+x^4}}$ 2) $\displaystyle\int\frac{\sqrt{x}dx}{(1+\sqrt[3]{x})^2}$

3) $\displaystyle\int\frac{dx}{x\sqrt[6]{1+x^6}}dx$ 4) $\displaystyle\int\frac{x^5dx}{\sqrt{1-x^2}}$

5) $\int\sqrt[3]{3x-x^3}\,dx$

27.4 Integrallarni toping.

1) $\int x\sqrt{a^2-x^2}dx$ 2) $\int x^2\sqrt{4-x^2}dx$

3) $\int\frac{dx}{\sqrt{(4+x^2)^3}}$ 4) $\sqrt{3+2x-x^2}dx$

5) $\int\frac{x^2dx}{(2-x^2)^3}$ 6) $\int\frac{\sqrt{x^2-a^2}}{x}dx$

§28 Trigonometrik ifodalarni integrallash.

$\int\sin^m x\cdot\cos^n xdx \quad m,n\in Z$ ko'rinishdagi integrallar trigonometrik almashtirishlar, daraja pasaytirish, ko'paytmani yig'indiga aylantirish, yangi o'zgaruvchi kiritish yordamida topiladi.

28.1 Integrallarni toping.

1) $\int\sin^2 3xdx$ 2) $\int\cos^4 xdx$

3) $\int\cos^3 xdx$ 4) $\int\sin^5 xdx$

5) $\int\sin^3 x\cos^3 xdx$ 6) $\int\frac{dx}{\sin 2x}$

7) $\int\frac{dx}{\cos x}$ 8) $\int tg^3xdx$

9) $\int ctg^3xdx$ 10) $\int\frac{\sin^3 xdx}{\cos^2 x}$

11) $\int\frac{\cos^3 xdx}{\sin^2 x}$ 12) $\int\sin 3x\sin xdx$

13) $\int\sin(5x-\frac{\pi}{4})\cos(x+\frac{\pi}{4})dx$

14) $\int\frac{dx}{\sin^4 x+\cos^4 x}$

28.2. Bo'laklab integrallash formulasidan foydalanib $\int \cos^n x dx$ uchun "Daraja pasaytirish " formulasini keltirib chiqaring va $\int \cos^6 x dx$, $\int \frac{dx}{\cos^3 x}$ larni hisoblang.

28.3. $\int \sin^6 x dx$ va $\int \frac{dx}{\sin^3 x}$ larni hisoblang.

Bobga doir misollar echish namunalari

1. Jadval yordamida integralni toping

1) $\int (2 + tg^2 x + 5x^4 + 2\sqrt[3]{x} + \frac{6}{x^3} + \frac{1}{\sqrt[3]{x^2}})dx =$

$= \int (1 + \frac{1}{\cos^2 x} + 5x^4 + 2x^{\frac{1}{3}} + 6x^{-3} + x^{-\frac{2}{3}})dx =$

$\int 1dx + \int \frac{dx}{\cos^2 x} + 5\int x^4 dx + 2\int x^{\frac{1}{3}} dx +$

$+6\int x^{-3} dx + \int x^{-\frac{2}{3}} dx = x + tgx + 5 \cdot \frac{x^5}{5} + 2\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 6 \cdot \frac{x^{-2}}{-2} +$

$+\frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C = x + tgx + x^5 + \frac{3}{2}x\sqrt[3]{x} - \frac{1}{x^2} + 3\sqrt[3]{x} + C$

2) $\int \left[\frac{1}{\sqrt{1-x^2}} + \frac{2^{x+1} - 5^{x-1}}{10^x} + \frac{x^2}{1+x^2} \right] dx =$

$= \int \left[\frac{1}{\sqrt{1-x^2}} + 2\left(\frac{1}{5}\right)^x - \frac{1}{5}\left(\frac{1}{2}\right)^x + \frac{1+x^2-1}{1+x^2} \right] dx =$

$$=\int\left[\frac{1}{\sqrt{1-x^2}}+2\left(\frac{1}{5}\right)^x-\frac{1}{5}\left(\frac{1}{2}\right)^x+\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right]dx=$$

$$=\arcsin x+2\left(\frac{1}{5}\right)^x\cdot\frac{1}{\ln\frac{1}{5}}-\frac{1}{5}\left(\frac{1}{2}\right)^x\cdot\frac{1}{\ln\frac{1}{2}}+x-arctgx+C$$

3) $\int\left[\frac{\cos 2x}{\cos^2 x\sin^2 x}+(\sin\frac{x}{2}-\cos\frac{x}{2})^2\right]dx=$

$$=\int\left[\frac{\cos^2 x-\sin^2 x}{\cos^2 x\cdot\sin^2 x}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}+\cos^2\frac{x}{2}\right]dx=$$

$$=\int[\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+1-\sin x]dx=$$

$$=-ctgx-tgx+x+\cos x+C$$

2. Bevosita integrallang.

1) $\int ctgxdx=\int\frac{\cos x}{\sin x}dx=\int\frac{d(\sin x)}{\sin x}=\ln|\sin x|+C$

2) $\int\frac{dx}{x+a}=\int\frac{d(x+a)}{x+a}=\ln|x+a|+C$

3. Yangi o'zgaruvchilar kiritib, integrallang.

1) $\int(x+1)^{10}dx=\int t^{10}dt=\frac{t^{11}}{11}+C=\frac{(x+1)^{11}}{11}+C$

$x+1=t,\qquad x=t-1\qquad dx=dt$

2) $\int\sqrt[3]{1-6x}dx=\int\sqrt[3]{t}(-\frac{1}{6}dt)=-\frac{1}{6}\int t^{\frac{1}{3}}dt=-\frac{1}{6}\frac{t^{\frac{4}{3}}}{\frac{4}{3}}=-\frac{1}{8}\sqrt[3]{(1-6x)^4}+C$

$1-6x=t,\quad 6x=1-t,\qquad x=-\frac{1}{6}t+\frac{1}{6},\quad dx=-\frac{1}{6}dt$

4. Bo'laklab integrallash

1) $\int \ln x dx = x\ln x - \int dx = x\ln x - x + C$

$u = \ln x \quad dv = dx \quad du = \frac{1}{x}dx \quad v = x$

2) $\int x^2 \cos x dx = \int x^2 d(\sin x) = x^2 \sin x +$

$+2\left\{x\cos x - \int \cos x dx\right\} =$

$= x^2 \sin x + 2x\cos x - 2\sin x + C$

3) $I = \int e^{ax} \cos bx dx = \frac{1}{a}\int \cos bx d(e^{ax}) =$

$= \frac{1}{a}[e^{ax} \cos bx + b\int e^{ax} \sin bx dx]$

$= \frac{1}{a}e^{ax} \cos bx + \frac{b}{x^2}\int \sin bx d(e^{ax}) = \frac{1}{a}e^{ax} \cos bx +$

$+\frac{b}{a^2}[e^{ax} \sin bx - b\int e^{ax} \cos bx dx] =$

$= \frac{1}{a}e^{ax} \cos bx + \frac{b}{a^2}e^{ax} \sin bx - \frac{b^2}{a^2}I;$

Demak, $I = \frac{e^{ax}}{a^2 + b^2}(b\sin bx + a\cos bx);$

5. $\int \frac{x^4 + 3x}{x^2 + 1}dx$ ni toping.

$$\int \frac{x^4 + 3x}{x^2 + 1}dx = \int x^2 dx - \int dx + \frac{3}{2}\int \frac{d(x^2 + 1)}{x^2 + 1} + \int \frac{dx}{x^2 + 1} =$$

$$=\frac{x^3}{3}-x+\frac{3}{2}\ln(x^2+1)+arctgx+C,$$ chunki

$$\frac{x^4+3x}{x^2+1}=x^2-1+\frac{3x+1}{x^2+1}.$$

6. $\int\frac{2x^2+x+2}{x^3+2x}dx$ ni toping.

$$\frac{2x^2+x+2}{x(x^2+2)}=\frac{A}{x}+\frac{Bx+c}{x^2+2}$$ dan

$2x^2+x+2=A(x^2+2)+Bx^2+Cx$ kelib chiqadi. Bundan A+B=2, C=1; 2A=2, ya'ni $A=B=C=1$

$$\int\frac{2x^2+x+2}{x^3+2x}dx=\int(\frac{1}{x}+\frac{x+1}{x^2+2})dx=$$

$$=\ln|x|+\frac{1}{2}\ln(x^2+2)+\frac{1}{\sqrt{2}}arctg\frac{x}{\sqrt{2}}+C$$

7. $\int\frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}dx$ ni toping.

$x+1=t^6;\ t=\sqrt[6]{x+1}\quad dx=6t^5dt$ almashtirishlar yordamida integral quyidagi ko'rinishga keladi.

$$\int\frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}dx=\int\frac{1-t^3}{1+t^2}6t^5dt=$$

$$=6\int\frac{-t^8+t^5}{1+t^2}dt=6\int(t^6+t^4+t^3-t^2-t+1+\frac{t-1}{t^2+1})dt=$$

$$=6[-\frac{t^7}{7}+\frac{t^5}{5}+\frac{t^4}{4}-\frac{t^3}{3}-\frac{t^2}{2}+t+\frac{1}{2}\ln(t^2+1)-arctgt]+C=$$

$$=6[-\frac{1}{7}\sqrt[6]{(x+1)^7}+\frac{1}{5}\sqrt[6]{(x+1)^5}+\frac{1}{4}\sqrt[3]{(x+1)^2}-$$

$$-\frac{1}{3}\sqrt{x+1}-\frac{1}{2}\sqrt[3]{x+1}++\sqrt[6]{x+1}+\frac{1}{2}\ln(\sqrt[3]{x+1}+1)-$$
$$-arctg\sqrt[6]{x+1}]+C$$

8. $\int\frac{dx}{x+\sqrt{x^2+3x+2}}$ ni toping.

$x^2+3x+2=(x+1)(x+2)$ dan $(x+1)t^2=x+2$ ko'rinishda yangi o'zgaruvchi kiritamiz. Undan

$$x=\frac{2-t^2}{t^2-1};\quad dx=\frac{-2tdt}{(t^2-1)^2}$$

Demak, berilgan integral quyidagi ko'rinishda bo'ladi:

$$\int\frac{-\frac{2t}{(t^2-1)^2}dt}{\frac{2-t^2}{t^2-1}+t(\frac{2-t^2}{t^2-1}+1)}=2\int\frac{t}{(t-1)(t+1)^2(t-2)}dt=$$

$$=2\int[\frac{1}{t-1}+\frac{4}{9}\frac{1}{t+1}+\frac{1}{3}\frac{1}{(t+1)^2}+\frac{5}{9}\frac{1}{(t-2)}]dt=$$

$$=2[\ln|1-t|+\frac{4}{9}\ln|t+1|-\frac{1}{3}\frac{1}{t+1}+\frac{5}{9}\ln|t-2|]+C=$$

$$=2[\ln\left|1-\sqrt{\frac{x+2}{x+1}}\right|+\frac{4}{9}\ln\left|1+\sqrt{\frac{x+2}{x+1}}\right|-\frac{1}{3}\frac{1}{(1+\sqrt{\frac{x+2}{x+1}})^2}+$$

$$+\frac{5}{9}\ln\left|\sqrt{\frac{x+2}{x+1}}-2\right|+C$$

9. $\int\sqrt{x^3+x^4}dx$ irratsionallikni yo'qoting:

$$\int\sqrt{x^3+x^4}\,dx=\int x^{\frac{2}{3}}(1+x)^{\frac{1}{2}}\,dx \text{ dan } \frac{m+1}{n}+p=3$$

ya'ni $x^{-1}+1=t^2$
almashtirish zarurligi kelib chiqadi.

$$x=\frac{1}{t^2-1};\quad dx=\frac{-2t}{(t^2-1)^2}dt$$

Natijada, berilgan integral quyidagi ko'rinishga keladi:

$$-2\int\frac{t^2}{(t^2-1)^4}dt$$

10. $\int\sin^2 x\cdot\cos^2 xdx$ da

$$\sin^2 x\cdot\cos^2 x=\frac{1-co2x}{2}\cdot\frac{1+co2x}{2}=\frac{1}{4}(1-\cos^2 2x)=$$

$$=\frac{1}{4}\sin^2 2x=\frac{1}{4}\cdot\frac{1-\cos 4x}{2}$$

dan

$$\int\sin^2 x\cdot\cos^2 xdx=$$

$$=\frac{1}{8}\int(1-\cos 4x)dx=\frac{1}{8}[x-\frac{\sin 4x}{4}]+C$$

11. $\int\sin^2 x\cdot\cos^3 xdx=\int\sin^2 x\cdot(1-\sin^2 x)d(\sin x)=$

$$=\int[\sin^2 x-\sin^4 x]d(\sin x)=\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$$

12.

$$\int\sin mx\sin nxdx=\frac{1}{2}\int[\cos(m-n)x-\cos(m+n)x]dx=$$

$$=\frac{1}{2}[\frac{\sin(m-n)x}{m-n}-\frac{\sin(m+n)x}{m+n}]+C$$

13. $\int \sin^n x dx$ integral uchun «daraja pasaytirish» formulasini chiqaring. Bo'laklab integrallash formulasidan

$$I_n = -\int \sin^{n-1} x d(\cos x) = -\cos x \sin^{n-1} x +$$

$$+\int \cos x (n-1) \sin^{n-2} x \cos x dx = -\cos x \sin^{n-1} x +$$

$$(n-1)\int (1-\sin^2 x)\sin^{n-2} x dx = -\cos x \sin^{n-1} x +$$

$$+(n-1)\int \sin^{n-2} x dx - (n-1) I_n ,$$

ya'ni $n \cdot I_n = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x dx$ kelib chiqadi.

Demak, $I_n = -\dfrac{1}{n}\cos x \sin^{n-1} x + \dfrac{n-1}{n}\int \sin^{n-2} x dx$.

7–bobga doir uy vazifalari.

Aniqmas integrallarni toping.

I

1) $\int e^{\sin^2 x} \sin 2x dx$ 2) $\int arctg\sqrt{x} dx$

3) $\int \dfrac{dx}{x^3+27}$ 4) $\int \dfrac{dx}{2+\sqrt[3]{x+1}}$

5) $\int \dfrac{xdx}{\sqrt{x^2+4x-5}}$

II

1) $\int \dfrac{xdx}{x^2+5}$ 2) $\int x \ln(x+1) dx$

3) $\int \dfrac{2x^2-1}{x^3+1} dx$ 4) $\int \dfrac{dx}{Sin\, x + tgx}$

5) $\int \dfrac{dx}{(1+\sqrt{x(1+x)})^2}$

III 1) $\int \frac{dx}{x(\ln x + 4}$ 2) $\int x \cdot e^{-4x} dx$

3) $\int \frac{3x-7}{x^3+4x^2+4x+16} dx$

4) $\int \frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}}$ 5) $\int \frac{dx}{\sin^5 x}$

IV 1) $\int \frac{dx}{sin^2 x(4ctgx+3)}$ 2) $\int \frac{x \arcsin x}{\sqrt{1-x^2}} dx$

3) $\int \frac{dx}{x^2+x^2+2x+2)}$ 4) $\int \frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}} dx$

5) $\int \frac{dx}{cos^6 x}$

V 1) $\int \frac{cos 2x}{1+\sin 3x} dx$ 2) $\int x^2 \cdot e^{3x} dx$

3) $\int \frac{xdx}{x^3+2x+x^2+2}$ 4) $\int \frac{cosxdx}{1+\cos x}$

5) $\int \frac{x^2 dx}{\sqrt{1-x^6}}$

VI 1) $\int \frac{cosxdx}{\sqrt{sin^2 x}}$ 2) $\int x \arcsin \frac{1}{x} dx$

3) $\int \frac{x+3}{x^3+x^2-2x}$ 4) $\frac{\sqrt[4]{x+1}}{(\sqrt{x}+4)\sqrt[4]{x^3}} dx$

5) $\int \frac{dx}{a+b\cos x}$

VII 1) $\int\frac{x+arctgx}{1+x^2}dx$ 2) $\int x\cdot\ln(x^2+1)dx$

3) $\frac{x^2-3}{x^4+5x^2+6}dx$

4) $\int\frac{\sqrt{x+5}}{1+\sqrt[3]{x+5}}dx$ 5) $\int\frac{\sin^3 x}{cos^5 x}dx$

VIII 1) $\int\frac{x+\arcsin x}{\sqrt{1-x^2}}dx$ 2) $x\sin x\cdot\cos \mathrm{dx}$

3) $\int\frac{x^2dx}{x^4-81}$ 4) $\frac{dx}{3\cos x+4\sin x}$

5) $\frac{dx}{\sqrt[3]{x^2+2\sqrt{x}}}$

IX 1) $\int\frac{\sin xdx}{\sqrt[3]{3+2\cdot\cos x}}$ 2) $\int x^2\cdot\sin 4xdx$

3) $\int\frac{x^2-x+1}{x^4+2x^2-3}dx$

4) $\int\frac{(\sqrt{x-1})(\sqrt[6]{x+1})}{\sqrt[3]{x^2}}dx$ 5) $\int\frac{dx}{e^{3x}-e^x}$

X 1) $\int\frac{\sqrt[3]{4+lmx}}{x}dx$ 2) $\int x\ln^2 xdx$

3) $\int\frac{x^3-6}{x^4+6x^2+8}dx$ 4) $\int\frac{dx}{2\sin x+\cos x+2}$

5) $\int\frac{dx}{(1+x^2)\sqrt{1-x^2}}$

XI 1) $\int \frac{x^2}{(1-x)^{100}}dx$ 2) $\int x \cdot shxdx$

3) $\int \frac{xdx}{(x+1)(x+4)(x-3)}$

4) $\int \frac{x+2}{x^2 \cdot \sqrt{1-x^2}}dx$ 5) $\int \frac{\sin^2 x}{1+\sin^2 x}dx$

XII 1) $\int \frac{x^5}{x+1}dx$ 2) $\int x^n \cdot \ln xdx$

3) $\int \frac{x^2dx}{x^4+5x^2+4}$ 4) $\int \sqrt{\frac{x}{1-x\sqrt{x}}}dx$

5) $\int \frac{\sin x \cos x}{\sin x + \cos x}dx$

XIII 1) $\int \frac{2^x \cdot 3^x}{9^x - 4^x}dx$ 2) $\int x^2 \sin 2x dx$

3) $\int \frac{x+1}{(x^2-4)(x^2+5)}dx$

4) $\int \frac{x\ln(1+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx$ 5) $\int \frac{dx}{\cos x + \cos\alpha}$

XIV 1) $\int \frac{x^2+1}{x^4+1}dx$ 2) $\int x^2 \arccos x dx$

3) $\int \frac{3x+1}{x(x^2+3)}dx$ 4) $\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}dx$

5) $\int \frac{dx}{\sin x - \sin\alpha}$

XV 1) $\int \frac{dx}{1-\cos x}$ 2) $\int \sqrt{x} \cdot \ln^2 x dx$

3) $\int \frac{x^2+5x+4}{x^4+5x^2+4}dx$

4) $\int x^2 \sqrt{\frac{x}{1-x}}dx$ 5) $\int \frac{dx}{\sqrt{tgx}}$

8–bob. Aniq integral va tadbiqlari.
§29.Aniq integral ta'rifi. N'yuton –Leybnits formulasi.

$[a,b]$ oraliqda $f(x)$ funktsiya aniqlangan va uzluksiz bo'lsin. Bu oraliqni bo'laklarga bo'lamiz: $a = x_0 < x_1 < x_2 < \ldots < x_i < x_{i+1} < \ldots < x_n = b$.

$\Delta x_i = x_{i+1} - x_i$; $i = \overline{0, n-1}$ ayirmalaridan eng kattasini λ deb belgilaymiz. Har bir $[x_i, x_{i+1}]$ oraliqlardan ixtiyoriy ravishda biror $x = \xi_i$ nuqta olib

$$\sigma = \sum_{i=0}^{n-1} f(\xi_i) \cdot \Delta x_i$$

yig'indini tuzamiz. Agar bu yig'indining λ→0 dagi chekli limiti mavjud bo'lsa, u holda bu limit $f(x)$ning a dan b gacha oraliqdagi aniq integrali deyiladi va quyidagicha belgilanadi:

$$\int_a^b f(x)dx = \lim_{\lambda \to 0} \sum_{k=0}^{n-1} f(\xi_k) \cdot \lambda$$

Agar $[x_i, x_{i-1}]$ oraliqdagi ξ_i o'rniga, shu oraliqdagi $f(x)$ ning aniq quyi va yuqori chegaralari m_i, M_i olinsa,

$$s = \sum_{k=0}^{n-1} m_k \Delta x_k, \quad S = \sum_{k=0}^{n-1} M_{ik} \Delta x_k$$

Darbuning quyi va yuqori integral yig'indilari hosil bo'ladi.

Aniq integral mavjud bo'lishi uchun

$$\lim_{\lambda \to 0}(S - s) = 0$$

o'rinli bo'lishi zarur va etarlidir.

Agar [a,b] oraliqda $f(x)$ ning biror boshlang'ich funktsiyasi $F(x)$ bo'lsa, quyidagi N'yuton – Leybnits formulasi o'rinlidir:

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$$

Aniq integralda ham yangi o'zgaruvchi kiritish mumkin, lekin, chegaralari yangi o'zgaruvchi chegaralari bilan almashtiriladi, bo'laklab integrallash formulasi esa quyidagi ko'rinishda bo'ladi:

$$\int_a^b u dv = u\cdot v\Big|_a^b - \int_a^b v du$$

29.1 Integral yig'indi yordamida hisoblang.

1) $\int_0^{\frac{\pi}{2}} \sin x dx$ 2) $\int_{-1}^{2} x^2 dx$ 3) $\int_a^b \frac{dx}{x^2}$ 4) $\int_0^{\frac{\pi}{2}} \cos x dx$

5) $\int_0^a x dx$ 6) $\int_0^a e^x dx$

29.2 Hisoblang:

1) $\int_1^3 x^3 dx$ 2) $\int_1^2 \left(x^2 + \frac{1}{x^4}\right) dx$ 3) $\int_1^4 \sqrt{x} dx$

4) $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$ 5) $\int_a^{a\sqrt{3}} \frac{dx}{a^2+x^2}$ 6) $\int_0^3 e^{\frac{x}{3}} dx$

7) $\int_0^1 \frac{dx}{\sqrt{x^2+1}}$ 8) $\int_0^{\frac{\pi}{4}} \sin 4x dx$ 9) $\int_4^9 \frac{dx}{\sqrt{x}-1}$

10) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+tg^2 x}{(1+tgx)^2} dx$ 11) $\int_0^4 \frac{dx}{1+\sqrt{2x+1}}$

12) $\int_0^1 \frac{x^2dx}{\sqrt{4-x^2}}$ 13) $\int_0^1 \frac{dx}{e^x+1}$ 14) $\int_0^{\frac{a}{2}} \sqrt{\frac{x}{a-x}}dx$

15) $\int_0^{\frac{\pi}{2}} \sin x\cos^2 xdx$ 16) $\int_0^1 \ln(x+1)dx$

17) $\int_0^{\sqrt{3}} xarctgxdx$ 18) $\int_0^{2\pi} x^2\cdot\cos xdx$

19) $\int_0^3 \arcsin\sqrt{\frac{x}{1+x}}dx$ 20) $I_n=\int_0^{\frac{\pi}{2}} \sin^n xdx$

21) $I_n=\int_0^{\frac{\pi}{2}} \cos^n xdx$

§30.Yuza va yoy uzunligini hisoblash.

$y_1(x)$, $y_2(x)$ $(y_2(x)\geq y_1(x))$ funktsiyalar va $x=a$; $x=b$ $(a\langle b)$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiya yuzi $S=\int_a^b [y_2(x)-y_1(x)]dx$ formuladan, qutb koordinatalarda

$r=r(\varphi)$, $\varphi=\alpha$, $\varphi=\beta$ $(\alpha<\beta)$ lar bilan chegaralangan sektor yuzi

$$S=\frac{1}{2}\int_\alpha^\beta \sqrt{r^2(\varphi)d\varphi}$$

formula yordamida topiladi.

$y=y(x)$ chiziq $[a,b]$ oralig'idagi yoyi uzunligi

$$l=\int_a^b\sqrt{1+y'^2(x)}dx$$

qutb koordinatalardagi $r=r(\varphi)$ chiziqning $[\alpha,\beta]$ oraliqdagi yoyi uzunligi esa

$$l=\int_\alpha^\beta\sqrt{r^2(\varphi)+r'^2(\varphi)}dx$$

formulalar yordamida topiladi.

30.1 Quyidagi chiziqlar bilan chegaralangan soha yuzini toping.

1) $y=4-x^2$; $y=0$ 2) $y^2=2px$, $x=h$

3) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

4) $xy=4$; $x=1$; $x=4$; $y=0$

5) $y=3-2x-x^2$, $y=0$ 6) $y=\ln x$, $x=l$; $y=0$

7) $y=x^2$; $y^2=4x$ 8) $y^2=x^3$, $y=8$; $x=0$

9) $4y=x^2$, $y^2=4x$ 10) $xy=1$, $xy=4$, $y=x$, $y=4x$

11) $r^2=a^2\cdot\cos 2\varphi$ 12) $r=3+2\cos\varphi$

13) $r=a\cdot\cos 3\varphi$ 14) $r=a\cdot(1+\sin^2 2\varphi)$, $r=a$

30.2 Egri chiziqlar yoy uzunligini hisoblang.

1) $x^2+y^2=a^2$ 2) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$

3) $y=x^{\frac{3}{2}}\,(0\le x\le 4)$ 4) $y^2=2px\,(0\le x\le x_0)$

5) $y=a\ln\frac{a^2}{a^2-x^2}$ $(0\le x\le b<a)$

6) $y=\ln x,\quad \frac{3}{4}\le x\le\frac{12}{5}$ 7) $r=a\varphi,\ 0\le\varphi\le 2\pi$

8) $r=al^{m\varphi}(m\ge 0)\ \ 0\le r\le a$ 9) $r=a\sin^3\frac{4}{3}$

10) $r=ath\frac{\varphi}{2}\quad (0\le\varphi\le 2\pi)$

§31. Aylanish figuralari hajmi, sirti. Momentlar va og'irlik markazi koordinatalarini topish.

Tekislikda $a\le x\le b,\ 0\le y\le y(x)$ egri chiziqli trapetsiyaning Ox o'qi (Oy o'qi) atrofida aylanishidan hosil bo'lgan jism hajmi

$$V_x=\pi\cdot\int_a^b y^2dx,\qquad \left[V_y=2\pi\int_a^b x\cdot y(x)dx\right]$$

$a\le\alpha\le b,\quad y_1(x)\le y\le y_2(x)$ egri chiziqli trapetsiyaning Ox o'qi (Oy o'qi) atrofida aylanishidan hosil bo'lgan xalqa hajmi esa

$$V=\pi\int_a^b[y_2^2(x)-y_1^2(x)]dx\qquad \left[V=2\pi\int_a^b x\cdot[y_2(x)-y_1(x)]dx\right]$$

formula yordamida topiladi.

Aylanish jismi V_x ning sirti

$S=2\pi\int_a^b y\cdot\sqrt{1+y'^2}dx$ formuladan topiladi.

Tekislikda $0\le\alpha\le\varphi\le\beta\le\pi,\quad 0\le r\le r(\varphi)$ silliq figuraning qutb o'qi atrofida aylanishidan hosil bo'lgan jism hajmi

$$V=\frac{2\pi}{3}\int_{\alpha}^{\beta} r^3(\varphi)\cdot \sin\varphi d\varphi$$ formuladan topiladi.

Zichligi $\rho(x)$ yuzi S bo'lgan figura massasi $M=\int_a^b \rho(x)dx$, statistik momentlari $M_x=\int_a^b xydy$;

$$M_y=\int_a^b xydx$$

og'irlik markazi koordinatalari esa

$$x=\frac{1}{s}M_y, \qquad y=\frac{1}{s}M_x$$

formuladan topiladi.

31.1 1) Silindr hajmi formulasini chiqaring.

2) Konus hajmi formulasini chiqaring .

3) Shar hajmi formulasini chiqaring.

4) $y^2=2px$ va $x=h$ bilan chegaralangan sohaning Ox o'qi atrofida aylanishidan hosil bo'lgan jism hajmini toping.

5) $y^2=(x+4)^3$, $x=0$ chiziqlar bilan chegaralangan sohaning Oy o'qi atrofida aylanishidan hosil bo'lgan jism hajmini toping.

6) $\frac{x^2}{a^2}-\frac{y^2}{в^2}=1; y=\pm в$ chiziqlar bilan chegaralangan sohaning Ox o'qi atrofida aylanishidan hosil bo'lgan jism hajmini toping.

31.2 Quyidagi chiziqlar aylanishidan hosil bo'lgan jism sirtini toping.

1) $x^2+y^2=R^2$ ning Ox o'qi atrofida.

2) $4x^2+y^2=4$ ning Oy o'qi atrofida .

3) $x^2+(y-a)^2=a^2$ ning Ox o'qi atrofida.

31.3 Qutb o'qi atrofida aylanishdan hosil bo'lgan jism sirtini toping .

1) $r=a(1+\cos\varphi)$

2) $r^2=a^2\cos 2\varphi$.

31.4 Quyidagi chiziqlar bilan chegaralangan sohalar inertsiya momentlari, og'irlik markazi koordinatalarini toping .

1) $bx+ay=ab,\ x=0,\ y=0$

2) $y=x^2,\ x=2;\ y=0$

3) $x^2+y^2=a^2;\ y\geq o$.

§32 Xosmas integrallar

Agar $\lim\limits_{A\to\infty}\int\limits_a^A f(x)dx$ mavjud va chekli bo'lsa, uni $[a;+\infty)$ oraliqdagi xosmas integral deyiladi va $\int\limits_a^{+\infty} f(x)\,dx$ tarzida yoziladi.

Shunga o'hshash $\int\limits_a^{+\infty} f(x)dx,\ \int\limits_{-\infty}^{+\infty} f(x)$ larni ham kiritiladi.

$[a,b]$ oraliqning S nuqtadan boshqa nuqtalarida uzluksiz, S nuqtada II$-$tur uzilishga ega funktsiya $f(x)$ dan $[a,b]$ da olingan xosmas integral deb $\lim\limits_{v\to 0}\int\limits_a^{c-v} f(x)dx+\lim\limits_{v\to 0}\int\limits_{c+v}^{b} f(x)dx$ yigindiga aytiladi.

Xosmas integrallar chekli chiqsa yaqinlashuvchi, aks xolda uzoqlashuvchi deyiladi.

32.1. Yaqinlashishini tekshiring.

1) $\int\limits_0^{+\infty}\frac{dx}{\sqrt{1+x^3}}$ 2) $\int\limits_2^{+\infty}\frac{dx}{\sqrt[3]{x^3-1}}$

3) $\int\limits_1^{+\infty}\frac{1}{xe^x}dx$ 4) $\int\limits_0^{2}\frac{dx}{\ln x}$

5) $\int\limits_0^{\frac{\pi}{2}}\frac{dx}{\sin^p x\cdot\cos^q x}$ 6) $\int\limits_0^{1}\frac{x^n dx}{\sqrt{1-x^4}}$

32.2. Xisoblang.

1) $\int\limits_0^1 \ln x dx$ 2) $\int\limits_a^{+\infty}\frac{dx}{x^2}$

3) $\int\limits_0^1\frac{dx}{(2-x)\sqrt{1-x}}$ 4) $\int\limits_{-1}^1\frac{dx}{\sqrt{1-x^2}}$

5) $\int\limits_{-\infty}^{+\infty}\frac{dx}{\left(x^2+x+1\right)^2}$ 6) $\int\limits_0^{+\infty}e^{-ax}\cdot\cos x dx$

7) $\int\limits_b^{\frac{\pi}{2}}\ln(\sin x)dx$

32.3. 1) $y=\frac{1}{1+x^2}$ va uning asimtotasi orasidagi yuzani hisoblang .

2) $x>0$ da $y=e^{-x}$ chiziq Ox o'qi atrofida aylanishidan hosil bulgan jism hajmini toping.

Integrallarni taqribiy hisoblash

$\int_a^b f(x)dx$ aniq integralni hisoblashda $f(x)$ funktsiyaning boshlang'ich funktsiyasi $F(x)$ ma'lum bo'lganda, N'yuton – Leybnits formulasi yordamida oson hisoblanadi. $F(x)$ – boshlang'ich funktsiyani topish mumkin bo'lmasa, aniq integralni taqribiy hisoblashga to'g'ri keladi, buning uchun $f(x)$ funktsiya soddaroq funktsiyalar bilan almashtiriladi.

2. To'g'ri to'rtburchaklar formulasi.

$[a;b]$ kesmani teng n ta bo'lakka $x_k = a + k \cdot \frac{b-a}{n}$ $(k = 0,1,...n)$ nuqtalar yordamida bo'lamiz va quyidagi

$$\int_a^b f(x)dx \approx \frac{b-a}{n} \cdot \sum_{k-0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

taqribiy tenglikka ega bo'lamiz. Uni aniq integralni to'g'ri to'rtburchaklar formulasi yordamida hisoblash formulasi deyiladi.

2. Trapetsiyalar formulasi.

$[a;b]$ kesmani teng n ta bo'lakka bo'lganda $S_k = \{(x,y) \in R : x_{k-1} \le x \le x_{k,}\ 0 \le y \le f(x)\}$ yuzani taqriban $x = x_{k-1}, x = x_k, y = 0,$ $y = f(x_{k-1}) + [f(x_k) - f(x_{k-1})]$ to'g'ri chiziqlar chegaralangan trapetsiya bilan almashtirish quyidagi

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)]$$

trapetsiyalar formulasi deb ataluvchi taqribiy tenglikni olib keladi.

3. Simpson (parabolik) formulasi:

Agar S_k yuza $x = x_{k-1}, x = x_k, y = 0,$ va uchi $f\left(\frac{x_{k-1} + x_k}{2}\right)$ nuqtada, $(x_{k-1}, f(x_{k-1})), (x_k, f(x_k))$ nuqatalardan o'tuvchi parabola bilan chegaralangan egri chiziqli trapetsiya bilan almashtirilsa, parabolik formula deb ataluvchi quyidagi Simpson formulasi hosil bo'ladi.

$$\int_a^b f(x)dx = \frac{b-a}{6n}\left[(y_0 + y_n) + 2(y_1 + \ldots + y_{n-1}) + 4\left(y_{\frac{1}{2}} + \ldots + y_{n-\frac{1}{2}}\right)\right]$$

Bu holda oraliq $2n$ qismga bo'linadi.

1. Trapetsiyalar formulasiga ko'ra $\ln 2 = \int_1^2 \frac{dx}{x}$ hisoblansin.
2. To'g'ri to'rtburchaklar formulasiga ko'ra $\int_0^{2\pi} x \sin x dx$ hisoblansin va aniq qiymati bilan solishtirilsin $(n = 12)$.
3. Simpson formulasiga ko'ra $\pi = 6\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ taqribiy qiymati hisoblansin.
4. $n = 10$ da Katalon doimiysi deb ataluvchi $G = \int_0^1 \frac{arctgx}{x} dx$ hisoblansin.
5. $\int_0^1 e^{x^2} dx$ ni 0,001 gacha aniqlikda hisoblang.
6. Yarim o'qlari $a = 10, b = 6$ bo'lgan ellips uzunligini taqriban hisoblang.

7. $\int_0^{+\infty} e^{-x^2}dx$ ni 0,001 gacha aniqlikda hisoblang.

Integrallash sohasini teng 20 qismga bo'lib, Simpson formulasi yordamida hisoblang.

1) $\int_{-2}^{8}\sqrt{x^3+16}dx$ 2) $\int_{-3}^{7}\sqrt{x^3+32}dx$

3) $\int_{-2}^{8}\sqrt{x^3+8}dx$ 4) $\int_{0}^{10}\sqrt{x^3+4}dx$

5) $\int_{-1}^{9}\sqrt{x^3+11}dx$ 6) $\int_{1}^{11}\sqrt{x^3+12}dx$

7) $\int_{-2}^{8}\sqrt{x^3+14}dx$ 8) $\int_{-4}^{6}\sqrt{x^3+1}dx$

9) $\int_{-1}^{9}\sqrt{x^3+5}dx$ 10) $\int_{-7}^{3}\sqrt{x^3+11}dx$

11) $\int_{-12}^{-2}\sqrt{x^3+13}dx$ 12) $\int_{-1}^{9}\sqrt{x^3+15}dx$

13) $\int_{-4}^{6}\sqrt{x^3+9}dx$ 14) $\int_{-2}^{8}\sqrt{x^3+22}dx$

15) $\int_{-1}^{9}\sqrt{x^3+7}dx$

Bob bo'yicha misollar echish namunalari

1. Integral yig'indi yordamida integralni hisoblang:

$$\int_a^b x^p dx$$

[a,b] oraliqni $q_n = \sqrt[n]{\frac{b}{a}}$ yordamida

$a, aq, aq^2, ..., aq^n = b$ bo'laklarga bo'lamiz: Har bir bo'lakdagi eng kichik sonni ξ_k sifatida olamiz:

$(p \neq -1)$

$$\sigma_n = \sum_{k=0}^{n-1}(a \cdot q^k)^p \cdot [aq^{k+1} - aq^k] = a^{p+1} \cdot (q-1) \cdot \sum_{k=0}^{n-1}(q^{p+1})^k =$$

$$= a^{p+1} \cdot (q-1) \cdot \frac{(\frac{b}{a})^{p+1} - 1}{q^{p+1} - 1} = (b^{p+1} - a^{p+1}) \cdot \frac{q-1}{q^{p+1} - 1}$$

Demak,

$$\int_a^b x^p dx = \lim_{n \to 0} \sigma_n = (b^{p+1} a^{b+1}) \lim_{q \to 1} \frac{q-1}{q^{p+1} - 1} = \frac{b^{p+1} - a^{p+1}}{p+1}$$

$p = -1$ bo'lgan holda

$$\sigma_n = n(q_n - 1) = n(\sqrt[n]{\frac{b}{a}} - 1) \quad \text{dan}$$

$$\int_a^b \frac{dx}{x} = \lim_{n \to \infty} \sigma_n = \lim_{n \to \infty} n(\sqrt[n]{\frac{b}{a}} - 1) = \ln b - \ln a$$

kelib chiqadi.

2. N'yuton – Leybnits formulasi yordamida hisoblang.

1)

$$\int_{-1}^{8} \sqrt[3]{x} dx = \frac{x^{\frac{3}{4}}}{\frac{3}{4}}\Bigg|_{-1}^{8} = \frac{3}{4}[\sqrt[3]{8^4} - \sqrt[3]{(-1)^4}] = \frac{3}{4}[16 - 1] = \frac{45}{4} = 11,25$$

2)

$$\int_0^2 |1-x|dx = \int_0^1 (1-x)dx + \int_1^2 (x-1)dx = (x-\frac{x^2}{2})\Big|_1^2 + (\frac{x^2}{2}-x)\Big|_1^2 =$$

$$= 1-\frac{1}{2}+\frac{2^2}{2}-2-(\frac{1}{2}-1)=1$$

3) $\int_b^a x^2 \cdot \sqrt{a^2-x^2}dx$

$x = a\sin t$ almashtirish o'tkazamiz. $x=0$ da $t=0$;

$x=a$ da esa $t=\frac{\pi}{2}$ ekanligini topamiz.

$dx = a\cos tdt$ ni hisobga olib:

$$\int_0^{\frac{\pi}{2}} a^2 \cdot \sin^2 t \cdot \sqrt{a^2-a^2\sin^2 t} \cdot a\cos tdt =$$

$$= a^4 \cdot \int_0^{\frac{\pi}{2}} \sin^2 t \cdot \cos^2 tdt = \frac{a^4}{4}\int_0^{\frac{\pi}{2}} \sin^2 2tdt =$$

$$\frac{a^4}{8}\int_0^{\frac{\pi}{2}} [1-\cos 4t]dt = \frac{a^4}{8}\left[t-\frac{\sin 4t}{4}\right]\Bigg|_0^{\frac{\pi}{2}} = \frac{\pi a^4}{16}$$

4) $\int_0^1 \arccos xdx = x\cdot\arccos x\Big|_0^1 + \int_0^1 \frac{x}{\sqrt{1-x^2}}dx =$

$$= -\frac{1}{2}\int_0^1 \frac{d(1-x^2)}{\sqrt{1-x^2}} = -1\sqrt{1-x^2}\Big|_0^1 = 1$$

$u = \arccos x$

$dv = dx$

$du = -\frac{1}{\sqrt{1-x^2}}dx$

$v = x$

3. $y=2x-x^2$ va $x+y=0$ chiziqlar bilan chegaralangan soha yuzini toping.

Dastlab bu chiziqlar kesishish nuqtalarini topamiz:

$$2x-x^2=-x\Rightarrow 3x-x^2=0 \quad x(3-x)=0$$

$x_1=0;\ x_2=3$

$$y=-x^2+2x=-\left[x^2-2x\right]=-\left[(x-1)^2-1\right]=-(x-1)^2+1$$

$[0;3]$ oraliqda $2x-x^2\geq -x$ ekanligidan

$$S=\int_0^3\left[2x-x^2+x\right]dx=\int_0^3\left[3x-x^2\right]dx=\left(3*\frac{x^2}{2}-\frac{x^3}{3}\right)\Bigg|_0^3=\frac{27}{2}-\frac{27}{3}=$$

$$=13{,}5-9=4{,}5 \text{ (kv.b)}$$

4. $r=a\cdot\sin 3\varphi$ (uch yaproqli gul) bilan chegaralangan soxa yuzini toping.

$\left[0;\frac{\pi}{6}\right]$ oraliqda uch yaproqli gulning $\frac{1}{6}$ qismi joylashadi. Demak,

$$S=\frac{1}{2}\cdot 6\int_0^{\frac{\pi}{6}}a^2\cdot\sin^3 3\varphi d\varphi=3a^2\int_0^{\frac{\pi}{6}}\frac{1-\cos 6\varphi}{2}d\varphi=$$

$$=\frac{3a^2}{2}\left(\varphi-\frac{\sin 6\varphi}{6}\right)\Bigg|_0^{\frac{\pi}{6}}=\frac{\pi a}{4}=\frac{3a^2}{2}\text{ (kv.b)}$$

5. $y=\ln\cos x$ $\left(0\leq x\leq a\langle\frac{\pi}{2}\right)$ chiziq yoyi uzunligini toping.

$$y^1=\frac{1}{\cos x}\cdot(-\sin x)=-tgx \quad \text{ekanligidan}$$

$$l=\int_0^a\sqrt{1+tg^2 x}d=\int_0^a\sqrt{\frac{1}{\cos^2 x}}dx=\int_0^a\frac{dx}{\cos x}=\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\Big|_0^a=$$

$$=\ln\left|tg\left(\frac{a}{2}+\frac{\pi}{4}\right)\right|+\ln\left|tg\left(0+\frac{\pi}{4}\right)\right|=\ln\left|tg\left(\frac{a}{2}+\frac{\pi}{4}\right)\right|$$

6. $r=a(1+\cos\varphi)$ (kardoida) yoyi uzunligini toping.

$[0;\pi]$ da bu chiziq yoyi yarmi joylashganligidan

$$l=2\int_0^\pi\sqrt{a^2(1+\cos\varphi)^2+a^2\cdot\sin^2\varphi}d\varphi=$$

$$=2a\int_0^\pi\sqrt{1+2\cos\varphi+\cos^2\varphi+\sin^2\varphi}d\varphi=$$

$$2\sqrt{2}\cdot a\int_0^\pi\sqrt{2\cos^2\frac{\varphi}{2}}d\varphi=4a\int_0^\pi\cos\frac{\varphi}{2}d\varphi=4a\cdot\frac{\sin\frac{\varphi}{2}}{\frac{1}{2}}\Big|_0^\pi=8a$$

7. Kesik konus hajmi formulasini keltirib chiqaring. Kesik konus asoslari radiuslari r,R, balandligi H bo'lsin. Uni $[0;H]$ kesmada $A(0;r)$ va $B(H;R)$ nuqtalardan o'tuvchi to'g'ri chiziq $x=0;\ x=H;\ y=0$ lar bilan chegaralangan trapetsiyani Ox o'qi atrofida aylantirib hosil qilish mumkin.

$$AB:\qquad y=r+\frac{R-r}{H}\cdot x$$

$$V_x=\pi\int_0^H[r+\frac{R-r}{H}\cdot x]^2dx=\frac{\pi H}{3(R-r)}[r+\frac{R-r}{H}\cdot x]^3\Big|_0^H=$$

$$=\frac{H}{3(R-r)}\left(R^3-r^3\right)=$$

$$=\frac{\pi H}{3}[R^2+rR+r^2]=\frac{H}{3}[S+\sqrt{S\cdot s}+s];$$

8. $x^2+(y-b)^2=a^2\quad(b\geq a)$ ning Ox o'qi atrofida hosil bo'lgan xalqa sirtini toping.

$y=b\pm\sqrt{a^2-x^2}$ bo'lgani uchun $[-a,\ a]$ da

$$S_1=4\pi\int_b^a\left(b+\sqrt{a^2-x^2}\right)\cdot\sqrt{1+\frac{x^2}{a^2-x^2}}dx=4\pi a\left(\frac{b\pi}{2}+a\right),$$

$S_2=4\pi a\left(\frac{b\pi}{2}-a\right)$ lardan

$S=4\pi^2\cdot ab$ kelib chiqadi.

9. $\int_1^{+\infty}\frac{1}{x^p}dx$ yaqinlashishini tekshiring.

$$\int_1^{+\infty}\frac{1}{x^p}dx=\lim_{A\to+\infty}\int_1^A\frac{1}{x^p}dx=\lim_{A\to+\infty}\frac{x^{-p+1}}{-p+1}\Big|_1^A=$$

$$=\lim_{a\to+\infty}\left[\frac{A^{1-\rho}}{1-\rho}-\frac{1}{1-\rho}\right]=\begin{cases}\frac{1}{p-1}; & p<1\\ +\infty, & p>1\end{cases}$$

$p=1$ holda alohida tekshiramiz:

$$\int_1^{+\infty}\frac{dx}{x}=\lim_{a\to\infty}\ln|x|=\lim_{A\to\infty}\ln|x|\Big|_1^A=+\infty.$$

Demak, r > 1ta yaqinlashuvchi; r ≤ 1 da uzoqlashuvchi.

10. $\int_a^b \frac{dx}{(b-x)^p}$ yaqinlashishga tekshirilsin

$$a\int_a^b \frac{dx}{(b-x)} = \lim_{\nu\to 0}\int_a^{b-\nu}\frac{dx}{(b-x)} = \lim_{\nu\to 0}\left[-\frac{(b-x)^{-p+1}}{-p+1}\right]\Big|_a^{b-\nu} =$$

$$= \lim_{\nu\to 0}\left[\frac{\nu^{-p+1}}{1-p} - \frac{(b-a)^{-p+1}}{1-p}\right] = \begin{cases} \frac{(b-a)^{1-p}}{p-1} & agar \quad p<1 \\ \infty; & agar \quad p>1. \end{cases}$$

r=1 da

$$\int_n^b \frac{dx}{b-x} = -\lim_{\nu-0}\int_a^{b-\nu}\frac{d(b-x)}{b-x} = -\lim_{\nu-0}\ln|b-x|\Big|_a^{b-\nu} =$$

$$\Big| = -\lim_{\nu-0}\left[\ln|\nu| - \ln|b-n|\right] = +\infty$$

Demak, bu xosmas integral p<1 da yaqinlashuvchi, $p \geq 1$ da uzoqlashuvchi ekan.

11. Xisoblang.

$$\int_1^\infty \frac{dx}{1+x^2} = \lim_{A\to\infty}\int_1^A \frac{dx}{1+x^2} = \lim_{A\to\infty} arctgx\Big|_1^A =$$

$$= \lim_{A\to\infty}\left[arctgA - arctg1\right] = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

12. Son uklari va $x>0$ da $y=e^{-2x}$ chizik bilan chegaralangan soha yuzini toping.

$$S = \int_0^{+\infty} e^{-2x}dx = \lim_{A\to\infty}\int_0^A e^{-2x}dx = \lim_{A\to\infty}\left[-\frac{e^{-2x}}{2}\right]\Big|_0^A = 0+\frac{1}{2} = \frac{1}{2}$$

(kv.b)

8–bobga doir uy vazifalari.

Quyidagi chiziqlar bilan chegaralangan figura yuzini hisoblang:

1) $y = 3x^2 + 1$ va $y = 3x + 7$

2) $y = x;\ \ y = x + \sin^2 x \qquad (0 \le x \le \Pi)$

3) $r = 3(1 - \cos\varphi)$

4) $r = \dfrac{p}{1 + \varepsilon\cos\varphi} \qquad (0\angle\sum\angle 1)$

5) $y = x;\ y = x + 2;\ y = 3;\ y = 4$

Yoy uzunligini hisoblang.

6) $y = \sqrt{(x-2)^3}$ $\ A(2;0)$ dan $B(6;8)$ gacha

7) $x = \dfrac{1}{4}y^2 - \dfrac{1}{2}ly \qquad (1 \le y \le e)$

8) $y^2 = \dfrac{x^3}{2a - x} \qquad (0 \le x \le \dfrac{5}{3}a)$

9) $\sqrt[3]{x^2} + \sqrt[3]{y^2} = \sqrt[3]{a^2}$

10) $y = ach\dfrac{x}{a},\ \ A(0;a)$ dan $B(b,h)$ gacha

Quyidagi chiziqlar bilan chegaralangan figura aylanishidan hosil bo'lgan jism hajmini toping

11) $xy = 4,\ x = 1, x = 4, y = 0$ Ox o'qi atrofida;

12) $y^2 = (x + 4)^3$ va $x = 0$ OY o'qi atrofida;

13) $y = x^2$ va $y = \sqrt{x}$ Ox o'qi atrofida;

14) $y = \dfrac{2}{1 + x^2}$ va $y = \sqrt{x^2}$ Ou o'qi atrofida;

Xosmas integralni hisoblang yoki uzoqlashishini hisoblang.

1) $\int_{0}^{+\infty} xe^{-x^2}dx$ 2) $\int_{-\infty}^{-3} \frac{xdx}{(x^2+1)^2}$

3) $\int_{-1}^{+\infty} \frac{dx}{x^2+x+1}$ 4) $\int_{0}^{1} \frac{x^2dx}{\sqrt{1-x^3}}$

5) $\int_{1}^{2} \frac{dx}{(x-1)^2}$ 6) $\int_{-3}^{2} \frac{dx}{(x+3)^2}$

7) $\int_{2}^{+\infty} \frac{dx}{x\ell nx}$ 8) $\int_{0}^{3} \frac{dx}{(x-2)^2}$

9) $\int_{0}^{1} \frac{dx}{\sqrt[3]{(x-3)^2}}$ 10) $\int_{-\infty}^{+\infty} \frac{dx}{x^2+4x+1}$

11) $\int_{1}^{+\infty} \frac{arctgx}{x^2}dx$ 12) $\int_{1}^{+\infty} \frac{dx}{x^2+2x}$

13) $\int_{2}^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$ 14) $\int_{1}^{+\infty} \frac{e^{-x}}{x}dx$

15) $\int_{0}^{+\infty} e^{-x^2}dx$

9–bob. Ko'p o'zgaruvchili funktsiyalar. Qatorlar.
§33. Ikki o'zgaruvchili funktsiyalar. Limit. Uzluksizlik.

Ikki x, y o'zgaruvchilarning x, y juftligiga biror qonun yordamida Z ning yagona qiymati mos qo'yilgan bo'lsa, bu qonun ikki x, y o'zgaruvchili funktsiya deyiladi va $Z = f(x, y)$ ko'rinishida yoziladi.

$\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x, y) = A$ bo'lsa, A soni $f(x, y)$ funktsiyaning $P(x, y)$ nuqta $P_0(x_0, y_0)$ nuqtaga intilgandagi limiti deyiladi.

$f(x, y)$ funktsiya $P_0(x_0, y_0)$ nuqtada uzluksiz deyiladi, agar

$\lim\limits_{P \to P_0} f(P) = f(P_0)$ tenglik o'rinli bulsa.

33.1. Aniqlanish sohalarini toping va tasvirlang.

1) $z = x + \sqrt{y}$ 2) $z = \sqrt{1 - x^2} + \sqrt{y^2 - 1}$

3) $z = \sqrt{\alpha^2 \cdot b^2 - bx^2 - \alpha y^2}$

4) $z = \dfrac{1}{\sqrt{x^2 + y^2 - 1}}$ 5) $z = \sqrt{\dfrac{x^2 + y^2 - x}{2x - x^2 - y^2}}$

6) $z = \arccos \dfrac{x}{x + y}$

7) $z = \sqrt{\sin(x^2 + y^2)}$ 8) $z = \ln xy$

9) $z = \ln(-x - y)$

33.2 Takroriy limitlarni xisoblang:

1) $\lim\limits_{x \to \infty} \left\{ \lim\limits_{y \to \infty} \dfrac{x^2 + y^2}{x^2 +^4} \right\}$ 2) $\lim\limits_{x \to \infty} \left\{ \lim\limits_{y \to +0} \dfrac{x^y}{1 + x^y} \right\}$

3) $\lim\limits_{x\to0}\left\{\lim\limits_{y\to\infty}\frac{1}{xy}\cdot tg\frac{xy}{1+xy}\right\}$ 4) $\lim\limits_{x\to1}\left\{\lim\limits_{y\to0}\log_x(x+y)\right\}$

33.3. Karrali limitlarni hisoblang.

1) $\lim\limits_{\substack{x\to\infty\\ y\to\infty}}\frac{x+y}{x^2-xy+y^2}$ 2) $\lim\limits_{\substack{x\to0\\ y\to a}}\frac{\sin xy}{x}$

2) 3) $\lim\limits_{\substack{x\to+\infty\\ y\to+\infty}}\left(\frac{xy}{x^2+y^2}\right)^{x^2}$

4) $\lim\limits_{\substack{x\to0\\ y\to0}}\left(x^2+y^2\right)^{x^2\cdot y^2}$ 5) $\lim\limits_{\substack{x\to1\\ y\to0}}\frac{\ln(x+e^y)}{\sqrt{x^2+y^2}}$

33.4 Quyidagi funktsiyalar uzilish nuqtalarini toping.

1) $z=\frac{1}{\sqrt{x^2+y^2}}$ 2) $z=\frac{xy}{x+y}$

3) $z=\frac{1}{x\cdot y\cdot z}$

4) $z=\ln\frac{1}{\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}}$

§34.Xususiy hosilalar. To'la diferentsial.

$z=f(x,y)$ funktsiyada y ni o'zgarmas son deb faraz qilib, x bo'yicha olingan hosila, (yoki aksincha) xususiy hosila deyiladi va mos ravishda $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ yoki z'_x, z'_y tarzida belgilanadi.

Ulardan olingan xususiy xosilalar ikkinchi tartibli xususiy xosilalar deyiladi va

$$\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y \partial x} \text{ yoki } z''_{xx}, z''_{yy},$$

$$z''_{xy}, z''_{yx}$$

ko'rinishda belgilanadi. SHunga o'xshash yuqori tartibli xususiy hosilalarni ham kiritiladi.

Agar funktsiya orttirmasini

$$\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y + 0(\sqrt{\Delta x^2 + \Delta y^2})$$

ko'rinishda yozish mumkin bo'lsa, funktsiya diferentsiallanuvchi deyiladi.

Orttirmaning chiziqli bosh qismi funktsiya to'la diferentsiali deyiladi:

$$dz = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y.$$

$z = x$ va $dx = \Delta x$, $z = y$ va $dy = \Delta y$

ekanligidan

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy.$$

34.1. Funktsiyalar xususiy hosilalarini toping.

1. $z = x^3 + 3x^2 y - y^2$
2. $y = \ln(x^2 + y^2)$
3. $z = \frac{y}{x}$
4. $z = arctg \frac{y}{x}$
5. $z = \frac{x \cdot y}{x - y}$
6. $z = x \cdot e^{-yx}$
7. $z = arctg \frac{x + y}{1 - xy}$
8. $z = \arcsin \frac{x}{\sqrt{x^2 + y^2}}$
9. $z = x^y$

34.2. Quyidagi funktsiyalar uchun $z''_{xy} = z''_{yx}$ tenglikni tekshiring:

1. $z = x^2 - 2xy - 3y^2$ 2. $z = x^{y^2}$

3. $z = \arccos\sqrt{\frac{x}{y}}$.

34.3 To'la diferentsialini toping;

1. $z = x^m \cdot y^n$ 2. $z = e^{xy}$ 3. $z = \frac{x}{y}$

4. $z = \sqrt{x^2 + y^2}$ 5. $z = \ln\sqrt{x^2 + y^2}$

6. $z = x \cdot \ln y$

34.4. Ko'rsatilgan xususiy hosilalarni toping.

1. $z = x\ln(xy)$ bo'lsa, $z = \frac{\partial^3 z}{\partial x^2 \partial y}$.

2. $z = e^{xy}$ bo'lsa, $\frac{\partial^2 z}{\partial x \partial y}$

3. $z = \frac{x+y}{x-y}$ bo'lsa, $\frac{\partial^{m+n} z}{\partial x^m \partial y^n}$.

4. $z = xy \cdot e^{x+y}$ bo'lsa, $\frac{\partial^{p+q} z}{\partial x^p + \partial y^n}$.

§35. Ikki o'zgaruvchili funktsiya ekstremumlari.

$z = f(x, y)$ funktsiya $z'_x = 0$, $z'_y = 0$ yoki $df = 0$ shartlar bajariladigan kritik nuqtalardagina ekstremumga erishishi mumkin.

$P(x_0, y_0)$ kritik nuqta uchun $A = f''_{xx}(x_0, y_0)$, $B = f''_{xy}(x_0, y_0)$, $C = f''_{yy}(x_0, y_0)$ belgilashlar kiritamiz; $P(x_0, y_0)$ nuqta

1. Minimum nuqta, agar $AC - B^2 > 0$, $A > 0$ $(C > 0)$ bo'lsa,
2. Maksimum nuqta, agar $AC - B^2 > 0$, $A < 0$ $(C < 0)$ bo'lsa,
3. Ekstremum mavjud emas, agar $AC - B^2 < 0$ bo'lsa.

Agar $AC - B^2 = 0$ bo'lsa, bu nuqtada ekstremum bo'lishi ham, bo'lmasligi ham mumkin.

$z = f(x; y)$ funktsiyaning $\varphi(x, y) = 0$ shart ostidagi ekstremumni topish uchun yordamchi

$$L(x, y, \lambda) = f(x, y) + \lambda\varphi(x, y)$$

Lagranj funktsiyasining ekstremumini topish kifoya.

35.1. Funktsiyalar ekstremumlarini toping.

1) $z = x^2 - xy + y^2 + 9x - 6y + 20$

2) $z = x^2 + (y-1)^2$

3) $z = x^3 + y^3 - 3xy$

4) $z = y\sqrt{x} - y^2 - x + 6y$

5) $z = e^{x^2 - y} \cdot (5 - 2x + y)$

35.2. Funktsiyalar shartli ekstremumlarini toping.

1. $z = \frac{1}{x} + \frac{1}{y}$, $\quad x + y = 2$

2. $z = x + y$, $\quad \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{2}$

3. $z = x \cdot y$, $\qquad x^2 + y^2 = 2$

4. $z = \frac{x}{a} + \frac{y}{b}$, $\qquad x^2 + y^2 = 1$

5. $z = x^2 + 12xy + 2y^2$, $4x^2 + y^2 = 25$

35.3. $z = x^2 - 9xy + 10$ funktsiyaning $D = \{x \geq 0, y \geq 0,\ x + y \leq 2\}$ sohadagi eng katta va eng kichik qiymatini toping.

§36 Ikki karrali integral yordamida yuzani hisoblash.

1⁰. Agar (S) soha $a \leq x \leq b,\ y_1(x) \leq y \leq y_2(x)$ tengsizlik bilan aniqlangan bo'lsa, shu soha yuzasi

$$S = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum\sum \Delta x \Delta y = \iint_{(S)} dxdy = \int_a^b dx \int_{y_1(x)}^{y_2(x)} dy$$ formula

yordamida hisoblanadi.

2⁰. Agar (S) soha $h \leq y \leq l,\ x_1(y) \leq x \leq x_2(y)$ tengsizlik bilan aniqlangan bo'lsa, shu soha yuzasi

$$S = \iint_{(S)} dxdy = \int_h^l dy \int_{x_1(y)}^{x_2(y)} dx$$

3⁰. Agar (S) soha qutb koordinatasida $\varphi_1 \leq \varphi \leq \varphi_2,\ r_1(\varphi) \leq r \leq r_2(\varphi)$ tengsizlik bilan aniqlangan bo'lsa, shu soha yuzasi

$$S = \iint_{(S)} rdd\varphi = \int_{\varphi_1}^{\varphi_2} d\varphi \int_{r_1(\varphi)}^{r_2(\varphi)} rdr.$$

Agar S soha $x = x(u;v)$ va $y = y(u;v)$ almashtirish yordamida sodda sohaga o'tkazilishi mumkin bo'lsa, u holda yuza $\iint |J| du dv$ yordamida aniqlanadi, bu erda

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial x} & \frac{\partial y}{\partial v} \end{vmatrix}$$

36.1 Ikki karrali integral ko'rinishida yozing va chiziqlar bilan chegaralangan yuzani hisoblang.

1. $xy = 4,\ y = x,\ x = 4$
2. $y = x^2,\ 4y = x^2,\ y = 4$
3. $y = x^2,\ 4y = x^2,\ x = \pm 2$
4. $xy = \frac{a^2}{2},\ xy = 2a^2,\ y = \frac{x}{2},\ y = 2x$

Egri chiziqli integrallar. Grin formulasi. Yuzalarni hisoblash.

1^0. Egri chiziqli integralning aniqlanishi. Silliq AB yoyda $P(x,y,z)$ uzluksiz funktsiya aniqlangan bo'lsin. AB yoyni $A(x_0;y_0;z_0)$ $M_1(x_1;y_1;z_1)$... , $M_{n-1}(x_{n-1};y_{n-1};z_{n-1})$ $B(x_n;y_n;z_n)$ nuqtalar yordamida qismlarga bo'lamiz, bunda $x_i - x_{i-1} = \Delta x_i$.

$\lim\limits_{\Delta x_i \to 0} \sum\limits_{i=1}^{n} P(x_i, y_i, z_i)\Delta x_i$ integral yig'indi limiti AB yoy bo'yicha olingan egri chiziqli integral deb ataladi va $\int\limits_{AB} P(x,y,z)dx$ tarzida yoziladi.

$$\int_{AB} Q(x,y,z)dy,\ \int_{AB} R(x,y,z)dz,$$

$\int_{AB} (Pdx + Qdy + Rdz)$ egri chiziqli integrallar ham yuqoridagiga o'hshash aniqlanadi.

Quyidagi ko'rinishdagi egri chiziqli integrallar ham uchrab turadi:

$\int_{AB} P(x,y,z)ds = \lim_{\Delta s_i \to 0} \sum_{i=1}^{n} P(x_i, y_i, z_i)\Delta s_i$, bu erda $\Delta s_i = M_{i-1}M_i$.

2[0]. Agar AB yoy $x = f(t),\ y = \varphi(t),\ z = \psi(t)$ tenglamalar bilan aniqlansa, t parametr esa $M(t)$ nuqta AB yoy bo'yicha bir yo'nalish bo'yicha harakatlanganda monoton o'zgarsa, u holda

$$\int_{AB} P(x,y,z)dx = \int_{t_A}^{t_B} P[f(t),\varphi(t),\psi(t)]f'(t)dt.$$

Yopiq L kontur bo'yicha olingan ikkinchi tur egri chiziqli integralni va shu kontur bilan chegaralangan D soha bo'yicha olingan ikki karrali integralni bog'lovchi formula Grin formulasi deb ataladi: $\oint_L Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy$

Bu formuladagi $P(x,y),\ Q(x,y)$ funktsiyalar va ularning birinchi tartibli hususiy hosilalari D sohada va L konturda uzluksiz bo'lishi kerak. Egri chiziqli integralda L kontur bo'yicha integrallash musbat yo'nalishda olinadi. Ikkinchi tur egri chiziqli integral orqali oddiy bo'lakli – silliq kontur bilan chegaralangan S yuzani hisoblash mumkin:

$$S = \oint_L xdy = -\oint_L ydx = \frac{1}{2}\oint_L xdy - ydx$$

Misollar:

1. Grin formulasi bo'yicha $J = \oint_L 2(x^2 + y^2)dx + (x+y)^2 dy$ integralni hisoblansin, bu erda $L - ABC$ uchburchak konturi: $A(1;1)$, $B(2;2;)$, $C(1;3)$.

Ecish: Ab, BC, CA to'g'ri chiziqlar tenglamasini $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$ tenglama yordamida topamiz: AB da $y = x$.

BC da $y = -x + 4$, AC da $x = 1$. ABC uchburchak konturi bilan chegaralangan D soha $x = 1$, $x = 2$, $y = x$, $y = 4 - x$ to'g'ri chiziqlar orasidadir. Bu ma'lumotlar ikki karrali integralni hisoblash uchun kerak. Endi $P = 2(x^2 + y^2)$, $Q = (x + y)^2$ larni topib, Grin formulasiga qo'yamiz va

$$J = \iint_D (2x + 2y - 4y)dxdy = 2\int_1^2 dx \int_x^{4-x} (x - y)dy =$$

$$= -\int_1^2 (x - y)\Big|_x^{4-x} dx = -\int_1^2 (2x - 4)^2 dx = -\frac{4}{3}$$

2. $\oint_L (e^{xy} + 2x\cos y)dx + (e^{xy} - x^2 \sin y)dy$ integralni L kontur bilan chegaralangan D soha bo'yicha olingan ikki karrali integralga keltiring.

Echish: Misolni echish uchun Grin formulasidan foydalamiz. Berilgan $P(x, y) = e^{xy} + 2x\cos y$,

$Q(x,y)=e^{xy}-x^2\sin y$ uchun $\dfrac{\partial Q}{\partial x}=ye^{xy}-2x\sin y$,

$\dfrac{\partial P}{\partial y}=xe^{xy}-2x\cdot\sin y$.

$$\oint\limits_L Pdx+Qdy=\iint\limits_D [(ye^{xy}-2x\sin y)-$$

$$-(x\cdot e^{xy}-2x\sin y)]dxdy=\iint\limits_D (y-x)e^{xy}dxdy$$ natijani hosil qilamiz.

36. 2. $A(2;2;)$, $B(2;0)$ nuqtalar berilgan. $\int\limits_{(C)}(x+y)dx$ integral 1) OA to'g'ri chiziq bo'yicha; 2) $y=\dfrac{x^2}{2}$ parabolaning OA yoyi bo'yicha; 3) OBA siniq chiziq bo'yicha hisoblansin.

36.3. 1). $\oint\limits_L (1-x^2)dx+x(1+y^2)dy$ integral L kontur $x^2+y^2=R^2$ aylana bo'yicha a) Grin formulasi orqali va b) bevosita hisoblansin.

2). $\oint\limits_L (xy+x+y)dx+(xy+x-y)dy$ integral a) Grin bo'yichava b) bevosita hisoblansin.

$1-$hol. L kontur $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ ellips;

$2-$hol. L kontur $x^2+y^2=ax$ aylana.

§37 Sonli qatorlar va ularning yaqinlashish alomatlari.

$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots\dots + a_n + \dots\dots$ qator yaqinlashuvchi deyiladi, agar $S_n = a_1 + a_2 + \dots\dots + a_n$ qismiy yigindilarining

limiti $\lim_{n\to\infty} S_n = S$ mavjud va chekli son bo'lsa, aks holda kator uzoqlashuvchi deyiladi.

Qator yaqinlashuvchi bulishi uchun $\lim_{n\to\infty} a_n = O$ bo'lishi zarur.

Yaqinlashuvchilikning etarli shartlarini quyidagi alomatlar bera oladi.

I. 1^0. $\sum_{n-1}^{\infty} a_n$ (1) va $\sum_{n-1}^{\infty} b_n$ (2) qatorlar uchun biror $n \geq n_0$ nomerdan boshlab $0 \leq a_n \leq b_n$ shart bajarilsa, (2) – qator yaqinlashishidan (1) – qator yaqinlashishi, (1) – qator uzoklashishidan (2) – qatorning uzoqlashishi kelib chiqadi.

2^0. Agar $a_n = O*\left(\frac{1}{n^p}\right)$, bo'lsa, $\sum_{n=1}^{\infty} a_n$ qator $p > 1$ da yaqinlashuvchi, $p \leq 1$ da uzoqlashuvchi bo'ladi.

Musbat hadli $\sum a_n$ qator berilgan bo'lsin.

II. 1^0. Dalamber alomati.

$D = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = q$ bo'lib, $q < 1$ bo'lsa, yaqinlashuvchi, $q > 1$ bo'lsa uzoqlashuvchidir.

2^0. Koshi alomati.

$K=\lim\limits_{n\to\infty}\sqrt[n]{a_n}=q$ bo'lib, $q<1$ bo'lsa, qator yaqinlashuvchi, $q>1$ bo'lsa uzoqlashuvchidir.

3^0. Koshining integral alomati. Agar $f(x)$ funktsiya $x\geq 1$ da manfiymas o'smaydigan uzluksiz funktsiya bo'lsa,

$$\sum_{m=1}^{\infty} f(x) \quad \text{va} \quad \int_1^{+\infty} f(x)dx$$

bir paytda yaqinlashadilar yoki uzoqlashadilar.

III. Ishorasi almashinuvchi qator $\sum\limits_{n=1}^{\infty}(-1)^{n+1}a_n$ yaqinlashuvchi bo'ladi,

agar $a_1>a_2>...$va $\lim\limits_{n\to\infty}a_n=0$ bajarilsa.

Agar $\sum\limits_{n=1}^{\infty}|b_n|$qator yaqinlashsa, $\sum\limits_{n=1}^{\infty}b_n$ qator absolyut yaqinlashuvchi deyiladi.

Agar $\sum|b_n|$ uzoqlashsa va $\sum b_n$

yaqinlashsa, u holda $\sum b_n$ shartli yaqinlashuvchi

deyiladi.

37.1. Yaqinlashuvchiligini isbotlang va yig'indilarni toping.

1) $\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{2^2}+\frac{1}{3^2}\right)+....+\left(\frac{1}{2^n}+\frac{1}{3^n}\right)+....$

2) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+...+\frac{1}{n(n+1)}+...$

3) $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+...+\frac{1}{n(n+1)}+...$

37.2 Alomatlar yordamida yaqinlashishini tekshiring.

1) $\sum_{n=1}^{\infty} \frac{(n^1)^2}{2n}$ 2) $\sum_{n=1}^{\infty} \frac{2^n \cdot n}{n^n}$ 3) $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$

4) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ 5) $\sum_{n=3}^{\infty} \frac{1}{n^2-1}$ 6) $\sum_{n=1}^{\infty} \frac{n}{1+n^2}$

7) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ 8) $\sum_{n=1}^{\infty} \frac{(n^1)^2}{2^{n^2}}$

36.3 Absolyut va shartli yaqinlashishni tekshiring.

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\rho}}$ 2) $\sum_{n=1}^{\infty} \frac{(-1)^n}{x+n}$

3) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ 4) $\sum_{n=1}^{\infty} \frac{(n1)^2}{2^{n^2}}$

§38 Funktsional va darajali qatorlar.

1^0. X to'plamda aniqlangan $f_1(x), f_2(x),....,f_n(x),....$ funktsiyalar ketma–ketligi uchun

a) $\lim_{n\to\infty} f_n(x) = f(x)$, b)

$\lim_{n\to\infty} \sup_{x\in X} |f_n(x) - f(x)| = 0$

shartlar bajarilsa, bu ketma–ketlik X to'plamda $f(x)$ funktsiyaga tekis yaqinlashadi deyiladi va $f_n \Rightarrow f$ tarzda yoziladi.

2^0. $\sum_{n=1}^{\infty} u_n(x) = u_1(x) + u_2(2) + ... + u_n(x) + ...$

funktsional qator yaqinlashadigan X nuqtalar to'plami bu qatorning yaqinlashish sohasi deyiladi.

$S(x) = \lim_{n\to\infty} S_n(x)$ funktsiya funktsional qator yigindisi,

$R_n(x) = S(x) - S_n(x)$ esa qator qoldig'i deyiladi.

$\sum_{n=1}^{\infty} u_n(x)$ qator $[a, b]$ kesmada tekis yaqinlashuvchi deyiladi, agar ixtiyoriy $\varepsilon>0$ uchun shunday N nomer topilsaki, n>N va ixtiyoriy $x \in [a,b]$ larda $|R_n(x) < \varepsilon|$ tengsizlik o'rinli bo'lsa,

3^0. **Veyershtrass alomati**: Agar shunday $\sum_{n=1}^{\infty} c_n$ yaqinlashuvi sonli qator mavjud bo'lib, $|U_n(x)| \le C_n$, $x \in [a,b]$, $n \in N$ shart bajarilsa, $\sum_{n=1}^{\infty} u_n(x)$ funktsionalqator [a,b] kesmada absolyut va tekis yaqinlashuvchi bo'ladi.

4^0.

$\sum_{n=0}^{\infty} = a_n x^n = a_0 + a_1 + a_2 x^2 + ... + a_n x^n + ...$ darajali qator uchun

$R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$ soni yakinlashish radiusi deyiladi:

Qator $|x| < R$ da yaqinlashuvchi, $|x| > R$ da esa uzoqlashuvchi bo'ladi.

$(-R, R)$ yaqinlashish intervali ichida qator absolyut va tekis yaqinlashuvchi buladi.

5^0. Berilgan sohada tekis yaqinlashuvchi funktsional, darajali qatorlarni hadma–had differentsiallash va integrallash mumkin.

38.1. Tekis yakinlashishini tekshiring.

a) $f_n(x) = x^n$, $0 \le x \le 1$

b) $f_n(x) = x^n - x^{2n}$, $0 \le x \le 1$

v) $f_n(x)=\dfrac{1}{x+n},\quad 0\le x<+\infty$

g) $f_n(x)=n\left(\sqrt{x+\dfrac{1}{n}}-\sqrt{x}\right),\quad 0<x<+\infty$

38.2. Tekis yaqinlashishini tekshiring.

a) $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{x+2^n},\quad -2<x<+\infty$

b) $\sum\limits_{n=1}^{\infty}\dfrac{x}{1+n^4x^2},\quad 0\le x<+\infty$

v) $\sum\limits_{n=1}^{\infty}\dfrac{nx}{1+n^5x^2},\quad |x|<+\infty$

g) $\sum\limits_{n=1}^{\infty}\dfrac{\cos nx}{n^2},\quad |x|<+\infty$.

38.3. Yaqinlashish intervalini toping.

a) $\sum\limits_{n=1}^{\infty}\left(1+\dfrac{1}{n}\right)^{n^2}x^n$ b) $\sum\limits_{n=1}^{\infty}\dfrac{3^n+(-2)^n}{n}(x+1)^n$

v) $\sum\limits_{n=1}^{\infty}\dfrac{x^{n^2}}{2^n}$ g) $\sum\limits_{n=1}^{\infty}\left(\dfrac{x}{\sin n}\right)^n$

38.4. Qator yig'indisini toping.

a) $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\ldots$

b) $\dfrac{x}{1\cdot2}+\dfrac{x^2}{2\cdot3}+\dfrac{x^3}{3\cdot4}+\ldots$

v) $1+\dfrac{1}{2}x+\dfrac{1\cdot3}{2\cdot4}x^2+\dfrac{1\cdot3\cdot5}{2\cdot4\cdot6}x^3+\ldots$

g) $x+2x^2+3x^3+\ldots$

d) $1\cdot2x-2\cdot3x^2+3\cdot4x^3-\ldots$

Bob bo'yicha misollar echish namunalari

1. $$Z=\sqrt{(x^2+y^2-1)(4-x^2-y^{2)}}$$

funktsiyaning aniqlanish sohasini toping.

$(x^2+y^2-1)(4-x^2-y^2)\geq\quad 0$ shartidan quyidagilarga egamiz;

$$\begin{cases} x^2+y^2-\geq 0 \\ 4-x^2-y^2\geq 0 \end{cases}$$ yoki

$$\begin{cases} x^2+y^2-1\leq 0 \\ 4-x^2-y^2\leq 0 \end{cases}$$

Bulardan $\begin{cases} x^2+y^2\geq 1 \\ x^2+y^2\leq 4 \end{cases}$ yoki

$\begin{cases} x^2+y^2\leq 1 \\ x^2+y^2\geq 4 \end{cases}$ kelib chiqadi.

Ikkinchi sistema echimga ega bo'la olmaydi. Demak, berilgan funktsiya $1\leq x^2+y^2\leq 4$ xalqada aniqlangan ekan.

2. $$\lim_{\substack{x\to\infty \\ y\to a}}(1+\frac{1}{x})^{\frac{x^2}{x+y}}=$$

$$\lim_{\substack{y\to a \\ x\to\infty}}(1+\frac{1}{x})^{x*\frac{1}{x}*\frac{x^2}{x+y}}=e^{\lim\limits_{\substack{x\to\infty \\ y\to a}}\frac{1}{1+\frac{y}{x}}}=e^{\lim\limits_{\substack{x\to\infty \\ y\to a}}\frac{1}{1+\frac{y}{x}}}=e^1=e.$$

3. $z=\ln(1-x^2-y^2)$ uzilish nuqtasini toping.

Funktsiya $1-x^2-y^2>0$ da uzluksiz, $x^2+y^2=1$ da esa, ya'ni birlik aylana har bir nuqtasida uzilishga ega.

4. Birinchi va ikkinchi tartibli xususiy hosilalarni toping.

$$z = y^4 + x^4 - 4x^2y^2;$$

$$z'_x = 4x^3 - 8x \cdot y^2; \; z'_y = 4y^3 - 8x^2 y;$$

$$z''_{xx} = 12x^2 - 8y^2; \quad z''_{yy} = 12y^2 - 8x^2.$$

$$z''_{xy} = -16xy = z''_{yx};$$

5. $z = (x^2 + y^2) \cdot e^{x+y}$ uchun $\dfrac{\partial^{m+n} z}{\partial x^m \partial y^n}$ ni toping.

$$z'_x = e^{x+y}\left[x^2 + y^2 + 2x\right]$$

$$z''_{xx} = e^{x+y}\left[x^2 + y^2 + 2x + 2x + 2\right] = e^{x+y}\left[x^2 + y^2 + 4x + 2\right]$$

$$z'''_{x^3} = e^{x+y}\left[x^2 + y^2 + 4x + 2 + 2x + 4\right] = e^{x+y}\left[x^2 + y^2 + 6x + 6\right]$$

$$\frac{\partial^{(4)} z}{\partial x^4} = e^{x+y}\left[x^2 + y^2 + 8x + 12\right]$$

Yuqoridagilardan :

$$\frac{\partial^m z}{\partial x^m} = \left[m(m-1) + 2mx + x^2 + y^2\right]e^{x+y}$$

$$\frac{\partial^{m+1} z}{\partial x^m \partial y} = \left[2y + m(m-1) + 2mx + x^2 + y^2\right]e^{x+y}$$

$$\frac{\partial^{m+n} n}{\partial x^m \partial x^n} = \left[n(n-1) + m(m-1) + 2(mx + ny) + x^2 + y^2\right]e^{x+y}$$

6. $z = \ln\sqrt{x^2 + y^2}$ ning to'liq differentsialini toping.

$$z'_x = \frac{1}{\sqrt{x^2 + y^2}} \cdot \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{x^2 + y^2} \quad \text{va}$$

$$z'_y = \frac{y}{x^2 + y^2}$$

ekanligidan $dz = \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy.$

7. $z = e^{\frac{x}{2}}(x+y^2)$ ekstremumini toping.

$z'_x = e^{\frac{x}{2}}(\frac{x}{2}+\frac{y^2}{2}+1) = 0$; $z'_y = 2ye^{\frac{x}{2}} = 0$ dan $x = -2$;

$y = 0$

kelib chiqadi. $P(-2,0)$ kritik nuqta.

$$z''_{x^2} = \frac{1}{2}e^{\frac{x}{2}}\left[\frac{x}{2}+\frac{y^2}{2}+2\right]; \qquad z''_{xy} = ye^{\frac{x}{2}};$$

$z''_{y^2} = e^{\frac{x}{2}}(2+y)$ lardan $A = \frac{1}{2e}$; $B = 0$; $C = \frac{2}{e}$

ekanligi kelib chiqadi.

$AB - C^2 = \frac{1}{e^2} > 0$; $A = \frac{1}{2e} > 0$ bo'lganligi uchun funktsiya $P(-2,0)$ nuqtada minimum qiymatiga erishadi:

$$z_{\min}(-2,0) = -\frac{2}{e};$$

8. $z = x^2 + y^2$ parabolaning $\frac{x}{a}+\frac{y}{b} = 1$ shartdagi ekstremumlarini toping.

Lagranj funktsiyasi

$L(x,y,\lambda) = x^2 + y^2 + \lambda(\frac{x}{a}+\frac{y}{b}-1)$ ko'rinishida bo'ladi.

$$L'_x = 2x + \frac{\lambda}{a} = 0, \quad L'_y = 2y + \frac{\lambda}{b} = 0 \text{ va}$$

$$\frac{x}{a} + \frac{y}{b} - 1 = 0$$

shartlaridan $\lambda = -\frac{2a^2 \cdot b^2}{a^2 + b^2}$; $x = \frac{ab^2}{a^2 + b^2}$;

$y = \frac{ba^2}{a^2 + b^2}$ kelib chiqadi.

$P\left(\frac{ab^2}{a^2 + b^2}; \frac{a^2b}{a^2 + b^2}\right)$ kritik nuqta ekan.

$L''_{x^2} = 2$; $L''_{xy} = 0$; $L''_{y^2} = 2$ lardan $A = 2$, $B = 0$,

$C = 2$ ekanligi, $AC - B^2 = 4 > 0$, $A = 2 > 0$ lardan esa

$$z_{min} = \left(\frac{ab^2}{a^2 + b^2}\right)^2 + \left(\frac{a^2b}{a^2 + b^2}\right)^2 = \frac{a^2b^4 + a^4b^2}{(a^2 + b^2)^2} = \frac{a^2b^2}{a^2 + b^2}.$$

9. $z = x^2 - xy + y^2 + 1$ funktsiyaning $D = \{|x| + |y| \le 1\}$ sohadagi eng katta va eng kichik qiymatlarini toping.

$z'_x = 2x - y = 0$, $z'_y = -x + 2y$ dan kritik nuqta $P(0;0)$ ekanligi kelib chiqadi.

$A = z''_{xx} = 2$, $B = z''_{xy} = -1$, $C = z''_{yy} = 2$ va

$D = AC - B^2 = 2 \cdot 2 - (-1)^2 = 3 \neq 0$ ekanligidan

$z_{min}(0;0) = 1$ kelib chiqadi.

Bundan tashqari, D soha chegaralari, masalan, $x + y = 1$ da $z = 3x^2 - 3x + 2$ ko'rinish oladi va $z \le 2$ bo'ladi.

$z(1;0)=z(0;1)=z(-1;0)=z(0;-1)=2$ ekanligidan esa $z_{e.kat}(1;0)=2,\quad z_{e.kich}(0;0)=1$ kelib chiqadi.

10. $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots+\frac{(-1)^{n-1}}{2^{n-1}}\ldots$ qator yaqinlashishini isbotlang va yig'indisining toping.

$$S=(1+\frac{1}{4}+\frac{1}{16}+\ldots.)-(\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\ldots.)=$$

$$=\frac{1}{1-\frac{1}{4}}-\frac{\frac{1}{2}}{1-\frac{2}{4}}=\frac{4}{3}-\frac{2}{3}=\frac{2}{3}$$

Berilgan qator yig'indisi chekli son bo'lganligi uchun, ta'rifga
ko'ra yaqinlashuvchidir.

11. 1) $\sum_{n=1}^{\infty}\frac{1000^n}{n!}$ yaqinlashishga tekshirilsin:

Dalamber alomatiga ko'ra;

$$D=\lim_{n\to\infty}\frac{\frac{1000^{n+1}}{(n+1)}}{\frac{1000^n}{n}}=\lim_{n=\infty}\frac{1000}{n+1}=0<1$$

Qator yaqinlashuvchi.

2) $\sum_{n=2}^{\infty}(\frac{n-1}{n+1})^{n(n-1)}$ yaqinlashishga tekshirilsin.

Koshi alomatiga ko'ra

$$K=\lim_{n\to\infty}\sqrt[n]{\left(\frac{n-1}{n+1}\right)^{n(n-1)}}=\lim_{n-\infty}(1-\frac{2}{n+1})^{-\frac{n+1}{2}\cdot(-\frac{2}{n+1})(n-1)}=$$

$$=e^{-2\lim\limits_{n\to\infty}\frac{n-1}{n+1}}=e^{-2}=\frac{1}{e^2}<1$$

Qator yaqinlashuvchi.

3) $\sum_{n=2}^{\infty}\frac{1}{n\cdot\ln^p n}$ yaqinlashishga tekshirilsin.

Koshining integral alomatiga ko'ra $p\neq 1$ da:

$$\int_2^{+\infty}\frac{dx}{x\ln^p x}=\lim_{A\to+\infty}\int_2^A\frac{d(\ln x)}{\ln^p x}=\lim_{A\to+\infty}\frac{\ln x}{-p+1}\Big|_2^A=$$

$$=\lim_{A\to+\infty}\left[\frac{\ln^{1-p}A}{1-p}-\frac{\ln^{1-p}2}{1-p}\right]$$

$$=\begin{cases} p>1 & da\ yaqinlashuvchi\\ p<1 & da\ uzoqlashuvchi\end{cases}$$

p=1

da $\int_2^{+\infty}\frac{dx}{x\ln x}=\lim_{A\to+\infty}\int_2^A d(\ln(\ln x))=\lim_{A\to\infty}\ln(x)\Big|_2^A=+\infty$

Demak berilgan qator ham $p\leq 1$ da uzoqlashuvchi, $p\geq 1$ da yaqinlashuvchidir.

12. $\sum_{n=1}^{\infty}\frac{n}{n+1}\left(\frac{x}{2x+1}\right)^n$ absolyut yaqinlashishga tekshirilsin.

Koshi alomatiga ko'ra:

$$K=\lim_{n\to\infty}\sqrt[n]{\left|\frac{n}{n+1}\cdot\left(\frac{x}{2x+1}\right)^n\right|}=\left|\frac{x}{2x+1}\right|$$

$\left|\frac{x}{2x+1}\right|<1$ da, ya'ni $x<-1$ va $x>-\frac{1}{3}$ bo'lganda berilgan qator absolyut yaqinlashuvchi.

13. Tekis yaqinlashishga tekshiring.

1. $f_n(x)=\sqrt{x^2+\frac{1}{n^2}}$; $\qquad -\infty<x<+\infty$

a) $\lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty}\sqrt{x^2+\frac{1}{n^2}} = \sqrt{x^2} = |x|$

b)

$$\lim_{n\to\infty}\sup_{x\in R}\left|\sqrt{x^2+\frac{1}{n^2}}-|x|\right| = \lim_{n\to\infty}\frac{1}{n^2}\sup_{x\in R}\frac{1}{\left|\sqrt{x^2+\frac{1}{n^2}}+|x|\right|} = \lim_{n\to\infty}\frac{n}{n^2} = 0$$

$$f_n(x) \Rightarrow |x|$$

2). $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}\cdot x^n}{n}$ qator $[0,1]$ kesmada tekis yaqinlashishini ko'rsating. n ning qanday qiymatlarida ixtiyoriy x uchun $|R_n(x)| < 0{,}1$ bo'ladi?

Ishora almashinuvchi qatorlarda har bir hadlari o'zidan keyingi hadlar yig'indisidan katta bo'ladi, ya'ni

$$|R_n(x)| < \frac{x^{n+1}}{n+1} < \frac{1}{n+1} \le 0{,}1$$

Demak, $n+1 \ge 10$ yoki $n \ge 9$ dan boshlab qoldik 0,1 dan kichik bo'ladi. Bu natija qator tekis yaqinlashishini ta'minlaydi.

3). $\sum\limits_{n+1}^{\infty}\frac{1}{x^2+n^2}$, $-\infty < x < +\infty$ qatorni tekis yakinlashishga tekshiring.

$\frac{1}{x^2+n^2} \le \frac{1}{n^2}$ ekanligidan Veyershtrass alomatiga ko'ra

$\sum\frac{1}{n^2+x^2}$ tekis yaqinlashuvchi, chunki $\sum\frac{1}{n^2}$ – yakinlashuvchidir.

14. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^n$ yaqinlashish intervalini ko'rsating.

$$R=\lim_{n\to\infty}\left|\frac{(n!)^2}{(2n)!}\cdot\frac{[2(n+1)]!}{[(n+1)!]^2}\right|=\lim_{n\to\infty}\left|\frac{(2n+1)(2n+2)}{(n+1)^2}\right|=4,$$

ya'ni yaqinlashish intervali $(-4,4)$ dir. $x=\pm 4$ da qator uzoqlashuvchi, masalan, $x=-4$ bo'lsa, uzoqlashuvchi qator.

$$\sum\frac{(-1)^n(n!)^2\cdot 2^{2n}}{2^n\cdot n!}=\sum(-1)^n n!2^n$$

kelib chiqadi.

15. $\sum_{n=1}^{\infty}nx^n$ qator yig'indisini toping.

$R=\lim_{n\to\infty}\frac{n}{n+1}=1$ ekanligidan $(-1,1)$ intervalda qator yig'indisi cheklilIgi kelib chikadi.

$$S(x)=x\cdot\sum_{n=1}^{\infty}n-x^{n-1}$$

$S_1(x)=\sum_{1}^{\infty}nx^{n-1}$ qatorni hadma-had integrallaymiz:

$$\int s_1(xdx)=\int\sum_{1}^{\infty}nx^{n-1}dx=\sum_{n=1}^{\infty}\int nx^{n-1}dx=\sum_{n=1}^{\infty}x^n=\frac{x}{1-x};$$

Demak, $S_1(x)=\left[\frac{x}{1-x}\right]'=\frac{1}{(1-x)^2}$ va

$$S(x)=\frac{x}{(1-x)^2}$$

9-bobga doir uy vazifalari

I. $z = f(x, y)$ **funktsiyaning yopiq D sohadagi eng katta va eng kichik qiymatlarini toping.**

1) $z = x^2 + y^2 - 9xy + 27; \quad D = \{0 \le x \le 3, \quad 0 \le y \le 3\}.$

2) $z = 3 - 2x^2 - xy - y^2; \quad D = \{x \le 1;\ y \ge 0;\ y \le x\}.$

3) $z = x^2 + 2y^2 + 1; \quad D = \{x \ge 0,\ y \ge 0, \quad x + y \le 3\}.$

4)
$z = x^2 + 3y^2 + x - y; \quad D = \{x \ge 1,\ y \ge -1; \ x + y \le 1\}.$

5) $z = x^2 + +2xy + 2y^2; \quad D = \{|x| \le 1; \quad 0 \le y \le 2\}.$

6) $z = 5x^2 - 3xy + y^2 + 4;\ D = \{x \ge -1,\ y \ge -1,\ x + y \le 1\}.$

7) $z = 10 + 2y - x^2; \quad D = \{0 \le y \le 4 - x^2\}.$

8) $z = x^2 + 2xy - y^2 + 4x;\ D = \{x \le 0;\ y \le 0,\ x + y + 2 \ge 0\}.$

9) $z = x^2 + xy - 2; \quad D = \{4x^2 - 4 \le y \le 0\}.$

10) $z = x^2 + xy; \quad D = \{|x| \le 1, \ 0 \le y \le 3\}.$

11) $z = x^2 + y^2 - 12x + 16y;\ D = \{x^2 + y^2 \le 25\}.$

12) $z = x^2 - xy + y^2; \quad D = \{|x| + |y| \le 1\}.$

13) $z = x^2 - 4xy + 4y^2;\ D = \{x \ge 0;\ y \ge 0;\ x + y \le 2\}.$

14) $z = x^2 + 4xy - 4y^2;\ D = \{x \ge -1;\ y \ge -1; \ x + y \le 1\}.$

15) $z = x^2 + 4xy; \quad D = \{x \le 2, \ y \le 2;\ y \ge x\}.$

II. $\sum_{n=1}^{\infty} a_n$ **sonli qatorni yaqinlashishini tekshiring:**

1) $a_n = \dfrac{n+3}{n^3 - 2}$ 2) $a_n = \dfrac{e^{-\sqrt{n}}}{\sqrt{n}}$

3) $a_n = \dfrac{1}{(2n+1)^2 - 1}$ 4) $a_n = \dfrac{3^n}{(2n)!}$

5) $a_n = \dfrac{n^3}{e^n}$

6) $a_n = \dfrac{1}{(n+1)[\ln(n+1)]^2}$

7) $a_n = \dfrac{2n+1}{\sqrt{n \cdot 2^n}}$

8) $a_n = \dfrac{n^2}{(3n)!}$

9) $a_n = \dfrac{1}{(n+1)\ln(n+1)}$

10) $a_n = \dfrac{n^{n+1}}{(n+1)!}$

11) $a_n = \dfrac{(n!)}{3^{n^2}}$

12) $a_n = \dfrac{1}{(3n+1)^2 - 2}$

13) $a_n = \dfrac{n-1}{n^4+1}$

14) $a_n = \dfrac{n^2}{e^n}$

15) $a_n = \dfrac{1}{n \ln n \ln(\ln n)}$

III. $\sum_{n=1}^{\infty} a_n \cdot x^n$ **darajali qator yaqinlashish intervalini toping.**

1) $a_n = \dfrac{\sqrt[3]{(n+1)^n}}{n!}$

2) $a_n = \dfrac{2^n}{n(n+1)}$

3) $a_n = \dfrac{(2n)!}{n^n}$

4) $a_n = \dfrac{3^n \cdot n!}{(n+1)^n}$

5) $a_n = \dfrac{n}{3^n \cdot (n+1)}$

6) $a_n = \dfrac{5^n}{\sqrt[n]{n}}$

7) $a_n = \left(1 + \dfrac{1}{n}\right)^n$

8) $a_n = \dfrac{n+1}{3^n \cdot (n+2)}$

9) $a_n = \dfrac{3^n}{\sqrt{2^n(3n-1)}}$ 10) $a_n = \dfrac{n+2}{n(n+1)}$

11) $a_n = \left(1+\dfrac{1}{n}\right)^{n^2}$ 12) $a_n = \dfrac{3^n + (-1)^n}{n}$

13) $a_n = \dfrac{(2n)!!}{(2n+1)!!}$ 14) $a_n = \dfrac{(-1)^n}{n!}\left(\dfrac{n}{e}\right)^n$

15) $a_n = \dfrac{n!}{3^{n2}}$

IV. Hadma–had differentsiallash, integrallash yordamida qator yig'indisini toping.

1) $x - \dfrac{x^7}{7} + \dfrac{x^{13}}{13} - \dfrac{x^{19}}{19} + \ldots.$

2) $x^2 + 2x^3 + 3x^4 + \ldots$

3) $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \ldots.$

4) $x^2 + \dfrac{x^3}{2} + \dfrac{x^4}{3} + \ldots.$

5) $x - 2x^2 + 3x^3 - \ldots$

6) $\dfrac{x}{1\cdot 2} + \dfrac{x^2}{2\cdot 3} + \dfrac{x^3}{3\cdot 4} + \ldots.$

7) $\dfrac{x^2}{1\cdot 2} - \dfrac{x^3}{2\cdot 3} + \dfrac{x^4}{3\cdot 4} - \ldots.$

8) $x - 4x^2 + 9x^3 - 16x^4 + \ldots$

9) $1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \ldots.$

10) $1\cdot 2\mathrm{x} - 2\cdot 3\cdot \mathrm{x}^2 + 3\cdot 4\cdot \mathrm{x}^3 - \ldots$

11) $1-\frac{x^4}{4}+\frac{x^8}{8}+\ldots$

12) $\frac{x^2}{1\cdot 2}-\frac{x^3}{2\cdot 3}+\frac{x^4}{3\cdot 4}-\ldots.$

13) $x+4x^2+9x^3+16x^4+\ldots$

14) $x+\frac{x^3}{3}+\frac{x^5}{5}+\ldots.$

15) $x+3x^3+5x^5+\ldots$

III–qism.
Differentsial tenglamalar.

Noma'lum funktsiya hosila yoki differentsial belgisi ostida qatnashgan tenglamalar differentsial tenglamalar deyiladi. Hosilaning eng yuqori tartibi differentsial tenglama tartibi deyiladi. n – tartibli differentsial tenglama

$$F(x, y, y' y'', ... y^{(n)}) = 0$$

tenglama bilan berilishi mumkin.

Bu tenglamani ayniyatga aylantiruvchi $y = \varphi(x)$ funktsiya differentsial tenglama echimi deyiladi. Tarkibida n ta o'zgarmas qatnashuvchi $\Phi(x, y, c_1, c_2, ..., c_n) = 0$ funktsiyalar oilasi differentsial tenglamani qanoatlantirsa, umumiy echim deyiladi. O'zgarmaslarning ma'lum bir qiymatida xususiy echimlar yuzaga keladi. Ma'lum shartlarda echimni topish Koshi masalasi deyiladi.

10–bob. Birinchi tartibli differentsial tenglamalar.

§39. Birinchi tartibli sodda differentsial tenglamalar.

Birinchi tartibli differentsial tenglamalar $F(x, y, \frac{dy}{dx}) = 0$ ko'rinishga ega. Bu tenglamani ko'p hollarda $\frac{dy}{dx}$ ga nisbatan uchib $\frac{dy}{dx} = f(x, y)$ ko'rinishga keltiriladi.

$\frac{dy}{dx} = f(x)$ ko'rinishdagi tenglamani $dy = f(x)dx$ ko'rinishda yozib, tomonlarni integrallasak $y = \int f(x)dx + c$ umumiy echim kelib chiqadi.

Shunga o'xshash $\frac{dy}{dx} = g(y)$ tenglama umumiy echimi $dx = \frac{dy}{g(y)}$ dan $x(y) = \int \frac{dy}{g(y)} + c$ yoki $\int \frac{dy}{g(y)} = x + c$ ko'rinishda bo'ladi.

39.1. Quyidagi umumiy echimlarga mos differentsial tenglamalarni tuzing.

1) $y = e^{Cx}$, 2) $y = (x-e)^3$

3) $y = Cx^3$ 4) $y = \sin(x+C)$

5) $y^2 + Cx = x^3$ 6) $y = C(x-C)^2$

7) $Cy = \sin Cx$

39.2. Quyidagi differentsial tenglamalar echilsin.

1) $y' = 3x^2$ 2) $y' = \cos x$ 3) $y' = 3e^{3x}$

4) $y' = y$ 5) $y' = \sin y$ 6) $y' = e^y$

39.3 Koshi masalasi echimini toping.

$$y' = \sin x, \qquad y(0) = 1$$

§40. O'zgaruvchilari ajraladigan differentsial tenglamalar.

$y' = f(x) \cdot g(y)$ yoki $M(x) \cdot N(y)dx + P(x) \cdot Q(y)dx = 0$ ko'rinishda yoziladigan differentsial tenglamalar o'zgaruvchilari ajraladigan differentsial tenglamalar deyiladi. Bunday tenglamalarni echish uchun ikkala tomonni shunday ifodalarga bo'lish (ko'paytirish) kerakki, natijada tenglamaning bir tomonida faqat y ga, ikkinchi tomonida faqat x ga bog'liq ifodalar hosil bo'lsin.

$$\frac{dy}{g(y)} = f(x)dx \quad \text{yoki} \quad \frac{Q(y)}{N(y)}dy = -\frac{M(x)}{P(x)}dx$$

So'ngra ikkala tomonni integrallab umumiy echim hosil qilinadi.

Ikkala tomon x, y qatnashgan ifodalarga bo'linganda, bu ifodalarni nolga aylantiradigan xususiy echimlar yo'qolishi mumkin.

$y' = f(ax + by + c)$ ko'rinishdagi differentsial tenglamalar

$z = ax + by + c$ yangi o'zgaruvchi kiritish yordamida o'zgaruvchilari ajraladigan differentsial tenglamalarga keltiriladi.

40.1. Quyidagi differentsial tenglamalarni eching.

1. $xy' - y = 0$
2. $xy' + y = 0$
3. $yy' + x = 0$
4. $y' = y$
5. $x^2 y' + y = 0$
6. $x + xy + y'(y + xy) = 0$
7. $\sqrt{y^2 + 1}dx = xydy$
8. $2x^2 yy' + y^2 = 0$
9. $y' - xy^2 = 2xy$
10. $y' = e^{x+y}$

40.2. Berilgan boshlang'ich shartni qanoatlantiruvchi xususiy echimlarni toping.

1. $2y'\sqrt{x} = y;\quad y(4) = 1.$
2. $y' = (2y + 1)ctgx;\quad y(\frac{\pi}{4}) = \frac{1}{2}.$
3. $x^2 y' + y^2 = 0;\quad y(-1) = 1.$
4. $y' = 2\sqrt{y}\ln x;\quad y(e) = 1.$
5. $(x^2 - 1)y' + 2xy^2 = 0;\quad y(0) = 1.$
6. $xy' + y = y^2,\quad y(1) = \frac{1}{2}.$

40.3. Yangi o'zgaruvchi kiritib o'zgaruvchilari ajraladigan differentsial tenglamaga keltiring va eching.

1. $y' = \sqrt{4x + 2y - 1},$
2. $y' = \cos(y - 1);$
3. $(x + 2y)y' = 1;\ y(0) = -1.$
4. $(2x - y + 1)y' = 1;$

§41. Bir jinsli differentsial tenglamalar.

$M(x,y)dx+N(x,y)dx=0$ tenglamada $M(\lambda x,\lambda y)$, $N(\lambda x,\lambda y)$ almashtirishlarda tenglama ko'rinishi o'zgarmasa, bunday tenglama bir jinsli deyiladi. Bunday tenglamalar

$\frac{dy}{dx}=\varphi(\frac{y}{x})$ ko'rinishga keladi va $\frac{y}{x}=u$ yoki $y=ux$ yangi o'zgaruvchi kiritish yordamida o'zgaruvchilari ajraladigan differentsial tenglamaga keltiriladi.

$y'=f\left(\frac{a_1x+b_1y+c_1}{ax+by+c}\right)$ ko'rinishdagi differentsial tenglamalar koorinatalar boshini $a_1x+b_1y+c_1=0$ va $a\ x+by+c=0$ to'g'ri chiziqlar kesishish nuqtasiga parallel ko'chirish yordamida bir jinsliga keltiriladi. Agar bu to'g'ri chiziqlar kesishmasa, $a_1x+b_1y=k(ax+by)$ bajarilib, $z=ax+by$ almashtirish yordamida o'zgaruvchilari ajraladigan differentsial tenglamaga keladi.

Ba'zi tenglamalarda $y=z^m$ almashtirish yordamida bir jinsliga keltirib olinadi. Buning uchun m soni differentsial tenglama bir jinsli bo'ladigan qilib tanlab olinadi. Bunday m soni mavjud bo'lmasa, bu usul bilan tenglamani bir jinsliga keltirib bo'lmaydi.

41.1. Bir jinsli ekanligini tekshiring va eching.

1. $yy'=2y-x$
2. $x^2+y^2-2xyy'=0$
3. $\frac{dy}{dx}=\frac{y}{x}-\frac{x}{y}$
4. $xy'+2\sqrt{xy}=y$
5. $(x-y)dx+(x+y)dy=0$
6. $(y^2-2xy)dx+x^2dy=0$
7. $xy'=y-x\cdot e^{\frac{y}{x}}$
8. $xy'=y\cos Ln\frac{y}{x}$

41.2. Parallel ko'chirish yordamida bir jinsliga keltiring va eching.

1. $(2x+y+1)dx-(4x+2y-3)dy=0$

2. $(x+4y)y'=2x+3y-5$

3. $(y+2)dx=(2x+y-4)dy$

4. $y'=2\left(\dfrac{x+2}{x+y-1}\right)^2$

5. $(y'+1)\ln\dfrac{y+x}{x+3}=\dfrac{y+x}{x+3}$

41.3. Yangi o'zgaruvchi kiritib bir jinsliga keltiring va eching.

1. $2x^2y'=y^3+xy$ 2.

$2xdy+(x^2y^4+1)ydx=0$

3. $ydx+x(2xy+1)dy=0$ 4. $2y'+x=4\sqrt{y}$

5. $y'=y^2-\dfrac{2}{x^2}$ 6.

$2y+(x^2\cdot y+1)xy'=0$

§42. Birinchi tartibli chiziqli tenglamalar.

Noma'lum funktsiya va uning hosilalari birinchi darajada qatnashgan differentsial tenglamalar chiziqli deyiladi. Birinchi tartibli chiziqli tenglama $y'+P(x)y=Q(x)$ ko'rinishda bo'ladi. Bunday tenglamani $y=u\cdot v$ almashtirish yordamida ikkita o'zgaruvchilari ajraladigan differentsial tenglamaga keltirish mumkin. O'zgarmas sonni variatsiyalash deb ataluvchi ikkinchi usulda bunday tenglamani echish uchun dastlab $y'+P(x)y=0$ tenglama umumiy echimi olinadi, undagi o'zgarmas S soni S(x) funktsiya bilan o'zgartiriladi, berilgan tenglamaga qo'yiladi va S(x) funktsiya topiladi.

Bu xususiy echim va bir jinsli tenglama umumiy echimi yig'indisi berilgan tenglama echimi hisoblanadi.

$y'+P(x)y=Q(x)\cdot y^n \quad (n\neq 1)$ ko'rinishdagi tenglama Bernulli tenglamasi deyiladi. Bu tenglamaning ikkala tomoni y^n ga bo'linib $\frac{1}{y^{n-1}}=z$ almashtirish o'tkazilsa, chiziqli tenglamaga ega bo'lamiz.

$y'+P(x)y+Q(x)\cdot y^2=R(x)$ ko'rinishdagi tenglama Rikkati tenglamasi deyiladi. Bunday tenglamaning biror xususiy $y_0(x)$ echimi ma'lum bo'lsagina, $y=y_0(x)+z$ almashtirish yordamida Bernulli tenglamasiga keltirish mumkin.

42.1. Chiziqli tenglamalarni eching.

1. $y'-\frac{3y}{x}=x$ 2. $(2x+1)y'=4x+2y$

3. $y'+ytgx=\frac{1}{\cos x}$ 4. $(xy+e^x)dx-xdy=0$

5. $2x(x^2+y)dx=dy$ 6. $x^2\cdot y'+(x+1)y=3x^2\cdot e^{-x}$

42.2. Izlanayotgan funktsiya va bog'liqsiz o'zgaruvchi «rollarini» almashtiring, hosil bo'lgan tenglamani eching.

1. $y=(2x+y^3)\cdot y'$ 2. $(x+y^2)dy=ydx$

3. $(2\cdot e^y-x)y'=1$ 4. $(\sin^2 y+xctgy)\cdot y'=1$

42.3. Bernulli tenglamalarini eching:

1. $x^2y'-xy=y^2$ 2. $y'-xy=-y^3\cdot e^{-x^2}$

3. $y'x+y=-xy^2$ 4. $xydy=(y^2+x)dx$

5. $xy'+2y+x^5y^3\cdot e^x=0$ 6. $y'x^3\sin y=xy'-2y.$

42.4. Xususiy echimi berilgan Rikkati tenglamalarini eching.

1. $x^2y'+xy+x^2y^2=4 \qquad y_0=\frac{2}{x}$

2. $3y'+y^2+\frac{2}{x^3}=0 \qquad y_0=\frac{1}{x}$

3. $xy' - (2x+1)y + y^2 = -x^2$, $y_0 = x$

4. $y' + 2ye^x - y^2 = e^{2x} + e^x$, $y_0 = e^x$

§43. To'la differentsial tenglamalar

$P(x,y)dx + Q(x,y)dy = 0$ tenglama chap tomoni biror $F(x,y)$ funktsiyaning to'la differentsiali bo'lsa, bu tenglama to'la differentsial tenglama deyiladi.

Masalan, $xdy + ydx = 0$, $\frac{xdy - ydx}{x^2} = 0$ tenglamalar chap tomoni mos ravishda $F(x,y) = x \cdot y$, $F(x,y) = \frac{y}{x}$ funktsiyalar to'la differentsiali bo'lib, umumiy echimlari $x \cdot y = C$; $\frac{y}{x} = C$ ko'rinishda bo'ladi.

$P(x,y)dx + Q(x,y)dy = 0$ tenglama chap tomoni to'la differentsial bo'lishi uchun $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ shart bajarilishi zarur. Bu shart bajarilsa, $dF = F_x' dx + F_y' dy = Pdx + Qdy = 0$ dan $F_x' = P$, $F_y' = Q$ kelib chiqadi.

$F = \int P(x,y)dx + \varphi(y)$ desak, (o'zgarmas son o'rniga $\varphi(y)$ olamiz.)

$F_y' = \left(\int P(x,y)dx\right)_y' + \varphi_y'(y) = Q(x,y)$ dan $\varphi(y)$, ya'ni $F(x,y)$ topiladi.

Agar $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$ bo'lsa, ba'zi hollarda shunday $\mu Pdx + \mu Qdy = 0$ tenglama to'la differentsial tenglama bo'ladi. Bu ko'paytuvchi integrallovchi ko'paytuvchi deyilib, quyidagi hollarda oson topiladi:

1) $\frac{P'_y - Q'_x}{Q} = \phi(x)$ bo'lsa, $\ln \mu = \int \phi(x)dx$.

2) $\frac{Q'_x - P'_y}{P} = \phi_1(y)$ bo'lsa, $\ln \mu = \int \phi_1(y)dy$.

Dastlabki paragraflardan differentsial tenglamalarning har biri to'la yoki to'la differentsial tenglamaga biror integrallovchi ko'paytuvchi yordamida keltiriluvchi tenglamalardir. Masalan, $y' + a(x)y = b(x)$ chiziqli tenglama uchun integrallovchi ko'paytuvchi

$\mu(x) = e^{\int a(x)dx}$ ko'rinishda bo'ladi.

43.1. To'la differentsialga keltirib eching.

1. $x^2dy + xydy = dx$
2. $y^2xdy - y^3dx = x^2dy$
3. $ydx + (x - y^3)dy = 0$
4. $ydx - (x - y^3)dy = 0$
5. $x^2y^2 + 1 + x^3yy' = 0$
6. $xdy - ydx = x^2dx$
7. $xy' + tgy = \frac{2x}{\cos y}$
8. $y(y \cdot e^{-\frac{x}{2}} + 1) = x \cdot y'$

43.2. To'la differentsial tenglama ekanligini tekshiring va eching.

1. $(4 - \frac{y^2}{x^2})dx + \frac{2y}{x}dy = 0$
2. $3x^2e^ydx + (x^3 \cdot e^y - 1)dy = 0$
3. $e^{-y}dx + (1 - xe^{-y})dy = 0$
4. $2x\cos^2 ydx + (2y - x^2 \sin 2y)dy = 0$
5. $(3x^2y - 4xy^2)dx + (x^3 - 4x^2y + 12y^3)dy = 0$
6. $(x\cos 2y + 1)dx - x^2 \sin 2ydy = 0$

7. $3x^2(1+\ln y)dx = (2y - \frac{x^3}{y})dy$

8. $2x(1+\sqrt{x^2 - y})dx - \sqrt{x^2 - y}dy = 0$.

43.3. Integrallovchi ko'paytuvchini toping va eching.

1. $(x^2 - y)dx + xdy = 0$ 2. $2xtgydx + (x^2 - 2\sin y)dy = 0$

3. $(e^{2x} - y^2)dx + ydy = 0$ 4. $(\sin x + e^y)dx + \cos xdy = 0$

5. $(1+3x^2 \cdot \sin y)dx - xctgydy = 0$ 6. $x(\ln y + 2nx - 1)dy = 2ydx$

7. $(x^2 - y)dx + x(y+1)dy = 0$ 8. $y^2(ydx - 2xdy) = x^3(xdy - 2ydx)$

§44. Hosilaga nisbatan echilmagan 1–tartibli differentsial tenglamalar. Lagranj va Klero tenglamalari.

Hosilaga nisbatan echilmagan $F(x, y, y') = 0$ tenglama asosan $y' = \frac{dy}{dx} = p$ parametr kiritish usuli bilan echiladi. Tenglamani $y = f(x, p)$ ko'rinishda yozib, ikkala tomondan to'liq differentsial olamiz.

$dy = pdx$ ekanligidan $M(x, p)dx + N(x, p)dp = 0$ ko'rinishdagi tenglamani hosil qilamiz. Bu tenglama echimi $x = \varphi(p)$ bo'lsa, berilgan tenglama echimi $y = f(\varphi(p), p)$ bo'ladi.

Differentsial tenglama $x = f(y, y')$ ko'rinishga kelsa ham, shu usulda umumiy echimdan tashqari maxsus echimlarni $F(x, y, p) = 0$, $F_p'(x, y, p) = 0$ tenglamalarda p ni yo'qotib topish mumkin.

$y = xf(y') + \varphi(y')$ tenglama Lagranj tenglamasi deyilib, $y' = p$ almashtirishdan quyidagi

$$p = f(p) + [xf'(p) + \varphi'(p)]\frac{dp}{dx}$$

x ga nisbatan chiziqli tenglamani hosil qilamiz.

$y = px + \varphi(p)$ tenglama Klero nomi bilan yuritilib, Lagranj tenglamasi xususiy holidir. Bunday tenglamalar maxsus echimga ham egadir.

44.1. Tenglamalar barcha echimlarini toping.

1) $y'^2 - y^2 = 0$ 2) $8y'^3 = 27y$

3) $(y'+1)^3 = 27(x+y)^2$ 4) $y^2(y'^2+1) = 1$

5) $y'^2 - 4y^3 = 0$ 6) $xy'^2 = y$

44.2. y' ga nisbatan echib, so'ngra umumiy echimlarini toping.

1) $xy'(xy'+y) = 2y^2$ 2) $xy'^2 - 2yy' + x = 0$

3) $xy'^2 = y(2y'-1)$ 4) $y'^2 + x = 2y$

5) $y'^2 - 2xy' = 8x^2$ 6) $(xy'+3y)^2 = 7x$

44.3. Yangi parametr kiritib eching.

1) $x = y'^3 + y'$ 2) $x(y'^2 - 1) = 2y'$

3) $y'^4 - y'^2 = y^2$ 4) $y'^2 - y'^3 = y^2$

5) $5y + y'^2 = x(x + y')$ 6) $y = 2xy' + y' \cdot y'^3$

44.4. Lagranj va Klero tenglamalarini eching.

1) $y = xy'^2 + y'^2$ 2) $y = 2xy' + \frac{1}{y'^2}$

3) $2y = \frac{xy'^2}{y'+2}$ 4) $y = xy' - y'^2$

5) $y = xy' - a\sqrt{1 + y'^2}$ 6) $y = xy' + \frac{1}{2y'^2}$

Bobga doir misollar echish namunalari

1. Echimi $x^2+cy^2=2y$ bo'lgan differentsial tenglamani tuzing.

Ikkala tomondan hosila olamiz:

$$2x+2c\cdot y\cdot y'=2y'$$

Bundan, $c=\frac{y'-x}{yy'}$. Berilgan tenglamaga qo'yib

$x^2+\frac{y'-x}{yy'}\cdot y^2=2y$ ni hosil qilamiz.

Soddalashtirib $x^2y'-xy=yy'$ tenglamani hosil qilamiz.

2. $y=Cx^3$ funktsiya $3y-xy'=0$ differentsial tenglama echimi ekanligini tekshiring va R(1;1) nuqtadan o'tuvchi xususiy echimini toping.

$y'=3Cx^2$ ni differentsial tenglamaga qo'ysak, $3Cx^3-x\cdot 3Cx^2=0$ ayniyat hosil bo'ladi. Demak, $y=Cx^3$ umumiy echim, $x=y=1$ ekanligidan $C=1$, ya'ni $y=x^3$ funktsiya R(1;1) nuqtadan o'tuvchi xususiy echimdir.

3. $\frac{dy}{dx}=\frac{1}{1+x^2}$, $x\in R$ tenglama umumiy echimi, $y(1)=\pi$ shartga bo'ysinuvchi xususiy echimini toping.

$dy=\frac{dx}{1+x^2}$ dan $y=arctgx+c$ umumiy echim.

$x=1$ da $y=\pi$ ekanligidan $\pi=arctg1+c$, ya'ni $c=\frac{3\pi}{4}$.

Koshi masalasi echimi $y=arctgx+\frac{3\pi}{4}$ dir.

4. $xydx+(x+1)dy=0$ tenglamani eching.

$(x+1)dy=-xydx$ ko'rinishda yozib olib, ikkala tomonni $y\cdot(x+1)$ ga bo'lamiz. Bunda tenglamani qanoatlantiruvchi $y=0$, $x=-1$ echimlar borligini yodda tutamiz.

Tenglama $\frac{dy}{y}=-\frac{x}{x+1}dx$ ko'rinishga keladi.

Ikkala tomonni integrallaymiz:

$$\int\frac{dy}{y}=-\int\frac{x}{x+1}dx$$

$\ln|y|=-x+\ln|x+1|+\ln C$ ya'ni $y=C\cdot(x+1)\cdot e^{-x}$ umumiy echimdir.

5. $y'\cdot ctgx+y=2$ tenglamaning $y(\frac{\pi}{3})=0$ shartni qanoatlantiruvchi echimini toping.

$\frac{dy}{dx}\cdot ctgx=2-y$ ko'rinishda yozib, tomonlarni $\frac{dy}{2-y}=tgxdx$ ko'rinishga keltiramiz. Ikkala tomonni integrallab $-\ln|2-y|=-\ln|\cos x|-\ln C$ yoki $-2+y=C\cdot\cos x$

Demak, $y=2+C\cdot\cos x$ umumiy echimdir.

Endi boshlang'ich shartni qanoatlantiruvchi echimni topamiz. $y(\frac{\pi}{3})=0$ dan $0=2+C\cdot\cos\frac{\pi}{3}$, ya'ni $0=2+C\cdot\frac{1}{2}$ dan $C=-4$.

Izlanayotgan echim $y=2-4Cosx$ bo'ladi.

6. $y'=y+2x-3$ tenglamani o'zgaruvchilari ajraladigan differentsial tenglamaga keltiring va eching.

$z=y+2x-3$ ko'rinishda yangi o'zgaruvchi kiritamiz.

$y=z-2x+3$ dan $y'=z'-2$

$z'-2=z$ ko'rinishdagi tenglamaga eǵa bo'lamiz.

$\frac{dz}{z+2}=dx$ dan

$\ln|z+2|=x+\ln C$ yoki $z+2=C\cdot e^{x}$

Eski o'zgaruvchilarga qaytib $y = C \cdot e^x - 2x + 1$ ekanligini topamiz.

7. $(x + 2y)dx - xdy = 0$ tenglamani eching.

$\lambda \neq 0$ uchun $(\lambda x + 2\lambda y)dx - \lambda xdy = 0$ tenglama berilgan tenglamaning aynan o'zi, demak, tenglama bir jinsli $y = u \cdot x$ almashtirish o'tkazamiz.

$$y' = u'x + u, \qquad dy = xdu + udx$$

ekanligidan $(x + 2 \cdot ux)dx - x \cdot (xdu + udx) = 0$

$x \cdot [(1 + 2u)dx - (xdu + udx)] = 0$ da $x = 0$ xususiy echim bo'ladi. Qavs ichini ixchamlab

$$(1 + u)dx = xdu$$

$$\int \frac{du}{1 + u} = \int \frac{dx}{x}$$

$$\ln|1 + u| = \ln x + \ln C$$

$$1 + u = C \cdot x$$

$\frac{y}{x} = Cx - 1$ dan $y = x \cdot (Cx - 1)$ umumiy echimdir.

8. $(2x - 4y + 6)dx + (x + y - 3)dy = 0$ tenglamani bir jinsliga keltiring va eching.

$2x - 4y + 6 = 0$ va $x + y - 3 = 0$ to'g'ri chiziqlar kesishish nuqtasi R(1;2)dir. Demak, $X = x - 1$, $Y = y - 2$ almashtirishlar o'tkazamiz.

$$[2(X + 1) - 4(Y + 2) + 6]dx + [(X + 1 + Y + 2 - 3)]dy = 0$$

$$(2X - 4Y)dX + (X + Y)dY = 0$$

hosil bo'lgan tenglama bir jinslidir.

$$(2x - 4 \cdot u \cdot X)dX + (X + u \cdot X)[udX + Xdu] = 0$$

$X = 0$, ya'ni $x - 1 = 0$ xususiy echim bo'lishi mumkin.

$$(2 - 4u)dX + (1 + u)(udX + xdu) = 0$$

$$\frac{1+u}{(u-1)(u-2)}=-\frac{dX}{X}$$

$$-\int\frac{2}{u-1}du+\int\frac{3}{u-2}du=-\int\frac{dX}{X}$$

$$-2\ln|u-1|+3\ln|u-2|=-\ln X+\ln C$$

$$3\ln|u-2|+\ln X=2\ln|u-1|+\ln C$$

$$(u-2)^3\cdot X=C(u-1)^2$$

$u=\frac{Y}{X}=\frac{y-2}{x-1}$ ekanligini hisobga olsak,

$$\left(\frac{y-2}{x-1}-2\right)^3\cdot(x-1)=C\left(\frac{y-2}{x-1}-1\right)^2$$

$(y+2x)^3=C(y-x-1)^2$ umumiy echimdir.

9. $x^3(y'-x)=y^2$ tenglamani bir jinsliga keltiring.

$y=z^m$, $y'=mz^{m-1}\cdot z'$ almashtirish o'tkazamiz.

$x^3(mz^{m-1}\cdot z'-x)=z^{2m}$

Bu tenglama bir jinsli bo'lishi uchun

$3+m-1=4=2m$ tengliklar bajarilishi, $m=2$ bo'lishi zarur.

Unda tenglama $x^3(2\cdot z\cdot z'-x)=z^4$ ko'rinishdagi bir jinsli differentsial tenglamaga aylanadi.

10. $y'-\frac{2}{x}\cdot y=2x^3$ chiziqli differentsial tenglamani eching.

$y'-\frac{2}{x}\cdot y=0$ tenglamaning umumiy echimi $y=Cx^2$.

$y=C(x)\cdot x^2$ deb olamiz va berilgan tenglamaga qo'yamiz:

$$C'(x)\cdot x^2+2x\cdot C(x)-\frac{2}{x}\cdot C(x)x^2=2x^3$$

Soddalashtirib $C'(x)=2x$, ya'ni $C(x)=x^2$ ekanligini topamiz. Xususiy echim $y=x^4$ ekan.

Berilgan tenglamaning umumiy echimi $y = Cx^2 + x^4$ ko'rinishda bo'ladi.

11. $y' - \frac{1}{x}y = \frac{x}{y^2}$ Bernulli tenglamasini eching.

$n = -2$ ekanligidan $z = \frac{1}{y^{-2-1}} = y^3$

almashtirish o'tkazamiz. $y = z^{\frac{1}{3}}$; $y' = \frac{1}{3}z^{-\frac{2}{3}} \cdot z'$

ekanligidan $\frac{1}{3}z^{-\frac{2}{3}} \cdot z' - \frac{1}{x} \cdot z^{\frac{1}{3}} = \frac{x}{z^{\frac{2}{3}}}$;

Tomonlarni $3z^{\frac{2}{3}}$ ga ko'paytirsak:

$z' - \frac{3}{x}z = 3x$ chiziqli tenglama hosil bo'ladi.

$z' - \frac{3}{x}z = 0$ ning umumiy echimi $z = C \cdot x^3$.

$z = C(x) \cdot x^3$ deb tenglamaga qo'yamiz: $C' = \frac{3}{x^2}$

dan $C(x) = -\frac{3}{x}$. Xususiy echim $z = -3x^2$ ko'rinishda, umumiy echim esa $z = C \cdot x^3 - 3x^2$ dir. Eski o'zgaruvchiga qaytib $y^3 = Cx^3 - 3x^2$

Bernulli tenglamasi echimi ekanligini topamiz.

12. $y' - 2xy + y^2 = 5 - x^2$ Rikkati tenglamasining xususiy echimi $y_1 = x + 2$ ma'lum bo'lsa, umumiy echimini toping.

$y = x + 2 + z$, $y' = 1 + z'$ almashtirish bajaramiz:

$1 + z' - 2x(x + 2 + z) + (x + 2 + z)^2 = 5 - x^2$.

Soddalashtirib

$z' + 4z = -z^2$ Bernulli tenglamasiga ega bo'lamiz.

$\frac{1}{z^{2-1}}=t;$ $z=\frac{1}{t};$ $z'=-\frac{1}{t^2}\cdot t'$ almashtirish yordamida $t'-4t=1$ chiziqli tenglamaga kelamiz.

$t'-4t=0$ tenglama echimi $t=C\cdot e^{4x}$,

$t'-4t=1$ tenglama echimi esa $t=\frac{4Ce^{4x}-1}{4}$ ekanligini topish mumkin.

Mos Bernulli tenglamasini echimi $z=\frac{4}{4Ce^{4x}-1}$ ko'rinishda, Rikkati tenglamasi umumiy echimi esa $y=x+2+\frac{4}{4Ce^{4x}-1}$ ko'rinishda bo'ladi.

13. $yCosxdx+\sin xdy=\cos 2xdy$ tenglamani to'la differentsialga keltirib eching.

Tenglama chap tomonini $d(y\sin x)$,

O'ng tomonini $d\left(\frac{\sin 2x}{2}\right)$ deyish mumkin.

Bundan $d(y\sin x-\frac{\sin 2x}{2})=0.$

Umumiy echim $y\sin x-\sin x\cos x=C$ yoki $y=\cos x+\frac{C}{\sin x}$ ko'rinishda bo'ladi.

14. $\frac{y}{x}dx+(y^3+\ln x)dy=0$ to'la differentsial tenglama ekanligini tekshiring va eching.

$P=\frac{y}{x};$ $Q=y^3+\ln x$ uchun $P_y'=Q_x'=\frac{1}{x}.$

Demak, berilgan tenglama to'la differentsial tenglamadir.

$F_x'=\frac{y}{x}$, $F_y'=y^3+\ln x$.

$F(x,y)=\int\frac{y}{x}dx+\varphi(y)=y\ln x+\varphi(y)\cdot$

$F_y'(x,y) = \ln x + \varphi_y'(y) = y^3 + \ln x$ tenglikdan $\varphi_y'(y) = y^3$, ya'ni $\varphi(y) = \frac{y^4}{4}$. Demak, umumiy echim quyidagi $y\ln x + \frac{y^4}{4} = C$ ko'rinishda bo'ladi.

15. $(x \cdot \sin y + y)dx + (x^2 \cdot \cos y + x\ln x)dy = 0$ tenglama uchun integrallovchi ko'paytuvchi toping va eching.

$P_y' = x\cos y + 1;\ Q_x' = 2x\cos y + \ln x + 1.$

$$\frac{P_y' - Q_x'}{Q} = \frac{-x\cos y - \ln x}{x^2\cos y + x\ln x} = -\frac{1}{x}.$$

$\ln\mu = \int(-\frac{1}{x})dx = -\ln x$ dan $\mu(x) = \frac{1}{x}.$

Demak, $(\sin y + \frac{y}{x})dx + (x\cos y + \ln x)dy = 0$ tenglama to'la differentsial bo'ladi. Haqiqatan, hosil bo'lgan tenglama uchun $P_y' = \cos y + \frac{1}{x} = Q_x'$

$F_x' = \sin y + \frac{y}{x};\ F_y' = x\cos y + \ln x.$

$F(x,y) = \int(\sin y + \frac{y}{x})dx = x\sin y + y\ln x + \varphi(y)$

$F_y'(x,y) = x\cos y + \ln x + \varphi_y'(y) = x\cos y + \ln x$

Bundan. $\varphi_y'(y) = 0,\ \varphi = C.$

Demak, umumiy echim $x\sin y + y\ln x = C_1$ ko'rinishda bo'ladi.

16. $yy'^2 + x = 1$ tenglamani eching.

y' ga nisbatan echamiz: $y' = -\sqrt[3]{\frac{x-1}{y}}$

O'zgaruvchilari ajraladigan differentsial tenglama hosil bo'ldi. Uning umumiy echimi

$y^{\frac{4}{3}} + (x-1)^{\frac{4}{3}} = \frac{4}{3}C$ ko'rinishda bo'ladi.

17. $y'^2+xy=y^2+xy'$ tenglamani y' ga nisbatan eching, so'ngra umumiy echimini toping.

$y'^2-xy'+xy-y^2=0$ tenglama y' ga nisbatan kvadrat tenglamadir.

$$(y')_{1,2}=\frac{x\pm\sqrt{x^2-4(xy-y^2)}}{2}=\frac{x\pm(x-2xy)}{2} \text{ dan}$$

a) $y'=x-y$ (chiziqli)

b) $y'=y$ (o'zgaruvchilari ajraladigan)

Bu tenglamalar mos ravishda $y=C\cdot e^{-x}+x-1,$ $y=Ce^x$ umumiy echimlarga egadir.

18. $y=y'^2+2y'^3$ tenglamani parametr kiritib eching.

$y'=p$ belgilash kiritsak, $y=p^2+2p^3$ dan $p=2p\cdot p'+6p^2\cdot p'$ Tomonlarni $p\neq 0$ ga qisqartirsak $1=2p'+6pp'$ yoki $x'=2+6p$. Bunda $x'=\dfrac{dx}{dp}$. Demak, $x=2p+3p^2+C,$ $y=p^2+2p^3$.

$p=0$ bo'lgan holda $y'=0,$ ya'ni $y=C$ lardan $y=0$ maxsus echim bo'ladi.

19. $y=xy'-y'^2$ Klero tenglamasini eching.

$y'=p$ belgilash kiritib, $p=x\cdot p'+p-2pp'$ ga egamiz. Undan $p'(x-2p)=0$ kelib chiqadi. Agar $p'=0$ bo'lsa $y'=C$ yoki $y=Cx+C_1$, $x-2p=0$ dan $y'=\dfrac{x}{2}$ yoki $4y=x^2+4c$ ga ega bo'lamiz.

10–bobga doir uy vazifalari.

I. Bir jinsli yoki unga keltiriluvchi differentsial tenglama umumiy echimlarini toping.

1) $(x^2 - y^2)y' = 2xy$ 2) $xy' = y\ln\frac{y}{x}$

3) $xy' + xe^{\frac{y}{x}} - y = 0$

4) $xy' - y = \sqrt{x^2 + y^2}$ 5) $xy' + y = \sqrt{x^2 + y^2}$

6) $xy' + y = x$

7) $y - xy' = 2(x + yy')$

8) $xy'(\ln y - \ln) = y$ 9) $y'\sqrt{x} = \sqrt{y - x} + \sqrt{x}$

10) $x^2 y' = y(x + y)$ 11) $(x + y)^2 \cdot y' = xy$

12) $xyy' = x^2 - y^2$

13) $(2x - y)y' = y - x - 1$

14) $2y' + x = 4\sqrt{y}$

15) $(x - y + 1)y' = y - x + 3$

II. Birinchi tartibli, chiziqli yoki chiziqliga keltiriluvchi differentsial tenglama umumiy echimini toping.

1) $y' + y = xy^3$ 2) $yy' + y^2 \text{ctg}x = \cos x$

3) $(1 + x^2)y' - 2xy = (1 + x^2)^2$ 4) $x^2 \cdot y' - 2xy + y^3 = 0$

5) $xy' + y = \frac{1}{y}$ 6) $xy' + y = (x + 1)y^2$

7) $(1 - x^2)y' - xy = 2x^2$ 8) $3y^2 y' + y^3 = x - 1$

9) $(2x + 1)y' + y = \frac{x}{y}$

10) $y' + y - y^2 = 1 - x - x^2,\ y_0 = x + 1$

11) $y' + xyy^2 = -2x - 3,\ y_0 = x + 2$

12) $y' + 4xy - 4y^2 = 12x - 5;\ y_0 = x - 3$

13) $y' - 3x^2y + xy^2 = 1 - 2x^3; \quad y_0 = x$

14) $y' - xy + 4y^2 = 5x^2 - 1; \quad y_0 = -x$

15) $xy' - y + y^2 = x^2; \quad y_0 = x$

III. Quyidagi to'la yoki to'laga keltiriluvchi differentsial tenglamalarni eching.

1) $2xydx + (x^2 - y^2)dy = 0$

2) $(2 - 9xy^2)xdx + (4y^2 - 6x^3)ydy = 0$

3) $e^{-y}dx - (2y + xe^{-y})dy = 0$

4) $\frac{x}{y}dx + (y^3 + \ln x)dy = 0$

5) $(1 + y^2 \sin 2x)dx - 2y\cos^2 xdy = 0$

6) $3x^2(1 + \ln y)dx = (2y - \frac{x^3}{y})dy$

7) $(1 - x^2)dy - xydx = 0$

8) $3dy + ydx = 0$

9) $(2x + 1)dy + ydx = 0$

10) $\cos xdy = (y + 1)\sin xdx$

11) $(3 + 2xy)dx - x^2dy = 0$

12) $(x + 1 - y)dx = xdy$

13) $(2y - x)dy - ydx = 0$

14) $dy + yctgxdx = 0$

15) $xdy - ydx = 0$

IV. Hosilaga nisbatan echilmagan quyidagi differentsial tenglamani eching.

1) $y = y'^2 + y'^3$

2) $y = 2y'x + \frac{x^2}{2} + y'^2$

3) $y = y'x + \frac{1}{x}$

4) $y = xy' + y' + y'^2$

5) $y'^2 + 4xy' - y^2 - 2x^2y = x^4 - 4x^2$

6) $y = xy' + y'^2$

7) $y = 4\sqrt{y'} - xy'$

8) $y = 2xy' - 4y'^3$

9) $y = xy' - (2 + y')$

10) $y'^3 = 3(xy' - y)$

11) $y = xy'^2 - 2y'^3$

12) $2xy' - y = \ln y'$

13) $xy' - y = \ln y'$

14) $xy'(y' + 2) = y$

15) $2y'^2(y - xy') = 1$

11–Bob.

Yuqori tartibli differentsial tenglama va sistemalar.

§45. Tartibi pasayadigan yuqori tartibli differentsial tenglamalar

$y^{(n)} = f(x)$ ko'rinishdagi differentsial tenglama ketma–ket n–marta integrallash yordamida umumiy echimi topiladi. Har bir integrallashda bittadan o'zgarmas qo'shiladi, natijada, umumiy echimda n ta o'zgarmas qatnashadi.

$F(x, y^{(k)}, y^{(k+1)}, \ldots, y^{(n)}) = 0$ ko'rinishidagi, noma'lum funktsiyaning o'zi qatnashmaydigan differentsial tenglamalar $y^{(k)} = z$ yangi o'zgaruvchi kiritish yordamida tartibi pasayadi.

Erkin o'zgaruvchi x qatnashmagan $F(x, y', y'', \ldots, y^{(n)}) = 0$ ko'rinishdagi differentsial tenglamalar $y' = p(y)$, $y'' = pp'$ almashtirishlar yordamida tartibi pasayadi.

Agar tenglama funktsiya va uning hosilalariga nisbatan bir jinsli bo'lsa ($(y, y', y'', \ldots, y^{(n)})$ lar $(ky, ky', ky'', \ldots, ky^{(n)})$ lar bilan almashtirganda tenglama o'zgarmasa), $y' = yz$ yangi o'zgaruvchi kiritish yordamida tartibi pasayishi mumkin.

Agar tenglama tomonlari to'la differentsiallar bo'lsa, integrallash yordamida tartibi pasayadi.

45.1. Tenglamalarni eching.

1) $y'' = 4\cos 2x$ 2) $y'' = \dfrac{1}{\cos^2 x}$

3) $y'' = \dfrac{1}{1+x^2}$ 4) $x^3 \cdot y'' + x^2 \cdot y' = 1$

5) $yy'' + y'^2 = 0$ 6) $y'' + y'tgx = \sin 2x$

7) $y'' + 2y \cdot y'^2 = 0$ 8) $y''x\ln x = y'$

8) $y''x\ln x = y'$ 10) $2yy'' = y'^2$

11) $2yy'' = 1 + y'^2$ 12) $y''tgx = y' + 1$

13) $xy''-y'=e^x\cdot x^2$

45.2. Tenglama tomonlarini to'la hosilaga keltirib eching.

1) $yy'''+3y'y''=0$ 2) $yy''=y'(y'+1)$ 3) $yy''+y'^2=1$ 4) $y''=xy'+y+1$ 5) $xy''+y'=2yy'$ 6) $xy''-y'=x^2\cdot yy'$

45.3 Bir jinsliligidan foydalanib eching:

1) $yy''=y'^2+15y^2\cdot\sqrt{x}$ 2) $(x^2+1)(y'^2-yy'')=xyy'$

3) $xyy''+xy'^2=2yy'$ 4) $x^2yy''=(y-xy')^2$

5) $x^2yy''+y'^2=0$ 6) $xyy''=y'(y+y')$

7) $x^2(y'^2-2yy'')=y^2$

§46. O'zgarmas koeffitsientli, chiziqli, bir jinsli differentsial tenglamalar

$y^{(n)}+a_1y^{(n-1)}+.......+a_ny=0$ (1) tenglamada $y=e^{kx}$ almashtirish yordamida $k^n+a_1k^{n-1}+.......+a_n=0$ (2) xarakteristik tenglamaga ega bo'lamiz.

1) Agar (2) tenglama o'zaro tengmas k_1, k_2, ...k_n – haqiqiy ildizlarga ega bo'lsa, e^{k_1x}, e^{k_2x}, ... e^{k_nx} funktsiyalar (1) ning xususiy, $y_0=C_1e^{k_1x}+C_2e^{k_2x}+...+C_ne^{k_nx}$ esa umumiy echim bo'ladi.

2) Agar (2) tenglama $k_1=k_2=....=k_m$, k_{m+1}, ...,k_n – haqiqiy ildizlarga ega bo'lsa, ya'ni k_1 – m karrali ildiz bo'lsa, u holda dastlabki m ta ildizga mos xususiy eichmlar e^{k_1x}, xe^{k_1x}, $...x^{m-1}e^{k_1x}$, ularga mos umumiy echim esa

$$y_0=(C_1+C_2x+C_3x^2+....+C_nx^{n-1})e^{k_1x}$$ ko'rinishda bo'ladi.

3) Har bir qo'shma kompleks $\alpha\pm\beta i$ ildizlarga $(C_1\cos\beta x+C_2\sin\beta x)e^{\alpha x}$ echim, agar bu ildizlar m – karrali bo'lsalar,

$$y_0=\left[(C_1+C_2x+...+C_mx^{m-1})\cos\beta x+(C_1+C_2x+....+C_mx^{m-1})\sin\beta x\right]e^{\alpha x}$$

echim mos keladi.

46.1. Tenglamalar umumiy echimlarini toping.

1. $y''-4y'=0$ 2. $y''-4y'+4y=0$
3. $y''-4y'+13y=0$ 4. $y''-4y'=0$
5. $y''+4y=0$ 6. $y''+4y'=0$
7. $y''+3y'-4y=0$ 8. $y''+2ay'+a^2y=0$
9. $y'''-5y''+8y'-4y=0$ 10. $y'''-3y''+4y=0$
11. $y'''+3ay''+3a^2y'+a^3y=0$ 12. $y^{IV}+4y=0$
13. $4y^{IV}-3y''-y=0$ 14. $y^{IV}-3y''-4y=0$
15. $y^{IV}+8y''+16y=0$

§47. O'zgarmas koefitsientli, chiziqli, bir jinsli bo'lmagan differentsial tenglamalar.

$$y^{(n)}+a_1y^{(n-1)}+....+a_ny=f(x)\quad (1)$$ va

$$y^{(n)}+a_1y^{(n-1)}+....+a_ny=0\quad (2)$$ tenglamalarni qaraymiz.

Agar y_1 (1) tenglama xususiy echimi, y_0 esa (2) tenglama umumiy echimi bo'lsa, (1) tenglama umumiy echimi $y=y_0+y_1$ ko'rinishda bo'ladi.

(1) tenglamaning xususiy echimi ikki xil usulda topilishi mumkin:

I. Aniqmas koeffitsientlar metodi.

(1) – tenglama xususiy echimi, bu metod yordamida quyidagi hollarda topiladi:

1) $f(x)=P_n(x)e^{mx}$ – ko'phad

2) $f(x)=e^{mx}(a\cos nx+b\sin nx)$

3) Funktsiya yuqoridagilarning yig'indisi yoki ko'paytmasi.

Bu hollarda y_1 – xususiy echim ham noma'lum koeffitsientli $f(x)$ funktsiya ko'rinishida izlanadi.

Agar 1) holda $k=m$, 2) holda $k=m\pm ni$ xarakteristik tenglamaning r – karrali ildizlari bo'lsa, izlanayotgan noma'lum koeffitsientli funktsiya $x^r\cdot f(x)$ ko'rinishda bo'ladi.

Ko'p hollarda $f(x)$ tarkibida sinus va kosinus qatnashganda Eylerning

$$\cos\beta x=\frac{1}{2}(e^{i\beta x}+e^{-i\beta x}),\quad \sin\beta x=\frac{1}{2i}(e^{i\beta x}-e^{-i\beta x})$$

formulalari yordamida yuqoridagi hollarga keltiriladi.

II. Lagranjning o'zgarmasni variatsiyalash usuli.

Agar $y_0=C_1y_1+C_2y_2+....+C_ny_n$ bir jinsli (2) tenglama umumiy echimi bo'lsa, (1) – tenglama umumiy echimi $y_0=C_1(x)y_1+C_2(x)y_2+....+C_n(x)y_n$ ko'rinishda izlanadi. Noma'lum C_ix funktsiyalar

$$C_1^1y_1+.....+C_n^1y_n=0$$

$$C_1^1y_1+.....+C_n^1y_n^1=0$$

....

$$C_1^1y_1^{(n-2)}+.....+C_n^1y_n^{(n-2)}=0$$

$$C_1^1y_1^{(n-1)}+.....+C_n^1y_n^{(n-1)}=f(x)$$

sistemadan topiladi.

47.1. Tenglamalarni eching.

1) $y''-2y'+y=e^{2x}$ 2) $y''-4y=8x^3$

3) $y''+3y'+2y=\sin 2x+2\cos 2x$

4) $y''+y=x+2e^x$ 5) $y''+3y'=9x$

6) $y''+4y'+5y=5x^2-32x+5$

7) $y''-3y'+2y=e^x$ 8) $y''-2y=x\cdot e^{-x}$

9) $y''-2y'=x^2-x$ 10) $y''+5y'+6y=e^{-x}+e^{-2x}$

11) $y'''+y''=6x+e^{-x}$ 12) $y^{IV}-81y=27e^{-3x}$

13) $y'''+8y=e^{-2x}$ 14) $y^{IV}-3y''+4y=3\sin x$

15) $y'''-3y''+3y'-y=e^x$.

47.2. O'zgarmasni variatsiyalash yordamida eching:

1) $y''+4y=\dfrac{1}{\sin 2x}$ 2) $y''-4y'-5y=\dfrac{e^{2x}}{\cos x}$

3) $y''-2y'+y=x^{-2}\cdot e^x$ 4) $y''+y=\text{tg}x$

5) $y''+y'=\dfrac{1}{1+e^x}$ 6) $y''+4y'+4=\dfrac{e^{-2x}}{x^3}$

7) $y''+4y'+4y=e^{-2x}\cdot\ln x$ 8) $y''+y=\dfrac{1}{\cos^3 x}$

9) $y''-2y'+y=\dfrac{e^x}{\sqrt{4-x^2}}$ 10) $y''+4y=\dfrac{1}{\sin^2 x}$

§48. O'zgarmas koeffitsientli, chiziqli differentsial tenglamalar sistemalari.

Noma'lumlarni ketma–ket yo'qotish yordamida murakkab bo'lmagan sistemalarni echish mumkin.

48.1. Bir jinsli sistemani eching:

1) $\begin{cases}\overset{\circ}{x}=x-y\\ \overset{\circ}{y}=y-4x\end{cases}$ 2) $\begin{cases}\overset{\circ}{x}+x-8y=0\\ \overset{\circ}{y}-x-y=0\end{cases}$

3) $\begin{cases} \overset{\circ}{x} = x + y \\ \overset{\circ}{y} = 3y - 2x \end{cases}$ 4) $\begin{cases} \overset{\circ}{x} = x + z - y \\ \overset{\circ}{y} = x + y - z \\ \overset{\circ}{z} = 2x - y \end{cases}$

5) $\begin{cases} \overset{\circ}{x} = x - 2y - z \\ \overset{\circ}{y} = y - x + z \\ \overset{\circ}{z} = x - z \end{cases}$

48.2. Bir jinsli bo'lmagan sistemani eching:

1) $\begin{cases} \overset{\circ}{x} = y + 2e^t \\ \overset{\circ}{y} = x + t^2 \end{cases}$ 2) $\begin{cases} \overset{\circ}{x} = y - 5\cos t \\ \overset{\circ}{y} = 2x + y \end{cases}$

3) $\begin{cases} \overset{\circ}{x} = 3x + 2y + 4e^{5t} \\ \overset{\circ}{y} = x + 2y \end{cases}$ 4) $\begin{cases} \overset{\circ}{x} = 2x - 4y + 4e^{-2t} \\ \overset{\circ}{y} = 2x - 2y \end{cases}$

5) $\begin{cases} \overset{\circ}{x} = 4x + y - e^{2t} \\ \overset{\circ}{y} = y - 2x \end{cases}$ 6) $\begin{cases} \overset{\circ}{x} = 2y - x + 1 \\ \overset{\circ}{y} = 3y - 2x \end{cases}$

Bobga doir misollar echish namunalari

1. $y''' = \dfrac{6}{x^3}$ tenglamaning $x = 1$ da $y = 2,\ \ y' = 1;\ \ y'' = 1$ shartlarga bo'ysunuvchi echimini toping.

Ketma-ket integrallab quyidagilarni topamiz:

$$y''=-\frac{3}{x^2}+C,\quad y'=\frac{x}{3}+C_1x+C_1,\quad y=3\ln x+C\frac{x^2}{2}+C_1x+C_2$$

$x=1$ da o'zgarmaslarni topish uchun quyidagi sistemaga ega bo'lamiz:

$$\begin{cases} 1=-3+C \\ 1=3+C+C_1 \\ 2=\frac{C}{2}+C_1+C_2 \end{cases}$$

Bundan esa $C=4;\ C_1=-6;\ C_2=6$. Xususiy echim $y=3\ln x+2x^2-6x+6$

2. $x^2\cdot y''=y'^2$ tenglamani eching.

$y'=z,\ y''=z'$ almashtirish yordamida $x^2\cdot z'=z^2$ o'zgaruvchilari ajraladigan differentsial tenglamaga ega bo'lamiz. Uning echimi quyidagi ko'rinishda bo'ladi: $z\neq 0,\ \frac{1}{z}=\frac{1}{x}-C$, яъни $\frac{1}{y'}=\frac{1}{x}-C$.

Bundan $y'=\frac{x}{1-Cx},\ Cy=C^2x+\ln|1-Cx|=C_1$ umumiy echimni topamiz. Agar $z=0$ bo'lsa, $y'=0,\ y=C.$

3. $2yy''-1=y'^2$ tenglamani eching.

$y'=p,\ y''=pp'$ almashtirishlardan $2ypp'-1=p^2,\ \frac{2pdp}{p^2+1}=\frac{dy}{y}$ va $\ln|p^2+1|=\ln y+\ln C.$

Bundan $p^2+1=C\cdot y$, yoki $y'=\pm\sqrt{Cy-1}$.

Bundan, $4(Cy-1)=C^2(x+C_2)$ umumiy echimni olamiz.

4. $y'\cdot y'''=2y''^2$ tenglama tomonlarini to'la hosilalar ko'rinishida keltirib eching.

Tomonlarni $y' \cdot y''$ ga bo'lib, $\frac{y'''}{y''} = 2\frac{y''}{y'}$ yoki $(\ln y'')' = (2\ln y')'$ dan $y'' = C \cdot y'^2$ ga ega bo'lamiz. Bu tenglamani ham $\frac{y''}{y'} = Cy'$ yoki $(\ln y')' = (C \cdot y)'$ ko'rinishda yozish mumkin. Demak, $\ln y' = Cy + LnC_1$ yoki $y' = C_1 e^{Cy}$ lardan $-\frac{1}{c} e^{-Cy} = C_1 x + C_2$ yoki $y = -\frac{1}{C} \ln|CC_2 - CC_1 x|$ kelib chiqadi.

5. Bir jinsliligidan foydalanib tarkibini pasaytiring va eching: $xyy'' - xy'^2 = yy'$

$$y' = y \cdot z;\ y'' = y' \cdot z + y \cdot z' = yz^2 + yz'$$

almashtirishlar o'tkazamiz: $xy(yz^2 + yz') - xy^2 \cdot z^2 = y \cdot yz$. $y^2 \neq 0$ deb tomonlarni qisqartirsak, $xz^2 + xz' - xz^2 = z$ yoki $xz' = z$ tenglama hosil bo'ladi. Bundan $z = Cx$ yoki $y' = C \cdot yx$. Bu tenglama echimi esa $y = C_1 \cdot e^{\frac{1}{2}Cx^2}$ ko'rinishda bo'ladi.

6. $y'' - 4y' + 3y = 0$ tenglama umumiy echimini toping.

Xarakteristik tenglama $k^2 - 4k + 3 = 0$ bo'lib, $k_1 = 1,\ k_2 = 3$ dir. Demak, $y_0 = C_1 e^x + C_2 e^{3x}$

45.2. $y''' - 3y'' + 3y' - y = 0$ tenglama umumiy echimini toping.

$k^3 - 3k^2 + 3k - 1 = 0$ tenglama $(k-1)^3 = 0$ ga ekvivalent tenglamadir. Demak, $k_{1,2,3} = 1$ va $y_0 = (C_1 + C_2 x + C_3 x^2)e^x$.

7. $y'' - 2y' + 5 = 0$ tenglamaning umumiy echimini toping.

$k^2-2k+5=0$ xarakteristik tenglama $k_{1,2}=1\pm 2i$ ildizlarga ega. Umumiy echim esa $(C_1\cos 2x+C_2\sin x)e^x$ ko'rinishda bo'ladi.

8. $y^{IV}+8y''+16=0$ tenglamaning umumiy echimini toping.

Xarakteristik tenglama $k^4+8k^2+16=(k^2+4)^2=0$ ko'rinishda bo'lib, $k_{1,2}=2i$, $k_{3,4}=-2i$ ildizlardir.

$y_0=[(C_1+C_2x)\cos 2x+(C_3+C_4x)\sin 2x]e^{0x}$ bo'ladi.

9. Berilgan differentsial tenglama xarakteristik tenglamasi

$k_1=2;\ k_2=3;\ k_{3,4}=4;\ k_{5,6}=-1\pm 5i;\ k_{7,8,9,10}=2\pm 7i$

ildizlarga ega. Umumiy echim ko'rinishini yozing.

Ildizlar barcha xususiy hollarni o'z ichiga oladi. Umumiy echim esa

$$y_0=C_1e^{2x}+C_2e^{3x}+(C_3+C_4x)e^{4x}+(C_5\cos 5x+C_6\sin 5x)e^{-x}+$$
$$+[(C_7+C_8x)\cos 7x+(C_9+C_{10}x)\sin 7x]e^{2x}$$

10. $y'''-3y''+3y'-y=e^x+x$ tenglamani aniqmas koeffitsientlar metodi bilan yozing.

Bir jinsli tenglama xarakteristik tenglamasi ildizlari $k_{1,2,3}=1$ ekanligidan $y_0=(C_1+C_2x+Cx^2)e^x$.

a) $y'''-3y''+3y'-y=e^x$ xususiy echim $y_1=Ax^3e^x$ ko'rinishda izlanadi.

$$y_1'=A[3x^2e^x+x^3e^x]=A(3x^2+x^3)e^x.$$
$$y_1''=A[6x+3x^2+3x^2+x^3]e^x=A[6x+6x^2+x^3]e^x.$$
$$y_1'''=A[6+12x+3x^2+6x+6x^2+x^3]e^x=A[6+18x+9x^2+x^3]e^x.$$

Topilganlarni o'rniga qo'yib:

$$A[6+18x+9x^2+x^3-18x-18x^2-3x^3+9x^2+3x^3-3x^2-x^3]e^x=e^x.$$

ya'ni $A[6-3x^2]=1$ dan $A=\frac{1}{6}$ ва $y_1=\frac{x^3}{6}\cdot e^x$ bo'ladi.

b) $y'''-3y''+3y'-y=x$ xususiy echimi $y_2=Ax+B$ tarzida izlanadi. $y_2'=A;\ y_2''=0\cdot$ Bundan: $3A-Ax-B=x$, ya'ni $A=-1; B=-3$ ва $y_2=-x-3$.

Umumiy echim esa $y=(C_1+C_2x+Cx^2+\frac{x^3}{6})e^x-x-x$ ko'rinishda bo'ladi.

11. $y''-3y'+2y=\frac{e^{3x}}{1+e^{2x}}$ tenglamani o'zgarmasni variatsiyalash yordamida eching.

$k^2-3k+2=0$ echimlari $k_1=1,\ k_2=2$ ekanligidan tenglama xususiy echimlari e^x va e^{2x} dir. Bundan $y_0=C_1e^x+C_2e^{2x}$ va

$$\begin{cases} C_1'\cdot e^x+C_2'e^{2x}=0 \\ C_1'e^x+2C_2'e^{2x}=\frac{e^{3x}}{1+e^{2x}} \end{cases}$$

sistemaga ega bo'lamiz. $C_1'=-C_2'e^x$ ni ikkinchi tenglamaga qo'yib $C_2^1e^{2x}=\frac{e^{3x}}{1+e^{2x}}$, ya'ni

$C_2^1=\frac{e^x}{1+e^{2x}}$ ga ega bo'lamiz. Bundan

$C_2=\text{arctg}e^x$.

$C_1^1=-\frac{e^{2x}+1-1}{1+e^{2x}}$ dan $C_1^1=-1+\frac{1}{1+e^{2x}}$ va

$C_1=-\ln\sqrt{1+e^{2x}}$.

Demak, umumiy echim

$y=C_1e^x+C_2e^{2x}-\ln\sqrt{1+e^{2x}}\cdot e^x+e^{2x}\text{arctg}e^x$.

12. $\begin{cases} \overset{\circ}{x} = 2x + y \\ \overset{\circ}{y} = 3x + 4y \end{cases}$ sistemani eching, bunda

$\overset{\circ}{x} = \dfrac{dx}{dt}, \quad \overset{\circ}{y} = \dfrac{dy}{dt}.$

Birinchi tenglamadan $y = \overset{\circ}{x} - 2x$ ekanligidan, uni ikkinchi tenglamaga qo'yib $\overset{\circ\circ}{x} - 2\overset{\circ}{x} = 3x + 4(\overset{\circ}{x} - 2x)$ yoki $\overset{\circ\circ}{x} - 6\overset{\circ}{x} + 5x = 0$ tenglamaga ega bo'lamiz. Xarakteristik tenglama ildizlari $k_1 = 1,\ k_2 = 5$ ekanligidan

$x = C_1e^t + C_2e^{5t}.\quad \overset{\circ}{x} = C_1e^t + 5C_2e^{5t}$ bo'lganligi uchun

$y = C_1e^t + 5C_2e^{2t} - 2C_1e^t - 2C_2e^{5t} = -C_1e^t + 3C_2e^{5t}$

kelib chiqadi.

Demak, $x = C_1e^t + C_2e^{5t}$

$y = -C_1e^t + 3C_2e^{5t}.$

13. $\begin{cases} \overset{\circ}{x} = x - y + 8t \\ \overset{\circ}{y} = 5x - y \end{cases}$ bir jinsli bo'lmagan sistemani eching.

Ikkinchi tenglamadan $x = \dfrac{\overset{\circ}{y}}{5} + \dfrac{y}{5},\quad \overset{\circ}{x} = \dfrac{\overset{\circ\circ}{y}}{5} + \dfrac{\overset{\circ}{y}}{5}$ larni topib birinchi tenglamaga qo'yamiz.

$\dfrac{\overset{\circ\circ}{y}}{5} + \dfrac{\overset{\circ}{y}}{5} = \dfrac{\overset{\circ}{y}}{5} + \dfrac{y}{5} - y + 8t$

$\overset{\circ\circ}{y} + 4y = 40t$ tenglama hosil bo'ladi.

$k^2+4=0$ dan $k_{1,2}=\pm 2i$ ya'ni $y_0=C_1\cos 2t+C_2\sin 2t$ $y_1=At+B$ ko'rinishda izlanadi. $4At+4B=40t$ dan $A=10; B=0$.

Demak,

$$y_1=C_1\cos 2t+C_2\sin 2t+10t,$$

$$x=\frac{1}{5}\left(-2C_1\sin 2t+2C_2\cos 2t+C_1\cos 2t+C_2\sin 2t+10t\right)$$

11-bobga doir uy vazifalari

I. Tartibini pasaytiring va yeching.

1) $2x'y''=y'^2-2$ 2) $y'^2+4yy''=0$

3) $yy''+3=y'^2$ 4) $y'''=4y''^2$

5) $y'''=2(y''-5)ctgx$ 6) $y''^3=xy''=6y'$

7) $y''+y'^2=7e^{-y}$ 8) $y''^2=y'^2+8$

9) $y''-xy'''+y'''^2=0$ 10) $y^4-y^3\cdot y''=10$

11) $y''(2y'+x)=11$ 12) $(1-x^2)y''+xy'=12$

13) $(y'+13y)y''=y'^2$ 14) $y'''\cdot y'^2=4y''^3$

15) $xy''=y'+x(y'^2+x^2)$

II. Bir jinsli bo'lmagan tenglamalarni yeching.

1) $y''+4y'-12y=8\sin 2x$

2) $y''-6y'+9y=x^2-x+3$

3) $y''+4y'=e^{-2x}$

4) $y''-2y'+5y=xe^{2x}$

5) $y''+5y'+6y=\cos 2x$

6) $y''-5y'+6y=(12x-7)e^{-x}$

7) $y''-4y'+13y=26x+5$

8) $y''-2y'+y=16e^{x}$

9) $y''-4y'=6x^2+1$

10) $y''+6y'+9y=10e^{-3x}$

11) $y''+4y'=e^{x}+x$

12) $y''-3y=x^2+5$

13) $y''+y'+y=e^{x}$

14) $y''+2y'+4y=e^{2x}$

15) $y''-4y=e^{2x}$

III. Sistemani yeching:

1) $\begin{cases} \overset{\circ}{x}=4x+6y \\ \overset{\circ}{y}=4x+2y \end{cases}$

2) $\begin{cases} \overset{\circ}{x}=-5x-4y \\ \overset{\circ}{y}=-2x-3y \end{cases}$

3) $\begin{cases} \overset{\circ}{x}=3x+y \\ \overset{\circ}{y}=8x+y \end{cases}$

4) $\begin{cases} \overset{\circ}{x}=6x+3y \\ \overset{\circ}{y}=-8x-5y \end{cases}$

5) $\begin{cases} \overset{\circ}{x}=-x+5y \\ \overset{\circ}{y}=x+3y \end{cases}$

6) $\begin{cases} \overset{\circ}{x}=3x-2y \\ \overset{\circ}{y}=2x+5y \end{cases}$

7) $\begin{cases} \overset{\circ}{x}=-4x-6y \\ \overset{\circ}{y}=-4x-2y \end{cases}$

8) $\begin{cases} \overset{\circ}{x}=5x-8y \\ \overset{\circ}{y}=-3x-3y \end{cases}$

9) $\begin{cases} \overset{\circ}{x}=-x-5y \\ \overset{\circ}{y}=-7x-3y \end{cases}$

10) $\begin{cases} \overset{\circ}{x}=-7x+5y \\ \overset{\circ}{y}=4x-8y \end{cases}$

11) $\begin{cases} \overset{\circ}{x} = 2x - y \\ \overset{\circ}{y} = x + 2e^t \end{cases}$ 12) $\begin{cases} \overset{\circ}{x} = 2x - 4y \\ \overset{\circ}{y} = x - 3y + 3e^t \end{cases}$

13) $\begin{cases} \overset{\circ}{x} = x + 2y \\ \overset{\circ}{y} = x - 5\sin t \end{cases}$ 14) $\begin{cases} \overset{\circ}{x} = 2x - y \\ \overset{\circ}{y} = y - 2x + 18t \end{cases}$

15) $\begin{cases} \overset{\circ}{x} = 5x - 3y + 2e^{3t} \\ \overset{\circ}{y} = x + y + 5e^{-t} \end{cases}$

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