O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA
MAXSUS TA'LIM VAZIRLIGI

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Oliy matematika fanidan mustaqil ishlarni bajarish uchun o'quv-uslubiy qo'llanma

2-qism

Maydonlar nazariyasi, kompleks o'zgaruvchili funktsiyalar nazariyasi, chiziqli tenglamalar sistemasini Jordan-Gauss usulida yechish, bir noma'lumli tenglamalarni urinma va vatarlar usulida yechish, aniq integrallarni to'rtburchaklar usulida taqriban yechish, oddiy differensial tenglamalarni yechishning sonli usullari.

Texnika oliy o'quv yurtlarining bakalavriat ta'lim yo'nalishlari talabalari uchun

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Oquv-uslubiy qo'llanma oliy texnika o'quv yurtlarining bakalavr talabalari uchun mo'ljallangan. Shuningdek, bu o'quv-uslubiy qo'llanmadan oliy o'quv yurtlarining o'qituvchilari ham ma'ruza va amaliy darslarda foydalanishlari mumkin.

Kafedra professori G'. Shodmonov umumiy tahriri ostida

Abu Rayhon Beruniy nomidagi Toshkent davlat texnika universiteti ilmiy-uslubiy kengashining qaroriga muvofiq chop etildi.

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So'z boshi.

Zamonaviy kadrlarni yetishtirish borasida respublikamiz oliy ta'limi tizimida tub o'zgarishlar amalga oshirilmoqda. Bunga sabab, «Ta'lim to'g'risida»gi qonun va «Kadrlar tayyorlash milliy dasturi»ning qabul qilinishi va ularda ilmiy-texnika taraqqiyoti yutuqlarini xalq xo'jaligiga tadbiq qilish, ijtimoiy-iqtisodiy rivojlanish bilan uzviy bog'liq ekanligining aniq ko'rsatilishidir.

O'quv uslubiy qo'llanma oliy matematika fanining asosiy bo'limlari bo'yicha mustaqil ish va laboratoriya ishi masalalaridan iborat bo'lib u 2- qismdan iborat. 2- qism o'z ichiga maydonlar nazariyasi, kompleks o'zgaruvchili funktsiyalar nazariyasi, chiziqli tenglamalar sistemasini Jordan-Gauss usulida yechish, bir noma'lumli tenglamalarni urinma va vatarlar usulida yechish, aniq integrallarni to'rtburchaklar usulida taqriban yechish, oddiy differensial tenglamalarni yechishning sonli usullari mavzularidan o'tkaziladigan tipik ishlari va laboratoriya ishlariga bag'ishlangan bo'lib, ularda nazariy ma'lumotlar va namuna uchun masalalar yechimi ko'rsatilgan. Talabalar mustaqil yechishlari uchun shahsiy variantlar yetarlicha keltirilgan

Qo'llanmani yozishda mualliflar Toshkent davlat texnika universitetida ko'p yillar davomida o'qigan ma'ruza va amaliy mashg'lot darslarini asos qilib oldilar.

Mualliflar.

MAYDONLAR NAZARIYASI

Skalyar maydon

Sath sirti va chizig'i.

Agar B fazoviy sohaning har bir M nuqtasiga to'liq aniqlangan $\varphi(M)$ son mos qo'yilgan bo'lsa, skalyar maydon berilgan deymiz. Fazoda $OXYZ$ dekart koordinatlari sistemasi berilgan bo'lsa, skalyar maydon $\varphi = \varphi(x, y, z)$ ko'rinishni oladi. Keyingi mulohazalarimizda B sohadagi φ maydon silliq deb, ya'ni φ funksiya B sohada o'zining barcha argumentlari bo'yicha uzluksiz xususiy hosilalarga ega deb faraz qilamiz. Agar skalyar maydon, tayinlangan π tekislikka perpendikulyar bo'lgan har bir to'g'ri chiziqda o'zgarmas qiymat qabul qilsa, u yassi maydon deyiladi. Dekart koordinatlar sistemasini shunday tanlasakki, bunda XOY koordinata tekisligi π tekislik bilan ustma-ust tushsa, yassi skalyar maydon $\varphi = \varphi(x, y)$ ko'rinishni oladi. Shuning uchun yassi maydon XOY tekisligidagi sohada aniqlangan deb tasavvur qilish mumkin. δ shunday sirt bo'lsaki, bu sirtning ustiga skalyar maydonning qiymatlari bir hil (o'zgarmas) bo'lsa, bu sirt sath sirti (yoki ekvipotentsial sirt) deyiladi. Sath sirti

$$\varphi(M) = C \qquad (C - \text{const}) \qquad (1)$$

tenglama bilan aniqlanadi. Yassi maydon uchun (1) tenglama (agar u π tekislikda qaralsa) sath chizig'ini aniqlaydi. M_o nuqtadan o'tadigan sath sirti

$$\varphi(M) = \varphi(M_o)$$

tenglama bilan aniqlanadi.

Skalyar maydonning gradienti

$\varphi(M)$ skalyar maydon dekart koordinatalar sistemasida

$\varphi = \varphi(x, y, z)$ tenglama bilan berilgan bo'lsin.

$$grad\ \varphi(M) = \frac{\partial\varphi}{\partial x}\bar{i} + \frac{\partial\varphi}{\partial y}\bar{j} + \frac{\partial\varphi}{\partial z}\bar{\kappa} \qquad (2)$$

vektor bu maydonning gradiyenti deyiladi, bunda xususiy hosilalar M nuqtada hisoblanadi. Shunday qilib, (2) formula, $\varphi(M)$ skalyar maydon aniqlangan har bir M nuqtaga, $grad\ \varphi(M)$ vektorni mos qo'yadi.

Gradiyentning differensiallash bilan bog'liq xossalari:

$$grad\ (C_1\varphi_1 + C_2\varphi_2) = C_1\ grad\varphi_1 + C_2\ grad\ \varphi_2,$$

$$grad\ (\varphi\psi) = \psi \cdot grad\varphi + \varphi grad\psi,$$

$$grad\ \frac{\varphi}{\Psi} = \frac{\Psi\ grad\ \varphi - \varphi\ grad\Psi}{\Psi^2},$$

$$grad\ F(u) = F'(u)\,\text{grad}\,u,$$

$$grad\ F(u,v) = \frac{\partial F}{\partial u} gradu + \frac{\partial F}{\partial v} gradv.$$

Yo'nalish buyicha hosila.

$\varphi(M)$ skalyar maydonning M nuqtadagi, shu M nuqtadan o'tuvchi l yo'nalish bo'yicha hosilasi deb

$$\frac{\partial\varphi}{\partial l} = \lim_{\Delta l \to o} \frac{\Delta\varphi}{\Delta l}$$

limitga aytiladi, bunda $\Delta\varphi = \varphi(M') - \varphi(M)$ - skalyar maydonning M nuqtadan M' nuqtaga o'tishdagi orttirmasi, $\Delta l = MM'$. $\frac{\partial\varphi}{\partial l}$ hosila φ maydonning M nuqtadagi l yo'nalish bo'ylab o'zgarish tezligini ifodalaydi va

$$\frac{\partial\varphi}{\partial l} = \vec{\tau}.grad\varphi(M) \qquad (3)$$

formula bilan hisoblanadi, bunda $\vec{\tau}$ - l yo'nalishining orti (birlik vektori).

$\varphi(M)$ skalyar maydonning M nuqtadagi, l chiziq bo'ylab olingan hosilasi deb

$$\frac{\partial\varphi}{\partial s} = \lim_{\Delta s \to o} \frac{\Delta\varphi}{\Delta s}$$

limitga aytiladi, bunda $\Delta\varphi$ – skalyar maydonning l chiziq bo'yicha orttirmasi, Δs esa yoy orttirmasi l chiziq bo'yicha xususiy hosila

$$\frac{\partial\varphi}{\partial s} = \vec{\tau}(M)\cdot grad\varphi(M) \qquad (4)$$

formula bilan hisoblanadi, bunda $\vec{\tau}(M)$ - chiziqning M nuqtasiga o‘tkazilgan urinmaning birlik vektori. (3) va (4) formulalarni taqqoslab, shunday xulosaga kelamizki, $\varphi(M)$ maydonning l chiziq bo‘ylab M nuqtadagi hosilasi, uning l chiziqqa shu nuqtada o‘tkazilgan urinma bo‘yicha olingan hosilasi bilan ustma-ust tushadi. Dekart koordinatalarida

$$\vec{\tau} = \vec{i}\cos\alpha + \vec{j}\cos\beta,$$

bunda α,β lar ℓ yo‘nalishning koordinata o‘qlari bilan hosil qilgan burchaklari.

Vektor maydon. Uning differensial xarakteristikalari. Vektor chiziqlari.

Agar biror B fazoviy sohaning har bir M nuqtasiga to‘la aniqlangan $\vec{a}(M)$ vektor mos qo‘yilgan bo‘lsa, vektor maydon berilgan deyiladi. Agar vektor maydon aniqlangan sohada $\{u;v;w\}$ egri chiziqli koordinatalar sistemasi berilgan bo‘lsa, $\vec{a}(M)$ vektor maydon V sohada aniqlangan uchta $a_u,\ a_v,\ a_w$ funksiya yordamida ifodalanadi:

$$\vec{a}(M) = a_u(u,v,w)\vec{e}_u + a_v(u,v,w)\vec{e}_v + a_w(u,v,w)\vec{e}_w .$$

Bunda $\vec{e}_u$, $\vec{e}_v$ ва $\vec{e}_w$ - bazis vektorlarlar.

Agar a_u, a_v, a_w -lar B sohada u,v,w argumentlar bo‘yicha uzluksiz xususiy hosilalarga ega bo‘lsa, $\vec{a}(M)$ maydon uzluksiz differensiyallanuvchi maydon deyiladi.

Xususan, dekart koordinatlari sistemasida $\vec{a}(M)$ maydon ortlar bo‘yicha

$$\vec{a}(M) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{\kappa} \qquad (5)$$

yoyilmasi orqali beriladi. Agar hamma $\vec{a}(M)$ vektorlar biror π tekislikka parallel bo‘lsa, va π ga o‘tkazilgan har bir perpendikulyarda o‘zgarmas qiymatlar qabul qilsa, $\vec{a}(M)$ vektor maydon - yassi maydon deyiladi. Agar π tekislik koordinata tekisliklaridan biri (masalan, XOY) bilan ustma-ust tushsa,

$$\vec{a}(M)=\vec{a}(x,y)=P(x,y)\vec{i}+Q(x,y)\vec{j},$$

ya'ni yassi maydonni tekislikda aniqlangan deb tasavvur qilish mumkin.Vektor maydon yo'nalishini xarakterlash uchun vektor chizig'i tushunchasi kiritilgan. Maydonning fizik tabiatiga qarab, vektor chizig'i , b'azan, kuch chizig'i yoki oqim chizig'i degan nomlar bilan xam atalishi mumkin. $\vec{a}(M)$ vektoı maydonning vektor chizig'i – shunday chiziqki, bu chiziqning har bir M nuqtasiga o'tkazilgan urinmaning yo'nalishi - $\vec{a}(M)$ vektorning yo'nalishi bilan ustma-ust tushadi.

(5) maydonning vektor chiziqlari oilasi

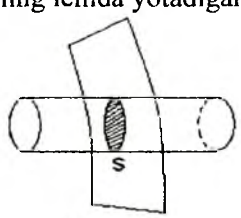
$$\frac{dx}{P(x,y,z)}=\frac{dy}{Q(x,y,z)}=\frac{dz}{R(x,y,z)}$$

differensial tenglamalar orqali topiladi.

Maydon yassi bo'lgan holda esa, vektor chiziqlar

$$\frac{dx}{P(x,y)}=\frac{dy}{Q(x,y)},\quad dz=0$$

tenglamalarni qanotlantiradi. $\vec{a}(M)$ va $\vec{b}(M)=F(M)\vec{a}(M)$ vektor maydonlar – kollinear maydonlar deyiladi, bunda $F(M)$ - skalyar funksiya. Kollinear maydonlar bir xil vektor chiziqlarga ega bo'ladilar. B sohada $\vec{a}(M)$ vektor maydon va S yopiq kontur berilgan bo'lsin. S konturning har bir nuqtasi orqali vektor chiziqlar o'tkazib, vektor nayi deb ataladigan sirtni hosil qilamiz. (15-rasm). Vektor nayining S kesimi deb vektor nayini kesib o'tadigan tekislikning vektor nayining ichida yotadigan qismiga aytiladi.



15-rasm

Vektor maydonning divergensiyasi.

$\vec{a}(M)$ vektor maydonning asosiy differensial xarakteristikalaridan biri – uning divergensiyasidir. (5) vektor maydonning divergensiyasi deb

$$div\vec{a} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

ifodaga aytiladi. Divergensiya quyidagi xossalarga ega:

$div(c_1\vec{a}_1 \pm c_2\vec{a}_2) = c_1 div\vec{a}_1 \pm c_2 div\vec{a}_2$ (chiziqliligi),

$$div(\varphi\vec{a}) = \varphi div\vec{a} + \vec{a} grad\varphi,$$

agar $\vec{a} = const$ bo'lsa, $div\vec{a} = 0$ va $div(\varphi\vec{a}) = \vec{a} \cdot grad\varphi$.

B sohada $div\,\vec{a}\,(M) = 0$ tenglikni qanoatlantiradigan $\vec{a}\,(M)$ vektor maydon bu sohada solenoidal (naysimon) maydon deyiladi.

Vektor maydon rotori.

Dekart koordinatalari sistemasida (5) formula bilan berilgan $\vec{a}\,(M)$ vektor maydonining rotori deb

$$rot\,\vec{a}\,(M) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} +$$

$$+\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k} \qquad (6)$$

ifodaga aytiladi. (6) formuladagi determinant birinchi satr elementlari bo'yicha yoyilayotganda, ikkinchi satr elementlarining uchinchi satr elementlariga ko'paytmasi sifatida tegishli xususiy hosila tushiniladi.

Masalan,

$$\frac{\partial}{\partial x} \cdot Q = \frac{\partial Q}{\partial x}.$$

Rotorning differensiallash bilan bog'liq xossalari ;

1) $rot(c_1\vec{a_1} + c_2\vec{a_2}) = c_1 rot\vec{a_1} + c_2 rot\vec{a_2}$,

2) $rot\,(\varphi\vec{a}) = [grad\varphi \cdot \vec{a}] + \varphi\, rot\vec{a}$,

3)agar $\vec{a} = const$ bo'lsa, u holda $rot\vec{a} = 0$ va $rot\,(\varphi\vec{a}) =$

$= [grad\varphi \vec{a}]$

Agar B sohada $rot\vec{a}(M) = 0$ bo'lsa, bu sohada $\vec{a}(M)$ uyurmasiz maydon deyiladi.

Nabla operatori. Ikkinchi tartibli differensiallash amallari

Skalyar maydondan gradiyent olish amalini, vektor maydondan divergensiya va rotor olish amallarini nabla (Gamelton) operatori deb ataladigan, quyidagi

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

simvolik vektor yordamida ifodalash mumkin. Aniqroq aytadigan bo'lsak,

$\nabla\varphi = grad\varphi, \quad \nabla\vec{a} = div\vec{a}, \quad [\nabla\vec{a}] = rot\vec{a}$

Nabla-birinchidan, chiziqli differensiallash operatordir, ya'ni uning tadbiqi chiziqlilik xossalariga ega hamda ko'paytmani differensiallash qonuniga bo'ysunadi. Ikkinchidan, u vektor operatordir, ya'ni ko'p hollarda ∇ ga vektorlar algebrasi formulalarini tadbiq qilish mumkin. Ammo, shuni unutmaslik lozimki , u yoki bu vektorni ∇ operator bilan almashtirish natijasida hamma vaqt ham to'g'ri munosabat hosil bo'lavermaydi. Masalan; $\vec{a}[\vec{a}\cdot\vec{b}] = 0$ tenglikdagi $\vec{b}$ vektor ∇ bilan almashtirilganda u to'g'ri tenglik bo'lmay qoladi. Shu sababli ∇ ga vektor sifatida qaralib , hosil qilinadigan har qanday formal operatsiyaning to'g'riligini tekshirib ko'rilishi lozim, ammo bir qator qoidalar mavjudki, ∇ operator bilan ish ko'rilayotganda bu qoidalarga rioya qilinsa, to'g'ri natijaga kelinadi. Shunday qoidalardan ba'zilarni keltiramiz ;

a) ∇ bilan ish ko'rilayotganda differensiallash qoidalariga rioya qilish kerak,

b) Vektorlar algebrasi qoidalariga ko'ra almashtirish bajarilayotganda, ∇ ko'paytmadagi oxirgi o'ringa tushib qolmasligi kerak. Ko'paytmadagi oxirgi o'rinda ∇ ta'sir ettirilayotgan ko'paytuvchi, undan oldin esa ∇ turishi mumkin, ∇ operator ikki marta ta'sir ettirilsa $[\nabla\,\nabla] = 0$ deb hisoblash lozim.

$$\nabla^2 = \Delta = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}$$

Laplas operatoridir. Agar φ –skalyar maydon bo'lsa,

$$\Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2},$$

agar $\vec{a} = P\vec{i} + Q\vec{j} + R\vec{k}$ – vektor maydon bo'lsa,

$$\Delta\vec{a} = \Delta P\vec{i} + \Delta Q\vec{j} + \Delta R\vec{k}$$

Ikki marta uzluksiz differensiallanuvchi skalyar va vektor maydonlar uchun hammasi bo'lib 5 ta 2-tartibli differensiallash amallari mavjud :

1. $div\ grad\varphi = \Delta\varphi$,
2. $rot\ grad\varphi = 0$
3. $grad\ div\ \vec{a} = \nabla(\nabla\vec{a})$
4. $div\ rot\ \vec{a} = 0$
5. $rot\ rot\ \vec{a} = grad\ div\ \vec{a} - \Delta\vec{a}$.

2. va 4. tengliklar $grad\varphi(M)$ uyurmasiz maydon ekanini, $rot\ \vec{a}(M)$ solenoidal maydon ekanligini ko'rsatadi.

Vektor maydonining integral xarakteristikalari.

Vektor maydon oqimi

Agar B sirtning normal vektori, shu sirtda yotgan har qanday yopiq kontur bo'ylab uzluksiz siljitilganda, siljish boshlangan dastlabki nuqtaga yo'nalishini o'zgartirmay qaytib kelsa, B ikki tomonli sirt deyiladi. Masalan, har qanday yopiq sirt ikki tomonli bo'ladi (bunday sirt tashqi va ichki tomonlarga ega).

Bir tomonli sirtga Myobius yaprog'i (to'g'ri to'rtburchak shaklidagi $ABCD$ sirtni bir marta burab, AB tomoni CD tomonga yelimlash orqali hosil qilingan sirt) misol bo'la oladi. Agar b ikki tomonli sirt bo'lib, uning tomonlaridan biri tanlab olingan bo'lsa, u oriyentirlangan sirt deyiladi. Oriyentirlangan sirtga o'tkazilgan normal deyilganda, sirtning tanlab olingan tomoniga o'tkazilgan normal nazarda tutiladi. Yopiq sirtning tashqi tomonini uning tabiiy oriyentiri sifatida olamiz.

$$\Pi(\vec{a}\ B) = \iint_B (\vec{a}, \vec{n})dB$$

(bunda $\vec{n}$ berilgan sirtga o'tkazilgan birlik vektor) ifoda $\vec{a}(M)$ vektor maydonining oriyentirlangan B sirt bo'yicha oqimi deyiladi. Vektor maydonining oqimi quydagi xossalarga ega:

1) $\iint_{B^+} \vec{a}(M)\vec{n}(M)dB = -\iint_{B^-} \vec{a}(M)\vec{n}(M)dB,$

bu yerda B^+ va B^- lar bitta B sirtning turli tomonlari.

2) Chiziqlilik xossasi:

$$\iint_B (c_1\vec{a}_1+c_2\vec{a}_2)\,\vec{n}\,dB = c_1\iint_B (\vec{a}_1\cdot\vec{n})\,dB + c_2\iint_B (\vec{a}_2\cdot\vec{n})\,dB$$

3) Additivlik xossasi. Agar B sirt $B_1, B_2, \dots B_k$ qismlarga ajratilsa, u holda

$$\iint_B (\vec{a}\cdot\vec{n})\,dB = \iint_{B_1} (\vec{a}\cdot\vec{n})\,dB + \iint_{B_2} (\vec{a}\cdot\vec{n})\,dB + \dots + \iint_{B_k} (\vec{a}\cdot\vec{n})\,dB$$

Maydon oqimini hisoblash masalasi – sirt integrallarini hisoblash masalasiga keltiriladi. Dekart koordinatalari sistemasida vektor maydon

$$\vec{a}(M) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$$

ko'rinishda, B sirtga o'tkazilgan normal esa

$$\vec{n}(M) = \vec{i}\cos\alpha + \vec{j}\cos\beta + \vec{k}\cos\gamma$$

ko'rinishida tasvirlangan bo'lsin. U holda maydon oqimi

$$\Pi(\vec{a}\,B) = \iint_B (P\cos\alpha + Q\cos\beta + R\cos\gamma)\,dB \qquad (7)$$

ko'rinishida yoziladi. (7) integralni hisoblash uchun B sirt koordinata tekisliklaridan biriga proyeksialanadi. Masalan B sirt XOY tekislikka o'zaro bir qiymatli proyeksiyalansin. U holda

$$dB = \frac{dxdy}{|\cos\gamma|}$$

B sirtning XOY tekislikdagi proeksiyasini B_{xy} orqali belgilab, maydon oqimi uchun

$$\Pi(\vec{a}\,B) = \pm\iint_{B_{xy}} \frac{(\vec{a},\vec{n})}{|\cos\gamma|}dxdy$$

integralni hosil qilamiz. Bu integralda z ning $x\ va\ y$

o'zgaruvchilar orqali ifodasi $z = z(x,y)$ B sirtning tenglamasidan topiladi. Bu formuladagi ishora $\vec{n}$ normal B sirtning ko'rsatilgan tomonga yo'naladigan qilib tanlanadi. Huddi shu usulda, agar B sirt YOZ yoki XOZ tekisligiga o'zaro bir qiymatli proeksiyalanadigan bo'lsa, maydon oqimi uchun

$$\varPi(\vec{a},B) = \mp \iint_{B_{yz}} \frac{(\vec{a},\vec{n})}{\cos\alpha} dydz \,, \qquad x = x(y,z)$$

yoki

$$\varPi(\vec{a},B) = \mp \iint_{B_{xz}} \frac{(\vec{a},\vec{n})}{\cos\beta} dxdz \,, \qquad y = y(x,z)$$

formulalarni hosil qilish mumkin. Ba'zan, maydon oqimini hisoblash uchun B sirt uchchala koordinata tekisligiga proeksiyalanadi va

$$\varPi(\vec{a}\,B) = \iint_B Pdydz + Qdxdz + Rdxdy$$

formula hosil qilinadi. Bu formuladagi har bir qo'shiluvchi B sirtni tegishli koordinata tekisligiga proeksiyalash vositasida, alo--hida hisoblanadi. Masalan,

$$\iint_B Pdydz = \mp \iint_{B_{yz}} P(x(y,z)y,z)\,dxdz \,,$$

Bunda $x = x(y,z)$ ifoda B sirtning tenglamasidan keltirib chiqariladi, B_{yz} bo'yicha olinayotgan integral oldidagi ishora, B sirtga o'tkazilgan normal OX o'qi bilan o'tmasligiga (yo'naltiruvchi kosinus - $\cos\alpha$ ning ishorasiga) qarab aniqlanadi.

Ostrogradskiy teoremasi.

Agar $P(x,y,z)$ $Q(x,y,z)$ $R(x,y,z)$ funksiyalar biror yopiq V sohada uzluksiz va uzluksiz xususiy hosilalarga ega bo'lib, V sohani chegaralovchi B sirt esa bo'lakli-silliq bo'lsa, Ostrogradskiy formulasi deb ataladigan quyidagi tenglik o'rinli bo'ladi.

$$\iiint_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right) dx\,dy\,dz =$$

$$= \oiint_B (P\cos\alpha + Q\cos\beta + R\cos\gamma)dB\,,$$

bunda $\cos\alpha, \cos\beta, \cos\gamma$ lar B sirtga o‘tkazilgan $\vec{n}$ birlik normalning yo‘naltiruvchi kosinuslari.

Ostrogradskiy formulasi vektor shaklida quyidagicha ifodalanadi;

$$\Pi(\vec{a}\,B) = \oiint_B \vec{a}\cdot\vec{n}\,dB = \iiint_V div\,\vec{a}\cdot dv$$

Ya’ni vektor maydoning yopiq B sirt bo‘yicha oqimi, uning divergensiyasidan B sirt chegaralab turgan V hajm bo‘yicha olingan integralga teng. Ostrogradskiy formulasini tadbiq qilish, ko‘pgina hollardda yopiq sirt bo‘yicha maydon oqimini hisoblashni soddalashtiradi. Xususan, bu formuladan solenoidal maydonning $(div\,\vec{a} = 0)$ har qanday yopiq sirt bo‘yicha oqimi 0 ga tengligi kelib chiqadi.Ostrogradskiy formulasi yordamida divergensiyaning mexanik ma’nosini aniqlashimiz mumkin M_0 – vektor maydon aniqlangan sohadagi tayinlangan nuqta, B - markazi M_0 da bo‘lgan sfera bo‘lsin. O‘rta qiymat haqidagi teoremaga ko‘ra

$$\iiint_V div\,\vec{a}\,dv = V(B)\,div\,\vec{a}(M),$$

Bunda $V(B) - B$ sfera bilan chegaralangan sharning hajmi, M – shardan olingan biror nuqta, bundan va Ostrogradskiy formulasidan

$$div\,\vec{a}(M) = \frac{\Pi(\vec{a},B)}{V(B)}.$$

Bu tenglikda B sferaning s radiusini 0 ga intiltirib limitga o‘tsak, divergensiyaning M_0 nuqtadagi qiymati uchun

$$div\vec{a}(M_0) = \lim_{s\to 0}\frac{\Pi(\vec{a},B)}{V(B)}$$

tenglik hosil bo‘ladi. Bundan ko‘rinadiki, $\vec{a}$ maydonning M_0 nuqtadagi divergensiyasi, shu maydoning M_0 nuqtaga kirayotgan $(div\ \vec{a}(M_0) < 0$ bo'lganda) oqimning hajmi bo‘yicha zichligini anglatar ekan.

Chiziqli integral va vektor maydonning sirkulyatsiyasi.

$\vec{a}(M)$ vektor maydondan l chiziq bo‘yicha olingan W chiziqli integral deb, $\vec{a}(M)$ vektorning, l chiziqqa o'tkazilgan $\vec{\tau}(M)$ birlik urinma vektorga skalyar ko'paytmasidan olingan egri chiziqli integralga aytiladi:

$$W = \int_l \vec{a}(M) \cdot \vec{\tau}(M) ds = \int_l \vec{a}\, d\vec{r},$$

bunda $ds - l$ chiziq yoyi differensiali. Agar $\vec{f}(M)$ –kuch maydoni bo'lsa

$$W = \int_l \vec{f}(M) \cdot \vec{\tau}(M) ds = \int_l \vec{f}\, d\vec{r},$$

bu maydonning l yo'l bo'ylab bajargan ishini anglatadi.

Chiziqli integralning asosiy xossalari:

I. Chiziqlilik xossasi:

$$\int_l (c_1\vec{a}_1 + c_2\vec{a}_2) dr = c_1 \int_l \vec{a}_1 \cdot d\vec{r} + c_2 \int_l \vec{a}_2 \cdot d\vec{r}$$

II. Additivlik xossasi:

$$\int_{l_1+l_2} \vec{a} d\vec{r} = \int_{l_1} \vec{a} d\vec{r} = \int_{l_2} \vec{a} d\vec{r}.$$

III. l chiziqdagi yo'nalish qarama-qarshiga o'zgartirilganda chiziqli integralning ishorasi qarama-qarshiga o'zgaradi:

$$\int_{AB} \vec{a} d\vec{r} = -\int_{BA} \vec{a} d\vec{r}$$

Dekart koordinatalari sistemasida vektor maydon

$$\vec{a}(M) = P(M)\vec{i} + Q(M)\vec{j} + R(M)\vec{k}$$

ko'rinishida, radius vektorining differensiali esa

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

ko'rinishida ifodalangani uchun

$$W = \int_l P(x,y,z)dx + Q(x,y,z)dy + P(x,y,z)dz.$$

Integral x,y,z larni L chiziqdagi ifodalari bilan almashtirib hisoblanadi. Bunda parametr bo'yicha aniq integral hosil bo'ladi. Bu integralni quyi chegarasi parametrning L chiziqning boshlang'ich nuqtasidagi qiymatidan , yuqori chegarasi esa oxirgi nuqtasidagi qiymatidan iborat boladi .

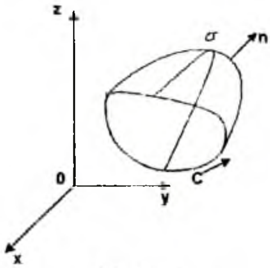
C yopiq satr boyicha hisoblangan

$$U(\vec{a}, c) = \int_{c} \vec{a} d\vec{r}$$

chiziqli integral $\vec{a}$ vektor maydonning C kontur bo'yicha olingan sirkulyatsiyasi deyiladi.

Stoks teoremasi.

$\vec{a}(m)$ – uzluksiz differensiallanuvchi vektor maydon aniqlangan V fazoviy sohada C bo'lakli silliq kontur bilan chegaralangan B bo'lakli –silliq sirt berilgan bo'lsin. (16-rasm). Sirtning shunday tomoni tanlanadiki, bu tomonga o'tkazilgan normalning uchidan qaraganda sirtni chegaralovchi konturdagi



16-rasm

yo'nalish soat strelkasiga teskari yo'nalishda bo'lishi kerak (o'ng vint qonuni). Bu holda Stoks formulasi deb ataladigan quyidagi tenglik o'rinli bo'ladi:

$$\iint_{B} (rot\,\vec{a} \cdot \vec{n}) db = \oint_{C} \vec{a} d\vec{r}$$

yoki dekart koordinatalarida

$$\iint_{B} \left[\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\cos\alpha + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\cos\beta + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\cos\gamma\right] db =$$

$$= \oint_{C} Pdx + Qdy + Rdz,$$

ya'ni $\vec{a}(M)$ vektor maydon rotorining B sirt bo'yicha oqimi, $\vec{a}(M)$ maydoning C chegara bo'yicha sirkulyatsiyasiga teng.

Potensial va solenoidal vektor maydonlar

Agar V sohada

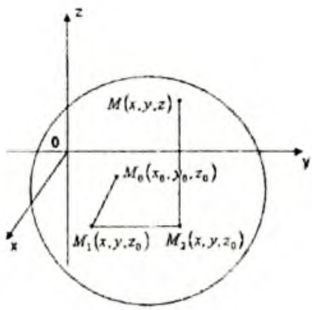
$$\vec{a}(M) = grad\varphi(M) \qquad (8)$$

tenglikni qanoatlantiruvchi $\varphi(M)$ skalyar maydon mavjud bo'lsa, $\vec{a}(M)$ vektor maydon V sohada potensial maydon deyiladi, $\varphi(M)$ maydon esa $\vec{a}(M)$ maydoning potensiali deyiladi. $rot\, grad\varphi(M) = 0$ bo'lgani uchun potensial maydon uchun $rot\vec{a} = 0$ tenglik o'rinli, ya'ni u uyurmasiz maydon bo'ladi. Agar V sohada yotgan har qanday yopiq konturni shu sohadan tashqariga chiqmay, uzluksiz ravishda nuqtaga qadar tortish mumkin bo'lsa, V soha - bir bog'lamli soha deyiladi. Butun fazo, yarim fazo, shar, kub, ellipsoid, sharning tashqarisi, bitta nuqtasi olib tashlangan fazo va shu kabilar bir bog'lamli sohaga misol bo'la oladi. Bitta to'g'ri chizig'i tashlab yuborilgan fazo, bitta diametrsiz shar, tor va shu kabilar bir bog'lamli bo'lmagan sohaga misol bo'la oladi.

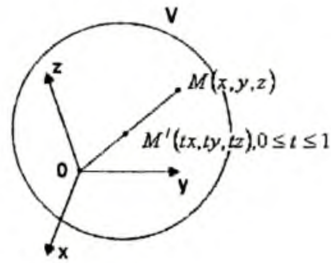
Bir bog'lamli sohada teskari tasdiq ham to'g'ri: bir bog'lamli V sohadagi har qanday uyurmasiz maydon potensial maydon bo'ladi. Shuni takidlab aytamizki, bir bog'lamli bo'lmagan sohada uyurmasiz maydon potensial maydon bo'lmasada sohadagi har bir nuqta shunday atrofga ega bo'ladiki, bu atrofda (8) shartni qanoatlantiradigan φ funksiya mavjud bo'ladi. Biroq, bu funksiya'ni butun sohaga davom ettirib bo'lmaydi. Odatda, bunday davom ettirishda y ko'p qiymatli bo'lib qoladi. Potensial vektor maydonning $\varphi(M)$ potensiali o'zgarmas qo'shiluvchi aniqligida topiladi. Uni topishda

$$\varphi(M) = \int_{M_0 M} \vec{a} d\vec{r} + const \qquad (9)$$

fo'rmuladan foydalaniladi, bunda $M_0 - V$ sohadagi tayinlangan nuqta, integrallash yo'li M_0M ixtiyoriy bo'lishi mumkin, faqatgina u V sohadan tashqariga chiqib ketmasa bas. Agar V shunday soha bo'lsaki, tayinlangan M_0 nuqtani sohadagi har bir M nuqta bilan bo'laklari koordinata o'qlariga parallel bo'lgan siniq chiziq orqali tutashtirish mumkin bo'lsa (17-rasm)



17-rasm



18-rasm

(9) formuladan

$$\varphi(x,y,z) = \int_{x_0}^{x} P(x,y_0,z_0)dx +$$
$$+ \int_{y_0}^{y} Q(x,y,z_0)dy + \int_{z_0}^{z} R(x,y,z)dz + c$$

tenglik kelib chiqadi, bunda $c = const.$

Bu formuladagi ikkinchi integral hisoblanayotganda x, uchinchi integral hisoblanayotganda esa x va y argumentlar o'zgarmaslar deb qaraladi. V dagi har bir M nuqta tayinlangan M_0 nuqta bilan to'g'ri chiziq orqali tutashtirilgan "yulduzli" fazoda, shu tog'ri chiziq kesmasi orqali amalga oshirib (bunda M_0 kordinta boshi sifatida olinadi (18-rasm),

$$\varphi(M) = \int_0^1 \vec{a}(M') \, \vec{r}\,(M)dt + const$$

fo'rmulani hosil qilamiz, bunda $r(M) = x\vec{i} + y\vec{j} + z\vec{k}, 0 \leq t \leq 1$ bo'lganda M' (tx, ty, tz) nuqta M_0 M kesmada xarakatlanadi.

Solenoidal vektor maydon .

Agar V sohada

$$div\vec{a}(M) = 0$$

tenglik o'rinli bo'lsa , *a(M)* vekto'r maydon V sohada solenoidal (vektor) maydon deyiladi. Solenoidal maydon

$$\vec{a}(M) = rot\vec{H}(M)$$

tenglikni qanoatlantiradigan $\vec{H}(M)$ vektor potensialga ega bo‘ladi. Vektor potensial qo‘shiluvchi sifatida olingan ixtiyoriy skalyar maydonning gradiyenti aniqligida topilada. Markazi koordinatalar boshi 0 da bo‘lgan “yulduzli” sohada vektor potensialining qiymatlaridan biri

$$\vec{H}(M) = \int_0^1 [\,\vec{a}(M')\vec{r}(M)]tdt$$

formula bilan topiladi,bunda

$$\vec{r}(M) = x\vec{i} + y\vec{j} + z\vec{k}, \quad M(x, y, z,)\ 0 \le t \le 1$$

bo‘lganda $M^1(tx, ty, tz)$ $0M$ kesmada o‘zgaradi

Garmonik maydon.

V sohada

$$\Delta\varphi(M) = 0$$

Laplas tenglamasini qanoatlantiradigan φ (M) skalyar maydon garmonik maydon deyiladi. $\varphi -$ garmonik maydon bo‘lganda $\vec{a} = grad\varphi(M)$ vector maydon ham garmonik maydon deyiladi. Garmonik vektor maydon bir vaqtda ham potensial,ham solenoidal maydon bo‘ladi. $f(M) -$ V sohada aniqlangan funksiya bo‘lsin. V sohada

$$\Delta\varphi(M) = f(M)$$

tenglik o‘rinli bo‘lsa , $\varphi(M)$ skalyar maydon Puasson tenglamasini qanoatlantiradi deyiladi.

Hisoblash topshiriqlari.

1 – vazifa. 1-masalada vektor maydonining potentsiali $u = u\ (x, y, z)$ berilgan:

a) Sirt sathi tenglamasi va ular orasidan M_0, M_1, M_2 nuqtalardan o‘tuvchi sirtlar tenglamasi topilsin.

b) Berilgan potensialga ko‘ra vektor maydon topilsin.

c) Vektor maydonning vektor chiziqlari tenglamasi topilsin.

d) M_0 nuqtada $\overline{M_1M_2}$ vektori yo‘nalishi bo‘ylab $u=u(x,y,z)$ skalyar maydon o‘zgarish tezligi va maydonning M_0 nuqtadagi eng katta o‘zgarishi topilsin.

e) O‘zgaruvchi M nuqta M_1 nuqtadan M_2 nuqtaga ko‘chganda vektor maydonning bajargan ishi hisoblansin.

2 – vazifa. 2-masalada M_0 nuqtada $\vec{a}(M)$ vektor maydonning divergensiyasi va uyurmasi topilsin.

3 – vazifa. 3- masalada $\vec{a}(M)$ vektor maydonning, S sirtning P tekislik bilan kesilgan qismi orqali o'tuvchi oqimi topilsin.
(Bu yerda normal berilgan sirtlar tashkil etuvchi yopiq sirtga nisbatan tashqi).

4 –vazifa. 4-masalada $\vec{b}(M)$ vektor maydonning $\Gamma-$ kontur bo'ylab, t parametr o'sishiga mos yo'nalishdagi sirkulatsiyasi topilsin. ($\Gamma-$ konturning chizmasi chizilsin).

5 – vazifa. 5-masalada $\vec{F}(M)$ maydonning potensialligi yoki solenoidalligi tekshirilsin. $\vec{F}(M)$ maydon potensial bo'lgan holda uning potensiali topilsin.

6 – vazifa. 6-masalada $U(M)$ maydonning garmonikligini tekshiring.

Namunaviy variant

1 – masala.

Vektor maydonning $u = x^2 + y^2 - 2z$ potensiali berilgan.

a)sirt sathi tenglamasi va ular orasidan $M_0\left(1, \frac{1}{2}, 5\right)$, $M_1(2, 1, -1)$,

$M_2(2, 4, 3)$ nuqtalardan o'tuvchi sirtlar tenglamasi yozilsin.

Yechish : Sath sirtlari uch o'lchovli Evklid fazosidagi har bir nuqtada

$$u\,(x, y, z) = C$$

tenglik bilan aniqlanuvchi sirtlar bo'ladi, bunda C – o'zgarmas son.

$$x^2 + y^2 - 2z = C \quad \text{yoki} \quad z = \frac{x^2+y^2-C}{2}.$$

Bu tenglama OZ o'qi atrofida aylanishdan hosil bo'lgan paraboloidlar oilasi tenglamasini aniqlaydi.

C– o'zgarmasning berilgan nuqtalarga mos keluvchi qiymatini topamiz

$-M_0$ nuqta uchun, $C = -\frac{35}{4}$ $\qquad z = \frac{x^2+y^2}{2} + \frac{35}{8};$

$-M_1$ nuqta uchun, $C = 7,$ $\qquad z = \frac{x^2+y^2}{2} - \frac{7}{2};$

$-M_2$ nuqta uchun, $C=14$, $z = \frac{x^2+y^2}{2} - 7$;

Bu paraboloidlar uchlari mos ravishda quyidagi nuqtalarda joylashgan:

$$A_1\left(O,O,\frac{35}{8}\right),\quad A_2\left(O,O,-\frac{7}{2}\right),\quad A_3(O,O,-7)$$

b). Berilgan potentsialga koʻra vektor maydon topilsin. Skalyar maydoning gradienti potentsial maydonni tashkil etadi:

$$u(x,y,z) = \frac{\partial u}{\partial X}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}.$$

Berilgan potentsial uchun

$$grad\, u = 2x\,\vec{i} + 2y\vec{j} - 2\,\vec{k};$$

Demak, izlangan vektor maydon

$$\vec{F} = 2\,x\,\vec{i} + 2y\vec{j} - 2\,\vec{k};$$

c). Bu vektor maydonning vektor chiziqlari tenglamasi topilsin. Vektor chiziqlar differensial tenglamalari qoʻyidagi koʻrinishda boʻladi;

$$\frac{dx}{F_x(x,y,z)} = \frac{dy}{F_y(x,\ y,z)} = \frac{dz}{F_z(x\,,y\,,z)}$$

Berilgan vektor maydon uchun

$$\frac{dx}{2x} = \frac{dy}{2y} = \frac{dz}{-2} \quad \text{ёки} \quad \begin{cases} \frac{dx}{x} = -dz \\ \frac{dy}{y} = -dz \end{cases}$$

Bundan esa

$$\begin{cases} Ln\ C_1x = -z \\ Ln\ C_2y = -z \end{cases}$$

yoki

$$\begin{cases} C_1x = e^{-z} \\ C_2y = e^{-z} \end{cases}$$

Ya'ni vektor chiziqlari uch oʻlchovli Evklid fazosida $C_1x = e^{-z}$ va $C_2y = e^{-z}$ sirtlari kesishidan hosil boʻlgan chiziqlardan iboratdir. $C_1 = C_2 = 1$ da vektor chiziqlar $x=y$ tekisligida joylashadilar.

d) $u = x^2 + y^2 - 2z$ skalyar maydonning $M_0\left(1, {}^1/_2, 5\right)$ nuqtadagi, $\overline{M_1M_2}$ yo'nalish bo'ylab o'zgarishi tezligini toping. Maydonning M_0 nuqtadagi eng katta o'zgarishi nimaga teng?

Yo'nalish bo'yicha hosilani topamiz:

$$\frac{\partial u}{\partial l} = \frac{\partial u}{\partial x} \cos\alpha + \frac{\partial u}{\partial y} \cos\beta + \frac{\partial u}{\partial z} \cos\gamma$$

Bu yerda $\cos\alpha, \cos\beta, \cos Z$ лар $\overline{M_1M_2}$ vektorning yo'naltiruvchi kosinuslari.

$$\overrightarrow{M_1M_2} = (2-2; 4-1; 3+1) = (0; 3; 4;),$$

$$|\overline{M_1M_2}| = \sqrt{0+9+16} = 5.$$

$$\cos\alpha = \frac{0}{5} = 0, \quad \cos\beta = \frac{3}{5}, \quad \cos\gamma = \frac{4}{5}$$

$$\frac{\partial u}{\partial l} = 2x \cdot 0 + 2y \cdot \frac{3}{5} + (-2) \cdot \frac{4}{5} = \frac{6}{5}y - \frac{8}{5},$$

$$\left.\frac{\partial u}{\partial l}\right|_{M_0} = \frac{6}{5}y - \frac{8}{5} = \frac{6}{5} \cdot \frac{1}{2} - \frac{8}{5} = -1$$

M_0 nuqtadagi eng katta o'zgarish:

$$|grad\ U\ (M_0)| = \left.\sqrt{4x^2 + 4y^2 + 4}\right|_{\left(1, \frac{1}{2}, 5\right)} = \sqrt{9} = 3$$

e) vektor maydonning M_1 nuqtadan M_2 nuqtagacha ko'chishda bajargan ishini hisoblang.

$\vec{F}$ vektor maydonning $\overset{\frown}{M_1M_2}$ chiziq bo'ylab bajargan ishi:

$$A = \int_l \vec{F} d\vec{l} = \int_l gradU \cdot d\vec{l} = \int_l \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial x} dy + \frac{\partial u}{\partial z} dz =$$

$$= \int_l du = U(M_2) - U(M_1)$$

Bundan esa $U\,(X, Y, Z) = X^2 + Y^2 - 2Z$ bo'lgani uchun vektor maydonning bajargan ishi

$A = (4 + 16 - 6) - (4 + 1 + 2) = 7.$

2 – masala.

$$\vec{a}(M) = (x + xy^2)\,\vec{\imath} + (y - yx^2)\,\vec{\jmath} + (z - 3)\,\vec{\kappa}$$

vektor maydonning divergensiyasi va uyurmasini $M_0(-1,1,2)$ nuqtada toping.
Yechish:

$$div\,\vec{a}(M) = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_F}{\partial z} = 1 + y^2 + 1 - x^2 + 1 =$$
$$= 3 + y^2 - x^2;$$
$$div\,\vec{a}(M_0) = 3 + 1 - 1 = 3$$
$$rot\,\vec{a}(M) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{\kappa} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + xy^2 & y - yx^2 & z - 3 \end{vmatrix} = -4xy\vec{\kappa};$$
$$rot\,\vec{a}(M_0) = 4\vec{\kappa}.$$

3 – masala.
$\vec{a}(M) = (x + xy^2)\vec{i} + (y - yx^2)\vec{j} + (z - 3)\vec{\kappa}$ vektor maydonning, $x^2 + y^2 = z^2, (z \geq o)$ sirtning $z=1$ tekislik kesgan qismidan tashqi normal yo'nalishi bo'ylab oqib chiqqan oqimi hisoblansin.
Yechish:

$$\Pi = \iint_{(s)} a_n(M)d\sigma = \oint\!\!\oint_{(\Sigma)} a_n(M)d\sigma - \iint_{(S_1)} a_n(M)d\sigma$$

$J_1 = \oint\!\!\oint_{(\Sigma)} a_n(M)d\sigma$, $J_2 = \iint_{(S_1)} a_n(M)d\sigma$ belgilashlar kiritamiz.

Bu yerda $\sum$ - konus sirti va $z = 1$ tekisliklar birgalikda tashkil etgan yopiq sirt S_1: $z = 1$ tekislikning konus bilan kesilgan qismi. Gauss – Ostrogradskiy formulasiga ko'ra

$$J_1 = \iiint_{(\Omega)} div\vec{a}(M)d\Omega = \iiint_{(\Omega)} (3 + y^2 - x^2)dxdydz =$$
$$= \int_0^{2\pi} d\varphi \int_0^1 rdr \int_r^1 (3 - r^2\cos 2\varphi)dz = \int_0^{2\pi} d\varphi \int_0^1 (3r - r^3\cos 2\varphi)(1 - r)dr =$$
$$= \int_0^{2\pi} \left(\frac{1}{2} - \frac{1}{20}\cos 2\varphi\right)d\varphi = \pi;$$
$$J_2 = \iint_D (1 - 3)dxdy = -2\iint_D dxdy = -2\pi$$

Chunki, $\iint\limits_D dxdy$ - S_1 sirtning XOY tekisligiga proyeksiyasi bo'lgan D: $x^2+y^2=1$ doiraning yuziga teng.

Topilgan J_1 va J_2 larning qiymatlarini oqimning formulasiga qo'yib

$$\Pi = J_1 - J_2 = \pi - (-2\pi) = 3\pi$$

ni topamiz.

4 - masala:

$\vec{b}(M) = y\vec{i} - x\vec{j} + z^2\vec{k}$ vektor maydonning

Γ: $x = \frac{\sqrt{2}}{2}\cos t, y = \frac{\sqrt{2}}{2}\cos t, z = \sin t$ konturi bo'ylab t parametr o'sishiga mos yo'nalishda sirkulyatsiyasini toping.

Yechish:

t parametr o'zgarishi chegaralarini topish uchun Γ chiziqning ko'rinishini aniqlab olamiz. Buning uchun Γ chiziqning tenglamasini quyidagicha yozib olamiz:

$$\begin{cases} y - x = 0, \\ x^2 + y^2 + z^2 = 1. \end{cases}$$

Bundan Γ: $x^2+y^2+z^2=1$ sfera bilan koordinatalar boshidan o'tuvchi $y-x=0$ tekislik kesishish chizig'i ekanligi ko'rinadi. Demak, Γ – aylana va $0 \le t \le 2\pi$.

$$\amalg = \oint\limits_\Gamma b_s ds = \oint\limits_\Gamma \vec{b}\cdot d\vec{s} = \oint\limits_\Gamma y dx - x dy + z^2 dz =$$

$$\int\limits_0^{2\pi}\left(\frac{\sqrt{2}}{2}\cos t\left(-\frac{\sqrt{2}}{2}\sin t\right) - \frac{\sqrt{2}}{2}\cos t\left(-\frac{\sqrt{2}}{2}\sin t\right) +\right.$$

$$\left. + \sin^2 t\cos t\right)dt = \frac{1}{2}\int\limits_0^{2\pi}\left(-\cos t\sin t + \cos t\sin t\right)dt +$$

$$+ \int\limits_0^{2\pi}\sin^2 t d(\sin t) = 0.$$

5 – masala.

$\vec{F} = (6x - 2yz)\,\vec{i} + (6y - 2xz)\vec{j} + (6z - 2xy)\,\vec{k}$ vektor maydon berilgan. Uning solenoidal va potentsialligini tekshiring. $\vec{F}$ *maydon* potentsial bo'lgan holda uning potentsialini toping.

Yechish: F maydoning divergensiyasini topamiz:

$$div\,\vec{F} = \frac{\partial(6x-2yz)}{\partial x} + \frac{\partial(6y-2xz)}{\partial y} + \frac{\partial(6z-2xy)}{\partial z} = 6+6+6 = 18.$$

$\vec{F}$ vektor maydon uchun $div\vec{F} \neq 0$ bo'lganligi uchun u solenoidal emas. $\vec{F}$ maydonning rotorini topamiz.

$$rot\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6x-2yz & 6y-2xz & 6z-2xy \end{vmatrix} = \left(\frac{\partial(6z-2xy)}{\partial y} - \frac{\partial(6y-2xz)}{\partial z}\right)\vec{i} -$$

$$-\left(\frac{\partial(6z-2xy)}{\partial x} - \frac{\partial(6x-2yz)}{\partial z}\right)\vec{j} + \left(\frac{\partial(6y-2xz)}{\partial x} - \frac{\partial(6x-2yz)}{\partial y}\right)\vec{k} =$$

$$= (-2x+2x)\vec{i} - (-2y+2y)\vec{j} + (-2z+2z)\vec{k} = 0$$

Bundan $\vec{F}$ maydoning potentsialligi kelib chiqadi. $\vec{F}$ vektor maydonning potensialini quyidagi formuladan topamiz:

$$\varphi(x,y,z) = \int_{x_0}^{x}(6x-2y_0z_0)dx + \int_{y_0}^{y}(6y-2xz_0)dy +$$

$$+\int_{z_0}^{z}(6z-2xy)dz + C = (3x^2-2y_0z_0x)\Big|_{x_0}^{x} +$$

$$+(3y^2-2xz_0y)\Big|_{y_0}^{y} + (3z^2-2xyz)\Big|_{z_0}^{z} + C =$$

$$= 3(x^2+y^2+z^2) - 2xyz + C$$

6 – masala.

$u = \rho^2 \cos 2\varphi$ maydon silindrik koordinatalarda berilgan.

Uning garmonikligini tekshiring.

Yechish: ΔU operatorini silindrik koordinatlarda yozib olamiz.

$$\Delta U=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{\partial^2 u}{\partial z^2}; \qquad \frac{\partial^2 u}{\partial\varphi^2}=-4\rho^2\cos 2\varphi,$$

$$\frac{\partial^2 u}{\partial z^2}=0. \quad \frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right)=\frac{\partial}{\partial\rho}(2\rho^2\cos 2\varphi)=4\rho\cos 2\varphi$$

ΔU operatorini silindrik koordinatlarda yozib olamiz:

$$\Delta U=\frac{1}{\rho}\cdot 4\rho\cos 2\varphi+\frac{1}{\rho^2}(-4\rho^2\cos 2\varphi)=0$$

Bundan $U=\rho^2\cos 2\varphi$ maydonning garmonikligi kelib chiqadi.

Variantlar

1-variant

1.

$U=\frac{x^2}{2}+\frac{y^2}{9}+\frac{z^2}{4}$, $\quad M_0(4,3,-2), M_1(2,0,2), M_2(1,-3,2)$.

2.

$\vec{a}(M)=2x\vec{i}+(y^2+z^2)\vec{j}+(x^2-2z)\vec{k}$, $\quad M_0=(-1,\sqrt{5},4)$.

3.

$S: 9y^2+z^2=x^2 \quad (x\geq 0)$, $\qquad P: x=2$.

4.

$\vec{b}(M)=x\vec{i}+2z\vec{j}+y\vec{k}$. $\qquad \Gamma:\begin{cases}x=3\cos t,\\ y=2\sin t,\\ z=6\cos t-4\sin t-1.\end{cases}$

5.

$\vec{F}=(5x^2-yz)\vec{i}+(5y^2-xz)\vec{j}+(5z^2-xy)\vec{k}$.

6.

$U=2\rho^2\cos\varphi \qquad (U=U(\rho,\varphi))$.

2-variant

1.

$$U = \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{2}, \quad M_0(2,2,2), M_1(4,0,0), M_2(0,4,0).$$

2.

$$\vec{a}(M) = xyz\vec{i} - x^2 z\vec{j} + 3\vec{k}, \qquad M_0 = (\sqrt{2}, \sqrt{2}, \frac{1}{\sqrt{2}}).$$

3.

$$S: x^2 + y^2 = z^2 \quad (z \geq 0), \qquad P: z = 2.$$

4.

$$\vec{b}(M) = (y - z)\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}, \quad \Gamma: \begin{cases} x = 3\cos t, \\ y = 3\sin t, \\ z = 2(1 - \cos t). \end{cases}$$

5.

$$\vec{F} = (x^2 + 7yz)\vec{i} + (y^2 + 7xz)\vec{j} + (z^2 + 7xy)\vec{k}.$$

6.

$$U = \rho^2 \sin 2\varphi \qquad (U = U(\rho, \varphi))$$

3-variant

1.

$$U = x^2 - y^2 + 2z^2, \quad M_0(1,1,2), M_1(1,0,1), M_2(2,1,1).$$

2.

$$\vec{a}(M) = x\vec{i} + (y + yz^2)\vec{j} + (z - zy^2)\vec{k}, \quad M_0 = (-1, -\sqrt{3}, \sqrt{2}).$$

3.

$$S: x^2 + y^2 + z^2 = 4 \quad (z \geq 0), \qquad P: z = 0.$$

4.

$$\vec{b}(M) = -z\vec{i} - x\vec{j} + xz\vec{k}, \qquad \Gamma: \begin{cases} x = 5\cos t, \\ y = 5\sin t, \\ z = 4. \end{cases}$$

5.

$$\vec{F} = (8x^2 - 5yz)\vec{i} + (8y^2 - 5xz)\vec{j} + (8z^2 - 5xy)\vec{k}.$$

6.

$$U = \rho^2 \cos\frac{\varphi}{2} \qquad (U = U(\rho,\varphi)).$$

4-variant

1.

$$U = x^2 - y^2 - z^2, \quad M_0(2,0,1), M_1(1,1,0), M_2(4,1,1).$$

2.

$$\vec{a}(M) = xz\vec{i} + yz\vec{j} + (z^2 - 1)\vec{k}.$$

3.

$$S : x^2 + y^2 = z \quad (z \geq 0), \qquad P : z = 4.$$

4.

$$\vec{b}(M) = x^2\vec{i} + y\vec{j} - z)\vec{k}, \qquad \Gamma : \begin{cases} x = \cos t, \\ y = \dfrac{\sqrt{2}}{2}\sin t, \\ z = \dfrac{\sqrt{2}}{2}\cos t. \end{cases}$$

5.

$$\vec{F} = \left(\frac{x^3}{12} - 3yz\right)\vec{i} + \left(\frac{y^3}{12} - 3xz\right)\vec{j} + \left(\frac{z^3}{12} - 3xy\right)\vec{k}.$$

6.

$$U = 4\rho^2 \sin\frac{\varphi}{2} \qquad (U = U(\rho,\varphi)).$$

5-variant

1.

$$U = x - y^2 - 2z^2, \quad M_0(4,2,0), M_1(1,1,0), M_2(1,0,0).$$

2.

$$\vec{a}(M) = y^2\vec{i} + (x + y)\vec{j} + 2z^2\vec{k}, \qquad M_0 = (2,-1,\frac{1}{8}).$$

3.

$$S: y^2 + z^2 = x^2 \quad (z \geq 0), \qquad P: x = 5$$

4.

$$\vec{b}(M) = x\vec{i} - \frac{1}{3}z^2\vec{j} - x\vec{k}, \quad \Gamma: \begin{cases} x = 2\cos t, \\ y = 3, \\ z = 3\sin t. \end{cases}$$

5.

$$\vec{F} = (3x^2y^2z + y^2z^3)\vec{i} + (2x^3yz + 2xyz^3)\vec{j} + (x^3y^2 + 3xy^2z^2)\vec{k}$$

6.

$$U = \frac{\rho^2}{z}\cos 2\varphi \qquad (U = U(\rho, \varphi, z)).$$

6-variant

1.

$$U = 4x^2 + y^2 + z^2, \quad M_0(1,0,2), M_1(1,2,-2), M_2(0,-2,0).$$

2.

$$\vec{a}(M) = x\vec{i} + (y + yz)\vec{j} + (z - y^2)\vec{k}, \quad M_0 = (\frac{1}{2}, -\frac{1}{6}, \frac{2}{3}).$$

3.

$$S: x^2 + y^2 + z^2 = 1 \quad (z \geq 0), \qquad P: z = 0.$$

4.

$$\vec{b}(M) = -x^2y^3\vec{i} + \vec{j} + z\vec{k}, \quad \Gamma: \begin{cases} x = \sqrt[3]{4}\cos t, \\ y = \sqrt[3]{4}\sin t, \\ z = 3. \end{cases}$$

5

$$\vec{F} = \left(\frac{3x^2y^2}{z} - 2x^3\right)\vec{i} + \left(\frac{2x^3y}{z} + 3y^3\right)\vec{j} + \left(z^3 - \frac{x^3y^2}{z^2}\right)\vec{k}.$$

6.

$U = r^2 \sin 2\varphi \qquad (U = U(r,\varphi,\theta))$.

7-variant

1.

$U = x^2 - 2y^2 + z^2$, $M_0(2,0,2), M_1(1,\frac{1}{\sqrt{2}},-1), M_2(-2,1,\sqrt{2})$.

2.

$\vec{a}(M) = (x^2 - y)\vec{i} - (x + y)\vec{j} + z^2\vec{k}$, $M_0 = (\frac{1}{3},1,-\frac{5}{6})$.

3.

$S: x^2 + z^2 = y$, $P: y = 4$.

4.

$\vec{b}(M) = xy\vec{i} + x\vec{j} + y^2\vec{k}$, $\Gamma: \begin{cases} x = cost, \\ y = sint, \\ z = -sint. \end{cases}$

5.

$$\vec{F} = \frac{x}{\sqrt{(x^2+y^2+z^2)^3}}\vec{i} + \frac{y}{\sqrt{(x^2+y^2+z^2)^3}}\vec{j} + \frac{z}{\sqrt{(x^2+y^2+z^2)^3}}\vec{k}.$$

6.

$U = r^2 \sin\theta \qquad (U = U(r,\varphi,\theta))$.

8-variant

1.

$U = x^2 - 3y^2 - z^2$, $M_0(2,0,1), M_1(0,0,0), M_2(4,0,1)$.

2.

$\vec{a}(M) = xy\vec{i} - x^2\vec{j} + 3\vec{k}$, $M_0 = (\sqrt{3},-\frac{1}{6},2)$.

3.

$S: x^2 + y^2 = z^2 \quad (z \geq 0)$, $P: z = 1$.

4.

$$\vec{b}(M) = (y-z)\vec{i} + (z-x)\vec{j} + (x-y)\vec{k}, \quad \Gamma: \begin{cases} x = \cos t, \\ y = \sin t, \\ z = 2(1-\cos t). \end{cases}$$

5.

$$\vec{F} = (3x - yz)\vec{i} + (3y - xz)\vec{j} + (3z - xy)\vec{k}.$$

6.

$$U = \frac{\rho^2}{z}\sin 2\varphi \qquad (U = U(\rho,\varphi,z)).$$

9-variant

1.

$$U = y - 4x^2 - 8z^2, \quad M_0(4,1,0), M_1(0,0,1), M_2(0,1,0).$$

2.

$$\vec{a}(M) = (x + xy)\vec{i} + (y - x^2)\vec{j} + z\vec{k}, \quad M_0 = (-3, \frac{1}{4}, 1).$$

3.

$$S: x^2 + y^2 + z^2 = 1 \quad (z \geq 0), \qquad P: z = 0.$$

4.

$$\vec{b}(M) = xz\vec{i} + x\vec{j} + z^2\vec{k}, \quad \Gamma: \begin{cases} x = \cos t, \\ y = \sin t, \\ z = \sin t. \end{cases}$$

5.

$$\vec{F} = (x + 2yz)\vec{i} + (y + 2xz)\vec{j} + (z + 2xy)\vec{k}.$$

6.

$$U = \frac{\rho}{z}\sin\varphi \qquad (U = U(\rho,\varphi,z)).$$

10-variant

1.

$$U = y^2 - 3x^2 - 3z^2, \quad M_0(1,3,-1), M_1(0,2,1), M_2(-1,4,2).$$

2.

$$\vec{a}(M) = (x-y)\vec{i} + (x^2+y)\vec{j} + 3z, \qquad M_0 = (3,-2,\frac{1}{5}).$$

3.

$$S: x^2 + y^2 + z^2 = 1 \quad (y \geq 0), \qquad P: y = 0.$$

4 .

$$\vec{b}(M) = y\vec{i} - x\vec{j} + z\vec{k}, \qquad \Gamma: \begin{cases} x = cost, \\ y = sint, \\ z = 5. \end{cases}$$

5.

$$\vec{F} = (x^2 - 3yz)\vec{i} + (y^2 - 3xz)\vec{j} + (z^2 - 3xy)\vec{k}.$$

6.

$$U = \rho^2 \cos 3\varphi \qquad (U = U(\rho,\varphi)).$$

11-variant

1.

$$U = x - z^2 - 3y^2, \quad M_0(1,0,-2), M_1(3,1,1), M_2(4,-1,3).$$

2.

$$\vec{a}(M) = x^3\vec{i} + (y+z)\vec{j} + (x-z)\vec{k}, \qquad M_0 = (-\sqrt{2},\frac{1}{3},\frac{1}{2}).$$

3.

$$S: y^2 + 4z^2 = x, \qquad P: x = 2.$$

4.

$$\vec{b}(M) = x\vec{i} - z^2\vec{j} + y\vec{k}, \qquad \Gamma : \begin{cases} x = 2\cos t, \\ y = 3\sin t, \\ z = 4\cos t - 3\sin t. \end{cases}$$

5.

$$\vec{F} = (3x^2 - 2yz)\vec{i} + (3y^2 - 2xz)\vec{j} + (3z^2 - 2xy)\vec{k}.$$

6.

$$U = r\cos\theta\sin\varphi + r^2\sin\theta\cos\varphi.$$

12-variant

1.

$$U = 2x^2 - y^2 - z^2, \quad M_0(4,1,1), M_1(2,0,0), M_2(1,1,0).$$

2.

$$\vec{a}(M) = (x + xz)\vec{i} + y\vec{j} + (z - x^2)\vec{k}, \qquad M_0 = \left(-\frac{1}{\sqrt{2}}, 2, \frac{1}{2}\right).$$

3.

$$S : x^2 + y^2 + z^2 = 4 \quad (z \ge 0), \qquad P : z = 0.$$

4.

$$\vec{b}(M) = (y - z)\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}, \qquad \Gamma : \begin{cases} x = 2\cos t, \\ y = 2\sin t, \\ z = 3(1 - \cos t). \end{cases}$$

5.

$$\vec{F} = (2xy + z)\vec{i} + (x^2 - 2y)\vec{j} + x\vec{k}.$$

6.

$$U = 2r^2\sin 3\theta \qquad (U = U(r,\varphi,\theta)).$$

13-variant

1.

$$U = x^2 + y^2 + z, \quad M_0(1,1,0), M_1(0,1,0), M_2(\sqrt{3},0,0).$$

2.

$$\vec{a}(M)=(x+z)\vec{i}+(y+z)\vec{j}+(z-x^2-y^2)\vec{k},\quad M_0=(2,-2,1).$$

3.

$$S: x^2+z^2=y,\qquad P: y=16.$$

4.

$$\vec{b}(M)=-x^2y^3\vec{i}+3\vec{j}+y\vec{k},\qquad \Gamma:\begin{cases} x=\cos t,\\ y=2\sin t,\\ z=5.\end{cases}$$

5.

$$\vec{F}=\frac{1}{x+y+z}\vec{i}+\frac{1}{x+y+z}\vec{j}+\frac{1}{x+y+z}\vec{k}.$$

6.

$$U=\rho\cos 3\varphi \qquad (U=U(\rho,\varphi)).$$

14-variant

1.

$$U=x^2+y^2+4z^2,\quad M_0(1,0,1), M_1(0,1,-2), M_2(1,-1,2).$$

2.

$$\vec{a}(M)=(x+y)\vec{i}+(y-2x^3)\vec{j}+(z^2-2)\vec{k},\quad M_0=\left(\frac{1}{3},\frac{1}{3},-\frac{1}{3}\right).$$

3.

$$S: x^2+y^2=z^2\quad (z\ge 0),\qquad P: z=2.$$

4.

$$\vec{b}(M)=-2z\vec{i}-x\vec{j}+x^2\vec{k},\qquad \Gamma:\begin{cases} x=\frac{1}{3}\cos t,\\ y=\frac{1}{3}\sin t,\\ z=8.\end{cases}$$

5.

$$\vec{F} = \frac{yz}{1+x^2y^2z^2}\vec{i} + \frac{xz}{1+x^2y^2z^2}\vec{j} + \frac{xy}{1+x^2y^2z^2}\vec{k}$$

6.

$$U = \frac{\rho}{z}\sin 2\varphi \qquad (U = U(\rho,\varphi,z))$$

15-variant

1.

$$U = x^2 + y^2 - z^2, \quad M_0(1,2,0), M_1(1,0,-1), M_2(0,2,1).$$

2.

$$\vec{a}(M) = (x + xz^2)\vec{i} + y\vec{j} + (z - zx^2)\vec{k}, \quad M_0 = (2,-\sqrt{3},1).$$

3.

$$S: x^2 + y^2 + z^2 = 9 \quad (z \geq 0), \quad P: z = 0.$$

4.

$$\vec{b}(M) = z\vec{i} + x\vec{j} + y\vec{k}, \quad \Gamma: \begin{cases} x = 2\cos t, \\ y = 3\sin t, \\ z = 0. \end{cases}$$

5.

$$\vec{F} = \frac{x}{\sqrt{x^2+y^2+z^2}}\vec{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\vec{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\vec{k}.$$

6.

$$U = r\sin 2\varphi\cos\theta \qquad (U = U(r,\varphi,\theta)).$$

16-variant

1.

$$U = y^2 - x^2 - z^2, \quad M_0(1,2,1), M_1(1,4,0), M_2(2,4,1).$$

2.

$$\vec{a}(M) = (x + z)\vec{i} + y\vec{j} + (z - x^2)\vec{k}.$$

3.

$S: y^2+z^2=x$, $P: x=1$.

4.

$\vec{b}(M)=-x^2y^3\vec{i}+4\vec{j}+x\vec{k}$, $\Gamma: \begin{cases} x=2cost, \\ y=2sint, \\ z=4. \end{cases}$

5.

$\vec{F}=e^x \sin y\vec{i}+e^x \cos y\vec{j}+\vec{k}$.

6.

$U=\rho^2 \sin\frac{\varphi}{2}$ $(U=U(r,\varphi,\theta))$.

17-variant

1.

$U=z^2+x^2-y$, $M_0(1,1,0), M_1(0,1,2), M_2(4,2,2)$.

2.

$\vec{a}(M)=(xz+y)\vec{i}+(yz-x)\vec{j}+(z^2-2)\vec{k}$, $M_0=(-1,\sqrt{2},6)$.

3.

$S: y^2+z^2=x^2 \quad (x\ge 0)$, $P: x=3$.

4.

$\vec{b}(M)=x\vec{i}+2z^2\vec{j}+y\vec{k}$, $\Gamma: \begin{cases} x=cos\,t, \\ y=3\,sint, \\ z=2\,cos\,t-3\,sin\,t-2. \end{cases}$

5.

$\vec{F}=y\vec{i}+x\vec{j}+e^z\vec{k}$.

6.

$U=2\rho \sin\frac{\varphi}{2}$ $(U=U(\rho,\varphi))$.

18-variant

1.

$$U = 4x^2 + 2y^2 + z^2, \quad M_0(1,0,2), M_1(0,1,2), M_2(2,1,0).$$

2.

$$\vec{a}(M) = (x + xy^2)\vec{i} + (y - yx^2)\vec{j} + z\vec{k}, \quad M_0 = (\frac{\sqrt{2}}{2}, -\sqrt{3}, 5).$$

3.

$$S: x^2 + y^2 + z^2 = 9 \quad (z \geq 0), \qquad P: z = 0.$$

4.

$$\vec{b}(M) = x\vec{i} + z^2\vec{j} + y\vec{k}, \qquad \Gamma: \begin{cases} x = \cos t, \\ y = 2\sin t, \\ z = 2\cos t - 2\sin t - 1. \end{cases}$$

5.

$$\vec{F} = \frac{1}{3}(x^3\vec{i} + y^3\vec{j} + xz^3\vec{k}).$$

6.

$$U = r\sin\theta + r^2\cos\varphi \qquad (U = U(r, \varphi, \theta)).$$

19-variant

1.

$$U = 2x^2 + y^2 - z^2, \quad M_0(1,-1,0), M_1(2,1,-1), M_2(-2,0,1).$$

2.

$$\vec{a}(M) = x\vec{i} + (y + z)\vec{j} + (z - y)\vec{k}, \quad M_0 = (-1,2,1).$$

3.

$$S: x^2 + z^2 = y, \qquad P: y = 9.$$

4.

$$\vec{b}(M) = z\vec{i} + y^2\vec{j} - x\vec{k}, \qquad \Gamma: \begin{cases} x = \sqrt{2}\cos t, \\ y = 3\sin t, \\ z = \sqrt{2}\cos t. \end{cases}$$

5.

$$\vec{F}=\left(\frac{z}{x^2}+\frac{1}{y}\right)\vec{i}+\left(\frac{x}{y^2}+\frac{1}{z}\right)\vec{j}+\left(\frac{y}{z^2}+\frac{1}{x}\right)\vec{k}.$$

6.

$$U=r\sin 2\theta \qquad (U=U(r,\varphi,\theta)).$$

20-variant

1.

$$U=2y^2-x^2-z^2, \quad M_0(-1,2,0), M_1(-1,2,1), M_2(0,1,-1).$$

2.

$$\vec{a}(M)=y^2x\vec{i}-yx^2\vec{j}+\vec{k}, \qquad M_0=(\sqrt{6},-\frac{\sqrt{2}}{2},5).$$

3.

$$S: x^2+y^2=z^2 \quad (z\geq 0), \qquad P: z=5.$$

4.

$$\vec{b}(M)=3y\vec{i}-3x\vec{j}+x\vec{k}, \quad \Gamma: \begin{cases} x=3\cos t, \\ y=3\sin t, \\ z=3-3\cos t-3\sin t. \end{cases}$$

5.

$$\vec{F}=yz\cos xy\vec{i}+xz\cos xy\vec{j}+\sin xy\vec{k}.$$

6.

$$U=r^2\sin\frac{\theta}{2} \qquad (U=U(r,\varphi,\theta)).$$

21-variant

1.

$$U=z-4y^2-x^2, \quad M_0(1,0,4), M_1(0,1,-5), M_2(-1,0,3).$$

2.

$$\vec{a}(M)=(x+xy)\vec{i}+(y-x^2)\vec{j}+(z-1)\vec{k}, \quad M_0=(-\sqrt{5},4,\frac{1}{2}).$$

3.

$S: x^2 + y^2 + z^2 = 1 \quad (z \geq 0)$, $\quad P: z = 0$.

4.

$\vec{b}(M) = -x^3 y^3 \vec{i} + 2\vec{j} + xz\vec{k}$, $\quad \Gamma: \begin{cases} x = \sqrt{2}\cos t, \\ y = \sqrt{2}\sin t, \\ z = 1. \end{cases}$

5.

$\vec{F} = xz\vec{i} + 2y\vec{j} + xy\vec{k}$.

6.

$U = z\rho\cos\frac{\varphi}{2} \quad (U = U(\rho, \varphi, z))$.

22-variant

1.

$U = \frac{x^2}{9} + \frac{y^2}{4} + z^2$, $\quad M_0(3,0,1), M_1(-3,2,0), M_2(0,4,2)$.

2.

$\vec{a}(M) = (y^2 - x^2)\vec{i} + y\vec{j} + (y^2 + z^2)\vec{k}$, $\quad M_0 = (\frac{1}{2}, -3, 1)$.

3.

$S: 4x^2 + z^2 = y$, $\quad P: y = 1$.

4.

$\vec{b}(M) = x\vec{i} - 2z^2\vec{j} + y\vec{k}$, $\quad \Gamma: \begin{cases} x = 3\cos t, \\ y = 3\sin t, \\ z = 6\cos t - 3\sin t + 1. \end{cases}$

5.

$\vec{F} = \frac{1}{2}(x^2\vec{i} + y^2\vec{j} + xz^2\vec{k})$

6.

$U = \rho^2 \sin\varphi \qquad (U = U(\rho,\varphi))$.

23-variant

1.

$U = x^2 + y^2 - 2z^2$, $M_0(4,1,1), M_1(1,-1,0), M_2(2,1,-1)$.

2.

$\vec{a}(M) = (x + y^2)\vec{i} + y\vec{j} + (x^2 + z - 2)\vec{k}$, $M_0 = (-1, \frac{1}{3}, \frac{1}{6})$.

3.

$S : x^2 + z^2 = y^2 \quad (y \geq 0)$, $P : y = 1$.

4.

$\vec{b}(M) = x\vec{i} - 3z^2\vec{j} + y\vec{k}$, $\Gamma : \begin{cases} x = \cos t, \\ y = \sin t, \\ z = 2\cos t - 4\sin t + 3. \end{cases}$

5.

$\vec{F} = (x^3 - yz)\vec{i} + (y^3 - xz)\vec{j} + (z^3 - xy)\vec{k}$.

6.

$U = r\cos\varphi \sin\theta \qquad (U = U(r,\varphi,\theta))$.

24-variant

1.

$u = \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25}$, $M_0(1,1,1), M_1(1,-1,1), M_2(0,0,0)$.

2.

$\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$, $M_0(0,1,0)$.

3.

$\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$, $S : z = 3(4 - x^2 - y^2), P : z = 2$.

4.

$$\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}, \qquad \Gamma: \begin{cases} x = \sqrt{2}\cos t, \\ y = \sin t, \\ z = \sin t. \end{cases}$$

5.

$$\vec{F} = (yz - xy)\vec{i} + (xz - \frac{x^2}{2} + yz^2)\vec{j} + (xy + y^2 z)\vec{k}.$$

6.

$$U = 2\rho\cos\varphi, \quad U = U(\rho,\varphi).$$

25-variant

1.

$$u = x^2 - y^2 - z^2, \quad M_0(0,0,0), M_1(2,1,2), M_2(-2,1,-2).$$

2.

$$\vec{a} = (y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2 + y^2)\vec{k}, \quad M_0(1,1,1).$$

3.

$$\vec{a} = (y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2 + y^2)\vec{k},$$

$$S: z^2 + 2x^2 = y^2, P: |y| = 1.$$

4.

$$\vec{a} = (y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2 + y^2)\vec{k}, \qquad \Gamma: \begin{cases} x = 4, \\ y = \sqrt{3}\sin t, \\ z = \sqrt{3}\cos t. \end{cases}$$

5.

$$\vec{F} = (\frac{1}{z} - \frac{y}{x^2})\vec{i} + (\frac{1}{x} - \frac{z}{y^2})\vec{j} + (\frac{1}{y} - \frac{x}{z^2})\vec{k}.$$

6.

$$U = 3\rho\sin\varphi, \quad U = U(\rho,\varphi).$$

26-variant

1.

$$u = x^2 + y^2 - z, \quad M_0(2,2,1), M_1(1,-1,1), M_2(0,1,3).$$

2.

$$\vec{a} = x^2yz\vec{i} + xy^2z\vec{j} + xyz^2\vec{k}, \quad M_0(2,1,2).$$

3.

$$\vec{a} = x^2yz\vec{i} + xy^2z\vec{j} + xyz^2\vec{k}, \quad S : x^2 + 9y^2 = z^2), P : |z| = 2$$

. 4.

$$\vec{a} = x^2yz\vec{i} + xy^2z\vec{j} + xyz^2\vec{k}, \quad \Gamma : \begin{cases} x = \cos t, \\ y = 5, \\ z = \sin t. \end{cases}$$

5.

$$\vec{F} = \left(\frac{z}{y^2} - \frac{y}{z^2} - \frac{2yz}{x^3}\right)\vec{i} + \left(\frac{z}{x^2} - \frac{x}{z^2} - \frac{2xz}{y^3}\right)\vec{j} + \left(\frac{y}{x^2} - \frac{x}{y^2} - \frac{2xy}{z^3}\right)\vec{k}.$$

6.

$$U = \frac{1}{2}\rho \sin\varphi \cos\varphi, \quad U = U(\rho, \varphi).$$

27-variant

1.

$$u = \frac{x^2}{2} - \frac{y^2}{2} + z, \quad M_0(2,1,1), M_1(0,2,0), M_2(3,2,1).$$

2.

$$\vec{a} = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}, \quad M_0(3,2,1).$$

3.

$$\vec{a} = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}, \quad S : x^2 + y^2 = z^2, P : |z| = 1.$$

4.

$$\vec{a}=2x\vec{i}+2y\vec{j}+2z\vec{k}, \quad \Gamma:\begin{cases} x=\cos t, \\ y=\sin t, \\ z=5. \end{cases}$$

5.

$$\vec{F}=2xy\vec{i}+(x^2-2yz)\vec{j}-y^2\vec{k}.$$

6.

$$U=2\rho^2\sin 2\varphi, \quad U=U(\rho,\varphi)$$

28-variant

1.

$$u=x^2+2y^2-z^2, \quad M_0(2,3,-1), M_1(1,-1,2), M_2(1,3,2).$$

2.

$$\vec{a}=xy\vec{i}+yz\vec{j}+xz\vec{k}, \quad M_0(2,3,1).$$

3.

$$\vec{a}=xy\vec{i}+yz\vec{j}+xz\vec{k}, \quad S: x^2+y^2+z^2=1, P: x+y+z=1.$$

4.

$$\vec{a}=xy\vec{i}+yz\vec{j}+xz\vec{k}, \quad \Gamma:\begin{cases} x=\cos t, \\ y=\cos t, \\ z=\sqrt{2}\sin t. \end{cases}$$

5.

$$\vec{F}=6x^2\vec{i}+3\cos(3x+2z)\vec{j}+\cos(3y+2z)\vec{k}.$$

6.

$$u=2\rho\sin 2\varphi, \quad U=U(\rho,\varphi).$$

29-variant

1.

$$u=x^2+y^2-2z^2, \quad M_0(-1,3,2), M_1(2,1,3), M_2(0,1,0).$$

2.

$\vec{a} = yz\vec{i} + xz\vec{j} + xy\vec{k}$, $M_0(1,2,3)$.

3.

$\vec{a} = yz\vec{i} + xz\vec{j} + xy\vec{k}$, $S: x^2 + y^2 = z, P: z = 3$.

4.

$\vec{a} = yz\vec{i} + xz\vec{j} + xy\vec{k}$,

$$\Gamma: \begin{cases} x = \cos t, \\ y = \sin t, \\ z = 2(1 - \cos t). \end{cases}$$

5.

$\vec{F} = (y + z)\vec{i} + (x + z)\vec{j} + (x + y)\vec{k}$.

6.

$U = \rho\cos\varphi + \rho^2 \sin\varphi$, $U = U(\rho, \varphi)$.

30-variant

1.

$u = 2x^2 - 4y^2 - z$, $M_0(2,1,4), M_1(4,1,2), M_2(3,4,1)$.

2.

$\vec{a} = y\vec{i} + z\vec{j} + x\vec{k}$, $M_0(3,2,2)$.

3.

$\vec{a} = y\vec{i} + z\vec{j} + x\vec{k}$, $S: z = 2(1 - x^2 - y^2), P: z = 0$.

4.

$\vec{a} = y\vec{i} + z\vec{j} + x\vec{k}$, $\Gamma: \begin{cases} x = \sin t, \\ y = \cos t, \\ z = 3(1 - \cos t). \end{cases}$

5.

$\vec{F} = xy(3x - 4y)\vec{i} + x^2(x - 4y)\vec{j} + 3xz^2\vec{k}$.

6. $U = \rho\cos 3\varphi$, $U = U(\rho, \varphi)$.

KOMPLEKS O‘ZGARUVCHILI FUNKSIYALAR NAZARIYASI

Kompleks funksiya tushunchasi.

Kompleks sonlar tekisligidagi E to‘plam bog‘langan deyiladi, agar uning ihtiyoriy ikki nuqtasini shu to‘plamga tegishili uzliksiz chiziq bilan tutashtirish mumkin bo‘lsa. Kompleks sonlar tekisligidagi E to‘plam ochiq deyiladi, agar uning har bir nuqtasi o‘zining biror atrofi bilan birgalikda E to‘plamga tegishli bo‘lsa. Kompleks sonlar tekisligdagi ochiq, bog‘langan E to‘plam soha deyiladi. Har bir $z = x + iy$ kompleks songa biror f qonun asosida bir yoki bir necha kompleks son $\omega = u + iv$ mos qo‘yilgan bo‘lsa, u holda

$$\omega = f(z) = u + iv = u(x,y) + iv(x,y)$$

kompleks funksiya berilgan deyiladi.Bu yerda

$$u(x,y) = \operatorname{Re} f(z), \qquad v(x,y) = \operatorname{Im} f(z).$$

Kompleks funksiyalar bir qiymatli va ko‘p qiymatli bo‘ladi. $\omega = z^2$ funksia bir qiymatli, chunki z ning har bir qiymatiga ω ning yagona qiymati mos keladi. $\omega = \sqrt{z}$ funksiya esa ikki qiymatliydir, chunki z ning har bir qiymatiga, kompleks sonning ildizi ta’rifiga ko‘ra ω ning ikkita qiymati mos keladi.

Analitik funksiyalar. Koshi-Riman shartlari.

Agar D sohaning ihtiyoriy z nuqtasida

$$\lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}, \qquad z + \Delta z \in D$$

limit mavjud bo‘lsa, bu limitga $f(z)$ funksiyaning z nuqtadagi hosilasi deyiladi va $f'(z)$ yoki $\frac{df(z)}{dz}$ ko‘rinishda belgilanadi. D sohaning har bir nuqtasida hosilasi mavjud bo‘lgan funksiya shu D sohada analitik funksiya deyiladi. $f(z) = u(x,y) + iv(x,y)$ funksiya D sohada analitik bo‘lishi uchun uning shu sohada quyidagi

$$\begin{cases} \dfrac{\partial u(x,y)}{\partial x} = \dfrac{\partial v(x,y)}{\partial y}, \\ \dfrac{\partial u(x,y)}{\partial y} = -\dfrac{\partial v(x,y)}{\partial x} \end{cases}$$

Koshi-Riman shartlarini qanoatlantiruvchi hususiy hosilalari mavfud bo'lishi zarur va yetarlidir.

Kompleks o'zgaruvchili funksiyadan olingan integrallar.

Faraz qilamiz l $z-$ tekislikning ihtiyoriy yo'naltirilgan, bo'lakli uzluksiz chizig'i bo'lsin va barcha $z \in l$ nuqtalarda $f(z)$ funksiya aniqlangan bo'lsin. $f(z) = u(x,y) + iv(x,y)$ ko'rinishda aniqlangan bo'lsa u holda

$$\int_l f(z)dz = \int_l u(x,y)dx - v(x,y)dy + i\int_l v(x,y)dx + u(x,y)dy$$

Koshi formulalari .

Agar $f(z)-$funksiya Γ-yopiq chiziq bilan chegaralangan D - bir bog'lamli sohada analitik bo'lsa, u holda

$$\int_\gamma f(\eta)d\eta = 0$$

Bunda γ - D sohaga to'liq tegishli yopiq chiziq. Agar bulardan tashqari $f(z)-$funksiya $\overline{D} = D + \Gamma$ yopiq sohada uzluksiz bo'lsa

$$\int_\Gamma f(n)dn = 0$$

Agar $f(z)-$funksiya bir bog'lamli D sohada uzluksiz bo'lsa va ihtiyoriy yopiq $\gamma \in D$ chiziq uchun $\oint_\gamma f(\eta)d\eta = 0$ tenglik bajarilsa u holda tayinlangan $z_0 \in D$ nuqtada

$\Phi(z) = \int_{Z_0}^{Z} f(\eta)d\eta$ funksiya D sohada analitik bo'ladi va

$\Phi'(z) = f(z)$ bo'ladi. $\Phi(z)$ funksiya $f(z)$ uchun boshlang'ich funksiya yoki aniqmas integral deyiladi. Agar $F(z)-$funksiya

$f(z)$ funksiyaning boshlangichlaridan biri bo'lsa, u holda

$$\int_{z_1}^{z_2} f(\eta)d\eta = F(z_2) - F(z_1)$$

D sohada analitik funksiya $f(z)$, $z_0 \in D$ nuqta va z_0 ni o'z ichiga oluvchi $\gamma \subset D$ chiziqlar uchun quyidagi Koshining integral formulasi o'rinli:

$$f(z_0) = \frac{1}{2\pi i}\oint_{\gamma} \frac{f(z)}{z - z_0} dz$$

Bundan tashqari $f(z)$ funksiya D ning barcha nuqtalarida ihtiyoriy tartibdagi hosilaga ega bo'ladi va bu hosilalar uchun quyidagi formula o'rinlidir

$$f^{(k)}(z) = \frac{k!}{2\pi \cdot i}\oint_{\gamma} \frac{f(\eta)}{(\eta - z)^{k+1}} d\eta \qquad k = 1,2,3,....$$

Loran qatori.

$$\sum_{n=-\infty}^{\infty} c_n (z - z_0)^n$$

qator- Loran qatori deyiladi. Bunda

$$f_1(z) = \sum_{n=0}^{\infty} c_n (z - z_0)^n$$

qatorga Loran qatorining to'g'ri qismi,

$$f_2(z) = \sum_{n=-\infty}^{-1} c_n (z - z_0)^n$$

qatorga esa Loran qatorining asosiy qismi deyiladi. Agar

$$\overline{\lim_{n\to\infty}} \sqrt[n]{|c_{-n}|} = r < R = \frac{1}{\overline{\lim_{n\to\infty}} \sqrt[n]{|c_n|}}$$

munosabat o'rinli bo'lsa, bu holda Loran qatorning yaqinlashish sohasi quyidagi halqa bo'ladi:

$$K = \{z / 0 \le r < |z - z_0| < R\}$$

Bu halqada $f(z) = f_1(z) + f_2(z)$ – analitik funksiya bo‘lib, c_n koeffitsiyent uchun quydagi formula o‘rinlidir.

$$c_n = \frac{1}{2\pi \cdot i} \int_{|\eta - z_0| = r'} \frac{f(\eta)}{(\eta - z_0)^{n+1}} d\eta,$$

$$n = 0, \pm 1, \ldots$$

bunda $r < r' < R$.

Hisoblash topshiriqlari

1- topshiriq. Kompleks ifodaning qiymatini toping.

2- topshiriq. Nuqtaning geometrik o‘rnini aniqlang.

3- topshiriq. Berilgan kompleks funksiyaning qiymatini , argumentning berilgan qiymatida toping yoki tenglamani yeching.

4- topshiriq. $f(z) = u(x, y) + iv(x, y)$ kompleks funksiyaning ko‘rinishini uning berilgan mavhum yoki haqiqiy qismi orqali toping:

5- topshiriq. Integralni hisoblang.

6- topshiriq. Qatorning yaqinlashish sohasini toping yoki uni berilgan sohada Loran qatoriga yoying.

Namunali variant.

1 - masala. a) Hisoblang: $\sqrt[3]{-2 + i2}$

Yechish:

$z = -2 + i2 = 2(-1 + i) = 2z_1$, bunda $z_1 = -1 + i$.

$z_1 = -1 + i$ kompleks sonning moduli quyidagiga teng:

$$|z_1| = r = \sqrt{1 + 1} = \sqrt{2};$$

z_1 ikkinchi chorakka tegishli bo‘lganligi uchun, uning argumenti quyidagiga teng:

$$\varphi = arctg\frac{1}{-1} + \pi = -\frac{\pi}{4} + \pi = \frac{3}{4}\pi.$$

Demak,

$$z_1 = -1 + i = \sqrt{2}\left(cos\frac{3}{4}\pi + i\,sin\frac{3}{4}\pi\right).$$

U holda:

$$\sqrt[3]{-2+2i}=\sqrt[3]{2\sqrt{2}(\cos\frac{3}{4}\pi+i\sin\frac{3}{4}\pi)}=$$

$$=(2^{3/2})^{1/3}\left(\cos\frac{\frac{3}{4}\pi+2\pi k}{3}+i\sin\frac{\frac{3}{4}\pi+2\pi k}{3}\right)=$$

$$=2^{\frac{1}{2}}\left(\cos\frac{\frac{3}{4}\pi+2\pi k}{3}+i\sin\frac{\frac{3}{4}\pi+2\pi k}{3}\right).$$

Ketma-ket $k=0,1,2$ deb, quyidagini olamiz:

$$k=0,\ \ (\sqrt[3]{-2+i2})_0=\sqrt{2}\left(\cos\frac{3\pi}{12}+i\sin\frac{3\pi}{12}\right)=$$

$$=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=1+i.$$

$$k=1,\qquad (\sqrt[3]{-2+i2})_1=\sqrt{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right).$$

$$k=2,\qquad (\sqrt[3]{-2+i2})_2=\sqrt{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)$$

b) Hisoblang:

$$w=(-1-i)^{1+i}$$

Yechish:

$$w=(-1-i)^{1+i}=e^{(1+i)\,Ln(-1-i)}=$$

$$=e^{(1+i)\left(ln\sqrt{2}+i\left(-\frac{3\pi}{4}+2k\pi\right)\right)}=e^{(1+i)\left(\frac{1}{2}ln2+i\left(-\frac{3\pi}{4}+2k\pi\right)\right)}=$$

$$=e^{\frac{1}{2}ln2+i\left(-\frac{3\pi}{4}+2k\pi\right)+i\frac{1}{2}ln2+i^2\left(-\frac{3\pi}{4}+2k\pi\right)}=$$

$$=e^{\frac{1}{2}ln2-\left(-\frac{3\pi}{4}+2k\pi\right)+i\left(-\frac{3\pi}{4}+2k\pi+\frac{1}{2}ln2\right)}=$$

$$=\left(e^{\frac{1}{2}ln2+\frac{3}{4}\pi-2k\pi}\right)\cdot\left(\cos\left(\frac{ln2}{2}-\frac{3\pi}{4}\right)+i\sin\left(\frac{ln2}{2}-\frac{3\pi}{4}\right)\right).$$

2 - masala. Tekislikning quyidagi shartlarni qanoatlantiruvchi nuqtalari to'plamini aniqlang.

$$\begin{cases} 3 \leq (z + 1 - 2i) \leq 4 \\ \dfrac{\pi}{2} \leq arg\, z \leq \pi \end{cases}$$

Yechish: $3 \leq (z + 1 - 2i) \leq 4$ shart radiuslari $r = 3$ va $R = 4$ bolgan va markazi $z_0 = -1 + 2i$ nuqtada yotuvchi konsentrik aylanalar orasidagi halqani beradi.

$$\frac{\pi}{2} \leq arg\, z \leq \pi$$

shart esa ikkinchi chorak nuqtalari to'plamini beradi. (19-rasm)

3 - masala. Berilgan tenglamani yeching:

$$sin\, z + cos\, z = i$$

Yechish:

$$\frac{e^{iz} - e^{-iz}}{2i} + \frac{e^{iz} + e^{-iz}}{2} = i$$

$$e^{2iz} - 1 + ie^{2iz} + i = -2e^{iz}$$

$$(1 + i)e^{2iz} + 2e^{iz} - 1 + i = 0$$

Quyidagi belgilashni kiritamiz: $e^{iz} = t$.

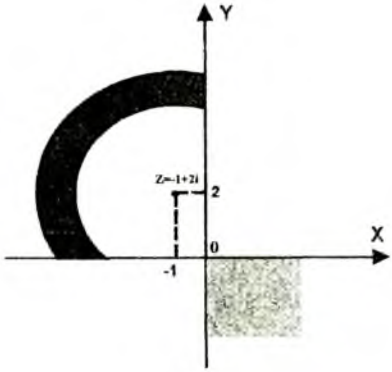
U holda quyidagiga ega bo'lamiz: $(1 + i)t^2 + 2t - 1 + i = 0$.

$$t_{1,2} = \frac{-1 \pm \sqrt{1 + 2}}{1 + i} = \frac{-1 \pm \sqrt{3}}{1 + i} = \frac{\left(-1 \pm \sqrt{3}\right)(1 - i)}{2}.$$

$$e^{iz} = \frac{\left(-1 \pm \sqrt{3}\right)(1 - i)}{2}.$$

$$iz = ln\left|\frac{\left(-1 \pm \sqrt{3}\right)(1 - i)}{2}\right| + i\left(\varphi_{1,2} + 2k\pi\right) \qquad k \in Z.$$

$$z_1 = -i \cdot ln\frac{-1 + \sqrt{3}}{\sqrt{2}} + \varphi_1 + 2k\pi, \qquad k\epsilon Z.$$



19-rasm

bunda

$$\varphi_1 = -\frac{\pi}{4};$$

$$z_2 = -i \cdot \ln\frac{1+\sqrt{3}}{\sqrt{2}} + \varphi_2 + 2k\pi, \quad k\epsilon Z.$$

bunda

$$\varphi_2 = \frac{3\pi}{4}.$$

4 – masala. Mavhum qismi $v(x,y) = x + y$ ga teng bo‘lgan

$$f(z) = u(x,y) + iv(x,y)$$

analitik funksiyani tiklang

Yechish: Masalaning shartiga ko‘ra:

$$\frac{dv}{dy} = 1.$$

Bilamizki,

$$\frac{dv}{dy} = \frac{du}{dx},$$

shuning uchun

$$u = \int dx = x + \varphi(y).$$

Bundan esa

$$u = x + \varphi(y); \quad \frac{du}{dy} = \varphi'(y); \quad \frac{dv}{dx} = 1.$$

$$\frac{du}{dy} = -\frac{dv}{dx}$$

bo'lgani uchun

$$\varphi'(y) = -1 \Rightarrow \varphi(y) = -y + c.$$

O'rniga qo'yib hosil qilamiz: $u = x - y + c.$

Demak:

$$f(z) = u(x,y) + iv(x,y) = (x - y + c) + i(x + y) =$$
$$= (1 + i)(x + iy) + c = (1 + i)z + c,$$

bunda c – const

5- masala. Hisoblang

$$\int\limits_{|z|=7} \frac{e^z}{z^3 - 5z^2} dz$$

Yechish:

Quyidagiga ega bo'lamiz

$$\int\limits_{|z|=7} \frac{e^z dz}{z^3 - 5z^2} = \int\limits_{\gamma_1} \frac{e^z}{z^3 - 5z^2} dz + \int\limits_{\gamma_2} \frac{e^z}{z^3 - 5z^2} dz =$$

$$= \int\limits_{\gamma_1} \frac{\frac{e^z}{z-5}}{z^2} dz + \int\limits_{\gamma_2} \frac{\frac{e^z}{z^2}}{z-5} dz = \frac{2\pi i}{1!} \left(\frac{e^z}{z-5}\right)' \Big|_{z=0} + 2\pi i \left(\frac{e^z}{z^2}\right) \Big|_{z=5} =$$

$$= 2\pi i \left(\frac{e^z(z-5) - e^z}{(z-5)^2}\right) \Big|_{z=0} + 2\pi i \frac{e^5}{25} = 2\pi i \left(-\frac{6}{25}\right) + 2\pi i \frac{e^5}{25} =$$

$$= 2\pi i \frac{e^5 - 6}{25} = \frac{2\pi i}{25}(e^5 - 6).$$

6- masala. Berilgan

$$f(z) = \frac{7z - 19}{z^2 - 6z + 5}$$

kompleks o'zgaruvchili funksiyani $z_0=1$ nuqta atrofida Loran qatoriga yoying.

Yechish:

$$f(z) = \frac{7z - 19}{z^2 - 6z + 5} = \frac{A}{z-5} + \frac{B}{z-1} = \frac{A(z-1) + B(z-5)}{(z-5)(z-1)}.$$

$$A(z-1)+B(z-5)=7z-19,$$
$$\left.\begin{matrix} z=1 \\ z=5 \end{matrix}\right| \left.\begin{matrix} -4B=-12 \\ 4A=16 \end{matrix}\right\} \Rightarrow \begin{matrix} B=3, \\ A=4. \end{matrix}$$

U holda
$$f(z)=\frac{4}{z-5}+\frac{3}{z-1}.$$
Ma'lumki $|z|<1$ atrofda
$$\frac{1}{1-z}=\sum_{n=0}^{\infty} z^n.$$
Bundan esa
$$\frac{4}{z-5}=\frac{4}{-4+z-1}=-\frac{4}{4-(z-1)}=-\frac{4}{4\left(1-\frac{z-1}{4}\right)}=$$
$$=-\frac{1}{1-\frac{z-1}{4}}=-\sum_{n=0}^{\infty}\left(\frac{z-1}{4}\right)^n=-\sum_{n=0}^{\infty}\frac{(z-1)^n}{4^n},$$
demak $0<|z-1|<4$ halqada,
$$f(z)=-\sum_{n=0}^{\infty}\frac{(z-1)^n}{4^n}+\frac{3}{z-1}.$$

Variantlar

1-variant

1. $(1-i)^{-1+i}$; 2. $1<\mathfrak{I}_m(3i-z)<3$;
3. $sh(iz)=i$; 4. $v(x,y)=2xy+y$;

5. $\mathfrak{I}=\int_\Gamma (x^2+iy^2)dz$, bunda $\Gamma-$ $z_0=1+i$ nuqtadan $z_1=2+3i$ nuqtaga o'tkazilgan to'g'ri chiziq.

6. $\sum_{n=1}^{\infty}\frac{1}{z^n}+\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$.

2-variant

1. $(3+4i)^i$; 2. $arg(z+2i)=\frac{\pi}{3}$; 3. $\cos z=2$;

4. $v(x,y)=e^x\sin y$, $f(0)=1$; 5. $\mathcal{J}=\oint_{|z|=2}\frac{e^z dz}{z^2-1}$;

6. $f(z)=\dfrac{2z+3}{z^2+3z+2}$

funksiya'ni $\frac{1}{4}<|z+2|<1$ halqada Loran qatoriga yoying.

3-variant

1. $(2+2i)^{i+1}$; 2. $Im(z^2-\bar{z})=2-\mathcal{J}_m z$;

3. $sh\left(ln4+\frac{\pi}{3}i\right)$; 4. $u(x,y)=\dfrac{x}{x^2+y^2}$, $f(\pi)=\frac{1}{5}$;

5. $\mathcal{J}=\int_\Gamma(1+i-z\bar{z})dz$, bunda $\Gamma-z_0=0$, $z_1=1+i$ nuqtalardan o'tuvchi $y=x^2$ parabola.

6. $f(z)=\dfrac{1}{z^2+2z-8}$ funksiyani $\frac{1}{3}<|z+4|<\frac{1}{2}$ halqada Loran qatoriga yoying.

4-variant

1. $(1-\sqrt{3}i)^{-1+i}$; 2. $|\bar{z}|-Imz=6$; 3. $ch3z=i$;

4. $v(x,y)=x^2-y^2+xy$;

5. $\mathcal{J}=\int_\Gamma(z\bar{z}-\bar{z}^2)dz$, bunda $\Gamma:|z|=1\ (-\pi\le arg\,z\le 0)$.

6. $f(z)=\frac{1}{z}\sin^2\frac{z}{2}$ funksiya'ni $z_0=0$ atrofda Loran qatoriga yoying.

5-variant

1. $(-1-i)^i$; 2. $|z-1|<|z-i|$; 3. $e^{z^2}=i$;

4. $V(x,y)=arctg\frac{y}{x}$, $(x>0)$ $f(1)=0$;

5. $\mathcal{J}=\oint_\Gamma\dfrac{zdz}{(z-2)^2(z+1)}$, bunda $\Gamma:|z-3|=6$;

6. $f(z)=\dfrac{z^2-z+3}{z^2-3z+2}$ funksiya'ni $\frac{1}{4}<|z-2|>\frac{1}{2}$ halqada Loran qatoriga yoying.

6-variant

1. $(-1+i)^{1+i}$; 2. $1 < Imiz < 3$; 3. $ch(ln4 + \frac{\pi}{3}i)$

4. $u = x^2y^2 + 5x + y - \dfrac{y}{x^2+y^2}$;

5. $\mathcal{J} = \oint\limits_{|z|=3} \dfrac{e^z \cos nz\, dz}{z^2+2z}$;

6. $f(z) = \dfrac{2z-3}{z^2-3z+2}$ funksiya'ni $0 < |z-1| < 1$ halqada Loran qatoriga yoying.

7-variant

1. $W = (\sqrt{3}-i)^{i+1}$; 2. $\frac{1}{4} < Im\left(\frac{1}{z}\right) < \frac{1}{2}$; 3. $\sin z = 3$;

4. $v(x,y) = 3x^2y - y^3$;

5. $\mathcal{J} \int\limits_{\Gamma} e^{|z|^2} \mathcal{J}mz\, dz$, bunda $\Gamma -$ $z_0 = 0$, $z_1 = 1+i$ nuqtalardan o'tuvchi to'g'ri chiziq.

6. $f(z) = \frac{1-e^{-z}}{z^3}$ funksiya'ni , $z_0 = 0$ nuqta atrofida Loran qatoriga yoying.

8-variant

1. $W = (3+\sqrt{3}i)^{-i+1}$; 2. $\dfrac{1}{4} < Re\dfrac{1}{z} < 1$; 3. $\sin z = \pi i$;

4. $v(x,y) = -\frac{y}{x^2+y^2} + 2x$; 5. $\mathcal{J} = \oint_{|z-1|} \frac{\sin \pi z}{(z^2+1)^2} dz$;

6. $f(z) = \dfrac{1-\cos z}{z^2}$ funksiya'ni $z_0 = 0$ nuqta atrofida Loran qatoriga yoying.

9-variant

1. $W = (5+4i)^{-2i+1}$; 2. $Im\left(\dfrac{1}{z}\right) < -\dfrac{1}{2}$; 3. $\cos(2+i)$;

4. $u(x,y) = (x^2 - y^2 + 2y)$, $f(i) = 2i - 1$.

5. $\mathcal{J} = \oint\limits_{|z|=2} \frac{sinz \cdot sin\,(z-1)}{z^2+z} dz$;

6. $f(z) = z^3 e^{\frac{1}{z}}$ funksiya'ni $0 < |z| < 1$ halqada Loran qatoriga yoying.

10-variant

1. $W = \left(-\sqrt{3}+3i\right)^{i+2}$; 2. $\left|\frac{z-3}{z-2}\right| \geq 1$; 3. $sh(2+i)$;

4. $v(x,y) = -2sin2x\,sh2y + y\,, f(0) = 2$; 5. $\mathcal{J} = \oint\limits_{|z|=3} \frac{z^2dz}{z-2i}$;

6. $f(z) = \frac{1}{(z-1)^2(z+2)}$ funksiya'ni $1 < |z-1| < 2$ halqada Loran qatoriga yoying.

11-variant

1. $\left(\frac{1-i}{\sqrt{2}}\right)^{i+1}$; 2. $-\frac{3\pi}{4} < ar\,g(z-1+i) < \frac{2\pi}{3}$;

3. $|z| + 2z = 3 - 4i$;

4. $v(x,y) = -\frac{1}{2}(x^2 - y^2) + 2xy$;

5. $\int\limits_C \frac{e^z}{(z^2+9)\left(z-\frac{1}{2}\right)} dz$, где $C: x^2 + y^2 = 2x + 2y + 4$;

6. $f(z) = \frac{2z-5}{z^2-5+6}$ $z_0 = 3$ nuqta atrofida;

12-variant

1. $(3-4i)^{1+i}$; 2. $\frac{1}{2} < Re(2iz) < 2$; 3. $|z| - 2z = 3i + 6$;

4. $u(x,y) = e^x(xcos\,y - ysin\,y) + 2sinx\,shy + x^3 - 3xy^2 + y$;

5. $\int\limits_C \frac{sin\,(cos2z)}{(z-1)(z^2-1)} dz$, $C: \left|z - \frac{1}{2}\right| = 1$;

6. $f(z) = \frac{3z-4}{3z^2-10z+3}$, $z_0 = 3$ nuqta atrofida;

13-variant

1. $\left(1+cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)^6$; 2. $Re\frac{1}{z}=5$, $\frac{1}{z}=5$;

3. $|z|+z=1+i$;

4. $v(x,y)=3+x^2-y^2-\frac{y}{2(x^2+y^2)}$; 5. $\int_{|z|=4}\frac{e^{2z}}{z^2+9}dz$;

6. $f(z)=\frac{5z+3}{z^2-5z+4}$, $z_0=1$ nuqta atrofida ;

14-variant

1. $(1+cos\frac{\pi}{4}+isin\frac{\pi}{4})^4$; 2. $Rez^2=5$; 3. $cosz=sinz$;

4. $v(x,y)=ln(x^2+y^2)+x-2y$; 5. $\int_{|z+5i|=2}\frac{e^{z^2+5z+6}}{(z^2+16)(z+4i)}dz$;

6. $f(z)=\frac{4z+5}{z^2-z-2}$, $z_0=2$ nuqta atrofida;

15-variant

1.$(-3=4i)^{1+i}$; 2. $Im\frac{z-2}{z-3}=0, Re\frac{z-2}{z-3}=0$; 3. $sinz=i\,shz$;

4. $u(x,y)=2arc\,tg\frac{y}{x}+5$; 5. $\int_{|z-2|=2}\frac{cosz}{(z-3)(z^2-9)}dz$;

6. $f(z)=\frac{3z+5}{z^2+2z-3}$, $z_0=1$ nuqta atrofida;

16-variant

1. $\sqrt[5]{-4+3i}$; 2. $|z-2|-|z+2|>3$; 3. $sin2z-\cos 2z=i$;

4. $u(x,y)=\frac{x}{x^2+y^2}-2y$; 5. $\int_{|z-i|=2}\frac{e^{sin\,(2z^2+5)}}{(z^2+4)(z-2i)}dz$;

6. $f(z)=\frac{7z+6}{z^2+5z+4}$, $z_0=-1$ nuqta atrofida;

17-variant

1. $\left(\frac{1+i}{\sqrt{2}}\right)^{1+i}$; 2. $Re\frac{z-4}{z+i}=0$, $Im\frac{z+4}{z+i}=0$; 3. $|z|-z=2+i$;

4. $v(x,y)=ln(x^2+y^2)-x^2+y^2$; 5. $\int_{|z-1|=2}\frac{sin(e^{z^7+3})}{(z-2)(z^2-4)}dz$;

6. $f(z)=\frac{6z+7}{2z^2-5z+2}$, $z_0=2$ nuqta atrofida ;

18-variant

1. $\left(\frac{-1+i}{\sqrt{2}}\right)^{1+i}$; 2. $|z-2i| = |z-3+2i|$; 3. $|z| - z = 2+4i$;

4. $u(x,y) = (x^2 - y^2) + e^x$;   5. $\int_{|z-8|=3} \frac{e^{z^3+4z}}{(z-7)(z^2-49)} dz$;

6. $f(z) = \frac{4z+7}{2z^2-5z+2}$, $z_0 = \frac{1}{2}$ nuqta atrofida;

19-variant

1. $\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right)^{1+i}$;   2. $|z-4+3i| = |z+2-3i|$;

3. $z^3 = -1 + i\sqrt{3}$; 4. $v(x,y) = e^x(xcosy - ysiny) - \frac{y}{x^2+y^2}$;

5. $\int_{|z+2|=3} \frac{3^{z^2-4z+3}}{(z+3)(z^2-9)} dz$; 6. $f(z) = \frac{3z+5}{z^2+4z-5}$,

$z_0 = -5$ nuqta atrofida;

20-variant

1. $(1+i)^8(1-i\sqrt{3})^6$; 2. $\frac{\pi}{3} < Argz < \frac{\pi}{2}$;   3. $sh(-3+i)$;

4. $v(x,y) = 2e^x siny$;   5. $\int_{|z|=4} \frac{sin\, iz}{z^2-4z+3} dz$;

6. $\sum_{n=1}^{\infty} \frac{n\, sinin}{3^n}$,   yaqinlashishga tekshiring.

21-variant

1. $\sqrt[4]{-16}$;   2. . $\frac{\pi}{2} < Argz < \frac{2\pi}{3}$;   3. $sh(i)$;

4. $u(x,y) = x^3 - 3xy^2$;   5. $\int_{|z|=3} \frac{cos\,(z+\pi i)}{z(e^z+2)} dz$;

6. $\sum_{n=1}^{\infty} \frac{\cos(in)}{2^n}$,   yaqinlashishga tekshiring.

22-variant

1. $\sqrt[3]{-25}$;   2. $|z-2+3i| < 3$;   3. $ch(i)$

4. $V(x,y) = 3x^2y - y^3$;

5. $\int_{|z|=e} \frac{sin(z+\pi i)}{(z-1)(e^z+2)} dz$

6. $\sum_{n=1}^{\infty} \frac{cos\,(2in)}{3^n}$, yaqinlashishga tekshiring.

23-variant

1. $\frac{(-1+i)^4}{(-\sqrt{3}+i^{17})^5}$; 2. $1 < |z-1-i| < 3$; 3. $sin\left(\frac{\pi}{6}+i\right)$;

4. $u(x,y) = \frac{1}{2}(x^2 - y^2)$;

5. $\int_l \bar{z}dz$, bunda $l -$ $|z| = 1$ yopiq kontur ;

6. $\sum_{n=1}^{\infty} \frac{(z-2i)^n}{2^n(n+1)} + \sum_{n=1}^{\infty} \frac{n^2+5}{(z-2i)^n}$.

24-variant

1. $\left(\frac{i^{13}+1}{i^{23}+1}\right)^3$; 2. $1 < |z-2i| < 2$; 3. $sin(1-5i)$;

4. $u(x,y) = 2xy + 3$;

5. $\oint_{|z-1|=3} \frac{sin\frac{\pi z}{2}}{z^2 - 2z - 3} dz$;

6. $\sum_{n=1}^{\infty} \frac{e^{in}}{n^2}$, yaqinlashishga tekshiring.

CHIZIQLI TENGLAMALAR SISTEMASINI JORDON-GAUSS USULIDA YECHISH

Chiziqli tenglamalar sistemasini Gauss usulida yechishda tekshiruv ustuniga ega bo'lgan matritsa usuli ko'rildiki, natijada berilgan tenglamalar sistemasi uchburchak ko'rinishiga keltirildi. Keyingi bayon uchun Jordan –Gaussning takomillashgan usuli bilan tanishish muhim ahamiyatga ega, bunda noma'lumlarning qiymatlari to'g'ridan-to'g'ri topiladi.Bizga quyidagi chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases} \qquad (1)$$

Bu sistemaning A matritsasidan 0 dan farqli a_{qp} elementini tanlaymiz.Bu element hal qiluvchi element deb ataladi. A matritsaning p- ustuni hal qiluvchi ustun deb, q- qatori hal qiluvchi qator deb ataladi.

Yangi tenglamalar sistemasini qaraymiz:

$$\begin{cases} a'_{11}x_1 + a'_{12}x_2 + \dots + a'_{1n}x_n = b'_1 \\ a'_{21}x_1 + a'_{22}x_2 + \dots + a'_{2n}x_n = b'_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a'_{m1}x_1 + a'_{m2}x_2 + \dots + a'_{mn}x_n = b'_m \end{cases} \qquad (2)$$

Bu sistemaning matritsasini A' orqali belgilaymiz. Uning koeffitsiyentlari va ozod hadlari quyidagi formulalardan aniqlanadi:

$$a'_{ij} = a_{ij} - \frac{a_{ip}a_{qj}}{a_{qp}},$$

agar $i \neq q$,

$$b'_i = b_i - \frac{a_{ip}b_q}{a_{qp}}$$

Xususan, agar i=q bo‘lsa $a'_{ip} = 0$ bo‘ladi. Agarda $i \neq q$ bo‘lsa, u holda $a'_{qj} = a_{qj}, b'_q = b_q$ deb qabul qilamiz. Shunday qilib (1) va (2) sistemalardagi q- tenglamalar bir hil bo‘lib, (2) sistemaning q- tenglamasidan boshqa barcha tenglamalaridagi x_p oldidagi koeffitsiyentlari 0 ga teng. Shuni ko‘zda tutish lozimki, (1) va (2) sistemalar bir vaqtda yoki birgalikda, yoki birgalikda emas. Ular birgalikda bo‘lgan holda teng kuchli sistemalardir (ularning yechimlari ustma-ust tushadi).

A' matritsaning a'_{ij} elementini aniqlashda «to‘rtburchak usuli» ni ko‘zda tutish foydalidir.

A matitsaning 4 elementini qaraymiz: a_{ij} (almashtirishga tanlangan element), a_{qp} (hal qiluvchi element) va a_{ip}, a_{qj} elementlar. a'_{ij} elementni topish uchun to'rtburchakning qarama-qarshi uchlaridagi a_{ip} va a_{qj} elementlar ko'paytmasini a_{qp} elementga bo'lib a_{ij} elementdan ayiramiz:

$$\begin{matrix} a_{ij} & \ldots\ldots\ldots\ldots & a_{ip} \\ & \ldots\ldots\ldots\ldots & \\ a_{qj} & \ldots\ldots\ldots\ldots & a_{qp} \end{matrix}$$

Xuddi shu tariqa (2) sistemani ham almashtirish mumkin, bunda A' matitsaning hal qiluvchi elementi sifatida $a'_{sl} \neq 0$ elementini qabul qilamiz $(s \neq q, l \neq p)$. Bu almashtirishdan so'ng x_p lar oldidagi barcha koeffitsiyentlar 0 ga teng bo'ladi. Hosil bo'lgan sistema yana almashtirilishi mumkin va xakozo. Agar $r=n$ (ya'ni sistemaning rangi r noma'lumlar soniga teng) bo'lsa, u holda bir qator almashtirishlardan so'ng quyidagi tenglamalar sistemasiga kelamiz:

$$k_1x_1 = l_1,$$
$$k_2x_2 = l_2,$$
$$\ldots\ldots\ldots$$
$$k_nx_n = l_n,$$

va bu tengliklardan noma'lumlarning qiymatlarini topamiz. Noma'lumlarni ketma-ket yo'qotishga asoslangan bu yechish usuli Jordan-Gauss usuli deb ataladi.

1.Quyidagi chiziqli tenglamalar sistemasini yeching:

$$\begin{cases} x_1+x_2-3x_3+2x_4=6 \\ x_1-2x_2 \quad\quad -x_4=-6 \\ \quad\quad x_2+x_3+3x_4=16 \\ 2x_1-3x_2+2x_3 \quad\quad =6 \end{cases}$$

Yechish:Bu sistemaning koeffitsiyentlarini, ozod hadlarini va koeffitsiyentlar bilan ozod hadlari yig'indilarini quyidagi jadvalga yozamiz (Σ-tekshiruv ustuni):

x_1	x_2	x_3	x_4	b	Σ
(1)	1	-3	2	6	7
1	-2	0	-1	-6	-8

0	1	1	3	16	21
2	-3	2	0	6	7

Biz hal qiluvchi element sifatida birinchi tenglamadagi x_1 oldidagi koeffitsiyentni oldik. Jadvalning bu element turgan qator elementlarini o'zgarishsiz yozamiz (hal qiluvchi qator), bu element turgan ustun elementlarining hal qiluvcisidan tashqari barchasini nollar bilan almashtiramiz. To'rtburchak qoidasini qo'llab boshqa katakda turgan elementlarni ham o'zgartiramiz (bu qoidani Σ turgan ustunga ham qo'llaymiz).

x_1	x_2	x_3	x_4	b	Σ
1	1	-3	2	6	7
0	-3	3	-3	-12	-15
0	1	1	3	16	21
0	-5	8	-4	-6	-7

Shuni takidlaymizki, tekshirish ustunida shu qator elementlari yig'indisiga teng element turadi.2- qator elementlarini -3 ga bo'lib quyidagi jadvalni hosil qilamiz:

x_1	x_2	x_3	x_4	b	Σ
1	1	-3	2	6	7
0	(1)	-1	1	4	5
0	1	1	3	16	21
0	-5	8	-4	-6	-7

2-qatorning 2- elementini hal qiluvchi deb olamiz, 1-ustun elementlarini o'zgarishsiz yozamiz, 2-ustun elementlarini hal qiluvchisidan tashqari barchasini nollar bilan almashtiramiz,2-hal qiluvchi qator elementlarini o'zgarishsiz yozamiz, qolgan barcha katak elementlarni to'rtburchak qoidasiga ko'ra o'zgartiramiz.

x_1	x_2	x_3	x_4	b	Σ
1	0	-2	1	2	2
0	1	-1	1	4	5
0	0	2	2	12	16
0	0	3	1	14	18

3- qator elementlarini 2 ga bo'lamiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	-2	1	2	2

0	1	-1	1	4	5
0	0	(1)	1	6	8
0	0	3	1	14	18

3-ustunning 3- elementini hal qiluvchi element deb jadvalni o'zgartiramiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	0	3	14	18
0	1	0	2	10	13
0	0	1	1	6	8
0	0	0	-2	-6	-6

4-qator elementlarini -2 ga bo'lamiz va to'rtburchak usulini qo'llaymiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	0	0	8	9
0	1	0	0	6	7
0	0	1	0	4	5
0	0	0	1	2	3

Natijada quyidagi tenglamalar sistemasiga kelamiz:

$$\begin{cases} 1 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 + 0 \cdot x_4 = 8, \\ 0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 0 \cdot x_4 = 6, \\ 0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + 0 \cdot x_4 = 4, \\ 0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 + 1 \cdot x_4 = 2, \end{cases}$$

Ya'ni $x_1 = 8$, $x_2 = 6$, $x_3 = 4$, $x_4 = 2$.

2.Tenglamalar sistemasini yeching:

$$\begin{cases} x_1 + x_2 - 2x_3 + x_4 = 1, \\ x_1 - 3x_2 + x_3 + x_4 = 0, \\ 4x_1 - x_2 - x_3 - x_4 = 1, \\ 4x_1 + 3x_2 - 4x_3 - x_4 = 2, \end{cases}$$

Jadval tuzamiz:

x_1	x_2	x_3	x_4	b	Σ
(1)	1	-2	1	1	2
1	-3	1	1	0	0
4	-1	-1	-1	1	2
4	3	-4	-1	2	4

1-ustunning 1-elementi hal qiluvchi:

x_1	x_2	x_3	x_4	b	Σ
1	1	-2	1	1	2
0	-4	3	0	-1	-2
0	-5	7	-5	-3	-6
0	-1	4	5	2	4

4-qator elementlari ishoralarini o'zgartiramiz:

x_1	x_2	x_3	x_4	b	Σ
1	1	-2	1	1	2
0	-4	3	0	-1	-2
0	-5	7	-5	-3	-6
0	(1)	-4	5	2	4

2-ustunning 4-elementi hal qiluvchi:

x_1	x_2	x_3	x_4	b	Σ
1	0	2	-4	-1	-2
0	0	-13	20	7	14
0	0	-13	20	7	14
0	1	-4	5	2	4

3-qator elementlaridan 2-qator elementlarini ayiramiz va 3-qatorni o'chiramiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	2	-4	-1	-2
0	0	-13	(20)	7	14
0	1	-4	5	2	4

2-qatorning 4-elementi hal qiluvchi:

x_1	x_2	x_3	x_4	b	Σ
1	0	-0,6	0	0,4	0,8
0	0	-13	20	7	14
0	1	-0,75	0	0,25	0,5

Matritsaning rangi 3 ga teng bo'lgani uchun sistema uchta x_1, x_2, x_4 bazis noma'lumlarga ega va bitta ozod noma'lum x_3 ga ega. Quyidagi sistemani hosil qilamiz:

$$\begin{cases} 1 \cdot x_1 + 0 \cdot x_2 - 0{,}6 \cdot x_3 + 0 \cdot x_4 = 0{,}4, \\ 0 \cdot x_1 + 0 \cdot x_2 - 13 \cdot x_3 + 20 \cdot x_4 = 7, \\ 0 \cdot x_1 + 1 \cdot x_2 - 0{,}75 \cdot x_3 + 0 \cdot x_4 = 0{,}25. \end{cases}$$

Bundan:

$$x_1 = 0{,}4 + 0{,}6x_3, x_2 = 0{,}25 + 0{,}75x_3, x_4 = 0{,}35 + 0{,}65x_3.$$

Demak sistemaning yechimi quyidagicha

$$x_1 = 0{,}4 + 0{,}6u, x_2 = 0{,}25 + 0{,}75u, x_3 = u, x_4 = 0{,}35 + 0{,}65u.$$

Bu yerda u – ihtiyoriy son.

3.Tenglamalar sistemasini yeching.

$$\begin{cases} 6x_1 - 5x_2 + 7x_3 + 8x_4 = 3, \\ 3x_1 + 11x_2 + 2x_3 + 4x_4 = 6, \\ 3x_1 + 2x_2 + 3x_3 + 4x_4 = 1, \\ x_1 + x_2 + x_3 = 0. \end{cases}$$

Jadval tuzamiz:

x_1	x_2	x_3	x_4	b	Σ
6	-5	7	8	3	19
3	11	2	4	6	26
3	2	3	4	1	13
(1)	1	1	0	0	3

1-ustunning 4- elementi hal qiluvchi:

x_1	x_2	x_3	x_4	b	Σ
0	-11	(1)	8	3	1
0	8	-1	4	6	17
0	-1	0	4	1	4
1	1	1	0	0	3

3-ustunning 1-elementi hal qiluvchi:

x_1	x_2	x_3	x_4	b	Σ
0	-11	1	8	3	1
0	-3	0	12	9	18
0	-1	0	4	1	4
1	12	0	-8	-3	2

3-qator elementlarining ishoralarini qarama-qarshisiga o‘zgartiramiz:

x_1	x_2	x_3	x_4	*b*	*Σ*
0	-11	1	8	3	1
0	-3	0	12	9	18
0	(1)	0	-4	-1	-4
1	12	0	-8	-3	2

2-ustunning 1- elementi hal qiluvchi:

x_1	x_2	x_3	x_4	*b*	*Σ*
0	0	1	-36	-8	-43
0	0	0	0	6	6
0	(1)	0	-4	-1	-4
1	0	0	40	9	50

Natijada quyidagi sistemaga kelamiz:

$$\begin{cases} 0\cdot x_1 - 0\cdot x_2 + 1\cdot x_3 - 36\cdot x_4 = -8, \\ 0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 + 0\cdot x_4 = \ \ 6, \\ 0\cdot x_1 + 1\cdot x_2 + 0\cdot x_3 + 4\cdot x_4 = -1, \\ 1\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 + 40\cdot x_4 = \ \ 9. \end{cases}$$

Osongina ko'rish mumkinki, ikkinchi tenglamani x_1, x_2, x_3, va x_4 larning hech qanday qiymatlari qanoatlantira olmaydi. Shunday qilib, hosil bo'lgan tenglamalar sistemasi birgalikda emas.

Variantlar:

1. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 6 \\ 2x_1 + x_2 + x_3 + 3x_4 = 1 \\ x_1 - x_2 - 3x_3 - x_4 = -6 \\ 2x_1 + x_2 + 3x_3 + x_4 = 3 \end{cases}$

6. $\begin{cases} x_1 - 2x_2 + 2x_3 + x_4 = 9 \\ 2x_1 - 2x_2 - 3x_3 + 2x_4 = 0 \\ -2x_1 + x_2 + x_3 - x_4 = -2 \\ 2x_1 - x_2 + 2x_3 + 2x_4 = 8 \end{cases}$

2. $\begin{cases} x_1 + 2x_2 + 2x_3 - x_4 = 5 \\ -x_1 + 2x_2 + x_3 + 2x_4 = 5 \\ 2x_1 - x_2 + 3x_3 - x_4 = 3 \\ x_1 + x_2 + 3x_3 + x_4 = 6 \end{cases}$

7. $\begin{cases} x_1 + 2x_2 - 2x_3 + x_4 = 2 \\ 3x_1 + 2x_2 + x_3 + 4x_4 = 9 \\ x_1 - 2x_2 + 3x_3 - x_4 = 3 \\ -2x_1 + 2x_2 + x_3 + 2x_4 = -1 \end{cases}$

3. $\begin{cases} x_1 + 2x_2 + x_3 - 2x_4 = -4 \\ 3x_1 + 2x_2 + 2x_3 + 3x_4 = 1 \\ x_1 - x_2 - 3x_3 - x_4 = 6 \\ 2x_1 + 4x_2 + 3x_3 + 2x_4 = -9 \end{cases}$

8. $\begin{cases} x_1 + 2x_2 + 3x_3 - x_4 = 3 \\ x_1 - 2x_2 - 2x_3 + 3x_4 = 1 \\ 3x_1 + x_2 + 2x_3 - x_4 = 5 \\ -2x_1 - 2x_2 + x_3 + x_4 = 4 \end{cases}$

4. $\begin{cases} x_1 + 2x_2 - 2x_3 - x_4 = -7 \\ 3x_1 + x_2 + x_3 + 2x_4 = 3 \\ 2x_1 + x_2 - 2x_3 - x_4 = -4 \\ x_1 + 3x_2 + 2x_3 + 5x_4 = -1 \end{cases}$

5. $\begin{cases} x_1 + 2x_2 - 2x_3 + 2x_4 = -3 \\ 3x_1 - 2x_2 + 2x_3 - 2x_4 = -1 \\ 2x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 + 2x_3 + 3x_4 = 1 \end{cases}$

11. $\begin{cases} x_1 + x_2 + 2x_3 - x_4 = 1 \\ 2x_1 + x_2 + x_3 - 3x_4 = -2 \\ 2x_1 - x_2 - x_3 + x_4 = 2 \\ 3x_1 + 2x_2 + x_3 - 2x_4 = -5 \end{cases}$

12. $\begin{cases} x_1 - 2x_2 + x_3 + 2x_4 = -8 \\ 3x_1 - x_2 + 4x_3 - 3x_4 = 0 \\ 2x_1 + x_2 + 3x_3 - x_4 = -1 \\ x_1 - 2x_2 + 2x_3 + x_4 = -9 \end{cases}$

13 $\begin{cases} x_1 - 2x_2 + x_3 + 2x_4 = -8 \\ 3x_1 - x_2 + 4x_3 - 3x_4 = 10 \\ 2x_1 + x_2 + 2x_3 - x_4 = -1 \\ x_1 - 2x_2 + 2x_3 + x_4 = -9 \end{cases}$

14. $\begin{cases} x_1 + 2x_2 - x_3 + 2x_4 = -7 \\ x_1 - x_2 + 2x_3 + x_4 = 6 \\ 3x_1 - x_2 - 3x_3 + 4x_4 = 0 \\ 4x_1 + 2x_2 - 2x_3 + x_4 = -6 \end{cases}$

15. $\begin{cases} x_1 + 2x_2 - 2x_3 + x_4 = -9 \\ x_1 + 3x_2 - x_3 + 2x_4 = -9 \\ 3x_1 - x_2 - 3x_3 - x_4 = -7 \\ -x_1 + 2x_2 + 3x_3 + 4x_4 = 3 \end{cases}$

9. $\begin{cases} x_1 - 2x_2 + x_3 + 2x_4 = 1 \\ 3x_1 - 2x_2 + 2x_3 - x_4 = 5 \\ 2x_1 - x_2 + 3x_3 + 4x_4 = 7 \\ x_1 - 2x_2 + x_3 - x_4 = 1 \end{cases}$

10. $\begin{cases} x_1 + 3x_2 + x_3 + 2x_4 = 10 \\ 2x_1 + 2x_2 + x_3 + 3x_4 = 9 \\ x_1 - x_2 + 2x_3 + x_4 = 5 \\ 3x_1 - 2x_2 - x_3 + x_4 = -4 \end{cases}$

17. $\begin{cases} x_1 + 2x_2 - x_3 - x_4 = 4 \\ x_1 + 3x_2 - 3x_3 + 2x_4 = 3 \\ 3x_1 - x_2 + 3x_3 - x_4 = 11 \\ 2x_1 - 2x_2 + x_3 + 2x_4 = 5 \end{cases}$

18. $\begin{cases} x_1 + 2x_2 + x_3 + 4x_4 = 4 \\ -2x_1 + x_2 + 2x_3 - 2x_4 = -2 \\ 3x_1 + x_2 + 2x_3 - 2x_4 = -3 \\ 2x_1 + 2x_2 + x_3 + 3x_4 = 5 \end{cases}$

19 $\begin{cases} x_1 + x_2 + 3x_3 - 2x_4 = 6 \\ 3x_1 + 2x_2 + x_3 - 2x_4 = 3 \\ 2x_1 - x_2 - 3x_3 - 4x_4 = -3 \\ x_1 + 3x_2 - 2x_3 + 2x_4 = -6 \end{cases}$

20 $\begin{cases} x_1 - 2x_2 + 2x_3 + x_4 = 3 \\ -2x_1 + x_2 - x_3 + 2x_4 = -3 \\ x_1 - x_2 + 3x_3 - x_4 = 0 \\ 3x_1 + x_2 + 2x_3 + x_4 = -1 \end{cases}$

21. $\begin{cases} x_1 + 2x_2 + x_3 - 2x_4 = 2 \\ -2x_1 - 2x_2 + x_3 + 3x_4 = 1 \\ x_1 + x_2 - 3x_3 - x_4 = -3 \\ 3x_1 + x_2 + 3x_3 + x_4 = 1 \end{cases}$

16. $\begin{cases} x_1 + 2x_2 + x_3 + x_4 = 6 \\ -2x_1 + x_2 - 3x_3 + 2x_4 = 8 \\ x_1 - x_2 - 3x_3 - x_4 = -9 \\ x_1 + 3x_2 - 2x_3 + x_4 = -2 \end{cases}$ 22. $\begin{cases} x_1 + 2x_2 - x_3 + 2x_4 = 2 \\ 2x_1 + 3x_2 + x_3 + 3x_4 = 0 \\ x_1 - x_2 + 2x_3 - x_4 = -4 \\ -2x_1 + x_2 + 2x_3 + 2x_4 = 1 \end{cases}$

23. $\begin{cases} x_1 - 3x_2 + 2x_3 - 4x_4 = 7 \\ 3x_1 + x_2 - 3x_3 + 2x_4 = -19 \\ 4x_1 + 2x_2 - x_3 + 3x_4 = -12 \\ 2x_1 - 3x_2 + 4x_3 - 2x_4 = 11 \end{cases}$ 24. $\begin{cases} 2x_1 + 3x_2 - x_3 + 2x_4 = 8 \\ x_1 - 2x_2 + 3x_3 - 4x_4 = -9 \\ 3x_1 + x_2 - 2x_3 - x_4 = 3 \\ 2x_1 - 5x_2 + x_3 - 2x_4 = 8 \end{cases}$

BIR NOMA'LUMLI TENGLAMALARNI YECHISHNING URINMA VA VATARLAR USULI

Bizdan quyidagi tenglamani taqriban yecish talab qilingan bo'lsin.

$$F(x) = 0 \qquad (1)$$

1. Agar $F(x)$ 5-tartibli yoki undan yuqori tartibli ko'phad bo'lsa, u holda (1) tenglamaning ildizlarini $F(x)$ ko'phadning koeffitsiyentlari orqali ifodalab bo'lmaydi. Bu tenglamani taqriban yechish uchun biz urinma yoki vatarlar usulini qo'llaymiz. (1) tenglamaning ildiziga boshlang'ich yaqinlashishni topish uchun ildizni ajratib olish lozim, ya'ni shunday $[a, b]$ kesmani topish kerakki, bu kesma ichida yagona aniq yechim mavjud bo'lsin.

Agar:

$$F(x) = \sum_{i=0}^{n} a_i x^{n-i} \ ,$$

bo'lsa, u holda ildizning ajralishi uchun quyidagi shartlar bajarilishi zarur:

$$F(a) \cdot F(b) < 0 \qquad (2)$$

(funksiyaning kesma uchlarida ishoralari turlicha),

$$F'(x) > 0, \qquad x \in (a,b)$$

yoki (3)

$$F'(x) < 0, \qquad x \in (a,b)$$

(funksiya kesmada monoton).

Kesmaning (2), (3) shartlarni bajaradigan boshlang'ich uchlarini topish uchun quyidagi tengliklardan foydalanish mumkin:

$$a = b_0 = \frac{1}{1 + \frac{1}{|a_n|} \cdot \max\limits_{0 \le k \le n-1} \{|a_i|\}};$$

$$b = b_1 = 1 + \frac{1}{|a_0|} \cdot \max\limits_{1 \le i \le n} \{|a_i|\};$$

Agar $F(b_0) \cdot F(b_1) > 0$ bo‘lsa, u holda $[b_0, b_1]$ kesma $b_2 = \frac{b_1 - b_0}{2}$ nuqtada teng ikkiga bo‘linadi va $[b_0, b_2]$ yoki $[b_2, b_1]$ kesmalardan biri, qaysi birining uchlarida (2) shart bajarilishiga qarab olinadi. Bundan keyin (3) shartning bajarilishi ham teng ikkiga bo‘lish yo‘li bilan ta’minlanadi.

2. (1) tenglamada $y = F(x)$ transendent funksiya bo‘lsin va bu tenglamani taqriban yechish talab etilgan bo‘lsin. Buning uchun biz urinma yoki vatarlar usulini qo‘llaymiz. Tenglamaning ildiziga boshlang‘ich yaqinlashishni topish uchun ildizni ajratib olish lozim, ya’ni shunday $[a, b]$ kesmani topish kerakki, bu kesma ichida yagona aniq yechim mavjud bo‘lsin. Buning uchun yoki grafik usulda yoki funksiyaning hosilasidan foydalanib uning o‘sish va kamayish oraliqlari topiladi. Oraliqlarni teng ikkiga bo‘lish yordamida, uchlarida quyidagi

$$F(a) \cdot F(b) < 0,$$

$$F'(x) > 0. \qquad x \in (a,b)$$

yoki (4)

$$F'(x) < 0, \qquad x \in (a,b)$$

(kesmalarda monotonligi) shartlar bajariladigan kesmalar topiladi. Agar biz funksiyaning o‘sish va kamayish oraliqlarini topsak u holda biz bu oraliqlarda (4) shartning bajarilishini tekshirmasak ham bo‘ladi. Ildizlarni grafik usulda ajratish uchun (1) tenglamani $\varphi_1(x) = \varphi_2(x)$ ko‘rinishda yozib olamiz va jadval yo‘li bilan yoki boshqa satrbilan $y = \varphi_1(x)$ va $y = \varphi_2(x)$ funksiyalarning grafiklariini chizamiz. Grafiklarning kesishgan nuqtalarining absisalari

(1) ning yechimlari bo'ladi. Taqriban ular qaysi oraliqlarda yotishlarini aniqlaymiz.

Urinmalar usuli

(1) tenglamani qaraymiz. Faraz qilamizki, yuqorida ko'rilgan usullardan biri yordamida orasida yagona yechim mavjud bo'lgan $[a,b]$ kesma aniqlangan va unda $F(a)\cdot F(b)<0$ shart bajariladi. $y=F(x)$ funksiya ikki marta differensiyallanuvchi va $F''(x)$ $[a,b]$ oraliqda ishorasini o'zgartirmaydi. U holda quyidagi 20-rasmda tasvirlangan to'rtta hol bo'lishi mumkin. Urinmalar usulida boshlang'ich yaqinlashishni topishda quyidagi

$$F(c)\cdot F''(c)>0 \qquad (5)$$

shartni tekshirish zarur, bunda $c=a$ yoki $c=b$.

Faraz qilamiz, (5) shart $c=b$ nuqtada bajariladi (21-rasm), ya'ni $F(b)\cdot F''(b)>0$,

$x_0=b$ deb olamiz. $B(x_0,F(x_0))$ nuqtadan o'tuvchi urinma quyidagi tenglamaga ega:

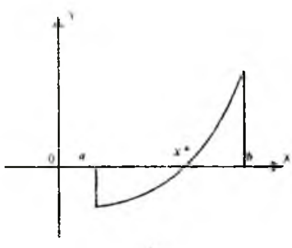
$$y-F(x_0)=F'(x_0)\cdot(x-x_0) \qquad (6)$$

Bu urinma OX o'qini $x_1=b_1$, $y=0$ nuqtada kesib o'tadi.

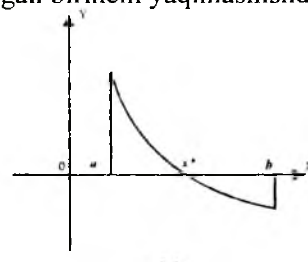
Bularni (6) ga qo'yib quyidagini hosil qilamiz:

$$x_1=x_0-\frac{F(x_0)}{F'(x_0)}.$$

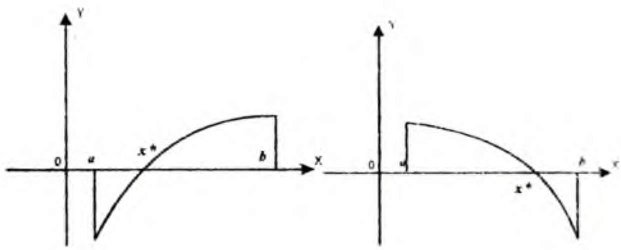
Bu esa urinmalar usuli bilan topilgan birinchi yaqinlashishdir.



a)



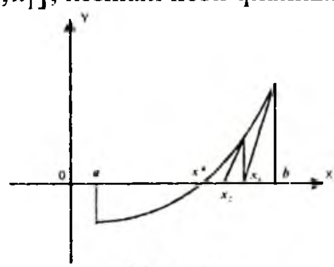
b)



c) d)

20-rasm

Yechim yotgan, $[a, x_1]$, kesmani hosil qilamiz.Huddi shuning kabi



21-rasm

x_2 ni topamiz va hokazo:

$$x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}, \ldots \quad x_{k+1} = x_k - \frac{F(x_k)}{F'(x_k)}$$

Vatarlar usuli

(1) tenglamani qaraymiz. Bu usulda boshlang‘ich yaqinlashishni tanlash uchun quyidagi shartni tekshirish zarur::

$$F(c) \cdot F''(c) < 0 \qquad (7)$$

Faraz qilamiz $c = a$ da (7) shart bajariladi, ya’ni

$$F(a) \cdot F''(a) < 0.$$

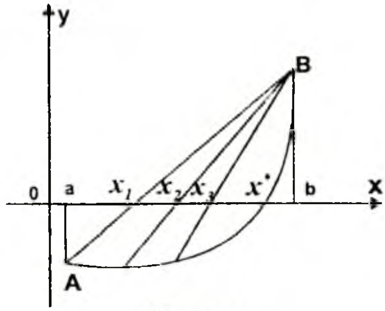
Bu holda, $x_0 = a$.

$\breve{AB}$ yoyning oxirlari vatar orqali tutashtiriladi (22-rasm) va bundan keyingi yaqinlashish sifatida bu vatarning OX o‘qi bilan kesishish nuqtasi $C[x_1, b]$ ning abssissasi qiymati olinadi. $A(a, F(a))$ va

$B(b, F(b))$ nuqtalardan o'tuvchi to'g'ri chiziqning tenglamasini yozamiz:

$$\frac{y - F(a)}{F(b) - F(a)} = \frac{x - a}{b - a};$$

$F(x) = 0$ tenglamaning ildiziga yaqinlashuvchi $C[x_1, b]$ nuqtaning abssissasi to'g'ri chiziqning tenglamasida $y = 0$ deb olib to-



22-rasm

pilishi mumkin. Bu holda quyidagini topamiz:

$$x_1 = x_0 - \frac{(b - x_0) \cdot F(x_0)}{F(b) - F(x_0)}.$$

Bundan keyin ildizni aniqlashtirish uchun vatarlar usuli bilan $[x_1; b]$ intervalni qaraymiz va x_2 ni topamiz:

$$x_2 = x_1 - \frac{(b - x_1) \cdot F(x_1)}{F(b) - F(x_1)}$$

Va hokazo:

$$x_{k+1} = x_k - \frac{(b - x_k) \cdot F(x_k)}{F(b) - F(x_k)};$$

Misol:

$$f(x) = x^5 + 2x^4 - 5x^3 + 6x^2 - 4x - 3 = 0 \qquad (8)$$

tenglamaning $\Delta = 0{,}01$ aniqlik bilan taqribiy yechimi umumlashgan usul yordamida topilsin.

Yecish: $f'(x) = 5x^4 + 8x^3 - 15x^2 + 12x - 4,$

$$f''(x) = 20x^3 + 24x^2 - 30x + 12.$$

Ildizlarni ajratamiz:

$$f(a)f(b) < 0$$

va

$$f'(x) > 0, \qquad x \in (a,b)$$

shartlar bajarilishi mumkin bo'lgan $[a;b]$ kesmaning (ya'ni $[a;b]$ kesmaning oxirlarida funksiya ishoralarini o'zgartirishi lozim va bu kesmada monoton bo'lisi lozim) boshlang'ich chegaralarini topish uchun quyidagi tengliklardan foydalanish mumkin:

$$a = \frac{1}{1 + \frac{1}{|a_n|} \cdot \max\limits_{0 \le i \le n-1} \{|a_i|\}},$$

$$b = 1 + \frac{1}{|a_0|} \cdot \max\limits_{1 \le i \le n} \{|a_i|\}.$$

Bundan va (8) dan topamiz:

$$a = \frac{1}{1 + \frac{1}{|-3|} \cdot 6} = \frac{1}{3}, \qquad b = 1 + \frac{1}{1} \cdot 6 = 7$$

$$f\left(\frac{1}{3}\right) < 0 \text{ va } f(7) >> 0.$$

Demak, $\left[\frac{1}{3};7\right]$ kesmada tenglamaning ildizi bor. $f(7)$ ning 0 dan farqi juda katta bo'lgani uchun $f(2)$ ning ishorasini tekshirib ko'ramiz: $f(2)$=32+32-40+24-8-3=37>0. Tenglamaning ildizini o'z ichiga olgan oraliqni qisqartirish maqsadida $f(1)$ ning ishorasini ham tekshirib ko'ramiz: f(1)=1+2-5+6-4-3=-3<0. Shunday qilib, tenglama [1;2] kesmada ildizga ega. Bu kesmada tenglama faqat bitta ildizga ega ekanini ko'rsatish uchun $f'''(x)$ ni va uning ildizlarini hisoblaymiz:

$$f'''(x) = 60x^2 + 48x - 30, \qquad x_{1,2} = \frac{-4 \mp \sqrt{66}}{10}.$$

Agar katta koeffitsiyenti musbat bo'lgan kvadrat uchhad o'z haqiqiy ildizlari orasidagina manfiy bo'lishi mumkinligini e'tiborga olsak, $x_1 < x_2 < 1$ tengsizlikdan [1;2] kesmada $f'''(x)$ ning musbat ekani kelib chiqadi. Demak, $f''(x)$ [1;2] kesmada o'suvchi va uning eng kichik qiymati $\min\limits_{x \in [1;2]} f''(x) = f''(1) = 26 > 0.$

Bundan esa $f'(x)$ ning ham [1;2] kesmada o'suvchi va musbat ekani kelib chiqadi.

Hisoblashni urinmalar usulidan boshlaymiz. $f(2) \cdot f''(2) > 0$ bo'lgani uchun boshlng'ich yaqinlashish sifatida $b_0 = 2$ ni olamiz. $f(2) = 37, f'(2) = 104.$

$$b_1 = b_2 - \frac{f(b_0)}{f'(b_0)} = 2 - \frac{37}{104} \approx 1{,}644.$$

Endi $[a_0; b_1]$ kesmaga vatarlar usulini qo'llaymiz, bunda $a_0 = 1.$

$$f(a_0) = f(1) = -3, \qquad f(b_1) = f(1{,}64) \approx 10{,}85445.$$

$$a_1 = a_0 - \frac{(b_1 - a_0) \cdot f(a_0)}{f(b_1) - f(a_0)} = 1 + \frac{(1{,}64 - 1) \cdot 3}{10{,}85445 + 3} \approx 1{,}1394497.$$

Tenglama ildizining topilgan taqribiy qiymatlarining uning aniq qiymatidan chetlanishini baholaymiz:

$$|b_1 - a_1| = 0{,}5047811 > 2\Delta.$$

Bundan ko'rinadiki, hali talab qilingan aniqlikka erishilgani yo'q. Shuning uchun hisoblashlarni davom ettiramiz. $f'(b_1) \cong 46{,}793292.$

$$b_2 = b_1 - \frac{f(b_1)}{f'(b_1)} \approx 1{,}4080341.$$

$$f(a_1) = -1{,}8772492, \qquad f(b_2) = 2{,}700234.$$

$$a_2 = a_1 - \frac{(b_2 - a_1) \cdot f(a_1)}{f(b_2) - f(a_1)} \approx 1{,}2496039.$$

$$|b_2 - a_2| = 0{,}1584302 > 2\Delta.$$

Bu bosqichda ham talab qilingan aniqlikka erishmadik. Shu sababdan keying bosqichga o'tamiz: $f'(2) = 25{,}140331.$

$$b_3 = b_2 - \frac{f(b_2)}{f'(b_2)} \approx 1{,}3005936.$$

$$f(a_2) = -0{,}4560549, \quad f(b_3) = 0{,}38013.$$

$$a_3 = a_2 - \frac{(b_3 - a_2) \cdot f(a_2)}{f(b_3) - f(a_2)} = 1{,}2829017,$$

$$|b_3 - a_3| = 0{,}0177 < 2\Delta.$$

Demak,

$$x^* \approx \frac{b_3 + a_3}{2} = 1{,}2917476$$

tenglama ildizining talab qilingan aniqlikdagi taqribiy qiymati bo'ladi.

Variantlar.

Berilgan algebraik tenglamaning 0,001 aniqlik bilan urinma va vatarlar umumlashgan usuli bilan taqribiy yechimi topilsin.

1. $2x^3 - 3x^2 - 12x - 5 = 0$
2. $x^3 - 3x^2 24x - 3 = 0$
3. $x^3 - 3x^2 + 3 = 0$
4. $x^3 - 12x + 6 = 0$
5. $x^3 + 3x^2 - 24x - 10 = 0$
6. $2x^3 - 3x^2 - 12x + 10 = 0$
7. $2x^3 + 9x^2 - 21 = 0$
8. $x^3 - 3x^2 + 2{,}5 = 0$
9. $x^3 + 3x^2 - 2 = 0$
10. $x^3 + 3x^2 - 3{,}5 = 0$
11. $x^3 + 3x^2 - 24x + 10 = 0$
12. $x^3 - 3x^2 - 24x - 8 = 0$
13. $2x^3 + 9x^2 - 10 = 0$
14. $x^3 - 12x + 10 = 0$
15. $x^3 + 3x^2 - 3 = 0$
16. $2x^3 - 3x^2 - 12x + 1 = 0$
17. $x^3 - 3x^2 - 24x - 5 = 0$
18. $x^3 - 4x^2 + 2 = 0$
19. $x^3 - 12x - 5 = 0$
20. $x^3 + 3x^2 - 24x + 1 = 0$

21. $2x^3 - 3x^2 - 12x + 12 = 0$
22. $2x^3 + 9x^2 - 6 = 0$
23. $x^3 - 3x^2 + 1{,}5 = 0$
24. $x^3 - 3x^2 - 24x + 10 = 0$
25. $x^3 + 3x^2 - 24x - 3 = 0$
26. $x^3 - 12x - 10 = 0$
27. $2x^3 + 9x^2 - 4 = 0$
28. $2x^3 - 3x^2 - 12x + 8 = 0$
29. $x^3 + 3x^2 - 1 = 0$
30. $x^3 - 3x^2 + 3{,}5 = 0$

ANIQ INTEGRALNI TAQRIBIY HISOBLASH UCHUN TO'G'RI TO'RTBURCHAK, TRAPETSIYA VA SIMPSON FORMULALARI

Ko'pgina amaliy va nazariy masalalarni yechish jarayonida biron *f(x)* funktsiyaning $[a,b]$ kesmadagi aniq integralini hisoblashga to'g'ri keladi. Agar *f(x)* funksiyaning boshlang'ich funksiyasi ma'lum bo'lsa, bu integralni hisoblashga Nyuton-Leybnis formulasini tatbiq qilish mumkin. Ammo ba'zi hollarda (hatto *f(x)* uzluksiz bo'lganda ham) boshlang'ich funksiyani elementar funksiyalarning chekli kombinatsiyasi shaklida ifodalab bo'lmaydi. Bundan tashqari amaliyotda *f(x)* jadval ko'rinishida berilgan bo'lishi ham mumkin, bunday holda boshlang'ich funksiya tushunchasining o'zi ma'noga ega bo'lmay qoladi. Shuning uchun ham aniq integrallarni taqribiy hisoblash usullari katta amaliy ahamiyatga ega.

To'g'ri to'rtburchak formulasi

Eng sodda taqribiy hisoblash formulasi bevosita aniq integralning ta'rifidan kelib chiqadi. $[a,b]$ kesmani

$$x_k = a + k\frac{b-a}{n} \qquad (k = 0,1,\dots,n)$$

nuqtalar yordamida, har birining uzunligi $h = \frac{b-a}{n}$ ga teng bo'lgan, *n* ta bo'lakka bo'lamiz va qaralayotgan aniq integralning taqribiy qiymati sifatida $\sigma_n = h\sum_{k=0}^{n} f(\xi_k)$ yig'indini olamiz (bunda $\xi_k = \frac{x_k + x_{k+1}}{2} - [x_k, x_{k+1}]$ kesmaning o'rtasida joylashgan nuqta), ya'ni

$$\int_a^b f(x)dx \approx h\sum_{k=0}^{n-1} f(\xi_k) \qquad (1)$$

formulani hosil qilamiz. Bu formula to'g'ri to'rtburchak formulasi deyiladi.

$f(x)$ funksiya $[a,b]$ kesmada $|f''(x)| \le M_2$ shartni qanoatlantiruvchi $f''(x)$ hosilaga ega bo'lsa, (1) formulaning $R_n(t) = \int_a^b f(x)dx - \sigma_n$ qoldiq hadi uchun

$$|R_n(f)| \le \frac{M_2(b-a)^3}{24n^2}$$

tengsizlik o'rinli.

Qoida: $\int_a^b f(x)dx$ integralning ε aniqlikdagi taqribiy qiymatini to'g'ri to'rtburchaklar formulasi yordamida hisoblash uchun

$$n = \left[\sqrt{\frac{M_2(b-a)^3}{24\varepsilon}}\right] + 1$$

tenglik orqali n natural son hisoblanadi. Topilgan n natural son yordamida

$$h = \frac{b-a}{n}, \quad \xi_0 = a + \frac{h}{2}, \quad \xi_k = \xi_{k-1} + h, \quad (k = 1,2,...,n-1)$$

formulalar orqali h qadam va ξ_k tugun nuqtalar hisoblanadi. Shundan so'ng $\int_a^b f(x)dx \approx h\sum_{k=0}^{n-1} f(\xi_k)$ formula yordamida qaralayotgan integralning ε aniqlikdagi taqribiy qiymati hisoblanadi.

Trapetsiya formulasi.

$[a,b]$ kesmani bo'lishni avvalgidek qoldiramiz, Lekin $y=f(x)$ chiziqning $[x_k, x_{k+1}]$ qismiy kesmaga mos keluvchi har bir yoyini bu yoyning chetki nuqtalarini tutashtiruvchi vatar bilan almashtiramiz. Bu - berilgan egri chiziqli trapetsiya yuzasi n ta chiziqli trapetsiyalar yuzalari yig'indisi bilan almashtirilganini bildiradi

Bunday figuraning yuzasi egri chiziqli trapetsiyaning yuzasini to'g'ri to'rtburchaklardan tuzilgan pog'onali figuraning yuzasiga qaraganda ancha aniq ifodalashi geometrik jihatdan ravshandir. Har bir xususiy trapetsiya'ning yuzasi

$$\frac{b-a}{n}\cdot\frac{f(x_k)+f(x_{k+1})}{2}$$

ga teng bo'lgani uchun qaralayotgan figuraning yuzasi

$$\frac{b-a}{n}\left[\frac{f(a)+f(x_1)}{2}+\frac{f(x_1)+f(x_2)}{2}+\ldots+\frac{f(x_{n-1})+f(x_n)}{2}\right]=$$

$$=\frac{b-a}{n}\left[\frac{f(a)+f(b)}{2}+\sum_{k=1}^{n-1} f(x_k)\right]$$

ga teng bo'ladi. Shunday qilib, ushbu

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left[\frac{f(a)+f(b)}{2}+\sum_{k=1}^{n-1} f(x_k)\right] \qquad (2)$$

formulani hosil qildik. Bu formula trapetsiyalar formulasi deyiladi. Agar $f(x)$ funksiya $[a,b]$ kesmada $|f''(x)| \le M_2$ shartni qanoatlantiruvchi uzluksiz ikkinchi tartibli hosilaga ega bo'lsa, (2) trapetsiyalar formulasining qoldiq hadi uchun $|R_n(f)| \le \frac{M_2(b-a)^3}{12n^2}$ tengsizlik o'rinli.

Qoida: $[a,b]$ kesmada $|f''(x)| \le M_2$ shartni qanoatlantiruvchi $f''(x)$ hosilaga ega bo'lsa, trapetsiyalar formulasi orqali $\int_a^b f(x)dx$ integralning ε aniqlikdagi taqribiy qiymatini hisoblash uchun

$$n = \left[\sqrt{\frac{M_2(b-a)^3}{12\varepsilon}} \right] + 1$$

formula yordamida n natural son topiladi. Topilgan n natural son orqali

$$h = \frac{b-a}{n}, \qquad x_k = a + kh, \qquad (k = 1,2,...,n-1)$$

formula yordamida h qadam va x_k tugun nuqtalar hisoblanadi. Shundan so'ng

$$\int_a^b f(x)dx \approx h \cdot \left[\frac{f(a)+f(b)}{2} + \sum_{k=1}^{n-1} f(x_k) \right]$$

Formula yordamida qaralayotgan integralning ε aniqlikdagi taqribiy qiymati hisoblanadi.

Simpson formulasi.

Biz yuqorida, to'g'ri to'rtburchaklar formulasini hosil qilishda $f(x)$ funksiya'ni qismiy intervallarda $f(\xi_k)$ o'zgarmaslar (0-tartibli ko'phad), trapetsiyalar formulasini hosil qilishda esa chiziqli funksiyalar (1-tartibli ko'phad) bilan almashtirdik. $f(x)$ funksiya almashtirilayotgan ko'phadning tartibi orttirilganda yanada aniqroq formula hosil bo'lishini kutish tabiiydir.

$$h = \frac{b-a}{2m} \qquad (3)$$

bo'lsin. $[a,b]$ kesmani $x_k = a + kh$ $\quad k = 0,1,...,2m$ nuqtalar yordamida $n=2m$ ta juft miqdordagi teng qismlarga bo'lamiz va $f(x)$ funksiyaning x_k nuqtalardagi qiymatlarini

$$y_k = f(x_k), k = 0,1,...,2m$$

orqali belgilaymiz. $f(x)$ funktsiyaning $[x_0;x_2]$ kesmadagi grafigini

$$M_0(x_0;y_0);M_1(x_1;y_1);M_2(x_2;y_2)$$

nuqtalardan o'tuvchi parabola yoyi bilan almashtiramiz. Bu parabolaning tenglamasini

$$y = A(x-x_1)^2 + B(x-x_1)+C \qquad (4)$$

ko'rinishda izlaymiz. (4) parabola M_0, M_1, M_2 nuqtalardan o'tish shartidan

$$y_0 = Ah^2 - Bh + C, \quad y_1 = C, \quad y_2 = Ah^2 + Bh + C.$$

tenglamalar hosil bo'ladi (bu yerda biz $x_0 - x_1 = -h, x_2 - x_1 = h$ tengliklardan foydalandik). Bu sistemadan A va C noma'lumlarni topamiz:

$$A = \frac{1}{2h^2}(y_0 - 2y_1 + y_2), \qquad C = y_1 \qquad (5)$$

Endi (4) funksiyadan $[x_0, x_2]$ kesmada olingan integralni hisoblaymiz:

$$S_1 = \int_{x_0}^{x_2}\left[A(x-x_1)^2 + B(x-x_1)+C\right]dx = \begin{vmatrix} t = x - x_1 & dt = dx \\ t_0 = -h & t_1 = h \end{vmatrix} =$$

$$= \int_{-h}^{h}\left(At^2 + Bt + C\right)dt = \frac{2A}{3}h^3 + 2Ch$$

A va C o'rniga ularning (5) ifodalarini qo'yib,

$$S_1 = \frac{h}{3}(y_0 + 4y_1 + y_2)$$

tenglikni hosil qilamiz va oxirgi ifodani $\int_{x_0}^{x_2} f(x)dx$ ning taqribiy qiymati sifatida olamiz:

$$\int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(y_0 + 4y_1 + y_2).$$

Xuddi shu kabi $f(x)$ funksiyaning $[x_2, x_4]$, $[x_4, x_6]$ va boshqa kesmalardagi grafigini tegishli parabolalarning mos yoylari bilan almashtirib,

$$\int_{x_2}^{x_4} f(x)dx \approx \frac{h}{3}(y_2 + 4y_3 + y_4),$$

$$\int_{x_4}^{x_6} f(x)dx \approx \frac{h}{3}(y_4 + 4y_5 + y_6),$$

....

$$\int_{x_{2m-2}}^{x_{2m}} f(x)dx \approx \frac{h}{3}(y_{2m-2} + 4y_{2m-1} + y_{2m})$$

formulalarga ega bo‘lamiz. Bu taqribiy tengliklarni qo‘shib quyidagini hosil qilamiz:

$$\int_a^b f(x)dx \approx$$

$$\approx \frac{h}{3}\left[y_0 + y_{2m} + 4(y_1 + y_3 + \ldots + y_{2m-1}) + 2(y_2 + y_4 + \ldots + y_{2m-2})\right]$$

Bundan va (3) dan,

$$\int_a^b f(x)dx \approx \frac{b-a}{6m}\left(y_0 + y_{2m} + 4\sum_{k=1}^{m} y_{2k-1} + 2\sum_{k=1}^{m-1} y_{2k}\right)$$

yoki

$$\int_a^b f(x)dx \approx \frac{b-a}{6m}\left[f(a) + f(b) + 4\sum_{k=1}^{m} f(x_{2k-1}) + 2\sum_{k=1}^{m-1}(x_{2k})\right]$$

Bu formula Simpson formulasi (yoki parabolalar formulasi) deyiladi.

$$\sum(h) = \frac{h}{3}\left(y_0 + y_{2m} + 4\sum_{k=1}^{m} y_{2k-1} + 2\sum_{k=1}^{m-1} y_{2k}\right).$$

Simpson yig‘indisining qoldiq hadi uchun

$$R(h) = \frac{\Sigma(h) - \Sigma(2h)}{15}$$

munosabat o'rinli. Aslida bu tenglik $f^{IV}(x)$ $[a;b]$ da o'zgarmas bo'lgandagina o'rinli. Biz $f^{IV}(x)$ ning $[a;b]$ kesmadagi o'zgarishi kichik deb faraz qilib,bu tenglik umumiy holda ham bajariladi deb hisoblaymiz.

Qoida: Agar $f(x)$ funksiya $[a,b]$ kesmada $|f^{IV}(x)| \le M_4$ shartni qanoatlantiruvchi $f^{IV}(x)$ hosilaga ega bo'lsa, integralning ε aniqlikdagi qiymatini hisoblash uchun biror m natural son olib $h_0 = \frac{b-a}{2m}$ va $h_1 = \frac{h_0}{2}$ qadamlarga mos $\sum(h_0)$ va $\sum(h_1)$ Simpson yig'indilarini hisoblaymiz va

$$R(h_1) = \frac{\sum(h_1) - \sum(h_0)}{15}$$

ifodani tekshiramiz. Agar bu ifoda absolyut qiymati bo'yicha ε dan kichik bo'lsa, I integralning ε aniqlikdagi taqribiy qiymati sifatida $\sum(h_1) + R(h_1)$ ifodani olamiz. Aks holda $h_2 = \frac{h_1}{2}$ qadamga mos keladigan $\sum(h_2)$ Simpson yig'indisini tuzamiz va

$$R(h_2) = \frac{\sum(h_2) - \sum(h_1)}{15}$$

ifodani tekshiramiz. Agar $|R(h_2)| < \varepsilon$ shart bajarilsa, $I \approx \sum(h_2) + R(h_2)$, aks holda $h_3 = \frac{h_2}{2}$ qadam bo'yicha $\sum(h_3)$ Simpson yig'indisini tuzamiz va hokazo... $\lim_{k\to\infty} R(h_k) = 0$ bo'lgani uchun bu jarayon cheksiz davom etmaydi, biror k qadamda $|R(h_k)| < \varepsilon$ shart bajariladi va $\sum(h_k) + R(h_k)$ I integralning ε aniqlikdagi taqribiy qiymati bo'ladi.

Misollar.

1) $\int_0^4 \frac{xdx}{\sqrt{x^2+9}}$ integralning Simpson usuli bilan $\varepsilon = 0,001$ aniqlikdagi taqribiy qiymati topilsin.

Yechish:

Simpson usuli bilan uning $\varepsilon = 0,001$ aniqlikdagi taqribiy qiymat-ni hisoblaymiz. $m = 1$ va $m = 2$ uchun $h_0 = \dfrac{4-0}{2\cdot 1} = 2$ va

$h_1 = \dfrac{4-0}{2\cdot 2} = 1$ qadamlarni va bu qadamlarga mos $\sum(h_0)$ ва $\sum(h_1)$ Simpson yig'indilarini jadval yordamida topamiz:

$$x_0 = 0; \quad x_1 = 0+2 = 2; \quad x_2 = 2+2 = 4.$$

$$y_0 = y(x_0) = \frac{0}{\sqrt{0^2+9}} = 0; \quad y_1 = y(x_1) = \frac{2}{\sqrt{2^2+9}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} =$$

$= 0{,}55470019622;$

$$y_2 = y(x_2) = \frac{4}{\sqrt{4^2+9}} = 0{,}8. \qquad h = 2$$

$h_0 = 2$

k	X_k	$Y_{0,2m}$	Y_{2k+1}	Y_{2k}	
0	0	0			
1	2		0,5547		
2	4	0,8			
		0,8	2,2188		3,0188
			$\sum(2) = 2{,}0125$		

$h_1 = 1$

k	X_k	$Y_{0,2m}$	Y_{2k+1}	Y_{2k}	
0	0	0			
1	1		0,3162		
2	2			0,5547	
3	3		0,7071		
4	4	0,8			
		0,8	1,0233	0,5547	
		0,8	4,0932	1,1094	6,0026
			$\sum(1) = 2{,}0008$		

Endi $\sum(1)$ uchun qoldiq hadni hisoblaymiz:

$$R(1)=\frac{2,0008-2,0125}{15}=-0,00078.$$

Simpson formulasi bo'yicha hisoblangan integralning taqribiy qiymati $J_1=2,0008-0,00078=2,00002$ ga teng.
Bunda

$$R_1=\int_0^4\frac{xdx}{\sqrt{x^2+9}}-J_1-0,00002$$

xatolik uchun $|R_1|<|R(1)|$ tensizlik o'rinli va $|R(1)|<\varepsilon$.

Shunday qilib, $\int_0^4\frac{xdx}{\sqrt{x^2+9}}$ integralning Simpson formulasi bo'yicha ε=0,001 aniqlikda hisoblangan taqribiy qiymati 2,00002 ga teng.
2)To'g'ri to'rtburchak, trapetsiya va Simpson formulalari orqali $\int_0^{0,5}\frac{dx}{1+x^4}$ integralning $\varepsilon=0,001$ aniqlikdagi taqribiy qiymati hisoblansin.
Yechish.
a)To'g'ri to'rtburchak formulasi bilan.
Avvalo berilgan integral $\varepsilon=0,001$ aniqlikda hisoblanishi uchun $[0;0,5]$ integrallash oralig'i qancha bo'lakka bo'linishi lozimligini hisoblaymiz. Buning uchun

$$n=\left[\sqrt{\frac{M_2(b-a)^3}{24\varepsilon}}\right]+1$$

formuladan foydalanamiz.

$$f(x)=\frac{1}{1+x^4};\quad f''(x)=-\frac{4x^2(3-5x^4)}{(1+x^4)^3}.$$

bundan, $0\le x\le 0,5$ bo'lgani uchun

$$|f''(x)| = 4|x^2| \cdot |3 - 5x^4| \cdot \frac{1}{|1 + x^4|^3} < 4 \cdot \frac{1}{4} \cdot 3 \cdot 1 = 3,$$

ya'ni $M_2 = 3$ va $n = \left[\sqrt{\frac{3 \cdot (0,5 - 0)^3}{24 \cdot 0.001}} \right] + 1 = 4$

h qadam va ξ_k tugun nuqtalarni hisoblaymiz. So'ngra jadval tuzib qaralayotgan integralning talab qilingan taqribiy qiymatini beradigan $h\sum_{k=0}^{n-1} f(\xi_k)$ yig'indini hisoblaymiz:

$$h = \frac{0,5 - 0}{4} = 0,125.$$

k	ξ_k	$1 + \xi_k^4$	$f(\xi_k)$
0	0,0625	1,0000152	0,9999848
1	0,1875	1,0012359	0,9987656
2	0,3125	1,0095367	0,9905533
3	0,4375	1,0366363	0,9646584
			$\sum f(\xi_k) = 3,9539621$
			$h\sum_{k=0}^{n-1} f(\xi_k) = 0,4942452$

Shunday qilib, $\int_0^{0,5} \frac{dx}{1 + x^4} \approx 0,4942$.

b) Trapetsiya formulasi bilan.

$$n = \left[\sqrt{\frac{M_2 (b - a)^3}{12\varepsilon}} \right] + 1$$

formula yordamida integrallash oralig'l qancha bo'lakka bo'linisi lozimligini topamiz:

$$n = \left[\sqrt{\frac{3(0,5-0)^3}{12\cdot 0,001}}\right] + 1 = 6$$

Endih qadam va x_k tugun nuqtalarni topamiz, so'ngra jadval yordamida integralning taqribiy qiymatini hisoblaymiz:

$$h = \frac{0,5-0}{6} = \frac{1}{12} \approx 0,0833333$$

k	x_k	$1+x_k$	$f(x_k)$	$^1/_2 f(x_k)$
0	0	1		0,5
1	$^1/_{12} = 0,0833333$	1,0000482	0,9999518	
2	$^2/_{12} = 0,1666666$	1,0007716	0,9992289	
3	$^3/_{12} = 0,2500000$	1,0039062	0,9961089	
4	$^4/_{12} = 0,3333333$	1,0123456	0,9878049	
5	$^5/_{12} = 0,4166666$	1,0301407	0,9707411	
6	$^6/_{12} = 0,5$	1,0625		0,4705882
			$\sum f(x_k) = 4,9538350$	$\sum f(x_k) = 0,9705882$
$h\left[\frac{f(a)+f(b)}{2} + \sum_{k=1}^{n-1} f(x_k)\right] = 0,4937017$				

Shunday qilib,

$$\int_0^{0,5} \frac{dx}{1+x^4} \approx 0,4937.$$

c) Simpson formulasi bilan. *m=2* da Simpson yig'indisi $\sum(h)$ ni hisoblaymiz:

$$h = \frac{0{,}5-0}{4} = 0{,}125.$$

k	x_k	$1+x_k$	$4\cdot y_{2k+1}$	$2\cdot y_{2k}$	y_k
0	0	1			1
1	0,125	1,000244	3,999024		
2	0,25	1,003962		1,9921072	
3	0,375	1,0197753	3,9224324		0,9411764
4	0,5	1,0625			
			$4\sum y_{2k+1} = 7{,}9214564$	$2\sum y_{2k} = 1{,}9921072$	$\sum y_k = 1{,}9411764$
					$\sum(h)$ =0,4939475

Hatolikni baholash uchun $\sum(2h)$ ni hisoblaymiz:

$$2h = 2\cdot 0{,}125 = 0{,}25.$$

k	x_k	$1+x_k$	$4\cdot y_{2k+1}$	$2\cdot y_{2k}$	y_k
0	0	1			1
1	0,25	1,003962	3,9842145		
2	0,5	1,0625			0,9411764
			$4\sum y_{2k+1} = 3{,}9842145$		$\sum y_k = 1{,}9411764$
					$\sum(2h)$ =0,4937825

Topilgan yig'indilardan foudalanib, $R(h)$ qoldiq hadni hisoblaymiz:

$$R(h) = \frac{\sum(h) - \sum(2h)}{15} = \frac{0{,}4939475 - 0{,}4937825}{15} = 0{,}000011.$$

Bundan ko'rinadiki,

$$\int_0^{0{,}5} \frac{dx}{1+x^4} \approx 0{,}4939475 + 0{,}000011 = 0{,}4939585,$$

bunda xatolik 0,000011 ga yaqin.

Quyidagi integrallarning taqribiy qiymatlarini to‘g‘ri to‘rtburchak, trapetsiya va Simpson usullarida ε=0,001 aniqlikda hisoblang.

1) $\int_0^2 \sqrt{20 - x^3}\,dx$;

2) $\int_0^6 \sqrt{5 + x^3}\,dx$;

3) $\int_1^7 \sqrt{1 + 2x^3}\,dx$;

4) $\int_0^1 \sqrt{4 - 3x^3}\,dx$;

5) $\int_0^6 \sqrt{4 + x^3}\,dx$;

6) $\int_2^4 \sqrt{x^3 - 7}\,dx$;

7) $\int_{-1}^5 \sqrt{x^3 + 16}\,dx$;

8) $\int_2^4 \sqrt{x^3 + 9}\,dx$;

9) $\int_{-3}^7 \sqrt{x^3 + 32}\,dx$;

10) $\int_6^{10} \sqrt{3x^3 + 2}\,dx$;

11) $\int_{-1}^9 \sqrt{x^3 + 2}dx$;

12) $\int_1^7 \sqrt{2x^3 + 3}\,dx$;

13) $\int_{-3}^7 \sqrt{x^3 + 36}\,dx$;

14) $\int_{-2}^8 \sqrt{x^3 + 8}\,dx$;

15) $\int_{-2}^8 \sqrt{x^3 + 11}\,dx$;

16) $\int_{-3}^7 \sqrt{40 + x^3}\,dx$;

17) $\int_2^{10} \sqrt{x^3 - 2}\,dx$;

18) $\int_2^6 \sqrt{x^3 - 3}\,dx$;

19) $\int_{-2}^6 \sqrt{x^3 + 11}\,dx$;

20) $\int_{-2}^4 \sqrt{x^3 + 9}\,dx$;

21) $\int_0^4 \sqrt{4x^3 + x}\,dx$;

22) $\int_0^1 \sqrt{4x^3 + 5}\,dx$;

23) $\int_0^1 \sqrt{3 - 2x^3}\,dx$;

24) $\int_{-3}^5 \sqrt{27 + x^3}\,dx$;

25) $\int_{-2}^6 \sqrt{16 + 2x^3}\,dx$;

26) $\int_0^1 \frac{dx}{\sqrt{2 - x^3}}$;

27) $\int_0^1 \frac{dx}{\sqrt{5 + x^3}}$;

28) $\int_0^1 \frac{dx}{\sqrt{1 + 2x^3}}$;

29) $\int_0^1 \frac{dx}{\sqrt{4 + x^3}}$;

30) $\int_2^{10} \frac{dx}{\sqrt{x^3 + 4}}$;

31) $\int_{-3}^{13} \frac{dx}{\sqrt{x^3 + 36}}$;

32) $\int_1^3 \frac{dx}{\sqrt{28 - x^3}}$;

33) $\int_1^2 \sqrt{18 - 2x^3}dx$.

Quyidagi integrallarning taqribiy qiymatlarini to'g'ri to'rtburchak, trapetsiya va Simpson usullarida ε=0,001 aniqlikda hiqoblang.

1) $\int_0^{\frac{\pi}{2}}\sqrt{4-\sin x}\,dx$; 11) $\int_0^{\frac{\pi}{2}}\frac{\cos x}{x+1}dx$; 21) $\int_0^{\sqrt{2ln10}} sin\sqrt{x^2+4}dx$;

2) $\int_0^{\frac{\pi}{2}}\sqrt{2-\sin x}\,dx$; 12) $\int_0^{\frac{\pi}{2}}\sqrt{2+cos\,x}\,dx$; 22) $\int_0^{\sqrt{ln16}} x^2e^{x^2}dx$;

3) $\int_0^{\pi}\frac{\sin x}{x+1}dx$; 13) $\int_0^{\frac{\pi}{2}}\sqrt{\sin x}\,dx$; 23) $\int_0^{\sqrt{ln4}} e^{\frac{x^2}{4}}dx$;

4) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\sin x}{x}dx$; 14) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}}\sqrt{5+sin\,x}\,dx$; 24) $\int_1^{ln4}\sqrt{xe^x}dx$;

5) $\int_0^{\pi}\sqrt{3+\sin x}\,dx$; 15) $\int_0^{\pi}\sqrt{8-\cos x}\,dx$; 25) $\int_1^{ln8}\frac{e^x}{x}dx$;

6) $\int_0^{\frac{\pi}{4}}sin\sqrt{x^2+1}dx$; 16) $\int_0^{\frac{\pi}{2}}\sqrt{2-\cos x}\,dx$; 26) $\int_0^{ln5}\sqrt{\frac{e^x}{x+1}}dx$;

7) $\int_{\frac{\pi}{2}}^{\pi}\frac{\cos x}{x}dx$; 17) $\int_0^{\frac{2\pi}{3}}\sqrt{\sin x}\,dx$; 27) $\int_0^{e-2}\frac{dx}{ln(2+x)}$;

8) $\int_0^{\frac{\pi}{2}}\sqrt{cos\,x}\,dx$; 18) $\int_0^{\pi}\sqrt{3+\cos x}\,dx$; 28) $\int_0^{\sqrt{3}}\frac{arctgx}{x+1}dx$

9) $\int_0^{\pi}\sqrt{2-\sin x}\,dx$; 19) $\int_0^{\frac{\pi}{4}}\sqrt{10-2\sin x}\,dx$; 29) $\int_1^{e}\frac{dx}{1+\ln x}$;

10) $\int_0^{\pi}\sqrt{2-\cos x}\,dx$; 20) $\int_{\frac{\pi}{4}}^{\pi}\sqrt{8+\cos x}\,dx$; 30) $\int_{0.1}^{\sqrt{3}-1}\frac{arctg(x+1)}{x}dx$

Variantlar.

1. $\int_0^4\frac{dx}{1+x^4}$; 2. $\int_0^{0,1}sin(100x^2)\,dx$; 3. $\int_0^{0,2}e^{-\frac{3x^2}{2}}dx$;

4. $\int_1^2\frac{sinx}{x}dx$; 5. $\int_1^{\pi}\frac{sinx}{x}dx$;

6. $\int_1^2\frac{cosx}{x}dx$; 7. $\int_0^1 e^{-x^2}dx$; 8. $\int_0^1\sqrt{1-x^3}dx$;

9. $\int_0^1\sqrt{1+x^4}dx$; 10. $\int_2^5\frac{dx}{lnx}$;

11. $\int_0^1\frac{dx}{\sqrt[4]{16+x^4}}$; 12. $\int_1^{\pi/3}\frac{sinx}{x}dx$; 13. $\int_0^{2,5}\frac{dx}{\sqrt[3]{125+x^3}}$;

14. $\int_0^{0,4}e^{\frac{-3x^2}{4}}dx$; 15. $\int_0^{0,5}sin(4x^2)\,dx$;

16. $\int_0^{0,4}cos\,(\frac{5x}{2})^2dx$; 17. $\int_0^2\frac{dx}{\sqrt[4]{256+x^4}}$; 18. $\int_0^{0,1}\frac{1-e^{-2x}}{x}dx$;

19. $\int_0^{0,5}\frac{dx}{\sqrt[3]{1+x^3}}$; 20. $\int_1^2\frac{sin3x}{x}dx$.

ODDIY DIFFERENSIAL TENGLAMALARNI YECHISHNING SONLI USULLARI

1.Masalaning qo'yilishi. Bir birlari bilan o'zaro ta'sirda bo'lgan nuqtalar sistemasining harakati haqidagi masalani,kimyoviy kinetika, elektr zanjirlari,materiallar qarshiligi masalalarini va boshqa ko'plab masalalarni oddiy differensial tenglamalar orqali ifodalash mumkin.Shunday qilib,oddiy differensial tenglamalarni yechimifizika, kimyo va texnikaning amaliy masalalari orasida muhim o'ringa ega.

Muayyan amaliy masala har qanday tartibli differensial tenglamaga yoki har qanday tartibli tenglamalar sistemasiga keltirilishi mumkin.

Oddiy differensial tenglamalar uchun uch xil tipdagi masala mavjud: Koshi masalasi,chegaraviy masala va xos qiymatlar haqidagi masala.

$$\frac{dy}{dx} = f(x,y) \qquad (1)$$

ko'rinishdagi birinchi tartibli oddiy differensial tenglamani qaraymiz. (1) tenglamani boshlang'ich shart deb ataladigan, quydagi

$$y(x_0) = y_0 \qquad (2)$$

shartni qanoatlantiruvchi yechimini topish masalasi - Koshi masalasi deyiladi. Agar (1) tenglamaning o'ng tomoni (x_0,y_0) boshlangich nuqtaning biror atrofda uzluksiz va chegaralangan bo'lsa, (1)-(2) Koshi masalasi,umuman aytganda, yagona bo'lmagan yechimga ega.Agar (1) tenglamani o'ng tamoni nafaqat uzluksiz,balki Lifshis shartini ham qanoatlantirsa, u holda Koshi masalasi boshlang'ich nuqta koordinatalariga uzluksiz bog'langan,yagona yechimga ega bo'ladi. Bunday holda masala korrekt qo'yilgan deyiladi.

2. Masalani yechish usullari. Ularni shartli ravishda aniq, taqribiy va sonli usullarga ajratish mumkin.

Aniq usullarga - differensial tenglama yechimlarini elementar funksiya ko'rinishida yoki elementar funksiyaning boshlang'ich funksiyasi ko'rinishida (kvadraturalarda) tasvirlashga imkon beradigan usullar kiradi. Bu usullar oddiy differensial tenglamalar kursida o'rganiladi. Koshi masalasi aniq yechimini topish,ayniqsa,(1) tenglamaning umumiy yechimini topish-yechimning sifat jihatidan tadqiq qilishni hamda yechim ustida qilinadigan keyingi amallarni osonlashtiradi. Ammo, aniq yechimlari topib bo'ladigan tenglamalar sinfi nisbatan tor va amalda uchraydigan masalalarning juda oz qismini qamrab oladi. Masalan,

$$\frac{du}{dx} = x^2+u^2$$

tenglamaning yechimi elementar funksiya shaklida ifodalanmasligi isbotlangan.

$$\frac{du}{dx} = \frac{u-x}{u+x} \qquad (3)$$

tenglama esa integrallanadi. Uning umumiy yechimi

$$\frac{1}{2} ln(x^2 + u^2) + arctg\frac{u}{x} = c \qquad (4)$$

ko'rinishda tasvirlanadi.Ammo $u(x)$ funksiyaning qiymatlari jadvalini tuzish uchun (4) trantsiendent tenglamani yechish kerak, bu ish esa (3) tenglamani sonli integrallashdan aslo oson emas. $u(x)$ yechimni biror y_n(x) ketma - ketlikni limiti shaklida topiladigan usul-taqribiy usul deyiladi,bunda $y_n(x)$ lar elementar funksiyalar shaklida yoki kvadraturalarda ifodalanadi. Chekli n soni bilan chegaralanib, $u(x)$ uchun $u(x) \approx y_n(x)$ taqribiy ifodaga ega bo'lamiz. Yechimni umumlashgan darajali qatorga yoyish usuli va boshqalar taqribiy usulga misol bo'la oladi.Ammo bu usul, qator koeffitsentlari oshkor ifodalash mumkin bo'lgan hollardagina samara beradi. Bunday holatlarni esa faqat sodda narsalar uchun mavjudligi-taqribiy usul tatbiq qilinadigan sohani ancha toraytirib yuboradi. Izlanayotgan $u(x)$ yechimning argumentning biror x_n to'ridagi qiymatlarinii taqribiy hisoblash algoritmlari – sonli usullardir. Bu usulda yechim jadval shaklida hosil qilinadi. Sonli usullar orqali (1) tenglamaning umumiy yechimini topishning imkoni yo'q. Bunday usullarda faqatgina qaysidir hususiy yechim topiladi. Sonli usulning asosiy kamchiligi shundadir. Biroq bu usulni tenglamalarning juda keng sinfining har qanday tipdagi masalalariga tatbiq qilish mumkin. Shu sababdan, tezkor EHM lar paydo bo'lishi bilan bu usul oddiy differensial tenglamalarning amaliy masalalarini yechishning asosiy usuliga aylandi.

3. Oddiy differensial tenglamalarni yechishning Eyler usuli. Garchi bu usul juda sodda bo'lsada, aniqlik darajasi past bo'lgani sababli amalda kam qo'llaniladi. Ammo bu usul misolida, sonli usullarning tuzilishi va tadbiq qilinishini tushuntirish qulay.

Birinchi tartibli differensial tenglama uchun quyidagi

$$\frac{dy}{dx} = f(x,y), \quad a = x_0 \le x \le b, \quad y(x_0) = y_0 \qquad (5)$$

Koshi masalasini qaraymiz. $[a,b]$ kesmadan argumentning

$$a = x_0 < x_1 > \cdots < x_N = b$$

shartni qanoatlantiradigan $\{x_n, \ 0 \le n \le N\}$ to'rini olamiz. To'rning $x_n \le x \le x_n$ kesmachasida $y(x)$ ni Teylor qatoriga yoyib va $y(x_n) = y_n$ belgilash kiritib,

$$y_{n+1} = y_n + hy'_n + \frac{1}{2}h^2 y''_n + \ldots \qquad h = x_{n+1} - x_n \qquad (6)$$

tenglikni hosil qilamiz. Bu ifodadagi hosilalarni (5) tenglikni kerakli marta differensiallab topish mumkin:

$$y' = \frac{dy}{dx} = f(x,y), \quad y'' = \frac{d}{dx} f(x,y) = f_x + f \cdot f_y \cdot \qquad (7)$$

(6) yoyilmada 2 ta qo'shiluvchi bilan cheklanib, y'_n ni uning (7) ifodasi bilan almashtirib Eyler sxemasi deb ataladigan

$$y_{n+1} = y_n + h_n f(x_n, y_n), \qquad h_n = x_{n+1} - x_n \qquad (8)$$

tenglikni hosil qilamiz. Hisoblashlarni Eyler sxemasi bo'yicha olib borish uchun $y(x_0) = y_0$ boshlang'ich qiymatning berilishi kifoya. Shundan so'ng (8) formula bo'yicha $y_1, y_2, \ldots, y_N$ qiymatlar ketma-ket hisoblanaveradi.

Topshiriq. $y' = 0.185(x^2 + \cos 0.7x) + 1.843y$

oddiy differensial tenglama uchun Koshi masalasi $[0.2, 1.2]$ kesmada $h = 0.1$ qadam va $y(0.2) = 0.25$ boshlang'ich shart bilan yechilsin. Hisoblashlar to'rtta raqam aniqligida bajarilsin.

Yechish: $y_{i+1} = y_i - h\, f(x_i, y_i)$

formuladan foydalanamiz va hamma hisoblashlarni jadvalda bajaramiz.

1	2	3	4	5	6	7	8	9	10	11
i	x_i	y_i	x_i^2	cos0,7-x	(4)-(5)	0,185-(6)	1,843y	(7)+(8)	h-(9)	(10)+(3)
0	0,2	0,25	0,04	0,99	1,03	0,19	0,46	0,65	0,06	0,32
1	0,3	0,32	0,09	0,98	1,07	0,19	0,58	0,78	0,07	0,39
2	0,4	0,39	0,16	0,96	1,12	0,21	0,72	0,93	0,09	0,49
3	0,5	0,49	0,25	0,94	1,19	0,22	0,89	1,12	0,11	0,59
4	0,6	0,59	0,36	0,91	1,27	0,24	1,10	1,34	0,13	0,73
5	0,7	0,73	0,49	0,88	1,37	0,25	1,35	1,60	0,16	0,89
6	0,8	0,89	0,64	0,85	1,49	0,28	1,64	1,92	0,19	1,08
7	0,9	1,08	0,81	0,81	1,62	0,29	1,99	2,29	0,23	1,31
8	1,0	1,31	1,0	0,76	1,76	0,33	2,42	2,75	0,27	1,59
9	1,1	1,59	1,21	0,72	1,93	0,36	2,93	3,28	0,33	1,92
10	1,2	1,92	1,44	0,67	2,11	0,39	3,53	3,92	0,39	2,31

Variantlar

1- variant

$y' = y^2 + x, \qquad y(0) = 0; \quad [0; 0,3].$

2- variant

$y' = -x^2 + y^2, \quad y(1) = 1; \quad [1; 2].$

3- variant

$y' = -\frac{y}{1-x}, \quad y(0) = 2; \quad [0; 1].$

4- variant

$y' = y - \frac{2x}{y}, \quad y(0) = 1; \quad [0; 1].$

5- variant

$y' = \frac{1}{x^2+y^2}, \qquad y(0,5) = 0,5; \quad [0,5; 3,5].$

6 -variant

$y' = \frac{x^2+y^2}{10}, \qquad y(1) = 1; \quad [1; 5].$

7- variant

$y' = xy^3 + x^2, \qquad y(0) = 0; \quad [0; 1].$

8- variant

$y' = \sqrt{x}y^2 + 1, \qquad y(1) = 0; \quad [1; 2].$

9- variant

$y' = \frac{3x^2}{x^3+y+1}, \qquad y(1) = 0; \quad [1; 2].$

10- variant

$y' = \frac{xy}{2}, \quad y(0) = 1; \quad [0; 0{,}9].$

11- variant

$y' = y^2 + xy + x^2, \quad y(0) = 1; \quad [0; 1].$

12-variant

$y' = xy^3 - 1, \quad y(0) = 0; \quad [0; 1].$

13- variant

$y' = x\ln y - y\ln x, \quad y(1) = 1; \quad [1; 1{,}6].$

14- variant

$y' = 0{,}1y^2 + 2xy - x, \quad y(0) = 1; \quad [0; 0{,}8].$

15- variant

$y' = \frac{x}{y} - x^2, \quad y(1) = 1; \quad [1; 1{,}5].$

16- variant

$y' = x^2y^2 + 1, \quad y(0) = 0; \quad [0; 1].$

17- variant

$y' = 2xy + 1, \quad y(0) = 0; \quad [0; 0{,}5].$

18- variant

$y' = y^2 - x, \quad y(0) = 1; \quad [0; 0{,}5].$

19- variant

$y' = 2xy + x^2, \quad y(0) = 0; \quad [1; 0{,}5].$

20- variant

$y' = x^3 + y^2, \quad y(0) = 0{,}5; \quad [0; 0{,}5].$

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