O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA
MAXSUS TA’LIM VAZIRLIGI

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TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

Oliy matematika fanidan mustaqil ishlarni bajarish uchun o‘quv-uslubiy qo‘llanma

1-qism

Chiziqli algebra va analitik geometriya, hosila, integral, differensial tenglamalar.

Texnika oliy o‘quv yurtlarining bakalavriat ta’lim yo‘nalishlari talabalari uchun

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Oliy matematika fanidan mustaqil ishlarni bajarish uchun o'quv-uslubiy qo'llanma.1-qism. Texnika oliy o'quv yurtlari talabalari uchun. Abzalimov R.R., Xolmuhamedov A.S., Xaldibayeva I.T., Abdikayimova G., Aralova M., Akberadjiyeva U.; Toshkent,ToshDTU, 2013.

Oquv-uslubiy qo'llanma oliy texnika o'quv yurtlarining bakalavr talabalari uchun mo'ljallangan. Shuningdek, bu o'quv-uslubiy qo'llanmadan oliy o'quv yurtlarining o'qituvchilari ham ma'ruza va amaliy darslarda foydalanishlari mumkin.

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Abu Rayhon Beruniy nomidagi Toshkent davlat texnika universiteti ilmiy-uslubiy kengashining qaroriga muvofiq chop etildi.

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So'z boshi.

Zamonaviy kadrlarni yetishtirish borasida respublikamiz oliy ta'limi tizimida tub o'zgarishlar amalga oshirilmoqda. Bunga sabab, «Ta'lim to'g'risida»gi qonun va «Kadrlar tayyorlash milliy dasturi»ning qabul qilinishi va ularda ilmiy-texnika taraqqiyoti yutuqlarini xalq xo'jaligiga tadbiq qilish, ijtimoiy-iqtisodiy rivojlanish bilan uzviy bog'liq ekanligining aniq ko'rsatilishidir.

O'quv uslubiy qo'llanma oliy matematika fanining asosiy bo'limlari bo'yicha mustaqil ish va laboratoriya ishi masalalaridan iborat bo'lib u 2- qismdan iborat. 1- qism o'z ichiga chiziqli algebra va analitik geometriya, hosila, integral, differensial tenglamalar mavzularidan o'tkaziladigan tipik ishlariga bag'ishlangan bo'lib, ularda nazariy ma'lumotlar va namuna uchun masalalar yechimi ko'rsatilgan. Talabalar mustaqil yechishlari uchun shahsiy variantlar yetarlicha keltirilgan

Qo'llanmani yozishda mualliflar Toshkent davlat texnika universitetida ko'p yillar davomida o'qigan ma'ruza va amaliy mashg'lot darslarini asos qilib oldilar.

Mualliflar.

1. CHIZIQLI ALGEBRA VA ANALITIK GEOMETRIYA

Ikkinchi tartibli determinantlar va chiziqli tenglamalar sistemasi.

$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ - jadvalga mos ikkinchi tartibli determinant

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

tenglik bilan aniqlanadi. Ikki noma'lumli ikkita chiziqli tenglamalar sistemasi

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases},$$

uning determinanti $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$ bo'lsa, u yagona yechimga ega bo'lib, yechim Kramer formulalaridan topiladi:

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}; \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}.$$

Agar D=0 bo'lsa, sistema yoki birgalikda emas, $(D_x \neq 0, D_y \neq 0)$ yoki aniqlanmagan $(D_x = D_y = 0)$. Oxirgi holda sistema bitta tenglamaga keltiriladi. Sistemaning birgalikda bo'lmaslik shartini $a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$, aniqmaslik shartini $a_1 / a_2 = b_1 / b_2 = c_1 / c_2$ kabi yozish mumkin. Chiziqli sistemaning ozod hadi nolga teng bo'lsa, u bir jinsli deb ataladi.

Uchinchi tartibli determinant va chiziqli tenglamalar sistemasi

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

elementlar jadvaliga mos uchinchi tartibli determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \cdot \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \cdot \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \cdot \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

tenglik bilan aniqlanadi.

Uchinchi tartibli determinantdagi berilgan elementni o'z ichiga olgan satr va ustunni o'chirishdan hosil bo'lgan ikkinchi tartibli determinant - uchinchi tartibli determinantning berilgan elementining minori deb ataladi. Minorning $(-1)^k$ ga ko'paytmasi berilgan elementning algebraik to'ldiruvchisi deyiladi. (k- berilgan elementni o'z ichiga olgan satr va ustun nomerlari yigindisi). Shunday qilib, determinantning elementiga mos minor ishorasi quyidagi

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

jadval bilan aniqlanadi.

Yuqoridagi 3 - tartibli determinantni ifodalovchi tenglikning o'ng tomoni 1 – satr elementlarini ularning o'z algebraik to'ldiruvchilariga ko'paytmalarining yig'indisidan iborat.

Teorema 1.

3 - tartibli determinant ixtiyoriy satr (ustun) elementlarini o'z algebraik to'ldiruvchilariga ko'paytmalarining yig'indisiga teng.

Bu teorema determinantni ixtiyoriy satr (ustun) elementlari bo'yicha yoyib hisoblashga imkon beradi.

Teorema 2.

Ixtiyoriy satr elementlarini boshqa satr elementlarining algebraik to'ldiruvchilariga ko'paytmalarining yig'indisi nolga teng.

Determinantning xossalari.

1. Agar determinantning satrlarini ustunlari bilan yoki ustunlarini satrlari bilan almashtirsak, determinantning qiymati o'zgarmaydi.
2. Determinantning biror satr elementlari umumiy ko'paytuvchiga ega bo'lsa, uni determinantning tashqarisiga chiqarish mumkin.
3. Determinantning biror satr elementlari boshqa satr elementlariga teng bo'lsa, unday determinant nolga teng.
4. Agar determinantning ikkita satrining o'rnini almashtirsak, uning ishorasi teskariga o'zgaradi.
5. Agar determinantning biror satr elementlariga boshqa satrining mos elementlarini biror o'zgarmas songa ko'paytirib qo'shsak, uning qiymati o'zgarmaydi.

Uch noma'lumli uchta chiziqli

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

tenglamalar sistemasini quyidagi Kramer formulalaridan foydalanib yechamiz.

$x=D_x/D,\ y=D_y/D,\ z=D_z/D,$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \quad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Bu yerda $D\neq 0$ deb faraz qilamiz (agar $D=0$ bo'lsa, sistema aniqlanmagan yoki birgalikda bo'lmaydi).

Ikkinchi va uchunchi tartibli kvadrat matritsalar.

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

jadval - matritsa deyiladi.

$$D_A=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

A matritsaga mos determinant deyiladi. Bundan so'ng $D_A \neq 0$ deb qaraladi. Agar $D_A \neq 0$ $(D_A = 0)$ bo'lsa, A matritsa xosmas (xos) deb ataladi.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

lar mos ravishda 2- va 3- tartibli kvadrat matritsalar deyiladi. Ko'p ta'riflami umumlashtirish uchun ulami 3- tartibli matritsa uchun beriladi. Ulami 2- tartibli matritsa uchun qo'llash qiyinchilik tug'dirmaydi. Agar kvadrat matritsaning elementlari $a_{mn} = a_{nm}$ shartni qanoatlantirsa, matritsa simmetrik deyiladi.

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad B=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

matritsalar teng bo'lishi uchun $a_{mn} = b_{nm}$ shartning bajarilishi zarur va yetarlidir. A, B matritsalar yig'indisi quyidagicha aniqlanadi:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}+\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{pmatrix}$$

A matritsani m soniga ko'paytirish uchun uning har bir elementini m ga ko'paytiramiz:

$$m\cdot\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} ma_{11} & ma_{12} & ma_{13} \\ ma_{21} & ma_{22} & ma_{23} \\ ma_{31} & ma_{32} & ma_{33} \end{pmatrix}$$

A, B matritsalar ko'paytmasi quyidagicha aniqlanadi:

$$AB = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} =$$

$$= \begin{pmatrix} \sum_{j=1}^{3} a_{1j}b_{j1} & \sum_{j=1}^{3} a_{1j}b_{j2} & \sum_{j=1}^{3} a_{1j}b_{j3} \\ \sum_{j=1}^{3} a_{2j}b_{j1} & \sum_{j=1}^{3} a_{2j}b_{j2} & \sum_{j=1}^{3} a_{2j}b_{j3} \\ \sum_{j=1}^{3} a_{3j}b_{j1} & \sum_{j=1}^{3} a_{3j}b_{j2} & \sum_{j=1}^{3} a_{3j}b_{j3} \end{pmatrix}$$

Ko'paytma matritsaning i- satr va k- ustunda turuvchi elementi, A matritsa i- satridagi elementlarini B matritsa k- ustuning mos elementlariga ko'paytmalari yig'indisiga teng.

Ikki matritsaning ko'paytmasi, umuman, o'rin almashtirish xossasiga bo'ysinmaydi. Ikki matritsa ko'paytmasining determinanti bu matritsalar determinantlari ko'paytmalariga teng.

Hamma elementlari nollardan iborat bo'lgan matritsa - nol matritsa deyiladi.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0.$$

Bu matritsa uchun $A+0=A$ tenglik o'rinli.

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ birlik matritsa deyiladi.

Bu matritsani A ga chapdan va o'ngdan ko'paytmasi A ga teng: $EA=AE=A$. Agar $AB=BA=E$ bo'lsa, B matritsa A-ga teskari matritsa deyiladi. A ga teskari matritsa A^{-1} bilan belgilanadi: $B=A^{-1}$. Har qanday xos emas matritsa teskari matritsaga ega. Teskari matritsa quyidagi formula orqali hisoblanadi:

$$A^{-1}=\begin{pmatrix} \frac{A_{11}}{D_A} & \frac{A_{21}}{D_A} & \frac{A_{31}}{D_A} \\ \frac{A_{12}}{D_A} & \frac{A_{22}}{D_A} & \frac{A_{32}}{D_A} \\ \frac{A_{13}}{D_A} & \frac{A_{23}}{D_A} & \frac{A_{33}}{D_A} \end{pmatrix}$$

Bu formulada A_{mn} A matritsa determinantidagi a_{mn} - elementning algebraik to'ldiruvchisi, ya'ni A_{mn} - A matritsa determinantidagi m –satr va n- ustunini o'chirishdan hosil bo'lgan ikkinchi tartibli determinant (minor) bilan $(-1)^{m+n}$ ifoda ko'paytmasidir.

$$X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

ustun matritsa deyiladi.

AX ko'paytma quyidagicha aniqlanadi:

$$AX=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\cdot\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} a_{11}x_1+a_{12}x_2+a_{13}x_3 \\ a_{21}x_1+a_{22}x_2+a_{23}x_3 \\ a_{31}x_1+a_{32}x_2+a_{33}x_3 \end{pmatrix}$$

$$\begin{cases} a_{11}x_1+a_{12}x_2+a_{13}x_3=b_1 \\ a_{21}x_1+a_{22}x_2+a_{23}x_3=b_2 \\ a_{31}x_1+a_{32}x_2+a_{33}x_3=b_3 \end{cases}$$

sistemani $AX=B$ ko'rinishda yozish mumkin, bu yerda

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},\quad X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\quad B=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Bu sistemaning yechimi $X = A^{-1} \cdot B \quad (D_A \neq 0)$ bo'ladi.

Vektorlar va ular ustida amallar

OXYZ koordinatlar fazosida berilgan $\bar{a}$ erkin vektorni $\bar{a} = a_x \cdot \bar{i} + a_y \cdot \bar{j} + a_z \cdot \bar{k}$ ko'rinishida tasvirlash mumkin. $\bar{a}$ vektorni bunday tasvirlash - uni koordinata o'qlari yoki ortlar bo'yicha yoyish deb ataladi. Bu yerda a_x, a_y, a_z lar - $\bar{a}$ vektorning mos o'qlardagi proyeksiyalari ($\bar{a}$ vektorning koordinatalari) deyiladi, $\bar{i}$, $\bar{j}$, $\bar{k}$ lar esa o'qlarning ortlari (mos o'qlarning musbat yo'nalishi bilan ustma-ust tushgan birlik vektorlar). $a_x\bar{i}$, $a_y\bar{j}$, $a_z\bar{k}$ lar $\bar{a}$ vektorning koordinat o'qlari bo'yicha tashkil etuvchilari (komponentalari) deb ataladi. $\bar{a}$ vektorning uzunligi $|\bar{a}|$ bilan belgilanib, $|\bar{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ formula orqali topiladi. $\bar{a}$ vektorning yo'nalishi uning koordinat o'qlari bilan tashkil qilgan α, β, γ burchaklar orqali aniqlanadi. Bu burchaklarning kosinuslari (vektorning yo'naltiruvchi kosinuslari)

$$\cos\alpha = \frac{a_x}{a} = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}; \quad \cos\beta = \frac{a_y}{a} = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}};$$

$$\cos\gamma = \frac{a_z}{a} = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

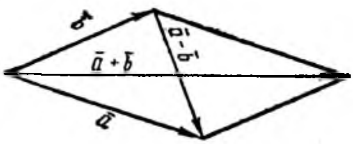
formulalar bilan aniqlanadi.Vektorning yo'naltiruvchi kosinuslari $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ munosabat bilan bog'langan. Agar $\bar{a}$ va $\bar{b}$ vektorlar ortlar bo'yicha yoyilmasi bilan berilgan bo'lsa, ularning yig'indisi va ayirmasi

$$\bar{a} + \bar{b} = (a_x + b_x)\cdot\bar{i} + (a_y + b_y)\cdot\bar{j} + (a_z + b_z)\cdot\bar{k}$$

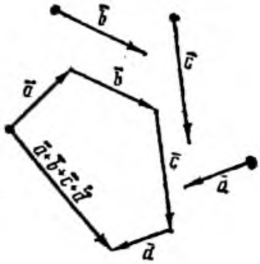
$$\bar{a} - \bar{b} = (a_x - b_x)\cdot\bar{i} + (a_y - b_y)\cdot\bar{j} + (a_z - b_z)\cdot\bar{k}$$

formulalardan aniqlanadi.

Boshlari ustma-ust tushadigan $\bar{a}$ va $\bar{b}$ vektorlar yig'indisi tomonlari $\bar{a}$ va $\bar{b}$ bo'lgan parallelogramm diagonali bilan ustma-ust tushadigan vektor orqali tasvirlanadi. $\bar{a}$ - $\bar{b}$ ayirma shu parallelogrammning ikkinchi diagonali bilan ustma-ust tushib, vektorning boshi $\bar{b}$ ning oxirida, oxiri $\bar{a}$ ning oxirida yotadi.



1-rasm



2-rasm

Ixtiyoriy sondagi vektorlar yig'indisi ko'pburchaklar qoidasi bo'yicha topiladi (1 va 2 – rasmlar). $\bar{a}$ vektorning m skalyarga ko'paytmasi $m \cdot \bar{a} = m \cdot a_x \cdot \bar{i} + m \cdot a_y \cdot \bar{j} + m \cdot a_z \cdot \bar{k}$ formuladan topiladi. Agar m>0 bo'lsa, $\bar{a}$ va $m \cdot \bar{a}$ vektorlar parallel (kollinear) va bir tomonga yo'nalgan, m<0 bo'lsa, qarama-qarshi tomonga yo'nalgan bo'ladi. Agar *m=1/a* bo'lsa, $\bar{a} / a$ vektor uzunligi birga teng bo'lib, yo'nalishi $\bar{a}$ ning yo'nalishi bilan ustma-ust tushadi. Bu vektor $\bar{a}$ vektorning birlik vektori (orti) deyilib, $\bar{a}_0$ bilan belgilanadi. $\bar{a}$ vektor yo'nalishidagi birlik vektorni toppish - $\bar{a}$ vektorni normallashtirish deyiladi. Shunday qilib, $\bar{a}_0 = \bar{a} / a$,yoki $\bar{a} = a\bar{a}_0$. Boshi koordinat boshida, oxiri *M* nuqtada yotgan $\overline{OM}$ vektor *M* nuqtaning radius - vektori deyilib, $\bar{r}(M)$ yoki $\bar{r}$ bilan belgilanadi. Uning koordinatalari *M* nuqtaning koordinatalari bilan ustma-ust tushgani uchun uning ort bo'yicha yoyilmasi $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ ko'rinishda bo'ladi. Boshi $A(x_1; y_1; z_1)$, oxiri $B(x_2; y_2; z_2)$ nuqtada bo'lgan $\overline{AB}$ vektor $\overline{AB} = \bar{r}_2 - \bar{r}_1$ ko'rinishda yoziladi, bu yerda $\bar{r}_2$ *B* nuqtaning, $\bar{r}_1$ *A* nuqtaning radius vektori.

Shuning uchun $\overline{AB}$ vektorning ortlar bo'yicha yoyilmasi

$$\overline{AB}=(x_2-x_1)\bar{i}+(y_2-y_1)\bar{j}+(z_2-z_1)\bar{k}$$

ko'rinishda bo'ladi. Uning uzunligi A va B nuqtalar orasidagi masofaga teng.

$$|\overline{AB}|=d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$\overline{AB}$ vektorning yo'nalishi

$$\cos\alpha=\frac{x_2-x_1}{d};\quad \cos\beta=\frac{y_2-y_1}{d};\quad \cos\gamma=\frac{z_2-z_1}{d}$$

yo'naltiruvchi kosinuslar bilan aniqlanadi.

Skalyar ko'paytma.

$\bar{a}$ va $\bar{b}$ vektorlarning skalyar ko'paytmasi deb shunday songa aytamizki, u vektorlar uzunliklarini ular orasidagi burchak kosinusiga ko'paytmasiga teng: $\bar{a}\cdot\bar{b}=ab\cos\varphi$

Skalyar ko'paytmaning xossalari.

1. $\bar{a}\cdot\bar{a}=|a|^2$ yoki $\bar{a}^2=|a|^2$;
2. Agar $\bar{a}=0$ yoki $\bar{b}=\bar{0}$, yoki $\bar{a}\perp\bar{b}$ (noldan farqli vektorlar ortogonalligi) bo'lsa, $\bar{a}\cdot\bar{b}=0$;
3. $\bar{a}\cdot\bar{b}=\bar{b}\cdot\bar{a}$ (o'rin almashtirish qonuni);
4. $\bar{a}(\bar{b}+\bar{c})=\overline{ab}+\overline{ac}$ (taqsimot qonuni);
5. $(m\bar{a})\bar{b}=\bar{a}(m\bar{b})=m(\overline{ab})$ (skalyar ko'paytuvchiga nisbatan guruhlash qonuni).

Koordinata o'qlari ortlarining skalyar ko'paytmasi:

$\bar{i}^2=\bar{j}^2=\bar{k}^2=1,\quad \bar{i}\cdot\bar{j}=\bar{j}\cdot\bar{k}=\bar{i}\cdot\bar{k}=0.$

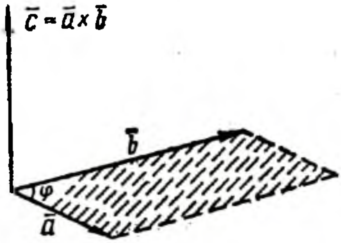
$\bar{a}=a_x\bar{i}+a_y\bar{j}+a_z\bar{k},\ \bar{b}=b_x\bar{i}+b_y\bar{j}+b_z\bar{k}$ lar berilgan bo'lsa, ularning skalyar ko'paytmasi $\overline{ab}=a_xb_x+a_yb_y+a_zb_z$ formuladan topiladi.

Vektor ko‘paytma.

Quyidagi shartlami qanoatlantiruvchi $\vec{c}$ vektor $\vec{a}$ vektoming $\vec{b}$ vektorga vector ko‘paytmasi deyiladi:

1. $\bar{c}$ ning uzunligi $\bar{a}$ va $\bar{b}$ vektorlardan yasalgan parallelogrammning yuziga teng ($c = ab\sin\varphi,\ \ \varphi = \bar{a}\wedge\bar{b}$);
2. c vektor $\bar{a}$ va $\bar{b}$ vektorlarga perpendikulyar;
3. $\bar{a},\bar{b},\bar{c}$ vektorlar bitta nuqtaga keltirilgandan so‘ng o‘ng sistemani tashkil etadi. (3-rasm)

$\bar{a}$ vektoming $\bar{b}$ vektorga vektor ko‘paytmasi $\bar{a}\times\bar{b}$ ko‘rinishda yoziladi.



3-rasm

Vektor ko‘paytmaning xossalari:

1. $\bar{b}\times\bar{a} = -\bar{a}\times\bar{b}$, o‘rin almashtirish xossasiga ega emas.
2. Agar $\bar{a} = 0$, yo $\bar{b} = 0$, yo $\bar{a} \parallel \bar{b}$ bo‘lsa, $\bar{a}\times\bar{b} = 0$ bo‘ladi.
3. $(m\bar{a})\times\bar{b} = \bar{a}\times(m\bar{b}) = m(\bar{a}\times\bar{b})$ (vektor ko‘paytmaning skalyar ko‘paytuvchiga nisbatan guruhlash qonuni)
4. $\bar{a}\times(\bar{b}+\bar{c}) = \bar{a}\times\bar{b}+\bar{a}\times\bar{c}$ (taqsimot qonuni)

$\bar{i},\ \bar{j},\ \bar{k}$ ortlaming vektor ko‘paytmasi uchun

$\bar{i}\times\bar{i} = \bar{j}\times\bar{j} = \bar{k}\times\bar{k} = 0,$

$\bar{i}\times\bar{j} = -\bar{j}\times\bar{i} = \bar{k};\ \ \bar{j}\times\bar{k} = -\bar{k}\times\bar{j} = \bar{i};\ \ \bar{k}\times\bar{i} = -\bar{i}\times\bar{k} = \bar{j}$ tengliklar o‘rinli. $\bar{a} = x_1\bar{i}+y_1\bar{j}+z_1\bar{k},\ \ \bar{b} = x_2\bar{i}+y_2\bar{j}+z_2\bar{k}$ vektorlaming vektor ko‘paytmasi

$$\bar{a}\times\bar{b}=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

formula yordamida topiladi.

Aralash ko‘paytma.

Uch $\bar{a},\bar{b},\bar{c}$ vektorning aralash ko‘paytmasi $\bar{a}\times\bar{b}$ ning $\bar{c}$ ga skalyar ko‘paytmasiga teng, ya’ni $\bar{a}\times\bar{b}\cdot\bar{c}$. Aralash ko‘paytmaning moduli shu vektorlarga qurilgan parallelepipedning hajmiga teng.

Aralash ko‘paytmaning xossalari:

1. Agar

a) ko‘paytiriluvchi vektorlardan biri nolga teng,

b) ikkitasi kolleniar,

d) uchta noldan farqli vektor bitta tekislikka parallel (komplanar) bo‘lsa, aralash ko‘paytma nolga teng.

2. Agar aralash ko‘paytmada vektor ko‘paytma (x) va skalyar ko‘paytma *()* larning o‘rnini almashtirsak aralash ko‘paytma o‘zgarmaydi, ya’ni $\bar{a}\times\bar{b}\cdot\bar{c}=\bar{a}\cdot\bar{b}\times\bar{c}$, shuni hisobga olib aralash ko‘paytma $\bar{a}\cdot\bar{b}\cdot\bar{c}$ kabi yoziladi.

3. Agar ko‘paytiriladigan vektorlar o‘rnini doiraviy shaklda almashtirsak, ko‘paytma o‘zgarmaydi: $\bar{a}\cdot\bar{b}\cdot\bar{c}=\bar{b}\cdot\bar{c}\cdot\bar{a}=\bar{c}\cdot\bar{a}\cdot\bar{b}$.

4. Ixtiyoriy ikkita qo‘shni vektor o‘rnini almashtirsak, aralash ko‘paytmaning ishorasi o‘zgaradi.

$\bar{b}\cdot\bar{a}\cdot\bar{c}=-\bar{a}\cdot\bar{b}\cdot\bar{c}$; $\bar{c}\cdot\bar{b}\cdot\bar{a}=-\bar{a}\cdot\bar{b}\cdot\bar{c}$; $\bar{a}\cdot\bar{c}\cdot\bar{b}=-\bar{a}\cdot\bar{b}\cdot\bar{c}$.

$\bar{a}=x_1\bar{i}+y_1\bar{j}+z_1\bar{k}$; $\bar{b}=x_2\bar{i}+y_2\bar{j}+z_2\bar{k}$; $\bar{c}=x_3\bar{i}+y_3\bar{j}+z_3\bar{k}$

larning aralash ko‘paytmasi

$$\bar{a}\cdot\bar{b}\cdot\bar{c}=\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

dan topiladi. Aralash ko‘paytmaning xossalaridan quyidagilar kelib chiqadi: uch vektor komplanarligining zarur va yetarli sharti

$\bar{a}\cdot\bar{b}\cdot\bar{c}=0$ dir. $\bar{a},\bar{b},\bar{c}$ larga qurilgan parallelepiped hajmi $V_1=|\bar{a}\cdot\bar{b}\cdot\bar{c}|$, uchburchakli piramidaning hajmi

$V_2=\frac{1}{6}V_1=\frac{1}{6}|\bar{a}\cdot\bar{b}\cdot\bar{c}|$ ga teng.

Tekislikdagi analitik geometriya.

To'g'ri chiziqdagi koordinatalar. Kesmani berilgan nisbatda bo'lish.

x absissaga ega bo'lgan OX koordinata o'qining M nuqtasi $M(x)$ bilan belgilanadi. $M_1(x_1)$ va $M_2(x_2)$ nuqtalar orasidagi masofa

$$d=|x_2-x_1|$$

formula bilan aniqlanadi.

Ixtiyoriy to'g'ri chiziqda AB (A-kesmaning boshi, B-oxiri) kesma berilgan bo'lsin; u holda bu to'g'ri chiziqning ixtiyoriy C nuqtasi AB kesmani qandaydir λ nisbatda bo'ladi, bu yerda $\lambda=\pm|AC|:|CB|$. Agar AC, CB kesmalar bir tomonga qarab yo'nalgan bo'lsa "+" ishora , qarama-qarshi tomonga yo'nalgan bo'lsa "-" ishora olinadi. Boshqacha qilib aytganda, agar C nuqta A va B nuqtalar orasida yotsa, λ musbat, tashqarida yotsa manfiy bo'ladi.

Agar $A(x_1)$ va $B(x_2)$ nuqtalar OX o'qida yotsa, AB kesmani λ nisbatda bo'luvchi $C(x)$ nuqtaning koordinatalari

$$x=\frac{x_1+\lambda x_2}{1+\lambda}$$

formula bilan aniqlanadi. Hususiy holda, agar $\lambda=1$ bo'lsa, kesma o'rtasining koordinatalari uchun

$$x=\frac{x_1+x_2}{2}$$

formula kelib chiqadi.

Tekislikdagi to'g'ri burchakli koordinatalar.

Agar berilgan tekislikda XOY dekart koordinatalar sistemasi berilgan bo'lsa, x, y kordinataga ega bo'lgan M nuqtani $M(x,y)$ bilan belgilaymiz, $M_1(x_1y_1)$, $M_2(x_2y_2)$ nuqtalar orasidagi masofa

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

formula bilan hisoblanadi. Xususiy holda koordinita boshidan *M(x,y)* nuqtagacha bo'lgan masofa

$$d=\sqrt{x^2+y^2}$$

formula bilan aniqlanadi.

$A(x_1,y_1)$, $B(x_2,y_2)$ nuqtalar orasidagi kesmani berilgan λ nisbatda bo'luvchi $C(\bar{x},\bar{y})$ nuqtaning koordinatalari

$$\bar{x}=\frac{x_1+\lambda x_2}{1+\lambda}; \qquad \bar{y}=\frac{y_1+\lambda y_2}{1+\lambda}$$

formulalar bilan aniqlanadi.

Xususiy holda, $\lambda=1$ bo'lganda kesma o'rtasining koordinatalari

$$\bar{x}=\frac{x_1+x_2}{2} \qquad \bar{y}=\frac{y_1+y_2}{2}$$

Uchlarining koordinatalari $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ bo'lgan uchburchak yuzasi

$$S=\frac{1}{2}\cdot|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|=$$

$$=\frac{1}{2}\cdot|(x_2-x_1)(y_3-y_1)-(x_3-x_1)(y_2-y_1)|$$

formula yordamida topiladi.

Uchburchak yuzasi

$$S=\frac{1}{2}\Delta$$

formula bilan hisoblanadi, bu yerda

$$\Delta=\left\|\begin{matrix}1 & 1 & 1\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{matrix}\right\|.$$

Qutb koordinatalari.

Qutb koordinatalarida *M* nuqtaning o'mi uning *O* qutbdan masofasi $/OM/=\rho$ (ρ - nuqtaning qutb radiusi) va *OM* kesmaning qutb o'qi OX bilan tashkil qilgan burchagi θ (θ - nuqtaning qutb burchagi) bilan aniqlanadi. Qutb o'qidan soat strelkasiga qarama - qarshi olingan θ burchak musbat hisoblanadi. Agar M nuqta qutb koordinatalariga ega bo'lsa ($\rho > 0$, $0 \le \theta < 2\pi$), u holda unga cheksiz ko'p ($\rho, \theta + 2k\pi$), $k \in Z$ qutb koordinatlari jufti to'g'ri keladi. Agar dekart koordinatalari sistemasining koordinatalar boshini qutbga keltirsak, OX o'qini qutb o'qi bo'yicha yo'naltirsak, u holda M nuqtaning to'g'ri burchakli (x,y) koordanatalari bilan (ρ, θ) qutb koordinatalari o'rtasida bog'lanish quyidagi

$$x = \rho\cos\theta, \qquad y = \rho\sin\theta;$$

$$\rho = \sqrt{x^2 + y^2}, \qquad tg\theta = \frac{y}{x}$$

formulalar bilan ainqlanadi.

To'g'ri chiziqning umumiy tenglamasi.

x, y larga nisbatan har qanday birinchi darajali tenglama, ya'ni

$$Ax+By+C=0$$

(A, B, C –lar o'zgarmas koeffitsientlar, $A^2+B^2 \neq 0$) tenglama tekislikda qandaydir to'g'ri chiziqni aniqlaydi. Bu tenglama to'g'ri chiziqning umumiy tenglamasi deb ataladi.

Xususiy hollar:

1.$C=0$; $A\neq0$; $B\neq0$. $Ax+By=0$ tenglama bilan aniqlanadigan to'g'ri chiziq koordinatalar boshidan o'tadi.

2.$A=0$; $B\neq0$; $C\neq0$. $By+C=0$ tenglama bilan aniqlanadigan ($y=-C/B$) to'g'ri chiziq OX o'qiga parallel.

3.$B=0$; $A\neq0$; $C\neq0$. $Ax+C=0$ tenglama bilan aniqlanadigan ($x=-C/A$) to'g'ri chiziq OY o'qiga parallel.

4.$B=C=0$; $A\neq0$; $Ax=0$ yoki $x=0$ tenglama bilan aniqlanadigan to'g'ri chiziq OY o'qi bilan ustma-ust tushadi.

5.$A=C=0$; $B\neq0$, $By=0$ yoki $y=0$ tenglama bilan aniqlanadigan to'g'ri chiziq OX o'qi bilan ustma-ust tushadi.

To'g'ri chiziqning burchak koeffitsientli tenglamasi

Agar umumiy tenglamada $B\neq0$ bo'lsa, uni y ga nisbatan yechib $y=kx+b$ tenglamani hosil qilamiz. (bu yerda $k=-A/B$, $b=-C/B$).

to‘g‘ri chiziqning burchak koeffitsientli tenglamasi deb atashadi, bu yerda $k = tg\alpha$, α – to‘g‘ri chiziq bilan OX o‘qining musbat yo‘nalishi orasidagi burchak. Tenglamaning ozod hadi b to‘g‘ri chiziqning OY o‘qi bilan kesishgan nuqtasining ordinatasi.

To‘g‘ri chiziqning kesmalarga nisbatan tenglamasi.

Agar to‘g‘ri chiziqning umumiy tenglamasida $C \neq 0$ bo‘lsa, tenglamani C ga bo‘lib,

$$\frac{x}{a} + \frac{y}{b} = 1$$

tenglikga ega bo‘lamiz (bu yerda $a=-C/A$, $b=-C/B$). Uni to‘g‘ri chiziqning kesmalarga nisbatan tenglamasi deb atashadi; bunda a – to‘g‘ri chiziqning OX o‘qi, b – esa OY o‘qi bilan kesishish nuqtasining shu o‘qlardagi koordinatalari. Shuning uchun, a, b larga to‘g‘ri chiziqning o‘qlardagi kesmalari deyiladi.

To‘g‘ri chiziqning normal tenglamasi.

Agar to‘g‘ri chiziqning umumiy tenglamasining ikki tomonini $\mu = 1/\pm\sqrt{A^2 + B^2}$ ga ko‘paytirsak (μ - normallashtiruvchi ko‘paytuvchi, ildiz oldidagi ishorani shunday tanlaymizki $\mu C < 0$ bo‘lsin) $x\cos\varphi + y\sin\varphi - p = 0$ ga ega bo‘lamiz. Bu tenglik to‘g‘ri chiziqning normal tenglamasi deyiladi. Bu yerda p koordinatalar boshidan to‘g‘ri chiziqqa tushirilgan perpendikulyarning uzunligi, φ - perpendikulyar bilan OX o‘qining musbat yo‘nalishi orasidagi burchak.

To‘g‘ri chiziqlar orasidagi burchak. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi.

$y=k_1x+b_1$, $y=k_2x+b_2$ to‘g‘ri chiziqlar orasidagi burchak

$$tg\alpha = \left|\frac{k_2 - k_1}{1 + k_1k_2}\right|$$

formula bilan aniqlanadi.

$k_1 = k_2$ ikki to‘g‘ri chiziqning parallellik sharti. $k_1=-1/k_2$ ikki to‘g‘ri chiziqning perpendikulyarlik sharti.

k burchak koeffitsientli va $M(x_1, y_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi quyidagi $y-y_1= k(x-x_1)$ ko‘rinishda yoziladi.

$M_1(x_1, y_1)$ va $M_2(x_2, y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

ko'rinishda yoziladi, to'g'ri chiziqning burchak koeffitsienti

$$k=\frac{y_2-y_1}{x_2-x_1}$$

formuladan topiladi. Agar $x_1=x_2$ bo'lsa, M_1, M_2 nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi $x=x_1$, agar $y_1=y_2$ bo'lsa $y=y_1$ bo'ladi.

To'g'ri chiziqlarning kesishuvi. Nuqtadan to'g'ri chiziqqacha masofa.

Agar $A_1/A_2 \neq B_1/B_2$ bo'lsa, $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlarning kesishgan nuqtasi koordinatalari ularning tenglamalarini birgalikda yechib topiladi. $M(x_0,y_0)$ nuqtadan $Ax+Bx+C=0$ to'g'ri chiziqqacha masofa

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}},$$

$A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlar orasidagi burchak bissektrisasining tenglamasi

$$\frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}}\pm\frac{A_2x+B_2y+C_2}{\sqrt{A_2^2+B_2^2}}=0$$

bo'ladi.

Tekislik.

1)Tekislikning vektor tenglamasi $\bar{r}\cdot\bar{n}=p$ ko'rinishda bo'ladi. Bu yerda $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$ vektor, tekislikdagi $M(x,y,z)$ nuqtaning radius-vektori; $\bar{n}=\bar{i}\cos\alpha+\bar{j}\cos\beta+\bar{k}\cos\gamma$ koordinat boshidan tekislikka tushirilgan perpendikulyar yo'nalishiga ega bo'lgan birlik vektor; α,β,γ lar shu perpendikulyarning OX, OY, OZ o'qlari bilan tashkil qilgan burchaklari; p– perpendikulyar uzunligi. Yuqoridagi tenglamani koordinata ko'rinishida yozsak,

$$x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0$$

ga ega bo'lamiz (tekislikning normal tenglamasi).

2) Ixtiyoriy tekislik tenglamasini $Ax+By+Cz+D=0$ ko'rinishda yozish mumkin.Bunda $A^2+B^2+C^2 \neq 0$ A, B, C lar tekislikka perpendikulyar $\bar{n}(A,B,C)$ vektorning koordinatalari. Umumiy tenglamani normal holga keltirish uchun uni normallashtiruvchi ko'paytuvchi

$$\mu = \pm\frac{1}{|\bar{n}|} = \pm\frac{1}{\sqrt{A^2+B^2+C^2}}$$

ga ko'paytirish kerak, bu yerdagi ishora D ning ishorasiga teskari bo'ladi.

3) $Ax+By+Cz+D=0$ umumiy tenglamaning xususiy hollari:

$A=0$; bu holda tekislik OX o'qiga parallel;

$B=0$; bu holda tekislik OY o'qiga parallel;

$C=0$; bu holda tekislik OZ o'qiga parallel;

$D=0$; bu holda tekislik koordinat boshidan o'tadi;

$A=B=0$; bu holda tekislik OZ o'qiga perpendikulyar (XOY tekisligiga parallel);

$A=C=0$; bu holda tekislik OY o'qiga perpendikulyar (XOZ tekisligiga parallel);

$B=C=0$; bu holda tekislik OX o'qiga perpendikulyar (YOZ tekisligiga parallel);

$A=D=0$; bu holda tekislik OX o'qidan o'tadi;

$B=D=0$; bu holda tekislik OY o'qidan o'tadi;

$C=D=0$; bu holda tekislik OZ o'qidan o'tadi;

$A=B=D=0$; bu holda tekislik XOY $(Z=0)$ tekisligi bilan ustma-ust tushadi;

$A=C=D=0$; bu holda tekislik XOZ $(Y=0)$ tekisligi bilan ustma-ust tushadi;

$B=C=D=0$; bu holda tekislik YOZ $(X=0)$ tekisligi bilan ustma-ust tushadi.

Agar umumiy tenglamada $D\neq0$ bo'lsa, tenglamani D ga bo'lib

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

ga ega bo'lamiz.(bu yerda $a=-D/A$, $b=-D/B$, $C=-D/C$.) Bu - tekislikning kesmalarga nisbatan tenglamasi deyiladi; a, b, c lar tekislikning OX, OY, OZ o'qlar bilan kesishgan nuqtalari. Tekislik koordinata o'qlarining ba'zilari bilan kesishmasligi ham mumkin, Masalan, agar $A=0$ bo'lsa, tenglamada x ishtirok etmaydi. Bu holda tekislikning kesmalarga nisbatan tenglamasi $\frac{y}{b}+\frac{z}{c}=1$ ko'rinishda yoziladi.

4) $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tekisliklar orasidagi burchak

$$\cos\varphi=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}$$

formula bilan aniqlanadi.

Ikki tekislikning parallellik sharti $A_1/A_2=B_1/B_2=C_1/C_2$,
Perpendikulyarlik sharti $A_1A_2+B_1B_2+C_1C_2=0$

5) $M_0(x_0, y_0, z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha masofa

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

formula bilan aniqlanadi.

Ya'ni tekislikning normal tenglamasiga $M_0(x_0, y_0, z_0)$ nuqtaning koordinatalarini qo'yib, natijaning absolut qiymati olingan. Natijaning musbat yoki manfiyligi nuqta va koordinata boshini berilgan tekislikka nisbatan joylanishini xarakterlaydi. Agar M_0 nuqta va koordinat boshi tekislikning turli tomonida yotsa,

$$\delta=\frac{Ax_0+By_0+Cz_0+D}{\sqrt{A^2+B^2+C^2}}$$

musbat, bir tomonida yotsa, manfiy bo'ladi,

6) $M_0(x_0, y_0, z_0)$ nuqtadan o'tib, $N=A\bar{i}+B\bar{j}+C\bar{k}$ vektorga perpendikulyar tekislik tenglamasi $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ ko'rinishda bo'ladi.

7) Berilgan $M_1(\bar{r}_1), M_2(\bar{r}_2), M_3(\bar{r}_3)$ uch nuqtadan o'tuvchi tekislik tenglamasini (bu yerda $\bar{r}_1 = x_1\bar{i} + y_1\bar{j} + z_1\bar{k},\ \bar{r}_2 = x_2\bar{i} + y_2\bar{j} + z_2\bar{k},\ \bar{r}_3 = x_3\bar{i} + y_3\bar{j} + z_3\bar{k}$), $\bar{r} - \bar{r}_1,\ \bar{r}_2 - \bar{r}_1,\ \bar{r}_3 - \bar{r}_1$ vektorlaming komplanarlik shartidan ($\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ radius - vektor) topamiz:

$$(\bar{r} - \bar{r}_1)(\bar{r}_2 - \bar{r}_1)(\bar{r}_3 - \bar{r}_1) = 0$$

yoki koordinatlar ko'rinishda

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Fazoda to'g'ri chiziq.

1) To'g'ri chiziqni ikki tekislikning kesishgan chizig'i deb qarash mumkin:

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases},$$

bunda $\dfrac{A_1}{A_2} \neq \dfrac{B_1}{B_2}$ yoki $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} \neq \dfrac{C_1}{C_2}$.

2) Bu tenglamada ketma-ket x va y ni yo'qotib, $x=az+c$, $y=bz+d$ ga ega bo'lamiz. Bu yerda to'g'ri chiziq, uni *XOZ*, *YOZ* tekisligiga proyeksiyalovchi ikkita tekislik bilan aniqlangan.

3) Ikki $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

4) $M_1(x_1, y_1, z_1)$ nuqtadan o'tib, $\bar{S} = \ell\bar{i} + m\bar{j} + n\bar{k}$ vektorga parallel to'g'ri chiziqning kanonik tenglamasi:

$$\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

Xususiy holda, uni quyidagicha

$$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\cos\beta}=\frac{z-z_1}{\cos\gamma}$$

yozish mumkin, bu yerda α, β, γ - to‘g‘ri chiziqning o‘qlar bilan tashkil qilgan burchaklari. To‘g‘ri chiziqning yo‘naltiruvchi kosinuslari

$$\cos\alpha=\frac{l}{\sqrt{\ell^2+m^2+n^2}},\quad \cos\beta=\frac{m}{\sqrt{\ell^2+m^2+n^2}},$$

$$\cos\gamma=\frac{n}{\sqrt{\ell^2+m^2+n^2}}$$

formulalar bilan aniqlanadi.

5) Kanonik tenglamalarda t parametr kiritib, parametrik tenglamalarga kelish mumkin:

$$\begin{cases} x=\ell t+x_1 \\ y=mt+y_1 \\ z=nt+z_1 \end{cases}$$

6) Kanonik tenglamalar bilan berilgan ikki to‘g‘ri chiziq orasidagi burchak

$$\cos\varphi=\frac{\ell_1\ell_2+m_1m_2+n_1n_2}{\sqrt{\ell_1^2+m_1^2+n_1^2}\cdot\sqrt{\ell_2^2+m_2^2+n_2^2}}$$

formula bilan hisoblanadi.

$\ell_1/\ell_2=m_1/m_2=n_1/n_2$ ikki to‘g‘ri chiziqning parallellik sharti,

$\ell_1\ell_2+m_1m_2+n_1n_2=0$ ikki to‘g‘ri chiziqning perpendikulyarlik sharti.

7) Kanonik tenglamalar bilan berilgan ikki to‘g‘ri chiziqning bir tekislikda yotish sharti

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix}=0.$$

Agar ℓ_1, m_1, n_1 lar ℓ_2, m_2, n_2 larga proportsioanal bo‘lmasa, u holda ko'rsatilgan munosabat ikki to'g'ri chiziqning fazoda kesishi shining zaruriy va yetarli shartidir.

8) $(x-x_1)/\ell=(y-y_1)/m=(z-z_1)/n$ to'g'ri chiziq va $Ax+By+Cz+D=0$ tekislik orasidagi burchak formulasi:

$$\sin\varphi=\frac{A\ell+Bm+Cn}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{\ell^2+m^2+n^2}}$$

$Al+Bm+Cn=0$ - to'g'ri chiziq va tekislikning parallellik sharti,

$A/\ell=B/m=C/n$ - to'g'ri chiziq va tekislikning perpendikulyarlik sharti.

9) To'g'ri chiziq va tekislik kesishgan nuqtasini topish uchun ularning tenglamasini birga yechish kerak.

a) Agar $A\ell+Bm+Cn\neq 0$ bo'lsa, to'g'ri chiziq tekislikni kesadi.

b) Agar $A\ell+Bm+Cn=0$, $Ax_1+By_1+Cz_1+D\neq 0$ bo'lsa, to'g'ri chiziq tekislikka parallel,

v) Agar $A\ell+Bm+Cn=0$, $Ax_1+By_1+Cz_1+D=0$ bo'lsa, to'g'ri chiziq tekislikda yotadi.

Hisoblash uchun vazifalar

1 – vazifa. Birinchi masalada tenglamalar sistemasining birgalikda ekanligi isbotlansin. Sistema Kramer va matritsa usuli bilan yechilsin.

2 – vazifa. Ikkinchi masalada $\bar{x}$ vektor $\bar{p}$, $\bar{q}$, $\bar{r}$ bazis vektorlari bo'yicha yoyilsin.

3 – vazifa. Uchinchi masalada $\overrightarrow{C_1}$ va $\overrightarrow{C_2}$ vektorlarning kollinearligi tekshirilsin.

4–vazifa. 4– masalada $\bar{a}$ va $\bar{b}$ vektorlarga qurilgan parallelogramm yuzi hisoblansin.

5 – vazifa. 5 – masalada $A(x_1,y_1,z_1)$, $B(x_2,y_2,z_2)$; $C(x_3,y_3,z_3)$ nuqtalar berilgan. A nuqtaga qo'yilgan $\overline{F}=\overline{AB}$ kuchining yo'naltiruvchi kosinuslari va C nuqtaga nisbatan moment miqdori topilsin.

6 – vazifa. 6 – masalada uchlari A_1, A_2, A_3, A_4 nuqtalarda yotgan tetraedrning hajmi va A_4 uchidan A_1, A_2, A_3 yoqqa tushirilgan balandligi hisoblansin

7 – vazifa . 7- masalada uchlari A, B, C nuqtalarda bo'lgan uchburchak berilgan :

a) AB to‘g‘ri chiziq tenglamasi yozilsin,

b) C uchidan tushirilgan balandlik tenglamasi yozilsin ,

c) B uchidan AC to‘g‘ri chiziqqacha bo‘lgan masofa topilsin ,

d) ichki A burchak bissektrisasining tenglamasi yozilsin.

8 – vazifa. 8– masalada A nuqtadan o‘tuvchi va $\overline{BC}$ vektorga perpendikulyar tekislik tenglamasi yozilsin .

9 – vazifa. 9– masalada M_4 nuqtadan M_1, M_2, M_3 – nuqtalar orqali o‘tuvchi tekislikkacha bo‘lgan masofa topilsin.

10 – vazifa.10 – masalada ikki tekislik orasidagi yoki tekislik bilan to‘g‘ri chiziq orasidagi yoki ikki to‘g‘ri chiziq orasidagi burchak topilsin.

11 – vazifa. 11 – masalada to‘g‘ri chiziqning umumiy tenglamasini normal va parametrik tenglama ko‘rinishiga keltirilsin.

12 – vazifa. 12 – masalada to‘g‘ri chiziq bilan tekislikning kesishish nuqtasi topilsin.

Ko‘rsatma variant

1. Sistema birgalikdaligi isbotlansin va yechimi Kramer hamda matritsa usuli yordamida topilsin:

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$$

Yechish: Noma'lumlar oldida turgan koeffitsentlardan tuzilgan Δ - aniqlovchini hisoblaymiz:

$$\Delta = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 3\cdot 8 - 4\cdot 2 + 1\cdot(-4) = 12 \neq 0$$

$\Delta \neq 0$ ekan, demak, sistema birgalikda va yagona yechimga ega bo‘ladi:

Kramer usuli:

$$\Delta x_1 = \begin{vmatrix} 5 & 2 & 1 \\ 1 & 3 & 1 \\ 11 & 1 & 3 \end{vmatrix} = 5\cdot 8 - 2\cdot(-8) + 1\cdot(-32) = 24,$$

$$\Delta x_2 = \begin{vmatrix} 3 & 5 & 1 \\ 2 & 1 & 1 \\ 2 & 11 & 3 \end{vmatrix} = 3\cdot(-8) - 5\cdot4 + 1\cdot 20 = -24,$$

$$\Delta x_3 = \begin{vmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \\ 2 & 1 & 11 \end{vmatrix} = 3\cdot32 - 2\cdot20 + 5\cdot(-4) = 36.$$

shunday qilib:

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{24}{12} = 2;$$

$$x_2 = \frac{\Delta x_2}{\Delta} = \frac{-24}{12} = -2;$$

$$x_3 = \frac{\Delta x_3}{\Delta} = \frac{36}{12} = 3;$$

Matritsa usuli:

sistemani quyidagi ko'rinishda yozamiz:

$AX = B$, bu yerda

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}; \quad B = \begin{pmatrix} 5 \\ 1 \\ 11 \end{pmatrix}.$$

Matritsa ko'rinishdagi tenglamaning yechimi ushbu ko'rinishga ega bo'ladi: $X = A^{-1}\cdot B$, bundagi A^{-1} ni ushbu formula yordamida topamiz:

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{22} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix},$$

bu yerda A_{ij} – element a_{ij} elementning algebraik to'ldiruvchisi $i = \overline{1,3}$ $j = \overline{1,3}$.

$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 8$ va x.k $|A| = \Delta = 12$.

shunday qilib

$$A^{-1} = \frac{1}{12}\begin{pmatrix} 8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix}$$

$$A^{-1}\cdot B=\frac{1}{12}\begin{pmatrix}8 & -5 & -1\\ -4 & 7 & -1\\ -4 & 1 & 5\end{pmatrix}\begin{pmatrix}5\\1\\11\end{pmatrix}=\frac{1}{12}\begin{pmatrix}24\\-24\\36\end{pmatrix}=\begin{pmatrix}2\\-2\\3\end{pmatrix}$$

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2\\-2\\3\end{pmatrix}$$

Javob $x_1=2,\quad x_2=-2,\quad x_3=3$

2. $\vec{x}$ vector $\vec{p},\ \vec{q},\ \vec{r}$ bazis vektorlar bo'yicha yoyilmasini yozing: bu yerda $\bar{x}=\{15,\quad -20,\quad -1\}$, $\vec{p}=\{0,\quad 2,\quad 1\}$,

$\vec{q}=\{0,\quad 1,\quad -1\},\qquad \vec{r}=\{5,\quad -3,\quad 2\}$

Yechish: Yoyilmani ushbu $\vec{x}=\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ ko'rinishda izlaymiz, bu yerda α,β,γ lar topilishi kerak bo'lgan noma'lumlar.

Bu tenglamani vektorial koordinata ko'rinishida yozamiz:.

$15\bar{i}-20\bar{j}-k=\alpha(2j+\quad k)+\beta(\bar{j}-\quad k)+\gamma(5\bar{\imath}-\quad 3j+2\bar{k})$

$15\bar{\imath}-20\bar{j}-k=5\gamma\cdot i+(2\alpha+\beta-3\gamma)\cdot j+(\alpha-\beta+2\gamma)\cdot k$

$$\begin{cases}15=5\gamma\\ -20=2\alpha+\beta-3\gamma\\ -1=\alpha-\beta+2\gamma\end{cases}\quad\Rightarrow\quad\begin{cases}\gamma=3\\ 2\alpha+\beta=-11\\ \alpha-\beta=-7\end{cases}$$

$\gamma=3\ ,\quad 3\alpha=-18\quad\Rightarrow\quad \alpha=-6\ ,\quad \beta=\alpha+7=1\ .$

Shunday qilib $\alpha=-6\ ,\ \beta=1\ ,\ \gamma=3$ qiymatlarga ega bo'lamiz

Javob: $\bar{x}=-6\bar{p}+\bar{q}+3\bar{r}$

3. $\bar{c_1}$ *ва* $\bar{c_2}$ vektorlar kolleniarmi?

$\bar{c_1}=4\bar{a}-3\bar{b}\ ,\qquad \bar{c_2}=9\bar{b}-12\bar{a}$

bu yerda

$\bar{a}=\{-1\ ,\quad 2\ ,\ 8\}$

$\bar{b}=\{3\ ,\quad 7\ ,\quad -1\}$

Yechish:

$4\bar{a}=\{-4,\ 8,\ 32\}\qquad 9\bar{b}=\{27,63,-9\}$

$-3\bar{b}=\{-9,-21,3\}\qquad -12\bar{a}=\{12,-24,-96\}$

$\bar{c_1}=4\bar{a}-3\bar{b}=\{-13,-13,35\}$

$\bar{c_2}=9\bar{b}-12\bar{a}=\{39,39,-105\}$

Vektorlarning kolleniarlik shartidan:

$$\frac{x_{c_1}}{x_{c_2}} = \frac{y_{c_1}}{y_{c_2}} = \frac{z_{c_1}}{z_{c_2}} = \lambda\ ; \qquad \frac{-13}{39} = \frac{-13}{39} = \frac{35}{-105} = -\frac{1}{3}$$

Javob: $\bar{c_1}$, $\bar{c_2}$ vektorlar kolleniar va qarama – qarshi yo'nalgan.

4. Tomonlari $\bar{a}$ va $\bar{b}$ vektorlardan iborat bo'lgan parallelogrammning yuzi hisoblansin:

$\bar{a} = 3\bar{p} + 2\bar{q}$, $\bar{b} = 2\bar{p} - \bar{q}$

$|\bar{p}| = 4$, $|\bar{q}| = 3$, $(\bar{p},\hat{}\ \bar{q}) = \frac{3\pi}{4}$.

Yechish: $S_{nap} = |\bar{a} \times \bar{b}|$

$\bar{a} \times \bar{b} = (3\bar{p} + 2\bar{q}) \times (2\bar{p} - \bar{q}) = 6\bar{p} \times \bar{p} - 3\bar{p} \times \bar{q} +$
$+2\bar{q} \times 2\bar{p} - 2\bar{q} \times \bar{q} = 2\bar{q} \times \bar{p}.$

Ma'lumki: $\bar{p} \times \bar{p} = 0,\quad \bar{q} \times \bar{q} = 0.\quad \bar{p} \times \bar{q} = -\bar{q} \times \bar{p}$

Demak $|\bar{a} \times \bar{b}| = 7|\bar{q}||\bar{p}|.sin\frac{3\pi}{4} = 7 \cdot 3 \cdot 4 \cdot \frac{\sqrt{2}}{2} = 42\sqrt{2}$

Javob: $S_{nap} = 42\sqrt{2} \cdot$ кв $\cdot$ б.

5. A nuqtaga qo'yilgan $\bar{F} = \overline{AB}$ kuchning yo'naltirovchi kosinuslari va C nuqtaga nisbatan moment miqdori topilsin, bu yerda

$A\,(3,\ 4,\ 5);\ B\ (-I,\ 7,\ 0);\ C\ (I, -I.\ I).$

Yechish: $\bar{F} = \overline{AB} = \overline{OB} - \overline{OA} = \{-4\ ,\ 3, -5\}.$

$\bar{M} = mom_c\,\bar{F} = \overline{CA} \times \overline{AB} = \overline{CA} \times \bar{F}, \overline{CA} = \{2, 5, 4\}.$

$$\bar{M} = mom_c\,\bar{F} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{x} \\ 2 & 5 & 4 \\ -4 & 3 & -5 \end{vmatrix} = -37\bar{\imath} - 6\bar{\jmath} + 26\bar{k}.$$

$|\bar{M}| = \sqrt{(-37)^2 + (-6)^2 + (26)^2} = \sqrt{2081}.$

$$Cos\ \alpha = \frac{M_x}{|\bar{M}|} = \frac{-37}{\sqrt{2081}}\ ; \quad cos\beta = \frac{M_x}{|\bar{M}|} = \frac{-6}{\sqrt{208}}$$

$$Cos\ j = \frac{M_z}{|\bar{M}|} = \frac{26}{\sqrt{2081}};$$

Javob : $|\bar{M}| = \sqrt{2081},\quad Cos\alpha = \frac{-37}{\sqrt{2081}},$

$$Cos\ \beta = \frac{-6}{\sqrt{2081}}, \quad Cos\ j = \frac{26}{\sqrt{2081}}.$$

6. Uchlari A_1, A_2, A_3 va A_4 nuqtalarda yotgan tetraedrining hajmi va A_4 uchidan $A_1\,A_2\,A_3$ yog'iga tushirilgan balandlik hisoblansin.

Bu yerda: $A_1(I, \quad -I, \quad 2) \qquad A_2(2, \quad I, \quad 2),$

$A_3(I, \quad I, \quad 4) \qquad A_4(6, \quad -3, \quad 8)$

Yechish: $V_{memp} = \frac{1}{3} \cdot h \cdot S_{\Delta A_1,A_2,A_3}. = \frac{1}{6}|\overline{A_1A_2} \cdot \overline{A_1A_3} \cdot \overline{A_1A_4}|.$

$$\overline{A_1A_2} = \overline{OA_2} - \overline{OA_1} = \{1,2,0\}$$

$$\overline{A_1A_3} = \overline{OA_3} - \overline{OA_1} = \{0,2,2\}, \quad \overline{A_1A_4} = \{5,-2,6\}$$

$$S_{\Delta_{A_1A_2A_3}} = \frac{1}{2}|\overline{A_1A_2} \times \overline{A_1A_3}| = \frac{1}{2}\left\| \begin{matrix} \vec{\imath} & \bar{\jmath} & \bar{k} \\ 1 & 2 & 0 \\ 0 & 2 & 2 \end{matrix} \right\| =$$

$$= \frac{1}{2}|4\bar{\imath} \quad -2\bar{\jmath} \quad +2\bar{\kappa}| = \frac{1}{2}\sqrt{4^2 + (-2)^2 + 2^2} = \sqrt{6}$$

$$V_{\text{тетр}} = \frac{1}{6}\begin{vmatrix} I & 2 & 0 \\ 0 & 2 & 2 \\ 5 & -2 & 6 \end{vmatrix} = \frac{1}{6}(I6 + \quad 20) = 6$$

$$h = \frac{3 \cdot V}{S} = \frac{3 \cdot 6}{\sqrt{6}} = 3\sqrt{6}$$

Javob: $V_{\text{тетр}} = 6$ kub birlik. $\quad h = 3\sqrt{6}$ birlik.

7. Uchlari $A\,(1, \quad -2)$; $B\,(7, \quad 6)$; $C\,(-11, \quad 3)$ nuqtalarda bo‘lgan uchburchak berilgan;

a) AB to‘g‘ri chiziq tenglamasi yozilsin ,

b) C uchidan tushirilgan balandlik tenglamasi yozilsin ,

c) B nuqtadan AC to‘g‘ri chiziqqacha bo‘lgan masofa topilsin ,

d) Ichki A burchak bissektrisasining tenglamasi yozilsin .

Yechish:

a) AB to‘g‘ri chiziq tenglamasini topish uchun ikki nuqta $A\,(1,-2)$ va $B(7,6)$ lardan o‘tuvchi to‘g‘ri chiziq tenglamasidan foydalanamiz:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - \quad y_1}{y_2 - \quad y_1} \Rightarrow \frac{x - \quad 1}{7 - \quad 1} = \frac{y + \quad 2}{6 + \quad 2} \Rightarrow$$

$$8x - \quad 8 = \quad 6y + 12 \Rightarrow \quad 4x - \quad 3y - 10 = 0 \ (AB)$$

Javob: $4x - \quad 3y - 10 = \quad 0$

b) $h_c \perp AB$ bo'lganligidan , burchak koeffitsiyentlari quyidagi munosobatda bo'ladi, ya'ni

$$K_{hc} = \frac{-1}{K_{AB}}$$

$$K_{hc} = -\frac{3}{4}$$

Berilgan C (-11 ,3) nuqtadan o'tgan to'g'ri chiziq tenglamasi

$$y - y_1 = \text{к}(x + x_1) \Rightarrow y - 3 = -\frac{3}{4}(x + 11)$$

yoki $3x + 4y + 21 = 0$

bu h_c balandlikning tenglamasi.

Javob: $3x + 4y + 21 = 0$

c) B nuqtadan AC to'g'ri chiziqqacha bo'lgan masofani topish uchun AC to'g'ri chiziq tenglamasini topamiz:

$$\frac{x - 1}{-11 - 1} = \frac{y + 2}{3 + 2}$$

$$5x + 12y + 19 = 0 \quad (AC)$$

Endi $B(7, 6)$ nuqtadan AC to'g'ri chiziqqacha bo'lgan masofa formulasidan foydalanamiz:

$$d = \frac{|A \cdot x_B + B \cdot y_B + C|}{\sqrt{A^2 + B^2}} \Rightarrow$$

$$d_{BM} = \frac{|5 \cdot 7 + 12 \cdot 6 + 19|}{\sqrt{25 + 144}} = \frac{126}{13},$$

Javob:

$$C_{BM} = \frac{126}{13}$$

d) Ichki A burchak bissektrisasining tenglamasini topish uchun bu bissektrisani AC va AB to'g'ri chiziqlaridan teng uzoqlikda yotgan nuqtalarning giometrik o'rni deb qaralib, uning tenglamasi

$$\frac{Ax + By + C}{\sqrt{A^2 + B^2}} = \mp \frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}}$$

ko'rinishda izlaymiz. AB va AC to'g'ri chiziqlar tenglamasini normal holga keltiramiz AB: $\frac{4}{5}x - \frac{3}{5}y - \frac{10}{5} = 0,$

$$AC: -\frac{5}{13}x - \frac{12}{13}y - \frac{19}{13} = 0$$

Bissektrisa tenglamasi

$$\left|\frac{4}{5}x - \frac{3}{5}y - \frac{10}{5}\right| = \left|-\frac{5}{13}x - \frac{12}{13}y - \frac{19}{13}\right|$$

B va *C* nuqtalar bissektrissaning turli tomonlarida yotishi shartidan

$$11x + 3y - 5 = 0.$$

Javob: $11x + 3y - 5 = 0.$

8. A $(5, 3 - I)$ nuqtadan o'tuvchi va $\overline{BC}$ vektorga perpendikulyar tekislik tenglamasi yozilsin. Bu yerda B(0, 0, -3) ; C (5, -I, 0).

Yechish: Bir nuqtadan o'tgan tekislik tenglamasidan foydalanamiz: ya'ni $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ bunda $\bar{n} = \{a, b, c,\}$ tekislikning normal vektori. Bu yerda $\bar{n} = \overline{BC} = \{5, -1, 3\}$. Izlanayotgan tekislik tenglamasi :

$$5(x - 5) - (y - 3) + 3(z + 1) = 0,$$

yoki

$$5x - y + 3z - 19 = 0,$$

Javob: $5x - y + 3z - 19 = 0,$

9. M_4 $(I, -I, 2)$ nuqtadan $M_1(I, 5, -7), M_2(-3, 6, 3)$ va $M_3(-2, 7, 3)$ nuqtalari orqali o'tuvchi tekislikkacha bo'lgan masofa topilsin.

Yechish: Uch nuqtadan o'tgan tekislik tenglamasi α ni vektor ko'rinishda izlaymiz, ya'ni $\overline{M_1M}\ \overline{M_1M_2}\ \overline{M_1M_3} = 0$

Koordinatlari bilan berilgan uch vektorning aralash ko'paymasi:

$$\alpha: \begin{vmatrix} x - 1 & y - 5 & z + 7 \\ -3 - 1 & 6 - 5 & 3 + 7 \\ -2 - 1 & 7 - 5 & 3 + 7 \end{vmatrix} = 0$$

yoki

$$\begin{vmatrix} x - 1 & y - 5 & z + 7 \\ -4 & 1 & 10 \\ -3 & 2 & 10 \end{vmatrix} = 0$$

$$\alpha: \quad (2x - 2y + z + 15) = 0$$

tekislikni normallaymiz.

$$\alpha: \quad -\frac{2}{3}x + \frac{2}{3}y - \frac{1}{3}z - 5 = 0:$$

Endi $M_\circ(1,-1,\ 2)$ nuqtalardan $\propto$ tekislikkacha bo'lgan masofani topamiz:

$$d = \left|-\frac{2}{3}x_0 + \frac{2}{3}y_0 - \frac{1}{3}z_0 - \frac{15}{3}\right| = \left|-\frac{2}{3}\cdot 1 - \frac{2}{3}\cdot 1 - \frac{1}{3}\cdot 2 - \frac{15}{3}\right|$$

$= 7.$

Javob: $d = 7.$

10. Ikki tekislik orasidagi burchak topilsin.

$\alpha \quad x + 2y - 2z - 7 = 0, \qquad \overline{n_1} = \{1, 2, -2\},$

$\beta: \quad x + y - 35 = 0, \qquad \overline{n_2} = \{1, 1, 0\}$

Yechish:

$$Cos\varphi = \frac{\overline{n_1}\cdot\overline{n_2}}{|\overline{n_1}||\overline{n_2}|} = \frac{1\cdot 1 + 2\cdot 1 - 0}{\sqrt{1+4+4}\cdot\sqrt{1+1+0}} = \frac{3}{3\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$$

ya'ni $cos\varphi = \frac{\sqrt{2}}{2}.$ Bundan $\varphi = \frac{\pi}{4}.$

Javob:

$$\varphi = \frac{\pi}{4}.$$

11. $\ell: \begin{cases} 2x - 3y - 2z + 6 = 0, \\ x - 3y + z + 3 = 0. \end{cases}$ to'g'ri chizikning umumiy tenglamasi kanonik va parametrik tenglama ko'rinishiga keltirilsin:

Yechish : To'g'ri chiziqning kononik ko'rinishdagi tenglamasi

$$\frac{x - x_\circ}{m} = \frac{y - y_\circ}{n} = \frac{z - z_\circ}{p} \qquad (1)$$

Yo'naltirovchi vektor $\vec{S} = \{m, n, p\}$ quyidagicha topiladi

$\vec{S} = \overline{n_1}x\overline{n_2}$ bunda $\overline{n_1} = \{2, -3, -2\},$ $\overline{n_2} = \{1, -3,\ 1\}$ ya'ni

$$\vec{S} = \overline{n_1}x\overline{n_2} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ 2 & -3 & 2 \\ 1 & -3 & 1 \end{vmatrix} = -9\bar{\imath} - 4\bar{\jmath} - 3\bar{k},$$

demak, $\vec{S} = -9\bar{\imath} - 4\bar{\jmath} - 3\bar{k} = \{-9, -4, -3\}$

$M_\circ(x_\circ\ \ y_\circ\ \ z_\circ)$ nuqtani topamiz. Buning uchun to'g'ri chiziqda $z_\circ = 0$ deb, ikki noma'lumli ikki tenglamalar sistemasini hosil qilamiz.

$$\begin{cases} 2x - 3y + 6 = 0 \\ x - 3y + 3 = 0. \end{cases}$$

Buni yechib, x, y noma'lumlarni topamiz :

$x_0 = -3, \quad y_0 = 0, \quad z_0 = 0, \qquad M_0(-3, 0, 0)$

Topilgan bu qiymatlarni

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{p}$$

ga qo'yib, to'g'ri chiziqning kanonik tenglamasini topamiz.

$$\frac{x + 3}{-9} = \frac{y - 0}{-4} = \frac{z - 0}{-3}$$

yoki

$$\frac{x + 3}{9} = \frac{y}{4} = \frac{z}{3},$$

bu tengliklarni t ga tenglab , to'g'ri chiziqning parametrik tenglamasini topamiz.

$$x = 9t - 3, \quad y = 4t, \quad z = 3t,$$

Javob

$$\frac{x + 3}{9} = \frac{y}{4} = \frac{z}{3}$$

$$x = 9t - 3, \quad y = 4t, \quad z = 3t.$$

12.

$$\frac{x - 7}{3} = \frac{y - 3}{1} = \frac{z + 1}{-2},$$

to'g'ri chiziq bilan

$$2x + y + 7z - 3 = 0$$

tekislikning kesishish nuqtasi $M_0(x_0 y_0 z_0)$ topilsin.

Yechish:

$$\frac{x - 7}{3} = \frac{y - 3}{1} = \frac{z + 1}{-2},$$

to'g'ri chiziqning parametrik tenglamasini yozamiz:

$$x = 3t + 7, \quad y = t + 3, \quad z = -2t - 1 \qquad (*)$$

bu yerdagi, x, y, z larning qiymatlarini

$$2x + y + 7z - 3 = 0.$$

tekislik tenglamasiga qo'yamiz , natijada t-ga nisbatan tenglama hosil bo'ladi .Bundan t ni topib, (*) ga qo'yamiz.

$$2(3t + 7) + (t + 3) + 7(-2t - 1) - 3 = 0 \quad 7t = 7, \quad t = 1$$

$$x_\circ = 3 \cdot 1 + 7 = 10, \quad y_\circ = 4, \quad z_\circ = -3,$$

Natijada *M (10,4,-3)* nuqtani topamiz.

Javob: *M (10,4,-3)*.

Variantlar

1-variant

1. $\begin{cases} 2x + y + 3z = 7 \\ 3x - 5y + z = 0 \\ 4x - 7y + z = -1 \end{cases}$

2. $\vec{x} = \{-2,4,7\}, \vec{p} = \{0,1,2\}, \quad \vec{q} = \{1,0,1\}, \quad \vec{r} = \{-1,2,4\}.$

3. $\vec{a} = \{1,-2,3\}, \quad \vec{b} = \{3,0,-1\}, \quad \vec{c}_1 = 2\vec{a} + 4\vec{b}, \quad \vec{c}_2 = 3\vec{b} - \vec{a}.$

4. $\vec{a} = \vec{p} + 2\vec{q}, \quad \vec{b} = 3\vec{p} - \vec{q} \quad |\vec{p}| = 1, \ |\vec{q}| = 2, \quad (\widehat{\vec{p},\vec{q}}) = \frac{\pi}{6}.$

5. $A(-1,2,1), \ B(3,-1,2), \ C(2,-1,3).$

6. $A_1(1,3,6), \ A_2(2,2,1), \ A_3(-1,0,1), \ A_4(-4,6,-3).$

7. $A(1,2); \ B(3,4); \ C(5,3).$

8. $A(1,0,-2); \ B(2,-1,3); \ C(0,-3,2).$

9. $M_1(-3,4,-7); \ M_2(1,5,-4); \ M_3(-5,-2,0);$ $M_0(-12,7,-1).$

10. $x - 3y + 5 = 0; \quad 2x - y + 5z - 11 = 0.$

11. $\begin{cases} 2x + y + z - 2 = 0 \\ 2x - y - 3z + 6 = 0 \end{cases}$ 12. $\begin{cases} \frac{x-1}{-1} = \frac{y+5}{4} = \frac{z-1}{2} \\ x - 2y + 7z - 24 = 0 \end{cases}$

2-variant

1. $\begin{cases} 3x + 4y + z = 10 \\ x + 3y - 2z = 9 \\ 3x - 2y + 2z = -3 \end{cases}$

2. $\vec{x} = \{6,12,-1\}, \vec{p} = \{1,3,0\} \quad \vec{q} = \{2,-1,1\}, \vec{r} = \{0,-1,2\}.$

3. $\vec{a} = \{1,0,1\}, \vec{b} = \{-2,3,5\} \quad \vec{c}_1 = \vec{a} + 2\vec{b}, \quad \vec{c}_2 = 3\vec{a} - \vec{b}.$

4. $\vec{a} = 3\vec{p} + \vec{q}, \quad \vec{b} = \vec{p} - 2\vec{q} \quad |\vec{p}| = 4, \ |\vec{q}| = 1, (\widehat{\vec{p},\vec{q}}) = \frac{\pi}{4}.$

5. $A(3,-1,4), \ B(1,2,-3), \ C(1,-1,2).$

6. $A_1(-4,2,6), A_2(2,-3,0), \ A_3(-10,5,8), \ A_4(-5,2,-4).$

7. $A(1,2); \ B(3,4); \ C(3,1).$

8. $A(-1,3,4)$; $B(-1,5,0)$; $C(2,6,1)$.
9. $M_1(-1,2,-3)$; $M_2(4,-1,0)$; $M_3(2,1,-2)$; $M_0(1,-6,-5)$.
10. $x-3y+z-1=0$; $x+z-1=0$.
11. $\begin{cases} x-3y+2z+2=0 \\ x+3y+z+14=0 \end{cases}$ 12. $\begin{cases} \frac{x+1}{3}=\frac{y-3}{-4}=\frac{z+1}{5} \\ x+2y-5z+20=0 \end{cases}$

3-variant

1. $\begin{cases} 2x+3y+5z=10 \\ 3x+7y+4z=3 \\ x+2y+2z=3 \end{cases}$
2. $\vec{x}=\{1,-4,4\}, \vec{p}=\{2,1,-1\}, \vec{q}=\{0,3,2\}, \vec{r}=\{1,-1,1\}$.
3. $\vec{a}=\{-2,4,1\}, \vec{b}=\{1,-2,7\}, \vec{c}_1=5\vec{a}+3\vec{b}, \vec{c}_2=2\vec{a}-\vec{b}$.
4. $\vec{a}=\vec{p}+3\vec{q}, \vec{b}=\vec{p}+2\vec{q}, |\vec{p}|=\frac{1}{5}, |\vec{q}|=1, (\widehat{\vec{p},\vec{q}})=\frac{\pi}{2}$
5. $A(1,-3,4)$, $B(3,-4,2)$, $C(-1,1,4)$.
6. $A_1(7,2,4)$, $A_2(7,-1,-2)$, $A_3(3,3,1)$, $A_4(-4,2,1)$.
7. $A(5,3)$; $B(3,4)$; $C(3,1)$.
8. $A(4,-2,1)$; $B(1,-1,5)$; $C(-2,1,-3)$.
9. $M_1(-3,-1,1)$; $M_2(-9,1,-2)$; $M_3(3,-5,4)$;
$M_0(-7,0,-1)$.
10. $4x-5y+3z-1=0$; $x-4y-z+9=0$.
11. $\begin{cases} x-2y+z-4=0 \\ 2x+2y-z-8=0 \end{cases}$ 12. $\begin{cases} \frac{x-2}{-1}=\frac{y-3}{-1}=\frac{z+1}{4} \\ x+2y+3z-14=0 \end{cases}$

4-variant

1. $\begin{cases} 5x-6y+4z=3 \\ 3x-3y+2z=2 \\ 4x-5y+2z=1 \end{cases}$
2. $\vec{x}=\{-9,5,5\}, \vec{p}=\{4,1,1\}, \vec{q}=\{2,0,-3\}, \vec{r}=\{-1,2,1\}$.
3. $\vec{a}=\{1,2,-3\}, \vec{b}=\{2,-1,-1\}, \vec{c}_1=4\vec{a}+3\vec{b}, \vec{c}_2=8\vec{a}-\vec{b}$.
4. $\vec{a}=3\vec{p}-2\vec{q}, \vec{b}=\vec{p}+5\vec{q}, |\vec{p}|=4, |\vec{q}|=\frac{1}{2}, (\widehat{\vec{p},\vec{q}})=\frac{5\pi}{6}$.
5. $A(3,-6,1)$, $B(1,4,-5)$, $C(3,-4,2)$.
6. $A_1(2,1,4)$, $A_2(-1,5,-2)$, $A_3(-7,-3,2)$, $A_4(-6,-3,6)$.
7. $A(-2,2)$; $B(1,5)$; $C(1,1)$.

8. $A(-8,0,7)$; $B(-3,2,4)$; $C(-1,4,5)$.
9. $M_1(1,-1,1)$; $M_2(-2,0,3)$; $M_3(2,1,-1)$; $M_0(-2,4,2)$.
10. $3x - y + 2z + 15 = 0$; $5x + 9y - 3z - 1 = 0$.
11. $\begin{cases} x + y + z - 2 = 0 \\ x - y - 2z + 2 = 0 \end{cases}$ 12. $\begin{cases} \frac{x+1}{-3} = \frac{y+2}{2} = \frac{z-3}{-2} \\ x + 3y - 5z + 9 = 0 \end{cases}$

5-variant

1. $\begin{cases} 4x - 3y + 2z = -4 \\ 6x - 2y + 3z = -1 \\ 5x - 3y + 2z = -3 \end{cases}$
2. $\vec{x} = \{-5,-5,5\}$, $\vec{p} = \{-2,0,1\}$, $\vec{q} = \{1,3,-1\}$, $\vec{r} = \{0,4,1\}$.
3. $\vec{a} = \{3,5,4\}$, $\vec{b} = \{5,9,7\}$, $\vec{c}_1 = -2\vec{a} + \vec{b}$, $\vec{c}_2 = 3\vec{a} - 2\vec{b}$.
4. $\vec{a} = \vec{p} - 2\vec{q}$, $\vec{b} = 2\vec{p} + \vec{q}$, $|\vec{p}| = 2$, $|\vec{q}| = 3$, $(\widehat{\vec{p},\vec{q}}) = \frac{3\pi}{4}$.
5. $A(-2,1,1)$, $B(1,5,0)$, $C(4,4,-2)$.
6. $A_1(-1,-5,2)$, $A_2(-6,0,-3)$, $A_3(3,6,-3)$, $A_4(10,6,7)$.
7. $A(-2,2)$; $B(1,5)$; $C(4,2)$.
8. $A(7,-5,1)$; $B(5,-1,-3)$; $C(3,0,-4)$.
9. $M_1(1,2,0)$; $M_2(1,-1,2)$; $M_3(0,1,-1)$; $M_0(2,-1,4)$.
10. $6x + 2y - 4z + 17 = 0$; $9x + 3y - 6z - 4 = 0$.
11. $\begin{cases} 2x + 3y + z + 6 = 0 \\ x - 3y - 2z + 3 = 0 \end{cases}$ 12. $\begin{cases} \frac{x-5}{1} = \frac{y-3}{-1} = \frac{z-2}{0} \\ 3x + y - 5z - 12 = 0 \end{cases}$

6-variant

1. $\begin{cases} 5x + 2y + 3z = -2 \\ 2x - 2y + 5z = 0 \\ 3x + 4y + 2z = -10 \end{cases}$
2. $\vec{x} = \{13,2,7\}$, $\vec{p} = \{-2,0,1\}$, $\vec{q} = \{1,3,-1\}$, $\vec{r} = \{0,4,1\}$.
3. $\vec{a} = \{1,4,-2\}$, $\vec{b} = \{1,1,-1\}$, $\vec{c}_1 = \vec{a} + \vec{b}$, $\vec{c}_2 = 4\vec{a} + 2\vec{b}$.
4. $\vec{a} = \vec{p} + 3\vec{q}$, $\vec{b} = \vec{p} - 2\vec{q}$, $|\vec{p}| = 2$, $|\vec{q}| = 3$, $(\widehat{\vec{p},\vec{q}}) = \frac{\pi}{3}$.
5. $A(-2,3,-4)$, $B(3,2,5)$, $C(4,2,-1)$.
6. $A_1(0,-1,-1)$, $A_2(-2,3,5)$, $A_3(1,-5,-9)$, $A_4(-1,-6,3)$.
7. $A(-1,1)$; $B(0,3)$; $C(1,1)$.

8. $A(-3,5,-2)$; $B(-4,0,3)$; $C(-3,2,5)$.
9. $M_1(1,0,2)$; $M_2(1,2,-1)$; $M_3(2,-2,1)$; $M_0(-5,-9,1)$.
10. $x - y\sqrt{2} + z - 1 = 0$; $x + 2y + 6z - 12 = 0$.
11. $\begin{cases} 3x + y - z - 6 = 0 \\ 3x - y + 2z = 0 \end{cases}$ 12. $\begin{cases} \frac{x-1}{1} = \frac{y}{0} = \frac{z+3}{2} \\ 2x - y + 4z = 0 \end{cases}$

Variant 7

1. $\begin{cases} 2x + 5y - 7z = 11 \\ x + 3y - z = -4 \\ 3x + 8y - 6z = 11 \end{cases}$
2. $\vec{x} = \{-19,-1,7\}$, $\vec{p} = \{0,1,1\}$, $\vec{q} = \{-2,0,1\}$, $\vec{r} = \{3,1,0\}$.
3. $\vec{a} = \{1,-2,5\}$, $\vec{b} = \{3,-1,0\}$, $\vec{c}_1 = 4\vec{a} - 2\vec{b}$, $\vec{c}_2 = \vec{b} - 2\vec{a}$.
4. $\vec{a} = 2\vec{p} - \vec{q}$, $\vec{b} = \vec{p} + 2\vec{q}$, $|\vec{p}| = 3$, $|\vec{q}| = 2$, $(\widehat{\vec{p},\vec{q}}) = \frac{\pi}{2}$.
5. $A(3,2,-4)$, $B(5,6,-1)$, $C(4,2,-1)$.
6. $A_1(5,2,0)$, $A_2(2,5,0)$, $A_3(1,2,4)$, $A_4(-1,1,1)$.
7. $A(-2,1)$; $B(1,3)$; $C(3,0)$.
8. $A(1,-1,8)$; $B(-4,-3,10)$; $C(-1,-1,7)$.
9. $M_1(1,2,-3)$; $M_2(1,0,1)$; $M_3(-2,-1,6)$; $M_0(3,-2,-9)$.
10. $3y - z = 0$; $2y + z = 0$.
11. $\begin{cases} x + 5y + 2z + 11 = 0 \\ x - y - z - 1 = 0 \end{cases}$ 12. $\begin{cases} \frac{x-1}{-2} = \frac{y-2}{1} = \frac{z+1}{-1} \\ x - 2y + 5z + 17 = 0 \end{cases}$

8-variant

1. $\begin{cases} x - 3y + 2z = 3 \\ 2x - y + 3z = -5 \\ x + 6y - z = 4 \end{cases}$
2. $\vec{x} = \{3,-3,4\}$, $\vec{p} = \{1,0,2\}$, $\vec{q} = \{0,1,1\}$, $\vec{r} = \{2,-1,4\}$.
3. $\vec{a} = \{3,4,-1\}$, $\vec{b} = \{2,-1,2\}$, $\vec{c}_1 = 6\vec{a} - 3\vec{b}$, $\vec{c}_2 = \vec{b} - 2\vec{a}$.
4. $\vec{a} = 4\vec{p} + \vec{q}$, $\vec{b} = \vec{p} - \vec{q}$, $|\vec{p}| = 7$, $|\vec{q}| = 2$, $(\widehat{\vec{p},\vec{q}}) = \frac{\pi}{4}$.
5. $A(3,2,-3)$, $B(5,1,-1)$, $C(1,-2,1)$.
6. $A_1(2,-1,-2)$, $A_2(1,2,1)$ $A_3(5,0,-6)$, $A_4(-10,9,-7)$.
7. $A(-5,2)$; $B(-2,5)$; $C(-1,2)$.
8. $A(-2,0,5)$; $B(2,7,-3)$; $C(1,10,-1)$.

9. $M_1(3,10,-1)$; $M_2(-2,3,-5)$; $M_3(-6,0,-3)$; $M_0(-6,7,-10)$.

10. $6x+3y-2z=0$; $x+2y+6z-12=0$.

11. $\begin{cases}3x+4y-2z+1=0\\2x-4y+3z+4=0\end{cases}$ 12. $\begin{cases}\frac{x-1}{2}=\frac{y-2}{0}=\frac{z-4}{1}\\x-2y+4z-19=0\end{cases}$

9-variant

1. $\begin{cases}x+3y-z=3\\2x-5y+3z=11\\6x-y-7z=17\end{cases}$

2. $\vec{x}=\{3,3,-1\}$, $\vec{p}=\{3,1,0\}$, $\vec{q}=\{-1,2,1\}$, $\vec{r}=\{-1,0,2\}$.

3. $\vec{a}=\{-2,-3,-2\}$, $\vec{b}=\{1,0,5\}$, $\vec{c}_1=3\vec{a}+9\vec{b}$, $\vec{c}_2=-\vec{a}-3\vec{b}$.

4. $\vec{a}=\vec{p}-4\vec{q}$, $\vec{b}=3\vec{p}+\vec{q}$, $|\vec{p}|=1$, $|\vec{q}|=2$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{6}$.

5. $A(-1,-2,4)$, $B(-4,2,0)$, $C(3,-2,-1)$.

6. $A_1(-2,0,-4)$, $A_2(-1,7,1)$, $A_3(4,-8,-4)$, $A_4(1,-4,6)$.

7. $A(-5,2)$; $B(-2,4)$; $C(-2,0)$.

8. $A(1,9,-4)$; $B(5,7,1)$; $C(3,5,0)$.

9. $M_1(-1,2,4)$; $M_2(-1,-2,-4)$; $M_3(3,0,-1)$; $M_0(-2,3,5)$.

10. $x+2y+2z-3=0$; $16x+12y-15z-1=0$.

11. $\begin{cases}5x+y-3z+4=0\\x-y+2z+2=0\end{cases}$ 12. $\begin{cases}\frac{x+2}{-1}=\frac{y-1}{1}=\frac{z+4}{-1}\\2x-y+3z+23=0\end{cases}$

10-variant

1. $\begin{cases}2x-5y+6z=-5\\x+3y+4z=4\\3x-2y+13z=2\end{cases}$

2. $\vec{x}=\{-1,7,-4\}$, $\vec{p}=\{-1,2,1\}$, $\vec{q}=\{2,0,3\}$, $\vec{r}=\{1,1,-1\}$.

3. $\vec{a}=\{-1,4,2\}$, $\vec{b}=\{3,-2,6\}$, $\vec{c}_1=2\vec{a}-\vec{b}$, $\vec{c}_2=3\vec{b}-6\vec{a}$.

4. $\vec{a}=\vec{p}+4\vec{q}$, $\vec{b}=2\vec{p}-\vec{q}$, $|\vec{p}|=7$, $|\vec{q}|=2$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{3}$.

5. $A(1,-2,2)$, $B(-4,1,1)$, $C(-5,-5,3)$.

6. $A_1(14,4,5)$, $A_2(-5,-3,2)$, $A_3(-2,-6,-3)$, $A_4(-2,2,-1)$.

7. $A(-1,1)$; $B(1,3)$; $C(1,10)$.

8. $A(-7,0,3)$; $B(1,-5,4)$; $C(2,-3,0)$.

9. $M_1(0,-3,1)$; $M_2(-4,1,2)$; $M_3(2,-1,5)$; $M_0(-3,4,-5)$.

10. $2x - y + 5z + 16 = 0$; $x + 2y + 3z + 8 = 0$.

11. $\begin{cases} x - y - z - 2 = 0 \\ x - 2y + z + 4 = 0 \end{cases}$ 12. $\begin{cases} \frac{x+2}{1} = \frac{y-2}{0} = \frac{z+3}{0} \\ 2x - 3y - 5z - 7 = 0 \end{cases}$

11-variant

1. $\begin{cases} x + 3y - z = 5 \\ 2x - 4y + 3z = 0 \\ x + 5y + 2z = 15 \end{cases}$

2. $\vec{x} = \{6,5,-14\}, \vec{p} = \{1,1,4\}$, $\vec{q} = \{0,-3,2\}$, $\vec{r} = \{2,1,-1\}$.

3. $\vec{a} = \{5,0,-1\}$, $\vec{b} = \{7,2,3\}$, $\vec{c}_1 = 2\vec{a} - \vec{b}$, $\vec{c}_2 = 3\vec{b} - 6\vec{a}$.

4. $\vec{a} = 3\vec{p} + 2\vec{q}$, $\vec{b} = \vec{p} - \vec{q}$, $|\vec{p}| = 10$, $|\vec{q}| = 1$, $(\widehat{\vec{p},\vec{q}}) = \frac{\pi}{2}$.

5. $A(1,-8,-7)$, $B(2,2,1)$, $C(1,-1,2)$.

6. $A_1(1,2,0)$, $A_2(3,0,-3)$ $A_3(5,2,6)$, $A_4(8,4,-9)$.

7. $A(-3,-3)$; $B(-2,0)$; $C(0,-2)$.

8. $A(0,-3,5)$; $B(-7,2,6)$; $C(-3,2,4)$.

9. $M_1(1,3,0)$; $M_2(4,-1,2)$; $M_3(3,0,1)$; $M_0(4,3,0)$.

10. $\begin{cases} 3x - 2y = 24 \\ 3x - z = -4 \end{cases}$, $6x + 15y - 10z + 31 = 0$.

11. $\begin{cases} 4x + y - 3z + 2 = 0 \\ 2x - y + z - 8 = 0 \end{cases}$ 12. $\begin{cases} \frac{x-1}{2} = \frac{y-1}{-1} = \frac{z+2}{3} \\ 4x + 2y - z = 11 \end{cases}$

12-variant

1. $\begin{cases} 5x - 2y + z = 4 \\ 4x - 3y + 2z = 4 \\ x + 2y + 3z = 14 \end{cases}$

2. $\vec{x} = \{6,-1,7\}$, $\vec{p} = \{1,-2,0\}$, $\vec{q} = \{-1,1,3\}$, $\vec{r} = \{1,0,4\}$.

3. $\vec{a} = \{0,3,2\}$, $\vec{b} = \{1,-2,1\}$, $\vec{c}_1 = 5\vec{a} - 2\vec{b}$, $\vec{c}_2 = 3\vec{a} + 5\vec{b}$.

4. $\vec{a} = 4\vec{p} - \vec{q}$, $\vec{b} = \vec{p} + 2\vec{q}$, $|\vec{p}| = 5$, $|\vec{q}| = 4$, $(\widehat{\vec{p},\vec{q}}) = \frac{\pi}{4}$.

5. $A(2,7,-6)$, $B(7,-9,9)$, $C(1,-3,1)$.

6. $A_1(2,-1,2)$, $A_2(1,2,-1)$, $A_3(3,2,1)$, $A_4(-4,2,5)$.

7. $A(-4,-4)$; $B(-2,0)$; $C(0,-3)$.

8. $A(5,-1,2)$; $B(2,-4,3)$; $C(4,-1,3)$.

9. $M_1(-2,-1,-1)$; $M_2(0,3,2)$; $M_3(3,1,-4)$;
$M_0(-21,20,-16)$.

10. $\frac{x}{2}=\frac{y-1}{-1}=\frac{z+1}{2}$, $x+2y+3z-29=0$.

11. $\begin{cases}3x+2y-2z-1=0\\2x-3y+z+6=0\end{cases}$ 12. $\begin{cases}\frac{x-1}{1}=\frac{y+1}{0}=\frac{z-1}{1}\\3x-2y-4z-8=0\end{cases}$

13-variant

1. $\begin{cases}6x-2y+z=4\\x-3y+4z=15\\2x-y+3z=6\end{cases}$
2. $\vec{x}=\{5,15,0\}$, $\vec{p}=\{1,0,5\}$, $\vec{q}=\{-1,3,2\}$, $\vec{r}=\{0,-1,1\}$.
3. $\vec{a}=\{-2,7,-1\}$, $\vec{b}=\{-3,5,2\}$, $\vec{c}_1=2\vec{a}+3\vec{b}$, $\vec{c}_2=3\vec{a}+2\vec{b}$.
4. $\vec{a}=5\vec{p}-\vec{q}, \vec{b}=\vec{p}+5\vec{q}$, $|\vec{p}|=2$, $|\vec{q}|=1$, $(\widehat{\vec{p},\vec{q}})=\frac{3\pi}{4}$.
5. $A(2,-1,-3)$, $B(3,2,-1)$, $C(-4,1,3)$.
6. $A_1(1,1,2)$, $A_2(-1,1,3)$, $A_3(2,-2,4)$, $A_4(-1,8,-2)$.
7. $A(-1,3)$; $B(2,0)$; $C(-2,1)$.
8. $A(-3,7,2)$; $B(3,5,-1)$; $C(4,5,3)$.
9. $M_1(-3,-5,6)$; $M_2(2,1,-4)$; $M_3(0,-3,-1)$; $M_0(3,6,8)$.
10. $\begin{cases}y=3x-1\\2z=-3x+2\end{cases}$, $2x+y+z-4=0$
11. $\begin{cases}6x-7y-4z-2=0\\x+7y-z-5=0\end{cases}$ 12. $\begin{cases}\frac{x+2}{-1}=\frac{y-1}{1}=\frac{z+3}{2}\\x+2y-z-2=0\end{cases}$

14-variant

1. $\begin{cases}3x+5y-7z=6\\2x-y+3z=3\\4x-7y+8z=-2\end{cases}$
2. $\vec{x}=\{2,-1,11\}$, $\vec{p}=\{1,1,0\}$, $\vec{q}=\{0,1,2\}$, $\vec{r}=\{1,0,3\}$.
3. $\vec{a}=\{3,7,0\}$, $\vec{b}=\{1,-3,4\}$, $\vec{c}_1=4\vec{a}-2\vec{b}$, $\vec{c}_2=\vec{b}-2\vec{a}$.
4. $\vec{a}=\vec{q}-5\vec{p}$, $\vec{b}=\vec{p}+\vec{q}$, $|\vec{p}|=7$, $|\vec{q}|=\frac{1}{2}$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{3}$.
5. $A(1,2,0)$, $B(3,0,-3)$, $C(5,2,6)$.
6. $A_1(2,3,1)$, $A_2(4,1,-2)$, $A_3(6,3,7)$, $A_4(7,5,-3)$.
7. $A(-1,1)$; $B(2,-2)$; $C(-4,-3)$.

8. $A(0,-2,8)$; $B(4,3,2)$; $C(1,4,3)$.

9. $M_1(2,-4,-3)$; $M_2(5,-6,0)$;
$M_3(-1,3,-3)$; $M_0(2,-10,8)$.

10. $\frac{x-2}{0}=\frac{y+1}{-1}=\frac{z+5}{1}$, $2x+y+7z-3=0$

11. $\begin{cases}8x-y-3z-1=0\\x+y+z+10=0\end{cases}$ 12. $\begin{cases}\frac{x+3}{1}=\frac{y-2}{-5}=\frac{z+2}{3}\\5x-y+4z+8=0\end{cases}$

15-variant

1. $\begin{cases}7x-y+z=6\\4x-2y+5z=14\\x+y-z=2\end{cases}$

2. $\vec{x}=\{11,5,-3\}$, $\vec{p}=\{1,0,2\}$, $\vec{q}=\{-1,0,1\}$, $\vec{r}=\{2,5,-3\}$.

3. $\vec{a}=\{-1,2,-1\}$, $\vec{b}=\{-2,-7,1,\}$ $\vec{c}_1=4\vec{a}-2\vec{b}$, $\vec{c}_2=\vec{b}-2\vec{a}$.

4. $\vec{a}=2\vec{p}+3\vec{q}$, $\vec{b}=\vec{q}-5\vec{p}$, $|\vec{p}|=2$, $|\vec{q}|=3$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{4}$.

5. $A(2,-4,5)$, $B(4,-2,3)$, $C(3,2,-1)$.

6. $A_1(1,1,-1)$, $A_2(2,3,1)$, $A_3(3,2,1)$, $A_4(5,9,-8)$.

7. $A(-3,1)$; $B(-2,3)$; $C(-1,1)$.

8. $A(1,-1,5)$; $B(0,7,8)$; $C(-1,3,8)$.

9. $M_1(1,-1,2)$; $M_2(2,1,2)$; $M_3(1,1,4)$; $M_0(-3,2,7)$.

10. $\frac{x-1}{-1}=\frac{y+5}{4}=\frac{z-1}{2}$, $x-2y+7z-24=0$.

11. $\begin{cases}6x-5y-4z+8=0\\6x+3y+3z+4=0\end{cases}$ 12. $\begin{cases}\frac{x-2}{2}=\frac{y-2}{-1}=\frac{z-4}{3}\\x+3y+5z-42=0\end{cases}$

16-variant

1. $\begin{cases}4x+y-2z=17\\x+3y+z=14\\3x-y-z=8\end{cases}$

2. $\vec{x}=\{8,0,5\}$, $\vec{p}=\{2,0,1\}$, $\vec{q}=\{1,1,0\}$, $\vec{r}=\{4,1,2\}$.

3. $\vec{a}=\{7,9,-2\}$, $\vec{b}=\{5,4,3\}$. $\vec{c}_1=4\vec{a}-\vec{b}$, $\vec{c}_2=4\vec{b}-\vec{a}$.

4. $\vec{a}=2\vec{p}-3\vec{q}$, $\vec{b}=3\vec{p}+\vec{q}$, $|\vec{p}|=4$, $|\vec{q}|=1$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{6}$.

5. $A(3,2,-4)$, $B(2,-1,1)$, $C(4,-2,3)$.

6. $A_1(1,5,-7)$ $A_2(-3,6,3)$, $A_3(-2,7,3)$, $A_4(-4,8,-12)$.

7. A(2,3); B(0,1); C(3,0).

8. A(−10,0,9); B(12,4,11); C(8,5,15).

9. $M_1(1,3,6)$; $M_2(2,2,1)$; $M_3(-1,0,1)$; $M_0(5,-4,5)$.

10. $\frac{x-5}{1}=\frac{y-3}{-1}=\frac{z-2}{0}$, $3x+y-5z-12=0$.

11. $\begin{cases} x+5y-z-5=0 \\ 2x-5y+2z+5=0 \end{cases}$ 12. $\begin{cases} \frac{x-3}{-1}=\frac{y-4}{5}=\frac{z-4}{2} \\ 7x+y+4z-47=0 \end{cases}$

17-variant

1. $\begin{cases} x+2y+z=7 \\ 3x-y+2z=1 \\ 4x+y-z=12 \end{cases}$

2. $\vec{x}=\{3,1,8\}$, $\vec{p}=\{0,1,3\}$, $\vec{q}=\{1,2,-1\}$, $\vec{r}=\{2,0,-1\}$.

3. $\vec{a}=\{5,0,-2\}$, $\vec{b}=\{6,4,3\}$, $\vec{c}_1=5\vec{a}-3\vec{b}$, $\vec{c}_2=6\vec{b}-10\vec{a}$.

4. $\vec{a}=5\vec{p}+\vec{q}$, $\vec{b}=\vec{p}-3\vec{q}$, $|\vec{p}|=1$, $|\vec{q}|=2$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{3}$.

5. $A(1,2,-1)$, $B(1,-1,3)$, $C(1,9,-11)$.

6. $A_1(-3,4,7)$, $A_2(1,5,-4)$, $A_3(-5,-2,0)$, $A_4(2,5,4)$.

7. $A(2,2)$; $B(-1,1)$; $C(1,-2)$.

8. $A(3,-3,-6)$; $B(1,9,-5)$; $C(6,6,-4)$.

9. $M_1(-4,2,6)$; $M_2(2,-3,0)$; $M_3(-10,5,8)$; $M_0(-12,1,8)$.

10. $\frac{x-1}{2}=\frac{y-2}{0}=\frac{z-4}{1}$, $x-2y+4z-19=0$.

11. $\begin{cases} 2x-3y+z+6=0 \\ x-3y-2z+3=0 \end{cases}$ 12. $\begin{cases} \frac{x+3}{2}=\frac{y-1}{3}=\frac{z-1}{5} \\ 2x+3y+7z-52=0 \end{cases}$

18-variant

1. $\begin{cases} 3x-y-2z=4 \\ 2x+3y+z=9 \\ 4x+y-4z=15 \end{cases}$

2. $\vec{x}=\{8,1,12\}$, $\vec{p}=\{1,2,-1\}$, $\vec{q}=\{3,0,2\}$, $\vec{r}=\{-1,1,1\}$.

3. $\vec{a}=\{8,3,-1\}$, $\vec{b}=\{4,1,3\}$, $\vec{c}_1=2\vec{a}-\vec{b}$, $\vec{c}_2=2\vec{b}-4\vec{a}$.

4. $\vec{a}=7\vec{p}-2\vec{q}$, $\vec{b}=\vec{p}+3\vec{q}$, $|\vec{p}|=\frac{1}{2}$, $|\vec{q}|=2$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{2}$.

5. $A(3,-2,1)$, $B(2,1,2)$, $C(3,-1,-2)$.

6. $A_1(-1,2,-3)$, $A_2(4,-1,0)$, $A_3(2,1,-2)$, $A_4(3,4,5)$.

7. $A(-3,-1)$; $B(3,3)$; $C(2,-3)$.
8. $A(2,1,7)$; $B(9,0,2)$; $C(9,2,3)$.
9. $M_1(7,2,4)$; $M_2(7,-1,-2)$; $M_3(-5,-2,-1)$; $M_0(10,1,8)$.
10. $\frac{x+2}{-1}=\frac{y-1}{1}=\frac{z+4}{-1}$, $2x-y+3z+23=0$.
11. $\begin{cases}5x+y+2z+4=0\\x-y-3z+2=0\end{cases}$ 12. $\begin{cases}\frac{x-3}{2}=\frac{y+1}{3}=\frac{z+3}{2}\\3x+4y+7z-16=0\end{cases}$

19-variant

1. $\begin{cases}3x+4y+2z=8\\2x-4y-3z=1\\x+5y+z=0\end{cases}$
2. $\vec{x}=\{-9,-8,-3\}$, $\vec{p}=\{1,4,1\}$, $\vec{q}=\{-3,2,0\}$, $\vec{r}=\{1,-1,2\}$.
3. $\vec{a}=\{3,-1,6\}$, $\vec{b}=\{5,7,10\}$, $\vec{c}_1=4\vec{a}-2\vec{b}$, $\vec{c}_2=\vec{b}-2\vec{a}$.
4. $\vec{a}=6\vec{p}-\vec{q}$, $\vec{b}=\vec{p}+\vec{q}$, $|\vec{p}|=3$, $|\vec{q}|=4$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{4}$.
5. $A(2,-1,1)$, $B(5,5,4)$, $C(3,2,-1)$.
6. $A_1(4,-1,3)$, $A_2(-2,1,0)$, $A_3(0,-5,1)$, $A_4(3,2,-6)$.
7. $A(4,2)$; $B(-2,-2)$; $C(2,-3)$.
8. $A(-7,1,-4)$; $B(8,11,-3)$; $C(9,9,-1)$.
9. $M_1(2,1,4)$; $M_2(3,5,-2)$; $M_3(-7,-3,2)$; $M_0(-3,1,8)$.
10. $\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z+2}{3}$, $4x+2y-z-11=0$
11. $\begin{cases}4x+y+z+2=0\\2x-y-3z-8=0\end{cases}$ 12. $\begin{cases}\frac{x-5}{-2}=\frac{y-2}{0}=\frac{z+4}{-1}\\2x-5y+4z+24=0\end{cases}$

20-variant

1. $\begin{cases}7x-5y=31\\4x+11y=-43\\2x-3y+4z=-20\end{cases}$
2. $\vec{x}=\{-5,9,-13\}$, $\vec{p}=\{0,1,-2\}$, $\vec{q}=\{3,-1,1\}$, $\vec{r}=\{4,1,0\}$.
3. $\vec{a}=\{1,-2,4\}$, $\vec{b}=\{7,3,5\}$, $\vec{c}_1=6\vec{a}-3\vec{b}$, $\vec{c}_2=\vec{b}-2\vec{a}$.
4. $\vec{a}=10\vec{p}+\vec{q}$, $\vec{b}=3\vec{p}-2\vec{q}$, $|\vec{p}|=4$, $|\vec{q}|=1$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{6}$.
5. $A(2,3,1)$, $B(4,1,-2)$, $C(6,3,7)$.
6. $A_1(1,-1,1)$, $A_2(-2,0,3)$, $A_3(2,1,-1)$, $A_4(2,-2,-4)$.

7. $A(-1,2)$; $B(3,1)$; $C(1,-1)$.
8. $A(1,0,-6)$; $B(-7,2,1)$; $C(-9,6,1)$.
9. $M_1(-1,-5,2)$; $M_2(-6,0,-3)$; $M_3(3,6,-3)$; $M_0(10,-8,-7)$.
10. $\frac{x-2}{2}=\frac{y-2}{-1}=\frac{z-4}{3}$, $x+3y+5z-42=0$.
11. $\begin{cases}2x+y-3z-2=0\\2x-y+z+6=0\end{cases}$ 12. $\begin{cases}\frac{x-1}{8}=\frac{y-8}{-5}=\frac{z+5}{12}\\x-2y-3z+18=0\end{cases}$

21-variant

1. $\begin{cases}5x+8y-z=7\\2x-3y+2z=9\\x+2y+3z=1\end{cases}$
2. $\vec{x}=\{-15,5,6\}$, $\vec{p}=\{0,5,1\}$, $\vec{q}=\{3,2,-1\}, \vec{r}=\{-1,1,0\}$.
3. $\vec{a}=\{3,7,0\}, \vec{b}=\{4,6,-1\}$, $\vec{c}_1=3\vec{a}+2\vec{b}, \vec{c}_2=5\vec{a}-7\vec{b}$.
4. $\vec{a}=6\vec{p}-\vec{q}, \vec{b}=\vec{p}+2\vec{q}$, $|\vec{p}|=8$, $|\vec{q}|=\frac{1}{2}, (\widehat{\vec{p},\vec{q}})=\frac{\pi}{3}$.
5. $A(1,4,-1)$, $B(-2,4,-5)$, $C(8,4,0)$.
6. $A_1(1,2,0)$, $A_2(1,-1,2)$, $A_3(0,1,-1)$, $A_4(-3,0,1)$.
7. $A(-1,2)$; $B(4,4)$; $C(1,-1)$.
8. $A(-3,1,0)$; $B(6,3,3)$; $C(9,4,-2)$.
9. $M_1(0,-1,-1)$; $M_2(-2,3,5)$; $M_3(1,-5,-9)$; $M_0(-4,-13,6)$.
10. $\begin{cases}2x-y-7=0\\2x-z+5=0\end{cases}$, $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z-0}{-1}$.
11. $\begin{cases}x+y-2z-2=0\\x-y+z+2=0\end{cases}$ 12. $\begin{cases}\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z+5}{0}\\x+7y+3z+11=0\end{cases}$

22-variant

1. $\begin{cases}3x+y+z=5\\x-4y-2z=-1\\5y-3x+6z=10\end{cases}$
2. $\vec{x}=\{8,9,4\}$, $\vec{p}=\{1,3,1\}$, $\vec{q}=\{0,-2,1\}$, $\vec{r}=\{1,3,0\}$.
3. $\vec{a}=\{2,-1,4\}$, $\vec{b}=\{3,-7,1\}$, $\vec{c}_1=2\vec{a}-3\vec{b}, \vec{c}_2=3\vec{a}-2\vec{b}$.
4. $\vec{a}=3\vec{p}+4\vec{q}, \vec{b}=\vec{q}-\vec{p}$, $|\vec{p}|=2{,}5$, $|\vec{q}|=2$, $(\widehat{\vec{p},\vec{q}})=\frac{\pi}{2}$.

5. $A(0,1,0)$, $B(0,2,1)$, $C(1,2,0)$.
6. $A_1(1,0,2)$, $A_2(1,2,-1)$, $A_3(2,-2,1)$, $A_4(2,1,0)$.
7. $A(-1,2)$; $B(-3,-2)$; $C(-1,-1)$.
8. $A(-4,-2,5)$; $B(3,-3,-7)$; $C(9,3,-7)$.
9. $M_1(5,2,0)$; $M_2(2,5,0)$; $M_3(1,2,4)$; $M_0(-3,-6,-8)$.
10. $\begin{cases} 2x - y - 7 = 0 \\ z = 2x + 5 \end{cases}$, $\begin{cases} y = \frac{3}{2}x + 8 \\ z = 3x \end{cases}$
11. $\begin{cases} x - 3y - z + 11 = 0 \\ x - 2y + 2z - 1 = 0 \end{cases}$ 12. $\begin{cases} \frac{x-5}{-1} = \frac{y+3}{5} = \frac{z-1}{2} \\ 3x + 7y - 5z - 11 = 0 \end{cases}$

23-variant

1. $\begin{cases} 2x - y + 5z = 4 \\ 5x + 2y + 13z = -21 \\ 3x - y + 5z = 0 \end{cases}$
2. $\vec{x} = \{23,-14,-30\}, \vec{p} = \{2,1,0\}$, $\vec{q} = \{1,-1,0\}$, $\vec{r} = \{-3,2,5\}$.
3. $\vec{a} = \{5,-1,-2\}$, $\vec{b} = \{6,0,7\}$ $\vec{c}_1 = 3\vec{a} - 2\vec{b}, \vec{c}_2 = 4\vec{b} - 6\vec{a}$.
4. $\vec{a} = 7\vec{p} + \vec{q}, \vec{b} = \vec{p} - 3\vec{q}$, $|\vec{p}| = 3$, $|\vec{q}| = 1$, $(\widehat{\vec{p},\vec{q}}) = \frac{3\pi}{4}$.
5. $A(-7,0,4)$, $B(-1,6,7)$, $C(1,10,9)$.
6. $A_1(1,2,-3)$ $A_2(1,0,1)$, $A_3(-2,-1,6)$, $A_4(0,-5,-4)$.
7. $A(-1,2)$; $B(3,1)$; $C(-3,-2)$.
8. $A(0,-8,10)$; $B(-5,5,7)$; $C(-8,0,4)$.
9. $M_1(2,-1,-2)$; $M_2(1,2,1)$; $M_3(5,0,-6)$; $M_0(4,-3,7)$.
10. $\begin{cases} 3x - 2y + 16 = 0 \\ 3x - z = 0 \end{cases}$, $\frac{x+0,5}{1} = \frac{y+1,5}{0} = \frac{z-0,5}{1}$.
11. $\begin{cases} x - y + 2z - 2 = 0 \\ x - 2y - z + 4 = 0 \end{cases}$ 12. $\begin{cases} \frac{x-1}{7} = \frac{y-2}{1} = \frac{z-6}{-1} \\ 4x + y - 6z - 5 = 0 \end{cases}$

24-variant

1. $\begin{cases} x + y - z = -2 \\ 4x - 3y + z = 1 \\ 2x + y - 5z = 1 \end{cases}$
2. $\vec{x} = \{3,1,3\}$, $\vec{p} = \{2,1,0\}$, $\vec{q} = \{1,0,1\}$, $\vec{r} = \{-3,2,5\}$.
3. $\vec{a} = \{-9,5,3\}$, $\vec{b} = \{7,1,-2\}$, $\vec{c}_1 = 2\vec{a} - \vec{b}, \vec{c}_2 = 3\vec{a} + 5\vec{b}$.

4. $\vec{a}=\vec{p}+3\vec{q}, \vec{b}=\vec{p}-3\vec{q}, \quad |\vec{p}|=3, \ |\vec{q}|=1, \ (\widehat{\vec{p},\vec{q}})=\frac{3\pi}{4}.$

5. $A(-2,4,-6), \qquad B(0,2,-4). \quad C(-6,8,-10).$

6. $A_1(3,10,-1), \quad A_2(-2,3,-5), \ A_3(-6,0,-3), \qquad A_4(1,-1,2).$

7. $A(-1,2); \ B(-3,-2); \ C(4,4).$

8. $A(-1,-5,-2); \ B(6,-2,1); \ C(2,-2,-2).$

9. $M_1(-2,0,-4); \quad M_2(-1,7,1); \quad M_3(4,-8,-4); \quad M_0(-6,5,5).$

10. $\begin{cases} x=3z-4 \\ y=z+2 \end{cases}, \quad \frac{x+3}{1}=\frac{y+2}{2}=\frac{z+1}{1}$

11. $\begin{cases} 6x-7y-z-2=0 \\ x+7y-4z-5=0 \end{cases}$ 12. $\begin{cases} \frac{x-3}{1}=\frac{y+2}{-1}=\frac{z-8}{0} \\ 5x+9y+4z-25=0 \end{cases}$

HOSILA

Hosila va differensial

x_1 va x_2 argumentning qiymatlari, $y_1=f(x_1)$ va $y_2=f(x_2)$ –esa $y=f(x)$ funksiyaning mos keluvchi qiymatlari, $\Delta x=x_2-x_1$ ayirma argumentning orttirmasi, $\Delta y=y_2-y_1=f(x_2)-f(x_1)$ esa funksiyaning $[x_1,x_2]$ kesmadagi orttirmasi deyiladi. $y=f(x)$ funksiyaning x argument bo'yicha hosilasi deb, argument orttirmasi nolga intilganda funksiya orttirmasining argument orttirmasiga nisbatining chekli limitiga aytiladi:

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

yoki

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

(hosila shunigdek $\frac{dy}{dx}$ bilan ham belgilanadi). $y=f(x)$ funksiyaning grafigining x nuqtasidagi urinmaning burchak koeffitsiyenti hosilaning geometrik ma'nosini bildiradi. Hosila funksiyaning x nuqtadagi o'zgarish tezligidir. Hosilani topish funksiyani differensiallash deyiladi.

Asosiy elementar funksiyalarni differensiallash formulalari:

1. $\left(x^m\right)' = m\cdot x^{m-1}$ 11. $\left(ctgx\right)' = -cosec^2 x$

2. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$. 12. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

3. $(\frac{1}{x})' = -\frac{1}{x^2}$. 13. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

4. $(e^x)' = e^x$ 14. $(arctgx)' = \frac{1}{1+x^2}$

5. $(a^x)' = a^x \cdot \ln a$ 15. $(arcctgx)' = -\frac{1}{1+x^2}$

6. $\left(\ln x\right)' = \frac{1}{x}$. 16. $(shx)' = (\frac{e^x - e^{-x}}{2})' = chx$

7. $(\log_a x)' = \frac{1}{x\cdot \ln a}$ 17. $(chx)' = (\frac{e^x + e^{-x}}{2})' = shx$

8. $\left(\sin x\right)' = \cos x$. 18. $(thx)' = (\frac{shx}{chx})' = \frac{1}{ch^2 x}$

9. $(\cos x)' = -\sin x$. 19. $(cthx)' = (\frac{chx}{shx})' = -\frac{1}{sh^2 x}$

10. $(tgx)' = sec^2 x$

Differensiallashning asosiy qoidalari:

C-o'zgarmas son, $u = u(x)$, $v = v(x)$ hosilaga ega bo'lgan funksiyalar.

1. $C' = 0$. 4. $(cu)' = cu'$

2. $x' = 1$ 5. $\left(uv\right)' = u'v + uv'$

3. $(u \pm v)' = u' \pm v'$ 6. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

7. Agar $y = f(u)$, $u = u(x)$, ya'ni $y = f[u(x)]$ bo'lib, $f(u)$ va $u(x)$ funksiyalar hosilaga ega bo'lsalar, u holda $y_x' = y_u' \cdot u_x'$ bo'ladi (murakkab funksiyani differensiallash qoidasi).

Oshkormas funksiyalarni differensiallash.

y x ning oshkormas funksiyasi sifatida $F(x,y) = 0$ tenglama bilan aniqlangan bo'lsin. Bundan keyin bu funksiyani differensiallanuvchi deb hisoblaymiz. $F(x,y) = 0$ tenglamaning ikki tomonini x bo'yicha differensiallab, y' ga nisbatan birinchi darajali tenglama hosil qilamiz. Bu tenglamadan y', ya'ni x va y ning barcha qiymatlari uchun oshkormas funksiyaning hosilasi oson topiladi, bunda tenglamadagi y' oldidagi ko'paytuvchi nolga aylanmaydi.

Parametrik ko'rinishda berilgan funksiyalarni differensiallash.

Agar argumentning funksiyasi $x = \varphi(t)$, $y = \psi(x)$ parametrik tenglamalar bilan berilgan bo'lsa, u holda

$$y_x' = \frac{y_t'}{x_t'}$$

yoki

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

bo'ladi.

Hosilaning geometriya va mexanika masalalariga tadbiqi.

Agar egri chiziq $y = f(x)$ tenglama bilan berilgan bo'lsa u holda $f'(x_0) = tg\alpha$ bo'lib, α - x_0 nuqtada egri chiziqqa o'tkazilgan urinma bilan OX o'qining musbat yo'nalishi orasidagi burchakdir.

$y=f(x)$ egri chiziqqa $M_0(x_0;y_0)$ nuqtada o'tkasilgan urinma tenglamasi quyidagi ko'rinishga ega: $y-y_0=y_0'(x-x_0)$, bu yerda y_0' - hosilaning $x=x_0$ nuqtadagi qiymati. Egri chiziqqa o'tkazilgan normal deb, urinmaga urinish nuqtasida perpendikulyar o'tkazilgan to'g'ri chiziqqa aytiladi. Normalning tenglamasi quyidagicha:

$$y-y_0=-\frac{1}{y_0'}(x-x_0)$$

$y=f_1(x)$ va $y=f_2(x)$ egri chiziqlarining $M_0(x_0;y_0)$ kesishish nuqtasida ular orasidagi burchak deb, ularga shu $M_0(x_0;y_0)$ nuqtada o'tkazilgan urinmalar orasidagi burchakka aytiladi. Bu burchak quyidagi formula yordamida topiladi:

$$tg\varphi=\frac{f_2'(x_0)-f_1'(x_0)}{1+f_1'(x_0)f_2'(x_0)}$$

Agar nuqtaning to'g'ri chiziq bo'ylab harakat qonuni $S=S(t)$ berigan bo'lsa, u holda xarakat tezligi deb yo'ldan t_0 vaqtda olingan hosilaga aytiladi: $V=S'(t_0)$.

Radius-vektor va chiziq orasidagi burchakni topish.

Dekart koordinatalarida $y=f(x)$ tenglama orqali yassi chiziq berilgan. Berilgan *M(x;y)* nuqtada chiziqning yo'nalishi, shu nuqtadagi urinma bilan, ya'ni urinma va *OX* o'qining musbat yo'nalishi orasidagi soat strelkasiga teskari yo'nalishdagi burchak bilan aniqlanadi, bunda $tg\alpha=y'$. *M* nuqtadagi radius vektorning burchak koeffitsienti $tg\varphi=y/x$ ni tashkil qiladi, radius vektor va bu nuqtadagi chiziqning urinmasi orasidagi burchak esa $\omega=\alpha-\varphi$ bo'ladi. Shunday qilib,

$$tg\omega=\frac{tg\alpha-tg\varphi}{1+tg\alpha\cdot tg\varphi}=\frac{y'-\frac{y}{x}}{1+y'\cdot\frac{y}{x}}=\frac{xy'-y}{x+yy'}=\frac{xdy-ydx}{xdx+ydy}.$$

Agar chiziq qutb koordinatalarida $r = r(\varphi)$ tenglama bilan berilan bo‘lsa, u holda

$$x = r\cos\varphi, \qquad y = r\sin\varphi.$$

Bundan $xdy - ydx = r^2 d\varphi$, $xdx + ydy = rdr$ ni hosil qilamiz. Demak,

$$tg\omega = \frac{r^2 d\varphi}{rdr} = \frac{r}{r'}.$$

Yuqori tartibli hosilalar.

$y=f(x)$ funksiyaning hosilasidan olingan hosila uning ikkinchi tartibli hosilasi (ikkinchi hosilasi) deyiladi. Ikkinchi hosila shunday belgililanadi: y'', yoki $\frac{d^2y}{dx^2}$, yoki $f''(x)$. Agar $s = f(t)$ - nuqtaning to‘gri chiziqli harakat qonuni bo‘lsa, u holda yo‘ldan vaqt bo‘yicha olingan ikkinchi hosila $\frac{d^2s}{dt^2}$ bu harakatning tezlanishi bo‘ladi. Xuddi shunga o‘xshash $y = f(x)$ funksiyaning uchinchi tartibli hosilasi ikkinchi tartibli hosiladan olingan hosila bo‘ladi: $y''' = (y'')'$. Umuman, $y = f(x)$ funksiyaning n- tartibli hosilasi deb (n-1)-tartibli hosiladan olingan hosilaga aytiladi: $y^{(n)}(x) = \left(y^{(n-1)}(x)\right)'$ n-hosila shunday belgilanadi: $y^{(n)}$ yoki $\frac{d^n y}{dx^n}$, yoki $f^{(n)}(x)$. Yuqori tartibli hosilalar (ikkinchi, uchinchi va h.k.), berilgan funksiyani ketma-ket differensiyalab hisoblanadi. Agar funksiya parametrik ko‘rinishda berilgan bo‘lsa: $x = \varphi(t), y = \phi(t)$, u holda $y'_x, y''_{xx}, y'''_{xxx}, \ldots$ hosilalar quyidagi formulalar bo‘yicha hisoblanadi;

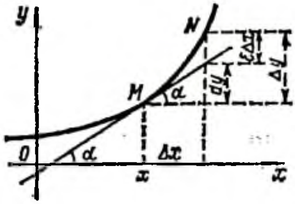
$$y'_x = \frac{y'_t}{x'_t}, \qquad y''_{xx} = \frac{(y'_x)'_t}{x'_t}, \qquad y''_{xx} = \frac{(y''_{xx})'_t}{x'_t}$$

Ikkinchi tartibli hosilani quyidagi formula bo‘yicha ham hisoblash mumkin:

$$y''_{xx} = \frac{(y''_{tt}x'_t - x''_{tt}y'_t)}{(x'_t)^3}.$$

Birinchi va yuqori tartibli diferensiallar.

$y = f(x)$ funksiyaning differensiali deb (birinchi tartibli) funksiya ortirmasining argument ortirmasiga nisbatan chiziqli bo‘lgan bosh qismiga aytiladi. Argument ortirmasi argument differensiali



4-rasm

deyiladi: $dx = \Delta x$. Funksiya differensiali , uning hosilasini argument differensialiga ko‘paytmasiga teng bo‘ladi: $dy = y'dx$. $M(x;y)$ nuqtada funksiya grafigiga o‘tkazilgan urinma ordinatasining ortirmasi differensialni geometrik tasvirlaydi (4-rasm).

Differensialning asosiy xossalari.

1. $dC = 0, \qquad C = const,$
2. $d(Cu) = Cdu,$
3. $d(u \pm v) = du \pm dv,$
4. $d(uv) = udv + vdu,$
5. $d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}, \qquad (v \neq 0),$
6. $df(u) = f'(u)du.$

Agar argument ortirmasi Δx absolut qiymati bo‘yicha yetarlicha kichik bo‘lsa, u holda $\Delta y \approx dy$ va

$$f(x+\Delta x)\approx f(x)+f'(x)\cdot\Delta x.$$

Shunday qilib, taqribiy hisoblashlarda funksiyaning differensialini qo'llash mumkin. $y=f(x)$ funksiyaning birinchi tartibli differensialidan olingan differensial uning ikkinchi tartibli differensiali deyiladi: $d^2y=d(dy)$. Xuddi shunga o'xshash uchinchi tartibli differensial aniqlanadi: $d^3y=d(d^2y)$ Umuman

$d^ny=d(d^{n-1}y)$. Agar $y=f(x)$, x- erkli o'zgaruvchi bo'lsa, u holda yuqori tartibli differensiallar quyidagi formulalar bo'yicha hisoblanadi:

$$d^2y=y''(dx)^2, d^3y=y'''(dx)^3,..., d^ny=y^{(n)}(dx)^n.$$

Funksiyani tekshirish.
Roll teoremasi.

Agar $f(x)$ funksiya $[a,b]$ kesmada uzluksiz, $]a,b[$ intervalda differensiallanuvchi va $f(a)=f(b)$ bo'lsa, u holda $]a,b[$ intervalda hech bo'lmaganda bitta $x=\xi$ qiymat topiladiki, unda $f'(\xi)=0$ bo'ladi. Agar, xususiy holda $f(a)=0, f(b)=0$ bo'lsa, u holda Roll teoremasi funksiyaning ikki ildizi orasida uning hosilasining hech bo'lmaganda bitta yechimi joylashishini bildiradi.

Lagranj teoremasi (chekli ortirmalar uchun).

Agar $f(x)$ funksiya $[a,b]$ kesmada uzluksiz, $]a,b[$ intervalda differensiyallanuvchi bo'lsa u holda $]a,b[$ intervalda hech bo'lmaganda bitta $x=\xi$ qiymat topiladiki, unda

$$f(b)-f(a)=(b-a)f'(\xi)$$

bo'ladi.

Bu teoremalar quyidagi geometrik ma'noga ega: $y=f(x)$ uzluksiz egri chiziqning har bir ichki nuqtasida aniq urinmaga ega bo'lgan AB yoyida (OY o'qiga parallel bo'lmagan), hech bo'lmaganda bitta ichki nuqta topiladiki , bu nuqtada urinma AB

vatarga parallel bo'ladi. (Roll teoremasi uchun, AB xorda va urinmalar OX o'qiga parallel).

Koshi teoremasi.

Agar $f(x)$ va $\varphi(x)$ funksiyalar $[a,b]$ kesmada uzluksiz, $]a,b[$ intervalda differensiallanuvchi bo'lib, undan tashqari $\varphi'(x) \neq 0$ bo'lsa, u holda $]a,b[$ intervalda hech bo'lmaganda bitta $x = \xi$ qiymat topiladiki, unda

$$\frac{f(b)-f(a)}{\phi(b)-\phi(a)} = \frac{f'(\xi)}{\phi'(\xi)}$$

tenglik bajariladi, bunda $a < \xi < b$.

Teylor formulasi.

a nuqtani o'z ichiga olgan biror intervalda $n+1$ marta differensiallanuvchi $f(x)$ funksiya n-tartibli ko'phad va R_n qoldiq hadning yigindisi ko'rinishida ifodalanishi mumkin:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n,$$

$R_n = \dfrac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$, Bunda ξ nuqta a va x nuqtalar orasida yotadi, ya'ni $\xi = a + \vartheta(x-a)$, shu bilan birga, $0 < \vartheta < 1$.

$a = 0$ da Makloren formulasi hosil bo'ladi:

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + R_n,$$

$$R_n = \frac{f^{(n+1)}(\theta x)}{(n+1)!}x^{n+1}, \qquad 0 < \theta < 1$$

Ba'zi funksiyalarning Makloren formulasi bo'yicha yoyilmasini keltiramiz.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + R_n; \qquad R_n = \frac{e^{\theta x}}{(n+1)!}x^{n+1};$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - ... + \frac{(-1)^{m+1} x^{2m-1}}{(2m-1)!} + R_{2m-1};$$

$$R_{2m-1} = (-1)^m \sin\theta x \frac{x^{2m}}{(2m)!};$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + \frac{(-1)^m x^{2m}}{(2m)!} + R_{2m};$$

$$R_{2m} = (-1)^{m+1} \sin\theta x \frac{x^{2m+1}}{(2m+1)!};$$

$$(1+x)^m = 1 + \frac{m}{1!}x + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + ...$$

$$... + \frac{m(m-1)...[m-(n-1)]}{n!}x^n + R_n;$$

$$R_n = \frac{m(m-1)\cdots(m-n)}{(m+1)}(1+\theta)^{n-m-1}x^{n+1};$$

(bu formulalarda $0 < \theta < 1$).

Noaniqliklarni ochishda Lopital qoidasi

x_0 nuqtaning biror atrofida (x_0 nuqtaning o‘zidan boshqa) $f(x)$ va $\varphi(x)$ funksiyalar differensiallanuvchi va $\varphi'(x) \neq 0$ bo‘lsin. Agar

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} \varphi(x) = 0$$

yoki

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} \varphi(x) = \infty$$

bo‘lsa, ya’ni $f(x)/\phi(x)$ nisbat $x = x_0$ nuqtada $0/0$ yoki ∞/∞ ko‘rinishidagi noaniqlikdan iborat bo‘lsa va $\lim\limits_{x \to x_0} \frac{f'(x)}{\varphi'(x)}$ mavjud bo‘lsa u holda

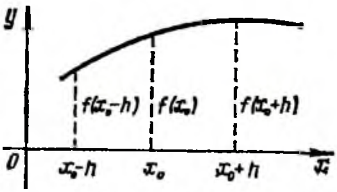
$$\lim_{x \to x_0} \frac{f(x)}{\varphi(x)} = \lim_{x \to x_0} \frac{f'(x)}{\varphi'(x)}$$

tenglik o'rinli.

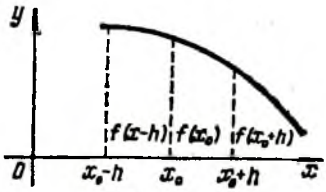
Agar $x = x_0$ nuqtada $\frac{f'(x)}{\varphi'(x)}$ nisbat $0/0$ yoki ∞/∞ ko'rinishidagi aniqmasliklar bo'lib va $f'(x)$ va $\varphi'(x)$ hosilalar mos shartlarni qanoatlantirsa, u holda ikkinchi tartibli hosilalar nisbatiga o'tish kerak va xakozo. $0 \cdot \infty$ yoki $\infty - \infty$ ko'rinishidagi noaniqlik hollarida, berilgan funksiyani shunday algebraik almashtiriladiki , bunda uni $0/0$ yoki ∞/∞ ko'rinishidagi noaniqliklarga keltiriladi va bundan keyin Lopital qoidasi qo'llaniladi. 0^0 , ∞^0 yoki 1^∞ ko'rinishidagi noaniqlik hollarida berilgan funksiyani logarifmlanadi va uning logarifmining limiti topiladi.

Funksiyaning o'sish va kamayishi

Agar har qanday yetarli kichik $h > 0$ uchun $f(x_0 - h) < f(x_0) < f(x_0 + h)$ shart bajarilsa (5-rasm), $f(x)$ funksiya x_0 nuqtada o'suvchi deyiladi.



5-rasm



6-rasm

Agar har qanday yetarli kichik $h > 0$ uchun $f(x_0 - h) > f(x_0) > f(x_0 + h)$ shart bajarilsa (6-rasm) $f(x)$ funksiya x_0 nuqtada kamayuvchi deyiladi. $]a, b[$ intervalda $f(x)$ funksiya o'suvchi deyiladi, agar ko'rsatilgan intervaldan olingan va $x_1 < x_2$ tengsizlikni qanoatlantiruvchi har qanday ikkita x_1 va x_2 nuqtalar uchun $f(x_1) < f(x_2)$ tengsizlik bajarilsa. $]a, b[$ intervalda $f(x)$ funksiya kamayuvchi deyiladi, agar ko‘rsatilgan intervalda $x_1 < x_2$ tengsizlikni qanoatlantiruvchi har qanday x_1 va x_2 nuqtalar uchun $f(x_1) < f(x_2)$ tengsizlik bajarilsa.

Funksiyaning o‘sish va kamayish belgilari

1) Agar $f'(x) > 0$ bo‘lsa, u holda $f(x)$ funksiya x_0 nuqtada o‘sadi.

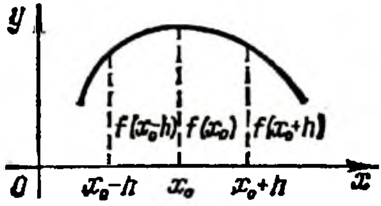
2) Agar $f'(x_0) < 0$ bo‘lsa, u holda $f(x)$ funksiya x_0 nuqtada kamayadi.

Funksiyaning ekstremumlari

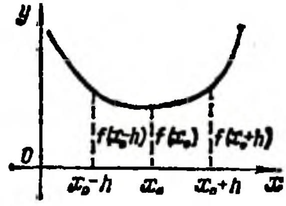
Agar har qanday yetarli kichik $h > 0$ uchun $f(x_0 - h) < f(x_0)$ va $f(x_0 + h) < f(x_0)$ shartlar bajarilsa, $f(x_0)$ qiymat $f(x)$ funksiyaning maksimumi deyiladi. Bu holda x_0 nuqta $f(x)$ funksiyaning maksimum nuqtasi deyiladi. (7-rasm).Agar har qanday yetarli kichik $h > 0$ uchun $f(x_0 - h) > f(x_0)$ va $f(x_0 + h) > f(x_0)$ shartlar bajarilsa, $f(x_0)$ qiymat $f(x)$ funksiyaning minimumi deyiladi. Bu holda x_0 nuqta $f(x)$ funksiyaning minimum nuqtasi deyiladi.(8-rasm). Funksiyaning maksimum va minimumlari funksiyaning ekstremumlari deyiladi. Funksiyaning maksimum va minimum nuqtalari uning ekstremum nuqtalari deyiladi.

Ekstremum mavjudligining zarur sharti

Agar $f(x)$ funksiya x_0 nuqtada ekstremumga ega bo‘lsa, u holda uning hosilasi $f'(x_0)$ nolga aylanadi yoki mavjud bo‘lmaydi. $f'(x_0) = 0$ bo‘lgan x_0 nuqta - statsionar nuqta deyiladi. $f'(x_0) = 0$ yoki $f'(x_0)$ mavjud bo‘lmagan nuqtalar - kritik nuqtalar deyiladi. Har qanday kritik nuqta ekstremum nuqtasi bo‘la olmaydi.



7-rasm



8-rasm

Ekstrmum mavjudligining yetarli shartlari

1-qoida. Agar x_0 nuqta $f(x)$ funksiyaning kritik nuqtasi bo‘lsa va ixtiyoriy yetarli kichik $h>0$ da $f'(x_0-h)>0$, $f'(x_0+h)<0$ tengsizlik bajarilsa, $f(x)$ funksiya x_0 nuqtada maksimumga ega (7-rasm), agar $f'(x_0-h)<0$, $f'(x_0+h)>0$ bo‘lsa, x_0 nuqtada $f(x)$ funksiya minimumga ega bo‘ladi (8-rasm). Agar $f'(x_0-h)$, $f'(x_0+h)$ larning ishoralari bir hil bo‘lsa, u holda x_0 nuqtada $f(x)$ funksiya ekstremumga ega bo‘lmaydi.

2-qoida. Agar $f'(x_0)=0$, $f''(x_0)\neq 0$ bo‘lsa, u holda $f(x)$ funksiya x_0 nuqtada ekstremumga ega, aniqroq aytadigan bo‘lsak, $f(x_0)$, agar $f''(x_0)<0$ bo‘lsa, maksimum bo‘ladi, va agar $f''(x_0)>0$ bo‘lsa, minimum bo‘ladi,.

3-**qoida.** Faraz qilaylik

$$f'(x_0)=0, f''(x_0)=0, f'''(x_0)=0, ..., f^{(n-1)}(x_0)=0, f^{(n)}(x_0)\neq 0$$

bo‘lsin. U holda agar *n*-juft son bo‘lsa x_0 nuqtada $f(x)$ funksiya ekstremumga ega bo‘ladi, ya’ni $f^{(n)}(x_0)<0$ da maksimum va $f^{(n)}(x_0)>0$ da minimumga ega bo‘ladi. Agar *n*-toq son bo‘lsa, u holda x_0 da $f(x)$ funksiya ekstremumga ega bo‘lmaydi.

$[a,b]$ kesmada $f(x)$ funksiyaning eng katta (eng kichik) qiymatini topish uchun, funksiyani oraliqning chegarasidagi va shu oraliqqa tegishli kritik nuqtalaridagi qiymatlaridan eng kattasini (eng kichikgini) tanlash kerak.

Qabariqlik, botiqlik. Burilish nuqtasi

$y = f(x)$ funksiyaning grafigi $]a,b[$ intervalda qabariq deyiladi, agar u bu intervaldagi har qanday nuqtaga o‘tkazilgan urinmadan pastda joylashgan bo‘lsa, $y = f(x)$ funksiyaning grafigi $]a,b[$ intervalda botiq deyiladi, agar u bu intervaldagi har qanday nuqtaga o‘tkazilgan urinmadan yuqorida joylashgan bo‘lsa,

Funksiya grafigining qabariqligining (botiqligining) yetarli sharti

Agar $]a,b[$ intervalda $f''(x) < 0$ bo‘lsa, u holda bu intervalda funksiya grafigi qabariq bo‘ladi (9-rasm).

9-rasm

10-rasm

11-rasm

Agar $f''(x) > 0$ bo‘lsa, u holda funksiya grafigi $]a,b[$ intervalda botiq (10-rasm). Funksiya grafigining qabariq qismidan botiqlikka o‘tadigan $(x_0, f(x_0))$ nuqta burilish nuqtasi deyiladi (11-rasm). Agar $x_0 - y = f(x)$ funksiya grafigi burilish nuqtasining absissasi bo‘lsa, u holda bu nuqtada ikkinchi hosila nolga teng yoki mavjud emas. $f''(x) = 0$ yoki $f''(x)$ mavjud bo‘lmagan nuqtalar, II turdagi kritik nuqtalar deyiladi.Agar x_0 II turdagi kritik nuqta va ixtiyoriy yetarli kichik $h > 0$ da

$$f''(x_0 - h) < 0, \quad f''(x_0 + h) > 0$$

yoki

$$f''(x_0 - h) > 0, \quad f''(x_0 + h) < 0$$

tengsizliklar bajarilsa, u holda $y = f(x)$ egri chiziqdagi absissasi x_0 bo‘lgan nuqta burilish nuqtasi deyiladi. Agar $f''(x_0 - h)$ va $f''(x_0 + h)$lar bir hil ishoraga ega bo‘lsa, u holda $y = f(x)$ egri chiziqning absissasi x_0 bo'lgan nuqtasi burilish nuqtasi bo'lmaydi.

Asimptotalar.

Agar egri chiziqning *M(x,y)* nuqtasidan *L* to'g'ri chiziqqacha bo'lgan masofa, bu nuqtani egri chiziq bo'yicha koordinata boshidan cheksiz uzoqlashtirilganda (ya'ni nuqtaning koordinatalaridan hech bo'lmaganda bittasi cheksizlikka intilsa) nolga intilsa , *L* to'g'ri chiziq $y = f(x)$ egri chiziqning asimptotasi deyiladi. Agar $\lim_{x \to a \pm 0} f(x) = +\infty$ yoki $\lim_{x \to a \pm 0} f(x) = -\infty$ bo'lsa, $x = a$ to'g'ri chiziq $y = f(x)$ egri chiziqning vertikal asimptotasi bo'ladi. Agar $\lim_{x \to +\infty} f(x) = b,$ yoki $\lim_{x \to -\infty} f(x) = b,$ limit mavjud bo'lsa, $y = b$ to'g'ri chiziq gorizontal asimptota bo'ladi.

Agar

$$k = \lim_{x \to +\infty} \frac{f(x)}{x} = b, \qquad b = \lim_{x \to +\infty} [f(x) - kx]$$

yoki

$$k = \lim_{x \to -\infty} \frac{f(x)}{x} = b, \qquad b = \lim_{x \to -\infty} [f(x) - kx]$$

limitlar mavjud bo'lsa, $y = kx + b$ to'g'ri chiziq og'ma asimptota bo'ladi.

Xarakterli nuqtalarga ko'ra funksiyalarning grafiklarini yasash

$y = f(x)$ funksiyaning grafigini yasashda uning xarakterli xususiyatlarini aniqlash foydali.

Buning uchun:

1. funksiyaning aniqlanish sohasini topish.
2. funksiya toq yoki juftligini tekshirish.
3. funksiya grafigining koordinata o'qlari bilan kesishish nuqtalarini topish.
4. funksiyaning uzluksizligini tekshirish.
5. funksiyaning o'sish va kamayish oraliqlarini va ekstremumlarini topish.

6. egri chiziqning qabariqlik va botiqlik oraliqlarini hamda uning burilish nuqtalarini topish kerak.

Hisoblash uchun vazifalar

1 – vazifa. Birinchi masalada berilgan chiziqning absissasi x_0 nuqtadan o‘tkazilgan urinma va normalning tenglamasi topilsin.

2 – vazifa. Ikkinchi masalada funksiya differensialini toping.

3 – vazifa. Uchinchi masalada funksiyaning ikkinchi tartibli hosilasi topilsin.

4 – vazifa. Logarifmik differensiallash qoidasiga ko‘ra funksiya hosilasini toping.

5 – vazifa. Oshkormas funksiyaning hosilasini toping.

6 – vazifa. Bu masalada parametrik ravishda berilgan funksiyaning ikkinchi tartibli hosilasini toping.

Namunali variant.

1 – masala. $y = ln\,(e^x + \sqrt{1+e^{2x}})$ funksiyaning $x_0 = 0$ nuqtasiga o‘tkazilgan urinma va normal tenglamasi yozilsin.

Yechish:

$$y'(x) = \frac{1}{e^x + \sqrt{1+e^{2x}}} \cdot \left(e^x + \frac{1}{2\sqrt{1+e^{2x}}} e^{2x} \cdot 2\right) = \frac{e^x}{\sqrt{1+e^{2x}}}$$

y – funktsiyaning va uning hosilasi y' ning $x_0 = 0$ nuqtadagi qiymatini topamiz:

$$y_0 = y(0) = ln\,(1+\sqrt{2}), \quad y'(0) = \frac{1}{\sqrt{2}}.$$

$y = y(x)$ egri chiziqning (x_0, y_0) nuqtadagi urinma tenglamasi

$$y = y_0 + y'(x_0)(x - x_0),$$

va normal tenglamasi

$$y = y_0 - \frac{1}{y'(x_0)}(x - x_0),$$

dan bizning misol uchun urinma tenglamasi

$$y = ln(1+\sqrt{2}) + \frac{1}{\sqrt{2}}x,$$

normal tenglamasi

$$y = ln(1+\sqrt{2}) - \sqrt{2}x.$$

Javobi: $x - \sqrt{2}\,y + \sqrt{2}\,ln(1+\sqrt{2}) = 0,$

$$\sqrt{2}x + y - ln(1+\sqrt{2}) = 0.$$

2 – masala.

$$y = ctgcos2 + \frac{1}{6}\cdot\frac{sin^2 6x}{cos12x}$$

funksiyaning differensialini toping.

Yechish: funksiya hosilasi $y'(x)$ ni topamiz:

$$y' = \frac{1}{6}\cdot\frac{2sin6x\cdot cos6x\cdot 6cos12x - sin^2 6x(-sin12x)\cdot 12}{cos^2 12x} =$$

$$= \frac{sin12x}{cos^2 12x}.$$

dy=y'dx formulaga asosan: $dy = \frac{sin12x}{cos^2 12x}dx$ ga ega bo'lamiz.

Javobi: $dy = \frac{sin12x}{cos^2 12x}dx.$

3 – masala. $y = arc\,tg(x + \sqrt{1+x^2})$ funksiyaning ikkinchi tartibli hosilasi topilsin.

Yechish:

$$y' = \frac{1}{1+(x+\sqrt{1+x^2})^2}\cdot\left(1 + \frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{2(1+x^2)}$$

$$y'' = (y')' = \left(\frac{1}{2(1+x^2)}\right)' = -\frac{1}{2}\cdot\frac{2x}{(1+x^2)^2} = -\frac{x}{(1+x^2)^2}$$

Javobi:

$$-\frac{x}{(1+x^2)^2}$$

4 – masala. $y = (x^3+1)^{tgx}$ funktsiyaning hosilasi topilsin.

Yechish: Tenglikning har ikkala tomonini logarifmlaymiz: $lny = tgx\,ln\,(x^3+1)$. So'ng $y(x)$ ni funksiya deb har ikkala tomondan hosila olamiz:

$$\frac{1}{y}\cdot y' = \frac{1}{cos^2 x}ln(x^3+1) + tgx\cdot\frac{1}{x^3+1}\cdot 3x^2\ ,$$

$$y' = y\left(\frac{ln(x^3+1)}{cos^2x} + \frac{3x^2tgx}{x^3+1}\right) =$$

$$= (x^3+1)^{tgx}\left(\frac{ln(x^3+1)}{cos^2x} + \frac{3x^2tgx}{x^3+1}\right).$$

5 – masala. $arctg\frac{y}{x} = \frac{1}{2}ln(x^2+y^2)$ funktsiyaning hosilasi topilsin.

Yechish: x – ni argument, $y(x)$ ni funktsiya ekanligini nazarga olib, tenglikning har ikki tomonini differentsiallaymiz:

$$\frac{xy'}{x^2+y^2} - \frac{yy'}{x^2+y^2} = \frac{x}{x^2+y^2} + \frac{y}{x^2+y^2}.$$

Bundan $y'(x-y) = x+y$ kelib chiqadi.

Javobi: $y' = \frac{x+y}{x-y}$.

6 – masala. y''_{xx} - hosilani toping.

$$\begin{cases} x = ln\,(1+t^2) \\ y = t - arc\,tgt \end{cases}$$

Bunda t – parametr.

Yechish:

$$y'_x = \frac{y'_t}{x'_t} = \frac{1 - \frac{1}{1+t^2}}{\frac{1}{1+t^2}\cdot 2t} = \frac{1+t^2-1}{2t} = \frac{t}{2}.$$

$$y''_{xx} = (y'_x)'_t \cdot \frac{1}{x'_t} = \frac{1}{2}\cdot\frac{1+t^2}{2t} = \frac{1+t^2}{4t}.$$

Variantlar

1-variant

1. $y = ctg\sqrt[3]{5} - \frac{1}{8}\frac{cos^2 4x}{sin8x}$, $x_0 = \frac{\pi}{16}$. 2. $y = \left(ln\sqrt[3]{e^{2x}+1}\right)^4$.

3. $y = \sqrt[3]{x^3} + \sqrt{x}$. 4. $y = (x^4+5)^{ctgx}$. 5. $e^{x+y} = arctgxy$.

6. $\begin{cases} x(t) = \frac{3t^2+1}{3t^3}, \\ y(t) = sin(t^3). \end{cases}$

2 -variant

1. $y=\frac{2}{3}\sqrt{(arctge^{x})^{3}},\ x_0=0.$

2. $y=ln\left(cos^{2}x+\sqrt{1+cos^{4}x}\right).$

3. $y=\frac{2x^{2}-x-1}{\sqrt[3]{2+4x}}.$ 4. $y=x^{cosx^{2}}.$ 5. $\sqrt{x^{2}+y^{2}}-tgxy=0.$

6. $\begin{cases} x(t)=\sqrt[3]{1-t^{2}}, \\ y(t)=tg\sqrt{1+t}. \end{cases}$

3-variant

1. $y=e^{2x}(2-sin2x-cos2x),\ x_0=0.$

2. $y=ln\,arcsin\sqrt{1-e^{2x}}.$

3. $y=\frac{2(3x^{3}+4x^{2}-x-2)}{15\sqrt{1+x}}.$ 4. $y=(x^{3}+4)^{tgx}.$ 5. $xy=sin(x+y).$

6. $\begin{cases} x(t)=\sqrt{2t-t^{2}}, \\ y(t)=\sqrt[3]{(t-1)^{2}}. \end{cases}$

4-variant

1. $y=\frac{1}{2}ln(e^{2x}+1)-2arctg(e^{x}),\ x_0=0.$ 2. $y=\frac{(2x^{2}-1)\sqrt{1+x^{2}}}{3x^{3}}.$

3. $y=cos(ctg2x)-\frac{1}{16}\cdot\frac{cos^{2}8x}{sin16x}.$ 4. $y=(sinx)^{5e^{x}}.$

5. $xy=3^{x}+xe^{y}.$ 6. $\begin{cases} x(t)=arccos(sint), \\ y(t)=arcsin(cost). \end{cases}$

5-variant

1. $y=\frac{x^{2}}{2\sqrt{1-3x^{4}}},\ x_0=\frac{1}{2}.$ 2. $y=arccos\frac{x^{2}-4}{\sqrt{x^{4}+16}}.$

3. $y=tg\left(ln\frac{1}{3}\right)+\frac{1}{4}\cdot\frac{sin^{2}4x}{cos8x}.$ 4. $y=(lnx)^{3^{x}}.$ 5. $x^{\frac{2}{3}}+y^{\frac{1}{3}}=a^{\frac{1}{3}}.$

6. $\begin{cases} x(t)=ln\left(t+\sqrt{t^{2}+1}\right), \\ y(t)=\sqrt{t^{2}+1}. \end{cases}$

6-variant

1. $y=\frac{x^{2}}{\sqrt{4+x^{2}}},\ x_0=0.$ 2. $y=ln^{3}\left(arccos\frac{1}{\sqrt{x}}\right).$

3. $y = 5^{-x^2}$.   4. $y = (tgx)^{4e^x}$.   5. $y \cdot sinx = cos(x - y)$.

6. $\begin{cases} x(t) = ctg(2e^t), \\ y(t) = lntg(e^t). \end{cases}$

7-variant

1. $y = \frac{cosln7 \cdot sin^2 7x}{7cos14x}$, $x_0 = \frac{\pi}{14}$.   2. $y = ln\, arccos\sqrt{1 - e^{4x}}$.

3. $y = x\sqrt{x^2 - 8}$.   4. $y = (x^2 - 1)^{sinx}$.   5. $3^y + 2^x = 2^{x+y}$.

6. $\begin{cases} x(t) = \sqrt{2t - t^2}, \\ y(t) = arcsin(t - 1). \end{cases}$

8-variant

1. $y = ln\left(e^x + \sqrt{e^{2x} + 1} + arcsine^{-x}\right), x_0 = 0$.

2. 2. $y = \frac{2}{3}\sqrt{(arctge^x)^3}$. 3. $y = x^2 lnx$. 4. $y = \left(sin\sqrt{x}\right)^{ctgx}$.

5. $x + y = 2^{x+y}$.   6. $\begin{cases} x(t) = lnctgt, \\ y(t) = \frac{4}{cos^2 t}. \end{cases}$

9-variant

1. $y = \sqrt[3]{x^3 + 2x + 1}$, $x_0 = 0$ 2. $y = \frac{1}{2} arctg\frac{e^x - 3}{2}$.

3. $y = ln\frac{x^2}{\sqrt{1 - ax^4}}$.   4. $y = (cos5x)^{2x}$.   5 $y^3 + 2xy + b^2 = 0$.

6. $\begin{cases} x(t) = arctg\, e^{\frac{t}{2}}, \\ y(t) = \sqrt{e^t + 1}. \end{cases}$

10-variant

1. $y = \frac{x-1}{(x^2+5)\sqrt{x^2+5}}$, $x_0 = 0$.   2. $y = arctg^3(e^x - e^{-x})$.

3. $y = x^4 ln3x$. 4. $y = (x^3 + 1)^{cosx}$.

5. $x - y = arcsinx - arcsiny$.   6. $\begin{cases} x(t) = tcost - 2sint, \\ y(t) = tsint + 2cost. \end{cases}$

11-variant

1. $y = xe^{-x} + arcsin(5^{2x})$, $x_0 = 0$.   2. $y = log_a \frac{1}{\sqrt{1 - x^4}}$.

3. $y = (1 - x^2)\sqrt[5]{x^3}$. 4. $y = (sinx)^{arctgx}$.

5. $xcosy - sin(y^2) = 0$. 6. $\begin{cases} x(t) = ln\sqrt{\frac{1-t}{1+t}}, \\ y(t) = \sqrt{1 - t^2}. \end{cases}$

12-variant

1. $y = (x^2 - 2)\sqrt{4 + x^2}$, $x_0 = 0$. 2. $y = arccos\sqrt{1 + 2x^3}$.

3. $y = ln\left(x + \sqrt{1 + x^2}\right)$. 4. $y = (x^7 + 1)^{tgx}$.

5. $x + y = sinxy$. 6. $\begin{cases} x(t) = \sqrt{1 - t^2}, \\ y(t) = \frac{t}{\sqrt{1-t^2}}. \end{cases}$

13-variant

1. $y = \frac{1}{ln4} \cdot ln\frac{1+2^x}{1-2^x}$, $x_0 = 0$. 2. $y = \frac{4+3x^2}{\sqrt[3]{2+x^4}}$.

3. $y = x\sqrt{4 - x^2} + 4arcsin\frac{x}{2}$. 4. $y = (1 - cosx)^{tgx}$.

5. $x + 1 = e^{xy}$. 6. $\begin{cases} x(t) = \sqrt{1 - t^2}, \\ y(t) = ln\left(1 + \sqrt{1 - t^2}\right). \end{cases}$

14-variant

1. $y = \frac{2}{3}(arctge^x)^3$, $x_0 = 0$. 2. $y = tg^3 2x$.

3. $y = x^2 ln_3 x$. 4. $y = x^{e^{cosx}}$. 5. $xy = sin(x + 4)$.

6. $\begin{cases} x(t) = arcsin\sqrt{1 - t^2}, \\ y(t) = (arccost)^2. \end{cases}$

15-variant

1. $y = \sqrt[5]{(x^3 + 7x - 7)^3}$, $x_0 = 1$. 2. $y = arccosx + lntge^x$.

3. $y = arctg\frac{x-1}{2}$. 4. $y = (cos2x)^{e^x}$. 5. $x^3 + y^3 = 3^{xy}$.

6. $\begin{cases} x(t) = ln^3 cost, \\ y(t) = e^{sint}. \end{cases}$

16-variant

1. $y = \sqrt[8]{(x^2 + x + 1)^3}, \quad x_0 = 0.$ 2. $y = e^{sin2x} - log_3(\sqrt{x} + 1).$

3. $y = arcsin\sqrt{\frac{x}{x+1}}.$ 4. $y = (tgx - 1)^{cosx}.$

5. $x - y = arctg\sqrt{y}.$ 6. $\begin{cases} x(t) = (1 + cos^2t), \\ y(t) = \frac{cost}{sin^2t}. \end{cases}$

17-variant

1. $y = \frac{3}{\sqrt{2}}arctg\frac{4x+3}{\sqrt{17}}, \ x_0 = 0.$ 2. $y = ln^2(x^3 + cos2x).$

3. $y = 3^x cos2x.$ 4. $y = (x^3 - 4)^{sin2x}.$ 5. $sinxy = y^2.$

6. $\begin{cases} x(t) = arccos\frac{1}{t}, \\ y(t) = \sqrt{t^2 - 1}. \end{cases}$

18-variant

1. $y = ln\frac{x+1}{x+2}, \ x_0 = 0.$ 2. $y = x^2 sin2x.$ 3. $y = \frac{2}{3}arcsin\sqrt{\frac{x^2-1}{3}}.$

4. $y = (sinx)^{\sqrt{x}}.$ 5. $cosxy = x + y.$ 6. $\begin{cases} x(t) = ln(1 + t^2), \\ y(t) = t - arctgt. \end{cases}$

19-variant

1. $y = ln(e^x + \sqrt{1 + e^{2x}}), \ x_0 = 0.$ 2. $y = arccos\sqrt{1 - x^2}.$

3. $y = 3^{\sqrt{x+1}}.$ 4. $y = (cosx)^{lnx}.$ 5. $x^3 + y^3 = 3^{xy}.$

6. $\begin{cases} x(t) = ln(1 - t^2), \\ y(t) = arcsin\sqrt{1 - t^2}. \end{cases}$

20-variant

1. $y = \sqrt[3]{x^3 + 4}, \ x_0 = 0.$ 2. $y = arctg\sqrt{sin\frac{5}{x}}.$ 3. $y = ln(1 + x^2).$

4. $y = (lnx)^{x^2-1}.$ 5. $arctg\frac{y}{x} = x.$ 6. $\begin{cases} x(t) = arcsin\sqrt{t}, \\ y(t) = \sqrt{1 + \sqrt{t}}. \end{cases}$

21-variant

1. $y = 1 - \sqrt[3]{x^2} + \frac{16}{x}$, $x_0 = -8$. 2. $y = ln\, sin(x^3 + 1)$.

3. $y = \sqrt{1 + x^2}$. 4. $y = (tgx)^{x+1}$. 5. $y = arctgy - y + x$.

6. $\begin{cases} x(t) = 2sint, \\ y(t) = 3cost. \end{cases}$

22-variant

1. $y = \frac{(1-\sqrt{x})^2}{x}$, $x_0 = 0{,}01$. 2. $y = arc\, sin\sqrt{1 - x^2}$.

3. $y = xe^{-\frac{1}{x}}$. 4. $y = (cos\sqrt{x})^{sin^2 3x}$. 5. $e^x - e^y = y - x$.

6. $\begin{cases} x(t) = e^{2t}(t^2 + 1), \\ y(t) = e^{3t}. \end{cases}$

23-variant

1. $y = \frac{cosx}{1-x}$, $x_0 = \frac{\pi}{6}$. 2. $y = sin^2\sqrt{\frac{1}{1-x}}$.

3. $y = x^2 lnx$. 4. $y = (ctg3x^4)^{\sqrt{x-3}}$. 5. $lnx + e^{\frac{-y}{x}} = 0$.

6. $\begin{cases} x(t) = arcsint, \\ y(t) = arccos\sqrt{t}. \end{cases}$

24-variant

1. $y = \frac{sinx+cosx}{sinx-cosx}$, $x_0 = \frac{\pi}{2}$. 2. $y = ln^5(tg3x)$.

3. $y = x\sqrt{1 + x^2}$. 4. $y = (ctg2x)^{sin\sqrt{x}}$. 5. $y = x + arctgy$.

6. $\begin{cases} x(t) = ln\sqrt{1 - t}, \\ y(t) = tlnt. \end{cases}$

25-variant

1. $y = arcsin\frac{2x}{1+x^2}$, $x_0 = 2$. 2. $y = 5^{cosx}$.

3 $y = \frac{arcsinx}{\sqrt{1-x^2}}$. 4.$y = (ln\,(7x - 5))^{arctg2x}$.

5. $y = x^2 + arcctgy$. 6. $\begin{cases} x(t) = 2cost - tsint, \\ y(t) = 3sint + tcost. \end{cases}$

26-variant

1. $y = (1 + 3x + 5x^2)^4,\ x_0 = 0.$
2. $y = sin(x^2 - 5x + 1) + tg\left(\frac{a}{x}\right).$ 3. $y = \frac{1+x}{\sqrt{1-x}}.$
4. $y = (arcsin\,(2 + x))^{ln\,(x+3)}.$ 5. $x^3 + x^2y = lny.$
6. $\begin{cases} x(t) = tge^t, \\ y(t) = lne^{2t}. \end{cases}$

27-variant

1. $y = \sqrt{x + \sqrt{x + \sqrt{x}}},\ x_0 = 4.$ 2. $y = \frac{1}{(1+cos4x)^5}.$
3. $y = arccos\sqrt{x}.$ 4. $y = (arcsin2x)^{ctg(x+1)}.$
5. $\sqrt{x^2 - y^2} - ln\sqrt{xy}.$ 6. $\begin{cases} x(t) = \sqrt{1 - t^2}, \\ y(t) = arcsin\sqrt{t}. \end{cases}$

28-variant

1. $y = \frac{cos^2x}{1+sin^2x},\ x_0 = \frac{\pi}{4}.$ 2. $y = ln(e^{-x} + xe^{-x}).$
3. $y = lnsin(x^3 + 1).$ 4. $y = (ln(5x - 4))^{arcctgx}.$
5. $e^{xy} - x^2 + y = 0.$ 6. $\begin{cases} x(t) = \sqrt{1 - 2t}, \\ y(t) = \frac{2}{\sqrt{1+t^2}}. \end{cases}$

29-variant

1. $y = arcsin\frac{x-1}{x},\ x_0 = 5.$ 2. $y = \frac{cosx}{sin^2x} + ln(tg\frac{x}{2}).$
3. $y = \frac{2x^2+x+1}{x^2-x+1},$ 4. $y = (tg3x^3)^{\sqrt{x+5}}.$ 5. $arctgy = \sqrt{x + y}.$
6. $\begin{cases} x(t) = t^2 + sin\sqrt{t}, \\ y(t) = arctgt. \end{cases}$

30-variant

1. $y = arctg\frac{x}{a} - ln\sqrt[4]{x^4 - a^4},\ x_0 = 2a.$ 2. $y = ln\sqrt{\frac{1+2x}{1-2x}}.$
3. $y = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x\sqrt[3]{x}}.$ 4. $y = (cos2x^2)^{arcsin\sqrt{x}}.$

5. $x^2 siny - cosy + ysinxy = 0.$ 6. $\begin{cases} x(t) = ln\frac{1}{\sqrt{1-t^2}}, \\ y(t) = cos\sqrt{t}. \end{cases}$

INTEGRAL

Aniqmas integral.

Agar $F'(x) = f(x)$ yoki $df(x) = f(x)dx$ bo'lsa, $F(x)$ funksiya $f(x)$ funksiya uchun boshlangich deyiladi. Agar $f(x)$ funksiya biror $F(x)$ boshlang'ichga ega bo'lsa, u cheksiz ko'p boshlangichlarga ega bo'ladi, shu bilan birga barcha boshlang'ichlar $F(x)+C$ ko'rinishda bo'ladi, bu yerda C - o'zgarmas. f(x) funksiyadan yoki $f(x)dx$ ifodadan olingan aniqmas integral deb uning barcha boshlang'ichlari to'plamiga aytiladi va quyidagicha belgilanadi

$$\int f(x)dx = F(x) + C,$$

bu yerda $\int$ - integral belgisi , f(x) integral ostidagi funksiya, $f(x)dx$ integral ostidagi ifoda, x- integrallash o'zgaruvchisi. Aniqmas integralni topish - funksiyani integrallash deyiladi.

Integrallash qoidasi.

1. $\left(\int f(x)dx\right)' = f(x)$,
2. $d\left(\int f(x)dx\right) = f(x)dx$,
3. $\int dF(x) = F(x) + C$,
4. $\int af(x)dx = a\int f(x)dx$ bu yerda a – o'zgarmas,
5. $\int [f_1(x) \pm f_2(x)]dx = \int f_1(x)dx \pm \int f_2(x)dx$,
6. Agar $\int f(x)dx = F(x) + C$ va $u = \varphi(x)$ bo'lsa

   $\int f(u)du = F(u) + C$ bo'ladi.

Asosiy integrallar jadvali.

I. $\int dx = x + C$,

II. $\int x^m dx = \frac{x^{m+1}}{m+1} + C$, $m \neq -1$,

III. $\int \frac{dx}{x} = ln|x| + C$,

IV. $\int \frac{dx}{1+x^2} = arctgx + C$,

V. $\int \frac{dx}{\sqrt{1-x^2}} = arcsin x + C$,

VI. $\int e^x dx = e^x + C$,

VII. $\int a^x dx = \frac{a^x}{ln a} + C$,

VIII. $\int sinxdx = -cosx + C$,

IX. $\int cosxdx = sinx + C$,

X. $\int sec^2 xdx = tgx + C$,

XI. $\int cosec^2 xdx = -ctgx + C$,

XII. $\int shxdx = chx + C$,

XIII. $\int chxdx = shx + C$,

XIV. $\int \frac{dx}{ch^2 x} = thx + C$,

XV. $\int \frac{dx}{sh^2 x} = -cthx + C$,

O'zgaruvchilarni almashtirish.

Aniqmas integralda o'zgaruvchini almashtirish ikki ko'rinishdagi o'rniga qo'yishlar yordamida bajariladi.

1. $x = \varphi(t)$, bunda $\varphi(t)$- yangi o'zgaruvchi t-ning monoton, uzluksiz differensiyalanuvchi funksiyasi. Bu holda o'zgaruvchini almashtirsh formulasi quyidagi ko'rinishga ega.

$\int f(x)dx = \int f[\varphi(t)]\varphi'(t)dt$

2. $u = \psi(x)$, bunda u-yangi o'zgaruvchi. Bunday o'rniga qo'yishda o'zgaruvchini almashtirish formulasi:

$$\int f[\psi(x)]\psi'(x)dx = \int f(u)du$$

Endi asosiy integrallar jadvalini quyidagi formulalar bilan to'ldiramiz:

XVI. $\int \frac{f'(x)}{f(x)}dx = ln|f(x)| + C$,

XVII. $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + C$,

XVIII. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctg \frac{x}{a} + C$,

XIX. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} ln\left|\frac{x-a}{x+a}\right| + C$,

XX. $\int \frac{dx}{\sqrt{a^2 - x^2}} = arcsin\frac{x}{a} + C$,

XXI. $\int \frac{dx}{\sqrt{x^2 + \lambda}} = ln\left|x + \sqrt{x^2 + \lambda}\right| + C$,

XXII. $\int \frac{dx}{sin\, x} = ln\left|tg\frac{x}{2}\right| + C = ln|cos\,ec\, x - ctgx| + C$,

XXIII. $\int \frac{dx}{cos\, x} = ln\left|tg\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| = ln|sec\, x + tgx|$,

XXIV. $\int tgxdx = -ln|cos\, x| + C$,

XXV. $\int ctgxdx = ln|sin\, x| + C$,

Bo'laklab integrallash.

Bo'laklab integrallash deb integralni

$$\int udv = uv - \int vdu$$

formula bo'yicha topilishiga aytiladi, bunda $u = \varphi(x)$, $v = \psi(x)$ lar uzluksiz differensialanuvchi funksiyalar. Bu formula yordamida $\int udv$ integralni topish - boshqa $\int vdu$ integralni qidirishga keltiriladi, bu formulaning qo'llanishi, oxirgi integral berilganidan soddaroq yoki unga o'xshash bo'lgan hollarda maqsadga muvofiq. Bunda u sifatida differensiallanganda soddalashadigan funksiya, dv sifatida esa, integrali ma'lum yoki topilishi mumkin bo'lgan integral ostidagi ifodaning qismi olinadi. Masalan, $\int P(x)e^{\alpha x}dx$, $\int P(x)\sin axdx$, $\int P(x)\cos axdx$ ko'rinishidagi, bunda $P(x)$ –ko'phad, integrallar uchun $u(x)$ sifatida mos ravishda $P(x)$ ni, $d(v(x))$ sifatida esa $e^{\alpha x}dx, \sin axdx, \cos axdx$ larni olish kerak,

$$\int P(x)\ln xdx, \quad \int P(x)\arcsin xdx, \quad \int P(x)\arccos xdx$$

ko'rinishdagi integrallar uchun $u(x)$ sifatida mos ravishda $\ln x, \arcsin x, \arccos x$ larni, $d(v(x))$ sifatida esa $P(x)dx$ ifodani olish kerak.

Eng sodda ratsional kasrlarni integrallash.

Ratsional kasr deb , $P(x)/Q(x)$ ko'rinishdagi kasrga aytiladi, bu yerda $P(x)$ va $Q(x)$ - ko'phadlar. Agar $P(x)$ ko'phadning darajasi $Q(x)$ ko'phadning darajasidan past bo'lsa, ratsional kasr to'g'ri kasr deyiladi, aks holda kasr noto'g'ri deyiladi. Eng sodda elementar kasr deb quyidagi ko'rinishdagi to'g'ri kasrlarga aytiladi:

I. $\dfrac{A}{x-a}$.

II. $\dfrac{A}{(x-a)^m}$ bunda m-birdan katta butun son.

III. $\dfrac{Ax+B}{x^2+px+q}$ bunda $\dfrac{p^2}{4}-q<0$ ya'ni x^2+px+q kvadrat uchhad haqiqiy ildizlarga ega emas.

IV. $\dfrac{Ax+B}{(x^2+px+q)^n}$ bu yerda n - birdan katta butun son va x^2+px+q kvadrat uchhad haqiqiy ildizlarga ega emas.

Bu kasrlaeda A,B,p,q,a koeffitsiyentlar – haqiqiy sonlar deb faraz qilinadi. Sanab o'tilgan kasrlarni mos ravishda I,II,III va IV turdagi eng sodda kasrlar deb ataymiz.

Birinchi uch turdagi eng sodda kasrlardan olingan integrallarni ko'rib chiqamiz. Quyidagilarga ega bo'lamiz:

I. $\int \frac{A}{x-a}dx = A\ln|x-a| + C$.

II. $\int \frac{Adx}{(x-a)^m} = -\frac{A}{m-1}\cdot\frac{1}{(x-a)^{m-1}} + C$

III. $\int \frac{Ax+B}{x^2+px+q}dx = \int \frac{\frac{A}{2}(2x+p) + \left(B - \frac{Ap}{2}\right)}{x^2+px+q}dx =$

$= \frac{A}{2}\int \frac{2x+p}{x^2+px+q}dx + \left(B - \frac{Ap}{2}\right)\int \frac{dx}{x^2+px+q} = \frac{A}{2}\ln|x^2+px+q| +$

$+\left(B - \frac{Ap}{2}\right)\int \frac{dx}{\left(x+\frac{p}{2}\right)^2 + \left(q - \frac{p^2}{4}\right)} = \frac{A}{2}\ln|x^2+px+q| +$

$+\frac{2B-Ap}{\sqrt{4q-p^2}}arctg\frac{2x+p}{\sqrt{4q-p^2}} + C$

Endi IV tur eng sodda kasrlarni qanday integrallashni ko'rsatamiz:

$\int \frac{Ax+B}{(x^2+px+q)^n}dx,\quad \frac{p^2}{4} - q < 0$ integralni hisoblash kerak. Suratda, maxrajda turgan kvadrat uchhadning hosilasini ajratamiz.

$$\int \frac{Ax+B}{(x^2+px+q)^n}dx = \int \frac{\frac{A}{2}(2x+p) + \left(B - \frac{Ap}{2}\right)}{(x^2+px+q)^n}dx = .$$

$$= \frac{A}{2}\int \frac{2x+p}{(x^2+px+q)^n}dx + \left(B - \frac{Ap}{2}\right)\int \frac{dx}{(x^2+px+q)^n}$$

Tenglikning o'ng tomonida turgan birinchi integral $x^2 + px + q = t$ o'rniga qo'yish yordamida oson topiladi, ikkinchisini esa quyidagicha o‘zgartiramiz:

$$\int \frac{dx}{\left(x^2+px+q\right)^n}=\int \frac{dx}{\left[\left(x+\frac{p}{2}\right)^2+\left(q-\frac{p^2}{4}\right)\right]^n}$$

Endi $x+\frac{p}{2}=t,\ dx=dt$ desak va $q-\frac{p^2}{4}=a^2$ deb belgilasak,

$\int \frac{dx}{\left(x^2+px+q\right)^n}=\int \frac{dt}{\left(t^2+a^2\right)^n}$ tenglikni hosil qilamiz. Shunday qilib, IV tur elementar kasrni integrallash rekurent formula yordamida bajarilishi mumkin.

$$\int \frac{dt}{\left(t^2+a^2\right)^n}=\frac{t}{2a^2(n-1)(t^2+a^2)^{n-1}}+\frac{2n-3}{2a^2(n-1)}\int \frac{dt}{\left(t^2+a^2\right)^{n-1}}$$

Ratsional kasrlarni eng sodda ratsional kasrlarga ajratish yordamida integrallash

$P(x)/Q(x)$ ratsional kasrni integrallashdan oldin quyidagi algebraik o‘zgartirishlar va hisoblashlar bajarilishi kerak.

1.Agar noto‘g‘ri ratsional kasr berilgan bo‘lsa, undan butun qismini ajratish, ya’ni kasrni

$$\frac{P(x)}{Q(x)}=M(x)+\frac{P_1(x)}{Q(x)}$$

ko‘rinishga keltirish , bunda $M(x)$ ko‘phad, $P_1(x)/Q(x)$- to‘g‘ri ratsional kasr.

2. Kasr maxrajini chiziqli va kvadrat ko‘paytuvchilarga ajratish kerak:

$Q(x)=(x-a)^m\ldots\left(x^2+px+q\right)^n\ldots,$ bu yerda $\frac{p^2}{4}-q<0$, ya’ni x^2+px+q ko‘phad kompleks qo‘shma ildizlarga ega.

3.To‘g‘ri ratsional kasrni eng soda kasrlarga ajratish kerak:

$$\frac{P_1(x)}{Q(x)}=\frac{A_1}{(x-a)^m}+\frac{A_2}{(x-a)^{m-1}}+\ldots+\frac{A_m}{x-a}+\ldots$$

$$\ldots+\frac{B_1x+C_1}{\left(x^2+px+q\right)^n}+\frac{B_2x+C_2}{\left(x^2+px+q\right)^{n-1}}+\ldots+\frac{B_nx+C_n}{x^2+px+q}+\ldots,$$

4. $A_1, A_2, \ldots, A_m, \ldots,\ B_1, C_1, B_2, C_2, \ldots, B_n, C_n \ldots,$ noma'lum koeffitsientlarni hisoblash uchun so'nggi tenglikni umumiy maxrajga keltirib, hosil bo'lgan ayniyatni chap va o'ng qismlaridagi x ning bir xil darajalari oldidagi koeffitsientlarini tenglash va hosil bo'lgan chiziqli tenglamalar sistemasini izlanayotgan koeffitsientlarga nisbatan yechish kerak. Koeffitsientlarni boshqa usul bilan ham topish mumkin, hosil bo'lgan ayniyatda x ga ixtiyoriy sonli qiymatlar berib, aniqlash mumkin. Ko'pincha koeffitsientlarni hisoblashni ikkala usulini qo'llash foydali. Natijada, ratsional kasrni integrallash masalasi ko'phadning va eng sodda ratsional kasrning integrallarini topishga keltiriladi.

$\int R(e^x)dx$ **ko'rinishdagi integrallar**, bunda R- ratsional funksiya.

$e^x=t$ o'rniga qo'yish yordamida, bundan

$$e^x dx=dt, \qquad dx=\frac{dt}{e^x}=\frac{dt}{t},$$

$\int R(e^x)dx$ ko'rinishdagi integral, ratsional funksiyadan olingan integralga keltiriladi.

Eng sodda irratsional funksiyalarni integrallash

$\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{m_1}{n_1}}, \left(\frac{ax+b}{cx+d}\right)^{\frac{m_2}{n_2}}, \ldots\right)dx$ **ko'rinishdagi integralalar,** bunda R-ratsional funksiya , $m_1, n_1,\ m_2, n_2, \ldots$- , butun sonlar($ad \neq bc$).

$\left(\frac{ax+b}{cx+d}\right)=t^s$ o'rniga qo'yish yordamida berilgan integral ratsional funksiyaning integraliga aylantiriladi, bu yerda s - $n_1, n_2 \ldots$ sonlarning eng kichik umumiy bo'linuvchisi.

Trigonometrik funksiyalarni integrallash

$\int R(sinx,\ cosx)\ dx$ **ko'rinishdagi integrallar**, bu yerda R ratsional funksiya. Bu ko'rinishdagi integrallar ratsional funksiyalarning integralariga universal trigonometric almashtirish deb nomlangan $tg(x/2)=t$ o'rniga qo'yish yordamida keltiriladi. Bu o'rniga qo'yish natijasida quyidagilarga ega bo'lamiz:

$$sin\,x=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{2t}{1+t^2},\quad cos\,x=\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{1-t^2}{1+t^2},\quad x=2arctgt,$$

$$dx=\frac{2dt}{1+t^2}$$

$tg(x/2)=t$ universal o'rniga qo'yish ko'p hollarda murakkab hisoblashlarga olib keladi, chunki uni qo'llaganda $sinx$ va $cosx$ lar t orqali t^2 ni o'z ichiga olgan ratsional kasrlar ko'rinishida ifodalanadi. Ba'zi xususiy hollarda $\int R(sin\,x, cos\,x)dx$ ko'rinishdagi integrallarni topish soddalashtirilishi mumkin.

1.Agar $R(sin\,x, cos\,x)$ – $sinx$ ga nisbatan toq funksiya, ya'ni $R(-sinx,\ cosx)=-R(sinx,\ cosx)$ bo'lsa, $cosx=t$ o'rniga qo'yish yordamida integral ratsionallashadi.

2. Agar $R(sin\,x, cos\,x)$ $cosx$ ga nisbatan toq funksiya , ya'ni $R(sin\,x,\ -cos\,x)=-R(sin\,x, cos\,x)$ bo'lsa, integral $sinx=t$ o'rniga qo'yish yordamida ratsionallashadi.

3.Agar $R(sin\,x, cos\,x)$- $sinx,\ cosx$ larga nisbatan juft funksiya, ya'ni $R(-sinx,-cosx)=R(sinx,cosx)$ bo'lsa, maqsadga $tgx=t$ o'rniga qo'yish olib keladi.

Trigonometrik o'rniga qo'yishlar.

$\int R\left(x,\sqrt{a^2-x^2}\right)dx$, $\int R\left(x,\sqrt{a^2+x^2}\right)dx$, $\int R\left(x,\sqrt{x^2-a^2}\right)dx$

ko'rinishdagi integrallar mos trigonometrik o'rniga qo'yish yordamida $sint$ va $cost$ larga nisbatan ratsional funksiyalarning integrallariga keltiriladi. Birinchi integral uchun $x=a\,sin t$ yoki $x=a\,cos\,t$, ikkinchi integral uchun $x=atgt$ yoki $x=actgt$ va uchinchi integral uchun $x=a\,sec\,t$ yoki $x=a\,cosec\,t$ almashti -

rishlar bajariladi.

Aniq integral

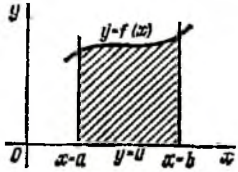
$f(x)$ funksiya $[a,b]$ kesmada aniqlangan bo‘lsin. $[a,b]$ kesmani

$$a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$$

nuqtalar bilan ixtiyoriy n-ta bo‘lakka bo‘lamiz, har bir elementar $[x_{k-1}, x_k]$ kesmadan ixtiyoriy ξ_k nuqta olamiz va har bir shunday kesmaning uzunligini topamiz: $\Delta x_k = x_k - x_{k-1}$. $f(x)$ funksiya uchun $[a,b]$ kesmada integral yig‘indi deb $\sigma = \sum_{k=1}^{n} f(\xi_k)\Delta x_k$ ko‘rinishdagi yigindiga aytiladi, agarda har bir $\varepsilon > 0$ son uchun shunday $\delta > 0$ son topilsaki, ξ_k ning ixtiyoriy tanlanishida $\Delta x_k < \delta$ dan $|\sigma - I| < \varepsilon$ tengsizlik bajarilishi kelib chiqsa, yig‘indi chekli limitga ega bo‘ladi, $f(x)$ funksiyadan $[a,b]$ kesmada (yoki a dan b-gacha bo‘lgan chegarada) olingan aniq integral deb integral yigindining elementar kesmalarning eng kattasining uzunligi $max\,\Delta x_k$ nolga intilgandagi limitga aytiladi.

$$I = \int_a^b f(x)dx = \lim_{max\,\Delta x_k \to 0} \sigma = \lim_{max\,\Delta x_k \to 0} \sum_{k=1}^{n} f(\xi_k)\Delta x_k$$

Agar $f(x)$ funksiya $[a,b]$ da uzluksiz bo‘lsa, integral yigindining limiti mavjud bo‘ladi (aniq integralning mavjudlik teoremasi).
a va b sonlar mos ravishda integrallashning yuqori va quyi chegaralari deyiladi. Agar $[a,b]$ da $f(x) > 0$ bo‘lsa, $\int_a^b f(x)dx$ aniq integral geometrik ma’noda egri chiziqli trapetsiyaning, ya’ni $y = f(x)$, $x = a$, $x = b$, $y = 0$ chiziqlar bilan chegaralangan shaklning yuzini ifodalaydi (12-rasm).



12-rasm

Aniq integralning asosiy hossalari

1. $\int_b^a f(x)dx = -\int_a^b f(x)dx$;

2. $\int_a^a f(x)dx = 0$;

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$;

4. $\int_a^b [f_1(x) \pm f_2(x)]dx = \int_a^b f_1(x)dx \pm \int_a^b f_2(x)dx$;

5. $\int_a^b C \cdot f(x)dx = C \cdot \int_a^b f(x)dx$; bu yerda C-o'zgarmas;.

6. Aniq integralni baholash:

agar $[a,b]$ da $m \le f(x) \le M$ bo'lsa, u holda

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

Aniq integralni hisoblash qoidalari

1. Nyuton-Leybnis formulasi:

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

bu yerda $F(x)$ - $f(x)$ uchun boshlangich, ya'ni $F'(x) = f(x)$.

2. Bo'laklab integrallash:

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$$

bu yerda $u=u(x)$, $v=v(x)$ lar $[a,b]$ kesmada uzluksiz differensiallanuvchi funksiyalar .

3. O'zgaruvchini almashtirish:

$$\int_a^b f(x)dx = \int_\alpha^\beta f[\varphi(t)]\,\varphi'(t)dt$$

bu yerda $x = \varphi(t)$ $\alpha \le t \le \beta$ kesmada o‘zining hosilasi $\varphi'(t)$ bilan birgalikda uzluksiz funksiya, $a = \varphi(\alpha)$, $b = \varphi(\beta)$, $f[\varphi(t)]$- $\alpha \le t \le \beta$ kesmada uzluksiz funksiya.

4. Agar $f(x)$–toq funksiya bo‘lsa, ya’ni $f(-x)=-f(x)$ bo‘lsa, u holda

$$\int_{-a}^{a} f(x)dx = 0$$

Agar $f(x)$–juft funksiya bo‘lsa, ya’ni $f(-x) = f(x)$, bo‘lsa, u holda

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

Xosmas integrallar

Xosmas integrallar deb:

1. chegaralari cheksiz bo‘lgan integralarga;
2. chegaralanmagan funksiyadan olingan integrallarga aytiladi.

a-dan $+\infty$-gacha $f(x)$ -funksiyadan olingan xosmas integral

$$\int_{a}^{+\infty} f(x)dx = \lim_{b \to +\infty} \int_{a}^{b} f(x)dx$$

tenglik bilan aniqlanadi. Agar bu limit mavjud va chekli bo‘lsa, xosmas integral yaqinlashuvchi deyiladi; agar limit mavjud bo‘lmasa yoki cheksizga teng bo‘lsa, uzoqlashuvchi deyiladi. Xuddi shuningdek,

$$\int_{\infty}^{b} f(x)dx = \lim_{a \to -\infty} \int_{a}^{b} f(x)dx$$

va

$$\int_{-\infty}^{+\infty} f(x)dx = \lim_{\substack{a \to +\infty \\ b \to +\infty}} \int_{a}^{b} f(x)dx$$

Agar $f(x)$ funksiya $[a,b]$ kesmaning c nuqtasida cheksiz uzilishga ega bo‘lsa va $a \le x < c$ va $c < x \le b$ larda uzluksiz bo‘lsa, u holda ta’rifga ko‘ra

$$\int_{a}^{b} f(x)dx = \lim_{\varepsilon \to 0+0} \int_{\alpha}^{c-\varepsilon} f(x)dx + \lim_{\beta \to 0+0} \int_{c+\beta}^{b} f(x)dx$$

deb olinadi. Agar tenglikning o‘ng tomonidagi ikkala limit mavjud bo'lsa, $\int_a^b f(x)dx$ (bu yerda $f(c)=\infty,\ a<c<b$) xosmas integral yaqinlashuvchi deyiladi, va agar yuqoridagi limitlardan xech bo'lmaganda bittasi mavjud bo'lmasa yoki cheksiz bo'lsa, xosmas integral uzoqlashuvchi deyiladi.

Taqqoslash belgilari.

Xosmas integrallami yaqinlashuvchiligini tekshirganda taqqoslash belgilarining biridan foydalanadilar.

1. Agar $f(x)$ va $\varphi(x)$ funksiyalar barcha $x \geq a$ -lar uchun aniqlangan va $[a,A]$ kesmada, bunda $A \geq a$, integrallanuvchi va barcha $x \geq a$ lar uchun $0 \leq f(x) \leq \varphi(x)$ bo'lsa, u holda

a) $\int_a^{+\infty} \varphi(x)dx$ integralning yaqinlashishidan $\int_a^{+\infty} f(x)dx$ integralning yaqinlashishi kelib chiqadi, shu bilan birga

$$\int_a^{+\infty} f(x)dx \leq \int_a^{+\infty} \varphi(x)dx$$

b) $\int_a^{+\infty} f(x)dx$ integrajning uzoqlashishidan $\int_a^{+\infty} \varphi(x)dx$ integralning ham uzoqlashishi kelib chiqadi.

2. (a) Agar $x \to +\infty$ da $f(x) \geq 0$ funksiya $1/x$ -ga nisbatan $p>0$ tartibli cheksiz kichik bo'lsa, $\int_a^{+\infty} f(x)dx$ integral $p>1$ bo'lganda yaqinlashadi va $p \leq 1$ bo'lganda uzoqlashadi.

(b) Agar $f(x) \geq 0$ funksiya $a \leq x < b$ oraliqda aniqlangan va uzluksiz va $x \to b-0$ da $\frac{1}{b-x}$ ga nisbatan p tartibli cheksiz katta bo'lsa, $\int_a^b f(x)dx$ integral $p<1$ bo'lganda yaqinlashadi, p>1 bo'lganda uzoqlashadi.

Yassi figuralarning yuzini hisoblash

$y = f(x)$ $[f(x) \geq 0]$ egri chiziq, $x=a$ va $x=b$ to'gri chiziqlar va OX o'qidagi $[a,b]$ kesma bilan chegaralangan egri ciziqli trapetsiya yuzi quyidagi formula yordamida hisoblanadi:

$$S = \int_a^b f(x)dx \cdot$$

$y = f_1(x)$, $y = f_2(x)[f_1(x) \leq f_2(x)]$ egri chiziqlar, $x=a$ va $x=b$ to'gri chiziqlar bilan chegaralangan figura yuzi quyidagi formula yordamida topiladi:

$$S = \int_a^b [f_2(x) - f_1(x)]dx$$

Agar egri chiziq $x = x(t)$, $y = y(t)$ ko'rinishdagi parametrik tenglama orqali berilgan bo'lsa, u holda bu egri chiziq bilan hamda $x=a$, $x=b$ to'gri chiziq va OX o'qidagi $[a,b]$ kesma bilan chegaralangan egri ciziqli trapetsiya yuzi quyidagi formula yordamida topiladi:

$$S = \int_{t_1}^{t_2} y(t)x'(t)dt$$

bunda t_1 va t_2 lar quyidagi

$$a = x(t_1), b = x(t_2), [t_1 \leq t \leq t_2 \text{ da } y(t) \geq 0]$$

tenglamalardan topiladi. Qutb koordinatalarida berilgan $\rho = \rho(\theta)$ egri chiziq va $\theta = \alpha$, $\theta = \beta$ $(\alpha < \beta)$ qutb radiuslari bilan chegaralangan egri chiziqli sektor yuzi quyidagi formula yordamida topiladi:

$$S = \frac{1}{2}\int_\alpha^\beta \rho^2 d\theta$$

Yassi egri chiziq yoyining uzunligini hisoblash

Agar $y = f(x)$ egri chiziq $[a,b]$ oraliqda silliq bo'lsa (ya'ni $y' = f'(x)$ hosila uzluksiz) u holda bu egri chiziqning mos yoyining uzunligi quyidagi formuladan topiladi:

$$L = \int_a^b \sqrt{1 + y'^2}\,dx \cdot$$

Agar L egri chiziq $x = x(t)$ va $y = y(t)$ parametrik tenglamalar bilan berilgan bo‘lsa ($x = x(t)$ va $y = y(t)$ funksiyalar uzluksiz differensiallanuvchi) t parametrning t_1 dan t_2 gacha monoton o‘zgarishiga mos keluvchi egri chiziq yoyining uzunligi quyidagi formuladan topiladi:

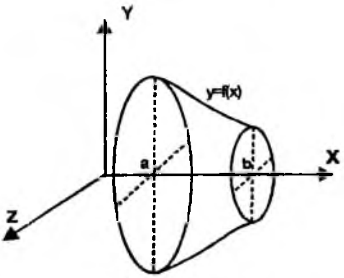
$$L = \int_{t_1}^{t_2} \sqrt{x'^2 + y'^2}\,dt \cdot$$

Agar L silliq egri chiziq qutb koordinatalar sistemasida berilgan bo‘lsa, ya’ni $\rho = \rho(\theta), \alpha \le \theta \le \beta$ bo‘lsa, u holda yoy uzunligi quyidagiga teng:

$$L = \int_\alpha^\beta \sqrt{\rho^2 + \rho'^2}\,d\theta$$

Aylanish sirtining yuzasini hisoblash

OXY tekisligidagi $y = f(x)$, $a \le x \le b$ chiziqni OX o‘qi atrofida aylantirishdan hosil bo‘lgan sirt-aylanish sirti deyiladi (13-rasm).



13-rasm

Aylanish sirtining yuzasi

$$S = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}\,dx$$

formula bilan, agar chiziq

$$x = x(t), \qquad y = y(t), \qquad \alpha \le t \le \beta \qquad (*)$$

parametric tenglamalar bilan berilgan bo'lsa,

$$S = 2\pi \int_a^b y(t)\sqrt{(x'(t))^2 + (y'(x))^2}\,dt$$

formula bilan hisoblanadi.

Aylanish jismining hajmini hisoblash

$y = f(x), \quad a \le x \le b$ ciziqni OX o'qi atrofida aylantirishdan hosil bo'lgan sirt, $x=a$ va $x=b$ tekisliklar bilan chegaralangan jism-aylanish jismi deyiladi.

Uning hajmi

$$V = \pi \int_a^b f^2(x)dx$$

formula bilan, agar ciziq (*) parametric tenglamalar orqali berilgan bo'lsa,

$$V = \pi \int_a^b y^2(t)x'(t)dt$$

formula bilan hisoblanadi.

Hisoblash uchun vazifalar

1 – vazifa. 1 – 4 – masalalarda aniqmas integrallami hisoblang.
2 - vazifa. 5,6 - masalalarda aniq integrallami hisoblang.
3 - vazifa. 7 – masalada hosmas integralni yaqinlashuvchanligini tekshiring.
4 - vazifa. 8 - masalada aniq integralni tatbiq qilib geometriyaga oid masaqlani yeching.

Namunali variant

1– masala. $\int arcsinx\, dx$ aniqmas integralni hisoblang.
Yechish: Bu integralni bo'laklab integrallaymiz. Buning uchun $\int udv = uv - \int vdu$ formuladan foydalanamiz:

$$\left[\begin{matrix} u = arcsinx \;, & du = \dfrac{dx}{\sqrt{1-x^2}} \\ dv = dx \;, & v = x \end{matrix}\right]$$

$$\mathfrak{I} = x\, arcsinx - \int x \cdot \frac{dx}{\sqrt{1-x^2}} =$$

$$= x \cdot arcsinx + \frac{1}{2}\int (1-x^2)^{-1/2}\, d(1-x^2) =$$

$$= x \cdot arcsinx + \sqrt{1-x^2} + C \quad .$$

Javobi: $\quad x \cdot arcsinx + \sqrt{1-x^2} + C \quad .$

2 – масала.

$$\mathfrak{I} = \int \frac{2x^5 + 6x^3 + 1}{x^4 + 3x^2} dx$$

aniqmas integralni hisoblang.

Yechish: Integral ostidagi noto‘g‘ri kasming butun qismini ajratib olib, to‘g‘ri kasrga keltiramiz:

$$\frac{2x^5 + 6x^3 + 1}{x^4 + 3x^2} = 2x + \frac{1}{x^4 + 3x^2}$$

hosil bo‘lgan to‘g‘ri kasmi noma’lum koeffitsientlar usuli bilan sodda kasrlarga ajratamiz:

$$\frac{1}{x^2(x^2+3)} = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx + D}{x^2 + 3}$$

$$1 = A(x^2+3) + Bx(x^2+3) + (Cx + D)x^2 =$$
$$= (B + C)x^3 + (A + D)x^2 + 3Bx + 3A$$

Bundan

$$\begin{cases} B + C = 0, \\ A + D = 0, \\ 3B = 0, \\ 3A = 1 \end{cases} \qquad A = \frac{1}{3}\,, \quad B = 0\,, \quad C = 0\,, \quad D = -\frac{1}{3}\;.$$

Demak,

$$\frac{1}{x^2(x^2+3)} = \frac{1}{3x^2} - \frac{1}{3(x^2+3)}\;.$$

Bu ifodani integrallasak,

$$\int \frac{2x^5 + 6x^3 + 1}{x^4 + 3x^2} dx == \int \left(2x + \frac{1}{3x^2} - \frac{1}{3(x^2+3)}\right) dx =$$

$$= x^2 - \frac{1}{3x} - \frac{1}{3\sqrt{3}} arctg \frac{x}{\sqrt{3}} + C ю$$

Javobi:

$$x^2 - \frac{1}{3x} - \frac{1}{3\sqrt{3}} arctg \frac{x}{\sqrt{3}} + C \ .$$

3 – masala. $\int \frac{1+\sqrt[4]{x}}{x+\sqrt{x}} dx$ integralni hisoblang .

Yechish: $x = t^4$ almashtirish bajaramiz. U holda $dx = 4t^3 dt$ bo'ladi.

$$\int \frac{1+\sqrt[4]{x}}{x+\sqrt{x}} dx = \int \frac{1+t}{t^4+t^2} \cdot 4t^3 dt = 4\int \frac{t^3(1+t)}{t^2(t^2+1)} dt =$$

$$= 4\int \frac{t(t+1)}{t^2+1} dt = 4\int \frac{t^2+t}{t^2+1} dt = 4\int \left(1 + \frac{t-1}{t^2+1}\right) dt =$$

$$= 4t - 2ln(t^2+1) - 4arctgt =$$

$$= 4\sqrt[4]{x} - 2\,ln\left(\sqrt{x}+1\right) - 4arctg\sqrt[4]{x} + C \ .$$

Javobi: $4\sqrt[4]{x} - 2\,ln\left(\sqrt{x}+1\right) - 4arctg\sqrt[4]{x} + C$.

4 – masala. $\int \frac{dx}{2sinx - cosx}$ aniqmas integralni hisoblang.

Yechish: Bu yerda $x = tg\frac{t}{2}$ belgilash kiritamiz, u holda

$$cosx = \frac{1-t^2}{1+t^2}, \quad sinx = \frac{2t}{1+t^2}, \quad dx = \frac{2dt}{1+t^2}$$ bo'ladi.

$$J = \int \frac{dx}{2sinx - cosx} = \int \frac{2dt}{(1+t^2)\left(\frac{4t}{1+t^2} - \frac{1-t^2}{1+t^2}\right)} =$$

$$= \int \frac{2dt}{t^2+4t-1} = 2\int \frac{dt}{(t+2)^2-5} = 2 \cdot \frac{1}{2\sqrt{5}} ln\left|\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right| =$$

$$= \frac{\sqrt{5}}{5} ln\left|\frac{tg\frac{x}{2}+2-\sqrt{5}}{tg\frac{x}{2}+2+\sqrt{5}}\right| + C \ .$$

Javobi: $\frac{\sqrt{5}}{5} ln \left| \frac{tg\frac{x}{2} + 2 - \sqrt{5}}{tg\frac{x}{2} + 2 + \sqrt{5}} \right| + C$.

5-masala.

$\mathcal{J} = \int_0^{\pi/4} x^2 cos2x\, dx$ aniq integralni hisoblang.

Yechish: Bu integralni bo'laklab integrallaymiz.

$$\begin{bmatrix} u = x^2 \;, & du = 2xdx \;, \\ dv = cos2xdx \;, & v = \frac{1}{2} sin2x \end{bmatrix},$$

$$\mathcal{J} = \int_0^{\pi/4} x^2 cos2x\, dx = \frac{x^2}{2} sin2x \Big|_0^{\pi/4} - \int_0^{\pi/4} xsin2xdx = \frac{\pi^2}{32} - \int_0^{\pi/4} xsin2x\, dx.$$

Yana bo'laklab integrallashni qo'llaymiz.

$$\begin{bmatrix} u = x \;, & du = dx \;, \\ dv = sin2xdx \;, & v = \frac{-1}{2} cos2x \end{bmatrix}, \quad \mathcal{J} = \int_0^{\pi/4} x^2 cos2x\, dx =$$

$$= \frac{\pi^2}{32} + \frac{x}{2} cos2x \Big|_0^{\pi/4} - \frac{1}{2} \int_0^{\pi/4} cos2x\, dx = \frac{\pi^2}{32} - \frac{1}{4} sin2x \Big|_0^{\pi/4} - \frac{\pi^2 - 8}{32}.$$

6 – masala.

$$\mathcal{J} = \int_0^{\sqrt{2}/2} \frac{e^{arcsinx} + x + 1}{\sqrt{1 - x^2}} dx$$ aniq integralni hisoblang.

Yechish: Berilgan integralni uchta integralning yig'indisiga ajratamiz, u holda

$$J=\int_0^{\sqrt{2}/2} e^{arcsinx}d(arcsinx)-\frac{1}{2}\int_0^{\sqrt{2}/2}(1-x^2)^{-\frac{1}{2}}d(1-x^2)+$$

$$+\int_0^{\sqrt{2}/2}\frac{dx}{\sqrt{1-x^2}}=e^{arcsinx}\Big|_0^{\sqrt{2}/2}-\sqrt{1-x^2}\Big|_0^{\sqrt{2}/2}+$$

$$+arcsinx\Big|_0^{\sqrt{2}/2}=e^{\pi/4}-1-\frac{1}{\sqrt{2}}+1+\frac{\pi}{4}=e^{\pi/4}+\frac{\pi-2\sqrt{2}}{4}.$$

Javobi:

$$e^{\pi/4}+\frac{\pi-2\sqrt{2}}{4}$$

7 – masala. $\int_{-1}^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}$ xosmas integralni yeching.

Yechish: Integral ostidagi funksiya $[-I;2]$ kesmaning ichidagi x_1 nuqtada uzilishga egadir. Shuning uchun quyidagi formulaga ko'ra

$$\int_a^b f(x)dx=\lim_{\substack{\varepsilon_1\to 0\\(\varepsilon_1>0)}}\int_a^{c-\varepsilon_1} f(x)dx+\lim_{\substack{\varepsilon_2\to 0\\(\varepsilon_2>0)}}\int_{c+\varepsilon_2}^{b} f(x)dx;$$

($x=c$ nuqtada funksiya uzilishga ega).

$$\int_{-1}^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}=\lim_{\varepsilon_1\to 0}\int_{-1}^{1-\varepsilon_1}\frac{dx}{\sqrt[3]{(x-1)^2}}+\lim_{\varepsilon_2\to 0}\int_{1+\varepsilon_2}^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}=$$

$$=\lim_{\varepsilon_1\to 0}3\sqrt[3]{x-1}\Big|_{-1}^{1-\varepsilon_1}+\lim_{\varepsilon_2\to 0}3\sqrt[3]{x-1}\Big|_{1+\varepsilon_2}^{2}=$$

$$=3\lim_{\varepsilon_1\to 0}(\sqrt[3]{-\varepsilon_1}-\sqrt[3]{-2})+\lim_{\varepsilon_2\to 0}(1-\sqrt[3]{\varepsilon_2})=3(\sqrt[3]{2}+1).$$

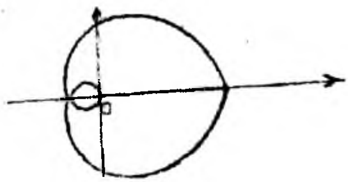
Berilgan xosmas integral yaqinlashadi.

Javobi: $3(\sqrt[3]{2}+1)$.

8 – masala. $r=acos^3\frac{\varphi}{3}$ egri chiziq yoyining uzunligini hisoblang (14-rasm0

Yechish: Berilgan funksiyaning shaklini chizamiz (14-rasm).

Shakldan ko'rinib turibdiki, $0 \le \varphi \le \frac{3}{2}\pi$ oralig'i o'zgarganda egri chiziqni yarmini yasaymiz. U holda
$L_{AB} = \int_{\varphi_A}^{\varphi_B} \sqrt{r^2(\varphi) + (r'(\varphi))^2}\, d\varphi$ formuladan,



14-rasm

$$L = 2\int_0^{\frac{3}{2}\pi} \sqrt{r^2(\varphi) + (r'(\varphi))^2}\, d\varphi \quad \text{bo'ladi.}$$

Bu yerda,

$$r(\varphi) = a\cos^3\frac{\varphi}{3},\ r'(\varphi) = -a\cos^2\frac{\varphi}{3}\cdot \sin\frac{\varphi}{3}\ .$$

$$L = 2\int_0^{\frac{3}{2}\pi} \sqrt{a^2\cos^6\frac{\varphi}{3} + a^2\cos^4\frac{\varphi}{3}\sin^2\frac{\varphi}{3}}\, d\varphi =$$

$$= 2a\int_0^{\frac{3}{2}\pi} \cos^2\frac{\varphi}{3}\, dv = a\int_0^{\frac{3}{2}\pi}\left(1 + \cos\frac{2\varphi}{3}\right) d\varphi =$$

$$= a\left(\varphi\Big|_0^{\frac{3}{2}\pi} + \frac{3}{2}\sin\frac{2\varphi}{3}\Big|_0^{\frac{3}{2}\pi}\right) = \frac{3}{2}a\pi.$$

Javobi: $\frac{3}{2}a\pi$.

VARIANTLAR

1-variant

1. $\int(4-3x)e^{-3x}dx$. 2. $\int\frac{x^3+1}{x^2-x}dx$. 3. $\int\frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx$.

4. $\int \sin2x\cos x dx$. 5. $\int_{e+1}^{e^2+1}\frac{1+\ln(x-1)}{x-1}dx$.

6. $\int_{-2}^{0}(x^2+5x+6)\cos2x dx$. 7. $\int_1^{\infty}\frac{arctgx}{x^2}dx$.

8. Quyidagi chiziqlar bilan chegaralangan tekis yuza hisoblansin $y=(x-2)^2$, $y=4x-8$.

2-variant

1. $\int arctg(\sqrt{4x-1})dx.$ 2. $\int \frac{dx}{\sqrt{x^2-4x+6}}.$ 3. $\int \frac{(2x-3)^{\frac{1}{2}}}{(2x-3)^{\frac{1}{3}}}dx.$

4. $\int sin5xcos7xdx.$ 5. $\int_0^1 \frac{x^2+1}{(x^3+3x+1)^2}dx.$ 6. $\int_{-2}^{0}(x^2-4)\,cos3xdx.$

7. $\int_1^{\infty} \frac{dx}{x\sqrt{x-1}}.$ 8. Quyidagi chiziqlar bilan chegaralangan tekis yuza hisoblansin $y=4-x^2,\quad y=x^2-2x.$

3-variant

1. $\int(3x+4)e^{3x}dx.$ 2. $\int \frac{x^3-17}{x^2-4x+3}dx.$ 3. $\int \frac{2}{(2-x)}\sqrt{\frac{2-x}{2+x}}dx.$

4. $\int sinxcos^3xdx.$ 5. $\int_0^1 \frac{4arctgx-x}{1+x^2}dx.$ 6. $\int_{-1}^{0} \frac{x^3+4x^2+3x}{x}cosxdx.$

7. $\int_0^2 \frac{dx}{\sqrt[3]{x^2}}.$ 8. Quyidagi chiziqlar bilan chegaralangan tekis yuza hisoblansin $y=arccosx,\ y=0,\ x=0.$

4-variant

1. $\int(4x-2)cos2xdx.$ 2. $\int \frac{2x^3+5}{x^2-x-2}dx.$ 3. $\int \frac{dx}{\sqrt{(x-1)^3(x+2)^5}}$

4. $\int sin2xcos^4xdx.$ 5. $\int_0^2 \frac{x^3}{x^2+4}dx.$ 6. $\int_{-2}^{0}(x+2)^2cos3x\,dx.$

7. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}.$ 8. Quyidagi chiziqlar bilan chegaralangan tekis yuza hisoblansin $y=(x+1)^2,\quad y^2=x+1.$

5-variant

1. $\int(4-16x)\sin4xdx.$ 2. $\int \frac{2x^3-1}{x^2+x-6}dx.$ 3. $\int \frac{\sqrt{x+1}+2}{(x+1)^2-\sqrt{x+1}}dx.$

4. $\int sin2xcos^3xdx.$ 5. $\int_{\pi}^{2\pi} \frac{x+cosx}{x^2+2sinx}dx.$

6. $\int_{-4}^{0}(x^2+7x+12)\,cosxdx.$ 7. $\int_{-\infty}^{+\infty} \frac{2xdx}{1+x^2}.$ 8. $y=e^{1-x}$ egri chizig'i va $y=0,\ x=0,\ x=1$ to'g'ri chiziqlari bilan chegaralangan figuraning *OX* o'qi atrofida aylanishidan hosil bo'lgan jismning hajmi topilsin.

6-variant

1. $\int ln(x^2+4)dx.$ 2. $\int \frac{3x^3+25}{x^2+3x+2}dx.$ 3. $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

4. $\int \frac{dx}{5+4sinx}.$ 5. $\int_0^{\frac{\pi}{4}} \frac{2cosx+3sinx}{(2sinx-3cos)^3}dx.$ 6. $\int_0^{\pi}(2x^2+4x+7)\,cos2xdx.$

7. $\int_{-\infty}^{\infty}\frac{dx}{x^2+2x+2}$. 8. $x = a(t - sint),\ y = a(1 - cost)$ sikloidaning yarmining OX o'qi atrofida aylanishidan hosil bo'lgan jism sirtining yuzasi hisoblansin.

7-variant

1. $\int(5x-2)e^{3x}dx$. 2. $\int\frac{(x^3+2x^2+3)}{(x-1)(x-2)(x-3)}$dx. 3. $\int\sqrt{(x-1)(x-2)}dx$.

4. $\int sin^3xcos2xdx$. 5. $\int_0^{\frac{1}{2}}\frac{8x-arctg2x}{x}dx$.

6. $\int_0^{\pi}(9x^2+9x+11)\,cos3xdx$. 7. $\int_1^{\infty}\frac{ln(x^2+1)dx}{x}$.

8. quyidagi chiziqlar bilan chegaralangan tekis yuza hisoblansin $y = e^x,\ y = e^{-x},\ x = 1$.

8-variant

1. $\int(1-6x)e^{2x}dx$. 2. $\int\frac{3x^3+2x^2+1}{(x^2-4)(x+1)}dx$. 3. $\int\frac{dx}{\sqrt[3]{(x-1)(x+1)^2}}$.

4. $\int sin4xcos5xdx$. 5. $\int_1^4\frac{1+\frac{1}{2\sqrt{x}}}{(\sqrt{x}+x)^2}dx$.

6. $\int_0^{\pi}(8x^2+16x+17)cos4x\,dx$. 7. $\int_0^{\infty}xe^{-x^2}dx$

8. quyidagi chiziqlar bilan chegaralangan tekis yuza hisoblansin $y = lnx \quad y = ln^2x$.

9-variant

1. $\int ln(1+4x^2)dx$. 2. $\int\frac{x^3-1}{(x^2-1)(x+2)}dx$. 3. $\int\frac{dx}{\sqrt{2x-1}-\sqrt[4]{2x-1}}dx$.

4. $\int\frac{2-sinx}{2+cosx}dx$. 5. $\int_0^1\frac{xdx}{1+x^4}$. 6. $\int_0^{2\pi}(3x^2+5)\,cos2xdx$.

7. $\int_0^{\infty}e^{-\sqrt{x}}dx$. 8. quyidagi chiziqlar bilan chegaralangan tekis yuza hisoblansin $x^2+y^2+6x-2y+8=0,\ y = x^2+6x+10$.

10-variant

1. $\int(2-4x)sin2xdx$. 2. $\int\frac{(2x-3)dx}{(x^2-4x+8)^3}$. 3. $\int\frac{\sqrt{x+5}}{x}dx$.

4. $\int sin^5xdx$. 5. $\int_{\sqrt{3}}^{\sqrt{8}}\frac{x+\frac{1}{x}}{\sqrt{x^2+1}}dx$. 6. $\int_0^{2\pi}(2x^2-15)\,cos3xdx$.

7. $\int_1^{\infty}\frac{dx}{x\sqrt{x^2+1}}$. 8. $r = a\sqrt{cos2\varphi}$ egri chizig'i, unga $x=0$ nuqtada o'tgan urinma va OX o'qi bilan chegaralangan tekis yuza hisoblansin.

11-variant

1. $\int xsin^2xdx$. 2. $\int \frac{dx}{x^4-x^2}$. 3. $\int \frac{xdx}{\sqrt[4]{x^2-4}}$.

4. $\int x^8sin^2xcos^6xdx$. 5. $\int_{\sqrt{2}}^{\sqrt{3}} \frac{xdx}{\sqrt{x^4-x^2-1}}$. 6. $\int_0^{\pi} x^2\, e^{3x}dx$.

7. $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$. 8. $y = lnx$ egri chizig‘i, unga $x=0$ nuqtada o‘tgan urinma va *OX* o‘qi bilan chegaralangan tekis yuza hisoblansin

12-variant

1. $\int e^{-2x}(4x-3)dx$. 2. $\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}dx$. 3. $\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x^5}-\sqrt[6]{x^7}}dx$.

4. $\int \frac{dx}{1-sinx}$. 5. $\int_0^{\sqrt{3}} \frac{arctgx+x}{1+x^2}dx$. 6. $\int_0^{2\pi}(1-8x^2)cos4x\,dx$.

7. $\int_0^2 \frac{x^3dx}{\sqrt{4-x^2}}$. 8. $r = asin3\varphi$ egri chizig‘i bilan chegaralangan tekis yuza hisoblansin.

13-variant

1. $\int e^{-3x}(2-9x)dx$. 2. $\int \frac{(5x^3+2)}{x^3+5x^2+4x}dx$. 3. $\int \frac{dx}{(1-x)\sqrt[5]{(1+x)^2}}$.

4. $\int cos^4xdx$. 5. $\int_0^{\sqrt{3}} \frac{x-arcctg^4x}{1+x^2}dx$. 6. $\int_{-4}^{0}(x^2+2x+1)\,sin3xdx$.

7. $\int_0^3 \frac{x^3dx}{\sqrt{9-x^2}}$. 8. $y = lnx$ egri chizig‘ining $\sqrt{3} \le x \le \sqrt{8}$ oraliqdagi yoy uzunligi hisoblansin.

14-variant

1. $\int arctg\sqrt{2x-1}dx$. 2. $\int \frac{32xdx}{(2x-1)(4x^2-16x+15)}$. 3. $\int \frac{dx}{\sqrt{(x+1)^2(x-1)^4}}$.

4. $\int sin^3xdx$. 5. $\int_0^1 \frac{x^3}{1+x^2}dx$. 6. $\int_0^3(x^2-3x)\,sin2xdx$.

7. $\int_{\frac{2}{\pi}}^{\infty} \frac{1}{x^2}sin\frac{1}{x}dx$. 8. $y = arcsin\,(e^{-x})$ egri chizig‘ining $0 \le x \le 1$ oraliqdagi yoy uzunligi hisoblansin.

15-variant

1. $\int arctg\sqrt{3x-1}dx$. 2. $\int \frac{(x^3-6x^2+9x+7)}{(x-2)^3(x-5)}dx$. 3. $\int \frac{dx}{x\left(2+\sqrt[3]{\frac{x-1}{x}}\right)}$.

4. $\int sin^5xdx$. 5. $\int_0^{sin1} \frac{1+(arcsinx)^2}{\sqrt{1-x^2}}dx$. 6. $\int_0^{\pi}(x^2-3x+2)\,sinxdx$.

7. $\int_0^{\infty} e^{-ax}sinbxdx$. 8. Quyidagi egri chiziqning berilgan oraliqdagi yoy uzunligi hisoblansin

$\begin{cases} x = 2(cost + tsint) \\ y = 2(sint - tcost) \end{cases}$ $0 \le t \le \frac{\pi}{2}$.

16-variant

1. $\int arctg\sqrt{5x-1}dx$. 2. $\int \frac{(x^3-2x^2+4)}{x^3(x-2)^3}dx$. 3. $\int \frac{x^3}{\sqrt{x-1}}dx$.

4. $\int cos^3xdx$. 5. $\int_1^3 \frac{1-\sqrt{x}}{\sqrt{x}(x+1)}dx$. 6. $\int_0^{\frac{t}{2}}(x^2-5x+6)sin3x\,dx$.

7. $\int_0^{\infty} e^{-x}sinxdx$. 8. $y = xe^x$ egri chizig'i hamda $y = 0$, $x = 1$ to'g'ri chiziqlari bilan chegaralangan figuraning OX o'qi atrofida aylanishidan hosil bo'lgan jismning hajmi hisoblansin.

17-variant

1. $\int(5x+6)cos2xdx$. 2. $\int \frac{xdx}{(x-1)(x+1)^2}$. 3. $\int \frac{xdx}{2+\sqrt{2x+1}}$.

4. $\int \frac{sinxdx}{sinx+3cosx}$. 5. $\int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x\sqrt{x^2+1}}$. 6. $\int_{-3}^{0}(x^2+6x+9)\,sin2xdx$.

7. $\int_1^{\infty} \frac{arctgx}{1+x^2}dx$ .8.Quyidagi egri chiziqning berilgan oraliqdagi yoy uzunligi hisoblansin $\begin{cases} x = 10cos^3t \\ y = 10sin^3t \end{cases}$ $0 \le t \le \frac{\pi}{2}$.

18-variant

1. $\int(7x-10)sin4xdx$. 2. $\int \frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}dx$. 3. $\int \frac{xdx}{\sqrt[3]{3x+8}}$.

4. $\int \frac{dx}{1+tgx}$. 5. $\int \frac{1-cosx}{(x-sinx)^2}dx$. 6. $\int_{-1}^{0}(x+2)^3ln^2(x+2)\,dx$.

7. $\int_0^1 \frac{xdx}{\sqrt{1-x^2}}$. 8. $\frac{x^2}{4}+\frac{y^2}{9} = 1$. ellipsning OX o'qi atrofida aylanishidan hosil bo'lgan jismning hajmi hisoblansin.

19-variant

1. $\int(x\sqrt{2}-3)cos2xdx$. 2. $\int \frac{(x+1)dx}{(x^2+4x+5)^2}$. 3. $\int \frac{xdx}{\sqrt[3]{2x+3}}$.

4. $\int 2^4sin^6\left(\frac{x}{2}\right)dx$. 5. $\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$. 6. $\int_0^{\frac{\pi}{2}}(1-5x^2)\,sinxdx$.

7. $\int_0^{\infty} x^3e^{-x^2}dx$. 8. $y= 2x - x^2$ parabolani y=-x to'g'ri chizig'i kesishdan hosil bo'lgan segmentning yuzasi hisoblansin.

20-variant

1. $\int(4x+7)cos3xdx$. 2. $\int \frac{dx}{(x+a)(x+b)}$. 3. $\int \frac{xdx}{\sqrt[3]{3x+4}}$.

4. $\int \frac{dx}{1+cos^2x}$. 5. $\int_1^e \frac{x^2+lnx^2}{x}dx$. 6. $\int_{\frac{\pi}{4}}^{3}(3x-x^2)sin2x\,dx$.

7. $\int_0^{\infty}\frac{xdx}{x^3+1}$. 8. $x^2+(y-3)^2=16$ doiraning OX o‘qi atrofida aylanishidan hosil bo‘lgan toming sirt yuzasi hisoblansin.

21-variant

1. $\int 2xe^{3x}dx$. 2. $\int\frac{3x+1}{(x+1)(x-2)}dx$ 3. $\int\frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$. 4.$\int ctg^6x\,dx$.

5.$\int_0^8(\sqrt[3]{x}-\sqrt{2}x)dx$. 6. $\int_0^{\frac{\pi}{2}}e^{2x}cosx\,dx$. 7.$\int_1^{\infty}\frac{2+cos\,x}{\sqrt{x}}dx$.

8 $y=lnx$, $x=\sqrt{3}$, $x=\sqrt{8}$, chiziqlar bilan chegaralangan yassi figuraning yuzini hisoblang.

22-variant

1.$\int ln\,(3x-5)dx$. 2.$\int\frac{3x+2}{2x^2+x-3}dx$. 3.$\int\frac{dx}{\sqrt[3]{x^2}+2\sqrt{x}}$ 4.$\int\frac{cos^4x}{sin^3x}dx$

5. $\int_0^1\frac{x\,dx}{(x^2+1)^2}$ 6. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{x\,dx}{sin^2x}$. 7. $\int_1^{\infty}\frac{arctgx}{x}dx$.

8 Quyidagi parabolalar bilan chegaralangan yassi figuraning yuzini hisoblang: $x=-2y^2$, $x=1-3y^2$.

23-variant

1.$\int(2x-1)ln\,(x+1)dx$. 2.$\int\frac{dx}{x^3+1}$. 3.$\int\sqrt[3]{\frac{1-x}{1+x}}dx$. 4.$\int tg^3\,x\,dx$.

5.$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{cosx}{sin^3x}dx$. 6.$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}sin2x\,sin7x\,dx$. 7.$\int_0^{+\infty}\frac{dx}{x^3+8}$.

8. $x=0, x=2$ to‘g‘ri chiziqlar va $y=2^x, y=2x-x^2$ egri chiziqlari bilan chegaralangan yassi figuraning yuzini hisoblang:

24-variant

1.$\int xarctg\,dx$. 2.$\int\frac{7x-15}{x^3+2x^2+5x}dx$. 3.$\int\frac{\sqrt{2x-3}}{\sqrt[3]{2x-3}+1}dx$ 4. $\int\frac{dx}{cos^4x}$.

5.$\int_0^{2\pi}cos^3\,x\,dx$. 6.$\int_0^{2\pi}\frac{dy}{1-4sin^2y}$. 7.$\int_a^{+}\frac{dx}{x\,lnx}$ $(a>1)$.

8. $y=x^2+1$ parabola va $x+y=3$ to‘g‘ri chizig‘i bilan chegaralangan yassi figuraning yuzini hisoblang.

25-variant

1.$\int e^{2x}sin5x\,dx$. 2.$\int\frac{x+1}{x^4+4x^2+4}dx$. 3.$\int\frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x^5}-\sqrt[6]{x^7}}$. 4.$\int tg^7x\,dx$.

5.$\int_0^{\pi}\sqrt{\frac{1+cos2x}{2}}dx$. 6.$\int_0^{\frac{\pi}{3}}\frac{x\,dx}{cos^2x}$. 7.$\int_{-\infty}^{\infty}\frac{dx}{x^2+2x+2}$. 8. $y^2=4x$, $x^2=4y$ va $x^2+y^2=5$ egri chiziqlari bilan chegaralangan yassi figuraning birinchi chorakdagi qismining yuzini hisoblang.

26-variant

1.$\int 2x^3 cosx\, dx$. 2.$\int \frac{5x-14}{x^3+x^2-4x+4} dx$. 2.$\int \frac{dx}{(1-x)\sqrt{1-x^2}}$. 4.$\int \frac{sin^2x}{cos^6x} dx$.

5.$\int_0^1 \frac{e^x}{1+e^{2x}} dx$. 6.$\int_0^{\frac{\pi}{3}} \frac{x\, dx}{cos^2x}$. 7.$\int_2^{\infty} \frac{arctg\frac{x}{2}}{4+x^2}$. 8. $y = (x-4)^2$, $y = 16 - x^2$ va OX o'qi bilan chegaralangan yassi figuraning yuzini hisoblang:

27-variant

1.$\int 3x^2 e^{4x} dx$. 2.$\int \frac{x^3}{x^2-5x+6} dx$. 3.$\int (x-2)\sqrt{\frac{1+x}{1-x}} dx$.

4.$\int sin^4x cos^6x\, dx$. 5.$\int_{-\pi}^{\pi} sin^3 \frac{x}{2} dx$ 6.$\int_0^1 x^5(1-x)^3 dx$

7.$\int_1^{\infty} \frac{nl(1+x^2)}{x} dx$. 8. Quyidagi parabolalar bilan chegaralangan yassi figuraning yuzini hisoblang: $x = y^2$, $x = \frac{3}{4}y^2 + 1$.

28-variant

1.$\int \sqrt{x}\, lnx\, dx$. 2.$\int \frac{x^3}{x^2-5x+6}$. 3.$\int \frac{(x+3)dx}{\sqrt{4x^2+4x-3}}$. 4.$\int \frac{sinx}{1+sinx} dx$.

5.$\int_0^{16} \frac{dx}{\sqrt{x+9}-\sqrt{x}}$. 6.$\int_0^{\frac{\pi}{2}} cos^5x sin\, 2x\, dx$. 7.$\int_0^{\infty} e^{-x} sinx\, dx$.

8.$r = cos\theta$ aylana yoyining uzunligini hisoblang.

29-variant

1.$\int 3x^2 e^{-3x} dx$. 2.$\int \frac{x^4-3}{x^3-x} dx$. 3.$\int \frac{5x+4}{\sqrt{x^2+2x+2}} dx$. 4.$\int \frac{1}{cos^4 sinx} dx$.

5.$\int_{ln3}^{ln7} \frac{e^x \cdot \sqrt{e^x-3}}{e^x+1} dx$. 6.$\int_0^{\frac{\pi}{2}} x\, cosx\, dx$. 7.$\int_{-2}^{\infty} \frac{dx}{x^2+\sqrt{x^4+3}}$.

8. $y = x^2 + 4x$ parabola va $y = x + 4$ to'g'ri chiziqlari bilan chegaralangan yassi figuraning yuzini hisoblang:

30-variant

1.$\int (2x^2+1) cosx\, dx$. 2.$\int \frac{x^3+3x^2+3x+1}{x^2-x} dx$. 3.$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}}$.

4.$\int \frac{sin^4x}{cosx} dx$ .5.$\int_1^e \frac{cos\,(lnx)}{x} dx$.6.$\int_0^{e-1} ln(x+1)\, dx$. 7.$\int_1^{\infty} \frac{dx}{x^2(1+x)}$.

8. $y = x^2 - 4x$ parabola va $y = 4 - 4x$ to'g'ri chiziqlari bilan chegaralangan yassi figuraning yuzini hisoblang.

DIFFERENSIAL TENGLAMALAR
Birinchi tartibli tenglamalar

$$F(x,y,y')=0 \qquad (1)$$

yoki

$$y'=f(x,y) \qquad (2)$$

ko'rinishdagi tenglamalar birinchi tartibli differensial tenglamalar deyiladi. Bu tenglamalarning (a,b) intervaldagi yechimi deb shunday $y=f(x)$ funksiyaga aytamizki, uni va uning hosilasini (1) va (2) tenglamlarga qo'yganda, bu tengliklar ixtiyoriy $x\in(a,b)$ larda ayniyatga aylanadi. Bu yechimni oshkormas ko'rinishda aniqlovchi $\Phi(x,y)=0$ funksiya differensial tenglamaning xususiy integrali deb ataladi. Uning grafigi esa differensial tenglamaning integral chizig'i deb ataladi. (1) yoki (2) tenglamaning G sohadagi umumiy yechimi deb x argumentga va C ihtiyoriy o'zgarmasga bog'liq bo'lgan shunday $y=\varphi(x,C)$ funksiyaga aytiladiki, u C o'zgarmasning har qanday qiymatida (1) yoki (2) tenglamaning yechimi bo'ladi va G sohsning har qanday (x_0,y_0) nuqtasi uchun C parametrning $y_0=\varphi(x_0,C_0)$ shartni qanoatlantiruvchi yagona C_0 qiymati topiladi. Umumiy yechimni oshkormas ko'rinishda aniqlovchi $\Phi(x,y,C)=0$ funksiya esa bu tenglamaning umumiy integrali deb ataladi

O'zgaruvchilari ajraluvchi tenglamalar

$y'=f(x,y)$ differensial tenglamada $f(x,y)$ funksiya $f_1(x)\cdot f_2(y)$ ko'rinishdagi ko'paytuvchilarga ajralgan bo'lsa, bu tenglama quyidagi ko'rinishga keladi:

$$\frac{dy}{dx}=f_1(x)\cdot f_2(y)$$

yoki

$$\frac{dy}{f_2(y)}=f_1(x)dx.$$

Bu oxirgi tenglikni integrallab, berilgan tenglamaning umumiy yechimini hosil qilamiz. Xuddi shuning kabi

$$M(x,y)dx + N(x,y)dy = 0$$

differensial tenglama berilgan bo'lsa va

$$M(x,y) = M_1(x) \cdot M_2(y)$$

$$N(x,y) = N_1(x) \cdot N_2(y)$$

ko'paytuvchilarga ajralsa, u holda berilgan tenglama quyidagi ko'rinishga keladi:

$$\frac{M_1(x)}{N_1(x)}dx = -\frac{N_2(y)}{M_2(y)}dy \cdot$$

Buni avallgidek integrallab tenglamaning umumiy yechimini topamiz.

Bir jinsli tenglamalar

Agar birinchi tartibli differensial tenglamani

$$y' = f(\frac{y}{x})$$

ko'rinishga keltirish mumkin bo'lsa u bir jinsli deb ataladi . Bu tenglamani $\frac{y}{x} = u(x)$ almashtirish yordamida o'zgaruvchilari ajraluvchi tenglamaga keltiriladi. Ya'ni $y = x \cdot u$ bo'lib, undan $y' = u + u' \cdot x$ ni hosil qilamiz. Ularni berilgan tenglamaga qo'yib $u + u' \cdot x = f(u)$ o'zgaruvchilari ajraluvchi tenglama hosil qilamiz. Uni yechib $u = u(x,C)$ yechimni topamiz va u o'zgaruvchidan $\frac{y}{x}$ larga qaytib $y = \varphi(x,C)$ yechimni olamiz.

Birinchi tartibli chiziqli tenglama

Agar birinchi tartibli differensial tenglama y va y' larni birinchi tartibda o'z ichiga olsa, yani

$$y' = P(x)y + Q(x)$$

ko'rinishda bo'lsa, u chiziqli deyiladi.

Agar $Q(x) \equiv 0$ bo'lsa, u holda berilgan tenglama

$$y' = P(x)y$$

ko'rinishdagi chiziqli bir jinsli tenglamaga keladi. Bu tenglamani o'zgaruvchilari ajraluvchi tenglama bo'lib uning yechimi.

$$y = C \cdot e^{\int P(x)dx}$$

ko'rinishda bo'ladi, bunda C – ixtiyoriy o'zgarmas son, $\int P(x)dx$ esa $P(x)$ funksiyaning biror boshlang'ichi. Berilgan chiziqli tenglama

$$y(x) = u(x) \cdot v(x)$$

almashtirish yordamida quyidagi ko'rinishga keltiriladi:

$$V\left(\frac{du}{dx} - P(x)u\right) + \left(\frac{dv}{dx}u - Q(x)\right) = 0$$

Birinchi qavsni nolga tenglab $u = u_1(x)$ – xususiy yechim topiladi. Ikkinchi qavsdagi $u(x)$ o'rniga $u_1(x)$ ni qo'yib va uni nolga tenglab $v = v_1(x, C)$ umumiy yechimini topamiz. Bulardan esa berilgan tenglamaning umumiy yechimini topamiz:

$$y = u_1(x) \cdot v_1(x, C)$$

To'la differensialli tenglamalar

$$P(x, y)dx + Q(x, y)dy = 0$$

tenglama to'la differensialli tenglama deyiladi agar uning chap qismi biror $u(x, y)$ funksiyaning to'la differensial bo'lsa, yani

$$P(x, y) = \frac{\partial u}{\partial x}, \quad Q(x, y) = \frac{\partial u}{\partial y}$$

bo'lsa.

Berilgan tenglamaning to'la differensialligini ko'rsatish uchun

$$\frac{\partial P(x, y)}{\partial y} = \frac{\partial Q(x, y)}{\partial x}$$

shartni tekshirish zarur va yetarlidir. Agar bu shart bajarilsa bu tenglamani $du(x,y)=0$ ko'rinishida yozish mumkin va uning umumiy integrali $u(x,y)=C$ ko'rinishda bo'ladi, bunda C - ixtiyoriy o'zgarmas son.

$u(x,y)$ ni topish uchun, avvalo

$$\frac{\partial u}{\partial x}=P(x,y)$$

ni x bo'yicha, y- tayinlangan deb integrallaymiz va integrallash o'zgarmasini y ning funksiyasi sifatida olamiz, va quyidagini hosil qilamiz:

$$u(x,y)=\int P(x,y)dx+\varphi(y)$$

So'ngra

$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\int P(x,y)dx\right)+\varphi'(y)=Q(x,y)$$

tenglikdan $\varphi(y)$ ni topib va uni yuqoridagi tenglikka qo'yib $u(x,y)$ ni hosil qilamiz.

Yuqori tartibli differensiyal tenglamalar

a) $y''=f(x)$ **ko'rinishdagi tenglama.**

Bu tenglamani 2 marta integrallash natijasida

$y=\int dx\int f(x)dx+Ax+B$ ko'rinishdagi yechimini topamiz. Bu yerda A va B lar ixtiyoriy o'zgamas sonlar.

b) $F(x,y',y'')=0$ **ko'rinishdagi tenglama.**

Bu tenglamada noma'lum y funksiya ishtirok etmaydi. Uning tartibi $y'=p(x)$ almashtirish yordamida 1 birlikka tushiriladi. $y''=p'(x)$ ni topib berilgan tenglamaga qo'yamiz va $F(x,p,p')=0$ ni hosil qilamiz. So'ngra $p=\varphi(x,C_1)$ ni topib $y'=p(x)$ ga qo'yib $y'=\varphi(x,C_1)$ dan $y=\int\varphi(x,C_1)dx+C_2$ yechimni hosil qilamiz.

c) $F(y,y',y'')=0$ **ko'rinishdagi tenglama.**

Bu tenglamada erkli o'zgaruvchi x – ishtirok etmaydi. $y' = p(y)$ almashtirish yordamida uning tartibi 1 birlikka tushiriladi. $y'' = p'y' = p'p$ ni topib berilgan tenglamaga qo'yamiz va $F(y,p,p') = 0$ tenglamaga kelamiz. Uni yechib $p = \varphi(y,C_1)$ yechimini hosil qilamiz va almashtirishga qo'yib
$y' = \varphi(y,C_1)$ ni hosil qilamiz . Undan esa $y = \psi(x,C_1,C_2)$ yechimni hosil qilamiz.

Chiziqli bir jinsli, o'zgarmas koeffitsientli differensial tenglamalar

$$y'' + a_1 y' + a_2 y = 0$$

ko'rinishdagi tenglamalar chiziqli bir jinsli, o'zgarmas koeffitsientli differensiyal tenglamalar deyiladi. Bu yerda a_1, a_2 lar ixtiyoriy o'zgarmas sonlar .

Uning hususiy yechimlarini $y = e^{\lambda x}$ ko'rinishda qidiramiz.

$y = e^{\lambda x}, y' = \lambda e^{\lambda x}, y'' = \lambda^2 e^{\lambda x}$ lami berilgan tenglamaga qo'yib va ikkala tomonini $e^{\lambda x}$ ga bo'lib

$$\lambda^2 + a_1\lambda + a_2 = 0$$

harakteristik tenglamani topamiz.

Uni yechib λ_1 va λ_2 ildizlarini topamiz.

3 xil hol bo'lishi mumkin:

1). $\lambda_1 \neq \lambda_2$ haqiqiy.

2) $\lambda = \alpha \pm i\beta$ qo'shma kompleks sonlar,

3) $\lambda_1 = \lambda_2$ 2 karrali haqiqiy ildiz .

1) Agar harakteristik tenglamaning ikkita turli haqiqiy ildizlari bo'lsa u holda berilgan tenglama $y_1 = e^{\lambda_1 x}$ va $y_2 = e^{\lambda_2 x}$ xususiy yechimlarga ega bo'ladi. Ulaming chiziqli kombinatsiyasi $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$ berilgan tenglamaning umumiy yechimi bo'ladi .

2) Agar $\lambda = \alpha \pm i\beta$ bo'lsa berilgan tenglama

$y_1 = e^{\alpha x} \sin \beta x, y_2 = e^{\alpha x} \cos \beta x$ xususiy yechimlarga ega bo‘ladi va ularning chiziqli kombinatsiyasi

$y = C_1 e^{\alpha x} \sin \beta x + C_2 e^{\alpha x} \cos \beta x$ berilgan tenglamaning umumiy yechimi bo‘ladi

3) $\lambda_1 = \lambda_2$ bo‘lsa berilgan tenglamaning bitta xususiy yechimi

$y_1 = e^{\lambda_1 x}$, ikkinchisi esa $y_2 = xe^{\lambda_1 x}$ bo‘ladi.

$y = C_1 e^{\lambda_1 x} + C_2 x e^{\lambda_2 x}$ esa umumiy yechim bo‘ladi .

Chiziqli bir jinslimas, o‘zgarmas koeffitsientli differensiyal tenglamalar

$$y'' + a_1 y' + a_2 y = f(x)$$

Bu tenglamaning umumiy yechimi $y = y_0 + \bar{y}$ ko‘rinishda izlanadi. Bunda y_0 – berilgan tenglamaga mos, bir jinsli differentsiyal tenglama $y'' + a_1 y' + a_2 y = 0$ ning umumiy yechimi. $\bar{y}$ esa berilgan tenglamani qanoatlantiruvchi biror xususiy yechim.

Differensiyal tenglamalar sistemasi

$$\begin{cases} \dfrac{dy_1}{dx} = a_1 x + b_1 y, \\ \dfrac{dy_2}{dx} = a_2 x + b_2 y \end{cases}$$

Bu sistemani yechishining usullaridan biri noma’lumlarni yo‘qotish usulidir. Bunda tenglama bir noma’lumli ikkinchi darajali tenglamaga keladi va uni yechib birinchi noma’lum topiladi so‘ngra ikkinchi noma’lum topiladi.

Koshi masalasi.

$F(x, y, y') = 0$ tenglama berilgan. Masala bu tenglamaning

$y(x_0) = y_0$ boshlang‘ish shartni qanoatlantiruvchi xususiy yechimini topish bo‘lsin. Avvalo berilgan tenglamani biror usul bilan yechib uning umumiy yechimini topamiz: $y = f(x, C)$.

So'ngra x va y o'zgaruvchilar o'miga x_0, y_0 larni qo'yib C ning mos qiymatini topamiz. Uni umumiy yechimiga qo'yib $y = f(x, C_0)$ xususiy yechimini topamiz.

Hisoblash uchun vazifalar

1 – vazifa. 1, 2, 3 – masalalarda birinchi tartibli differensial tenglamaning umumiy yechimini toping.
2 – vazifa. 4 – masalada birinchi tartibli differensial tenglama uchun Koshi masalasini yeching.
3 – vazifa. 5 – masalada (masalani shartiga qarab) differensial tenglamaning xususiy yoki umumiy yechimini toping.
4 – vazifa. 6 – masalada o'zgarmas koeffitsientli chiziqli bir jinslimas differensial tenglamaning umumiy yechimini toping.
5 – vazifa. 7 – masalada o'zgarmas koeffitsientli chiziqli bir jinsli differensial tenglamalar sistamasini yeching:
a) yuqori tartibli bitta tenglamaga keltirish usulida;
b) matritsalar usulida;
c) sistemalar yechimini turg'unligini tekshiring.

Namunaviy variant

1 – masala. $xy' - y = y^3$ tenglamani umumiy yechimi topilsin.
Yechish: Bu tenglama o'zgaruvchilari ajraladigan differensial tenglamadir. Uni quyidagicha yozamiz:

$$x\frac{dy}{dx} = y^3 + y$$

So'ngra o'zgaruvchilarini ajratamiz:

$$\frac{dy}{y(y^2+1)} = \frac{dx}{x} \qquad \text{ёки} \qquad \left(\frac{1}{y} - \frac{y}{y^2+1}\right)dy = \frac{dx}{x}.$$

tenglamani ikkala tomonini integrallaymiz:

$$ln|y| - \frac{1}{2}ln(y^2+1) = ln|x \cdot c|\ , \quad c \neq 0\ .$$

Natijada $y = Cx\sqrt{y^2+1}$.
Javobi: $y = Cx\sqrt{y^2+1}$.
2 – masala. $ydx + \left(2\sqrt{xy} - x\right)dy = 0$ tenglamani umumiy yechimi topilsin.

Yechish: Berilgan tenglama bir jinsli tenglama bo'lganligidan uni quyidagicha yozamiz:

$$\frac{dy}{dx}=\frac{y}{x-2\sqrt{xy}} \quad \text{yoki} \quad \frac{dy}{dx}=\frac{\frac{y}{x}}{1-2\sqrt{\frac{y}{x}}}.$$

So'ng $y=u\cdot x$ алмаштириш бажарамиз,

$$\frac{dy}{dx}=x\cdot\frac{du}{dx}+u$$

U holda berilgan tenglama:

$$x\frac{du}{dx}+u=\frac{u}{1-2\sqrt{u}} \quad \text{yoki} \quad \frac{1-2\sqrt{u}}{2u^{3/2}}du=\frac{dx}{x} \quad \text{bo'ladi}.$$

Buni integlallab $ln|x|=-\frac{1}{\sqrt{u}}-ln|u|+C$ ni hosil qilamiz.

Bundan $ln|ux|=-\frac{1}{\sqrt{u}}+C, y=x\cdot u$ edi, unda $ln|y|=-\sqrt{\frac{x}{y}}+C$

$$\text{Javobi:} \quad ln|y|=C-\sqrt{\frac{x}{y}}$$

3 – masala.

$$\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0$$

tenglamaning umumiy yechimi topilsin.

Yechish: Berilgan tenglama to'la differensial tenglama ekanini ko'rsatamiz. Buning uchun, $M(x;y)=\frac{2x}{y^3}$ ва $N(x;y)=\frac{y^2-3x^2}{y^4}$ deb olamiz:

$$\frac{dM}{dy}=-\frac{6x}{y^4}, \quad \frac{dN}{dx}=-\frac{6x}{y^4}, \text{demak } \frac{dM}{dy}=\frac{dN}{dx}$$

bo'lganligidan

$$\int_{x_0}^{x} M(x;y)dx+\int_{y_0}^{y} N(x;y)dy=C \qquad x_0=0,\ y_0=1 \qquad \text{desak}$$

$$\int_0^x \frac{2x}{y^3}dx+\int_1^y \frac{y^2}{y^4}dy=C \quad \text{yoki} \quad \frac{x^2}{y^3}-\frac{1}{y}=C.$$

Javobi: $x^2 - y^2 = Cy^3$.

4 – masala.

$$y' - y\,tgx = \frac{1}{cosx} \;; \quad y_0 = 0.$$

Koshi masalasi yechilsin.

Yechish: Bu differensial tenglama chiziqli bo'lganligi uchun Bernulli usulida yechamiz. Yechimni $y = u \cdot v$ ko'rinishda izlaymiz.

$$u'v + v'u - uv\,tgx = \frac{1}{cosx} \text{ yoki } u'v + u(v' - v\,tgx) = \frac{1}{cosx}.$$

Bundan

$$\begin{cases} v' - v\,tgx = 0 \\ u'v = \dfrac{1}{cosx} \end{cases}$$

ni hosil qilamiz. Sistemaning birinchi tenglamasini integrallab:

$$v = \frac{1}{cosx}$$

ni hosil qilamiz. $v = \frac{1}{cosx}$ – ni qiymatini ikkinchi tenglamaga qo'yib:

$$u'\frac{1}{cosx} = \frac{1}{cosx}$$

ni hosil qilamiz. Bundan $u=x+c$ bo'ladi. u va v ning topilgan qiymatlarini $y=u\cdot v$ ga qo'ysak, umumiy yechim

$$y = (x + c)\frac{1}{cosx}$$

bo'ladi. $y_{(0)} = 0$ boshlang'ich shartni qo'llasak, $C=0$ hosil bo'ladi. Unda Koshi masalasini yechimi

$$y = x \cdot \frac{1}{cosx}$$

Javobi:

$$y = \frac{x}{cosx} \quad .$$

5 – masala. $y''y^3 + 25 = 0$ tenglamaning $y(2)=-5$, $y'(2)=-1$

boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.
Yechish: Bu tenglamani tartibini pasaytirish yo'li bilan yechamiz: $y' = p(y)$, $y'' = p \cdot p'$. Bu qiymatlarni berilgan tenglamaga qo'ysak:

$$pp' \cdot y^3 + 25 = 0$$

yoki

$$y^3 p \frac{dp}{dy} + 25 = 0.$$

O'zgaruvchilarini ajratamiz:

$$pdp = -\frac{25}{y^3} dy$$

bundan

$$\frac{p^2}{2} = \frac{25}{2y^2} + \frac{C_1}{2}$$

yoki

$$p(y) = \pm \frac{\sqrt{25 + C_1 y^2}^{\,2}}{y}.$$

$y' = p(y)$ ni e'tiborga olsak: $y' = \pm \frac{\sqrt{25 + C_1 y^2}^{\,2}}{y}$ ni hosil qilamiz. C_1 – ixtiyoriy o'zgarmasni boshlang'ich shartdan topamiz:

$$-1 = \pm \frac{\sqrt{25 + C_1 \cdot 25}}{-5}$$

yoki $5 = 5\sqrt{1 + C_1}$. Bundan $C_1=0$. C_1 ning qiymatini oxirgi tenglamaga qo'ysak

$$y' = \pm \frac{5}{y}$$

hosil bo'ladi.
Buni integrallasak:

$$\frac{y^2}{2} = \pm 5x + C_2$$

ga ega bo'lamiz. Bu tenglikni integrallab va $y(2) \cdot y'^{(2)} = 5$ ni hisobga olib $\frac{y^2}{2} = 5x + C_2$ ni hosil qilamiz. $y(2) = 5$ shartdan C_2 ni topamiz:

$$\frac{25}{2} = 5 \cdot 2 + C_2, \quad C_2 = \frac{5}{2}.$$

Demak, $\frac{y^2}{2} = 5x + \frac{5}{2}$ yoki $y^2 = 10x + 5$.

Javobi: $y^2 = 10x + 5$.

6 – masala. $y''+9y=cos2x$ tenglamaning umumiy yechimi topilsin.

Yechish: Bu yerda xarakteristik tenglama $k^2 + 9 = 0$ qo'shma kompleks ildizga ega bo'ladi: $k_{1,2} = \pm 3i$. U holda bir jinsli tenglamaning umumiy yechimi

$$y = C_1 cos3x + C_2 sin3x$$

bo'ladi . Bir jinsli bo'lmagan tenglamaning xususiy yechimini quyidagicha izlaymiz:

$$y = Acos2x + Bsin2x$$

A va B o'zgarmas sonlarni aniqmas koeffitsientlar usulidan foydalanib topamiz:

$$y' = -2Asin2x + 2Bcos2x \ ,$$
$$y'' = -4Acos2x - 4Bsin2x \ .$$

Bularni berilgan tenglamaga qo'yamiz

$$5Acos2x + 5Bsin2x = cos2x$$

Bundan

$$\begin{cases} 5A = 1 \ , \\ B = 0 \ . \end{cases} \qquad \begin{cases} A = \frac{1}{5} \\ B = 0 \end{cases}$$

larni hosil qilamiz.

Berilgan tenglamaning xususiy yechimi

$$y = \frac{1}{5} cos2x$$

bo'ladi .Umumiy yechim esa:

$$y = C_1 cos3x + C_2 sin3x + \frac{1}{5} cos2x$$

Javobi. $y = C_1 \cos 3x + C_2 \sin 3x + \frac{1}{5}\cos 2x$.

7 –masala

$$\begin{cases} \dot{x} = -y, \\ \dot{y} = x \end{cases}$$

tenglamalar sistemasini umumiy yechimi topilsin va turg'unligi tekshirilsin.

Yechish:

a) Yechimni bitta bir jinsli tenglamaga keltirib toping;

Sistemani birinchi tenglamasini differensiallaymiz.

$\ddot{x} = -\dot{y}$, bunda $\dot{y}$ ning qiymatini ikkinchi tenglamaga qo'yamiz: $-\ddot{x} = x$ ёки $\ddot{x} + x = 0$.

Bu tenglamaning umumiy yechimi

$$x(t) = C_1 cost + C_2 sint$$

bo'ladi.

Endi

$$y = -\dot{x}$$

dan

$$y(t) = -(-C_1 sint + C_2 cost)$$

yoki $y(t) = C_1 sint - C_2 cost$ ni hosil qilamiz.

Shunday qilib C_1, C_2 ixtiyoriy o'zgarmaslar uchun sistemaning umumiy yechimi:

$$\begin{cases} x(t) = C_1 cost + C_2 sint \\ y(t) = C_1 sint - C_2 cost \end{cases}$$

b) Sistemani matritsa usulida yechamiz.

Berilgan sistemani matritsa ko'rinishida yozamiz:

$\frac{d\overline{x}}{dt} = A\overline{x}$, bu yerda $\overline{x} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

U holda sistemani umumiy yechimi $\overline{x}(t) = \Phi(t) \cdot C$ bo'ladi.

Bu yerda: $\Phi(t)$ - haqiqiy xususiy yechimining fundamental matritsasi: $C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$ – ixtiyoriy o'zgarmasining matritsasi.

Xarakteristik tenglama tuzamiz: $det(A - \lambda E) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$

Uning ildizi $\lambda_{1,2} = \pm i$ dir.

Endi B matritsa tuzamiz (xarakteristik tenglamaning ildizlari har xil haqiqiy bo'lganda B matritsa A matritsaning kanonik ko'rinishiga teng bo'ladi):

$$B = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

Fundamental kanonik matritsa tuzamiz:

$$H(t) = e^{Bt} = \begin{pmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{pmatrix}.$$

$$S = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

matritsa tuzamiz, u $A{\cdot}S{=}S{\cdot}B$ shartni qanoatlantiradi.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

a_{11} , a_{12}, a_{21}, a_{22} sonlami topish uchun 4ta tenglamadan 2ta har xil tenglamani olamiz.

$$\begin{cases} -a_{21} = ia_{11} \\ --a_{22} = -ia_{12} \end{cases}$$

$a_{11} = 1$, $a_{12} = 1$ deb olib $a_{21} = -i$, $a_{22} = i$ lami hosil qilamiz. Demak,

$$S = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$

Berilgan sistemaning yechimini fundamental matritsasi,

$$\Phi(t) = S \cdot H(t)$$

yoki

$$\Phi(t) = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{pmatrix}$$

bo'ladi. Haqiqiy va mavhum qismlarini ajratib

$$\Phi(t) = \begin{pmatrix} cost & sint \\ sint & -cost \end{pmatrix}$$

ni topamiz. Sistemaning umumiy yechimi

$$\overline{x}(t) = \begin{pmatrix} cost & sint \\ sint & -cost \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} C_1 cost + C_2 sint \\ C_1 sint - C_2 cost \end{pmatrix}$$

Javobi:

$$\begin{cases} x(t) = C_1 cost + C_2 sint \\ y(t) = C_1 sint - C_2 cost \end{cases}$$

c) Berilgan tenglamalar sistemasini *x(t)=0, y(t)=0* tinch nuqtadagi trivial yechimini turg'unligini tekshiramiz.

Xarakteristik tenglamalarni ildizi qo'shma komleks bo'lganligi $\lambda=\pm i$ ва $Re\lambda=0$, $\Im m\,\lambda = \pm 1 \neq 0$ dan tinch nuqtada sistema turg'undir va tinch nuqta – markazdir.

Variantlar

1-variant

1. $4xdx - 3ydy = 3x^2ydy - 2x^2ydx$. 2. $y' = \frac{y^2}{x^2} + 4\frac{y}{x} + 2$.

3. $3x^2e^ydx + (x^3e^y - 1)dy = 0$. 4. $y' - \frac{y}{x} = x^2,\ y(1) = 0$.

5. $4y^3y'' = y^4 - 16, \quad y(0) = 2\sqrt{2},\ \ y'(0) = \frac{1}{\sqrt{2}}$.

6. $y'' - 2y' = e^{2x} + x$. 7. $\begin{cases} x = 2x + y, \\ y = 3x + 4y. \end{cases}$

2-variant

1. $\sqrt{4 + y^2}dx - ydy = x^2ydy$. 2. $y' = \frac{x+y}{x-y}$.

3$\left(3x^2 + \frac{2}{y}cos\frac{2x}{y}\right)dx = \frac{2x}{y^2}cos\frac{2x}{y}dy$.

4. $y' + ycosx = \frac{sin2x}{2}, y(0) = 0$.

5. $y'' = 128y\ \ y(0) = 1, y'(0) = 8$. 6. $y''' - y' = 2e^x + cosx$.

7. $\begin{cases} x = x - y, \\ y = y - 4x. \end{cases}$

3-variant

1.$6xdx - 6ydy = 2x^2ydy - 3xy^2d$.2. $xy' = \frac{3y^3+2yx^2}{2y^2+x^2} + 4\frac{y}{x} + 2$.

3. $(3x^2 + 4y^2)dx + (8xy + e^y)dy = 0$. 4. $y' - yctgx =$
$= 2xsinx,\ \ y\left(\frac{\pi}{2}\right) = 0$. 5. $y''y^3 + 64 = 0\ \ y(0) = 4, y'(0) = 2$.

6. $y'' + y = 2sinx$. 7. $\begin{cases} x = 8y - x, \\ y = x + y. \end{cases}$

4-variant

1. $x\sqrt{1 + y^2} + yy^1\sqrt{1 + x^2} = 0$. 2. $xy' = \sqrt{x^2 + y^2} + y$.

3. $\left(2x-1-\frac{y}{x^2}\right)dx-\left(2y-\frac{1}{x}\right)dy=0$ 4. $y'+ytgx=cos^2x,$ $y\left(\frac{\pi}{4}\right)=\frac{1}{2}.$ 5. $y''=32sin^3y\cdot cos5y,\quad y(1)=\frac{\pi}{2},\quad y'(1)=4.$

6. $y''+y=2e^x.$ 7. $\begin{cases} x=x+y, \\ y=3y-2x. \end{cases}$

5-variant

1. $\sqrt{3+y^2}dx-ydy=x^2ydy.$ 2. $2y'=\frac{y^2}{x^2}+6\frac{y}{x}+3.$

3. $(y^2+ysec^2x)dx+(2xy+tgx)dy=0.$

4. $y'-\frac{y}{x+2}=x^2+2x,\ y(0)=0.$

5. $y''=2y^3,\qquad y(-1)=1,\ \ y'(-1)=1.$

6. $y'''-4y'=8sin2x.$ 7. $\begin{cases} x=x-3y, \\ y=3x+y. \end{cases}$

6-variant

1. $x\sqrt{3+y^2}dx+y\sqrt{2+x^2}dy=0.$ 2. $xy'=\frac{3y^3+4yx^2}{2y^2+2x^2}.$

3. $(3x^2y+2y+3)dx+(x^3+2x+3y^2)dy=0.$

4. $y'-\frac{y}{x+1}=e^x(x+1),\ y(0)=1.$ 5. $x^2y''+xy'=1.$

6. $y'''-4y'=24e^{2x}.$ 7. $\begin{cases} x=-5y-x, \\ y=x+y. \end{cases}$

7-variant

1. $(e^{2x}+5)dy+ye^{2x}dx=0.$ 2. $y'=\frac{x+2y}{2x-y}.$

3. $\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)dy=0.$

4. $y'-\frac{y}{x}=xsinx,\ y\left(\frac{\pi}{2}\right)=1.5.$ $tgx\cdot y'''=2y''.$

6. $y''+16y=16cos4x.$ 7. $\begin{cases} x=2x+y, \\ y=2y-x. \end{cases}$

8-variant

1. $y'y\sqrt{\frac{1-x^2}{1-y^2}}=-1.$ 2. $xy'=2\sqrt{x^2+y^2}+y.$

3. $[sin2x-2cos(x+y)]dx-2cos(x+y)dy=0.$

4. $y'+\frac{y}{x}=sinx,\ y(\pi)=\frac{1}{\pi}.$ 5. $(1+x^2)y''+2xy'=x^3.$

6. $y'' + 3y' = -e^{-3x}$. 7. $\begin{cases} x = 3x - y, \\ y = 4x - y. \end{cases}$

9-variant

1. $x\sqrt{5 + y^2}dx + y\sqrt{4 + x^2}dy = 0$. 2. $xy' = \frac{3y^3 + 6yx^2}{2y^2 + 3x^2}$.
3. $(x^2 + y^2 + 2x)dx + 2xydy = 0$. 4. $y' + \frac{2x}{1+x^2}y = \frac{2x^2}{1+x^2}, y(0) = \frac{2}{3}$.
5. $y'' + \frac{2x}{1+x^2}y' = 2x$. 6. $y''' - 36y' = 36e^{6x}$. 7. $\begin{cases} x = 7x + 3y, \\ y = 6x + 4y. \end{cases}$

10-variant

1. $6xdx - 6ydy = 3x^2ydy - 2xy^2dx$. 2. $3y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4$.
3. $(x + y)dx + (e^y + x + 2y)dy = 0$. 4. $y' + \frac{y}{2x} = x^2, \quad y(1) = 1$
5. $x^4y'' + x^3y' = 4$. 6. $y'' + 3y' = e^{3x}$. 7. $\begin{cases} x = 4x - 3y, \\ y = 3x + 4y. \end{cases}$

11-variant

1. $y(4 + e^x)dy - e^xdx = 0$. 2. $y' = \frac{x^2 + xy - y^2}{x^2 - 2xy}$. 3. $xy^2dx +$
$+y(x^2 + y^2)dy = 0$. 4. $y' - \frac{2x-5}{x^2}y = 5, \quad y(2) = 4$. 5. $y''' -$
$36y' = 299(cos7x + sin7x)$. 6. $y'''tgx = y''$. 7. $\begin{cases} x = x + 4y, \\ y = 2x + 3y. \end{cases}$

12-variant

1. $\sqrt{4 - x^2}y' + xy^2 + x = 0$. 2. $xy' = \sqrt{2x^2 - y^2} + y$.
3. $\left(\frac{x}{\sqrt{x^2+y^2}} + y\right)dx + \left(x + \frac{y}{\sqrt{x^2+y^2}}\right)dy = 0$.
4. $y' + \frac{y}{x} = \frac{x+1}{x}e^x, y(1) = e$. 5. $2xy''' = y''$.
6. $y'' + 25y = 20cos5x$. 7. $\begin{cases} x = x + 5y, \\ y = x - 3y. \end{cases}$

13-variant

1. $2xdx - 2ydy = x^2ydy - xy^2dx$. 2. $y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 6$.
3. $\frac{1+xy}{x^2y}dx + \frac{1-xy}{xy^2}dy = 0$. 4. $y' + \frac{y}{x} = \frac{lnx}{x}, \quad y(1) = 1$.
5. $xy''' + xy'' = 1$. 6. $y''' + 3y'' + 2y' = 1 - x^2$. 7. $\begin{cases} x = 3x + 2y, \\ y = -2x + y. \end{cases}$

14-variant

1. $x\sqrt{4+y^2}dx + y\sqrt{1+x^2}dy = 0$. 2. $xy' = \frac{3y^3-6yx^2}{2y^2+4x^2}$.

3. $\frac{dx}{y} - \frac{x+y^2}{y^2}dy = 0$. 4. $y' - \frac{y}{x} = -\frac{12}{x^3}$, $y(1) = 4.5$. $xy''' + 2y'' = 0$.

6. $3y^{IV} + y''' = 6x - 1$. 7. $\begin{cases} x = x + 4y, \\ y = x + y. \end{cases}$

15-variant

1. $(e^x + 8)dy - ye^x dx = 0$. 2. $y' = \frac{x^2+2xy-y^2}{2x^2-2xy}$.

3. $\frac{x}{y^2}dx - \frac{xy+1}{x}dy = 0$. 4. $y' - \frac{2}{x}y = x^3$, $y(1) = -\frac{5}{6}$.

5. $y'' = 32y^3$, $y(4) = 1$, $y'(4) = 4$.

6. $y''' - 4y'' + 4y' = (x-1)e^x$. 7. $\begin{cases} x = 3x + y, \\ y = x + 3y. \end{cases}$

16-variant

1. $\sqrt{5+y^2} + y'y\sqrt{1-x^2} = 0$. 2. $xy' = 3\sqrt{x^2+y^2} + y$.

3. $\left(xe^x + \frac{y}{x^2}\right)dx - \frac{dy}{x} = 0$. 4. $y' + \frac{y}{x} = 3x$, $y(1) = 1$.

5. $y''y^3 + 16 = 0$, $y(1) = 2$, $y'(1) = 2$.

6. $y^{IV} + y''' = 12x + 6$. 7. $\begin{cases} x = x - 2y, \\ y = x - y. \end{cases}$

17-variant

1. $6xdx - ydy = yx^2dy - 3xy^2dx$. 2. $2y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4$.

3. $\left(10xy - \frac{1}{siny}\right)dx + \left(5x^2 + \frac{xcosy}{sin^2y} - y^2siny^3\right)dy = 0$.

4. $y' = \frac{2xy}{1+x^2} + 1 + x^2$, $y(1) = 3$.

5. $y'' = 2y^3$, $y(-1) = 1$, $y'(-1) = 1$.

6. $y'' + 2y' = 4e^x(sinx + cosx)$. 7. $\begin{cases} x = x - 3y, \\ y = 3x + y. \end{cases}$

18-variant

1. $ylny + xy' = 0$. 2. $xy' = \frac{3y^3+10yx^2}{2y^2+5x^2}$.

3. $\left(\frac{y}{x^2+y^2}+e^x\right)dx-\frac{xdy}{x^2+y^2}=0.$ 4. $y'=\frac{2x-1}{x^2}y+1,\ y(1)=1$
5. $y''y^3+y=0, \quad y(0)=-1,\ y'(0)=-2.$
6. $y''+y'+5y=-sin2x.$ 7. $\begin{cases}x=x-2y,\\ y=3x+6y.\end{cases}$

19-variant

1. $(1+e^x)y'=ye^x.$ 2. $y'=\frac{x^2+3xy-y^2}{3x^2-2xy}.$
3. $e^ydx+(cosy+xe^y)dy=0.$ 4. $y'+\frac{3y}{x}=\frac{2}{x^3},\ y(1)=1.$
5. $y^3y''=4(y^4-1),\ y(0)=\sqrt{2},\ y'(0)=\sqrt{2}.$
6. $y''+y=2cos4x+3sin4x.$ 7. $\begin{cases}x=5x+y,\\ y=-3x+3y.\end{cases}$

20-variant

1. $\sqrt{1-x^2}y'+xy^2+x=0.$ 2. $xy'=3\sqrt{2x^2+y^2}+y.$
3. $(y^3+cosx)dx+(3xy^2+e^y)dy=0.$
4. $y'+2xy=-2x^3,\ y(1)=e^{-1}.$
5. $xy'''=2.$ 6. $y''-4y'+4y=e^{2x}sin5x.$
7. $\begin{cases}x=x+6y,\\ y=-2x+9y.\end{cases}$

21-variant

1. $\sqrt{3+y^2}dx-ydy=x^2ydy.$ 2. $\dot{y}=\frac{x+4y-5}{6x-y-6}$
3. $2xydx+(x^2-y^2)dy=0.$ 4. $y'+\frac{y}{x}=3x, \quad y(1)=1.$
5. $y''-2y'+10y=74sin3x,\ y(0=6,\ y'0)=3.$
6. $y''-2xy+2y=e^x+xcosx.$ 7. $\left(\frac{\dot{x}=2x+y}{\dot{y}=4y-x}\right.$.

22-variant

1. $x(4+y^2)dx+y(1+3x^2)dy=0.$ 2. $y'=\frac{x+2y-3}{x-1}.$
3. $y^2(ydx-2xdy)=x^3(xdy-2ydx).$
4. $y'+\frac{y}{2x}=x,y(1)=1.$

5. $y''-\frac{y'}{x-1}=x(x-1),y(2)=1,\ y'(2)=-1.$

$6. y'' + 6y' + 10y = 3xe^{-3x} - 2e^{3x}cosx.$ 7. $\left(\frac{\dot{x}=2y-3x}{\dot{y}=y-2x}\right.$.

23-variant

1.$(6x+3xy^2)dx = ydy + 2x^2ydy$. 2. $y' = \frac{x+y-4}{x-2}$.

3. $(2 - 9xy^2)xdx + (4y^2 - 6x^3)ydy = 0$.

4. $y' - \frac{y}{x} = x\ sinx,\ y\left(\frac{\pi}{2}\right) = 1$. 5. $yy'' - y'^2 = 0, y(0) = 1,$

$y'(0) = 2$. 6. $y'' = 8y' + 20y = 5xe^{4x}sin2x$.

7. $\left(\frac{\dot{x} = 2x + y}{\dot{y} = x + 2y}\right.$.

24-variant

1 .$3(x^2y + y)dy + \sqrt{2 + y^2}dx = 0$. 2. $y' = \frac{4y-8}{3x+2y-7}$.

3. $(2 - 9xy^2)xdx + (4y^2 - 6x^3)ydy = 0$.

4. $y' = \frac{2xy}{1 + x^2} = 1 + x^2,\ \ y(1) = 3$.

5. $y'' + 3y' = 0, y(0) = 1, y'(0) = 3$.

6. $y'' + 7y' + 10y = xe^{-2x}cos5x$. 7. $\frac{\dot{x}=y-7x}{\dot{y}=-2x-5y}$.

25-variant

1. $2xdx - 2ydy = yx^2dy - 3xy^2dx$. 2. $y' = \frac{x+3x+4}{3x+6}$.
3. $\frac{y}{x} = dx + (y^3 + lnx)dy = 0$. 4. $y' + yx = -x, y(0) = 3$.

5. $y'' = 4x^3 - 2x + 1, y(1) = \frac{11}{30},\ \ y'(1) = 2$.

6. $y''' + y' = sinx + xcosx$. 7. $\frac{\dot{x}=5x+4y}{\dot{y}=-2x+11y}$.

26-variant

1. $2xdx + \sqrt{2 - x^2}dy = -2xy^2dx$. 2. $y' = \frac{3x+3}{2x+y-1}$.

3. $3x^2(1 + lny)dx = \left(2y - \frac{x^3}{y}\right)dy$.

4. $y' + ytgx = cosx, y\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

5. $y'' = e^{2x}$, $y(0) = \frac{1}{4}, y'(0) = \frac{1}{2}$.

6. $y'' + 2y' + y = x(e^{-x} - cosx)$. 7. $\frac{\dot{x}=-5x-8y}{\dot{y}=-3x-3y}$.

27-variant

1. $(3 + e^x)dy = e^x dx$. 2. $y' = \frac{x+7y-8}{9x-y-8}$.

3. $x(lny + +2lnx - 1)dy = 2ydx$. 4. $y' - y\,cosx = -sin2x$, $y(0) = 3$. 5. $y'' = sinx - 1, y(0) = -1, y(0) = 1$.

6. $y'' - 6y' + 8y = 5xe^{2x} + 2e^{4x}sinx$. 7. $\frac{\dot{x}=-4x-6y}{\dot{y}=-4x-2y}$.

28-variant

1. $ylnxdx + xdy = 0$. 2. $y' = \frac{x+y-3}{2x-2}$.

3. $(x^2 + y^2 + x)dx + y\,dy = 0$. 4. $y' + \frac{2y}{x+1} =$ $= e^x(x + 1), y(0) = 1$. 5. $y'' = x - cos2x$, $y(0)\frac{9}{4}$, $y'(0) = -1$. 6. $y'' - 9y = c^{-3x}(x^2 + sin3x)$. 7. $\frac{\dot{x}=-x-5y}{\dot{y}=-4x-2y}$.

29-variant

1. $\sqrt{5 + y^2}dx + 4y(x^2 + 1)dy = 0$. 2. $y' = \frac{y}{2x+2y-2}$.

3. $ydy = (xdx + ydx)\sqrt{1 + y^2}$. 4. $y' + \frac{y}{x} = \frac{(x+1)e^x}{x}, y(e) = e$.

5. $y'' = \frac{1}{x}, y(e) = 2e\,, y'(e) = 3$.

6. $y'' - 8y' + 17y = e^{4x}(x^2 - 3xsinx)$. 7. $\frac{\dot{x}=-7x+5y}{\dot{y}=4x+8y}$.

30-variant

1. $y\sqrt{1 - x^2}dy + \sqrt{5 + y^2}dx = 0$. 2. $y' = \frac{2x+y-3}{2x-2}$.

3. $(x^2 + 3lny)ydx = xdy$. 4. $y' + y\,ctgy = 2xsinx, y\left(\frac{\pi}{2}\right) = 0$.

5. $y'' = 2x - 3x^2, y(1) = 0\,, y'(1) = 1$.

6. $y'' - 2y' + 5y = 2xe^x + e^x sinx$. 7. $\frac{\dot{x}=-x-5y}{\dot{y}=-7x-3y}$.

Adabiyotlar

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Mundarija

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