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OLIY MATEMATIKA FANIDAN LABORATORIYA ISHLARINI MAPLE DASTURIDA BAJARISH

"Tafakkur Bo'stoni"
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fanidan

LABORATORIYA ishlarini MAPLE dasturida bajarish

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Ushbu o‘quv qo‘llanma texnika yo‘nalishi talabalari uchun, “Oliy matematika” fanidan o‘quv rejadagi 18 soatli, laboratoriya ishlarini MAPLE dasturlaridan foydalanib kompyuterda bajarish uchun mo‘ljallangan.

Qo‘llanmadagi laboratoriya ishlarida amaliyot masalalarni taqribiy yechishda ko‘p qo‘llaniladigan sonli usullari yordamida, chiziqli tenglamalar sistemalarini yechish, algebraik va transtsendent tenglamalarning ildizini aniqlash, aniq integrallarni taqribiy hisoblash, oddiy va xususiy hosilali differensial tenglamalarni taqribiy yechish, Lagrang va Nyuton iterpolyatsiya ko‘phadlarini topish, chiziqli va chiziqsiz regressiya tenglamalarini kichik kvadratlar usulida topish yo‘llari ko‘rsatilgan.

Laboratoriya ishlari bo‘yicha hisoblash usullari va ularga mos masalalarni yechish uchun zaruriy nazariy ma’lumotlar berilgan. Masalalarni Maple tizimida yechish dasturlari tuzilgan.

Mustaqil ishlar uchun har bir mavzuga mos topshiriqlar berilgan.

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SO'ZBOSHI

O'zbekiston mustaqillikka erishgandan so'ng, o'z taraqqiyotining muhim shartlaridan biri bo'lgan xalqning boy ma'naviy salohiyati va umuminsoniy qadriyatlariga hamda hozirgi zamon madaniyati, iqtisodiyoti, ilmi, texnikasi va texnologiyasining so'nggi yutuqlariga asoslangan mukammal ta'lim tizimi barpo etilmoqda.

"Ta'lim to'g'risida" gi qonun va "Kadrlar tayyorlash milliy dasturi" ning qabul qilinishi natijasida ilmiy–texnika taraqqiyoti yutuqlarini xalq xo'jaligiga tadbiq qilish ijtimoiy–iqtisodiy rivojlanish bilan uzviy bog'liq ekanligining ahamiyati tobora ortib bormoqda

Oliy o'quv yurtlarining texnika yo'nalishi bo'yicha bakalavrlar tayyorlashning yangi o'quv rejasi va dasturlarida kompyuter va axborot texnologiyalari bilan ishlashga, axborotlarga zamonaviy texnik vositalar yordamida ishlov berishga va uni tahlil qilishga, amaliy masalalarni yechishda sonli usullarni tadbiq qilinishiga katta e'tibor qaratilgan.

Ushbu o'quv qo'llanma, 2011-yili tasdiqlangan o'quv rejasi asosida 5320200 "Mashinasozlik tehnalogiyasi, mashinasozlik ishlab chiqarishni jihozlash va avtomatlashtirish" ta'lim yo'nalishi, shuningdek 6 ta texnika yo'nalish talabalari uchun belgilangan 18 soatli reja asosida 3–semestrda o'tiladigan laboratoriya mashg'ulotlari uchun tayorlangan bo'lib, u Toshkent davlat texnika universitetida ishlab chiqarilgan, Oliy va o'rta maxsus ta'lim vazirligi tamonidan 2012-yilgi 14-martdagi 102 sonli buyrug'i bilan tasdiqlangan "Oliy matematika" faning namunaviy o'quv dasturidagi "Laboratoriya ishlari mazmuni va tashkil etish bo'yicha ko'rsatmalar" dagi tavsiya etilgan mavzularni o'z ichiga olgan.

Ushbu o'quv qo'llanma laboratoriya ishlaridagi sonli hisoblash masalalarini Maple dasturidan foydalanib kompyuterda yechish uchun mo'ljallangan.

KIRISH

Ushbu o'quv qo'llanma "Oliy matematika" fanidan laboratoriya ishlarini bayon qilishda undagi husoblash usullarini qat'iy matematik asoslashni maqsad qilib qo'yilmagan holda misol va masalalarni yechisli usullari ko'rsatilgan va kompyuterdan foydalanish uchun Maple dasturlari tuzilgan.

Qo'llanma "Oliy matematika" faning namunaviy o'quv dasturidagi "Laboratoriya ishlari mazmuni va tashkil etish bo'yicha ko'rsatmalar" dagi tavsiya etilgan mavzular bo'yicha 9 ta laboratoriya ishidan iborat bo'lib, unda quyidagi masalalar yoritilgan:

1) Chiziqli tenglamalar sistemasini yechimini, determinantning qiymatini va teskart matritsani Gauss usulida topish;

2) Algebraik va trantsendent tenglamalarning ildizini taqribiy hisoblash usullari,

3) Tajriba natijalarida topilgan qiymatlarning o'zgaruvchilari orasidagi bog'lamshni Lagranj va Nyuton interpolyatsiya ko'phadlari yordamida topish;

4) Tajriba natijalarinig chiziqli va parabolik bog'laninshini aniqlashda kichik kvadratlar usuli;

5) Aniq integrallarni taqribiy hisoblash usullari;

6) Birinchi va ikkinchi tartibli oddiy differensial tnglama va differensial tnglama sistemasi uchun Koshi masalasini yechimini taqribiy hisobiash;

7) Xususiy hosilali differensial tenglamalarni taqribiy yechimini to'r usulida topish;

8) Kuzatilgan tajriba ma'lumotlariga asoslanib korrelyatsion jadvalni tuzish;

9) Korrelyatsion jadval bo'yicha to'g'ri chiziqli va ikkinch darajali regressiya tenlamalarini kichik kvadratlar usulida aniqlash.

Har bir laboratoriya ishidagi hisoblashlarda foydalaniladigan Maple dasturi amallarining glossariysi tuzilgan.

Mustaqil ishlash uchun topshiriqlar bo'limida har bir laboratoriya ishi uchun topshiriqlar berilgan.

1–LABORATORIYA ISHI

Chiziqli tenglamalar sistemasini yechish
Maple dasturining buyruqlari:

with(Student[LinearAlgebra])– *student paketidan chiziqli algebra amallarini chaqirish.*

A:=<<1,2,–2>|<3,0,1>|<–2,3,2>> – *A matritsani elementlarini ustunlari bo'yicha yozilishi;*

A[2,1] – *A matritsaning 2–satr 1–ustunda joylashgan elementini aniqlash;*

Minor(A,2,1)– *A matritsaning a_{21} elementiga mos minorini hisoblash;*

Determinant(A)– *A matritsani determinantini hisoblash;*

A.B – *A va B matritsalarning ko'paytmasini;*

A^(–1) –*A matitsaga teskari matritsani topish;*

solve–*tenglama, tengsizlik va tenglamalar sistemasini yechimini topish;*

A^(–1)– *A matritsaga teskari matritsa topish;*

InverseTutor(A)– *Tutor oynasida A matritsaga teskari matritsa topish;*

GaussianElimination(A)– *kengaytirilgan matritsa uchun Gauss usulini qo'llash;*

LinearSolveTutor(A)–*Tutor oynasida Gauss usulini ketma–ket bajarish*;

with(linalg) –*paketidagi chiziqli algebra amallarini chaqirish;*

A:=matrix(3,3,[2,2,1,3,2,–1,1,–1,1]) – **with(linalg)** *paketida matritsani satrlar bo'yicha yozilishi;*

with(linalg):addrow(A,1,2,x) – *A matritsaning 1–satr elementlarini x ga ko'paytirib 2–satrga qo'shish;*

mulrow(A,1,1/A[1,1]) – *A matritsaning 1–satr elementlarini a_{11} elementiga bo'lish;*

with(linalg): det(A) –*A matritsani determinantini hisoblash.*

Maple dasturining ishchi oynasida Ctrl+K tugmalari bilan qo'yiladigan "> " taklif belgisidan so'ng buyruqni yozib, uni oxiriga " ; " ni qo'yamiz. Buyruqni bajarish uchun Enter tugmasini bosish kerak. Yangi satr uchun taklif " > " belgisini qo'yish uchun [>] piktogrammani bosamiz. Bu satrga buyruq yozish uchun F5 tugmani bosamiz, matn terish uchun bu tugmani qayta bosamiz.

Maqsad: Gauss usulida ko'p noma'lumli chiziqli tenglamalar sistemasini yechish, yuqori tartibli determinantlarni hisoblash va matritsaga teskari matritsa topishni o'rganish.

Reja:

1.1. Gauss usulida chiziqli tenglamalar sistemasini yechish.

1.2. Gauss usulida determinantni hisoblash.

1.3. Jardan-Gauss usulida matritsaga teskari matritsa topish

1.1. Chiziqli tenglamalar sistemasini Gauss usulida yechish

Chiziqli algebraik tenglamalar sistemasini yechishda keng tarqalgan Gauss usuli aniq yechish usullari guruhiga mansub bo'lib, uning mohiyati shundan iboratki, nomahlumlarni ketma – ket yo'qotish yo'li bilan berilgan sistema o'ziga ekvivalent bo'lgan pog'onali (uch burchakli) sistemaga keltiriladi. Bu kompyuter xotirasidan samarali ravishda foydalanish imkonini beradi .

Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \\ \qquad a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2 \\ \qquad \ldots \quad \ldots \quad \ldots \\ \qquad a_{kk}\ x_k + a_{kn}x_n = b_k \end{cases} \qquad (1.1)$$

ko'rinishdagi chiziqli tenglamalar sistemasi *pog'onali sistema* deyiladi, bu yerda $k \le n$, $a_{ii} \ne 0$, $i=1, 2, \ldots, k$.

Agar $k=n$ bo'lsa, u holda (1.1) sistema *uch burchakli* deyiladi.

Noma'lumlarni ketma-ket yo'qotib borish, asosan, sistemada elementar almashtirishlar qilish yordamida amalga oshiriladi. Bu elementar almashtirishlarga quyidagilar kiradi:

1) sistemaga tegishli istalgan ikkita tenglamaning o'rnini almashtirish;

2) tenglamalardan birining har ikkala qismini noldan farqli istalgan songa ko'paytirish;

3) biror tenglamaning har ikkala qismiga, biror songa ko'paytirilgan ikkinchi tenglamaning mos qismlarini qo'shish.

Berilgan tenglamalar sistemasidagi elementar almashtirishlar natijasida hosil bo'lgan sistemani berilgan sistemaga ekvivalent bo'lishini isbotlash mumkin.

Oddiylik uchun quyidagi chiziqli tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = a_{15}, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = a_{25}, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = a_{35}, \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = a_{45}. \end{cases}$$

Berilgan chiziqli tenglamalar sistemasi yechimga ega bo'lishi uchun sistemaning noma'lumlarining koeffisentlaridan tuzilgan A matrisa va barcha koeffisentlaridan, ya'ni ozod hadlarni hisobga olib tuzilgan Ab kengaytirilgan matrisa ranglari teng bo'lishi zarur, yani bu matrisalarning har biridan tuzilgan to'rtinchi tartibli determinattan biriotsi noldan farqli bo'lishi kerak: $r(A)=r(Ab)$.

Aytaylik, berilgan sistemada $a_{11}\neq 0$ (yetakchi element) bo'lsin, aks holda x_1 oldidagi koeffitsienti noldan farqli bo'lgan tenglamani birinchi tenglama o'ringa ko'chiramiz.

Sistemaning birinchi tenglamasining barcha koeffitsientlarini a_{11} ga bo'lib,

$$x_1+b_{12}x_2+b_{13}x_3+b_{14}x_4=b_{15} \qquad (1.2)$$

tenglamani hosil qilamiz, bu yerda.

$$b_{1j}=\frac{a_{1j}}{a_{11}}, \quad j=2,3,4,5.$$

Bu topilgan (1.2) tenglamadan foydalanib, yuqoridagi sistemaning qolgan tenglamalaridagi x_1 qatnashgan hadni yo'qotish mumkin. Buning uchun (1.2) tenglamani ketma-ket a_{21}, a_{31} va a_{41} larga ko'paytirib, mos ravishda sistemaning ikkinchi, uchinchi va to'rtinchi tenglamalaridan ayiramiz.

Natijada quyidagi uchta tenglamalar sistemasini hosil qilamiz.

$$\begin{cases} a_{22}^{(1)}x_2+a_{23}^{(1)}x_3+a_{24}^{(1)}x_4=a_{25}^{(1)}, \\ a_{32}^{(1)}x_2+a_{33}^{(1)}x_3+a_{34}^{(1)}x_4=a_{35}^{(1)}, \\ a_{42}^{(1)}x_2+a_{43}^{(1)}x_3+a_{44}^{(1)}x_4=a_{45}^{(1)}. \end{cases} \qquad (1.3)$$

bu sistemadagi $a_{ij}^{(1)}$ koeffitsientlar

$$a_{ij}^{(1)}=a_{ij}-a_{i1}b_{1j} \quad (i=2,3,4;\ j=2,3,4,5) \qquad (1.4)$$

formula yordamida hisoblanadi. Endi (1.3) sistemaning birinchi tenglamasini $a_{22}^{(1)}$ ga bo'lib,

$$x_2+b_{23}^{(1)}x_3+b_{24}^{(1)}x_4=b_{25}^{(1)} \qquad (1.5)$$

tenglamani hosil qilamiz, bu yerda

$$b_{2j}^{(1)}=\frac{a_{2j}^{(1)}}{a_{22}}, \qquad (j=3,4,5)$$

(1.5) tenglama yordamida (1.3) sistemaning keyingi tenglamalaridan x_2 ni, yuqoridagidek qoida asosida yo'qotamiz va quyidagi tenglamalar sistemasini topamiz:

$$\begin{cases} a_{33}^{(2)}x_3+a_{34}^{(2)}x_4=a_{35}^{(2)} \\ a_{43}^{(2)}x_3+a_{44}^{(2)}x_4=a_{45}^{(2)} \end{cases} \qquad (1.6)$$

bu yerda

$$a_{ij}^{(2)} = a_{ij}^{(1)} - a_{i2}^{(1)} b_{2j}^{(1)} \qquad (i = 3,4; \quad j = 3,4,5) \qquad (1.7)$$

(1.6) sistemaning birinchi tenglamasini $a_{33}^{(2)}$ ga bo‘lib,

$$x_3 + b_{34}^{(2)} x_4 = b_{35}^{(2)} \qquad (1.8)$$

tenglamani hosil qilamiz, bu yerda

$$b_{3j}^{(2)} = \frac{a_{3j}^{(2)}}{a_{33}^{(2)}}, \qquad (j = 4,5)$$

Bu (1.8) tenglama yordamida (1.6) sistemaning ikkinchi tenglamasidan x_3 ni yo‘qotamiz. Natijada

$$a_{44}^{(3)} x_4 = a_{45}^{(3)}$$

tenglamani hosil qilamiz, bu yerda

$$a_{4j}^{(3)} = a_{4j}^{(2)} - a_{43}^{(2)} b_{3j}^{(2)} \qquad (j = 4,5) \qquad (1.9)$$

Shunday qilib biz qaralayotgan sistemasini unga ekvivalent bo‘lgan quyidagi *uchburchakli chiziqli* tenglamalar sistemasiga olib keldik.

$$\left.\begin{array}{lllll} x_1 + b_{12}x_2 & + b_{13}x_3 & + b_{14}x_4 & = & b_{15} \\ \quad x_2 & + b_{23}^{(1)}x_3 & + b_{24}^{(1)}x_4 & = & b_{25}^{(1)} \\ & \quad x_3 & + b_{34}^{(2)}x_4 & = & b_{35}^{(2)} \\ & & a_{44}^{(3)}x_4 & = & b_{45}^{(3)} \end{array}\right\} \qquad (1.10)$$

Bu (1.10) sistemadan foydalanib nom’lumlarni, ketma–ket quyidagicha topamiz:

$$\left\{\begin{array}{l} x_4 = \dfrac{a_{45}^{(3)}}{a_{44}^{(3)}} \\ x_3 = b_{35}^{(2)} - b_{34}^{(2)}x_4 \\ x_2 = b_{25}^{(1)} - b_{24}^{(1)}x_4 - b_{23}^{(1)}x_3 \\ x_1 = b_{15} - b_{14}x_4 - b_{13}x_3 - b_{12}x_2 \end{array}\right. \qquad (1.11)$$

Demak, yuqorida keltirilgan Gauss usulida sistemaning yechimini topish 2 qismdan iborat bo‘lar ekan.

Olg‘a borish – (1.1) sistemani uchburchakli (1.10) sistemaga keltirish

Orqaga qaytish – (1.11) formulalar yordamida noma’lumlarni topish.

Gauss usuli bilan noma’lumli n ta chiziqli algebraik tenglamalar sistemasini yechish uchun bajariladigan arifmetik amallarning miqdori quyidagidan iborat:

$(n^3+3n^2-n)/3$ ta ko'paytirish va bo'lish,

$(2n^3+3n^2-5n)/6$ ta qo'shish.

Xususan:

$n=2$ da, $(2^3+3\cdot2^2-2)/3=6$. ko'paytirish va bo'lish

$(2\cdot2^3+3\cdot2^2-5\cdot2)/6=3$. qo'shish,

$n=3$ da, $(3^3+3\cdot3^2-3)/3=17$ ko'paytirish va bo'lish

$(2\cdot3^3+3\cdot3^2-5\cdot3)/6=11$. qo'shish,

$n=4$ da, $(4^3+3\cdot4^2-a)/3=36$ ko'paytirish va bo'lish

$(2\cdot4^3+3\cdot4^2-5\cdot4)/6=26$ qo'shish.

1.1–masala. Berilgan quyidagi sistemani Gauss usilida yechamiz. Buning uchun nomahlumlarni ketma–ket yo'qotamiz. Yetakchi satr uchun birinchi tenglamani tanlasak bo'ladi, chunki $a_{11}=2\neq0$.

$$\begin{cases} 2x_1 + 7x_2 + 13x_3 = 0 \\ 3x_1 + 14x_2 + 12x_3 = 18 \\ 5x_1 + 25x_2 + 16x_3 = 39 \end{cases} \qquad (1.12)$$

Gauss usili yordamida yechish uchun sistemaning satrlar bo'yicha koeffitsientlarini quyidagicha belgilaymiz:

$$\begin{array}{l} a_{11}=2, a_{12}=7, a_{13}=13 b_1=0[1] \\ a_{21}=3, a_{22}=14, a_{23}=12 b_2=18[2] \\ a_{31}=5, a_{32}=25, a_{33}=16 b_3=39[3] \end{array} \qquad (1.13)$$

Hisoblash jarayoni quyidagicha bo'ladi.

Olg'a borish.

1) (1.13) dagi 1–satr elementlarini $a_{11}=2$ ga bo'lamiz, ya'ni [1]/2:

$$(1,\ a_{12}/a_{11},\ a_{13}/a_{11},\ b_1/a_{11}) = (1,\ 7/2,\ 13/2,\ 0/2) \quad (1.14)$$

2) (1.13) ning 2– satridagi $a_{21}=3$ elementni nolga aylantirish uchun, (1.14) ni $a_{21}=3$ ga ko'paytirib, [2] satr elementlaridan mos ravishda ayiramiz. ya'ni [2] –(1.14) a_{21}:

$$a^{(0)}_{21}= a_{21}-a_{21}=0$$

$$a^{(0)}_{22}= a_{22}-a_{21}a_{12}/a_{11} = 14-3(2/2) = 7/2$$

$$a^{(0)}_{23}= a_{23}-a_{21}a_{13}/a_{11} = 12-3(6/2) = -15/2$$

$$b^{(0)}_{1} = b_1-a_{21}b_1/a_{11} = 18-3(0/2) = 18$$

Demak, 2– tenglama koeffitsentlari:

$$(0,\ 7/2,\ -15/2,\ 18) \qquad (1.15)$$

bo'ladi.

3) (1.13) ning 3– satridagi $a_{31}=5$ elementni nolga aylantirish uchun (1.14) ni $a_{31}=5$ ga ko'paytirib, [3] satr elementlaridan mos ravishda ayiramiz, ya'ni [3] –(1.14) a_{31}:

$$a^{(0)}_{31}= a_{31}-a_{31}=0$$

$$a^{(0)}_{32}= a_{32}-a_{31}a_{12}/a_{11} = 25-5(7/2) = 15/2$$

$a^{(0)}_{33} = a_{33} - a_{31}a_{13}/a_{11} = 16 - 5(6/2) = -33/2$

$b^{(0)}_3 = b_3 - a_{31}b_1/a_{11} = 39 - 5(0/2) = 39$

Demak, 3 - tenglama koeffitsentlari:

$$(0,\ 15/2,\ -33/2,\ 39) \qquad (1.16)$$

bo'ladi. Natijada topilgan yangi koeffitsientlar asosida quyidagi sistemani hosil qilamiz:

$$\begin{cases} x_1 + (7/2)x_2 + (13/2)x_3 = 0 \\ (7/2)x_2 - (15/2)x_3 = 18 \\ (15/2)x_2 - (33/2)x_3 = 39 \end{cases} \qquad (1.17)$$

Bu sistemaning koeffitsentlari:

$$\begin{aligned} &a_{11} = 1,\ a_{12} = 7/2,\ a_{13} = 13/2, b_1 = 0[1] \\ &a_{21} = 0, a_{22} = 7/2, a_{23} = -15/2, b_2 = 18[2] \\ &a_{31} = 0. a_{32} = 15/2. a_{33} = -33/2, b_3 = 39[3] \end{aligned} \qquad (1.13)$$

(1.13) ni [2] – satrini 7/2 ga bo'lamiz. Bu tenglama koeffitsentlari:

$$(0,\ 1,\ -15/7,\ 36/7) \qquad (1.18)$$

bo'ladi. (1.17) sistemaning 3–tenglamalaridan x_2 noma'lumni yo'qotish

uchun (1.18) ni 15/2 ga ko'paytirib 3–satr koeffitsentlardan mos ravishda ayirib, quyidagi koeffitsentlar topamiz, ya'ni [3]– (1.18) a_{32}:

$$(0,\ 0,\ -3/7,\ 3/7) \qquad (1.19)$$

Natijada berilgan sistemani quyidagicha yozamiz:

$$\begin{cases} x_1 + (7/2)x_2 + (13/2)\,x_3 = 0 \\ x_2 - (15/7)x_3 = 36/7 \\ -(3/7)\,x_3 = 3/7 \end{cases}$$

Orqaga qaytish.

Bu oxirgi sistemadagi 3–tenglamadan x_3 qiymatini topib bu asosida 2–tenglamadan x_2 ni topamiz. Topilgan x_2 va x_3 asosida 1– tenglamadan x_1 ni topamiz:

$x_3 = -1$

$x_2 = 36/7 + (15/7)(-1) = 21/7 = 3$

$x_1 = (-7/2)(3) - (13/2)(-1) = -8/2 = -4$

Berilgan chiziqli tenglamalar sistemasining yechimi:

$$x_1 = -4,\ x_2 = 3,\ x_3 = -1$$

1.1.1–M a p l e d a s t u r i:

1) *Gauss usilida yechish:*

> **with(LinearAlgebra):**

A := <<2,3,5>|<7,14,25>|<13,12,16>>; $A := \begin{bmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{bmatrix}$

> **b := <0,18,39>;** $b := \begin{bmatrix} 0 \\ 18 \\ 39 \end{bmatrix}$

2)*kengaytirilgan matritsani tuzish:*

> **Ab:= <<2,3,5>|<7,14,25>|<13,12,16>|<0,18,39>>;**

$$Ab := \begin{bmatrix} 2 & 7 & 13 & 0 \\ 3 & 14 & 12 & 18 \\ 5 & 25 & 16 & 39 \end{bmatrix}$$

Sistema yechimga ega bo'lishini asosiy va kengaytirilgan matritsalarning rangini tengligidan aniqlaymiz:

> **Rank(A);** 3

> **Rank(Ab);** 3

asosiy matritsaga Gauss usulini qo'llash:

> **GaussianElimination(A);** $\begin{bmatrix} 2 & 7 & 13 \\ 0 & \frac{7}{2} & \frac{-15}{2} \\ 0 & 0 & \frac{-3}{7} \end{bmatrix}$

> **GaussianElimination(A,'method'='FractionFree');**

$$\begin{bmatrix} 2 & 7 & 13 \\ 0 & 7 & -15 \\ 0 & 0 & -3 \end{bmatrix}$$

> **ReducedRowEchelonForm(`<|>`(A, b));**

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

2)*KENGAYTIRILGAN matritsa yordamida yechimni topish*

> **restart; with(Student[LinearAlgebra]):**

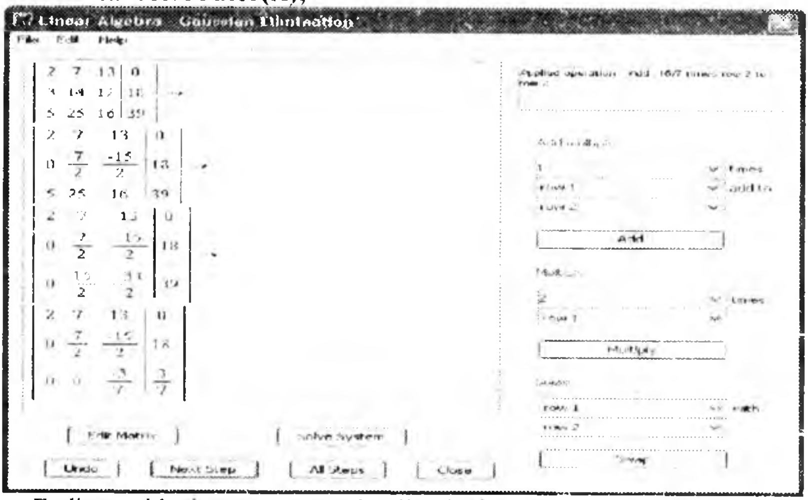
> **A:=<<2,3,5>|<7,14,25>|<13,12,16>|<0,18,39>>;**

$$A := \begin{bmatrix} 2 & 7 & 13 & 0 \\ 3 & 14 & 12 & 18 \\ 5 & 25 & 16 & 39 \end{bmatrix}$$

> **X:=LinearSolve(A);** $X := \begin{bmatrix} -4 \\ 3 \\ -1 \end{bmatrix}$

Chiziqli tenglamalar sistemasini, kengaytirilgan matritsa asosida Tutor oynasida yechimini topish:

> **LinearSolveTutor(A);**



Endi quyidagi to‘rt noma’lumli chiziqli tenglamalar sistemaning yechimni kengaytirilgan matritsasi asosida topishdagi amallar ketma-ketligini Maple dasturida bajarishni ko‘rsatamiz.

$$\begin{cases} x_1 - 5x_2 - x_3 + 3x_4 = -5, \\ 2x_1 + 3x_2 + x_3 - x_4 = 4, \\ 3x_1 - 2x_2 + 3x_3 + 4x_4 = -1, \\ 5x_1 + 3x_2 + 2x_3 + 2x_4 = 0. \end{cases}$$

1. Gauss usulini qo‘llashda amallar ketma-ketligini bajarish.

1.1.2–M a p l e d a s t u r i:

```
> restart;with(Student[LinearAlgebra]):
> A := <<1,2,3,5>|<-5,3,-2,3>|<-1,1,3,2>|<3,-1,4,2>>;
```

$$A := \begin{bmatrix} 1 & -5 & -1 & 3 \\ 2 & 3 & 1 & -1 \\ 3 & -2 & 3 & 4 \\ 5 & 3 & 2 & 2 \end{bmatrix}$$

a)*asosiy matritsa determinantini hisoblash:*

```
> d:=Determinant(A);
```
$d := 67$

b) *kengaytirilgan matritsa uchun Gauss usulining amallar ketma-ketligini bajarish:*

```
> with(linalg):
> B:=matrix([[1,-5,-1,3,-5],[2,3,1,-1,4],[3,-2,3,4,
-1], [5,3,2.2,0]]);
```

$$B := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 2 & 3 & 1 & -1 & 4 \\ 3 & -2 & 3 & 4 & -1 \\ 5 & 3 & 2 & 2 & 0 \end{bmatrix}$$

```
> B[1,1]; 1
```

```
> B1:=mulrow(B,1,1/B[1,1]);
```

$$B1 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 2 & 3 & 1 & -1 & 4 \\ 3 & -2 & 3 & 4 & -1 \\ 5 & 3 & 2 & 2 & 0 \end{bmatrix}$$

```
> B2:=addrow(B1,1,2,-B1[2,1]);
```

$$B2 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 13 & 3 & -7 & 14 \\ 3 & -2 & 3 & 4 & -1 \\ 5 & 3 & 2 & 2 & 0 \end{bmatrix}$$

```
> B3:=addrow(B2,1,3,-B1[3,1]);
```

$$B3 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 13 & 3 & -7 & 14 \\ 0 & 13 & 6 & -5 & 14 \\ 5 & 3 & 2 & 2 & 0 \end{bmatrix}$$

> **B4:=addrow(B3,1,4,–B1[4,1]);**

$$B4 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 13 & 3 & -7 & 14 \\ 0 & 13 & 6 & -5 & 14 \\ 0 & 28 & 7 & -13 & 25 \end{bmatrix}$$

> **B3[2,2];** 13

> **B5:=mulrow(B4,2,1/B4[2,2]);**

$$B5 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 1 & \frac{3}{13} & -\frac{7}{13} & \frac{14}{13} \\ 0 & 13 & 6 & -5 & 14 \\ 0 & 28 & 7 & -13 & 25 \end{bmatrix}$$

> **B6:=addrow(B5,2,3,–B5[3,2]);**

$$B6 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 1 & \frac{3}{13} & -\frac{7}{13} & \frac{14}{13} \\ 0 & 0 & 3 & 2 & 0 \\ 0 & 28 & 7 & -13 & 25 \end{bmatrix}$$

> **B7:=addrow(B6,2,4,– B5[4,2]);**

$$B7 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 1 & \frac{3}{13} & -\frac{7}{13} & \frac{14}{13} \\ 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & \frac{7}{13} & \frac{27}{13} & -\frac{67}{13} \end{bmatrix}$$

> **B7[3,3];** 3

> **B8:=mulrow(B7,3,1/B7[3,3]);**

$$B8 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 1 & \frac{3}{13} & -\frac{7}{13} & \frac{14}{13} \\ 0 & 0 & 1 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{7}{13} & \frac{27}{13} & -\frac{67}{13} \end{bmatrix}$$

> **B9:=addrow(B8,3,4,-B8[4,3]);**

$$B9 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 1 & \frac{3}{13} & -\frac{7}{13} & \frac{14}{13} \\ 0 & 0 & 1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{67}{39} & -\frac{67}{13} \end{bmatrix}$$

> **B9[4,4];** $\frac{67}{39}$

> **B10:=mulrow(B9,4,1/B9[4,4]);**

$$B10 := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 0 & 1 & \frac{3}{13} & -\frac{7}{13} & \frac{14}{13} \\ 0 & 0 & 1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

Yetakchi elementlar asosida asosiy matritsa determinantini hisoblash:

> **d:= B[1,1]*B3[2,2]*B7[3,3]*B9[4,4]*1;** $d := 67$

2. Berilgan sistemaning kengaytirilgan matritsasiga Gauss usulini qo'llash ketma–ketni Tutor oynasida ko'rsatamiz.

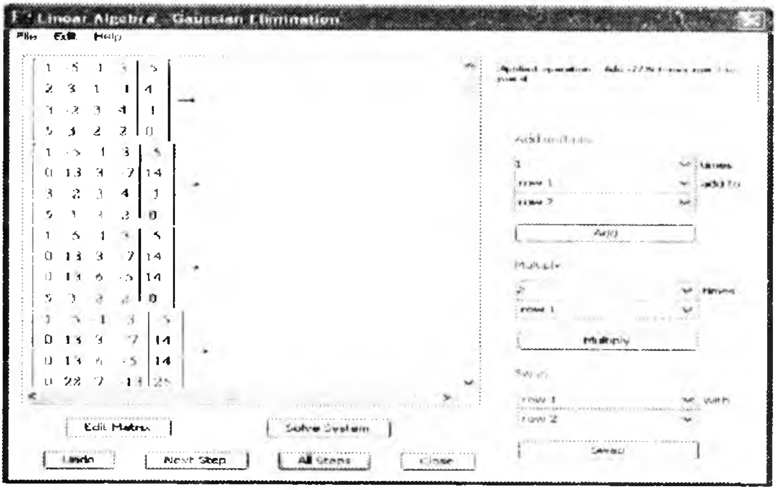
1.1.3–M a p l e d a s t u r i:

> **restart; with(Student[LinearAlgebra]):**

> **Ab := <<1,2,3,5>|<–5,3,–2,3>|<–1,1,3,2>|<3,–1,4,2>|<–5,4,–1,0>>;**

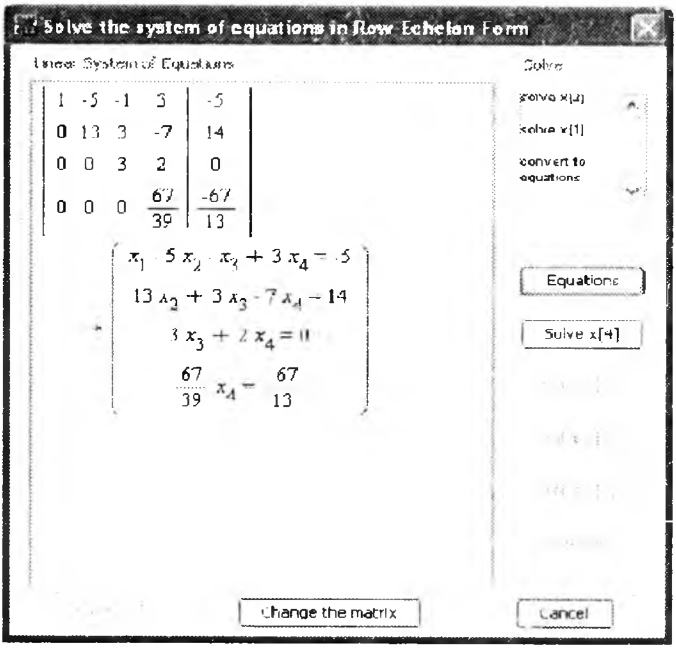
$$A := \begin{bmatrix} 1 & -5 & -1 & 3 & -5 \\ 2 & 3 & 1 & -1 & 4 \\ 3 & -2 & 3 & 4 & -1 \\ 5 & 3 & 2 & 2 & 0 \end{bmatrix}$$

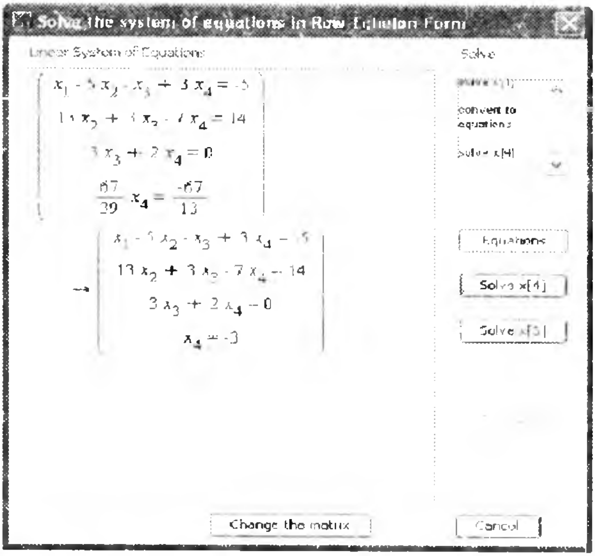
> **LinearSolveTutor(Ab);** (1.1– rasm)

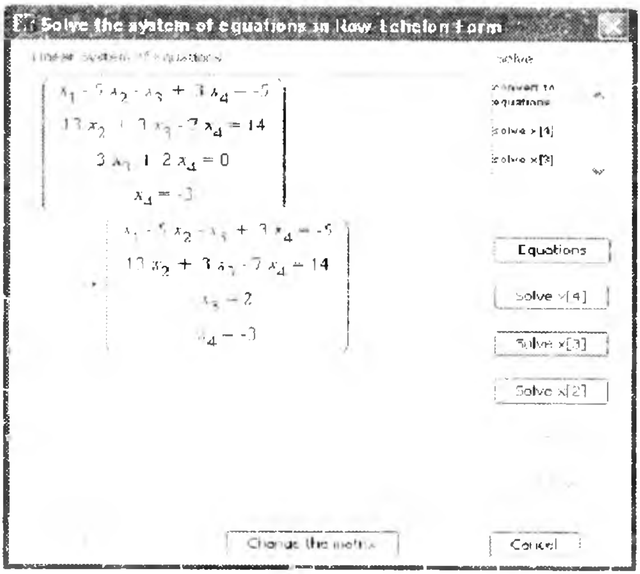


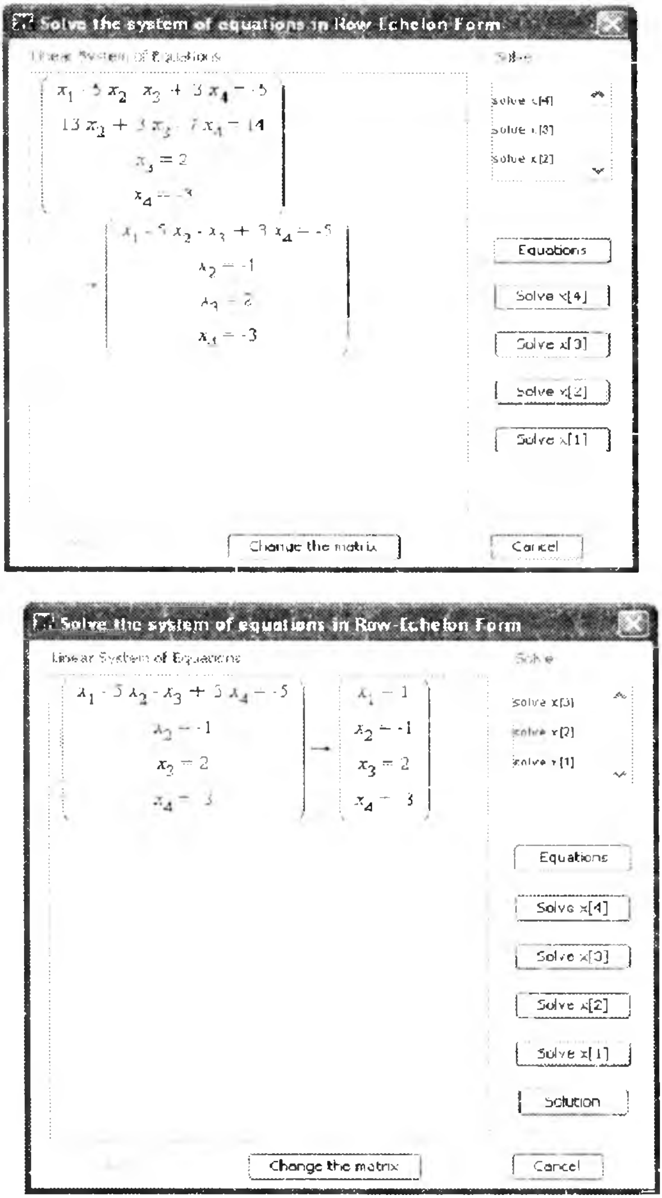
1.1– rasm.

Gauss usulida topilgan oxirgi matritsa asosida tuzilgan ekvivalent sistemani Tutor oynasida yechimini topish(1.2– rasm):




Solve the system of equations in Row Echelon Form
Linear System of Equations
Solve
convert to equations
Equations
Solve x[4]
Solve x[3]
Change the matrix
Cancel


Solve the system of equations in Row Echelon Form
Solve
convert to equations
solve x[4]
solve x[3]
Equations
Solve x[4]
Solve x[3]
Solve x[2]
Change the matrix
Cancel


Solve the system of equations in Row-Echelon Form
Equations
Solve x[4]
Solve x[3]
Solve x[2]
Solve x[1]
Change the matrix
Cancel
Solve the system of equations in Row-Echelon Form
Linear System of Equations
Solve
Equations
Solve x[4]
Solve x[3]
Solve x[2]
Solve x[1]
Solution
Change the matrix
Cancel

1.2– rasm.

1.2. Gauss usulida determinantni hisoblash

Determinantlarning tartibi (satr va ustunlar soni) katta bo'lganda determinantlarni hisoblash qiyin bo'ladi. Shuning uchun bu determinantlarni Gauss usuli asosida hisoblash qulay. Bu usulni namuna sifatida quyidagi determinant uchun bajaramiz.

1.2–masala. Quyidagi determinantni Gauss usuli asosida hisoblang.

$$d=\begin{vmatrix}2 & 7 & 13\\ 3 & 14 & 12\\ 5 & 25 & 16\end{vmatrix}$$

Yechish. Gauss usuli bo'yicha uchburchak determinant hosil qilish uchun, determinantning bosh diogonal elementlarini 1 ga va ostidagi elementlarini nolga aylantiramiz.

Berilgan determinantdagi birinchi satrning yetakchi $a_{11}=2\neq 0$ elementini determinant belgisidan tashqariga chiqaramiz:

$$d=2\begin{vmatrix}1 & 7/2 & 13/2\\ 3 & 14 & 12\\ 5 & 25 & 16\end{vmatrix}$$

hosil bo'lgan determinantda birinchi satr elementlarini ketma–ket 3 va 5 larga ko'paytirib, mos ravishda 2– va 3– satrlarning elementlaridan ayiramiz:

$$d=2\begin{vmatrix}1 & 7/2 & 13/2\\ 0 & 7/2 & -15/2\\ 0 & 15/2 & -33/2\end{vmatrix}$$

Bu determinantning ikkinchi satridagi yetakchi $a_{22}^{(1)}=7/2$ elementini determinant belgisidan tashqariga chiqaramiz:

$$d=2\cdot(7/2)\begin{vmatrix}1 & 7/2 & 13/2\\ 0 & 1 & -15/7\\ 0 & 15/2 & -33/2\end{vmatrix}$$

hosil bo'lgan determinantda ikkinchi satr elementlarini 15/2 ga ko'paytirib, mos ravishda 3– satrdan ayiramiz:

$$d=2\cdot(7/2)\begin{vmatrix}1 & 7/2 & 13/2\\ 0 & 1 & -15/7\\ 0 & 0 & -3/7\end{vmatrix}$$

hosil bo'lgan determinantning oxirgi satridagi yetakchi $a_{33}^{(2)}=-3/7$ elementini determinant belgisidan tashqariga chiqaramiz:

$$d = 2\cdot(7/2)\cdot(-3/7)\begin{vmatrix} 1 & 7/2 & 13/2 \\ 0 & 1 & -15/7 \\ 0 & 0 & 1 \end{vmatrix}$$

hosil bo'lgan determinant diagonal elementlari 1 sonidan va diagonal ostidagi elementlari 0 dan iborat bo'lgani uchun uning qiymati 1 ga teng. Natijada asosiy determinant qiymati yetakchi elementlar ko'paytmasidan iborat bo'ladi:

$$d = a_{11}\, a_{22}^{(1)}\, a_{33}^{(2)} \cdot 1 = 2\cdot(7/2)\cdot(-3/7)\cdot 1 = -3.$$

Xuddi shuningdek Gauss usuli bilan qolgan determinantlarni ham hisoblash mumkin.

2. Yuqoridagi Gauss usulini $n\text{x}n$ tartibli determinant uchun hisoblash formulasini beramiz:

$$d = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Bu determinant qiymati Gauss usulini qo 'llash jarayonida aniqlanadigan yetakchi elementlar ko'paytmasidan iborat bo'ladi:

$$d = \det A = a_{11} a_{22}^{(1)} a_{33}^{(2)} \cdots a_{nn}^{(n-1)}$$

Bu yetakchi elementlarni quyidagi formulalar asosida hisoblaymiz:

$i=1$,

$$b_{1j} = a_{1j}/a_{11}, \qquad j = 2,3,\dots,n$$

$$a_{ii}^{(1)} = a_{ii} - a_{i1}\, b_{1j}, \qquad i = 2,3,\dots,n$$

$i=2$,

$$b_{2i}^{(1)} = a_{2i}^{(1)}/a_{22}^{(1)}, \qquad j = 2,3,\dots,n$$

$$a_{ii}^{(2)} = a_{ii}^{(1)} - a_{i2}^{(1)}\, b_{2i}^{(1)}, \qquad i = 2,3,\dots,n$$

. . . .

Agar berilgan determinant yetakchi satridagi yetakchi element $a_{11}=0$ bo'lsa, bu satrni yetakchi elementi noldan farqli bo'lgan satr bilan almashtiramiz.

Bu determinantni Gauss usuli asosida Maple dasturida hisoblash ketma–ketligini ko'rsatamiz.

1.2–M a p l e d a s t u r i:

1)*misolda ko'rsatilgan tartibi bo'yicha hisoblash* :

```
> restart;with(linalg):
> A:=matrix([[2,7,13],[3,14,12],[5,25,16]]);
```

$$A := \begin{bmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{bmatrix}$$

```
> A[1,1];
```

$$2$$

```
> A1:=mulrow(A,1,1/A[1,1]);
```

$$A1 := \begin{bmatrix} 1 & \frac{7}{2} & \frac{13}{2} \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{bmatrix}$$

```
> A2:=addrow(A1,1,2,-3);
```

$$A2 := \begin{bmatrix} 1 & \frac{7}{2} & \frac{13}{2} \\ 0 & \frac{7}{2} & -\frac{15}{2} \\ 5 & 25 & 16 \end{bmatrix}$$

```
> A3:=addrow(A2,1,3,-5);
```

$$A3 := \begin{bmatrix} 1 & \frac{7}{2} & \frac{13}{2} \\ 0 & \frac{7}{2} & -\frac{15}{2} \\ 0 & \frac{15}{2} & -\frac{33}{2} \end{bmatrix}$$

```
> A3[2,2];
```

$$\frac{7}{2}$$

```
> A4:=mulrow(A3,2,1/A3[2,2]);
```

$$A4 := \begin{bmatrix} 1 & \frac{7}{2} & \frac{13}{2} \\ 0 & 1 & -\frac{15}{7} \\ 0 & \frac{15}{2} & -\frac{33}{2} \end{bmatrix}$$

> **A5:=addrow(A4,2,3,–15/2);** $A5 := \begin{bmatrix} 1 & \frac{7}{2} & \frac{13}{2} \\ 0 & 1 & -\frac{15}{7} \\ 0 & 0 & -\frac{3}{7} \end{bmatrix}$

> **A5[3,3];** $-\frac{3}{7}$

> **A6:=mulrow(A5,3,1/A5[3,3]);** $A6 := \begin{bmatrix} 1 & \frac{7}{2} & \frac{13}{2} \\ 0 & 1 & -\frac{15}{7} \\ 0 & 0 & 1 \end{bmatrix}$

> **d:= A[1,1]*A3[2,2]*A5[3,3]*det(A6);** $d := -3$

2) GaussianElimination *amali asosida topilgan matritsa determinantini hisoblash*:

> **restart; with(LinearAlgebra):**

A := <<2,3,5>|<7,14,25>|<13,12,16>>; $A := \begin{bmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{bmatrix}$

> **A:=GaussianElimination(A);** $A := \begin{bmatrix} 2 & 7 & 13 \\ 0 & \frac{7}{2} & -\frac{15}{2} \\ 0 & 0 & -\frac{3}{7} \end{bmatrix}$

> **d:=Determinant(A);** $d := -3$

1.3. Matritsaga teskari matritsa topish

Teskari matritsa topishning ikki xil usulini beramiz.

1.3.2. Formula bo'yicha topish.

1.3.2. Jardan–Gauss usulida teskari matritsa topish.

1.3.2. Chiziqli tenglamalar sistemasini teskari matritsa topish asosida yechish.

1.3–masala. Quyidagi berilgan A matitsaga teskari A^{-1} matritsani toping.

$$A=\begin{pmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{pmatrix}$$

1.3.1. Formula asosida topish

A matitsaga teskari A^{-1} matritsani quyidagi formula asosida topiladi.

$$A^{-1}=\frac{1}{\Delta}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \qquad (**)$$

Bu uchunchi tartibli A matritsaga teskari matritsa topish formulasi bo'lib, bunda $\Delta=\det(A)$ – A matritsa determnanti, $A_{ij}(i,j=1,2,3)$ elementlar Δ determinantning a_{ij} $(i,j=1,2,3)$ elementlariga mos keluvchi algebraik to'ldiruvchilari.

Teskari matritsani topish uchun A matritsa detreminanti Δ ni tuzamiz va hisoblaymiz, so'ngra uning algebraik to'ldiruvchilarini topamiz.

1) A matritsaning determinanti hisoblaymiz:

$$\Delta=det(A)=\begin{vmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{vmatrix}=22,\ \Delta=detA=22\neq 0.$$

2)Bu holda A^{-1} matritsaning elementlarini $det(A)$ determinantning a_{ij} elementlariga mos kelgan A_{ij} algebraik to'ldiruvchilarni quyidagicha topamiz.

$$A_{11}=(-1)^{1+1}\begin{vmatrix} -1 & -1 \\ 1 & -3 \end{vmatrix}=4,\quad A_{21}=(-1)^{2+1}\begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix}=10,$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix} 3 & 1 \\ -1 & -1 \end{vmatrix}=-2$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix} 2 & -1 \\ 7 & -3 \end{vmatrix}=-1,\quad A_{22}=(-1)^{2+2}\begin{vmatrix} 4 & 1 \\ 7 & -3 \end{vmatrix}=-19,$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix} 4 & 1 \\ 2 & -1 \end{vmatrix}=6$$

$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & -1\\ 7 & 1\end{vmatrix}=9$, $A_{23}=(-1)^{2+3}\begin{vmatrix}4 & 3\\ 7 & 1\end{vmatrix}=17$,

$A_{33}=(-1)^{3+3}\begin{vmatrix}4 & 3\\ 2 & -1\end{vmatrix}=-10$

A matritsaning determinanti va A_{ij} algebraik to'ldiruvchilarining qiymatlari asosida quyidagi A^{-1} matritsani yozamiz:

$$A^{-1}=\begin{pmatrix}4/22 & 10/22 & -2/22\\ -1/22 & -19/22 & 6/22\\ 9/22 & 17/22 & -10/22\end{pmatrix}=\begin{pmatrix}2/11 & 5/11 & -1/11\\ -1/22 & -19/22 & 3/11\\ 9/22 & 17/22 & -5/11\end{pmatrix}$$

Matritsaga teskari matritsa topish formulasi yordamida hisoblashning Maple dasturini beramiz.

1.3.1–Maple dasturi:

> restart; with(Student[LinearAlgebra]):

> A := <<4,2,7>|<3,–1,1>|<1,–1,–3>>; $A:=\begin{bmatrix}4 & 3 & 1\\ 2 & -1 & -1\\ 7 & 1 & -3\end{bmatrix}$

Berilgan matrisaning determinantini hisoblash :

> d:=Determinant(A); $d:=22$

A^{-1} *teskari matrisaning elementlari* :

> A11:=(–1)^(1+1)*Minor(A,1,1); $A11:=4$

> A12:=(–1)^(1+2)*Minor(A,1,2); $A12:=-1$

> A13:=(–1)^(1+3)*Minor(A, 1, 3); $A13:=9$

> A21:=(–1)^(2+1)*Minor(A, 2, 1); $A21:=10$

> A22:=(–1)^(2+2)*Minor(A, 2, 2); $A22:=-19$

> A23:=(–1)^(2+3)*Minor(A, 2, 3); $A23:=17$

> A31:=(–1)^(3+1)*Minor(A, 3, 1); $A31:=-2$

> A32:=(–1)^(3+2)*Minor(A, 3, 2); $A32:=6$

> A33:=(–1)^(3+3)*Minor(A, 3, 3); $A33:=-10$

teskari matrisani topsh :

> A := <<A11,A12,A13>|<A21,A22,A23>|<A31,A32,A33>>/d;

$$A := \begin{vmatrix} \frac{2}{11} & \frac{5}{11} & -\frac{1}{11} \\ -\frac{1}{22} & -\frac{19}{22} & \frac{3}{11} \\ \frac{9}{22} & \frac{17}{22} & -\frac{5}{11} \end{vmatrix}$$

1.3.2. Jardan–Gauss usulida teskari matritsa topish

Berilgan yuqori tartibli A matritsaga teskari $B=A^{-1}$ matritsani Jordan–Gauss usulida topish uchun quyidagicha kengaytirilgan matritsani tuzamiz.

$$\begin{pmatrix} a_{11} \; a_{12} \ldots a_{1n} & \vdots & b_{11} \; b_{12} \ldots b_{1n} \\ a_{21} \; a_{22} \ldots a_{2n} & \vdots & b_{21} \; b_{22} \ldots b_{2n} \\ \ldots\ldots\ldots\ldots & & \ldots\ldots\ldots\ldots \\ a_{n1} \; a_{n2} \ldots a_{nn} & \vdots & b_{n1} \; b_{n2} \ldots b_{nn} \end{pmatrix}$$

Bu matritsadagi $b_{ij} - i,j=1,2,3,..,n$ elementlar boshlang'ich holatda birlik matritsa o'rnida bo'lib, A matritsani birlik matritsaga aylantirish bilan teskari matritsa elementlariga aylanadi.

$$\begin{pmatrix} 1 \; 0 \ldots 0 \; a_{1,k+1}^{(k)} \ldots a_{1n}^{(k)} & \vdots & b_{11}^{(k)} \; b_{12}^{(k)} \ldots b_{1n}^{(k)} \\ 0 \; 1 \ldots 0 \; a_{2,k+1}^{(k)} \ldots a_{2n}^{(k)} & \vdots & b_{21}^{(k)} \; b_{22}^{(k)} \ldots b_{2n}^{(k)} \\ \ldots\ldots\ldots\ldots\ldots\ldots & & \ldots\ldots\ldots\ldots \\ 0 \; 0 \ldots 1 \; a_{n,k+1}^{(k)} \ldots a_{nn}^{(k)} & \vdots & b_{n1}^{(k)} \; b_{n2}^{(k)} \ldots b_{nn}^{(k)} \end{pmatrix} \Rightarrow \ldots \Rightarrow$$

$$\begin{pmatrix} 1 \; 0 \ldots 0 & \vdots & b_{11}^{(n)} \; b_{12}^{(n)} \ldots b_{1n}^{(n)} \\ 0 \; 1 \ldots 0 & \vdots & b_{21}^{(n)} \; b_{22}^{(n)} \ldots b_{2n}^{(n)} \\ \ldots\ldots\ldots & & \ldots\ldots\ldots\ldots \\ 0 \; 0 \ldots 1 & \vdots & b_{n1}^{(n)} \; b_{n2}^{(n)} \ldots b_{nn}^{(n)} \end{pmatrix}$$

Bu almashtirish elementlari quyidagicha bog'lash mumkin:

$$a_{kj}^{(k)} = a_{kj}^{(k-1)} / a_{kk}^{(k-1)}, \qquad k=1,2,..,n; \quad j=k+1,...,n$$

$$b_{kj}^{(k)} = b_{kj}^{(k-1)} / b_{kk}^{(k-1)}, \qquad k=1,2,..,n; \quad j=1,2,...,n$$

$$a_{ij}^{(k)} = a_{ij}^{(k-1)} - a_{kj}^{(k-1)} \, a_{ik}^{(k-1)} / a_{kk}^{(k-1)},$$

$$i=1,...,k-1, \; k+1,...,n, \; j=k+1,...,n, a_{ik}^{(0)} = a_{ik}$$

$$b_{ij}^{(k)} = b_{ij}^{(k-1)} - b_{kj}^{(k-1)}\, a_{ik}^{(k-1)} / a_{kk}^{(k-1)}, \qquad i = 1,\dots,k-1,\ \kappa+1,\dots,n;$$

$$j = \kappa+1,\dots,n, b_{ij}^{(0)} = b_{ij}$$

1.4–masaladagi matrisaga Jardano–Gauss usuli bilan teskari matritsni toping.

$$A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{pmatrix}$$

Yechish. Teskari matritsa topish jarayonini matrits yonida ko'rsatib boramiz. Berilgan matritsaga teskari matritsani Jardano–Gauss usulida topish:

$$AE = \begin{pmatrix} 4 & 3 & 1 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 7 & 1 & -3 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

Bu AE matritsaning satrlarini mosravishda [1], [2], [3] kabi belgilab, A matritsani birlik matritsaga, E matritsani A ning teskari matritsiga aylantirish uchun quyidagi amallarni **Jardano–Gauss usulida** bajaramiz.

$$\begin{array}{l} [1]/4 \\ \\ \\ \end{array} \qquad \begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 7 & 1 & -3 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

$$\begin{array}{l} [1] \\ [2]-[1]*2 \\ [3]-[1]*7 \end{array} \quad \begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & -5/2 & -3/2 & -1/2 & 1 & 0 \\ 0 & -17/2 & -19/4 & -7/4 & 0 & 1 \end{pmatrix} \Rightarrow$$

$$\begin{array}{l} [1] \\ [2]*(-2/5) \\ \\ \end{array} \quad \begin{pmatrix} & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & -17/2 & -19/4 & -7/4 & 0 & 1 \end{pmatrix} \Rightarrow$$

$$\begin{array}{l} [1] \\ [2] \\ [3]+[2]*(17/4) \end{array} \quad \begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & -11/5 & -9/10 & -17/10 & 1 \end{pmatrix} \Rightarrow$$

[1]

[2]

[3]*(-5/11)

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix} \Rightarrow$$

[1]+ [2]*(-3/4)
[2]
[3]

$$\begin{pmatrix} 1 & 0 & -1/5 & 1/10 & 3/10 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix} \Rightarrow$$

[1]+ [3]*(-1/5)
[2]+[3]*(-3/5)
[3]

$$\begin{pmatrix} 1 & 0 & 0 & 2/11 & 5/11 & -1/11 \\ 0 & 1 & 0 & -1/22 & -19/22 & 3/11 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 2/11 & 5/11 & -1/11 \\ -1/22 & -19/22 & 3/11 \\ 9/22 & 17/22 & -5/11 \end{pmatrix}$$

Jardano–Gauss usulida matritsaga teskari matritsa topishning Maple dasturini tuzamiz.

1.3.2–M a p l e d a s t u r i:

1) *GAUSS usulida teskari matritsa topish:*

> restart; with(Student[LinearAlgebra]):

> A := <<4,2,7>|<3,–1,1>|<1,–1,–3>>;

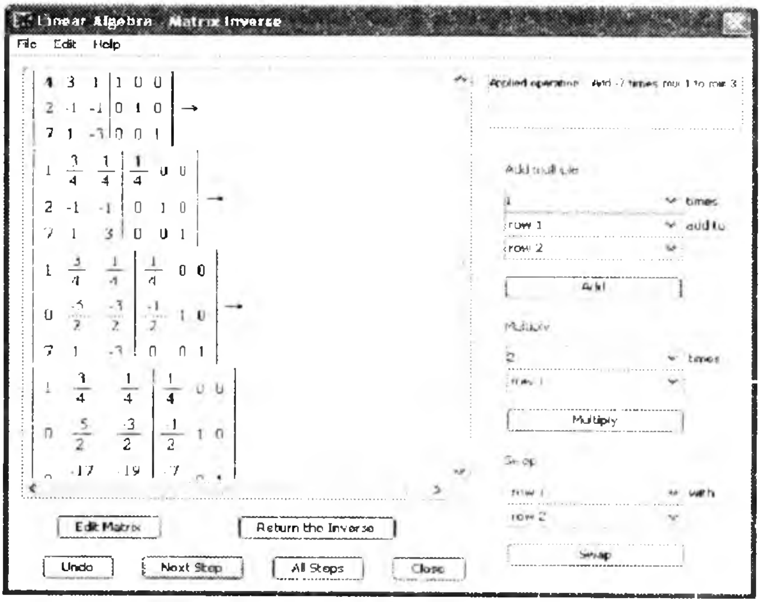
$$A := \begin{bmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{bmatrix}$$

> A^(–1);

$$\begin{bmatrix} \frac{2}{11} & \frac{5}{11} & -\frac{1}{11} \\ -\frac{1}{22} & -\frac{19}{22} & \frac{3}{11} \\ \frac{9}{22} & \frac{17}{22} & -\frac{5}{11} \end{bmatrix}$$

2) *GAUSS usulida teskari matritsa topishni* **Tutor** *oynasida bajarish:*

> InverseTutor(A); (1.3– rasm)



1.3– rasm.

1.3.3. Chiziqli tenglamalar sistemasini teskari matritsa asosida yechish

1.5–masala. Quyidagi chiziqli tenglamalar sistemasini teskari matritsa yordamida yeching.

$$\begin{cases} 4x_1 + 3x_2 + x_3 = 1, \\ 2x_1 - x_2 - x_3 = 2, \\ 7x_1 + x_2 - 3x_3 = 3. \end{cases}$$

Tenglamalar sistemani matritsa ko‘rinishida quyidagicha yozamiz:

$$A \cdot X = B \qquad (*)$$

(*) tenglamadagi noma’lum X matritani quyidagicha topamiz:

$$X = A^{-1} \cdot B$$

bu yerda:
$$A^{-1} = \frac{1}{\Delta}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \qquad (**)$$

Bu uchunchi tartibli A matritsaga teskari matritsa topish formulasi bo‘lib, bunda $\Delta=\det(A)$ – A matritsa determnanti, teskari matritsa A^{-1} dagi $A_{ij}(i,j=1,2,3)$ elementlar Δ determinantning a_{ij} elementiga mos keluvchi algebraik to‘ldiruvchilari. Teskari matritsani topish uchun A matritsa detreminanti Δ ni tuzamiz va uning algebraik to‘ldiruvchlarini topamiz.

Demak, $X = A^{-1}\cdot B$ dan sistema yechimi quyidagich topiladi:

$$x_1 = \frac{A_{11}b_1 + A_{21}b_2 + A_{31}b_3}{\Delta},\quad x_2 = \frac{A_{12}b_1 + A_{22}b_2 + A_{32}b_3}{\Delta},$$

$$x_3 = \frac{A_{13}b_1 + A_{23}b_2 + A_{33}b_3}{\Delta}$$

Quyidagi Maple dasturida chiziqli tenglamalar sistemani teskari matritsa yordamida yechishning ikki xil usulini ko‘rsatamiz:

1.3.3–M a p l e d a s t u r i:

Chiziqli tenglamalar sistemanining matritsalari:

> **restart; with(Student[LinearAlgebra]):**

> **A := <<4,2,7>|<3,–1,1>|<1,–1,–3>>;** $A := \begin{bmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{bmatrix}$

> **B:=<1,2,3>;** $B := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

1) teskari matritsani hisoblash formulasi yordamida yechish:

Berilgan matrisaning determinantini hisoblash:

> **d:=Determinant(A);**

(**) *teskari matrisaning elementlarini hisoblash:*

> **A11:=(–1)^(1+1)*Minor(A,1,1);** $A11 := 4$

> **A12:=(–1)^(1+2)*Minor(A,1,2);** $A12 := -1$

> **A13:=(–1)^(1+3)*Minor(A, 1, 3);** $A13 := 9$

> **A21:=(–1)^(2+1)*Minor(A, 2, 1);** $A21 := 10$

> **A22:=(–1)^(2+2)*Minor(A, 2, 2);** $A22 := -19$

> **A23:=(–1)^(2+3)*Minor(A, 2, 3);** $A23 := 17$

> **A31:=(–1)^(3+1)*Minor(A, 3, 1);** $A31 := -2$

> **A32:=(–1)^(3+2)*Minor(A, 3, 2);** $A32 := -6$

> **A33:=(–1)^(3+3)*Minor(A, 3, 3);** $A33 := -10$

> **x1:=(A11*B[1]+A21*B[2]+A31*B[3])/d;** $x1 := \frac{9}{11}$

> **x2:=(A12*B[1]+A22*B[2]+A32*B[3])/d;** $x2 := -\frac{21}{22}$

> **x3:=(A13*B[1]+A23*B[2]+A33*B[3])/d;** $x3 := \frac{13}{22}$

2) teskari matritsa topish buyrug'i **A^(–1)** asosida yechish:

> **A^(–1);** $\begin{bmatrix} \frac{2}{11} & \frac{5}{11} & -\frac{1}{11} \\ -\frac{1}{22} & -\frac{19}{22} & \frac{3}{11} \\ \frac{9}{22} & \frac{17}{22} & -\frac{5}{11} \end{bmatrix}$

> **X:=A^(–1).B;** $X := \begin{bmatrix} \frac{9}{11} \\ -\frac{21}{22} \\ \frac{13}{22} \end{bmatrix}$

O'z-o'zini tekshirish uchun savollar

1. Chiziqli tenglama ta'rifini bering.
2. Qanday chiziqli tenglamalar sistemasi birgalikda deyiladi?
3. Chiziqli tenglamalar sistemasining tuzilishi va yozilishi qanday?
4. Sistema yechimining yagonaligi.
5. Aniq va taqribiy yechimlar farqini tushuntiring.
6. Chiziqli tenglamalar sistemasini yechishning Gauss usuli nimalardan iborat?
7. Yetakchi element va yetakchi tenglamaning vazifasi.
8. Noma'lumlarni ketma-ket yo'qotishda yangi koeffitsientlarni aniqlash.
9. Gauss usulida chiziqli tenglamalar sistemasining yechimini topishda bajariladigan ko'paytirish, bo'lish va qo'shish amallari sonini aniqlash.
10. Chiziqli tenglamalar sistemasini Gauss usulida yechish.
11. Chiziqli tenglamalar sistemasining kengaytirilgan matritsasi nima?
12. 4. Kengaytirilgan matritsa uchun elementar almashtirishlar.

13. Chiziqli tenglamalar sistemasining kengaytirilgan matritsasiga Gauss usulini qo'llash bilan ekvivalent matritsalarga o'tish.

14. Chiziqli tenglamalar sistemasining kengaytirilgan matritsasiga Jordan–Gauss usulini qo'llash.

15. Gauss va Jordan–Gauss usullarining farqi.

16. Teskari matritsaga ta'rif bering.

17. Maxsus bo'lmagan matritsani tushuntiring.

18. Teskari matritsa elementlarini topish qoidasi.

19. Chiziqli tenglamalar sistemasini matritsa usulida yozish.

20. Qanday shartda teskari matritsani topish mumkin?

21. Algebraik to'ldiruvchini aniqlash.

22. Teskari matritsa elementlarini topish qoidasi.

23. Chiziqli tenglamalar sistemasini yechishda teskari matritsa usuli.

1-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

1) Quyidagi chiziqli tenglamlar sistemasidan birinchisini Gauss va ikkinchisini Kramer usulida yeching undagi determinatlarni Gauss usulida hisoblang;

2) Matritsaviy tenglamani teskari matritsa topish usulida yeching.

1. 1) $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 - x_2 - x_3 - 2x_4 = -4 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \\ x_1 + x_2 + x_3 - x_4 = 6 \end{cases}$ 2) $\begin{cases} 5x + 8y - z = -7 \\ x + 2y + 3z = 1 \\ 2x - 3y + 2z = 9 \end{cases}$

3) $\begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 5 & 3 & 0 \end{pmatrix} X = \begin{pmatrix} 2 & 7 & 13 \\ -1 & 0 & 5 \\ 5 & 13 & 21 \end{pmatrix}$

2. 1) $\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 2x_1 + 2x_2 - x_3 + 2x_4 = 4 \\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8 \end{cases}$ 2) $\begin{cases} x + 2y + z = 4 \\ 3x - 5y + 3z = 1 \\ 2x + 7y - z = 8 \end{cases}$

3) $\begin{pmatrix} -1 & -2 & 3 \\ 2 & 3 & 5 \\ 1 & 4 & -1 \end{pmatrix} X = \begin{pmatrix} 4 & 11 & 3 \\ 1 & 6 & 1 \\ 2 & 2 & 16 \end{pmatrix}$

3. 1) $\begin{cases} x_1 + 2x_2 + 3x_3 - 4x_4 = 5 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$ 2) $\begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases}$

3) $\begin{pmatrix} 4 & -2 & 0 \\ 1 & 1 & 2 \\ 3 & -2 & 0 \end{pmatrix} X = \begin{pmatrix} 0 & -2 & 6 \\ 2 & 4 & 3 \\ 0 & -3 & 4 \end{pmatrix}$

4. 1) $\begin{cases} x_1 - 3x_3 + 4x_4 = -5 \\ x_1 - 2x_3 + 3x_4 = -4 \\ 3x_1 + 2x_2 - 5x_4 = 12 \\ 4x_1 + 3x_2 - 5x_3 = 5 \end{cases}$ 2) $\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 29 \\ 3x_1 - x_2 + x_3 = 10 \end{cases}$

3) $\begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 0 \\ 4 & -3 & 0 \end{pmatrix} X = \begin{pmatrix} 22 & -14 & 3 \\ 6 & -7 & 0 \\ 11 & 3 & 15 \end{pmatrix}$

5. 1) $\begin{cases} x_1 + 3x_2 + 5x_3 - 7x_4 = 12 \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0 \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4 \\ 7x_1 + x_2 + 3x_3 + 5x_4 = 16 \end{cases}$ 2) $\begin{cases} 4x - 3y + 2z = 9 \\ 2x + 5y - 3z = 4 \\ 5x + 6y - 2z = 18 \end{cases}$

3) $\begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix} X = \begin{pmatrix} 9 & 8 & 7 \\ 2 & 7 & 3 \\ 4 & 3 & 5 \end{pmatrix}$

6. 1) $\begin{cases} x_1 + 5x_2 + 3x_3 - 4x_4 = 20 \\ 3x_1 + x_2 - 2x_3 = 9 \\ 5x_1 - 7x_2 + 10x_4 = -9 \\ 3x_2 - 5x_3 = 1 \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 - x_3 = 4 \\ 3x_1 + 4x_2 - 2x_3 = 11 \\ 3x_1 - 2x_2 + 4x_3 = 11 \end{cases}$

3) $\begin{pmatrix} 5 & 1 & 2 \\ -1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 8 & 1 & 5 \\ -2 & 2 & -1 \\ 17 & 1 & 7 \end{pmatrix}$

7. 1) $\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8 \\ x_1 - 3x_2 - 6x_4 = 9 \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0 \end{cases}$ 2) $\begin{cases} x_1 + x_2 + 2x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases}$

3) $\begin{pmatrix} 4 & 2 & 1 \\ 3 & -2 & 0 \\ 0 & -1 & 2 \end{pmatrix} X = \begin{pmatrix} 2 & 0 & 2 \\ 5 & -7 & -2 \\ 1 & 0 & -1 \end{pmatrix}$

8. 1) $\begin{cases} 2x_1 - x_2 + 3x_3 + 2x_4 = 4 \\ 3x_1 + 3x_2 + 3x_3 + 2x_4 = 6 \\ 3x_1 - x_2 - x_3 + 2x_4 = 6 \\ 3x_1 - x_2 + 3x_3 - x_4 = 6 \end{cases}$ 2) $\begin{cases} 3x_1 - x_2 = 5 \\ -2x_1 + x_2 + x_3 = 0 \\ 2x_1 - x_2 + 4x_3 = 15 \end{cases}$

3) $\begin{pmatrix} 1 & 4 & 2 \\ 2 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 4 & 6 & -2 \\ 4 & 10 & 1 \\ 2 & 4 & -5 \end{pmatrix}$

9. 1) $\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 8 \\ 2x_1 + x_2 + x_3 + x_4 = 5 \\ x_1 - x_2 + 2x_3 + x_4 = -1 \\ x_1 + x_2 - x_3 + 3x_4 = 10 \end{cases}$ 2) $\begin{cases} 3x_1 - x_2 + x_3 = 4 \\ 2x_1 - 5x_2 - 3x_3 = -17 \\ x_1 + x_2 - x_3 = 0 \end{cases}$

3) $\begin{pmatrix} 3 & 2 & -5 \\ 4 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} X = \begin{pmatrix} -1 & 2 & 4 \\ 0 & 3 & 2 \\ -1 & -3 & 4 \end{pmatrix}$

10. 1) $\begin{cases} 4x_1 + x_2 - x_4 = -9 \\ x_1 - 3x_2 + 4x_3 = -7 \\ 3x_2 - 2x_3 + 4x_4 = 12 \\ x_1 + 2x_2 - x_3 - 3x_4 = 0 \end{cases}$ 2) $\begin{cases} x_1 + x_2 + x_3 = 2 \\ 2x_1 - x_2 - 6x_3 = -1 \\ 3x_1 - 2x_2 = 8 \end{cases}$

3) $\begin{pmatrix} 5 & 3 & -1 \\ -2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

11. 1) $\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 1 \\ 2x_1 - x_2 - 3x_4 = 2 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6 \end{cases}$ 2) $\begin{cases} 2x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 + x_3 = 6 \\ 3x_1 - x_2 + x_3 = 4 \end{cases}$

3) $\begin{pmatrix} 5 & -1 & 3 \\ 0 & 2 & -1 \\ -2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 3 & 7 & -2 \\ 1 & 1 & -2 \\ 0 & 1 & 3 \end{pmatrix}$

12. 1) $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_4 = 0 \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 - 3x_3 = 3 \\ 3x_1 + 4x_2 - 5x_3 = 8 \\ 2x_2 + 7x_3 = 17 \end{cases}$

3) $\begin{pmatrix} 4 & 5 & -2 \\ 3 & -1 & 0 \\ 4 & 2 & 7 \end{pmatrix} X = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 5 & 7 & 3 \end{pmatrix}$

13. 1) $\begin{cases} 5x_1 + x_2 - x_4 = -9 \\ 3x_1 - 3x_2 + x_3 + 4x_4 = -7 \\ 3x_1 - 2x_3 + x_4 = -16 \\ x_1 - 4x_2 + x_4 = 0 \end{cases}$ 2) $\begin{cases} x_1 + 5x_2 + x_3 = -7 \\ 2x_1 - x_2 - x_3 = 0 \\ x_1 - 2x_2 - x_3 = 2 \end{cases}$

3) $\begin{pmatrix} 2 & -8 & 5 \\ -1 & 1 & 1 \\ -2 & -2 & -3 \end{pmatrix} X = \begin{pmatrix} 10 & -2 & 6 \\ 0 & 4 & -2 \\ -4 & -2 & 0 \end{pmatrix}$

14. 1) $\begin{cases} 2x_1 + x_3 + 4x_4 = 9 \\ x_1 + 2x_2 - x_3 + x_4 = 8 \\ 2x_1 + x_2 + x_3 + x_4 = 5 \\ x_1 - x_2 + 2x_3 + x_4 = -1 \end{cases}$ 2) $\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 16 \\ 3x - 2y - 5z = 12 \end{cases}$

3) $\begin{pmatrix} 5 & 3 & -1 \\ 2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

15. 1) $\begin{cases} 2x_1 - 6x_2 + 2x_3 + 2x_4 = 12 \\ x_1 + 3x_2 + 5x_3 + 7x_4 = 12 \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0 \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4 \end{cases}$ 2) $\begin{cases} 3x + 4y + 2z = 8 \\ 2x - y - 3z = -1 \\ x + 5y + z = 0 \end{cases}$

3) $\begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 2 \\ -1 & 0 & 7 \end{pmatrix} X = \begin{pmatrix} -1 & 0 & 5 \\ 2 & 1 & 3 \\ 0 & -2 & 4 \end{pmatrix}$

16. 1) $\begin{cases} x_1 + 5x_2 = 2 \\ 2x_1 - x_2 + 3x_3 + 2x_4 = 4 \\ 3x_1 - x_2 - x_3 + 2x_4 = 6 \\ 3x_1 - x_2 + 3x_3 - x_4 = 6 \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 + 3x_3 = 7 \\ x_1 + 3x_2 - 2x_3 = 0 \\ 2x_2 - x_3 = 2 \end{cases}$

3) $\begin{pmatrix} 12 & 15 & -6 \\ 0 & -3 & 0 \\ 12 & 0 & 21 \end{pmatrix} X = \begin{pmatrix} 8 & 7 & -4 \\ 3 & 1 & 6 \\ 16 & 16 & 13 \end{pmatrix}$

17. 1) $\begin{cases} x_1 - 4x_2 - x_4 = 2 \\ x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 2x_1 + 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$ 2) $\begin{cases} 2x_1 + x_2 + 4x_3 = 20 \\ 2x_1 - x_2 - 3x_3 = 3 \\ 3x_1 + 4x_2 - 5x_3 = -8 \end{cases}$

3) $\begin{pmatrix} 1 & 3 & 4 \\ 6 & 6 & 5 \\ -1 & -2 & 11 \end{pmatrix} X = \begin{pmatrix} 4 & -3 & 11 \\ 0 & -3 & 4 \\ 1 & -4 & 1 \end{pmatrix}$

18. 1) $\begin{cases} 5x_1 - x_2 + x_3 + 3x_4 = -4 \\ x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ 2x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4 \end{cases}$ 2) $\begin{cases} x_1 - x_2 = 4 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$

3) $\begin{pmatrix} 8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix} X = \begin{pmatrix} 5 & 4 & -1 \\ 10 & 12 & -3 \\ 0 & 1 & 1 \end{pmatrix}$

19\. 1) $\begin{cases} 4x_1 - 2x_2 + x_3 - 4x_4 = 3 \\ 2x_1 - x_2 + x_3 - x_4 = 1 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6 \end{cases}$ 2) $\begin{cases} x_1 + 5x_2 - x_3 = 7 \\ 2x_1 - x_2 - x_3 = 4 \\ 3x_1 - 2x_2 + 4x_3 = 11 \end{cases}$

3) $\begin{pmatrix} 3 & 2 & -5 \\ 4 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} X = \begin{pmatrix} -1 & 2 & 4 \\ 0 & 3 & 2 \\ -1 & -3 & 4 \end{pmatrix}$

20\. 1) $\begin{cases} 2x_1 - x_3 - 2x_4 = -1 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$ 2) $\begin{cases} 11x + 3y - z = 2 \\ 2x + 5y - 5z = 0 \\ x + y + z = 2 \end{cases}$

3) $\begin{pmatrix} 1 & 2 & 1 \\ 3 & -5 & 3 \\ 2 & 7 & -1 \end{pmatrix} X = \begin{pmatrix} 4 & 2 & 1 \\ 1 & -5 & 3 \\ 8 & 7 & -1 \end{pmatrix}$

21\. 1) $\begin{cases} -x_1 + x_2 + x_3 + x_4 = 4 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$ 2) $\begin{cases} 7x + 5y + 2z = 18 \\ x - y - z = 3 \\ x + y + 2z = -2 \end{cases}$

3) $\begin{pmatrix} -1 & 2 & 0 \\ -3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} X = \begin{pmatrix} 5 & -1 & 3 \\ 4 & 2 & 1 \\ -1 & 0 & 2 \end{pmatrix}$

22\. 1) $\begin{cases} 5x_1 + 3x_2 - 7x_3 + 3x_4 = 1 \\ x_2 - 3x_3 + 4x_4 = -5 \\ x_1 - 2x_3 - 3x_4 = -4 \\ 4x_1 + 3x_2 - 5x_3 = 5 \end{cases}$ 2) $\begin{cases} 2x + 3y + z = 1 \\ x + z = 0 \\ x - y - z = 2 \end{cases}$

3) $\begin{pmatrix} 1 & 1 & -1 \\ 4 & -3 & 1 \\ 0 & 2 & 1 \end{pmatrix} X = \begin{pmatrix} 7 & 0 & -5 \\ 4 & 11 & 2 \\ 1 & 3 & 1 \end{pmatrix}$

23. 1) $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_1 + 2x_2 - 2x_4 = 1 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$ 2) $\begin{cases} x - 2y - 2z = 3 \\ x + y - 2z = 0 \\ x - y - z = 1 \end{cases}$

3) $\begin{pmatrix} 2 & 3 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 7 & 5 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & 2 \end{pmatrix}$

24. 1) $\begin{cases} 2x_1 + x_2 - x_3 + 3x_4 = -6 \\ 3x_1 - x_2 + x_3 + 5x_4 = 3 \\ x_1 + 2x_2 - x_3 + 2x_4 = 28 \\ 2x_1 + 3x_2 + x_3 - x_4 = 0 \end{cases}$ 2) $\begin{cases} 3x_1 + x_2 - 5x_3 = -7 \\ 2x_1 - 3x_2 + 4x_3 = -1 \\ 5x_1 - x_2 + 3x_3 = 0 \end{cases}$

3) $\begin{pmatrix} 2 & 3 & 1 \\ 3 & -1 & 0 \\ 1 & 2 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 5 & 0 \\ 3 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

25. 1) $\begin{cases} 2x_1 - x_2 + 2x_3 + 2x_4 = -3 \\ 3x_1 + 2x_2 + x_3 - x_4 = 3 \\ x_1 - 3x_2 - x_3 - 3x_4 = 0 \\ 4x_1 + 2x_2 + 2x_3 + 5x_4 = -15 \end{cases}$ 2) $\begin{cases} x_1 - x_2 - 5x_3 = -7 \\ 2x_1 - 3x_2 + 4x_3 = -1 \\ 5x_1 - x_2 + 3x_3 = 0 \end{cases}$

3) $\begin{pmatrix} -2 & 1 & 2 \\ 3 & 0 & 4 \\ 2 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

26. 1) $\begin{cases} x_1 + 3x_2 - x_3 - 4x_4 = 6 \\ x_1 + 2x_2 - 3x_4 = 3 \\ 2x_1 - x_2 - x_4 = -1 \\ x_1 + 3x_2 - 2x_3 = 5 \end{cases}$ 2) $\begin{cases} x - y - 2z = 3 \\ x + 2y - 3z = 4 \\ x - 5y - z = -1 \end{cases}$

3) $\begin{pmatrix} 2 & 3 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 7 & -5 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix}$

27. 1) $\begin{cases} 2x_1 + x_2 - 3x_3 + 3x_4 = 7 \\ 3x_1 - x_2 + 2x_3 + 5x_4 = 9 \\ x_1 + 2x_2 - x_3 + 2x_4 = 8 \\ 2x_1 + 3x_2 + x_3 - x_4 = 5 \end{cases}$ 2) $\begin{cases} 3x_1 + x_2 - x_3 = -7 \\ 2x_1 - 4x_2 + x_3 = -1 \\ 5x_1 - 2x_2 + 3x_3 = 2 \end{cases}$

3) $\begin{pmatrix} 5 & 3 & 2 \\ 3 & -1 & 1 \\ 2 & 2 & -1 \end{pmatrix} X = \begin{pmatrix} 1 & 5 & 2 \\ 3 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

28. 1) $\begin{cases} 2x_1 - 3x_2 + 2x_3 + 5x_4 = 7 \\ 3x_1 + 2x_2 + x_3 - 4x_4 = 3 \\ x_1 - 3x_2 - x_3 - 3x_4 = 5 \\ 4x_1 + 2x_2 + 2x_3 + 5x_4 = 8 \end{cases}$ 2) $\begin{cases} x_1 - x_2 - 6x_3 = -7 \\ 2x_1 - 3x_2 - 4x_3 = -2 \\ 5x_1 - x_2 + 3x_3 = 2 \end{cases}$

3) $\begin{pmatrix} 4 & 1 & 2 \\ 3 & 5 & 4 \\ 2 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 4 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

2-LABORATORIYA ISHI

Chiziqsiz tenglamalarini yechish. Transendent va algebraik tenglamalarini taqribiy yechish

Maple dasturining buyruqlari:

with(plots)– *funksiyalarning grafiklarini qurish paketidagi amallar;*

implicitplot(y=f(x),x=a..b) –*tekislikda oshkormas funksiyaning grafigini qurish;*

implicitplot3d(u=f(x,y,z),x=a..b,y=c..d,z=m..n)–*fazoda oshkormas funksiyaning grafigini qurish;*

solve(f(x),x) – *tenglamani x ga nisbatan ildizlarini hisoblash;*

coeffs(p,x)– **p** *ko'phadning koeffisentlarini aniqlash;*

max(coeffs(p,x) – *ko'phadning koeffisentlarining eng kattasini aniqlash;* **min(coeffs(p,x)** – *ko'phadning koeffisentlarining eng kichigini aniqlash;*

realroot(p,1)– *ko'phadning ildizlari yotgan 1 birlik kenglikdagi oraliqlarni aniqlash;*

with(Student[Calculus1]):NewtonsMethod(f(x),x=–1)– *Nyuton (urinmalar) usulida* $f(x)=0$ *tenglamaning* $x=-1$ *dan o'ngdagi ildizini aniqlash;*

> **fsolve({f,g},{x=–2..–1,y=–1..1})**– *tenglamalar sistemasining ko'rsatilgan sohalardagi yechimni hisoblash;*

with(Student[MultivariateCalculus]):Jacobian([u(x,y,z),v(x,y,z),w(x,y,z)],[x,y,z])– *Yakobiyanni hisoblash,*

Maqsad: Chiziqsiz bo'lgan murakkab transtsendent tenglama va ko'phadning ildizi yotgan oraliqni aniqlash usullarini o'rganish.

Reja:

2.1. Tenglama ildizini ajratish.
2.2. Transendent tenglama ildizini ajratish.
2.3. Algebraik tenglama ildizlari yotgan oraliqlarni aniqlash.
2.4. Tenglama ildizini urinmalar (Nyuton) usulida hisoblash.

2.1. Tenglama ildizini ajratish

Amaliyotda, ba'zi masalalarda

$$f(x)=0 \qquad (2.1)$$

ko'rinishdagi tenglamalarni yechishga to'g'ri keladi. Bunda $f(x)$ $[a,b]$ oraliqda aniqlangan, uzluksiz funksiya bo'lib, $f(t)=0$ bo'lganda, $x=t$ ni (2.1) tenglamaning *yechimi–ildizi* deyiladi. Tenglamaning aniq yechimini topish qiyin bo'lgan hollarda, uning taqribiy yechimini topishni quyidagi ikki bosqichga bo'lishi mumkin.

1) Yechimni ajratish(yakkalash), ya'ni yagona yechim yotgan intervalni aniqlash;

2) Taqribiy yechimni berilgan aniqlikda hisoblash.

Tenglamaning yagona yechimi yotgan oraliqni aniqlash uchun quyidagi teoremadan foydalaniladi.

2.1–teorema . *Aytaylik,*

1) $f(x)$ *funksiya* $[a,b]$ *kesmada uzluksiz va* (a,b) *intervalda hosilaga ega bo'lsin;*

2) $f(a)f(b)<0$, *ya'ni* $f(x)$ *funksiya kesmaning chetlarida har xil ishoraga ega bo'lsin;*

3) $f'(x)$ *hosila* (a,b) *intervalda o'z ishorasini saqlasin.*

U holda, (2.1) *tenglama* $[a,b]$ *oraliqda yagona yechimga ega bo'ladi.*

2.2. Transtsendent tenglama ildizini ajratish

Tarkibida algebraik, trigonometrik, logorifmik, ko'rsatkichli funksiyalar ishtrok etgan murakkab tenglamalarni *transendent* tenglamalar deb ataladi.

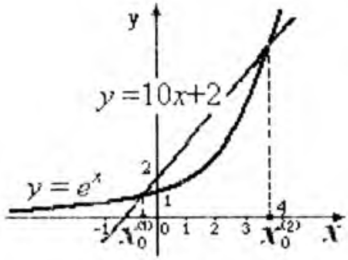
Tenglama ildizi yotgan $[a,b]$ kesmani topishda, ba'zan grafik usuldan foydalanamiz. Bu usulga asosan (2.1) tenglamaning ildizini ajratish uchun $y=f(x)$ funksiyaning $[a,b]$ oraliqdagi egri chizig'ining grafigini quramiz. Bu egri chizig'ining Ox o'qi bilan kesishish nuqtasining abstsissasi (2.1) tenglamaning yechimi bo'ladi. Ba'zan $y=f(x)$ funksiyaning grafigini chizish qiyin bo'lsa, $f(x)=0$ tenglamani, grafigini chizish mumkin bo'lgan funksiyalarga aratamiz, masalan

$$f_1(x)=f_2(x) \qquad (2.2)$$

ko'rinishga keltiramiz va $y=f_1(x)$, $y=f_2(x)$ funksiyalarning grafiklarini chizamiz. Bu grafiklar kesishish nuqtasining absissasi x_0 $f(x_0)=0$ tenglamaning yechimi bo'ladi, chunki $f(x)$ ning grafigi x_0 nuqtada Ox o'qi bilan kesishadi. Bu yechimni o'z ichiga oluvchi (a,b) oraliqda yuqoridagi teorema shartlarini tekshirish asosida tanlaymiz.

2.1–masala. $e^x-10x-2=0$ tenglamaning yagona ildizi yotgan oraliq topilsin

Yechish. Berilgan tenglamani $e^x=10x+2$ ko'rinishda yozamiz. So'ngra, $y=e^x$, $y=10x+2$ funksiyalarning grafiklarini quramiz.



2.1.1–rasm.

2.1.1–rasmdan ko'rinadiki, $e^x-10x-2=0$ tenglamaning ikkita ildizi bo'lib, 1–ildizi $x_0^{(1)}$ ni o'z ichiga olgan oraliq $(-1,0)$ va ikkinchisi $x_0^{(2)}$ $(3,4)$ oraliqda yotadi.

Biz $(-1,0)$ oraliqdagi ildizini aniqlaymiz va hisoblaymiz. Bu $[-1,0]$ kesmada teorema shartlarini tekshiramiz.

$f(x)=e^x-10x-2$ funksiya $[-1,0]$ oraliqda uzluksiz, $(-1,0)$ intervalda

$f'(x)=e^x-10$ hosilaga ega.

1) $[-1,0]$ kesma chetlarida:

$f(-1)=e^{-1}-10(-1)-2\approx 3.368>0$,

$f(0)=e^0-10\cdot 0-2=-1<0$ bo'ladi, bundan: $f(-1)\cdot f(0)<0$

3) $x\in(-1,0)$ bo'lganda $f'(x)=e^x-10<0$.

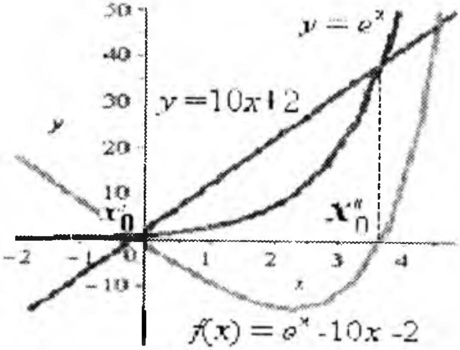
Demak, 2.1–teoremaning barcha shartlari $[-1,0]$ oraliqda bajariladi. Bu $[-1,0]$ oraliqda tenglama yagona yechimga ega ekanligini bildiradi.

Tenglamaning ildizi yotgan oraliqni topish va ildizni hisoblashning Maple dasturini tuzamiz.

2.1–M a p l e d a s t u r i:

Berilgan funksiyalarning grafiklarini qurish:

```
> with(plots):
> implicitplot([y=exp(x),y=10*x+2,y=exp(x)-10*x-2],
x=-2..10,y=-6..50,color=[blue,blue,red],thickness=3);
```



2.1.2–rasm.

2.2–M a p l e d a s t u r i:

> restart;

a)*tenglamaning barcha ildizini aniqlash.*

> solve(exp(x)=10*x+2,x);

$$\left\{x=-\mathrm{LambertW}\left(-\frac{1}{10}e^{-\frac{1}{5}}\right)-\frac{1}{5}\right\},\ \left\{x=-\mathrm{LambertW}\left(-1,-\frac{1}{10}e^{-\frac{1}{5}}\right)-\frac{1}{5}\right\}$$

> evalf(%);

$$\{x=-.1104575676\},\ \{x=3.650889167\}$$

b)*tenglamaning manfiy ildizini aniqlash.*

> _EnvExplicit:= true:

solve({exp(x)=10*x+2,x<0},x): evalf(%);

$$\{x=-.1104575676\}$$

c)*tenglamaning musbat ildizini aniqlash.*

> solve({exp(x)=10*x+2,x>0},x): evalf(%);

$$\{x=3.650889167\}$$

d)*tenglamaning [–5,5] oraliqdagi ildizlarini aniqlash.*

> _EnvExplicit:= true:

solve({exp(x)=10*x+2,x>–5,x<5},x): evalf(%);

$$\{x=-.1104575676\},\ \{x=3.650889167\}$$

> with(Student[Calculus1]):

> x:=Roots(exp(x)–10*x–2,x=–5..5,numeric);

$$x:=[-.1104575676,\ 3.650889167]$$

> x[1]; $-.1104575676$

> x[2]; 3.650889167

2.3. Algebraik tenglama ildizlari yotgan oraliqlarni aniqlash

Aytaylik, bizga

$$f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0 \qquad (2.3)$$

n –darajali algebraik tenglama berilgan bo'lsin.

1. Algebraik tenglama ildizlarining chegarasini topishda, tenglama $a_0, a_1, \ldots, a_{n-1}, a_n$ koeffitsientlari asosida, quyidagi teorema va qoidalardan foydalanamiz.

2.2–teorema. *Agar*

$$A = \max\left\{\left|\frac{a_1}{a_0}\right|, \left|\frac{a_2}{a_0}\right|, \ldots, \left|\frac{a_n}{a_0}\right|\right\}, \quad A_1 = \max\left\{\left|\frac{a_0}{a_n}\right|, \left|\frac{a_1}{a_n}\right|, \ldots, \left|\frac{a_{n-1}}{a_n}\right|\right\}$$

bo'lsa, (2.3) *tenglamaning barcha ildizlari*

$r = 1/(1+A_1) < |x| < 1 + A = R$

halqada yotadi.

Musbat ildizlar chegarasi: $r < x^{+} < R$

Manfiy ildizlar chegarasi: $-R < x^{-} < -R$

Agar (2.3) tenglamani

$f_1(x) = x^n f(1/x) = 0,$

$f_2(x) = f(-x) = 0,$

$f_3(x) = x^n f(-1/x) = 0$

ko'rinishlardan biriga keltirib, ulardan topilgan musbat ildizlarining yuqori chegaralari mos ravishda R_1, R_2, R_3 bo'lsa, (2.3) tenglama ildizlarining chegaralari quyidagicha bo'ladi:

$$1/R_1 < x^{+} < R, \quad \text{va} \ -R_2 < x^{-} < -1/R_3$$

2. Koeffitsentlarining ishorasi almashinuvchi algebraik tenglamaning musbat ildizlarining yuqori chegarasini topishda quyidagi Lagranj teoremasidan foydalanamiz:

2.3-teorema. (2.3) *tenglamada* $a_0 > 0$ *va* a_k ($k \geq 1$ *–tartib raqami*) *– birinchi uchragan manfiy koeffitsient bo'lib, B manfiy koeffitsientlar ichida modul bo'yicha eng kattasi bo'lsa, musbat ildizla rining yuqori chegarasi*

$$R = 1 + \sqrt[k]{\frac{B}{a_0}} \qquad (2.4)$$

formula bilan topiladi.

Berilgan (2.3) tenglamaning manfiy ildizlarining quyi chegarasini aniqlash uchun tenglamani

$f(-x) = 0 \quad (2.5)$

ko'rinishga keltirib, hosil bo'lgan (2.5) tenglamaga Lagranj teoremasini qo'llab, uning musbat ildizlarining yuqori chegarasi R_1 topamiz, $R1$ (2.3) tenglama manfiy ildizlarining quyi chegarasi uchun $-R_1$ bo'lishi ayondir.

Demak, berilgan (2.3) tenglamaning barcha haqiqiy ildizlarining chegarasi:

$$-R_1 < x < R.$$

2.2–masala. $2x^3-9x^2-60x+1=0$ tenglama ildizlari yotgan oraliqning chegarasini aniqlang.

Yechish.

1) Teorema bo'yicha:

$$A=\max\left\{\left|\frac{a_1}{a_0}\right|,\left|\frac{a_2}{a_0}\right|,\ldots,\left|\frac{a_n}{a_0}\right|\right\}=\max\left\{\left|\frac{-9}{2}\right|,\left|\frac{-60}{2}\right|,\left|\frac{1}{2}\right|\right\}=30.$$

$$A_1=\max\left\{\left|\frac{a_0}{a_n}\right|,\left|\frac{a_1}{a_n}\right|,\ldots,\left|\frac{a_{n-1}}{a_n}\right|\right\}=\max\left\{\left|\frac{2}{1}\right|,\left|\frac{-9}{1}\right|,\left|\frac{-60}{1}\right|\right\}=60$$

$$r=\frac{1}{1+60}<|x|<1+30=R,\ r=0.016,\ R=31.$$

Musbat ildizlarining chegarasi: $0.016 < x^+ < 31$

Manfiy ildizlarining chegarasi: $-31 < x^- < -0.016$

Barcha ildizlarining chegarasi: $-31 < x < 31$

Bu masalani Maple dasturida quyidagich yechamiz.

2.3.1–M a p l e d a s t u r i:

2.2–teorema asosida berilgan ko'phad ildizlarining chegarasini aniqlash:

```
> C := proc(p,x) local i;
[seq(coeff(p,x,i), i=0..degree(p,x) )];
end proc:
C( 2*x^3–9*x^2–60*x+1, x );  [1, –60, –9, 2]
> A := proc(p,x) local i;
[max(seq(abs(coeff(p.x,i)/lcoeff(p)),
i=0..degree(p,x)))];
end proc:
A:=A(2*x^3–9*x^2–60*x+1, x );  A := [30]
> A1 := proc(p,x) local i; # indeks haqiqiy
[max(seq(abs(coeff(p,x,i)/tcoeff(p)),
i=0..degree(p,x))]);
end proc:
A1:=A1(2*x^3–9*x^2–60*x+1,x);  A1 := [60]
```

> **A:=30:A1:=60:R1:=1/(1+A1);** $R1 := \frac{1}{61}$

> **R2:=1+A;** $R2 := 31$

2)Berilgan tenglamadan $a_0=2$, $B=60$, $k_1(a_1=-9)=1$ larni aniqlab, Lagranj
formulasini quyidagicha hisoblaymiz:

$$R = 1 + \sqrt[k]{\frac{B}{a_0}} = 1 + \frac{60}{2} = 31$$

Budan musbat ildizlarining yuqori chegarasi $R=31$ekanini topamiz.

Manfiy ildizlarining quyi chegarasini topaish uchun berilgan tenglamada x ni $-x$ *bilan almashtirib,* quyidagi ishlarni bajaramiz.

$f(-x)=2(-x)^3-9(-x)^2-60(-x)+1=0$

$f(-x)=2x^3+9x^2-60x-1=0$

bu tenglamadan: $a_0=2$, $B_2=60$, $k_2=2$ va Lagranj formulasi:

$$R_1 = 1 + \sqrt[k]{\frac{B_2}{a_0}} = 1 + \sqrt{\frac{60}{2}} \approx 6.77$$

dan manfiy ildizlar quyi chegarasini $R_1=-6.77$ bo'ladi.

2.3.2a–M a p l e d a s t u r i:

2.3–teorema asosida berilgan ko'phadning musbat ildizlarining yuqori chegarasini aniqlash:

> **p:=2*x^3–9*x^2–60*x+1;**

$$p := 2\,x^3 \text{ K } 9\,x^2 \text{ K } 60\,x \text{ C } 1$$

> **coeffs(p,x);** 1, 2, K 9, K 60

> **M1:=max(coeffs(p,x));** $M1 := 2$

> **B:=min(coeffs(p,x));** $B :=$ K 60

> **R:=1+(abs(B)/a0)^1;** $R := 31$

Ildizlarining quyi chegarasi:

> **p1:=2*(–x)^3–9*(–x)^2–60*(–x)+1;**

$$p1 := \text{K } 2\,x^3 \text{ K } 9\,x^2 \text{ C } 60\,x \text{ C } 1$$

> **p:=(–1)^3*p1;** $p := 2\,x^3 \text{ C } 9\,x^2 \text{ K } 60\,x \text{ K } 1$

> a0:=lcoeff(p); $a0 := 2$

> coeffs(p,x); K 1, 2, 9, K 60

> B1:=min(coeffs(p,x)); $B1 :=$ K 60

> R1:=-1-(abs(B1)/a0)^(1/2); $R1 :=$ K 1 K $\sqrt{30}$

> evalf(R1); – 6.477225575

Ildizlari yotgan oraliqni va ildizlarni hisoblash.

2.3.2b–Maple dasturi:

Ildizlari yotgan oraliq uning kengligini tanlash bilan aniqlash.

> f:= 2*x^3–9.*x^2–60*x+1=0;

$$f := 2\,x^3 - 9.\,x^2 - 60\,x + 1 = 0$$

> readlib(proot);

$$\mathbf{proc}(p, r)\ \ldots\ \mathbf{end\ proc}$$

Ildizlari yotgan oraliqlarni 1, 2, 0.1,0.01 *ga teng kengliklar bo'yicha aniqlash:*

> realroot(2*x^3–9*x^2–60*x+1,1);

$$[[0, 1], [8, 9], [-4, -3]]$$

> realroot(2*x^3–9*x^2–60*x+1,2);

$$[[0, 2], [8, 10], [-4, -2]]$$

> realroot(2*x^3–9*x^2–60*x+1,1/10);

$$\left[\left[0, \frac{1}{16}\right], \left[\frac{65}{8}, \frac{131}{16}\right], \left[-\frac{59}{16}, -\frac{29}{8}\right]\right]$$

> realroot(2*x^3–9*x^2–60*x+1,1/100);

$$\left[\left[\frac{1}{64}, \frac{3}{128}\right], \left[\frac{1045}{128}, \frac{523}{64}\right], \left[-\frac{59}{16}, -\frac{471}{128}\right]\right]$$

ildizlarni hisoblash:

> sols:=solve(f,x);

$$sols := 0.01662535946,\ 8.166187279,\ -3.682812638$$

> sols[1]; 0.01662535946

> sols[2]; 8.166187279

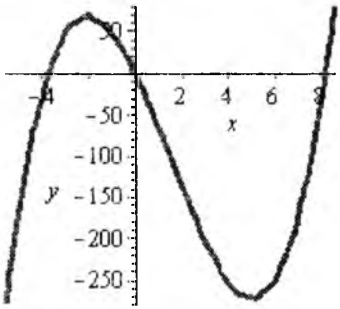
> sols[3]; -3.682812638

berilgan tenglama–ko'phadning grafigini qurish:

> with(plots):

> implicitplot([y=2*x^3–9*x^2–60*x+1],x=–10..10,

y=–280..80,color=[blue, blue,red],thickness=3); (2.1.3–rasm)



2.1.3–rasm.

3. Dekart qoidasi. (2.3) tenglamaning berilish tartibida koeffitsientlari ketma–ketligida, ularning isoralarining almashinishi soni qancha bo'lsa, tenglamaning shuncha ildizlari mavjud yoki musbat ildizlar soni isora almasinishlar sonidan juft songa kam.

4. Agar berilgan (2.3) tenglamaning barcha koeffitsientlari musbat bo'lsa, ildizlarining chegarasini

$$m<|x|<M$$

tengsizlikka asosan aniqlaymiz, bunda

$$m=\min(a_k / a_{k-1}),\ M=\max(a_k / a_{k-1}),\ 1<k<n$$

5. (2.3) tenglamaning barcha koeffitsientlari musbat bo'lib, ular:

a) $a_0>a_1>...>a_n$ bo'lganda, barcha ildizlar $|x|>1$ doiradan tashqarida yotadi;

b) $a_0<a_1<...<a_n$ bo'lganda, barcha ildizlar $|x|<1$ doira ichida yotadi.

6. Toq darajali algebraik tenglama hech bo'lmaganda bitta ildizga ega bo'ladi.

2.4. Tenglama ildizini hisoblash

Maqsad: Transtsendent tenglama ildizi yotgan oraliqda ildizini hisoblash vatarlar va urinmalar usulini o'rganish.

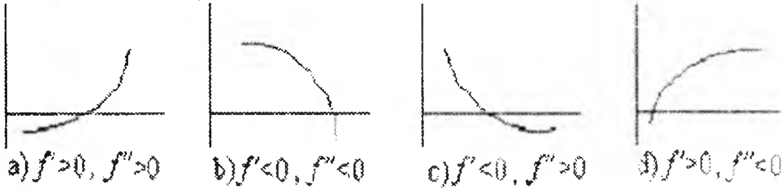
Reja:2.4.1. Vatarlar usuli.

2.4.2. Urinmalar –Nyuton usuli.

2.4.3. Birgalashgan usul.

Aytaylik, berilgan $f(x)=0$ tenglamadagi $f(x)$ funksiya grafik usulda aniqlangan $[a,b]$ oraliqda 2.1–teoremaning hamma shartlarini qanoatlantirsin. Bundan tashqari $f(x)$ funksiya $[a,b]$ oraliqda ikkinchi tartibli $f''(x)$ uzluksiz hosilaga ega bo'lib, bu hosila shu oraliqda o'z ishorasini saqlasin, yahni 2.1–teorema sharlari o'rinli bo'lsin.

Bu teorema shartlarining mazmunini quyidagi shakllarda ko'rish mumkin (2.2–rasm).



2.2–rasm.

Bu holatlardan birortasiga mos kelgan oraliqdagi ildizni hisoblash uchun oraliqning chetlaridagi nuqtalarda birida $f(x)f''(x)$ ko 'paytmanig ishoralariga qarab, quyidagi vatarlar yoki urinmalar usullaridan birini qo'llaymiz.

2.4.1. Vatarlar usuli

Aniqlangan oraliqdagi ildizga vatarlar usuli bilan yaqinlashish ketma-ketligini qurishda, bu oraliqning chetki nuqtalaridan birida

$$f(x)f''(x)<0 \qquad (2.6)$$

shartni bajarilishiga qarab, quyidagi ikki holni keltiramiz.

1)Agar $[a,b]$ oraliqning chap chetida

$$f(a)f''(a)<0$$

shart bajarilsa, vatarlar usuli bilan ildizga yaqinlashish ketma–ketligini chap tomondan qo'llaymiz (2.3–rasm):

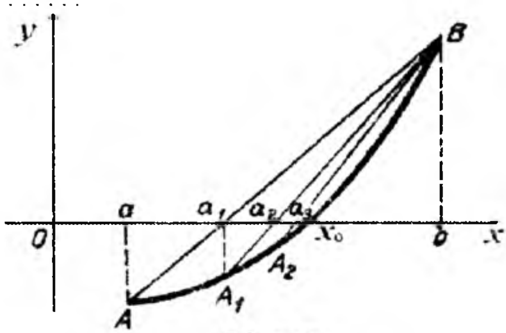
$a_0= a$

$a_1= a_0 - (b–a_0) f(a_0)/ (f(b)–f(a_0))$

. . . . (2.7)

$a_n= a_{n-1} - (b - a_{n-1}) f(a_{n-1})/ (f(b)–f(a_{n-1}))$

. . . .

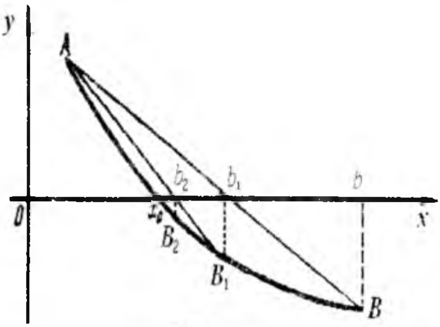


2.3–rasm.

Bu ketma–ketlik hadlarini hisoblash jarayonini $|a_n - a_{n-1}| < \varepsilon$ shart bajarilguncha davom ettiramiz va ildizning taqribiy qiymati uchun $x \approx a_n$ ni qabul qilamiz, bu yerda ε taqribiy ildiz aniqligini belgilaydi.

2)Agar $[a,b]$ oraliqning o'ng tomonida $f(b)f''(b) < 0$ shart bajarilsa, vatarlar usuli bilan ildizga yaqinlashish ketma–krtligini o'ng tomondan qo'llaymiz (2.4–rasm).

$$b_0 = b,$$
$$b_1 = b_0 - (a - b_0) f(b_0) / (f(a) - f(b_0)),$$
$$\ldots\ldots\ldots\ldots\ldots \qquad (2.8)$$
$$b_n = b_{n-1} - (a - b_{n-1}) f(b_{n-1}) / (f(a) - f(b_{n-1})),$$
$$\ldots\ldots\ldots\ldots\ldots$$



2.4–rasm.

Bu ketma–ketlik hadlarini hisoblash jarayonini $|b_n - b_{n-1}| < \varepsilon$ shart bajarilguncha davom ettiramiz va ildizni taqribiy qiymati uchun $x \approx b_n$ ni qabul qilamiz.

2.3–masala. $e^x - 10x - 2 = 0$ tenglamaning $\varepsilon = 0.01$ aniqlikdagi ildizini vatar usulida taqribiy hisoblang.

Yechish. Berilgan tenglamaning ildizi yotgan $(-1,0)$ oraliqni grafiklar usulida aniqlaymiz va oraliqda $f(x) = e^x - 10x - 2$ funksiya 2.1–teoremaning barcha shartlarini qanoatlantirishini tekshiramiz.

$x \in [-1,0]$ kesma chetlarida: $f(0) = -1$, $f(-1) = 8.368$ bo'lib, bulardan faqat $f(0) = -1$ ni $f''(x) = e^x > 0$ ga ko'paytmasi manfiy bo'ladi, yani $x = 0$ nuqtada (2.6) shart bajariladi:

$$f(0) f''(0) < 0$$

bundan ildizga vatar usulida yaqinlashish ketma–ketligi $\{b_n\}$ o'ngdan (2.8) jarayon bilan quyidagicha quriladi.

Berilganlar: $a = -1$, $b = 0$, $\varepsilon = 0.01$:

$$f(a) = f(-1) = e^{-1} - 10(-1) - 2 = 8.386,$$

$$f(b_0)=f(0)=e^0-10\cdot 0-2=\ -1$$

$b_0=0,$

$b_1=b_0-(a-b_0)f(b_0)/(f(a)-f(b_0))=-0.107$

yaqinlashish sharti $|b_1-b_0|>\varepsilon$ bajarilmaganligi uchun b_2 yaqinlashishni hisoblaymiz.

$$b_1=-0.107,$$

$$f(b_1)=f(-0.107)=e^{-0.107}-10(-0.107)-2=-0.038,$$

$$f(a)=f(-1)=8.386.$$

$b_2=b_1-(a-b_1)f(b_1)/(f(a)-f(b_1))=-0,111$

$|b_2-b_1|=|-0.111+0.107|=0.004<\varepsilon=0.01$

Demak, 0.01 aniqlikdagi taqribiy yechim uchun $x\approx b_2=-0.11$ ni olish mumkin.

Aniqlangan oraliqda tenglama ildizini vatarlar usuli asosida yaqinlashishning Maple dasturini tuzamiz, bunda hisoblashlar jarayonida ildiz qiymatining takrorlanishiga qarab ildizni aniqlaymiz.

1. Birinchi (–1,0) oraliqdagi ildizning qiymatini hisoblash. Hisoblashni yechim qiymti takrorlanguncha davom ettiramiz.

2.4.1a–M a p l e d a s t u r i:

> restart;

Izlanayotgan ildiz yotgan oraliqning chetlari kiritish :

> a:=–1: b:=0:

Oraliqning tashqarisidagi o'ng chetiga yaqin sonni tanlash :

> c:=1:

Hisoblashlar soni chapdan n va o'ngdan m larni tamlash :

> n:=11;m:=10; $n:=11\ m:=10$

Vatar usulini qo'llash :

> XORD:=proc(f,x) local iter;
iter:=x–(c–x)*f/(fc–f): unapply(iter,x) end;

XORD := **proc**(*f*, *fb*, *x*)

local *iter*;

a; *c*; *fb*; *iter* := *x* − (*c* − *x*) * *f* / (*fc* − *f*); *unapply*(*iter*, *x*)

end proc

> f:=exp(x)–10*x–2; $f:=e^x-10x-2$

> fc:=exp(c)–10*c–2; $fc:=e-12$

> **F:=XORD(f,x);**

$$F := x \to x - \frac{(1-x)\left(e^x - 10x - 2\right)}{e - 10 - e^x + 10x}$$

Chapdan yaqinlashish :

> **to n do a:=evalf(F(a)); od;**

Ongdan yaqinlashish :

> **to m do b:=evalf(F(b)); od;**

$$a := -1 b := 0$$
$$a := -0.0517767458 b := -.1207478906$$
$$a := -.1158150426 b := -.1095425961$$
$$a := -.1099803100 b := -.1105392704$$
$$a := -.1105001775 b := -.1104502746$$
$$a := -.1104537641 b := -.1104582185$$
$$a := -.1104579071 b := -.1104575094$$
$$a := -.1104575373 b := -.1104575728$$
$$a := -.1104575703 b := -.1104575671$$
$$a := -.1104575673 b := -.1104575676$$
$$a := -.1104575675 b := -.1104575676$$
$$a := -.1104575675 b := -.1104575676$$

> **x0:=(a+b)/2;** $x0 := -.110457567($

Ildizning 0.0001 aniqlikdagi taqribiy qiymati:

> **x0:=evalf(%,5);** $x0 := -.1104($

2.Ikkinchi (3.2.3.8) oraliqdagi ildizning qiymatini hisoblash.

2.4.1b–M a p l e d a s t u r i:

> **restart;**

Izlanayotgan ildiz yotgan oraliqning chetlari kiritish :

> **a:=3.2: b:=3.8:**

Oraliqning tashqarisidagi o'ng chetiga yaqin sonni tanlash :

> **c:=4:**

Hisoblashlar soni chapdan n va o'ngdan m larmi tamlash :

> **n:=15;m:=14;** $n := 15$ $m := 14$

Vatar usulini qo'llash :

> **XORD:=proc(f,x) local iter;**
> **iter:=x–(c–x)*f/(fc–f); unapply(iter,x) end;**

```
XORD := proc(f, fb, x)
    local iter;
    a; c; fb: iter := x − (c − x)*f/(fc − f); unapply(iter, x)
end proc
```

> **f:=exp(x)–10*x–2;** $f := e^{x} - 10\,x - 2$

> **fc:=exp(c)–10*c–2;** $fc := e^{4} - 42$

> **F:=XORD(f,x);**

Chapdan yaqinlashish :

> **to n do a:=evalf(F(a)); od;**

Ongdan yaqinlashish :

> **to m do b:=evalf(F(b)); od;**

Ildizga har ikki tomondan yaqinlashish:

> **a:=3.2; b:=3.8; to n do**
a:=evalf(F(a)); b:=evalf(F(b)):od;

$$a := 3.2 b := 3.8$$
$$a := 3.543247817 b := 3.680936938$$
$$a := 3.627595964 b := 3.657146243$$
$$a := 3.645966191 b := 3.652200758$$
$$a := 3.649854013 b := 3.651164477$$
$$a := 3.650671741 b := 3.650946973$$
$$a := 3.650843509 b := 3.650901305$$
$$a := 3.650879580 b := 3.650891716$$
$$a := 3.650887154 b := 3.650889702$$
$$a := 3.650888744 b := 3.650889279$$
$$a := 3.650889078 b := 3.650889191$$
$$a := 3.650889148 b := 3.650889172$$
$$a := 3.650889163 b := 3.650889168$$
$$a := 3.650889166 b := 3.650889167$$
$$a := 3.650889167 b := 3.650889167$$

Ildizning qiymati:

> **x0:=(a+b)/2;** $x0 := 3.65088916$

Ildizning 0.0001 *aniqlikdagi taqribiy qiymati:*

> **x0:=evalf(%,5);** $x0 := 3.6509$

Hisoblash natijasiga qarab ildiz uchun $x \approx 3.650889167$ ni olamiz.

2.4.2. Urinmalar – Nyuton usuli

Berilgan tenglamaning ildizi yotgan oraliqda teorema shartlari asosida ildizni hisoblash uchun urinmalar usulini qo'llash shart

$$f(x)\,f''(x)>0$$

ni oraliqning qaysi chetida bajarilishiga qarab ildizga yaqinlashishni aniqlaymiz.

Bundan:

$f(a)f''(a)>0$ bo'lganda, boshlang'ich yaqinlashishni chapdan $a_0=a$, aks holda o'ngdan $b_0=b$ deb olinadi.

Urinmalar usulida chapdan ildizga yaqinlashish ketma-ketligi $\{a_n\}$ quyidagicha topiladi.

$y=f(x)$ funksiya grafigining $A(a,f(a))$ nuqtasiga o'tkazilgan urinma (2.5–rasm), tenglamasini tuzamiz.

$$y-f(a)=f'(a)(x-a),\quad f'(a)\neq 0$$

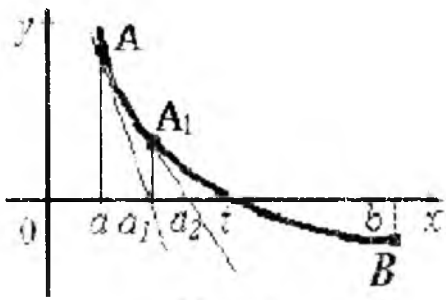
Urinmaning Ox o'qi bilan kesishish nuqtasi $x=a_1$ –desak, bu nuqtada $y=0$ ekanligidan

$$0-f(a)=f'(a)(a_1-a)$$

ni olamiz. Budan esa

$$a_1=a-f(a)/f'(a)$$

formula topiladi. Bu chapdan ildizga birinchi yaqinlashish qiymati bo'ladi.



2.5–rasm.

Ildizga ikkinchi yaqinlashishni topish uchun $[a_1,b]$ oraliqqa yuqoridagi jarayonni takrorlab,

$$a_2=a_1-f(a_1)/f'(a_1)$$

formulani olamiz va hokazo, jarayonning n– takrorlanishida (n–qadamda)

$$a_n=a_{n-1}-f(a_{n-1})/f'(a_{n-1}) \qquad (2.9)$$

formulaga ega bo'lamiz. Bu jarayonni ko'p takrorlash (davom ettirish) natijasida $\{a_n\}$ ketma–ketlikni tuzamiz.

Olingan $\{a_n\}$ ketma–ketlik 2.1–teoremaning shartlari bajarilganda aniq yechim x_0 ga yaqinlashadi. (2.9) jarayon $|a_n - a_{n-1}| < \varepsilon$ shart bajarilguncha davom ettiriladi va taqribiy ildiz uchun $x \approx a_n$ ni qabul qilinadi.

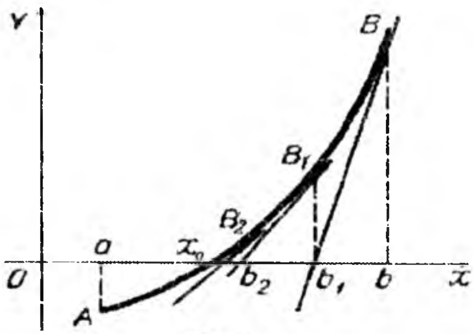
Agar

$$f(b)f''(b)>0$$

bo'lsa, $b_0=b$ deb olib,

$$b_n = b_{n-1} - \frac{f(b_{n-1})}{f'(b_{n-1})}, \quad f'(b_{n-1}) \neq 0$$

formula asosida ildizga yaqinlashishning $\{b_n\}$ ketma-ketlikni (2.6–rasm) hisoblaymiz.



2.6–rasm.

2.4–masala. $e^x - 10x - 2 = 0$ tenglama taqribiy yechimini $\varepsilon = 0.01$ aniqlik bilan toping.

Yechish. Grafiklar usulida aniqlangan $[-1,0]$ oraliqda $f(x)=e^x - 10x - 2$ funksiya 2.1–teoremaning barcha shartlarini qanoatlantiradi.

$$f''(x)=e^x>0,\ x \in [-1,0] \quad \text{va}\ f(-1)=8.386>0$$

dan

$$f(-1)f''(-1)>0$$

bo'lgani uchun yaqinlashish chapdan bo'lib, unda $a_0=-1$ deb olinadi.

$f'(-1) = e^{-1} - 10 = -9.632$ ni e'tiborga olib, birinchi yaqinlashish a_1 ni hisoblaymiz:

$$a_1 = a_0 - f(a_0)/f'(a_0) = -1 - f(-1)/f'(-1) = -1 - 8.386/(-9.632) = -0.131.$$

Yaqinlashish shartini tekshiramiz:

$|a_1 - a_0| = |-0.131 + 1| = 0.869 > \varepsilon = 0.01.$

Teorema sharti bajarilmaganligi uchun hisoblashni davom ettiramiz. Ikkinchi yaqinlashish a_2 ni

$$a_2=a_1-f(a_1)/f'(a_1)$$

formulaga asosan hisoblaymiz:

$$f(a_1)=e^{-0.131}+10(0.131)-2=0.1895,$$

$$f'(a_1)=e^{-0.131}-10=-9.123$$

lar asosida: $a_2=-0.131-0.1895/(-9.123)=-0.1104$.

Yana $|a_2-a_1|=0.0214>\varepsilon$ bajarilmaganligi uchun a_3 ni hisoblamiz:

$$a_2=-0.1104,\ f(a_2)=0.0006,\ f'(a_2)=-9.1046$$

lar asosida:

$$a_3=a_2-f(a_2)/f'(a_2)=-0.1104-0.0006/(-0.1046)=-0.1104;$$

yaqinlashish sharti $|a_3-a_2|<\varepsilon=0.01$ bajarilganligi uchun tenglamaning $\varepsilon=0.01$ aniqlikdagi taqribiy yechimi:

$$x\approx a_3=-0.11$$

bo'ladi. Aniqlangan oraliqda ildizni aniqlash va Nyuton usulida hisoblash uchun oraliqni kengroq olib, unda yotgan ildizga chpdan yoki o'ngdan yaqinlashishni hisoblash va grafigini qurish dasturini tuzamiz.

2.4.2a–Maple dasturi:

```
> restart;
```

Izlanayotgan ildiz yotgan oraliqning chetlari kiritish :

```
> a:=-1: b:=0:
```

Oraliqning tashqarisidagi o'ng chetiga yaqin sonni tanlash :

```
> c:=1:
```

Hisoblashlar soni chapdan n va o'ngdan m larni tamlash :

```
> n:=4:m:=4:
```

Urinmalar usulini qo'llash :

```
> Ur:=proc(f,x) local iter;
iter:=x-f/diff(f,x); unapply(iter,x) end;
```

$Ur :=$ **proc**(f, x) **local** $iter$; $iter := x - f/diff(f, x)$; $unapply(iter, x)$ **end proc**

```
> f:=exp(x)-10*x-2;
```

$f := e^x - 10x - 2$

```
> F:=Ur(f,x);
```

$F := x \to x - \dfrac{e^x - 10x - 2}{e^x - 10}$

Chapdan yaqinlashish :

```
> to n do a:=evalf(F(a)); od:
```

Ongdan yaqinlashish :

```
> to m do b:=evalf(F(b)); od;
```

$a := -.1312526261\ b := -.1111111111$

$a := -.1104784974\ b := -.1104575885$

$a := -.1104575675 b := -.1104575675$

$a := -.1104575675 b := -.1104575675$

> **x0:=(a+b)/2;** $x0 := -.110457567$;

2.4.2b–M a p l e d a s t u r i:

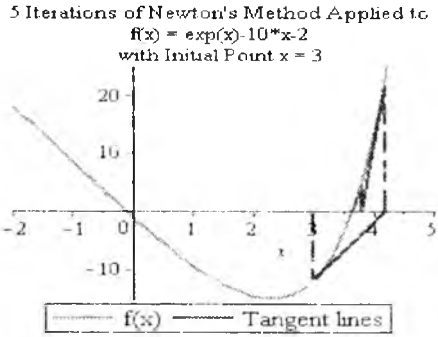
Urinmalar (Nyuton) usulida $e^x-10x-2=0$ *tenglama ildizini aniqlash*
1–ildiz: x = -1 dan o'ngdagi:

```
> with(Student[Calculus1]):
  NewtonsMethod(exp(x)-10*x-2,x=-1); -.1104575675
> NewtonsMethod(exp(x)-10*x-2,x=-1,output=sequence);
```

K 1. K .1312526261 . K .1104784974 . K .1104575675

2–ildiz: x = 3 dan o'ngdagi:

```
> with(Student[Calculus1]):
  NewtonsMethod(exp(x)-10*x-2, x=3); 3.650889174
> NewtonsMethod(exp(x)-10*x-2,x=3,output=sequence):
3,4.181341477,3.791101988,3.663011271,3.650987596,  3.650889174
> NewtonsMethod(exp(x)-10*x-2,x=3, thickness=2,
  view=[-2..5.DEFAULT], output=plot); (2.7-rasm)
```



2.7–rasm.

2.4.3. Birgalashgan usul

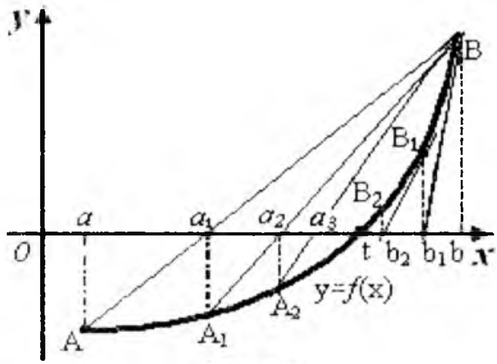
Berilgan tenglamaning aniqlangan $[a,b]$ oraliqdagi ildizini hisoblashda vatarlar va urinmalar usulini bir vaqtda qo'llash uchun, oraliqning chetki *a va b* nuqtalarida $f(x)f''(x)$ ko'pqytmanmg ishorasiga qarab ildizga yaqinlashish ketma–ketliklarini tuzamiz.

1. $x=a$ nuqtada urinmani qo 'llash shartiga asosan $f(a)f''(a)>0$ bo'lganda, chapdan urinmalar. o'ngdan esa vatarlar usullarini qo'llash mumkin:

$$a_1 = a - f(a)/f'(a),$$
$$b_1 = b - (a-b)f(b)/(f(a) - f(b)) \qquad (2.10)$$

2. $x=b$ nuqtada urinmani qo'llash shartiga asosan $f(b)f''(b)>0$ bo'lganda, chapdan vatarlar, o'ngdan esa urinmalar usullarini qo'llash mumkin (2.8–rasm.):

$$a_1 = a-(b-a)f(a)/(f(b)-f(a)), \qquad (2.11)$$
$$b_1 = b - f(b)/f'(b)$$



2.8–rasm.

Agar $b_1-a_1 < \varepsilon$ tengsizlik bajarilsa tenglamaning $\varepsilon=0.0001$ aniqlikdagi yechimi deb $t=(a_1+b_1)/2$ olinadi. Aks holda yana $[a_1,b_1]$ oraliqda urinmalar va vatarlar usulini qo'llab, aniq yechim t ga yanada yaqinroq bo'lgan a_2 va b_2 qiymatlarni hosil qilamiz.

Agar $b_2-a_2 < \varepsilon$ bo'lsa, taqribiy yechim deb $t=(a_2+b_2)/2$ ni olinadi. Aks holda, yuqoridagi jarayon yana takrorlanadi va hokazo.

$e^x-10x-2=0$ tenglamaning $(-1,0)$ oraliqdagi ildizining taqribiy qiymatini $\varepsilon=0.0001$ aniqlikda birgalashgan usulda hisoblashning Maple dasturini tuzamiz.

2.4.3–Maple dasturi:

```
> restart;
> a:=-1:b:=0:c:=1:n:=11:m:=10:
> XORD:=proc(f,x) local iter;
iter:=x-(c-x)*f/(fc-f): unapply(iter,x) end;
```

$XORD := \mathbf{proc}(f, fb, x)$

local *iter*;

a; b; fb; $iter := x - (c - x)*f/(fc - f)$; $unapply(iter, x)$

end proc

> Ur:=proc(f,x) local iter;
iter:=x–f/diff(f,x); unapply(iter,x) end;

$Ur := \mathbf{proc}(f, x)$

local *iter*;

$iter := x - f/diff(f, x)$; $unapply(iter, x)$

end proc

> f:=exp(x)–10*x–2; $f := e^x - 10x - 2$

> fc:=exp(c)–10*c–2; $fc := e - 12$

> Fvat:=XORD(f,x);

$$Fvat := x \to x - \frac{(1 - x)\left(e^x - 10x - 2\right)}{e - 10 - e^x + 10x}$$

> Fur:=Ur(f,x); $Fur := x \to x - \dfrac{e^x - 10x - 2}{e^x - 10}$

1) *Ildizga chapdan vataralar usulida yaqinlashish:*

> to n do a:=evalf(Fvat(a)); od;

$$a := -0.0517767458$$
$$a := -.1158150426$$
$$a := -.1099803100$$
$$a := -.1105001775$$
$$a := -.1104537641$$
$$a := -.1104579071$$
$$a := -.1104575373$$
$$a := -.1104575703$$
$$a := -.1104575673$$
$$a := -.1104575675$$
$$a := -.1104575675$$

2) *Ildizga o'ngdan urinmalar usulida yaqinlashish:*

> to m do b:=evalf(Fur(b)); od;

$$b := -.1111111111$$
$$b := -.1104575885$$
$$b := -.1104575675$$
$$b := -.1104575675$$
$$b := -.1104575675$$
$$b := -.1104575675$$
$$b := -.1104575675$$
$$b := -.1104575675$$
$$b := -.1104575675$$
$$b := -.1104575675$$

> **x0:=(a+b)/2;** $x0 := -.1104575676$

> **x0:=evalf(%,5);** $x0 := -.11046$

3) *Ildizga chapdan vatаralar va o'ngdan urinmalar usulida yaqinlashish:*

> **a:=-1;b:=0; to n do**
a:=evalf(Fvat(a)); b:=evalf(Fur(b)):od;

$$a := -1 b := 0$$
$$a := -0.0517767458 b := -.1111111111$$
$$a := -.1158150426 b := -.1104575885$$
$$a := -.1099803100 b := -.1104575675$$
$$a := -.1105001775 b := -.1104575675$$
$$a := -.1104537641 b := -.1104575675$$
$$a := -.1104579071 b := -.1104575675$$
$$a := -.1104575373 b := -.1104575675$$
$$a := -.1104575703 b := -.1104575675$$
$$a := -.1104575673 b := -.1104575675$$
$$a := -.1104575675 b := -.1104575675$$

$2x^3-9x^2-60x+1=0$ algebraik tenglama ildizlari yotgan oraliqlarni aniqlash va ulardagi ildizlarni hisoblash.

2.4.4–M a p l e d a s t u r i:

1) *ildizlari yotgan oraliqlarni aniqlash:*

> **f:=2*x^3–9.*x^2–60*x+1=0;** $f := 2x^3 - 9.x^2 - 60x + 1 = 0$

> **readlib(proot);** **proc**(p, r) ... **end proc**

> **realroot(2*x^3–9*x^2–60*x+1,3);**

$$[[0, 2], [8, 10], [-4, -2]]$$

> sols:=solve(f,x);

$$sols := 0.01662535946,\ 8.166187279,\ -3.682812638$$

2)[8,10] *oraliqdai yotgan ildizni hisoblash:*

> restart;

> a:=8:b:=10:c:=11:n:=23:m:=6:

> XORD:=proc(f,x) loeal iter;
iter:=x–(c–x)*f/(fc–f); unapply(iter,x) end;

XORD := **proc**(*f*, *x*)

local *iter*;

iter := *x* − (*c* − *x*) * *f* / (*fc* − *f*); *unapply*(*iter*, *x*)

end proc

> Ur:=proc(f,x) local iter;
iter:=x–f/diff(f,x); unapply(iter,x) end;

Ur := **proc**(*f*, *x*)

local *iter*;

iter := *x* − *f* / *diff*(*f*, *x*); *unapply*(*iter*, *x*)

end proc

> f:=2*x^3–9*x^2–60*x+1; $f := 2x^3 - 9x^2 - 60x + 1$

> fc:=2*c^3–9*c^2–60*c+1; $fc := 914$

> Fvat:=XORD(f,x);

$$Fvat := x \rightarrow x - \frac{(11 - x)\left(2x^3 - 9x^2 - 60x + 1\right)}{913 - 2x^3 + 9x^2 + 60x}$$

> Fur:=Ur(f,x); $Fur := x \rightarrow x - \dfrac{2x^3 - 9x^2 - 60x + 1}{6x^2 - 18x - 60}$

1) *Ildizga chapdan vataralar usulida yaqinlashish :*

> a:=8:to n do a:=evalf(Fvat(a)); od;

$a := 8.098412698 a := 8.138813932 a := 8.15517522$

$a := 8.161764315 a := 8.164411952 a := 8.165474867$

$a := 8.165901427 a := 8.166072587 a := 8.166141262$

$a := 8.166168815 a := 8.166179871 a := 8.166184306$

$a := 8.166186087 \quad a := 8.166186800 \quad a := 8.166187087$

$a := 8.166187202 \quad a := 8.166187248 \quad a := 8.166187268$

$a := 8.166187276 a := 8.166187277 a := 8.166187278$

$a := 8.166187280a := 8.166187280$

2) *Ildizga o'ngdan urinmalar usulida yaqinlashish* :

> **b:=10:to m do b:=evalf(Fur(b)); od;**

$$b := 8.608333333$$
$$b := 8.201737828$$
$$b := 8.166446130$$
$$b := 8.166187290$$
$$b := 8.166187280$$
$$b := 8.166187280$$

> **x0:=(a+b)/2;** $x0 := 8.16618728$

> **x0:=evalf(%,5);** $x0 := 8.1662$

O'z–o'zini tekshirish uchun savollar

1. Tenglamalarning qanday turlari bor?
2. Ildiz yotgan oraliqni ajratish.
3. Trantsendent tenglama ildizini ajratish qoidasi.
4. Algebraik tenglama ildizlarini aniqlashda Dekart qoidasi.
5. Algebraik tenglamaning barcha ildizlari oralig'ini aniqlash teoremasini tushuntiring.
6. Algebraik tenglama musbat ildizlarini ajratish haqidagi teorema.
7. Qanday tenglamalar musbat ildizlarining chegarasini topishda Lagranj usulini qo'llaymiz?
8. Manfiy ildizlar quyi chegarasini aniqlash.
9. Musbat koeffitsientli algebraik tenglama ildizlarining chegarasini qanday aniqlanadi?
10. Tenglama ildiziga yaqinlashish sharti.
11. Ildizga ketma–ket yaqinlashish haqidagi teorema.
12. Ildizni hisoblashda vatarlar usulini qo'llashning asosiy sharti.
13. Vatarlar usuli bilan ildizga chapdan yaqinlashish sharti.
14. Vatarlar usuli bilan ildizga o'ngdan yaqinlashish sharti.
15. Vatarlar usulini qo'llashda boshlang'ich yaqinlashishni tanlash.
16. Ildizni hisoblashda urinmalar usulini qo'llashning asosiy sharti.
17. Urinmalar usuli bilan ildizga chapdan yaqinlashish sharti.
18. Urinmalar usuli bilan ildizga o'ngdan yaqinlashish sharti.
19. Urinmalar usulini qo'llashda boshlang'ich yaqinlashishni tanlash.

2.1–laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi tenglamalarning:

1) Ildizlarning qisqa atrofini analitik yoki grafik usulda aniqlang:
2) Aniqlangan oraliqda ildizni vatarlar va urinmalar usulida hisoblang.

1.	1) $2^x + 5x - 3 = 0$ 2) $3x^4 - 4x^3 - 12x^2 - 5 = 0$ 3) $0.5^x + 1 = (x-2)^2$ 4) $(x-3)cosx = 1$. $(-2\pi \le x \le 2\pi)$	2.	1) $arctgx - 1/(3x^3) = 0$. 2) $2x^3 - 9x^2 - 60x + 1 = 0$. 3) $[\log_2(-x)](x+2) = -1$. 4) $sin(x + p/3) - 0.5x = 0$.
3.	1) $5^x + 3x = 0$, 2) $x^4 - x - 1 = 0$, 3) $0.5^x + x^2 = 2$, 4) $(x-1)^2 ln(x+1) = 1$.	4.	1) $2e^x = 2 + 5x$, 2) $2x^4 - x^2 - 10 = 0$, 3) $xlog_3(x+1) = 1$, 4) $cos(x + 0.5) = x^3$.
5.	1) $3^{x-1} - 2 - x = 0$, 2) $3x^4 + 8x^3 + 6x^2 - 10 = 0$, 3) $(x-4)^2 \log_{0.5}(x-3) = -1$, 4) $5sinx = x$.	6.	1) $arctgx - 1/(2x^3) = 0$. 2) $x^4 - 18x^2 + 6 = 0$. 3) $x^2 2^x = 1$. 4) $tgx = x + 1$, $(-\pi/2 \le x \le \pi/2)$.
7.	1) $e^{-2x} - 2x + 1 = 0$, 2) $x^4 + 4x^3 - 8x^2 - 17 = 0$, 3) $0.5^x - 1 = (x+2)^2$, 4) $x^2 cos2 = -1$.	8.	1) $5^x - 6x - 3 = 0$, 2) $x^4 - x^3 - 2x^2 + 3x - 3 = 0$, 3) $0.5^x - 2x^2 - 3 = 0$, 4) $xlog(x+1) = 1$.
9.	1) $arctg(x-1) + 2x = 0$, 2) $3x^4 + 4x^3 - 12x^2 + 1 = 0$, 3) $(x-2)^2 2^x = 1$, 4) $x^2 - 20sinx = 0$	10.	1) $2arctgx - x + 3 = 0$. 2) $3x^4 - 8x^3 - 18x^2 + 3 = 0$, 3) $2sin(x + \pi/3) = 0.5x^2 - 1$, 4) $2lgx - x/2 + 1 = 0$

11	1) $3^x + 2x - 2 = 0$, 2) $2x^4 - 8x^3 + 8x^2 - 1 = 0$, 3) $\left[(x-2)^2 - 1\right]2^x = 1$, 4) $(x-2)Cosx = 1$.	12.	1) $2arctgx - 3x + 2 = 0$, 2) $2x^4 + 8x^3 + 8x^2 - 1 = 0$, 3) $sin(x - 0.5) - x + 0.8 = 0$, 4) $(x-1)log_2(x+2) = 1$.
13.	1) $3^x + 2x - 5 = 0$, 2) $x^4 - 4x^3 - 8x^2 + 1 = 0$, 3) $0.5^x + x^2 - 3 = 0$, 4) $(x-2)^2 Lg(x+1) = 1$.	14.	1) $2e^x + 3x + 3x + 1 = 0$, 2) $3x^4 + 4x^3 - 12x^2 - 5 = 0$, 3) $cos(x + 0.3) = x^2$, 4) $xlog_3(x+1) = 2$.
15.	1) $3^{x-1} - 4 - x = 0$, 2) $2x^3 - 9x^2 - 60x + 1 = 0$. 3) $(x-3)^2 log_{0.5}(x-2) = -1$, 4) $sinx = x - 1$.	16.	1) $arctgx - 1/(3x^3) = 0$, 2) $x^4 - x - 1 = 0$, 3) $(x-1)^2 2^x = 1$, 4) $tg^3 x = x - 1$.
17.	1) $e^x + x + 1 = 0$, 2) $2x^3 - x^2 - 1 = 0$. 3) $0.5^x - 3 = (x+2)^2$. 4) $x^2 cos2x = -1$, $(-2\pi \le x \le 2\pi)$	18.	1) $3^x - 2x + 5 = 0$, 2) $3x^4 + 8x^3 + 6x^2 - 10 = 0$, 3) $2x^2 - 0.5^x = 0$, 4) $x\lg(x+1) = 1$.
19.	1) $arctg(x-1) + 3x - 2 = 0$, 2) $x^4 - 18x^2 + 6 = 0$, 3) $x^2 - 20sinx = 0$, 4) $(x-2)^2 2^x = 1$.	20.	1) $2arctgx - x + 3 = 0$, 2) $x^4 + 4x^3 - 8x^2 - 17 = 0$, 3) $2Sin(x + \pi/2) = x^2 - 0.8$, 4) $2\lg x - x/2 + 1 = 0$.
21.	1) $2^x - 3x - 2 = 0$, 2) $x^4 - x^3 - 2x^2 + 3x - 3 = 0$, 3) $(0.5)^x + 1 = (x-2)^2$. 4) $(x-3)sosx = -1, -2\pi \le x \le 2\pi$.	22.	1) $arctgx + 2x - 1 = 0$, 2) $3x^4 + 4x^3 - 12x^2 + 1 = 0$, 3) $(x+2)log_2(x) = 1$, 4) $sin(x+1) = 0.5x$.

23.	1) $3^x + 2x - 3 = 0,$ 2) $3x^4 - 8x^3 - 18x^2 + 2 = 0,$ 3) $(0.5)^x = 4 - x^2,$ 4) $(x+2)^2 Lg(x+11) = 1.$	24.	1) $2e^x - 2x - 3 = 0,$ 2) $3x^4 + 4x^3 - 12x^2 - 5 = 0,$ 3) $xlog_2(x+1) = 1,$ 4) $\cos(x+0.5) = x^3.$
25.	1) $3^x + 2 + x = 0,$ 2) $2x^3 - 9x^2 - 60x + 1 = 0,$ 3) $(x-4)^2 log_{0.5}(x-3) = -1,$ 4) $5Sinx = x - 0.5.$	26.	1) $arctg(x-1) + 2x - 3 = 0.$ 2) $x^4x - 1 = 0,$ 3) $(x-1)^2 2^x = 1,$ 4) $tg^3x = x - 1, (-\pi/2 \le x \le \pi/2).$
27.	1) $2e^x - 2x - 3 = 0,$ 2) $2x^4 - x^2 - 10 = 0,$ 3) $(0.5)^x - 3 = -(x+1)^2,$ 4) $x^2cos2x = 1.$	28.	1) $3^x - 2x - 5 = 0,$ 2) $3x^4 + 8x^3 + 6x^2 - 10 = 0,$ 3) $2x^2 - 0.5^x - 3 = 0,$ 4) $xlg(x+1) = 1.$
29.	1) $arctg(x-1) + 2x = 0.$ 2) $x^4 - 18x^2 + 6 = 0,$ 3) $(x-2)^2 2^x = 1,$ 4) $x^2 - 10\sin x = 0.$	30.	1) $3^x + 5x - 2 = 0.$ 2) $3x^4 + 4x^3 - 12x^2 + 1 = 0,$ 3) $(x-2)^2 = 0.5^x + 1,$ 4) $(x+3)cosx = 1, -2\pi \le x \le 2\pi$

2.5. Chiziqsiz tenglamalar sistemasini yechish

2.5.1. Nyuton usuli

1.Chiziqsiz ikki noma'lumli tenglamalardan tuzilgan

$$\begin{cases} F(x,y) = 0 \\ G(x,y) = 0 \end{cases} \qquad (2.12)$$

sistema berilgan bo'lsin.

Bu sistemaning yechimlari yotgan oraliqlarni aniqlashda grafik usuldan foydalanamiz.

$F(x,y)=0$ va $G(x,y)=0$ funksiyalar grafiklari kesishgan nuqtani o'z ichiga oluvchi kesmani taqriban aniqlaymiz:

$$D = \{a \le x \le b,\ c \le x \le d\}$$

Bu kesmada yechimga mos keluvchi nuqtaga iloji boricha yaqin bo'lgan (x_0, y_0) nuqtani tanlaymiz. Bu $x=x_0$, $y=y_0$ qiymatlardan foydalanib

$\varepsilon=0.001$ aniqlikda hisoblash algoritmini tuzamiz.

$n=1,2,3,...$ lar uchun berilgan sistemadagi funksiya va ularning xususiy hosilalarini hisoblab sistema yechimini topamiz:

1) $F=F(x_{n-1},y_{n-1}),\ F'_x=F'_x(x_{n-1},y_{n-1}),\ F'_y=F'_y(x_{n-1},y_{n-1});$

$G=G(x_{n-1},y_{n-1}),\ G'_x=G'_x(x_{n-1},y_{n-1}),\ G'_y=G'_y(x_{n-1},y_{n-1});$

2) $J=F'_x G'_y-G'_x F'_y;\quad \Delta_1=F G'_y-G F'_y,\quad \Delta_2=F'_x G-G'_x F;$

3) $x_n=x_{n-1}+\Delta_1/J,\ y_n=y_{n-1}+\Delta_2/J;$

4) $|x_n-x_{n-1}|<\varepsilon,\ |y_n-y_{n-1}|<\varepsilon.$

bo'lsa, taqribiy yechimni: $x\approx x_n,\ x\approx y_n$ deb olamiz.

2.5.–masala. Ushbu

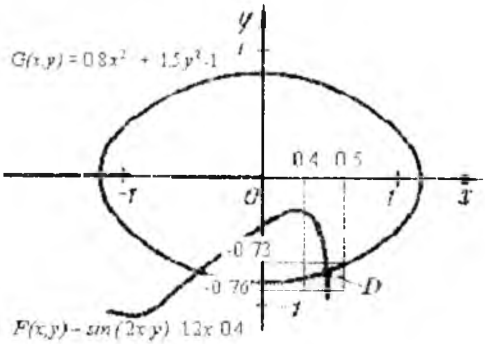
$$\begin{cases} F(x,y)=\sin(2x-y)-1.2x-0.4 \\ G(x,y)=0.8x^2+1.5y^2-1 \end{cases}$$

chiziqsiz tenglamalar sistemasining yechimini Nyuton usuli bilan 0.1 aniqlikda toping.

Yechish. Yechim yotgan kesmani (2.8–rasm)

$$D=\{0.4<x<0.5,\ -0.76<y<-0.73\}$$

deb olsa bo'ladi (bunga ishonch hosil qilishni o'quvchining o'ziga havola qilamiz). U holda, boshlang'ich yaqinlashishni: $x_0=0.4,\ y_0=-0.75$ deb olsak bo'ladi.



2.8–rasm.

Xususiy hosilalarni topamiz:

$F'_x=2\cos(2x-y)-1.2$, $G'_x=1.6x$,

$F'_y=-\cos(2x-y)$, $G'_y=3y$

boshlang'ich yaqinlashish $x_0=0.4$, $y_0=-0.75$ dagi funksiya va hosilalarning qiymatlari:

$F=F(0.4,-0.75)=0.1198$,

$F'_x=F'_x(0.4,-0.75)=-1.1584$, $F'_y=F'_y(0.4,-0.75)=-0.0208$,
$G=G(0.4,-0.75)=-0.0282$,
$G'_x=G'_x(0.4,-0.75)=0.64$, $G'_y=G'_y(0.4,-0.75)=-2.25$,
$J=2.6197$, $\Delta_1=0.2701$, $\Delta_2=0.044$.

$$x_1=x_0+\Delta_1/J=0.5\ ,\ y_1=y_0+\Delta_2/J=-0.733.$$

$$|x_1-x_0|=0.1=0.1,\ |y_1-y_0|=0.02<0.1.$$

Aniqlik sharti bajarilmagani uchun, birinchi yaqinlashish qiymatlari $x_1=0.5$, $y_1=-0.733$ asosan ikkinchi yaqinlashishni hisoblaymiz.

$$F=-0.0131\ ,\ F'_x=0.8\ ,\quad F'_y=-1.4502$$

$G=0.059$, $G'_x=-2.191$, $G'_y=0.1251$, $J=3.2199$, $\Delta_1=-0.0293$, $\Delta_2=0.0749$

$x_2=x_1+\Delta_1/J=0.491$, $y_2=y_1+\Delta_2/J=-0.710$
$|x_2-x_1|=0.009<0.1$, $|y_2-y_1|=0.023<0.1$
bo'lganidan, yechimni quyidagicha olamiz:

$$x\approx 0.5,\ y\approx -0.71$$

Chiziqsiz tenglamalar sistemasini Maple dasturida sohalardagi yechimlarni topish va sistemaning tenglamalari funksiyalarning grafigini qurish (2.5.1–masala).

2.5.1–M a p l e dasturi

> f:=sin(2*x–y)–1.2*x=0.4: g:=0.8*x^2+1.5*y^2=1:

1) $\{-2<x<-1,\ -1<y<1\}$ *sohalardagi yechim:*

> fsolve({f,g},{x=–2..–1,y=–1..1});

$$\{x=-1.090593921, y=-.1797849074\}$$

2) $\{-1<x<-0.7,\ -1<y<1\}$ *sohalardagi yechim:*

> fsolve({f,g},{x=–1..–0.7,y=–2..2});

$$\{x=-.9415815625, y=0.4402569923\}$$

3) $\{-0.5<x<0,\ -1<y<1\}$ *sohalardagi yechim:*

> fsolve({f,g},{x=–0.5..0,y=–2..2});

$$\{x=-.4390572805, y=-.7509029957\}$$

4) $\{0<x<2,\ -1<y<1\}$ *sohalardagi yechim:*

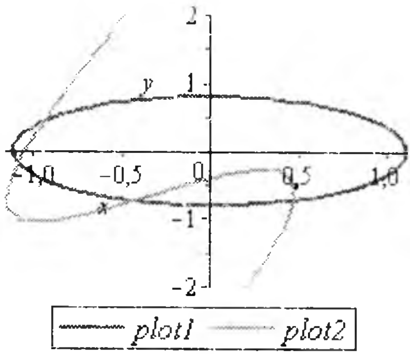
> fsolve({f,g},{x=0..2,y=–2..2});

$$\{x=0.4912379505, y=-.7334613013\}$$

Sistemaning tenglamalari funksiyalarning grafigini qurish:

> with(plots):

> implicitplot([0.8*x^2+1.5*y^2=1,sin(2*x–y)–1.2*x=0.4], x=–2..2,y=–2..2, color=[blue,green], thickness=2, legend=[plot1,plot2]);(2.9–rasm)



2.9–rasm.

2. Endi Nyuton usulini n ta noma'lumli n ta chiziqsiz tenglamalar sistemasini yechish uchun qo'llaymiz.

Buning uchun quyidagi chiziqsiz tenglamalar sistemasini olamiz

$$\begin{cases} f_1(x_1, x_2, \dots, x_n) = 0, \\ f_2(x_1, x_2, \dots, x_n) = 0, \\ \dots \quad \dots \quad \dots \\ f_n(x_1, x_2, \dots, x_n) = 0. \end{cases} \qquad (2.13)$$

Bu sistemasini yechimini topish uchun ketma–ket yaqinlashish (iteratsiya) usulidan foydalanamiz. Bu ketma–ketlikni yechimga p–yaqinlashishini quyidagicha yozamiz:

$$x^{(p+1)} = x^{(p)} - W^{-1}\left(x^{(p)}\right) f\left(x^{(p)}\right) \qquad (2.14)$$

bu formulada:

$-x^{(p)} = \left(x_1^{(p)}, x_2^{(p)}, \dots, x_n^{(p)}\right)$–boshlang'ich yoki p–yaqinlashishini bildiradi;

$-W^{-1}(x^{(p)})$ (2.13) sistemaning chap tamonidagi funksiyalarning har bir argumenti bo'yicha olingan 1–tartibli xususiy hosilalarning $x^{(p)}$ p–yaqinlashish qiymati bo'yicha topilgan sonlardan tuzilgan quyidagi Yakobiyan matritsa

$$W=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial fn}{\partial x_1} & \frac{\partial fn}{\partial x_2} & \cdots & \frac{\partial fn}{\partial x_n} \end{pmatrix}=\frac{\partial f_k}{\partial x_i},\ k,i=1,2,3,\ldots,n \qquad (2.15)$$

ga teskari matritsa:

– $f(x^{(p)})$ (2.13) sistemaning chap tamonidagi funksiyalarning $x^{(p)}$ dagi qiymatilaridan tuzilgan matritsa.

(2.14) ketma–ketlikni yechimga yaqinlashishining asosiy sharti:

$$\sum_{i=1}^{n}\left|\frac{\partial f_k}{\partial x_i}\right|<1, \quad k=1,2,\ldots,n$$

2.6–masala. Quyidagi chiziqsiz tenglamalar sistemasi yechimining musbat qimatlarini Nyuton usulida toping.

$$\begin{cases} x^2+y^2+z^2=1 \\ 2x^2+y^2-4z=0 \\ 3x^2-4y+z^2=0 \end{cases}$$

Sistemasi yechimining boshlang'ich qimatlarini $x_0=y_0=z_0=0.5$ bo'lsin.

Yechish.

1. Sistemaning yechimga 1–yaqinlashishining qimatlarini topamiz.

$$\begin{cases} f_1(x,y,z)=x^2+y^2+z^2-1 \\ f_2(x,y,z)=2x^2+y^2-4z \\ f_3(x,y,z)=3x^2-4y+z^2 \end{cases} \qquad f(x)=\begin{pmatrix} x^2+y^2+z^2-1 \\ 2x^2+y^2-4z \\ 3x^2-4y+z^2 \end{pmatrix}$$

boshlang'ich yaqinlashish qimatlari $x_0=y_0=z_0=0.5$ asosida

$$f(x^{(0)})=\begin{pmatrix} 0.25+0.25+0.25-1 \\ 2\cdot 0.25+0.25-4\cdot 0.5 \\ 3\cdot 0.25-4\cdot 0.5+0.25 \end{pmatrix}=\begin{pmatrix} -0.75 \\ -1\cdot 25 \\ -1\cdot 00 \end{pmatrix}$$

Yakobi W matritsasini tuzamiz:

$$W=\begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{pmatrix}=\begin{pmatrix} 2x & 2y & 2z \\ 4x & 2y & -4 \\ 6x & -4 & 2z \end{pmatrix}$$

boshlang'ich yaqinlashish qimatlari asosida Yakobiyan matritsasi:

$$W(x^{(0)})=\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -4 \\ 3 & -4 & 1 \end{pmatrix}$$

$$\det(W(x^{(0)}))=\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -4 \\ 3 & -4 & 1 \end{vmatrix}=-40$$

$W(x^{(0)})$ matrisaga teskari matrisani topamiz:

$$W^{-1}(x^{(0)})=-\frac{1}{40}\begin{pmatrix} -15 & -5 & -5 \\ -14 & -2 & 0 \\ -11 & 7 & -1 \end{pmatrix}=\begin{pmatrix} 3/8 & 1/8 & 1/8 \\ 7/20 & 1/20 & -3/20 \\ 11/40 & -7/40 & 1/40 \end{pmatrix}$$

Ketma-ket yaqinlashish formulasiga asosan 1-yaqinlashishining qimatlarini topamiz.

$$x^{(1)}=x^{(0)}-W^{-1}\left(x^{(0)}\right)f\left(x^{(0)}\right)=$$

$$=\begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}-\begin{pmatrix} 3/8 & 1/8 & 1/8 \\ 7/20 & 1/20 & -3/20 \\ 11/40 & -7/40 & 1/40 \end{pmatrix}\begin{pmatrix} -0.25 \\ -1.25 \\ -1.00 \end{pmatrix}=$$

$$=\begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}+\begin{pmatrix} 0.375 \\ 0 \\ -0.125 \end{pmatrix}=\begin{pmatrix} 0.875 \\ 0.500 \\ 0.375 \end{pmatrix}$$

1. Endi sistemaning yechimga 2-yaqinlashishining qimatlarini topamiz.

$$f(x)=\begin{pmatrix} x^2+y^2+z^2-1 \\ 2x^2+y^2-4z \\ 3x^2-4y+z^2 \end{pmatrix}$$

1–yaqinlashish qimatlari $x^{(1)}=\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0.875 \\ 0.500 \\ 0.375 \end{pmatrix}$ asosida, quyidagilarni hisoblaymiz:

$$f(x^{(1)})=\begin{pmatrix} 0.875^2+0.5^2+0.375^2-1 \\ 2\cdot 0.875^2+0.5^2-4\cdot 0.375^2 \\ 3\cdot 0.875^2-4\cdot 0.5^2+0.375^2 \end{pmatrix}=\begin{pmatrix} 0.15625 \\ 0\cdot 28125 \\ 0\cdot 43750 \end{pmatrix}$$

Yakobi W matritsasini tuzamiz:

$$W=\begin{pmatrix} 2x & 2y & 2z \\ 4x & 2y & -4 \\ 6x & -4 & 2z \end{pmatrix}$$

$$W(x^{(1)})=\begin{pmatrix} 2\cdot 0.875 & 2\cdot 0.5 & 2\cdot 0.375 \\ 4\cdot 0.875 & 2\ 0.5 & -4 \\ 3\cdot 0.875 & -4 & 2\cdot 0.375 \end{pmatrix}=\begin{pmatrix} 1\cdot 75 & 1 & 0.75 \\ 3.5 & 1 & -4 \\ 5.25 & -4 & 0.75 \end{pmatrix}$$

$$\det(W(x^{(1)})=\begin{vmatrix} 1.75 & 1 & 0.75 \\ 3.5 & 1 & -4 \\ 5.25 & -4 & 0.75 \end{vmatrix}=64.75$$

$$W^{-1}(x^{(1)})=-\frac{1}{64.75}\begin{pmatrix} -15.25 & -3.75 & -4.75 \\ -23.625 & -2.625 & 9.625 \\ -19.25 & 12.25 & -1.75 \end{pmatrix}$$

Ketma–ket yaqinlashish formulasiga asosan 2–yaqinlashishining qimatlarini topamiz:

$$x^{(2)}=x^{(1)}-W^{-1}(x^{(1)})f(x^{(1)})=$$

$$=\begin{pmatrix} 0.875 \\ 0.500 \\ 0.375 \end{pmatrix}-\frac{1}{64.75}\begin{pmatrix} -15.25 & -3.75 & -4.75 \\ -23.625 & -2.625 & 9.625 \\ -19.25 & 12.25 & -1.75 \end{pmatrix}\begin{pmatrix} 0.15625 \\ 0\cdot 28125 \\ 0\cdot 43750 \end{pmatrix}=$$

$$=\begin{pmatrix} 0.875 \\ 0.500 \\ 0.375 \end{pmatrix}-\begin{pmatrix} 0.08519 \\ 0.00338 \\ 0.00507 \end{pmatrix}=\begin{pmatrix} 0.78981 \\ 0.49662 \\ 0.36993 \end{pmatrix}$$

$$x^{(2)}=\begin{pmatrix}0.78981\\0.49662\\0.36993\end{pmatrix}$$

$x^{(2)}$ 2–yaqinlashishining qimatlarini sistemaga qo'yib tekshiaramiz.

$$f(x^{(2)})=\begin{pmatrix}0.00001\\0\cdot 00004\\0\cdot 00005\end{pmatrix}$$

bu qiymatlar nolga yaqinligidan yechimning qiymatlari 2–yaqinlashish bo'yicha quyidagicha olinadi:

$x=0.7852$, $y=0.49662$, $z=0.36992$.

2.5.2–M a p l e d a s t u r i:

Chiziqsiz tenglamalar sistemasini yechish(4.2–masala).

1)> restart;with(Student[MultivariateCalculus]):

$1-$ *yaqinlashish* :

> **Digits:= 5;**

$$Digits := 5$$

> **W:=Jacobian([x^2+y^2+z^2–1, 2*x^2+y^2–4*z, 3*x^2–4*y+z^2],[x,y,z]);**

$$W := \begin{bmatrix}2x & 2y & 2z\\4x & 2y & -4\\6x & -4 & 2z\end{bmatrix}$$

> **W0:=Jacobian([x^2+y^2+z^2–1, 2*x^2+y^2–4*z, 3*x^2–4*y+z^2],[x,y,z]=[0.5,0.5,0.5]);**

$$W0 := \begin{bmatrix}1.0 & 1.0 & 1.0\\2.0 & 1.0 & -4\\3.0 & -4 & 1.0\end{bmatrix}$$

> **Jacobian([x^2+y^2+z^2–1, 2*x^2+y^2–4*z, 3*x^2–4*y+z^2],[x,y,z]=[0.5,0.5,0.5],output=determinant);**

$$-40.000$$

> **F0:=<x^2+y^2+z^2–1, 2*x^2+y^2–4*z, 3*x^2–4*y+z^2>;**

$$F0 := \begin{bmatrix}x^2+y^2+z^2-1\\2x^2+y^2-4z\\3x^2-4y+z^2\end{bmatrix}$$

> **WT:=W0^(-1): evalm(WT);** $\begin{bmatrix} 0.37500 & 0.12500 & 0.12500 \\ 0.35000 & 0.050000 & -.15000 \\ 0.27500 & -.17500 & 0.025000 \end{bmatrix}$

> **x:=0.5:y:=0.5:z:=0.5:**

> **F0;** $\begin{bmatrix} -25 \\ -1.25 \\ -1.00 \end{bmatrix}$

> **X0:=<0.5,0.5,0.5>;** $X0 := \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$

> **X:=X0–W0^(–1).F0;** $X := \begin{bmatrix} 0.8750000000000000(\\ 0.5000000000000000(\\ 0.3750000000000000(\end{bmatrix}$

$2 - \textit{yaqinlashish}$:

> **x:=X[1];y:=X[2];z:=X[3]; F0;W0:=W; W0^(–1);**

$x := 0.8750000000000000($
$y := 0.5000000000000000($
$z := 0.3750000000000000($

> **X0:=X;** $X0 := \begin{bmatrix} 0.8750000000000000(\\ 0.5000000000000000(\\ 0.3750000000000000(\end{bmatrix}$

> **F0;** $\begin{bmatrix} 0.1562 \\ 0.2812 \\ 0.43752 \end{bmatrix}$

> **W0:=W;** $W0 := \begin{bmatrix} 1.7500 & 1.0000 & 0.75000 \\ 3.5000 & 1.0000 & -4 \\ 5.2500 & -4 & 0.75000 \end{bmatrix}$

> **W0^(–1);**

$$\begin{bmatrix} 0.235520000000000008 & 0.057915000000000012 & 0.073358999999999937 \\ 0.364860000000000018 & 0.040541000000000008 & -.148650000000000004 \\ 0.297300000000000008 & -.189189999999999996 & 0.027026999999999989 \end{bmatrix}$$

> **X:=X0–W0^(–1).F0;** $X := \begin{bmatrix} 0.789830000000000030 \\ 0.496645999999999976 \\ 0.369937000000000014 \end{bmatrix}$

$3 - yaqinlashish$:

> **x:=X[1];y:=X[2];z:=X[3];**

$x := 0.78983000000000003$

$y := 0.49664599999999997$

$z := 0.36993700000000001$

> **X0:=X;** $X0 := \begin{bmatrix} 0.789830000000000030 \\ 0.496645999999999976 \\ 0.369937000000000014 \end{bmatrix}$

> **F0;** $\begin{bmatrix} 0.0074 \\ 0.0146 \\ 0.02176 \end{bmatrix}$

> **W0:=W;** $W0 := \begin{bmatrix} 1.5797 & 0.99330 & 0.73988 \\ 3.1593 & 0.99330 & -4 \\ 4.7390 & -4 & 0.73988 \end{bmatrix}$

> **W0^(–1);**

$$\begin{bmatrix} 0.262747533691878587 & 0.063589965687895031 & 0.081037759533482300 \\ 0.366510781085204574 & 0.040233869104152900 & -.148995134741727792 \\ 0.298538360511171440 & -.189783979805269536 & 0.027006431588793095 \end{bmatrix}$$

> **X:=X0–W0^(–1).F0;**

$$X := \begin{bmatrix} 0.785193800000000054 \\ 0.496588599999999992 \\ 0.369910950000000014 \end{bmatrix}$$

2)*Chiziqsiz tenglamalar sistemasining yuqorida topilgan yechimini to'g'ridan–to'g'ri hisoblash*

> solve({x^2+y^2+z^2=1,2*x^2+y^2–4*z=0,3*x^2–4*y+z^2=0},[x,y,z]);

$$x = \mathrm{RootOf}\big(2\_Z^2 \, K \, 4\,\mathrm{RootOf}(K\,23 C\,36\_Z^2 C\,4\_Z^4 C\,24\_Z \; C\,16\_Z^3)\,K$$
$$\mathrm{RootOf}(K\,23 C\,36\_Z^2 C\,4\_Z^4 C\,24\_Z C\,16\_Z^3)^2 C\,1\big)$$
$$, y = \mathrm{RootOf}(K\,23 C\,36\_Z^2 C\,4\_Z^4 C\,24\_Z \; C\,16\_Z^3),\; z = \frac{1}{2}$$
$$\mathrm{RootOf}(K\,23 C\,36\_Z^2 C\,4\_Z^4 C\,24\_Z C\,16\_Z^3)^2\,K\,\frac{1}{4}$$
$$C\;\mathrm{RootOf}(K\,23 C\,36\_Z^2 C\,4\_Z^4 C\,24\_Z C\,16\_Z^3)$$

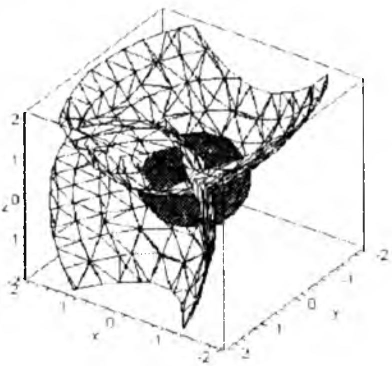
\> **evalf(%,5);** $\{y = 0.49664,\ x = 0.78520,\ z = 0.36992\}$

chiziqsiz tenglamalar sistemasidagi sfera va paraboloidlarining kesishishini aniqlash grafigini qurish:

> with(plots):

implicitplot3d([x^2+y^2+z^2=1, 2*x^2+y^2–4*z=0, 3*x^2–4*y+z^2=0], x=–2..2, y=–2..2,z=–2..2,

color=[blue,green,yellow]); (2.10–rasm)



2.10–rasm.

2.5.2. Ketma-ket yaqinlashish (iteratsiya) usuli

1. Chiziqsiz tenglamalardan tuzilgan

$$\begin{cases} F(x,y) = 0 \\ G(x,y) = 0 \end{cases} \tag{2.16}$$

sistema berilgan bo'lsin.

Bu sistema yechimini o'z ichiga oluvchi sohani topamiz:

$$D = \{a \le x \le b, c \le x \le d\}$$

(2.16) ga tengkuchli bo'lgan quyidagi sistemani tuzamiz:

$$\begin{cases} x = \varphi_1(x, y) \\ y = \varphi_2(x, y) \end{cases} \tag{2.17}$$

Teorema. D sohada

1) $\varphi_1(x,y)$, $\varphi_2(x,y)$ funksiyalar aniqlangan va uzluksiz xususiy hosilalarga ega;

2) boshlang'ich (x_0, y_0) nuqta D sohaga tegishli;

3) D sohada $\left|\frac{\partial\varphi_1}{\partial x}\right| + \left|\frac{\partial\varphi 2}{\partial x}\right| \le q_1 < 1$, $\left|\frac{\partial\varphi_1}{\partial y}\right| + \left|\frac{\partial\varphi_2}{\partial y}\right| \le q_2 < 1$

tengsizliklar o'rinli bo'lsa, u holda

$x_n = \varphi_1(x_{n-1}, y_{n-1})$

$y_n = \varphi_2(x_{n-1}, y_{n-1})$, $(n=1,2,3, \ldots)$ (2.18)

formulalar yordamida tuzilgan $\{(x_n, y_n)\}$ nuqtalar ketma–ketligining barcha hadlari D sohada yotadi va u (2.17) sistema-ning yechimi bo'lgan (ξ,η) nuqtaga yaqinlashadi.

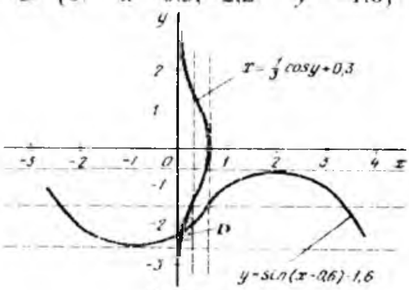
2.7–masala. Chiziqsiz tenglamalar sistemasi yechimini

$$\begin{cases} \sin(x - 0.6) - y = 1.6, \\ 3x - \cos y = 0.9. \end{cases} \tag{2.19}$$

$=0.01$ aniqlikda ketma–ket yaqinlashish (iteratsiya) usulida topamiz.

Yechish. 1) Sistema funksiyalarining grafiklarining bitta kesishgan nuqtasi (2.11–rasm) bo'lib, bu sistema yechimini o'z ichiga olgan sohani quyidagicha tanlaymiz:

$$D=\{0 \quad x \quad 0.3, -2.2 \quad y \quad -1.8\}$$



2.11–rasm.

Berilgan (2.19) sistemaga iteratsiya usulini qo'llash qulay bo'lishi uchun, un quyidagich ko'rinishga keltiramiz:

$$\begin{cases} x = \varphi_1(x,y) = \frac{1}{3}\cos y + 0.3, \\ y = \varphi_2(x,y) = \sin(x-0.6) - 1.6. \end{cases}$$

funksiyalar uchun teoremaning yaqinlashish shartlarini tekshiramiz:

$$\frac{\partial\varphi_1}{\partial x} = 0, \quad \frac{\partial\varphi_1}{\partial y} = -\sin(y)/3, \quad \frac{\partial\varphi_2}{\partial x} = \cos(x-0.6), \quad \frac{\partial\varphi_2}{\partial y} = 0.$$

D sohada

$$\left|\frac{\partial\varphi_1}{\partial x}\right| + \left|\frac{\partial\varphi_2}{\partial x}\right| = |\cos(x-0.6)| \le \cos(0.3) = 0.2935 < 1,$$

$$\left|\frac{\partial\varphi_1}{\partial y}\right| + \left|\frac{\partial\varphi_2}{\partial y}\right| = \left|-\frac{1}{3}\sin(y)\right| \le \left|\frac{1}{3}\sin(-1.8)\right| < \frac{1}{3} < 1,$$

yaqinlashish shartlarini bajarilishini ko'ramiz.

Demak, boshlang'ich qiymatlarni $x_0=0.15$, $y_0=-2$ deb qabul qilib,

$$\begin{cases} x_n = \varphi_1(x_{n-1}, y_{n-1}) = \cos(y_{n-1})/3 + 0.3, \\ y_n = \varphi_2(x_{n-1}, y_{n-1}) = \sin(x_{n-1} - 0.6) - 1.6, \; n = 1,2,3,\ldots \end{cases}$$

ketma–ketlik bilan yechimga yaqinlashish qiymatlarini topish mumkin.

$x_0=0.15$, $y_0=-2$

$x_1=0.1616$, $y_1=-2.035$

$x_2=0.1508$, $y_2=-2.0245$

$x_3=0.1538$, $y_3=-2.0342$

$$|x_3 - x_2| = 0.003 < \varepsilon; \quad |y_3 - y_2| = 0{,}0097 < \varepsilon$$

Demak, $=0.01$ aniqlik bilan taqribiy yechim deb quyidagilarni olamiz:

$$x \approx 0.15, y \approx -2.03.$$

Maple dasturida 2.7–masalani yechish va tenglamalar sistemasidagi funksiyalarning grafigini qurish.

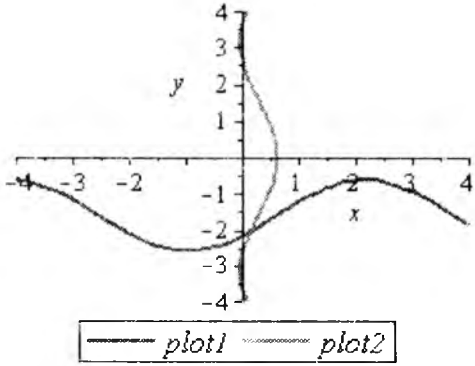
2.5.3–M a p l e d a s t u r i:

```
> with(plots):
> solve({sin(x-0.6)-y=1.6,-cos(y)+3*x=0.9},[x,y]);
```

$$[\,[x = 0.1510571926\;,\; y = -2.034013345\;]\,]$$

```
> implicitplot([sin(x-0.6)-y=1.6,-cos(y)+3*x=0.9],
x=-4..4,y=-4..4,color=[blue,red], thickness=2, legend=[plot1,plot2]);
```

(2.11a–rasm)



2.11a–rasm.

O'z–o'zini tekshirish uchun savollar

1. Chiziqsiz tenglamalar sistemasini Nyuton usulida yechishda xatolik.
2. Chiziqsiz tenglamalar sistemasini Nyuton usulida yechishda yaqinlashish sharti.
3. Chiziqsiz tenglamalar sistemasida iteratsiya qurish.
4. Nyuton usulini chiziqli sistema bo'lgan hol uchun qo'llash mumkinmi?

2.2–laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi chiziqsiz tenglamalar sistemasining

1. Ildizlarining qisqa atrofini – grafik usulda aniqlang.
2. Aniqlangan kesmada yechimni Nyuton usuli yordamida hisoblang.

1. $\begin{cases} 0.6x^2 + 2y^2 = 1, \\ x^2 - 0.8y = 0. \end{cases}$ 2. $\begin{cases} x^2 + y^2 = 1, \\ y^2 - 0.5x = 0. \end{cases}$ 3. $\begin{cases} x^2 + 2y^2 = 1, \\ 0.6x^2 + y = 0. \end{cases}$

4. $\begin{cases} 0.7x^2 + 2y^2 = 1, \\ x^2 + y = 0. \end{cases}$ 5. $\begin{cases} x^2 + y^2 = 2 \\ y - \ln x = 0 \end{cases}$ 6. $\begin{cases} 0.8x^2 + 2y^2 = 1 \\ tgy = x^2 \end{cases}$

7. $\begin{cases} 0.9x^2 + 2y^2 = 1 \\ y^2 - x^2 = 1 \end{cases}$ 8. $\begin{cases} x^2 + y^2 = 1 \\ 2y + 0.5x^2 = 0 \end{cases}$ 9. $\begin{cases} x^2 + 0.5y^2 = 1 \\ y = 2^x \end{cases}$

10. $\begin{cases} x^2 + y^2 = 3 \\ xy = 0.8 \end{cases}$ 11. $\begin{cases} x^2 + 0.2y^2 = 3 \\ y^2 = x^3 \end{cases}$ 12. $\begin{cases} x^2 + y^2 = 1 \\ x^2 - 0.8y^2 = 1 \end{cases}$

13. $\begin{cases} x^2 - y^2 = 1 \\ x^2 + 3y^2 = 6 \end{cases}$ 14. $\begin{cases} 6x^2 + 0.3y^2 = 8 \\ 3x + y^2 = 3 \end{cases}$ 15. $\begin{cases} x^2 + y^2 = 1 \\ 0.5x^2 + 2y^2 = 1 \end{cases}$

16. $\begin{cases} x^2 + 3y^2 = 6 \\ y = 3^x \end{cases}$ 17. $\begin{cases} x^2 + y^2 = 3 \\ 0.5x^2 + 2y^2 = 1 \end{cases}$ 18. $\begin{cases} 0.5x^2 + 3y^2 = 3 \\ y = 0.3^x \end{cases}$

19. $\begin{cases} x^2 - y^2 = 1 \\ 0.8x^2 + 2y^2 = 1 \end{cases}$ 20. $\begin{cases} 2x^2 + 3y^2 = 1 \\ y = 5^x \end{cases}$ 21. $\begin{cases} 2x^2 + y^2 = 1 \\ y = 2^x \end{cases}$

22. $\begin{cases} x^2 + y^2 = 1,\ x > 0,\ y > 0 \\ \sin(x + y) - 1.6x = 0 \end{cases}$ 23. $\begin{cases} x^2 + y^2 = 1, \\ \cos(x + y) - 1.2x = 0.2 \end{cases}$

24. $\begin{cases} x^2 + 2y^2 = 1, \\ tg(xy + 0.1) = x^2 \end{cases}$

25. $\begin{cases} 0,9x^2 + 2y^2 = 1, \\ tgxy = x^2 \end{cases}$ 27. $\begin{cases} 0,9x^2 + 2y^2 = 3, \\ \sin(xy) = x^2 \end{cases}$

28. $\begin{cases} 0.9x^2 + 2y^2 = 2, \\ \cos(xy) = x^2. \end{cases}$

3–LABORATORIYA ISHI

Interpolyatsiyalash formulalari

Maple dasturining buyruqlari:

with(CurveFitting)– *egrichiziqlarni moslashtirish amallarini chaqirish;*

PolynomialInterpolation([2,3,4,5],[0.6,1.09,1.3,1.6], x,form=Lagrange)– *jadvallarga mos Lagranj interpolyatsiya ko'phadini topish.*

PolynomialInterpolation([2,3,4,5],[0.6,1.09,1.3,1.6], x,form=Newton)– *jadvallarga mos Nyuton interpolyatsiya ko'phadini topish.*

Maqsad: Tajriba natijalarida topilgan qiymatlarning o'zgaruvchilari orasidagi bog'lanishni Lagranj interpolyatsiya ko'phadi yordamida topishni o'rganish.

Reja:

3.1. Interpolyatsiya masalasini qo'yilishi.

3.2. Lagranjning interpolyatsiya ko'phadini topish.

3.3. Nyuton interpolyatsiya ko'phadini topish.

3.1. Interpolyatsiya masalasini qo'yilishi

Agar $y=f(x)$ funksiya $[a,b]$ kesmaning x_k, $k=0,1,2,.., n$ nuqtalarda $f(x_k)=y_k$ qiymatlarga ega bo'lsa, quyidagi jadvalni tuzish mumkin:

x	x_0	x_1	x_2	...	x_n
y	y_0	y_1	y_2	...	y_n

Bu jadvalni asosida berigan funksiyani ko'phadini quyidagi ko'rinishda

$$P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n \quad (3.1)$$

topish uchun quyidagicha shart qo'yamiz: jadvalning har bir x_k, $(k=0,1,2,...,n)$ nuqtasida

$$P_n(x_k) \approx f(x_k)=y_k \quad (3.2)$$

munosabat urinla bo'lsin. Bunday masala *interpolyatsiyalash* deyiladi.

Topilgan ko'phadini *interpolyatsiya* ko'phadi deyiladi. Topilgan interpolyatsiya ko'phadi asosida biror $[x_k, x_{k+1}]$ oraliqqa tegishli x ning taqribiy qiymatini topish masalasini ham yechamiz.

Ikkinchi tartibli

$$P_2(x)=a_0x^2+ a_1x+ a_2 \quad (3.3)$$

bu ko'phadining koeffitsentlarini

$$P_2(x_i)=y_i, \ i=0,1,2 \quad (3.4)$$

shart saosida topish masalasini qo'yamiz.

Haqiqatan ham $x=x_0$, $x=x_1$, $x=x_2$ larda (3.4) shart va (3.3) ko'phad asosida quyidagi sistemani tuzamiz:

$$\begin{cases} a_0x_0^2 + a_1x_0 + a_2 = y_0 \\ a_0x_1^2 + a_1x_1 + a_2 = y_1 \\ a_0x_2^2 + a_1x_2 + a_2 = y_2 \end{cases}$$

Bu sistemadagi koeffitsentlari dan tuzilgan determinant

$$\Delta = \begin{vmatrix} x_0^2 & x_0 & 1 \\ x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \end{vmatrix} = (x_1 - x_0)(x_2 - x_0)(x_3 - x_0) \neq 0$$

bo'lganda a_0, a_1, a_2 noma'lumlarni topish mumkin. Lekin (3.1) yuqori tartibli ko'phadilarni topishda tuziladigan sistemalarni yechish qiyinlashadi. Bu masalani yechish uchun jadval asosida ko'phadni topishda Lagranj ko'phadidan foydalanamiz.

3.2. Lagranjning interpolyatsiya ko'phadini topish

Yuqoridagi jadval asosida topiladigan ko'phadini quyidagicha tanlaymiz:

$$\begin{aligned} P_n(x) = {} & a_0(x-x_1)(x-x_2)(x-x_3)...(x-x_n) + \\ & + a_1(x-x_0)(x-x_2)(x-x_3)...(x-x_n) + \\ & + ... + a_n(x-x_1)(x-x_2)(x-x_3)...(x-x_{n-1}) \end{aligned} \quad (3.6)$$

bunda $n=2$ uchun ikkinchi darajali ko'phadini topamiz:

$$P_2(x) = a_0(x-x_1)(x-x_2) + a_1(x-x_0)(x-x_2) + a_2(x-x_0)(x-x_1) \quad (3.7)$$

Bu a_0, a_1, a_2 koeffitsentlarini topish uchun (3.4) shartga asosan:

$$P_2(x_0)=y_0, \; P_2(x_1)=y_1, \; P_2(x_2)=y_2$$

Bo 'lganda, x_0, x_1, x_2 larni (3.7) ga ketma-ket qo 'yib quyidagi sistemani topamiz:

$$a_0(x_0-x_1)(x_0-x_2)=y_0$$
$$a_1(x_1-x_0)(x_1-x_2)=y_1$$
$$a_2(x_2-x_0)(x_2-x_1)=y_2$$

hundan:
$$a_0 = y_0/(x_0-x_1)(x_0-x_2),$$
$$a_1 = y_1/(x_1-x_0)(x_1-x_2),$$
$$a_2 = y_2/(x_2-x_0)(x_2-x_1).$$

Endi bu topilgan a_0, a_1, a_2 larni (3,7) ga qo'yib izlanayotgan Lagranjning 2–darajali interpolyatsiya ko'phadini yozamiz:

$$P_2(x) = y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

Shuningdek $n=3$ bo'lganda:

$$P_3(x)=y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}+y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}+$$
$$+y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}+y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}.$$

Bu ko'phadlardan ko'ramizki ko'phadning darajasi jadvalda berilgan qiymatlar sonidan bitta kam bo'lar ekan.

Demak, Lagranj interpolyatsiya ko'phadini umumiy holda quyidagicha yozamiz:

$$P_n(x)=\sum_{j=0}^{n} y_j\prod_{i\neq j}\frac{(x-x_i)}{(x_j-x_i)}. \qquad (3.8)$$

Lagranj interpolyatsiya ko'phadi yordamida $y=f(x)$ funksiyaning qiymatini $[a\ b]$ kesmada quyidagicha baholanadi:

$$|R_n(x)|\leq\frac{f^{(n+1)}(\xi)}{(n+1)!}|(x-x_0)(x-x_1)\cdots(x-x_n)|,\quad a<\xi<b \qquad (3.9)$$

3.1–masala. Quyidagi, $y=ln\,x$ funksiya asosida tuzilgan

x	2	3	4	5
y	0.6931	1.0986	1.3863	1.6094

Jadvaldan foydalanib Lagranj interpolyatsiya ko'phadini toping va bu ko'phadilar yordamida ln 3.5 ni hisoblang.

Yechish.

$$L_3(x)=\frac{(x-3)(x-4)(x-5)}{(2-3)(2-4)(1-5)}0.6981+\frac{(x-2)(x-4)(x-5)}{(3-2)(3-4)(3-5)}1.0986+$$
$$+\frac{(x-2)(x-3)(x-5)}{(4-2)(4-3)(4-5)}1.3865+\frac{(x-2)(x-3)(x-4)}{(5-2)(5-3)(5-4)}1.6094=$$
$$=0.0089\,x^3-0.1387\,x^2+0.9305\,x-0.6841$$

Hosil bo'lgan ko'phadga asosan

$$\ln 3{,}5\approx L(3{,}5)=0.0089\cdot(3.5)^3-0.1387\cdot(3{,}5)^2+0.9805\cdot(3.5)-0.684=$$
$$=0.31-1.701+3.2567-0.6841=1.25145$$

bo'ladi.

Topilgan interpolyatsiya polinomining qiymatini baholaymiz.

Polinom darajasi $n=3$ bo'lganligi uchun (3.9) formulaga asosan:

$$f^{(IV)}(x)=-\frac{6}{x^4},\quad f^{(IV)}(3.5)=-\frac{6}{(3.5)^4}=-0.03998334028$$

$$|R_3(3.5)| \le \left| \frac{f^{(IV)}(3.5)}{4}(3.5-2)(3.5-3)(3.5-4)(3.5-5) \right| =$$

$$= \left| -\frac{6}{(3.5)^4\, 4!} \cdot 0.5625 \right| = 0.005512409046$$

Haqiqtan ham hatolik 0.005512409046 dan katta bo'lmaydi:

$ln(3.5) - L(3,5) = 1.252762968 - 1.251450000 = 0.004312968$

Lagranj interpolyatsiya ko'phadini aniqlash va grafigini qurish hamda uining $x=3.5$ bo'lgandagi qiymatni hisoblashning Maple dasturini tuzamiz.

3.1 – Maple dasturi

Jadvalga asosan ko'phadni topish:

1)> with(CurveFitting):

> PolynomialInterpolation([2,3,4,5], [0.6931,1.0986.1.3865,1.6094], x, form=Lagrange);

$-0.1155166667\,(x-3)\,(x-4)\,(x-5) + 0.5493000000\,(x-2)\,(x-4)\,(x-5) - 0.6932500000\,(x-2)\,(x-3)\,(x-5) + 0.2682333333\,(x-2)\,(x-3)\,(x-4)$

> evalf(%,3);

$-0.116\,(x-3.)\,(x-4.)\,(x-5.) + 0.549\,(x-2.)\,(x-4.)\,(x-5.) - 0.693\,(x-2.)\,(x-3.)\,(x-5.) + 0.268\,(x-2.)\,(x-3.)\,(x-4.)$

2)> with(CurveFitting):

> PolynomialInterpolation([[2,0.6931],[3,1.0986], [4,1.3865],[5,1.6094]],x);

$0.008766666667\,x^3 - 0.1377000000\,x^2 + 0.9274333334\,x - 0.6811000000$

> p:=evalf(%,3); $p := 0.00877x^3 - 0.138x^2 + 0.927x - 0.681$

> x:=3.4:p:=p; $p := 1.22021608$

To'g'ridan–to'g'ri jadvalga asosan ko'phadning $x=3.5$ dagi qiymatini hisoblash:

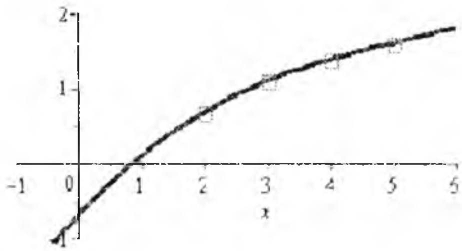
> p:=PolynomialInterpolation([2,3,4,5],[0.6971, 1.0986,1.3863,1.6094],3.5,form=Lagrange);

$$p := 1.253600000$$

Jadvalga asosan topilgan ko'phadni grafigini qurish

> with(stats):with(plots):

> plot([p,[[2,0.6931],[3,1.0986],[4,1.3863], [5,1.6094]]], x=-1..6,-1..2,style=[line,point], color=[blue,red],symbol=BOX, symbolsize=30, thickness=3);



3.1–rasm.

3.3. Nyuton interpolyatsiya ko'phadini topish

Maqsad: Tajriba natijalarida topilgan qiymatlarning o'zgaruvchilari orasidagi bog'lanishni Nyuton interpolyatsiya ko'phadi yordamida topishni o'rganish.

Reja:

3.2.1. Chekli ayirmalar masalasini qo'yilishi.

3.2.2. Nyuton interpolyatsiya ko'phadini topish.

3.3.1. Chekli ayirmalar masalasini qo'yilishi

Berilgan jadvaldagi x_i, $i=0,1,2,...,n$ nuqtalar bir xil h uzoqlikda bo'lsa, ularga mos $y_i=f(x_i)$ $i=0,1,2,...,n$ lar asosida quyidagi ayirmalarni tuzamiz:

$y_1-y_0=f(x_1)-f(x_0)$

$y_2-y_1=f(x_2)-f(x_1)$

....

$y_n-y_{n-1}=f(x_n)-f(x_{n-1})$

Bu ayirmalarni *birinchi tartibli chekli ayirmalar* deb ataladi. Ikkinch, uchinchi va undan yuqori tartibli chekli ayirmalarni quyidagich topamiz:

1– tartibli2– tartibli

$\Delta y_0=y_1-y_0 \Delta^2 y_0=\Delta y_1-\Delta y_0$

$\Delta y_2=y_2-y_1 \Delta^2 y_1=\Delta y_2-\Delta y_1$

... ...

$\Delta y_k=y_{k+1}-y_k \quad \Delta^2 y_k=\Delta y_{k+1}-\Delta y_k$

... ...

$\Delta y_{n-1}=y_n-y_{n-1} \Delta^2 y_{n-1}=\Delta y_n-\Delta y_{n-1}$

3 – tartibli: $\Delta^3 y_0=\Delta^2 y_1-\Delta^2 y_0$, $\Delta^3 y_1=\Delta^2 y_2-\Delta^2 y_1$, ...

p – tartibli:$\Delta^p y_k=\Delta^{p-1} y_{k+1}-\Delta^p y_k$, $k=1,2,...,n$

Bu topilgan ayirmalarni quyidagi jadvalga joylashtiramiz:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$

x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
$x_1=x_0+h$	y_1	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4 y_1$	
$x_2=x_0+2h$	y_2	Δy_2	$\Delta^2 y_2$	$\Delta^3 y_2$		
$x_3=x_0+3h$	y_3	Δy_3	$\Delta^2 y_3$			
$x_4=x_0+4h$	y_4	Δy_4				
$x_5=x_0+5h$	y_5					
... ...	...					

3.3.2. Nyuton interpolyatsiyalash formulasi

1. Berilgan jadvalda mos $y_i=f(x_i)$, $i=0,1,2,\ldots,n$ larga mos x_i, $i=0,1,2,\ldots,n$ nuqtalar bir xil h uzoqlikda bo'lganda, bu qiymatlar bog'lanishini ifodalovchi interpolyatsiya ko'phadini quyidagicha topamiz.

Bu ko'phadini quyidagi ko'rinishda izlaymiz:

$$P_n(x)=A_0+A_1(x-x_0)+A_2(x-x_0)(x-x_1)+A_3(x-x_0)(x-x_1)(x-x_2)+ \quad (13.1)$$
$$+\ldots+A_n(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1})$$

bu yerdagi A_i, $i=1,2,..n$ koeffitsentlarni topish uchun jadvaldagi mos x va y larning qiymatlarini izlanayotgan ko'phadiga qo'yamiz.

$x=x_0$ da: $y_0=A_0$;

$A_0=y_0$

$x=x_1$ da: $y_1=A_0+A_1(x_1-x_0)=A_0+A_1h=y_0+A_1h$,

$y_1=y_0+A_1h$; $A_1=(y_1-y_0)/h$,

$$A_1=\frac{y_1-y_0}{1!h}=\frac{\Delta y_0}{1!h}$$

$x=x_2$ da: $y_2=A_0+A_1(x_2-x_0)+A_2(x_2-x_0)(x_2-x_1)$

$y_2=A_0+A_1 2h+A_2\, 2h\, h$

A_0 va A_1 larning qiymatlarini hisobga olib,

$$y_2=y_0+\Delta y\, 2h/h+A_2\, 2h^2\, ,$$
$$A_2\, 2h^2=y_2-y_0-2\Delta y=\Delta y_1-\Delta y_0=\Delta^2 y_0\, ,$$
$$A_2=\frac{\Delta^2 y_0}{2!h^2}$$

Demak, ketma-ket koeffitsentlarni topish formulasi:

$$A_0=y_0\, ,\quad A_1=\frac{\Delta y_0}{1!h}\quad ,$$

$$A_2=\frac{\Delta^2 y_0}{2!h^2}\, ,\ A_3=\frac{\Delta^3 y_0}{3!h^3}\ \ldots,\ A_k=\frac{\Delta^k y_0}{k!h^k}\ \ldots$$

Topilgan koeffitsentlar asosida izlanayotgan interpolyatsiya ko'phadini quyidagicha topamiz:

$$P_n(x) = y_0 + \frac{\Delta y_0}{1!h}(x-x_0) + \frac{\Delta^2 y_0}{2!h^2}(x-x_0)(x-x_1) + \frac{\Delta^3 y_0}{3!h^3}(x-x_0)(x-x_1)(x-x_2) + \dots \quad (13.2)$$

Bu Nyutonning *birinchi interpolyatsiya ko'phadi* deyiladi.

2. Nyutonning birinchi interpolyatsiya ko'phadida quyidagicha almashtirish qilamiz:

$$\frac{x-x_0}{h} = t,$$

$$\frac{x-x_1}{h} = \frac{x-(x_0+h)}{h} = \frac{x-x_0}{h} - 1 = t-1,$$

$$\frac{x-x_2}{h} = \frac{x-(x_0+2h)}{h} = \frac{x-x_0}{h} - 2 = t-2$$

va hakazo

$$\frac{x-x_k}{h} == t-k$$

Bu almashtirishlarni hisobga olib (13.2) formulani quyidagicha yozamiz:

$$P_n(x) = P_n(x_0+ht) = y_0 + \frac{\Delta y_0}{1!}t + \frac{\Delta^2 y_0}{2!}t(t-1) + \dots + \frac{\Delta^n y_0}{n!}t(t-1)(t-2)\dots[t-(n-1)] \quad (13.3)$$

Bu Nyutoning 2– *interpolyatsiya ko'phadi* deyiladi.

3.2–masala. Quyidagi, $y=\ln x$ funksiya asosida tuzilgan

x	2	3	4	5
y	0.6931	1.0986	1.3863	1.6094

jadvaldan foydalanib Nyuton interpolyatsiya ko'phadilarini toping va bu ko'phadilar yordamida ln 3.5 ni hisoblang.

Nyutonning interpolyatsiya ko'phadini tuzish uchun chekli ayrimalarning jadvalini tuzamiz:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
2	0.6931	0.1055	–0.1178	0.053 2
3	1.0986	0.2877	–0.0646	
4	1.3863	0.2231		
5	1.6094			

(13.2) formulaga asosan , n=3, h=1 bo'lgada:

$$P_3(x) = y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{2!h^2}(x-x_0)(x-x_1) + \frac{\Delta^3 y_0}{3!h^3}(x-x_0)(x-x_1)(x-x_2) =$$

$$= 0.6941 + 0.4055(x-2) - \frac{0.1178}{2}(x-2)(x-3) + \frac{0.0532}{6}(x-2)(x-3)(x-4) =$$

$$= -0.6841 - 0.930\,x - \frac{0.1178}{2}(x-2)(x-3) + \frac{0.0532}{6}(x-2)(x-3)(x-4) =$$

$$= -0.6841 - 0.930\,x - 0.1387x^2 + 0.0089x^3$$

Bu ko'phadidan foydalanib $\ln 3{,}5 \approx P_3(3.5)=1.2552$ ekanligini hisoblab topamiz.

Nyuton interpolyatsiya ko'phadini aniqlash va grafigini qurish hamda uining x=3.5 bo'lgandagi qiymati hisoblashning Maple dasturini tuzamiz.

3.2 – Maple dasturi

Nyuton interpolyatsiya ko'phadini topish:

> **restart; with(CurveFitting):**

> **PolynomialInterpolation([2,3,4,5], [0.6931,1.0986,1.3865,1.6094],x,form=Newton);**

$((0.008766666667\,x - 0.09386666667)\,(x-3) + 0.4055)\,(x-2) + 0.6931$

> **p:=evalf(%,3);**

$p := ((0.00877x - 0.0939)\,(x - 3.) + 0.406)\,(x - 2.) + 0.693$

> **p:=simplify(p);**

$p := 0.008770000000\,x^3 - 0.1377500000\,x^2 + 0.9281200000\,x - 0.6824000000$

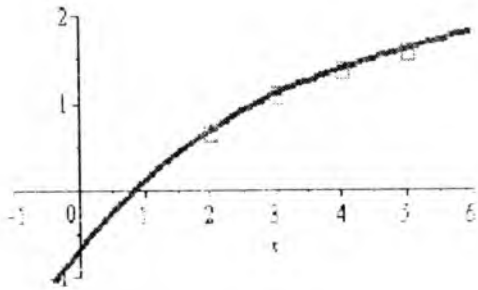
> **p:=evalf(%,4);** $p := 0.008770x^3 - 0.1378x^2 + 0.9281\,x - 0.6824$

> **#x:=3.5:P[3.5]:=p;** $P_{3.5} := 1.25391375$

Nyuton interpolyatsiya ko'phadining grafigini qurish:

> **with(stats):with(plots):**

> **plot([p,[[2,0.6931],[3,1.0986],[4,1.3863], [5,1.6094]]], x=–1..6,–1..2,style=[line,point],color =[blue,red], thickness=3,symbol=BOX,symbolsize=30);**



3.2–rasm.

O'z–o'zini tekshirish uchun savollar

1. Interpolyatsiya masalasini kuyilish moxiyatini tushintiring.

2. Lagranj interpolyatsiyalash ko'phadini tanlash qoidasi va uning ahamiyati.

3. Qanday xollarda Lagranj interpolyatsiyalash ko'phadini qo'llash mumkin.

4. Ikkinchi va uchunchi tartibli Lagranj ko'phadini yozing.

5. Chekli ayirmalar.

6. Nyuton interpolyatsiyalash ko'phadini tanlash qoidasi va uning ahamiyati.

7. Chekli ayirmalar asosida Nyuton interpolyatsiyalash ko'phadining koeffitsentlarini topish.

8. Ikkinchi va uchunchi tartibii Nyuton ko'phadini yozing.

9. Lagranj va Nyuton interpolyatsiyalash ko'phadini tanlash qoidalarining farqi

10. Sonli differentsiyalashda Nyuton interpolyatsiyalash formulasidan foydalanish.

3-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi jadval uchun:

1) Lagranj interpolyatsiya ko'phadini toping(1–jadval bo'yicha):

2) Nyuton interpolyatsiya ko'phadini toping(2–jadval bo'yicha).

Jadvalda berilgan (x_i, y_i) nuqtalar yordamida x qiymatlari teng uzoqlikda bo'lmagan 1–jadval uchun Lagranj, x qiymatlari teng uzoqlikda bo'lgan 2–jadval uchun Nyuton interpolyatsion ko'phadini tuzing.

Variant 1

1-jadval	**X**	0,43	0,48	0,55	0,62	0,70	0,75
	Y	1.63597	1,7323	1.8768	2.0334	2,2284	2.35973
2-jadval	**X**	1	7	13	19	25	
	Y	0,702	0.512	0,645	0.736	0,608	

Variant 2

Jadval 1	**X**	0,02	0,08	0,12	0.17	0,23	0,30
	Y	1,0231	1,0959	1.14725	1,2148	1.3012	1,4097
Jadval 2	**X**	2	8	14	20	26	
	Y	0,102	0,114	0,125	0,203	0,154	

Variant 3

Jadval 1	**X**	0.35	0.41	0,47	0,51	0,56	0,64
	Y	2,739	2,300	1,968	1,787	1,595	1,345
Jadval 2	**X**	3	9	15	21	27	
	Y	0,526	0,453	0,482	0.552	0,436	

Variant 4

Jadval 1 X	0,41	0,46	0,52	0,60	0,65	0,72
Y	2,574	2,325	2,093	1,862	1,749	1,620
Jadval 2 X	4	10	16	22	28	
Y	0,616	0, 478	0,665	0,537	0,673	

Variant 5

Jadval 1 X	0,68	0,73	0,80	0,88	0,93	0,99
Y	0.808	0,894	1,029	1.209	1,340	1,523
Jadval 2 X	5	11	17	23	29	
Y	0,896	0.812	0,774	0,955	0,715	

Variant 6

Jadval 1 X	0.11	0.15	0,21	0,29	0,35	0,40
Y	9.054	6,616	4,691	3.351	2,739	2,365
Jadval 2 X	6	12	18	24	30	
Y	0,314	0,235	0,332	0,275	0,186	

Variant 7

Jadval 1 X	1,375	1,380	1,385	1,390	1,395	1,400
Y	5,041	5,177	5,320	5,470	5,629	5,797
Jadval 2 X	1	7	13	19	25	

Y	1,3832	1,3926	1,3862	1,3934	1,3866

Variant 8

Jadval 1 X	0,115	0,120	0,125	0,130	0,135	0,140
Y	8,657	8,293	7,958	7.648	7,362	7,096
Jadval 2 X	2	8	14	20	16	
Y	0,1264	0,1315	0,1232	0,1334	0,1285	

Variant 9

Jadval 1 X	0,150	0,155	0,160	0,165	0,170	0,175
Y	6.616	6,399	6,196	6,005	5,825	5,655
Jadval 2 X	3	9	15	21	27	
Y	0,1521	0,1611	0,1662	0,1542	0,1625	

Variant 10

Jadval 1 X	0.180	0,185	0.190	0,195	0,200	0,205
Y	5,615	5,466	5,326	5,193	5,066	4,946
Jadval 2 X	4	10	16	22	28	
Y	0,183 8	0,187 5	0.194 4	0.197 6	0,203 8	

Variant 11

Jadval 1 X	0,210	0,215	0,220	0,225	0,230	0,235
Y	4.831	4.722	4,618	4,519	4,424	4,333
Jadval 2 X	5	11	17	23	29	
Y	0,2121	0,2165	0,2232	0,2263	0,2244	

Variant 12

Jadval 1 X	1,415	1,420	1,425	0,430	0,435	0,440
Y	0,888	0,889	0,890	0.891	0,892	0,893
Jadval 2 X	6	12	18	24	30	
Y	1,4179	1,4258	1,4396	1,4236	1,4315	

Variant 13

Jadval 1 X	0,33	0,38	0,45	0,52	0,60	0,65
Y	1.63597	1,73234	1.87686	2,03345	2,22846	2.35973
Jadval 2 X	1	5	9	14	18	
Y	0.702	0,512	0,645	0,736	0.608	

Variant 14

Jadval 1 X	0,03	0,09	0,13	0,18	0,24	0,31
Y	1,02316	1,0959	1,14725	1,21483	1,3012	1,4097
Jadval 2 X	2	6	10	14	18	
Y	0,102	0,114	0,125	0,203	0,154	

Variant 15

Jadval 1 X	0,25	0,31	0,37	0,41	0,46	0,54
Y	2,739	2,300	1,968	1,787	1,595	1,345
Jadval 2 X	3	6	9	12	15	
Y	0,526	0,453	0,482	0,552	0,436	

Variant 16

Jadval 1 X	0,21	0,26	0,32	0,40	0,45	0,52
Y	2,574	2,325	2,093	1,862	1,749	1,62
Jadval 2 X	4	7	10	13	16	
Y	0,616	0, 478	0,665	0,537	0,673	

Variant 17

Jadval 1 X	0,38	0,43	0,50	0.58	0,63	0,69
Y	0,808	0,894	1,029	1,209	1,340	1,523
Jadval 2 X	5	11	17	23	29	
Y	0.896	0,812	0,774	0.955	0.715	

Variant 18

Jadval 1 X	0,31	0,35	0,41	0,49	0,55	0.60
Y	9,054	6,616	4,691	3,351	2,739	2,365
Jadval 2 X	6	7	8	9	10	
Y	0,314	0,235	0,332	0,275	0,186	

Variant 19

Jadval 1 X	1,175	1,180	1,185	1,190	1,195	1,200
Y	5,041	5,177	5,320	5,470	5,629	5,797
Jadval 2 X	1	6	10	14	18	
Y	1,3832	1,3926	1,3862	1,3934	1,3866	

Variant 20

Jadval 1 X	0,215	0,220	0,225	0,230	0,235	0,240
Y	8,657	8,293	7,958	7,648	7,362	7,096
Jadval 2 X	2	7	12	17	22	
Y	0.1264	0,1315	0,1232	0,1334	0,1285	

Variant 21

Jadval 1 X	0,250	0,255	0,260	0,265	0,270	0,275
Y	6,616	6,399	6,196	6,005	5,825	5,655
Jadval 2 X	3	9	15	21	27	
Y	0.1521	0,1611	0,1662	0,1542	0,1625	

Variant 22

Jadval 1 X	0.280	0,285	0,290	0,295	0,300	0,305
Y	5,615	5,466	5,326	5,193	5.066	4,946
Jadval 2 X	4	10	16	22	28	
Y	0.1838	0,1875	0,1944	0,1976	0,2038	

Variant 23

Jadval 1 X	0,310	0,315	0,320	0,325	0,330	0,335
Y	4,831	4.722	4,618	4,519	4,424	4,333
Jadval 2 X	5	11	17	23	29	
Y	0,2121	0.2165	0,2232	0,2263	0,2244	

Variant 24

Jadval 1 X	1.315	1,320	1,325	0,330	0,335	0,340
Y	0,888	0,889	0,890	0,891	0,892	0,893
Jadval 2 X	6	12	18	24	30	
Y	1,4179	1,4258	1,4396	1,4236	1,4315	

Variant 25

Jadval 1 X	0.315	0,320	0,325	0.330	0,335	0,340
Y	8.657	8.293	7,958	7,648	7,362	7,096
Jadval 2 X	2	4	6	8	10	
Y	0,1264	0.1315	0,1232	0.1334	0.1285	

Variant 26

Jadval 1 X	0,450	0,455	0,460	0,465	0,470	0,475
Y	6,616	6,399	6,196	6,005	5,825	5,655
Jadval 2 X	3	7	11	15	19	
Y	0,1521	0,1611	0,1662	0,1542	0,1625	

Variant 27

Jadval 1 X	0,580	0,585	0,590	0,595	0,600	0,605
Y	5,615	5,466	5,326	5,193	5,066	4,946
Jadval 2 X	4	9	14	19	24	
Y	0,1838	0,1875	0,1944	0,1976	0,2038	

Variant 28

Jadval 1 X	0,410	0,415	0,420	0,425	0,430	0,435
Y	4,831	4,722	4,618	4,519	4,424	4,333
Jadval 2 X	3	10	17	24	31	
Y	0,2121	0,2165	0,2232	0,2263	0,2244	

Variant 29

Jadval 1 X	0,315	0,320	0,325	0,330	0,335	0,340
Y	0,888	0,889	0,890	0,891	0,892	0,893
Jadval 2 X	6	11	16	21	26	
Y	1,4179	1,4258	1,4396	1,4236	1,4315	

Variant 30

Jadval 1 X	2,315	2,320	2,325	2,330	2,335	2,340
Y	0,888	0,889	0,890	0,891	0,892	0,893
Jadval 2 X	3	7	11	15	19	
Y	1,4179	1,4258	1,4396	1,4236	1,4315	

4-LABORATORIYA ISHI

Kichik kvadratlar usuli
Tajriba natijalarining chiziqli va parabolik bog'laninshini aniqlash.
Maple dasturining buyruqlari:

with(stats)– *statistika paketidagi amallarni chaqirish;*

Vector([0.5,1,1.5,2,2.5,3],datatype=float)– *qiymatlarni vektorini aqlash;*

add(X[k],k=1..n)– *qiymatlar yig'indisini topish;*

Fit(a+b*t,X,Y,t)– *qiymatlar asosida ko'rsatilgan tenglamani aniqlash funksiyasi;*

fit[leastsquare[[x,y],y=a*x+b]]([[0.5,1,1.5,2,2.5,3],[6,5,3.7, 2.6,1.6, 0.6]])– *kichik kvadratlar usuli asosida ko'rsatilgan qiymatlar orasidagi chiziqli bog'lanish tenglamani aniqlash funksiyasi;*

fit[leastsquare[[x,y],y=a*x^2+b*x+c]]([[0.5,1,1.5,2, 2.5,3],[6,5,3.7,2.6,1.6,0.6]])]])– *kichik kvadratlar usuli asosida ko'rsatilgan qiymatlar orasidagi parabolik bog'lanish tenglamani aniqlash funksiyasi;*

with(CurveFitting):Interactive([[0.5,6],[1,5],[1.5,3.7],[2,2.6],[2.5,1.6], [3,0.6]],t)– *ko'rsatilgan nuqtalar orasidagi bog'lanishning grafigini Tutor muloqat oynasida qurish.*

Maqsad: Kichik kvadratlar usulida tajriba natijalarida topilgan qiymatlar orasidagi chiziqli va parabolik bog'laninshini aniqlash.

Reja:

4.1. Kichik kvadratlar usuli

4.2. To'g'ri chiziqli bog'laninsh tenglamasini aniqlash.

4.3. Ikkinchi darajali bog'laninsh tenglamasini topish.

4.4. Chiziqsiz bog'laninsh tenglamasini topish.

4.1. Kichik kvadratlar usuli

Aytaylik tajriba natijalari quyidagi jadval asosida berilgan bo'lsin.

x	x_1	x_2	x_3	...	x_n
y	y_1	y_2	y_3	...	y_n

Bu ikki o'zgaruvchilar orasidagi bog'lanish formulasini kichik kvadratlar usuli bilan analinik usulda aniqlash masalasini yechamiz. Buning uchun bog'laninshni ifodalovchi funksiyalar turini tanlaymiz.

Masalan:

1) chiziqli bog'laninsh: $y=ax+b$

2) parabolik bog'laninsh: $y=ax^2+bx+c$

Bu bog'lanishlarni aniqlashda ularning koeffitsentlarini aniqlash asosiy masala hisoblanadi. Umumiylik uchun izlanayotgan funksiyani

$$y=f(x,a,b,c)$$

ko'rinishda izlaymiz. Bu bog'lanishning a,b,c koeffitsentlarini aniqlash uchun berilgan jadval asocida

$$f(x_i,a,b,c)=y_i,\ i=1,2,..,n$$

shartni yozamiz. Bu izlanayotgan funksiya qiymatlari bilan jadvaldagi y_i lar orasidagi farq minimum yoki yetarlicha kichik bo'lish shartini topish uchun quyidagi funksianalni tuzamiz:

$$F(a,b,c)=\sum_{i=1}^{n}\left[y_i-f(x_i,a,b,c)\right]^2,\ i=1,2,...,n$$

Bu ko'p o'zgaruvchili $F(a,b,c)$ funksianing minimumini topish uchun quyidagi zaruriy sharttan foydalanamiz.

$$\begin{cases} F'_a(a,b,c)=0, \\ F'_b(a,b,c)=0, \\ F'_c(a,b,c)=0 \end{cases} \quad (*)$$

ya'ni

$$\begin{cases} \sum\limits_{i=1}^{n}\left[y_i-f(x_i,a,b,c)\right]\cdot f'_a(x_i,a,b,c)=0. \\ \sum\limits_{i=1}^{n}\left[y_i-f(x_i,a,b,c)\right]\cdot f'_b(x_i,a,b,c)=0, \\ \sum\limits_{i=1}^{n}\left[y_i-f(x_i,a,b,c)\right]\cdot f'_c(x_i,a,b,c)=0. \end{cases}$$

Ushbu sistemeni yechish bilan a,b,c larni topamiz va jadvalni ifodalovchi bog'lanish funktsiasini topamiz.

4.2. To'g'ri chiziqi bog'laninsh tenlamasini aniqlash

Chiziqli bog'lanish $f(x_i,a,b)=a\,x_i+b$, uchun $f'_a=x_i$, $f'_b=1$ bo'lganda. (*) zaruriy shatga asosan quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} \sum\limits_{i=1}^{n}\left[y_i-ax_i-b\right]\cdot x_i=0, \\ \sum\limits_{i=1}^{n}\left[y_i-ax_i-b\right]\cdot 1=0. \end{cases}$$

$$\begin{cases}\left(\sum_{i=1}^{n} x_i^2\right)a+\left(\sum_{i=1}^{n} x_i\right)b=\sum_{i=1}^{n} x_i y_i\\ \left(\sum_{i=1}^{n} x_i\right)a+nb=\sum_{i=1}^{n} y_i\end{cases}$$

Bu sistemani a,b larga nisbatan yechamiz:

$$a=\frac{n\sum_{i=1}^{n} x_i y_i-\sum_{i=1}^{n} x_i\sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2-\left(\sum_{i=1}^{n} x_i\right)^2},\quad b=\frac{\sum_{i=1}^{n} x_i^2\cdot\sum_{i=1}^{n} y_i-\sum_{i=1}^{n} x_i\cdot\sum_{i=1}^{n} x_i y_i}{n\sum_{i=1}^{n} x_i^2-\left(\sum_{i=1}^{n} x_i\right)^2}\quad (**)$$

4.1–masala. Tajriba natijasida topilgan quyidagicha o‘lchov natijalarining bog‘lanishini aniqlang.

3.1–jadval

x	0.5	1.0	1.5	2.0	2.5	3.0
y	6.0	5.0	3.7	2.6	1.6	0.6

Masalada berilgan 3.1– jadval asosida yuqoridagi kichik kvadratlar usuli bilan chiziqli bog‘lanishni aniqlash uchun (**) formuladan foydalanamiz:

1)bundagi yig‘indilarni hisoblaymiz: n=6

$$\sum_{i=1}^{6} x_i = 0.5+1+1.5+2+2.5+3=10.5$$

$$\sum_{i=1}^{6} x^2 = 0.5^2+1^2+1.5^2+2^2+2.5^2+3^2=22.75$$

$$\sum_{i=1}^{6} y_i = 6+5+3.7+2.6+1.6+0.6=19.5$$

$$\sum_{i=1}^{6} x_i y_i = 0.5\cdot 6+1\cdot 5+1.5\cdot 3.7+2\cdot 2.6+2.5\cdot 1.6+3\cdot 0.6=24.55$$

2) a va b larni hisoblaymiz:

$$a=\frac{6\cdot 24.55-10.5\cdot 19.5}{6\cdot 22.75-(10.5)^2}=\frac{147.3-204.75}{136.5-110.25}=\frac{-57.45}{26.25}=-2.18857$$

$$b=\frac{19.5\cdot 22.75-10.5\cdot 24.55}{6\cdot 22.75-(10.5)^2}=\frac{443.625-257.775}{136.5-110.25}=\frac{185.85}{26.25}=7.08;$$

3)$y=ax+b$ bog‘lanishni yozamiz:

y=–2.18857 x+ 7.08.

Chiziqli bog ‘lanishni aniqlovchi dasturlarini tuzamiz:

4.1a–M a p l e d a s t u r i:

y =a+bx chiziqli bog'lanishni aniqlash.

1. Bog'lanishni aniqlash.

a) *To'g'ri chiziqi bog'laninshni yuqorida ko 'rsatilgan qoida asosida:*

> restart; with(stats):

> X:=Vector([0.5,1,1.5,2,2.5,3],datatype=float):

> Y:=Vector([6,5,3.7,2.6,1.6,0.6],datatype=float):

> n:=6:

> SX:=add(X[k],k=1..n); *SX* := 10.5000000

> SY:=add(Y[k],k=1..n); *SY* := 19.5000000

> SXX:=add(X[k]^2,k=1..n); *SXX* := 22.7500000

> SXY:=add(X[k].Y[k],k=1..n); *SXY* := 24.5500000

> ab:=solve([a*SX+n*b=SY,a*SXX+b*SX=SXY],{a,b});

$$ab := \{a = -2.188571429, b = 7.080000000\}$$

> y:=ab[1]*x+ab[2];

$$y := x\,a + b = -2.188571429x + 7.080000000$$

b) *To'g'ri chiziqi bog'laninshni* **Fit** *funksiyasi asosida:*

> with(Statistics):

> X := Vector([0.5,1,1.5,2,2.5,3], datatype=float):

Y := Vector([6,5,3.7,2.6,1.6,0.6], datatype=float):

> Fit(a+b*t, X, Y, t);

7.08000000000000274 K 2.18857142857142950 *t*

> evalf(Fit(a+b*t,*X*,*Y*,t),5); 7.0800 − 2.1886*t*

c) *nuqtalardan o'tuvchi chiziqni kichik kvadratlar usulida topish.*

> fit[leastsquare[[x,y]]]([[0.5, 1, 1.5, 2, 2.5,3], [6,5, 3.7, 2.6. 1.6, 0.6]]);

$$y = 7.080000000 - 2.188571429x$$

2. Bog'lanishni grafigini qurish.

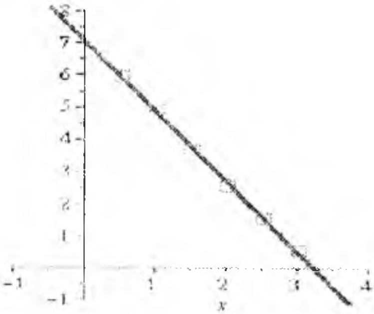
> with(stats):with(plots):

> r2:=rhs(fit[leastsquare[[*x*,*y*],y=a*x+b,{a,b}]]
([[0.5,1.0,1.5,2.0,2.5,3.0],[6.0,5.0,3.7,2.6,1.6, 0.6]]));

$$r2 := -2.188571429x + 7.080000000$$

> with(stats):with(plots):

> plot([r2,[[0.5,6],[1,5],[1.5,3.7],[2,2.6], [2.5,1.6],[3,0.6]]],x=-1..4,-1..8,style=[line,point], thickness=3,red],symbol=BOX,symbolsize=30, color=[blue);

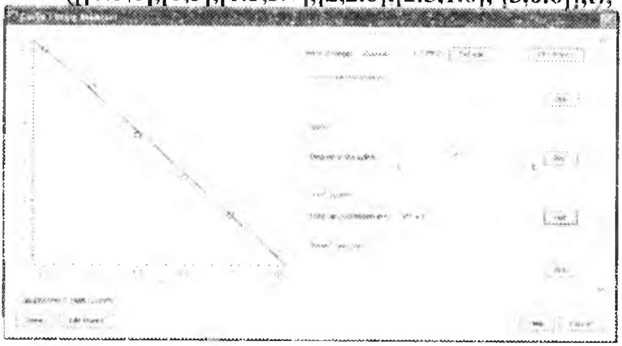


4.1--rasm.

3. Bog'lanishning grafigini Tutor muloqat oynasida qurish(4.2–rasm).

4.1b–M a p l e d a s t u r i:

```
> with(CurveFitting):
Interactive([[0.5,6],[1,5],[1.5,3.7],[2,2.6],[2.5,1.6], [3,0.6]],t);
```



4.2--rasm.

4.3. Ikkinchi darajali(parabolik) bog'lanish tenglamasini topish

Parabolik bog'laninsh $f(x_i,a,b,c)=ax_i^2+bx_i+c$ uchun $f'_a=x_i^2$, $f'_b=x_i$, $f'_c=1$ bo'lganda, (*) zaruriy shartga asosan quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} \sum_{i=1}^{n}[y_i - ax_i^2 - bx_i - c]x_i^2 = 0, \\ \sum_{i=1}^{n}[y_i - ax_i^2 - bx_i - c]x_i = 0, \\ \sum_{i=1}^{n}[y_i - ax_i^2 - bx_i - c]\cdot 1 = 0. \end{cases}$$

Bu sistemani quyidagicha yozamiz

$$\begin{cases} \left(\sum x_i^4\right)a + \left(\sum x_i^3\right)b + \left(\sum x_i^2\right)c = \sum y_i x_i^2 \\ \left(\sum x_i^3\right)a + \left(\sum x_i^2\right)b + \left(\sum x_i\right)c = \sum y_i x_i \\ \left(\sum x_i^2\right)a + \left(\sum x_i\right)b + nc = \sum y_i \end{cases} \qquad (***)$$

va uni biror usul bilan yechib a, b, c larni topamiz.

3.1–jadval asosida $y=ax^2+bx+c$ parabolik bog'lanishni aniqlash uchun (***) formuladagi yig'indilarni hisoblab, quyidagi sistemani topamiz:

$$\begin{cases} 142.187a + 55.125b + 22.75c = 40.625 \\ 55.125a + 22.75b + 10.5c = 24.55 \\ 22.75a + 10.5b + 6c = 19.5 \end{cases}$$

Bu sistemani yechib, $a=0.0857$, $b=-2.288$, $c=7.28$ larni topamiz va parabolik bog'lanishni yozamiz:

$$y=0.0857x^2-2.288x+7.28.$$

$y=ax^2+bx+c$ parabolik bog'lanishni aniqlash dasturini tuzamiz:

1)Bog'lanishni aniqlash.

4.2a–Maple dasturi:

a) *parabolik bog'lanishni yuqorida ko'rsatilgan qoida asosida:*

```
> restart; with(stats):
> X:= Vector([0.5,1.0,1.5,2,2.5,3]):
  Y:= Vector([6,5,3.7,2.6,1.6,0.6]):
> n:=6:
> SX:=add(X[k],k=1..n);  SX := 10.5
> SX2:=add(X[k]^2,k=1..n); SX2 := 22.75
> SX3:=add(X[k]^3,k=1..n); SX3 := 55.125
> SX4:=add(X[k]^4.k=1..n); SX4 := 142.1875
> SY:=add(Y[k],k=1..n);  SY := 19.5
> SYX2:=add(Y[k].X[k]^2,k=1..n); SYX2 := 40.625
> SYX:=add(X[k].Y[k],k=1..n); SYX := 24.55
> abc:=solve([a*SX4 + b*SX3 + c*SX2=SYX2,
    a*SX3 + b*SX2 + c*SX=SYX,
    a*SX2 + b*SX + c*n=SY],{a,b,c});
```

$abc := \{a = 0.08571428571, b = -2.488571429, c = 7.280000000\}$

```
> y:=abc[1]*x^2+abc[2]*x+abc[3];
```

$$y := x^2\,a + x\,b + c = 0.08571428571x^2 - 2.488571429x + 7.280000000$$

b) *To'g'ri chiziqi bog'laninshni* **Fit** *funksiyasi asosida:*

```
> with(Statistics):
> X:=Vector([0.5,1,1.5,2,2.5,3],datatype=float):
Y := Vector([6,5,3.7,2.6,1.6,0.6],datatype=float):
Fit(a+b*t+c*t^2, X, Y, t);
```

$$7.28000000000000380 K\ 2.48857142857143244 t\ C\ 0.0857142857142865894 t^2$$

c) *nuqtalardan o'tuvchi chiziqni kichik kvadratlar usulida topish:*

```
> restart; with(stats):
> fit[leastsquare[[x,y],y=a*x^2+b*x+c]]([[0.5,1,1.5, 2, 2.5,3],
[6,5,3.7,2.6,1.6, 0.6]]);
```

$$y = 0.08571428571 x^2 - 2.488571429 x + 7.28000000($$

2)Bog'lanishni grafigini qurish(4.3 rasm).

4.2b–M a p l e d a s t u r i:

```
> with(stats):with(plots):
> r3:=rhs(fit[leastsquare[[x,y],y=a*x^2+b*x+c]]
([[0.5,1.0,1.5,2.0,2.5,3.0],[6.0,5.0,3.7,2.6,1.6,0.6]]));
```

$$r3 := 0.08571428571 x^2\ K\ 2.488571429 x\ C\ 7.28000000($$

```
> plot([r3,[[0.5,6],[1.5],[1.5,3.7],[2,2.6],
[2.5,1.6],[3,0.6]]],x=0..28,-12..10, thickness=3, style
=[line,point],color=[blue,red], symbol=BOX, symbolsize=30);
```

(4.3–rasm)

4.1–masala bo'yicha bog'lanishlarning yaqinlashishini aniqlash uchun ularning grafiklarini bitta koordinatalar sistemasida quramiz (4.4–rasm).

4.3–M a p l e d a s t u r i:

```
> restart;
> with(stats):with(plots):with(CurveFitting):
> r2:=rhs(fit[leastsquare[[x,y], y=a*x+b,{a,b}]]
([[0.5,1.0,1.5,2.0,2.5.3.0],[6.0,5.0,3.7.2.6,1.6, 0.6]]));
```

$$r2 := -2.188571429 x + 7.08000000($$

```
> r3:=rhs(fit[leastsquare[[x,y], y=a*x^2+b*x+c]](
[[0.5,1.0,1.5,2.0,2.5,3.0],[6.0,5.0,3.7,2.6,1.6, 0.6]]));
```

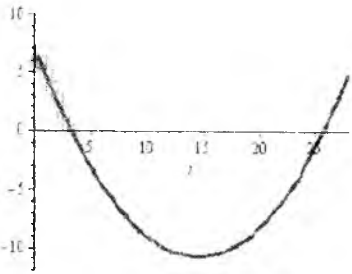
$$r3 := 0.08571428571 x^2 - 2.488571429 x + 7.28000000($$

```
> r4:=rhs(fit[leastsquare[[x,y], y=a*x^3+b*x^2+c*x+d] ]([[0.5, 1.0,
1.5, 2.0,2.5,3.0],[6.0, 5.0, 3.7, 2.6, 1.6, 0.6]])):
```

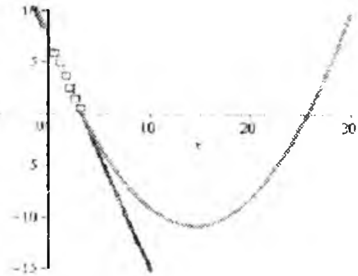
$$r4 := 0.08888888889 x^3 - 0.3809523810 x^2 - 1.784126984 x + 7.$$

> plot([r2,r3,r4,[[0.5, 6],[1,5],[1.5, 3.7],[2,2.6], [2.5, 1.6],[3,0.6]]],x=–6..30,–6..8,style=[line,line, line, point],color=[blue,red,green],thickness=3, symbol=BOX,symbolsize=20, view=[–6..30,–15..10]);

(4.4–rasm).



4.3–rasm.



4.4–rasm.

4.4. Chiziqsiz bog'laninsh tenglamasini topish

Tajriba natijasida topilgan x va y o'zgaruvchilar orasida bog'lanish quyidagi jadval ko'rinishida berilgan bo'lsin.

3.2–jadval

x	1	2	3	4	5	6	7	8
y	12.2	6.8	5.2	4.6	3.9	3.7	3.5	3.2

3.2–jadval uchun quyidagi bog'lanishlarning parametr (koeffitent)larini aniqlovchi formulalarni topamiz.

$y = a + \frac{b}{x}$ giperbolik bog'lanishni a, b parametrlarini kichik kvdratlar usuli asosida aniqlovchi quyidagi sistemani yozamiz:

$$\begin{cases} a\,n + b\sum_{i=1}^{n} \frac{1}{x_i} = \sum_{i=1}^{n} y_i \\ a\sum_{i=1}^{n} \frac{1}{x_i} + b\sum_{i=1}^{n} \frac{1}{x_i^2} = \sum_{i=1}^{n} \frac{y_i}{x_i} \end{cases}$$

Giperbolik bog'lanishni a,b parametrlarini aniqlash va bog'lanishning grafigini qurishning Maple dasturini tuzamiz (3.2–jadval uchun).

4.4–M a p l e d a s t u r i:

1) *Bog'lanishni aniqlash.*

> with(Statistics):

> X:= Vector([1,2, 3, 4, 5,6,7,8]):

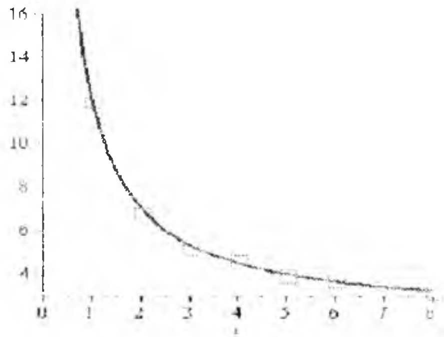
Y:= Vector([12.2,6.8,5.2,4.6,3.9,3.7,3.5,3.2]):
> Fit(a+b/t, X, Y, t);

$$1.93576189703930290C\ \frac{10.160175230791024\text{:}}{t}$$

2) *Bog'lanishni grafigini qurish.*
> with(plots):
> r4:=rhs(fit[leastsquare[[x,y],y=a+b/x]]([[1,2,3,4, 5,6,7,8],[12.2,6.8, 5.2, 4.6,3.9,3.7, 3.5, 3.2]]));

$$r4 := 1.935761897C\ \frac{10.16017523}{x}$$

> plot([r4,[[1,12],[2,6.8],[3,5.2],[4,4.6],[5,3.9], [6,3.7],[7,3.5],[8,3.2]]],x=0..8,3..16,symbol=BOX, symbolsize=30,style=[line,point],color=[blue,red], thickness=2);



4.4–rasm.

Quyidagi 3.3-jadvalda ko'rsatilgan bog'lanishlarning parametrlarini aniqlash uchun kichik kvadratlar usuli asosida tuzilgan sistemalarni beramiz.

3.3–jadval

T/r	Bog'lanish tenglamasi	Kichik kvadratlar usulida bog'lanish koeffitsentlarini aniqlovchi tenglamalar sistemasi
1	$y=a+bx$	$an+b\Sigma x=\Sigma yma,\ \Sigma x+b\Sigma x^2=\Sigma(xy)$
2	$lgy=a+bx$	$an+b\Sigma x=\Sigma lgym,\ a\Sigma x+b\Sigma x^2=\Sigma(xlgy)$
3	$y=a+blgx$	$an+b\Sigma lgx=\Sigma y,\ a\Sigma lgx+b\Sigma(lgx)^2=\Sigma(ylgx)$
4	$lgy=a+blgx$	$an+b\Sigma lgx=\Sigma y,\ \Sigma lgx+b\Sigma(lgx)^2=\Sigma(lgxlgy)$
5	$y=ab^x$ yoki $lgy=lga+blgx$	$an+b\Sigma lgx=\Sigma lgy$ $lga\Sigma lgx+lgb\Sigma x^2=\Sigma(lgxlgy)$
6	$y=a+bx+cx^2$	$an+b\Sigma x+c\Sigma x^2=\Sigma y$

		$a\Sigma x + b\Sigma x^2 + c\Sigma x^3 = \Sigma(xy)$ $a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 = \Sigma(x^2y)$
7	$y = a + bx + cx^2 + dx^3$	$an + b\Sigma x + c\Sigma x^2 + d\Sigma x^3 = \Sigma y$ $a\Sigma x + b\Sigma x^2 + c\Sigma x^3 + d\Sigma x^4 = \Sigma(xy)$ $a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 + d\Sigma x^5 = \Sigma(x^2y)$ $a\Sigma x^3 + b\Sigma x^4 + c\Sigma x^5 + d\Sigma x^6 = \Sigma(x^3y)$
8	$y = a + bx + c\sqrt{x}$	$an + b\Sigma x + c\Sigma\sqrt{x} = \Sigma y$ $a\Sigma x + b\Sigma x^2 + c\Sigma\sqrt{x^3} = \Sigma(xy)$ $a\Sigma\sqrt{x} + b\Sigma\sqrt{x^3} + c\Sigma x = \Sigma(\sqrt{xy})$
9	$y = ab^x c^{x^2}$ yoki $lgy = lga + xlgx + x^2lgc$	$nlga + lgb\Sigma x + lgc\Sigma x^2 = \Sigma lgy$ $lga\Sigma x + lgb\Sigma x^2 + lgc\Sigma x^3 = \Sigma(xlgy)$ $lga\Sigma x^2 + lgb\Sigma x^3 + lgc\Sigma x^4 = \Sigma(x^2lgy)$

O‘z–o‘zini tekshirish uchun savollar

1. Kichik kvadratlar usulining mohiyatini tushintring
2. Kichik kvadratlar usulida bog‘lanish koeffitsentlarini topish sistemasini tuzish
3. Kichik kvadratlar usulida chiziqli va parabolik bog‘lanishlarni topish qoidasini tushintiring
4. Chiziqliqi bog‘lanish koeffitsentlarini topish formulasi
5. Parabolik bog‘lanish koeffitsentlarini topish formulasi
6. Bog‘lanishlar tenglamalarini aniqlashda koeffitsentlarni topish usullari

4–laboratoriya ishi bo‘yicha mustaqil ishlash uchun topshiriqlar

Quyidagi jadval uchun kichik kvadratlar usulida to‘g‘ri chiziqli va ikkinch darajali bog‘lanishlarni aniqlang.

Variant 1

X	1,43	3,48	4,55	5,62	6,70	8,75
Y	1,635	1,732	1.876	2,033	2,228	2,359

Variant 2

X	0,02	1,08	0,12	3,17	4,23	0,30
Y	1,02316	1,095	1,147	1.214	1.301	1,409

Variant 3

X	0,35	3,41	0,47	4,51	0,56	7.64
Y	2,739	2,300	1,968	1,787	1.595	1,345

Variant 4

X	1,41	3,46	5,52	6,60	7,65	8,72
Y	2.574	2,325	2,093	1,862	1,749	1,620

Variant 5

X	0,68	0,73	0,80	0,88	0,93	0,99
Y	0,808	0,894	1,029	1,209	1,340	1,523

Variant 6

X	0,11	5,15	0,21	0,29	7,35	0.40
Y	9,054	6,616	4,691	3,351	2,739	2,365

Variant 7

X	1,375	1,380	1,385	1,390	1,395	1,400
Y	5,041	5,177	5,320	5,470	5,629	5,797

Variant 8

X	8.115	0,120	5,125	0,130	0,135	2.140
Y	8,657	8,293	7,958	7.648	7,362	7,096

Variant 9

X	0,150	0,155	8,160	0,165	0,170	3,175
Y	6,616	6,399	6,196	6,005	5.825	5,655

Variant 10

X	0,180	3,185	0,190	0,195	7,200	0,205
Y	5,615	5,466	5,326	5,193	5,066	4,946

Variant 11

X	0,210	1,215	0,220	8,225	0,230	0,235
Y	4,831	4,722	4,618	4,519	4,424	4.333

Variant 12

X	1.415	1,420	1,425	0,430	0,435	0.440
Y	0,888	0,889	0,890	0.891	0,892	0,893

Variant 13

X	0,33	4,38	0,45	9,52	0,60	0,65
Y	1,635	1,732	1,876	2,033	2,228	2,359

Variant 14

X	1,03	5,09	0,13	1,18	0,24	6,31
Y	1,023	1,095	1,147	1,214	1,301	1,409

Variant 15

X	0,25	0,31	0,37	0,41	0,46	0,54
Y	2,739	2,300	1,968	1,787	1,595	1,345

Variant 16

X	0,21	5,26	0,32	4,40	0,45	0,52
Y	2,574	2,325	2,093	1,862	1,749	1,620

Variant 17

X	0,38	7,43	0,50	0,58	2,63	1,69
Y	0,808	0,894	1,029	1,209	1,340	1,523

Variant 18

X	0,31	0,35	0,41	0,49	0,55	0,60
Y	9,054	6,616	4,691	3,351	2,739	2,365

Variant 19

X	1,175	1,180	1,185	1,190	1,195	1,200
Y	5,041	5,177	5,320	5,470	5,629	5,797

Variant 20

X	0,215	0,220	0,225	0,230	0,235	0,240
Y	8,657	8,293	7,958	7,648	7,362	7,096

Variant 21

X	0,250	0,255	0,260	0,265	0,270	0,275
Y	6,616	6,399	6,196	6,005	5,825	5,655

Variant 22

X	0,280	0,285	0,290	0,295	0,300	0,305
Y	5,615	5,466	5,326	5,193	5,066	4,946

Variant 23

X	0,310	0,315	0,320	0,325	0,330	0,335
Y	4,831	4.722	4,618	4,519	4,424	4,333

Variant 24

X	1,315	1,320	1,325	0,330	0,335	0,340
Y	0,888	0,889	0,890	0,891	0,892	0,893

Variant 25

X	0,315	0,320	0,325	0,330	0,335	0,340
Y	8,657	8,293	7,958	7,648	7,362	7,096

Variant 26

X	0,450	0,455	0,460	0,465	0,470	0,475
Y	6,616	6,399	6,196	6,005	5,825	5,655

Variant 27

X	0,580	0,585	0,590	0,595	0,600	0,605
Y	5,615	5,466	5,326	5,193	5,066	4,946

Variant 28

X	0,410	0,415	0,420	0,425	0,430	0,435
Y	4,831	4,722	4,618	4,519	4,424	4.333

Variant 29

X	0,315	0.320	0,325	0.330	0,335	0,340
Y	0,888	0,889	0.890	0,891	0.892	0,893

Variant 30

X	2,315	2,320	2,325	2,330	2,335	2,340
Y	2,888	3,889	4,890	5,891	6,892	7,893

5–LABORATORIYA ISHI

Aniq integralni taqribiy hisoblash

Maple dasturining buyruqlari:

with(Student[Calculus1])– *hisoblash paketidagi amallarni chaqirish;*

Int(f(x),x)– *aniqmas integralni ko'rinishini yozish;*

int(f(x),x)– *aniqmas integralni hisoblash;*

Int(f(x),x=a..b)– *aniq integralni ko'rinishini yozish;*

int(f(x),x=a..b)– *aniq integralni hisoblash;*

RiemannSum(f(x),x=a..b,method=left)– *chap(ostki) Riman integral yig'indilarini hisoblash;*

RiemannSum(f(x),x=a..b,method=left,output=plot)– *ostki to'rburchaklar grafigini qurish;*

ApproximateInt(f(x), a..h,method =trapezoid)– *aniq integralni trapetsiyalar usulida hisoblash;*

ApproximateInt(f(x), a..b, method= trapezoid, output=plot,tbickness=2)– *aniq integralni trapetsiyalar usulida hisoblashdagi yuzani grafigini qurish,*

ApproximateInt(f(x), a..b,method =simpson, thickness=2)– *aniq integralni trapetsiyalar usulida hisoblash;*

> ApproximateInt(f(x), a..b, method=simpson, output=plot, thickness=2)– *aniq integralni Simpson usulida hisoblashdagi yuzani grafigini qurish;*

Maqsad: Aniq integralni taqribiy hisoblash usullarini o'rganish.

Reja:

5.1. To'g'ri to'rtburchaklar formulasi.

5.2. Trapetsiyalar formulasi.

5.3. Simpson yoki parabola formulasi.

Integrallanuvchi $f(x)$ funksiyaning boshlang'ichini $F(x)$ funksiyani bizga ma'lum funksiyalar orqali ifodalash mumkin bo'lmaganda hamda $f(x)$ funksiya jadval yoki grafik usul bilan berilganda integrallni taqribiy hisoblashga to'gri keladi.

Demak, aniq integralni geometrik ma'nisidan kelib chiqib, yassi yuzani taqribiy hisoblashning bir necha usullsrini keltiramiz.

Aytaylik $[a,b]$ oraliqda $f(x)$ funksiya grafigi yordamida $x=a$, $x=b$ hamda $y=0(Ox)$ to'g'ri chiziqlar bilan chegaralangan yuzani hisoblash kerak bo'lsin.

Berilgan $[a,b]$ oraliqda qadami $h=(b-a)/n$ bo'lgan bo'linish nuqtalarida integral ostidagi funksiya qiymatlarini hisoblaymiz.

$$x_0=a,\ x_i=x_{i-1}+h,\ y_i=f(x_i),\ i=0,1,2,\dots,n.$$

Hosil bo'lgan bo'linishlar bo'yicha asosi h, balandligi

$$y_i = f(x_i),\quad i=0,1,2,\ldots,n$$

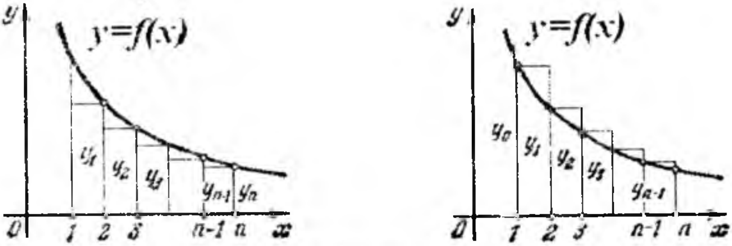
funksiya qiymatlaridan iborat bo'lgan yuzalarning integral yig'indilarini tuzamiz:

$$s = \sum_{i=1}^{n} hf(x_i) = \sum_{i=1}^{n} hy_i$$

Quyida bunday yuzalarni taqribiy hisoblash formulalarini ko'ramiz.

5.1. To'g'ri to'rtburchaklar formulasi

Aniq integralni taqribiy hisoblashda ichki va tashqi to'g'ri to'rtburchaklar(5.1–rasm) bo'yicha (chap va o'ng yig'indilar) hisoblash formulasi quyidagicha bo'ladi.



5.1–rasm.

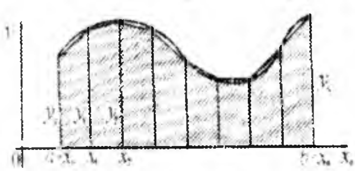
$$\int_a^b f(x)dx \approx h\sum_{i=1}^{n} f(x_i) = h(y_1 + y_2 + \ldots y_n)$$

$$\int_a^b f(x)dx \approx h\sum_{i=0}^{n-1} f(x_i) = h(y_0 + y_1 + y_2 + \ldots y_{n-1}),$$

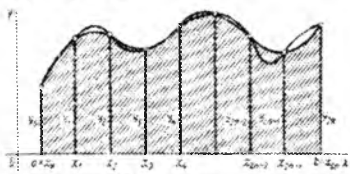
5.2. Trapetsiyalar formulasi

Aniq integralni taqribiy hisoblash formulasi quyidagicha bo'ladi.

$$\int_a^b f(x)dx \approx h\left[\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f(x_i)\right] = h\left[\frac{y_0 + y_n}{2} + y_1 + y_2 + \ldots + y_{n-1}\right]$$



5.2–rasm.



5.3–rasm.

5.3. Simpson yoki parabola formulasi

Berilgan kesmadagi bo'linish nuqtalariga mos egri chiziqning har uch nuqtasiga parabola uch hadini(5.3–rasm) qo'llash bilan, aniq integralni taqribiy hisoblashning Simson formulasi quyidagicha bo'ladi($h=(b-a)/2n$).

$$\int_a^b f(x)dx \approx \frac{h}{3}[f(a)+f(b)+4\sum_{k=1}^{n} f(x_{2k-1})+2\sum_{k=1}^{n-1} f(x_{2k})]$$

$$\int_a^b f(x)\mathrm{d}x \approx \frac{h}{3}[y_0+y_n+4(y_1+y_3+\ldots+y_{2k-1})+2(y_2+y_4+\ldots+y_{2k})]$$

Yuqoridagi formulalarning integral yig'indilari $h\to 0$ dagi integralning qiymatini beradi. Bu qiymat, tanlangan h uchun hisoblangan yig'indi qiymatidan $R_n(f)$ miqdorga farq qiladi. Bu farq–hatolikni ε ($0<\varepsilon<1$) aniqlikda

$$|R_n(f)|<\varepsilon$$

shart bo'yicha baholashni oraliqni bo'linishlar sono n yoki h qadamlarni tanlash bilan aniqlaymiz. Aniq integralni hisoblashning taqribiy formulalar bo'yich hatoliklar quyidagicha:

1) to'rt burchaklar usuli uchun: $R_n(f)=\dfrac{b-a}{24}f''(\xi)h^2,\ \xi\in[a,b]$;

2) trapetsiyalar usuli uchun: $R_n(f)=\dfrac{b-a}{12}f''(\xi)h^2,\ \xi\in[a,b]$;

3) Simpson usuli uchun: $R_n(f)=\dfrac{b-a}{180}f^{(IV)}(\xi)h^4,\ \xi\in[a,b]$.

5.1–masala. Ushbu $\displaystyle\int_2^{3.5}\frac{dx}{\sqrt{5+4x-x^2}}$ aniq integralda [2.3.5] oraliqning bo'linishlar soni $n=10$ bo'lganda to'g'ri to'rtburchaklar, trapetsiyalar va Simpson formulalari bilan $e=0.001$ aniqlikda hisoblang.

Yechish.

1.Aniq integralni bevosita integrallash va hisoblashning Maple dasturi.

5.1–Maple dasturi:

1) *Boshlang'ich funksiyasini topish:*

```
> Int( 1/sqrt(5+4*x–x^2),x)=int(1/sqrt(5+4*x–x^2),x);
```

$$\int\frac{1}{\sqrt{5+4x-x^2}}\,dx=\arcsin\left(-\frac{2}{3}+\frac{1}{3}x\right)$$

2) 10 xona aniqlikda taqribiy hisoblash.

```
> f:= 1/sqrt(5+4*x-x^2):
> Int(f,x=2..3.5)=evalf(int(f,x=2..3.5, digits=10, method=_Dex));
```

$$\int_{2}^{3.5} \frac{1}{\sqrt{5+4x-x^2}}\,dx = 0.523598775$$

```
> evalf(Int( 1/sqrt(5+4*x-x^2),x=2..3.5)); 0.523598775
> evalf[25](Int(1/sqrt(5+4*x-x^2),x=2..3.5 ));
```

$$0.5235987755982988730771$$

2.Berilgan aniq integralni taqribiy hisoblash. Berilgan [2,3.5] oraliqning bo'linish qadami

$$h=(b-a)/n=(3.5-2)/10=0.15$$

bo'lganda, bo'linish nuqtalari

$$x_i=a+i\,h,\ i=1,2,\ldots,10$$

bo'lsa, nuqtalarni [2,3.5] oraliqda aniqlab, bu nuqtalarda integral ostidagi funksiya qiymatlarini topamiz.

$$x_0=2.00\ y_0=f(2)=\frac{1}{\sqrt{5+4\cdot 2-3^2}}=0{,}3333$$

$$x_1=2.15\ y_1=f(2.15)=0{,}3388$$

$x_2=2.30$ $y_2=f(2{,}30)=0{,}3350$

$x_3=2.45$ $y_3=f(2{,}45)=0{,}3371$

$x_4=2.60$ $y_4=f(2{,}60)=0{,}3402$

$x_5=2.75$ $y_5=f(2{,}75)=0{,}3443$

$x_6=2.90$ $y_6=f(2{,}90)=0{,}3494$

$x_7=3.05$ $y_7=f(3{,}05)=0{,}3558$

$x_8=3.20$ $y_8=f(3{,}20)=0{,}3637$

$x_9=3.35$ $y_9=f(3{,}35)=0{,}3733$

$x_{10}=3.50$ $y_{10}=f(3.50)=0{,}3849$

Topilgan x va y larning qiymatlarini integralni taqribiy hisoblash formulalariga qo'yib integralning qiymatini hisoblaymiz.

To'g'ri to'rtburchaklar formulasiga asosan hisoblash

$$\int_{2}^{3.5}\frac{dx}{\sqrt{5+4x-x^2}}\approx 0.15(0.3333+0.3388+0.3350+0.3371+0.3402+$$
$$+0.3443+0.3494+0.3858+0.3637+0.3733+0.3849)=0.5755$$

Bu to'g'ri to'rtburchaklar usulida hisoblashning Maple dasturining ikkita variantini ko'rsatamiz.

1.Yuqoridagi hisoblash algoritimi asosida:

5.1a–Maple d a s t u r i:

```
> restart;
> f:=x->1/sqrt(5+4*x-x^2);
```

$$f := x \to \frac{1}{\sqrt{5+4x-x^2}}$$

```
> n:=10: a:=2: b:=3.5: h:=(b-a)/n:
> x:= array(1..10):y:= array(1..10):
> S1:=0:
> for i to n do
x[i]:=evalf(a+(i-1)*h,5): y[i]:=evalf(f(x[i]),5):
S1:=S1+y[i]: end do:
print(x,y),print("Tort burchak usulida S1=",S1*h);
```

[2., 2.1500, 2.3000, 2.4500, 2.6000, 2.7500, 2.9000, 3.0500, 3.2000, 3.3500], [0.33333, 0.33376, 0.33501, 0.33714, 0.34021, 0.34427, 0.34943, 0.35585, 0.36370, 0.37325]

"Tort burchak usulida S1=",0.5198925000

2. Integral yeg'indilar bo'yicha **RiemannSum** funksiyasi yordamida hisoblash va yuzaning grafigini qurish:

5.1b–M a p l e d a s t u r i:

```
> restart; with(Student[Calculus1]):
```

1)*ostki to'g'ri to'rtburchaklar bo'yicha hisoblash va grafigini qurish:*

```
> RiemannSum(1/sqrt(5+4*x-x^2),x=2..3.5,method=left);
```

0.5198915129

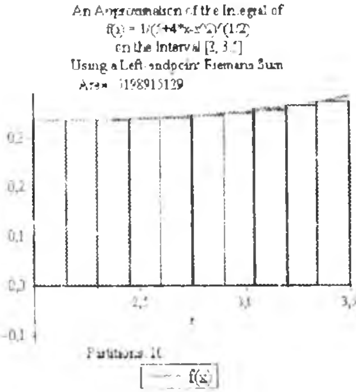
```
> RiemannSum(1/sqrt(5+4*x-x^2),x=2..3.5,method=left,
output=plot);
```
(5.1a–rasm)

2) *ustki to'g'ri to'rtburchaklar bo'yicha hisoblash va grafigini qurish:*

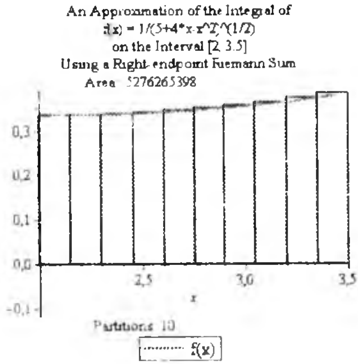
```
> RiemannSum(1/sqrt(5+4*x-x^2),x=2..3.5,method= right);
```

0.5276265393

```
> RiemannSum(1/sqrt(5+4*x-x^2),x=2..3.5,method=
right,output=animation);
```
(5.1b–rasm)



5.1a–rasm.



5.1b–rasm.

Trapetsiya formulasiga asosan hisoblash

$$\int_{2}^{3.5}\frac{dx}{\sqrt{5+4x-x^2}} \approx 0{,}15\left(\frac{0.3333+0.3849}{2}+0.3388+0.3350+0.3371+0.3402+\right.$$
$$\left.+0.3443+0.3494+0.3858+0.3637+0.3733\right)=0.15\cdot 3.49178=0{,}52376$$

Bu trapetsiya usulida hisoblashning Maple dasturi:

1.Yuqoridagi hisoblash algoritimi asosida:

5.2a–Maple d a s t u r i:

```
> restart;
> f:=x->1/sqrt(5+4*x-x^2);
```

$f := x \rightarrow \frac{1}{\sqrt{5+4x-x^2}}$

```
> n:=10: a:=2: b:=3.5: h:=(b-a)/n:
> x:= array(1..10):y:= array(1..10):
> S2:=(f(a)+f(b))/2:
```

$S2 := 0.359116756$

```
> for i to n-1 do
x[i]:=evalf(a+i*h,5): y[i]:=evalf(f(x[i]),5):
S2:=S2+y[i]: end do:
print(x,y),print("Trapetsiya usulida S2=".S2*h);
```

[2.1500 2.3000 2.4500 2.6000 2.7500 2.9000 3.0500 3.2000 3.3500 3.3500], [0.33376 0.33501 0.33714 0.34021 0.34427 0.34943 0.35585 0.36370 0.37325 0.37325]

"Trapetsiya usulida S2="0.523760513

2.Integralni **ApproximateInt** funksiyasi yordamida hisoblash va yuzaning grafigini qurish:

5.2b–M a p l e d a s t u r i:

```
> restart;with(Student[Calculus1]):
> ApproximateInt(1/sqrt(5+4*x–x^2), 2..3.5,
method = trapezoid);
```
0.523759026

```
> ApproximateInt(1/sqrt(5+4*x–x^2), 2..3.5, method=trapezoid,
output=plot,thickness=2);
```
(5.4–rasm)

Simpson formulasiga asosan hisoblash.

$$\int_2^{3.5}\frac{dx}{\sqrt{5+4x-x^2}} \approx \frac{0.15}{3}[0.3333+0.3849+4(0.3338+0.3371+0.3443+0.3558+0.3733)+2(0.3350+0.3402+0.3494+0.3637)] = 0.54265.$$

Bu Simpson usulida hisoblashning Maple dasturi:

1.Yuqoridagi hisoblash algoritimi asosida:

5.3a–Maple d a s t u r i:

```
> restart;f:=x–>1/sqrt(5+4*x–x^2);
```
$f := x \to \frac{1}{\sqrt{5+4x-x^2}}$

```
> n:=10: a:=2: b:=3.5: h:=(b–a)/n:
> x:= array(1..10):y:= array(1..10):
> S3:=f(a)+f(b);c:=1:
```
$S3 := 0.718233512$

```
> for i to n–1 do
x[i]:=evalf(a+i*h,5): y[i]:=evalf(f(x[i]),5):
S3:=S3+(c+3)*y[i]:c:=–c: end do: print(x,y),print("Simpson usulida
S3=",S3*h/3);
```

[2.1500 2.3000 2.4500 2.6000 2.7500 2.9000 3.0500 3.2000 3.3500 3.3500], [0.33376 0.33501 0.33714 0.34021 0.34427 0.34943 0.35585 0.36370 0.37325 0.37325]

"Trapetsiya usulida S3=","0.523600675

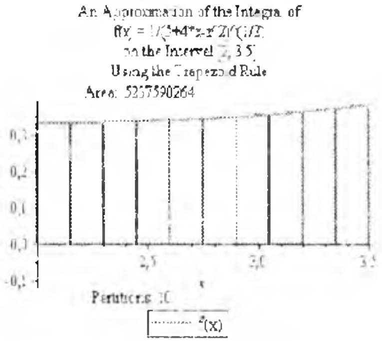
2.Integralni **ApproximateInt** funksiyasi yordamida hisoblash va yuzaning grafigini qurish:

5.3b–M a p l e d a s t u r i:

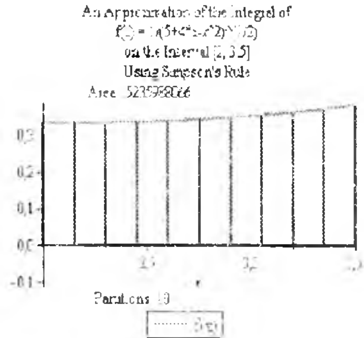
```
> ApproximateInt(1/sqrt(5+4*x–x^2),2..3.5,
```

method=simpson, thickness=2); 0.5235988061

> ApproximateInt(1/sqrt(5+4*x−x^2),2..3.5, method=simpson, output=plot,thickness=2); (5.3a–rasm)



5.4–rasm.

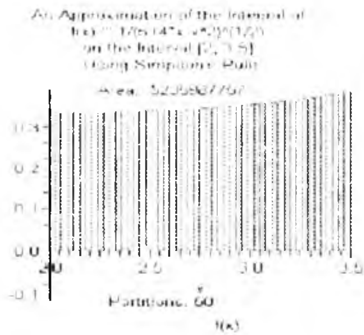


5.4a–rasm.

Yuza grafigini bo'linishlarni animatsiyasi asosida qurish.

> with(Student[Calculus1]):

> ApproximateInt(1/sqrt(5+4*x −x^2), 2..3.5, method=simpson, output=animation); (5.5–rasm)



5.5–rasm.

Yuqorida topilgan integrallarning taqribiy qiymatlarini baholash.

1) To'g'ri to'rtburchaklar formulasi xatoligini bahosi:

$$f(x)=\frac{1}{\sqrt{5+4x-x^2}}=\frac{1}{\sqrt{9-(x-2)^2}}\Rightarrow x\in[2{,}3.5]\Rightarrow\frac{1}{3}\le f(x)\le 0{,}3849$$

$x\in[2{,}3.5]$ кесма учун

$$f'(x)=-\frac{4-2x}{2\left(\sqrt{5+4x-x^2}\right)^3}=\frac{2(x-2)}{2(5+4x-x^2)\sqrt{5+4x-x^2}}=\frac{x-2}{(5+4x-x^2)\sqrt{5+4x-x^2}}=(x-2)f^3$$

$$f'(x)=(x-2)f^3(x);\qquad |f'(x)|=|x-2|\cdot|f^3(x)|<1.5\cdot 0.3849^3=0.0855\Rightarrow M_1\le 0.0855;$$

$$|R(h)|\le\frac{(b-a)M_1}{2}h<\frac{1.5\cdot 0.0855}{2}(0.15)=0.0096\approx 0.01.$$

2) Trapetsiyalar formulasi xatoligining bahosi:

$$f''(x)=f^3(x)+(x-2)\cdot 3f^2(x)\cdot f'(x)=f^3(x)+(x-2)\cdot 3f^2(x)\cdot(x-2)f^3(x)=$$

$$=(1+3(x-2)^2f^2(x))f^3(x)\Rightarrow|f''(x)|<(1+3\cdot 2.25\cdot 0.3849^2)\cdot 0.3849^3=0.1140\Rightarrow M_2=0.1140$$

$$M_2<0.11,\qquad |R(h)|<\frac{1.5\cdot 0.1140}{12}\cdot 0.15^2=0.0003$$

3) Simpson formulasi xatoligining bahosi:

$$f^{IV}(x)=(9+(90+105(x-2)^2f^2(x))(x-2)^2f^2(x))f^5(x)\Rightarrow$$

$$\Rightarrow f^{IV}(x)<(9+(90+105\cdot 1.5^2\cdot 0.3849^2))1.5^2\cdot 0.3849^2)0.3849^5=0.4256\Rightarrow$$

$$\Rightarrow M_4=0.4256$$

$$|R(h)|<\frac{1.5\cdot 0.4256}{180}\cdot 0.15^4\approx 0.000002.$$

Bu baholashni qaralayotgan integralning aniq qiymati bo'lgan $\pi/6$ soni bilan taqqoslash natijasi ham tasdiqlaydi. Haqiqatdan ham $\pi=3.1416$ (0.0001 aniqlikda) deb olsak integralning aniq qiymatining 0.0001 aniqlikdagi qiymati 0.5236 bo'lishini ko'ramiz. bu esa yuqorida Simpson formulasi yordamida olingan taqribiy qiymat bilan bir xildir.

Olingan xatoliklarni baholashlardan ko'rinadiki, Simpson formulasining aniqligi sezilarli yuqori ekan.

O'z–o'zini tekshirish uchun savollar

1. Qanday hollarda aniq integralni taqribiy hisoblanadi?
2. Bo'linish qadamini toping.
3. Oraliqning bo'linish nuqtalari qanday topiladi?
4. To'g'ri to'rtburchaklar usuli va formulasini tushuntiring.
5. Trapetsiyalar usuli va formulasini tushuntiring.
6. Simpson usuli va formulasini tushuntiring.
7. Aniq integralni taqribiy hisoblashlardagi xatoliklarini qanday baholaymiz?
8. Simpson usulini boshqa usullardan farqi.
9. Simpson usulida bo'linish qadamini aniqlash.

5–laboratoriya ishi
bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi integrallarni to'g'ri to'rtburchaklar, trapetsiyalar, Simpson usullarida hisoblang.

1. $\int_1^{3.5} \frac{\ln x}{x\sqrt{1+\ln x}}dx$

2. $\int_{\pi/6}^{\pi/3} (tg^2 x + ctg^2 x)dx$

3. $\int_1^4 \frac{1}{x}\ln^2 x dx$

4. $\int_2^3 \frac{1}{x\lg x}dx$

5. $\int_0^{\ln 2} \sqrt{e^x - 1}dx$

6. $\int_0^1 xe^x \sin x dx$

7. $\int_0^2 \frac{1}{\sqrt{9+x^3}}dx$

8. $\int_1^{2.5} \frac{1}{x^2}\sin\frac{1}{x}dx$

9. $\int_0^3 x\,arctgx\,dx$

10. $\int_0^3 \arcsin\sqrt{\frac{x}{1+x}}dx$

11. $\int_1^3 x^x(1+\ln x)dx$

12. $\int_0^1 \frac{dx}{\sqrt{1+3x+2x^2}}$

13. $\int_1^2 \frac{1}{x}\sqrt{x^2+0.16}dx$

14. $\int_0^1 \frac{x\,arctgx}{\sqrt{1+x^2}}dx$

15. $\int_0^2 \frac{e^{3x}+1}{e^x+1}dx$

16. $\int_0^{1.99} x^2\sqrt{4-x^2}dx$

17. $\int_0^{\pi} e^x \cos^{-2} x dx$

18. $\int_1^e (x\ln x)^2 dx$

19. $\int_{-1}^{2} \arccos\sqrt{\frac{x}{1+x}}dx$

20. $\int_0^1 \frac{(x^2+4)dx}{(x^2+1)\sqrt{x^4+1}}$

21. $\int_1^{1.5} \sin x \ln(tgx)dx$

22. $\int_0^{1.5} \frac{e^x(1+\sin x)}{1+\cos x}dx$

23. $\int_0^{3/4} (x+1)/\sqrt{x^2+1}dx$

24. $\int_0^1 \frac{dx}{(3\sin x + 2\cos x)^2}$

25. $\int_1^2 \left(\frac{\ln x}{x}\right)^3 dx$

26. $\int_1^2 \frac{x^3}{\sqrt{x+3}} dx$

27. $\int_1^2 \frac{x}{x^4+3x^2+2} dx$

28. $\int_0^{\frac{\pi}{2}} \sqrt{1-\frac{1}{4}\sin 2x}\, dx$

29. $\int_0^{\frac{\pi}{2}} \sqrt{2+\cos x}\, dx$

30. $\int_{\ln 2}^{\ln 3} \frac{e^{2x}}{e^x-e^{-x}} dx$

6-LABORATORIYA ISHI

Birinchi tartibli oddiy differensial tenglama uchun Koshi masalasini taqribiy yechish

Maple dasturining buyruqlari:

diff(y(x),x)=cos(y(x)/sqrt(5))+x *–differensial tenglamani ifodalash*

Bsh1 := y(1.8)=2.6– *boshlang'ich shartni kiritish*

with(DEtools):DEplot(Odt1,y(x),x=–5..3,y=–1..5, [y(1.8)=2.8],linecolor=[red])– *ko'rsatilgan sohada differensial tenglamaning boshlang'ich sharti asosida yechim grafifini qurish;*

dsolve({Odt1,Bsh1},numeric,method=classical)– *differensial tenglamaning yechimini Eyler usulida topish;*

dsolve({dsol1.init1}, numeric, method=rkf45, output= procedurelist)– *differensial tenglamaning yechimini Runge–Kutta usulida topish;*

dsolve(dsys1,numeric,method=rkf45,output=procedurelist)– *differensial tenglamalar sistemasinig yechimini Runge–Kutta usulida topish;*

Maqsad: Birinchi tartibli oddiy differensial tenglama uchun Koshi masalasini taqribiy yechish usullarini o'rganish.

Reja: 6.1. Eyler usuli.

6.2. Runge – Kutta usuli.

6.3. Birinchi tartibli differensial tneglamalar sistemasi uchun Koshi masalasini Eyler usulida taqribiy yechish.

6.1. Eyler usuli

Aytaylik bizga birinchi tartibli

$$\frac{dy}{dx} = f(x, y) \text{ yoki } y_0 = f(x_0) \quad (6.1)$$

differensial tenglama berilgan bo'lib, $[x,b]$ kesmada

$$x=x_0,\ y=y_0 \quad (6.2)$$

boshlang'ich shartni qanoatlantiruvchi yechimni taqribiy hisoblash masalasi qo'yilgan bo'lsin. Bu masala *Koshi masalasi* deyiladi. Bu masalani taqribiy yechishning bir necha usullarini ko'ramiz.

Berilgan $[x_0,b]$ kesmani n ta teng bo'lakka bo'lib bo'linish nuqtalari orasidagi qadam

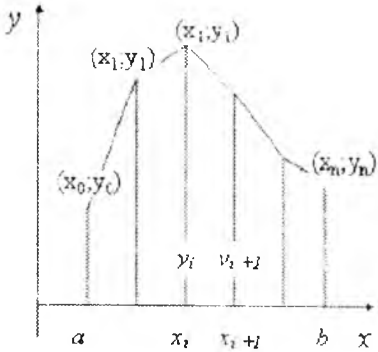
$$h=(b-x_0)/n \quad (6.3)$$

bo'lganda, bu nuqtalar koordinatalari

$$x_i=x_{i-1}+h,\ i=1, 2, \dots n \quad (6.4)$$

bo'ladi. (6.2) boshlang'ich shartlardagi x_0 va y_0 lardan foydalanib tenglama yechimining qiymatlarini taqriban quyidagicha hisoblaymiz.

$$y_1 = y_0 + hf(x_0, y_0).$$



6.1-rasm.

$$y_2 = y_1 + hf(x_1, y_1),$$

$$y_3 = y_2 + hf(x_2, y_2),$$

... ...

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1}).$$

natijada izlanaetgan yechimni qanoatlantiruvchi

$(x_0;y_0)$, $(x_1;y_1)$, $(x_2;y_2)$, ..., $(x_n;y_n)$

nuqtalarni aniqlaymiz. Bu nuqtalarni tutashtiruvchi sinik chiziq Eyler chiziqi (6.1–rasm) deb ataladi va u tenglama yechimining taqribiy grafigini ifodalydi.

6.1–masala. $y' = x + \cos\left(y/\sqrt{5}\right)$ birinchi tartibli differentsial tenglamaning [1.8,2.8] oraliqda $x_0=1.8$, $y_0=2.6$ boshlang'ich shartni qanoatlantiruvchi yechimini Eyler usulida $h=0.1$ qadam bilan $\varepsilon=0.0001$ aniqlikda hisoblang.

1. Berilgan differensial tenglamani Eyler usulida yechamiz, [1.8,2.8] oraliqni $h=0.1$ qadam bilan

$$n = \frac{b-a}{h} = \frac{2.8-1.8}{0.1} = 10$$

$n=10$ ta bo'lakka ajratamiz. Bo'linish nuqtalarini

$x_i = x_{i-1} + hi = 1,2,...,10$

formulaga asosan topamiz:

$x_1 = x_0 + h = 1.8 + 0.1 = 1.9$

$$x_2=x_1+h=1.9+0.1=2.0$$

shuningdek

$x_3=2.1$, $x_4=2.2$, $x_5=2.3$, $x_6=2.4$, $x_7=2.5$, $x_8=2.6$, $x_9=2.7$. $x_{10}=2.8$

Berilgan tenglamaning o‘ng tomonidagi

$$f(x,y)=x+\cos(y/\sqrt{5})$$

funksiyaga asosan, Eyler qoidasi bilan quyidagi

$$y_i = y_{i-1} + h f\left(x_{i-1}, y_{i-1}\right),\ i=1,2,...,10$$

formulaga asosan berilgan differitsial tenglama yechimining qiymatlarini quyidagicha hisoblaymiz.

$y_1=y_0+hf(x_0,y_0)=y_0+h(x_0+\cos(y_0/\sqrt{5}))=2.6+0.1(1.8+$

$+\cos(2.6/\sqrt{5}))=2.6+0.1(1.8+0.3968)=2.81968;$

$y_2=y_1+h\,f(x_1,y_1)=y_1+h(x_1+\cos(y_1/\sqrt{5}))=$

$=2.819+0.1(1.9+\cos(2.819/\sqrt{5}))=2.819+0.1(1.9+0.3968)=3.03948$

Shuningdek, qolgan qimatlarini topamiz:

$y_3=3.261$, $y_4=3.4831$, $y_5=3.7045$, $y_6=3.926$,

$y_7=4.1478$, $y_8=4.3701$, $y_9=4.5931$, $y_{10}=4.8173$

6.1–masalani Eyler usulida taqribiy yechimni boshlang‘ich shart bo‘yicha berilgan oraliqdagi grafigini qurish va taqribiy qiymatlarini hisoblashning Maple dasturini tuzamiz.

6.1–M a p l e d a s t u r i:

Berilgan differensial tenglamani aniqlash:

> Odt1 := diff(y(x),x) = cos(y(x)/sqrt(5)) + x;

$$Odt1 := \frac{d}{dx}\,y(x) = x + \cos\left(\frac{1}{5}\,y(x)\,\sqrt{5}\right)$$

Boshlang‘ich shartni kiritish:

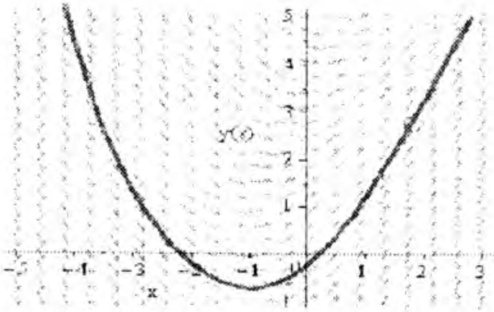
> Bsh1 := y(1.8)=2.6; $Bsh1 := y(1.8) = 2.6$

Berilgan tenglama umumiy yechimning egri chiziqlari oilasidan boshlang‘ich shartni qanoatlantiruvchi yechim egri chiziqning grafigini qurish:

```
> with(DEtools):with(plots):
> Odt1:= diff(y(x),x)=x+cos(y(x)/sqrt(5));
UYG:=DEplot(Odt1,y(x),x=-5..3,y=-1..5,[y(1.8)=2.8],
linecolor=[red]):
point1:=pointplot({[1.8,2.8]},symbol=circle,
color=blue, symbolsize=35,thickness=3):    display([UYG,point1]);
```

(6.2–rasm)

$$Odt1 := \frac{d}{dx}\,y(x) = x + \cos\left(\frac{1}{5}\,y(x)\,\sqrt{5}\right)$$



6.2–rasm.

Eyler usulida taqribiy yechim qiymatlarini hisoblash(1–matritsa):

> Eyler1:=dsolve({Odt1,Bsh1},numeric,method= classical);

$$Eyler1 := \mathbf{proc}(x\_classical)\ \dots\ \mathbf{end\ proc}$$

> Eyler1:=dsolve({Odt1,Bsh1},numeric,method= classical[heunform],output=array([1.8,1.9,2.0,2.1, 2.2,2.3,2.4,2.5,2.6,2.7,2.8]),stepsize=0.1);

Eyler usulida taqribiy yechim qiymatlarini $\varepsilon = 0.0001$ *aniqlikda hisoblash*

(2–matritsa):

> Eyler1[0.0001]:=evalf(%,5)

$Eyler1 :=$

x	y(x)
1.8	2.6
1.9	2.8200838397748433
2.0	3.040791359104897 8
2.1	3.2618526584035305 6
2.2	3.4831002966532529 4
2.3	3.7044675479690782 9
2.4	3.9259851022942826 6
2.5	4.1477768878609540 0
2.6	4.3700556484277903 8
2.7	4.5931188339506707 6
2.8	4.8173452760642403 5

1–matritsa

$Eyler1_{0.0001} :=$

x	y(x)
1.8	2.6
1.9	2.8201
2.0	3.0408
2.1	3.2619
2.2	3.4831
2.3	3.7045
2.4	3.9260
2.5	4.1478
2.6	4.3700
2.7	4.5931
2.8	4.8172

2–matritsa

2. Endi berilgan differensial tenglamaning taqribiy yechimi qiymatlari bo‘yicha interpolyasiya polinomini aniqlaymiz va uning grafigining ko‘rinishi qulay bo‘lgan [–8,8] kesmaga mos bo‘lagini ajratamiz. Berilgan tenglama yechimining Maple dasturida topilgan grafigi bilan taqribiy yechim grafiklarini qurib, ularni yaqinlashishini ko‘rsatamiz (6.3–rasm):

6.2–Maple dasturi:

> restart; with(plots):with(DEtools):

Umumiy yechimning [–8,8] kesmadagi grafigi:

> p1:=DEplot(diff(y(x),x$1)=x+cos(y(x)/sqrt(5)),y(x), x=–9..9,[[y(1.8)=2.6]],y=–2..40,stepsize=.005, linecolour=red);

$p1 := PLOT(...)$

Taqribiy yechimning qiymatlari asosida [–8,8] *kesmaga mos nuqtalarni aniqlash:*

> points1:=[[–8,28.345],[–5,9.456],[–3,1.106], [–1,–0.981], [0,–0.556],[1,0.924],[2,3.040], [3,5.270], [5,12.041], [8,31.936]]:

Taqribiy yechimning qiymatlari asosida uning [–8,8] *kesmadagi mos nuqtalarni qurish:*

> pointplot1:=pointplot(points1,symbol=BOX, color=blue,symbolsize=30):

Taqribiy yechimning qiymatlari asosida [–8,8] *kesmadagi polinomni ajratish:*

> polycurve:=PolynomialInterpolation(points1,x);

$$polycurve := -1.022970837\,10^{-8}\,x^9 - 0.000011162904\,76x^8 + 0.00000524713553x^7 + 0.00106564954\,4x^6 + 0.000038471728x^5 - 0.02357817007x^4 - 0.03316631771x^3 + 0.5500236749x^2 + 0.985622607x - 0.55599999$$

Taqribiy yechimning qiymatlari asosida [–8,8] *kesmada polinomning grafigini qurish:*

> polyplot:=plot(polycurve,x=–9..9,color=red, thickness=3):

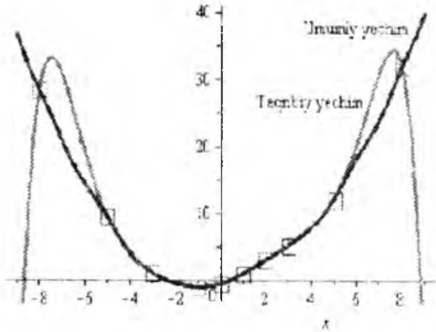
Grafikda chziqlar nomini ko'rsatish:

> tp1:=textplot([6,36,typeset("Umumiy yechim")], align=above):

> tp2:=textplot([4,25,typeset("Taqribiy yechim")], align=above):

Umumiy va taqribiy yechimning [–8,8] *kesmadagi grafigini qurish:*

> display([pointplot1,polyplot,p1,tp1,tp2]);(6.3–rasm)



6.3–rasm.

6.2. Runge – Kutta usuli

Maqsad: Birinchi tartibli oddiy differensial tenglama uchun Koshi masalasini Runge – Kutta usulida taqribiy yechishni topishni o'rganish

Reja: 1. Runge – Kutta usuli

2. Maple dasturida hisoblash.

Yuqoridagi birinchi tartibli (6.6) tenglamani (6.7) shartni qanoatlantiruvchi taqribiy yechimni Runge – Kutta usuli bilan quyidagicha topamiz.

$$x_i = x_{i-1} + h,\ (x_i = x_0,\ y_i = y_0)$$

$$k_1^{(i)} = hf(x_i, y_i),$$

$$k_2^{(i)} = hf(x_i + \frac{h}{2},\ y_i + k_1^{(i)}/2),$$

$$k_3^{(i)} = hf(x_i + \frac{h}{2},\ y_i + k_2^{(i)}/2), \qquad (6.8)$$

$$k_4^{(i)} = hf(x_i + h,\ y_i + k_3^{(i)}/2),$$

$$\Delta y_i = (k_1^{(i)} + 2k_2^{(i)} + 2k_3^{(i)} + k_4^{(i)})/6,$$

$$y_{i+1} = y_i + \Delta y_i,\ i = 0,1,2,...,n.$$

Bu *Runge–Kutta usuli* Koshi masalasining yechimni to'rtinchi darajali aniqlikda hisoblaydi. Bu (68) formulalar asosida hisoblab topilgah qiymatning aniqligini ortirish uchun h qadamni kichraytirish nbilan (6.8) formula bo'yicha qiymatni qayta hisoblaymiz va uni yechim qiymati uchun olamiz.

6.1–masalani Runge – Kutta usuli bilan yeching.

Yechish. Bu usulda tenglamaning yechimini topish uchun quyidagi hisoblash ketma-ketligini bajaramiz.

$i=0$ bo'lganda $x_0=1.8$ $y_0=2.6$ larda yechimning birinchi qiymatini (6.8) formulaga asosan hisoblaymiz.

$k_1^0=hf(x_0,y_0)=0.1(x_0+cos(y_0/\sqrt{5}))=0.1(1.8+cos(2.6/\sqrt{5}))=0.21968119;$

$k_2^0=hf(x_0+h/2,\ y_0+k_1^0/2)=0.220126211;$

$k_3^0=hf(x_0+h/2,\ y_0+k_2^0/2)=0.22116893;$

$k_4^0=hf(x_0+h,\ y_0+k_3^0)=0.22046793;$

$y_1=y_0+(k_1^0+2k_2^0+2k_3^0+k_4^0)/6=2.82010588.$

Demak, berilgan tenglama yechimining birinchi qiymati

$$y_1=2.82010588$$

bo'ladi.

$i=1$, $x_1=1.9$, $y_1=2.82010588$ larda yechimning ikkinchi qiymatini topish uchun yuqoridagi qoidani quyidagicha qo'llaymiz:

$$k_1^1=hf(x_1,y_1)$$

$=0.1(x_1+cos(y_1/\sqrt{5}))=0.1(1.9+cos(2.8201/\sqrt{5}))=0.21968119;$

$$k_2^1=hf\left(x_1+h/2,\ y_1+k_1^1/2\right)=0.220126211;$$

$$k_3^1=hf\left(x_1+h/2,\ y_1+k_2^1/2\right)=0.220116893;$$

$$k_4^1=hf\left(x_1+h,\ y_1+k_3^1\right)=0.220467930;$$

$$y_2=y_1+\left(k_1^1+2k_2^1+2k_3^1+k_4^1\right)/6=3.04021177.$$

$$y_2=3.04021177$$

Shuningdek, $i=2,3,\ldots,10$ lar uchun tenglama yechimini qolgan qiymatlarini topamiz.

$y_3=3.2603$ $y_4=3.4804$

$y_5=3.7005$ $y_6=3.9206$

$y_7=4.1407$ $y_8=4.3608$

$y_9=4.5931$ $y_{10}=4.9172$

Runge–Kutta usulida topiladigan yechimning qiymatlarini ketma-ket hisoblashning Maple dasturini quyidagicha tuzamiz (6.2–masala uchun).

6.3a–Maple dasturi:

```
> restart;
> f:=(x,y)->cos(y(x)/sqrt(5))+x;
```

$$f := (x, y) \to \cos\left(\frac{y(x)}{\sqrt{5}}\right) + x$$

> dsol1:=diff(y(x),x)=f(x,y);

$$dsol1 := \frac{d}{dx}\, y(x) = \cos\left(\frac{1}{5}\, y(x)\, \sqrt{5}\right) + x$$

> k1:=(x,y)->h*f(x,y); $k1 := (x, y) \to h\, f(x, y)$

> k2:=(x,y)->h*f(x+h/2,y+k1(x,y)/2);

$$k2 := (x, y) \to h\, f\left(x + \frac{1}{2}\, h, y + \frac{1}{2}\, k1(x, y)\right)$$

> k3:=(x,y)->h*f(x+h/2,y+k2(x,y)/2);

$$k3 := (x, y) \to h\, f\left(x + \frac{1}{2}\, h, y + \frac{1}{2}\, k2(x, y)\right)$$

> k4:=(x,y)->h*f(x+h,y+k3(x,y));

$$k4 := (x, y) \to h\, f(x + h, y + k3(x, y))$$

> h:=0.1:x:=1.8:y:=2.6:

> k1:=evalf(k1(x,y)); $k1 := 0.219681190$

> k2:=evalf(k2(x,y)); $k2 := 0.220126211$

> k3:=evalf(k3(x,y)); $k3 := 0.220116893$

> k4:=evalf(k4(x,y)); $k4 := 0.220467930$

> y1:=y+(k1+2*k2+2*k3+k4)/6; $y1 := 2.82010588$

> x:=1.9:y:=y1:

> k1; 0.219681190

> k2; 0.220126211

> k3; 0.220116893

> k4; 0.220467930

> y2:=y+(k1+2*k2+2*k3+k4)/6; $y2 := 3.04021177$

> x:=2.0:y:=y2:y3:=y+(k1+2*k2+2*k3+k4)/6; $y3 := 3.26031766$

> x:=2.1:y:=y3:y4:=y+(k1+2*k2+2*k3+k4)/6; $y4 := 3.48042355$

> x:=2.2:y:=y4:y5:=y+(k1+2*k2+2*k3+k4)/6; $y5 := 3.70052944$

> x:=2.3:y:=y5:y6:=y+(k1+2*k2+2*k3+k4)/6; $y6 := 3.92063533$

> x:=2.4:y:=y6:y7:=y+(k1+2*k2+2*k3+k4)/6; $y7 := 4.14074122$

> x:=2.5:y:=y7:y8:=y+(k1+2*k2+2*k3+k4)/6; $y8 := 4.36084711$

> x:=2.6:y:=y8:y9:=y+(k1+2*k2+2*k3+k4)/6; $y9 := 4.58095300$

> x:=2.7:y:=y8:y9:=y+(k1+2*k2+2*k3+k4)/6; $y9 := 4.58095300$

> x:=2.8:y:=y9:y10:=y+(k1+2*k2+2*k3+k4)/6; $y10 := 4.80105889$

method=rkf45 funksiyasida hisoblashning Maple dasturini quyidagicha tuzamiz (6.2–masala uchun).

\6.3b–M a p l e d a s t u r i:

```
> restart;
> dsol1 := diff(y(x),x) = cos(y(x)/sqrt(5)) + x;
```

$$dsol1 := \frac{d}{dx}\,y(x) = \cos\left(\frac{1}{5}\,y(x)\,\sqrt{5}\right) + x$$

```
> init1 := y(1.8)=2.6;
```

$$init1 := y(1.8) = 2.6$$

```
> Yechim:= dsolve({dsol1, init1}, numeric,
    method=rkf45,output=procedurelist):
```

> Yechim(2.1); $[x = 2.1,\ y(x) = 3.26190171988390883]$

> Yechim(2.2); $[x = 2.2,\ y(x) = 3.48315051875561643]$

> Yechim(2.3); $[x = 2.3,\ y(x) = 3.70451100671142083]$

> Yechim(2.4); $[x = 2.4,\ y(x) = 3.92601463954131973]$

6.1–laboratoriya ishi
bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi birinchi tartibli differetsial tenglamalar uchun Koshi masalasinining taqribiy yechimini h=0.1 qadam bilan Eyler va Runge–Kutta usullarida toping.

1) $\acute{o}' = x/(x+y)$ y(0)=1,[0,1].

2) $\acute{o}' - 2\acute{o} = 3\tilde{a}^{\tilde{o}}$, y(0,3)=1,415,[0,1;0,5].

3) $\acute{o}' = x + y^2$,y(0)=0, [0;0,3].

4) $y'=y^2-x^2$,y(0)=1, [1;2].

5) $y'=x^2+y^2$,y(0)=0.27, [0;1].

6) $y'+xy(1-y^2)=0$,y(0)=0.5,[0;1].

7) $y'=x^2-xy+y^2$,y(0)=0.1, [0;1].

8) $y'=(2y-x)/y$,y(0)=2, [1;2].

9) $y'=x^2+xy+y^2+1$,y(0)=0,[0;1].

10) $y'+y=x^3$,y(0)=–1, [1;2].

11) $y'=xy+e^y$,y(0)=0, [0;0.1].

12) $y'=2xy+x^2$,y(0)=0, [0;0.5].

13) $y'=e^x-y^2$,y(0)=0. [0;0.4].

14) $y'=x+\sin\frac{y}{3}$,y(0)=1, [0;1].

15) $y'=2x+\cos y$, y(0)=0, [0;0.1].
16) $y'=x^3+y^2$, y(0)=0.5, [0;0.5].
17) $y'=xy^3-y$, y(0)=1, [0;1].
18) $y'=y^2e^x-2y$, y(0)=1, [0;1].
19) $y'=\dfrac{1}{y^2-x}$, y(0)=0, [1;2].
20) $y'=\dfrac{x^2+1}{e^x}$ y(0)=1, [1;2].
21) $y'=e^x\cos y/x$ y(0)=1, [1,2].
22) $y'=e^x\sin y/x$, y(0)=1 , [1;2].
23) $y'\cos x-y\sin x=2x$, y(0)=0, [0;1].
24) $y'=y\operatorname{tg}x-\dfrac{1}{\cos^3 x}$, y(0)=0, [0;1].
25) $y'+y\cos x=\cos x$, y(0)=0, [0;1].
26) $y'=\dfrac{y}{x}+tg\dfrac{x}{y}$, y(0)=0, [0;1].
27) $y'=(1+\dfrac{y-1}{2x})^2$ y(0)=1, [1;2].
28) $xy'-\dfrac{y}{x+1}-x=0$, y(0)=1/2, [1;2].
29) $y'=\dfrac{y}{x}(1+\ln y-\ln x)$, y(0)=e, [1;2].
30) $y^3xdx=(x^2y+2)dy$, y(0.348)=2, [0;1].

6.3. Birinchi tartibli differensial tneglamalar sistemasi uchun Koshi masalasini taqribiy yechish

Maqsad: Birinchi tartihli differensial tneglamalar sistemasi uchun Koshi masalasini taqribiy yechishda Eyler va Runge–Kutta usulini qo‘llashni o‘rganish.

Reja: 6.3.1. Eyler usuli.

6.3.2. Runge–Kutta usuli.

6.3.1. Eyler usuli

Quyidagi

$$y'=f_1(x,y,z),\quad z'=f_2(x,y,z,) \qquad (6.9)$$

birinchi tartibli diffirintsial tenglamalar sistemasiga qo'yilgan

$$y(x_0)=y_0,\quad z(x_0)=z_0 \tag{6.10}$$

boshlang'ich shartlarni qanoatlantiruvchi $[a,b]$ oraliqdagi yechimning qiymatlarini topish uchun Eyler usulini qo'llaymiz.

(6.9) sistemasining $[a,b]$ oraliqdagi yechimini topish uchun oraliqni bo'linish nuqtalari

$$x_i=x_0+ih\ ,\ i=0,1,2,\ldots,n$$

ni topib, har bir tenglama uchun Eyler usulini qo'llaymiz.

$$y_{i+1}=y_i+hf_1(x_i,\ y_i,\ z_i), \tag{6.11}$$

$$z_{i+1}=z_i+hf_1(x_i,\ y_i,\ z_i),$$

Natijada differensial tenglamalar sistemasi yechimining taqribiy qiymatini topamiz.

$$y(x_i)=y_i z(x_i)=z_i i=1,2,3,\ldots,n$$

Quyidagi 6.2–masalada berilgan ikkinchi tartibli differensial tenglamani birinchi tartibli differensial tenglamalar sistemasiga keltirib yechimini topishni ko'rsatamiz.

6.2–masala. Quyidagi

$$y''+y'/x+y=0$$

differensial tenglamani

$$y(1)=0.77,\ y'(1)=-0.44$$

boshlang'ich shartlarini qanoatlantiruvchi $[1,1.5]$ oraliqdagi yechimi $h=0.1$ qadam bilan, Eyler usulida topilsin.

Yechish. Berilgan differensial tenglamada

$$y'=z,\quad y''=z'$$

almashtirish qilib, quyidagi birinchi tartibli differensial tenglamalar sistemasiga o'taamiz:

$$\begin{cases} y'=z \\ z'=-z/x-y \end{cases} \tag{6.12}$$

va boshlangich shartlari esa

$$y(1)=0.77,\ z(1)=-0.44$$

kabi yoziladi

Bu holda (6 9) differensial tenglamalar sistemasiga asosan (6.12) dan.

$$\begin{cases} f_1(x,y,z)=z=0.x+0.y+z \\ f_2(x,y,z)=-z/x-y \end{cases} \tag{*}$$

Endi hosil bulgan (*) differensial tenglamalar sistemaning yechimini Eyler usulida topish uchun quyidagi formulalar

$x_i = x_0 + ih,$

$y_{i+1} = y_i + hf_1(x_i, y_i, z_i);$

$z_{i+1} = z_i + hf_2(x_i, y_i, z_i);$

$i = 0, 1, 2, 3, \ldots$

bo'yicha quyidagilarni topamiz. Bu (*) tenglamalar sistemasi bo'yicha $x \neq 0$ b olganligi uchun $x_0 = 1$ deb olamiz:

$i = 1,\ x_0 = 1.05,\ y_0 = 0.77,\ z_0 = -0.44;$

$y_1 = y_0 + hf_1(x_0, y_0, z_0) = 0.77 + 0.05(z_0) = 0.748;$

$z_1 = z_0 + hf_2(x_0, y_0, z_0) = -0.44 + 0.05(-z_0/x_0 - y_0) = -0.455.$

$i = 2,\ x_1 = 1.1,\ y_1 = 0.748,\ z_1 = -0.455;$

$y_2 = y_1 + hf_1(x_1, y_1, z_1) = 0.748 + 0.05(-0.455) = 0.725;$

$z_2 = z_1 + hf_2(x_1, y_1, z_1) = -0.455 + 0.05(-0.455/1.1 - 0.748) = -0.470.$

Bu qoidani ketma–ket takrorlab tenglamalar sistemasi yechimining $i = 3,4,5$ –qiymatlarini hisoblab quyidagilarni topamiz:

$$y_3 = 0.702,\ z_3 = -0.484,$$
$$y_4 = 0.678,\ z_4 = -0.497,$$
$$y_5 = 0.658,\ z_5 = -0.508,$$
$$\ldots\ \ldots\ \ldots$$

Differensial tenglamalar sistemasiga qo'yilgan Koshi masalasi yechimini Eyler usuli bilan topishning Maple dasturini beramiz.

6.4–Maple dasturi:

```
> restart;
> dsys1:= {diff(y(x),x$1) = z(x),
    diff(z(x),x$1) = -z(x)/x-y(x),
   y(1) = 0.77, z(1) =-0.44};
```

$$dsys1 := \left\{ y(1) = 0.77, z(1) = -.44, \frac{d}{dx}\,y(x) = z(x), \frac{d}{dx}\,z(x) = -\frac{z(x)}{x} - y(x) \right\}$$

```
> dsol1:=dsolve(dsys1,numeric,output=listprocedure, range=1..2):
dsol1y:=subs(dsol1,y(x)):dsol1z:=subs(dsol1,z(x)):
> x:=1: dsol1y(x); dsol1z(x);
```

$$0.7700000000000 - .4400000000000$$

`> evalf(%,5);` $-.4400($

> x:=1.1: dsol1y(x); dsol1z(x);

0.7244058886424937 −.4713142811991297

> x:=1.2: dsol1y(x); dsol1z(x);

0.6758539649217231 −.4991223242264403

> x:=1.3: dsol1y(x); dsol1z(x);

0.6247043854650459 −.5232417771331504

> x:=1.4: dsol1y(x); dsol1z(x);

0.5713337128502281 −.5435179689664931

> x:=1.5: dsol1y(x); dsol1z(x);

0.5161330236349788 −.5598268829618819

6.3.2. Runge – Kutta usuli

Berilgan (6.9) differentsial tenglamalar sistemasini taqribiy yechimini topish uchun Runge-Kutta usulini sistemaning har bir tenglamasi uchun ko'llaymiz.

$$k_{1y} = hf_1(x_i, y_i, z_i),$$
$$k_{1z} = hf_2(x_i, y_i, z_i),$$
$$k_{2y} = hf_1(x_i + h/2, y_i + k_{1y}/2, z_i + k_{1z}/2),$$
$$k_{2z} = hf_2(x_i + h/2, y_i + k_{1y}/2, z_i + k_{1z}/2);$$
$$k_{3y} = hf_1(x_i + h/2, y_i + k_{2y}/2, z_i + k_{2z}/2),$$
$$k_{3z} = hf_2(x_i + h/2, y_i + k_{2y}/2, z_i + k_{2z}/2); \qquad (6.13)$$
$$k_{4y} = hf_1(x_i + h, y_i + k_{3y}, z_i + k_{3z}),$$
$$k_{4z} = hf_2(x_i + h, y_i + k_{3y}, z_i + k_{3z});$$
$$y_{i+1} = y_i + (k_{1y} + 2k_{2y} + 2k_{3y} + k_{4y})/6,$$
$$z_{i+1} = z_i + (k_{1z} + 2k_{2z} + 2k_{3z} + k_{4z})/6;$$
$$x_i = x_0 + ih; \quad i = 0,1,2,3,\ldots,n.$$

Bu qoida bilan tenglamalar sistemasini yechishda $i=1,2,\ldots,n$ lar uchun yuqoridagi usulni ketma-ket takrorlab sistema yechimining taqribiy qiymatlarini topamiz:

$$y_{i+1};\ z_{i+1};\ i=0,1,2,\ldots,n$$

Runge –Kutta usuli bilan yechim 0.001 aniqlikda topiladi.

6.2–masalani Runge – Kutta usulida yechish. Bu qoida bilan (6.12) sistemaning yechimini topish uchun (6.13) formulaga asosan:

$x_0=1.0$, $y_0=0.77$, $z_0=-0.44$, $i=0$ bo'lganda:

$$k_{1y} = hf_1(x_0, y_0, z_0) = 0.1(z_0) = 0.044$$

$$k_{1z} = hf_2(x_0, y_0, z_0) = 0.1(-z_0 / x_0 - y_0) = -0.0726$$

$$k_{2y} = hf_1(x_0 + h/2, y_0 + k_{1y}/2, z_0 + k_{1z}/2) = 0.1 f_1(0.05, 0.55, -0.4565) =$$
$$= 0.1(-0.44 - 0.0126/2) = -0.04763$$

$$k_{2z} = hf_2(x_0 + h/2, y_0 + k_{1y}/2, z_0 + k_{1z}/2) = 0.1 f_1(0.05, 0.55, -0.4565) =$$
$$= 0.1(0.4565/1.05 - 0.55) = 0.0115$$

$$k_{3z} = hf_2(x_0 + h/2, y_0 + k_{2y}/2, z_0 + k_{2z}/2) = 0.1 f_2(0.05, 0.7462, -0.4457) =$$
$$= 0.1(0.4457/1.05 - 0.7462) = -0.03217$$

$$k_{4y} = hf_1(x_0 + h, y_0 + k_{3y}, z_0 + k_{3z}) 0.1 f_1(0.1;\ 0.72543;\ -0.47217) =$$
$$= 0.1(-0.47217) = -0.047217$$

$$k_{4z} = hf_2(x_1 + h, y_1 + k_{3y}, z_1 + k_{3z}) = 0.1 f_2(0.1,\ 0.72543, -0.47217) =$$
$$= 0.1(0.47217/1.1 - 0.72543) = -0.029618$$

$$y_1 = y_0 + (k_{1y} + 2k_{2y} + 2k_{3y} + k_{4y})/6 =$$
$$= 0.77 + (-0.044 - 2\cdot 0.0476 - 2\cdot 0.04457 - 0.472)/6 =$$
$$= 0.77 - 0.02288 = 0.747$$

$$z_1 = z_0 + (k_{1z} + 2k_{2z} + 2k_{3z} + k_{4z})/6 =$$
$$= -0.44 + (-0.033 - 2(0.0115 + 0.03217) - 0.029618)/6 =$$
$$= -0.44 - 0.024993 = -0.464993$$

Demak berilgan differentsial tenglamalar sistemasi yechimining birinchi qiymatlari $y_1 = 0.747$, $z_1 = -0.4649$ bo`lar ekan.

Yechimni keyingi qiymatlarini topish uchun $i = 1$ bo`lganda $x_1 = 1.1$; $y_1 = 0.747$; $z_1 = -0.4649$ lar uchun yuqoridagi qoidani takrorlab y_2, z_2 larni topamiz va x.k.

Hisob $n = (b-a)/h = (1.5-1)/0.1 = 5$ bo`lganidan, $i = 5$ gacha davom etadi.

Differentsial tenglamalar sistemasiga qo`yilgan Koshi masalasi (9.3 – masala) yechimini Runge – Kutta usuli bilan topishning Maple dasturi.

6.5–M a p l e d a s t u r i:

1.*Masalani qo'yilishi*:

> diff(y(x),x$1)= z(x),
diff(z(x),x$1)=–z(x)/x–y(x);

$$dsys1 := \frac{d}{dx} y(x) = z(x),\ \frac{d}{dx} z(x) = K\ \frac{z(x)}{x}\ K\ y(x)$$

> init1 := y(1) = 0.77, z(1) =–0.44;

$$init1 := y(1) = 0.77,\ z(1) = K\ .44$$

2.*Masalani yechilishi:*

```
1)> dsol1 := dsolve(dsys1,numeric,
output=listprocedure,range=1..2):
dsol1y:= subs(dsol1,y(x)):
dsol1z:= subs(dsol1,z(x)):
> evalf(dsol1y(1),5), evalf(dsol1z(1),5);
```

$$0.77000, -.44000$$

```
> evalf(dsol1y(1.1),5), evalf(dsol1z(1.1),5);
```

$$0.72441, -.47131$$

```
> evalf(dsol1y(1.2),5), evalf(dsol1z(1.2),5);
```

$$0.67585, -.49912$$

```
> evalf(dsol1y(1.3),5), evalf(dsol1z(1.3),5);
```

$$0.62470, -.52324$$

```
> evalf(dsol1y(1.4),5), evalf(dsol1z(1.4),5);
```

$$0.57133, -.54352$$

```
> evalf(dsol1y(1.5),5), evalf(dsol1z(1.5),5);
```

$$0.51613, -.55983$$

```
2)> dsol2 := dsolve(dsys1, numeric, method=rkf45,
output=procedurelist):
> evalf(dsol2(1),5);
```

$$[x=1., y(x)=0.77000, z(x)=-.44000]$$

```
> evalf(dsol2(1.2),5);
```

$$[x=1.2, y(x)=0.67585, z(x)=-.49912]$$

```
> evalf(dsol2(1.3),5);
```

$$[x=1.3, y(x)=0.62470, z(x)=-.52324]$$

```
> evalf(dsol2(1.4),5);
```

$$[x=1.4, y(x)=0.57133, z(x)=-.54352]$$

```
> evalf(dsol2(1.5),5);
```

$$[x=1.5, y(x)=0.51613, z(x)=-.55983]$$

O'z–o'zini tekshirish uchun savollar

1. Birinchi tartibli oddiy, differentsial tenglamalar sistemasi uchun Koshi masalasining taqribiy yechimi Eyler usuli yordamida qanday topiladi?

2. Birinchi tartibli oddiy, differentsial tenglamalar sistemasi uchun Koshi masalasining taqribiy yechimi Runge–Kutta usuli yordamida qanday topiladi?

2. Taqribiy yechim xatoligini baholashni tushintirib byering.

3. Yuqori tartibli differentsial tenglama yoki tenglamalar sistemasi uchun Koshi masalasining taqribiy yechimi uchun yaqinlashishlarni hisoblash formulalarini yozing.

4. Rung-Kut usuli bilan tenglamaga kuyilgan Koshi masalasining taqribiy yechimi uchun yakinlashishlar qaysi formulalar yordamida xisoblanadi?

5.Taqribiy yechim xatoligini baholashning takroriy hisoblash qoidasini tushintirib bering.

6. Yuqori tartibli differentsial tenglama yoki tenglamalar sistemasi uchun Koshi masalasining taqribiy yechimi uchun yaqinlashishlarni Eyler usulida hisoblash formulalarini yozing.

6.2-laboratoriya ishi
bo'yicha mustaqil ishlash uchun topshiriqlar

1.Quyidagi birinchi tartibli differensial tenglamalar sistemasi uchun Koshi masalasini Eyler usulida taqribiy yechimini toping.

1. $\begin{cases} y' = \cos(y + 2z) + 3, \\ z' = 2/(x + 3x^2) + y + x, \end{cases}$ $y(0)=1,\ z(0)=0.05$

2. $\begin{cases} x' = \sin(2x^2) + t + y \\ y' = t + x - 3y^2 + 1 \end{cases}$ $x(0)=1,\ y(0)=0.5$

3. $\begin{cases} x' = \sqrt{(t^2 + 2x^2)} + y \\ y' = \cos(3y + x), \end{cases}$ $x0)=0.5,\ y(0)=1$

4. $\begin{cases} x' = \ln(6t + \sqrt{2t^2 + y^2}) \\ y' = (2t^2 + x^2) \end{cases}$ $x(0)=1, y(0)=0.5$

5. $\begin{cases} x' = e^{-(x^2 + y^2)} + 0.15t \\ y' = 6x^2 + y \end{cases}$ $x(0)=0.5, y(0)=1$

6. $\begin{cases} y' = z/x + \sqrt{(x + y)} \\ x' = 2z^2/(x(y-1)) + z/x \end{cases}$ $x(0)=1/3,\ y(0)=0$

7. $\begin{cases} y' = (z-y)x \\ z' = (z + y)x, \end{cases}$ $y(0)=1,\ z(0)=1$

8. $\begin{cases} y' = \cos(y + 2z) + 2 \\ z' = 2/(x + 2y^2) + x + 1 \end{cases}$ $y(0)=1,\ z(0)=1$

9. $\begin{cases} y' = e^{-(y^2+z^2)} + 2x \\ z' = 2y^2 + z, \end{cases}$ $y(0)=0.5,\ z(0)=1$

10. $\begin{cases} y' = y + 2z - \sin z^2 \\ z' = -y - 3z + x(e^{(x^2/2)} - 1) \end{cases}$ $y(0)=0,\ z(0)=0$

11. $\begin{cases} y' = -z + xy \\ z' = z^2/y \end{cases}$ $y(0)=1,\ z(0)=-0.5$

12. $\begin{cases} y' = (z-1)/z \\ z' = 1/(y-x), \end{cases}$ $y(0)=-1,\ z(0)=1$

13. $\begin{cases} y' = 2xy/(x^2-y^2-z^2) \\ z' = 2xz/(x^2-y^2-z^2), \end{cases}$ $y(0)=2,\ z(0)=1$

14. $\begin{cases} y' = z/(z-y)^2 \\ z' = y/(z-y)^2 \end{cases}$ $y(0)=1,\ z(0)=2$

15. $\begin{cases} y' = -y/x + xz \\ z' = -2y/x^3 + z/x \end{cases}$ $y(0)=1,\ z(0)=2$

16. $\begin{cases} dx/dt = x - 2y \\ dy/dt = x - y, \end{cases}$ $x(0)=1,\ z(0)=1$

17. $\begin{cases} dy/dx = z - y \\ dz/dx = -y - z \end{cases}$ $y(0)=2.23,\ z(0)=1.05$

18. $\begin{cases} dy/dx = 1 - 1/z \\ dz/dx = 1/(y-x) \end{cases}$ $y(0)=2.12,\ z(0)=1.13$

19. $\begin{cases} dy/dx = x/yz \\ dz/dx = x/y^2 \end{cases}$ $y(0)=1,\ z(0)=2$

20. $\begin{cases} dy/dx = -z \\ dz/dx + 4y = 0, \end{cases}$ $y(0)=1.2,\ z(0)=-2$

21. $\begin{cases} y' = z/x \\ z' = 2z^2/(x(y-1)) + z/x \end{cases}$ $y(0)=0,\ z(0)=1/3$

22. $\begin{cases} y' = (z-y)x \\ z' = (z+y)x \end{cases}$ $y(0)=1 z(0)=1$

23. $\begin{cases} y' = \cos(y+2z)+2 \\ z' = 2/(x+2y^2)+x+1 \end{cases}$ $y(0)=1,\ z(0)=0.05$

24. $\begin{cases} y' = e^{-(y^2+z^2)} + 2x \\ z' = 2y^2 + z \end{cases}$ $y(0)=0.5,\ z(0)=1$

25. $\begin{cases} y' = (z-y)y \\ z' = (z+y)z \end{cases}$ $y(0)=1.05\ \ z(0)=2$

2. Quydagi ikkinchi tartibli differensial tenglamalar uchun Koshi masalasining yechimini topishda, ikkinchi tartibli differensial tenglamani birinchi tartibli differensial tenglamalar sistemasiga keltirib Eyler usulida taqribiy yechimini toping.

T/p	Tenglama	$y(0)$	$y'(0)$	oraliq	qadam
1	$y''=1/\cos x-y$	1	0	[0,0.5]	0.1
2	$(1+x^2)y''+(y')^2+1=0$	1	1	[0,0.5]	0.05
3	$y''+2y'+2y=2e^{-x}\cos x$	1	0	[0,0.5]	0.05
4	$y''+4y=e^{3x}(13x-7)$	0	–1	[0,1]	0.1
5	$y''+4y'+4y=0$	1	–1	[0,1]	0.1
6	$y''-y=\sin x+\cos 3x$	1.8	–0.5	[0,2]	0.2
7	$y''-3y'=e^{5x}$	2.2	0.8	[0,02]	.02
8	$y''+y=\cos x$	0.8	2	[0,1]	0.8
9	$y''-y'-6y=2e^{4x}$	1.433	0.367	[0,1]	0.1
10	$y-2y'+y=5xe^x$	1	2	[0,1]	0.1
11	$y''+y'-6y=3x^2-x$	–0.9	3.2	[0,1]	0.1
12	$8y''+2y'-3y=x+5$	1/9	–7/12	[0,1]	0 1
13	$y''-4y'+5y=3x$	1.48	3.6	[0,0.5]	0.05
14	$y''-5y'+6y=e^x$	0	0	[0,0.2]	0.02
15	$y''-3y'+2y=x^2+3x$	5.1	4.2	[0,1]	0.1
16	$y''+(1/x)y'-(1/x)y=8x$	4	4	[1, 1.5]	0.05
17	$x^2y''+xy'=0$	5	–1	[1, 1.5]	0.05

18	$y''-2y'+y=xe^x$	1	2	[0,05]	0.05
19	$y''-3y'+2y=2sinx$	2	3.2	[0,1]	0.1
20	$x^2y''+2.5y'x-y=0$	2	3.5	[0,1]	0.1
21	$4xy''+2y'+y=0$	1.3817	–0.1505	[1,2]	0.1
22	$x^2y''-4xy'+6y=2$	1.43	2.3,	[1,2]	0.1
23	$y''-y=e^{2x}(x-1)$	11/9	–11/9	[0,1]	0.1
24	$y''-3y'-2y=cos2x$	1.95	2.7	[0,05]	0.05
25	$y''-0.5y'-0.5y=3e^{x^2}$	–4	–2.5	[0,1]	0.1
26	$y''+4y'=sinx+sin2x$	1	–23/12	[0,1]	0.1
27	$y''+y=x^2-x+2$	1	0	[0,1]	0.1
28	$x^2y''-2y=0$	5/6	2/3	[1,2]	0.1
29	$y''+4y'+4y=2x-3$	–1/4,	–1/2	[0,05]	0.05
30	$y''+y=x^2-x+2$	1	0	[0,1]	0.1

7-LABORATORIYA ISHI
Xususiy hosilali differensial tenglamalarni taqribiy yechimini topish
Maple dasturining buyruqlari:

> u:=array(1..6,1..6)– yechimi funksiya matritsasining o'lchami

> for i to n do for j to m+1 do x:=a+(j–1)*h;
u[n+1,j]:=fAD(x); u[1,j]:=fBC(x); od; od; evalm(u)– yechimi funksiyasining chegaraviy shartlar bo'yicha qiymatlarini hisoblash;

> with(linalg):transpose(UN) –yechimi funksiya matritsasini transponirlash.

Maqsad: Xususiy hosilali differensial tenglamalarni taqribiy yechimini topishni o'rganish.

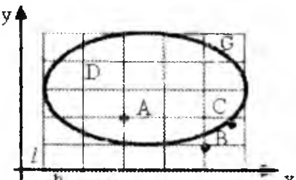
Reja: 7.1. Chekli ayirmalar yoki to'r usuli.
7.2. Elliptik turdagi tenglamaga qo'yilgan Dirixle masalasi uchun to'r usuli.

7.1. Chekli ayirmalar yoki to'r usuli

Chekli ayirmalar usuli xususiy hosilali tenglamalarning sonli yechimini topishda eng qulay usullardan biridir.

Biz quyida eng sodda xususiy hosilali tenglamalar uchun qo 'yilga aralash masalalarni to'r usulida taqribiy yechimini topishni o 'rganamiz.

Bu usul asosida xususiy hosilalarni chekli ayirmalar nisbati bilan almashtirish qoidasi yotadi.



7.1-rasm

Aytaylik, Oxy koordinatalar tekisligida G chiziq bilan chegaralangan yopiq D soha berilgan bo'lsin. D sohani kesib o'tuvchi o'qlarga parallel bo'lgan to'g'ri chiziqlar oilasini quramiz :

$$x_i = x_0 + ih,$$
$$i = 0, \pm 1, \pm 2, \ldots$$
$$y_k = y_0 + kh$$
$$k = 0, \pm 1, \pm 2, \ldots$$

Bu to'g'ri chiziqlarning kesishishidan hosil bo'lgan to'rdagi nuqtalarni *tugunlar* deb ataladi. Hosil bo'lgan to'rda Ox yoki Oy koordinata o'qlari yo'nalishida h yoki l masofada joylashgan ikki tugunni *qo'shni tugun* deb ataladi.

$D+G$ sohaga tegishli bo'lgan va sohaning chegarasi G dan, bir qadamdan kichik masofada turgan tugunlarni ajratamiz.

Sohaning biror tuguni va unga qo'shni bo'lgan to'rtta tugun, ajratilgan tugunlar to'plamiga tegishli bo'lsa, bundiy tugunlarni *ichki tugunlar* deb ataladi. (7.1–rasm, *A* tugun). Ajratilganndan qolganlari *chegara tugunlari* deb ataladi(7.1–rasm, *B*, *C* tugunlar).

To'rning tugunlaridagi тоma'lum $u = u(x, y)$ funksiyaning qiymatini

$u_{ik} = u(x_0 + ih, y_0 + kl)$ kabi belgilaymiz. Har bir $(x_0 + ih, y_0 + kl)$ ichki nuqtalardagi xususiy hosilalarni chekli ayirmalar nisbati bilan quyidagicha almashtiramiz:

$$\left(\frac{\partial u}{\partial x}\right)_{ik} \approx \frac{u_{i+1,k} - u_{i-1,k}}{2h} \qquad (7.1)$$

$$\left(\frac{\partial u}{\partial y}\right)_{ik} \approx \frac{u_{i,k+1} - u_{i,k-1}}{2l}$$

Chegaraviy nuqtalarda esa aniqligi kamroq bo'lgan quyidagi formular bilan almashtiramiz:

$$\left(\frac{\partial u}{\partial x}\right)_{ik} \approx \frac{u_{i+1,k} - u_{ik}}{h} \qquad (7.2)$$

$$\left(\frac{\partial u}{\partial y}\right)_{ik} \approx \frac{u_{ik,k+1} - u_{ik}}{l}$$

Xuddi shuningdek, ikkinchi tartibli xususiy hosilarni quyidagicha almashtpramiz:

$$\left(\frac{\partial^2 u}{\partial x}\right)_{ik} \approx \frac{u_{i+1,k} - 2u_{ik} + u_{i-1,k}}{h^2} \qquad (7.3)$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{ik} \approx \frac{u_{i,k+1} - 2u_{ik} + u_{i,k-1}}{l^2}$$

Yuqorida ketirilgan almatiririshlar xususiy hosilali tenglamalarning o'rniga chekli ayrimali tenglamalar sistemasini yechishga olib keladi.

7.2. Elliptik tipdagi tenglamaga qo'yilgan Dirixle masalasi uchun to'r usuli.

Birinchi chegaraviy masala yoki *Puasson tenglamasi*:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \qquad (7.4)$$

uchun *Dirixle masalasi* quyidagicha qo'yiladi. (7.4) tenglamani va *D* sohaning ichki nuqtalarida va uning *G*– chegarasida esa

$$u|_G = \varphi(x,y)$$

shartni qanoatlantiruvchi $u=u(x,y)$ funksiya topilsin.

Mos ravishda x va y o'qlarida h va l qadamlarni tanlab,

$$x_i = x_0 + ih, \quad (i = 0, \pm 1, \pm 2, \ldots)$$

$$y_k = y_0 + kl, \quad (k = 0, \pm 1, \pm 2, \ldots)$$

to'g'ri chiziqlar yordamida to'r quramiz va sohaning ichki tugunlaridagi

$$\frac{\partial^2 u}{\partial x^2}, \ \frac{\partial^2 u}{\partial y^2}$$

hosilarni (7.3) formula asosida almashtirib (6.4) tenglamani quyidagi chekli ayirmali tenglamalar ko'rinishga keltiramiz:

$$\frac{u_{i+1,k} - 2u_{ik} + u_{i-1,k}}{h^2} + \frac{u_{i,k+1} - 2u_{ik} + u_{i,k-1}}{l^2} = f_{ik} \qquad (7.5)$$

bu yerda $f_{ik} = f(x_i, y_k)$ (7.5) tenglama sohaning chegaraviy nuqtalaridagi u_{ik} qiymatlari bilan birgalikda (x_i, y_k) tugunlaridagi $u(x,y)$ funksiya qiymatlariga nisbatan chiziqli algebraik tenglamalar sistemasini hosil qiladi. Bu sistema to'g'ri to'rtburchakli sohada va $l=k$ bulganda eng sodda ko'rinishga keladi Bu holda (7.5) tenglama quyidagicha yoziladi.

$$u_{i+1,k} + u_{i-1,k} + u_{i,k+1} + u_{i,k-1} - 4u_{ik} = h^2 f_{ik} \qquad (7.6)$$

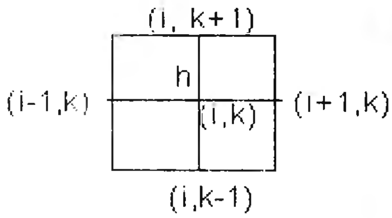
Chegaraviy tugunlardagi qiymatlar esa chegaraviy funksiya qiymatlariga teng bo'ladi. Agar (7.4) tenglamada $f(x,y)=0$ bo'lsa,

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

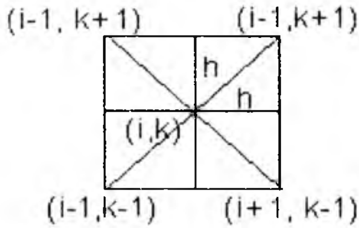
Laplas tenglamasi hosil bo'ladi. Bu tenglamaning chekli ayirmalar tenglamasi quyidagicha:

$$u_{i,k} = \frac{1}{4}(u_{i+1,k} + u_{i-1,k} + u_{i,k+1} + u_{i,k-1}) \qquad (7.7)$$

Bu (7.6) va (7.7) tenglamalarni 7.2–rasmdagi tugunlar siemasidan foydaniladi. Bundan buyon rasmlardarda (x_i, y_j) tugunlarni ularning indekslari bilan almashtirib yozamiz.



7.2–rasm.



7.3–rasm.

Ba'zan 7.3–rasmdagi kabi tugunlar sxemasidan foydalanish qulay bo'ladi. Bu holda chekli ayrimalar bo'yicha Laplas tenglamasi quydagicha yoziladi.

$$u_{i,k} = \frac{1}{4}(u_{i-1,k-1} + u_{i+1,k-1} + u_{i-1,k+1} + u_{i+1,k+1}) \tag{7.8}$$

(7.4) tenglamasi uchun esa:

$$u_{i,k} = \frac{1}{4}(u_{i-1,k-1} + u_{i+1,k-1} + u_{i-1,k+1} + u_{i+1,k+1}) + \frac{h^2}{2} f_{i,k} \tag{7.9}$$

Differensial tenglamalarni chekli ayrimalar bilan almatirish xatoligi ya'ni (7.6) tenglama uchun qoldiq xad $R_{i,k}$ quyidagicha baholanadi.

$$R_{i,k} < \frac{h^2}{6} M_4$$

bu yerda

$$M_4 = \max_G \left\{ \left| \frac{\partial^4 u}{\partial x^4}, \frac{\partial^4 u}{\partial y^4} \right| \right\} \tag{7.10}$$

Ayrimalar usuli bilan topilgan taqribiy yechim xatoligi uchta xatoligidan kelib chiqadi:

1) differensial tenglamalarni ayrimalar bilan almashtirishdan;
2) chegaraviy shartni approksimasiya qilishdan;
3) hosil bo'lgan ayrimali tenglamalarni taqribiy yechishlardan.

7.1–**masala.** Quyidagi Laplas tenglamasi

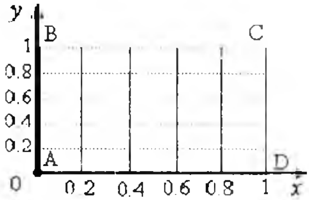
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

uchun uchlari $A(0;0)$, $B(0;1)$, $C(1;1)$, $D(1;0)$ nuqtalarda bo'lgan kvadratga Dirixle masalasi shartlari:

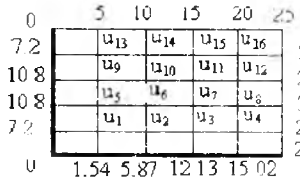
$$u|_{AB} = 45y(1-y);\ u|_{BC} = 25x;\ u|_{CD} = 25;\ u|_{AD} = 25x\sin\frac{\pi x}{2};$$

bo'lganda, $h=0.2$ qadam bilan to'r usulida yechimini 0.01 aniqlikda toping .

Yechish: 1. Yechim sohasini $h=0.2$ qadam bilan kataklarga ajratamiz va sohaning chegara(7.4–rasm) nuqtalarida (7.10) ga asosan noma'lum $u(x,y)$ funksiya qiymatlarini hisoblaymiz.



7.4–rasm.



7.5–rasm.

$u(x,y)$ funksiya qiymatlarini soha chegaralarida hisoblash:

1) AB tomondagi $u(x,y)=45y(1-y)$ funksiyaning qiymatlari:
$u(0,0)=0$,$u(0,0.2)=7.2$, $u(0,0.4)=10.8$,
$u(0,0.6)=10.8$, $u(0,0.8)=7.2$, $u(0,1)=0$.

2) AB tomondagi $u(x,y)=25x$ funksiyaning qiymatlari:
$u(0.2,1)=5$, $u(0.4,1)=10$, $u(0.6,1)=15$,
$u(0.8,1)=20$. $u(1,1)=17$.

3) CD tomondagi $u(x,y)=25$ funksiyaning qiymatlari:
$u(1,0.8)=u(1,0.6)=u(1,04)$ AD tomondagi $u(x,y)=25\sin\frac{\pi x}{2}$ funksiyaning qiymatlari:
$u(0.2,0)=1,545$, $u(0.4,0)=5,878$.
$u(0.6,0)=12,135$, $u(0.8,0)=19,021$.

2. Yechim soha ichidagi nuqtalarda(7.5–rasm) izlanayotgan funksiya qiymatlarini topish uchun, Laplas tenglamasi uchun chekli ayirmalarni qo'llashdan hosil bo'lgan (7.7):

$$u_{ij}=u(x_i y_i)=\frac{1}{4}(u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1})$$

formuladan quyidagicha foydalanamiz:

$$u_1=\frac{1}{4}(7,2+1,545+u_2+u_5);\qquad u_2=\frac{1}{4}(5,878+u_1+u_3+u_6).$$

$$u_3=\frac{1}{4}(12,135+u_2+u_4+u_7);\qquad u_4=\frac{1}{4}(19,021+25+u_3+u_8)$$

$$u_5 = \frac{1}{4}(10{,}8 + u_1 + u_6 + u_9); \qquad u_6 = \frac{1}{4}(u_2 + u_5 + u_7 + u_{10}),$$

$$u_7 = \frac{1}{4}(u_3 + u_6 + u_8 + u_{11}); \qquad u_8 = \frac{1}{4}(25 + u_4 + u_7 + u_{10}),$$

$$u_9 = \frac{1}{4}(10{,}8 + u_5 + u_{10} + u_{13}); \qquad u_{10} = \frac{1}{4}(u_6 + u_9 + u_{11} + u_{14}),$$

$$u_{11} = \frac{1}{4}(u_7 + u_{10} + u_{12} + u_{15}); \qquad u_{12} = \frac{1}{4}(25 + u_8 + u_{11} + u_{16}),$$

$$u_{13} = \frac{1}{4}(7{,}2 + 5 + u_9 + u_{16}); \qquad u_{14} = \frac{1}{4}(10 + u_{10} + u_{13} + u_{15}),$$

$$u_{15} = \frac{1}{4}(15 + u_{11} + u_{14} + u_{16}); \qquad u_{16} = \frac{1}{4}(20 + 25 + u_{12} + u_{15})$$

Bu hosil bo‘lgan sistemani Zeydelning iterasiya usuli bilan yechib

$$u_i^{(0)},\ u_i^{(1)},\ u_i^{(2)},\ldots,u_i^{(k)},\ldots$$

ketma -ketlikni tuzamiz va yaqinlashishni 0,01 aniqlik bilan olamiz . Bu ketma -ketlik elementlarini quyidagi bog‘lanishlardan topamiz:

$$u_1^{(k)} = \frac{1}{4}(8{,}745 + u_2^{(k-1)} + u_5^{(k-1)}); \qquad u_2^{(k)} = \frac{1}{4}(5{,}878 + u_1^{(k)} + u_3^{(k-1)} + u_6^{(k-1)})$$

$$u_3^{(k)} = \frac{1}{4}(12{,}135 + u_2^{(k)} + u_4^{(k-1)} + u_7^{(k-1)}); \qquad u_4^{(k)} = \frac{1}{4}(44{,}021 + u_3^{(k)} + u_8^{(k-1)})$$

$$u_5^{(k)} = \frac{1}{4}(10{,}8 + u_1^{(k)} + u_6^{(k)} + u_9^{(k-1)}); \qquad u_6^{(k)} = \frac{1}{4}(u_2^{(k)} + u_5^{(k)} + u_7^{(k-1)} + u_{10}^{(k-1)}),$$

$$u_7^{(k)} = \frac{1}{4}(u_3^{(k)} + u_6^{(k)} + u_8^{(k-1)} + u_{11}^{(k-1)}); \qquad u_8^{(k)} = \frac{1}{4}(25 + u_4^{(k)} + u_7^{(k)} + u_{12}^{(k-1)}),$$

$$u_9^{(k)} = \frac{1}{4}(10{,}8 + u_5^{(k)} + u_{10}^{(k-1)} + u_{13}^{(k-1)}); \qquad u_{10}^{(k)} = \frac{1}{4}(u_6^{(k)} + u_9^{(k)} + u_{11}^{(k-1)} + u_{14}^{(k-1)}),$$

$$u_{11}^{(k)} = \frac{1}{4}(u_7^{(k)} + u_{10}^{(k)} + u_{12}^{(k-1)} + u_{15}^{(k-1)}); \qquad u_{12}^{(k)} = \frac{1}{4}(25 + u_8^{(k)} + u_{11}^{(k)} + u_{16}^{(k-1)})$$

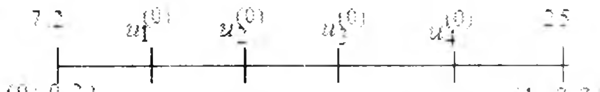
$$u_{13}^{(k)} = \frac{1}{4}(12{,}2 + u_9^{(k)} + u_{14}^{(k-1)}); \qquad u_{14}^{(k)} = \frac{1}{4}(10 + u_{10}^{(k)} + u_{13}^{(k)} + u_{15}^{(k-1)}),$$

$$u_{15}^{(k)} = \frac{1}{4}(15 + u_{11}^{(k)} + u_{14}^{(k)} + u_{16}^{(k-1)}); \qquad u_{16}^{(k)} = \frac{1}{4}(45 + u_{12}^{(k)} + u_{15}^{(k)}).$$

Yuqoridagi formulalar yordamida yechimni topish uchun boshlang'ich $u_i^{(0)}$ qiymatlarni aniqlash kerak bo'ladi. Shu boshlang'ich taqribiy yechimni aniqlash uchun $u(x,y)$ funksiya soha gorizantallari bo'yicha tekis taqsimlangan deb hisoblaymiz. Chegara nuqtalari (0;0.2) va (1:0.2) bo'lgan gorizantal kesmani 5 ta bo'lakka bulib, ularni boshlsng'ich va oxirgi nuqtalardagi $u(x,y)$ funksiya qiymatlari bo'yicha

$$K_1=(25-7.2)/5=3.56$$

qadam bilan uning ichki nuqtalardagi funksiya qiymatlari $u_1^{(0)}, u_2^{(0)}, u_3^{(0)}, u_4^{(0)}$ ni quyidagicha topamiz.



$$u_1^{(0)} = 7,2 + K_1 = 7,2 + 3,56 = 10,76$$

$$u_2^{(0)} = u_1^{(0)} + K_1 = 10.76 + 3,56 = 14,32$$

$$u_3^{(0)} = u_2^{(0)} + K_1 = 14,32 + 3,56 = 17,88$$

$$u_4^{(0)} = u_3^{(0)} + K_1 = 17,88 + 3.56 = 21,44$$

Shuningdek qolgan gorizontallarda ham ularga mos $K_2=K_3=2.84$, $K_4=K_1=3.56$ qadamlarini aniqlab ichki nuqtalardagi funksiya qiymatlarini topamiz va quyidagi boshlang'ich yaqinlashish bo'yicha yechim jadvalni tuzamiz:

1	0	5	10	15	20	25
0,8	7,2	10,76	14.32	17,88	21,44	25
0,6	10,8	13,64	16,48	19,32	22,16	25
0.4	10,8	13,64	16,48	19,32	22,16	25
0,2	7,2	10,76	14,32	17,88	21,44	25
0	0	1,545	5,878	12,135	19,021	25
y/x	0	0,2	0,4	0,6	0.8	1

Bu boshlang'ich yaqinlashishdan foydalanib hisoblash jarayonidagi birinchi, ikkinchi va xokazo yaqinlashishlarni aniqlash va jadvalini tuzish mumkin. Natija 0.01 aniqlik bilan 15-yaqinlashish bo'yicha hisoblangan quyidagi yechim jadvalini topamiz:

1	0	5	10	15	20	25
0,8	7,2	8,63	11,77	15,80	20,30	25
0,6	10,8	10,56	12,64	16,14	20,40	25
0,4	10,8	10,17	12,10	15,69	20,18	25
0,2	7,2	7,20	9,88	14,34	19,64	25
y/x	0	0,2	0,4	0,6	0,8	1

Laplas tenglamasi uchun Dirixle masalasini chekli ayirmalar usulida yechishning Maple dasturini quyidagicha tuzamiz.

7.1–Maple dasturi:

```
> restart;
> fAB:=y->45*y*(1-y);
```
$$fAB := y \to 45\, y\, (1 - y)$$
```
> fCD:=y->25+0*y;
```
$$fCD := y \to 25 + 0\, y$$
```
> fBC:=x->25*x;
```
$$fBC := x \to 25\, x$$
```
> fAD:=x->25*x*sin(3.14159*x/2);
```
$$fAD := x \to 25\, x \sin\left(\frac{3.14159x}{2}\right)$$
```
> n:=5:  m:=5: a:=0: b:=1: c:=0: d:=1:
h:=(b-a)/n: g:=(d-c)/m: e:=0.01:
>  u:= array(1..6,1..6):
>  for i to n do for j to m+1  do
x:=a+(j-1)*h;  u[n+1,j]:=fAD(x); u[1,j]:=fBC(x);
od;  od;  evalm(u);
```
$$\begin{bmatrix} 0 & 5 & 10 & 15 & 20 & 25 \\ u_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & u_{2,5} & u_{2,6} \\ u_{3,1} & u_{3,2} & u_{3,3} & u_{3,4} & u_{3,5} & u_{3,6} \\ u_{4,1} & u_{4,2} & u_{4,3} & u_{4,4} & u_{4,5} & u_{4,6} \\ u_{5,1} & u_{5,2} & u_{5,3} & u_{5,4} & u_{5,5} & u_{5,6} \\ 0. & 1.545083710 & 5.877848230 & 12.13524790 & 19.02112376 & 25.00000000 \end{bmatrix}$$
```
> for i  to n do for j to m+1 do
  y:=c+(i-1)*g : u[i,1]:=fAB(y): u[i,m+1]:=fCD(y):
od; od; evalm(u);
```

0	5	10	15	20	25
$\frac{36}{5}$	$u_{2,2}$	$u_{2,3}$	$u_{2,4}$	$u_{2,5}$	25
$\frac{54}{5}$	$u_{3,2}$	$u_{3,3}$	$u_{3,4}$	$u_{3,5}$	25
$\frac{54}{5}$	$u_{4,2}$	$u_{4,3}$	$u_{4,4}$	$u_{4,5}$	25
$\frac{36}{5}$	$u_{5,2}$	$u_{5,3}$	$u_{5,4}$	$u_{5,5}$	25
0.	1.545083710	5.877848230	12.13524790	19.02112376	25.00000000

> for i from 2 by 1 to n do
for j from 2 by 1 to m+1 do
u[i,j]:=u[1,j]-(u[n+1,j]-u[1,j])*i/n; od; od; evalm(u); evalf(%,4);

0	5	10	15	20	25
$\frac{36}{5}$	6.381966516	11.64886071	16.14590084	20.39155050	25.
$\frac{54}{5}$	7.072949774	12.47329106	16.71885126	20.58732574	25.
$\frac{54}{5}$	7.763933032	13.29772142	17.29180168	20.78310099	25.
$\frac{36}{5}$	8.454916290	14.12215177	17.86475210	20.97887624	25.
0.	1.545083710	5.877848230	12.13524790	19.02112376	25.00000000

0.	5.	10.	15.	20.	25.
7.200	6.382	11.65	16.15	20.39	25.
10.80	7.073	12.47	16.72	20.59	25.
10.80	7.764	13.30	17.29	20.78	25.
7.200	8.455	14.12	17.86	20.98	25.
0.	1.545	5.878	12.14	19.02	25.00

> for i from 2 by 1 to n–1 do
for j from 2 by 1 to m–1 do u[i,j]:=(u[i–1,j]+u[i+1,j]+u[i,j–1]+u[i,j+1])/4; od; od; evalf(evalm(u),4);

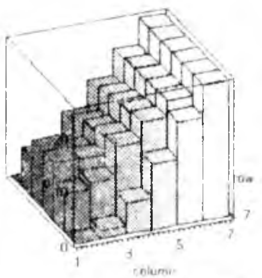
0.	5.	10.	15.	20.	25.
7.200	8.370	11.78	15.96	20.39	25.
10.80	10.64	13.19	16.75	20.59	25.
10.80	10.90	13.87	17.32	20.78	25.
7.200	8.455	14.12	17.86	20.98	25.
0.	1.545	5.878	12.14	19.02	25.00

Dirixle masalasini chekli ayirmalar usulidagi yechishning gistogrammasi va sirt grafigi:

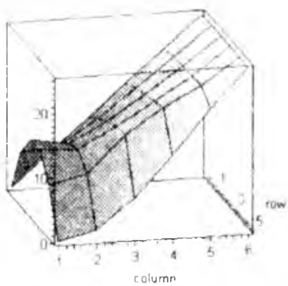
> **with(plots):with(LinearAlgebra):**

> **matrixplot(u,heights=histogram,axes=boxed);** (7.6–rasm)

> **matrixplot(u,axes=boxed);** (7.7–rasm)



7.6–rasm.



7.7–rasm

O'z-o'zini tekshirish uchun savollar

1. Berilgan sohani to'r bilan ko'lash, to'r tugunlarining turlari, tugun nuqtalar aniqlash.

2. Xususiy hosilalarni chekli ayirmalar nisbati bilan almashtirishlar asosida to'r usuli moxiyatini tushuntiring.

3. Laplas yoki Puasson tenglamasi uchun Dirixle masalasining taqribiy yechimi to'r usuli yordamida qanday topiladi?

4. Taqribiy yechim xatoligini baholash formulasini yozing.

7.1–laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi Laplas tenglamasi $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ uchun учун Dirixli masalasini to'r usulida, uchlari $A(0;0)$, $B(0;1)$, $C(1;1)$, $D(1;0)$ nuqtalarda bo'lgan kvadratdagi taqribiy yechimni, $h=0.2$ qadam bilan toping.

№	$u\|_{AB}$	$u\|_{BC}$	$u\|_{CD}$	$u\|_{AD}$
1	$30y$	$30(1-x^2)$	0	0
2	$20y$	$30\cos(\pi x/2)$	$30\cos(\pi y/2)$	$20x^2$
3	$50y(1-y^2)$	0	0	$50\sin \pi x$
4	$20y$	20	$20y^2$	$50x(1-x)$
5	0	$50x(1-x)$	$50y(1-y^2)$	$50x(1-x)$
6	$30\sin \pi y$	$20x$	$20y$	$30x(1-x)$
7	$30(1-y)$	$20\sqrt{x}$	$20y$	$30(1-x)$
8	$30\sin \pi y$	$30\sqrt{x}$	$30y^2$	$50\sin \pi x$
9	$40y^2$	40	40	$40\sin(\pi x/2)$
10	$50y^2$	$50(1-x)$	0	$60x(1-x^2)$
11	$20y^2$	20	$20y$	$10x(1-x)$
12	$40\sqrt{y}$	$40(1-x)$	$20y(1-y)$	0
13	$20\cos(\pi y/2)$	$30x(1-x)$	$30y(1-y^2)$	$20(1-x^2)$
14	$30y^2(1-y)$	$50\sin \pi x$	0	$10x^2(1-x)$
15	$20y$	$20(1-x^2)$	$30\sqrt{y(1-y)}$	0
16	$30(1-x^2)$	$30x$	30	30
17	$30\cos(\pi y/2)$	$30x^2$	$30y$	$30\cos(\pi x/2)$
18	0	$50\sin \pi x$	$50y(1-y^2$	0
19	$20\sqrt{y}$	20	$20y^2$	$40x(1-x)$
20	$50y(1-y$	$20x^2(1-x)$	0	$40x(1-x^2)$
21	$20\sin \pi y$	$30x$	$30y$	$20x(1-x)$
22	$40(1-y)$	$30\sqrt{x}$	$30y$	$40(1-x)$
23	$20\sin \pi y$	$50\sqrt{x}$	$50y^2$	$20\sin \pi x$
24	40	40	$40y^2$	$40\sin(\pi x/2)$
25	$30y^2$	$30(1-x)$	0	$40x^2(1-x)$
26	$25y^2$	25	$25y$	$20x(1-x)$

27	$15\sqrt{y}$	$15(1-x)$	$30y(1-y)$	0
28	$30\cos\frac{\pi y}{2}$	$20x(1-x)$	$25y(1-y^2)$	$30(1-x^2)$
29	$10y^2(1-y)$	$30\sin \pi x$	0	$15x(1-x^2)$
30	$25y$	$25(1-x^2)$	$30\sqrt{y}(1-y)$	0

7.3. Parabolik turdagi xususiy hosilali differensial tenglama uchun aralash masalani to'r usulida yechish

7.3.1. Parabolik turdagi tenglamasi uchun to'r usuli.

7.3.2. Bir jinisli bo'lmagan parabolik tenglama uchun to'r usuli.

7.3.1. Parabolik turdagi tenglamasi uchun to'r usuli.

Parabolik turdagi issiklik o'tkazuvchanlik tenglamasi uchun aralash masalani ko'ramiz. Ya'ni

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}. \tag{7.11}$$

tenglamani

$u(x,0)=f(x),(0<x<s)$ (7.12)

boshlang'ich shartni va

$u(0,t)=\varphi(t)$, $u(s,t)=\psi(t)$ (7.13)

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ funksiyani topish masalasi bilan shug'ullanamiz.

Yuqoridagi (7.11)-(7.13) masalaga, xususan uzunligi s bo'lgan bir jinsli sterjenda issiqlik tarqalish masalasini ko'rish mumkin.

(7.11) tenglamada $a=1$ deb uni

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

ko'rinishga keltirish mumkin.

Yarim tekislik $t\geq 0, 0\leq x\leq s$ da (7.8-rasm) koordinata o'qlariga parallel to'g'ri chiziqlar:

$$x=ih,\ i=0,1,2,\ldots \quad t=jl,\ j=0,1,2,\ldots$$

oilasini quramiz. $x_i=ih$ va $t_j=jl$ deb, $u_{ij}=u(i,j)=u(x_i,t_j)$ belgilash bilan va har bir ichki (x_i,t_j) tugundagi $\frac{\partial^2 u}{\partial x^2}$ hosilani taqribiy ayrimalar nisbatida quydagicha yozamiz:

$$\left(\frac{\partial^2 u}{\partial x^2}\right)\approx\frac{u_{i+1,j}-2u_{ij}+u_{i-1,j}}{h^2} \tag{7.14}$$

$\frac{\partial u}{\partial t}$ hosilani esa, quyidagi nisbatlardan biri bilan almashtiramiz:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} \approx \frac{u_{i,j+1} - u_{ij}}{e} \qquad (7.15)$$

$$\left(\frac{\partial u}{\partial u}\right)_{ij} \approx \frac{u_{ij} - u_{i,j-1}}{e}. \qquad (7.16)$$

Bu holda (7.11) tenlamani($a=1$ bo'lganda) quyidagi 2 turdagi chekli ayrimali tenglamalar ko'rinishda yozish mumkin.

$$\frac{u_{i,j+1} - u_{ij}}{e} = \frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{h^2} \qquad (7.17)$$

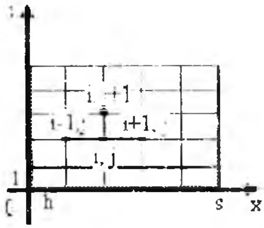
$$\frac{u_{ij} - u_{i,j-1}}{e} = \frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{h^2} \qquad (7.18)$$

Bu tenglamalarda $\sigma = l/h^2$ kabi belgilab, ularni quydagicha yozamiz:

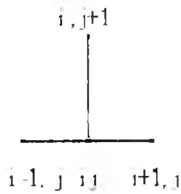
$$u_{i,j+1} = (1-2\sigma)u_{ij} + \sigma(u_{i+1,j} + u_{i-1,j}) \qquad (7.19)$$

$$(1+2\sigma)u_{ij} - \sigma(u_{i+1,j} + u_{i-1,j}) - u_{i,j-1} = 0 \qquad (7.20)$$

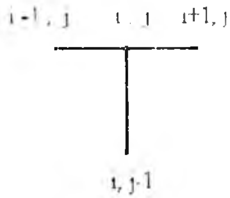
(7.17) dagi tenglamani tuzishda 7.9–rasmdagi oshkor sxemadan, (7.18) dagi tenglamani tuzishda 7.10–rasmdagi oshkormas sxemadan foydalanamiz.



7.8–rasm.



7.9–rasm.



7.10–rasm.

(7.19), (7.20) tenglamalarda σ sonini tanlashda ikkita holatni hisobga olish kerak :

1) differentsial tenglamani ayirmalar bilan almashtirishdagi xatolik eng kichik bo'lishi kerak;

2) ayirmalar tenglamalari turg'un bo'lishi kerak. (7.19) tenglamani $0<\sigma \leq 1/2$ da, (7.20) tenglamani esa ixtieriy σ da turg'un bo'lishi isbotlangan.

(7.17) tenglamaning eng qulay ko'rinishi

$$\sigma=\frac{1}{2} \text{ da: } \quad u_{i,j+1}=\frac{1}{2}(u_{i-1,j}+u_{i+1,j}) \qquad (7.21)$$

$$\sigma=\frac{1}{6} \text{ da: } \quad u_{i,j+1}=\frac{1}{6}(u_{i-1,j}+4u_{i,j}+u_{i+1,j}) \qquad (7.22)$$

(7.20), (7.21), (7.22) tenglamalardan topilgan taqribiy yechimning $0\leq x\leq s$, $0\leq t\leq T$ sohadagi hatoligini boholash tenglamalarga mos ravishda quydagicha:

$$|u-\bar{u}|\leq TM_1h^2/3 \qquad (7.23)$$

$$|u-\bar{u}|\leq TM_2h^4/135 \qquad (7.24)$$

$$|u-\bar{u}|\leq T\left(\frac{l}{2}+\frac{h^2}{12}\right)M_1 \qquad (7.25)$$

bu yerda $\bar{u}$ (7.11)–(7.13) masalani aniq yechimi, $0\leq x\leq s$, $0\leq t\leq T$ sohada:

$$M_1=\max\left\{|f^{(4)}(x)|,\ |\varphi''(t)|,\ |\psi''(t)|\right\}$$

$$M_2=\max\left\{\left|f^{(6)}(x)\right|,\ \left|\varphi^{(4)}(t)\right|,\ \left|\psi^{(4)}(t)\right|\right\}$$

Yuqoridagi xatoliklarni boholashda tanlanadigan h argumentning qadami (7.22) tenglama uchun yetarlich kichik bo'lishi kerak. l va h larni bir–biriga bog'liqsiz tanlaymiz.

7.2–masala: (7.21) ayirmalar tenglamasidan foydalanib, $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ tenglamaning

$$u(x,0)=\sin\pi x, \quad (0\leq x\leq 1)$$

$$u(0,t)=u(1,t)=0 \quad (0\leq t\leq 0.025)$$

chegaraviy shartni qanoatlantiruvchi taqribi yechimini topamiz.

Yechish: Uzgaruvchi argument x uchun $h=0.1$ qadam tanlaymiz. $\sigma=\frac{1}{2}$ bo'lganligidan t argumen uchun qadam $l=h^2/2=0.005$ 7.1–jadvalni boshlang'ich va chegaraviy qiymatlari bilan hamda simmetiriklikni eg'tiborga olib faqat $x=0$, 0.1, 0.2, 0.3 ,0.4, 0.5 lar uchun to'ldiramiz. $u(x,t)$ funtsiya birinchi qatlamdagi qiymatlarini boshlang'ich va chegaraviy shartlardan foydalanib, $j=0$, bo'lganda (7.21) formuladan foydalanamiz:

$$u_{i1} = \frac{1}{2}(u_{i+1,0} + u_{i-1,0})$$

Bu holda

$$u_{11} = \frac{1}{2}(u_{20} + u_{00}) = \frac{1}{2}(0.5878 + 0) = 0.2939$$

$$u_{21} = \frac{1}{2}(u_{30} + u_{10}) = \frac{1}{2}(0.8090 + 0.3090) = 0.5590$$

va hokazo u_{i1} ning i=2,3,4,5 larda ham qiymatlarini to'ib 7.1–jadvalni to'ldiramiz

7.1–javal

j	T	x	0	0,1	0.2	0,3	0.4	0,5
0	0	0	0,3090	0,5878	0.8090	0.9511	1.0000	
1	0.005	0	0,2939	0,5590	0,7699	0,9045	0,9511	
2	0,010	0	0.2795	0,5316	0,7318	0.8602	0.9045	
3	0,015	0	0,2558	0.5056	0.6959	0,8182	0.8602	
4	0.020	0	0.2528	0.4808	0,6619	0,7780	0,8182	
5	0.025	0	0,2404	0,4574	0,6294	0,7400	0.7780	
$u(x,t)$	0.025	0	0,2414	0.4593	0,6321	0,7431	0,7813	
$\lvert u - u\rvert$	0,025	0	0,0010	0,0019	0.0027	0.0031	0,0033	

asosan: ikkinchi qatlamda j=1 bo'lganda (7.21) formulaga

$$u_{i2} = \frac{1}{2}(u_{i+1,1} + u_{i-1,1})$$

bo'ladi. Xuddi shuningdek, u_{ij} ning qiymatlarini 0.010, 0.015, 0.020, 0.025 lar uchun ham hisoblaymiz. Jadvalning oxirida aniq yechim

$$u(t,x) = e^{-\pi t} \sin \pi x$$

va ayirma $|\tilde{u} - u|$ ning qiymatlarini t=0.005 uchun berilgan xatolikni taqqoslash uchun (7.23) formuladan foydalanib quyidacha baholashni ko'ramiz. Berilgan masala uchun $\varphi(t)$=$\psi(t)$=0

$$f^{(4)}(x) = \pi^4 \sin \pi x \quad дан \quad M_1 = \pi^2$$

bu yerda

$$|\tilde{u} - u| \le \frac{0,025}{3}\pi^4 h^2 = \frac{00,25}{3} 97,22 * 0.01 = 0,0081$$

Parabolik turdagi tenglamasi uchun to'r usulida (7.2.1) formula asosida hisoblashning Maple dasturi tuzishda matritsa indikslarini 1 dan boshlanishini e'tiborga olib, uni o'lchovini $u(i,j)$, i=1,2,...,n; j=1,2,...,n; kabi tamlaymiz.

7.2.1–M a p l e d a s t u r i:

```
> restart;
```

Boshlang'ich va chegaraviy funksiyalarini kiritish:

```
> f:=x–> sin(3.14*x);
```

$f := x \to \sin(3.14\,x)$

```
> phi:=t–>0*t;
```

$\phi := t \to 0\,t$

```
> psi:=t–>0*t;
```

$\psi := t \to 0\,t$

Sohani bo'linishlar soni va qadamlarini kiritish:

```
> n:=5:  m:=5: a:=0: b:=1.: c:=0: d:=0.025:
> h:=(b–a)/(10); g:=(d–c)/(10): e:=0.01:k:=h*h/2;
```

$h := 0.1000000000\; k := 0.00500000000$

Funksiya matritsasining o'lchamini belgilash

```
> u:=array(1..10,1..10);
```

Chegaraviy shart funksiyalarining qiymatlarini hisoblash:

```
> for j  to 2*m do t:=(j)*k:
u[1,j]:=phi(t): u[2*m,j]:=psi(t):
 od;  evalm(u): evalf(%,4);
```

$t := 0.005000000000 u_{1,1} := 0. u_{10,1} := 0.$

$t := 0.01000000000 u_{1,2} := 0. u_{10,2} := 0.$

$t := 0.01500000000 u_{1,3} := 0. u_{10,3} := 0.$

$t := 0.02000000000 u_{1,4} := 0. u_{10,4} := 0.$

$t := 0.02500000000 u_{1,5} := 0. u_{10,5} := 0.$

$t := 0.03000000000 u_{1,6} := 0. u_{10,6} := 0.$

$t := 0.03500000000 u_{1,7} := 0. u_{10,7} := 0.$

$t := 0.04000000000 u_{1,8} := 0. u_{10,8} := 0.$

$t := 0.04500000000 u_{1,9} := 0. u_{10,9} := 0.$

$t := 0.05000000000 u_{1,10} := 0. u_{10,10} := 0.$

$$
\begin{bmatrix}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
u_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & u_{2,5} & u_{2,6} & u_{2,7} & u_{2,8} & u_{2,9} & u_{2,10} \\
u_{3,1} & u_{3,2} & u_{3,3} & u_{3,4} & u_{3,5} & u_{3,6} & u_{3,7} & u_{3,8} & u_{3,9} & u_{3,10} \\
u_{4,1} & u_{4,2} & u_{4,3} & u_{4,4} & u_{4,5} & u_{4,6} & u_{4,7} & u_{4,8} & u_{4,9} & u_{4,10} \\
u_{5,1} & u_{5,2} & u_{5,3} & u_{5,4} & u_{5,5} & u_{5,6} & u_{5,7} & u_{5,8} & u_{5,9} & u_{5,10} \\
u_{6,1} & u_{6,2} & u_{6,3} & u_{6,4} & u_{6,5} & u_{6,6} & u_{6,7} & u_{6,8} & u_{6,9} & u_{6,10} \\
u_{7,1} & u_{7,2} & u_{7,3} & u_{7,4} & u_{7,5} & u_{7,6} & u_{7,7} & u_{7,8} & u_{7,9} & u_{7,10} \\
u_{8,1} & u_{8,2} & u_{8,3} & u_{8,4} & u_{8,5} & u_{8,6} & u_{8,7} & u_{8,8} & u_{8,9} & u_{8,10} \\
u_{9,1} & u_{9,2} & u_{9,3} & u_{9,4} & u_{9,5} & u_{9,6} & u_{9,7} & u_{9,8} & u_{9,9} & u_{9,10} \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{bmatrix}
$$

Boshlang'ich shart funksiyasining qiymatlarini hisoblash:

```
> for  i to 2*n–2 do
x:=i*h: u[i+1,1]:=f(x):
 od; evalm(u): evalf(%,4);
```

$$x := 0.100000000 \quad u_{2,1} := 0.308865520$$
$$x := 0.200000000 \quad u_{3,1} := 0.587527525$$
$$x := 0.300000000 \quad u_{4,1} := 0.808736060$$
$$x := 0.400000000 \quad u_{5,1} := 0.950859460$$
$$x := 0.500000000 \quad u_{6,1} := 0.999999682$$
$$x := 0.600000000 \quad u_{7,1} := 0.951351376$$
$$x := 0.700000000 \quad u_{8,1} := 0.809671788$$
$$x := 0.800000000 \quad u_{9,1} := 0.588815562$$

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0.3089 & u_{2,2} & u_{2,3} & u_{2,4} & u_{2,5} & u_{2,6} & u_{2,7} & u_{2,8} & u_{2,9} & u_{2,10} \\ 0.5875 & u_{3,2} & u_{3,3} & u_{3,4} & u_{3,5} & u_{3,6} & u_{3,7} & u_{3,8} & u_{3,9} & u_{3,10} \\ 0.8087 & u_{4,2} & u_{4,3} & u_{4,4} & u_{4,5} & u_{4,6} & u_{4,7} & u_{4,8} & u_{4,9} & u_{4,10} \\ 0.9509 & u_{5,2} & u_{5,3} & u_{5,4} & u_{5,5} & u_{5,6} & u_{5,7} & u_{5,8} & u_{5,9} & u_{5,10} \\ 1.000 & u_{6,2} & u_{6,3} & u_{6,4} & u_{6,5} & u_{6,6} & u_{6,7} & u_{6,8} & u_{6,9} & u_{6,10} \\ 0.9514 & u_{7,2} & u_{7,3} & u_{7,4} & u_{7,5} & u_{7,6} & u_{7,7} & u_{7,8} & u_{7,9} & u_{7,10} \\ 0.8097 & u_{8,2} & u_{8,3} & u_{8,4} & u_{8,5} & u_{8,6} & u_{8,7} & u_{8,8} & u_{8,9} & u_{8,10} \\ 0.5888 & u_{9,2} & u_{9,3} & u_{9,4} & u_{9,5} & u_{9,6} & u_{9,7} & u_{9,8} & u_{9,9} & u_{9,10} \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

Boshlang'ich va chegaraviy shart funksiyasining qiymatlari asosida izlanayotgan u(x,t) funksiyasining qiymatlarini qatlamlar bo'yicha (7.21) *formulasi asosida hisoblash:*

```
> for j to 2*m–1 do
for i from 2 by 1 to 2*n–1 do
u[i,j+1]:=(u[i–1,j]+u[i+1,j])/2:
od; od; UN:=evalm(u): evalf(%,4);
```

UN :=

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.3089	0.2938	0.2794	0.2657	0.2527	0.2404	0.2286	0.2175	0.2056	0.1956
0.5875	0.5588	0.5315	0.5055	0.4808	0.4573	0.4349	0.4112	0.3911	0.3677
0.8087	0.7692	0.7316	0.6958	0.6618	0.6294	0.5938	0.5648	0.5299	0.5040
0.9509	0.9044	0.8601	0.8181	0.7781	0.7303	0.6946	0.6486	0.6168	0.5746
1.000	0.9511	0.9046	0.8604	0.7989	0.7598	0.7033	0.6689	0.6192	0.5889
0.9514	0.9048	0.8606	0.7797	0.7416	0.6762	0.6432	0.5899	0.5611	0.5167
0.8097	0.7701	0.6548	0.6228	0.5536	0.5265	0.4765	0.4532	0.4141	0.3938
0.5888	0.4048	0.3850	0.3274	0.3114	0.2768	0.2633	0.2383	0.2266	0.2070
0.	0.	0.	0.	0.	0	0.	0.	0.	0.

```
> with(linalg):transpose(UN);
```

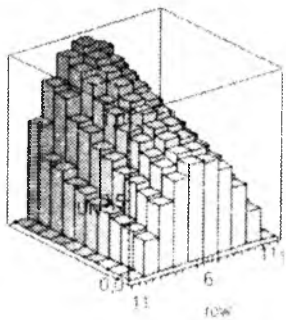
0.	0.3089	0.5875	0.8087	0.9509	1.000	0.9514	0.8097	0.5888	0
0.	0.2938	0.5588	0.7692	0.9044	0.9511	0.9048	0.7701	0.4048	0
0.	0.2794	0.5315	0.7316	0.8601	0.9046	0.8606	0.6548	0.3850	0
0	0.2657	0.5055	0.6958	0.8181	0.8604	0.7797	0.6228	0.3274	0
0	0.2527	0.4808	0.6618	0.7781	0.7989	0.7416	0.5536	0.3114	0
0.	0.2404	0.4573	0.6294	0.7303	0.7598	0.6762	0.5265	0.2768	0
0.	0.2286	0.4349	0.5938	0.6946	0.7033	0.6432	0.4765	0.2633	0
0.	0.2175	0.4112	0.5648	0.6486	0.6689	0.5899	0.4532	0.2383	0
0.	0.2056	0.3911	0.5299	0.6168	0.6192	0.5611	0.4141	0.2266	0.
0.	0.1956	0.3677	0.5040	0.5746	0.5889	0.5167	0.3938	0.2070	0.

Parabolik turdagi tenglama uchun aralash masala yechimining grafigi:

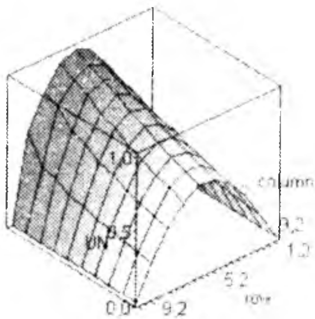
> **with(plots):with(LinearAlgebra):**

> **matrixplot(UN,heights=histogram,axes=boxed);**
(7.11–rasm)

> **matrixplot(UN,axes=boxed);** (7.12–rasm)



7.11–rasm.



7.12–rasm.

Chekli ayirmalar tenglamasi $u_{i,j+1}=\frac{1}{6}(u_{i-1,j}+4u_{i,j}+u_{i+1,j})$ bo'lganda (7.11)–(7.13) masalaning yechimi 7.2.1–M a p l e dasturi asosida quyidagicha bo'ladi:

0.	0.3089	0.5875	0.8087	0.9509	1.000	0.9514	0.8097	0.5888	0.
0.	0.3038	0.5780	0.7956	0.9354	0.9837	0.9358	0.7965	0.5275	0.
0.	0.2989	0.5685	0.7826	0.9201	0.9677	0.9206	0.7749	0.4844	0.
0.	0.2940	0.5593	0.7698	0.9051	0.9519	0.9042	0.7507	0.4521	0
0.	0.2892	0.5502	0.7573	0.8904	0.9361	0.8865	0.7265	0.4265	0.
0.	0.2845	0.5412	0.7449	0.8758	0.9202	0.8681	0.7032	0.4054	0
0.	0.2799	0.5324	0.7328	0.8614	0.9042	0.8495	0.6811	0.3875	0.
0	0.2753	0.5237	0.7208	0.8471	0.8879	0.8304	0.6602	0.3718	0.
0.	0.2708	0.5151	0.7090	0.8329	0.8715	0.8116	0.6405	0.3579	0.
0.	0.2664	0.5067	0.6973	0.8187	0.8551	0.7931	0.6219	0.3454	0.

7.3.2. Bir jinisli bo'lmagan parabolik tenglama uchun aralash masala

To'r usuli bilan bir jinisli bo'lmagan parabolik turdagi

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + F(x,t)$$

tenglama uchun aralash masalani yechish mumkin .

Bu holda tugunlarning oshkor holdagi sxemasida foydalangan holda ayirmalar tenglamasi quydagicha bo'ladi:

$$u_{i,j+1} = (1-2\sigma)u_{ij} + \sigma(u_{i+1,j} + u_{i-1,j}) + lF_{ij}$$

bunda $\sigma = \frac{1}{2}$ bo'lsa ,

$$u_{i,j+1} = \frac{1}{2}(u_{i+1,j} + u_{i-1,j}) + lF_{ij} \qquad (7.26)$$

bo'ladi. $\sigma = \frac{1}{6}$ bo'lsa.

$$u_{i,j+1} = \frac{1}{6}(u_{i+1,j} + 4u_{i,j} + u_{i-1,j}) + lF_{ij} \qquad (7.27)$$

bo'ladi. Bu holda xatolikni quydagicha baholash o'rinlidir.

(7.26) tenglama uchun:

$$|\bar{u} - u| \le \frac{T}{4}(M_2 + \frac{1}{3}M_4)h^2$$

(7.27) tenglama uchun:

$$|u - u| \le \frac{T}{12}(\frac{1}{3}M_3 + \frac{1}{5}M_6)h^4$$

Bu yerda

$$M_k = \max\left|\frac{\partial^k u}{\partial x^k}\right|, \quad k = 2,3,4,6$$

Bir jinisli bo'lmagan parabolik turdagi tenglama uchun aralash masalani (7.27) formula asosida yechim qiymatlarini hisoblashning Maple dasturi.

7.2.2–M a p l e d a s t u r i:

```
> restart; Digits:=3:
> f:=x->sin(3.14*x); f := x → sin( 3.14 x )
> phi:=t->0*t; φ := t → 0 t
> psi:=t->0*t; ψ := t → 0 t
> F0:=(x,t)->3*t*sin(x); F0 := (x, t) → 3 t sin(x)
> n:=5:  m:=5: a:=0: b:=1.: c:=0: d:=0.025:
> h:=(b-a)/(10); g:=(d-c)/(10): e:=0.01:k:=h*h/2;
                    h := 0.100  k := 0.00500
> u:=array(1..10,1..10):
> for j  to 2*m do
t:=j*k:  u[1,j]:=phi(t): u[2*m,j]:=psi(t):
 od;  evalm(u): evalf(%,4);
```

$$t := 0.00500 \quad u_{1,1} := 0. \quad u_{10,1} := 0.$$
$$t := 0.0100 \quad u_{1,2} := 0. \quad u_{10,2} := 0.$$
$$t := 0.0150 \quad u_{1,3} := 0. \quad u_{10,3} := 0.$$
$$t := 0.0200 \quad u_{1,4} := 0. \quad u_{10,4} := 0.$$
$$t := 0.0250 \quad u_{1,5} := 0. \quad u_{10,5} := 0.$$
$$t := 0.0300 \quad u_{1,6} := 0. \quad u_{10,6} := 0.$$
$$t := 0.0350 \quad u_{1,7} := 0. \quad u_{10,7} := 0.$$
$$t := 0.0400 \quad u_{1,8} := 0. \quad u_{10,8} := 0.$$
$$t := 0.0450 \quad u_{1,9} := 0. \quad u_{10,9} := 0.$$
$$t := 0.0500 \quad u_{1,10} := 0. \quad u_{10,10} := 0.$$

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ u_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & u_{2,5} & u_{2,6} & u_{2,7} & u_{2,8} & u_{2,9} & u_{2,10} \\ u_{3,1} & u_{3,2} & u_{3,3} & u_{3,4} & u_{3,5} & u_{3,6} & u_{3,7} & u_{3,8} & u_{3,9} & u_{3,10} \\ u_{4,1} & u_{4,2} & u_{4,3} & u_{4,4} & u_{4,5} & u_{4,6} & u_{4,7} & u_{4,8} & u_{4,9} & u_{4,10} \\ u_{5,1} & u_{5,2} & u_{5,3} & u_{5,4} & u_{5,5} & u_{5,6} & u_{5,7} & u_{5,8} & u_{5,9} & u_{5,10} \\ u_{6,1} & u_{6,2} & u_{6,3} & u_{6,4} & u_{6,5} & u_{6,6} & u_{6,7} & u_{6,8} & u_{6,9} & u_{6,10} \\ u_{7,1} & u_{7,2} & u_{7,3} & u_{7,4} & u_{7,5} & u_{7,6} & u_{7,7} & u_{7,8} & u_{7,9} & u_{7,10} \\ u_{8,1} & u_{8,2} & u_{8,3} & u_{8,4} & u_{8,5} & u_{8,6} & u_{8,7} & u_{8,8} & u_{8,9} & u_{8,10} \\ u_{9,1} & u_{9,2} & u_{9,3} & u_{9,4} & u_{9,5} & u_{9,6} & u_{9,7} & u_{9,8} & u_{9,9} & u_{9,10} \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

> for i to 2*n–2 do x:=i*h: u[i+1,1]:=f(x):
od; evalm(u): evalf(%,4);

$$x := 0.100 u_{2,1} := 0.309$$

$$x := 0.200 u_{3,1} := 0.588$$

$$x := 0.300 u_{4,1} := 0.809$$

$$x := 0.400 u_{5,1} := 0.952$$

$$x := 0.500 u_{6,1} := 1.00$$

$$x := 0.600 u_{7,1} := 0.953$$

$$x := 0.700 u_{8,1} := 0.808$$

$$x := 0.800 u_{9,1} := 0.590$$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.3089	$u_{2,2}$	$u_{2,3}$	$u_{2,4}$	$u_{2,5}$	$u_{2,6}$	$u_{2,7}$	$u_{2,8}$	$u_{2,9}$	$u_{2,10}$
0.5875	$u_{3,2}$	$u_{3,3}$	$u_{3,4}$	$u_{3,5}$	$u_{3,6}$	$u_{3,7}$	$u_{3,8}$	$u_{3,9}$	$u_{3,10}$
0.8087	$u_{4,2}$	$u_{4,3}$	$u_{4,4}$	$u_{4,5}$	$u_{4,6}$	$u_{4,7}$	$u_{4,8}$	$u_{4,9}$	$u_{4,10}$
0.9509	$u_{5,2}$	$u_{5,3}$	$u_{5,4}$	$u_{5,5}$	$u_{5,6}$	$u_{5,7}$	$u_{5,8}$	$u_{5,9}$	$u_{5,10}$
1.000	$u_{6,2}$	$u_{6,3}$	$u_{6,4}$	$u_{6,5}$	$u_{6,6}$	$u_{6,7}$	$u_{6,8}$	$u_{6,9}$	$u_{6,10}$
0.9514	$u_{7,2}$	$u_{7,3}$	$u_{7,4}$	$u_{7,5}$	$u_{7,6}$	$u_{7,7}$	$u_{7,8}$	$u_{7,9}$	$u_{7,10}$
0.8097	$u_{8,2}$	$u_{8,3}$	$u_{8,4}$	$u_{8,5}$	$u_{8,6}$	$u_{8,7}$	$u_{8,8}$	$u_{8,9}$	$u_{8,10}$
0.5888	$u_{9,2}$	$u_{9,3}$	$u_{9,4}$	$u_{9,5}$	$u_{9,6}$	$u_{9,7}$	$u_{9,8}$	$u_{9,9}$	$u_{9,10}$
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

```
> for j  to 2*m do for i  to 2*n do
 x:=i*h: t:=j*k: uF[i,j]:=F0(x,t);
 od; od;  evalm(uF): evalf(%,4);
```

0.001498	0.002995	0.004493	0.005990	0.007488	0.008985	0.01048	0.01198	0.01348	0.01498
0.002980	0.005960	0.008940	0.01192	0.01490	0.01788	0.02086	0.02384	0.02682	0.02980
0.004433	0.008866	0.01330	0.01773	0.02216	0.02660	0.03103	0.03546	0.03990	0.04433
0.005841	0.01168	0.01752	0.02337	0.02921	0.03505	0.04089	0.04673	0.05257	0.05841
0.007191	0.01438	0.02157	0.02877	0.03596	0.04315	0.05034	0.05753	0.06472	0.07191
0.008470	0.01694	0.02541	0.03388	0.04235	0.05082	0.05929	0.06776	0.07623	0.08470
0.009663	0.01933	0.02899	0.03865	0.04832	0.05798	0.06764	0.07731	0.08697	0.09663
0.01076	0.02152	0.03228	0.04304	0.05380	0.06456	0.07532	0.08608	0.09684	0.1076
0.01175	0.02350	0.03525	0.04700	0.05875	0.07050	0.08225	0.09400	0.1057	0.1175
0.01262	0.02524	0.03787	0.05049	0.06311	0.07573	0.08835	0.1010	0.1136	0.1262

```
> for j  to 2*m-1 do
for i from 2 by 1 to 2*n-1 do
 u[i,j+1]:=(u[i+1,j]+4*u[i,j]+u[i-1,j])/6+k*uF[i,j]:
od;  od:
UN:=evalm(u):  evalf(%,4);
```

UN =

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.3089	0.3038	0.2989	0.2941	0.2894	0.2847	0.2801	0.2757	0.2713	0.2670
0.5875	0.5780	0.5686	0.5594	0.5504	0.5415	0.5328	0.5243	0.5159	0.5077
0.8087	0.7956	0.7827	0.7700	0.7576	0.7454	0.7334	0.7216	0.7101	0.6986
0.9509	0.9354	0.9202	0.9053	0.8907	0.8764	0.8622	0.8481	0.8341	0.8203
1.000	0.9837	0.9678	0.9522	0.9366	0.9209	0.9050	0.8891	0.8730	0.8570
0.9514	0.9359	0.9207	0.9044	0.8870	0.8689	0.8503	0.8318	0.8133	0.7952
0.8097	0.7965	0.7750	0.7511	0.7271	0.7040	0.6821	0.6616	0.6423	0.6242
0.5888	0.5275	0.4846	0.4524	0.4270	0.4061	0.3884	0.3731	0.3594	0.3472
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

> **with(linalg):transpose(UN);**

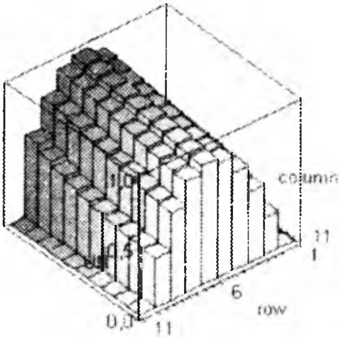
0.	0.3089	0.5875	0.8087	0.9509	1.000	0.9514	0.8097	0.5888	0.
0.	0.3038	0.5780	0.7956	0.9354	0.9837	0.9359	0.7965	0.5275	0.
0.	0.2989	0.5686	0.7827	0.9202	0.9678	0.9207	0.7750	0.4846	0.
0.	0.2941	0.5594	0.7700	0.9053	0.9522	0.9044	0.7511	0.4524	0.
0.	0.2894	0.5504	0.7576	0.8907	0.9366	0.8870	0.7271	0.4270	0.
0.	0.2847	0.5415	0.7454	0.8764	0.9209	0.8689	0.7040	0.4061	0.
0.	0.2801	0.5328	0.7334	0.8622	0.9050	0.8503	0.6821	0.3884	0.
0.	0.2757	0.5243	0.7216	0.8481	0.8891	0.8318	0.6616	0.3731	0.
0.	0.2713	0.5159	0.7101	0.8341	0.8730	0.8133	0.6423	0.3594	0.
0.	0.2670	0.5077	0.6986	0.8203	0.8570	0.7952	0.6242	0.3472	0.

Bir jinisli bo'lmagan parabolik turdagi tenglama uchun aralash yechimining grafigi:

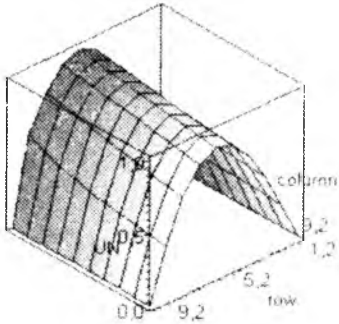
> **with(plots): with(LinearAlgebra):**

> **matrixplot(UN,heights=histogram,axes=boxed);** (7.13–rasm)

> **matrixplot(UN,axes=boxed);** (7.14–rasm)



7.13–rasm.



7.14–rasm.

O'z-o'zini tekshirish uchun savollar

1. Bir jinsli issiklik utkazuvchanlik tenglamasi uchun aralash masalani tur usuli yordamida taqribiy yechimi qanday topiladi?
2. Taqribiy yechim kaysi formulalar yordamida baxolanadi?
3. Bir jinsli bulmagan issiklik tarkalish tenglamasi va unga mos chekli ayirmali tenglamani xamda xatolikni baxolash formulalarini yozing.
4. Issiklik tarkalish tenglamasi uchun aralash masalani va unga mos chekli ayirmali sistemani yozing.
5. Aralash masalani xaydash usuli bilan taqribiy yechish tartibi tugri berish va orkaga kaytish jarayonlarini tushintirib bering.

7.2-laboratoriya ishi
bo'yicha mustaqil ishlash uchun topshiriqlar

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

parabolik tenglamani

$$u(x,0)=f(x),\ (0\le x\le 0.6)$$

boshlang'ich va

$$u(0,t)=\varphi(t),\ u(0.6,t)=\psi(t),\ 0\le t\le 0.05$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ yechimini $h=0.1$, $l=0.005$ qadamlar bilan to'r usilida toping.

№	$f(x)$	$\varphi(t)$	$\psi(t)$
1	$\cos 2x$	$1-6t$	0,3624
2	$x(x+1)$	0	$2t+0.96$
3	$1,2+\lg(x+0,4)$	$0,8+t$	1,2
4	$\sin 2x$	$2t$	0,932
5	$3x(2-x)$	0	$t+2,52$
6	$\lg(x+0.4)$	1,4	$t+1$
7	$\sin(0.55x+0.03)$	$t+0,03$	0,354
8	$2x(1-x)+0,2$	0,2	$t+0,68$
9	$\sin x+0,08$	$0,08+2t$	0,6446
10	$\cos(2x+0,19)$	0,932	0.1798
11	$2x(x+0,2)+0,4$	$2t+0,4$	1,36
12	$\lg(x+0,26)+1$	$0,415+t$	0,9345
13	$\sin(x+0,45)$	$0,435-2t$	0,8674
14	$0,3+x(x+0,4)$	0,3	$6t+0,9$
15	$(x-0,4)(x+1)+0,2$	$6t$	0,84
16	$x(0,3+2)$	0	$6t+0,9$
17	$\sin(x+0,48)$	0,4618	$3t+0,882$
18	$\sin(x+0,02)$	$3t+0,02$	0,581
19	$\cos(x+0,48)$	$6t+0,887$	0,4713
20	$\lg(2,53-x)$	$3(0,14-t)$	0,3075
21	$1.5-x(1-x)$	$3(0.5-t)$	1,26
22	$\cos(x+0,845)$	$6(t+0,11)$	0,1205
23	$\lg(2,42+x)$	0,3838	$6(0,08-t)$
24	$0,6+x(0,8-x)$	0,6	$3(0,24+t)$
25	$\cos(x+0,66)$	$3t+0,79$	0,3058
26	$\lg(1,43+2x)$	0.1553	$3(t+0,14)$
27	$0,9+2x(1-x)$	$3(0,3-2t)$	1,38
28	$\lg(1,95+x)$	$0,29-6t$	0,4065
29	$2\cos(x+0,55)$	1,705	$0,817+3t$
30	$x(1-x)+0.2$	0.2	$2(t+0.22)$

7.4. Giperbolik turdagi differentsial tenglamani taqriy yechishda to'r usuli

7.4.1. Tor tebranish tenglamasi uchun aralash masalani taqribiy yechish.

7.4.2. Yechimni boshlang'ich qatlamdagi yechim qiymatlari asosida hisoblash.

7.4.1. Tor tebranish tenglamasi uchun aralash masalani taqribiy yechishda to'r usuli.

Tor tebranishini ifodalovchi quyidagi giperbolik tenglamasi uchun aralash masalani ko'ramiz. Ya'ni

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (7.28)$$

tenglamani

$$u(x,0) = f(x),\ u_t(x,0) = \Phi(x),\ 0 \le x \le s \qquad (7.29)$$

boshlang'ich shartlarni va

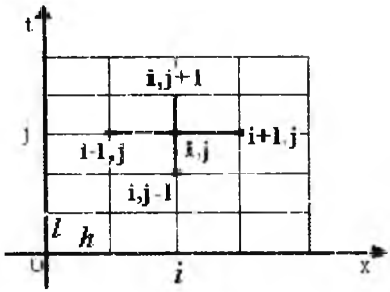
$$u(0,t) = \varphi(t),\ u(s,t) = \psi(t),\ \ 0 \le t < \infty \qquad (7.30)$$

chegaraviy shartlarni qanoatlantiruvchi funksiyasi topish masalasini yechamiz.

(7.28) tenglamada $\tau = a^2 t$ belgilash qilib, uni quyidagi ko'rinishiga keltiramiz:

$$\frac{\partial^2 u}{\partial \tau^2} = \frac{\partial^2 u}{\partial x^2} \qquad (7.31)$$

Keyinchalik $a = 1$ deb olsak bo'ladi.



7.15-rasm.

$t>0,\ 0 \le x \le s$ yarim qatlamda

$$x = x_i = ih,\quad i = 0, 1, 2, \ldots, n,$$
$$t = t_j = j\,h,\ j = 0, 1, 2, \ldots, n$$

to'g'ri chiziqlar oyilasini quramiz. (7.31) tenglamadagi hosilalarni ayirmalar nisbati bilan almashtiramiz. Hosilalar uchun simmetrik formulalardan foydlanib,

$$\frac{u_{i,j+1} - 2u_{ij} + u_{i,j-1}}{l^2} = \frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{h^2} \qquad (7.32)$$

ayirmalar tenglamasini topamiz. Bu yerda $\alpha = l/h$ belgilash qilib, (7.32) tenglamani quyidagicha yozamiz:

$$u_{i,j+1} = 2u_{i,j} - u_{i,j-1} + \alpha^2\left(u_{i+1,j} - 2u_{ij} + u_{i-1,j}\right) \quad (7.33)$$

(7.33) tenglamaning $\alpha \leq 1$ bo'lganda turg'un ekanligi isbotlangan.

(7.33) tenglamada $\alpha=1$ bo'lganda tenglamaning soddalashgan holini topamiz:

$$u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1} \quad (7.34)$$

(7.33) tenglama bilan $0 \leq x \leq s$, $0 \leq t \leq T$ qatlamda topilgan taqribiy echimning hatoligi quyidagicha boholanadi:

$$|\tilde{u} - u| \leq \frac{h^2}{12}\left[(M_4 h + 2M_3)T + T^2 M_4\right]$$

bu yerda, $\tilde{u}$ – aniq yechim,

$$M_k = \max\left\{\left|\frac{\partial^k u}{\partial x^k}\right|, \left|\frac{\partial^k u}{\partial t^k}\right|\right\}, \ k = 3,4.$$

(7.33) tenglamani hosil qilish uchsun 7.15–rasmdagi tugunlar sxemasidan foydalanilganini ko'ramiz. Bu oshkor sxema bo'lib, agar oldingi ikki qatlamdagi qiymtalr ma'lum belsa, (7.33) tenglama t_{j+1} qatlamdagi $u(x,t)$ funksiyaning qiymatini topishga imkon beradi.

7.4.2. Yechimni boshlang'ich qatlamdagi yechim qiymatlari asosida hisoblash

Demak (7.28)–(7.30) masalaning taqribiy yechimini topish uchun yechimning birinchi ikki boshlang'ich qatlamdagi qiymatini bilish zarur. Bularni boshlang'ich shartlardan topishning quyidagicha usullaridan foydalanimiz:

B i r i n c h i u s u l: (7.29) boshlang'ich shartda $u_t(x,0)$ hosilani quyidagicha ayirmalar nisbati bilan almashtiramiz.

$$\frac{u_{i1} - u_{i0}}{l} = \Phi(x_i) = \Phi_i$$

$u(x,t)$ funksiyaning $j=0$, $j=1$ qatlamdagi qiymatlarini topish uchun

$$u_{i0} = f_i, \ u_{i1} = f_i + l\Phi_i \quad (7.35)$$

ga ega bo'lamiz.

Bu holda u_{i1} qiymatlarining xatoligini baholash quyidagicha bo'ladi.

$$|\tilde{u}_{i1} - u_{i1}| \leq \frac{lh}{2} M_2 \quad (7.36)$$

bu yerda $M_2 = \max\left\{\left|\frac{\partial^2 u}{\partial t^2}\right|, \left|\frac{\partial^2 u}{\partial x^2}\right|\right\}$.

I k k i n c h i u s u l: $u_t(x,t)$ hosilani $(u_{i1}-u_{i,-1})/(2m)$ ayirmalar nisbati bilan almashtiramiz. bu yerda $u_{i,-1}$, $j=-1$ qatlamdagi $u(x,t)$ funksiyaning qiymatlari. Bu holda (7.39) boshlang'ich shartdan

$$u_{i0}=f_i, \qquad \frac{u_{i1}-u_{i,-1}}{2l}=\Phi_i \tag{7.37}$$

larni topamiz. (7.44) ayirmalar tenglamasini $j=0$ qatlam uchun quyidagicha yozamiz:

$$u_{i1}=u_{i+1,0}+u_{i-1,0}-u_{i,-1} \tag{7.38}$$

(7.37), (7.38) tenglamalardan $u_{i,-1}$ qiymatlarni yo'qotib.

$$u_{i0}=f_i,\ u_{i1}=\frac{1}{2}(f_{i+1}+f_{i-1})+l\Phi_i \tag{7.39}$$

ga ega bo'lamiz. Bu holda u_{i1} qiymatlarning xatoligini baholash quyidagicha bo'ladi:

$$|\tilde{u}_{i1}-u_{i1}|\le\frac{h^4}{12}M_4+\frac{h^3}{6}M_3 \tag{7.40}$$

bu yerda $$M_k=\max\left\{\left|\frac{\partial^k u}{\partial x^k}\right|,\left|\frac{\partial^k u}{\partial t^k}\right|\right\},\ k=3,4.$$

U c h i n c h i u s u l: Agar $f(x)$ funksiya ikkinchi tartibli chekli hosilaga ega bo'lsa, u_{i1} qiymatlarni Teylor formulasi yordamida quyidagicha aniqlash mumkin.

$$u_{i1}\approx u_{i0}+l\frac{\partial u_{i0}}{\partial t}+\frac{l^2}{2}\frac{\partial^2 u_{i0}}{\partial t^2} \tag{7.41}$$

(7.31) tenglamadan va (7.29) boshlang'ich shartlardan foydalanimiz, quyidagilarni yozish mumkin:

$$u_{i0}=f_i,\ \frac{\partial u_{i0}}{\partial t}=\Phi_i,\ \frac{\partial^2 u_{i0}}{\partial t^2}=\frac{\partial^2 u_{i0}}{\partial x^2}=f_i''$$

Bu holda (7.41) formulaga asosan

$$u_{i1}\approx f_i+l\Phi_i+\frac{l^2}{2}f_i'' \tag{7.42}$$

ekanligini topamiz. u_{i1} ning bu formula yordamida topilgan qiymatlarining xatoligining tartibi $0(h^3)$ bo'ladi.

Shuningdek,

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=F(x,t)$$

bir jinsli bo'lmagan tenglama uchun aralash masala yuqoridagidek yechiladi. Bu holda ayirmalar tenglamasi quyidagicha bo'ladi:

$$u_{i,j+1} = 2u_{ij} - u_{i,j-1} + \alpha^2(u_{i+1,j} - 2u_{ij} + u_{i-1,j}) + l^2h^2F_{ij}$$

7.3–masala. Quyidagi

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = 0.2x(1-x)\sin\pi x,\ u_t(x,0) = 0, \qquad (7.43)$$

$$u(0,t) = u(1,t) = 0.$$

aralash masalani to‘r usulida yechimini toping.

Yechish: Qadami $h=l=0.05$ bo‘lgan kvadrat to‘r olamiz. Boshlang‘ich ikki qatlamdagi $u(x,t)$ ning qiymatlarini ikkinchi usul bilan topamiz:

$\Phi(x)=0$ va $f(x)=0.2x(1-x)\sin\pi x$ ekanligini e’tiborga olib, (7.39) formulaga asosan:

$$u_{i0} = f_i = f(x_i), \qquad (7.44)$$

$$u_{i1} = \frac{1}{2}(f_{i+1} + f_{i-1}) = \frac{1}{2}[f(x_{i+1}) + f(x_{i-1})] + l\Phi(x_i),$$

$$i = 0,1,2,3,\ldots,10.$$

larni topamiz.

Jadvalni tulgizish tartibi:

1) $x_i=ih$ larda $u_{i0}=f(x_i)$ qiymatlarni hisoblaymiz ($t_0=0$ dagi qiymtalarga mos keladi) va ularni birinchi satrga yozamiz. (7.2– jadval) jadvalni masalani simmetrikligi asosida, $0\le x\le0.5$ ga, mos to‘ldiramiz. Birinchi ustunga ($x_0=0$ ga mos) chegaraviy qiymatlarni yozamiz.

2) (7.44) formula asosida u_{i1} larni u_{i0} ning birinchi satridagi qiymatlari asosida topamiz. Natijalarni 7.2 jadvalning ikkinchi satriga yozamiz.

3) (7.44) formula asosida u_{ij} ning keyingi qatlamalaridagi qiymatlarini hisoblaymiz.

$j=1$ bo‘lganda

$u_{12}=u_{21}+u_{01}-u_{10}=0.0065+0-0.0015=0.005,$

$u_{22}=u_{31}+u_{11}-u_{20}=0.0122+0.005-0.0056=0.0094,$

. . . .

$u_{10,2}=u_{11,1}+u_{91}-u_{10,0}=0.8478+0.0478-0.05=0.456.$

Shuningdek, $j=2,3,\ldots,10$ lar uchun ham hisoblab, quyidagi jadvalni to‘ldiramiz. Jadvalning oxirigi satrida $t=0.5$ bo‘lgandagi yechimning aniq qiymatlari yozilgan.

7.2–jadval

$t \backslash x$	0	0.05	0,10	0.15	0.20	0,25
0	0	0,0015	0,0056	0,0116	0,0188	0,0265
0,05	0	0,0028	0,0065	0,0122	0,0190	0,0264
0.10	0	0.0050	0,0094	0.0139	0.0198	0.0260

0,15	0	0,0066	0,0224	0,0170	0,0209	0,0256
0,20	0	0,0074	0,0142	0,0194	0,0228	0,0251
0,25	0	0,0076	0,0144	0,0200	0,0236	0,0249
0,30	0	0,0070	0,0134	0,0186	0,0221	0,0236
0,35	0	0,0058	0,0112	0,0155	0,0186	0,0199
0,40	0	0,0042	0,0079	0,0112	0,0133	0,0144
0,45	0	0,0021	0,0042	0,0057	0,0070	0,0074
0,50	0	0,0001	-0,0001	0,0000	-0,0002	0,0000
$\hat{u}(x,0.5)$	0	0	0	0	0	0

Giperbolik turdagi differentsial tenglamani to'r usulida taqriy yechishda (7.44) formulasi asosida hisoblashning Maple dasturini tuzaniz.

7.3.1–Maple dasturi:

> restart; Digits:=3:

Boshlang'ich funksiyalarini kiritish:

> f:=x->0.2*x*(1-x)*sin(3.14*x);

$$f := x \to 0.2\,x\,(1-x)\sin(3.14\,x)$$

> Fix:=x->x*0; $Fix := x \to x \cdot 0$

Chegaraviy funksiyalarini kiritish:

> phi:=t->0*t; $\phi := t \to 0\,t$

> psi:=t->0*t; $\psi := t \to 0\,t$

Sohani bo'linishlar soni va qadamlarini kiritish:

> n:=5: m:=5: h:=0.05: l:=0.05; c:=l/h; $l := 0.05$ $c := 1.00$

u(x,t) funksiya matritsasining o'lchamini belgilash

> u:=array(1..10,1..10); $u := array(1..10, 1..10, [\,])$

Chegaraviy shart funksiyalarining qiymatlarini hisoblash:

```
> for j from 1 by 1 to 2*m do
 t:=j*l; u[1,j]:=phi(t); u[2*m,j]:=psi(t);
od; evalm(u): evalf(%,4);
```

$t := 0.05 u_{1,1} := 0. u_{10,1} := 0.$

$t := 0.10 u_{1,2} := 0. u_{10,2} := 0.$

$t := 0.15 u_{1,3} := 0. u_{10,3} := 0.$

$t := 0.20 u_{1,4} := 0. u_{10,4} := 0.$

$t := 0.25 u_{1,5} := 0. u_{10,5} := 0.$

$t := 0.30 u_{1,6} := 0. u_{10,6} := 0.$

$t := 0.35 u_{1,7} := 0. u_{10,7} := 0.$

$t := 0.40 u_{1,8} := 0. u_{10,8} := 0.$

$t := 0.45 u_{1,9} := 0. u_{10,9} := 0.$

$t := 0.50 u_{1,10} := 0. u_{10,10} := 0.$

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ u_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & u_{2,5} & u_{2,6} & u_{2,7} & u_{2,8} & u_{2,9} & u_{2,10} \\ u_{3,1} & u_{3,2} & u_{3,3} & u_{3,4} & u_{3,5} & u_{3,6} & u_{3,7} & u_{3,8} & u_{3,9} & u_{3,10} \\ u_{4,1} & u_{4,2} & u_{4,3} & u_{4,4} & u_{4,5} & u_{4,6} & u_{4,7} & u_{4,8} & u_{4,9} & u_{4,10} \\ u_{5,1} & u_{5,2} & u_{5,3} & u_{5,4} & u_{5,5} & u_{5,6} & u_{5,7} & u_{5,8} & u_{5,9} & u_{5,10} \\ u_{6,1} & u_{6,2} & u_{6,3} & u_{6,4} & u_{6,5} & u_{6,6} & u_{6,7} & u_{6,8} & u_{6,9} & u_{6,10} \\ u_{7,1} & u_{7,2} & u_{7,3} & u_{7,4} & u_{7,5} & u_{7,6} & u_{7,7} & u_{7,8} & u_{7,9} & u_{7,10} \\ u_{8,1} & u_{8,2} & u_{8,3} & u_{8,4} & u_{8,5} & u_{8,6} & u_{8,7} & u_{8,8} & u_{8,9} & u_{8,10} \\ u_{9,1} & u_{9,2} & u_{9,3} & u_{9,4} & u_{9,5} & u_{9,6} & u_{9,7} & u_{9,8} & u_{9,9} & u_{9,10} \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

Boshlang'ich ikki qatlamdagi u(x,t) ning qiymatlarini hisoblashning usullari:

> #1–usul: u[i,1]:=f(h*i)+l*Fix(h*i);

> #2–usul: u[i,1]:=(f(h*(i+1))+f(h*(i–1)))/2+ l*Fix(h*i);

> #3–usul: u[i,1]:=f(h*i)+l*Fix(h*i)+l*l*f2(h*i)/2;

Boshlang'ich ikki qatlamdagi u(x,t) ning qiymatlarini hisoblashning ikkinch usulida hisoblash:

```
> for i from 2 by 1 to 2*n–1 do x:=i*h;
u[i,1]:=f(x): u[i,2]:=(f(h*(i+1))+f(h*(i–1)))/2; #+l*Fix(h*i): od;
evalm(u):evalf(%,4);
```

$x := 0.10\ u_{2,1} := 0.0055\ u_{2,2} := 0.0065$

$x := 0.15\ u_{3,1} := 0.0116\ u_{3,2} := 0.0122$

$x := 0.20\ u_{4,1} := 0.0188\ u_{4,2} := 0.0190$

$x := 0.25\ u_{5,1} := 0.0265\ u_{5,2} := 0.0264$

0,15	0	0,0066	0,0224	0,0170	0,0209	0,0256
0,20	0	0,0074	0,0142	0,0194	0,0228	0,0251
0,25	0	0,0076	0,0144	0,0200	0,0236	0,0249
0,30	0	0,0070	0,0134	0,0186	0,0221	0,0236
0,35	0	0,0058	0,0112	0,0155	0,0186	0,0199
0,40	0	0,0042	0,0079	0,0112	0,0133	0,0144
0,45	0	0,0021	0,0042	0,0057	0,0070	0,0074
0,50	0	0,0001	-0,0001	0,0000	-0,0002	0,0000
$\bar{u}(x,0.5)$	0	0	0	0	0	0

Giperbolik turdagi differentsial tenglamani to'r usulida taqriy yechishda (7.44) formulasi asosida hisoblashning Maple dasturini tuzamiz.

7.3.1–M a p l e d a s t u r i:

> restart; Digits:=3:

Boshlang'ich funksiyalarini kiritish:

> f:=x–>0.2*x*(1–x)*sin(3.14*x);

$$f := x \to 0.2\,x\,(1-x)\sin(3.14\,x)$$

> Fix:=x–>x*0; $Fix := x \to x \cdot 0$

Chegaraviy funksiyalarini kiritish:

> phi:=t–>0*t; $\phi := t \to 0\,t$

> psi:=t–>0*t; $\psi := t \to 0\,t$

Sohani bo'linishlar soni va qadamlarini kiritish:

> n:=5: m:=5: h:=0.05: l:=0.05; c:=l/h; $l := 0.05\quad c := 1.00$

u(x,t) funksiya matritsasining o'lchamini belgilash

> u:=array(1..10,1..10); $u := array(1..10, 1..10, [\,])$

Chegaraviy shart funksiyalarining qiymatlarini hisoblash:

```
> for j from 1 by 1 to 2*m do
 t:=j*l; u[1,j]:=phi(t); u[2*m,j]:=psi(t);
od; evalm(u): evalf(%,4);
```

$$t := 0.05\,u_{1,1} := 0.\,u_{10,1} := 0.$$

$$t := 0.10\,u_{1,2} := 0.\,u_{10,2} := 0.$$

$$t := 0.15\,u_{1,3} := 0.\,u_{10,3} := 0.$$

$$t := 0.20\,u_{1,4} := 0.\,u_{10,4} := 0.$$

$$t := 0.25\,u_{1,5} := 0.\,u_{10,5} := 0.$$

$$t := 0.30\,u_{1,6} := 0.\,u_{10,6} := 0.$$

$t := 0.35 u_{1,7} := 0. u_{10,7} := 0.$

$t := 0.40 u_{1,8} := 0. u_{10,8} := 0.$

$t := 0.45 u_{1,9} := 0. u_{10,9} := 0.$

$t := 0.50 u_{1,10} := 0. u_{10,10} := 0.$

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ u_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & u_{2,5} & u_{2,6} & u_{2,7} & u_{2,8} & u_{2,9} & u_{2,10} \\ u_{3,1} & u_{3,2} & u_{3,3} & u_{3,4} & u_{3,5} & u_{3,6} & u_{3,7} & u_{3,8} & u_{3,9} & u_{3,10} \\ u_{4,1} & u_{4,2} & u_{4,3} & u_{4,4} & u_{4,5} & u_{4,6} & u_{4,7} & u_{4,8} & u_{4,9} & u_{4,10} \\ u_{5,1} & u_{5,2} & u_{5,3} & u_{5,4} & u_{5,5} & u_{5,6} & u_{5,7} & u_{5,8} & u_{5,9} & u_{5,10} \\ u_{6,1} & u_{6,2} & u_{6,3} & u_{6,4} & u_{6,5} & u_{6,6} & u_{6,7} & u_{6,8} & u_{6,9} & u_{6,10} \\ u_{7,1} & u_{7,2} & u_{7,3} & u_{7,4} & u_{7,5} & u_{7,6} & u_{7,7} & u_{7,8} & u_{7,9} & u_{7,10} \\ u_{8,1} & u_{8,2} & u_{8,3} & u_{8,4} & u_{8,5} & u_{8,6} & u_{8,7} & u_{8,8} & u_{8,9} & u_{8,10} \\ u_{9,1} & u_{9,2} & u_{9,3} & u_{9,4} & u_{9,5} & u_{9,6} & u_{9,7} & u_{9,8} & u_{9,9} & u_{9,10} \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

Boshlang'ich ikki qatlamdagi u(x,t) ning qiymatlarini hisoblashning usullari:

> #1–usul: u[i,1]:=f(h*i)+l*Fix(h*i);

> #2–usul: u[i,1]:=(f(h*(i+1))+f(h*(i–1)))/2+ l*Fix(h*i);

> #3–usul: u[i,1]:=f(h*i)+l*Fix(h*i)+l*l*f2(h*i)/2;

Boshlang'ich ikki qatlamdagi u(x,t) ning qiymatlarini hisoblashning ikkinch usulida hisoblash:

```
> for i from 2 by 1 to 2*n–1 do x:=i*h;
u[i,1]:=f(x): u[i,2]:=(f(h*(i+1))+f(h*(i–1)))/2; #+l*Fix(h*i): od;
evalm(u):evalf(%,4);
```

$x := 0.10 \; u_{2,1} := 0.0055(\; u_{2,2} := 0.0065$

$x := 0.15 \; u_{3,1} := 0.011(\; u_{3,2} := 0.0122$

$x := 0.20 \; u_{4,1} := 0.0188 \; u_{4,2} := 0.019($

$x := 0.25 \; u_{5,1} := 0.0265 \; u_{5,2} := 0.0264$

$$x := 0.30 \quad u_{6,1} := 0.0340 \quad u_{6,2} := 0.0334$$

$$x := 0.35 \quad u_{7,1} := 0.0405 \quad u_{7,2} := 0.0398$$

$$x := 0.40 \quad u_{8,1} := 0.0457 \quad u_{8,2} := 0.0446$$

$$x := 0.45 \quad u_{9,1} := 0.0489 \quad u_{9,2} := 0.0478$$

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.00556	0.00654	0.00664	0.00736	0.00756	0.00694	0.00584	0.00406	0.00206	−0.00106
0.0116	0.0122	0.0139	0.0142	0.0143	0.0134	0.0110	0.0079	0.0030	−0.0457
0.0188	0.0190	0.0198	0.0208	0.0200	0.0184	0.0155	0.0099	−0.0399	−0.0405
0.0265	0.0264	0.0259	0.0256	0.0249	0.0221	0.0173	−0.0323	−0.0336	−0.0340
0.0340	0.0334	0.0322	0.0300	0.0277	0.0238	−0.0257	−0.0262	−0.0264	−0.0265
0.0405	0.0398	0.0375	0.0343	0.0289	−0.0201	−0.0197	−0.0198	−0.0191	−0.0188
0.0457	0.0446	0.0419	0.0364	−0.0135	−0.0146	−0.0142	−0.0126	−0.0122	−0.0116
0.0489	0.0478	0.0435	−0.0059	−0.0071	−0.0076	−0.0075	−0.0066	−0.0051	−0.0056
0.0489	0.0478	0	0.	0.	0.	0.	0	0.	0.

Boshlang'ich va chegaraviy shart funksiyasining qiymatlarini asosida izlanayotgan u(x t) funksiyasining qiymatlarini qatlamlar bo'yicha (7.44) formulasi asosida hisoblash:

```
> for j from 2 by 1 to 2*m–1 do
for i from 2 by 1 to 2*n–1 do
  u[i,j+1]:=u[i+1,j]+u[i–1,j]--u[i,j–1];
 od;od; evalm(u):UN:=evalf(%,3);
```

UN =

0	0.	0.	0	0	0.	0.	0.	0	0.
0.00556	0.00654	0.00664	0.00736	0.00756	0.00694	0.00584	0.00406	0.00206	−0.00106
0.0116	0.0122	0.0139	0.0142	0.0143	0.0134	0.0110	0.0079	0.0030	−0.0457
0.0188	0.0190	0.0198	0.0208	0.0200	0.0184	0.0155	0.0099	−0.0399	−0.0405
0.0265	0.0264	0.0259	0.0256	0.0749	0.0221	0.0173	−0.0323	−0.0336	−0.0340
0.0340	0.0334	0.0322	0.0300	0.0277	0.0238	−0.0257	−0.0262	−0.0264	−0.0265
0.0405	0.0398	0.0375	0.0343	0.0289	−0.0201	−0.0197	−0.0198	−0.0191	−0.0188
0.0457	0.0446	0.0419	0.0364	−0.0135	−0.0146	−0.0142	−0.0126	−0.0122	−0.0116
0.0489	0.0478	0.0435	−0.0059	−0.0071	−0.0076	−0.0075	−0.0066	−0.0051	−0.0056
0.0489	0.0478	0	0.	0.	0	0	0.	0.	0

```
> with(linalg):transpose(UN);
```

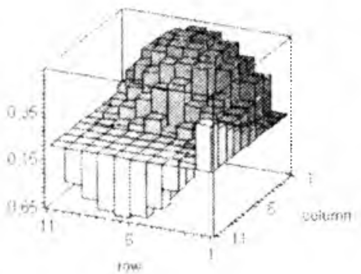
0	0.00556	0.0116	0.0188	0.0265	0.0340	0.0405	0.0457	0.0489	0.0489
0.	0.00654	0.0122	0.0190	0.0264	0.0334	0.0398	0.0446	0.0478	0.0478
0	0.00664	0.0139	0.0198	0.0259	0.0322	0.0375	0.0419	0.0435	0.
0	0.00736	0.0142	0.0208	0.0256	0.0300	0.0343	0.0364	−0.0059	0.
0	0.00756	0.0143	0.0209	0.0249	0.0277	0.0289	−0.0135	−0.0071	0.
0	0.00694	0.0134	0.0184	0.0221	0.0238	−0.0201	−0.0146	−0.0076	0.
0	0.00584	0.0110	0.0155	0.0173	−0.0257	−0.0197	−0.0142	−0.0075	0.
0	0.00406	0.0079	0.0099	−0.0323	−0.0262	−0.0198	−0.0126	−0.0066	0
0.	0.00206	0.0030	−0.0399	−0.0336	−0.0264	−0.0191	−0.0122	−0.0051	0.
0.	−0.00106	−0.0457	−0.0405	−0.0340	−0.0265	−0.0188	−0.0116	−0.0056	0.

Giperbolik turdagi differentsial tenglamani to'r usulida topilgan taqriy yechimining grafigini qurish:

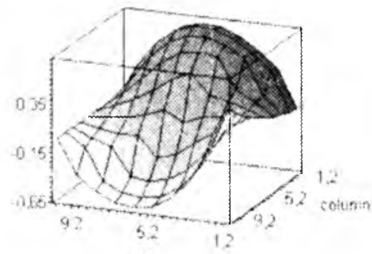
> **with(plots):with(LinearAlgebra):**

> **matrixplot(UN,heights=histogram,axes=boxed);** (7.16–rasm)

> **matrixplot(UN,axes=boxed);** (7.17–rasm)



7.16–rasm.



7.17–rasm.

7.4-masala. Endi yuqorida tuzilgan dasturni bir jinsli bo'lmagan tenglama uchun tadbiq qilamiz. Masalan,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \frac{1}{2} t x^2$$

tenglamani

$$u(x,0) = x^2,\ u_t(x,0) = \sin(x),\ 0 \le x \le 2$$

boshlang'ich va

$$u(0,t) = e^t - 1,\ u(2,t) = 2\cos(t),\quad 0 \le t < 1$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ funksiyasini to'r usilida $h = 0.1$, $l = 0.1$ qadamlar bilan taqribiy yechimini topish masalasini Maple dasturi yordamida yechamiz.

7.3.2–M a p l e d a s t u r i:

> **restart;**

```
> f:=x->x*x;
```
$f := x \to x\,x$

```
> Fxt:=(x,t)->t*x*x/2;
```
$Fxt := (x, t) \to t\,x\,x\,\frac{1}{2}$

```
> Fix:=x->sin(x);
```
$Fix := x \to \sin(x)$

```
> phi:=t->exp(t)-1;
```
$\phi := t \to e^t - 1$

```
> psi:=t->4*cos(t);
```
$\psi := t \to 4\cos(t)$

```
> h:=0.2:l:=0.2;c:=l/h;n:=5: m:=5:
```
$l := 0.2 \quad c := 1.000000000$

```
> u:=array(1..10,1..10); F0:=array(1..10,1..10);
```
$$u := array(1..10, 1..10, [\ ])$$
$$F0 := array(1..10, 1..10, [\ ])$$

Chegaraviy shartlar bo'yicha izlanayotgan u(x,t) funksiyasining qiymatlari:

```
> for j from 1 by 1 to 2*m do
 t:=j*l; u[1,j]:=phi(t); u[2*n,j]:=psi(t);
od; evalm(u):evalf(%,4);
```
$t := 0.2 \quad u_{1,1} := 0.221402758 \quad u_{10,1} := 3.920266311$

$t := 0.4 \quad u_{1,2} := 0.491824698 \quad u_{10,2} := 3.684243976$

$t := 0.6 \quad u_{1,3} := 0.822118800 \quad u_{10,3} := 3.301342460$

$t := 0.8 \quad u_{1,4} := 1.225540928 \quad u_{10,4} := 2.786826837$

$t := 1.0 \quad u_{1,5} := 1.718281828 \quad u_{10,5} := 2.161209224$

$t := 1.2 \quad u_{1,6} := 2.320116923 \quad u_{10,6} := 1.449431018$

$t := 1.4 \quad u_{1,7} := 3.055199967 \quad u_{10,7} := 0.6798685716$

$t := 1.6 \quad u_{1,8} := 3.953032424 \quad u_{10,8} := -.1167980892$

$t := 1.8 \quad u_{1,9} := 5.049647464 \quad u_{10,9} := -.9088083788$

$t := 2.0 \quad u_{1,10} := 6.389056099 \quad u_{10,10} := -1.664587346$

0.2214	0.4918	0.8221	1.226	1.718	2.320	3.055	3.953	5.050	6.389
$u_{2,1}$	$u_{2,2}$	$u_{2,3}$	$u_{2,4}$	$u_{2,5}$	$u_{2,6}$	$u_{2,7}$	$u_{2,8}$	$u_{2,9}$	$u_{2,10}$
$u_{3,1}$	$u_{3,2}$	$u_{3,3}$	$u_{3,4}$	$u_{3,5}$	$u_{3,6}$	$u_{3,7}$	$u_{3,8}$	$u_{3,9}$	$u_{3,10}$
$u_{4,1}$	$u_{4,2}$	$u_{4,3}$	$u_{4,4}$	$u_{4,5}$	$u_{4,6}$	$u_{4,7}$	$u_{4,8}$	$u_{4,9}$	$u_{4,10}$
$u_{5,1}$	$u_{5,2}$	$u_{5,3}$	$u_{5,4}$	$u_{5,5}$	$u_{5,6}$	$u_{5,7}$	$u_{5,8}$	$u_{5,9}$	$u_{5,10}$
$u_{6,1}$	$u_{6,2}$	$u_{6,3}$	$u_{6,4}$	$u_{6,5}$	$u_{6,6}$	$u_{6,7}$	$u_{6,8}$	$u_{6,9}$	$u_{6,10}$
$u_{7,1}$	$u_{7,2}$	$u_{7,3}$	$u_{7,4}$	$u_{7,5}$	$u_{7,6}$	$u_{7,7}$	$u_{7,8}$	$u_{7,9}$	$u_{7,10}$
$u_{8,1}$	$u_{8,2}$	$u_{8,3}$	$u_{8,4}$	$u_{8,5}$	$u_{8,6}$	$u_{8,7}$	$u_{8,8}$	$u_{8,9}$	$u_{8,10}$
$u_{9,1}$	$u_{9,2}$	$u_{9,3}$	$u_{9,4}$	$u_{9,5}$	$u_{9,6}$	$u_{9,7}$	$u_{9,8}$	$u_{9,9}$	$u_{9,10}$
3.920	3.684	3.301	2.787	2.161	1.449	0.6799	−.1168	−.9088	−1.665

Tenglama o'ng tomonidagi F(x,t) funksiyasining qiymatlarini:

```
> for i from 1 by 1 to 2*n do
  for j from 1 by 1 to 2*m do
   x:=i*h; t:=j*l:F0[i,j]:=Fxt(x,t):
od; od; evalm(F0):evalf(%,4);
```

0.004000	0.008000	0.01200	0.01600	0.02000	0.02400	0.02800	0.03200	0.03600	0.04000
0.01600	0.03200	0.04800	0.06400	0.08000	0.09600	0.1120	0.1280	0.1440	0.1600
0.03600	0.07200	0.1080	0.1440	0.1800	0.2160	0.2520	0.2880	0.3240	0.3600
0.06400	0.1280	0.1920	0.2560	0.3200	0.3840	0.4480	0.5120	0.5760	0.6400
0.1000	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	1.000
0.1440	0.2880	0.4320	0.5760	0.7200	0.8640	1.008	1.152	1.296	1.440
0.1960	0.3920	0.5880	0.7840	0.9800	1.176	1.372	1.568	1.764	1.960
0.2560	0.5120	0.7680	1.024	1.280	1.536	1.792	2.048	2.304	2.560
0.3240	0.6480	0.9720	1.296	1.620	1.944	2.268	2.592	2.916	3.240
0.4000	0.8000	1.200	1.600	2.000	2.400	2.800	3.200	3.600	4.000

```
> for i from 2 by 1 to 2*n-1 do
x:=i*h: u[i,1]:=f(x):
u[i,2]:=(f(h*(i+1))+f(h*(i-1)))/2+l*Fix(h*i):
od; evalm(u):evalf(%,4);
```

$$x := 0.4 \quad u_{2,1} := 0.16 \quad u_{2,2} := 0.2778836685$$

$$x := 0.6 \quad u_{3,1} := 0.36 \quad u_{3,2} := 0.5129284947$$

$$x := 0.8 \quad u_{4,1} := 0.64 \quad u_{4,2} := 0.8234712182$$

$$x := 1.0 \quad u_{5,1} := 1.00 \quad u_{5,2} := 1.208294197$$

$$x := 1.2\ u_{6,\,1} := 1.44\ u_{6,\,2} := 1.666407817$$
$$x := 1.4\ u_{7,\,1} := 1.96\ u_{7,\,2} := 2.197089946$$
$$x := 1.6\ u_{8,\,1} := 2.56\ u_{8,\,2} := 2.799914721$$
$$x := 1.8\ u_{9,\,1} := 3.24\ u_{9,\,2} := 3.474769526$$

0.2214	0.4918	0.8221	1.226	1.718	2.320	3.055	3.953	5.050	6.389
0.16	0.2779	0.7310	1.180	1.701	2.311	3.031	3.887	4.911	6.350
0.36	0.5129	0.4755	1.204	1.770	2.408	3.139	3.984	5.182	5.627
0.64	0.8235	0.7465	1.060	1.904	2.588	3.351	4.422	4.687	4.867
1.00	1.208	1.089	1.436	1.866	2.831	3.853	4.034	4.084	4.045
1.44	1.666	1.501	1.879	2.343	3.107	3.486	3.483	3.355	3.144
1.96	2.197	1.981	2.385	3.092	2.964	2.696	2.761	2.492	2.153
2.56	2.800	2.527	3.162	2.967	2.634	2.184	1.642	1.488	1.063
3.24	3.475	3.347	3.067	2.653	2.125	1.508	0.8293	0.1207	−.1331
3.920	3.684	3.301	2.787	2.161	1.449	0.6799	−.1168	−.9088	−1.665

```
> for j from 2 by 1 to 2*m-1 do
  for i from 2 by 1 to 2*n-1 do
  u[i,j+1]:=u[i+1,j]+u[i-1,j]-u[i,j-1]+l*l*F0[i,j];
  od;od; evalm(u):UN:=evalf(%,3);
```

$UN :=$

0.221	0.492	0.822	1.23	1.72	2.32	3.06	3.95	5.05	6.39
0.16	0.278	0.846	1.29	1.81	2.41	3.12	3.97	4.99	5.67
0.36	0.513	0.744	1.42	1.98	2.60	3.32	4.15	4.58	5.03
0.64	0.823	1.09	1.43	2.21	2.88	3.62	3.92	4.18	4.35
1.00	1.21	1.50	1.87	2.32	3.22	3.46	3.63	3.66	3.60
1.44	1.67	1.98	2.37	2.85	2.88	3.20	3.17	3.02	2.78
1.96	2.20	2.52	2.94	2.90	2.79	2.54	2.54	2.24	1.87
2.56	2.80	3.13	3.02	2.84	2.52	2.08	1.55	1.32	0.863
3.24	3.47	3.27	3.00	2.59	2.07	1.46	0.784	0.0788	−.253
3.92	3.68	3.30	2.79	2.16	1.45	0.680	−.117	−.909	−1.66

izlanayotgan u(x,t) funksiya qiymatlarining matritsasi:

```
> with(linalg):transpose(UN);
```

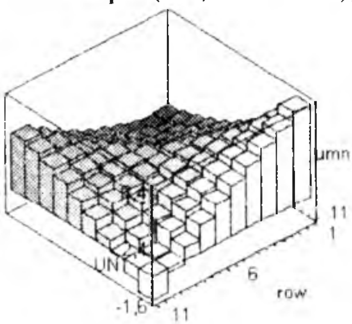
0.221	0.16	0.36	0.64	1.00	1.44	1.96	2.56	3.24	3.92
0.492	0.278	0.513	0.823	1.21	1.67	2.20	2.80	3.47	3.68
0.822	0.846	0.744	1.09	1.50	1.98	2.52	3.13	3.27	3.30
1.23	1.29	1.42	1.43	1.87	2.37	2.94	3.02	3.00	2.79
1.72	1.81	1.98	2.21	2.32	2.85	2.90	2.84	2.59	2.16
2.32	2.41	2.60	2.88	3.22	2.88	2.79	2.52	2.07	1.45
3.06	3.12	3.32	3.62	3.46	3.20	2.54	2.08	1.46	0.680
3.95	3.97	4.15	3.92	3.63	3.17	2.54	1.55	0.784	−.117
5.05	4.99	4.58	4.18	3.66	3.02	2.24	1.32	0.0788	−.909
6.39	5.67	5.03	4.35	3.60	2.78	1.87	0.863	−.253	−1.66

Giperbolik turdagi bir jinsli bo'lmagan differentsial tenglamani, to'r usulida topilgan, taqriy yechimining grafigini qurish:

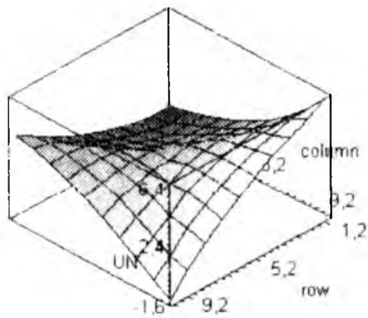
> **with(plots):with(LinearAlgebra):**

> **matrixplot(UN,heights=histogram,axes=boxed);** (7.18–rasm)

> **matrixplot(UN,axes=boxed);** (7.19–rasm)



7.18–rasm.



7.19–rasm.

O'z-o'zini tekshirish uchun savollar

1. Berilgan sohani to'r bilan qoplash, to'r tugunlarining turlari, tugun nuqtalarni aniqlash.
2. Xususiy hosilalarni chekli ayirmalar nisbati bilan almashtirishlar asosida to'r usuli moxiyatini tushuntiring.
3. Laplas yoki Puasson tenglamasi uchun Dirixle masalasining taqribiy yechimi to'r usuli yordamida qanday topiladi?
4. Taqribiy yechim xatoligini baxolash formulasini yozing.
5. Bir jinsli issiklik utkazuvchanlik tenglamasi uchun aralash masalani tur usuli yordamida taqribiy yechimi qanday topiladi?

6. Taqribiy yechim kaysi formulalar yordamida baxolanadi?

7. Bir jinsli bulmagan issiklik tarkalish tenglamasi va unga mos chekli ayirmali tenglamani xamda xatolikni baxolash formulalarini yozing.

8. Issiklik tarkalish tenglamasi uchun aralash masalani va unga mos chekli ayirmali sistemani yozing.

7.3-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

To'r usulida $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$ giperbolik tenglama yechimi $u(x,t)$ ning qiymatlarini,

$$u(x,0)=f(x),\ u'_t(x,0)=\Phi(x),\ (0\le x\le 1)$$

boshlang'ich va

$$u(0,t)=\varphi(t),\ u(s,t)=\psi(t),\ 0\le t\le 0.5$$

chegaraviy shartlari asosida $h=0.1$, $l=0.01$ qadamlar bilan hisoblang.

№	f (x)	Φ(x)	φ(t)	ψ(t)
1	$x(x+1)$	$\cos x$	0	$2(t+1)$
2	$x\cos\pi x$	$x(2-x)$	$2t$	-1
3	$\cos(\pi x/2)$	x^2	$1+2t$	0
4	$x(x+0,5)$	$\sin(x+0,2)$	$t-0,5$	$3t$
5	$2x(x+1)+0,3$	$2\sin x$	0,3	$4,3+t$
6	$(x+0,2)\sin(\pi x/2)$	$1+x^2$	0	$1,2(t+1)$
7	$x\sin\pi x$	$(x+1)^2$	$2t$	0
8	$3(1-x)x$	$\cos(x+0,5)$	$2t$	0
9	$x(2x-0,5)$	$\cos 2x$	t^2	1,5
10	$(x+1)\sin\pi x$	x^2+x	0	0,5
11	$(1-x)\cos(\pi x/2)$	$2x+1$	$2t+1$	0
12	$0,5x(x-1)$	$x\cos x$	$2t^2$	1
13	$0,5(x^2+1)$	$x\sin 2x$	$0,5+3t$	1
14	$(x+1)\sin(\pi x/2)$	$1-x^2$	$0,5t$	2
15	$x^2\cos\pi x$	$x^2(x+1)$	$0,5t$	$t-1$
16	$(1-x^2)\cos\pi x$	$2x+0,6$	$1+0,4t$	0
17	$(x+0,5)^2$	$(x+1)\sin x$	$0.5(0,5+t)$	2,25
18	$1,2x-x^2$	$(0,5+x)\sin x$	0	$0,2+0,5t$
19	$(0,6+x)x$	$\cos(x+0,3)$	0,5	$3-2t$
20	$0,5(x+1)^2$	$(0,5+x)\cos\pi x$	0,5	$2-3t$
21	$(x+0,4)\sin\pi x$	$(x+1)^2$	$0,5t$	0
22	$(2-x)\sin\pi x$	$(0,6+x)^2$	$0,5t$	0

23	$x\cos(\pi x/2)$	$2x^2$	0	t^2
24	$(0,4+x)\cos(\pi x/2)$	$0,3(x^2+1)$	0,4	$1,2t$
25	$1-x^2+x$	$2\sin(x+0,4)$	1	$(t+1)^2$
26	$0,4(x+0,6)^2$	$x\sin(x+0,6)$	$0,5+5t$	0,9
27	$(x^2+0,5)\cos\pi x$	$(0,7+x^2)$	0,5	$2t-1,5$
28	$(x+2)(0,5x+1)$	$2\cos(x+\pi/6)$	2	$4,5-3t$
29	$(x^2+1)(1-x)$	$1-\sin x$	1	$0,5t$
30	$(0,2+x)\sin(\pi x/2)$	$1+x^2$	$0,6t$	1,2

8-LABORATORIYA

Kuzatilgan tajriba ma'lumotlariga asoslanib korrelyasion jadvalni tuzish

Maple dasturining buyruqlari:

> **with(stats[statplots])**– statistika amallarini nchaqirish;

> **transform[statsort](X)**– *X* vektor qiymatlarini saralash;

> **describe[count](X)**– *X* vektor qiymatlari sonini sanash;

> **max(X)**– *X* vektor qiymatlarining eng kattasini aniqlash;

> **transform[tallyinto](X,[90..94,94..98,98..102, 102..106])**– *X ning intervallardagi qiymatlari soni–chastotasini aniqlash;*

> **transform[classmark](X)**– *Intervallardagi X ning qiymatlar soni–chastotasini va o'rta qiymatlarini aniqlash;*

> **transform[statvalue](X)**– *X ning o'rta qiymatlarini aniqlash;*

> **transform[frequency](X)**– *X ning qiymatlar soni–chastotasini aniqlash;*

Maqsad: Korrelyatsion bog'lanishni va kuzatilgan tajriba ma'lumotlariga asoslanib korrelyasion jadvalni tuzishni o'rganish

Reja:

8.1. Korrelyatsion bog'lanish haqida

8.2. Tanlanmaning korrelyasion jadvalni tuzish.

8.3. Ko'paytmalar usuli yordamida korrelyasiya koeffisientini hisoblash

8.4. *Y* ning *X* ga regressiya to'g'ri chizig'ining tanlanma tenglamasini yozish.

8.5. Tanlanma korrelyasion nisbatini hisoblash.

8.1. Korrelyatsion bog'lanish haqida

Biror *y* miqdorning faktorga nisbatan bog'liqligini umumiy holda

$$y=f(x) \qquad (8.1)$$

ko'rinishda ifodalash mumkin. Bunday bog'lanish funksional yoki stoxastik holda uchrashi mumkin.

Agar *x* faktorning har bir qiymatida *y* miqdorning aniq bir qiymati topilgan bo'lsa, bunday bog'lanish funksional bog'lanish deyiladi.

Korrelyatsiya so'zi lotin tilidan olingan bo'lib, u "munosabat" yoki "o'zaro aloqa" degan ma'noni anglatadi. Yuqoridagi (8.1) bog'lanish korrelyatsion bog'lanish deyiladi, bu tenglama "*x*" miqdorga ko'ra "*y*" ning regressiya tenglamasi ham deyiladi.

Statistik bog'lanish deb shunday bog'lanishga aytiladiki, unda miqdorlardan birining o'zgarishi ikkinchisining taqsimoti o'zgarishga olib keladi. Xususan, statistik bog'liqlik miqdorlaridan birining o'zgarishi ikkinchisining o'rtacha qiymatini o'zgarishida ko'riladi. Bu holda statistik bog'lanish korrelyatsion bog'lanish deb aytiladi.

Korrelyatsion bog'liqlik ta'sirini aniqlashtiramiz. Buning uchun shartli o'rtacha qiymati tushunchasini kiritamiz.

Aytaylik, y va x tasodifiy miqdorlar orasidagi bog'lanish o'rganilayotgan bo'lsin. x ning har bir qiymatiga y ning bir nechta qiymati mos kelsin.

Masalan, $x_1=2$ da, y miqdor $y_1=5$, $y_2=6$, $y_3=10$ qiymatlar olgan bo'lsin.

Bu sonlarning arifmetik o'rtacha qiymatini topamiz:

$$\bar{y}_i = \frac{5+6+10}{3} = 7 \tag{8.2}$$

son *shartli o'rtacha* qiymat deyiladi. y harfi ustidagi chiziqga arifmetik o'rtacha qiymat belgisi bo'lib xizmat qiladi. 2 soni esa y ning $x_1=2$ ga mos qiymatlari qaralayotganini ko'rsatadi. Yuqoridagi misolga nisbatan olganda, bu maolumotlarni quyidagicha tahmin qilish mumkin. Uchta bir xil uchastkaning har biriga y birlikdan o'g'it solindi va mos ravishda 5, 6 va 10 birlikdan paxta hosili olindi; o'rtacha hosil 7 birlik bo'ladi.

Shartli o'rtacha qiymat deb uning $x=x_0$ qiymatga mos qiymatlarning *arifmetik o'rtacha* qiymatiga aytiladi.

Agar har bir x qiymatga shartli o'rtacha qiymatning bitta qiymati mos kelsa, u holda, ravshanki shartli o'rtacha qiymat x ning funksiyasidir. Bu holda y tasodifiy miqdor x miqdorga *korrelyatsion bog'liq* deyiladi.

Korrelyatsiya nazariyasining **birinchi masalasi** –korrelyatsion bog'lanish formasini aniqlash, ya'ni regressiya funksiyasining ko'rinishini topishdir.

Regressiya funksiyalari ko'p hollarda chiziqli bo'ladi.

Korrelyatsiya nazariyasining **ikkinchi masalasi** –korrelyatsion bog'lanishning zichligini aniqlashdir.

y ning x ga korrelyatsion bog'liqlikning zichligi y ning qiymatlarini shartli o'rtacha qiymat atrofida tarqoqligining kattaligi bo'yicha baholanadi.

Ko'p tarqoqlik y ning x ga kuchsiz bog'liqligidan yoki bog'liqlik yo'qligidan darak beradi. Kam tarqoqlik ancha kuchli bog'liqlik borligini ko'rsatadi; bu holda y va x xatto funksional bog'langan bo'lib, lekin ikkinchi darajali tasodifiy faktorlar ta'sirida bu bog'lanish kuchsizlangan, buning natijasida esa x ning bitta qiymatida y turli qiymatlar qabul qilishi mumkin.

8.2. Tanlanmaning korrelyasion jadvalni tuzish

Quyidagi jadvalda ma'lum bir shahardagi 20 ta erkakning ko'krak aylanasi uzunligi X (sm.da) va bo'yi Y (sm.da) berilgan.

1 – jadval

X	91	95	97	99	92	96	100	100	97	101
Y	160	169	162	168	164	164	165	169	159	170
X	97	95	102	98	101	99	103	104	104	103
Y	171	185	171	166	172	175	170	181	176	175

1. Korrelyasion jadvalni tuzamiz. Buning uchun X va Y belgilarning umumiy o'zgarish intervallarini topamiz:

$R_1 = x_{max} - x_{min} = 104 - 91 = 13;$

$R_2 = y_{max} - y_{min} = 181 - 159 = 22;$

Eng katta qiymatlarni biroz o'ngga va eng kichik qiymatlarni biroz chapga surib, o'zgarish intervallarini qulay holga keltirib olish mumkin.

Masalan,

$x_{max} = 106, x_{min} = 90, y_{max} = 185, y_{min} = 155$

kabi tanlasak

$R_1 = 16; R_2 = 30$

bo'ladi.

Bu holda intervallar sonini $k_1 = 4; k_2 = 5$ deb olib, X va Y belgilar qismiy intervallarining uzunliklarini topamiz:

$$h_1 = \Delta x = R_1 / k_1 = 16/4 = 4,\ h_2 = \Delta y = R_2 / k_2 = 30/5 = 6.$$

Korrelyasiya jadvalini quyidagicha tuzamiz:

1 – qatorga uzunligi $h_1 = 4$ bo'lgan X ning qismiy intervallarini;

2 – qatorga bu intervallarning o'rtalari x_i larni yozamiz.

1 – ustunga uzunligi $h_2 = 6$ bo'lgan Y ning qismiy intervallarini;

2 – ustunga bu intervallarning o'rtalari y_i larni topib yozamiz.

X ning qismiy intervallari va Y ning qismiy intervallari kesishgan qismga
tushuvchi (x_i, y_i) qiymatlarni sanab. (Bunda intervallarning chegaralariga to'g'ri kelgan
qiymatlarni faqat oldingi intervallarga tushadi deb sanaymiz).

2 – jadval

Y \ X	$h_1 = 4$	90 – 94	94 – 98	98 – 102	102 – 106	
$h_2 = 6$	Y \ X	$X_1 = 92$	$X_2 = 96$	$X_3 = 100$	$X_4 = 104$	n_y
155 – 161	$Y_1 = 158$	1	1			2
161 – 167	$Y_2 = 164$	1	4	1		6
167 – 173	$Y_3 = 170$		2	5	1	8
173 – 179	$Y_4 = 176$			1	2	3
179 – 185	$Y_5 = 182$				1	1
	n_x	2	7	7	4	$n = 20$

Qatorlar bo'yicha chastotalarni jamlab, n_x larni topamiz va oxirgi ustunga yozamiz.

Ustunlar bo'yicha chastotalarni jamlab, n_y larni topamiz va oxirgi qatorga yozamiz.

n_x larning yig'indisi ham , n_y larning yig'indisi ham tanlanma hajmi $n=20$ ga teng bo'ladi.

8.1.1–M a p l e d a s t u r i:

> restart;with(stats[statplots]): Digits:=3:

> X:=[91,95,97,99,92,96,100,100,97,101.97,95,102,96, 101,90, 103,104,104,103];

$X := [91, 95, 97, 99, 92, 96, 100, 100, 97, 101, 97, 95, 102, 96, 101, 90, 103, 104, 104, 103]$

> Y:=[160,169,162,168,164,164,165,169,159,170,171, 165, 171, 166,172, 175,170,181,176,175];

$Y := [160, 169, 162, 168, 164, 164, 165, 169, 159, 170, 171, 165, 171, 166, 172, 175, 170, 181, 176, 175]$

Saralash :

> X:=transform[statsort](X);

$X := [90, 91, 92, 95, 95, 96, 96, 97, 97, 97, 99, 100, 100, 101, 101, 102, 103, 103, 104, 104]$

> Y:=transform[statsort](Y);

$Y := [159, 160, 162, 164, 164, 165, 165, 166, 168, 169, 169, 170, 170, 171, 171, 172, 175, 175, 176, 181]$

Tanlanma hajmi :

> N1:=describe[count](X); $N1 := 20$

> N2:=describe[count](Y); $N2 := 20$

Intervallar soni :

> k1:=1+3.2*log[10](20);k1:=evalf(%,2);

$$k1 := 1 + \frac{3.2 \ln(20)}{\ln(10)} \quad k1 := 5.2$$

> k2:=1+3.2*log[10](20);k2:=evalf(%,2);

$$k2 := 1 + \frac{3.2 \ln(20)}{\ln(10)} \quad k2 := 5.2$$

Tuzatilgan intervallar sonini:

> **k1:=4;** $k1 := 4$

> **k2:=5;** $k2 := 5$

Eng katta va eng kichik qiymatni aniqlash :

> **Xmax:=max(90,91,92,95,95,96,96,97,97,97,99,100, 100, 101, 101, 102, 103, 103, 104, 104);**

$$Xmax := 104$$

> **Xmin:=min(90,91,92,95,95,96.96,97,97,97,99,100, 100, 101, 101, 102, 103, 103, 104, 104);**

$$Xmin := 90$$

> **Ymax:=max(159,160,162,164,164,165,165,166,168,169, 169, 170, 170, 171, 171, 172, 175, 175, 176, 181);**

$$Ymax := 181$$

> **Ymin:=min(159,160,162,164.164.165,165,166,168,169, 169,170,170,171,171,172,175,175,176,181);**

$$Ymin := 159$$

Qiymatlar qulachi :

> **R1:=Xmax–Xmin;** $R1 := 14$

> **R2:=Ymax–Ymin;** $R2 := 22$

Tuzatilgan qiymatlar qulochi:

> **R1:=16;** $R1 := 16$

> **R2:=30;** $R2 := 30$

Interval qadami :

> **h1:=R1/k1;h2:=R2/k2;** $h1 := 4 \; h2 := 6$

Birinchi intervalning chap qiymatini aniqlash :

> **x0:=X[1]–(h1*k1–R1);x0:=evalf(%,3);** $x0 := 90 \;\; x0 := 90.$

X ning qismiy intervallarni aniqlash:

> **for i to k1 do x[i]:=x0+(i–1)*h1; print(x[i],x[i]+h1) od;**

$$x_1 := 90. \quad 90., 94.$$

$$x_2 := 94. \quad 94., 98.$$

$$x_3 := 98. \quad 98., 102.$$

$$x_4 := 102. \quad 102., 106.$$

Intervallarga tushuvchi X ning qiymatlari soni–chastotasini aniqlash:

> **transform[tallyinto]((X,[90..94,94..98,98..102, 102..106]);**

[*Weight*(90..94, 3), *Weight*(94..98, 7), *Weight*(98..102, 5),
Weight(102..106, 5)]

> **X:=transform[statsort](%);**

$X := [Weight(90..94, 3), Weight(94..98, 7), Weight(98..102, 5), Weight(102..106, 5)]$

Intervallardagi X ning o'rta qiymatlari soni–chastotasini aniqlash:

> **X:=transform[classmark](X);**

$X := [Weight(92, 3), Weight(96, 7), Weight(100, 5), Weight(104, 5)]$

> **X1:=transform[statvalue](X);** $X1 := [92, 96, 100, 104]$

> **nx:=transform[frequency](X);** $nx := [3, 7, 5, 5]$

> **nx:=[2,7,7,4];** $nx := [2, 7, 7, 4]$

Y ning qismiy intervallarni aniqlash:

> **Y[1]:=155:**

> **y0:=Y[1]–(h2*k2–R2);y0:=evalf(%,3);** $y0 := 155$ $y0 := 155.$

> **for i to k2 do y[i]:=y0+(i–1)*h2; print(y[i],y[i]+h2) od;**

$$y_1 := 155. \quad 155., 161.$$
$$y_2 := 161. \quad 161., 167.$$
$$y_3 := 167. \quad 167., 173.$$
$$y_4 := 173. \quad 173., 179.$$
$$y_5 := 179. \quad 179., 185.$$

Intervallarga tushuvchi Y ning qiymatlari soni–chastotasini aniqlash:

> **transform[tallyinto](Y,[155..161,161..167,167..173, 173..179, 179..185]);**

$[Weight(155..161, 2), Weight(161..167, 6), Weight(167..173, 8), Weight(173..179, 3), 179..185]$

> **Y:=transform[statsort](%);**

$Y := [Weight(155..161, 2), Weight(161..167, 6), Weight(167..173, 8), Weight(173..179, 3), 179..185]$

Intervallardagi X ning o'rta qiymatlari soni–chastotasini aniqlash:

> **Y:=transform[classmark](Y);**

$Y := [Weight(158, 2), Weight(164, 6), Weight(170, 8), Weight(176, 3), 182]$

> **Y1:=transform[statvalue](Y);** $Y1 := [158, 164, 170, 176, 182]$

> **ny:=transform[frequency](Y);** $ny := [2, 6, 8, 3, 1]$

8.3. Ko‘paytmalar usuli yordamida korrelyasiya koeffisientini hisoblash

Agar X va Y belgilar ustida kuzatish ma'lumotlari teng uzoqlikdagi variantali korrelyasion 2-jadvai ko‘rinishda berilgan bo'lsa,

$$u_i = \frac{x_i - C_1}{h_1}, \quad v_i = \frac{y_i - C_2}{h_2} \qquad (*)$$

shartli variantlarga o'tamiz. Bunda $C_1 - x_i$ variantlarni «soxta noli» bo'lib, uni korrelyasion jadvaldagi eng katta chastotaga mos ravishda olamiz. Tanlanma qadami $h_1 = x_{i+1} - x_i$. $C_2 - y_i$ variantlarni «soxta noli», $h_2 = y_{i+1} - y_i$.

Bu holda tanlanma korrelyasiya koeffisenti quyidagicha bo‘ladi.

$$r_t = \frac{\sum n_{uv} uv - n\bar{u}\bar{v}}{n\sigma_u \sigma_v}$$

Bunda $\bar{u}, \bar{v}$, σ_u, σ_v lar ko‘paytmalar usuli bilan yoki bevosita quyidagi formulalar bilan hisoblanadi

$$\bar{u} = \frac{\sum n_u u}{n}, \quad \bar{\vartheta} = \frac{\sum n_\vartheta \vartheta}{n},$$

$$\sigma_u = \sqrt{\overline{u^2} - (\bar{u})^2}, \quad \sigma_\vartheta = \sqrt{\overline{\vartheta^2} - (\bar{\vartheta})^2}$$

3 – j a d v a l

$v \setminus u$	–2	–1	0	1	n_ϑ
–2	1	1			2
–1	1	4	1		6
0		2	5	1	8
1			1	2	3
2				1	1
n_u	2	7	7	4	$n = 20$

Coxta nollar sifatida $C_1 = 100$ va $C_2 = 170$ ni tanlab (bu variantalar eng katta chastota $n_{xy} = 5$ ning to‘g‘risida joylashgan), $h_1 = 4$ va $h_2 = 6$ ekanligini e’tiborga olib (*) shartli variantalarga asosan 3–jadvalni tuzamiz, masalan

$$u_1 = \frac{x_1 - C_1}{h_1} = \frac{92 - 100}{4} = \frac{-8}{4} = -2,$$

$$v_1 = \frac{y_1 - C_2}{h_2} = \frac{158 - 170}{6} = \frac{-12}{6} = -2$$

kabi hisoblashlar bilan 3–jadvalni 1–satrini va 1– ustunini to'ldiramiz.

Bu 3–jadvaldagi ma'lumotlarga asoslanib korrelyasiya koeffisentini topish uchun, quyidagilarni hisoblaymiz:

$$\bar{u}=\frac{\sum n_u u}{n}=\frac{2(-2)+7(-1)+7\cdot 0+4\cdot 1}{20}=-\frac{7}{20}=-0,35$$

$$\bar{\vartheta}=\frac{\sum n_\vartheta \vartheta}{n}=\frac{2(-2)+6(-1)+8\cdot 0+3\cdot 1+1\cdot 2}{20}=-\frac{5}{20}=-0,25$$

$$\overline{u^2}=\frac{\sum n_u u^2}{n}=\frac{2(-2)^2+7(-1)^2+7\cdot 0^2+3\cdot 1^2+1\cdot 2^2}{20}=\frac{19}{20}=0,95$$

$$\overline{\vartheta^2}=\frac{\sum n_\vartheta \vartheta^2}{n}=\frac{2(-2)^2+6(-1)^2+8\cdot 0^2+3\cdot 1^2+1\cdot 2^2}{20}=\frac{21}{20}=1,05$$

$$\sigma_u=\sqrt{\overline{u^2}-(\bar{u})^2}=\sqrt{0,95-0,35^2}=\sqrt{0,8275}=0,911$$

$$\sigma_\vartheta=\sqrt{\overline{\vartheta^2}-(\bar{\vartheta})^2}=\sqrt{1,05-0,25^2}=\sqrt{0,9875}=0,991$$

$\sum n_{u\vartheta}u\vartheta$ ni topish uchun quyidagi 4 – jadvalni tuzamiz:

1)3–jadvaldagi har bir chastotani unga mos keluvchi u va ϑ larga ko'paytirib, shu kattalikni o'ng va chap burchagiga yozamiz;

2)o'ng burchakdagi sonlar yig'indisini $U=\sum n_{u\vartheta}u$ ustunga yozamiz va uni shu satrga mos ϑ ga ko 'paytirib $u\vartheta$ ustunga yozamiz;

3)chap burchakdagi sonlar yig'indisini $V=\sum n_{u\vartheta}\vartheta$ satrga yozamiz va uni shu ustunga mos u ga ko'paytirib uV ustunga yozamiz.

Hisoblashlarni tekshirish maqsadida oxirgi qator va ustundagi sonlar yig'indisini taqqoslaymiz:

$$\sum_U uV=\sum n_{u\vartheta}u\,\vartheta=16,\quad \sum_\vartheta \vartheta V=\sum n_{u\vartheta}u\,\vartheta=16$$

4 – j a d v a l

ϑ \ u	−2	−1	0	1	$U=\sum n_{u\vartheta}u$	ϑU
−2	$_{-2}$\ 1 \ $^{-2}$	$_{-2}$\ 1\ $^{-1}$			−3	6
−1	$_{-1}$\ 1 \ $^{-2}$	$_{-4}$\ 4\ $^{-4}$	$_{-1}$\ 1\ ^{0}		−6	6
0		$_{0}$\ 2 \ $^{-2}$	$_{0}$\ 5 \ ^{0}	$_{0}$\ 1\ ^{1}	−1	0
1			$_{1}$\ 1\ ^{0}	$_{2}$\ 2 \ ^{2}	2	2
2				$_{2}$\ 1\ ^{1}	1	2
$V=\sum n_{u\vartheta}\vartheta$	−3	−6	0	4		$\sum_\vartheta \vartheta V=16$
uV	6	6	0	4	$\sum_U uV=16$	Tekshir.

Yig'indilarning bir xilligi hisoblashlar to'g'riligini ko'rsatadi. Tanlanma korrelyasiya koeffisientini hisoblaymiz:

$$r_l = \frac{\sum n_{u\vartheta} u\vartheta - n\bar{u}\bar{\vartheta}}{n\sigma_u\sigma_\vartheta} = \frac{16 - 20\cdot(-0.35)\cdot(-0.25)}{20\cdot 0.911\cdot 0.991} = 0.78$$

Bundan $r_l = 0.78 > 0.5$ bo'lishi regression bog'lanish zichligining katta ekanligini ko'rsatadi.

8.1.2-Maple dasturi:

$3 - jadvalni\ \ tuzish$:

```
> restart;with(stats[statplots]): Digits:=3:
> N1:=20:N2:=20:k1:=4:k2:=5:h1:=4:h2:=6:
> nx:=[2,7,7,4]; ny:=[2,6,8,3,1];
                    nx := [2, 7, 7, 4]  ny := [2, 6, 8, 3, 1]
> X1:= [92,96,100,104]; X1 := [92, 96, 100, 104]
> Y1:=[158,164,170,176,182]; Y1 := [158, 164, 170, 176, 182]
> C1:=100: u:=[seq((X1[i]-C1)/h1,i=1..4)];
u := [ -2, -1, 0, 1 ]
> C2:=170: v:=[seq((Y1[i]-C2)/h2,i=1..5)];
v := [ -2, -1, 0, 1, 2 ]
```

$\bar{u}$ ni hisoblash:

```
> u0:=seq(u[i]*nx[i]/N1,i=1..4);
```
$u0 := -\frac{1}{5}, -\frac{7}{20}, 0, \frac{1}{5}$

```
>u0:=add(u[i]*nx[i]/N1,i=1..4);u0:=evalf(%);
```
$u0 := -\frac{7}{20} \quad u0 := -.350$

```
> u20:=add(u[i]^2*nx[i]/N1,i=1..4);
```
$u20 := \frac{19}{20}$

$\bar{v}$ ni hisoblash:

```
> v0:=seq(v[i]*ny[i]/N2,i=1..5);
```
$v0 := -\frac{1}{5}, -\frac{3}{10}, 0, \frac{3}{20}, \frac{1}{10}$

```
> v0:=add(v[i]*ny[i]/N2,i=1..5);v0:=evalf(%);
```
$v0 := -\frac{1}{4} \quad v0 := -.250$

```
> v20:=add(v[i]^2*ny[i]/N2,i=1..5); evalf(%);
```

$$v20 := \frac{21}{20} \quad 1.05$$

σ_u, σ_v *larni hisoblash:*

```
> sigma[1]:=sqrt(u20-u0^2);evalf(%);
```
$\sigma_1 := 0.910 \quad 0.910$

```
> sigma[2]:=sqrt(v20-v0^2);evalf(%);
```
$\sigma_2 := 0.994 \quad 0.994$

```
> nuv:=matrix([[1,1,0,0],[1,4,1,0],[0,2,5,1], [0,0,1,2],[0,0,0,1]]);
```

$$nuv := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$V = \sum n_{u\vartheta}\vartheta$ *larni hisoblash:*

```
> V[1]:=seq(v[i]*nuv[i,1],i=1..5);
```
$$V_1 := -2, -1, 0, 0, 0$$

```
V[1]:=add(v[i]*nuv[i,1], i=1..5);
```
$V_1 := -3$

```
> V[2]:=seq(v[i]*nuv[i,2],i=1..5);
```
$$V_2 := -2, -4, 0, 0, 0$$

```
V[2]:=add(v[i]*nuv[i,2], i=1..5);
```
$V_2 := -6$

```
> V[3]:=seq(v[i]*nuv[i,3],i=1..5);
```
$$V_3 := 0, -1, 0, 1, 0$$

```
V[3]:=add(v[i]*nuv[i,3], i=1..5);
```
$V_3 := 0$

```
> V[4]:=seq(v[i]*nuv[i,4],i=1..5);
```
$$V_4 := 0, 0, 0, 2, 2$$

```
V[4]:=add(v[i]*nuv[i,4],i=1..5);
```
$V_4 := 4$

```
> S2:=add(V[i]*u[i],i=1..4);
```
$S2 := 16$

$U = \sum n_{u\vartheta} u$ *larni hisoblash:*

```
> U[1]:=seq(u[j]*nuv[1,j],j=1..4);
```
$U_1 := -2, -1, 0, 0$

```
U[1]:=add(u[j]*nuv[1,j],j=1..4);
```
$U_1 := -3$

```
> U[2]:=seq(u[j]*nuv[2,j],j=1..4);
```
$U_2 := -2, -4, 0, 0$

```
U[2]:=add(u[j]*nuv[2,j], j=1..4);
```
$U_2 := -6$

```
> U[3]:=seq(u[j]*nuv[3,j],j=1..4);
```
$U_3 := 0, -2, 0, 1$

Yig'indilarning bir xilligi hisoblashlar to'g'riligini ko'rsatadi. Tanlanma korrelyasiya koeffisientini hisoblaymiz:

$$r_I = \frac{\sum n_{u\vartheta} u\vartheta - n\bar{u}\bar{\vartheta}}{n\sigma_u \sigma_\vartheta} = \frac{16 - 20\cdot(-0.35)\cdot(-0.25)}{20\cdot 0.911\cdot 0.991} = 0.78$$

Bundan $r_I = 0.78 > 0.5$ bo'lishi regression bog'lanish zichligining katta ekanligini ko'rsatadi.

8.1.2–Maple dasturi:

3 – jadvalni tuzish :

> restart;with(stats[statplots]): Digits:=3:

> N1:=20:N2:=20:k1:=4:k2:=5:h1:=4:h2:=6:

> nx:=[2,7,7,4]; ny:=[2,6,8,3,1];

$nx := [2, 7, 7, 4]$ $ny := [2, 6, 8, 3, 1]$

> X1:= [92,96,100,104]; $X1 := [92, 96, 100, 104]$

> Y1:=[158.164,170,176,182]; $Y1 := [158, 164, 170, 176, 182]$

> C1:=100: u:=[seq((X1[i]–C1)/h1,i=1..4)];

$u := [-2, -1, 0, 1]$

> C2:=170: v:=[seq((Y1[i]–C2)/h2,i=1..5)];

$v := [-2, -1, 0, 1, 2]$

$\bar{u}$ *ni hisoblash:*

> u0:=seq(u[i]*nx[i]/N1,i=1..4); $u0 := -\frac{1}{5}, -\frac{7}{20}, 0, \frac{1}{5}$

>u0:=add(u[i]*nx[i]/N1,i=1..4);u0:=evalf(%);

$u0 := -\frac{7}{20}$ $u0 := -.350$

> u20:=add(u[i]^2*nx[i]/N1,i=1..4); $u20 := \frac{19}{20}$

$\bar{v}$ *ni hisoblash:*

> v0:=seq(v[i]*ny[i]/N2,i=1..5);

$v0 := -\frac{1}{5}, -\frac{3}{10}, 0, \frac{3}{20}, \frac{1}{10}$

> v0:=add(v[i]*ny[i]/N2,i=1..5);v0:=evalf(%);

$v0 := -\frac{1}{4}$ $v0 := -.250$

> v20:=add(v[i]^2*ny[i]/N2,i=1..5); evalf(%);

$$v20 := \frac{21}{20} \quad 1.05$$

σ_u, σ_v larni hisoblash:

> **sigma[1]:=sqrt(u20–u0^2);evalf(%);** $\sigma_1 := 0.910 \quad 0.910$

> **sigma[2]:=sqrt(v20–v0^2);evalf(%);** $\sigma_2 := 0.994 \quad 0.994$

> **nuv:=matrix([[1,1,0,0],[1,4,1,0],[0,2,5,1], [0,0,1,2],[0,0,0,1]]);**

$$nuv := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$V = \sum n_{u\vartheta}\vartheta$ larni hisoblash:

> **V[1]:=seq(v[i]*nuv[i,1],i=1..5);**

$$V_1 := -2, -1, 0, 0, 0$$

V[1]:=add(v[i]*nuv[i,1], i=1..5); $V_1 := -3$

> **V[2]:=seq(v[i]*nuv[i,2],i=1..5);**

$$V_2 := -2, -4, 0, 0, 0$$

V[2]:=add(v[i]*nuv[i,2], i=1..5); $V_2 := -6$

> **V[3]:=seq(v[i]*nuv[i,3],i=1..5);**

$$V_3 := 0, -1, 0, 1, 0$$

V[3]:=add(v[i]*nuv[i,3], i=1..5); $V_3 := 0$

> **V[4]:=seq(v[i]*nuv[i,4],i=1..5);**

$$V_4 := 0, 0, 0, 2, 2$$

V[4]:=add(v[i]*nuv[i,4],i=1..5); $V_4 := 4$

> **S2:=add(V[i]*u[i],i=1..4);** $S2 := 16$

$U = \sum n_{u\vartheta}u$ larni hisoblash:

> **U[1]:=seq(u[j]*nuv[1,j],j=1..4);** $U_1 := -2, -1, 0, 0$

U[1]:=add(u[j]*nuv[1,j],j=1..4); $U_1 := -3$

> **U[2]:=seq(u[j]*nuv[2,j],j=1..4);** $U_2 := -2, -4, 0, 0$

U[2]:=add(u[j]*nuv[2,j], j=1..4); $U_2 := -6$

> **U[3]:=seq(u[j]*nuv[3,j],j=1..4);** $U_3 := 0, -2, 0, 1$

U[3]:=add(u[j]*nuv[3,j], j=1..4); $U_3 := -1$

> U[4]:=seq(u[j]*nuv[4,j],j=1..4); $U_4 := 0, 0, 0, 2$

U[4]:=add(u[j]*nuv[4,j], j=1..4); $U_4 := 2$

> U[5]:=seq(u[j]*nuv[5,j],j=1..4); $U_5 := 0, 0, 0, 1$

U[5]:=add(u[j]*nuv[5,j],j=1..4); $U_5 := 1$

> S1:=add(U[i]*v[i],i=1..5); $S1 := 16$

r_T –korrelyasiya koeffisientini hisoblash:

> rT:=(S1–N1*u0*v0)/(N1*sigma[1]*sigma[2]); rT:=evalf(%);

$rT := 0.785 \quad rT := 0.785$

8.4. *Y* ning *X* ga regressiya to'g'ri chizig'ining tanlanma tenglamasini aniqlash

Y ning *X* ga regressiya to'g'ri chizig'ining tenglamasi

$$\bar{y}_x - \bar{y} = r_T \frac{\sigma_y}{\sigma_x}(x - \bar{x}) \qquad (*)$$

ni aniqlash uchun 3–jadvalda $h_1 = 4$, $h_2 = 6$, $C_1 = 100$, $C_2 = 170$ ekanligini e'tiborga olib, quyidagilarni topamiz:

$$\bar{x} = \bar{u}\, h_1 + C_1 = -0.35 \cdot 4 + 100 = 98.6$$

$$\bar{y} = \bar{\vartheta}\, h_2 + C_2 = -0.25 \cdot 6 + 170 = 168.5$$

$$\sigma_x = h_1\sigma_u = 4 \cdot 0.991 = 3.64$$

$$\sigma_y = h_2\sigma_\vartheta = 6 \cdot 0.991 = 5.95$$

Topilgan kattaliklarni (*) ga qo'yib, *Y* ning *X* ga regressiya to'g'ri chizig'ining tenglamasini hosil qilamiz:

$\bar{y}_x - 168.5 = 0.78\,(5.95/3.64)\,(x - 98.6)$

$\bar{y}_x - 168.5 = 1.27x - 125.2$

$$\bar{y}_x = 1.27x + 43.3$$

Endi bu tenglama bo'yicha shartli o'rtacha qiymatlarni hisoblaymiz:

$$\bar{y}_{92} = 1.27 \cdot 92 + 43.3 = 160.14$$

$\bar{y}_{96} = 1.27 \cdot 94 + 43.3 = 165.22$

$\bar{y}_{100} = 1.27 \cdot 100 + 43.3 = 170.3$

$\bar{y}_{104} = 1.27 \cdot 104 + 43.3 = 175.38$

2 – jadvaldagi ma'lumotlar bo'yicha shartli o'rtacha qiymatlarni topamiz:

$\bar{y}_{92} = (1 \cdot 158 + 1 \cdot 164)/2 = 161.0$

$\bar{y}_{96} = (1 \cdot 158 + 4 \cdot 164 + 2 \cdot 170)/7 = 164.8$

$\bar{y}_{100} = (1 \cdot 164 + 5 \cdot 170 + 1 \cdot 176)/7 = 170.0$

$\bar{y}_{104} = (1 \cdot 170 + 2 \cdot 176 + 1 \cdot 182)/4 = 176.0$

Ko'rinib turibdiki, topilgan regressiya to'g'ri chizig'ining tenglamasi bo'yicha hisoblangan va kuzatilgan shartli o'rtacha qiymatlarning mos kelishi qoniqarlidir.

8.5. Tanlanma korrelyasion nisbatini hisoblash

η_{yx} tanlanma korrelyasion nisbatini hisoblaymiz. U Y ning X ga bog'lanish zichligini aniqlaydi.

Buning uchun 2 – korrelyasion jadvaldagi ma'lumotlar bo'yicha quyidagilarni hisoblaymiz. Umumiy o'rtacha qiymat:

$$\bar{y} = (\sum n_y y)/n = \frac{1}{20}(2 \cdot 158 + 6 \cdot 164 + 8 \cdot 170 + 3 \cdot 176 + 1 \cdot 182) = 168,5$$

Umumiy o'rtacha kvadratik chetlanish:

$$\sigma_y = \sqrt{\frac{1}{n}\sum n_y(y-\bar{y})^2} = \{\frac{1}{20}[2 \cdot (158-168,5)^2 + 6(164-168,5)^2 +$$
$$+ 8(170-168,5)^2 + 3(176-168,5)^2 + 1(182-168,5)^2]\}^{1/2} = 5,95$$

Guruxlar aro o'rtacha kvadratik chetlanish:

$$\sigma_{\bar{y}_x} = \sqrt{\frac{1}{n}\sum n_x(\bar{y}_x - \bar{y})^2} = \{\frac{1}{20}[2 \cdot (161,0 - 168,5)^2 + 7 \cdot (164,8- 168,5)^2 +$$
$$+ 7 \cdot (170,0 - 168,5)^2 + 4 \cdot (176,0 - 168,5)^2]\}^{1/2} =$$

$$= \{\frac{1}{20}[2 \cdot 56,25 + 7 \cdot 13,69 + 7 \cdot 3,25 + 4 \cdot 56,25)\}^{1/2} =$$

$$= \{\frac{1}{20}[112.50 + 95,83 + 22,75 + 225,0)\}^{1/2} = \sqrt{22,80} = 4,78$$

Endi tanlanma korrelyasion nisbatni topamiz:

$$\eta_{\bar{y}_x} = \frac{\sigma_{\bar{y}_x}}{\sigma_y} = 4,78/5,95 = 0,803$$

Y ning X ga regressiya to'g'ri chizig'ining tenglamasini aniqlash va tanlanma korrelyasion nisbatini hisoblash dasturi.

8.1.3–M a p l e d a s t u r i:

Regressiya togri chizigini aniqlash :

```
> restart; Digits:=4:
> u0 :=-.350:v0:=-.250: rT :=.789:
> C1:=100:C2:=170:h1:=4:h2:=6:N1:=20:N2:=20:
> nx := [2, 7, 7, 4];ny := [2, 6, 8, 3, 1];
```

$$nx := [2, 7, 7, 4]\ ny := [2, 6, 8, 3, 1]$$

```
> sigma[1]:= .910: sigma[2]:= .994:
> x1:=u0*h1+C1;
```
$x1 := 98.60$

```
> x1:=evalf(%);
```
$x1 := 98.60$

```
> y1:=v0*h2+C2;
```
$y1 := 168.5$

```
> y1:=evalf(%);
```
$y1 := 168.5$

```
> Gx:=h1*sigma[1];
```
$Gx := 3.640$

```
> Gx:=evalf(%);
```
$Gx := 3.640$

```
> Gy:=h2*sigma[2];
```
$Gy := 5.964$

```
> Gy:=evalf(%);
```
$Gy := 5.964$

```
> Yx:=y1+rT*Gy*(x-x1)/Gx;
```
$Yx := 41.0 + 1.293\,x$

Tekshirish

```
> x:=92:Yx;x:=96:Yx;x:=100:Yx;x:=104:Yx;
```

$$160.0\quad 165.1\quad 170.3\quad 175.5$$

```
> Yx:=[160,165.1,170.3,175.5]:
```

η_{ux} tanlanma korrelyasion nisbatini hisoblash:

```
> Y1:=[158,164,170,176,182];
```
$Y1 := [158, 164, 170, 176, 182]$

```
> Yt:=add(ny[i]*Y1[i]/N2,i=1..5);Yt:=evalf(%);
```

$$Yt := \frac{337}{2}\quad Yt := 168.5$$

```
> sigmay:=add(ny[i]*(Y1[i]-Yt)^2/N2,i=1..5);
```

$$sigmay := 35.55$$

```
> sigma[y]:=sqrt(evalf(%));
```
$\sigma_y := 5.962$

```
> sigmaYx:=add(nx[i]*(Yx[i]-Yt)^2/N2,i=1..4);
```

$$sigmaYx := 22.20$$

```
> sigma[yx]:=sqrt(evalf(%));
```
$\sigma_{yx} := 4.712$

```
> eta[yx]:=sigma[yx]/sigma[y];
```
$\eta_{yx} := 0.7903$

8–laboratoriya ishi

bo'yicha mustaqil ishlash uchun topshiriqlar

1. Tanlanmaning korreiyasion jadvalni tuzish.
2. Ko'paytmalar usuli yordamida korrelyasiya koeficentini hisoblash.

3. Y ning X ga regressiya to'g'ri chizig'ining tanlanma tenglamasini yozish.

4 Tanlanma korrelyasion nisbatini hisoblash.

Quyidagi jadvaldagi 20 ta qiymatlarni talaba V–variantiga bog'liq holda $x_i=X_i$+butun(i/V) va $y_i=Y_i$+butun(i/V), i=1,2,...,30 kabi oladi.

1 – j a d v a l

X	91	95	97	99	92	96	100	100	97	101
Y	160	169	162	168	164	164	165	169	159	170
X	97	95	102	98	101	99	103	104	104	103
Y	171	185	171	166	172	175	170	181	176	175

Masalan, V=2 da tuziladigan 1-jadval qiymatlarini quyidagicha topamiz:

i=1, $x_1=X_1$+butun(i/V)=91+ butun(1/2)=93,

$y_1=Y_1$+butun(i/V)= 160+butun(1/2)=162

....

i=10, $x_{10}=X_{10}$+butun(i/V)=101+ butun(10/2)=106,

$y_{10}=Y_{10}$+butun(i/V)= 170+butun(10/2)=175

9-LABORATORIYA ISHI

Korrelyasion jadval bo'yicha to'g'ri chiziqli va ikkinch darajali regressiya tenlamalarini kichik kvadratlar usulida aniqlash

Maqsad: Korrelyasion jadval bo'yicha qiymatlar orasidagi bog'lanishini ifodalovchi, Y ning X ga to'g'ri chiziqli va ikkinchi darajali, tenglamalarini kichik kvadratlar usulida aniqlash.

Reja:

1. Rregressiya bog'laninshining to'g'ri chiziqli tenlamasini aniqlash.
2. Regressiya bog'laninshining ikkinch darajali tenlamasini aniqlash.

Quyidagi korrelyasion jadval berilgan bo'lsin:

1–jadval

Y/X	92	96	100	104	n_y
158	1	1			2
164	1	4	1		6
170		2	5	1	8
176			1	2	3
182				1	1
n_x	2	7	7	4	$N=20$
$\bar{y}_x$	161	164.8	170	176	

9.1. To'g'ri chiziqli bog'laninsh regressiya tenglamasini topish

Berilgan jadvaldagi ma'lumotlar bo'yicha y ning x ga regressiya to'g'ri chizig'ining tanlanma tenglamasini

$$y_x = ax + b \qquad (9.1)$$

ko'rinishda izlaylik.

Buning uchun a, b parametrlarni topish uchun, quyidagi

$$F(a,b) = \sum\left(y_{xi} - \bar{y}_{xi}\right)^2 n_{xi} = \sum\left(ax_i + b - \bar{y}_{xi}\right)^2 n_{xi}$$

farqlarning kvadratlari minimal bo'ladigan qilib tanlab olish imkonini beruvchi quyidagi tenglamalar sistemasini hosil qilamiz:

$$\frac{\partial F(a,b)}{\partial a} = 2\sum\left(ax_i + b - \bar{y}_{xi}\right)x_i n_{xi} = 0$$

$$\frac{\partial F(a,b)}{\partial b} = 2\sum_i \left(ax_i + b - \overline{y}_{x_i}\right) n_{x_i} = 0$$

bu sistemadan:

$$\left.\begin{aligned} \left(\sum n_x x^2\right)a + \left(\sum n_x x\right)b &= \sum n_x x \cdot \overline{y}_x \\ \left(\sum n_x x\right)a + nb &= \sum n_x \overline{y}_x \end{aligned}\right\} \qquad (9.2)$$

Bu sistemani echib, a, b – parametrlarni aniqlovchi munosabatlarga ega bo'lamiz.

$$a = \frac{n\sum n_x x \cdot \overline{y}_x - \sum n_x x \cdot \sum n_x \overline{y}_x}{n\sum n_x x^2 - \left(\sum n_x x\right)^2} \qquad (9.3)$$

$$b = \frac{\sum n_x \overline{y}_x \cdot \sum n_x x^2 - \sum n_x x \cdot \sum n_x x \overline{y}_x}{n\sum n_x x^2 - \left(\sum n_x x\right)^2} \qquad (9.4)$$

9.1–masala. Berilgan 1–korrelasion jadvaldagi ma'lumotlar asosida quyidagi 2–jadvalni ko'paytmalar usulida tuzamiz:

2–jadval.

n_x	x	$\overline{y}_x$	$n_x x$	$n_x x^2$	$n_x \overline{y}_x$	$n_x x \overline{y}_x$
2	92	161	164	16928	316	29624
7	96	164.8	672	64512	1154	40746
7	100	170	700	70000	1190	119000
4	104	176	416	43264	704	73216
20			1972	1947004	3370	332586

2-jadvaldagi oxirgi qatorga yozilgan qiymatlarni (9.3) va (9.4) ga qo'yib,

$$a = \frac{20 \cdot 332586 - 1972 \cdot 3370}{20 \cdot 19477004 - 1972^2} = 1{,}3,$$

$$b = \frac{3370 \cdot 194704 - 1972 \cdot 332586}{20 \cdot 194704 - 1972^2} = 40{,}8$$

topilgan a va b larning qiymatlari asosida izlanayotgan regressiya tenglamasi:

$$y_x = ax + b = 1.3x + 40.8$$

bu tenglama bo'yicha hisoblanadigan y_{xi} qiymatlar kuzatilgan $\overline{y}_{xi}$ qiymatalarga qanchalik mos kelishlni topish uchun, y_{xi} va $\overline{y}_{xi}$ qiymatlari orasidagi farqlarni aniqlash maqsadida quyidagi jadvalni tuzamiz:

3-jadval

x_i	y_{xi}	$\overline{y}_{xi}$	$y_{xi} - \overline{y}_{xi}$
92	160.4	161	-0.6
96	165.4	164.8	0.8
100	170.8	170	0.8
104	176	176	0

Jadvaldagi farqlar bog'lanishining aniqligini ifodalab beradi. Bu jadvaldan ko'rinadiki chetlanishlarning hammasi ham yetarlicha kichik emas. Bu kuzatishlar sonining kamligi bilan izoxlanadi.

1.Berilgan korrelasion jadval asosida Y ning X ga regressiya to'g'ri chizig'ining tenglamasi topishda kichik kvadratlar usulida tuzilgan sistema koeffisentlarini ko'paytmalar usulida topishning Maple dasturini tuzamiz.

9.1.1–Maple dasturi:

> restart; with(stats):

1)*korrelasion jadval asosida X va Y larini kiritish:*

> X:= Vector([92,96,100,104]);

$$X := \begin{bmatrix} 92 \\ 96 \\ 100 \\ 104 \end{bmatrix}$$

Y:= Vector([158,164,170,176,182]);

$$Y := \begin{bmatrix} 158 \\ 164 \\ 170 \\ 176 \\ 182 \end{bmatrix}$$

2)*korrelasion jadval asosida n_x va n_{xy} chastotalarni kiritish.*

```
> nx:=Vector([2,7,7,4]);
```

$$nx := \begin{bmatrix} 2 \\ 7 \\ 7 \\ 4 \end{bmatrix}$$

```
> nxy:=matrix([[1,1,0,0],[1,4,1,0],[0,2,5,1], [0,0,1,2],[0,0,0,1]]);
```

$$nxy := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3)*korrelasion jadval asosida shartli o 'rta qiymatlarni hisoblash:*

```
> Yx[1]:=(Y[1]*nxy[1,1]+Y[2]*nxy[2,1]+Y[3]*nxy[3,1]+
Y[4]*nxy[4,1]+Y[5]*nxy[5,1])/nx[1]:
```

$$Yx_1 := 161$$

```
> Yx[2]:=(Y[1]*nxy[1,2]+Y[2]*nxy[2,2]+Y[3]*nxy[3,2]+
Y[4]*nxy[4,2]+Y[5]*nxy[5,2])/nx[2]:
```

$$Yx_2 := \frac{1154}{7}$$

```
> evalf(%,4);
```

$$164.9$$

```
> Yx[3]:=(Y[1]*nxy[1,3]+Y[2]*nxy[2,3]+Y[3]*nxy[3,3]+
Y[4]*nxy[4,3]+Y[5]*nxy[5,3])/nx[3];
```

$$Yx_3 := 170$$

```
> Yx[4]:=(Y[1]*nxy[1,4]+Y[2]*nxy[2,4]+Y[3]*nxy[3,4]+
Y[4]*nxy[4,4]+Y[5]*nxy[5,4])/nx[4];
```

$$Yx_4 := 176$$

4)*korrelasion jadval asosida X ning qiymatlar soni n va tanlanma xajmi N qiymatlarni kiritish:*

```
> n:=4:N:=20:
```

5)2–*jadvalning qiymatlarni ko'paytmalar usulidagi hisoblash:*

```
> Sx:=add(X[k]*nx[k],k=1..n);
```
$Sx := 1972$

```
> Sxx:=add(nx[k].X[k]^2,k=1..n);
```
$Sxx := 194704$

```
> SYx:=add(nx[k].Yx[k],k=1..n);
```
$SYx := 3370$

```
> SxYx:=add(nx[k].X[k].Yx[k],k=1..n);
```
$SxYx := 332624$

6)*kichik kvadratlar usulida tuzilgan sistemani yechish:*

```
> ab:=solve({a*Sxx+b*Sx=SxYx,a*Sx+b*N=SYx},{a,b});
```

$$ab := \left\{a = \frac{855}{662}, b = \frac{13622}{331}\right\}$$

`> evalf(%,4);` $\{a = 1.292, b = 41.15\}$

7)*regressiya to'g'ri chizig'ining tenglamasini yozish.*

```
> y:=ab[1]*x+ab[2];evalf(%,4);
```

$$y := x\,a + b = \frac{855}{662}\,x + \frac{13622}{331}$$

$$x\,a + b = 1.292\,x + 41.15$$

2.Berilgan korrelasion jadval asosida Y ning X ga regressiya to'g'ri chizig'ining tenglamasi topishda **fit** asfunksiyasidan foydalanib Maple dasturini tuzamiz.

9.1.2–M a p l e d a s t u r i:

```
> restart;with(stats):
```

1)1*–korrelasion jadval asosida X va Y larining qiymatlarini chastotalari bilan satr bo'yicha kiritish:*

```
> W:=[[Weight(92,1),Weight(96,1),Weight(92,1),
Weight(96,4),Weight(100,1),Weight(96,2),Weight(100,5),
Weight(104,1),Weight(100,1),Weight(104,2),Weight(104,1)],
[Weight(158,1),Weight(158,1),Weight(164,1),Weight(164,4),
Weight(164,1),Weight(170,2),Weight(170,5), Weight(170,1),
Weight(176,1),Weight(176,2), Weight(182,1)]]:
```

$W := [[Weight(92, 1), Weight(96, 1), Weight(92, 1), Weight(96, 4), Weight(100, 1), Weight(96, 2), Weight(100, 5), Weight(104, 1), Weight(100, 1), Weight(104, 2), Weight(104, 1)], [Weight(158, 1), Weight(158, 1), Weight(164, 1), Weight(164, 4), Weight(164, 1), Weight(170, 2), Weight(170, 5), Weight(170, 1), Weight(176, 1), Weight(176, 2), Weight(182, 1)]]$

2) *X va Y larining qiymatlari bo'yicha* (x,y) *larni koordinatalar sistemasida aniqlash:*

```
> statplots[scatterplot](W[1],W[2],color=blue,
symbol=BOX,symbolsize=20);
```
(9.1–rasm)

3)*regressiya to'g'ri chizig'ining tenglamasini aniqlash:*

```
> x:=vector(transform[statvalue](W[1]));
```

$$x := [\,92\;\;96\;\;92\;\;96\;\;100\;\;96\;\;100\;\;104\;\;100\;\;104\;\;104\,]$$

```
> y:=vector(transform[statvalue](W[2]));
```

$y := [\ 158\ 158\ 164\ 164\ 164\ 170\ 170\ 170\ 176\ 176\ 182\ ]$

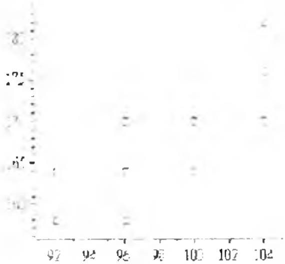
> fit[leastsquare[[x,y]]](W);evalf(%,5);

$$y = \frac{13622}{331} + \frac{855}{662}x \quad y = 41.154 + 1.2915x$$

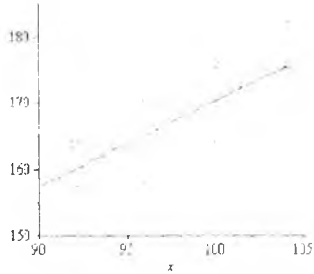
4)*regressiya to'g'ri chizig'ini qurish:*

> with(plots):

> plot([[x[i],y[i]],i=1..11],41.154+1.2915*x], x=90..104, 156..182,style=[point,line],symbol=BOX, color=[red,blue], view=[90..105,150..185], symbolsize=20); (9.2–rasm)



9.1–rasm.



9.2–rasm.

9.2. Ikkinchi darajali bog'lanishning regressiya tenglamasini topish

Maqsad: Ikkinchi darajali regressiya bog'lanishning tenglamani topishni o'rganish

Reja: Ikkinchi darajali regressiya bog'lanishning tenglamasini aniqlash.

Ikkinch darajali regressiya tenglamasini topishni quyidagi misol orqali izohlaymiz. Soddaroq bo'lishi uchun kichikroq jadval, hamda chiziqli bo'lmagan eng ommalashgan holi–kvadrat uchhad ko'rinishi bilan chegaralanamiz.

9.2–masala. Quyidagi korrelyasion jadvalda keltirilgan ma'lumotlar bo'yicha $y = ax^2 + bx + c$ regressiya tenglamasini eng kichik kvadratlar usuli yordamida topamiz.

4–jadval

$y \setminus x$	2	3	5	n_y
25	20			20
45		30	1	31
110		1	48	49
n_x	20	31	49	N=100

Yechish. Buning uchun a, b,c parametrlarni

$$F(a,b,c)=\sum\left(y_{x_i}-\bar{y}_{x_i}\right)^2 n_{x_i}=\sum\left(ax_i^2+bx_i+c-\bar{y}_{x_i}\right)^2 n_{x_i}$$

farqlarning kvadratlari minimal bo'ladigan qilib tanlab olish imkonini beruvchi quyidagi tenglamalar sistemasini hosil qilamiz:

$$\frac{\partial F(a,b,c)}{\partial a}=2\sum\left(ax_i^2+bx_i+c-\bar{y}_{x_i}\right)x_i^2 n_{x_i}=0$$

$$\frac{\partial F(a,b,c)}{\partial b}=2\sum\left(ax_i^2+bx_i+c-\bar{y}_{x_i}\right)x_i n_{x_i}=0$$

$$\frac{\partial F(a,b,c)}{\partial c}=2\sum\left(ax_i^2+bx_i+c-\bar{y}_{x_i}\right)n_{x_i}=0$$

bu sistemadan:

$$\begin{cases}\left(\sum n_x x^4\right)a+\left(\sum n_x x^3\right)b+\left(\sum n_x x^2\right)c=\sum n_x \bar{y}_x x^2\\ \left(\sum n_x x^3\right)a+\left(\sum n_x x^2\right)b+\left(\sum n_x x\right)c=\sum n_x \bar{y}_x x\\ \left(\sum n_x x^2\right)a+\left(\sum n_x x\right)b+nc=\sum n_x \bar{y}_x\end{cases} \qquad (*)$$

Bu sistemadagi yig'indilarni quyidagicha topamiz:
4–jadval asosida shartli o'rta qiymatlarni topamiz.

$$\bar{y}_2=\frac{25\cdot 20}{20}=25$$

$$\bar{y}_3=\frac{45\cdot 30+110\cdot 1}{31}=47{.}1$$

$$\bar{y}_5=\frac{45\cdot 1+110\cdot 48}{49}=108{,}67$$

5–jadval.

x	n_x	$\bar{y}_x$	$n_x x$	$n_x x^2$	$n_x x^3$	$n_x x^4$	$n_x \bar{y}_x$	$n_x \bar{y}_x x$	$n_x \bar{y}_x x^2$
2	20	25	40	80	160	320	500	1000	2000
3	31	47.1	93	279	837	2511	4380	13140	13141
5	49	108.67	245	12285	6125	30625	5325	26625	133121
$\sum$	100		378	1584	7122	33456	7285	32004	148262

5–jadval oxirida turgan yig'indilarni (*) sistemaga qo'yib, quyidagi sistemani hosil qilamiz:

$$\begin{cases} 33456\,a + 7122\,b + 1584\,c = 148262 \\ 7122\,a + 1584\,b + 378\,c = 32004 \\ 1584\,a + 378\,b + 100\,c = 7285 \end{cases}$$

Sistemani echib, $a=2.94$, $b=7.27$, $c=-1{,}25$ qiymatlarni topamiz va bu qiymatlarni regressiya tenglamasi:

$$\bar{y}_x = ax^2 + bx + c$$

ga qo'yib,

$$y_x = 2.94x^2 + 7.27x - 1.25$$

regressiya tenglamasiga ega bo'lamiz.

1. Berilgan korrelasion jadval asosida Y ning X ga regressiya chizig'i $y_x = ax^2 + bx + c$ ning tenglamasini topishda kichik kvadratlar usulida tuzilgan sistema koeffisentlarini ko'paytmalar usulida topishning Maple dasturini tuzamiz.

9.2.2a–M a p l e d a s t u r i:

> **restart;with(stats):**

1)4–*korrelasion jadval asosida X va Y larini kiritish:*

> **X:=Vector([2,3,5]);**

$$X := \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

> **Y:=Vector([158,164,170,176,182]);**

$$Y := \begin{bmatrix} 158 \\ 164 \\ 170 \\ 176 \\ 182 \end{bmatrix}$$

2)*korrelasion jadval asosida n_x va n_{xy} chastotalarni kiritish:*

> **nx:=Vector([20,31,49]);**

$$nx := \begin{bmatrix} 20 \\ 31 \\ 49 \end{bmatrix}$$

> **nxy:=matrix([[20,0,0],[0,30,1],[0,1,48]]);**

$$nxy := \begin{bmatrix} 20 & 0 & 0 \\ 0 & 30 & 1 \\ 0 & 1 & 48 \end{bmatrix}$$

3)*korrelasion jadval asosida shartli o'rta qiymatlarni hisoblash:*

> Yx[1]:=(Y[1]*nxy[1,1]+Y[2]*nxy[2,1]+Y[3]*nxy[3,1])/nx[1];

$$Yx_1 := 25$$

> Yx[2]:=(Y[1]*nxy[1,2]+Y[2]*nxy[2,2]+Y[3]*nxy[3,2])/nx[2];

$$Yx_2 := \frac{1460}{31}$$

> evalf(%,4); 47.10

> Yx[3]:=(Y[1]*nxy[1,3]+Y[2]*nxy[2,3]+Y[3]*nxy[3,3])/ nx[3];

$$Yx_3 := \frac{5325}{49}$$

> evalf(%,4); 108.7

4)*korrelasion jadval asosida X ning qiymatlar soni n va tanlanma xajmi N qiymatlarni kiritish:*

> n:=3:N:=100:

5)5–*jadvalning qiymatlarni ko'paytmalar usulidagi hisoblash:*

> Sx:=add(X[k]*nx[k],k=1..n); $Sx := 378$

> Sxx:=add(nx[k]*X[k]^2,k=1..n); $Sxx := 1584$

> Sxxx:=add(nx[k]*X[k]^3,k=1..n); $Sxxx := 7122$

> Sxxxx:=add(nx[k]*X[k]^4,k=1..n); $Sxxxx := 33456$

> SYx:=add(nx[k]*Yx[k],k=1..n); $SYx := 7285$

> SxYx:=add(nx[k]*X[k]*Yx[k],k=1..n); $SxYx := 32005$

> SxxYx:=add(nx[k]*X[k]^2*Yx[k],k=1..n); $SxxYx := 148265$

6)*kichik kvadratlar usulida tuzilgan sistemani yechish:*

> abc:=solve([a*Sxxxx+b*Sxxx+c*Sxx=SxxYx,
a*Sxxx+b*Sxx+c*Sx=SxYx,
a*Sxx+b*Sx+c*N=SYx],{a,b,c});

$$abc := \left\{a = \frac{26405}{9114}, b = \frac{69365}{9114}, c = -\frac{2750}{1519}\right\}$$

> evalf(%,4);

$$\{b = 7.611, c = -1.810, a = 2.897\}$$

7)*regressiya egri chizig'ining tenglamasini yozish:*

> y:=abc[1]*x^2+abc[2]*x+abc[3];

$$y := x^2 a + x b + c = \frac{26405}{9114}x^2 + \frac{69365}{9114}x - \frac{2750}{1519}$$

> y:=evalf(%,4);

$$y := x^2 a + x b + c = 2.897x^2 + 7.611x - 1.810$$

2 Berilgan korrelasion jadval asosida Y ning X ga regressiya chizig'i $\overline{y}_x = ax^2 + bx + c$ ning tenglamasini topishda fit asfunksiyasidan foydalanib Maple dasturini tuzamiz.

9.2.2b–M a p l e d a s t u r i:

> restart; with(stats):

1)4–korrelasion jadval asosida X va Y larining qiymatlarini chastotalari bilan satr bo'yicha kiritish:

> W:=[[Weight(2,20),Weight(3,30),Weight(5,1), Weight(3,1), Weight(5,48)], [Weight(25,20), Weight(45,30), Weight(45,1), Weight(110,1), Weight(110,48)]];

$$W := [[Weight(2, 20), Weight(3, 30), Weight(5, 1), Weight(3, 1), Weight(5, 48)], [Weight(25, 20), Weight(45, 30), Weight(45, 1), Weight(110, 1), Weight(110, 48)]]$$

2) X va Y larining qiymatlari bo 'yicha (x,y) larni koordinatalar sistemasida aniqlash:

> statplots[scatterplot](W[1],W[2],color=blue, symbol=BOX, symbolsize=20); (9.3–rasm)

3)regressiya eg'ri chizig'ining tenglamasini aniqlash:

> x:=vector(transform[statvalue](W[1]));

$$x := [\,2\ \ 3\ \ 5\ \ 3\ \ 5\,]$$

> y:=vector(transform[statvalue](W[2]));

$$y := [\,25\ \ 45\ \ 45\ \ 110\ \ 110\,]$$

> fit[leastsquare[[x,y],y=a*x^2+b*x+c]](W);

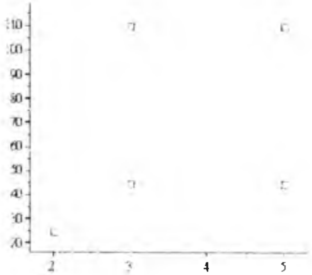
$$y = \frac{26405}{9114}x^2 + \frac{69365}{9114}x - \frac{2750}{1519}$$

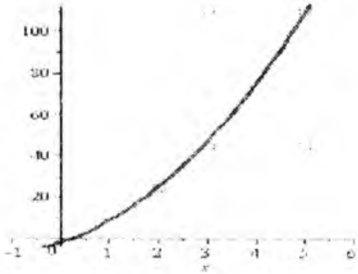
> evalf(%,5); $y = 2.8972x^2 + 7.6108x - 1.8104$

4)regressiya eg'ri chizig'ini qurish:

> with(plots):

> plot([[x[i],y[i],i=1..5],2.8972*x^2+7.6108*x–1.8104], x=–1..6,–4..112,style=[point,line], color=[red,blue],symbol=BOX,symbolsize=25, view=[–1..6,–4..112],thickness=3); (9.4–rasm)





9.3–rasm. 9.4–rasm.

9-laboratoriya ishi
bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi korrelyatsion jadval asosida kichik regression bog'lanishining to'g'ri chiziqi va ikkinch darajali tenlamasini kichik kvadratlar usulida aniqlang.

1.1–15 variantlar uchun 1–korrelyasiya jadvai (o'rta qavs ichidagi sonning butun qismi, v–talaba varianti).

1–korrelyasya jadval

Y/X	92	96	100	104	n_y
158	[(30–v)/10]	[(30–v)/7]	0	0	
164	[(30–v)/5]	[(30–v)/6]	[(30–v)/15]	0	
170	[(30–v)/8]	[(30–v)/5]	[(30–v)/7]	[(30–v)/9]	
176	0	[(30–v)/7]	[(30–v)/4]	[(30–v)/3]	
182	0	0	[(30–v)/3]	[(30–v)/6]	
n_x					N=
$\overline{y}_x$					

3. 16–30 variantlar uchun 2–korrelyasya jadval (o'rta qavs ichidagi sonning butun qismi olinadi)

2–korrelyasiya jadval

Y/X	92	96	100	104	n_y
158	[(35–v)/2]	[(35–v)/7]	0	0	
164	[(35–v)/5]	[(35–v)/2]	[(35–v)/3]	0	
170	[(35–v)/3]	[(35–v)/5]	[(35–v)/4]	[(35–v)/2]	
176	0	[(35–v)/7]	[(35–v)/3]	[(35–v)/3]	

182	0	0	[(35–v)/2]	[(35–v)/6]	
n_x					N=
$\bar{y}_x$					

Masalan, korrelyatsion jadvalni hosil qilish. V=1 bo'lsa, bu jadval quyidagicha bo'ladi:

Y/X	92	96	100	104	n_y
158	2	4			6
164	5	4	1		10
170	3	5	4	3	15
176		4	7	9	20
182			9	4	13
n_x	10	17	21	16	N=64
$\bar{y}_x$	164.4	167,2	176.8	176.4	

Bunda $\bar{y}_x$ –shartli o'rtachalarni topish:

X=92 ga mos :

$\bar{y}_{92}$ =(158*2+164*5+170*3+176*0+182*0)/10=164.4

X=96 ga mos: $\bar{y}_{96}$ =(158*4+164*4+170*5+176*4+182*0)/17=167.2

X=100 ga mos:

$\bar{y}_{100}$ =(158*0+164*1+170*4+176*7+182*9)/21=176.8

X=104 ga mos:

$\bar{y}_{104}$ =(158*0+164*0+170*3+176*9+182*4)/16=176.4

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Mundarija

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OLIY MATEMATIKA

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