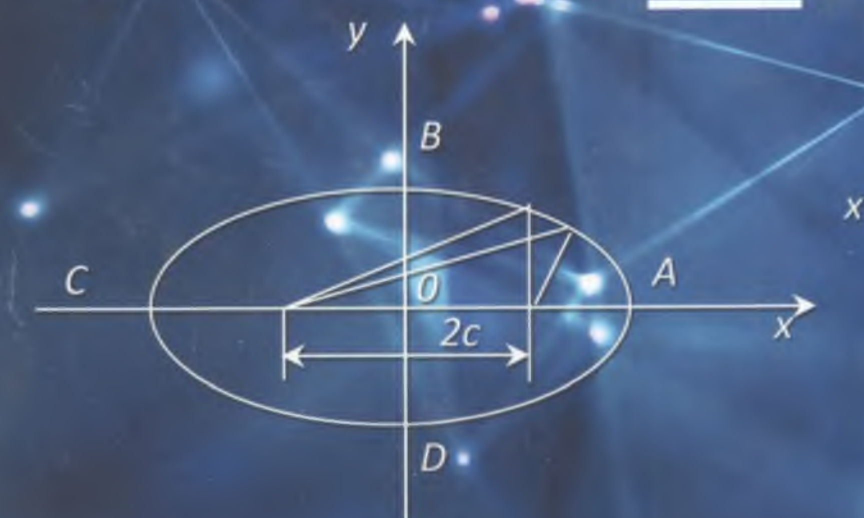
Sharipova N.O'

OLIY MATEMATIKA

Darslik



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INSTITUTI

Sharipova N.O'.

OLIY MATEMATIKA

310000 – Muhandislik ishi ta`lim sohasining barcha ta`lim yo`nalishida tahsil olayotgan talabalar uchun darslik

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Oliy matematika" darsligi Oliy o'quv yurtlarning 310000 – Muhandislik ishi ta`lim sohasining barcha ta`lim yo'nalishlari bo'yicha bakalavrlar tayyorlash uchun bu fanning tasdiqlangan namunaviy o'quv dasturi asosida yaratilgan.

Shuni qayd etish lozimki, ushbu darslik "Oliy matematika" faniga doir asosiy tushunchalar, amaliy mashg'ulotlardan, testlar va foydalanilgan adabiyotlar ro'yxatidan iborat. Darslikda amaliy mashg'ulotlar sodda va tushunarli qilib bayon qilingan.

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Kirish

Respublikamizda ta'lim-tarbiya tizimining bugungi kundagi asosiy vazifasi o'sib kelayotgan yosh avlodni har tomonlama odobli, axloqli, ijodkor, vatanparvar, zamonaviy bilim, ko'nikma va malakalarni o'zlashtirgan hamda jamiyatda o'z munosib o'rnini egallashga qodir bo'lgan – komillikka intiladigan barkamol avlodni voyaga yetkazishdir. Xalqimizning shunday ezgu maqsadlarini ro'yobga chiqarish yo'lida mustaqillik yillarida ta'lim-tarbiya sifati va samaradorligini zamon talablari darajasiga ko'tarish davlat siyosatining ustuvor yo'nalishlaridan biriga aylandi.

Yuqori malakali, raqobatbardosh, zamonaviy talablarga javob bera oladigan kadrlar tayyorlashda ularga chuqur matematik bilimlar berish va bu bilimlarni hayotga tatbiq eta olishga o'rgatish katta ahamiyatga ega. Shu sababli muhandis-texnolog yo'nalishlari bo'yicha ta'lim oluvchi bakalavrlarning o'quv rejalarida "Oliy matematika" fanini o'qitish ko'zda tutilgan.

Bu kitobda muhandis-texnologlar uchun " Oliy matematika" fanining namunaviy dasturida rejalashtirilgan barcha mavzular o'z o'rnini topgan.

Ushbu kitob oliy ta'limning Davlat ta'lim standartiga ko'ra Muhandislik-texnologiya ta'lim sohalarida o'qitiladigan "Oliy matematika" fani dasturi bakalavr yo'nalishlarida ta'lim oladigan talabalarni malakaviy bilimga ega bo'lishi uchun zarur bo'ladigan: Chiziqli algebra, analitik geometriya elementlari, matematik analiz, oddiy differensial tenglamalar nazariyasi, qatorlar, operatsion hisob, kompleks sonlar va kompleks o'zgaruvchili funksiyalar nazariyasi, matematik fizika tenglamalari, ehtimollar nazariyasidan boshlang'ich tushunchalarni o'z ichiga olgan bo'limlardan tashkil topgan.

Bu bo'limlar bo'yicha nazariy ma'lumotlar yoritilgan, amaliy misol va masalalar berilgan, mavzularga oid testlar va savollar keltirilgan. Ushbu kitobda muhandis-texnologlar yo'nalishi xususiyatidan kelib chiqib misol va masalalar tuzilgan. Talabalarning bilimini boyitish maqsadida imkon qadar ko'proq misollar berilgan.

Ushbu kitobni yozishda ko'p yillar davomida Buxoro muhandislik texnologiya institutida "Oliy matematika" fanini o'qitish tajribasidan va bu fan bo'yicha mavjud bo'lgan o'zbek va rus tilidagi o'quv adabiyotlaridan ijodiy ravishda foydalanildi. Mavzularning nechog'lik yoritilganligini baholash o'quvchiga havola qilinadi.

Ushbu o'quv adabiyoti kamchiliklardan xoli degan fikrdan uzoqda bo'lganligimiz tufayli uni takomillashtirish bo'yicha kitobxonlarning taklif va mulohazalarini minnatdorchilik bilan qabul etamiz.

Muallif

Agar matematika go'zal bo'lmaganda edi,
ehtimol matematikaning o'zi ham mavjud bo'lmasdi.
Aks holda qanday kuch, insoniyatning buyuk daholarini
bu qiyin fanga torta olardi.
Chaykovskiy

I-BOB . MATRITSALAR VA ANIQLOVCHILAR

§ 1.1. Matritsalar va ular ustida amallar
§ 1.2 Aniqlovchi(determinant)larva ular ustida amallar
§ 1.3. Teskari matritsa va matritsa rangi

§ 1.1. Matritsalar va ular ustida amallar

Matritsa bir qator matematik va iqtisodiy masalalarni yechishda juda ko'p qo'llaniladigan tushuncha bo'lib, uning yordamida bu masalalar va ularning yechimlarini sodda hamda ixcham ko'rinishda ifodalanadi.

Matritsa ta'rifi: *m* ta satr va *n* ta ustundan iborat to'g'ri to'rtburchak shaklidagi $m\cdot n$ ta sondan tashkil topgan jadval $m\times n$ tartibli ***matritsa,*** uni tashkil etgan sonlar esa ***matritsaning elementlari*** deb ataladi.

Matritsalar $A,B,C,\ldots$ kabi bosh harflar bilan, ularning *i*-satr va *j*-ustunida joylashgan elementlari esa odatda a_{ij}, b_{ij}, c_{ij} kabi mos kichik harflar bilan belgilanadi. Masalan,

$$A=\begin{pmatrix} 1 & -3 & 1.2 \\ 0 & 7.5 & -1 \end{pmatrix}$$

matritsa 2×3 tartibli, ya'ni 2 ta satr va 3 ta ustun ko'rinishidagi $2\cdot 3=6$ ta sondan tashkil topgan. Uning 1-satr elementlari $a_{11}=1$, $a_{12}=-3$, $a_{13}=1.2$ va 2-satr elementlari $a_{21}=0$, $a_{22}=7.5$, $a_{23}=-1$ sonlardan iborat. Bu matritsaning 1-ustuni $a_{11}=1$ va $a_{21}=0$, 2-ustuni $a_{12}=-3$ va $a_{22}=7{,}5$, 3-ustuni esa $a_{13}=1.2$ va $a_{23}=-1$ elementlardan tuzilgan.

Agar biror *A* matritsaning tartibini ko'rsatishga ehtiyoj bo'lsa, u $A_{m\times n}$ ko'rinishda yoziladi va umumiy holda

$$A_{m\times n}=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

yoki qisqacha $A_{m\times n}=(a_{ij})$ ko'rinishda ifodalanadi.

$A_{m\times n}$ matritsada $m = n \neq 1$ bo'lsa, u ***kvadrat matritsa***, $m\neq n$ ($m\neq1$, $n\neq1$) bo'lsa ***to'g'ri burchakli matritsa*** , $m=1$, $n\neq1$ holda ***satr matritsa*** va $m\neq1$, $n=1$ bo'lganda ***ustun matritsa*** deb ataladi.

$A_{n\times n}$ kvadrat matritsa qisqacha A_n kabi belgilanadi va n-tartibli kvadrat matritsa deyiladi.

Masalan, xalq xo'jaligining n ta tarmoqlari orasidagi o'zaro mahsulot ayirboshlash $A_n=(a_{ij})$ kvadrat matritsa yordamida ifodalanadi. Bunda $a_{ij}(i,j=1,2, \ldots , n$ va $i\neq j)$ i-tarmoqda ishlab chiqarilgan mahsulotning j-tarmoq uchun mo'ljallangan miqdorini, $a_{ii}(i=1,2, \ldots , n)$ esa i-tarmoqning o'zi ishlab chiqargan mahsulotga ehtiyojini bildiradi.

Shuni ta'kidlab o'tish kerakki, $m=1$ va $n=1$ bo'lganda $A_{1\times1}$ matritsa bitta sonni ifodalaydi va shu sababli ma'lum bir ma'noda matritsa son tushunchasini umumlashtiradi.

A va B matritsalar bir xil tartibli va ularning mos elementlari o'zaro teng bo'lsa, ya'ni $a_{ij}= b_{ij}$ shart bajarilsa, ular ***teng matritsalar*** dir

A va B matritsalarning tengligi $A=B$ yoki $(a_{ij})= (b_{ij})$ ko'rinishda belgilanadi. Masalan, ixtiyoriy $a\neq0$ soni uchun

$$A=\begin{pmatrix} a+a & a-a \\ a:a & a\cdot a \end{pmatrix}, \qquad B=\begin{pmatrix} 2a & 0 \\ 1 & a^2 \end{pmatrix}$$

matritsalar o'zaro teng, ya'ni $A = B$ bo'ladi.

$A=\{a_{ij}\}$ matritsada i=j bo'lgan a_{ii} elementlar ***diagonal elementlar***

Masalan, yuqorida ko'rilgan $A_{2\times3}$ matritsaning diagonal elementlari $a_{11}=1$ va $a_{22}=7.5$ bo'ladi.

Diagonal matritsa diagonal elementlaridan boshqa barcha elementlari nolga teng bo'lgan ($a_{ij}=0$, $i\neq j$) kvadrat matritsadir.

Diagonal matritsaning diagonal elementlari nolga ham teng bo'lishi mumkin.

Masalan,

$$A_{2\times2} = A_2 = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}, \quad B_{3\times3} = B_3 = \begin{pmatrix} 15 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

diagonal matritsalar bo'ladi.

Barcha diagonal elementlari a_{ii}=1 bo'lgan n-tartibli diagonal matritsa n-tartibli birlik matritsa yoki qisqacha ***birlik matritsadir***

Odatda n-tartibli birlik matritsa E_n yoki qisqacha E kabi belgilanadi. Masalan,

$$E_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

mos ravishda ikkinchi va uchinchi tartibli birlik matritsalardir.

Barcha elementlari nolga teng (a_{ij}=0) bo'lgan ixtiyoriy $m\times n$ tartibli matritsa ***nol matritsa*** deyiladi.

$m\times n$ tartibli nol matritsa $O_{m\times n}$ yoki qisqacha O kabi belgilanadi. Masalan,

$$O_{2\times3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad O_{3\times2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad O_{3\times3} = O_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

ko'rsatilgan tartibli nol matritsalar bo'ladi.

Matritsalar ustida amallar.

Endi matritsalar ustida algebraik amallar kiritib, matritsalar algebrasini hosil etamiz.

Ixtiyoriy tartibli $A_{m\times n}=(a_{ij})$ matritsaning istalgan λ***songa ko'paytmasi*** deb $C_{m\times n}=\{\lambda a_{ij}\}$ kabi aniqlanadigan matritsaga aytiladi.

Bunda A matritsaning λ songa ko'paytmasi λA deb belgilanadi. Masalan,

$$A = \begin{pmatrix} 5 & 4 & -1 \\ 0 & 2 & 7 \end{pmatrix} \Rightarrow 6A = \begin{pmatrix} 6\cdot5 & 6\cdot4 & 6\cdot(-1) \\ 6\cdot0 & 6\cdot2 & 6\cdot7 \end{pmatrix} = \begin{pmatrix} 30 & 24 & -6 \\ 0 & 12 & 42 \end{pmatrix}.$$

Bir xil tartibli $A_{m\times n}=(a_{ij})$ va $B_{m\times n}=(b_{ij})$ ***matritsalar yig'indisi*** deb elementlari $c_{ij} = a_{ij} + b_{ij}$ kabi aniqlanadigan $C_{m\times n}=(c_{ij})$ matritsaga aytiladi.

Bunda A va B matritsalarning yig'indisi $A+B$ ko'rinishda belgilanadi va ularning mos elementlarini qo'shish orqali hisoblanadi. Masalan,

$$A = A_{2\times3} = \begin{pmatrix} 5 & 3 & -1 \\ 0 & 7 & 2 \end{pmatrix}, \quad B = B_{2\times3} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -3 & 4 \end{pmatrix},$$

matritsalar uchun

$$A + B = \begin{pmatrix} 5+1 & 3+0 & -1+1 \\ 0+2 & 7+(-3) & 2+4 \end{pmatrix} = \begin{pmatrix} 6 & 3 & 0 \\ 2 & 4 & 6 \end{pmatrix}.$$

Matritsalarni songa ko'paytirish va o'zaro qo'shish amallari quyidagi qonunlarga bo'ysunishi bevosita ularning ta'riflaridan kelib chiqadi:

I. $A+B=B+A$ (qo'shish uchun kommutativlik qonuni);

II. $A+(B+C) = (A+B)+C$ (qo'shish uchun assotsiativlik qonuni);

III. $\lambda\ (A+B) = \lambda A + \lambda B$, $(\lambda + \mu)A = \lambda A + \mu A$ (distrubutivlik qonuni)

Bundan tashqari yuqoridagi ta'riflar orqali bu amallar ushbu xossalarga ham ega bo'lishini ko'rsatish qiyin emas:

$$A + O = A, \quad A+A = 2A, \quad 0 \cdot A = O, \quad \lambda \cdot O = O.$$

Bir xil tartibli $A_{m\times n}=(a_{ij})$ va $B_{m\times n}=(b_{ij})$ ***matritsalar ayirmasi*** deb $A_{m\times n}$ va $(-1)\ B_{m\times n}$ matritsalarning yig'indisiga, ya'ni $A_{m\times n}+(-1)B_{m\times n}$ matritsaga aytiladi.

Bunda A va B matritsalarning ayirmasi $A-B$ ko'rinishda belgilanadi va ularning mos elementlarini o'zaro ayirish orqali hisoblanadi. Masalan,

$$A = A_{2\times3} = \begin{pmatrix} 5 & 3 & -1 \\ 0 & 7 & 2 \end{pmatrix}, \quad B = B_{2\times3} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -3 & 4 \end{pmatrix},$$

matritsalar uchun

$$A - B = \begin{pmatrix} 5-1 & 3-0 & -1-1 \\ 0-2 & 7-(-3) & 2-4 \end{pmatrix} = \begin{pmatrix} 4 & 3 & -2 \\ -2 & 10 & -2 \end{pmatrix}.$$

$A_{m\times p}=(a_{ij})$va $B_{p\times n}=(b_{ij})$***matritsalarning ko'paytmasi*** deb shunday $C_{m\times n}=(c_{ij})$ matritsaga aytiladiki, uning c_{ij} elementlari ushbu

$$c_{ij}=\sum_{k=1}^{p}a_{ik}b_{kj}\,, \qquad i=1,2,\cdots,m;\quad j=1,2,\cdots,n$$

yig'indilar kabi aniqlanadi.

Shunday qilib, $A_{m\times p}=(a_{ij})$va $B_{q\times n}=(b_{ij})$matritsalar uchun $p=q$, ya'ni A matritsaning ustunlari soni B matritsaning satrlari soniga teng bo'lgandagina ularning ko'paytmasi mavjud bo'ladi va AB kabi belgilanadi. Bunda $AB=C_{m\times n}=(c_{ij})$ matritsaning satrlar soni m birinchi A ko'paytuvchi matritsa, ustunlar soni n esa ikkinchi B ko'paytuvchi matritsa orqali aniqlanadi. Bundan tashqari $AB=C_{m\times n}=(c_{ij})$ ko'paytma matritsaning c_{ij}elementi A matritsaning i – satr elementlarini B matritsaning j-ustunidagi mos elementlariga ko'paytirib, hosil bo'lgan ko'paytmalarni qo'shish orqali hisoblanadi. Bu "satrni ustunga ko'paytirish" qoidasi deb aytiladi. Masalan,

$$A_{3\times 2}=\begin{pmatrix}3 & 1\\ 0 & -2\\ 4 & 5\end{pmatrix},\qquad B_{2\times 2}=\begin{pmatrix}6 & -4\\ 1 & 2\end{pmatrix}$$

matritsalar uchun $m=3$, $p=q=2$, $n=2$ bo'lgani uchun ularning ko'paytirish mumkin va ko'paytma matritsa $AB=C_{3x2}$ quyidagicha bo'ladi:

$$C_{3\times 2}=\begin{pmatrix}3\cdot 6+1\cdot 1 & 3\cdot(-4)+1\cdot 2\\ 0\cdot 6+(-2)\cdot 1 & 0\cdot(-4)+(-2)\cdot 2\\ 4\cdot 6+5\cdot 1 & 4\cdot(-4)+5\cdot 2\end{pmatrix}=\begin{pmatrix}19 & -10\\ -2 & -4\\ 29 & -6\end{pmatrix}.$$

Matritsalar ko'paytmasi uchun $AB\neq BA$, ya'ni kommutativlik qonuni o'rinli bo'lmaydi. Masalan, $A_{m\times q}B_{q\times n}=C_{m\times n}$ ko'paytma mavjud, ammo $B_{q\times n}\,A_{m\times q}$ko'paytma har doim ham mavjud emas va mavjud bo'lgan taqdirda, ya'ni $n=m$ holda ham ular teng bo'lishi shart emas. Masalan,

$$A=\begin{pmatrix}3 & -1\\ 4 & 5\end{pmatrix},\quad B=\begin{pmatrix}2 & 7\\ 0 & -1\end{pmatrix}$$

matritsalar uchun $AB\neq BA$, chunki

$$AB=\begin{pmatrix}3 & -1\\ 4 & 5\end{pmatrix}\begin{pmatrix}2 & 7\\ 0 & -1\end{pmatrix}=\begin{pmatrix}6 & 22\\ 8 & 23\end{pmatrix}, \quad BA=\begin{pmatrix}2 & 7\\ 0 & -1\end{pmatrix}\begin{pmatrix}3 & -1\\ 4 & 5\end{pmatrix}=\begin{pmatrix}34 & 33\\ -4 & -5\end{pmatrix}.$$

Matritsalar ko'paytmasi va yig'indisi quyidagi qonunlarga bo'ysunadi hamda ushbu xossalarga ega bo'ladi:

I. $A(BC)=(AB)C$, $(\lambda A)B=A(\lambda B)$ (ko'paytirish uchun assotsiativlik qonuni);

II. $A(B+C) = AB + AC$ (ko'paytirish va qo'shish amallari $(A+B)C = AC + BC$ uchun distributivlik qonunlari);

III. $AE = EA = A$, $O\cdot A = O$, $A\cdot O = O$, $0\cdot A= O$.

Bunda E va O mos ravishda tegishli tartibli birlik va nol matritsalarni ifodalaydi.

Matritsa ko'paytmasi ta'rifidan ko'rinadiki, har qanday n-tartibli A kvadrat matritsani o'ziga–o'zini ko'paytirish mumkin va natijada yana n-tartibli kvadrat matritsa hosil bo'ladi.

Akvadrat matritsani o'zaro $m$marta (m – birdan katta ixtiyoriy natural son) ko'paytirishnatijasida hosil bo'lgan kvadrat matritsa ***A*matritsaning *m*- darajasi** deyiladi.

Amatritsaning m- darajasi A^m kabi belgilanadi. Bunda $A^0=E$ va $A^1=A$ deb olinib, A^m daraja ixtiyoriy nomanfiy butun m soni uchun aniqlanadi. Bu holda A^m daraja ta'rifdan uning quyidagi xossalari bevosita kelib chiqadi (m,k-natural sonlar, λ-haqiqiy son):

$$1.\ A^m\cdot A^k = A^{m+k}\ ; \quad 2.\ (A^m)^k = A^{mk}\ ; \quad 3.\ (\lambda A)^m = \lambda^m A^m\ ;$$

$$4.\ E^m = E; \quad 5.\ O^m = O.$$

Shunday qilib, har qanday kvadrat matritsa uchun natural darajaga ko'tarish amalini kiritish mumkin ekan. Masalan,

$$A=\begin{pmatrix}2 & 5\\ 1 & -3\end{pmatrix}\Rightarrow A^2=\begin{pmatrix}2 & 5\\ 1 & -3\end{pmatrix}\begin{pmatrix}2 & 5\\ 1 & -3\end{pmatrix}=\begin{pmatrix}9 & -5\\ -1 & 14\end{pmatrix},$$

$$A^3=A^2A=\begin{pmatrix}9 & -5\\ -1 & 14\end{pmatrix}\begin{pmatrix}2 & 5\\ 1 & -3\end{pmatrix}=\begin{pmatrix}13 & 60\\ 12 & -47\end{pmatrix}.$$

Shuni ta'kidlab o'tish kerakki, 5-xossaning teskarisi o'rinli emas, ya'ni A^m=O tenglikdan har doim ham A=O ekanligi kelib chiqmaydi. Masalan,

$$A=\begin{pmatrix}1 & 1\\ -1 & -1\end{pmatrix}\neq O\Rightarrow A^2=\begin{pmatrix}1 & 1\\ -1 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & -1\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}=O.$$

Kelgusida matritsani darajaga ko'tarish amalini ixtiyoriy m butun son uchun umumlashtiramiz.

$B=(b_{ij})$ matritsa $A=(a_{ij})$ matritsaning ***transponirlangani*** deyiladi, agar i va j indekslarning barcha mumkin bo'lgan qiymatlarida $a_{ij}=b_{ji}$ shart bajarilsa.

A matritsaning transponirlangani A^T kabi belgilanadi. Agar A matritsa $m\times n$ tartibli bo'lsa, uning transponirlangani $A^T n\times m$ tartibli bo'ladi.Masalan,

$$A_{2\times 3}=\begin{pmatrix}2 & -4 & 1\\ 3 & 0 & -5\end{pmatrix}\Rightarrow A^T_{3\times 2}=\begin{pmatrix}2 & 3\\ -4 & 0\\ 1 & -5\end{pmatrix}.$$

Matritsani transponirlanganini topish ***transponirlash amali*** deyiladi va u quyidagi xossalarga ega bo'lishini ko'rsatish mumkin:

1. $(A^T)^T=A$; 2. $(\lambda A)^T=\lambda A^T$ (λ– ixtiyoriy haqiqiy son);
3. $(A\pm B)^T=A^T\pm B^T$; 4. $(A\cdot B)^T=B^T\cdot A^T$.

Agar A kvadrat matritsa uchun $A^T=A$ bo'lsa, u ***simmetrik matritsa,*** $A^T=-A$ bo'lganda esa ***kososimmetrik matritsa*** dir.

Ta'rifdan har qanday simmetrik matritsaning elementlari $a_{ij}=a_{ji}$, kososimmetrik matritsaning elementlari esa $a_{ij}=-a_{ji}$shartni qanoatlantirishi bevosita kelib chiqadi. Bundan kososimmetrik matritsaning barcha diagonal elementlari nolga teng bo'lishi kelib chiqadi.

Masalan,

$$A=\begin{pmatrix}3 & 6 & 1\\ 6 & 0 & -4\\ 1 & -4 & -2\end{pmatrix},\quad B=\begin{pmatrix}0 & 3 & 2\\ -3 & 0 & -4\\ -2 & 4 & 0\end{pmatrix}$$

matritsalardan A simmetrik, B kososimmetrik bo'ladi.

Matritsalarning iqtisodiy tadbiqlari.

1-misol. Xalq xo'jaligining tarmoqlari o'rtasida ayrim ishlab chiqarish resurslarining taqsimoti quyidagi jadval orqali berilgan bo'lsin (umumiy hajmga nisbatan foiz hisobida, raqamlar shartli):

Resurslar	Xalq xo'jaligi tarmoqlari		
	Sanoat	Qishloq xo'jaligi	Boshqa tarmoqlar
1.Yoqilg'i	45	30	25
2. Elektr energiyasi	53	27	20
3. Mehnat resurslari	38	21	41
4. Suv resurslari	40	48	12

Bu jadvalni matritsa yordamida quyidagi qulay ko'rinishda ifodalash mumkin:

$$A_{4\times3}=\begin{pmatrix}45 & 30 & 25\\ 53 & 27 & 20\\ 38 & 21 & 41\\ 40 & 48 & 12\end{pmatrix}$$

Bu yozuvda A matritsaning har bir elementi aniq iqtisodiy ma'noga ega. Masalan, a_{11}=45 va a_{21}=53 sanoat tarmoqlari yoqilg'ining 45 foizini va elektr energiyasining 53 foizini iste'mol qilishini ko'rsatadi; a_{22}=27 qishloq xo'jaligi elektr energiyasining 27 foizini sarflashini, a_{33}=41 esa mehnat resurslarining 41 foizi boshqa tarmoqlarda band ekanligini ifodalaydi va hokazo.

2-Misol. Korxona M_1,M_2 , M_3 va M_4 kabi belgilangan 4 xil mahsulot ishlab chiqaradi. Bu mahsulotlarni ishlab chiqarish uchun 3 xil S_1 , S_2 va S_3 xom ashyolardan foydalaniladi. Bunda a_{ij} (i=1,2,3,4; j=1,2,3) orqali M_i mahsulot birligini ishlab chiqarish uchun S_j xom ashyodan qancha birlik sarflanishini belgilab, mahsulotlar birligini ishlab chiqarish uchun xom ashyolar sarfi me'yorini ushbu $A_{4\times3}=(a_{ij})$ matritsa orqali ifodalaymiz:

$$A=\begin{pmatrix}4 & 3 & 2\\ 1 & 0 & 5\\ 3 & 2 & 4\\ 5 & 3 & 1\end{pmatrix}.$$

Bunda M_i (i=1,2,3,4) mahsulotlarni ishlab chiqarish rejasini ifodalovchi C satr va S_j (j=1,2,3) xom ashyo birligining bahosini ko'rsatuvchi B ustun matritsalar quyidagicha bo'lsin:

$$C=\begin{pmatrix}90 & 110 & 80 & 100\end{pmatrix},\quad B=\begin{pmatrix}7\\4\\5\end{pmatrix}.$$

Bu holda CA matritsalar ko'paytmasi mavjud va u rejalangan mahsulotlarni ishlab chiqarish uchun sarflanadigan S_1, S_2 va S_3 xom ashyolar miqdorini ifodalovchi quyidagi D satr matritsadan iboratdir:

$$D=C\cdot A=\begin{pmatrix}90 & 110 & 80 & 100\end{pmatrix}\begin{pmatrix}4 & 3 & 2\\1 & 0 & 5\\3 & 2 & 4\\5 & 3 & 1\end{pmatrix}=\begin{pmatrix}1210 & 730 & 1150\end{pmatrix}.$$

Demak biz ishlab chiqarish rejasini bajarishimiz uchun S_1, S_2 va S_3 xom ashyolardan mos ravishda 1210, 730 va 1150 birlik miqdorda ega bo'lishimiz kerak.

Xom ashyo miqdorini ifodalovchi topilgan D matritsani xom ashyo birligi bahosini ko'rsatuvchi B matritsaga ko'paytmasi DB ham mavjud va u bizga zarur miqdordagi xom ashyolarni sotib olish xarajatimizni determinant sondan iborat bo'ladi:

$$DB=\begin{pmatrix}1210 & 730 & 1150\end{pmatrix}\begin{pmatrix}7\\4\\5\end{pmatrix}=1210\cdot 7+730\cdot 4+1150\cdot 5=17140.$$

Mustaqil yechish uchun savollar.

Berilgan A va B matritsalar uchun C maritsani toping.

1.1. *a)* $A=\begin{pmatrix}-2 & 3 & 3\\1 & 0 & -4\\-3 & 1 & -1\end{pmatrix},\ B=\begin{pmatrix}1 & -3 & 0\\-1 & 2 & 2\\6 & -1 & 5\end{pmatrix},\ C=2A+B$

в) $A=\begin{pmatrix}-4 & 5 & 1\\2 & -3 & 8\end{pmatrix},\ B=\begin{pmatrix}-7 & 4\\10 & -5\\3 & 16\end{pmatrix},\ C=B-2A^T$

1.2. $A=\begin{pmatrix}4 & 1 & 3\\0 & -1 & 2\\2 & 1 & -1\end{pmatrix},\ B=\begin{pmatrix}1 & -1 & 3\\2 & 1 & 0\\1 & 1 & -1\end{pmatrix},\ C=(3A)^T-B$

1.3. $A=\begin{pmatrix}5 & -2\\4 & 3\\0 & 2\end{pmatrix},\ B=\begin{pmatrix}-4 & -1 & 5\\2 & 3 & -5\end{pmatrix},\ C=A+B^T$

1.4. $A=\begin{pmatrix} 2 & 3 & 5 \\ -1 & 2 & -4 \end{pmatrix},\ B=\begin{pmatrix} 1 & -2 & -1 \\ 4 & 3 & 2 \end{pmatrix},\ C=2A+3B$

1.5. $A=\begin{pmatrix} 3 & 0 \\ -3 & 1 \\ 1 & 3 \end{pmatrix},\ B=\begin{pmatrix} 4 & 1 \\ -3 & 1 \\ 2 & 1 \end{pmatrix},\ C=A^T-B^T$

1.6. $A=\begin{pmatrix} 2 & -4 \\ 3 & 7 \\ 6 & -1 \end{pmatrix},\ B=\begin{pmatrix} 4 & 0 & 1 \\ 6 & 1 & -3 \end{pmatrix},\ C=2A-B^T$

1.7. $A=\begin{pmatrix} 3 & 1 & 0 \\ -2 & -1 & 4 \\ 7 & 2 & 1 \end{pmatrix},\ B=\begin{pmatrix} 0 & -1 & 3 \\ 2 & -1 & 4 \\ 4 & 6 & 2 \end{pmatrix},\ C=(2B)^T+A$

1.8. $A=\begin{pmatrix} 3 & -4 & 2 \\ 7 & 5 & 1 \end{pmatrix},\ B=\begin{pmatrix} -2 & 4 \\ 6 & 6 \\ -3 & 1 \end{pmatrix},\ C=A-B^T$

1.9. $A=\begin{pmatrix} 7 & -4 \\ 0 & -3 \\ 1 & 2 \end{pmatrix},\ B=\begin{pmatrix} -1 & 4 \\ -1 & 2 \\ 6 & 3 \end{pmatrix},\ C=3B-2A$

1.10. $A=\begin{pmatrix} 6 & 1 & -3 \\ 2 & -1 & 0 \end{pmatrix},\ B=\begin{pmatrix} 7 & 3 & 1 \\ -5 & 1 & 4 \end{pmatrix},\ C=A^T+B^T$

A va B matritsani ko`paytmasini hisoblang

1.11.*a*) $A_{3\times 2}=\begin{pmatrix} 2 & 0 \\ -1 & -2 \\ 3 & -5 \end{pmatrix}\quad B_{2\times 2}=\begin{pmatrix} -2 & 3 \\ 4 & -1 \end{pmatrix}$

б) $A=\begin{pmatrix} 1 & -1 & 2 & 3 \end{pmatrix},\ B=\begin{pmatrix} 1 & 2 \\ 3 & 6 \\ 5 & -1 \\ 1 & -3 \end{pmatrix}$

1.12. $A=\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 5 & 7 & 1 \\ 1 & 4 & 3 \end{pmatrix},\ B=\begin{pmatrix} 5 & -1 & 2 & 1 \\ 2 & 2 & 0 & 1 \\ 1 & -1 & 5 & 4 \end{pmatrix}$

1.13. $A=\begin{pmatrix} -2 & 3 & 0 & 1 \\ 1 & 1 & 2 & -1 \end{pmatrix},\ B=\begin{pmatrix} 2 & 0 \\ 1 & -1 \\ -1 & 2 \\ 1 & 3 \end{pmatrix}$

1.14. $A=\begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix},\ B=\begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 2 \end{pmatrix}$

1.15. $A=\begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{pmatrix},\ B=\begin{pmatrix} 3 & 2 & 1 \\ 5 & 1 & 1 \end{pmatrix}$

1.16. $A=\begin{pmatrix} 2 & 0 \\ 1 & -1 \\ -1 & 2 \\ 1 & 3 \end{pmatrix},\ B=\begin{pmatrix} -2 & 3 & 0 & 1 \\ 1 & 1 & 2 & -1 \end{pmatrix}$

1.17. $A=\begin{pmatrix}5&-1&3&1\\2&0&-1&4\end{pmatrix}$, $B=\begin{pmatrix}-1&3&0\\-2&1&1\\3&0&-2\\4&1&2\end{pmatrix}$ 1.18. $A=\begin{pmatrix}1\\2\\3\end{pmatrix}$, $B=(3\ \ 2\ \ 1)$

1.19. $A=\begin{pmatrix}1&2&3&4\\2&1&-2&3\end{pmatrix}$, $B=\begin{pmatrix}1&5&3\\6&8&2\\1&2&-1\\3&0&1\end{pmatrix}$ 1.20. $A=\begin{pmatrix}3&2&1\\5&1&1\end{pmatrix}$, $B=\begin{pmatrix}1&2\\1&1\\0&1\end{pmatrix}$

1.21. $A=\begin{pmatrix}3&2&1&2\\4&1&1&3\end{pmatrix}$, $B=\begin{pmatrix}3&4\\2&1\\1&1\\2&3\end{pmatrix}$ 1.22. $A=\begin{pmatrix}3&1&1\\2&1&2\\1&2&3\end{pmatrix}$, $B=\begin{pmatrix}1&1&-1\\2&-1&1\\1&0&1\end{pmatrix}$

1.23. $A=(5\ \ 1\ \ 0\ -3)$, $B\begin{pmatrix}2&0\\1&-4\\3&1\\0&-1\end{pmatrix}$ 1.24. $A=\begin{pmatrix}2&1&1\\3&0&1\end{pmatrix}$, $B=\begin{pmatrix}3&1\\2&1\\1&0\end{pmatrix}$

1.25. $A=\begin{pmatrix}1&2&3\\-2&3&1\\4&2&4\end{pmatrix}$, $B=\begin{pmatrix}1&1&1&1\\2&3&-1&3\\-2&-1&5&-2\end{pmatrix}$ 1.26. $A=\begin{pmatrix}1&1&1&1\\1&2&1&-2\end{pmatrix}$, $B=\begin{pmatrix}1&1\\-3&5\\3&-5\\-1&1\end{pmatrix}$

1.27. $A=\begin{pmatrix}2&1&1\\1&2&1\\1&1&2\end{pmatrix}$, $B=\begin{pmatrix}2\\-1\\1\end{pmatrix}$ 1.28. $A=\begin{pmatrix}3&1\\2&1\\1&0\end{pmatrix}$, $B=\begin{pmatrix}2&1&1\\3&0&1\end{pmatrix}$

1.29. $A=(1\ \ 2\ -1)$, $B=\begin{pmatrix}2&4&-2&1\\1&2&1&2\\3&1&-3&1\end{pmatrix}$ 130.

$$A=\begin{pmatrix}1&2&3\\2&4&6\\3&6&9\end{pmatrix},\quad B=\begin{pmatrix}-1&-2&-4\\-1&-2&-4\\1&2&4\end{pmatrix}$$

§ 1.2 Aniqlovchi(determinant)larva ular ustida amallar

Matritsaning bir qator xususiyatlarini ta'riflash va o'rganish uchun uning determinant tushunchasi kerak bo'ladi.

Determinant ta'rifi: n - tartibli A kvadrat matritsaning elementlaridan ma'lum bir qoida asosida hosil qilinadigan son n – ***tartibli determinant*** deyiladi.

A kvadrat matritsaning determinanti $|A|$ yoki det$A$ kabi belgilanadi. Ayrim o'quv adabiyotlarida determinant atamasi aniqlovchi deb aytiladi. Umumiy holda n-tartibli determinant quyidagi ko'rinishda bo'ladi:

$$|A| = \det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}.$$

Berilgan $|A|$ determinantni tashkil etgan a_{ij} (i,j=1,2, … ,n) sonlar ***determinantning elementlari***, gorizontal ko'rinishda joylashgan a_{ij} (j=1,2, … ,n) elementlar ***determinantning i- satri*** (i=1,2, … ,n), vertikal ko'rinishda joylashgan a_{ij} (i=1,2, … ,n) elementlar esa ***determinantning j ustuni*** (j=1,2, … ,n) deyiladi.

Endi I, II va III tartibli determinantlarni hisoblash qoidasini formula ko'rinishida aniq ifodalaymiz.

I tartibli $|A|$ determinant faqat bitta a_{11} sondan iborat bo'lib, uning qiymati shu sonni o'ziga teng, ya'ni $|A|=|a_{11}|= a_{11}$ deb olinadi.

II tartibli determinantning qiymati quyidagi formula bilan aniqlanadi:

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21} \qquad (1)$$

Masalan,

$$\begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} = 3 \cdot 6 - 4 \cdot 5 = 18 - 20 = -2, \qquad \begin{vmatrix} 5 & 4 \\ -2 & 10 \end{vmatrix} = 5 \cdot 10 - 4 \cdot (-2) = 50 + 8 = 58$$

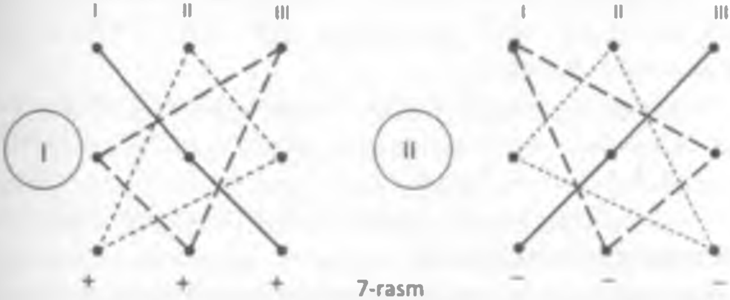
III tartibli determinant esa quyidagi formula bilan hisoblanadi:

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{21} \cdot a_{32} \cdot a_{13} -$$

$$- a_{13} \cdot a_{22} \cdot a_{31} - a_{12} \cdot a_{21} \cdot a_{33} - a_{23} \cdot a_{32} \cdot a_{11} \qquad (2)$$

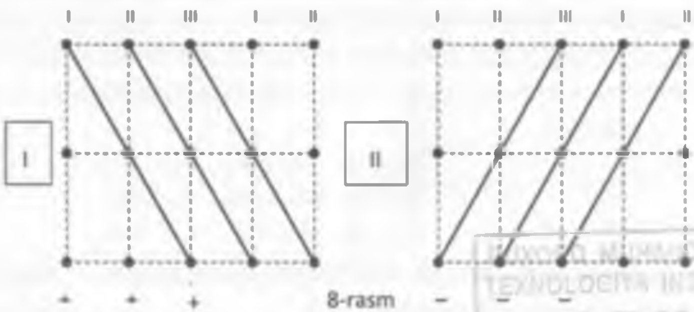
Bu yerda (2) formulani eslab qolish oson emas va shu sababli III tartibli determinantlarni quyidagi usullarda hisoblash mumkin.

❖ **Uchburchaklar usuli.** Bu usulda determinantning elementlari sxematik ko'rinishda nuqtalar singari ifodalanadi (7-rasmga qarang). So'ngra asoslari determinantning diagonallariga parallel bo'lgan uchburchaklar qaraladi. Bu uchburchaklarning uchlarida va diagonallarda joylashgan elementlarning ko'paytmalari hosil qilinadi. I holda chiziqlar bilan tutashtirilgan elementlarning ko'paytmalari o'z ishorasi, II holda esa qarama-qarshi ishora bilan olinadi.



7-rasm

❖ **Sarrius usuli.** Bu usulda determinantning o'ng tomoniga uning I va II ustunlari takroran yozilib, 3×5 tartibli matritsa hosil qilinadi. Bu matritsaning elementlari sxematik ko'rinishda nuqtalar singari ifodalanadi (8-rasmga qarang) va chiziqlar bilan tutashtirilgan elementlarning ko'paytmasi I holda o'z ishorasi, II holda esa qarama-qarshi ishora bilan olinadi.



8-rasm

III tartibli determinantni hisoblashga doir misol keltiramiz:

$$\Delta=\begin{vmatrix}2 & 6 & -2\\ 1 & 5 & 4\\ 3 & -1 & 0\end{vmatrix}=2\cdot5\cdot0+6\cdot4\cdot3+1\cdot(-1)\cdot(-2)-$$

$$-(-2)\cdot5\cdot3-6\cdot1\cdot0-4\cdot(-1)\cdot(2)=112.$$

Determinantlar va matritsalar orasida quyidagi o'xshashlik va farqlar mavjud:

1) matritsa sonlar jadvalini ifodalaydi. Determinant esa sonlar jadvalidan hosil qilinadigan sonli ifoda bo'lib, uning qiymati sondan iboratdir;

2) matritsa sonlar jadvalini dumaloq qavslar ichiga olish bilan belgilansa, determinant bu jadvalni vertikal chiziqlar orasiga olish bilan belgilanadi;

3) A matritsa va $|A|$ determinantni tashkil etuvchi sonlar ularning elementlari deyiladi;

4) matritsa va determinanant satrlar va ustunlardan iborat;

5) determinantlarda ustun va satrlar soni teng bo'lishi kerak, matritsalarda esa bunday bo'lishi shart emas.

Determinantlarning asosiy xossalari.

Endi ixtiyoriy tartibli determinantlar uchun o'rinli bo'lgan xossalar bilan tanishamiz. Aniqlik va soddalik uchun umumiy holda ifodalangan bu xossalarni uchinchi tartibli determinantlar misolida ko'rsatamiz va isbotlaymiz.

1-xossa. Agar determinantda har bir i-satr (i=1,2,3, ···,n) i-ustun bilan almashtirilsa, uning qiymati o'zgarmaydi.

Masalan,

$$\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=\begin{vmatrix}a_{11} & a_{21} & a_{31}\\ a_{12} & a_{22} & a_{32}\\ a_{13} & a_{23} & a_{33}\end{vmatrix}$$

Bu tenglik bevosita III tartibli determinantni (2) hisoblash formulasidan kelib chiqadi.

Demak determinantning satr va ustunlari teng kuchlidir, ya'ni satr (ustun) uchun o'rinli bo'lgan tasdiq ustun (satr) uchun ham o'rinli bo'ladi. Bundan tashqari bu xossadan matritsani transponirlashda uning determinanti o'zgarmay qolishi, ya'ni $\det A=\det A^T$ bo'lishi kelib chiqadi. Shu sababli determinantning keyingi xossalarini faqat satrlar uchun ifodalaymiz.

2-xossa. Determinantda ixtiyoriy ikkita satrlar o'rni o'zaro almashtirilsa, determinantning qiymati faqat ishorasini o'zgartiradi.

Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix}.$$

Bu tasdiq ham bevosita (2) formuladan kelib chiqadi.

3-xossa. Agar determinantda ikkita satr elementlari bir xil bo'lsa, uning qiymati nolga teng bo'ladi.

Isbot: Berilgan determinantning qiymatini Δ , uning bir xil elementli satrlarining o'rinlarini almashtirishdan hosil bo'lgan determinantning qiymatini esa Δ' deb belgilaymiz. Unda, 2-xossaga asosan, $\Delta'= -\Delta$ bo'ladi. Ammo determinantda bir xil elementli satrlarning o'rinlari almashtirilganligi uchun uning ko'rinishi o'zgarmay qoladi va shu sababli $\Delta'= \Delta$ bo'ladi. . Bu tengliklardan $\Delta = -\Delta$ natijani olamiz va undan $\Delta=0$ ekanligi kelib chiqadi.

Masalan, hozircha biz IV tartibli determinantni hisoblash formulasini bilmasakda, 3-xossaga asosan, birinchi va uchinchi satrlari bir xil bo'lgan ushbu determinantning qiymatini yozishimiz mumkin:

$$\begin{vmatrix} 1 & -4 & 3 & 6 \\ 0 & 7 & 1 & -2 \\ 1 & -4 & 3 & 6 \\ 8 & 0 & 4 & 5 \end{vmatrix} = 0$$

4-xossa. Determinantda biror satr elementlari umumiy λ ko'paytuvchiga ega bo'lsa, uni determinant belgisidan tashqariga chiqarib yozish mumkin.

Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \lambda \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Isbot: III tartibli determinantni (2) hisoblash formulasidagi yig'indining har bir qo'shiluvchisida λ umumiy ko'paytuvchi qatnashadi. Bu λ umumiy ko'paytuvchini qavsdan tashqariga chiqarib, 4-xossadagi tasdiqning o'rinli ekanligiga ishonch hosil etamiz.

5-xossa. Agar determinantda biror satr faqat nollardan iborat bo'lsa, uning qiymati nolga tang bo'ladi.

Bu xossaning isboti oldingi xossadan $\lambda=0$ bo'lgan holda kelib chiqadi.

Masalan, quyidagi III tartibli determinantning qiymatini (2) formula bilan hisoblab o'tirmay, 4-xossaga asosan to'g'ridan-to'g'ri

$$\begin{vmatrix} 11 & 20 & 401 \\ -8 & 37 & 139 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

deb ta'kidlay olamiz.

6-xossa. Agar determinantning ixtiyoriy ikkita satr elementlari o'zaro proporsional bo'lsa, uning qiymati nolga teng bo'ladi.

Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

Isbot: 4-xossaga asosan λ proporsionallik koeffitsiyentini determinant belgisi oldiga umumiy ko'paytuvchi sifatida chiqarish mumkin. Bu holda ikkita satri bir xil bo'lgan determinant hosil bo'ladi va uning qiymati, 3-xossaga asosan, nolga teng. Bundan berilgan determinantning ham qiymati nol ekanligi kelib chiqadi.

Masalan,

$$\begin{vmatrix} 2 & 5 & -3 \\ 4 & 1 & 8 \\ 3 & 7.5 & -4.5 \end{vmatrix} = 0,$$

chunki bu determinantda I va III satrlar proporsional va proporsionallik koeffitsiyenti λ=1.5 ga teng.

7-xossa. Agar determinantning biror i-satri ikkita qo'shiluvchi yig'indisidan iborat, ya'ni $a_{ij}+ b_{ij}$ ko'rinishda bo'lsa, bu determinantni ikkita determinantlar yig'indisi ko'rinishida yozish mumkin. Bunda bu determinantlarning i-satri mos ravishda a_{ij} va b_{ij} elementlardan iborat bo'lib, qolgan satrlari berilgan determinantniki singari bo'ladi.

Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ b_{21} & b_{22} & b_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

Bu xossaning o'rinli ekanligiga bevosita (2) formula orqali ishonch hosil qilish mumkin.

8-xossa. Agar $|A|$ determinantning a_{ii} diagonal elementlaridan yuqorida yoki pastda joylashgan barcha elementlari nolga teng bo'lsa, uning qiymati diagonal elementlar ko'paytmasiga teng bo'ladi.

Masalan,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}.$$

Isbot: Bu determinantlar uchun ularni (2) hisoblash formulasidagi $a_{11}a_{22}a_{33}$qo'shiluvchidan boshqa hamma qo'shiluvchilari nolga teng bo'ladi va shuning uchun ularning yig'indisi, ya'ni determinantning qiymati shu ko'paytmaga teng bo'ladi.

Masalan, ushbu IV tartibli determinantni hisoblaymiz:

$$\begin{vmatrix} -3 & 2 & 4 & -5 \\ 0 & 5 & 11 & 9 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & -1 \end{vmatrix} = (-3)\cdot 5\cdot 2\cdot(-1) = 30.$$

9-xossa. Diagonal matritsaning determinanti uning diagonal elementlari ko'paytmasiga teng bo'ladi.

Bu xossa isboti bevosita oldingi xossadan kelib chiqadi. Jumladan har qanday birlik matritsaning determinanti birga tengdir.

Navbatdagi xossani ifodalash uchun ikkita yangi tushuncha kiritamiz.

Ixtiyoriy *n*-tartibli determinantning a_{ij} (i,j=1,2, ... , n) elementining *minori* deb bu determinantdan shu element joylashgan *i*-satr va *j*- ustunni o'chirishdan hosil bo'lgan (n–1)-tartibli determinant qiymatiga aytiladi.

Determinantning a_{ij} elementining minori M_{ij} deb belgilanadi va ularning soni n^2 ta bo'ladi. Masalan,

$$\Delta=\begin{vmatrix}1 & -5 & 2\\ 2 & -3 & 7\\ 0 & -2 & 1\end{vmatrix} \qquad (3)$$

determinantning II satr elementlarining minorlarini yozamiz va hisoblaymiz:

$$M_{21}=\begin{vmatrix}-5 & 2\\ -2 & 1\end{vmatrix}=-1,\quad M_{22}=\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix}=1,\quad M_{23}=\begin{vmatrix}1 & -5\\ 0 & -2\end{vmatrix}=-2.$$

Bunda III tartibli determinantning minorlari II tartibli determinantlar ekanligini yana bir marta ta'kidlab o'tamiz.

Ixtiyoriy *n*-tartibli determinantning a_{ij} (i,j=1,2, ... , n) elementining *algebraik to'ldiruvchisi* deb $(-1)^{i+j}M_{ij}$ kabi aniqlanadigan songa aytiladi.

Determinantning a_{ij} (i,j=1,2, ... , n) elementining algebraik to'ldiruvchisi A_{ij} kabi belgilanadi va, ta'rifga asosan,

$$A_{ij}=\begin{cases}M_{ij}, & i+j \text{ - juft bo'lsa;}\\ -M_{ij}, & i+j \text{ - toq bo'lsa.}\end{cases}$$

formula bilan hisoblanadi. Masalan, (3) determinantning II satr elementlarining algebraik to'ldiruvchilari quyidagicha bo'ladi:

$$A_{21}=-M_{21}=1,\quad A_{22}=M_{22}=1,\quad A_{23}=-M_{23}=2. \qquad (4)$$

10-xossa (Laplas teoremasi). **Determinantning ixtiyoriy bir *i*-satrida joylashgan a_{ij} (*j*=1,2, ... , *n*) elementlarini ularning A_{ij} (*j*=1,2, ... , *n*) algebraik to'ldiruvchilariga ko'paytmalarining yig'indisi shu determinantning qiymatiga teng bo'ladi. bo'lsa**

Isbot: Bu xossa III tartibli $|A|$ determinantning birinchi satri uchun quyidagi ko'rinishda bo'ladi:

$$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A| \qquad (5)$$

Bu tenglikni isbotlash uchun algebraik to'ldiruvchi ta'rifidan va determinantlarni hisoblashning (1), (2) formulalaridan quyidagicha foydalanamiz:

$$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13} =$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}) =$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} =$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} = |A|.$$

Xuddi shunday tarzda determinantning ikkinchi va uchinchi satrlari uchun

$$a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = |A|, \quad a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = |A| \qquad (6)$$

tengliklar o'rinli bo'lishi isbotlanadi

Yuqoridagi (5) va (6) tengliklar determinantning ***satrlar bo'yicha yoyilmasi*** deb ataladi.

Shunga o'xshash determinantning ***ustunlar bo'yicha yoyilmasini*** ham quyidagicha yozish mumkin:

$$a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31} = |A|, \quad a_{12}A_{12} + a_{22}A_{22} + a_{23}A_{23} = |A|, \quad a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33} = |A|. \qquad (7)$$

Masalan, yuqorida keltirilgan (3) determinant qiymatini uning II satrining (4) algebraik to'ldiruvchilari yordamida hisoblaymiz:

$$\Delta = 2A_{21} + (-3)A_{22} + 7A_{23} = 2 \cdot 1 + (-3) \cdot 1 + 7 \cdot 2 = 13.$$

Laplas teoremasidan foydalanib yuqori tartibli determinantlarni hisoblash mumkin. Bunda *n*-tartibli determinantni hisoblash n ta (n–1) - tartibli determinantni (A_{ij}algebraik to'ldiruvchilarni) hisoblash va uning ixtiyoriy satr yoki ustuni bo'yicha yoyilmasidan foydalanishga keltiriladi. Jumladan, I tartibli determinant qiymati $|A|=|a_{11}|=a_{11}$ ekanligidan foydalanib, (1) va (2) formulalarni keltirib chiqarish mumkin. Determinant qiymatini Laplas teoremasi yordamida hisoblash uchun uning ixtiyoriy satr yoki ustun bo'yicha yoyilmasidan foydalanish mumkin. Ammo, amaliy nuqtai nazardan, ko'proq elementlari nolga teng bo'lgan satr yoki ustunni tanlash (agar shundaylar mavjud bo'lsa) maqsadga muvofiqdir. Bu holda nolga teng elementlarning algebraik to'ldiruvchilarini topishga hojat bo'lmaydi va hisoblashlar hajmi ancha kamayadi.

Misol. Ushbu IV tartibli determinantni hisoblang:

$$|A|=\begin{vmatrix} 2 & -3 & 4 & 5 \\ 1 & 0 & 2 & 10 \\ 6 & 7 & 5 & -2 \\ 3 & 0 & 9 & 1 \end{vmatrix}$$

Yechish: Bu determinantni II ustun bo'yicha yoyilmasidan foydalanib hisoblash qulaydir. Bunga sabab shuki, bu ustunda nol elementlar boshqa satr va ustunlarga qaraganda ko'proq hamda $a_{22}=0$, $a_{42}=0$ elementlarning A_{22} , A_{42} algebraik to'ldiruvchilarini hisoblash shart emas.

Dastlab A_{12} va A_{32} algebraik to'ldiruvchilarni hisoblab, $A_{12}=-389$ va $A_{32}=45$ ekanligini aniqlaymiz. Endi determinant qiymatini II ustunga Laplas teoremasini tatbiq etib hisoblaymiz:

$$|A|=a_{12}A_{12}+a_{22}A_{22}+a_{32}A_{32}+a_{42}A_{42}=$$
$$=(-3)\cdot(-389)+0\cdot A_{22}+7\cdot 45+0\cdot A_{42}=1482.$$

11-xossa. **Agar $|A|$ determinantni biror i-satrining algebraik to'ldiruvchilari A_{ij} (j=1,2, ... , n) va b_j (j=1,2, ... , n) ixtiyoriy sonlar bo'lsa, unda**

$$b_1A_{i1}+b_2A_{i2}+b_3A_{i3}+\cdots+b_nA_{in}=|B|$$

tenglik o'rinli bo'ladi. Bunda $|B|$ determinant $|A|$ determinantdan faqat i-satri bilan farq qilib, uning i-satri b_j

(j=1,2, … , n) sonlardan tashkil topgan bo'ladi.

Isbot: Bu xossa isbotini III tartibli $|A|$ determinantning, masalan, birinchi satri uchun keltiramiz. Bu holda

$$b_1A_{11}+b_2A_{12}+b_3A_{13}=|B|$$

yig'indining qo'shiluvchilari $|A|$ determinantning birinchi satr bo'yicha yoyilmasini ifodalovchi

$$a_1A_{11}+a_2A_{12}+a_3A_{13}=|A|$$

yig'indi qo'shiluvchilaridan faqat birinchi ko'paytuvchilari, ya'ni birinchi satr elementlari bilan farq qiladi. Shu sababli $|A|$, $|B|$ determinantlar bir-biridan faqat birinchi satri bilan farq qiladi va $|B|$ determinantning birinchi satri b_1, b_2 va b_3 sonlardan iborat bo'ladi.

Masalan, A_{11}, A_{12} va A_{13}

$$|A|=\begin{vmatrix} 2 & -4 & 1 \\ 0 & 7 & 3 \\ 9 & 12 & -4 \end{vmatrix}$$

determinantni birinchi satri elementlarining algebraik to'ldiruvchilari bo'lsa, unda

$$11A_{11}+12A_{12}+13A_{13}=|B|=\begin{vmatrix} 11 & 12 & 13 \\ 0 & 7 & 3 \\ 9 & 12 & -4 \end{vmatrix}.$$

12-xossa. **Agar $|A|$ determinantni biror i-satrining a_{ij} (j=1,2, … , n) elementlari boshqa bir k-satr ($i\neq k$) mos elementlarining A_{kj} (j=1,2, … , n) algebraik to'ldiruvchilariga ko'paytirilgan bo'lsa, bu ko'paytmalar yig'indisi nolga teng bo'ladi.**

Isbot: Oldingi xossada $b_j=a_{ij}$ (j=1,2, … , n) deb olsak, unda

$$b_1A_{k1}+b_2A_{k2}+b_3A_{k3}+\cdots+b_nA_{kn}=a_{i1}A_{k1}+a_{i2}A_{k2}+a_{i3}A_{k3}+\cdots+a_{in}A_{kn}=|B|$$

Bu yerda $|B|$ determinant berilgan $|A|$ determinantning k-satriga i-satrining a_{ij} elementlarini qo'yish bilan hosil qilinadi. Shu sababli $|B|$ determinantning i-satri va k-satri bir xil bo'lib, 3-xossaga asosan uning qiymati nolga teng bo'ladi, ya'ni

$$a_{i1}A_{k1}+a_{i2}A_{k2}+a_{i3}A_{k3}+\cdots+a_{in}A_{kn}=0, \quad i,k=1,2,\cdots,n; \quad i\neq k. \quad (8)$$

13-xossa. Agar A va B bir xil tartibli kvadrat matritsalar bo'lsa ularning ko'paytmasining determinanti har birining determinantlari ko'paytmasiga teng bo'ladi, ya'ni $|A\cdot B|=|A|\cdot|B|$ tenglik o'rinlidir.

Bu xossani isbotsiz keltiramiz.

Ko'rib o'tilgan bu xossalar determinantlarni hisoblash va ularning turli tatbiqlarida qo'llaniladi.

Mustaqil yechish uchun misollar

Quyidagi ikkinchi tartibli aniqlovchilarni hisoblang:

1.31. $\begin{vmatrix} -3 & -5 \\ 4 & 5 \end{vmatrix}$ 1.32. $\begin{vmatrix} 0,(3) & -0,(42) \\ 99 & 3 \end{vmatrix}$ 1.33. $\begin{vmatrix} 3 & -6 \\ -5 & 7 \end{vmatrix}$

1.34. $\begin{vmatrix} \sqrt{2} & -3 \\ \frac{1}{3} & \sqrt{2} \end{vmatrix}$ 1.35. $\begin{vmatrix} \sqrt{a} & -1 \\ a & 1+\sqrt{2} \end{vmatrix}$ 1.36. $\begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix}$

1.37. $\begin{vmatrix} a^{\frac{3}{4}} & a^{\frac{1}{2}} \\ -a^{\frac{1}{2}} & a^{\frac{1}{4}} \end{vmatrix}$ 1.38. $\begin{vmatrix} 2\sin^2\alpha & \cos^2\alpha \\ 2\sin^2\beta & \cos^2\beta \end{vmatrix}$

1.39. $\begin{vmatrix} \sqrt[4]{5} & \sqrt{2} \\ \sqrt{2} & \sqrt[4]{125} \end{vmatrix}$ 1.40. $\begin{vmatrix} \cos\frac{x}{2} & \sin\frac{x}{2} \\ \sin\frac{x}{2} & \cos\frac{x}{2} \end{vmatrix}$

Quyidagi uchunchi tartibli aniqlovchilarni hisoblang:

1.41. a) $\begin{vmatrix} 3 & -1 & -1 \\ -3 & 1 & 5 \\ 2 & -2 & 4 \end{vmatrix}$ в) $\begin{vmatrix} 2 & 0 & 4 \\ 5 & -2 & 1 \\ -1 & -2 & 3 \end{vmatrix}$ 1.42. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}$ 1.43. $\begin{vmatrix} 3 & 2 & -1 \\ 1 & 2 & 9 \\ 1 & 1 & 2 \end{vmatrix}$

1.44. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}$ 1.45. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix}$ 1.46. $\begin{vmatrix} 1 & 2 & 3 \\ 3 & -4 & 7 \\ -3 & 12 & -15 \end{vmatrix}$ 1.47. $\begin{vmatrix} 1 & -3 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & -3 \end{vmatrix}$

1.48. $\begin{vmatrix} 2 & -1 & -3 \\ 1 & 3 & 2 \\ 3 & 2 & 2 \end{vmatrix}$ 1.49. $\begin{vmatrix} 1 & 2 & 0 \\ 5 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$ 1.50. $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ 1.51. $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$

1.52. $\begin{vmatrix} 4 & 2 & 1 \\ 5 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}$ 1.53. $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$ 1.54. $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$ 1.55. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 0 \end{vmatrix}$

1.56. $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}$ 1.57. $\begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$ 1.58. $\begin{vmatrix} 5 & 6 & 3 \\ 0 & 1 & 0 \\ 7 & 4 & 5 \end{vmatrix}$ 1.59. $\begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}$

1.60. $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}$ 1.61. $\begin{vmatrix} 1 & 5 & 25 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \end{vmatrix}$ 1.62. $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ 1.63. $\begin{vmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{vmatrix}$

1.64. $\begin{vmatrix} \cos\alpha & \sin\alpha\cos\beta & \sin\alpha\sin\beta \\ -\sin\alpha & \cos 3\alpha\cos\beta & \cos\alpha\sin\beta \\ 0 & -\sin\beta & \cos\beta \end{vmatrix}$ 1.65. $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$

Tenglamani yeching:

1.66. $\begin{vmatrix} x^2-4 & -1 \\ x-2 & x+2 \end{vmatrix}=0$ 1.67. $\begin{vmatrix} 4\sin x & 1 \\ 1 & \cos x \end{vmatrix}=0$

1.68. $\begin{vmatrix} 2 & -1 & 2 \\ 3 & 5 & 3 \\ 1 & 6 & x+5 \end{vmatrix}=0$ 1.69. $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3-x & 3 \\ 1 & 2 & 5+x \end{vmatrix}=0$

1.70. $\begin{vmatrix} \cos^2\varphi-\sin^2\varphi & 2\cos\varphi\sin\varphi \\ 2\sin\varphi\cos\varphi & \cos^2\varphi-\sin^2\varphi \end{vmatrix}=0$ 1.71 $\begin{vmatrix} 3-x & 2 \\ 2 & 4-x \end{vmatrix}=0$

1.72 $\begin{vmatrix} 5-x & 2 \\ 2 & 8-x \end{vmatrix}=0$ 1.73 $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}=0$

1.74 $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix}=0$ 1.75 $\begin{vmatrix} 3-x & -1 & 1 \\ -1 & 5-x & -1 \\ 1 & -1 & 3-x \end{vmatrix}=0$

Tengsizlikni hisoblang:

1.76 $\begin{vmatrix} x-1 & 3 \\ 2x-3 & -2 \end{vmatrix}<0$ 1.77 $\begin{vmatrix} 2x-3 & 4x \\ -5 & 3 \end{vmatrix}>0$ 1.78 $\begin{vmatrix} x & 3x-4 \\ 6 & -2 \end{vmatrix}<0$

1.79 $\begin{vmatrix} 3-x & 4 \\ 2 & 4-x \end{vmatrix}<0$ 1.80 $\begin{vmatrix} x^2+7 & 2 \\ 3x & 1 \end{vmatrix}>-1$ 1.81 $\begin{vmatrix} x^2+3 & x \\ 2 & 5 \end{vmatrix}>5$

1.82 $\begin{vmatrix} 2 & -1 & 2 \\ 3 & 5 & 3 \\ 1 & 6 & x+5 \end{vmatrix} > 0$ 1.83 $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3-x & 3 \\ 1 & 2 & 5+x \end{vmatrix} < 0$ 1.84 $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix} > 0$

1.85 $\begin{vmatrix} x\cos x & x \\ -x & x \end{vmatrix} > 0$

Aniqlovchilarni hisoblang:

1.86 *a)* $\begin{vmatrix} 1 & 2 & -1 & 2 \\ 2 & -1 & 2 & 3 \\ -1 & -3 & -2 & 4 \\ 3 & -4 & -3 & 6 \end{vmatrix}$ *б)* $\begin{vmatrix} -1 & 6 & 5 & 1 \\ -2 & 8 & 6 & 2 \\ 2 & 16 & 7 & 3 \\ -3 & 9 & 3 & 4 \end{vmatrix}$

1.87.*a)* $\begin{vmatrix} 3 & 0 & 2 & 0 \\ 2 & 3 & -1 & 4 \\ 0 & 4 & -2 & 3 \\ 5 & 2 & 0 & 1 \end{vmatrix}$ *б)* $\begin{vmatrix} 5 & 62 & -79 & 4 \\ 0 & 2 & 3 & 0 \\ 6 & 183 & 201 & 5 \\ 0 & 3 & 4 & 0 \end{vmatrix}$

1.88. $\begin{vmatrix} -5 & 6 & 10 & 6 \\ -9 & 8 & 8 & 5 \\ -8 & 5 & 9 & 5 \\ -11 & 7 & 7 & 4 \end{vmatrix}$ 1.89. $\begin{vmatrix} 9 & 7 & 9 & 7 \\ 8 & 6 & 8 & 6 \\ -9 & -7 & 9 & 7 \\ -8 & -6 & 8 & 6 \end{vmatrix}$

1.90. $\begin{vmatrix} 6 & 8 & -9 & -12 \\ 4 & 6 & -6 & -9 \\ -3 & -4 & 6 & 8 \\ -2 & -3 & 4 & 6 \end{vmatrix}$ 1.91. $\begin{vmatrix} 6 & -5 & 1 & 2 \\ -4 & 7 & -1 & 4 \\ 5 & -9 & 2 & 7 \\ 4 & -6 & 1 & 2 \end{vmatrix}$

1.92. $\begin{vmatrix} -3 & 9 & 3 & 6 \\ -5 & 8 & 2 & 7 \\ 4 & -5 & -3 & -2 \\ 7 & -8 & -4 & -5 \end{vmatrix}$ 1.93. $\begin{vmatrix} 2 & -5 & 4 & 3 \\ 3 & -4 & 7 & 5 \\ 4 & -9 & 8 & 5 \\ -3 & 2 & -5 & 3 \end{vmatrix}$

1.94. $\begin{vmatrix} 3 & -5 & 2 & -4 \\ -3 & 4 & -5 & 3 \\ -5 & 7 & -7 & 5 \\ 8 & -8 & 5 & -6 \end{vmatrix}$ 1.95. $\begin{vmatrix} 7 & 6 & 3 & 7 \\ 3 & 5 & 7 & 2 \\ 5 & 4 & 3 & 5 \\ 5 & 6 & 5 & 4 \end{vmatrix}$

1.96. $\begin{vmatrix} 3 & -3 & -5 & 8 \\ -3 & 2 & 4 & -6 \\ 2 & -5 & -7 & 5 \\ -4 & 3 & 5 & -6 \end{vmatrix}$ 1.97. $\begin{vmatrix} 3 & 2 & 2 & 2 \\ 9 & -8 & 5 & 10 \\ 5 & -8 & 5 & 8 \\ 6 & -5 & 4 & 7 \end{vmatrix}$

1.98. $\begin{vmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 0 & 1 \\ -1 & 1 & -2 & 2 \\ 1 & 4 & 1 & 1 \end{vmatrix}$

1.99. $\begin{vmatrix} 1 & -1 & 2 & 1 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 3 & 1 \\ -5 & 0 & 1 & -2 \end{vmatrix}$

1.100. $\begin{vmatrix} -1 & 3 & 1 & 2 \\ -5 & 8 & 2 & 7 \\ 3 & -2 & -2 & 0 \\ 1 & 0 & -1 & 1 \end{vmatrix}$

1.101. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 4 \\ 3 & 1 & 3 & 3 \\ 4 & 4 & 3 & 4 \end{vmatrix}$

1.102. $\begin{vmatrix} 4 & 3 & -1 & 2 \\ 2 & 1 & 2 & -1 \\ 3 & 1 & -1 & 1 \\ 1 & 2 & 1 & 2 \end{vmatrix}$

1.103. $\begin{vmatrix} 1 & 1 & 3 & 2 \\ 2 & -3 & 1 & 2 \\ 2 & 2 & 4 & 4 \\ -1 & 1 & 5 & 3 \end{vmatrix}$

1.104. $\begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -2 \\ 6 & 0 & 3 & -3 \\ 1 & -5 & 3 & -3 \end{vmatrix}$

1.105. $\begin{vmatrix} 2 & 2 & -3 & -3 \\ 4 & 6 & -6 & -9 \\ -3 & -4 & 6 & 8 \\ -5 & -7 & 10 & 14 \end{vmatrix}$

1.106. $\begin{vmatrix} -4 & 10 & 0 & -2 & 6 \\ 1 & 0 & 3 & 7 & -2 \\ 3 & -1 & 0 & 5 & -5 \\ 2 & 6 & -4 & 1 & 2 \\ 0 & -3 & -1 & 2 & 3 \end{vmatrix}$

1.107. $\begin{vmatrix} 6 & 6 & 10 & -5 \\ 5 & 8 & 8 & -9 \\ 5 & 5 & 9 & -8 \\ 4 & 7 & 7 & -11 \end{vmatrix}$

1.108. $\begin{vmatrix} 2 & 4 & -1 & 8 & 3 \\ 4 & 8 & -2 & 16 & 6 \\ 5 & -1 & -3 & 0 & 1 \\ 1 & 1 & 0 & 1 & 2 \\ -1 & 7 & 0 & 8 & 1 \end{vmatrix}$

1.109. $\begin{vmatrix} -1 & 2 & -2 & -1 \\ 3 & -4 & 7 & 5 \\ 4 & -9 & 8 & 5 \\ -3 & 2 & -5 & 3 \end{vmatrix}$

1.110. $\begin{vmatrix} 3 & 5 & -4 & 4 \\ 1 & 3 & 2 & 1 \\ 4 & 7 & 3 & 5 \\ 1 & -2 & 4 & 2 \end{vmatrix}$

1.111. $\begin{vmatrix} 9 & 7 & 9 & 7 \\ 4 & 3 & 4 & 3 \\ 9 & 7 & -9 & -7 \\ -8 & -6 & 8 & 6 \end{vmatrix}$

1.112. $\begin{vmatrix} 0 & 0 & 0 & 4 \\ 0 & 0 & 2 & 1 \\ 0 & 7 & 3 & 5 \\ 1 & -2 & 4 & 2 \end{vmatrix}$

1.108. $\begin{vmatrix} 1 & 3 & 0 & 1 \\ 3 & 3 & 1 & 2 \\ -1 & 1 & -2 & 2 \\ 1 & 4 & 1 & 1 \end{vmatrix}$

1.109. $\begin{vmatrix} 2 & 1 & 3 & 4 \\ -1 & 3 & -2 & 5 \\ 0 & 5 & 4 & 2 \\ 1 & 2 & -3 & 1 \end{vmatrix}$ 1.110. $\begin{vmatrix} 2 & 3 & 1 & -1 \\ 7 & 8 & 2 & -5 \\ 0 & -2 & -2 & 3 \\ 1 & 0 & -1 & 1 \end{vmatrix}$

§ 1.3. Teskari matritsa va matritsa rangi

Biz matritsalar ustida qo'shish, ayirish va ko'paytirish amallarini ko'rib o'tgan edik. Matritsalar son tushunchasini umumlashtirishdan hosil qilinganligi haqida ham aytilgan edi. Sonlar uchun qo'shish, ayirish va ko'paytirish amallaridan tashqari bo'lish amali ham mavjud. Bunda ikkita b va a ($a \neq 0$) sonlar bo'linmasini quyidagi ko'rinishda ifodalash mumkin:

$$\frac{b}{a} = \frac{1}{a} b = a^{-1} b.$$

Bundan ko'rinadiki, bo'lish amalini $a \neq 0$ songa teskari bo'lgan a^{-1} son yordamida ko'paytirish amali orqali ifodalash mumkin. Bunda ixtiyoriy $a \neq 0$ va unga teskari a^{-1} sonlar orasida $aa^{-1} = a^{-1}a = 1$ munosabat o'rinli bo'ladi. Bu tenglik faqat o'zaro teskari sonlar uchun o'rinlidir va shu sababli teskari sonni ta'rifi sifatida qabul qilinishi mumkin. Shu sababli bu tenglikdan berilgan A matritsaga teskari matritsa tushunchasini kiritish uchun foydalaniladi.

Teskari matritsa ta'rifi: Berilgan n- tartibli A kvadrat matritsaga ***teskari matritsa*** deb $AB = BA = E$ (E–n- tartibli birlik matritsa) shartni qanoatlantiruvchi n-tartibli B kvadrat matritsaga aytiladi.

Berilgan A matritsaga teskari matritsa A^{-1} kabi belgilanadi va, ta'rifga asosan, ular uchun $AA^{-1} = A^{-1}A = E$ munosabat o'rinli bo'ladi.

Ma'lumki, ixtiyoriy $a \neq 0$ soni uchun a^{-1} teskari son mavjud va yagonadir. $a = 0$ sonining esa teskarisi mavjud emas. Shu sababli matritsalar uchun quyidagi savollar paydo bo'ladi:

1. Qanday matritsalar uchun ularning teskarisi mavjud?

2. Teskari matritsa yagonami va uni qanday topish mumkin?

3. Qanday matritsalarning teskarisi mavjud emas?

Agar A matritsaning determinanti $|A|=0$ bo'lsa u ***maxsus matritsa*** , aks holda, ya'ni $|A|\neq0$ bo'lsa ***maxsusmas matritsa*** dir.

Masalan,

$$A=\begin{pmatrix} -3 & 1 & 4 \\ 2 & 5 & -1 \\ 1 & 11 & 2 \end{pmatrix}, \quad B=\begin{pmatrix} 2 & 1 & 0 \\ 0 & 5 & -1 \\ 3 & 6 & 1 \end{pmatrix}$$

matritsalardan A maxsus (chunki $|A|=0$), B esa maxsusmas (chunki $|B|=19\neq0$) matritsa bo'ladi.

1-TEOREMA: Maxsus A matritsa uchun teskari A^{-1} matritsa mavjud emas.

Isbot: Teskarisini faraz qilamiz, ya'ni teskari A^{-1} matritsa mavjud deymiz. Ta'rifga asosan $AA^{-1}=A^{-1}A=E$ va, teorema sharti bo'yicha, $|A|=0$ tengliklar o'rinlidir. Bu holda, determinantning oldin ko'rib o'tilgan 13-xossasiga asosan,

$$|E|=|A^{-1}A|=|A\,A^{-1}|=|A|\,|A^{-1}|=0\cdot|A^{-1}|=0$$

natijani olamiz. Ammo E birlik matritsaning determinanti $|E|=1$ bo'ladi. Hosil bo'lgan bu ziddiyat bizning farazimiz noto'g'ri ekanligini ko'rsatadi va maxsus matritsa uchun uning teskarisi mavjud emasligini ifodalaydi.

Berilgan n- tartibli

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

kvadrat matritsa determinantinig A_{ij} algebraik to'ldiruvchilaridan tuzilgan ushbu

$$\bar{A}=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{pmatrix}^T=\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}$$

matritsani kiritamiz.

<u>*3-TA'RIF:*</u> $\overline{A}$ berilgan A matritsaga ***birkitilgan matritsa*** deb ataladi.

Masalan,

$$A=\begin{pmatrix} 2 & -3 & 1 \\ 0 & 5 & -3 \\ 4 & 0 & 1 \end{pmatrix} \qquad (1)$$

matritsa determinantining algebraik to'ldiruvchilari

$A_{11}=5,\quad A_{12}=-12,\quad A_{13}=-20,\quad A_{21}=3,\quad A_{22}=-2,\quad A_{23}=-12,$
$A_{31}=4,\quad A_{32}=6,\quad A_{33}=10$

bo'lgani uchun unga birkitilgan matritsa quyidagicha bo'ladi:

$$\overline{A}=\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}=\begin{pmatrix} 5 & 3 & 4 \\ -12 & -2 & 6 \\ -20 & -12 & 10 \end{pmatrix}. \qquad (2)$$

<u>*2-TEOREMA:*</u> Agar A maxsusmas matritsa bo'lsa unga teskari A^{-1} matritsa mavjud va u

$$A^{-1}=\frac{\overline{A}}{|A|}=\frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix} \qquad (3)$$

formula bilan topiladi.

<u>*Isbot:*</u> Dastlab

$$A\overline{A}=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}$$

ko'paytmani topamiz. Determinantlarning 10 va 12-xossalariga asosan

$$a_{i1}A_{1k}+a_{i2}A_{2k}+a_{i3}A_{3k}+\cdots a_{in}A_{nk}=\begin{cases} |A|, & i=k \\ 0, & i\neq k \end{cases}$$

tengliklar o'rinli bo'ladi. Bundan, matritsalar ko'paytmasining ta'rifiga asosan,

$$A\overline{A}=\begin{pmatrix} |A| & 0 & 0 & \cdots & 0 \\ 0 & |A| & 0 & \cdots & 0 \\ 0 & 0 & |A| & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & |A| \end{pmatrix}$$

tenglikni hosil etamiz. Unda, matritsalarni songa ko'paytirish amalining ta'rifi va xossalariga asosan,

$$A\frac{1}{|A|}\overline{A}=\frac{1}{|A|}A\overline{A}=\frac{1}{|A|}\begin{pmatrix} |A| & 0 & 0 & \cdots & 0 \\ 0 & |A| & 0 & \cdots & 0 \\ 0 & 0 & |A| & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & |A| \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}=E$$

natijaga kelamiz. Shu tarzda

$$\frac{1}{|A|}\overline{A}A=E$$

tenglik ham o'rinli bo'lishini ko'rsatish mumkin. Bu yerdan, teskari matritsa ta'rifiga asosan, (3) tenglik o'rinli ekanligiga ishonch hosil qilamiz.

Misol sifatida yuqorida ko'rilgan (1) matritsa uchun $|A|=26\neq 0$ ekanligidan va unga birkitilgan (2) matritsadan foydalanib, A^{-1} teskari matritsani topamiz:

$$A^{-1}=\frac{1}{|A|}\overline{A}=\frac{1}{26}\begin{pmatrix} 5 & 3 & 4 \\ -12 & -2 & 6 \\ -20 & -12 & 10 \end{pmatrix}=\begin{pmatrix} \frac{5}{26} & \frac{3}{26} & \frac{2}{13} \\ -\frac{6}{13} & -\frac{1}{13} & \frac{3}{13} \\ -\frac{10}{13} & -\frac{6}{13} & \frac{5}{13} \end{pmatrix}.$$

3-TEOREMA: Maxsusmas A matritsa uchun A^{-1} teskari matritsa yagona ravishda aniqlanadi.

Isbot: Teskarisini faraz qilamiz. C va B matritsalar A matritsaga teskari va $C\neq B$ bo'lsin. Unda, teskari matritsa ta'rifiga asosan,

$AC=CA=E$, $AB=BA=E$

tengliklar o'rinli bo'ladi. Bu tengliklar va birlik matritsa xossasidan foydalanib, quyidagi natijalarni olamiz:

$$C=CE=CAB, \quad B=BE=EB=CAB.$$

Bu ikkala tenglikning o'ng tomonlarini taqqoslab, $C=B$ natijaga kelamiz. Hosil bo'lgan bu ziddiyat farazimiz noto'g'ri ekanligini ko'rsatadi va teskari matritsa yagona bo'ladi.

Shunday qilib, A maxsusmas matritsa uchun A^{-1} teskari matritsa mavjud va u yagonadir.

Teskari matritsa tushunchasidan foydalanib bir xil tartibli A va B kvadrat matritsalarning bo'linmasini $A^{-1}B$ $(|A|\neq 0)$ ko'rinishda kiritish mumkin. Shuningdek A^{-1} teskari matritsani m marta o'zaro ko'paytirib, hosil bo'lgan matritsani A maxsusmas matritsaning $-m$- darajasi deb olishimiz va A^{-m} (m-ixtiyoriy natural son) kabi belgilashimiz mumkin. Bundan A^k daraja (A-maxsusmas matritsa) ixtiyoriy k butun son uchun (§1 ga qarang) aniqlangan bo'ladi.

Teskari matritsalar quyidagi xossalarga ega:

1-xossa. $E^{-1}=E$.

Isbot: Bu xossa (3) teskari matritsani topish formulasidan bevosita kelib chiqadi.

2-xossa. $(A^{-1})^{-1}=A$.

Isbot: Bu xossa bevosita teskari matritsaning ta'rifidan, ya'ni $AA^{-1}=A^{-1}A=E$ tenglikdan kelib chiqadi.

3-xossa. $(AB)^{-1}=B^{-1}A^{-1}$.

Isbot: Teskari matritsa ta'rifi va matritsalar ko'paytmasining assosiativlik xossasiga asosan quyidagi tengliklarni olamiz:

$$(AB)(B^{-1}A^{-1})=A(BB^{-1})A^{-1}=AEA^{-1}=AA^{-1}=E,$$

$$(B^{-1}A^{-1})(AB)=B^{-1}(A^{-1}A)B=B^{-1}EB=B^{-1}B=E.$$

Bu tengliklardan AB va $B^{-1}A^{-1}$ o'zaro teskari matritsalar ekanligi ko'rinadi.

4-xossa. $(A^{-1})^T=(A^T)^{-1}$.

Isbot: Matritsalarni transponirlash amalining $(AB)^T=B^TA^T$ va $E^T=E$ xossalaridan foydalanib ushbu tengliklarga ega bo'lamiz:

$$A^T(A^{-1})^T=(A^{-1}A)^T=E^T=E, \quad (A^{-1})^TA^T=(AA^{-1})^T=E^T=E.$$

Bu tengliklardan, teskari matritsa ta'rifiga asosan, $(A^{-1})^T=(A^T)^{-1}$ ekanligi kelib chiqadi.

5-xossa. $|A^{-1}|=1/|A|=|A|^{-1}$.

Isbot: Matritsalar ko'paytmasining determinanti uchun $|AB|=|A||B|$ formula va $|E|=1$ tenglik hamda teskari son ta'rifidan foydalanib, ushbu natijaga kelamiz:

$$|A^{-1}|\cdot|A|=|A^{-1}A|=|E|=1\Rightarrow|A^{-1}|=\frac{1}{|A|}=|A|^{-1}.$$

Matritsaning rangi. Endi kelgusida kerak bo'ladigan matritsa rangi tushunchasini kiritamiz. Dastlab oldin kvadrat matritsalar uchun aniqlangan minor tushunchasini ixtiyoriy to'rtburchakli matritsa uchun umumlashtiramiz.

Ta'rif: Har qanday $A_{m\times n}$ matritsaning ixtiyoriy ravishda tanlangan k ta ($k\leq\min(m,n)$) satr va ustunlarining kesishmasida joylashgan elementlaridan tuzilgan k-tartibli determinant bu matritsaning ***k-tartibli minori*** deyiladi.

Masalan, $A_{4\times3}=\begin{pmatrix}-2&3&1\\3&1&4\\0&1&1\\3&2&5\end{pmatrix}$

matritsaning har bir elementi uning I tartibli ,

$$\begin{vmatrix}-2&3\\3&1\end{vmatrix},\ \begin{vmatrix}-2&3\\0&1\end{vmatrix},\ \begin{vmatrix}3&1\\1&1\end{vmatrix},\ \begin{vmatrix}1&1\\2&5\end{vmatrix},\ \begin{vmatrix}0&1\\3&2\end{vmatrix},\ \begin{vmatrix}1&4\\2&5\end{vmatrix}$$

kabi determinantlar II tartibli,

$$\begin{vmatrix}-2&3&1\\3&1&4\\0&1&1\end{vmatrix},\ \begin{vmatrix}-2&3&1\\3&1&4\\3&2&5\end{vmatrix},\ \begin{vmatrix}3&1&4\\0&1&1\\3&2&5\end{vmatrix},\ \begin{vmatrix}-2&3&1\\0&1&1\\3&2&5\end{vmatrix}$$

determinantlar esa berilgan matritsaning III tartibli minorlariga misol bo'ladi.

Berilgan ***Amatritsaning rangi*** deb uning noldan farqli minorining eng katta tartibiga aytiladi.

Matritsaning rangi r(A) kabi belgilanadi va uni quyidagicha topish mumkin. Agar matritsaning barcha elementlari nolga teng, ya'ni u nol matritsa bo'lsa, uning rangi r(A)=0 bo'ladi.

Matritsaning noldan farqli elementi mavjud bo'lsa, uning rangi $r(A) \geq 1$ bo'ladi. Bu noldan farqli elementni o'z ichiga olgan barcha II tartibli minorlarni hisoblaymiz. Agar barcha II tartibli minorlar nolga teng bo'lsa, unda $r(A)=1$ bo'ladi. Aks holda $r(A) \geq 2$ bo'ladi va noldan farqli biror II tartibli minorni o'z ichiga olgan barcha III tartibli minorlarni qaraymiz. Ularning hammasi nolga teng bo'lsa $r(A)=2$, aks holda $r(A) \geq 2$ bo'ladi. Bu jarayonni shunday tarzda davom ettiramiz. Natijada, biror qadamda noldan farqli k-tartibli minorni o'z ichiga oluvchi barcha $(k+1)$-tartibli minorlar nolga teng bo'lgan holga kelamiz va bundan matritsaning rangi $r(A)=k$ ekanligini topamiz.

Masalan, $A_{4\times 3} = \begin{pmatrix} -2 & 3 & 1 \\ 3 & 1 & 4 \\ 0 & 1 & 1 \\ 3 & 2 & 5 \end{pmatrix}$

matritsaning rangini aniqlaymiz. Bu matritsaning noldan farqli elementi mavjud va shu sababli $r(A) \geq 1$. Endi noldan farqli ixtiyoriy bir, masalan $a_{11}= -2$ elementni, o'z ichiga olgan va noldan farqli bo'lgan II tartibli minor mavjud yoki yo'qligini aniqlaymiz:

$$\begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = -2-9=-11 \neq 0.$$

Demak, $r(A) \geq 2$ bo'ladi. Bu noldan farqli II tartibli minorni o'z ichiga olgan ikkita III tartibli minorlarni qaraymiz:

$$\begin{vmatrix} -2 & 3 & 1 \\ 3 & 1 & 4 \\ 0 & 1 & 1 \end{vmatrix} = 0, \quad \begin{vmatrix} -2 & 3 & 1 \\ 3 & 1 & 4 \\ 3 & 2 & 5 \end{vmatrix} = 0.$$

Bu yerdan ko'rilayotgan matritsaning rangi $r(A)=2$ ekanligi kelib chiqadi.

Shuni ta'kidlab o'tish lozimki, n-tartibli maxsusmas A matritsaning rangi $r(A)=n$ bo'ladi.

Quyidagi matritsalarning teskari matritsalarini toping

1.111 *a)* $A=\begin{pmatrix}-2 & 2 & -3\\ 2 & -1 & 2\\ 3 & -1 & 3\end{pmatrix}$ *б)* $A=\begin{pmatrix}1 & 2 & -1\\ 2 & 1 & -1\\ 1 & -7 & 3\end{pmatrix}$

1.112. $A=\begin{pmatrix}2 & 1 & -1\\ 5 & 2 & 4\\ 7 & 3 & 2\end{pmatrix}$ 1.113. $A=\begin{pmatrix}1 & 2 & -3\\ 0 & 1 & 2\\ 1 & 0 & 4\end{pmatrix}$ 1.114. $A=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & 1\\ 1 & 3 & 1\end{pmatrix}$

1.115. $A=\begin{pmatrix}1 & -1 & 1\\ 2 & 1 & 1\\ 1 & 1 & 2\end{pmatrix}$ 1.116. $A=\begin{pmatrix}1 & 2 & -1 & -2\\ 3 & 8 & 0 & -4\\ 2 & 2 & -4 & -3\\ 3 & 8 & -1 & -6\end{pmatrix}$ 1.117. $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$

1.18. $A=\begin{pmatrix}3 & 4\\ 5 & 7\end{pmatrix}$ 1.119. $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ 1.120. $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$

1.121. $A=\begin{pmatrix}2 & 5 & 7\\ 6 & 3 & 4\\ 5 & -2 & -3\end{pmatrix}$ 1.122. $A=\begin{pmatrix}3 & -4 & 5\\ 2 & -3 & 1\\ 3 & -5 & -1\end{pmatrix}$

1.123. $A=\begin{pmatrix}2 & 7 & 3\\ 3 & 9 & 4\\ 1 & 5 & 3\end{pmatrix}$ 1.124. $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{pmatrix}$

Quyidagi matritsalarni rangini toping

1.125. *a)* $A=\begin{pmatrix}1 & 3 & 2 & -7 & -4\\ -5 & -15 & -10 & 35 & 20\\ 0 & 1 & 21 & 67 & 3\\ 1 & 4 & 23 & 60 & -1\end{pmatrix}$ *б)* $\begin{pmatrix}2 & -1 & 3 & -2 & 4\\ 4 & -2 & 5 & 1 & 7\\ 2 & -1 & 1 & 8 & 2\end{pmatrix}$

1.126.: $\begin{pmatrix}1 & 3 & 5 & -1\\ 2 & -1 & -3 & 4\\ 5 & 1 & -1 & 7\\ 7 & 7 & 9 & 1\end{pmatrix}$ 1.127. $\begin{pmatrix}3 & -1 & 3 & 2 & 5\\ 5 & -3 & 2 & 3 & 4\\ 1 & -3 & -5 & 0 & -7\\ 7 & -5 & 1 & 4 & 1\end{pmatrix}$

Matritsali tenglamalarni yeching

1.128 $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\cdot x=\begin{pmatrix}3 & 5\\ 5 & 9\end{pmatrix}$ 1.129 $x\cdot\begin{pmatrix}3 & -2\\ 5 & -4\end{pmatrix}=\begin{pmatrix}-1 & 2\\ -5 & 6\end{pmatrix}$

1.130 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot x = \begin{pmatrix} 3 & 0 \\ 7 & 2 \end{pmatrix}$

1.131 $x \cdot \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$

1.132 $\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} \cdot x = \begin{pmatrix} 2 & 11 \\ 2 & 6 \end{pmatrix}$

1.133 $x \cdot \begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 11 & -19 \end{pmatrix}$

1.134 $\begin{pmatrix} -3 & 1 \\ 2 & -1 \end{pmatrix} \cdot x = \begin{pmatrix} -8 & -5 \\ 5 & 3 \end{pmatrix}$

1.135 $x \cdot \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & -12 \\ -4 & 7 \end{pmatrix}$

1.136 $\begin{pmatrix} -2 & 2 \\ -1 & 3 \end{pmatrix} \cdot x = \begin{pmatrix} -2 & -10 \\ 1 & -7 \end{pmatrix}$

1.137 $x \cdot \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 11 & 16 \end{pmatrix}$

1.138 $\begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix} \cdot x = \begin{pmatrix} -3 & 2 \\ 3 & -1 \end{pmatrix}$

1.139 $x \cdot \begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -1 & -2 \end{pmatrix}$

1.140 $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \cdot x = \begin{pmatrix} 2 & 5 \\ 6 & 9 \end{pmatrix}$

Takrorlash uchun savollar

1. Matritsa deb nimaga aytiladi?
2. Matritsaning tartibi qanday aniqlanadi?
3. Matritsaning elementi deb nimaga aytiladi?
4. Matritsalar qanday turlarga ajratiladi?
5. Qachon ikkita matritsa teng deyiladi?
6. Matritsaning qanday elementi diagonal deyiladi?
7. Birlik matritsa qanday ta'riflanadi?
8. Qachon matritsa nol matritsa deyiladi?
9. Matritsani songa ko'paytirish qanday aniqlanadi?
10. Qaysi shartda matritsalarni qo'shish yoki ayirish mumkin?
11. Qaysi shartda matritsalarni ko'paytirish mumkin?
12. Determinant deyilganda nima tushuniladi?
13. II tartibli determinant qanday hisoblanadi?
14. III tartibli determinant uchburchak usulida qanday hisoblanadi?
15. III tartibli determinant Sarrius usulida qanday hisoblanadi?
16. Determinant va matritsa o'rtasida qanday o'xshashlik va farqlar bor?

17. Determinantda satr va ustunlar o'zaro qanday xususiyatga ega?

18. Determinantda ikkita satr yoki ikkita ustun

19. o'rni almashtirilsa nima bo'ladi?

20. Qaysi hollarda hisoblamasdan determinantning qiymati nol bo'lishini aytish mumkin?

MATRITSA VA ANIQLOVCHILARGA DOIR NAZORAT TESTLARI

1.Matritsalar va ular ustida amallar

1. Matritsa mazmuni qayerda to'g'ri ko'rsatilgan?

A) sonlar yig'indisi B) sonlar ko'paytmasi.
C) sonlar to'plami. D) sonlar jadvali.
E) sonlar birlashmasi.

2. $\begin{pmatrix} -1 & 0 & 5 \\ 2 & 7 & 3 \end{pmatrix}$ matritsaning tartibini aniqlang.

A) 2×2. B) 2×3. C) 3×2. D) 3×3. E) 3×2=6.

3. $A=\begin{pmatrix} -2 & 5 & 0 \\ 4 & 1 & -3 \\ 6 & -1 & 7 \end{pmatrix}$ matritsaning elementlari bo'yicha $a_{13}+a_{2}$ yig'indini toping.

A) 7. B) 4. C) 6. D) −5. E) 5.

4. Elementlari a_{ij} bo'lgan matritsa qachon nol matritsa deyiladi?

A) Barcha a_{ij} elementlarning yig'indisi nolga teng bo'lsa.
B) Barcha a_{ij} elementlari nolga teng bo'lsa.
C) Barcha a_{ij} elementlarning ko'paytmasi nolga teng bo'lsa.
D) Biror satridagi barcha a_{ij} elementlar nolga teng bo'lsa.
E) Biror ustundagi barcha a_{ij} elementlar nolga teng bo'lsa.

5. Quyidagi matritsalarning qaysi biri nol matritsa bo'lmaydi?

A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. B) $(0 \quad 0 \quad 0)$. C) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. D) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

E) Keltirilgan barcha matritsalar nol matritsa bo'ladi.

6. Elementlari a_{ij} bo'lgan kvadrat matritsa qachon birlik matritsa deyiladi?

A) Barcha a_{ij} elementlar birga teng bo'lsa.

B) $a_{ii}=1$ va $a_{ij}=0$ $(i\neq j)$ bo'lsa.

C) Barcha a_{ii} diagonalelementlar birga teng bo'lsa.

D) Biror satrdagi barcha a_{ij} elementlar birga teng bo'lsa.

E) Biror ustundagi barcha a_{ij} elementlar birga teng bo'lsa.

7. Birlik matritsani ko'rsating.

A) $\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$. C) $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. B) $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$.

D) $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. E) $\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$.

8. Birlik matritsani ko'rsating.

A) $\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$. B) $\begin{pmatrix}1 & 1 & 1\\ 0 & 0 & 0\end{pmatrix}$. C) $\begin{pmatrix}0 & 0 & 0\\ 1 & 1 & 1\end{pmatrix}$.

D) $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$. E) bu yerda birlik matritsa keltirilmagan

9. Qaysi shartda A va B matritsalar teng deyiladi?

A) A va B bir xil tartibli matritsalar bo'lsa.

B) A va B bir xil tartibli kvadrat matritsalar bo'lsa.

C) A va B bir xil tartibli kvadrat matritsalar va ularning mos elementlari o'zaro teng bo'lsa.

D) A va B matritsalarning diagonal elementlari o'zaro teng bo'lsa.

E) A va B matritsalarning tartiblari bir xil va mos elementlari teng bo'lsa.

10. $A=\begin{pmatrix}1 & 3 & 7\\ 5 & 2 & 9\end{pmatrix}$ matritsa bo'yicha 2A matritsani toping.

A) $\begin{pmatrix}2 & 6 & 14\\ 10 & 4 & 18\end{pmatrix}$. B) $\begin{pmatrix}1 & 3 & 7\\ 10 & 4 & 18\end{pmatrix}$. C) $\begin{pmatrix}2 & 3 & 7\\ 10 & 2 & 9\end{pmatrix}$

D) $\begin{pmatrix}1 & 6 & 7\\ 5 & 4 & 9\end{pmatrix}$ E) $\begin{pmatrix}2 & 3 & 14\\ 10 & 2 & 18\end{pmatrix}$.

2. Determinantlar va ularning xossalari

1. Quyidagi |A| determinantning a_{12} va a_{32} elementlari yig'indisini toping:

$$|A| = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & 5 \\ -2 & 6 & 3 \end{vmatrix}.$$

A) 5. B) 2. C) 7. D) – 6. E) 6.

2. Quyidagi |A| determinantning diagonal elementlari yig'indisini toping:

$$|A| = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 9 & 5 \\ -2 & 6 & -10 \end{vmatrix}.$$

A) 14. B) 0. C) 20. D) –6. E) 4.

3. $\begin{vmatrix} 5 & -2 \\ 3 & 4 \end{vmatrix}$ determinantni hisoblang.

A) 14. B) –26. C) 26. D) –14. E) 0.

4. $\begin{vmatrix} 3 & -3 \\ 2 & -2 \end{vmatrix}$ determinantni hisoblang.

A) 0. B) –12. C) 12. D) 2. E) 3.

5. $\begin{vmatrix} x-1 & 3 \\ 2 & 1 \end{vmatrix} = 0$ tenglamani yeching.

A) $x=7$. B) $x=-1$. C) $x=2$. D) $x=4$. E) $x=8$.

6. $\begin{vmatrix} 3 & x+1 \\ x & -2 \end{vmatrix} = 1$ tenglamani yeching.

A) $x_1=4$. $x_2=1$. B) $x_1=-2$. $x_2=3$. C) $x_1=1$. $x_2=-1$.
D) tenglama yechimga ega emas. E) tenglama cheksiz ko'p yechimga ega

7. Ushbu determinantni hisoblang:

$$\begin{vmatrix} 1 & 3 & 2 \\ 2 & 2 & 4 \\ 3 & 1 & 6 \end{vmatrix}$$

A) 1. B) 0. C) –2. D) 4. E) 12.

8. Ushbu determinantni hisoblang:

$$\begin{vmatrix} 3 & -5 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & -1 \end{vmatrix}$$

A) 0. B) 12. C) 10. D) –10. E) –12.

9. Quyidagi tenglamani yeching:

$$\begin{vmatrix} x & 1 & 2 \\ x & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = 0$$

A) x=1 . B) x=2.5 . C) x=0.5 . D) x=0 . E) x= –1 .

10. Ushbu tenglamani yeching:

$$\begin{vmatrix} x & 1 & -7 \\ 5 & 3 & 2 \\ 10 & 6 & 4 \end{vmatrix} = 0$$

A) x=0. B) x=1. C) x=2. D) $x \in \varnothing$.

E) tenglama cheksiz koʻp yechimga ega.

3. Teskari matritsa. Matritsa rangi

1. A va unga teskari A^{-1} matritsalar koʻpaytmasi uchun qaysi tasdiq oʻrinli?

A) AA^{-1} faqat 0 lardan iborat matritsa boʻladi.

B) AA^{-1} faqat 1 lardan iborat matritsa boʻladi.

C) AA^{-1} diagonal elementlari 0, qolgan barcha elementlari 1 boʻlgan matritsa boʻladi.

D) AA^{-1} diagonal elementlari 1, qolgan barcha elementlari 0 boʻlgan matritsa boʻladi.

E) AA^{-1} ixtiyoriy kvadrat matritsa boʻladi.

2. Qaysi shartda A matritsa maxsus deyiladi ?

A) $|A|<0$. B) $|A|>0$. C) $|A|\neq 0$. D) $|A|=0$. E) $|A|\leq 0$.

3. Qaysi shartda A matritsa maxsusmas deyiladi ?

A) $|A|<0$. B) $|A|>0$. C) $|A|\neq 0$. D) $|A|=0$. E) $|A|\leq 0$.

4. Quyidagi matritsalardan qaysi biri maxsusmas?

A) $\begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$. B) $\begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$. C) $\begin{pmatrix} 1 & 0 \\ 5 & 0 \end{pmatrix}$.

D) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$. E) $\begin{pmatrix} 0 & 7 \\ 0 & 3 \end{pmatrix}$.

5. Quyidagi matritsalardan qaysi biri maxsus?

A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. B) $\begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$. C) $\begin{pmatrix} 1 & 2 \\ 5 & 0 \end{pmatrix}$.

D) $\begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}$. E) $\begin{pmatrix} 0 & 7 \\ 3 & 0 \end{pmatrix}$.

6. A va unga teskari A^{-1} matritsalar uchun quyidagi tengliklardan qaysi biri o'rinli emas (E– birlik matritsa, O– nol matritsa)?

A) $A \cdot A^{-1} = A^{-1} \cdot A$. B) $A \cdot A^{-1} = E$. C) $A^{-1} \cdot A = E$.

D) $A A^{-1} - A^{-1} A = O$. E) $A - A^{-1} = O$.

7. Agar A kvadrat matritsaning determinanti Δ bo'lsa, qaysi shartda A^{-1} teskari matritsa mavjud bo'lmaydi?

A) $\Delta=0$. B) $\Delta<0$. C) $\Delta>0$. D) $\Delta\neq0$. E) $\Delta=1$.

8. Qanday matritsaga teskari matritsa mavjud?

A) har qanday matritsaga.

B) har qanday kvadrat matritsaga.

C) determinanti 0 ga teng bo'lgan matritsaga.

D) faqat determinanti musbat bo'lgan matritsaga.

E) determinanti 0 ga teng bo'lmagan kvadrat matritsaga.

9. Teskari matritsaga doir xossa qayerda xato ko'rsatilgan?

A) $(A^{-1})^{-1}=A$. B) $(A^{-1})^{T}=(A^{T})^{-1}$. C) $(AB)^{-1} = B^{-1} A^{-1}$.

D) $E^{-1}=E$. E) $(A+B)^{-1}=A^{-1}+ B^{-1}$.

10. $A=\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ matritsaga teskari A^{-1} matritsani toping.

A) $\begin{pmatrix} 1/2 & 1/3 \\ 1/4 & 1/6 \end{pmatrix}$. B) $\begin{pmatrix} 4 & 6 \\ 2 & 3 \end{pmatrix}$. C) $\begin{pmatrix} 6 & 4 \\ 3 & 2 \end{pmatrix}$. D) $\begin{pmatrix} 6 & -4 \\ -3 & 2 \end{pmatrix}$.

E) A^{-1} mavjud emas .

Hayotning ikkita bezagi bor :
matematika bilan shug'ullanish va o'qitish.
Puasson

II-BOB. CHIZIQLI TENGLAMALAR SISTEMALARI VA ULARNI YECHISH USULLARI

§ 2.1.Bir jinsli bo'lmagan chiziqli tenglamalar sistemalari va ularni yechish usullari

Chiziqli tenglamalar sistemasi umumiy ko'rinishdaquyidagicha yoziladi:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \quad (2.1)$$

bu yerda m – tenglamalar soni; n – no'malumlar soni; a_{ij} $(i = 1,2,\ldots m; \quad j = 1,2\ldots n)$ -koeffitsiyentlar; $b_1, b_2 \ldots b_m$ - erkin hadlar; $x_1, x_2, \ldots x_n$ - noma'lumlar.

Chiziqli tenglamalar sistemasi (2.1)-ni matritsa ko'rinishda quyidagicha yozish mumkin.

$A \cdot X = B$,

bu yerda $A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$,

Sistemaning asosiy matritsasi bo'lib $(m \times n)$o'lchovli,

$X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}$- noma'lum kattaliklar ustun matritsasi, $B = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_m \end{pmatrix}$- ozod hadlar ustun matritsasi.

Agar $m = n$ bo'lsa, ko'p holda $\Delta = |A| = \det A$ (2.1) sistemaning aniqlovchisi deyiladi.

1- rasm

Chiziqli tenglamalar sistemasini yechish usullari(1- rasm). Chiziqli tenglamalar sistemasi (2.1) Kramer, teskari matritsalar va Gauss usullari yordamida yechiladi. Kramer va matritsalar usuli sistemaning asosiy matritsasi maxsus bo`lmagan va kvadrat bo'lgan holda (chiziqli tenglamalar sistema uchun) qo`llaniladi. Gauss va Gauss-Jordan usullari esa noma'lumlarni ketma- ket yo'qotishga asoslangan.

Kramer formulasi. Agar (2.1) sistema uchun $n = m$ bo`lsa, ya`ni n noma`lumli$n$ ta chiziqli tenglamalar sistemasi bo`lsa,

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \quad (2.2)$$

bu yerda

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix} \neq 0, \quad \Delta_1 = \begin{vmatrix} b_1 & a_{12} & \ldots & a_{1n} \\ b_2 & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ b_n & a_{n2} & \ldots & a_{nn} \end{vmatrix}, \ldots, \Delta_n = \begin{vmatrix} a_{11} & a_{12} & \ldots & b_{1n} \\ a_{21} & a_{22} & \ldots & b_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & b_n \end{vmatrix}$$

Sistemaning yechimi:

$$x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta}, \ldots, x_n = \frac{\Delta_n}{\Delta}, \quad (2.3)$$

Ko'rinishda yoziladi va bu holda sistema yagona yechimga ega bo`lishi uchun $\Delta \neq 0$ bo'lishi zarur va yetarli.

Agar Δ aniqlovchining qandaydir ikkita satr elementlari bir-biriga proportsional bo`lsa, $\left(\frac{a_{11}}{a_{21}}=\frac{a_{12}}{a_{22}}=...=\frac{a_{1n}}{a_{2n}}=p\right)$ $p\in R$ , u holda $\Delta=0$ bo`ladi. Bu holda Kramer formulasini qo`llab bo`lmaydi.

Misol: Ushbu uch noma'lumli chiziqli tenglamalar sistemasini Kramer usulida yeching:

$$\begin{cases} x_1+2x_2+3x_3=1 \\ 2x_1+3x_2+x_3=0 \\ 2x_1+x_2-2x_3=0 \end{cases}$$

Yechish: Asosiy va yordamchi determinantlarni hosil etamiz va hisoblaymiz:

$$\Delta=\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 2 & 1 & -1 \end{vmatrix}=18, \qquad \Delta_1=\begin{vmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 1 & -2 \end{vmatrix}=-5,$$

$$\Delta_2=\begin{vmatrix} 1 & 1 & 3 \\ 2 & 0 & 1 \\ 2 & 0 & -2 \end{vmatrix}=-1, \qquad \Delta_3=\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 2 & 1 & 0 \end{vmatrix}=7.$$

Kramer formulalariga asosan sistema yechimini topamiz:

$$x_1=\frac{\Delta_1}{\Delta}=-\frac{5}{18}, \quad x_{21}=\frac{\Delta_2}{\Delta}=-\frac{1}{18}, \quad x_3=\frac{\Delta_{31}}{\Delta}=\frac{7}{18}.$$

Teskari matritsa usuli. n - noma`lumli tenglamalar sistemasini quyidagi ko`rinshda yozamiz. $A\cdot X=B$

bu yerda

$$A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ \_\_\_ & \_\_\_ & \_\_\_ \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}, \quad \det A\neq 0. \quad B=\begin{pmatrix} b_1 \\ b_2 \\ b_n \end{pmatrix}, \qquad X=\begin{pmatrix} x_1 \\ x_2 \\ x_n \end{pmatrix}$$

Agar A - matritsa maxsus matritsa bo`lmasa, u holda yechim $X=A^{-1}B$ ko'rinishida bo'ladi.

Misol: Ushbu tenglamalar sistemasini matritsa usulida yeching:

$$\begin{cases} 2x_1-3x_2+4x_3=20 \\ 3x_1+4x_2-2x_3=-11 \\ 4x_1+2x_2+3x_3=9 \end{cases}$$

Yechish: Dastlab sistemaning A matritsasini yozib, uning determinantini hisoblaymiz:

$$A=\begin{pmatrix}2&-3&4\\3&4&-2\\4&2&3\end{pmatrix},\quad \Delta=|A|=\begin{vmatrix}2&-3&4\\3&4&-2\\4&2&3\end{vmatrix}=43\neq 0.$$

Demak A matritsa maxsusmas, unga teskari matritsa mavjud va uni §1.3 dagi formulaga asosan topamiz:

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix}A_{11}&A_{21}&A_{31}\\A_{12}&A_{22}&A_{32}\\A_{13}&A_{23}&A_{33}\end{pmatrix}=\frac{1}{43}\begin{pmatrix}16&17&-10\\-17&-10&16\\10&-16&17\end{pmatrix}$$

Endi

$$X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=A^{-1}B=\frac{1}{43}\begin{pmatrix}16&17&-10\\-17&-10&16\\10&-16&17\end{pmatrix}\begin{pmatrix}20\\-11\\9\end{pmatrix}=\frac{1}{43}\begin{pmatrix}43\\-86\\129\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}$$

ekanini aniqlaymiz

Demak, sistemaning yagona yechimi $x_1=1$, $x_2=-2$, $x_3=3$ bo'ladi.

Shunday qilib matritsalar usuli har qanday n noma'lumli n ta tenglamali aniq sistema yechimini oddiy va ixcham ko'rinishdagi

$$AX=B\Rightarrow A^{-1}AX=A^{-1}B\Rightarrow EX=A^{-1}B\Rightarrow X=A^{-1}B$$

formula bilan ifodalash imkonini beradi. Bu formula nazariy tadqiqotlar uchun qulaydir, ammo n oshib borishi bilan uning amaliy tatbig'i murakkablashib boradi. Bunga sabab shuki, bu holda A^{-1}teskari matritsani topish uchun yuqori tartibli determinantlarni hisoblashga to'g'ri keladi.

Gauss usuli. Yuqorida keltirilgan (2.1) n-noma'lumli m-tenglamalar sistemasi berilgan bo'lsa, uning koeffitsientlari qatoriga ozod haldlar ustunni ham kiritamiz. Olingan matritsaga kengaytirilgan matritsa deyiladi:

$$B = \left(\begin{array}{cccc|c} x_1 & x_2 & \dots & x_n & \\ \hline a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array}\right)$$

Kengaytirilgan B matritsa ustida bajariladigan elementar almashtirishlar tenglamalar sistemasi yechimlar tuzimiga ta`sir etmaydi. Elementar almashtirishlar bajarilganda "~" belgi qo'yiladi.

Faraz qilaylik, chiziqli algebraik tenglamalar sistemasi (2.1) n noma`lumli $m$ ta tenglamadan iboart bo`lsasin. Tenglamalar sistemasi hech bo`lmaganda bitta yechimga ega bo`lishi uchun sistema asosiy matritsa (A) ning rangi kengaytirilgan matritsa (B) ning rangiga teng bo`lishi zarur va yetarlidir (Kronkera -Kapelli teoremasi).

(2.1) sistema yagona yechimga ega bo`lishi uchun $r = n$ bo'lishi kerak. Agar $r < n$ bo`lsa, u holda sistema cheksiz ko`p yechimga ega bo`ladi.

Misol: Ushbu sistemani Gauss usulida yeching:

$$\begin{cases} 2x_1 - 3x_2 + 4x_3 = 20 \\ 3x_1 + 4x_2 - 2x_3 = -11 \\ 4x_1 + 2x_2 + 3x_3 = 9 \end{cases}$$

Yechish: Bu sistemadan noma'lumlarni birin-ketin yo'qotamiz.

1-qadam. Sistemaning ikkinchi va uchinchi tenglamalardan x_1 noma'lumni yo'qotamiz. Kasr sonlarga kelmaslik va bu orqali hisoblashlarni soddalashtirish maqsadida buni quyidagicha amalga oshiramiz. Dastlab 1-tenglamani ikkala tomonini –3 soniga, 2-tenglamani esa 2 soniga ko'paytirib, ularni o'zaro qo'shamiz. So'ngra 1-tenglamani ikkala tomonini –2 soniga ko'paytirib, hosil bo'lgan tenglamani 3-tenglamaga qo'shamiz. Natijada quyidagi ekvivalent sistemaga kelamiz:

$$\begin{cases} 2x_1 - 3x_2 + 4x_3 = 20 \\ 17x_2 - 16x_3 = -82 \\ 8x_2 - 5x_3 = -31 \end{cases}$$

2-qadam. Oldingi qadamda hosil qilingan sistemaning 2-tenglamasini –8 soniga, 3-tenglamasini 17 soniga ko'paytirib o'zaro qo'shamiz:

$$\begin{cases} 2x_1 - 3x_2 + 4x_3 = 20 \\ 17x_2 - 16x_3 = -82 \\ 43x_3 = 129 \end{cases}$$

Dastlab bu uchburchakli sistemaning 3- tenglamasidan $x_3=3$ ekanligini topamiz.

So'ngra bu natijani sistemaning 2- tenglamasiga qo'yib, undan $x_2= -2$ ekanligini aniqlaymiz. Yakuniy qadamda $x_2= -2$ va $x_3= 3$ natijalarni sistemaning 1- tenglamasiga qo'yib, undan $x_1=1$ ekanligini topamiz. Demak berilgan sistemaning yagona yechimi $x_1=1$, $x_2= -2$ va $x_3=3$ ekan.

Quyidagi 2 noma'lumli ikkita tenglamadan tashkil topgan sistemani Kramer va matrisalar usulida yeching.

2.1 $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ **2.2** $\begin{cases} 2x + 5y = 153 \\ x - 2y = 3 \end{cases}$ **2.3** $\begin{cases} 3x + 5y = 21 \\ 2x - y = 1 \end{cases}$

2.4 $\begin{cases} \frac{1}{2}x - \frac{1}{3}y = 1 \\ 3x - 5y = -3 \end{cases}$ **2.5** $\begin{cases} 2x - 3y = -1 \\ y = 0{,}75x \end{cases}$ **2.6** $\begin{cases} \frac{1}{4}x - y = -5 \\ \frac{1}{2}x - \frac{1}{7}y = 3 \end{cases}$

2.7. $\begin{cases} 4x - 3y = -4 \\ 4y - 10x = 3 \end{cases}$ **2.8** $\begin{cases} 3x - 2y = 1/2 \\ 4y - x = 2/3 \end{cases}$ **2.9** $\begin{cases} 11x - 5y = 37 \\ 4y - x = 25 \end{cases}$

2.10 $\begin{cases} 3y - x = -17 \\ 5x + 3y = -5 \end{cases}$ **2.11** $\begin{cases} 2x_1 + 3x_2 + 1 = 0 \\ 5x_1 + 4x_2 - 1 = 0 \end{cases}$ **2.12** $\begin{cases} 2x_1 - 3x_2 - 5 = 0 \\ 4x_1 - 5x_2 - 7 = 0 \end{cases}$

2.13 $\begin{cases} 4x - 6y + 7 = 0 \\ x + 3y - 5 = 0 \end{cases}$ **2.14** $\begin{cases} 2x_1 - 3x_2 = 22 \\ -3x_1 - 4x_2 = 1 \end{cases}$ **2.15** $\begin{cases} 3x + 2y = 1/6 \\ 9x + 6y = 1/2 \end{cases}$

Quyidagi 2 noma'lumli ikkita tenglamadan tashkil topgan sistemani Kramer va matrisalar usulida yeching.

2.16 $\begin{cases} ax - by = a^2 + b^2 \\ bx + ay = a^2 + b^2 \end{cases}$

2.17 $\begin{cases} x\cos\alpha - y\sin\alpha = \cos 2\alpha \\ x\sin\alpha + y\cos\alpha = \sin 2\alpha \end{cases}$

2.18 $\begin{cases} (a+b)x - (a-b)y = 4ab \\ (a-b)x + (a+b)y = 2(a^2 - b^2) \end{cases}$

2.19 $\begin{cases} 2x + 5y = 15 \\ 3x + 8y = -3 \end{cases}$

"*a*" ning qanday qiymatida tenglamalar sistemasi yechimga ega bo'lmaydi.

2.20. $\begin{cases} x + ay = 1 \\ x - 3ay = 2a + 3 \end{cases}$

2.21 $\begin{cases} -4x + ay = 1 + a \\ (6+a)x + 2y = 3 + a \end{cases}$

2.22 $\begin{cases} 16x + ay = 4 \\ ax + 9y - 3 = 0 \end{cases}$

2.23 $\begin{cases} x + ay = 1 \\ ax + y = 2a \end{cases}$

2.24 $\begin{cases} (a+1)x - 3y - 4 = 0 \\ 2x - ay - 3 = 0 \end{cases}$

"*a*" ning qanday qiymatida sistema cheksiz ko'p yechimga ega bo'ladi.

2.25 $\begin{cases} (a+1)x + 8y = 4a \\ ax + (a+3)y = 3a - 1 \end{cases}$

2.26 $\begin{cases} x + ay = 1 \\ ax - 3ay = 2a + 3 \end{cases}$

2.27 $\begin{cases} 3x + ay = 3 \\ ax + 3y = 3 \end{cases}$

2.28 $\begin{cases} ax - (a+1)y = 6 \\ 7ax - 28y = 6(a+4) \end{cases}$

2.29 $\begin{cases} (a+1)x - y = a + 2 \\ x + (a-1)y = 2 \end{cases}$

2.30 $\begin{cases} 2x + y - 5z = 3 \\ 3x - 5y + 2z = 1 \\ 5x - 6y + 3z = 6 \end{cases}$

2.31 $\begin{cases} 4x + 3y + 2z + 3 = 0 \\ 7x + 9y - 9z + 8 = 0 \\ 2x - 5y + 6z + 3 = 0 \end{cases}$

2.32 $\begin{cases} 5x + 2y - 3z + 3 = 0 \\ 8x - 3y + 2z + 7 = 0 \\ 2x + 3y - 5z - 4 = 0 \end{cases}$

2.33 $\begin{cases} 7x + 2y - 8z - 1 = 0 \\ 5x - 3y + 13z - 14 = 0 \\ x + 2y - 9z + 5 = 0 \end{cases}$

2.34 $\begin{cases} x + 2x_3 = 4 \\ 3x_1 + 2x_2 + 4x_3 = 19 \\ 2x_1 + 5x_2 + x_3 = -5 \end{cases}$

Quyidagi uch noma'lumli uchta tenglamadan tashkil topgan sistemani Kramer, Gauss va matrisa usulida yeching.

2.35 *a*) $\begin{cases} x_1 + x_2 + x_3 = 3 \\ 2x_1 - x_2 + x_3 = 2 \\ -3x_1 + 2x_2 + x_3 = 0 \end{cases}$ *в*) $\begin{cases} 3x_1 + x_2 + x_3 - 4 = 0 \\ x_1 + 2x_2 - x_3 - 4 = 0 \\ 2x_1 + x_2 + 2x_3 - 16 = 0 \end{cases}$

2.36 *a*) $\begin{cases} x_1 + x_2 + 4x_3 = 1 \\ 2x_1 + x_2 + 6x_3 = 2 \\ 3x_1 + 3x_2 + 13x_3 = 2 \end{cases}$ *в*) $\begin{cases} x_1 + 2x_2 - x_3 = 7 \\ 2x_1 - x_2 + x_3 = 2 \\ 3x_1 + 5x_2 + 2x_3 = -7 \end{cases}$

2.37 *a*) $\begin{cases} 2x_1 + x_2 - x_3 + 3x_4 = 20 \\ 5x_1 - x_2 + 2x_3 - x_4 = 17 \\ -3x_1 + 2x_2 - x_3 + 2x_4 = 1 \\ x_1 - x_2 + 4x_3 - 2x_4 = -4 \end{cases}$ *в*) $\begin{cases} x_1 + 2x_2 + 3x_3 - 13 = 0 \\ 3x_1 + 2x_2 + 2x_3 - 16 = 0 \\ 4x_1 - 2x_2 + 5x_3 - 5 = 0 \end{cases}$

2.38 $\begin{cases} x_1 - 2x_2 + 4x_3 = 6 \\ 2x_1 - x_2 + 3x_3 = 11 \\ 4x_1 + x_2 - 5x_3 = 9 \end{cases}$ **2.39** $\begin{cases} 2x_1 - 3x_2 + x_3 = 2 \\ 2x_1 + x_2 - 4x_3 = 9 \\ 6x_1 - 5x_2 + 2x_3 = 17 \end{cases}$

2.40 $\begin{cases} x_1 + 2x_2 - x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 13 \\ 3x_1 + 2x_2 - 5x_3 = -1 \end{cases}$ **2.41** $\begin{cases} 2x_1 + x_2 - 3x_3 = -1 \\ x_1 - 3x_2 + 2x_3 = 10 \\ 3x_1 - 4x_2 - x_3 = 5 \end{cases}$

2.42 $\begin{cases} 2x_1 + x_2 - x_3 = 6 \\ 3x_1 - x_2 + 2x_3 = 5 \\ 4x_1 + 2x_2 - 5x_3 = 9 \end{cases}$ **2.43** $\begin{cases} 6x_1 + 2x_2 - x_3 = 2 \\ 4x_1 - x_2 + 3x_3 = -3 \\ 3x_1 + 2x_2 - 2x_3 = 3 \end{cases}$

2.44. $\begin{cases} 2x_1 + x_2 + 3x_3 = 13 \\ x_1 + x_2 + x_3 = 6 \\ 3x_1 + x_2 + x_3 = 8 \end{cases}$ **2.45.** $\begin{cases} 2x_1 + x_2 + x_3 = 7 \\ x_1 + 2x_2 + x_3 = 8 \\ x_1 + x_2 + 2x_3 = 9 \end{cases}$ **2.46.** $\begin{cases} x_1 + 2x_2 + 3x_3 = 3 \\ 3x_1 + x_2 + 2x_3 = 7 \\ 2x_1 + 3x_2 + x_3 = 3 \end{cases}$

Chiziqli tenglamalar sistemasini yeching

2.47. *a*) $\begin{cases} x_1 + 2x_2 - 3x_3 - 2x_4 = 1 \\ -2x_1 - 3x_2 + x_3 + 3x_4 = 3 \\ 5x_1 + 9x_2 - 10x_3 - 9x_4 = 0 \end{cases}$ *в*) $\begin{cases} -2x_1 + 3x_2 - 2x_3 = 2 \\ 3x_1 - 2x_2 + x_3 = 2 \\ -5x_1 + 10x_2 - 7x_3 = 10 \end{cases}$

2.48 $\begin{cases} 2x_1 - x_2 - x_3 = 7 \\ -4x_1 + 2x_2 = -2 \\ 6x_1 - 3x_2 + x_3 = -3 \end{cases}$

2.49 $\begin{cases} x_1 + 2x_2 = 2 \\ 2x_1 + x_2 + 3x_3 = 1 \\ 2x_1 + 7x_2 - 3x_3 = 31 \end{cases}$

2.50 $\begin{cases} 5x_1 + 3x_2 + 5x_3 + 12x_4 = 10 \\ 2x_1 + 2x_2 + 3x_3 + 5x_4 = 4 \\ x_1 + 7x_2 + 9x_3 + 4x_4 = 2 \end{cases}$

2.51 $\begin{cases} x_1 + 2x_2 - 2x_3 + 5x_4 = 3 \\ -3x_1 - 2x_2 + 12x_3 - 7x_4 = -5 \\ x_2 - 3x_3 + 4x_4 = 2 \end{cases}$

§ 2.2. Bir jinsli chiziqli tenglamalar sistemalari

Bir jinsli chiziqli tenglamalar sistemalari

Agar(2.1) tenglamalar sistemasida $b_1 = b_2 = ... = b_m = 0$ bo'lsa, u holda har doim nol (triviol) yechimga ega bo'ladi.

$$x_1 = 0, \quad x_2 = 0 \quad ... \quad x_n = 0.$$

Har qanday bir jinsli tenglama uchun $r(A) = r(B)$ shart bajariladi.

Agar bir jinsli sistemada $r(A) = r(B) = n$ bo'lsa, u holda sistema faqat nol yechimga ega bo'ladi. Agar bir jinsli sistema $r(A) = r(B) < n$, u holda sistema noldan farqli yechimga ega.

Uch noma`lumli ikkita bir jinsli tenglamalardan tashkil topgan sistemani ko`ramiz.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0 \end{cases} \qquad (2.4)$$

Bu (2.4) sistema birgalikda bo`lib, nol yechimga ega $x_1 = x_2 = x_3 = 0$.Sistemani noldan farqli yechimi topish talb etiladi. Faraz qilaylik, $x_3 \neq 0$ bo'lsin. U holda

$$\begin{cases} a_{11}\dfrac{x_1}{x_3} + a_{12}\dfrac{x_2}{x_3} = -a_{13} \\ a_{21}\dfrac{x_1}{x_3} + a_{22}\dfrac{x_2}{x_3} = -a_{23} \end{cases} \qquad (2.5)$$

Bu (2.5) sistemani asosiy aniqlovchisi $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0.$

$$\frac{x_1}{x_3}=\frac{1}{D}\begin{vmatrix}-a_{13} & a_{12}\\ -a_{23} & a_{22}\end{vmatrix}=\frac{1}{D}\begin{vmatrix}a_{12} & a_{13}\\ a_{22} & a_{23}\end{vmatrix}$$
$$\frac{x_2}{x_3}=\frac{1}{D}\begin{vmatrix}a_{11} & -a_{13}\\ a_{21} & -a_{23}\end{vmatrix}=-\frac{1}{D}\begin{vmatrix}a_{11} & a_{13}\\ a_{21} & a_{23}\end{vmatrix} \qquad (2.6)$$

Bir jinsli tenglamalar sistemasi (2.4) uchun kengaytirilgan matritsani ko'ramiz:

$$\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{vmatrix} \qquad (2.7)$$

Ikkinchi tartibli aniqlovchilar (2.7) matritsani qator va ustunlarini o'chirish bilan topiladi.

$$D_1=\begin{vmatrix}a_{11} & a_{13}\\ a_{21} & a_{22}\end{vmatrix},\quad D_2=\begin{vmatrix}a_{11} & a_{13}\\ a_{21} & a_{23}\end{vmatrix},\quad D_3=\begin{vmatrix}a_{11} & a_{21}\\ a_{21} & a_{22}\end{vmatrix}=D$$

Bu belgilashlardan va (2.6) foydalanib quyidagi tenglamani olamiz.

$$\frac{x_1}{x_3}=\frac{D_1}{D_3},\quad \frac{x_2}{x_3}=\frac{D_2}{D_3}$$

Bu yerdan

$$\frac{x_1}{D_1}=\frac{x_2}{-D_2}=\frac{x_3}{D_3} \qquad (2.8)$$

tenglamani olamiz.

Agar proportsionallik koeffitsientini t bilan belgilasak, u holda (2.8) dan foydalanib, (2.6) sistema yechimini quyidagicha yozamiz

$$x_1=D_1t;\quad x_2=-D_2t;\quad x_3=D_3t \quad (-\infty<t<\infty)$$

Chiziqli tenglamalar sistemasining iqtisodiy tatbiqlari.

Chiziqli tenglamalar sistemasi iqtisodiy masalalarni yechishda juda keng qo'llanishini aytib o'tgan edik. Masalan, xom ashyolardan foydalanishni eng katta foyda beradigan yo'lini topish, transportda yuk tashishni tashkil etishda eng kam xarajatga erishish, chorvachilikda mollar ozuqasi ratsionini oqilona tuzish va hokazo. Bunday masalalarni o'rganish va yechish natijasida "Chiziqli dasturlash" deb ataladigan matematikaning yangi bir yo'nalishi yaratildi va unda chiziqli tenglamalar sistemasi ko'p ishlatiladi. Bunga misol sifatida ikkita masalani ko'ramiz.

Masala. Qandolat firmasida holva, pecheniye va vafli ishlab chiqariladi. Bu mahsulotlarni tayyorlash uchun xom ashyo sifatida un, shakar va margarin yog'i ishlatiladi. Bir birlik mahsulot ishlab chiqarish uchun sarflanadigan xom ashyo normasi va sex omboridagi xom ashyolarning zaxirasi bo'yicha ma'lumotlar quyidagi jadvalda berilgan:

Xom-ashyo turi	Bir birlik konditer mahsulotini tayyorlash uchun sarflanadigan xom ashyo normasi			Ombordagi xom ashyolar zaxirasi
	Holva	Pecheniye	Vafli	
Un	2	5	2	2350
Shakar	7	3	5	2750
Margarin	1	2	3	1400

Bu ma'lumotlar asosida xom ashyodan to'liq foydalanish maqsadida har bir konditer mahsulotining ishlab chiqarish hajmini toping.

Yechish: Holva, pecheniye va vaflining ishlab chiqarish hajmlarini mos ravishda x_1, x_2 va x_3 deb belgilaymiz. Jadvaldagi ma'lumotlar asosida uchala mahsulotni ishlab chiqarish uchun sarflanadigan har bir xom-ashyo miqdorini topamiz va uni ombordagi zaxirasi bilan tenglashtiramiz. Natijada quyidagi chiziqli tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} 2x_1 + 5x_2 + 2x_3 = 2350 \\ 7x_1 + 3x_2 + 5x_3 = 2750 \\ x_1 + 2x_2 + 3x_3 = 1400 \end{cases}$$

Bu chiziqli tenglamalar sistemasini yuqorida ko'rib o'tilgan usullardan biri yordamida yechib, $x_1=100$, $x_2=350$ va $x_3=200$ ekanligini topamiz. Demak, qandolat firmasida 200 birlik holva, 350 birlik pecheniye va 200 birlik vafli ishlab chiqarilsa xom ashyo to'liq sarflanadi.

Bir jinsli tenglamalar sistemasini yeching. $(-\infty < t < \infty)$

2.52 $\begin{cases} x_1 - 2x_2 + 3x_3 = 0 \\ 4x_1 + 5x_2 - 6x_3 = 0 \end{cases}$ **2.53** $\begin{cases} 3x_1 + 2x_2 + 2x_3 = 0 \\ 5x_1 + 2x_2 + 3x_3 = 0 \end{cases}$

2.54 $\begin{cases} 2x_1 - 5x_2 + 2x_3 = 0 \\ x_1 + 4x_2 - 3x_3 = 0 \end{cases}$ **2.55** $\begin{cases} x_1 + 2x_2 - 3x_3 = 0 \\ 2x_1 + 3x_2 + x_3 = 0 \end{cases}$

2.56 $\begin{cases} x_1 - 3x_2 + 5x_3 = 0 \\ 7x_1 - 9x_2 - 11x_3 = 0 \end{cases}$ **2.57** $\begin{cases} 3x_1 + 4x_2 + 5x_3 = 0 \\ x_1 + 2x_2 - 3x_3 = 0 \end{cases}$

2.58 $\begin{cases} 5x_1 + 3x_2 + 4x_3 = 0 \\ 6x_1 + 5x_2 + 6x_3 = 0 \end{cases}$ **2.59** $\begin{cases} 4x_1 - 6x_2 + 5x_3 = 0 \\ 6x_1 - 9x_2 + 10x_3 = 0 \end{cases}$

2.60 $\begin{cases} x_1 + x_2 - 7x_3 = 0 \\ x_1 - 6x_2 + x_3 = 0 \end{cases}$ **2.61** $\begin{cases} x_1 - 6x_2 + x_3 = 0 \\ 5x_1 - x_2 - x_3 = 0 \end{cases}$

2.62 $\begin{cases} x_1 + x_2 - 7x_3 = 0 \\ 5x_1 - x_2 - x_3 = 0 \end{cases}$ **2.63** $\begin{cases} x_1 - x_2 - x_3 = 0 \\ x_1 + 4x_2 + 2x_3 = 0 \end{cases}$

2.64 $\begin{cases} x_1 + 4x_2 + 2x_3 = 0 \\ 3x_1 + 7x_2 + 3x_3 = 0 \end{cases}$ **2.65** $\begin{cases} x_1 - x_2 - x_3 = 0 \\ 3x_1 + 7x_2 + 3x_3 = 0 \end{cases}$ **2.66** $\begin{cases} x_1 - x_2 - x_3 = 0 \\ x_1 + x_2 - 7x_3 = 0 \end{cases}$

Bir jinsli tenglamalar sistemasini yeching $(-\infty < t < \infty)$.

2.67 $\begin{cases} x_1 + x_2 - 7x_3 = 0 \\ x_1 - 6x_2 + x_3 = 0 \\ 5x_1 - x_2 - x_3 = 0 \end{cases}$ **2.68** $\begin{cases} x_1 - x_2 - x_3 = 0 \\ x_1 + 4x_2 + 2x_3 = 0 \\ 3x_1 + 7x_2 + 3x_3 = 0 \end{cases}$

2.69 $\begin{cases} 2x_1 - x_2 - 3x_3 + x_4 = 0 \\ x_1 + 3x_2 + 2x_3 - 2x_4 = 0 \\ 3x_1 + 2x_2 + 2x_3 + 3x_4 = 0 \end{cases}$ **2.70** $\begin{cases} -5x_1 + x_2 + x_3 = 0 \\ x_1 - 6x_2 + x_3 = 0 \\ x_1 + x_2 - 7x_3 = 0 \end{cases}$

2.71 $\begin{cases} x_1 + x_2 + x_3 = 0 \\ 3x_1 + 6x_2 + 5x_3 = 0 \\ x_1 + 4x_2 + 3x_3 = 0 \end{cases}$

2.72 $\begin{cases} ax_1 + bx_2 + (a+b)x_3 = 0 \\ bx_1 + ax_2 + (a+b)x_3 = 0 \\ x_1 + x_2 + 2x_3 = 0 \end{cases}$

2.73 $\begin{cases} x_1 + 4x_2 + 2x_3 + x_4 = 0 \\ 2x_1 - x_2 - 3x_3 + x_4 = 0 \\ 3x_1 + 2x_2 + 2x_3 + 3x_4 = 0 \end{cases}$

2.74 $\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 0 \\ 2x_1 + x_2 + x_3 - x_4 = 0 \\ x_1 + 2x_2 - x_3 + x_4 = 0 \end{cases}$

2.75 $\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 0 \\ x_1 + x_2 - x_3 + 2x_4 = 0 \\ x_1 - x_2 - x_3 + x_4 = 0 \end{cases}$

2.76 $\begin{cases} x_1 - x_2 - 3x_3 + x_4 = 0 \\ x_1 + 3x_2 + 4x_3 - x_4 = 0 \\ 3x_1 - x_2 - 3x_3 + x_4 = 0 \end{cases}$

2.77 $\begin{cases} 4x_1 + x_2 - x_3 + 2x_4 = 0 \\ x_1 + 3x_2 - x_3 + 2x_4 = 0 \\ 2x_1 + 2x_2 - x_3 - x_4 = 0 \end{cases}$

2.78 $\begin{cases} 3x_1 + x_2 - 3x_3 + x_4 = 0 \\ x_1 + 3x_2 + x_3 - 3x_4 = 0 \\ x_1 + x_2 - x_3 - x_4 = 0 \end{cases}$

2.79 $\begin{cases} x_1 + x_2 - 3x_3 + 2x_4 = 0 \\ 2x_1 - 2x_2 - 2x_3 + 3x_4 = 0 \\ x_1 + x_2 + x_3 - x_4 = 0 \end{cases}$

2.80 $\begin{cases} 2x_1 - 2x_2 + 3x_3 - 2x_4 = 0 \\ x_1 + x_2 - 2x_3 + 4x_4 = 0 \\ 3x_1 - x_2 + 2x_3 + x_4 = 0 \end{cases}$

Takrorlash uchun savollar

1. Chiziqli tenglamalar sistemasi qanday ko'rinishda bo'ladi?
2. Sistemaning koeffitsiyentlari, noma'lumlari va ozod hadlari deb nimaga aytiladi?
3. Sistemaning yechimlari qanday ta'riflanadi?
4. Qachon sistema birgalikda yoki birgalikda emas deyiladi?
5. Qachon sistema aniq va qachon aniqmas deyiladi?
6. Qaysi shartda chiziqli tenglamalar sistemasi yagona yechimga ega?
7. Qaysi shartda chiziqli tenglamalar sistemasi cheksiz ko'p yechimga ega?
8. Chiziqli tenglamalar sistemasi matritsa ko'rinishda qanday yoziladi?
9. Sistema matritsa usulida qanday yechiladi?
10. Matritsalar usulining qanday qulayliklari va kamchiliklari bor?
11. Sistemani Kramer usulida yechishning mohiyati nimadan iborat?

12. Sistemaning asosiy determinanti deb nimaga aytiladi?

13. Sistemaning yordamchi determinantlari qanday hosil qilinadi?

14. Sistema yechimi uchun Kramer formulalari qanday ko'rinishda bo'ladi?

15. Qachon tenglamalar sistemasi ekvivalent deyiladi?

16. Gauss usulining mohiyati nimadan iborat?

CHIZIQLI TENGLAMALAR SISTEMASIGA DOIR NAZORAT TESTLARI

1. Chiziqli tenglamalar sistemasi

1. Quyidagi sistemalardan qaysi biri chiziqli tenglamalar sistemasini ifodalaydi?

A) $\begin{cases} a_{11}x_1^2 + a_{12}x_2^2 = b_1 \\ a_{21}x_1^2 + a_{22}x_2^2 = b_2 \end{cases}$. B) $\begin{cases} \frac{a_{11}}{x_1} + \frac{a_{12}}{x_2} = b_1 \\ \frac{a_{21}}{x_1} + \frac{a_{22}}{x_2} = b_2 \end{cases}$.

C) $\begin{cases} a_{11}x_1^2 + a_{12}x_2^2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$.

D) $\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1^2 + a_{22}x_2^2 = b_2 \end{cases}$. E) $\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$.

2. Ushbu chiziqli tenglamalar sistemasi koeffitsientlarining yig'indisini toping:

$$\begin{cases} 3x_1 - 2x_2 = 5 \\ 2x_1 - 3x_2 = 0 \end{cases}$$

A) 10. B) 0. C) 5. D) 15.
E) to'gri javob keltirilmagan.

3. Ushbu chiziqli tenglamalar sistemasi ozod hadlarining ko'paytmasini toping:

$$\begin{cases} 4x_1 + 7x_2 = 6 \\ 3x_1 - 5x_2 = 3 \end{cases}$$

A) 28. B) –15. C) 18. D) 12. E) –35.

4. *Ta'rifni to'ldiring:* α, β va γ sonlari uch noma'lumli chiziqli tenglamalar sistemasining yechimi deyiladi, agarda ular sistemaning ...... tenglamasini ayniyatga aylantirsa.

A) birinchi . B) ikkinchi. C) birorta.

D) kamida bitta . E) uchala.

5. Qachon chiziqli tenglamalar sistemasi birgalikda deb ataladi?

A) yechimga ega bo'lmasa.

B) kamida bitta yechimga ega bo'lsa.

C) yagona yechimga ega bo'lsa.

D) cheksiz ko'p yechimga ega bo'lsa.

E) keltirilgan barcha hollarda.

6. Qachon chiziqli tenglamalar sistemasi birgalikda emas deb ataladi?

A) yechimga ega bo'lmasa.

B) kamida bitta yechimga ega bo'lsa.

C) yagona yechimga ega bo'lsa. D) cheksiz ko'p yechimga ega bo'lsa.

E) to'gri javob keltirilmagan.

7. Qachon chiziqli tenglamalar sistemasi aniq deb ataladi?

A) yechimga ega bo'lmasa. B) kamida bitta yechimga ega bo'lsa.

C) yagona yechimga ega bo'lsa. D) cheksiz ko'p yechimga ega bo'lsa.

E) keltirilgan barcha hollarda.

8. Qachon chiziqli tenglamalar sistemasi aniqmas deb ataladi?

A) yechimga ega bo'lmasa. B) yechim ega bo'lsa.

C) yagona yechimga ega bo'lsa. D) cheksiz ko'p yechimga ega bo'lsa.

E) keltirilgan barcha hollarda.

9. ***Kroniker-Kapelli teoremasi shartini ko'rsating:*** n noma'lumli chiziqli tenglamalar sistemasi $AX=B$ birgalikda

bo'lishi uchun uning matritsasi A va kengaytirilgan A^B matritsa ranglari r(A) va r(A^B) …….. shartni qanoatlantirishi zarur va yetarlidir.

A) r(A) > r(A^B). B) r(A) < r(A^B). C) r(A) = r(A^B).

D) r(A) ≠ r(A^B). E) r(A) = r(A^B)=n.

10. Qaysi shartda uch noma'lumli uchta chiziqli tenglamalar sistemasi $AX=B$ birgalikda bo'lmaydi?

A) r(A) = r(A^B). B) r(A) =3, r(A^B)=3 . C) r(A) =2, r(A^B)=3

D) r(A) =2, r(A^B)=2 . E) r(A) =1, r(A^B)=1 .

2.Chiziqli tenglamalar sistemasini Kramer va Gauss usullarida yechish

1. Ushbu ikki noma`lumli chiziqli tenglamalar sistemasining asosiy determinanti qayerda to'gri ko'rsatilgan?

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}.$$

A) $\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$. B) $\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$. C) $\begin{vmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{vmatrix}$.

D) $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$. E) $\begin{vmatrix} a_{21} & a_{12} \\ b_1 & b_2 \end{vmatrix}$.

2. Chiziqli tenglamalar sistemasinining asosiy Δ determinantining qiymati qanday bo'lganda uni yechish uchun matritsalar usulini qo'llab bo'lmaydi?

A) Δ≠0 . B) Δ=0 . C) Δ>0 . D) Δ<0 .

E) ko'rsatilgan barcha hollarda qo'llab bo'ladi.

3. Matritsaviy ko'rinishda yozilgan $AX=B$ chiziqli tenglamalar sistemasi yechimining formulasi qayerda to'gri ko'rsatilgan?

A) $X = B \cdot A$. B) $X = A \cdot B^{-1}$. C) $X = B^{-1} \cdot A$.

D) $X = A^{-1} \cdot B$. E) $X = B \cdot A^{-1}$.

4. Matritsaviy ko'rinishdagi $AX=B$ tenglamani yeching. Bunda

$$A=\begin{pmatrix}2 & 3\\ 5 & 4\end{pmatrix},\qquad X=\begin{pmatrix}x_1\\ x_2\end{pmatrix},\qquad B=\begin{pmatrix}18\\ 31\end{pmatrix}.$$

A) $X=\begin{pmatrix}1\\ 0\end{pmatrix}$. B) $X=\begin{pmatrix}3\\ 4\end{pmatrix}$. C) $X=\begin{pmatrix}2\\ 3\end{pmatrix}$. D) $X=\begin{pmatrix}1\\ 5\end{pmatrix}$

E) sistema yechimga ega emas.

5. Chiziqli tenglamalar sistemasinining asosiy Δ determinantining qiymati qanday bo'lganda uni yechish uchun Kramer usulini qo'llab bo'lmaydi?

A) $\Delta\neq 0$. B) $\Delta=0$. C) $\Delta>0$. D) $\Delta<0$.

E) ko'rsatilgan barcha hollarda qo'llab bo'ladi.

6. Ushbu ikki noma'lumli ikkita chiziqli tenglamalr sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

Quyidagilardan qaysi biri sistemaning asosiy determinantiga teng emas?

A) $\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}$. B) $\begin{vmatrix}a_{11} & a_{22}\\ a_{21} & a_{12}\end{vmatrix}$. C) $\begin{vmatrix}a_{11} & a_{21}\\ a_{12} & a_{22}\end{vmatrix}$. D) $-\begin{vmatrix}a_{21} & a_{22}\\ a_{11} & a_{12}\end{vmatrix}$.

E) Hamma ko'rsatilgan determinantlar asosiy determinantga teng.

7. Ushbu ikki noma'lumli ikkita chiziqli tenglamalr sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}.$$

Agar bu sistemaning asosiy determinanti

$$\Delta=\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}\neq 0$$

bo'lsa, quyidagi tasdiqlardan qaysi biri to'gri?

A) sistema yechimga ega emas.

B) sistema yagona yechimga ega.

C) sistema cheksiz ko'p yechimga ega.

D) sistema faqat nol yechimga ega.

E) sistemaning yechimi yo'q yoki cheksiz ko'p .

8. Ushbu ikki noma'lumli ikkita chiziqli tenglamalr sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}.$$

Quyidagilardan qaysi biri sistemaning yordamchi determinantiga teng emas?

A) $\begin{vmatrix} b_1 & b_2 \\ a_{12} & a_{22} \end{vmatrix}$. B) $\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$. C) $\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$; D) $\begin{vmatrix} a_{11} & a_{21} \\ b_1 & b_2 \end{vmatrix}$.

E) Hamma ko'rsatilgan determinantlar yordamchi determinantdir .

9. Agar

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

ikki noma'lumli ikkita chiziqli tenglamalar sistemasining asosiy determinanti $\Delta \neq 0$ va yordamchi determinantlari Δ_1, Δ_2 bo'lsa, sistemaning yechimi uchun Kramer formulasi qayerda to'g'ri ko'rsatilgan?

A) $x_1 = \Delta_1 \cdot \Delta, x_2 = \Delta_2 \cdot \Delta$. B) $x_1 = \Delta_1 + \Delta, x_2 = \Delta_2 + \Delta$.

C) $x_1 = \Delta_1 - \Delta, x_2 = \Delta_2 - \Delta$. D) $x_1 = \Delta - \Delta_1, x_2 = \Delta - \Delta_2$.

E) $x_1 = \Delta_1 / \Delta, x_2 = \Delta_2 / \Delta$.

10. Agar

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

ikki noma'lumli ikkita chiziqli tenglamalar sistemasining asosiy determinanti $\Delta=0$ va yordamchi determinantlari $\Delta_1=\Delta_2=0$ bo'lsa sistemaning yechimi to'grisidagi qaysi tasdiq o'rinli bo'ladi?

A) yechim yagona.

B) yechim cheksiz ko'p.

C) yechim mavjud emas.

D) yechimlar o'zaro teng.

E) yechim nolga teng.

3. Chiziqli tenglamalar sistemasini umumiy holda yechish. Bir jinsli tenglamalar sistemasi

1. Chiziqli sistemada tenglamalar soni m ta va noma'lumlar soni n ta bo'lsa, ular orasida qanday munosabat o'rinli bo'la olmaydi?

A) $m<n$. B) $m>n$. C) $m = n$. D) $m \neq n$.

E) keltirilgan barcha munosabatlar o'rinli bo'la oladi.

2. Umumiy holda n noma'lumli m ta tenglamali chiziqli sistemani yechishda qaysi shart bajariladi deb hisoblanadi?

A) $m = n$. B) $m \geq n$. C) $m \leq n$. D) $m \neq n$. E) $m>n$

3. n noma'lumli m ta ($m \leq n$) tenglamali $AX=B$ chiziqli sistemada $r(A)=r(A^B)=r$ bo'lsa, uning asosiy o'zgaruvchilarining soni s qaysi shartni qanoatlantiradi?

A) $s = r$. B) $s \geq r$. C) $s \leq r$. D) $s \neq r$. E) $s> r$.

4. n noma'lumli m ta ($m \leq n$) tenglamali $AX=B$ chiziqli sistemada $r(A)=r(A^B)=r$ bo'lsa, uning erkli o'zgaruvchilarining soni t qaysi shartni qanoatlantiradi?

A) $t = r$. B) $t=n+ r$. C) $t=n - r$. D) $t=n \cdot r$. E) $t=r - n$.

5. Ushbu 3 noma'lumli 2 ta chiziqli tenglamalar sistemaning umumiy yechimini toping:

$$\begin{cases} 2x_1 - 2x_2 + x_3 = 1 \\ x_1 + x_2 - x_3 = 1 \end{cases}$$

A) $\begin{cases} x_1 = 1 \\ x_2 = C + 1 \\ x_3 = C \end{cases}$. B) $\begin{cases} x_1 = (C+3)/4 \\ x_2 = (3C+1)/4 \\ x_3 = C \end{cases}$. C) $\begin{cases} x_1 = (C-3)/2 \\ x_2 = (C+1)/2 \\ x_3 = C \end{cases}$.

D) $\begin{cases} x_1 = 1 + C_1 - C_2 \\ x_2 = C_2 \\ x_3 = C_1 \end{cases}$ E) $\begin{cases} x_1 = 1 + C \\ x_2 = -2C \\ x_3 = C \end{cases}$.

6. Ushbu 3 noma'lumli 2 ta chiziqli tenglamalar sistemaning umumiy yechimini toping:

$$\begin{cases} x_1 - x_2 - x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \end{cases}.$$

A) $\begin{cases} x_1 = 1 \\ x_2 = C+1 \\ x_3 = C \end{cases}$. B) $\begin{cases} x_1 = C+1 \\ x_2 = C \\ x_3 = 0 \end{cases}$. C) $\begin{cases} x_1 = C+1 \\ x_2 = 0 \\ x_3 = C \end{cases}$.

D) $\begin{cases} x_1 = 1 + C_1 + C_2 \\ x_2 = C_2 \\ x_3 = C_1 \end{cases}$. E) $\begin{cases} x_1 = 1 + C \\ x_2 = -2C \\ x_3 = C \end{cases}$.

7. Ushbu 2 noma'lumli 3 ta chiziqli tenglamalar sistemaning yechimini toping:

$$\begin{cases} 2x_1 - 2x_2 = 1 \\ x_1 + x_2 = 1 \\ 3x_1 - x_2 = 2 \end{cases}$$

A) $\begin{cases} x_1 = C/4 \\ x_2 = C \end{cases}$. B) $\begin{cases} x_1 = 3C/4 \\ x_2 = C \end{cases}$. C) $\begin{cases} x_1 = 3C/4 \\ x_2 = C/4 \end{cases}$. D) $\begin{cases} x_1 = 3/4 \\ x_2 = 1/4 \end{cases}$.

E)sistema yechimga ega emas.

8. Ushbu 2 noma'lumli 3 ta chiziqli tenglamalar sistemaning yechimini toping:

$$\begin{cases} 2x_1 - 2x_2 = 1 \\ x_1 + x_2 = 1 \\ 3x_1 - x_2 = 1 \end{cases}$$

A) $\begin{cases} x_1 = C/4 \\ x_2 = C \end{cases}$. B) $\begin{cases} x_1 = 3C/4 \\ x_2 = C \end{cases}$. C) $\begin{cases} x_1 = 3C/4 \\ x_2 = C/4 \end{cases}$. D) $\begin{cases} x_1 = 3/4 \\ x_2 = 1/4 \end{cases}$.

E)sistema yechimga ega emas.

9. Agarda b_1 va b_2 chiziqli tenglamalar sistemasining ozod hadlari bo'lsa, qaysi shartda sistema bir jinsli bo'lmasligi mumkin?

A) $b_1=b_2=0$. B) $b_1^2+b_2^2=0$. C) $|b_1|+|b_2|=0$. D) $|b_1|-|b_2|=0$.

E) keltirilgan barcha hollarda sistema bir jinsli bo'ladi.

10. Agarda b_1 va b_2 chiziqli tenglamalar sistemasining ozod hadlari bo'lsa, qaysi shartda sistema bir jinsli bo'ladi?

A) $|b_1|\cdot|b_2|=0$. B) $|b_1|/|b_2|=0$. C) $|b_2|/|b_1|=0$.

D) $|b_1|+|b_2|=0$. E) $|b_1|-|b_2|=0$.

Insonning qimmati emas siymu zar.
Insonning qimmati ilm ham hunar.
Bedil

III-BOB. VEKTORLAR ALGEBRASI

3.1. Vektorlarni qo'shish va ularni fazodagi tasviri

Hayotda uchraydigan barcha kattaliklar matematikada ikki turga, skalyar va vektor kattaliklarga ajratiladi.

T A' R I F 1: Faqat sonli qiymatlari bilan aniqlanadigan kattaliklar skalyarlar deb ataladi.

Masalan, massa, hajm, uzunlik, modda zichligi, guruhdagi talabalar soni skalyarlar bo'ladi. Skalyarlar a, $в, c$ kabi belgilanadi.

T A' R I F 2: Sonli qiymati va yo'nalishi bilan aniqlanadigan kattaliklar vektorlar deyiladi.

Masalan, kuch, tezlik, bosim, harakat, oqim vektor kattaliklar bo'ladi. Vektorlar $\bar{a}, \bar{в}, \bar{c}$ kabi belgilanadi.

T A' R I F 3: $\bar{a}$ vektorning sonli qiymati uning **moduli** yoki **uzunligi** deb ataladi va $|\bar{a}|$ kabi belgilanadi.

Geometrik nuqtai-nazardan vektorlar yo'naltirilgan kesmalar singari qaraladi. Yo'naltirilgan kesmaning boshi A va oxiri B nuqtada bo'lsa, tegishli vektor $\overline{AB}$ kabi belgilanadi. Bunda A nuqta vektorning boshi, B nuqta esa vektorning uchi, kesma uzunligi vektor uzunligi deyiladi, ya'ni $|AB| = |\overrightarrow{AB}|$.

T A' R I F 4: Boshi va uchi bitta nuqtadan iborat bo'lgan vektor nol vektor deyiladi.

Nol vektor $\bar{0}$ kabi belgilanib, uning moduli $|\bar{0}| = 0$ bo'ladi. Bu vektor yo'nalishi to'g'risida so'z yuritib bo'lmaydi.

<u>*T A' R I F 5:*</u> Bir to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda joylashgan vektorlar **<u>kollinear</u>** vektorlar deb ataladi.

Masalan, ABCD parallelogramm bo'lsa, $\overline{AD}$ va $\overline{BC}$, $\overline{AB}$ va $\overline{CD}$ vektorlar kollinear, $\overline{AD}$ va $\overline{AB}$, $\overline{AC}$ va $\overline{AB}$ vektorlar esa kolllinear bo'lmaydi.

<u>*I z o h.*</u> Nol vektor $\bar{0}$ har qanday $\bar{a}$ vektorga kollinear deb hisoblanadi.

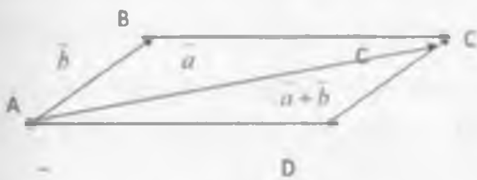
<u>*T A' R I F 6:*</u> Ikkita $\bar{a}$, $\bar{в}$ vektorlar teng deyiladi va $\bar{a} = \bar{в}$ kabi belgilanadi, agarda quyidagi uchta shart bajarilsa:

1. $\bar{a}$ va $\bar{в}$ vektorlar kollinear;
2. $\bar{a}$ va $\bar{в}$ vektorlar bir xil uzunlikka ega, ya'ni $|\bar{a}| = |\bar{в}|$;

$\bar{a}$ va $\bar{в}$ bir xil yo'nalishga ega.

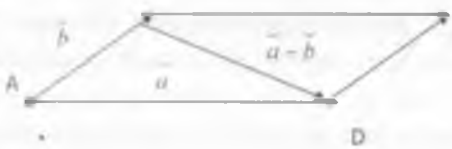
Masalan, ABCD parallelogrammda $\overline{AD} = \overline{BC}$, $\overline{AB} = \overline{DC}$ bo'ladi. Bu erdan vektorlarni parallel ko'chirish mumkinligi kelib chiqadi.

Endi ikkita $\bar{a}$ va $\bar{в}$ vektorlarni qo'shish va ayirish amalini kiritamiz. Buning uchun parallel ko'chirish orqali ularning boshlarini bitta A nuqtaga keltiramiz. Unda bu vektorlarni $\bar{a} = \overline{AD}$, $\bar{в} = \overline{AB}$ kabi belgilab, ABCD parallelogrammni hosil qilamiz.



$\bar{a}$ **3.1-rasm**

Bu holda $\vec{a}$ va $\vec{в}$ vektorlarning yig'indisi deb parallelogrammning A uchidan chiquvchi diagonalidan hosil qilingan $\overline{AC}$ vektorga aytiladi va $\vec{a} + \vec{в}$ kabi belgilanadi. Bu vektorlarning $\bar{a}-\bar{в}$ ayirmasi parallelogrammning B uchidan chiquvchi diagonalidan hosil qilingan $\overline{BD}$ vektorga aytiladi.



3.2-rasm

Vektorlarni qo'shish amali quyidagi xossalarga ega:

1. $\bar{a}+\bar{в}=\bar{в}+\bar{a}$ **2.**$(\bar{a}+\bar{в})+\bar{c}=\bar{a}+(\bar{в}+\bar{c})$ **3.** $\bar{a}+0=\bar{a}$

<u>*T A' R I F 7:*</u> $\bar{a}$ vektorni λ songa (skalyarga) ko'paytmasi deb, $\lambda\bar{a}$ kabi belgilanadigan va quyidagi shartlar bilan aniqlanadigan vektorga aytiladi:

1. $|\lambda\bar{a}|=|\lambda|\cdot|\bar{a}|$, ya'ni vektorning uzunligi. |λ| marta o'zgaradi;

2. $\lambda\bar{a}$ va $\bar{a}$ vektorlar kollinear;

3. λ>0 bo'lsa $\lambda\bar{a}$ va $\bar{a}$ bir xil yo'nalgan,

λ<0 bo'lsa $\lambda\bar{a}$ va $\bar{a}$ qarama-qarshi yo'nalgan.

Masalan, ABCD trapetsiya bo'lib, uning asoslari AD=8 vaBC=4 bo'lsa, unda $\overline{AD}=2\overline{BC}$ va $\overline{AD}=-2\overline{CB}$ tengliklar o'rinli bo'ladi.

<u>*I z o h.*</u>λ=0 bo'lsa, har qanday $\bar{a}$ vektor uchun $0\cdot\bar{a}=0$ bo'ladi.

Vektorning songa ko'paytirish amali quyidagi xossalarga ega:

1. $\lambda(\beta\bar{a})=\beta(\lambda\bar{a})$ 2. $(\lambda\pm\beta)\bar{a}=\lambda\bar{a}\pm\beta\bar{a}$
3. $\lambda(\bar{a}\pm\bar{в})=\lambda\bar{a}\pm\lambda\bar{в}$

Bu yerda α vaβ ixtiyoriy sonlar, $\bar{a}$ va $\bar{b}$ ixtiyoriy vektorlardir.

T A' R I F 8: (-1) $\bar{a}$ vektor $\bar{a}$ vektorga qarama-qarshi vektor deyiladi va $-\bar{a}$ kabi belgilanadi. Bunda doimo $\bar{a}+(-\bar{a})=0$ bo'ladi.

Endi bir tekislikda joylashgan vektorlarning koordinatalari tushunchasini kiritamiz. Buning uchun bu tekislikda XOY koordinatalar sistemasini olamiz. OX(OY) koordinata uqida joylashgan, musbat yo'nalishda yo'nalgan va uzunligi birga teng bo'lgan $\bar{i}(\bar{j})$ vektorni kiritamiz. Kiritilgan $\bar{i}$ va $\bar{j}$ vektorlar ort vektorlar yoki qisqacha ortlar deb ataladi. Endi berilgan $\bar{a}$ vektorni yo'naltirilgan kesma sifatida qarab, uning OX vaOY o'qdagi proektsiyalarini qaraymiz. Bu proektsiyalar ham yo'naltirilgan kesma bo'lib, ular $\bar{a}$ vektorning OX vaOY o'qdagi proektsiyalari deb ataladi va $\overline{a_x}$, $\overline{a_y}$ kabi belgilanadi. Unda, vektorlarni qo'shish ta'rifidan foydalanib, $\bar{a}=\overline{a_x}+\overline{a_y}$ tenglikni yozish mumkin.

Endi $\bar{a}$ vektor proektsiyalarining uzunligini $|\overline{a_x}|=|x|$, $|\overline{a_y}|=|y|$ kabi belgilaymiz. $\overline{a_x}$ va$\bar{i}$ ort ($\overline{a_y}$ va $\bar{j}$ ort) kollinear vektorlar bo'ladi, chunki ular OX(OY) koordinata o'qida joylashgan. Unda $|\bar{i}|=|\bar{j}|=1$ bo'lgani uchun, vektorlarni songa ko'paytmasi ta'rifiga asosan, $\overline{a_x}=x\bar{i}$ va $\overline{a_y}=y\bar{j}$ deb yozish mumkin. Bu erda $\overline{a_x}$ va $\bar{i}$ ort bir xil yo'nalgan bo'lsa, $x=|\overline{a_x}|$ deb, qarama-qarshi yo'nalgan bo'lsa, $x=-|\overline{a_x}|$ deb olinadi. Xuddi shunday tarzda u qiymati $\pm|\overline{a_y}|$ kabi olinadi.Bu holda tekislikdagi ixtiyoriy $\bar{a}$ vektorini $\bar{i}$ va $\bar{j}$ ortlar orqali

$$\bar{a}=x\bar{i}+y\bar{j} \tag{3.1}$$

ko'rinishda yozish mumkin.

T A' R I F 9: (3.1) tenglik $\bar{a}$ vektorning ortlar bo'yicha yoyilmasi, x va y sonlari esa uning koordinatalari deb ataladi va $\vec{a}(x,y)$ kabi ifodalanadi.

Masalan, $\vec{a}=2\vec{i}-3\vec{j}$ vektorning koordinatalari $x=2$, $y=-3$ bo'ladi.

Nol vektor uchun $\vec{0}=0\cdot\vec{i}+0\cdot\vec{j}$ bo'lgani uchun uning koordinatalari $x=0$, $y=0$ bo'ladi.

Har qanday $\vec{a}$ vektor uzining x va y koordinatalari bilan (3.1) tenglik orqali to'liq aniqlanadi. Koordinatalari bilan berilgan vektorlarning tengligi, kollinearligi va ular ustidagi qo'shish, ayirish, songa ko'paytirish amallarining natijalari oson aniqlanadi.

<u>TEOREMA 1:</u> $\vec{a}(x_1,y_1)$ va $\vec{e}(x_2,y_2)$ vektorlar teng bo'lishi uchun ularning mos koordinatalari teng, ya'ni $x_1=x_2$, $y_1=y_2$ bo'lishi zarur va etarli.

<u>TEOREMA 2:</u> $\vec{a}(x_1,y_1)$ va $\vec{e}(x_2,y_2)$ vektorlar kollinear bo'lishi uchun ularning mos koordinatalari proportsional, ya'ni

$$\frac{x_1}{x_2}=\frac{y_1}{y_2}=k \Rightarrow x_1=kx_2, y_1=ky_2$$

bo'lishi zarur va etarli.

Masalan, $\vec{a}(3,-2)$ va $\vec{e}(9,-6)$ kollinear vektorlar, chunki

9/3=(-6/-2)=3.

<u>TEOREMA 3:</u> $\vec{a}(x_1,y_1)$ va $\vec{e}(x_2,y_2)$ vektorlar yig'indisi yoki ayirmasining koordinatalari mos koordinatalarning yig'indisi yoki ayirmasiga teng bo'ladi, ya'ni

$\vec{a}(x_1,y_1)\pm\vec{e}(x_2,y_2)=\vec{c}(x_1\pm x_2, y_1\pm y_2)$

Masalan, $\vec{a}(4,-2)$ va $\vec{e}(5,9)$ bo'lsa, $\vec{a}+\vec{e}=(4+5,-2+9)=(9,7)$, $\vec{a}-\vec{e}=(4-5,-2-9)=(-1,-11)$ koordinatali vektorlardan iborat bo'ladi.

<u>TEOREMA 4:</u> $\vec{a}(x,y)$ vektorning λ songa ko'paytmasining koordinatalari uning har bir koordinatasini λ songa ko'paytirishidan hosil bo'ladi, ya'ni $\lambda\cdot\vec{a}(x,y)=\vec{c}(\lambda x,\lambda y)$

Masalan, $\vec{a}(4,-7)$ bo'lsa, $3\vec{a}=(3\cdot4,3\cdot(-7))=(12,-21)$ koordinatali vektor bo'ladi.

Bu teoremalarning isboti talabalarga mustaqil ish sifatida beriladi.

Fazodagi vektorlarning koordinatalari tushunchasini kiritish uchun OX, OY vaOZ o'qlari bo'yicha $\bar{i}, \bar{j}$ va $\bar{k}$ ort vektorlarni kiritamiz. Unda yo'qorida ko'rsatilgan singari, fazodagi ixtiyoriy $\bar{a}$ vektorni

$$\bar{a} = x\bar{i} + y\bar{j} + z\cdot\bar{k}$$

ko'rinishda yozish mumkin bo'ladi. Bu tenglik $\bar{a}$ vektorning ortlar bo'yicha yoyilmasi deb atalib, undagi x, y vaz sonlari uning koordinatalari deyiladi va $\bar{a}\,(x,y,z)$ kabi yoziladi. Fazodagi vektorlar uchun ham yuqorida ko'rilgan teoremalardagi tasdiqlar o'rinli bo'ladi.

3.1-3. 30. $\overline{AB}$ vektorning koordinatasi va uzunligini toping.

3.1. $A(1;1;3)$, $B(2;2;3)$

3.2. $A(0;1;3)$, $B(1;2;3)$

3.3. $A(0;1;-1)$, $B(1;2;0)$

3.4. $A(2;2;3)$, $B(3;2;4)$

3.5. $A(2;1;2)$, $B(3;2;2)$

3.6. $A(0;1;1)$, $B(1;2;2)$

3.7. $A(0;1;4)$, $B(1;2;4)$

3.8. $A(1;1;1)$, $B(1;2;2)$

3.9. $A(0;-4;3)$, $B(1;-3;4)$

3.10. $A(1;2;1)$, $B(0;1;2)$

3.11. $A(2;1;3)$, $B(3;2;4)$

3.12. $A(0;1;1)$, $B(1;2;2)$

3.13. $A(2;1;3)$, $B(3;2;4)$

3.14. $A(2;0;7)$, $B(0;2;4)$

3.15. $A(8;2;-5)$, $B(7;1;4)$

3.16. $A(-2;1;3)$, $B(5;1;2)$

3.17. $A(2;-1;4)$, $B(5;2;3)$

3.18. $A(3;1;3)$, $B(2;2;-1)$

3.19. $A(2;2;3)$, $B(3;1;3)$

3.20. $A(0;7;3)$, $B(4;7;-5)$

3.21. $A(4;-3;2)$, $B(1;2;3)$

3.22. $A(5;1;1)$, $B(6;2;1)$

3.23. $A(0;4;2)$, $B(3;6;-4)$

3.24. $A(1;3;2)$, $B(4;6;-5)$

3.25. $A(0;-2;1)$, $B(2;0;3)$

3.26. $A(2;-2;3)$, $B(2;1;7)$

3.27. $A(1;3;3)$, $B(2;4;2)$

3.28. $A(2;0;-1)$, $B(4;2;0)$

3.29. $A(1;3;-2)$, $B(3;2;0)$ 3.30. $A(1;3;-1)$, $B(3;1;0)$

3.31.Koordinatalar boshidan $M(12;-3;4)$ nuqtagacha bo'lgan masofani aniqlang.

3.32. $\vec{r}(0;2;-3)$ radius vektorning ortlar bo'yicha yoyilmasini yozing va modulini hisoblang.

3.33. $A(-2;+1;+3)$ va $B(0;-1;+2)$ nuqtalar orasidagi masofani toping.

3.34. $\vec{a}\{3;2;7\}$ va $\vec{b}\{4;1;-5\}$ vektorlar yig'indisini va ayirmasini toping.

3.35. Uchlari $A(5;2;6)$, $B(6;4;4)$, $C(4;3;2)$ va $D(3;1;4)$ nuqtalarda bo'lgan to'rtburchakning kvadrat ekanligini tekshiring.

3.36. α va β larning qanday qiymatlarida $\vec{a}=2\vec{i}+\alpha\vec{j}+\vec{k}$, $\vec{b}=3\vec{i}-6\vec{j}+\beta\vec{k}$ vektorlar kollinear bo'ladi?

3.37.Uchlari $A(2;1;-4)$, $B(1;3;5)$, $C(7;2;3)$ va $D(8;0;-6)$ nuqtalarda bo'lgan to'rtburchakning parallelogramm ekanligini isbotlang va parallelogramm tomonlarining uzunliklarini toping.

3.38. $A(-1;2;3)$, $B(2;-1;1)$, $C(1;-3;-1)$ va $D(-5;3;3)$ nuqtalar trapetsiyaning uchlari bo'lishini tekshiring.

3.39. AB kesmaning boshlang'ich nuqtasi $A(-1;2;4)$ va uni 1/2 nisbada bo'luvchi $C(2;0;2)$ nuqta berilgan. B uchining koordinatalarini toping.

3.40.Uchlari $A(1;2;3)$ va $B(4;2;-1)$ bo'lgan AB kesmani teng ikkiga bo'luvchi M nuqtaning koordinatalarini toping.

3.41. AB kesmaning boshlang'ich nuqtasi $A(-1;3;2)$. Uni teng ikkiga bo'luvchi nuqta esa $C(2;0;2)$ bo'lsin. B uchning koordinatalarini toping.

3.42. Boshlang'ich nuqtasi $A(-1;3;2)$, oxirgi nuqtasi $B(0;1;4)$ bo'lgan AB vektorning yo'naltiruvchi kosinuslarini toping.

3.43. $\vec{a}$ vektor Ox o'q bilan $\alpha=45^\circ$, Oy o'q bilan $\beta=60^\circ$ burchak hosil qiladi. Agar $|\vec{a}|=5$ bo'lsa, uning koordinatalarini aniqlang.

3.44. Uchlari $A(5;3;-10)$, $B(0;1;4)$ va $C(-1;3;2)$ nuqtalarda bo'lgan uchburchak berilgan. B ichki burchak bissektrissasining yo'naltiruvchi kosinuslarini toping.

3.45. a va b vektorlar orasidagi buchak $\varphi = \frac{\pi}{4}$ ga teng va $|\bar a| = \sqrt{2}$; $|\bar b| = 3$ ekanligi ma'lum. $\bar c = 2\bar a + 3\bar b$ vektorning uzunligini hisoblang.

3.46. $\bar a$ va $\bar b$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{3}$ ga teng. $|\bar a| = 3$; $|\bar b| = 4$. $\bar c = 3\bar a + 2\bar b$ vektorning uzunligini hisoblang.

3.47. Agar $|\bar a| = 7\sqrt{2}$, $|\bar b| = 4$ va $\left(\widehat{\bar a, \bar b}\right) = \frac{\pi}{4}$ bo'lsa? $3\bar a + \alpha \bar b$ va $\bar a - 2\bar b$ vektorlar α ning qanday qiymatlarida o'zaro perpendikulyar bo'ladi?

3.48. $\bar a$ va $\bar b$ vektorlarning koordinatalari berilgan:

$$\bar a = 7\bar i + 2\bar j + 3\bar k, \quad \bar b = 2\bar i - 2\bar j + 4\bar k.$$

Bu vektorlarning skalyar ko'paytmasini toping.

3.49. Uchalri $A(-1;5;1)$, $B(1;1;-2)$ va $C(-3;3;2)$ nuqtalarda bo'lgan uchburchak berilgan. AC tomonni davom ettirishdan hosil bo'lgan tashqi burchakni aniqlang.

3.50. Uchalri $A(-2;3;1)$, $B(-2;-1;4)$ va $C(-2;-4;0)$ nuqtalarda bo'lgan uchburchak berilgan. Bu uchburchakning C ichki burchagini hisoblang.

3.51. $\bar a, \bar b$ va $\bar c$ vektorlarning koordinatalari berilgan:

$$\bar a\{1;-4;8\} = \bar i - 4\bar j + 8\bar k, \quad \bar b\{4;4;-2\} = 4\bar i + 4\bar j - 2\bar k, \quad \bar c\{2;3;6\} = 2\bar i + 3\bar j + 6\bar k.$$

$(\bar b + \bar c)$ vaktor $\bar a$ vektordagi proyeksiyasini toping.

3.52. $|\bar a| = 8$, $|\bar b| = 15$, $\bar a \cdot \bar b = 96$ berilgan. $\bar a$ vektorning $\bar b$ vektor bilan vektor ko'paytmasining uzunligini toping.

3.53. Uchalari $A(1;2;0)$, $B(3;0;-3)$ va $C(5;2;6)$ nuqtalarda bo'lgan uchburchak yuzini hisoblang.

3.54. $\overline{AB} = -3\bar i - 2\bar j + 6\bar k$ va $\overline{BC} = -2\bar i + 4\bar j + 4\bar k$ vektorlar ΔABC tomonlari. $\overline{AD}$ balandlikning uzunligini hisoblang.

3.55. $\vec{a}\{6;0;2\}$, $\vec{b}\{1,5;2;1\}$ vektorlardan tuzilgan parallelogrammning yuzini va diagonallarining uzunliklarini toping.

3.56. Vektorning koordinatalarini ayting: 1) $3\vec{i}+2\vec{j}-5\vec{k}$;
2) $2\vec{i}-\vec{k}$; 3) $0,5\vec{i}+\sqrt{2}\vec{j}$;
4) $3\vec{k}$; 5) $-4\vec{i}$; 6) $\vec{0}$.

3.57. Quyidagi vektorlar berilgan: 1) $\vec{a}=2\vec{i}+3\vec{j}-5\vec{k}$; 2) $\vec{b}=-\vec{i}-2\vec{j}+3\vec{k}$. Ularning koordinatalarini toping.

3.58. *A(4;-3;2)* va *B(-2;4;-3)*, *M(0;5;1)* va *N(-4;0;-3)* nuqtalarning koordinatalarini bilgan holda $\overline{AB}$ va $\overline{MN}$ vektorlarning koordinatalarini toping.

3.59. $\vec{a}=(2;3;-4)$, $\vec{b}=(-1;2;1)$ va $\vec{c}=(3;0;2)$ vektorlarning koordinatalarini bilgan holda quyidagi vektorlarning koordinatalarini toping: 1) $\vec{a}+\vec{b}$; 2) $\vec{a}+\vec{c}$; 3) $\vec{a}+\vec{b}-\vec{c}$; 4) $3\vec{a}$;
5) $-\vec{a}+2\vec{c}$; 6) $2\vec{a}+3\vec{b}-2\vec{c}$.

3.60. Ikki vektorning kolleniarlik shartidan foydalanib, quyidagi vektorlarning kolleniar emasligini tekshiring: 1) $\vec{a}=(2/5;-1/3;4/5)$ va $\vec{b}=(3/5;-1/2;6/5)$;
2) $\vec{c}=(-6;1/3;3)$ va $\vec{d}=(-2;1/9;-1/3)$.

3.61. n va p ning qanday qiymatlarida $\vec{a}=(-3;n;4)$ va $\vec{b}=(-2;4;p)$ vektorlar kolleniar bo'ladi?

3.62. Vektorning uzunligini hisoblang: 1) $\vec{a}=-\vec{i}-2\vec{j}+2\vec{k}$;
2) $\vec{b}=\vec{i}+2\vec{j}-3\vec{k}$; 3) $\vec{c}=\vec{i}-\vec{k}$; 4) $\vec{d}=-3\vec{k}$.

3.63. $\vec{a}+\vec{b}$ vektorning uzunligini hisoblang; bunda: 1) $\vec{a}=(-1;2;1)$; $\vec{b}=(-2;2;-1)$;
2) $\vec{a}=(1;-2;3)$; $\vec{b}=(-1;2;-3)$.

3.64. Agar $\vec{a}=(2;0;0)$, $\vec{b}=(1;1;-1)$ bo'lsa, $3\vec{a}+2\vec{b}$ vektorning uzunligini hisoblang.

3.65. Agar *A(5;3;1)* va *B(4;5;-1)* bo'lsa, $\overline{AB}$ vektorning uzunligini hisoblang.

3.66. Agar *A(8;0;6)*, *B(8;-4;6)*, *C(6;-2;5)* bo'lsa, $\overline{AB}, \overline{BC}$ va $\overline{CA}$ vektorlardan hosil bo'lgan uchburchakning perimetrini toping.

3.67. $|AB|$ kesma (bunda $A(7;2;-3)$, $B(-5;0;4)$ C nuqta bilan $\lambda=|AC|:|CB|=1:5$ nisbatda bo'linadi. C nuqtaning koordinatalarini toping.

3.68. $|AB|$ kesma uchlarining koordinatalari bilan berilgan: $A(4;2;-3)$, $B(6;-4;-1)$. Bun kesmani teng ikkiga bo'luvchi C nuqtaning koordinatalarini toping.

3.69. $|AB|$ kesma uchlarining koordinatalari berilgan: $A(3;-2;-5)$ va $B(7;6;-1)$. Kesmani $\lambda=|AC|:|CB|=1:3$ nisbatda bo'luvchi C nuqtaning koordinatalarini toping.

3.70. Agar uchburchakning uchlari $A(7;-4;5)$, $B(-1;8;-2)$ va $C(-12;-1;6)$ bo'lsa, uchburchak medianalarining kesishgan nuqtasini toping.

3.71. $\vec{a}=\vec{i}-2\vec{j}+2\vec{k}$ vektor bazis vektorlar bilan tashkil etgan burchaklarning kosinusini toping.

3.72. Quyidagi vektorlarning bazis vektorlar bilan tashkil etgan burchaklarining kosinuslarini toping: 1) $\vec{a}=\vec{i}+\vec{j}+\vec{k}$; 2) $\vec{b}=(4;3;0)$; 3) $\vec{c}=-\vec{j}-3\vec{k}$; 4) $\vec{d}=3\vec{i}$.

3.73. $\vec{a}=(4;-3;1)$ va $\vec{b}=(5;-2;-3)$ vektorlarning skalyar ko'paytmasini toping.

3.74. Vektorlarning skalyar ko'paytmasini toping:

1) $\vec{a}=(3;-2;1)$ va $\vec{b}=(4;-7;-3)$;

2) $\vec{c}=(2/3;-5/6;1/4)$ va $\vec{d}=(3/2;6/5;4/3)$.

3.75. $\vec{a}=\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=-2\vec{i}-4\vec{j}+3\vec{k}$ va $\vec{c}=4\vec{i}-2\vec{j}-3\vec{k}$ vektorlar berilgan. Dastlabki ikki vektor yig'indisining uchinchisiga skalyar ko'paytmasini toping.

3.76. $\vec{a}=-4\vec{i}-3\vec{j}+5\vec{k}$ va $\vec{b}=-2\vec{i}+3\vec{j}+\vec{k}$ vektorlar berilgan. Ular orasidagi burchakni toping.

3.77. $\vec{a}=3\vec{i}-4\vec{k}$ va $\vec{b}=5\vec{i}-12\vec{k}$ vektorlar orasidagi burchakni toping.

3.78. $\vec{a}=(-2;2;-1)$ va $\vec{b}=(-6;3;6)$ vektorlar orasidagi burchakni toping.

3.79. Agar $\vec{a}=(1;-1;2)$ va $\vec{b}=(0;2;1)$ bo'lsa, $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlar orasidagi burchakni toping

3.80. Uchburchakning uchlari berilgan: $A(-1;4;1)$, $B(3;4;-2)$ va $C(5;2;-1)$. $\cos(\overline{BA},\hat{\ }\overline{BC})$ ni toping.

3.81. ABC uchburchakda $\cos(\overline{CA},\hat{\ }\overline{CB})$ ni toping, bunda $A(1;1;5)$,$B(-2;0;7)$,$C(-3;-2;5)$.

3.82. Vektorlar perpendikulyarni, tekshiring. 1) $\vec{a}=(3;0;-6)$ va $\vec{b}=(4;7;2)$;

2) $\vec{c}=(-3;2;5)$ va $\vec{d}=(6;-3;1)$.

3.83. Uchburchak berilgan: $A(2;4;5)$, $B(-3;2;2)$, $C(-1;0;3)$ $\overline{CA}\perp\overline{BC}$ ekanini ko'rsating.

3.84. Agar $\vec{a}=\vec{p}+2\vec{q}$ va $\vec{b}=5\vec{p}-4\vec{q}$ ekani ma'lum bo'lsa, $\vec{p}$ va $\vec{q}$ birlik vektorlar qanday burchak tashkil qiladi?

3.85. Berilgan: $|\vec{a}|=4$, $|\vec{b}|=6$, $\left(\vec{a},\hat{\ }\vec{b}\right)=\varphi$. $\vec{a}\times\vec{b}$ ni toping, bunda 1) $\varphi=0$;

2) $\varphi=90^\circ$; 3) $\varphi=150^\circ$.

3.86. $\vec{a}=2\vec{i}+3\vec{j}-4\vec{k}$ va $\vec{b}=\vec{i}-\vec{j}+3\vec{k}$ vektorlarning vektor ko'paytmasini toping.

3.87. $\vec{a}=2\vec{i}+\vec{j}+2\vec{k}$ va $\vec{b}=3\vec{i}+2\vec{j}+2\vec{k}$ vektorlarda yasalgan paralellogrammning yuzini toping.

3.88. $\vec{a}=\vec{i}+\vec{j}-\vec{k}$ va $\vec{b}=2\vec{i}-\vec{j}+2\vec{k}$ vektorlarda yasalgan paralellogrammning yuzini toping.

3.89. Uchining koordinatalariga ko'ra uchburchakning yuzini toping: $A(2;-3;4)$, $B(1;2;-1)$ va $C(3;-2;1)$.

3.90. Ordinata o'qida $A(1;-3;7)$ va $B(5;7;-5)$ nuqtalardan baravar uzoqlikdagi nuqtani toping.

3.91. a)Ikkita vektor berilgan: $\vec{a}=3\vec{i}+2\vec{j}-5\vec{k}$ va $\vec{b}=-2\vec{i}+3\vec{j}+4\vec{k}$ $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlarning koordinatalarini toping.

b) Parallelogramning uchlari $A(1;-2;3)$, $B(3;2;1)$, $C(6;4;4)$ va $D(x;y;z)$. D uchining koordinatalarini toping.

3.92. a) $\overline{AB}=\vec{a}$ vektor uchlarining koordinatalari berilgan: $A(-4;1;3)$, $B(2;-5;6)$. $\vec{a}$ vektorning bazis vektorlar bilan tashkil qilgan burchaklarning kosinuslarini hisoblang.

b) $\vec{d}(0;7;23)$ vektor $\vec{a}(5;2;1)$, $\vec{b}(-1;4;2)$ va $\vec{c}(-1;-1;6)$ vektrolarning chiziqli kombinatsiya koeffisiyentlarini toping.

3.93. a) $\vec{a}=(2;2;-1)$ va $\vec{b}=(-3;6;-6)$ vektorlar berilgan. Bu vektorlar orasidagi burchakning kosinusini hisoblang.

b) Parallelogramning ikkita uchining $A(1;3;-3)$, $B(2;-5;5)$ va dioganallari kesishish nuqtasining koordinatalari $O(1;1;1)$. C va D uchining koordinatalatini toping.

3.94. a) $\vec{a}=2\vec{i}+\vec{j}$ va $\vec{b}=-\vec{j}+\vec{k}$ vektorlarda parallelogramm yasalgan. Uning diagonallari orasidagi o'tkir burchakni toping.

b) $\vec{d}$ vektor $\vec{a},\vec{b},\vec{c}$ vektorlarning chiziqli kombinatsiyasi bo'ladimi? Chiziqli kombinatsiya koeffisiyentlarini toping.

$$\vec{a}(1;3;5),\ \vec{b}(0;4;5),\ \vec{c}(7;-8;4),\ \vec{d}(2;-1;3)$$

3.2.Skalyar va vektor ko'paytma va uning xossalari

***Ikki vektorning skalyar ko`paytmasi*** deb shu vektorlar modullarining ular orasidagi burchak kosinusi bilan ko`paytmasiga aytiladi. Quyidagi $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko`paytmasi $\vec{a}\cdot\vec{b}$ ko`rinishida belgilanadi. Demak,

$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos\varphi \qquad (3.2)$$

Skalyar ko'paytmaning xossalari

1) $(\alpha\vec{a},\vec{b})=(\vec{a},\alpha\vec{b})=\alpha(\vec{a},\vec{b})$, $\alpha=const$;

2) $(\vec{a},\vec{b})\geq 0$;

3) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ o`rin almashtirish qonuni.

4) $\vec{a}(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ tarqatish qonuni.

5) Agar $\vec{a}\parallel\vec{b}$ bo`lsa $\vec{a}\cdot\vec{b}=\pm a\cdot b$ xususiy holda $|\vec{a}|^2=\vec{a}\cdot\vec{a}=|\vec{a}|\cdot|\vec{a}|\cos^0=a^2$, $|\vec{a}|=\sqrt{\vec{a}^2}$;

6) agar $a\perp b$ bo`lsa $a\cdot b=a\cdot b\cdot\cos 90^0=0$;

7) ortlari skalyar ko`paytmasi

$$\vec{i}\cdot\vec{j}=0,\ \vec{j}\cdot\vec{k}=0,\ \vec{i}\cdot\vec{k}=0,\ \vec{i}\cdot\vec{i}=1,\ \vec{j}\cdot\vec{j}=1,\ \vec{k}\cdot\vec{k}=1$$

8) agar vektorlar $\vec{a}\{a_x, a_y, a_z\}$ va $\vec{b}\{b_x, b_y, b_z\}$ koordinatalar orqali berilgan bo`lsa, $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$.

Ikki vektor orasidagi burchak

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} + \sqrt{b_x^2 + b_y^2 + b_z^2}} \qquad (3.3)$$

Parallelik sharti: $\vec{b} = m\vec{a}$ yoki $\frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z} = m$;

Perpendikulyar sharti $\vec{a}\cdot\vec{b} = 0$ yoki $a_x b_x + a_y b_y + a_z b_z = 0$

<u>1-masala.</u> $\vec{a}(x,y,z)$ vektorning modulini toping.

<u>Yechish.</u> Skalyar ko'paytmaning 5) xossasiga asosan

$$|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}} = \sqrt{x\cdot x + y\cdot y + z\cdot z} = \sqrt{x^2 + y^2 + z^2} \qquad (3.4)$$

Masalan, $\vec{a}(3,4,12)$ vektorning moduli

$$|\vec{a}| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

<u>2-masala.</u> $\vec{a}$ (x_1,y_1,z_1) va $\vec{b}$ (x_2,y_2,z_2) vektorlar orasidagi φ burchakni toping.

<u>Yechish.</u> (3.3) formulaga asosan ikki vektor orasidagi burchak

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} + \sqrt{b_x^2 + b_y^2 + b_z^2}}$$ teng

Masalan, $\vec{a}(1,0,1)$ va $\vec{b}$ (0,1,1) vektorlar orasidagi φ burchak uchun

$$\cos\varphi = \frac{1\cdot 0 + 0\cdot 1 + 1\cdot 1}{\sqrt{1^2 + 0^2 + 1^2}\sqrt{1^2 + 0^2 + 1^2}} = \frac{1}{2}$$

natijani olamiz va undan $\varphi = 60^0$ ekanligini topamiz.

<u>3-masala.</u> $\vec{a}(x_1,y_1,z_1)$ va $\vec{b}$ (x_2,y_2,z_2) vektorlarning ortogonallik shartini toping.

<u>Yechish.</u> $\vec{a}\perp\vec{b}$ bo'lgani uchun ular orasidagi burchak $\varphi = 90^0$ bo'ladi va shu sababli $\cos\varphi = 0$

$$x_1x_2 + y_1y_2 + z_1z_2 = 0$$

tenglikni hosil qilamiz. Bu ikki vektorning ortogonallik shartidir.

Masalan, $\vec{a}$ (3,-2,1) va $\vec{b}$ (5,7,-1) vektorlar ortogonaldir, chunki $x_1x_2+y_1y_2+z_1z_2 = 3\cdot 5+(-2)\cdot 7+1\cdot(-1) = 15-14-1=0$

4-masala. Fazodagi $A(x_1,y_1,z_1)$ va $B(x_2,y_2,z_2)$ nuqtalar orasidagi d masofani toping.

Yechish. Bu nuqtalarni kesma bilan tutashtirib, $\overline{AB}$ vektorni hosil qilamiz. Ma'lumki, bu vektorning koordinatalari uning uchi bilan boshi koordinatalari ayirmasiga teng bo'ladi, ya'ni $\overline{AB}$ $(x_1-x_2, y_1-y_2, z_1-z_2)$. Unda (4) formulaga asosan,

$$d = \left|\overline{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \qquad (3.5)$$

tenglikka ega bo'lamiz.

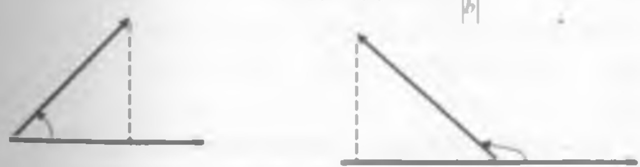
Masalan, A(5,-3,1) va B(8,1,13) nuqtalar orasidagi masofa

$$d = \sqrt{(8-5)^2 + (1-(-3))^2 + (13-1)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

bo'ladi.

Vektorni boshqa vektor yo'nalishidagi proyeksiyasi. Berilgan $\bar{a}$ vektorni $\bar{b}$ vektor yo'nalishdagi proyeksiyasi (3.2 - chizma) quyidagi formula orqali topiladi.

$$\text{Пр}_{\bar{b}}\bar{a} = |\bar{a}|\cos\alpha = \frac{(\bar{a},\bar{b})}{|\bar{b}|}$$



3.3–chizma. $\bar{a}$ vektorni $\bar{b}$ yo'nalishidagi proyeksiyasi.

Vektorlarni vektor ko'paytmasi. Ikkita $\bar{a}$ va $\bar{b}$ vektorlarning vektor ko`paytmasi deb shunday uchinchi $\bar{c}$ vektorga aytiladiki (3.3 - chizma), $\bar{c} = [\bar{a},\bar{b}]$ yoki $\bar{c} = \bar{a}\times\bar{b}$

1) $\bar{c}$ vektor son qiymati bo`yicha berilgan $\bar{a}$ va $\bar{b}$ vektorlardan yasalgan parallellogram yuziga teng moduliga ega;

2) u parallellogram tekisligiga perpendikulyar;

Ikkita vektordan qo'shilgan parallelogramning yuzi quyidagicha topiladi.

$$|\bar{c}| = |\bar{a}||\bar{b}|\sin\alpha = \left|[\bar{a},\bar{b}]\right| \qquad (3.6)$$

Bu $\bar{a}$ va $\bar{b}$ vektorlardan yasalgan uchburchakning yuzi

$$S_{uch} = \frac{1}{2}\left|\left[\vec{a},\vec{b}\right]\right| \qquad (3.7)$$

3) $\vec{c}$ vektorning yo'nalishi

Uch o'lchovli fazoda (R^3) $\vec{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ va $\vec{b} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ vektorlarni vektor ko'paytmasi

$$\left[\vec{a},\vec{b}\right] = (y_1 z_2 - y_2 z_1)\vec{i} - (x_1 z_2 - x_2 z_1)\vec{j} + (x_1 y_2 - x_2 y_1)\vec{k}$$

formula orqali yoki

$$\left[\vec{a},\vec{b}\right] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} \qquad (3.8)$$

topiladi.

3.4 –chizma. Vektorlarni vektor ko'paytirishga doir.

Vektor ko'paytma quyidagi xossalarga ega

1) $\left[\vec{a},\vec{b}\right] = -\left[\vec{b},\vec{a}\right]$ 2) $\left[\vec{a}+\vec{b},\vec{c}\right] = \left[\vec{a},\vec{c}\right] + \left[\vec{b},\vec{c}\right]$ 3) $\left[\alpha\,\vec{a};\vec{b}\right] = \left[\vec{a},\alpha\,\vec{b}\right] = \alpha\left[\vec{a};\vec{b}\right]$

4)Agar $\vec{a},\vec{b}$ nol bo'lingan vektorlar bo'lsa, $\left[\vec{a},\vec{b}\right] = 0$, ekanligidan, bu vektorlarni parallel $(\vec{a}\parallel\vec{b})$ ekanligi kelib chiqadi

Quyidagi $\bar{a}$ va $\vec{b}$ vektorlarni skalyar ko'paytmasini toping

3.95. $\vec{a}\{-2;1;1\}$, $\vec{b}\{3;-2;4\}$

3.96. $\vec{a}\{0;1;1\}$, $\vec{b}\{-1;-3;0\}$

3.97. $\vec{a}\{-2;1;1\}$, $\vec{b}\{0;-2;-5\}$

3.98. $\vec{a}\{0;1;1\}$, $\vec{b}\{3;-1;0\}$

3.99. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;-3;8\}$

3.100. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{2,0,2\}$

3.101. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;2;-1\}$

3.102. $c_1 \cdot c_2 = -40$, $\vec{c}\times\vec{c} = -5\vec{k}$

3.103. $\vec{a}\{-2;1;2\}$, $\vec{b}\{1;0;-1\}$

3.104. $\vec{a}\{0;1;1\}$, $\vec{b}\{-3;-1;1\}$

3.105. $\vec{a}\{-2;1;-2\}$, $\vec{b}\{-1;0;3\}$

3.106. $\vec{a}\{1;-1;-1\}$, $\vec{b}\{-2;3;-1\}$

3.107. $\vec{a}\{2;-1;3\}$, $\vec{b}\{0;1;1\}$

3.108. $\vec{a}\{2;1;-2\}$, $\vec{b}\{-1;0;-2\}$

3.109. $\vec{a}\{2;0;0\}$, $\vec{b}\{-3;1;1\}$

3.110. $\vec{a}\{2;1;0\}$, $\vec{b}\{1;1;3\}$

3.111. $\vec{a}\{1;-1;0\}$, $\vec{b}\{0;3;2\}$

3.112. $\vec{a}\{2;1;-2\}$, $\vec{b}\{0;1;1\}$

3.113. $\vec{a}\{1;0;-1\}$, $\vec{b}\{0;3;-1\}$

3.114. $\vec{a}\{2;-1;4\}$, $\vec{b}\{-1;0;0\}$

3.115. $\vec{a}\{1;0;-1\}$, $\vec{b}\{-1;-3;0\}$

3.116. $\vec{a}\{1;0;-1\}$, $\vec{b}\{-1;-3;0\}$

3.117. $\vec{a}\{5;2;-2\}$, $\vec{b}\{3;3;4\}$

3.118. $\vec{a}\{-1;-1;-1\}$, $\vec{b}\{0;0;-1\}$

Quyidagi $\bar{a}$ va $\bar{b}$ vektorlarni vektor ko'paytmasini toping

3.119. $\vec{a}\{-2;1;1\}$, $\vec{b}\{3;-2;4\}$

3.120. $\vec{a}\{0;1;1\}$, $\vec{b}\{-1;-3;0\}$

3.121. $\vec{a}\{-2;1;1\}$, $\vec{b}\{0;-2;-5\}$

3.122. $\vec{a}\{0;1;1\}$, $\vec{b}\{3;-1;0\}$

3.123. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;-3;8\}$

3.124. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{2;0;2\}$

3.125. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;2;-1\}$

3.126. $c_1 \cdot c_2 = -40$, $\bar{c} \times \bar{c} = -5\bar{k}$

3.127. $\vec{a}\{-2;1;2\}$, $\vec{b}\{1;0;-1\}$

3.128. $\vec{a}\{0;1;1\}$, $\vec{b}\{-3;-1;1\}$

3.129. $\vec{a}\{-2;1;-2\}$, $\vec{b}\{-1;0;3\}$

3.130. $\vec{a}\{1;-1;-1\}$, $\vec{b}\{-2;3;-1\}$

3.131. $\vec{a}\{2;-1;3\}$, $\vec{b}\{0;1;1\}$

3.132. $\vec{a}\{2;1;-2\}$, $\vec{b}\{-1;0;-2\}$

3.133. $\vec{a}\{2;0;0\}$, $\vec{b}\{-3;1;1\}$

3.134. $\vec{a}\{2;1;0\}$, $\vec{b}\{1;1;3\}$

3.135. $\vec{a}\{1;-1;0\}$, $\vec{b}\{0;3;2\}$

3.136. $\vec{a}\{2;1;-2\}$, $\vec{b}\{0;1;1\}$

3.137. $\vec{a}\{1;0;-1\}$, $\vec{b}\{0;3;-1\}$

3.138. $\vec{a}\{2;-1;4\}$, $\vec{b}\{-1;0;0\}$

3.139. $\vec{a}\{1;0;-1\}$, $\vec{b}\{-1;-3;0\}$

3.140. $\vec{a}\{1;0;-1\}$, $\vec{b}\{-1;-3;0\}$

3.141. $\vec{a}\{5;2;-2\}$, $\vec{b}\{3;3;4\}$

3.142. $\vec{a}\{-1;-1;-1\}$, $\vec{b}\{0;0;-1\}$

3.143. a) $\vec{a}\{2;2;1\}$, $\vec{b}\{-2;-3;0\}$ b) $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ vektor yo'nalishidagi birlik vektorni toping.

c) $\bar{a} = 2\bar{i} + 3\bar{j} + 5\bar{k}$ va $\bar{b} = \bar{i} + 2\bar{j} + \bar{k}$ vektorlarni vektor ko'paytmasini toping.

d) $\vec{a} = 6\bar{i} + 3\bar{j} - 2\bar{k}$ va $\bar{b} = 3\bar{i} - 2\bar{j} + 6\bar{k}$ vektorlardan yasalgan parallellogram yuzini hisoblang.

e) Uchlari A(1;1;1), B(2;3;4) va C(4;3;2) nuqtalarda bo'lgan uchburchakni yuzini toping.

k) $\bar{a} + 3\bar{b}$ *va* $3\bar{a} + \bar{b}$ vektorlardan $\left(|\bar{a}| = |\bar{b}| = 1, \left(\hat{\vec{a}, \bar{b}}\right) = 30^\circ\right)$ qurilgan parallelogramm yuzini toping.

m) Agar $\vec{a} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ va $\vec{b} = \begin{pmatrix} 1,5 \\ -1 \\ -2,4 \end{pmatrix}$ bo'lsa, $\vec{c}_1 = 2\vec{a}_1 + 3\vec{b}$ va $\vec{c}_2 = -4\vec{a} + \vec{b}$ vektorlarning skalyar ko'paytmasini toping.

l) m ning qanday qiymatda $\vec{a} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ va $\vec{b} = \begin{pmatrix} m \\ 4 \\ 0,5 \end{pmatrix}$ vektorlar ortogonal bo'ladi.

n) Vektor koordinata o'qlari bilan 60^0, 120^0 va 135^0 burchak hosil qilishi mumkinmi?

o) Agar $\vec{a} = \begin{pmatrix} -4 \\ 3 \\ -1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 2 \\ -6 \\ 2 \end{pmatrix}$, $M(3;-2;-5)$, $N(3;4;3)$ berilgan bo'lsa, $\vec{a} + 2\vec{b}$ vektorni $\overline{MN}$ vektorga proyeksiyasini toping.

3.144. *a)* $\vec{a}\{2;-4;1\}$, $\vec{b}\{3;1;-2\}$ b) $(5\vec{a} + 3\vec{b})\cdot(2\vec{a} - \vec{b})$ ni $a = 2, b = 3$ va $a \perp b$ bo'lgan hol uchun hisoblang.

c) $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ *va* $\vec{b} = 6\vec{i} + 4\vec{j} - 2\vec{k}$ vektorlar orasidagi burchakni toping.

Quyidagi $\vec{c}_1 = 2\vec{a} - \vec{b}$ va $\vec{c}_2 = -\vec{a} + 3\vec{b}$ vektorlarni vektor ko'paytmasini toping.

3.145. $\vec{a}\{-2;1;1\}$, $\vec{b}\{3;-2;4\}$	3.146. $\vec{a}\{0;1;1\}$, $\vec{b}\{-1;-3;0\}$
3.147. $\vec{a}\{-2;1;1\}$, $\vec{b}\{0;-2;-5\}$	3.148. $\vec{a}\{0;1;1\}$, $\vec{b}\{3;-1;0\}$
3.149. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;-3;8\}$	3.150. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{2;0;2\}$
3.151. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;2;-1\}$	3.152. $c_1 \cdot c_2 = -40$, $\vec{c} \times \vec{c} = -5\vec{k}$
3.153. $\vec{a}\{-2;1;2\}$, $\vec{b}\{1;0;-1\}$	3.154. $\vec{a}\{0;1;1\}$, $\vec{b}\{-3;-1;1\}$
3.155. $\vec{a}\{-2;1;-2\}$, $\vec{b}\{-1;0;3\}$	3.156. $\vec{a}\{1;-1;-1\}$, $\vec{b}\{-2;3;-1\}$
3.157. $\vec{a}\{2;-1;3\}$, $\vec{b}\{0;1;1\}$	3.158. $\vec{a}\{2;1;-2\}$, $\vec{b}\{-1;0;-2\}$
3.159. $\vec{a}\{2;0;0\}$, $\vec{b}\{-3;1;1\}$	3.160. $\vec{a}\{2;1;0\}$, $\vec{b}\{1;1;3\}$
3.161. $\vec{a}\{1;-1;0\}$, $\vec{b}\{0;3;2\}$	3.162. $\vec{a}\{2;1;-2\}$, $\vec{b}\{0;1;1\}$
3.163. $\vec{a}\{1;0;-1\}$, $\vec{b}\{0;3;-1\}$	3.164. $\vec{a}\{2;-1;4\}$, $\vec{b}\{-1;0;0\}$
3.165. [illegible]	3.166. $\vec{a}\{1;0;-1\}$, $\vec{b}\{-1;-3;0\}$
3.167. $\vec{a}\{5;2;-2\}$, $\vec{b}\{3;3;4\}$	3.168. $\vec{a}\{-1;-1;-1\}$, $\vec{b}\{0;0;-1\}$

§ 3.3. Aralash ko'paytma ва uning xossalari

$\bar{a},\bar{b}$ va $\bar{c}$ vektorlarni R^3 fazoda aralash ko'paytmasi sondan iborat bo'lib, $\bar{a}\cdot\bar{b}\cdot\bar{c}$ ko'rinishda belgilanadi.

$$\bar{a}\cdot\bar{b}\cdot\bar{c}=\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \qquad (3.9)$$

bu yerda $\bar{a}=\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$; $\bar{b}=\begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$; $\bar{c}=\begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}$

Vektorlar $\bar{a},\bar{b}$ va $\bar{c}$ vektorni aralash ko'paytmasi $\bar{a}$ vektorni $[\bar{b},\bar{c}]$ vektorga yoki $[\bar{a},\bar{b}]$ vektorni $\bar{c}$ vektorga skalyar ko'paytmasidan iborat bo'ladi $\bar{a}\cdot\bar{b}\cdot\bar{c}=(\bar{a},[\bar{b},\bar{c}])=([\bar{a},\bar{b}],\bar{c})$

Aralash ko'paytma quyidagi xossalarga ega:

1. Aralash ko'paytmaning ikkita ko'paytmasini o'rni o'zaro almashtirilsa, ko'paytmaning ishorasi o'zgaradi

$$\bar{a}\cdot\bar{b}\cdot\bar{c}=-\bar{a}\bar{c}\bar{b}=-\bar{b}\bar{a}\bar{c}=-\bar{c}\bar{b}\bar{a}.$$

2. Aralash ko'paytma hadlarini aylanma usulda almashtirishda o'z ishorasini o'zgartirmaydi $\bar{a}\cdot\bar{b}\cdot\bar{c}=\bar{c}\bar{a}\bar{b}=\bar{b}\bar{c}\bar{a}$.

3. Agar $\bar{a},\bar{b}$ va $\bar{c}$ vektorlar noldan farqli vektorlar bo'lsa, u holda $\bar{a}\cdot\bar{b}\cdot\bar{c}=0$ bo'lishi uchun bu vektorlar chiziqli bog'liq (yoki komplanar) bo'lishi kerak. $\bar{a},\bar{b}$ va $\bar{c}$ vektorlardan qurilgan parallelipipedning hajmi $V_{par}=|\bar{a}\cdot\bar{b}\cdot\bar{c}|$ teng. Xuddi shunday shu uchta vektordan qurilgan tetrayedrning hajmi $V_T=\frac{1}{6}|\bar{a}\cdot\bar{b}\cdot\bar{c}|$.

Quyidagi $\bar{a},\bar{b},\bar{c}$ vektorlarni komplanarligini tekshiring.

3.169. $\bar{a}\{-2;1;1\}$, $\bar{b}\{0;-2;-5\}$, $\bar{c}\{2;-1;-1\}$

3.170. $a\{0;1;1\}$, $\bar{b}\{0;4;-2\}$, $\bar{c}\{2;1;0\}$

3.171. $\bar{a}\{2;0;1\}$, $\bar{b}\{2;0;-1\}$, $\bar{c}\{-2;-1;4\}$

3.172. $\bar{a}\{1;-1;-1\}$, $\bar{b}\{-2;3;-1\}$, $\bar{c}\{0;1;0\}$

3.173. $\bar{a}\{1;1;1\}$, $\bar{b}\{2;3;0\}$, $\bar{c}\{3;-1;-1\}$

3.174. $\bar{a}\{-1;0;-2\}$, $\bar{b}\{-3;2;-1\}$, $\bar{c}\{2;0;-2\}$

3.175. $\bar{a}\{1;0;3\}$, $\bar{b}\{0;1;1\}$, $\bar{c}\{2;-1;3\}$

3.176. $\vec{a}\{-3;1;4\}$, $\vec{b}\{2;0;0\}$, $\vec{c}\{-3;1;1\}$

3.177. $\vec{a}\{1;0;-1\}$, $\vec{b}\{0;-1;-1\}$, $\vec{c}\{0;0;-2\}$

3.178. $\vec{a}\{-1;0;-2\}$, $\vec{b}\{0;0;-1\}$, $\vec{c}\{-1;0;3\}$ $\vec{a}\{-1;0;-2\}$, $\vec{b}\{1;0;-4\}$, $\vec{c}\{2;0;-2\}$

3.179. 3.180. $\vec{a}\{1;0;-2\}$, $\vec{b}\{-3;2;-1\}$, $\vec{c}\{4;2;-3\}$

3.181. $\vec{a}\{1;2;4\}$, $\vec{b}\{-3;6;4\}$, $\vec{c}\{3;-6;4\}$ 3.182. $\vec{a}\{1;-1;1\}$, $\vec{b}\{1;1;1\}$, $\vec{c}\{2;3;4\}$

3.183. $\vec{a}\{5;3;-1\}$, $\vec{b}\{1;-2;3\}$, $\vec{c}\{2;0;-4\}$

3.184. $\vec{a}\{-3;3;3\}$, $\vec{b}\{2;1;1\}$, $\vec{c}\{19;11;17\}$

3.185. $\vec{a}\{1;6;5\}$, $\vec{b}\{3;-2;4\}$, $\vec{c}\{7;-18;2\}$

3.186. $\vec{a}\{7;-3;2\}$, $\vec{b}\{3;-7;8\}$, $\vec{c}\{1;-1;1\}$

3.187. $\vec{a}\{2;1;-1\}$, $\vec{b}\{1;-4;1\}$, $\vec{c}\{3;-2;2\}$

3.188. *a*) $\vec{a}\{3;1;-1\}$, $\vec{b}\{-2;-1;0\}$, $\vec{c}\{5;2;-1\}$

в) Uchlari A(2;2;2), B(4;3;3), C(4;5;4) va D(5;5;6) bo`lgan uch burchakli piramidani xajmini toping.

c) Quyidagi to`rtta vektor berilgan bo'lsa, $\vec{a}=2\vec{i}+\vec{j}$, $\vec{b}=\vec{i}+\vec{j}+2\vec{k}$, $\vec{c}=2\vec{i}+2\vec{j}-\vec{k}$, $\vec{d}=3\vec{i}+7\vec{j}-7\vec{k}$ quyidagilarni topish kerak.

1. $\vec{d}$ vektorini $\vec{a},\vec{b},\vec{c}$ lar bo`yicha yozing.
2. $\vec{m}=5\vec{a}+2\vec{b}$ vektor uzunligi va yo'nalishini toping.

§3.4 Vektor (chiziqli) fazo va n-o'lchovli vektor. Yevklidfazosi.

Chiziqli operatorlar ва kvadrat formalar

1. Tartiblangan n-ta haqiqiy sondan tuzilgan $\vec{x}=(x_1,x_2,...x_n)$ vektor n- o'lchovli vektor deyiladi. Agar $x_i=y_i (i=1,2,3,...,n)$ bo'lsa, $\vec{x}=\vec{y}$, ya'ni $\vec{x}=(x_1,x_2,...x_n)$ *va* $\vec{y}=(y_1,y_2,...y_n)$ vektorlar teng bo'ladi. Vektorni $\vec{x}=(x_1,x_2,...x_n)$ biror o'zgarmas λ soniga ko'paytirsak, u holda $\vec{u}=\lambda\cdot\vec{x}$, ya'ni $\vec{u}_i=\lambda\cdot\vec{x}_i (i=1,2,3,...,n)$ bo'ladi. Ikkita vektor yig'indisi $\vec{x}=(x_1,x_2,...x_n)$ *va* $\vec{y}=(y_1,y_2,...y_n)$ lar uchun $\vec{z}=\vec{x}+\vec{y}$, ya'ni $\vec{z}_i=\vec{x}_i+\vec{y}_i$ $(i=1,2,3,...,n)$ bo'ladi.

2. Vektorli (chiziqli) fazo deganda haqiqiy kompanentli vektorlar to'plami tushunilib, bu fazoda vektorlarni qo'shish va vektorlarni biror songa ko'paytirish amalidan iborat operatsiyalar bajariladi.

3. Agar a_m vektor $\bar{a}_1,\bar{a}_2,...\bar{a}_{m-1}$ vektorlarni chiziqli kombinatsiyasi bo'lsa, u holda

$$\bar{a}_m=\lambda_1\bar{a}_1+\lambda_2\bar{a}_2+...+\lambda_{m-1}\bar{a}_{m-1} \quad (3.14)$$

bu yerda $\lambda_1,\lambda_2,...,\lambda_{m-1}$ - ixtiyoriy o'zgarmas sonlar.

4. Bir vaqtda nolga teng bo'lmagan $\lambda_1,\lambda_2,...,\lambda_n$ sonlar mavjud bo'lsa va ular uchun

$$\lambda_1\bar{a}_1+\lambda_2\bar{a}_2+...+\lambda_m\bar{a}_m=0 \quad (3.15)$$

tenglik o'rinli bo'lsa, u holda $\bar{a}_1,\bar{a}_2,...\bar{a}_m$ vektorlar chiziqli bog'liq deyiladi. Agar (2) shart $\lambda_1=\lambda_2=...=\lambda_n=0$, bo'lganda bajarilsa, u holda $\bar{a}_1,\bar{a}_2,...\bar{a}_m$ vektorlar chiziqli bog'liq bo'lmagan vektorlar deyiladi yoki chiziqli ekrli vertorlar deyiladi.

5. Chiziqli bog'liq bo'lmagan vektorlarning maksimal soni fazo o'lchami deyiladi. n-o'lchovli fazo bazisi uchun n-chiziqli bog'liq bo'lmagan vektorlar majmuasi xizmat qiladi.

6. Bir bazisdan $(\bar{e}_1,\bar{e}_2,...,\bar{e}_n)$ boshqa bazis $(\bar{e}_1^*,\bar{e}_2^*,...,\bar{e}_n^*)$ ga o'tish o'tish matrisasi yordamida amalga oshiriladi.

$$A=\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \text{ya'ni } \begin{pmatrix} \bar{e}_1^* \\ \bar{e}^* \\ \cdots \\ \bar{e}^* \end{pmatrix}=A^T\begin{pmatrix} \bar{e}_1 \\ \bar{e}_2 \\ \cdots \\ \bar{e}_n \end{pmatrix}$$

Koordinatalari $x_1,x_2,...x_n$ bo'lgan $\bar{x}$ vektorini eski bazisdan yangi $x_1^*,x_2^*,...x_n^*$ koordinatalarning bazisga o'tish uchun

$$\begin{pmatrix} x_1^* \\ x_2^* \\ \cdots \\ x_n^* \end{pmatrix}=A^{-1}\begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix} \text{ yoki } \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix}=A\begin{pmatrix} x_1^* \\ x_2^* \\ \cdots \\ x_n^* \end{pmatrix} \quad (3.10)$$

bu yerda A-o'tish matrisasi hisoblanadi.

7. Ikki $\bar{x}=(x_1,x_2,...x_n)$ va $\bar{y}=(y_1,y_2,...y_n)$ skalyar vektorlarni ko'paytmasi deb

$$(\bar{x},\bar{y})=x_1y_1+x_2y_2+...+x_ny_n=\sum_{i=1}^{n}x_iy_i .$$

8. Skalyar ko'paytma ma'lum xossalarni qanoatlantirishsa, chiziqli (vektor) fazo Yevklid fazosi deyiladi. Bu fazoda $\bar{x}$ vektorni uzunligi

$$|\bar{x}| = \sqrt{(\bar{x},\bar{y})} = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$$

9. Ikkita $\bar{x}$ va $\bar{y}$ vektorlar orasidagi burchak φ quyidagicha aniqlanadi.

$$\cos\varphi = \frac{(\bar{x},\bar{y})}{|\bar{x}||\bar{y}|}, \quad 0 < \varphi < \pi$$

10. Ikkita $\bar{x}$ va $\bar{y}$ vektorlar ortogonal vektorlar deyiladi, agarda

$$(\bar{x},\bar{y}) = 0$$

Agar $(\bar{e}_i,\bar{e}_j) = 0 \quad i \neq j$ bo'lsa, n-o'lchovli Yevklid fazosida $\bar{e}_1, \bar{e}_2, \ldots, \bar{e}_n$ vektorlar ortogonal bazislarni hosil qiladi. Ba'zi masalalarni yechimini keltiramiz.

11. Agar operator R^n fazodagi har qanday $\bar{x},\bar{y}$ vektorlarni ixtiyoriy λ soni uchun

$$A(\bar{x}+\bar{y}) = A(\bar{x}) + A(\bar{y}), \quad A(\lambda\bar{x}) = \lambda A(\bar{x})$$

munosabat o'rinli bo'lsa, A operator chiziqli deyiladi.

12. Vektor $\bar{y} = A(\bar{x})$ $\bar{x}$ vektorning tarsi deyiladi. Vektor $\bar{x}$ va uning tarsi $\bar{y}$ orasidagi bog'lanish $\bar{y} = A(\bar{x})$ ko'rinishida bo'ladi. Bu yerda A- chiziqli operator matritsasi bo'lib,

$\bar{x} = (x_1, x_2, \ldots x_n)^T$, $\bar{y} = (y_1, y_2, \ldots y_n)^T$ bir ustunli matritsa.

13. Chiziqli operator yig'indisi va ko'paytmasi quyidagicha bo'ladi,

$$(\bar{A} + \bar{B})(\bar{x}) = \bar{A}(\bar{x}) + \bar{B}(\bar{x})$$

$$(\bar{A}\bar{B})(\bar{x}) = A(B(\bar{x}))$$

$$\lambda\bar{A}(\bar{x}) = \lambda(A(\bar{x}))$$

Chiziqli operatorga doir quyidagi misollarni ko'ramiz.

14. Agar shunday λ soni mavjud bo'lsaki,

$$\bar{A}(\bar{x}) = \lambda x \quad yoki \quad A\bar{x} = \lambda\bar{x} \tag{3.11}$$

$\bar{x} \neq 0$ vektor chiziqli operator $\bar{A}$(yoki matrisa A) ning xos vektori deyiladi. Bu yerda λ soni $\bar{x}$ ga mos keluvchi A operatorning (yoki A₁ matrissaning) xos soni deyiladi. Bu (4.21) formulani quyidagi ko'rinishga olib keladi.

$$(A-\lambda E)\bar{x}=0 \tag{3.12}$$

15. Operator $\bar{A}$ ning (yoki matrisa A) xarakteristik tenglamasi deb quyidagi tenglamaga aytiladi.

$$|A-\lambda E| = \begin{vmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\lambda & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn}-\lambda \end{vmatrix} = 0$$

Bu yerda $|\lambda-\lambda E|$ - xarakteristik ko'p had hisoblanadi. Chiziqli operatorning xarakteristik ko'p hadi bazis tanlashga bog'liq emas.

16. Xos vektor va unga mos keluvchi xos son $\lambda_1,\lambda_2,...,\lambda_n$ bazisda operator matrisasi $\tilde{A}$ diagonal matrisa bo'ladi.

$$A^* = diog(\lambda_1,\lambda_2,...,\lambda_n).$$

17. Bir necha darajali bir jinsli $x_1,x_2,...,x_n$ o'zgaruvchidan tashkil topgan kvadrat forma deb quyidagi ifodaga aytiladi.

$$L(x_1,x_2,...,x_n)=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}x_ix_j \tag{3.13}$$

$(a_{ij}=a_{ji})$ kvadratik farqi deyiladi.

Kvadratik formadagi har bir o'zgaruvchilarini kvadrati, yoki ikkita har xil o'zgaruvchilarini ko'paytmasi bo'ladi.

Kvadratik formani matritsa ko'rinishda quyidagicha yozish mumkin $L=X^TAX$

bu yerda

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}; \quad X=\begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}$$

18. Kvadratik forma matritsasiga maxsus bo'lmagan $X=CY$ (bu yerda $Y=(y_1,y_2,...,y_n)^T$, C- chiziqli almashtirish matritsasi) chiziqli almashtirish bajarilsa, u holda $A^*=C^TAC$.

19. Agar $a_{ij}=0$ $(i\neq j)$ bo'lsa, u holda $L=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}x_ix_j$ kanonik kvadrat forma bo'ladi va quyidagicha ko'rinishda yoziladi

$$L = a_{11}x_1^2 + a_{22}x_2^2 + ... + a_{nn}x_n^2 = \sum_{i=1}^{n} a_{ii}x_i^2$$

Kvadratik forma rangi kanonik formasining noldan farqli koeffisientlar soniga teng, hamda chiziqli almashtirishlar natijasida o'zgarmaydi.

20.Agar o'zgaruvchilari barcha qiymatlarida hech bo'lmaganda birortasini hadi noldan farqli bo'lsa, ya'ni $L(x_1, x_2, ..., x_n) > 0$, u holda kvadratik forma $L(x_1, x_2, ..., x_n)$ musbat aniqlangan deyiladi.

Kvadratik forma $L = X^T AX$ musbat aniqlangan bo'lishi uchun quyidagi shartlar bajarilishi kerak:

a)A matritsani barcha xos qiymatlari λ_i musbat bo'lishi;

b) A matritsani barcha bosh minorlari musbat (Silvesstr kriteriyasi).

Kvadratikforma $L = X^T AX$ manfiyaniqlanganbo'lishiuchunquyidagishartlarbajarilishikerak:

a)A matritsani barcha xos qiymatlari λ_i musbat bo'lishi;

b) A matritsani juft bo'lmagan tartibdagi manfiy, juft tartibdagilari esa musbat (Silvestr kriteriyasi).

A matritsani xarakteristik tenglamasini ildizlari kvadratik formani xos sonlari ularga mos kelishi xos vektorlar esa kvadratik formani bosh yo'nalishi deyiladi.

Quyidagichiziqni $\overline{A}$ operatorning (Amatritsa) ning xos son va xos vektorlarini toping.

3.189. $A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$ 3.190. $A = \begin{pmatrix} 3 & -4 \\ -2 & 1 \end{pmatrix}$

3.191. $A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$ 3.192. $A = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 3 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

3.193. $A = \begin{pmatrix} 1 & 1 & 8 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ 3.194. $A = \begin{pmatrix} 2 & 0 & -6 \\ 1 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}$

3.195. $A=\begin{pmatrix}1&0&1&0\\0&2&0&3\\0&0&0&-1\\0&0&0&0\end{pmatrix}$ 3.196. $A=\begin{pmatrix}1&0&0&0\\1&1&-4&-8\\2&-4&7&-4\\1&-8&-4&1\end{pmatrix}$

Chiziqli operator $\overline{A}$ ning A- matritsasini diagonal ko'rinishga keltiring.

3.197. $A=\begin{pmatrix}2&1\\3&4\end{pmatrix}$ 3.198. $A=\begin{pmatrix}2&-1\\3&4\end{pmatrix}$ 3.199. $A=\begin{pmatrix}2&5&3\\0&3&4\\0&0&0\end{pmatrix}$

Kvadratik formani matritsa ko'rinishida yozing.

3.200. $L=3x_1^2+x_2^2-x_1x_2$

3.201. $L=2x_1^2-x_2^2+3x_3^2+2x_1x_2+6x_1x_2$

3.202. $L=2x_1^2+3x_2^2-2x_3^2+x_1x_2+2x_1x_3+3x_2x_3$

Kvadratik forma rangini toping

3.203. $L=x_1^2+x_2^2+x_3^2+2x_1x_2$

3.204. $L=2x_1^2-x_2^2+x_3^2+2x_1x_2+5x_1x_2$

3.205. $L=2x_1^2+4x_2^2+9x_3^2+4x_1x_2+6x_1x_3+12x_2x_3$

3.206. Kvadratik formani kanonik ko'rinishga olib keling.

$$L=2x_1^2-3x_3^2-4x_1x_2+4x_1x_3-8x_2x_3.$$

3.207. Kvadratik formani kanonik ko'rinishga olib keling.

$$L=x_1^2-4x_2x_3+x_3^2$$

3.208. Kvadratik formani kanonik ko'rinishga olib keling.

$$L=x_1^2+3x_2^2+4x_3^2+2x_1x_2+2x_1x_3+6x_2x_3$$

<u>Takrorlash uchun savollar</u>

1. Qanday kattaliklar skalyar deyiladi?
2. Qanday kattaliklar vektorlar deyiladi?
3. Vektorlarga qanday misollar bilasiz?
4. Vektorlarning songa ko'paytmasi qanday aniqlanadi?
5. Vektorlarning koordinatalari qanday topiladi?
6. Koordinatalari bilan berilgan vektorlar ustida arifmetik amallar qanday bajariladi?
7. Vektorlarning skalyar ko'paytmasi qanday aniqlaniladi?

8.Skalyar ko'paytma qanday xossalarga ega?
9.Vektorial ko'paytma qanday aniqlaniladi?
10.Vektorlarning aralash ko'paytmasi qanday aniqlaniladi?

VEKTORLAR ALGEBRASIGA DOIR NAZORAT TESTLARI

1. Vektorlar va ular ustida amallarga doir testlar.

1. Skalyar deb nimaga aytiladi?

A)Faqat yo'nalishi bilan aniqlanadigan kattalikka skalyar deb aytiladi.

B) Faqat son qiymati bilan aniqlanadigan katalikka skalyar deb aytiladi.

C) Ham son qiymati, ham yo'nalishi bilan aniqlanadigan kattalikka skalyar deb aytiladi.

D) Yo'nalgan kesmaga skalyar deb aytiladi.

E) Har qanday kattalik skalyar deyiladi.

2. Quyidagi kattaliklardan qaysi biri skalyar emas?

A) Sirt yuzasi .

B) Kesma uzunligi .

C) Jism hajmi.

D) Burchak kattaligi.

E) kuch momenti.

3. Vektor deb nimaga aytiladi?

A)Faqat yo'nalishi bilan aniqlanadigan kattalikka vektor deb aytiladi.

B) Faqat son qiymati bilan aniqlanadigan katalikka vektor deb aytiladi.

C) Ham son qiymati, ham yo'nalishi bilan aniqlanadigan kattalikka vektor deb aytiladi.

D) Har qanday kesmaga vektor deb aytiladi.

E) Har qanday kattalik vektor deyiladi.

4. Quyidagilardan qaysi biri vektor bo'ladi ?

A) Sirt yuzasi.

B) Jism hajmi.

C) Kesma uzunligi.

D) Moddiy nuqta harakati.

E) Modda massasi.

5. Kollinear vektorlar ta'rifi qayerda to'g'ri ifodalangan ?

A) Bir xil yo'nalgan vektorlar kollinear deb aytiladi.

B) Har qanday $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar kollinear vektorlar deb aytiladi.

C) Bir xil yo'nalgan va uzunliklari teng bo'lgan vektorlar kollinear deb aytiladi.

D) Bitta to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotuvchi $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarga kollinear vektor deb aytiladi.

E) Bitta tekislikda yoki parallel tekisliklarda yotuvchi $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarga kollinear vektor deb aytiladi.

6. Agar ABCD trapetsiya (AD||BC) bo'lsa, unda quyidagi juftliklardan qaysi biri kollinear vektorlarni ifodalaydi?

A) $\overrightarrow{AB}$ va $\overrightarrow{BC}$. B) $\overrightarrow{AC}$ va $\overrightarrow{BD}$. C) $\overrightarrow{AB}$ va $\overrightarrow{DC}$.
D) $\overrightarrow{AD}$ va $\overrightarrow{CB}$. E) $\overrightarrow{AD}$ va $\overrightarrow{DC}$.

7. Vektorlar tengligini ifodalovchi ta'rifni ko'rsating.

A) Bir xil yo'nalgan $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar teng deb aytiladi.

B) Bir xil uzunlikli $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar teng deb aytiladi.

C) Turli yo'nalgan, ammo uzunliklari teng bo'lgan $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar teng deb aytiladi.

D) $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar kollinear, bir xil yo'nalgan va uzunliklari teng bo'lsa, ular teng deb aytiladi.

E) Kollinear va bir xil yo'nalgan $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar teng deb aytiladi.

8. Agar ABCD parallelogramm (AD||BC) bo'lsa, unda quyidagi juftliklardan qaysi biri teng vektorlarni ifodalaydi?

A) $\overrightarrow{AB}$ va $\overrightarrow{BC}$. B) $\overrightarrow{AC}$ va $\overrightarrow{BD}$. C) $\overrightarrow{AB}$ va $\overrightarrow{DC}$.

D) $\overline{AD}$ va $\overline{CB}$.   E) $\overline{AD}$ va $\overline{DC}$.

9. Vektorlar yig'indisi uchun qaysi qoida bo'lmaydi ?
A) Parallelogramm qoidasi.
B) Uchburchak qoidasi.
C) Trapetsiya qoidasi.
D) Ko'pburchakqoidasi;
E) Parallelepiped qoidasi.

10. Vektorlar yig'indisi uchun qaysi tenglik o'rinli bo'lmaydi ?
A) $a+b = b+a$.B) $a+(b+c) =(a+b)+c$ .C) $a+0 =a$.
D) $a+(-a) =0$ .E) $| a+b |=|a|+|b|$.

2. Vektorlarning koordinatalari va ular ustida amallarga doir testlar

1. Fazoda joylashgan quyidagi vektorlar uchliklarining qaysi biri ort vektorlar deb olinishi mumkin?

A) O'zaro perpendikulyar joylashgan, musbat yo'nalishga ega va uzunliklari birga teng bo'lgan uchta vektorlar.

B) Uzunliklari birga teng bo'lgan uchta vektorlar.

C) O'zaro perpendikulyar bo'lgan uchta vektorlar.

D) O'zaro kollinear va uzunliklari birga teng bo'lgan uchta vektorlar.

E) Bir xil yo'nalishga ega uchta birlik vektorlar.

2. Fazodagi i, j va k ort vektorlar uchun quyidagi tasdiqlardan qaysi biri o'rinli emas ?

A) Bu vektorlarning modullari birga teng.

B) Bu vektorlar o'zaro kollinear.

C) Bu vektorlar o'zaro perpendikulyar.

D) Bu vektorlar koordinata o'qlarida joylashgan.

E) Bu vektorlar koordinata o'qlari bo'yicha musbat yo'nalishga ega.

3. Tekislikdagi $\boldsymbol{a}=(x,\ y)$ vektorning $\bar{i}$ va $\bar{j}$ ortlar bo'yicha yoyilmasi qayerda to'g'ri ko'rsatilgan ?

A) $\boldsymbol{a}=(x+y)(\boldsymbol{i}+\boldsymbol{j})$. B) $\boldsymbol{a}=x\boldsymbol{j}+y\boldsymbol{i}$. C) $\boldsymbol{a}=x\boldsymbol{i}+y\boldsymbol{j}$.

D) $\boldsymbol{a}=xy(\boldsymbol{i}+\boldsymbol{j})$. E) To'g'ri javob keltirilmagan .

4. $\boldsymbol{a}=(2,\ -5)$ vektorning $\bar{i}$ va $\bar{j}$ ortlar bo'yicha yoyilmasi qayerda to'g'ri ko'rsatilgan ?

A) $\boldsymbol{a}=2\boldsymbol{i}-5\boldsymbol{j}$. B) $\boldsymbol{a}=-5\boldsymbol{i}+2\boldsymbol{j}$. C) $\boldsymbol{a}=-2\boldsymbol{i}+5\boldsymbol{j}$.

D) $\boldsymbol{a}=5\boldsymbol{i}-2\boldsymbol{j}$. E) $\boldsymbol{a}=2\boldsymbol{i}+5\boldsymbol{j}$.

5. ***Tasdiqni to'ldiring:*** $\boldsymbol{a}=(x,\ x)$ vektor $(x\neq 0)$ XOY koordinata tekisligining ··· joylashgan .

A) OX o'qida .

B) OX o'qiga parallel .

C) OX o'qiga perpendikulyar .

D) diagonalida ;

E) OY o'qida yoki unga parallel .

6. Fazodagi $\boldsymbol{a}=(x,\ y,\ z)$ vektorning $\bar{i},\ \bar{j},\ \bar{k}$ ortlar bo'yicha yoyilmasi qayerda to'g'ri ko'rsatilgan ?

A) $\boldsymbol{a}=(x+y+z)(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$. B) $\boldsymbol{a}=x\boldsymbol{j}+y\boldsymbol{i}+z\boldsymbol{k}$. C) $\boldsymbol{a}=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}$.

D) $\boldsymbol{a}=xyz(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$. E) $\boldsymbol{a}=x\boldsymbol{k}+y\boldsymbol{i}+z\boldsymbol{j}$.

7. $\boldsymbol{a}=(2,\ -5,\ 1)$ vektor ning $\bar{i},\ \bar{j},\ \bar{k}$ ortlar bo'yicha yoyilmasi qayerda to'g'ri ko'rsatilgan ?

A) $a=i-5j+2k$. B) $a=-5i+j+2k$. C) $a=2i-5j+k$.

D) $a=i+2j-5k$. E) $a=2i+j-5k$.

8. $\boldsymbol{a}=(0,\ y,\ z)$ vektor qanday xususiyatga ega ?

A) Bu vektor OX koordinata o'qiga parallel joylashgan.

B) Bu vektor OX koordinata o'qida yotadi.

C) Bu vektor OX koordinata o'qiga perpendikulyar joylashgan.

D) Bu vektor koordinata boshidan o'tadi.

E) To'g'ri tasdiq keltirilmagan.

9. $\boldsymbol{a}=(x,\ y,\ 0)$ vektor qanday xususiyatga ega ?

A) Bu vektor OZ koordinata o'qiga parallel joylashgan.

B) Bu vektor OZ koordinata o'qida yotadi.

C) Bu vektor OZ koordinata o'qiga perpendikulyar joylashgan.

D) Bu vektor koordinata boshidan o'tadi.

E) To'g'ri tasdiq keltirilmagan.

10. a=(0, 0, z) vektor qanday xususiyatga ega ?

A) Bu vektor OZ koordinata o'qiga parallel joylashgan.

B) Bu vektor OZ koordinata o'qida yotadi.

C) Bu vektor OZ koordinata o'qiga perpendikulyar joylashgan.

D) Bu vektor koordinata boshidan o'tadi.

E) To'g'ri tasdiq keltirilmagan.

3. Vektorlarning skalyar ko'paytmasi, uning xossalari va tatbiqlariga doir testlar

1. a va b vektorlarning skalyar ko'paytmasi qayerda to'g'ri ifodalangan ?

A) $a\cdot b=|a|\cdot|b|$. B) $a\cdot b=|a|\cdot|b|\cos\phi$. C) $a\cdot b=|a|\cdot|b|\sin\phi$.

D) $a\cdot b=|a|\cdot|b|\,\mathrm{tg}\phi$. E) $a\cdot b=|a|\cdot|b|\,\mathrm{ctg}\phi$.

2. Qaysi holda a va b vektorlarning skalyar ko'paytmasi $|a\cdot b|=\pm|a|\cdot|b|$ shartni qanoatlantiradi ?

A) a va b bir xil uzunlikka ega bo'lsa.

B) a va b ort vektorlar bo'lsa.

C) a va b ortogonal bo'lsa.

D) a va b kollinear bo'lsa.

E) hech qaysi a va b vektorlar uchun bu shart bajarilmaydi.

3. a va b vektorlarning skalyar ko'paytmasining xossasi qayerda noto'g'ri ifodalangan ?

A) $a\cdot b=b\cdot a$. B) $a\cdot a=|a|^2$. C) $a\cdot(b+c)=a\cdot b+a\cdot c$.

D) $(\lambda a,b)=(a,\lambda b)=\lambda(a,b)$. E) Barcha xossalar to'g'ri.

4. i, j, k ort vektorlarning skalyar ko'paytmalari bo'yicha quyidagi tasdiqlardan qaysi biri o'rinli emas ?

A) $i\cdot i=1$, $i\cdot j=0$, $i\cdot k=0$.

B) $j\cdot j=1$, $j\cdot i=0$, $j\cdot k=0$.

C) $k\cdot k=1$, $k\cdot i=0$, $k\cdot j=0$.

D) $j\cdot(i+k)=0$, $i\cdot(k+j)=0$, $k\cdot(i+j)=0$.

E) $j\cdot(i+j)=0$, $i\cdot(k+i)=0$, $k\cdot(j+k)=0$.

5. Tekislikda koordinatalari bilan berilgan $a=(x_1,y_1)$ va $b=(x_2,y_2)$ vektorlarning $a\cdot b$ skalyar ko'paytmasini hisoblash formulasini ko'rsating.

A) $a\cdot b=(x_1+y_1)\cdot(x_2+y_2)$. B) $a\cdot b=x_1y_1+x_2y_2$.
C) $a\cdot b=x_1x_2+y_1y_2$. D) $a\cdot b=x_1y_2+x_2y_1$.
E) $a\cdot b=(x_1+y_1)+(x_2+y_2)$.

6. Tekislikda koordinatalari bilan berilgan $a=(3,4)$ va $b=(-5, 2)$ vektorlarning $a\cdot b$ skalyar ko'paytmasini hisoblang.

A) 2. B) 23. C) −14. D) −7. E) 0.

7. Tekislkdagi $a=(x,y)$ vektorning $|a|$ modulini hisoblash formulasini ko'rsating.

A) $|x+y|$. B) $|x|+|y|$. C) x^2+y^2. D) $\sqrt{x^2+y^2}$. E) $\sqrt{|x|+|y|}$

8. $a=(-8, 6)$ vektorning $|a|$ modulini hisoblang.

A) $|a|=2$. B) $|a|=14$. C) $|a|=48$. D) $|a|=10$. E) $|a|=\sqrt{14}$

9. Fazoda koordinatalari bilan berilgan $a=(x_1, y_1, z_1)$ va $b=(x_2, y_2, z_2)$ vektorlarning $a\cdot b$ skalyar ko'paytmasini hisoblash formulasini ko'rsating.

A) $a\cdot b=(x_1+y_1+z_1)\cdot(x_2+y_2+z_2)$. B) $a\cdot b=x_1y_1z_1+x_2y_2z_2$
C) $a\cdot b=x_1x_2+y_1y_2+z_1z_2$. D) $a\cdot b=(x_1+y_1+z_1)/(x_2+y_2+z_2)$.
E) $a\cdot b=x_1/x_2+y_1/y_2+z_1/z_2$.

10. Fazoda koordinatalari bilan berilgan $a=(3, 4,-1)$ va $b=(-5, 2, 6)$ vektorlarning $a\cdot b$ skalyar ko'paytmasini hisoblang.

A) 31. B) 54. C) −13. D) −29. E) 0.

4. Vektorial ko'paytma, uning xossalalri va tatbiqlariga doir testlar

1. Agar $c=a\times b$ bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli emas ?

A) $|c|=|a||b|\sin\phi$ (ϕ– a va b vektorlar orasidagi burchak).
B) $c\perp a$ .C) $c\perp b$. D) c, a va b vektorlar bir tekislikda yotadi
E) Keltirilgan barcha tasdiqlar o'rinli .

2. Qanday a va b vektorlarning skalyar va vektorial ko'paytmalari o'zaro teng bo'ladi ?

A) a va b vektorlar teng bo'lsa.
B) a va b vektorlar kollinear bo'lsa.

C) a va b vektorlar ortogonal bo'lsa.
D) a va b vektorlar qarama-qarshi bo'lsa.
E) Bunday vektorlar mavjud emas.

3. Qaysi shartda a va b vektorlar uchun $|a\times b|=|a||b|$ tenglik o'rinli ?
A) a va b vektorlar teng bo'lsa.
B) a va b vektorlar kollinear bo'lsa.
C) a va b vektorlar ortogonal bo'lsa.
D) a va b vektorlar qarama-qarshi bo'lsa.
E) Bunday vektorlar mavjud emas.

4. Agar $|a|=4$, $|b|=5$ va $\phi=30^0$ bo'lsa, $|a\times b|=?$
A) 20 . B) 10 . C) $10\sqrt{3}$. D) 41 . E) $5\sqrt{2}$.

5. Agar $|a|=4$, $|b|=5$ va $a\cdot b=10$ bo'lsa, $|a\times b|=?$
A) 12 . B) 6 . C) $6\sqrt{3}$. D) $10\sqrt{3}$. E) $6\sqrt{2}$.

6. Vektorial ko'paytmaning xossasi qayerda xato yozilgan ?
A) $a\times b=b\times a$. B) $a\times\lambda b=\lambda a\times b$ (λ- o'zgarmas son).
C) $a\times(b+c)=a\times b+a\times c$. D) $a\times a=0$. E) $(a+b)\times c=a\times c+b\times c$.

7. $(2a+b)\times(2a-b)$ ko'paytmani hisoblang.
A) $4a\times a-b\times b$. B) 0 . C) $4a\times b$. D) $4|a|^2-|b|^2$. E) $4b\times a$.

8. Agar $a+b+c=0$ bo'lsa, quyidagi tengliklarning qaysi biri o'rinli emas ?
A) $a\times b=b\times c$. B) $a\times b=c\times a$. C) $c\times a=b\times c$. D) $a\times b=b\times c=c\times a$
E) Keltirilgan barcha tengliklar o'rinli.

9. Agar i , j va k ortlar bo'lsa, ularning vektorial ko'paytmasi qayerda to'g'ri ko'rsatilgan ?
A) $i\times i=1$. B) $i\times k=j$. C) $i\times j=k$. D) $k\times j=i$. E) $k\times k=k$.

10. Koordinatalari bilan berilgan $a=(x_1, y_1, z_1)$ va $b=(x_2, y_2, z_2)$ vektorlar uchun $a\times b=(x, y, z)$ vektorial ko'paytmaning koordinalarini topish formulasi qayerda xato ko'rsatilgan ?
A) $x=\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}$. B) $y=-\begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix}$. C) $z=\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$.

D) $y=\begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix}$. E) Keltirilgan barcha formulalar to'g'ri.

5. Vektorlarning aralash ko'paytmasi , uning xossalari va tatbiqlariga doir testlar

1. Ta'rif bo'yicha uchta vektorning aralash ko'paytmasi *abc* qanday hisoblanadi ?

A) Dastlab $a \cdot b$ skalyar ko'paytma, so'ngra $(a \cdot b)c$ ko'paytma hisoblanadi.

B) Dastlab $a \times b$ vektorial ko'paytma, so'ngra $(a \times b) \cdot c$ skalyar ko'paytma hisoblanadi.

C) Dastlab $a \times b$ vektorial ko'paytma, so'ngra $(a \times b) \times c$ vektorial ko'paytma hisoblanadi.

D) Dastlab $|a||b|$ ko'paytma, so'ngra $(|a||b|)|c|$ ko'paytma hisoblanadi.

E) To'g'ri javob keltirilmagan.

2. Uchta vektorning aralash ko'paytmasi *abc* qanday geometrik ma'noga ega ?

A) *a, b* va *c* vektorlardan hosil qilingan piramida hajmi .

B) *a, b* va *c* vektorlarga yasalgan parallelepiped hajmi .

C) *a* va *b* vektorlardan hosil qilingan parallelogramm yuzasini $|c|$ modulga ko'paytmasi.

D) *a* va *c* vektorlardan hosil qilingan parallelogramm yuzasini $|b|$ modulga ko'paytmasi.

E) *c* va *b* vektorlardan hosil qilingan parallelogramm yuzasini $|a|$ modulga ko'paytmasi.

3. *Ta'rifni yakunlang:* Fazodagi uchta vektor komplanar deyiladi, agar ...

A) ular o'zaro kollinear bo'lsa.

B) ular o'zaro perpendikulyar bo'lsa.

C) ular koordinata o'qlariga parallel bo'lsa.

D) ular koordinat tekisliklarida joylashgan bo'lsa.

E) ular bitta yoki parallel tekisliklarda joylashgan bo'lsa.

4. Fazodagi uchta *a, b* va *c* vektorlar komplanar bo'lishining zaruriy va yetarli sharti nimadan iborat ?

A) $abc>0$. B) $abc<0$. C) $abc\neq0$. D) $abc=0$; E) $abc=\pm1$.

5. Aralash ko'paytmaning xossasi qayerda xato ko'rsatilgan?

A) $abc = cab$. B) $abc = -bac$. C) $(a\times b)\cdot c = a\cdot(b\times c)$.
D) $a \parallel b \Rightarrow abc = 0$. E) $|abc| = |a|\cdot|b|\cdot|c|$.

6. Fazodagi uchta noldan farqli *a, b* va *c* vektorlar komplanar bo'lishining zaruriy va yetarli sharti nimadan iborat ?

A) $abc>0$. B) $abc<0$. C) $abc\neq0$. D) $abc=0$. E) $abc=\pm1$

7. Fazodagi *i, j* va *k* ort vektorlarning aralash ko'paytmasi qayerda noto'g'ri topilgan ?

A) $ijk=1$. B) $ikj=-1$. C) $kij=1$. D) $jik=1$. E) $kji=-1$.

8. *a, b* va *c* vektorlar o'zaro perpendikulyar va $|a|=6$, $|b|=3$ va $|c|=4$ bo'lsa, *abc* aralash ko'paytma qiymatini hisoblang.

A) 18 . B) 24 . C) 12 . D) 72 . E) 1 .

9. Qachon $|abc| = |a||b||c|$ tenglik o'rinli bo'ladi ?

A) Agar *a, b* va *c* vektorlar o'zaro perpendikulyar bo'lsa.
B) Agar $|a|=|b|=|c|$ shart bajarilsa.
C) Agar *a, b* va *c* vektorlar kollinear bo'lsa.
D) Agar *a, b* va *c* vektorlar komplanar bo'lsa.
E) Agar $a=b=c$ bo'lsa.

10. Quyidagi hollardan qaysi birida $abc=0$ tenglik bajarilmaydi?

A) *a* , *b* va *c* vektorlardan kamida bittasi nol vektor.
B) *a* , *b* va *c* vektorlardan kamida ikkitasi kollinear.
C) *a* , *b* va *c* vektorlardan kamida ikkitasi o'zaro teng.
D) *a* , *b* va *c* vektorlardan kamida ikkitasi o'zaro perpendikulyar.
E) *a* , *b* va *c* vektorlar komplanar.

Aylana –geometriyaning qalbidir. Aylanani o'rganing,
shunda siz nafaqat geometriyaning qalbini,
balki o'z qalbingizni ham o'rgangan bo'lasiz.
Sharigin

IV-BOB. ANALITIK GEOMETRIYA

§ 4.1. To'g'ri chiziqli tekislikdagi tenglamasi va oddiy masalalar.

§ 4.2. Ikkinchi tartibli chiziqlar.

§4.3. Fazodagi to'g'ri chiziq va tekislik

§ 4.4. Ikkinchi tartibli sirtlar

§ 4.1. To'g'ri chiziqli tekislikdagi tenglamasi va oddiy masalalar

1.Koordinata o'qidagi $M(x_1)$ va $M(x_2)$ nuqtalar orasidagi masofa d quyidagicha aniqlanadi

$$d = |x_2 - x_1|. \tag{4.1}$$

2. Tekislikdagi $M(x_1, y_1)$ va $M(x_2, y_2)$ nuqtalar orasidagi masofa d quyidagi formula yordamida aniqlanadi:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4.2}$$

3. Oxirlari $M(x_1, y_1)$ va $M(x_2, y_2)$ nuqtalar bilan berilgan kesimni $M(x, y)$ nuqta yordamida $\lambda(\lambda = |M_1M| : |MM_2|)$ nisbatda bo'lsak, u holda

$$x = \frac{x_1 + \lambda x_2}{1+\lambda}; \quad y = \frac{y_1 + \lambda y_2}{1+\lambda} \,. \tag{4.3}$$

4. Agar $M(x, y)$ nuqta M_1M_2 kesmani o'rtasidagi nuqtani belgilasa, u holda

$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2} \tag{4.4}$$

5. To'g'ri chiziq tenglamasi:

- boshlang'ich ordinatasi b va burchak koeffisienti k bo'lgan chiziq tenglamasi

$$y = kx + b \tag{4.5}$$

- berilgan $M(x_1, y_1)$ nuqtadan o'tuvchi (burchak koeffisienti κ) to'g'ri chiziq tenglamasi

$$y - y_1 = \kappa(x - x_1) \qquad (4.6)$$

- ikki $M(x_1, y_1)$ va $M(x_2, y_2)$ nuqtalaridan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \qquad (4.7)$$

(burchak koeffisienti $\kappa = \frac{y_2 - y_1}{x_2 - x_1}$) yoki $y = \frac{y_2 - y_1}{x_2 - x_1}x + \frac{y_1x_2 - y_2x_1}{x_2 - x_1}$ yoki

$$\begin{vmatrix} 1 & 1 & 1 \\ x & x_1 & x_2 \\ y & y_1 & y_2 \end{vmatrix} = 0$$

- kesimdagi to'gri chiziq tenglamasi

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad (4.8)$$

(a va b lar mos ravishda to'g'ri chiziqli OX va OY o'qlarini kesib o'tgan nuqtalarini koordinata boshigacha bo'lgan masofa $a \neq 0, \quad b \neq 0$)

- to'g'ri chiziqni umumiy tenglamsi

$$Ax + By + C = 0 \qquad (4.9)$$

6. A(x0;y0) nuqtadan o'tuvchi $Ax + By + C = 0$ to'gri chiziqgacha bo'lgan masofa

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \text{ yoki } d = |x_0 \cos\alpha + y_0 \sin\alpha - p|\,, \qquad (4.10)$$

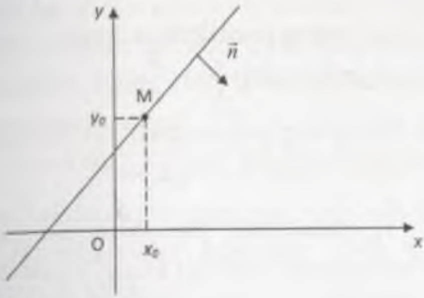
hamda koordinata boshidan to'g'ri chiziqgacha bo'lgan masofa quyidagicha topiladi:

$$d = \frac{|C|}{\sqrt{A^2 + B^2}} \text{ yoki } d = |-p|.$$

7. Ixtiyoriy M(x0;y0) nuqtadan o'tuvchi va $\bar{n} = \begin{pmatrix} A \\ B \end{pmatrix}$ vektorga perpendikulyar bo'lsa, to'g'ri chiziq tenglamasi

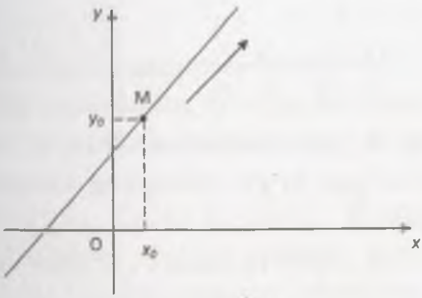
$$A(x - x_0) + B(y - y_0) = 0$$

bo'ladi (4.1-chizma).



4.1-chizma. To'g'ri chiziq va uning normal bo'lgan vektori

8. To'g'ri chiziqning kanonik tenglamasi $\frac{x-x_0}{m}=\frac{y-y_0}{n}$ bo'ladi. Bu yerda x_0, y_0 to'g'ri chiziqqa tegishli M nuqtani koordinatasini ifoda qiladi hamda $\vec{s}=\begin{pmatrix} m \\ n \end{pmatrix}$ -to'g'ri chiziqni yo'naltiruvchi vektori (ixtiyoriy nolga teng bo'lmagan vektor) (4.2-rasm).



4.2-chizma. To'g'ri chiziq va yo'naltiruvchi vektor

9.Ikki $y=k_1x+b_1$ va $y=k_2x+b_2$ yoki $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlar orasidagi burchak quyidagi formulalar yordamida aniqlanadi

$$tg\,\varphi=\frac{k_1-k_2}{1+k_1k_2}, \tag{4.11}$$

yoki

$$\cos\varphi=\frac{\pm(A_1A_2+B_1B_2)}{\sqrt{A_1^2+B_1^2}\sqrt{A_2^2+B_2^2}} \tag{4.12}$$

10. Ikki to'g'ri chiziqning parallellik sharti:

$$k_1 = k_2 \text{ yoki } \frac{A_1}{A_2} = \frac{B_1}{B_2} \qquad (4.13)$$

11. Ikki to'g'ri chiziqning perpendikulyarlik sharti

$$k_2 = -\frac{1}{k_1} \text{ yoki } A_1A_2 + B_1B_2 = 0 \qquad (4.14)$$

12. Ikki to'g'ri chiziqning kesishish nuqtasi quyidagi chiziqli tenglamalar sistemasini yechish orqali topiladi:

$$\begin{cases} y = k_1x + b_1 \\ y = k_2x + b_2 \end{cases} \text{ yoki } \begin{cases} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{cases} \qquad (4.15)$$

Agar $\Delta = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} \neq 0$ bo'lsa, u holda to'g'ri chiziqning kesishish nuqtasini koordinatasi $x = \dfrac{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}{\Delta}$ bo'ladi.

13. Uchta $A(x_1y_1), B(x_2,y_2)$ va $C(x_3,y_3)$ nuqtalarni L to'g'ri chiziqqa tegishli $(A,B,C \in L)$ bo'lishi uchun quyidagi shartni bajarilishi kerak.

$$\frac{x_2 - x_1}{x_3 - x_2} = \frac{y_2 - y_1}{y_3 - y_2} \text{ yoki } \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = 0 \qquad (4.16)$$

14. Berilgan $M_0(x_0;y_0)$ nuqtadan o'tuvchi va $\vec{s} = \{S_1,S_2\} \neq 0$ vektorga parallel bo'lgan to'g'ri chiziqning kanonik tenglamasi quyidagicha bo'ladi:

$$\frac{x - x_0}{S_1} = \frac{y - y_0}{S_2} \quad (S_1 = 0 \Rightarrow x = x_0; S_2 = 0 \Rightarrow y = y_0) \qquad (4.17)$$

Quyidagi masalalarni yeching.

4.1. *a*) $2x - y + 3 = 0$ tenglama bilan berilgan to'g'ri chiziqning burchak koeffisienti va ordinata o'qidan kesgin kesishishni aniqlang.

b). *Oy* o'qidan $b = 3$ birlik kesma ajratuvchi hamda *Ox* o'qi bilan 150^0 burchak hosil qiluvchi to'g'ri chiziq tenglamasini tuzing.

c)Oy o'qdan miqdori 7 birlikka teng kesma kesib, Ox o'qning musbat yo'nalishi bilan: 1) 30^0; 2) 120^0; 3) 135^0 li burchak hosil qiluvchi to'g'ri chiziq tenglamasini tuzing.

4.2. a)Oy o'qdan miqdori 2 birlikka teng kesma, Ox o'qning musbat yo'nalishi bilan 150^0 li burchak tashkil qiluvchi to'g'ri chiziq tenglamasini tuzing.

b) 1. $y=3x+1$ va $y=-\frac{1}{2}x+2$ to`g'ri chiziqlari qaysi biri Ox o`qi bilan 60^0 burchakdan katta burchak hosil qiladi.

4.3.a) Koordinatalar boshidan o'tib Ox o'qning musbat yo'nalishi bilan 1) 45^0 li;

2) 135^0 li burchak hosil qiluvchi to'g'ri chiziq tenglamasini tuzing.

b) $y=3x+9$ chiziqning koordinata o`qlari bilan kesimini nuqtalarini koordinatasini toping.

4.4. a)Quyidagi to'g'ri chiziqlar tenglamalari berilgan. Bu to'g'ri chiziqlar koordinata o'qlariga nisbatan joylashganini aytib bering.1) $2x-y=0$; 2) $x-3=0$; 3) $x-5=0$; 4) $x=0$;

5) $y=0$

b) $y=2x-1$, $y=\frac{x-3}{3}$, $y=\frac{3x+1}{2}$, $y=x+2$, $y=4x-2$ chiziqlarning qaysi birlari Oy o`qi bilan nuqtani kesib o`tadi.

4.5.a) To'g'ri chiziqning umumiy ko'rinishdagi tenglamalari berilgan. Ularni burchak koeffisiyentli tenglamalar shakliga keltiring:

1) $x-2y+1=0$, 2) $3x+5y-1=0$, 3) $8x+3y=0$.

4.6.a) Abssissalar o'qidan ajratgan kesmaning miqdori 2 birlik, ordinatalar o'qidan ajratgan kesmaning miqdori 3 birlik bo'lgan chiziqning tenglamasini tuzing.

4.7.a) Abssissalar o'qidan ajratgan kesmaning miqdori -5 birlik, ordinatalar o'qidan ajratgan kesmasining miqdori 5 birlik bo'lgan to'g'ri chiziqning tenglamasini tuzing.

b). C(2;-6) nuqta orqali $y=2x+7$ chiziqg'iga parallel chiziq o`tkazing.

4.8.a) Ushbu

1) $2x+3y-6=0$; 2) $3x-5y-15=0$; 3) $ax+by-ab=0$;

4) $y=5x-2$; 5) $y=x+1$; 6) $ay=-bx+c$.

To'g'ri chiziqlarning tenglamalarini ularning kesmalarga nisbatan tenglamalari shaklida yozing.

b). Quyidagicha chiziqlarning burchak koeffisiyentlarini aniqlang. a) $2x-y+7=0$ b) $x+3y-4=0$ v) $2x+5y=6$

g) $3x-2y=1$ d) $5x+5y-8=0$ e) $\frac{1}{2}x-y+1=0$

4.9.a)*O*(0;0) va *A*(-5,0) nuqtalar berilgan bo'lib, *OA* kesmada diagonallari *B*(0,3) nuqtada kesishuvchi parallelogramm yasalgan. Bu parallelogrammning tomonlari hamda diagonallarining tenglamalarini tuzing.

b)Uchburchakning tomonlarining tenglamasini

$2x+y-4=0$, $3x-y-1=0$, $x+y-1=0$ bo`lsa, uning uchlarining koordinatalarini toping.

4.10. *a*) *M*(5,2) nuqtadan o'tib, koordinata burchagidan yuzi 20 kv. birlikka uchburchak kesuvchi to'g'ri chiziq tenglamasini tuzing.

b).Uchburchak uchlarining koordinatalari A(7;9), B(2;-3), C(3;6) berilgan bo'lsa , quyidagilarni topish kerak:

1. mediananing kesish nuqtasi M ning koordinatalari;

2. BC tomon AE bissektrisani kesish nuqtasi E ning koordinatalari;

c)A(3;2) nuqtadan o'tuvchi va quyidagi keltirilgan shartlarni qanoatlantiruvchi to'g'ri chiziq tenglamasini tuzing:

1. *Ox* o'qi bilan 135^0 burchak hosil qiluvchi; 2. *Oy* o'qiga parallel; 3. B(-2;-1) nuqtadan o'tuvchi; a) va b) punktlarda keltirilgan to'g'ri chiziqlar orasidagi burchakni toping.

4.11. Rombning diagonallari mos ravishda 6 va 4 birlikka teng bo'lib, ular koordinata o'qlari uchun qabul qilngan. Rombning tomonlari tenglamalari yozilsin.

b) $x+3y-1=0$, $2x-5y-13=0$ chiziqlar va absissa o`qi hosil qilingan uchburchakning perimetrini toping

4.12. Quyidagi tengsizliklarning geometrik ma'nosini aniqlang:

1) $y>2x+1$, 2) $y<2x+1$, 3) $x>3$, 4) $x<3$.

4.13. $M(x,y)$ nuqta harakati davomida A(-3,3) va B(3,-3) nuqtalarga masofalar kvadratlarining ayirmasi hamma vaqt 36 ga teng bo'lib qoladi. Bu nuqta trayektoriyasi tenglamasini tuzing.

4.14. Tomonlari $x-2y=0$, $4x-3y-3=0$, $3x-y-7=0$ to'g'ri chiziqlardan iborat uchburchakning yuzini toping.

4.15. Quyidagi to'g'ri chiziqlarning tenglamasini normal ko'rinishga keltiring:

1) $3x+4y-20=0$, 2) $2x-3y=6$.

4.16. Koordinatalar boshidan to'g'ri chiziqqa o'tkazilgan perpendikulyar bilan Ox o'qi orasidagi burchak: 1) 45^0, 2) 135^0, 3) 60^0. To'g'ri chiziqning tenglamasini tuzing va berilgan ma'lumotlarga binoan to'g'ri chiziqni yasang.

4.17. A(2,3) va B(3,0) nuqtalarning har biridan $3x+4y-20=0$ to'g'ri chiziqqa bo'lgan masofani toping.

4.18. $y=kx+3$ to'g'ri chiziq koordinatalar boshidan $\sqrt{3}$ masofa uzoqdan o'tgan. Burchak koeffisiyent k topilsin.

4.19.a) $4x-3y=0$ to'g'ri chiziqdan 5 birlik uzoqda yotuvchi tekislik nuqtalarining geometrik o'rnining tenglamasi tuzilsin.

4.20. Quyidagi berilgan to'g'ri chiziqlar orasidagi burchakni toping.

1) $5x-y-7=0$ va $3x+2y=0$ 2) $x-2y-4=0$ va $2x-4y+3=0$,

3) $3x+2y+7=0$ va $2x-3y-3=0$ 4) $3x-2y-1=0$ va $5x+2y+3=0$

b). $\frac{x}{4}+\frac{y}{-3}=1$ kesmalar orqali berilgan umumiy tenglamani toping.

4.21.a) M(1,2) nuqtadan o'tib, $3x+2y-5=0$ to'g'ri chiziq bilan 45^0 li burchaktashkil qiluvchi chiziqning tenglamasini tuzing.

b) M(-5;1) nuqta orqali $y=3x-1$ chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.

4.22. a)A(-4,5) nuqta diagonali $7x-y+8=0$ to'g'ri chiziq yotgan kvadratning bitta uchidir. Kvadratning tomonlari va ikinchi diagonalida yotgan kvadrat tomonlarining tenglamalarni tuzing.

b) $x-y+3=0$, $2x+y-6=0$, $x+5y-21=0$ chiziqlar 1 ta chiziq dastasiga to'g'ri keladimi?

4.23.Kvadratning ikkita qarama – qarshi uchi $A(-1,3)$ va C(6,2) nuqtalarda yotadi. Kvadrat tomonlarining tenglamalarni tuzing.

b)Uchlari A(6;5), B(3;1), C(9;1)bo`lgan uchburchakning perimetri va Ah medianasini uzunligini toping.

4.24. $A(-2,3)$ nuqtadan Ox o'q bilan α burchak tashkil qiluvchi yorug'lik nuri yuborilgan. Nur Ox o'q o'qqa yetib borib, undan qaytgan. Tushgan va qaytgan nurlar tenglamalarini tuzing.

b) Uchburchakning A(-3;4), B(-4;1) va C(-1;2) uchlari bo`lsa, eng katta balandlikni toping.

4.25. $x-2y+5=0$ to'g'ri chiziq bo'yicha yo'naltirilgan yorug'lik nuri $3x-2y+7=0$ to'g'ri chiziqdan qaytgan. Qaytgan nur tenglamasini tuzing.

4.26. $M(x,y)$ nuqtadan o'tib, $Ax+By+C=0$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqning tenglamasini tuzing.

4.27. 21 – mashqning natijasiga asoslanib $M(1,4)$ nuqtadan o'tib:

1) $x-3y+5=0$, 2) $3x-4y+5=0$, 3) $8x-12y+3=0$,

4) $5x-2=0$, 5) $6y-5=0$

To'g'ri chiziqlarning har biriga parallel bo'lgan to'g'ri chiziqning tenglamasi tuzing.

4.28. Quyidagi to'g'ri chiziqlarning o'zaro joylashishini 27 mashqning natijasiga muvofiq tekshiring:

1) $x+2y+5=0$ va $3x-6y+8=0$

2) $9x-12y-4=0$ va $8x+6y+1=0$

3) $9x-12y-4=0$ va $8x+6y+1=0$

4) $4x+6y-7=0$ va $12x+18y-21=0$.

4.29. $M(-1,2)$ nuqtadan o'tib:

1) $5x-2y+3=0$, 2) $x+4y-1=0$, 3) $3x+7y-8=0$,

4) $2x+3y+5=0$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqning tenglamasini tuzing.

4.30. Ikkita to'g'ri chiziq orasidagi burchak aniqlansin:

1) $3x-y+5=0$ 2) $\frac{y}{12}-\frac{x}{3}=1$ $2x+y-7=0$ $\frac{x}{25}-\frac{y}{15}=1$

4.31. Uchlari A(2,1), B(3,1) va C(1,2) nuqtada bo'lgan uchburchak tomonlarining uzunliklari va ichki burchaklarini toping.

4.32. koordinatalar boshidan o'tib: 1) $y=2x+3$ to'g'ri chiziqqa parallel bo'lgan, 2) $x-3y-1=0$ to'g'ri chiziqqa perpendikulyar bo'lgan, 3) $y=3x-5$ to'g'ri chiziq bilan 45^0 li burchak hosil qiladigan to'g'ri chiziq tenglamasini tuzing.

4.33. Uchburchakning ikkita uchi *A*(3,4) va *B*(5,1) nuqtada bo'lib, unin balandliklari (4,1) nuqtada kesishadi. Uchburchakning uchlari C uchini toping.

4.34. $2x+3y-6=0$ to'g'ri chiziq *Ox*, *Oy* o'qlarni *A*,*B* nuqtalarda kesib o'tadi. *C* nuqta *AB* kesmani *AC*:*CB*=1:2 nisbatda bo'ladi. C nuqtadan *AB* to'g'ri chiziqqa tushirilgan perpendikulyarning tenglamasini tuzing.

4.35. Uchburchakning $3x+4y-12=0$ va $3x-7y+21=0$ tomonlari berilgan. *M*(4;3) nuqta uning medianalarining kesishish nuqtasi ekanligi ma'lum. Uchburchakning uchinchi tomoni tenglamasi tuzing.

4.36. Birinchi chorakda joylashgan va tomonining uzunligi 5 uzunlik birligiga teng bo'lgan rombning ikki tomoni absissa o'qi va $4x-3y=0$ to'g'ri chiziq bilan ustma – ust tushadi. Rombning qolgan tomonlarining tenglamalari va dioganallari tenglamalari tuzilsin.

4.37. Koordinatalari mos ravishda (1;-1), (5;2) va (4;5) bo'lgan *A*, *B*, *C* nuqtalar berilgan. *AC* to'g'ri chiziqqa nisbatan *B* nuqtaga simmetrik bo'lgan *D* nuqtaning koordinatalari topilsin va shu nuqtadan *A*, *C* nuqtalar orqali o'tkazilgan to'g'ri chiziqlarning tenglamalari tuzilsin.

4.38.Parallelogramm ikki uchining koordinatalari mos ravishda (1;1) va (2;-2) nuqtalarda bo'lib, dioganallari (-1;0) nuqtada kesishadi. Parallelogramm tomonlarining tenglamalari tuzilsin.

4.39. $2x+3y-12=0$ to'g'ri chiziqda (4;5) va (1;-2) nuqtalardan teng uzoqlikda yotgan nuqtaning koordinatalari topilsin.

4.40. *A*(-3;5) nuqtadan o'tuvchi va ordinata hamda abscissa o'qlaridan kesgan kesmalarning uzunliklari nisbati 1:2 kabi bo'lgan to'g'ri chiziqning tenglamasi tuzilsin.

4.41. *A*(-1;3) nuqtadan o'tuvchi shunday to'g'ri chiziqning tenglamasi tuzilsinki, bu to'g'ri chiziqning $x+2y+5=0$ va $x+2y-2=0$ parallel to'g'ri chiziqlar orasidagi kesmaning o'rta nuqtasi 2x-3y-11=0 to'g'ri chiziqda yotsin.

4.42. Tekislikdagi $x-2y+2=0$ to'g'ri chiziqqa $x+2y-6=0$ to'g'ri chiziqqa nisbatan ikki marta yaqin jaolashgan nuqtalar geometrik o'rinlarining tenglamalari tuzilsin.

4.43. Uchlari *A*(-2;1), *B*(2;5) va *C*(2;-1) nuqtalarda joylashgan uchburchakka tashqi chizilgan aylana markazining koordinatalari topilsin va radiusining uzunligi hisoblansin.

4.44. Koordinata boshidan o'tuvchi va A(1;3) nuqtaga B(4;2) nuqtaga nisbatan 4 marta yaqin masofadan to'g'ri chiziqning tenglamasi tuzilsin.

4.45. Gipotenuzasining tenglamasi $3x+2y-6=0$ bo'lgan va uni *A*(-1;-2) nuqtada joylashgan to'g'ri burchakli teng tomoni uchburchakning qolgan ikki tomoni tenglamalari tuzilsin.

4.46. Birinchi chorakda joylashgan rombning ikki tomoni $x-3y+4=0$ va $3x-y-4=0$ tenglamalar bilan ifodalangan. Shu tomonlarning kesishish nuqtasidan o'tgan diagonalining uzunligi $4\sqrt{2}$ uzunlik birligiga teng. Rombning qolgan tomonlarining va diagonallarining tenglamalari tuzilsin.

4.47. Quyidagi to'g'ri chiziqlar orasidagi burchak aniqlansin:

1) $\begin{cases} y = 2x - 3 \\ y = \frac{1}{2}x + 1 \end{cases}$ 2) $\begin{cases} 5x - y + 7 = 0 \\ 2x - 3y + 1 = 0 \end{cases}$ 3) $\begin{cases} 2x + y = 0 \\ y = 3x - 4 \end{cases}$

4) $\begin{cases} 3x + 2y = 0 \\ 6x + 4y + 9 = 0 \end{cases}$ 5) $\begin{cases} 3x - 4y = 6 \\ 8x + 6y = 11 \end{cases}$ 6) $\begin{cases} \frac{x}{a} + \frac{y}{b} = 1 \\ \frac{x}{b} + \frac{y}{a} = 1 \end{cases}$

4.48. $3x - 2y + 7 = 0$, $6x - 4y - 9 = 0$, $6x + 4y - 5 = 0$, $2x + 3y - 6 = 0$ to'g'ri chiziqlardan parallel va perpendikulyar bo'lganlari ko'rsatilgan.

4.49. $A(2,3)$nuqtadan o'tuvchi to'g'ri chiziqlar dastasining tenglamasi yozilsin. Shu dastadan 1) 45^0, 2) 60^0, 3) 135^0, 4) 0^0 burchak tashkil etuvchi to'g'ri chiziqlar tanlab olinsin va ular yasalsin.

4.49. $A(-2;5)$ nuqta va $2x - y = 0$ to'g'ri chiziq yasalsin. A nuqtadan o'tuvchi to'g'ri chiziqlar dastasining tenglamasi yozilsin va o'sha dastadan berilgan to'g'ri chiziqqa: 1) parallel; 2) perpendikulyar bo'lgan to'g'ri chiziq tanlab olinsin.

4.50. $2x - 5y - 10 = 0$ to'g'ri chiziqning koordinata o'qlari bilan kesishgan nuqtalaridan bu to'g'ri chiziqqa perpendikulyarlar chiqarilgan. Ularning tenglamalari yozilsin.

4.51. $A(-1,3)$ va $B(4,-2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi yozilsin.

4.52. Uchlari $A(-2,0)$, $B(2,6)$ va $C(4,2)$ nuqtalarda bo'lgan uchburchakning BD balandligi va BE medianasi o'tkazilgan. AC tomon, BE mediana va BD balandlikning tenglamalari tuzilsin.

4.53. Uchburchak tomonlari $x + 2y = 0$, $x + 4y - 6 = 0$, $x - 4y - 6 = 0$ tenglamalar bilan berilgan. Uning ichki burchklari topilsin.

4.54. Koordinatalar boshidan o'tib, $y = 4 - 2x$ to'g'ri chiziq bilan 45^0 burchak tashkil etuvchi to'g'ri chiziq tenglamasi yozilsin.

4.55. $A(-1,1)$ nuqtadan o'tib, $2x + 3y = 6$ to'g'ri chiziq bilan 45^0 burchak to'g'ri chiziq tenglamasi yozilsin.

4.56.Ox o'q bilan $\varphi = arctg2$ burchak tashkil etuvchi yorug'lik nuri $A(5,4)$ nuqtadan chiqadi va shu o'qdan qaytadi. Tushuvchi va qaytuvchi nurlarning tenglamalari yozilsin.

4.57. Uchlari $A(-2,0)$, $B(2,4)$ va $C(4,0)$ nuqtalarda bo'lgan uchburchak berilgan. Uning tomonlari, AE mediana, AO balandligining tenglamalari yozilsin va AE mediana uzunligi topilsin.

4.58. Uchlari $A(0,7)$, $B(6,-1)$ va $C(2,1)$ nuqtalarda bo'lgan uchburchak tomonlarining tenglamalari yozilsin va burchaklari topilsin.

4.59. $2x-y+8=0$ to'g'ri chiziq Ox va Oy o'qlarni A va B nuqtalarda kesib o'tadi. N nuqta AB ni $AN : NB = 3:1$ nisbatda bo'ladi. AB to'g'ri chiziqqa N nuqtadan chiqarilgan perpendikulyarning tenglamasi yozilsin.

4.58.Tomonlari $x+y=4$, $3x-y=0$, $x-3y-8=0$ tenglamalar bilan berilgan uchburchak yasalsin, uning burchaklari va yuzi topilsin.

4.59.Uchlari $A(-4,2)$, $B(2,-5)$ va $C(5,0)$ nuqtalarda bo'lgan uchburchak medianalarining kesishgannuqtasi va balandliklarining kesishgan nuqtasi topilsin.

4.60.1) $3x4y-20=0$, 2) $x+y+3=0$, 3) $y=kx+b$ to'g'ri chiziqlarning tenglamalari normal ko'rinishga keltirilsin.

4.61.Normal uzunligi $p=2$ va uning Ox o'qqa og'ish burchagi β: 1) 45^0; 2) 135^0; 3) 225^0, 4) 315^0 bo'lgan to'g'ri chiziqlar yasalsin. Bu to'g'ri chiziqlarning tenglamalari yozilsin.

4.62.$A(4,3)$, $B(2,1)$ va $C(1,0)$ nuqtalardan $3x+4y-10=0$ to'g'ri chiziqqacha bo'lgan masofalar topilsin. Nuqtalar va to'g'ri chiziq yasalsin.

4.63.Koordinatalar boshidan $12x-5y+39=0$ to'g'ri chiziqqacha bo'lgan masofa topilsin.

4.64. $2x-3y=6$ va $4x-6y=25$ to'g'ri chiziqlar o'zaro parallel ekanligi ko'rsatilgan va ular orasidagi masofa aniqlansin.

4.65. $y=kx+5$ to'g'ri chiziqkoordinatalar boshidan $d=\sqrt{5}$ masofa uzoqlikda bo'lsa, k topilsin.

4.66. $4x-3y=0$ to'g'ri chiziqdan 4 birlik uzoqlikdagi nuqtalar geometrik o'rnining tenglamasi yozilsin.

4.67. $8x-15y=0$ to'g'ri chiziqqa parallel bo'lib, A(4,-2) nuqtadan 4 birlik uzoqlikdagi to'g'ri chiziqning tenglamasi yozilsin.

4.68. $2x+3y=12$ va $3x+2y=12$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalari yozilsin.

4.69. $3x+4y=12$ va $y=0$ to'g'ri chiziqlar orasidgi burchaklar bissektrissalarining tenglamalari yozilsin.

4.70. $M(x,y)$ nuqta $y=4-2x$ to'g'ri chiziqqa nisbatan $y=2x-4$ to'g'ri chiziqdan uch marta uzoqroqda harakat qiladi. o'sha nuqta trayektoriyasining tenglamasi yozilsin.

4.71. $2x+y+6=0$ va $3x+5y-15=0$ to'g'ri chiziqlarning kesishish nuqtasi M va $N(1,-2)$ nuqtadan o'tuvchi to'g'ri chiziq englamasi (M nuqtani topmasdan) yozilsin.

4.72. $5x-y+10=0$ va $8x+4y+9=0$ to'g'ri chiziqlarning kesishish nuqtasi M dan o'tib $x+3y=0$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasi (M nuqtani topmasdan) yozilsin.

4.73.Uchlari $A(-3,0)$, $B(2,5)$ va $C(3,2)$ nuqtalarda bo'lgan uchburchak BD balandligining uzunligi topilsin.

4.74. $A(2,4)$ nuqtadan o'tuvchi va koordinatalar boshidan $d = 2$ uzoqlikda bo'lgan to'g'ri chiziq tenglamasi yozilsin.

4.75.$A(-4,-3)$, $B(-5,0)$, $C(5,6)$ va $D(1,0)$ nuqtalar trapetsiyaning uchlari bo'lishi tekshirilsin va uning balandligi topilsin.

4.76. Koordinatalar boshidan $A(2,2)$ va $B(4,0)$ nuqtalargacha masofalari bir xil bo'lgan to'g'ri chiziq o'tkazilgan. Bu masofa topilsin.

4.77. $x+2y-5=0$ to'g'ri chiziq $\sqrt{5}$ masofa uzoqlikda bo'lgan geometric o'rnining tenglamasi yozilsin.

4.78. $y=-x$ to'g'ri chiziqqa nisbatan $y=x$ to'g'ri chiziqdan ikki marta uzoqroqda harakat qiluvchi $M(x,y)$ nuqta trayektoriyasining tenglamasi yozilsin.

4.79. $2x-3y+5=0$ va $3x+y-7=0$ to'g'ri chiziqlarning kesishish nuqtasi $M(x,y)$ dan o'tuvchi va $y=2x$ to'g'ri chiziqqa perpendikulyar to'g'ri chiziq tenglamasi (M nuqtani topmasdan) yozilsin.

4.80. $A(-4,0)$ va $B(0,6)$ nuqtalar berilgan. AB kesma o'rtasidan Oy o'qidagiga qaraganda Ox o'qdan ikki baravar katta kesma ajratuvchi chiziq o'tkazilsin.

4.81. $A(-2,0)$ va $B(2,-2)$ nuqtalar berilgan. OA kesmani tomon deb olib, diagonallari B nuqtada kesishuvchi $OACD$ parallelogramm yasalgan. Parallelogramm tomonlarining, diagonallarining tenglamalari yozilsin va CAD burchak topilsin.

§ 4.2. Ikkinchi tartibli chiziqlar

1. Ikkinchi tartibli chiziqning umumiy tenglamasi

$$Ax^2 + Bxy + Cy^2 + Ey + F = 0 \qquad (4.18)$$

2. Radiusi R va markazi $C(x_0;y_0)$ va O(0;0) nuqtada bo'lgan aylana tenglamasi quyidagicha ko'rinishda bo'ladi:

$$(x - x_0)^2 + (y - y_0)^2 = R^2 \qquad (4.19)$$

$$x^2 + y^2 = R^2 \qquad (4.20)$$

3. Ellipsning kanonik tenglamasi (koordinata o'qlari ellips o'qi bilan ustma-ust tushadi)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (4.21)$$

bu yerda a va b –ellips o'qlari:

$$b^2 = a^2 - c^2 \qquad (4.22)$$

$F_1(-c_1;0)$ va $F_2(c;0)$ - ellips fokuslarini ifodalaydi $(a > b)$. Ellipsning eksttisiteti ($\varepsilon < 1$-ellips uchun)

$$\varepsilon = \frac{c}{a} \qquad (4.23)$$

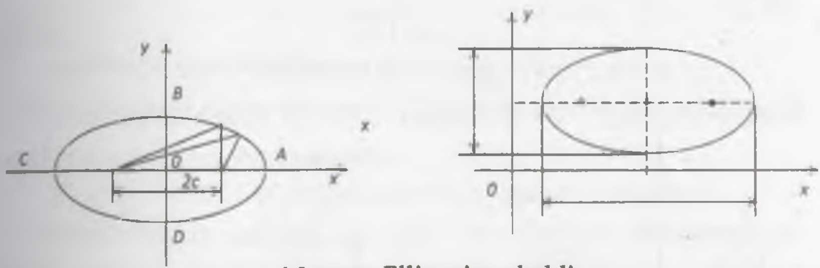
1. Umumiy holda aylana tenglamasi

$$Ax^2 + Ay^2 + Bx + Cy + D = 0 \qquad (1)$$

Ba`zi bir xususiy hollarni ko`rib chiqamiz. a) $B^2 + C^2 > 4AD$, u holda $R > 0$. (1) tenglama ayniyat ifoda qoladi. б) $B^2 + C^2 = 4AD$, $R = 0$. Ko`rilayotgan tenglamaga faqat (a,b) nuqta mos keladi. b) $B^2 + C^2 < 4AD$, u holda $R$ - mavhum son bo`lib, (1) tenglama ma`nosini yo`qotadi. Bu tenglamani markazi $F(a,b)$ va radiusi R bo'lgan aylana $(x-a)^2+(y-b)^2=R^2$ markazi absissa o'qida joylashgan aylana tenglamasi a) $(x-a)^2+y^2=R^2$ joylashgan aylana tenglamasi б) $x^2+(y-b)^2=R^2$ -markazi ordinata o'qida bo'lgan aylana tenglamasi

в) $x^2+y^2=R^2$ - markazi koordinatalar boshida bo'lgan aylana tenglamasi

$$a=-\frac{B}{2A},\quad b=-\frac{C}{2A},\quad R=\frac{\sqrt{B^2+C^2-4AD}}{2A}$$



4.3-rasm. Ellipsning shakli

3. Markazi (α,β) nuqtada bo'lgan ellips ko'rib chiqaylik. $\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$.Ellipsning uchlarining koordinatalari

$$(\alpha-a,\beta),(\alpha+a,\beta),(\alpha,\beta+b),(\alpha,\beta-b).$$

Agar fokuslar orasidagi masofa 2c bo`lsa, u holda koordinatasi $(\alpha-c,\beta),(\alpha+c,\beta)$bo'ladi$(a>b)$. Agar $a<b$ bo`lsa, u holda fokus nuqtalarining koordinatalar $(\alpha,\beta+c),(\alpha,\beta-c)$ bo`ladi.

Ellipsning umumiy tenglamasini

$$Ax^2+By^2+Cx+Dy+E=0 \qquad (2)$$

deb yozish mumkin. Bu tenglamani $\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=-1$ olamiz. U holda mavhum ellipsni olamiz, ya`ni tenglamani o`z ma`nosini yo`qotadi.

Xususiy hollar a) $\alpha=0,\beta\neq 0,$ $\frac{x^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$ $a>b$ bo'lsa.

б) $\beta=0,$ $\alpha\neq 0$, u holda kanonik tenglama $\frac{(x-\alpha)^2}{a^2}+\frac{y^2}{b^2}=1$ bo'ladi.

Ellipsning ixtiyoriy M(x;y) nuqtasidan fokus masofagacha bo'lgan masofa quyidagi formula bilan topiladi.

$$r_1 = a - \varepsilon x, \quad r_2 = a + \varepsilon x \qquad (4.24)$$

4. Giperbolani kanonik tenglamasi (giperbola o'qi koordinata o'qlari ustma-ust tushadi)

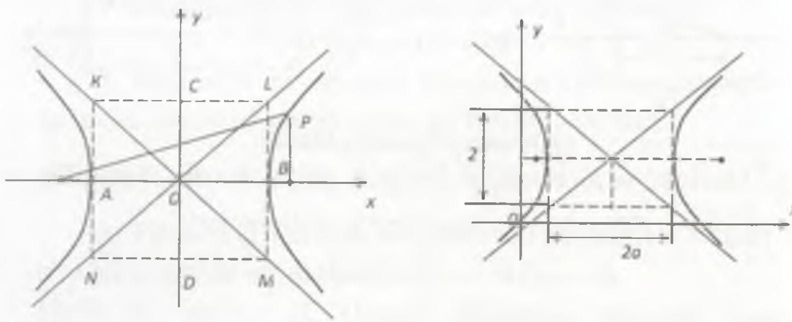
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (4.25)$$

bu yerda a,b mos ravishda haqiqiy va mavhum yarim o'qlari,

$$c^2 = a^2 + b^2, \qquad (4.26)$$

$F_1(-c_1;0)$ va $F_2(c;0)$ - giperbola fokuslari, $c > a$.

Giperbolaning ekstrentsiteti (4.23) formula orqali topiladi ($\varepsilon > 1$).



4.4-rasm. Giperbolaning shakli

Giperbolaning M(x;y) nuqtadan fokusgacha bo'lgan masofa (4.27) quyidagi formula yordamida aniqlanadi.

$$r_1 = |\varepsilon x - a|, \quad r_2 = |\varepsilon x + a| \qquad (4.27)$$

Giperbolaning ikkita asimptotasi quyidagi formula bilan aniqlanadi.

$$y = \pm\frac{b}{a}x \qquad (4.28)$$

5. Koordinata boshida joylashgan parabolaning kanonik tenglamasi

$$y^2 = 2px \qquad (4.29)$$

(agar OX o'qiga nisbatan simmetrik bo'lsa), yoki

$$x^2 = 2px \qquad (4.30)$$

$$y = Ax^2 \quad (4.31)$$

(agar Oy o'qiga nisbatan simmetrik bo'lsa), bu yerda p yoki $A = \frac{1}{2p}$ -parabola parametri.

Parabolaning fokus nuqtasi $F\left(\frac{p}{2};0\right)$ dan OX (fokus radius) bo'lgan masofa quyidagi formula bilan aniqlanadi.

$$r = x + \frac{p}{2} \quad (4.32)$$

Parabola direktrissasining tenglamasi

$$x = -\frac{p}{2} \quad (4.33)$$

Aylanaga doir misollar:

4.82. *a*) Markazi *C*(-4,3), rasiusi $R = 5$ bo'lgan aylana tenglamasi yozilsin va u yasalsin.

A(-1,-1), *B*(3,2), *O*(0,0) nuqtalar bu aylanada yotadimi?

b)Aylananing ushbu $2x^2 + 2y^2 - 3x + 4y + 2 = 0$ tenglamasiga ko'ra, uning makazi va radiusini aniqlang. c). Aylana diametrlaridan biri uchlarining koordinatalari berilgan: A(1;4) va B(-3;2). Shu aylananing tenglamasini tuzing.

4.83. *A*(-4,6) nuqta berilgan. Diametrik *OA* kesmadan iborat aylana tenglamasi yozilsin.

4.84. 1) $x^2 + y^2 - 4x + 6y - 3 = 0$; 2) $x^2 + y^2 - 8x = 0$;

3) $x^2 + y^2 + 4y = 0$ aylanalar yasalsin.

4.85. $x^2 + y^2 + 5x = 0$ aylana $x + y = 0$ to'g'ri chiziq yasalgan va ularning kesishgan nuqtalari topilsin.

4.86. *A*(1,2) nuqtadan o'tuvchi va koordinata o'qlariga urinuvchi aylana tenglamasi yozilsin.

4.87. $x^2 + y^2 + 4x - 6y = 0$ aylananing *Oy* o'q bilan kesishgan nuqtalariga o'tkazilgan radiuslari orasidagi burchak topilsin.

4.88. *A*(-1,3), *B*(0,2) va *C*(1,-1)nuqtalardan o'tuvchi aylana tenglamasi yozilsin.

4.89. A(4,4) nuqtadan va $x^2 + y^2 + 4x - 4y = 0$ aylana bilan $y = -x$ to'g'ri chiziqning kesishgan nuqtalaridan o'tuvchi aylana tenglamasi yozilsin.

4.90. $y=-\sqrt{-x^2-4x}$ egri chiziqning joylashish sohasi aniqlab, shakli chizilsin.

4.91. $x^2+y^2-8x-4y+16=0$ aylanaga koordinatalar boshidan o'tkazilgan urinmalarning tenglamalari yozilsin.

4.92. $A(a,0)$ nuqta berilgan. M nuqta shunday harakat qiladiki, ΔOMA da OMA burchak doimo to'g'ri burchak bo'lib qoladi. M nuqta trayektoriyasining tenglamasi yozilsin.

4.93. A(-6,0) va B(2,0) nuqtalar berilgan. Shunday nuqtalarning geometrik o'rni topilsinki, ulardan OA va OB kesmalar teng burchaklar ostida ko'rinsin.

4.94. $M(x,y)$ nuqta shunday harakarlanadiki, undan $A(-a,0)$, $B(0,a)$ va $C(a,0)$ nuqtalargacha bo'lgan masofalar kvadratlarining yig'indisi $3a^2$ ga teng bo'lib qolaveradi. Nuqta trayektoriyasining tenglamasi yozilsin.

4.95. $M(x,y)$ nuqta shunday harakarlanadiki, undan koordinat burchaklarining bissektrisalarigacha bo'lgan masofalar kvadratlarining yig'indisi a^2 ga teng bo'lib qolaveradi. Nuqta trayektoriyasining tenglamasi yozilsin.

4.96. $x^2+y^2=a^2$ aylana berilgan. Uning $A(a,0)$ nqutasidan mumkin bo'lgan barcha vatarlar o'tkazilgan. Bu vatarlar o'rtalarining geometrik o'rni aniqlansin.

4.97. A(-3;0) va B(3,6) nuqtalar berilgan. Diametrik AB kesmadan iborat aylana tenglamasi yozilsin.

Ellipsga doir misollar:

4.98. a) $x^2+4y^2=16$ ellips yasalsin, uning fokuslari va ekssentrisiteti topilsin.

b) Uchlari (±5; 0) va (0; ±3) bo`lgan ellipsning kanonik tenglamasi va fokus nuqtalarining koordinatlarini toping?

4.99. a) Agar ellipsning: 1) fokuslari orasidagi masofa 8 ga teng bo'lib, kichik yarim o'qi $b=3$; 2) katta yarim o'qi $a=6$, ekstrensiteti $\varepsilon=0{,}5$ bo'lsa, uning kanonik tenglamasi yozilsin.

b) Ellipsning fokuslari abtsissa o`qida hamda o`qlari 16 va 10 bo`lsa kanonik tenglamani tuzing.

c)Ellipsning fokus nuqtalarining koordinatalari (±3; 5) va katta o`qi 10 ga teng kanonik tengalamasini yozing.

4.100. a) Ellipsning katta yarimo'qi $a=5$ va c parametri: 1) 4,8; 2) 4; 3) 3; 4) 1,4; 5) 0 ga teng bo'lsa, uning kichik yarim o'qi b va ekstrensiteti ε topilsin.

b) Ellipsning $4x^2+9y^2=36$ tenglamasini kanonik ko`rinishiga olib keling va o`qlar uzunliklarini hisoblang.

c) Ellips a) $x^2+4y^2-4x+8y-8=0$, b) $9x^2+4y^2+54x-40y+145=0$ larning markazi fokus nuqtalarining koordinatlari hamda o`qlarining uzunliklarini toping.

4.101. a)Yer fokuslaridan birida quyosh joylashgan ellips bo'yicha harakat qiladi. quyoshdan yergacha bo'lgan eng kichik masofa taxminan 147,5 million kilometrga, eng katta masofa 152,5 million kilometrga teng bo'lsa, yer orbitasining katta yarim o'qi va eksentrisiteti topilsin.

b) Ellipsning $64x^2+9y^2-576=0$ tenglamasini kanonik ko`rinishga olib keling va fokus nuqtasini koordinatalarini toping

4.102. a) Koordinata o'qlariga nisbatan simmetrik bo'lgan ellips $M(2,\sqrt{3})$ va $B(0,2)$ nuqtalardan o'tadi. Uning tenglamasi yozilsin va M nuqtadan fokuslargacha bo'lgan masofa topilsin.

b) Ellipsning $16x^2+25y^2-400=0$ tenglamasini kanonik ko`rinishga olib keling va o`q uchlarini koordinatalarini toping

4.103. a) Fokuslati Ox o'qda yotuvchi ellips koordinata o'qlariga nisbatan simetrik bo'lib,

$M(-4,\sqrt{21})$ nuqtadan o'tadi va $\varepsilon=\frac{3}{4}$ ekstrensitetiga ega. Ellips tenglamasi yozilsin va M nuqtaning fokal radiuslari topilsin.

b) Ellipsning fokus nuqtalari koordinatalari $(\pm 8,0)$ va ektsentrisiteti $e=0,8$.Kanonik tenglamasini tuzing.

4.104. *a*) $x^2+2y^2=18$ ellipsning o'qlari orasidagi burhckani teng ikkiga bo'luvchi vatar uzunligi topilsin.

b) Quyidagi egri chiziq tenglamasi qanday chiziqqa tegishli ekanligini aniqlang va grafini chizing. $2x^2+2xy+2y^2+\sqrt{2}x-\sqrt{2}y-5=0$

c) Ellipsning fokuslari abtsissa o`qida koordinat boshiga nisbatan simmetrik joylashgan. O`qlar aylanasi 6 va ularning nisbati 2. Ellipsning kanonik teng tuzing.

4.105. a) $9x^2+25y^2=225$ ellipsda shunday $M(x,y)$ nuqta topilsinki, undan o'ng fokusgacha bo'lgan masofa chap fokusgacha bo'lgan masofadan 4 marta katta bo'lsin.

b) Ellips o`qlarining qanday nisbatida ektsentrensitet a) $\frac{1}{2}$,

b) $\frac{1}{4}$, v) $\frac{1}{10}$, g) $\frac{1}{12}$ teng bo`ladi.

Giperbola ga doir misollar:

4.106. a) $x^2-4y^2=16$ giperbola va uning asimptotalari yasalsin. Giperbolaning fokuslari, ekstrensiteti va asimptotalari orasidagi burchka topilsin. b) Quyidagi tenglamalarni kanonik ko`rinishga olib kelitiring. a) $9x^2-16y^2-144=0$ b) $2x^2-y^2=6$ c) Giperbolaning fokusi $(0,\pm 3)$ va asimptotasi $y=\pm 2x$ berilgan. Giperbolaning tenglamasi va ekstsentritentini toping

4.107 a) $x^2-4y^2=16$ giperbolada ordinatasi 1 ga teng M nuqta olingan. Undan fokuslargacha bo'lgan masofalar topilsin.

b)Giperbola $\frac{x^2}{144}+\frac{y^2}{25}=1$ ning fokus nuqtaning koordinatalari va o`q uzunligini toping?

c)Quyidagi tenglamalardan qaysilari giperbolani tenglamasini ifoda qiladi:

a) $3x^2-5y^2+10x+12y-78=0$ v) $3x^2+7y^2-140x+11x-500=0$

b) $13x^2-17x+10y-24=0$ g) $4x^2-4y^2-4x+7y-120=0$

4.108.a) Giperbola koordinata o'qlariga nisbatan simmetrik bo'lib, M(6,2$\sqrt{2}$) nuqtadan o'tadi va $b=2$ mavhum yarim o'qqa ega. Uning tenglamasi yozilsin hamda M nuqtadan fokuslargacha bo'lgan masofalar topilsin.

b) Teng tomonli giperbolaning ekstsentristenti toping?

4.109.a)Uchlari $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsning fokuslari esa uning uchlarida bo'lgan giperbolaning tenglamasi yozilsin. b)

$\frac{y^2}{64}-\frac{x^2}{17}=1$ giperbolaning ekstretsentritenti fokus koordinatasini toping.

4.110. a) $y^2=a^2+x^2$ giperbola yasalsin, uning fokuslarining koordinatalari va asimptotalari orasidagi burchak topilsin. b) $4x^2-25y^2-100=0$ giperbolaning asimptotalarini toping.

c) Giperbola $\frac{(y+7)^2}{25}-\frac{(x-3)^2}{16}=1$ ni 3 birlikga abtsissa o`qini manfiy yo`nalishida va 4 birlik ordinata o`qini musbat yo`nalishi bo`yicha parallel ko`chiring va tenglamasini tuzing.

4.111 a) $x^2-4y^2=16$ giperbolaga A(0,-2) nuqtadan io'tkazilgan urinmalarning tenglamalari yozilsin.

b) Giperbola $3x^2-5y^2=28$ va to'g'ri chiziq $2x-y=8$ning kesim nuqtasining koordinatalarini toping.

c) Giperbola $\frac{(x+1)^2}{4}-\frac{(y-2)^2}{3}=1$ ni ordinata o`qi musbat yo`nalishi parallel 2 birlikka siljiting.

4.112.a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning fokusidan asimptotalarigacha bo'lganmasofalar va asimptotalari orasidagi burchak topilsin.

b) $\frac{x^2}{25}-\frac{y^2}{4}=1$ giperbola A(5,0), B($\frac{5\sqrt{5}}{2}$,-1), C($5\sqrt{2}$, -2) va D(2, -3) nuqtalar orqali o`tadimi?.

4.113.a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaga ichki chizilgan kvadratning tomoni topilsin va qanday giperbolalarga kvadartni ichki chizish mumkinligi tekshirilsin.

b)Giperbolaning fokus nuqtasini koordinatalari $(\pm 2\sqrt{2};0)$ va asimptotasi $y=\pm x$ berilgan. Giperbolaning tenglamasini toping.

Parabolaga doir misollar:

4.114.a) $F(0,2)$ nuqtadan va $y = 4$ chiziqdan bir xil uzoqlashgan nuqtalar geometrik o'rnining tenglamasi tuzilsin. Bu egri chiziqning koordinata o'qlari bilan kesishgan nuqtalari topilsin va u yasalsin. b) $y^2=12x$ parabola grafigi A(3; -6), B(0.5, $\sqrt{6}$) va C(1;3)nuqtalar orqali o`tadimi?

4.115. a)Koordinatalar boshidan va x = -4 to'g'ri chiziqdan bir xil uzoqlikda bo'lgan nuqtalar geometrik o'rnining tenglamasi tuzilsin. Bu egri chiziqning koordinata o'qlari bilan kesishgan nuqtalari topilsin va u yasalsin.

b) Har bir parabolaning fokus va direkrissachini toping:

c) $y^2=-12x$, b) $y^2+x=0$, d) $3x-y^2=0$

4.116. a)1) (0,0) va (1,-3) nuqtalardan o'tuvchi va Ox o'qqa nisbatan simmetrik;b) 2) (0,0) va (2,-4) nuqtalardan o'tuvchi Oy o'qqa simmetrik bo'lgan parabola tenglamasi yozilsin. c) Uchi koordinata boshida va fokusi a) (3,0), b) (-2,0), c) (0,4),d) (0; $-\frac{1}{4}$)bo`lgan parabolalarning tenglamasini toping.

4.117. a)Markazi $y^2= 2px$ parabolaning fokusida bo'lib, parabola direktrissasiga urinuvchi aylana tenglamasi yozilsin parabola va aylananing kesishgan nuqtalari topilsin.

b) Uchi koordinata boshida va direkrisisasi a) $x=2$, b) $x=-5$, v) $y=3$, g) $y=-2$bo`lgan parabola tenglamaning toping.

4.118.a) $x^2+y^2+4y=0$ aylana $x+y=0$ to'g'ri chiziqning kesishgan nuqtalaridan o'tib, Oy o'qqa nisbatan simmetrik bo'lgan parabolaning va uning direkttissasining tenglamalari yozilsin. Aylana, to'g'ri chiziq va parabola yasalsin.

b) Radius vektori 10 bo`lgan $y^2=8x$ parabolaning fokus nuqtasini koordinatasi topilsin.

4.119.a) $y^2= 6x$ parabolada fokal radius – vektori 4,5 ga teng bo'gan nuqta topilsin. b) $y^2=36x$ parabolaning (4; y) nuqtasini fokal radius vektorini toping.

4.120a)Projektorning oynali sirti parabolaning o'z simmaterik o'qi atrofida aylanishidan hosil bo'lgan. Oynaning diametric 80 sm, chuqurligi 10 sm. Nurlarning parallel dasta sharlida qaytishi uchun yorug'lik manbai parabolaning fokuslida o'rnatilishi kerak bo'lsa, yorug'lik manbai parabola uchidan qanday masofada o'rnatilishi kerak?

b) Parabolani uchi va o`qini toping. a) $y=x^2+5$, b) $y=12-x^2$, v) $y=x^2+2x-3$ g) $x=y^2+8y$, d) $y=x^2-x+4$,

4.121. $y=-\sqrt{x}$ egri chiziqning joylashish sohasi aniqlansin. bu egri chiziq yasalsin.

4.122. $y^2=6x$ parabola uchidan o'tishi mumkin bo'lgan barcha vatarlar o'tkazilgan, bu vatarlar o'rtalari geometriko'rnining tenglamasi yozilsin.

Parametrik formada berilgan ikkinchi tartibli chiziqlar

1. Parametrik formada berilgan egri chiziq tenglamasi $x=\sqrt{4-t^2}$, $y=t+3$ Dekart koordinatalar sistemasida qanday chiziqni ifodalaydi.

2. $x=a+r\cos t$, $y=b+r\sin t$.

3. $x=5\cos t$, $y=4\sin t$

4. $x=a\cos t$, $y=b\sin t$

5.Material nuqta a) $x=3\cos^2 t$, $y=4\sin^2 t$ b) $x=\cos t$, $y=\sin t$ v) $x=5\cos\frac{t}{2}$, $y=2\sin\frac{t}{2}$

g) $x=4+t^2$, $y=\frac{t}{2}$ qonuniyat bilan harakatlansa, material nuqtaning harakat trayektoriyasini toping.

b) $x^2+y^2=1$ (aylana) v) $\frac{x^2}{25}+\frac{y^2}{4}=1$ (ellips) g) $y^2=\frac{1}{4}x-1$ (parabola)

6. v_0 boshlang'ich tezlik bilan α burchak ostida otilgan jismning harakat tenglamasi $x=v_0 t\cos t$; $y=v_0 t\sin t-\frac{gt^2}{2}$ bu yerda t -vaqt, g -erkin tushish tezlanishi $\left(g\approx 10\frac{м}{c^2}\right)$. Jismning harakat trayektoriyasini $\alpha=\frac{\pi}{4}$ va $v_0\approx 10\frac{м}{c}$ bo`lganda toping.

§4.3. Fazodagi to'g'ri chiziq va tekislik

1. Tekislik tenglamalari:

- umumiy tenglamasi:

$$Ax+By+Cz+D=0\ , \qquad (4.34)$$

bu yerda A,B,C,D lar haqiqiy sonlar;

-$M_0(x_0;y_0;z_0)$ nuqtadan o'tuvchi va $\vec{n}=(A,B,C)$ vektorlarga perpendikulyar bo'lgan tekislik tenglamasi

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0; \qquad (4.35)$$

-koordinata o'qlarini mos ravishda a,b,c, nuqtalardan kesib o'tgan tekislik tenglamasi

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1; \qquad (4.36)$$

2. $A(x_0;y_0;z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikgacha bo'lgan d masofa

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}} \qquad (4.37)$$

3. Berilgan ikkita $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tekisliklar orasidagi burchak (φ) quyidagicha topiladi

$$\cos\varphi=\pm\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}. \qquad (4.38)$$

4. Ikki tekislikka parallelik

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2} \qquad (4.39)$$

va perpendikulyarlik shartlari:

$$A_1A_2+B_1B_2+C_1C_2=0 \qquad (4.40)$$

5. Tekislikdagi to'g'ri chiziq ikki tekislikni kesishidan to'g'ri chiziq hosil bo'ladi

$$\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases} \qquad (4.41)$$

6. $M(x_1;y_1;z_1)$ nuqtadan o'tuvchi va $\vec{s}=(m,n,p)$ vektorlarga parallel bo'lgan to'g'ri chiziq tenglamasi

$$\frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p} \qquad (4.42)$$

to'g'ri chiziqning kanonik tenglama yoki

$$x=x_1+mt; \quad y=y_1+nt; \quad z=z_1+pt \qquad (4.43)$$

to'g'ri chiziqning parametrik tenglamasi deyiladi.

7. $M_1(x_1;y_1;z_1)$ va $M_2(x_2;y_2;z_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (4.44)$$

8. agar ikki to'g'ri chiziqning yo'naltiruvchi vektori berilgan bo'lsa, $\vec{S}_1=(m_1,n_1,p_1)$ va $\vec{S}_2=(m_2,n_2,p_2)$, u holda

$$\cos\varphi=\pm\frac{m_1m_2+n_1n_2+p_1p_2}{\sqrt{m_1^2+n_1^2+p_1^2}\sqrt{m_2^2+n_2^2+p_2^2}} \qquad (4.46)$$

9. Ikki to'g'ri chiziqning parallelligi

$$\frac{m_1}{m_2}=\frac{n_1}{n_2}=\frac{p_1}{p_2} \qquad (4.47)$$

bo'ladi.

10. Ikki to'g'ri chiziqning perpendikulyarlik sharti

$$m_1m_2+n_1n_2+p_1p_2=0 \qquad (4.48)$$

11. Berilgan $\frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p}$ to'g'ri chiziq va $Ax+By+Cz+D=0$ tekisliklar orasidagi burchak

$$\sin\varphi=\frac{|Am+Bn+Cp|}{\sqrt{A^2+B^2+C^2}\sqrt{m^2+n^2+p^2}} \qquad (4.49)$$

12.To'g'ri chiziq va tekislik ikkinchi parallelik sharti

$$Am+Bn+Cp=0 \qquad (4.50)$$

13. To'g'ri chiziq va tekislikning parallelik sharti

$$\frac{A}{m}=\frac{B}{n}=\frac{C}{p} \qquad (4.51)$$

bo'ladi.

Parametrik formada berilgan ikkinchi tartibli chiziqlar

1. Aylana, $x^2+(y-2)^2=4$

2. $(x-a)^2+(y-b)^2=r^2$ aylana.

3. Ellips $\frac{x^2}{25}+\frac{y^2}{16}=1$

4. Ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

5. $\frac{x}{3}+\frac{y}{4}=1$ (to'g'ri chiziq)

b) $x^2+y^2=1$ (aylana)

v) $\frac{x^2}{25}+\frac{y^2}{4}=1$ (ellips)

g) $y^2=\frac{1}{4}x-1$ (parabola)

6.: $y\approx 1-0,1x^2$

Quyidagi masalarni yeching

4.123. $A\left(a;-\frac{a}{2};a\right)$ va $B\left(0;\frac{a}{2};0\right)$ nuqtalardan teng uzoqlikda bo'lgan nuqtalar geometrik o'rnining tenglamasi yozilsin.

4.124. *a*) $M_1(-4;0;4)$ nuqtadan o'tuvchi *Ox* va *Oy* o'qlardan $a=4$ va $b=3$ kesmalar ajratuvchi tekislikning tenglamasi yozilsin.

b) Oz o'q hamda M(4;-2;7) nuqta orqali o'tuvchi tekislik tenglamasini tuzing.

c) N(4;-5;7) nuqta orqali o'tib, *Oyz* tekislikka parallel bo'lgan tekislik tenglamasini tuzing.

4.125. *a*) 1) $x-2y+2z-8=0$ va $x+z-6=0$; 2) $x+2z-6=0$ va $x+2y-4=0$ tekisliklar orasidagi burchak topilsin.

в) $M_1(2;3;-1)$ va $M_2(1;5;3)$ nuqtalardan o'tib $3x-y+3z+15=0$ tekislikka perpendikulyar bo'lgan tekislik tenglamasini tuzing.

c) $2x+3y-5z+30=0$ tekislikning koordinata o'qlari bilan kesishgan nuqtalarining koordinatalarini toping.

4.126. (-1; -1; 2) nuqtadan o'tuvchi va $x-2y+z-4=0$ hamda $x+2y-2z+4=0$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

4.127. $4x+3y-5z-8=0$ va $4x+3y-5z+12=0$ parallel tekisliklar orasidagi masofa topilsin.

4.128. $2x-y+3z-9=0$; $x+2y+2z-3=0$; $3x+y-4z+6=0$ tekislikning kesishgan nuqtasi topilsin.

4.129. 1) $\begin{cases} x=z+5 \\ y=4-2z \end{cases}$ va 2) $\left\{\frac{x-3}{1}=\frac{y-2}{2}=\frac{z-3}{1}\right.$ to'g'ri chiziqlarni ung *xOy* va *xOz* tekisliklardagi izlari topilsin va to'g'ri chiziqlar yasalsin.

4.130. $\left.\begin{matrix} x+2y+3z-13=0 \\ 3x+y+4z-14=0 \end{matrix}\right\}$ to'g'ri chiziq tenglamalarini:

1) proyeksiyalari bo'yicha; 2) kanonik ko'rinishda yozilsin. To'g'ri chiziqning koordinata tekisliklaridagi izlari topilsin hamda to'g'ri chiziq va uning proyeksiyalari yasalsin.

4.131. 1) $\begin{cases} y=3 \\ z=2, \end{cases}$ 2) $\begin{cases} y=2 \\ z=x+1 \end{cases}$ 3) $\begin{cases} x=4 \\ z=y \end{cases}$ to'g'ri chiziq yasalsin va ularning vektorlari aniqlansin.

4.132. $A(-1;\ 2;\ 3)$ va $B(2;\ 6;\ -2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari yozilsin va uning yo'naltiruvchi kosinuslari topilsin.

4.133. $A(2;\ -1;\ 3)$ va $B(2;\ 3;\ 3)$ nuqtalardan o'tuvchi to'g'ri chiziq yasalsin va uning tenglamalari yozilsin.

4.134. $\begin{cases} x-y+z-4=0 \\ 2x+y-2z+5=0 \end{cases}$ va $\begin{cases} x+y+z-4=0 \\ 2x+3y-z-6=0 \end{cases}$ to'g'ri chiziqlar orasidagi burchak topilsin.

4.135. $\frac{x}{2}=\frac{y}{3}=\frac{z}{1}$ to'g'ri chiziqning $x=z+1,\ \ y=1-z$ to'g'ri chiziqqa perpendikulyar ekanini ko'rsating.

4.136. (-4; 3; 0) nuqtadan o'tuvchi va $\left.\begin{matrix} x-2y+z=4 \\ 2x+y-z=0 \end{matrix}\right\}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamalari yozilsin.

4.137. (2; -3; 4) nuqtadan Oz o'qqa tushilgan perpendikulyarning tenglamalari yozilsin.

4.138. $N(2;\ -1;\ 3)$ nuqtadan $\frac{x+1}{3}=\frac{y+2}{4}=\frac{z-1}{5}$ to'g'ri chiziqqacha bo'lgan masofa topilsin.

4.139. $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+3}{2}$ va $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z+1}{2}$ parallel to'g'ri chiziqlar orasidagi masofa topilsin.

4.139. $y=3x-1,\ \ 2z=-3x+2$ to'g'ri chiziq bilan $2x+y+z-4=0$ tekislik orasidagi burchak topilsin.

4.140. $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z-1}{3}$ to'g'ri chiziq $2x+y-z=0$ tekislikka parallel ekanligi, $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z+3}{3}$ to'g'ri chiziq esa shu tekislik ustida yotishi ko'rsatilgan.

4.141. (-1; 2; -3) nuqtadan o'tuvchi va $x=2,\ \ y-z=1$ to'g'ri chiziqqa perpendikulyar tekislikning tenglamasi yozilsin.

4.142. $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{3}$ to'g'ri chiziqdan va (3; 4; 0) nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

4.143. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+2}{2}$ to'g'ri chiziqdan o'tuvchi va $2x+3y-z=4$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

4.144. $\frac{x-3}{2}=\frac{y}{1}=\frac{z-1}{2}$ va $\frac{x+1}{1}=\frac{y-1}{1}=\frac{z}{2}$ parallel to'g'ri chiziqlardan o'tuvchi tekislikning tenglamasi yozilsin.

4.145. Koordinatalar boshidan o'tuvchi va $4y=3x, y=0$ va $z=0$ tekisliklar bilan teng burchaklar tashkil etuvchi to'g'ri chiziq tenglamalari yozilsin va o'sha burchaklar topilsin.

4.146. $x=2t-1,\ y=t+2,\ z=1-t$ to'g'ri chiziqning $3x-2y+z=3$ tekislik bilan kesishgan nuqtasi topilsin.

4. 147(2; 3; 4) nuqtaning $x=y=z$ to'g'ri chiziqdagi proyeksiyasi topilsin.

§ 4.4. Ikkinchi tartibli sirtlar

Agar L egri chiziqning har bir nuqtasi orqali berilgan $\bar{a}$ vektorga parallel to'g'ri chiziq o'tkazsak, u holda silindrik sirt deb ataladigan sirtni hosil qilamiz. $\bar{a}$ vektorga parallel va silindrik sirtga tegishli to'g'ri chiziqlar bu sirtning yasovchilari deb ataladi, L egri chiziq esa silindrik sirtning yo'naltiruvchisi deb ataladi.

Silindrik sirtni uning yasovchilariga perpendikulyar tekislik bilan kesilganda (normal kesim) aylana hosil bo'lsa, u holda silindrik sirt doiraviy sirt, agar ellips hosil bo'lsa, elliptik sirt, agar parabola hosil bo'lsa, u holda parabolik sirt deyiladi.

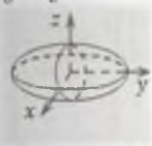
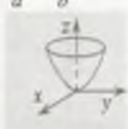
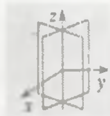
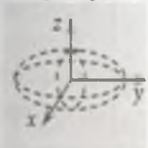
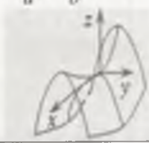
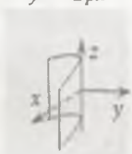
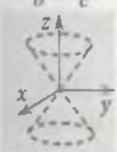
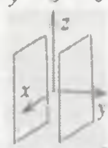
Berilgan egri chiziqning har bir nuqtasidan va fazoning bu egri chiziqda yotmaydigan belgilangan nuqtasidan o'tuvchi barcha to'g'ri chiziqlar birlashmasi konus sirt deb ataladi. Mazkur egri chiziq konus sirtning yo'naltiruvchisi, oldindan belgilangan nuqta uning uchi, to'g'ri chiziqlar esa konus sirtning yasovchilari deb ataladi.

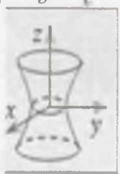
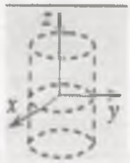
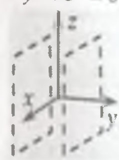
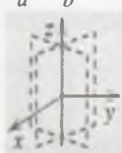
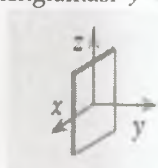
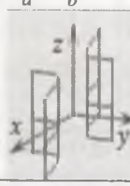
Fazodagi egri chiziqni harakatlanayotgan M nuqtaning trayektoriyasi sifatida tasavvur qilaylik, bunda vaqtning har bir t momentida bu nuqtaning x, y, z koordinatalari oldindan tanlangan koordinatalar sistemasiga nisbatan ma'lum bo'lsin:

$$x = f_1(t), \quad y = f_2(t), \quad z = f_3(t) \qquad (4.51)$$

(1) ko'rinishdagi tenglamalar fazodagi egri chiziqning parametrik tenglamalari deb ataladi.

4.1-jadval

1.	Ellipsoid tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 	7.	Paraboloid tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ 	13.	Kesishuvchi tekisliklar jufti tenglamasi $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$ 
2.	Mavhum ellipsoid tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$ 	8.	Giperbolik paraboloid sirt tenglamasi $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ 	14.	Parabolik silindt sirti tenglamasi $y^2=2px$ 
3.	Mavhum konus tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$ 	9.	Elliptik silindr tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ 	15.	Parallel tekisliklar jufti tenglamasi $y^2-b^2=0$ 

4.	Bir pallali giperboloid tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ 	10.	Mavhum elliptik silind tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1$ 	16.	Mavhum parallel tekisliklar jufti tenglamasi $y^2+b^2=0$ 
5.	Ikki pallali giperboloid tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ 	11.	Mavhum kesishuvchi tekisliklar jufti tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$ 	17.	O'zaro ustma-ust tushadigan tekisliklar tenglamasi $y^2=0$ 
6.	Konus tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ 	12.	Giperbolik silindr tenglamasi $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ 	18.	barcha tenglamalar uchun $a>0, b>0, c>0, p>0$ 1 va 2 tenglamalar uchun $a\ge b\ge c$ 3, 4, 5, 6, 7, 9, 10 tenglamalar uchun $a\ge b$

α tekislikdagi chegaralangan D figurani va α tekislikka parallel bo'lmagan biror a vektorni qaraylik. U holda $M\in D$ va $\overline{MN}=a$ bo'lgan barcha MN kesmalarning birlashmasi asosi D bo'lgan silindr deyiladi.

Silindrik sirtlar tenglamasini olish uchun yo'naltiruvchilarning tenglamalarini bilish talab etiladi.

$$F_1(x,y,z)=0 \text{ va } F_2(x,y,z)=0$$

yasovchi tenglamasini

$$\frac{X-x}{m}=\frac{Y-y}{n}=\frac{Z-z}{p}$$

bu yerda $M(x,y,z)$ yo'naltiruvchiga tegishli nuqta, X,Y,Z - silindrik sirtning koordinatalari, m,n,p -yasovchiga tegishli tashkil etuvchilar (o'zgarmas sonlar)

Quyida berilgan masalalarni yeching.

4.148. Uchi koordinata boshida bo'lgan konus sirti tenglamasini tuzing

$$\left.\begin{aligned}\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\\ z=c\end{aligned}\right\} \tag{4.52}$$

Yechish. (0;0;0) va (x,y,z) nuqtalardan o'tuvchi tashkil etuvchini tenglamasi

$$\frac{X}{x}=\frac{Y}{y}=\frac{Z}{z}, \tag{4.53}$$

bo'ladi. Bu(4.52) va (4.53) tenglamalardan

$$x=c\frac{x}{z};\quad y=c\frac{y}{z}$$

topiladi.U holda quyidagi konus sirtini olamiz.

$$\frac{c^2}{a^2}\frac{x^2}{z^2}+\frac{c^2}{b^2}\frac{y^2}{z^2}=1 \text{ yoki } \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

4.149. Quyidagi tenglamalar qanday sirtlarni aniqlaydi:

a) $x^2+y^2=25$; b) $x^2+y^2+z^2=25$; v) $\frac{x^2}{9}+\frac{y^2}{4}=1$; g) $\frac{x^2}{4}-\frac{y^2}{9}=1$?

4.150. Yarim o'qlari $a=6$ va $b=4$ va markazi koordinatalar boshida bo'lgan ellips o'zining Oz o'qda yotadigan kichik o'qi atrofida aylanadi. Ellipsning bu aylanishida chizadigan sirtning tenglamasini tuzing. Masalaga doir chizma chizing.

4.151. Yarim o'qlari $a=6$, $b=4$ va markazi koordinatalar boshida bo'lgan ellips o'zining Oz o'qda yotadigan katta o'qi atrofida aylanadi. Aylanish sirti tenglamasini tuzing. Masalaga doir chizma chizing.

4.152. $\frac{x^2}{25}+\frac{y^2}{14}=1$ ellipsning Oy o'q atrofida aylanish sirti tenglamasini tuzing.

4.153. Giperbolaning o'zining Ox o'q bilan ustma-ust tushadigan katta o'qi atrofida aylanishida chiziladigan sirtning tenglamasini tuzing. Giperbolaning yarim o'qlari $a=8$ va $b=6$, uning markazi esa koordinatalar boshi bilan ustma-ust tushadi. Chizma chizing.

4.154.Yarim o'qlari $a=3$ va $b=4$ bo'lgan ellips o'zining Oz o'q bilan ustma-ust tushadigan mavhum o'qi atrofida aylanadi. Giperbolaning markazi koordinatala boshi bilan ustma-ust tushadi. Bu giperbolaning aylanishidan hosil bo'lgan sirtning tenglamasini tuzing. Masalaga doir chizma chizing.

4.155. $y^2=2x$ parabolaning Ox o'q atrofida aylanishidan hosil bo'lgan sirtning tenglamasini tuzing.

4.156. $2x=3z$ to'g'ri chiziqning Ox o'q atrofida aylanishidan hosil bo'lgan sirtning tenglamasini tuzing. Masalaga doir chizma chizing.

4.157. $\frac{x^2}{9}-\frac{z^2}{4}=1$ giperbolaning Oz o'q atrofida aylanish sirtning tenglamasini tuzing. Chizma chizing.

4.158. $y^2=6z$ parabolaning Oz o'q atrofida aylanish sirtning tenglamasini tuzing. Masalaga doir chizma chizing.

4.159. $x-3=0$ to'g'ri chiziqning Oz o'q atrofida aylanish sirtning tenglamasini tuzing. Masalaga doir chizma chizing.

4.160. $14x=y^2+z^2$ sirtning turini aniqlang va uning tasvirini yasang.

4.161. $\frac{x^2}{48}-\frac{y^2}{12}-\frac{z^2}{12}=1$ tenglama bilan qanday sirt belgilanganligi va u $z-1=0$ tekislik bilan qanday chiziq bo'ylab kesishishini aniqlang.

4.162. $\frac{x^2}{8}-\frac{y^2}{18}+\frac{z^2}{8}=1$ tenglama bilan qanday sirt berilganligini va bu sirt $z+2=0$ tekislik bilan qanday sirt bo'ylab kesishishini aniqlang.

4.163. $x^2+y^2-4z=0$ tekislik bilan qanday sirt berilganligini va u $x-2=0$ tekislik bilan qanday chiziq bo'yicha kesishishini aniqlang.

4.164. Diametri 6 m ga, balandligi esa 12 m ga teng bo'lgan silindrik bak sirtning yuzini aniqlang.

4.165. Ikkita silindrik bak berilgan birinchi bakning diametri 3 m, balandligi esa 4 m, ikkinchi bakning diametri 4 m, balandligi esa 3m. qaysi bak sirtning yuzi kattaligini aniqlang.

4.166. Sfera markazi $z=4$ tekislikda joylashgan, sferaning o'zi esa xOy tekislikka M(2;3;0) nuqtada urinadi. Sferaning tenglamasini tuzing va uning markazi koordinatalarini aniqlang.

4.167. A(3;-5;6) va B(5;7;-1) nuqtalar sfera diametrlaridan birining oxirlaridir. Bu sferaning tenglamasini tuzing.

4.168.Quyidagi tenglamalar bilan qanday sirtlar aniqlanadi:
a) $x^2+y^2+z=9$; b) $x^2+y^2+z^2=6z$; v) $x^2+y^2+z^2=4y+5$;
g) $x^2+y^2+z^2-2x+4y-6z=2$?

4.169. $x^2+2y^2-3z^2+2x+8y+18z-54=0$ tenglama qanday sirtni ifoda qiladi.

4.170. Quyidagi tenglamalar qanday sirtlarni ifoda qiladi?

a) $x^2-4y^2+4(z-2)^2-16=0$ b) $4x^2+3z^2-12y+24=0$

v) $x^2+2(y-2)^2+3(z-1)^2=18$ g) $4x^2-y^2-z^2=4$

4.171. Koordinata boshini ko'chirish yordamida sirt tenglamalarini sodda ko'rinishga keltiring.

a) $x^2+y^2+z^2-4x+8y-6z+20=0$

b) $4x^2+y^2-8z^2+8x-4y+16z-32=0$

v) $9x^2-4y^2-36z^2-18x-16y-216z-367=0$

g) $3x^2+y^2+2z^2-12x-6y+4z-13=0$

d) $2x^2-3z^2+4x+2y+6z+1=0$.

Takrorlash uchun savollar

1. Analitik geometriya predmeti nimadan iborat?
2. To'g'ri chiziqning umumiy tenglamasi qanday ko'rinishda?
3. To'g'ri chiziqning burchak koeffitsiyenti deb nimaga aytiladi?
4. Ikkinchi darajali tenglamaning umumiy ko'rinishi qanday bo'ladi?
5. Aylana tenglamasi qanday ko'rinishda bo'ladi?
6. Aylana qanday ta'riflanadi?
7. Ellips qanday ta'riflanadi?
8. Ellips ekssentrisiteti, direktrisalari qanday ifodalaniladi?
9. Giperbola va parabolaning kanonik tenglamasi qanday ko'rinishda bo'ladi?

ANALITIK GEOMETRIYAGA DOIR TESTLAR

1. To'g'ri chiziq va uning tenglamalari

1. Analitik geometriyada chiziq nima asosida o'rganiladi ?

A) tenglama. B) chizma. C) proektsiya.

D) ta'rif. E) nuqtalar.

2. Tekislikdagi to'g'ri chiziqning umumiy tenglamasini ko'rsating:

A) $y = kx + b$. B) $\frac{x}{a} + \frac{y}{b} = 1$. C) $Ax + By + C = 0$.

D) $x\cos\alpha + y\sin\alpha - p = 0$. E) $\frac{x - x_0}{m} = \frac{y - y_0}{n}$.

3. Tekislikdagi to'g'ri chiziqning umumiy $Ax+By+C=0$ tenglamasidagi A va B koeffitsientlar qanday shartni qanoatlantirishi kerak ?

A) $A \cdot B>0$. B) $A+B=0$. C) $A-B<0$.

D) $A^2+B^2\neq0$. E) $A^2-B^2\neq0$.

4. *Tasdiqni yakunlang:* Tekislikdagi to'g'ri chiziqning umumiy $Ax+By+C=0$ tenglamasi bo'yicha tuzilgan n=(A,B) vektor bu to'g'ri chiziqqa ··· .

A) parallel bo'ladi. B) tegishli bo'ladi.

C) perpendikulyar bo'ladi.

D) perpendikulyar bo'lmaydi . E) og'ma bo'ladi

5. Tekislikdagi to'g'ri chiziqning umumiy $3x+5y+2=0$ tenglamasi bo'yicha uning n normal vektorini toping.

A) n =(5,2) . B) n =(3,5) . C) n =(3,2) .

D) n =(2,5) . E) n =(5,3) .

6. $Ax+By=0$ $(A,B\neq0)$ tenglama qanday to'g'ri chiziqni ifodalaydi ?

A) OX koordinata o'qiga parallel bo'lgan.

B) OX koordinata o'qiga perpendikulyar bo'lgan.

C) OY koordinata o'qiga parallel bo'lgan.

D) OY koordinata o'qiga perpendikulyar bo'lgan.

E) koordinata boshidan o'tuvchi.

7. $Ax+C=0$ $(A,C\neq0)$ tenglama qanday to'g'ri chiziqni ifodalaydi ?

A) OX koordinata o'qiga parallel bo'lgan.

B) OX koordinata o'qiga perpendikulyar bo'lgan.

C) OY koordinata o'qiga parallel bo'lmagan.

D) OY koordinata o'qiga perpendikulyar bo'lgan.

E) koordinata boshidan o'tuvchi.

8. $By+C=0$ $(B,C\neq0)$ tenglama qanday to'g'ri chiziqni ifodalaydi ?

A) OX koordinata o'qiga parallel bo'lmagan.

B) OX koordinata o'qiga perpendikulyar bo'lgan.

C) OY koordinata o'qiga parallel bo'lgan.

D) OY koordinata o'qiga perpendikulyar bo'lgan.

E) koordinata boshidan o'tuvchi.

9. Tekislikdagi to'g'ri chiziqning burchak koeffitsientli tenglamasini ko'rsating.

A) $y = kx + b$. B) $\frac{x}{a} + \frac{y}{b} = 1$. C) $Ax + By + C = 0$

D) $x\cos\alpha + y\sin\alpha - p = 0$. E) $\frac{x - x_0}{m} = \frac{y - y_0}{n}$.

10. Tekislikdagi to'g'ri chiziqning burchak koeffitsientli $y=kx+b$ tenglamasida k parametr nimani ifodalaydi ?

A) bu to'g'ri chiziqning OX koordinata o'qidan ajratgan kesmasini.

B) bu to'g'ri chiziqning OY koordinata o'qidan ajratgan kesmasini.

C) bu to'g'ri chiziqning OX koordinata o'qi bilan hosil etgan burchak tangensini.

D) bu to'g'ri chiziqning OY koordinata o'qi bilan hosil etgan burchak tangensini.

E) bu to'g'ri chiziqdan O koordinata boshigacha bo'lgan masofani.

2. To'g'ri ciziqlarga doir asosiy masalalar

1. Qaysi tenglama $M(x_0,y_0)$ nuqtadan o'tuvchi to'g'ri chiziqlar dastasini ifodalamaydi ?

A) $y - y_0 = k(x - x_0)$. B) $\frac{y - y_0}{x - x_0} = k$. C) $\frac{x - x_0}{y - y_0} = \frac{1}{k}$.

D) $A(x - x_0) + B(y - y_0) = 0$. E) $y + y_0 = k(x + x_0)$.

2. M(–1,4) nuqtadan o'tuvchi barcha to'g'ri chiziqlar tenglamasini ko'rsating.

A) $(x+1)^2+(y+4)^2=0$. B) $y=k(x-1)+4$.

C) $y=k(x-1)-4$. D) $y=k(x+1)+4$. E) $y=k(x+1)-4$

3. Qaysi tenglama M(–3,1) va N(0,7) nuqtalardan o'tuvchi to'g'ri chiziqda yotuvchi $P(1,y)$ nuqtaning ordinatasi toping.

A) $y=0$. B) $y=-4$. C) $y=9$.

D) $y=-7$. E) $y=\pm 5$.

4. Ikki $y=k_1x+b_1$ va $y=k_2x+b_2$ to'g'ri chiziqlar orasidagi α burchak tangensi formulasini toping.

A) $tg\alpha = \frac{k_2 - k_1}{1 + k_1k_2}$.   B) $tg\alpha = \frac{k_1 + k_2}{1 - k_1k_2}$.

C) $tg\alpha = \frac{k_1 \cdot k_2}{1 + k_1k_2}$.   D) $tg\alpha = \frac{k_1 - k_2}{1 - k_1k_2}$.   E) $tg\alpha = \frac{1 + k_1k_2}{k_2 - k_1}$.

5. $y=2x-3$ va $y = -3x +5$ to'g'ri chiziqlar orasidagi burchakni toping.

A) 90^0.   B) 60^0.   C) 45^0.   D) 30^0.   E) 120^0.

6. k parametrning qanday qiymatida $y=2x+b$ va $y = kx +c$ to'g'ri chiziqlar orasidagi burchak $\alpha=45^0$ bo'ladi?

A) bc.   B) -3.   C) $b+c$.   D) 1.   E) $\pm1/3$.

7. $y=k_1x+b_1$ va $y=k_2x+b_2$ to'g'ri chiziqlarning parallellik shartini ko'rsating.

A) $k_1 \cdot k_2 = -1$.   B) $k_1+k_2=0$.   C) $k_1 \cdot k_2=1$.

D) $k_1-k_2=0$.   E) $k_1 \cdot k_2=0$.

8. $3x+\alpha y+C_1=0$ va $\alpha x+12y+C_2=0$ to'g'ri chiziqlar α parametrning qanday qiymatlarida parallel bo'ladi ?

A) $\pm C_1 \cdot C_2$.   B) ±4.   C) $C_1 \pm C_2$.

D) ±6.   E) ±7.

9. $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlarning perpendikulyarlik shartini ko'rsating.

A) $A_1B_1+A_2B_2=0$.   B) $A_1A_2+B_1B_2=0$.

C) $A_1B_1-A_2B_2=0$.   D) $A_1A_2-B_1b_2=0$.

E) $A_1A_2+B_1B_2+C_1C_2=0$.

10. $3\alpha x-4y+C_1=0$ va $\alpha x+12y+C_2=0$ to'g'ri chiziqlar α parametrning qanday qiymatlarida perpendikulyar bo'ladi ?

A) $C_1 \pm C_2$.   B) ±4.   C) ±5.

D) $\pm C_1 \cdot C_2$.   E) ±7.

3. II tartibli tenglama va chiziqlar. Aylana va ellips

1. Markazi $M(a,b)$ nuqtada va radiusi R bo'lgan aylana tenglamasini ko'rsating.

A) $(x+a)^2+(y+b)^2=R^2$.   C) $(x-a)^2+(y-b)^2=R^2$.

B) $(x+b)^2+(y+a)^2=R^2$. D) $(x-b)^2+(y-a)^2=R^2$.
E) $(x-a)+(y-b)=R$.

2. Markazi $M_0(x_0,y_0)$ nuqtada joylashgan R radiusli aylana tenglamasini ko'rsating.

A) $x_0+y_0=R$.

B) $|x-x_0|+|y-y_0|=R$.

C) $(x-x_0)^2+(y-y_0)^2=R^2$.

D) $(x+x_0)^2+(y+y_0)^2=R^2$.

E) $xx_0+yy_0=R^2$.

3. Umumiy ko'rinishdagi II tartibli $Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0$ tenglama aylanani ifodalashi uchun B koeffitsient qanday shartni qanoatlantirishi kerak ?

A) $B>0$. B) $B<0$. C) $B\neq0$. D) $B=0$. E) $B\geq0$.

4. Umumiy tenglamasi $x^2+y^2-4x+2y+1=0$ bo'lgan aylananing R radiusini toping.

A) R=2. B) R=3. C) R=4. D) R=5. E) R=6.

5. Umumiy tenglamasi $x^2+y^2-6x-4y-3=0$ bo'lgan aylananing $M_0(x_0,y_0)$ markazini toping.

A) $M_0(-2,-3)$. B) $M_0(2,3)$. C) $M_0(-3,-2)$. D) $M_0(3,2)$. E) $M_0(-3,2)$.

6. II tartibli $x^2+y^2-4x+2y+F=0$ tenglama aylanani ifodalashi uchun ozod had F qanday shartni qanoatlantirishi kerak ?

A) F=5. B) F<5. C) F>5. D) $F\neq5$. E) $|F|=5$.

7. Qaysi shartda $y=kx+b$ to'g'ri chiziq $x^2+y^2=R^2$ aylanani kesib o'tadi ?

A) $(k^2+1)R^2=b^2$. B) $(k^2+1)R^2>b^2$. C) $(k^2+1)R^2<b^2$.
D) $(k^2+1)R^2\neq b^2$. E) $(k^2+1)R^2+b^2=0$.

8. Qaysi shartda $y=kx+b$ to'g'ri chiziq $x^2+y^2=R^2$ aylanaga urinib o'tadi ?

A) $(k^2+1)R^2=b^2$. B) $(k^2+1)R^2>b^2$. C) $(k^2+1)R^2<b^2$.
D) $(k^2+1)R^2\neq b^2$. E) $(k^2+1)R^2+b^2=0$.

9. Qaysi shartda $y=kx+b$ to'g'ri chiziq $x^2+y^2=R^2$ aylana bilan kesishmaydi ?

A) $(k^2+1)R^2=b^2$. B) $(k^2+1)R^2>b^2$. C) $(k^2+1)R^2<b^2$.

D) $(k^2+1)R^2\neq b^2$. E) $(k^2+1)R^2+b^2=0$.

10. $y=x+4$ to'g'ri chiziq va $x^2+y^2=8$aylanani urinish nuqtasi koordinatalarining yig'indisini toping .

A) –1 . B) 0 . C) 1 . D) 2 . E) –2 .

4. Giperbola va parabola

1. *Ta'rifni to'ldiring:* Berilgan ikkita F_1 va F_2 nuqtalargacha masofalari ... o'zgarmas son bo'lgan tekislikdagi nuqtalarning geometrik o'rni giperbola deyiladi.

A) ayirmasining moduli. B) ko'paytmasi.

C) yig'indisi.

D) bo'linmasi. E) kvadratlarining yig'indisi.

2. Yarim o'qlari a va $b(a, b>0)$ bo'lgangiperbolaning kanonik tenglamasi qayerda to'g'ri ko'rsatilgan ?

A) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. B) $\frac{x^2}{a}-\frac{y^2}{b}=1$. C) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

D) $a^2x^2 + b^2y^2 = 1$. E) $ax^2-by^2 = 1$.

3. Quyidagi tenglamalardan qaysi biri giperbolani ifodalaydi $(a, b>0)$?

A) $a^2x^2-b^2y^2=\pm a^2b^2$.

B) $x^2-y^2=\pm a^2b^2$.

C) $a^2x^2-b^2y^2=\pm 1$.

D) barcha tenglamalar giperbolani ifodalaydi.

E) barcha tenglamalar giperbolani ifodalamaydi.

4. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning asimptotalari tenglamasini ko'rsating.

A) $y=\pm\frac{ab}{a+b}x$. B) $y=\pm\frac{a}{b}x$. C) $y=\pm\frac{b}{a}x$.

D) $y = (a\pm b)\,x$. E) $y= \pm abx$.

5. Agar $x^2-4y^2=4$ giperbolaga tegishli nuqtaning ordinatasi 0 ga teng bo'lsa, uning abssissasini toping.

A) $x=1$. B) $x=1$. C) $x=2$.

D) $x=\pm2$. E) $x=\pm1$

6. $9x^2-4y^2=36$ giperbola yarim o'qlarining yig'indisini toping.

A)5. B) 13. C) 9. D) 45. E) 32 .

7. $9x^2-16y^2=144$ giperbolaning fokuslari orasidagi $2c$ masofani toping.

A) $2c=5$. B) $2c=10$. C) $2c=15$.

D) $2c=20$. E) $2c=25$.

8. Agar giperbolaning tenglamasi $x^2-4y^2=16$ ko'rinishda bo'lsa, uning asimptotalari tenglamalarini toping.

A) $y=\pm\frac{1}{2}x$. B) $y=\pm\frac{1}{4}x$. C) $y=\pm4x$.

D) $y=\pm2x$. E) $y=\pm x$

9. Kanonik tenglamasi $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ bo'lgan giperbolaning ekstsentrisiteti ε qaysi formula bilan hisoblanadi ?

A) $\varepsilon=\sqrt{1+\frac{b}{a}}$. B) $\varepsilon=\sqrt{1-\frac{b}{a}}$. C) $\varepsilon=\sqrt{1+\frac{b^2}{a^2}}$.

D) $\varepsilon=\sqrt{1-\frac{b^2}{a^2}}$. E) $\varepsilon=\sqrt{\frac{b^2}{a^2}-1}$.

10. Giperbolaning ekstsentrisiteti ε qanday shartni qanoatlantiradi ?

A) $\varepsilon>1$. B) $\varepsilon<1$. C) $\varepsilon\neq1$. D) $\varepsilon=1$. E) $0<\varepsilon<1$.

5. Tekislik va uning tenglamalari

1. Tekislikning umumiy tenglamasi qayerda to'liq va to'g'ri ifodalangan?

A) $Ax+By+Cz+D=0$. B) $Ax+By+CDz=0$.

C) $Ax+By+(C+D)z=0$. D) $Ax^{-1}+By^{-1}+Cz^{-1}+D=0$.

E) $Ax^2+By^2+Cz^2+D=0$.

2. Umumiy tenglamasi $2x-5y+4z+9=0$ bo'lgan tekislikka tegishli va koordinatalarining yig'indisi 15 bo'lgan $M(x,y,5)$ nuqtaning abssisasini toping.

A) 4. B) –7. C) 3. D) 0. E) –1.

3. Umumiy tenglamasi $2x-5y+4z-9=0$ bo'lgan tekislikka tegishli, OZ o'qda yotuvchi va koordinatalarining yig'indisi birga teng bo'lgan nuqtaning ordinatasini toping.

A) 4. B) –7. C) 3. D) 0. E) –1.

4. Quyidagilardan qaysi biri $Ax+By+Cz+D=0$ tenglamali tekislikning $\boldsymbol{n}$ normal vektori bo'ladi ?

A) $\boldsymbol{n}$=(B,C,D). B) $\boldsymbol{n}$=(A,C,D). C) $\boldsymbol{n}$=(A,B,C).
D) $\boldsymbol{n}$ =(A,B,D). E) $\boldsymbol{n}$=(C,A,B).

5. ***Tasdiqni yakunlang:*** Umumiy tenglamasi $Ax+By+Cz+D=0$ bo'lgan tekislikning $\boldsymbol{n}$=(A,B,C) normal vektori shu ··· .

A) tekislikda yotadi .
B) tekislikka parallel bo'ladi .
C) tekislikka perpendikulyar bo'ladi .
D) tekisliikka og'ma bo'ladi.
E) to'g'ri javob keltirilmagan .

6. $3x+4y+7z-81=0$ tenglama bilan berilgan tekislik normalini aniqlang.

A) $\boldsymbol{n}$ =(3,4,–81). B) $\boldsymbol{n}$ =(3,–4,–81). C) $\boldsymbol{n}$ =(4,7,–81).
D)$\boldsymbol{n}$ =(–3,–4,81). E) $\boldsymbol{n}$ =(3,4,7).

7. Quyidagi tenglamalardan qaysi biri koordinatalar boshidan o'tuvchi tekislikni ifodalaydi ?

A) $Ax+By+D=0$. B) $Ax+By + Cz=0$.
C) $By+Cz+D=0$. D) $Ax+By+Cz+D=0$. E) $Ax+Cz+D=0$.

8. $x+y-z=0$ tenglamali P tekislik to'g'risidagi quyidagi tasdiqlardan qaysi biri o'rinli ?

A) P koordinatalar boshidan o'tadi.
B) P OXY tekisligiga parallel.
C) P OXZ tekisligiga parallel .
D) P OYZ tekisligiga parallel.
E) P tekislik OZ koordinata o'qiga perpendikulyar.

9. $Ax+By+Cz+D=0$ tenglama A=D=0 holda qanday P tekislikni ifodalaydi?

A) P OX o'qiga parallel.
B) P OX o'qiga perpendikulyar.
C) P OX o'qi orqali o'tadi.
D) P OY o'qiga perpendikulyar.

E) P OY o'qiga parallel .

10. Quyidagi tenglamalardan qaysi biri OZ koordinata o'qidan o'tuvchi tekislikni ifodalaydi ?

A) $Ax+By+Cz=0$. B) $Ax+Cz+D=0$. C) $Ax+By=0$.
D) $By+Cz+D=0$. E) $By+D=0$.

6. Tekislikka doir asosiy masalalar

1. Berilgan $M_0(x_0,y_0,z_0)$ nuqtadan o'tuvchi tekisliklar dastasining tenglamasi qayerda to'g'ri ifodalangan?

A) $A(x+x_0)+B(y+y_0)+C(z+z_0)=0$. B) $Axx_0+Byy_0+Czz_0=0$.

C) $\frac{x-x_0}{A}+\frac{y-y_0}{B}+\frac{z-z_0}{C}=0$. D) $\frac{x+x_0}{A}+\frac{y+y_0}{B}+\frac{z+z_0}{C}=0$.

E) $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$.

2. Fazoning M(1,2,–3) nuqtasidan o'tuvchi tekisliklar dastasi tenglamasini ko'rsating.

A) $Ax+2By-3Cz=0$. B) $A(x-1)+B(y-2)+C(z+3)=0$.
C) $A(x+1)+B(y+2)+C(z-3)=0$. D) $Ax+2By-3Cz+D=0$.
E) $A(x-1)+B(y-2)+C(z-3)+D=0$.

3. $M_1(3,2,-1)$, $M_2(0,3,1)$ va $M_3(4,5,0)$ nuqtalardan o'tuvchi tekislik tenglamasini ko'rsating.

A) $2x-y+z-3=0$. B) $x-2y+z+2=0$. C) $x-y+2z+1=0$.
D) $x-y+z=0$. E) $x-2y+2z+3=0$.

4. $M_1(3,2,-1)$, $M_2(0,3,1)$ va $M_3(4,5,0)$ nuqtalardan o'tuvchi tekislikda yotuvchi $M_0(x,-4,7)$ nuqtaning abssissasini toping.

A) $x_0=-3$. B) $x_0=5$.
C) $x_0=-1,5$. D) $x_0=9$. E) $x_0=0$

5. Berilgan $M_0(x_0,y_0,z_0)$ nuqtadan o'tuvchi va $n=(A,B,C)$ vektorga
perpendikulyar tekislik tenglamasini ko'rsating.

A) $A(x+x_0)+B(y+y_0)+C(z+z_0)=0$. B) $Axx_0+Byy_0+Czz_0=0$.

C) $\frac{x-x_0}{A}+\frac{y-y_0}{B}+\frac{z-z_0}{C}=0$. D) $\frac{x+x_0}{A}+\frac{y+y_0}{B}+\frac{z+z_0}{C}=0$.

E) $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$.

6. M(1,2,3) nuqtadan o'tuvchi va $n=(3,2,1)$ normal vektorga ega tekislik tenglamasini yozing.

A) $3x+2y+z-10=0$. B) $x+2y+3z-14=0$.

C) $2x+3y+z-11=0$. D) $x+3y+2z-13=0$.

E) $3x+y+2z-11=0$.

7. Berilgan $M_0(3,-4,0)$ nuqtadan o'tuvchi va $n=(1,2,-3)$ normal vektorga ega bo'lgan tekislikda yotuvchi $N(-3,8, z)$ nuqtaning aplikatasini toping.

A) $z=0$. B) $z=5$. C) $z=-1$.

D) $z=6$. E) $z=-3{,}5$.

8. $M_0(x_0,y_0,z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha bo'lgan masofani topish formulasini ko'rsating.

A) $d=|Ax_0+By_0+Cz_0+D|$.

B) $d=\sqrt{|Ax_0+By_0+Cz_0+D|}$.

C) $d=\dfrac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$.

D) $d=\dfrac{|A+x_0+B+y_0+C+z_0+D|}{\sqrt{A^2+B^2+C^2}}$.

E) $d=\sqrt{\dfrac{Ax_0+By_0+Cz_0+D}{A^2+B^2+C^2}}$.

9. $3x+4y-2\sqrt{6}z+14=0$ tekislikdan koordinata boshigacha bo'lgan masofani toping.

A) 14. B) 7. C) 2. D) 1. E) $2\sqrt{6}$.

10. Ushbu $4x+3y-5z-8=0$ va $4x+3y-5z-12=0$ parallel tekisliklar orasidagi masofani toping.

A) 4 . B) 20 . C) $\sqrt{2}$. D) $\dfrac{2\sqrt{2}}{5}$. E) $2\sqrt{2}$.

7. Fazodagi to'g'ri chiziq tenglamalari

1. $M_0(x_0,y_0,z_0)$ nuqtadan o'tuvchi va $a=(m,n,p)$ vektorga parallel to'g'ri chiziqning kanonik tenglamasini ko'rsating.

A) $\dfrac{x-m}{x_0}=\dfrac{y-n}{y_0}=\dfrac{z-p}{z_0}$. B) $\dfrac{x+m}{x_0}=\dfrac{y+n}{y_0}=\dfrac{z+p}{z_0}$.

C) $\dfrac{x-x_0}{m}=\dfrac{y-y_0}{n}=\dfrac{z-z_0}{p}$. D) $\dfrac{x+x_0}{m}=\dfrac{y+y_0}{n}=\dfrac{z+z_0}{p}$.

E) To'g'ri javob keltirilmagan.

2. Kanonik tenglamasi $\frac{x}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ ko'rinishda bo'lgan L to'g'ri chiziq qanday xususiyatga ega ?

A) L to'g'ri chiziq YOZ koordinata tekisligiga parallel joylashgan.

B) L to'g'ri chiziq YOZ koordinata tekisligiga perpendikulyar joylashgan.

C) L to'g'ri chiziq YOZ koordinata tekisligini kesib o'tadi.

D) L to'g'ri chiziq YOZ koordinata tekisligini kesib o'tmaydi.

E) To'gri javob keltirilmagan.

3. Fazodagi L to'g'ri chiziqning $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ kanonik tenglamasida m=0 bo'lsa, L qanday xususiyatga ega bo'ladi ?

A) L to'g'ri chiziq OX koordinata o'qiga parallel joylashgan.

B) L to'g'ri chiziq OX koordinata o'qiga perpendikulyar joylashgan.

C) L to'g'ri chiziq OX koordinata o'qiga parallel emas.

D) L to'g'ri chiziq OX koordinata o'qiga perpendikulyar emas.

E) To'gri javob keltirilmagan.

4. Kanonik tenglamasi $\frac{x-3}{2}=\frac{y+1}{5}=\frac{z}{-3}$ bo'lgan to'g'ri chiziqning yo'naltiruvchi vektorining koordinatalarini toping.

A) (–3, 1, 0). B) (3, –1, 0). C) (2,5, –3).
D) (–2, –5, 3). E) (3, 1, 0).

5. Kanonik tenglamasi $\frac{x-3}{12}=\frac{y}{-4}=\frac{z+4}{3}$ bo'lgan to'g'ri chiziq yo'naltiruvchi vektorining modulini toping.

A) 1 B) $\sqrt{7}$. C) 5. D) 13. E) $\sqrt{11}$.

6. Kanonik tenglamasi $\frac{x-3}{2}=\frac{y+1}{5}=\frac{z}{-3}$ bo'lgan to'g'ri chiziq boshlang'ich nuqtasining koordinatalarini toping.

A) (–3, 1, 0). B) (3, –1, 0). C) (2,5, –3).
D) (–2, –5, 3). E) (3, 5, 0).

7. Quyidagilardan qaysi biri fazodagi $M_0(x_0,y_0,z_0)$ boshlang'ich nuqta va $a=(m,n,p)$ yo'naltiruvchi vektorga ega to'g'ri chiziqning parametrik tenglamasiga teng kuchli emas?

A) $xx_0=mt,\ yy_0=nt,\ zz_0=pt$.

B) $x-mt=x_0,\ y-nt=y_0,\ z-pt=z_0$.

C) $x-x_0=mt,\ y-y_0=nt,\ z-z_0=pt$.

D) $x-x_0-mt=0,\ y-y_0-nt=0,\ z-z_0-pt=0$.

E) $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}=t$.

8. Kanonik tenglamasi $\frac{x-3}{2}=\frac{y+1}{5}=\frac{z}{-3}$ bo'lgan to'g'ri chiziqning parametrik tenglamasini toping.

9.

A) $x=2-3t,\ y=5+t,\ z=-3$.

B) $x=2+3t,\ y=5-t,\ z=-3$.

C) $x=3+2t,\ y=-1+5t,\ z=-3t$.

D) $x=3-2t,\ y=-1-5t,\ z=3t$.

E) $x=3+2t,\ y=5+t,\ z=-3t$.

10. To'g'ri chiziqning $x=3+2t,\ y=2+5t,\ z=4-3t$ parametrik tenglamasi bo'yicha uning kanonik tenglamasini toping.

A) $\frac{x-3}{2}=\frac{y+1}{5}=\frac{z}{-3}$.

B) $\frac{x-3}{2}=\frac{y-5}{3}=\frac{z+1}{-3}$.

C) $\frac{x-3}{2}=\frac{y-2}{5}=\frac{z-4}{-3}$.

D) $\frac{x-3}{2}=\frac{y-2}{5}=\frac{z-4}{-3}$.

E) $\frac{x-2}{3}=\frac{y+1}{5}=\frac{z+4}{3}$.

11. Fazodagi to'g'ri chiziqning umumiy tenglamasini ko'rsating .

A) $Ax+By+C=0$.

B) $Ax+By+Cz+D=0$.

C) $A_1x+B_1y+C_1z+D_1=A_2x+B_2y+C_2z+D_2$.

D) $\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$.

E) $\begin{cases} A_1x+B_1y+C_1z=0 \\ A_2x+B_2y+C_2z=0 \end{cases}$.

Odamning ulug'vorligi uning bo'yi bilan o'lchanmaganidek, xalqning ulug'ligi ham, uning soni bilan o'lchanmaydi, yagona o'lchovi-uning aqliy kamoloti va axloqiy barkamolligidir.

V.Gyugo

V-BOB. TO'PLAMLAR NAZARIYASI ELEMENTLARI

§5.1. To'plamlar nazariyasining asosiy tushunchalari
§ 5.2. To'plamlar ustida amallar

5.1.To'plamlar nazariyasining asosiy tushunchalari

To'plamlar va ularga doir tushunchalar. To'plam deyilganda biror bir xususiyati bo'yicha umumiylikka ega bo'lgan ob'yektlar majmuasi tushuniladi. Masalan, I kurs talabalari to'plami, [0,1] kesmadagi nuqtalar to'plami, natural sonlar to'plami, firma xodimlari to'plami, korxonada ishlab chiqarilgan mahsulotlar to'plami va hokazo. Matematikada to'plamlar A,B,C,D,... kabi bosh harflar bilan belgilanadi. A,B,C,D,... to'plamlarga kiruvchi obyektlar ularning ***elementlari*** deyiladi va odatda mos ravishda kichik $a,b,c,d,\ldots$ kabi harflar bilan belgilanadi. Bunda «a element A to'plamga tegishli (tegishli emas)» degan tasdiq $a\in A$ ($a\notin A$) kabi yoziladi.Birorta ham elementga ega bo'lmagan to'plam ***bo'sh to'plam*** deyiladi va $\varnothing$ kabi belgilanadi. Masalan, {sin$x$ = 2 tenglamaning yechimlari}=$\varnothing$, {perimetri 0 bo'lgan kvadratlar}=$\varnothing$, {kvadrati manfiy bo'lgan haqiqiy sonlar}=$\varnothing$. Algebrada 0 soni qanday vazifani bajarsa, to'plamlar nazariyasida $\varnothing$ bo'lgan shunga o'xshash vazifani bajaradi.

Agar A to'plamga tegishli har bir a element boshqa bir B to'plamga ham tegishli bo'lsa ($a\in A\Rightarrow a\in B$), u holda A to'plam B ***to'plamining qismi*** deyiladi va $A\subset B$ (yoki $B\supset A$) kabi belgilanadi. Masalan, korxonada ishlab chiqarilayotgan oliy navli mahsulotlar to'plamini A, barcha mahsulotlar to'plamini esa B deb olsak , unda $A \subset B$ bo'ladi.

Ta'rifdan ixtiyoriy A to'plam uchun $A \subset A$ va $\emptyset \subset A$ tasdiqlar o'rinli bo'lishi kelib chiqadi. Shu sababli to'plamlar uchun $\subset$ belgisi sonlar uchun $\leq$ belgiga o'xshash ma'noga egadir. Agarda A vaBto'plamlar uchun $A \subset B$ va$B \subset A$ shartlar bir paytda bajarilsa, bu to'plamlar ***teng*** deyiladi va A=B kabi yoziladi.

Masalan,A={–1;1} va B={x^2–1=0 tenglama ildizlari},

C={badiiy asarni yozish uchun ishlatilgan harflar} va D={alfavitdagi harflar} to'plamlari uchun A=B, C=D bo'ladi.

To'plamlar ustida amallar va ularning xossalari. Algebrada *a* va *b* sonlar ustida qo'shish va ko'paytirish amallari kiritilgan bo'lib, ular

$a+b=b+a$ va $ab=ba$ (kommutativlik , ya'ni o'rin almashtirish),

$a+(b+c)=(a+b)+c$ va $a(bc)=(ab)c$ (assotsiativlik, ya'ni guruhlash),

$a(b+c)=ab+ac$ (distributivlik, ya'ni taqsimot)

qonunlariga bo'ysunadilar. Bulardan tashqari har qanday *a* soni uchun $a+0=a$ va $a\cdot 0=0$ tengliklar ham o'rinli bo'ladi.

A to'plamdan B to'plamlarning ***birlashmasi (yig'indisi)*** deb shunday C to'plamga aytiladiki, u A va B to'plamlardan kamida bittasiga tegishli bo'lgan elementlardan tashkil topgan bo'ladi va $A \cup B$ kabi belgilanadi.

Shunday qilib $A \cup B$ to'plam yoki A to'plamga , yoki B to'plamga, yoki A va B to'plamlarning ikkalasiga ham tegishli elementlardan iboratdir.

Masalan, A={1,2,3,4,5} va B={2,4,6,8} bo'lsa $A \cup B$={1,2,3,4,5,6,8},

C={I navli mahsulotlar} va D={II navli mahsulotlar } bo'lsa, unda

$C \cup D$={I yoki II navli mahsulotlar} to'plamni ifodalaydi.

To'plamlarni birlashtirish amali , sonlarni qo'shish amali singari, $A \cup B = B \cup A$ (kommutativlik),

$(A \cup B) \cup C = A \cup (B \cup C)$ (assosiativlik) qonunlarga bo'ysunadi. Bulardan tashqari $A\cup\varnothing=A$ va, sonlardan farqli ravishda, $A\cup A=A$, $B\subset A$ bo'lsa $A\cup B=A$ tengliklar ham o'rinli bo'ladi. Bu tasdiqlarning barchasi to'plamlar tengligi ta'rifidan foydalanib isbotlanadi. Misol sifatida, oxirgi tenglikni isbotlaymiz:

$$x\in A\cup B\Rightarrow x\in A \text{ yoki } x\in B\Rightarrow x\in A\Rightarrow (A\cup B)\subset A;$$

$$x\in A\Rightarrow x\in A\cup B\Rightarrow A\subset(A\cup B)$$

Demak, $(A\cup B)\subset A$, $A\subset(A\cup B)$ va , ta'rifga asosan, $A\cup B=A$.

Bir nechta $A_1, A_2, A_3, \ldots, A_n$ to'plamlarning yig'indisi

$$A_1\cup A_2\cup A_3\cup\ldots\cup A_n=\bigcup_{k=1}^{n}A_k$$

kabi belgilanadi va ulardan kamida bittasiga tegishli bo'lgan elementlar to'plami sifatida aniqlanadi.

A va B to'plamlarning ***kesishmasi (ko'paytmasi)*** deb shunday C to'plamga aytiladiki, u A va B to'plamlarning ikkalasiga ham tegishli bo'lgan elementlardan tashkil topgan bo'ladi va $A\cap B$ kabi belgilanadi.

Shunday qilib $A\cap B$ to'plam A va B to'plamlarning umumiy elementlaridan tashkil topgan bo'ladi. Shu sababli agar ular umumiy elementlarga ega bo'lmasa, ya'ni kesishmasa, unda $A\cap B=\varnothing$ bo'ladi.

Masalan, A={1,2,3,4,5} va B={2,4,6,8} bo'lsa $A\cap B$={2,4},

C={Tekshirilgan mahsulotlar} va D={Sifatli mahsulotlar} bo'lsa, unda

$C\cap D$={Tekshirishda sifatli deb topilgan mahsulotlar} to'plamni ifodalaydi.

To'plamlarni kesihmasi amali quyidagi qonunlarga bo'ysunadi:

$A\cap B=B\cap A$ (kommutativlik),

$(A\cap B)\cap C=A\cap(B\cap C)$ (assotsiativlik),

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$,

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ (distributivlik)

Shu bilan birga $A \cap A = A$, $A \cap \varnothing = \varnothing$ va $B \subset A$ bo'lsa $A \cap B = B$ tengliklar ham o'rinli bo'ladi. Bu tasdiqlarning o'rinli ekanligiga yuqorida ko'rsatilgan usulda ishonch hosil etish mumkin.

Bir nechta $A_1, A_2, A_3, \dots, A_n$ to'plamlarning kesishmasi

$$A_1 \cap A_2 \cap \dots \cap A_n = \bigcap_{k=1}^{n} A_k$$

kabi belgilanadi va barcha A_k ($k=1,2, \cdots, n$) to'plamlarga tegishli bo'lgan umumiy elementlardan tuzilgan to'plam kabi aniqlanadi.

A va B to'plamlarning ***ayirmasi*** deb A to'plamga tegishli, ammo B to'plamga tegishli bo'lmagan elementlardan tashkil topgan to'plamga aytiladi va $A \setminus B$ kabi belgilanadi.

Masalan, A={1,2,3,4,5} va B={1,3,7,9} bo'lsa, unda $A \setminus B = \{2,4,5\}$, $B \setminus A = \{7,9\}$;

C={Korxonada ishlab chiqarilgan mahsulotlar}va D={Sifatli mahsulotlar} bo'lsa,

C\D={ Korxonada ishlab chiqarilgan sifatsiz mahsulotlar }.

Demak, $A \setminus B$ to'plam A to'plamning B to'plamga tegishli bo'lmagan elementlaridan hosil bo'ladi. To'plamlar ayirmasi uchun

$$A \setminus A = \varnothing, \quad A \setminus \varnothing = A, \quad \varnothing \setminus A = \varnothing$$

va $A \subset B$ bo'lsa $A \setminus B = \varnothing$ munosabatlar o'rinlidir.

Agar ko'rilayotgan barcha to'plamalarni biror Ω to'plamning qism to'plamlari kabi qarash mumkin bo'lsa, unda Ω ***universal to'plam*** deb ataladi.

Masalan, sonlar bilan bog'liq barcha to'plamlar uchun $\Omega = (-\infty, \infty)$, insonlardan iborat to'plamlar uchun Ω={Barcha odamlar} universal to'plam bo'ladi. Agar A to'plam Ω universalto'plamning qismi bo'lsa, unda $\Omega \setminus A$ to'plam ***A to'plamning to'ldiruvchisi*** deb ataladi va C(A) kabi belgilanadi.

Demak, C(A) to'plam A to'plamga kirmaydigan elementlardan tashkil topgan bo'ladi, ya'ni

$$x \in A \Rightarrow x \notin C(A), \quad x \notin A \Rightarrow x \in C(A).$$

Masalan, Ω={Barcha korxonalar}, A={Rejani bajargan korxonalar} bo'lsa, unda C(A)={ Rejani bajarmagan korxonalar} to'plami bo'ladi;

Ω={1,2,3, · · ·, n, · · ·}–natural sonlar to'plami, A={2,4,6, · · ·, 2n, · · ·}–juft sonlar to'plami, B={5,6,7, · · ·, n, · · ·}– 4dan katta natural sonlar to'plami bo'lsa, unda

C(A)={1,3,5, · · ·, 2n–1, · · ·}– toq sonlar, C(B)={1,2,3,4}–5dan kichik natural sonlar to'plamlarini ifodalaydi.A va B to'plamlarning ***Dekart ko'paytmasi*** deb A×B kabi belgilanadigan va (x, y) (x∈A, y∈B) ko'rinishdagi juftliklardan tuzilgan yangi to'plamga aytiladi.

Agar C={Tajribali ishchilar} va D={Yosh ishcilar} bo'lsa, unda C×D tajribali va yosh ishchidan iborat bo'lgan turli "ustoz-shogird" juftliklaridan iborat to'plamni ifodalaydi.

Umuman olganda to'plamlarning Dekart ko'paytmasi uchun A×B≠B×A, ya'ni kommutativlik qonuni bajarilmaydi. Masalan, A=[0,2] va B=[0,1] to'plamlar uchun A×B asosining uzunligi 2, balandligi 1 bo'lgan to'g'ri to'rtburchakni , B×A esa asosining uzunligi 1, balandligi 2 bo'lgan to'g'ri to'rtburchakni ifodalaydi va bunda A×B≠B×A bo'ladi.

X chekli to'plam elementlari sonini $n(X)$ orqali belgilaymiz. k ta elementli X to'plamni k elementli to'plam deb ataymiz.

Berilgan masalalarni yeching

5.1. $A=\{x|x\in N,\quad x^2>7\}$ to'plam 2 dan katta bo'lgan barcha natural sonlardan tuzilgan, ya'ni $A=\{3,4,5,6,7,8,9....\}$.

5.2. $x^2+3x+2=0$ tenglamaning ildizlari $X=\{-2;-1\}$ cheksiz to'plamni tashkil etadi.

5.3. $X=\{x|x\in N,\quad x\leq 3\}$ va $Y=\{x|(x-1)(x-2)(x-3)=0\}$ to'plamlar tengmi?

5.4. A-ikki xonali sonlar to'plami, B-ikki xonali juft sonlar to'plami bo'lsin. Har bir ikki xonali juft son A to'plamda ham mavjud. $A = B$ tenglik o'rinlimi?

5.5. $A = \{1,2,3,4\}$, $B = \left\{1, \frac{4}{2}, \sqrt{9}, 2^2\right\}$ bo'lsa, $A \subset B, B \subset A$ munosabat o'rinlimi?

5.6. 100 kishidan iborat sayyohlar guruhida 70 kishi ingliz tilini, 41 kishi fransuz tilini, 23 kishi esa ikkala tilni biladi. Sayyohlar guruhidagi necha kishi ingliz tilini ham, fransuz tilini ham bilmaydi?

5.7. O'zbekiston Respublikasining Davlat Gerbi qabul qilingan yilni ifodalovchi sonda qatnashgan raqamlar to'plamini tuzing.

5.8. $B = \left\{10; 12\frac{3}{4}; 17,3; -7; 136\right\}$ to'plam berilgan. Qaysi natural sonlar bu to'plamga kiradi? Shu to'plamga tegishli bo'lmagan uchta son ayting. Javobni $\in, \notin$ belgilari yordamida yozing.

5.9. S to'plam -3;-2;-1;4 elementlaridan tuzilgan. Shu sonlarga qarama-qarshi sonlarning S_1 to'plamini yozing.

5.10. "Bo'sh vaqtdan unumli foydalan" jumlasidagi harflar to'plamini tuzing.

5.11. Quyidagi har bir to'plamning elementlarni ko'rsating:

a) $E = \{x | x \in N, \ -1 < x < 5\}$ b) $F = \{x | 5x = x - 7\}$

d) $Q = \{x | x(x + 12) = 0\}$ e) $U = \{x | x \in R, x^2 = 2\}$

f) $V = \{x | x \in N, x^2 < 9\}$ g) $U = \{x | x \in N, x^2 \leq 9\}$

5.12. Quyidagi to'plamlarni son o'qida belgilang:

a) $\{x | x \in N, \ x \leq 3\}$ b) $\{x | x \in Z, \ -2 \leq x \leq 2\}$

d) $\{x | x \in R, \ x > 4,1\}$ e) $\{x | x \in R, \ -2,7 \leq x \leq 1\}$

f) $\{x | x \in R, x < 6\}$ g) $\{x | x \in R, 3,4 < x \leq 8\}$

h) $\left\{x | x \in R, \ -3\frac{1}{4} \leq x \leq -1\right\}$ i) $U = \{x | x^2 = 4\}$ j) $\{x | x(x^2 - 1)(x^2 - 4) = 0\}$

5.13. Quyidagi to'plam qaysi elementlardan tuzilgan:

a) 1 va 3 bilangina yoziladigan barcha uch xonali sonlar to'plami,

b) 1,3,5 raqamlardan (faqat bir marta) foydalanib yoziladigan barcha uch xonali sonlar to'plami;

d) raqamlarining yig'indisi 5 ga teng bo'lgan uch xonali sonlar to'plami;

e) 100 dan kichik va oxirgi raqami 1 bo'lgan barcha natural sonlar to'plami?

5.14. Quyidagi to'plamlardan qaysilari bo'sh to'plam:

a) simmetriya markaziga ega bo'lmagan kvadratlar to'plami;

b) $\{x | x^2 + 1 = 0\}$ d) $\{x | x^2 = 4\}$ e) $\{x | x \in R, \quad x^3 = 1\}$

5.15. Quyidagi to'plamning nega bo'sh to'plam ekanligini tushuntiring:

a) $\{x | x \in N, \quad x < -1\}$ b) $\{x | x \in N, \quad 15 < x < 16\}$

d) $\left\{x | x \in N, \quad x = \frac{3}{5}\right\}$ e) $\{x | x > 7, \quad x < 5\}$

5.16. To'plamning haqiqiy ildizlari to'plamini toping. Bu to'plamlarning qaysilari bo'sh to'plam ekanligini aniqlang:

a) $3x + 15 = 4(x - 8)$; b) $2x + 4 = 4$; d) $2(x - 5) = 3x$;

e) $x^2 - 4 = 0$; f) $x^2 + 16 = 0$; g) $(2x + 7)(x - 2) = 0$.

5.17. Quyidagi to'plam elementlarini va elementlar sonini ko'rsating:

a) $\{l, f, g\}$ b) $\{a\}$ d) $\{\{a\}\}$ e) $\varnothing$ f) $\{\varnothing\}$

g) $\{\{a, b\}, \{c, d\}\}$ h) $\{\{a, b, c\}, a\}$

5.18. 5 ta elementi bor bo'lgan to'plam tuzing.

5.19. 5 ta natural son qatnashgan sonly to'plam tuzing.

5.20 $A = \{a, b, c, d, e, f, g, k\}$ $B = \{a, l, k\}$, $C = \{b, a, g, k, t\}$, $D = \{a, l\}$, $E = \{e, f, k, g\}$ to'plamlar berilgan.

a) ularning qaysilari A to'plamning xos qism to'plami bo'ladi?

b) D to'plam C to'plamning qism to'plamimi?

d) B to'plam qaysi to'plamning qism to'plami bo'ladi?

e) $n(A), n(B), n(C), n(D), n(E)$ sonlarni o'sish tartibida joylashtiring.

5.21. $A = \{3, 6, 9, 12\}$ to'plamning barcha qism to'plamlarini tuzing.

5.22. To'plamlar jufti berilgan:

a) $A = \{Navoiy, Bobur, Furqat, Nodirabegim\}$ va B-barcha shoir va shoiralar to'plami;

b) C-qavariq to'rtburchaklar to'plami va D – to'rtburchaklar to'plami;

d) E-Samarqand olimlari to'plami; F-O'zbekiston olimlari to'plami;

e) K-barcha tub sonlar to'plami, M-manfiy sonlar to'plami

juftlikdagi to'plamlardan qaysi biri ikkinchisining qism to'plami bo'lishini aniqlang.

5.23. Quyidagi to'plamlar uchun $A \subset B$ yoki $B \subset A$ munosabatlardan qaysi biri o'rinli:

a) $A = \{a, b, c, d\}$, $B = \{a, c, d\}$; b) $A = \{a, b\}$, $B = \{a, c, d\}$;

d) $A = \emptyset$, $B = \emptyset$; e) $A = \emptyset$, $B = \{a, b, c\}$;

f) $A = \emptyset$, $B = \{\emptyset\}$; g) $A = \{\{a\}, a, \emptyset\}$, $B = \{a\}$;

h) $A = \{\{a, b\}, \{c, d\}, c, d\}$, $B = \{\{a, b\}, c\}$; i) $A = \{\{0\}, 0\}$, $B = \{\emptyset, \{0\}, 0\}$?

5.24. Munosabatning to'g'ri yoki noto'g'ri ekanligini aniqlang:

a) $\{1; 2\} \subset \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$ b) $\{1; 2\} \in \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$

d) $\{1; 3\} \subset \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$ e) $\{1; 3\} \in \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$

5.25. Quyidagi to'plamlar tengmi:

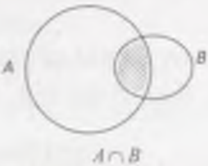
a) $A = \{2; 4; 6\}$ va $B = \{6; 4; 2\}$; b) $A = \{1; 2; 3\}$ va $B = \{1; 11; 111\}$;

d) $A = \{\{1; 2\}; \{2; 3\}\}$ va $B = \{2; 3; 1\}$; e) $\{\sqrt{256}; \sqrt{81}; \sqrt{16}\}$ va $B = \{2^2; 3^2; 4^2\}$?

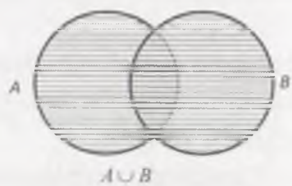
5.26. $x = \{x | x^2 - 5x + 6 = 0\}$ to'plamlar elementlari va $A = \{2; 3\}$ to'plamlar haqida nim deyish mumkin?

§ 5.2.To'plamlar ustida amallar

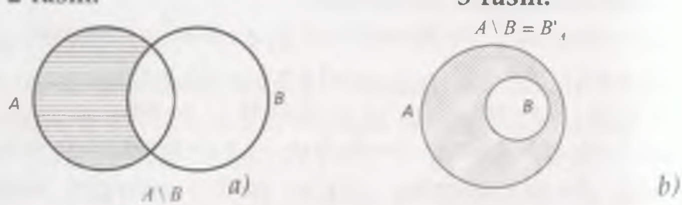
To'plamlar ustida amallar. A va B to'plamlarning ikkalasida ham mavjud bo'lgan x elementga shu to'plamlarning umumiy elementi deyildi. A va B to'plamlarning kesishmasi (yoki ko'paytmasi) deb, ularning barcha umumiy elementlaridan tuzilgan to'plamga aytiladi. A va B to'plamlarning kesishmasi $A \cap B$ ko'rinishda belgilanadi: $A \cap B = \{x | x \in A \quad va \quad x \in B\}$. 2-rasmda Eyler – Venn diagrammasi nomi bilan ataladigan chizmada A va B shakllarning kesishmasi $A \cap B$ ni beradi (chizmada shtrixlab ko'rsatilgan).



2-rasm.



3-rasm.



5.1-rasm

A va B to'plamlarning birlashmasi (yoki yig'indisi), deb ularning kamida bittasida mavjud bo'lgan barcha elementlardan tuzilgan to'plamga aytiladi. A va B to'plamlarning birlashmasi $A \cap B$ ko'rinishida belgilanadi: $A \cup B = \{x | x \in A \quad yoki \quad x \in B\}$ (3-rasm).

A to'plamdan B to'plamlarning ayirmasi, deb A da mavjud va B da mavjud bo'lmagan barcha elementlardan tuzilgan to'plamga aytiladi. A to'plamdan B to'plamlarning ayirmasi $A - B$ ko'rinishida belgilanadi: $A - B = \{x | x \in A \quad va \quad x \notin B\}$ (4-rasm). To'plamlar ustida bajariladigan amallarning xossalari sonlar ustida bajariladigan amallarning xossalariga o'xshash. Har qanday X,Y va Z to'plamlar uchun:

1) $X \cup Y = Y \cup X$;   1') $X \cap Y = Y \cap X$;

2) $(X \cup Y) \cup Z = X \cup (Y \cup Z) = (X \cup Z) \cup Y$;

2') $(X \cap Y) \cap Z = (X \cap Z) \cap Y = X \cap (Y \cap Z)$;

3) $(X \cup Y) \cap Z = (X \cup Z) \cup (Y \cup Z)$;   3') $(X \cap Y) \cup Z = (X \cup Z) \cap (Y \cup Z)$.

1) $\emptyset = U$; 2) $U' = \emptyset$; 3) $(X')' = X$, 4) U dan olingan har qanday X va Y to'plam uchun $(X \cap Y)' = X' \cap Y'$; $(X \cup Y)' = X' \cap Y'$.Shuningdek, agar $X \subset Y$ bo'lsa, $X \cap Y = X$, $X \cup Y = Y$ bo'ladi. Xususan, $\emptyset \subset X$ va $X \subseteq X$ bo'lganda, $\emptyset \cup X = \emptyset$, $\emptyset \cup X = X$, $X \cap X = X$, $X \cup X = X$ bo'ladi.

To'plamlarga doir masalalarni yeching

5.27. $A = \{a, b, c, d, e, f\}$ va $B = \{b, d, e, g, f\}$ to'plamlar berilgan. Ularning kesishmasi va birlashmasini Eyler-Venn diagrammasida talqin eting?

5.28 $A = \{1,2,3,4\}$, $B = \{1,3,5\}$, $C = \{1,5,9\}$ to'plam berilgan. $D = \{1,2,3,4,5,9\}$ to'plam universal to'plam bo'ladimi? $E = \{1,2,3,4,5,9,15\}$ va $M = \{1,3,4,5,9\}$ to'plamlar-chi?

5.29. $M = \{36; 29; 15; 68; 27\}$, $P = \{4; 15; 27; 47; 36; 90\}$, $Q = \{90; 4; 47\}$ to'plamlar berilgan. $M \cap P, M \cap Q, P \cap Q, M \cap P \cap Q$ larni toping.

5.30. A-18 ning hamma natural bo'linuvchilari to'plami, B-24 ning hamma natural bo'linuvchilari to'plami. $A \cap B$ to'plam elementlarini ko'rsting.

5.31. P ikki xonali natural sonlar to'plami, S barcha toq natural sonlar to'plami bo'lsa, $K = P \cap S$ to'plamga qaysi sonlar kiradi?

a) $21 \in K$; b) $32 \in K$; d) $7 \notin K$; e) $17 \notin K$ deyish to'g'rimi?

5.32. "Matematika" va "grammatika" so'zlaridagi harflar to'plamini tuzing. Bu to'plamlar kesishmasini toping.

5.33. [1; 5] va [3; 7] kesmalarning kesishmasini toping.

5.34. $P = \{a, b, c, d, e, f\}$ va $E = \{a, g, z, e, k\}$ to'plamlar birlashmasini toping.

5.35. $A = \{n | n \in N, n < 5\}$ va $B = \{n | n \in N, n > 7\}$ to'plamlar birlashmasini toping.

a) $4 \in A \cup B$; b) $-3 \in A \cup B$; d) $6 \in A \cup B$ deyish to'g'rimi?

5.36. Agar a) $A = \{x|x = 8k, k \in Z\}$, $B = \{x|x = 8l - 4, l \in Z\}$;

b) $A = \{x|x = 6k - 1, k \in Z\}$, $B = \{x|x = 6l + 4, l \in Z\}$ bo'lsa, $A \cup B$ ni toping.

5.37. $A = \{2; 4; 6; 8; ...; 40\}, B = \{1; 3; 5; 7; ...; 37\}$, $C = \{\{a; b\}, \{c; d\}, \{e; f\}, g, h\}$ to'plamlarning har biridagi elementlar sonini aniqlang. $A \cup B$ da nechta element mavjud?

5.38. $A = \{2; 3; 4; 5; 7; 10\}, B = \{3; 5; 7; 9\}, C = \{4; 9; 11\}$ bo'lsin. Quyidagi to'plamlarda nechtadan element mavjud:

a) $A \cup (B \cup C)$ b) $(C \cup B) \cup A$ d) $A \cap (B \cup C)$

e) $A \cup (B \cap C)$ f) $A \cap (B \cap C)$ g) $B \cap (A \cup C)$?

5.39. $A = \{x| -5 \le x \le 10\}$, $B = \{x|x \in N, 3 \le x \le 15\}$ bo'lsin. $A/_B$ va $B/_A$ to'plam elementlarini toping.

5.40. P-ikki xonali natural sonlar to'plami, Q-juft natural sonlar to'plami bo'lsin. P/Q va Q/P to'plamlarni tuzing.

5.41. C va D kesishuvchi to'plamlar bo'lsin. Eyler –Venn diagrammalari yordamida $C/D, D/C, C/D \cup D/C$ larni tasvirlang.

5.42. N' bilan natural sonlar to'plami N ning butun sonlar to'plami Z ga to'ldiruvchisini belgilaymiz. Quyidgilar to'g'rimi:

a) $-4 \in N'$; b) $0 \in N'$; d) $13 \in N'$; e) $-8 \notin N'$;

f) $-5{,}3 \notin N'$; g) $0 \notin N'$.

5.43. $A = \{x|x = 2k + 1, k \notin Z\}$ to'plamning Z to'plamga to'ldiruvchisini toping.

5.44. $A = \{x|x = 3k, k \notin Z\}$ to'plamning Z to'plamga to'ldiruvchisini toping.

5.45. Agar $A \subset U, B \subset U$ bo'lsa, quyidagi tengliklar o'rinli bo'lishini isbotlang:

a) $(A \cup B)^{\prime} = A^{\prime} \cap B^{\prime}$, b) $(A \cap B)^{\prime} = A^{\prime} \cup B^{\prime}$.

5.46. Agar A to'plam $x^2 - 7x + 6 = 0$ tenglamaning ildizlari to'plami va $B = \{1; 6\}$ bo'lsa, $A = B$ bo'lishini isbotlang.

5.47. $A \backslash B = A \backslash (A \cap B)$ tenglikni isbotlang.

5.48. $A \cap (B \backslash A) = \emptyset$ tenglikni isbotlang.

5.49. $A \subset U, B \subset U, A \cap B = \emptyset$ bo'lsin. Quyidagilarni Eyler-Benn diagrammalari yordami bilan tasvirlang va ulardan tenglarini ko'rsating:

1) $(A'\cap B)$; 2) $A\cap B'$; 3) $A\cap B$; 4) $A\cup B'$; 5) $(A'\cup B)$; 6) $A\cup B$

5.50. a) Munosabatlarni isbot qiling:

1) $(A\cup B)\setminus B = A$; 2) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

b) A va B lar U universal to'plamning qism-to'plamlari. Isbot qiling:

1) $(A\cap B)' = A'\cup B'$;

5.51. Ifodalarni soddalashtiring:

1) $B\cap(A\cup B)$; 2) $(A\cap B)\cap(A'\cap B)$.

5.52. Sinfda bir necha o'quvchi marka yig'dilar. 15 o'quvchi O'zbekiston markalarini, 11 kishi chet el markalarini, 6 kishi ham O'zbekiston markalarini, ham chet el markalarini yig'di. Sinfda necha o'quvchi marka to'plagan?

5.53. 32 o'quvchining 12 tasi voleybol seksiyasiga, 15 tasi basketbol seksiyasiga, 8 kishi esa ikkala seksiyaga ham qatnashadi. Sinfdagi necha o'quvchi hech bir seksiyaga qatnashmaydi?

5.54. 30 o'quchidan 18 tasi matematikaga, 17 tasi esa fizikaga qiziqadi. Ikkala fanga ham qiziqadigan o'quvchilar soni nechta bo'lishi mumkin? (**Ko'rsatma**. Ikkala fanga ham qiziqmaydigan o'quvchilar soni $k\in\{0,1,2,3,\dots,12\}$).

5.55. 100 odamdan iborat sayyohlar guruhida 10 kishi nemis tilini ham, fransuz tilini ham bilmaydi, 75 tasi nemis tilini, 83 tasi esa fransuz tilini biladi. Ikkala tilni ham biladigan sayyohlar sonini toping.

5.56. 26 o'quvchining 14 tasi shaxmatga, 16 tasi shashkaga qiziqadi. Ham shashkaga, ham shaxmatga qiziqadigan o'quvchilar nechta?

Takrorlash uchun savollar

1. To'plam deb nimaga aytiladi?
2. Universal to'plam deb nimaga aytiladi?
3. Qism to'plam nima?
4. To'plamning qanday xossalari bor?
5. To'plamning birlashmasi va kesishmasining farqi nimada?
6. Bo'sh to'plam nima?
7. Qachon to'plamlar teng deyiladi?

TO'PLAMLARGA DOIR NAZORAT TESTLARI

1. To'plamlar va ular ustida amallar

1. To'plamlar nazariyasining asoschisi kim ?

A) Pifagor . B) Dekart . C) Kantor .

D) Ferma . E) Gauss .

2. Quyidagi to'plamlardan qaysi biri bo'sh to'plam emas?

A) Kvadrati manfiy bo'lgan haqiqiy sonlar.

B) sinx = 2 tenglama yechimlari to'plami.

C) Ikkita burchagi o'tmas bo'lgan uchburchaklar to'plami.

D) Kubi manfiy bo'lgan sonlar to'plami.

E) Ikkiga bo'linmaydigan juft sonlar to'plami.

3. Qachon A to'plam B to'plamning qismi deyiladi?

A) Agar A va B bir xil elementlardan tashkil topgan bo'lsa.

B) Agar A va B har xil elementlardan tashkil topgan bo'lsa.

C)Agar B to'plamning har bir elementi A to'plamga tegishli bo'lsa.

D)Agar A to'plamning har bir elementi B to'plamga tegishli bo'lsa.

E) To'g'ri javob keltirilmagan.

4. Quyidagi tasdiqlardan qaysi biri noto'g'ri?

A) bo'sh to'plam barcha to'plamlarning to'plam ostisi bo'ladi.

B) har bir to'plam o'zining to'plam ostisi bo'ladi.

C)Agar $A \subset B$ va $C \subset A$ bo'lsa, unda $C \subset B$ bo'ladi.

D)Agar $B \subset A$ bo'lsa, unda $A \cap B=B$ bo'ladi.

E)Agar $B \subset A$ bo'lsa, unda $A \cup B=B$ bo'ladi.

5. A va B to'plamlar birlashmasi amali qayerda ifodalangan

A) $A \cup B$. B) $A \cap B$. C) $A \subset B$.

D) $A \supset B$. E) $A \setminus B$.

6. Agar $x \in A \cup B$ bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli emas ?

A) $x \in A$, $x \notin B$. B) $x \notin A$, $x \in B$. C) $x \notin A$, $x \notin B$.

D) $x \in A$, $x \in B$. E) barcha tasdiqlar o'rinli bo'ladi .

7. To'plamlar birlashmasi amalining xossasi qayerda noto'g'ri ko'rsatilgan ? (Ω – universal to'plam, $\varnothing$ – bo'sh to'plam)

A) $A \cup B = B \cup A$. B) $A \cup \varnothing = A$. C) $A \cup A = A$.

D) $A \cup \Omega = \Omega$. E) Barcha xossalar to'g'ri ko'rsatilgan.

8. A = [–3, 0] va B = (–1, 5] to'plamlar birlashmasi qayerda to'g'ri ko'rsatilgan?

A) [–3, 5]. B) [–3, –1]. C) (–1, 0). D) (0, 5]. E) [–1, 5].

9. A va B to'plamlar kesishmasi amali qayerda ifodalangan?

A) $A \cup B$. B) $A \cap B$. C) $A \subset B$. D) $A \supset B$. E) $A \setminus B$.

10. Agar $x \in A \cap B$ bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli bo'ladi ?

A) $x \in A$, $x \notin B$. B) $x \notin A$, $x \in B$. C) $x \notin A$, $x \notin B$.

D) $x \in A$, $x \in B$. E) barcha tasdiqlar o'rinli emas .

2.Chekli va cheksiz to'plamlar

1. Chekli to'plamni ko'rsating.

A) Moduli 2 dan kichik bo'lgan haqiqiy sonlar to'plami.

B) Moduli 2 dan kichik ratsional sonlar to'plami.

C) Moduli 2 dan kichik irratsional sonlar to'plami.

D) Moduli 2 dan kichik butun sonlar to'plami.

E) Moduli 2 dan kichik kasr sonlar to'plami.

2. Quyidagi to'plamlardan qaysi biri cheksiz to'plam bo'ladi?

A) $ax^2+bx+c=0$ kvadrat tenglama ildizlari to'plami .

B) $ax+b=c$ $(a\neq0)$ chiziqli tenglama ildizlari to'plami .

C) $\sin x=a$ $(|a|\leq1)$ trigonometrik tenglama ildizlari to'plami .

D) $\log_a x=b$ ($a>0$, $a\neq 1$) logarifmik tenglama ildizlari to'plami .

E) $a^x = b$ ($a>0$, $a\neq 1$) ko'rsatgichli tenglama ildizlari to'plami .

3. Quyidagi to'plamlaridan qaysi biri sanoqli emas?
A) butun sonlar to'plami . B) ratsional sonlar to'plami .
C) juft sonlar to'plami . D) toq sonlar to'plami .
E) irratsional sonlar to'plami.

4. Quyidagi to'plamlaridan qaysi biri sanoqsiz?
A) butun sonlar to'plami . B) ratsional sonlar to'plami .
C) irratsional sonlar to'plami . D) toq sonlar to'plami .
E) juft sonlar to'plami.

5. Quyidagi to'plamlaridan qaysi biri sanoqli?
A) (a,b) oraliqdagi sonlar to'plami.
B) ratsional sonlar to'plami .
C) irratsional sonlar to'plami .
D) haqiqiy sonlar to'plami .
E) musbat sonlar to'plami.

6. Quyidagi to'plamlardan qaysi biri chekli emas?
A) kitobdagi varaqlar to'plami.
B) alfavitdagi harflar to'plami.
C) tub sonlar to'plami.
D) ko'pburchak tomonlari to'plami.
E) Buxorodagi talabalar to'plami .

7. Quyidagi tenglamalardan qaysi birining ildizlari cheksiz to'plam hosil qiladi?
A) $a^x=b$ ($a>0$, $a\neq 1$) . B) $\log_a x=b$ ($a>0$, $a\neq 1$).
C) $ax^3+bx^2+cx+d=0$. D) $\sin ax=b$ ($|b|\leq 1$).
E) Barcha tenglama ildizlari chekli to'plam hosil qiladi .

8. Quyidagi to'plamlardan qaysi biri cheksiz emas?
A) 5 ga karrali natural sonlar to'plami.
B) to'g'ri burchakli uchburchaklar to'plami.
C) $\cos x=0.7$ tenglama ildizlari to'plami.

D) Yer yuzidagi barcha odamlar to'plami.

E) $[a,b]$ kesmadagi nuqtalar to'plami.

9. Qaysi javobdagi A va B to'plamlar ekvivalent ($A \sim B$) emas?

A) $A=Z=$ {butun sonlar to'plami}, $B=N=${natural sonlar to'plami}.

B) $A=[0,1]$, $B=[0,10]$.

C) $A=\{1,2,3,4,5\}$, $B=\{11,12,13,14,15\}$.

D) $A=\{10,15,20,25\}$ $B=\{31,32,33,34,35\}$.

E) $A=N=${natural sonlar to'plami}, $B=Q=${ratsional sonlar to'plami}.

10. *Sanoqli to'plam ta`rifini to'ldiring:* A to'plam sanoqli deyiladi, agarda u .... to'plamiga ekvivalent bo'lsa.

A) (0,1) oraliqdagi nuqtalar. B) irratsional sonlar .

C) [0,1] kesmadagi nuqtalar.

D) natural sonlar. E) haqiqiy sonlar.

3. Qavariq to'plamlar

1. Quyidagi to'plamlardan qaysi biri nuqtaviy emas?

A) $[a,b]$ kesma . B) (a,b) oraliq . C) doira . D) shar .

E) keltirilgan barcha to'plamlar nuqtaviy .

2. *Ta'rifni to'ldiring:* Nuqtaviy A to'plam qavariq deyiladi, agarda uning ixtiyoriy ikkita nuqtasini tutashtiruvchi kesmaning ..... A to'plamga tegishli bo'lsa.

A) faqat chegaraviy nuqtalari . B) kamida bitta nuqtasi .

C) kamida ikkita nuqtasi . D) barcha nuqtalari .

E) bir qism nuqtalari .

3. Tekislikdagi ushbu to'plamlardan qaysi biri qavariq bo'lmasligi mumkin?

A) trapetsiya. B) uchburchak.

C) to'rtburchak. D) kesma. E) romb.

4. Fazodagi ushbu to'plamlardan qaysi biri qavariq bo'lmasligi mumkin?

A) shar. B) piramida. C) prizma. D) konus.

E) keltirilgan barcha to'plamlar doimo qavariq bo'ladi.

5. *Tasdiqni to'ldiring:* A to'plamning ichki nuqtasini o'z ichiga oluvchi shunday yetarli kichik kesma mavjudki, uning ··· A to'plamga tegishli bo'ladi.

A) faqat bitta nuqtasi . B) faqat biror nuqtasi .

C) barcha nuqtalari D) faqat bir qism nuqtalari .

E) faqat ikkita nuqtasi .

6. Agar *l* kesma A to'plamning biror chetki nuqtasini o'z ichiga olsa, quyidagi tasdiqlardan qaysi biri o'rinli bo'la olmaydi?

A) *l* kesmaning bitta nuqtasi A to'plamga tegishli.

B) *l* kesmaning biror nuqtasi A to'plamga tegishli.

C) *l* kesmaning barcha nuqtalari A to'plamga tegishli.

D) *l* kesmaning bir qism nuqtalari A to'plamga tegishli.

E) barcha tasdiqlar o'rinli bo'ladi.

7. Trapetsiya nechta chetki nuqtaga ega ?

A) 0 . B) 1 . C) 2 . D) 3 . E) 4 .

8. Tekislikdagi ushbu to'plamlardan qaysi biri cheksiz ko'p chetki nuqtaga ega?

A) romb. B) kesma. C) uchburchak.

D) doira. E) trapetsiya.

9. Kub nechta chetki nuqtaga ega ?

A) 0 . B) 2 . C) 4 . D) 6 . E) 8 .

10. Fazodagi ushbu to'plamlardan qaysi biri cheksiz ko'p chetki nuqtaga ega?

A) piramida. B) prizma. C) kub. D) shar. E) parallelepiped.

Matematikaning asosiy vazifasi bizni o'rab turgan tartibsizliklarda yashiringan tartibni topishdan iborat.

N.Viner

VI-BOB. FUNKSIYA, UNING LIMITI VA UZLUKSIZLIGI

§ 6.1. Funksiya va u bilan bog'liq tushunchalar

1. Agar X to'plamning har bir x elementiga Y to'plamning ma'lum bir y elementi biror f qonun-qoida asosida mos qo'yilgan bo'lsa, u holda X to'plamda $y = f(x)$ ***funksiya*** berilgan deyiladi. Bunda ***x-erkli o'zgaruvchi yoki argument, y-erksiz o'zgaruvchi yoki funksiya*** deyiladi.

$y = f(x)$ funksiyada x argument qabul qila oladigan barcha qiymatlar to'plami funksiyaning ***aniqlanish sohasi*** deyiladi va D{*f*} kabi belgilanadi. $x \in D\{f\}$ bo'lganda $y = f(x)$ funksiya qabul qiladigan qiymatlar to'plami funksiyaning ***o'zgarish sohasi*** deb ataladi va E{*f*} kabi belgilanadi.

2.Agar $y = f(x)$ funksiya uchun $f(-x) = f(x)$ yoki $f(-x) = -f(x)$ shart bajarilsa u mos ravishda ***juft yoki toq funksiya*** deyiladi.

3. Agar $y = f(x)$ funksiyada argumentning har qanday $x_1 < x_2$ qiymatlari uchun $f(x_1) < f(x_2)$ yoki $f(x_1) > f(x_2)$ shart bajarilsa, u mos ravishda ***o'suvchi yoki kamayuvchi funksiya*** deyiladi. O'suvchi va kamayuvchi funksiyalar birgalikda ***monoton funksiyalar*** deyiladi.

4. Agarda ixtiyoriy $x \in D\{f\}$ va biror chekli M soni uchun $|f(x)| < M$ tengsizlik bajarilsa, unda $y = f(x)$ ***chegaralangan funksiya*** deyiladi. Aks holda $y = f(x)$ chegaralanmagan funksiyani ifodalaydi.

5. Quyidagilar ***asosiy elementar funksiyalar*** deb ataladi:

1) $y = x^{\alpha}$ - ***darajali funksiya***. Bunda daraja ko'rsatkichi α ixtiyoriy haqiqiy son bo'lishi mumkin. Darajali funksiyaning aniqlanish D{*f*} va o'zgarish E{*f*} sohalari α qiymatiga qarab topiladi.

2) $y = a^x; a > 0\ a \neq 1$ - ***ko'rsatkichli funksiya***. Bunda $D\{f\} = (-\infty, \infty)$ va $E\{f\} = (0, +\infty)$ bo'ladi.

3) $y = \log_a x \quad a > 0, a \neq 1$ -***logarifmik funksiya***. Bunda $D\{f\} = (0, +\infty)$ va $E\{f\} = (-\infty, +\infty)$ bo'ladi.

4) $y = \sin x, \quad y = \cos x, \quad y = tgx, \quad y = ctgx$ - ***trigonometrik funksiyalar***. Bu yerda

$$D\{\sin x\} = D\{\cos x\} = (-\infty, +\infty), \quad E\{\sin x\} = E\{\cos x\} = [-1, 1],$$

$$D\{tgx\} = \{x : x \neq \frac{\pi}{2}(2k+1), k = 0, \pm 1, \pm 2, \cdots\}, \quad E\{tgx\} = (-\infty, +\infty),$$

$$D\{ctgx\} = \{x : x \neq \pi k, k = 0, \pm 1, \pm 2, \cdots\}, \quad E\{ctgx\} = (-\infty, +\infty).$$

5) $y = \arcsin x, \ y = \arccos x, \quad y = arctgx, \quad y = arcctgx$ - ***teskari trigonometrik funksiyalar***. Bu funksiyalar uchun

$$D\{\arcsin x\} = [-1, 1], \quad E\{\arcsin x\} = [-\pi/2, \pi/2],$$

$$D\{\arccos x\} = [-1, 1], \quad E\{\arccos x\} = [0, \pi],$$

$$D\{arctgx\} = (-\infty, +\infty), \quad E\{arctgx\} = (-\pi/2, \pi/2),$$

$$D\{arcctgx\} = (-\infty, +\infty), \quad E\{arcctgx\} = (0, \pi).$$

6. *y*=*f*(*x*) va *y*=*g*(*x*) funksiyalar berilgan bo'lib, $E\{g\} \subseteq D\{f\}$ shart bajarilgan bo'lsin. Bu holda *y*=*f*(*g*(*x*)) funksiya aniqlangan bo'lib, u ***murakkab funksiya*** deb ataladi. Bunda *y*=*f*(·) – ***tashqi***, *y*=*g*(*x*)- ***ichki funksiya*** deyiladi. Murakkab funksiya hosil etish amali funksiyalarni kompozitsiyalash deb yuritiladi.

7. Chekli sondagi asosiy elementar funksiyalar ustida arifmetik va kompozitsiyalash amallarini bajarish orqali hosil qilingan funksiyalar ***elementar funksiyalar*** deyiladi.

8. Agar $y = f(x)$ funksiya uchun biror *T*>0 soni va x argumentning barcha qiymatlarida $f(x+T) = f(x)$ tenglik o'rinli bo'lsa, unda *y*=*f*(*x*) ***davriy funksiya***, *T* esa uning ***davri*** deyiladi.

9. *y*=*f*(*x*) funksiya D{*f*} aniqlanish sohasiga tegishli har bir x nuqtaga E{*f*} o'zgarish sohasiga tegishli bitta *y* nuqtani mos qo'yadi. Bu yerda har bir $y \in E\{f\}$ nuqtaga bitta $x \in D\{f\}$ nuqtani mos qo'yuvchi $x=g(y)$ funksiya mavjud bo'lsin. Bu holda $x=g(y)$ funksiya $y=f(x)$ funksiyaga ***teskari funksiya*** deyiladi. Odatda $y=f(x)$ funksiyaga teskari funksiya $y=f^{-1}(x)$ kabi belgilanadi. Bunda $[f^{-1}(x)]^{-1}=f(x)$ munosabat o'rinli bo'ladi va shu sababli $f(x)$ hamda $f^{-1}(x)$ o'zaro teskari funksiyalar deb yuritiladi. Bunda $D\{f\}=E\{f^{-1}\}, E\{f\}=D\{f^{-1}\}$ munosabatlar o'rinli bo'ladi.

10. Grafiklarni almashtirish:

a) $y=f(x+a)$ funksiya grafigi $y=f(x)$ funksiya grafigini OX o'qi bo'yicha a birlik chapga ($a>0$) yoki o'ngga ($a<0$) parallel ko'chirishdan hosil bo'ladi.

b) $y=f(x)+b$ funksiya grafigi $y=f(x)$ funksiya grafigini OY o'qi bo'yicha b birlik yuqoriga ($b>0$) yoki pastga ($b<0$) parallel siljishitishdan hosil bo'ladi.

c) $y=Cf(x)$ $(C\neq 0)$ funksiya grafigi $y=f(x)$ funksiya grafigini OY o'qiga nisbatan C marta cho'zish ($|C|>1$) yoki qisish ($0<|C|<1$) orqali hosil qilinadi.

d) $y=f(kx)$ funksiya grafigi $y=f(x)$ funksiya grafigini OX o'qiga nisbatan k marta kengaytirish ($k>1$) yoki qisish ($0<k<1$)orqali hosil qilinadi.

11. Haqiqiy x sonning absolut qiymati $|x|$ kabi belgilanadi va

$$|x|=\begin{cases} x, & agar \quad x\geq 0 \\ -x, & agar \quad x<0 \end{cases}$$

tenglik blan aniqlanadi.

1. Quyidagi funksiyalarning aniqlanish sohasini toping:

6.1) $y=\dfrac{\sqrt[5]{\lg(x+1)}}{x-1}+2^{\sqrt{10-x}}$; 6.2) $y=\dfrac{\sqrt[6]{16-x^2}}{\lg(x-1)^2}$;

6.3) $y=\sqrt{4-x^2}\,tgx$;

6.4) $y=\dfrac{\arcsin(x-1)}{\lg x}$; 6.5) $y=\dfrac{\sqrt{\sin x-0.5}}{\sqrt[3]{x-2}}-\lg(x-1)\ln(4-x)$;

6.6) $y = \dfrac{x}{\sqrt[4]{25 - x^2}}$; 6.7) $y = \dfrac{3^{\sqrt{x}}}{\lg(3 - x)}$;

6.8) $y = \sqrt{x + 2} - \ln(4 - x)$; 6.9) $y = \dfrac{\sqrt{1 - x^2}\ln(x + 1)}{(x^2 + 1)\sqrt{5^x}} - \dfrac{\sqrt[4]{x - 1}}{x}$;

6.10) $y = \dfrac{\arcsin x}{\sin 5x}$. 6.11) $y = \dfrac{\sqrt{x^2 - 4}}{2^x(x - 6)} + \ln(x + 10)$

2.Quyidagi funksiyalarning o'zgarish sohasini toping:

6.12) $y = 3\sin x + 4\cos x$; 6.13) $y = e^{-\frac{x^2}{2}}$;

6.14) $y = \dfrac{3x}{1 + x^2}$; 6.15) $y = \dfrac{3}{(\sin x + \cos x)^2 + 2}$;

6.16) $y = \log_\pi(\arccos x)$; 6.17) $y = \dfrac{2\sqrt{2x} - 1}{x^2 + 1}$;

6.18) $y = 6\sin x - 8\cos x$; 6.19) $y = 2 \cdot 5^{-2x^2}$;

6.20) $y = -3x^2 + 10x - 3$; 6.21) $y = -5x^2 + 26x + 5$.

6.22) $y = \dfrac{1}{3\sin 2x + 4\cos 2x}$.

6.23). $y = 10^{-2x^2}$ funksiyaning E(f) o'zgarish sohasini toping.

3. Quyidagi funksiyalarni juft va toqlikka tekshiring:

6.24) $y = x + \sin x$; 6.25) $y = x\sin^3 x$;

6.26) $y = x\cos x$; 6.27) $y = \dfrac{\lg(1 - x^2)}{\sqrt[3]{\cos x}}$; 6.28) $y = \dfrac{x^3\cos x}{2^{x^2}}$;

6.29) $y = \lg\left(\dfrac{2 - x^2}{2 + x^3}\right)$; 6.30) $y = \dfrac{\sin x}{x^3}$;

6.31) $y = (\sin^2 x + \cos x)x^3$; 6.32) $y = x^2 \ln x$;

6.33) $y = 3^{4x} \cdot x^2 + \cos x$.

6.34)*a*) $y = \dfrac{x^4}{\cos x} - \sqrt{1 - x^2}$; *b*) $y = |x| tgx + x^3$; *c*) $y = 3^x \sin x$

d) $y = 2\sin 4x$ funksiyani eng kichiik davrini toping.

4.Murakkab funksiyaga doir quyidagi masalalarni yeching:

6.35) $y(x)=\frac{1+x}{1-x}$, $y\left(\frac{4-x}{2+x}\right)=?$; 6.36) $y(x)=2^x$, $y(\log_{\frac{1}{2}} x)=?$;

6.37) $y(x)=\frac{3-x}{2+x}$, $y\left(\frac{1+z(x)}{2}\right)=\frac{1}{x}$, $z(x)-?$;

6.38) $y=3^x$, $y(4z(x))=\frac{1}{x^2}$, $z(x)-?$.

6.39) $f(g(x))=tg\sqrt{1+x^2}$. $f(x)=?$, $g(x)=?$

6.40) $f(g(x))=\sqrt{1+tg^2x}$. $f(x)=?$, $g(x)=?$

6.41) $y(x)=\frac{x+2}{x-2}$ funksiya bo'yicha $y\left(\frac{1}{x}\right)$ murakkab funksiyani toping.

6.42) $y(x)=2x+5$ va $y(3-2z(x))=10-6x$ bo'lsa, $z(x)$ funksiyani toping.

§ 6.2. Funksiyaning limiti

1. Agar ixtiyoriy kichik $\varepsilon>0$ soni uchun shunday $\delta=\delta(\varepsilon)>0$ son topilsaki, $|x-a|<\delta$ shartni qanoatlantiruvchi barcha x va biror chekli A soni uchun $|f(x)-A|<\varepsilon$ tengsizlik bajarilsa, u holda A soni $y=f(x)$ ***funksiyaning*** $x\to a$ ***holdagi limiti*** deyiladi va $\lim\limits_{x\to a} f(x)=A$ kabi ifodalanadi.

2. Agar ixtiyoriy kichik $\varepsilon>0$ soni uchun shunday $N=N(\varepsilon)>0$ son topilsaki, $|x|>N$ shartni qanoatlantiruvchi barcha x va biror chekli A soni uchun $|f(x)-A|<\varepsilon$ tengsizlik bajarilsa, u holda A soni $y=f(x)$ ***funksiyaning*** $x\to\pm\infty$ ***holdagi limiti*** deyiladi va $\lim\limits_{x\to\pm\infty} f(x)=A$ kabi ifodalanadi.

3. Agar $\lim\limits_{x\to a}\alpha(x)=0$ yoki $\lim\limits_{x\to\pm\infty}\alpha(x)=0$ bo'lsa, unda $\alpha(x)$ funksiya mos ravishda $x\to a$ yoki $x\to\pm\infty$ holda ***cheksiz kichik miqdor*** deyiladi.

4. Cheksiz kichik miqdorlar quyidagi xossalarga ega:

a) Agar $\alpha(x)$ cheksiz kichik miqdor bo'lsa, unda ixtiyoriy C o'zgarmas son uchun $C\alpha(x)$ ham cheksiz kichik miqdor bo'ladi.

b) Agar $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlar bo'lsa, unda $\alpha(x)\pm\beta(x), \alpha(x)\cdot\beta(x)$ ham va $\beta(x)$ cheksiz kichik miqdorlar bo'ladi.

c) $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlarning nisbati $\alpha(x)/\beta(x)$ cheksiz kichik miqdor bo'lishi shart emas. Agar $\lim\limits_{x\to a}\frac{\alpha(x)}{\beta(x)}=1$ (a – chekli son yoki cheksiz) bo'lsa , $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlar ***ekvivalent*** deb ataladi va $\alpha(x)\sim\beta(x)$ kabi belgianadi. Masalan, agar $\alpha(x)$ cheksiz kichik miqdor bo'lsa

$$\sin\alpha(x)\sim\alpha(x);\quad \arcsin\alpha(x)\sim\alpha(x);\ tg\,\alpha(x)\sim\alpha(x);\quad arctg\,\alpha(x)\sim\alpha(x);$$

$$1-\cos\alpha(x)\sim\frac{\alpha^2(x)}{2};\ \log_a(1+\alpha(x))\sim\frac{\alpha(x)}{\ln a};\quad \ln(1+\alpha(x))\sim\alpha(x);$$

$$a^{\alpha(x)}-1\sim\alpha(x)\ln a;\quad e^{\alpha(x)}-1\sim\alpha(x),\quad (1+\alpha(x))^n\sim1+n\alpha(x).$$

d) Agar $\alpha(x)$ cheksiz kichik miqdor bo'lsa, unda $\lim\limits_{x\to a}\frac{1}{\alpha(x)}=\infty$, ya'ni $\frac{1}{\alpha(x)}$ funksiya cheksiz katta bo'ladi.

5. Turli funksiyalarning limitlarini hisoblashda
$\lim\limits_{x\to a}Cf(x)=C\lim\limits_{x\to a}f(x)=C\cdot A\ (C-const),\quad \lim\limits_{x\to a}C=C$
formulalardan va

$$\lim_{x\to0}x^{\alpha}=\begin{cases}0, & \text{agar } \alpha>0 \text{ bo'lsa;}\\ 1, & \text{agar } \alpha=0 \text{ bo'lsa;}\\ \infty, & \text{agar } \alpha<0 \text{ bo'lsa.}\end{cases},\qquad \lim_{x\to\infty}x^{\alpha}=\begin{cases}\infty, & \text{agar } \alpha>0 \text{ bo'lsa;}\\ 1, & \text{agar } \alpha=0 \text{ bo'lsa;}\\ 0, & \text{agar } \alpha<0 \text{ bo'lsa.}\end{cases}$$

ekanligidan foydalanish mumkin.

6. Limitlarni hisoblashda quyidagi tengliklardan foydalanish mumkin:

1) $\lim\limits_{x\to0}\frac{\sin x}{x}=1$ - ***birinchi ajoyib limit;***

2) $\lim\limits_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=\lim\limits_{x\to0}(1+x)^{1/x}=e=2.71828\cdots$ - ***ikkinchi ajoyib limit;***

3) $\lim\limits_{x\to0}\frac{\log_a(1+x)}{x}=\log_a e;$ 4) $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=1;$ 5) $\lim\limits_{x\to0}\frac{a^x-1}{x}=\ln a;$

6) $\lim\limits_{x\to0}\frac{e^x-1}{x}=1;$ 7) $\lim\limits_{x\to0}\frac{(1+x)^{\alpha}-1}{x}=\alpha;$ 8) $\lim\limits_{x\to0}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}.$

7. $[f(x)]^{g(x)}$ ko'rinishdagi murakkab darajali-ko'rsatkichli funksiyaning limiti $f(x)>0$ shartda umumiy holda

$$\lim_{x\to a}[f(x)]^{g(x)} = \exp\{\lim_{x\to a}[g(x)\ln f(x)]\},\ \exp\{*\} = e^{\{*\}}$$

tenglik orqali hisoblanishi mumkin.

Quyidagi xususiy hollarda bu limit javobini darhol yozish mumkin:

7.1. Agar $\lim_{x\to a} f(x) = A > 0$, $\lim_{x\to a} g(x) = B$ va A, B chekli sonlar bo'lsa, unda

$$\lim_{x\to a}[f(x)]^{g(x)} = A^B \ ;$$

7.2. Agar $0<A<1$ va $B=+\infty$ bo'lsa, unda $\lim_{x\to a}[f(x)]^{g(x)} = [A^{+\infty}] = 0$;

7.3. Agar $A>1$ va $B=+\infty$ bo'lsa, unda $\lim_{x\to a}[f(x)]^{g(x)} = [A^{+\infty}] = +\infty$;

7.4. Agar $0<A<1$ va $B=-\infty$ bo'lsa, unda $\lim_{x\to a}[f(x)]^{g(x)} = [A^{-\infty}] = +\infty$

7.5. Agar $A>1$ va $B=-\infty$ bo'lsa, unda $\lim_{x\to a}[f(x)]^{g(x)} = [A^{-\infty}] = 0$;

$A=1$ va $B=\pm\infty$, $A=0$ va $B=0$, $A=+\infty$ va $B=0$ hollarda $\lim_{x\to a}[f(x)]^{g(x)}$ limitni hisoblash masalasi mos ravishda 1^{∞} , 0^0 , ∞^0 ko'rinishdagi aniqmasliklarni ochish orqali yechiladi.

8. $\lim_{x\to a} f(x) = A$, $\lim_{x\to a} g(x) = B$ bo'lganda $f(x)$ va $g(x)$ funksiyalar nisbatlarining limiti A va B chekli sonlar ($B\neq 0$) holda

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{A}{B}$$

formula bilan aniqlanadi.

Quyidagi hollarda bu limit qiymatini darhol yozish mumkin:

8.1. $A\neq 0;\quad B=0 \Rightarrow \lim_{x\to x_0}\frac{f(x)}{g(x)} = \left[\frac{A}{0}\right] = \infty$;

8.2. A – chekli son va $B=\pm\infty \Rightarrow \lim_{x\to x_0}\frac{f(x)}{g(x)} = \left[\frac{A}{\pm\infty}\right] = 0$;

8.3. $A=\infty$ va B – chekli son $\Rightarrow \lim_{x\to x_0}\frac{f(x)}{g(x)} = \left[\frac{\infty}{B}\right] = \infty$;Agar $A=0$ va $B=0$, $A=\infty$ va $B=\infty$ bo'lsa, $\lim_{x\to a}\frac{f(x)}{g(x)}$ limitni hisoblash masalasi mos ravishda $\frac{0}{0}, \frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarni ochish orqali yechiladi.

9. $\lim\limits_{x\to a} f(x)=A, \lim\limits_{x\to a} g(x)=B$ bo'lganda $f(x)$ va $g(x)$ funksiyalar ko'paytmasining limiti A va B chekli sonlar bo'lgan holda $\lim\limits_{x\to a}[f(x)\cdot g(x)]=\lim\limits_{x\to a} f(x)\cdot \lim\limits_{x\to a} g(x)=A\cdot B$ formula bilan aniqlanadi.

Quyidagi hollarda bu limit qiymatini darhol yozish mumkin:

9.1. $A\neq 0;\quad B=\infty \Rightarrow \lim\limits_{x\to x_0}[f(x)\cdot g(x)]=[A\cdot\infty]=\infty$;

9.2. $A=\infty;\quad B=\infty \Rightarrow \lim\limits_{x\to x_0}[f(x)\cdot g(x)]=[\infty\cdot\infty]=\infty$

Agar A=0 va B=∞ bo'lsa, $\lim\limits_{x\to a}[f(x)\cdot g(x)]$ limitni hisoblash masalasi $\infty\cdot\infty$ ko'rinishdagi aniqmasliklarni ochish orqali yechiladi.

10. $\lim\limits_{x\to a} f(x)=A, \lim\limits_{x\to a} g(x)=B$ bo'lganda $f(x)$ va $g(x)$ funksiyalar algebraik yig'indisining limiti A va B chekli sonlar bo'lgan holda $\lim\limits_{x\to a}[f(x)\pm g(x)]=\lim\limits_{x\to a} f(x)\pm \lim\limits_{x\to a} g(x)=A\pm B$ formula bilan aniqlanadi.

Quyidagi hollarda bu limit qiymatini darhol yozish mumkin:

10.1. $A=+\infty;\quad B=+\infty \Rightarrow \lim\limits_{x\to x_0}[f(x)+g(x)]=[+\infty+(+\infty)]=+\infty$;

10.2. $A=-\infty;\quad B=-\infty \Rightarrow \lim\limits_{x\to x_0}[f(x)+g(x)]=[-\infty+(-\infty)]=-\infty$

10.3. A – chekli son, $B=\pm\infty \Rightarrow \lim\limits_{x\to x_0}[f(x)+g(x)]=[A\pm\infty)]=\pm\infty$

Agar A=+∞ va B=–∞ bo'lsa, $\lim\limits_{x\to a}[f(x)+g(x)]$ limitni hisoblash masalasi $\infty-\infty$ ko'rinishdagi aniqmasliklarni ochish orqali yechiladi.

11. Ikkita ko'phad nisbatining $x\to\infty$ holdagi limiti

$$\lim_{x\to\infty}\frac{P_n(x)}{Q_m(x)}=\lim_{x\to\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+...+a_1x+a_0}{b_mx^m+b_{m-1}x^{m-1}+...+b_1x+b_0}=\begin{cases}\infty, & \text{agar } n>m \text{ bo'lsa;}\\ a_n/b_m, & \text{agar } n=m \text{ bo'lsa;}\\ 0, & \text{agar } n<m \text{ bo'lsa.}\end{cases}$$

tenglik bilan topiladi.

5. Ko'phadlar nisbatidan iborat funksiyaning limitini hisoblang.

6.43) $\lim\limits_{x\to -2}\dfrac{x^3+2x^2-4x-8}{x^3+8}$;

6.44) $\lim\limits_{x\to 6}\dfrac{x^2-8x+12}{x^3-7x^2+6x}$;

6.45) $\lim\limits_{x\to -2}\dfrac{2x^2+x-1}{x^3-3x-2}$;

6.46) $\lim\limits_{x\to 2}\dfrac{x^3-3x-2}{x^2-5x+6}$;

6.47) $\lim\limits_{x\to 0}\dfrac{(1+x)^3-(1+3x)}{x+x^5}$;

6.48) $\lim\limits_{x\to -1}\dfrac{x^3+2x^2-x-2}{x^2+x}$

6.49) $\lim\limits_{x\to \infty}\dfrac{(x^2+2x-3)^2}{x^3+4x^2+3x}$;

6.50) $\lim\limits_{x\to 1}\dfrac{(2x^2-x-1)^2}{x^3+2x^2-x-2}$;

6.51) $\lim\limits_{x\to 1}\dfrac{x^4-x^3+x-1}{2x^2-x-1}$;

6.52) $\lim\limits_{x\to -2}\dfrac{x^3-3x+2}{x^3-x^2-x+1}$

6.53) $\lim\limits_{x\to -2}\dfrac{x^2+2x-3}{x^3+4x^2+3x}$;

6.54) $\lim\limits_{x\to -3}\dfrac{(x^2+2x-3)^2}{x^3+4x^2+3x}$;

6.55) $\lim\limits_{x\to 1}\dfrac{x^4-1}{2x^4-x^2-1}$;

6.56) $\lim\limits_{x\to \infty}\dfrac{x^3-3x-2}{x^3+2x^2-x-2}$;

6.57) $\lim\limits_{x\to -2}\dfrac{x^3+4x^2+3x}{x^2+x-6}$;

6.58) $\lim\limits_{x\to 1}\dfrac{x^4-1}{x^3-x^2-x+1}$;

6.59) $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^3+2x^2-4x-8}$;

6.60) $\lim\limits_{x\to -3}\dfrac{x^3-2x-1}{x^4+2x+1}$

6.61) $\lim\limits_{x\to -1}\dfrac{x^4+2x+1}{x^2-x-2}$;

6.62) $\lim\limits_{x\to 2}\dfrac{x^3-3x^2+4}{x^4-4x^2}$;

6.63) $\lim\limits_{x\to -2}\dfrac{3x^3-4x^2-2}{5x^3+8x^2+1}$;

6.64) $\lim\limits_{x\to 2}\dfrac{x^3-3x-2}{x^3-2x^2-x+2}$;

6.65) $\lim\limits_{x\to 2}\dfrac{4x^2-5x-6}{3x^3-7x^2+2x}$;

6.66) $\lim\limits_{x\to -1}\dfrac{x^4+2x^3+x^2}{x^4+2x+1}$;

6.67) $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^3-3x-2}$;

6.68) $\lim\limits_{x\to \infty}\dfrac{(1+x)^3-(1+3x)}{x^2+x^5}$;

6.69) $\lim\limits_{x\to 2}\dfrac{3x^3-7x^2+2x}{4x^2-5x-6}$;

6.70) $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x+x^2}$;

6.71) $\lim\limits_{x\to \infty}\dfrac{x^3-2x-1}{(x^2-x-2)^2}$;

6.72) $\lim\limits_{x\to -1}\dfrac{x^3-3x-2}{x^2-x-2}$.

6.73) $\lim\limits_{x\to 0}\dfrac{\sqrt{4-x}-\sqrt{4+x}}{2x}$

6.74) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+2x}-1}{x}$

6.75) $\lim\limits_{x\to\infty}\dfrac{4x^3-3x^2+2x-1}{x^3+3x^2-5x+4}$ 6.76) $\lim\limits_{x\to 1}\dfrac{x^2-3x+2}{x^3+4x^2+3x}$

6.77) $\lim\limits_{x\to 1}\left[\dfrac{1}{x-1}-\dfrac{2x}{x^2-1}\right]$ 6.78) $\lim\limits_{x\to -\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$

6.79) $\lim\limits_{x\to 0}(x\,ctgx)$ 6.80) $\lim\limits_{x\to \pi/2}(1+ctgx)^{tgx}$

6.81) $\lim\limits_{x\to\infty}\left(\dfrac{x-1}{x+1}\right)^{2x}$ 6.82) $\lim\limits_{x\to 0}\dfrac{e^{x^2}-e^{2x}}{\cos 4x-\cos 2x}$

6. Irratsional ifodali funksiya limitini hisoblang.

6.83) $\lim\limits_{x\to\pm\infty}\left(\sqrt{x^2+2x-1}-\sqrt{x^2-5x+3}\right)$;

6.84) $\lim\limits_{x\to 5}\dfrac{x^2-25}{2-\sqrt{x-1}}$; 6.85) $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}$; 6.86) $\lim\limits_{x\to\infty}\left(x+\sqrt[3]{1-x^3}\right)$;

6.87) $\lim\limits_{x\to 3}\dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}$; 6.88) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+4}-\sqrt{x+3}\right)$;

6.89) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x-x^2}}{\sqrt[3]{x^2+x^3}}$; 6.90) $\lim\limits_{x\to 0}\dfrac{\sqrt{1-2x+x^2}-(1+x)}{x}$

6.91) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x+1}-\sqrt{1-x}}{\sqrt[3]{x}}$; 6.92) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{1+x}-\sqrt{2x}}$;

6.93) $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x+2}$; 6.94) $\lim\limits_{x\to +\infty}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)$;

6.95) $\lim\limits_{x\to 0}\dfrac{\sqrt{25-2x}-3}{\sqrt[3]{x}-2}$; 6.96) $\lim\limits_{x\to -\infty}\left(\sqrt[3]{(x+2)^2}-\sqrt[3]{(x-2)^2}\right)$;

6.97) $\lim\limits_{x\to -8}\dfrac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$; 6.98) $\lim\limits_{x\to\infty}\left(\sqrt{(x+3)(x-2)}-x\right)$;

6.99) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x^2}-(1+x)}{\sqrt[3]{x}}$; 6.100) $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x^3+8}$;

6.101) $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$; 6.102) $\lim\limits_{x\to +\infty}\left(\sqrt{x^2+5x+4}-\sqrt{x^2+x}\right)$

6.103) $\lim\limits_{x\to\infty}\left(x+\sqrt[3]{1-x^3}\right)$; 6.104) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-1}{\sqrt{1+x}-1}$;

6.105) $\lim\limits_{x\to +\infty}x^{\frac{3}{2}}\left(\sqrt{x^3+2}-\sqrt{x^3-2}\right)$; 6.106) $\lim\limits_{x\to +\infty}\left(\sqrt{x^2-5x+6}-x\right)$;

6.107) $\lim_{x\to 1}\left(\frac{3}{1-\sqrt{x}}-\frac{2}{1-\sqrt[3]{x}}\right)$;

6.108) $\lim_{x\to 1}\frac{\sqrt[4]{x}-1}{\sqrt[3]{x}-1}$;

6.109) $\lim_{x\to 0}\frac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{\sqrt[3]{x^2}-\sqrt[5]{x}}$;

6.110) $\lim_{x\to+\infty} x\left(\sqrt{x^2+2}-x\right)$;

6.111) $\lim_{x\to 8}\frac{\sqrt{9+2x}-5}{\sqrt[3]{x^2}-4}$;

6.112) $\lim_{x\to 0}\frac{\sqrt[3]{8+3x+x^2}-2}{x+x^2}$

7. Trigonometrik ifodali funksiya limitini hisoblang.

6.113) $\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{\sin 6x}$;

6.114) $\lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\pi-x}$;

6.115) $\lim_{x\to 0}\frac{\ln(1-7x)}{\sin(\pi(x+7))}$;

6.116) $\lim_{x\to\frac{\pi}{2}}\frac{2^{\cos^2 x}-1}{\ln\sin x}$;

6.117) $\lim_{x\to 0}\frac{\sin(\alpha+x)-\sin(\alpha-x)}{x}$;

6.118) $\lim_{x\to 0}\frac{\sin 3x-\sin 2x}{\ln(1+2tg3x)}$;

6.119) $\lim_{x\to 0}\frac{\cos 2x-\cos x}{1-\cos x}$;

6.120) $\lim_{x\to 0}\frac{e^{\sin 2x}-e^{\sin x}}{tg3x}$;

6.121) $\lim_{x\to 0}\frac{\cos\left(x+\frac{5\pi}{2}\right)\cdot tgx}{\arcsin 2x^2}$;

6.122) $\lim_{x\to 0}\frac{1-\cos 2x}{\cos 7x-\cos 3x}$;

6.123) $\lim_{x\to 0}\frac{x^2-2x}{tg[2\pi(x+0.5)]}$;

6.124) $\lim_{x\to 0}\frac{\ln\cos 2x}{\sin^2 3x}$;

6.125) $\lim_{x\to 2}\frac{\sin 7\pi x}{tg8\pi x}$;

6.126) $\lim_{x\to\frac{\pi}{3}}\frac{2\sin x-\sqrt{3}}{\cos\frac{3x}{2}}$;

6.127) $\lim_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}$;

6.128) $\lim_{x\to 1}\frac{1+\cos\pi x}{tg^2\pi x}$;

6.129) $\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{x\sin^2\sqrt{x}}$;

6.130) $\lim_{x\to 1}\frac{1-\sqrt{\cos x}}{1-\cos\sqrt{x}}$;

6.131) $\lim_{x\to 0}\frac{1-\cos 2x+tg^2 x}{x\sin^2 3x}$;

6.132) $\lim_{x\to\frac{\pi}{3}}\frac{1-2\cos x}{\pi-3x}$;

6.133) $\lim_{x\to\frac{\pi}{3}}\frac{1-2\cos x}{\sin(\pi-3x)}$;

6.134) $\lim_{x\to 0}\frac{\sqrt{\cos 4x}-1}{\sin^2 8x}$;

6.135) $\lim\limits_{x\to\frac{1}{2}}\frac{(2x-1)^2}{e^{\sin\pi x}-e^{-\sin 3\pi x}}$;

6.136) $\lim\limits_{x\to\frac{\pi}{2}}\frac{2^{\cos^2 x}-1}{\ln\sin x}$;

6.137) $\lim\limits_{x\to 0}\frac{\sin 4x-2\sin 2x}{x\ln\cos 6x}$;

6.138) $\lim\limits_{x\to\pi}\frac{tg(3^{\frac{\pi}{x}}-3)}{3^{\cos\frac{3x}{2}}-1}$;

6.139) $\lim\limits_{x\to 0}\frac{\sin 2x-\sin x}{x\ln(1-x\sin x)}$;

6.140) $\lim\limits_{x\to\pi}\frac{\cos 5x-\cos 3x}{\sin^2 x}$;

6.141) $\lim\limits_{x\to 0}\frac{\sqrt[3]{\cos^2 x}-\sqrt[3]{\cos x}}{\sin^2 x}$;

6.142) $\lim\limits_{x\to 0}\frac{1-\cos^3 x}{x^2+\sin^3 x}$.

8. Berilgan limitlarni hisoblang.

6.143) $\lim\limits_{x\to\frac{1}{2}}\frac{\ln(4x-1)}{\sqrt{1-\cos\pi x}-1}$;

6.144) $\lim\limits_{x\to\infty}\frac{\ln(x^2+2x)-\ln(x^2+3)}{e^{\frac{x}{x^2-1}}-1}$;

6.145) $\lim\limits_{x\to 0}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$;

6.146) $\lim\limits_{x\to\pi}\frac{\sin(x^2/\pi)}{2^{\sqrt{\sin x+1}}-2}$;

6.147) $\lim\limits_{x\to -1}\frac{tg(x+1)}{e^{\sqrt[3]{x^3-4x^2+6}}-e}$;

6.148) $\lim\limits_{x\to 1}\frac{\sqrt{x^2-3x+3}-1}{\sin\pi x}$;

6.149) $\lim\limits_{x\to\infty}\frac{\ln(x^2-1)-\ln(x^2+1)}{\sqrt[3]{x-1}-1}$;

6.150) $\lim\limits_{x\to 3}\frac{\ln(x+2)-\ln(2x-1)}{\sin\pi x}$;

6.151) $\lim\limits_{x\to 2}\frac{\ln(5-2x)}{\sqrt{10-3x}-2}$;

6.152) $\lim\limits_{x\to 0}\frac{\ln(1+xe^x)}{\ln(x+\sqrt{1+x^2})}$;

9.Murakkab ko'rsatkichli-darajali funksiya limitini hisoblang.

6.153) $\lim\limits_{x\to 0}\left(6-\frac{5}{\cos x}\right)^{ctg^2 x}$;

6.154) $\lim\limits_{x\to 0}(2-\cos 3x)^{\frac{1}{\ln(1+x^2)}}$;

6.155) $\lim\limits_{x\to 2\pi}(\cos x)^{\frac{ctg 2x}{\sin 3x}}$;

6.156) $\lim\limits_{x\to 1}(2-x)^{\frac{\sin\frac{\pi x}{2}}{\ln(2-x)}}$;

6.157) $\lim\limits_{x\to 1}\left(\frac{2x-1}{x}\right)^{\frac{1}{\sqrt[3]{x-1}}}$;

6.158) $\lim\limits_{x\to 2}(2e^{x-2}-1)^{\frac{1}{x-2}}$;

6.159) $\lim\limits_{x\to 0+0}(\cos\sqrt{x})^{\frac{1}{x}}$;

6.160) $\lim\limits_{x\to 0}[1-\ln(1+x^3)]^{\frac{3}{x^2\arcsin x}}$;

6.161) $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x}}$;

6.162) $\lim\limits_{x\to 0}\left(2-3^{\operatorname{arctg}^2\sqrt{x}}\right)^{\frac{2}{\sin x}}$;

6.163) $\lim\limits_{x\to 0}\left(6-\frac{5}{\cos x}\right)^{\frac{1}{\operatorname{tg}^2 x}}$;

6.164) $\lim\limits_{x\to 0}\left(5-\frac{4}{\cos x}\right)^{\frac{1}{\sin^2 3x}}$;

6.165) $\lim\limits_{x\to 0}\sqrt[x^2]{2-\cos x}$;

6.166) $\lim\limits_{x\to 0}\left(2-e^{x^2}\right)^{\frac{1}{1-\cos \pi x}}$

6.167) $\lim\limits_{x\to \infty}\left(\frac{x^2+5}{x^2+3}\right)^{4x^2}$;

6.168) $\lim\limits_{x\to 0}(3-2\cos x)^{-\operatorname{cosec}^2 x}$;

§ 6.3. Funksiyaning uzluksizligi va uzulish nuqtalari

1.Quyidagi shartlar bajarilsa, $y=f(x)$ funksiya x_0 nuqtada ***uzluksiz*** deyiladi:

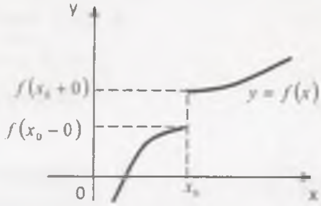
- funksiya x_0 nuqta va uning atrofida aniqlangan;
- $f(x)$ funksiya x_0 nuqtada chekli chap $\lim\limits_{x\to x_0-0} f(x)=f(x_0-0)$ va o'ng $\lim\limits_{x\to x_0+0} f(x)=f(x_0+0)$ limitlarga ega;
- $f(x)$ funksiyaning x_0 nuqtadagi chap va o'ng limitlari o'zaro teng, ya'ni $\lim\limits_{x\to x_0-0} f(x)=\lim\limits_{x\to x_0+0} f(x)\Rightarrow f(x_0-0)=f(x_0+0)$.

2. Agar x_0 nuqtada yuqorida keltirilgan shartlardan kamida bittasi bajarilmasa, $f(x)$ funksiya bu nuqtada ***uzlukli***, x_0 esa uning ***uzilish nuqtasi*** deyiladi.

Funksiya quyidagi ko'rinishdagi uzulishlarga ega bo'lishi mumkin.

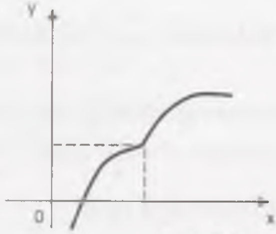
a) Agar x_0 nuqtada $f(x)$ funksiyaning chap va o'ng limitlari mavjud va chekli, ammo $f(x_0-0)\neq f(x_0+0)$ bo'lsa, unda funksiya ***birinchi tur uzulishga*** ega deb ataladi. Bu holda

$\delta = |f(x_0 - 0) - f(x_0 + 0)|$ ifoda $f(x)$ funksiyaning x_0 nuqtadagi *sakrashi* deyiladi (6.1 - chizma).



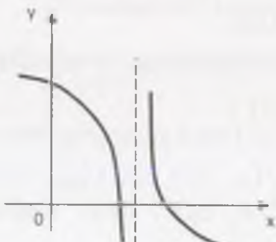
6.1- chizma.

b) Agar $f(x_0 - 0) = f(x_0 + 0) \neq f(x_0)$ yoki $f(x_0 - 0) = f(x_0 + 0)$ bo'lib, $f(x)$ funksiya x_0 nuqtada aniqlanmagan bo'lsa, unda funksiya ***tuzatib bo'ladigan uzulishga*** egadeyiladi (6.2- chizma). Buholda $f(x_0) = f(x_0 - 0) = f(x_0 + 0)$ debolinsa, $f(x)$ funksiya x_0 nuqtadauzluksizbo'ladi.



6.2- chizma

c)Agar x_0 nuqtada $f(x_0 - 0), f(x_0 + 0)$ bir tomonlama limitlardan kamida bitta sicheksiz qiymatga ega yoki mavjud bo'lmasa, unda funksiya ***ikkinchi turuzulishga*** ega deyiladi (6.3- chizma).



6.3- chizma

Quyidagi funksiyalarning uzilish nuqtalarini topib, unda uzulish turini aniqlang va funksiyaning sxematik grafigini chizing:

6.169) $y = arctg\frac{1}{x-4}$; 6.170) $y = 3^{\frac{1}{x}}$; 6.171) $y = \frac{x^3-1}{x^2-1}$;

6.172) $y = 0.5^{\frac{1}{x}}$; 6.173) $y = \begin{cases} 2x, & \text{agar } x \le 1 \text{ bo'lsa} \\ 2-x, & \text{agar } x > 1 \text{ bo'lsa.} \end{cases}$

6.174) $y = E(x)$. Bunda $E(x)$ ifoda x sonning butun qismini, ya'ni undan katta bo'lmagan eng katta butun sonni ifodalaydi.

6.175) $y = \begin{cases} 0.5x^2, & |x| < 2 \\ 2.5, & |x| = 2 \\ 3, & |x| > 2 \end{cases}$; 6.176) $y = \begin{cases} \sin x, & x \le 0; \\ 1+\cos x, & 0 < x \le 2\pi; \\ x, & x > 2\pi. \end{cases}$

6.177) $y = \begin{cases} \sin\frac{\pi}{2x}, & x \ne 0; \\ 1, & x = 0. \end{cases}$; 6.178) $y = \begin{cases} \cos\frac{\pi}{2x}, & x \ne 0; \\ 0, & x = 0. \end{cases}$

11. Funksiyalarni uzluksizlikka tekshiring va uzilish nuqtalarini aniqlang.

6.179) $y = \frac{2^{\frac{1}{x-2}}-1}{2^{\frac{1}{x-2}}+1}$; 6.180) $y = \frac{1}{(x-1)(x-5)}$;

6.181) $y = \frac{\ln(x+1)}{x}$; 6.182) $y = xtgx$; 6.183) $y = \cos(1+\ln x)$;

6.184) $y = \ln(1+\cos x)$.

12. Quyidagi funksiyalar barcha nuqtalarda uzkuksiz bo'ladigan αparametrning qiymatlarini toping:

6.185) $y = \begin{cases} \alpha\sin x, & x < \frac{\pi}{2}; \\ x+\pi, & x \ge \frac{\pi}{2}. \end{cases}$; 6.186) $y = \begin{cases} (\alpha^2+3\alpha-3)e^x, & x < 0; \\ \ln(x^2+x+e), & x \ge 0. \end{cases}$

Takrorlash uchun savollar

1. Qanday miqdorlar o'zgarmas deyiladi
2. Funksiya qanday ta'riflanadi?
3. Funksiyaning aniqlanish sohasi deb nimaga aytiladi?
4. Davriy funksiya deb nimaga aytiladi?

5. Teskari funksiya qanday aniqlanadi?
6. Murakkab funksiya qanday ta'riflanadi?
7. Funksiyaning cheksiz limiti qanday ta'riflanadi?
8. Qanday shartda funksiya limiti mavjud bo'ladi?
9. Ajoyib limitlarni yozing.
10. Limitlarning asosiy xossalari nimalardan iborat?
11. Cheksiz kichik miqdor deb nimaga aytiladi?

FUNKSIYA, UNING LIMITI VA UZLUKSIZLIGIGA DOIR TESTLAR

1. Sonli to'plamlar. Sonning absolut qiymati va uning xossalari

1. Umumiy holda sonli to'plam elementlari ... sonlardan iborat bo'ladi.

A)butun. B)natural. C) haqiqiy.
D) ratsional. E) irratsional.

2. N (natural sonlar), Z (butun sonlar), Q (ratsional sonlar), R (haqiqiy sonlar) sonli to'plamlar orasidagi munosabat qayerda to'g'ri ko'rsatilgan ?

A) $N\subset Z\subset Q\subset R$. B) $Z\subset N\subset Q\subset R$.
C) $R\subset Q\subset Z\subset N$. D) $Q\subset Z\subset N\subset R$. E) $n\subset Z\subset R\subset Q$.

3. Asosiy sonli to'plamlar uchun quyidagi munosabatlardan qaysi biri to'g'ri?

A) $N\cap Z=N$. B) $Q\cap Z=Q$. C) $Z\cap R=R$.
D) barcha munosabatlar to'g'ri.
E) barcha munosabatlar noto'g'ri .

4. Asosiy sonli to'plamlar uchun to'g'ri munosabatni ko'rsating.

A) $N\cup Z=Z$. B) $Q\cup Z=Q$. C) $Z\cup R=R$.
D)barcha munosabatlar to'g'ri.
E) barcha munosabatlar noto'g'ri.

5. Quyidagi sonli to'plamlardan qaysi biri oqaliq deb ataladi ?

A) (a,b) . B) $[a,b)$. C) $(a,b]$. D) $[a,b]$. E) $\{a,b\}$.

6. Quyidagi sonli to'plamlardan qaysi biri kesma deb ataladi ?

A) (a,b) . B) $[a,b)$. C) $(a,b]$. D) $[a,b]$. E) $\{a,b\}$.

7. Quyidagi sonli to'plamlardan qaysi birlari yarim oqaliq deb ataladi ?

1) (a,b) 2) $[a,b)$ 3) $(a,b]$ 4) $[a,b]$ 5) $\{a,b\}$

A) 1) va 2) . B) 2) va 3) . C) 3) va 4) .

D) 4) va 5) . E) 1) va 4) .

8. (a,b) , $[a,b)$, $(a,b]$ va $[a,b]$ sonli to'plamlarning d uzunligi qayerda to'g'ri ko'rsatilgan ?

A) $a-b$. B) $a+b$. C) $b-a$. D) $a\cdot b$. E) $\sqrt{b^2-a^2}$.

9. $[-3,5)$ yarim oraliq uzunligini toping.

A) 2. B) 5. C) 3. D) 4. E) 8.

10. $(-4.5,5.5)$ oraliq uzunligini toping.

A) 1. B) 4.5. C) 5.5. D) 9. E) 10.

2. Sonli ketma-ketlik va uning limiti

1. Quyidagalardan qaysi biri sonli ketma-ketlikni tashkil etmaydi ?

A) Barcha juft sonlar. B) Barcha toq sonlar.

C)Barcha tub sonlar.

D) Barcha natural sonlar. E) Barcha musbat sonlar .

2. Ushbu $-\frac{1}{2}+\frac{3}{4}-\frac{5}{8}+\frac{7}{16}-\frac{9}{32}+\cdots$ sonli ketma-ketlikning umumiy a_n hadi qayerda to'g'ri ko'rsatilgan?

A) $a_n=\pm\frac{2n+1}{2^n}$. B) $a_n=\pm\frac{2n-1}{2^n}$. C) $a_n=(-1)^n\frac{2n+1}{2^n}$.

D) $a_n=(-1)^n\frac{2n-1}{2^n}$.E) $a_n=(-1)^n\frac{2n\pm1}{2^n}$.

3. Umumiy hadi $a_n=\frac{n}{n+1}$ ko'rinishda bo'lgan ketma-ketlikning yuqori M va quyi m chegaralarini toping.

A) M=10, m=1/2. B) M=1, m=1/2. C) M=3, m=1/2.

D) A) M=1/2, m=0. E)M=2, m=1.

4. Quyidagi sonli ketma-ketliklardan qaysi biri chegaralangan ?

A) $\{n^2+3\}$. B) $\{(-1)^n\cdot n\}$.C) $\left\{n^{(-1)^n}\right\}$. D) $\left\{\frac{n^2-1}{n}\right\}$.E) $\left\{\frac{(-1)^n}{3}\right\}$.

5. Quyidagi ketma-ketliklardn qaysi birlari yaqinlashuvchi?

$x_n = \frac{1}{n^2+1}$, $y_n = \frac{1}{2n^2-1}$, $z_n = (-1)^n$.

A) faqat x_n. B)faqat x_n va y_n. C) faqat z_n. D) faqat y_n va z_n.

E)uchalaketma-ketlik ham yaqinlashuvchi.

6. Quyidagi ketma-ketliklardan qaysi birlari yaqinlashuvchi?

$x_n = \frac{n^2+1}{2n}$, $y_n = (-1)^{n^2}$, $z_n = \frac{1}{n+1}$

A) y_n . B) x_n . C) z_n . D) x_n va y_n .

E)uchalaketma-ketlik ham uzoqlashuvchi.

7. Limitlar haqidagi quyidagi tasdiqlardan qaysi biri to'g'ri?

A) Har qanday yuqoridan chegaralangan ketma-ketliik limitga ega.

B) Har qanday quyidan chegaralangan ketma-ketliik limitga ega.

C) Har qanday chegaralangan ketma-ketliik limitga ega.

D) Keltirilgan barcha tasdiqlar to'g'ri

E) Keltirilgan barcha tasdiqlar noto'g'ri.

8. Limitlar haqidagi quyidagi tasdiqlardan qaysi biri noto'g'ri?

A) Har qanday monoton o'suvchi va yuqoridan chegaralangan ketma-ketlik limitga ega.

B) Har qanday monoton kamayuvchi va quyidan chegaralangan ketma-ketlik limitga ega.

C) Har qanday monoton va chegaralangan ketma-ketlik limitga ega.

D) Keltirlgan barcha tasdiqlar to'g'ri

E) Keltirilgan barcha tasdiqlar noto'g'ri.

9. Quyidagi shartlardan qaysi birida ikkita sonli ketma-ketliklarning algebraik yig'indisi chekli limitga ega bo'ladi ?

A) Ikkala ketma-ketlik chegaralangan.

B) Ikkala ketma-ketlik monoton.

C) Ikkala ketma-ketlik musbat hadli.

D) Ikkala ketma-ketlik chekli limitga ega;

E) Ikkala ketma-ketlikdan kamida bittasi chekli limitga ega.

10. Ikkita $\{x_n\}$ va $\{y_n\}$ sonli ketma-ketliklarning nisbati $\{x_n/y_n\}$ chekli limitga ega bo'lishi uchun quyidagi shartlardan qaysi biri talab etilmaydi ?

A) $\{x_n\}$ ketma-ketlik chekli limitga ega.

B) $\{y_n\}$ ketma-ketlik chekli limitga ega.

C) $\{x_n\}$ ketma-ketlikning limiti noldan farqli.

D) $\{y_n\}$ ketma-ketlikning limiti noldan farqli.

E) Keltirilgan barcha shartlar talab etiladi.

3. Funksiya va u bilan bog'liq tushunchalar

1. ***Ta'rifni to'ldiring:*** $y=f(x)$ funksiya deb x o'zgaruvchining har bir $x \in D$ qiymatiga y o'zgaruvchining .... $y \in E$ qiymatini mos qo'yilishiga aytiladi.

A) bir nechta. B) kamida bitta.

C) faqat bitta. D) ikkita. E) kamida ikkita.

2. ***Ta'rifni to'ldiring:*** $y=f(x)$ funksiyaning aniqlanish sohasi deb x argumentning $y=f(x)$ funksiya .... bo'ladigan qiymatlar to'plamiga aytiladi.

A) Musbat. B) manfiy. C) nol.

B) D) ma'noga ega. E) cheksiz.

3. $f(x)=\sqrt{2x+1}-\lg x$ funksiyaning aniqlanish sohasini toping.

A) $(-1,+\infty)$. B) $(0,+\infty)$. C) $(2,11)$. D) $(-\infty,+\infty)$. E) $(1,+\infty)$.

4. $f(x)=\ln\sqrt{x-1}$ funksiyaning aniqlanish sohasini toping.

A) $[1,+\infty)$. B) $[0,+\infty)$. C) $(-\infty,+\infty)$. D) $(-\infty,1)$. E) $(1,+\infty)$.

5. $f(x)=\arcsin\frac{x-1}{2}$ funksiyaning aniqlanish sohasini toping.

A) $[0,1]$. B) $[1,2]$. C) $(-\infty,+\infty)$. D) $[-1,3]$. E) $[-1,1]$.

6. $f(x)=\log_3 x$ va $g(x)=\sqrt{3-x}$ funksiyalarning aniqlanish sohalari bo'yicha $D\{f\} \cap D\{g\}$ to'plamni toping.

A) $(3,\infty)$. B) $(0,\infty)$. C) $(-\infty,3]$. D) $(0,3]$. E) $[0,3]$.

7. $y=\sqrt{4-5\sin x}$ funksiyaning qiymatlar sohasini toping.

A) [0,4]. B)[0,2].

C) [0,5]. D) [–1,3]. E)[0,3].

8. $y=\sqrt{4-x^2}$ funksiyaning qiymatlar sohasini toping.

A) [0,4]. B)[0,2]. C) [0,5].

D) [–1,3]. E) [0,3].

9. Qaysi shartda $y=f(x)$ funksiya albatta juft bo'ladi ?

A) $f(x)f(-x)>1$. B) $f(x)-f(-x)=0$. C) $f(x)+f(-x)=0$.

D) $f(x)f(-x)\leq 0$. E) $|f(x)|=|f(-x)|$.

10. Quyidagi funksiyalardan qaysi biri juft emas ?

A) $y=\cos^3 x$. B) $y=-x^4$. C) $y=|x|$.

D) $y=\sin^3 x$. E) $y=\sin x^2$.

4. Funksiya limiti va uning xossalari

1. ***Ta'rifini to'ldiring:*** $y=f(x)$ funksiya $x\to a$ bo'lganda A soniga teng limitga ega deyiladi, agarda ixtiyoriy kichik $\varepsilon>0$ soni uchun shunday $\delta>0$ son topilsaki, $|x-a|<\delta$ shartni qanoatlantiruvchi barcha x uchun ... bo'lsa.

A) $|f(x)+A|<\varepsilon$. B) $||f(x)-A|>\varepsilon$.

C) $|f(x)+A|>\varepsilon$. D) $|f(x)-A|<\varepsilon$. E) $|f(x)-A|=\varepsilon$

2. ***Teoremani yakunlang:*** Agarda $y=f(x)$ funksiya $x\to a$ bo'lganda limitga ega bo'lsa, bu limit........... .

A) kamida bitta qiymatga ega bo'ladi.

B) cheksiz ko'p qiymatga ega bo'ladi.

C) faqat bitta qiymatga ega bo'ladi.

D) kamida ikkita qiymatga ega bo'ladi

E) bittadan ortiq qiymatga ega bo'ladi.

3. $y=2x^2+5x-1$ funksiyaning $x\to 2$ bo'lgandagi limiti toping.

A) 10 B) 12. C) 17. D) 21. E) –1.

4. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4}$ limitni hisoblang.

A) 0. B) ∞. C) $-\infty$. D) 3. E) mavjud emas.

5. $\lim\limits_{x\to 1}\frac{x-1}{1-\sqrt{2-x}}$ limitni hisoblang.

A) 0. B) ∞. C) – ∞.

D) 2. E) mavjud emas.

6. $\lim\limits_{x\to 1}\frac{x-1}{1+\sqrt{2-x}}$ limitni hisoblang.

A) 0. B) ∞. C) – 2. D) 2 E) 1.

7. $\lim\limits_{x\to\infty}(\sqrt{x+1}-\sqrt{x})$ limitni hisoblang.

A) 0. B) ∞. C) – 2. D) 2. E) mavjud emas.

8. Ushbu funksiyaning x=1 nuqtadagi chap limitini toping:

$$y=\begin{cases}3x-1, & x<1;\\ 2x+1, & x\geq 1.\end{cases}$$

A) –2. B) –1. C) 1. D) 2. E) 3.

9. Ushbu funksiyaning x=0 nuqtadagi o'ng limitini toping:

$$y=\begin{cases}3x^2+2x-1, & x\leq 0;\\ 2x^2+1, & x>0.\end{cases}$$

A) –2. B) –1. C) 1. D) 3. E) ∞.

10. $f(x)=0.5^{1/x}$ funksiyaning x=0 nuqtadagi o'ng limitini toping.

A) 0. B) 1. C) e. D) 0.5. E) +∞.

5. Uzluksiz funksiyalar va ularning xossalari

1. $y=f(x)$ funksiyaning x_0 nuqtadagi uzluksizlik sharti qayerda noto'g'ri ifodalangan ?

A) $\lim\limits_{x\to x_0} f(x)=f(x_0)$. B) $\lim\limits_{\Delta x\to 0} f(x_0+\Delta x)=f(x_0)$.

C) $\lim\limits_{\Delta x\to 0}\Delta f=0$ D) $\lim\limits_{x\to x_0}[f(x)-f(x_0)]=0$.

E) Barcha javoblarda to'g'ri ifodalangan.

2. Agar $y=f(x)$ funksiya x_0 nuqtada uzluksiz bo'lsa quyidagi munosabatlardan qaysi biri o'rinli bo'la oladi?

A) $\lim\limits_{x\to x_0} f(x)=f(x_0)$. B) $\lim\limits_{x\to x_0} f(x)>f(x_0)$. C) $\lim\limits_{x\to x_0} f(x)<f(x_0)$.

D) $\lim\limits_{x\to x_0} f(x)\neq f(x_0)$. E) Barcha munosabatlar o'rinli bo'lishi mumkin.

3. ***Teoremani yakunlang:*** Asosiy elementar funksiyalar … uzluksiz .

A) $(-\infty,\infty)$ oraliqda.

B) $(0,\infty)$ oraliqda.

C) $(-\infty,0)$ oraliqda.

D) aniqlanish sohasiga tegishli har bir nuqtada.

E) aniqlanish sohasidagi noldan farqli har bir nuqtada .

4. Quyidagi funksiyalardan qaysi biri x=0 nuqtada uzluksiz?

A) $y=|x|$. B) $y=|x|^{-2}$. C) $y=ctg|x|$. D) $y=\log_2|x|$.

E) keltirilgan barcha funksiyalar x=0 nuqtada uzluksiz.

5. Agarda $f(x)$ va $g(x)$ funksiyalar $x=x_0$ nuqtada uzluksiz bo'lsa, shu nuqtada quyidagi funksiyalardan qaysi biri uzluksiz bo'lmasligi mumkin?

A) $f(x)+g(x)$. B) $f(x)-g(x)$.

C) $f(x)\cdot g(x)$. D) $f(x)/g(x)$.

E) keltirilgan barcha funksiyalar doimo uzluksiz bo'ladi.

6. Agar $f(x)$ va $g(x)$ funksiyalar x_0 nuqtada uzluksiz va $g(x_0)\neq 0$ bo'lsa, shu nuqtada quyidagi funksiyalarning qaysi biri uzluksiz bo'lmasligi mumkin ?

A) $f(x)+g(x)$. B) $f(x)-g(x)$.

C) $f(x)\cdot g(x)$. D) $f(x)/g(x)$.

E) Ko'rsatilgan barcha funksiyalar x_0 nuqtada uzluksiz bo'ladi.

7. Elementar funksiyalar qaysi sohada uzluksiz bo'ladi?

A) $(-\infty,\infty)$ oraliqda.

B) $(0,\infty)$ oraliqda.

C) $(-\infty,0)$ oraliqda.

D) aniqlanish sohasiga tegishli har bir nuqtada.

E) aniqlanish sohasidagi noldan farqli har bir nuqtada .

8. Qaysi shartda $y=f(x)$ funksiya $x=a$ nuqtada chapdan uzluksiz bo'ladi ?

A) $f(a+0)=f(a)$. B) $f(a-0)=f(a)$. C) $f(a-0)=f(a+0)$.

D) $f(a-0)\neq f(a+0)$. E) $f(a-0)\neq f(a+0)$.

9. Qaysi shartda $y=f(x)$ funksiya $x=a$ nuqtada o'ngdan uzluksiz bo'ladi ?

A) $f(a+0)=f(a)$. B) $f(a-0)=f(a)$. C) $f(a-0)=f(a+0)$.

D) $f(a-0)\neq f(a+0)$. E) $f(a-0)\neq f(a+0)$.

10. Agar uzluksiz $y=f(x)$ funksiyaning x_0 nuqtadagi chap va o'ng limitlari mavjud va mos ravishda A va B bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli bo'la oladi ?

A) $A>B$. B) $A<B$. C) $A=B$. D) $A\neq B$.

E) Barcha tasdiqlar o'rinli bo'lishi mumkin.

6. Funksiyaning uzilish nuqtalari va ularning turlari

1. ***Ta'rifni to'ldiring:*** $y=f(x)$ funksiya x_0 nuqtada uzlukli deyiladi, agar ... shart bajarilsa.

A) $\lim\limits_{x\to x_0} f(x)>f(x_0)$. B) $\lim\limits_{x\to x_0} f(x)<f(x_0)$. C) $\lim\limits_{x\to x_0} f(x)\neq f(x_0)$.

D) $\lim\limits_{x\to x_0} f(x)=f(x_0)$. E) $\lim\limits_{x\to x_0} f(x)=\pm\infty$.

2. Agar $y=f(x)$ funksiya x_0 nuqtada uzlukli va bu nuqtada uning chap va o'ng limitlari mos ravishda A va B bo'lsa, unda quyidagi munosabatlardan qaysi biri o'rinli bo'la olmaydi?

A) $A>B$. B) $A<B$. C) $A=B$. D) $A\neq B$.

E) Barcha munosabatlar o'rinli bo'la oladi.

3. Funksiyaning $x=a$ uzilish nuqtasining qanday turi mavjud emas ?

A) I tur uzilish nuqtasi. B) II tur uzilish nuqtasi.

C) yo'qotib bo'ladigan turdagi uzilish nuqtasi.

D) o'zgartirib bo'ladigan turdagi uzilish nuqtasi.

E) barcha ko'rsatilgan turdagi uzilish nuqtalari mavjud .

4. Agar $y=f(x)$ funksiyaning x_0 nuqtadagi chap va o'ng limitlari mavjud hamda mos ravishda A va B bo'lsa, unda qaysi shartda x_0 I tur uzilish nuqtasi bo'ladi ?

A) A va B limitlardan kamida bittasi chekli son.

B) A va B limitlardan kamida bittasi cheksiz.

C) A va B limitlarning ikkalasi ham cheksiz.

D) A va B llimitlarning ikkalasi ham chekli son va $A\neq B$.

E) A va B limitlarning ikkalasi ham chekli son va A=B.

5. Quyidagi funksiyalardan qaysi biri uzilishga ega ?

A) $y=|x\pm 1|$. B) $y=|x|+1$. C) $y=\frac{1}{|x|+1}$.

D) $y=|x|-1$. E) $y=\frac{1}{|x|-1}$.

6. Agar $y=f(x)$ funksiya uchun x_0 I tur uzilish nuqtasi va bu nuqtadagi funksiyaning chap va o'ng limitlari mos ravishda A va B bo'lsa, unda funksiyaning bu nuqtadagi sakrashi Δ qanday aniqlanadi ?

A) $\Delta=A+B$. B) $\Delta=A\cdot B$. C) $\Delta=A/B$.

D) $\Delta=B/A$. E) $\Delta=B-A$.

7. $y=3\text{sgn}x+1$ funksiya $x=0$ nuqtada qanday sakrashga ega ?

A) 2. B) 3. C) 7. D) 5. E) 6.

8. Quyidagi funksiyaning $x=2$ nuqtadagi Δ sakrashini toping:

$$f(x)=\begin{cases} x^3, & x<2, \\ x^2-2, & x\geq 2 \end{cases}$$

A) $\Delta=8$. B) $\Delta=6$. C) $\Delta=-8$. D) $\Delta=-6$. E) $\Delta=2$.

9. Quyidagi funksiyalardan qaysi biri $x=0$ nuqta I tur uzilishga ega ?

A) $y=\text{ctg}x$. B) $y=1/x$. C) $y=\text{sgn}x$. D) $y=\ln|x|$. E) $y=2^{1/x}$.

10. Quyidagi funksiyalardan qaysi biri $x=0$ nuqtada II tur uzilishga ega?

A) $y=\text{sgn}x$. B) $y=[x]$. C) $y=\{x\}$. D) $y=x^{-1}$.

E) keltirilgan barcha funksiyalar uchun $x=0$ I tur uzilish nuqtasi bo'ladi.

Matematikda insonning hayratini
keltiradigan nimadir bor.
Xausdorf

VII-BOB. DIFFERENSIAL HISOB

§ 7.1 Funksiya hosilasi ta'rifi, uning mexanik va geometrik ma'nosi

Umumiy holda y=f(x) funksiyaning hosilasini topish, ya'ni uni differensiallash,quyidagi algoritm bo'yicha amalga oshiriladi:

1)x argumentga $\Delta x \neq 0$ orttirma berib, $x+\Delta x$ nuqtani topamiz;

2)funksiya orttirmasini $\Delta f= f(x+\Delta x)-f(x)$ tenglik o'yicha hisoblaymiz;

3)$\Delta f/\Delta x$ nisbatni topamiz va uning $\Delta x \to 0$ bo'lgandagi limitini hisoblaymiz. Bu limit mavjud bo'lsa, uning qiymati $f'(x)$ hosilani aniqlaydi.

Misol sifatida f(x)=sinx funksiya hosilasini yuqoridagi algoritm bo'yicha topamiz:

1)x va $x+\Delta x$ nuqtalarda funksiyani hisoblaymiz;

2)trigonometrik formuladan foydalanib, funksiya orttirmasini quyidagicha yozamiz:

$\Delta f=\sin(x+\Delta x)-\sin x= 2\sin(\Delta x/2)\cos(x+\Delta x/2)$

3) $\Delta f/\Delta x$ nisbatni tuzamiz va uning limitini hisoblaymiz:

$$\lim_{\Delta x \to 0}(\Delta f/\Delta x)= \lim_{\Delta x \to 0} 2\sin(\Delta x/2)\cos(x+\Delta x/2)/\Delta x=$$

$$= \lim_{\Delta x \to 0} \sin(\Delta x/2)/(\Delta x/2)\cdot \lim_{\Delta x \to 0} \cos(x+\Delta x/2)=1\cdot\cos x=\cos x.$$

Bu yerda ko'paytmaning limiti, $\lim_{x \to 0} \sin x/x=1$ ajoyib limitdan va u=cosx funksiya uzluksizligidanfoydalanildi.

Demak, (sinx)′=cosx buladi. Xuddi shunday usulda (cosx)′= -sinx ekanligianiqlanadi. Bundan tashqari

$$(a^x)' = a^x \ln a\ , \quad (\log_a x)' = \frac{1}{x}\log_a e$$

Ekanligini isbotlash mumkin.

Ammo, har qanday funksiya hosilasini bu algoritm bo'yicha hisoblash oson emas va muhim ishart ham emas. Umumiy holda funksiya hosil asini hisoblashni quyidagi differensiallash qoidalari bo'yicha amalga oshirish mumkin.

1- *qoida:* O'zgarmas S soning hosilasi nolga teng , ya'ni (S)′=0

<u>*I s b o t :*</u> O'zgarmas S sonni x argumentning har qanday qiymatida bir xil qiymat qabul qiluvchi f(x)=S funksiya deb qarash mumkin. Bu holda,

$$\Delta f = f(x+\Delta x) - f(x) = S - S = 0, \qquad \Delta f/\Delta x = 0,$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} 0 = 0.$$

2-*qoida:* *u=u(x), v=v(x)* funksiyalar *x* nuqtada differensiallanuvchi bo'lsa, bu nuqtada $u \pm v$, $u \cdot v$ va $v(x) \neq 0$ shartda u/v funksiyalar ham differensiallanuvchi bo'lib, ularni hisoblash uchun

$$(u \pm v)' = u' \pm v', \quad (u \cdot v)' = u' \cdot v + u \cdot v', \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

formulalar o'rinli bo'ladi.

<u>*Isbot :*</u>Funksiya orttirmasi ta'rifidan foydalanib, har qanday Δx argumentort tirmasida $\Delta(u \pm v) = \Delta u \pm \Delta v$ ekanligini ko'rsatish mumkin. Bu holda limit xossasi va hosila ta'rifiga asosan

$$(u+v)' = \lim_{\Delta x \to 0} \frac{\Delta(u+v)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u \pm \Delta v}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} \pm \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = u' + v'.$$

Xuddi shunday,

$$\Delta(u \cdot v) = u \cdot \Delta v + \Delta u \cdot v + \Delta u \cdot \Delta v, \qquad \Delta\left(\frac{u}{v}\right) = \frac{u\Delta v - u\Delta v}{v(v + \Delta v)}$$

munosobatlardan foydalanib, 2-qoidadagi qolgan formulalarni ham isbotlash mumkin.

Natija 1: Funksiyaga ixtiyoriy S o'zgarmas sonni qo'shsak, uning hosilasi o'zgarmaydi.

Haqiqatdan ham $(f(x)+S)'=f'(x)+S'=f'(x)+0=f'(x)$.

Natija 2 : O'zgarmas S ko'paytuvchini hosila belgisidan tashqariga chiqarish mumkin.

Haqiqatdan ham, ko'paytmaning hosilasi formulasi va 1-qoidaga asosan

$$(S\cdot f(x))'=S'\cdot f'(x)+S\cdot f'(x)=0\cdot f(x)+S\cdot f'(x)=S\cdot f'(x)$$

Natija 3 : $(tgx)'=1/\cos^2x$, $(ctgx)'=-1/\sin^2x$.

Haqiqatdan ham, bo'linmaning hosilasi formulasiga ko'ra

$$(tgx)'=\left(\frac{\sin x}{\cos x}\right)'=\frac{(\sin x)'\cdot\cos x-\sin x\cdot(\cos x)'}{\cos^2 x}=$$

$$=\frac{\cos x\cdot\cos x-\sin x\cdot(-\sin x)}{\cos^2 x}=\frac{1}{\cos^2 x}.$$

Xuddi shunday ravishda $(ctgx)'$ hosila topiladi.

Shunday qilib, barcha asosiy elementar funksiyalar aniqlanish sohasida differensiallanuvchi va ularning hosilalari quyidagi formulalar bilan hisoblanadi:

1) $(x^{\alpha})'=\alpha\cdot x^{\alpha-1}$, α- ixtiyoriy haqiqiy son;

2) $(a^x)'=a^x\cdot\ln a$, $(e^x)'=e^x$; 3) $(\log_a x)'=\frac{1}{x}\log_a e$, $(\ln x)'=\frac{1}{x}$.

4) $(\sin x)'=\cos x$, $(\cos x)'=-\sin x$, $(tgx)'==\frac{1}{\cos^2 x}$ $(ctgx)'=-\frac{1}{\sin^2 x}$

5) $(\arcsin x)'=-(\arccos x)'=\frac{1}{\sqrt{1-x^2}}$, $(arctg\ x)'=-(arcctg\ x)'=\frac{1}{1+x^2}$

Bu hosilalar jadvalidan va ko'rib o'tilgan hosila olish qoidalaridan foydalanib, har qanday elementar funksiyaning hosilasini hisoblash mumkin.

I. $(S)'=0$ II. $(u\pm v)'=u'\pm v'$, III. $(u\cdot v)'=u'\cdot v+u\cdot v'$,

IV. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ V. $[f(u)]' = f'_u \cdot u'_x$ VI. $x'_y = \frac{1}{y'_x}$

Differentsiallashning oddiy qoidalaridan foydalanib quyidagi funktsiyalarning hosilalari topilsin.

7.1. $y=3x^3-4x^2+7x-9$;

7.2. $y=\frac{2}{x}+\ln x$;

7.3. $y=3\sin x+5^x$;

7.4. $y=-2\sin x+\mathrm{tg}x$;

7.5. $y=\frac{1}{x}-1000$;

7.6. $y=25t^{10}+2\sqrt{t}$;

7.7. $y=\sin x+\cos x$;

7.8. $y=6x^{15}-9x^{20}$;

7.9. $y=100x^2-100$;

7.10. $y=e-3^x+4$.

7.11. $y=3x^7-6x^6+5x^2-7$;

7.12. $y=(x^3-2x^2-1)(x^5+x^2)$;

7.13. $y=\sqrt[3]{3x}+\sqrt{2x}+10$;

7.14. $y=\frac{1}{x\sqrt{2x}}$;

7.15. $y=10^{x+3}-7$;

7.16. $y=5\sqrt[5]{x^2}+7$;

7.17. $y=\frac{11-5x+x^6}{\sqrt{x}}$;

7.18. $y=e^{2x^2+3x-5}$;

7.19. $y=\frac{1}{2x}-\ln x+e^{x+2}$;

7.20. $y=e^{2x^2-\frac{1}{\sqrt{x}}+1}$.

7.21. $y=(ax^2+вx+c)^3$;

7.22. $y=\left(e^{2x}-\frac{2}{x^3}+\ln\sqrt{x}\right)^2$;

7.23. $y=\sin 3(ax+в)$;

7.24. $y=\cos(e^x-e^{-x})$;

7.25. $y=\mathrm{tg}^2(\ln 2x)$;

7.26. $y=ctg^3(tg2x)$;

7.27. $y=\ln^2(\ln 4x)$;

7.28. $y=(e^{\sin x}+3^{4x})^5$;

7.29. $y=7^{\cos^3 4x}$;

7.30. $y=\sin^m x\cdot\sin mx$.

7.31. 1) $y=\frac{x^3}{3}-2x^2+4x-5$ 2) $y=\frac{bx+c}{a}$

7.32. 1) $y=\frac{x^5}{5}-\frac{2x^3}{3}+x$ 2) $y=\left(1-\frac{x^2}{2}\right)^2$

7.33. 1) $y=x+2\sqrt{x}$ 2) $y=\left(\sqrt{a}-\sqrt{x}\right)^2$

7.34. 1) $y=\frac{10}{x^3}$ 2) $y=\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}$

7.35. 1) $y=x+\frac{1}{x^2}-\frac{1}{5x^5}$ 2) $y=3x-6\sqrt{x}$

7.36. 1) $y=6\sqrt[3]{x}-4\sqrt[4]{x}$ 2) $y=\left(1-\frac{1}{\sqrt[3]{x}}\right)^2$

7.37. 1) $y=\frac{1}{2x^2}-\frac{1}{3x^3}$ 2) $y=\frac{8}{\sqrt[4]{x}}-\frac{6}{\sqrt[3]{x}}$

7.38. 1) $y=x-\sin x$ 2) $y=x-tgx$

7.39. 1) $y=x^2\cos x$ 2) $y=y^2ctgx$

7.40. 1) $y=\frac{\cos x}{x^2}$ 2) $y=\frac{x^2}{x^2+1}$

Ko'rsatkichli va logarifmik funksiyalarning hosilalari topilsin

7.41. 1) $y=x\ln x$ 2) $y=\frac{1+\ln x}{x}$;

7.42. 1) $y=\ln x-\frac{2}{x}-\frac{1}{2x^2}$ 2) $y=\ln(x^2+2x)$

7.43. 1) $y=\ln(1+\cos x)$ 2) $y=\ln\sin x-\frac{1}{2}\sin^2 x$

7.44. $y=\ln(\sqrt{x}+\sqrt{x+1})$

Quyidagi funksiyalarning hosilalari topilsin

7.45. 1) $y=x^2+3^x$ 2) $y=x^2 2^x$ 3) $y=x^2e^x$

7.46. 1) $y=a^{\sin x}$ 2) $y=e^{-x^2}$ 3) $y=x^2e^{-2x}$

7.47. $y=2\left(e^{\frac{x}{2}}-e^{-\frac{x}{2}}\right)$ 7.48. $y=\sqrt{x}e^{\sqrt{x}}$

7.49. $y=\frac{1+e^x}{1-e^x}$ 7.50. $y=e^{\frac{x}{a}}\cos\frac{x}{a}$

7.51 $y=2x^3-x$ egri chiziq Oy o'qini Ox o'qiga nisbatan qanday burchak ostida kesib o'tadi?

7.52 $y=(4x-x^2)/3$ parabolaga uning (0;0), (2;2), (4;0) nuqtalaridan urinmalar o'tkazilgan. Ularning Ox o'qiga og'ish burchaklarini toping.

7.53 $y=(x^3+1)/3$ funktsiya grafigiga uning abtsissa o'qi bilan kesilgan nuqtasida o'tkazilgan urinma tenglamasini yozing.

7.54 $y=1/x$ giperbola uning $M(1;1)$ nuqtadan o'tkazilgan urinmaning Ox o'qiga og'ish burchagini toping.

7.55 $y=3x^2-x$ parabolaga uning abtsissasi $x=-1$ bo'lgan nuktasidan urinma va normal o'tkazilgan. Ularning t englamalarini tuzing.

7.56 $y=3x-4$ to'g'ri chiziq $y=x^3-2$ egri chiziqqa urinma bo'ladimi?

7.57 $xy=1$ giperbola uning $M(-1;3)$ nuqtadan o'tkazilgan urinmaning tenglamasini tuzing.

7.58 $y=x^2-2x-8$ parabolada shunday bir M nuqta topish kerakki, undan o'tkazilgan urinma $4x+y+4=0$ to'g'ri chiziqqa parallel bo'lsin.

7.59 Nuqta $S=2t^3+t^2-4$ qonun bo'yicha harakat qiladi. $t=4$ sek. vaqtdagi tezlik va tezlanish qiymati topilsin.

7.60 Nuqta $S=6t-t^2$ qonun bo'yicha to'g'ri chiziqli harakat qiladi. Qaysi bir daqiqada nuqta tezligi nolga teng bo'ladi?

Teskari trigonometrik funksiyalarning hosilalari topilsin

7.61. $y=\sqrt{1-x^2}+\arcsin x$ 7.62. $y=x-arctgx$

7.63. $y=\arcsin\sqrt{1-4x}$ 7.64. $y=\arcsin\frac{x}{a}$

7.65. $y=arctg\frac{x}{a}$ 7.66. $y=\arccos(1-2x)$

7.67. $y=arctg\frac{1+x}{1-x}$ 7.68. 1) $y=x\sqrt{1-x^2}+\arcsin x$ 2) $y=\arcsin(e^{3x})$

Quyidagi funksiyalarning hosilalari topilsin

7.69. 1) $y=\frac{\sqrt{x^2-1}}{x}+\arcsin\frac{1}{x}$ 2) $y=\frac{tg^2x}{2}+\ln\cos x$

7.70. $y=\sqrt{4x-1}+arcctg\sqrt{4x-1}$ 6.71. $x=\ln(e^{2t}+1)-2arctg(e^t)$

7.72. $y=4\ln(\sqrt{x-4}+\sqrt{x})+\sqrt{x^2-4x}$

§ 7.2. Murakkab va oshkormas funksiyalarning hosilalari

3-qoida: $u=f(u)$ murakkab funksiyada $f(u)$ va $u(x)$ funksiyalar argumentlari bo'yicha differensiallanuvchi bo'lsin. Bu holda

$u=f(u)$murakkab funksiya x bo'yicha differensiallanuvchi bo'lib, uning hosilasi

$$f'_x = f'(u)\cdot u'(x)$$

formula bilan topiladi.

I s b o t :u(x) funksiya differensiallanuvchi bo'lganligidan uning uzluksizligi kelib chiqadi va shu sababli $\Delta x\to 0$ bo'lganda $\Delta u\to 0$ bo'ladi. Hosila ta'rifiga asosan

$$f'_x = \lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta f}{\Delta u}\frac{\Delta u}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta f}{\Delta u}\cdot\lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x} = f'(u)\cdot u'(x).$$

Masalan, $(\sin x^2)' = (i=x^2) = (\sin i)'_x = \cos i\cdot i' = 2x\cos x^2$.

Bu qoidaning tadbiqi sifatida $u=x^\alpha$ darajali funksiyaning u' hosilasini topamiz. Bu xolda

$\ln u = \ln x^\alpha = \alpha\ln x \Rightarrow (\ln u)'_x = (\alpha\ln x)' \Rightarrow$

$$\frac{y'}{y} = \frac{\alpha}{x} \Rightarrow y' = \frac{\alpha}{x}y = \frac{\alpha}{x}x^\alpha = \alpha\cdot x^{\alpha-1}.$$

***4-qoida:*.** $u=f(x)$ differensiallanuvchi va $f'(x)\neq 0$ bo'lsa, $x=f^{-1}(u)$ teskari funksiya ham differensiallanuvchi bo'ladi va uning hosilasi $x'_y = \frac{1}{y'_x}$ formula bo'yicha topiladi.

I s b o t : $x=f^{-1}(u)$ teskari funksiyaning argument orttirmasi $\Delta u\neq 0$ bo'lgandagi orttirmasi Δx bo'lsin. Berilgan f(x) funksiya differensiallanuvchi bo'lgani uchun uzluksizdir va shu sababli unga teskari $f^{-1}(u)$ funksiya ham uzluksiz bo'ladi. Demak, $\Delta u\to 0$ bo'lganda $\Delta x\to 0$ bo'ladi. Bu holda, hosila ta'rifiga asosan,

$$x_u' = \lim_{\Delta y\to 0}\frac{\Delta x}{\Delta y} = \lim_{\Delta x\to 0}\left(\frac{\Delta y}{\Delta x}\right)^{-1} = \frac{1}{y'_x}.$$

Misol sifatida u=arcsinx funksiya hosilasini topamiz. Bu yerda

D{f}=[-1;1] , E{f}=[-π/2, π/2] bo'lgani uchun, x=siny teskari funksiyaning hosilasi $x'_y = \cos y \neq 0$, u ∈(-π/2 π/2), shartni qanoatlantiradi. Bu holda (arcsin x)'= $y'_x = \frac{1}{x'_y} = \frac{1}{\cos y}$

Ammo u∈(-π/2 π/2) bo'lganda cosy > 0 va

cosy= $\sqrt{1-\sin^2 y} = \sqrt{1-x^2}$, x∈(-1,1)

tenglik o'rinli. Bu natijani oldingi tenglikka qo'yib,

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

formulani hosil qilamiz. Xuddi shunday usulda

$$(\text{arcsos } x)' = -\frac{1}{\sqrt{1-x^2}}, \quad (\text{arctg } x)' = \frac{1}{1+x^2}, \quad (\text{arcctg } x)' = -\frac{1}{1+x^2}$$

formulalarni hosil qilish mumkin.

Logarifmik funksiyaning hosilasi:

$y = \ln y$ funksiyani hosilasi

$(\ln y)' = \frac{y'}{y} = \frac{f'(x)}{f(x)}$ ko'rinishida bo'ladi.

Parametrik ko'rinishida berilgan funksiyaning hosilasi. Agar $y = f(x)$ funksiya parametrik ko'rinishda $x = \varphi(t)$ va $y = \psi(t)$ berilgan bo'lsa, u holda uning hosilasi

$$y'_x = \frac{y'_t}{x'_t};$$

ko'rinishida bo'ladi.

Agar y ga nisbatan yechilmagan $F(x;y)=0$ tenglama $y$ni $x$ning bir qiymatli funksiyasi sifatida aniqlasa, uholda y o'zgaruvchi x ning oshkormas funksiyasi deyiladi. Bu oshkormas funksiyadan y' hosilani topish uchun y ni x ning funksiyasi deb, $F(x;y)=0$ tenglamaning ikki tomonini x bo'yicha differensiallash kerak. Hosil bo'lgan tenglamadan izlangan y' ni topamiz. y'' ni topish uchun $F(x;y)=0$ tenglamani x bo'yicha ikki marta differensiallash kerak va hokazo.

Misol: $(e^x \cdot \sin 2x)' = (e^x)' \sin 2x + e^x (\sin 2x)' = e^x \cdot \sin 2x + e^x \cdot \cos 2x \cdot (2x)' = (\sin 2x + 2 \cdot \cos 2x)\, e^x$,

$$(\ln \sin x)' = \frac{1}{\sin x} \cdot (\sin x)' = \frac{\cos x}{\sin x} = ctgx.$$

Quyidagi funksiyalarning hosilalari topilsin.

7.73. 1) $y = \sin 6x$ 2) $y = \cos(a - bx)$

3) Funksiyaning hosilasini toping.

a) $y=2x^5-5\cdot 2^x+4x-7\log_2 x-\ln 2$;

b) $y=(1+x^2)\cdot arctgx$;

v) $y=\dfrac{\sin x+\cos x}{\sin x-\cos x}$; g) $y=\log_3(2x^3+1)$.

7.74. 1) $y=\sin\dfrac{x}{2}+\cos\dfrac{x}{2}$ 2) $y=6\cos\dfrac{x}{3}$

7.75. 1) $y=(1-5x)^4$ 2) $y=\sqrt[3]{(4+3x)^2}$

7.76. 1) $y=\dfrac{1}{(1-x^2)^5}$ 2) $y=\sqrt{1-x^2}$ 3) $y=\sqrt{\cos 4x}$

7.77. $y=\sqrt{2x-\sin 2x}$ 7.78. $y=\sin^4 x=(\sin x)^4$

7.79. $y=\sqrt{4x+\sin 4x}$ 7.80. $y=x^2\sqrt{1-x^2}$

7.81. $y=\sin^4 x+\cos^4 x$ 7.82. $y=\sqrt[3]{1+\cos 6x}$

7.83. 1) $y=tgx+\dfrac{2}{3}tg^3x+\dfrac{1}{5}tg^5x$ 2) $y=\sin^2 x^3$

3). $y=x^2+e^x\ln x$ funksiyaga teskari bo'lgan funksiya uchun x'_y ni toping.

4) Hosilani tekshiring. a) $y=x^{x^3}$; b) $y=\dfrac{(3x+2)^4\sqrt[3]{5x-1}}{(1-2x)^3\sqrt{1-x^2}}$.

Quyidagi tenglamalardan y' topilsin.

7.84. 1) $x^2+y^2=a^2$ 2) $y^2=2px$ 3) $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

4). $x^2+9y^2=16$ oshkormas funksiya uchun $y'_x=\dfrac{dy}{dx}$ ni toping.

7.85. 1) $x^2+xy+y^2=6$ 2) $x^2+y^2-xy=0$

Quyidagi funksiyalarning differensiallari topilsin:

7.86. 1) $y=x^n$ 2) $y=x^3-3x^2+3x$

3) $x=a^2\cos^2 t,\quad y=a\sin^2 t$ berilgan bo'lsa, $y'_x=\dfrac{dy}{dx}$ ni toping.

7.87. 1) $y=\sqrt{1+x^2}$ 2) $S=\dfrac{gt^2}{2}$

7.88. 1) $r=2\varphi-\sin 2\varphi$ 2) $x=\dfrac{1}{t^2}$

7.89 $y=|x|$ funksiyani $x_0=0$ nuqtadagi hosilasini toping.

7.90. $y=\cos^2 x\cdot\sqrt{1-x^2}$ funksiyaning ixtiyoriy x nuqtasi uchun differensialini hisoblang.

7.91 $y=\ln\cos\dfrac{1}{x}$ 7.92 $y=\dfrac{x^2-1}{2(1+2x)}$

7.93 $y=\cos 2x+4\cos x$

7.94 $f(x)=e^{x^2}+e^{-x^2}$

7.95 $f(x)=\ln(x\sin x)$

7.96 $f(x)=\ln\left(x+\sqrt{x^2+1}\right)$

7.97 $y=arctg\frac{1}{x}$

7.98 $y=arctg\frac{2}{x}$

7.99 $y=\arccos\sqrt{1-x^2}$

7.100 $y=x^2ctg2x$

7.101 $y=\frac{3ctg2x}{\sin 4x}$

7.102 $y=\ln\ln x$

7.103 $y=\frac{x^3-x^2}{\sqrt{1-x}}$

7.104 $y=\arcsin x+\sqrt{1-x^2}$

7.105 $y=\ln\sqrt{4x^2+1}+\frac{1}{2}arctg2x$

7.106 $y=\arcsin\left(\frac{x}{2}-1\right)-\sqrt{x(4-x)}$

7.107 $y=\frac{\cos x}{\sin x+\cos x}$

7.108 $y=\frac{\ln\cos x}{\cos x}$

7.109 $y=e^x\ln x$ berilgan. $y'(1)$ ni toping.

7.110 $y=2^x\cos 2^x$ berilgan. $y'(0)$ ni toping.

7.111 $y=\ln^3\sin^2\left(\frac{\pi}{4}-2x\right)$ berilgan. $y'\left(\frac{\pi}{4}\right)$ ni toping.

7.112 $y=3^{\ln x}+3\ln x+\frac{3}{\ln x}$ berilgan. $y'(e)$ ni toping.

7.113 $y=ch^3x^2-sh^3x^2$ berilgan. $y'(0)$ ni toping.

7.114 $z=\frac{1+chu}{1+shu}$ berilgan. $z'(0)$ ni toping.

7.115 $y=\sin^2x+sh^2x$ berilgan. $y'(1)$ ni toping.

7.116 $\ln x=\sin y$ 7.117 $tgx=e^y$

7.118 $\ln(x-y)=\cos x$ 7.119 $2x^3y^2-y^3-3x=0$

7.120 $\arcsin y=e^x+\ln 3$

§ 7.3. Silliq egri chiziqqa o'tkazilgan urinma va normal

Hosilaning geometrik ma'nosi. Agar $y=f(x)$ yoki $F(x;y)=0$ egri chiziq tenglamasi berilgan bo'lsa, $f'(x_0)=tg\alpha$, ya'ni egri chiziqqa x_0 nuqtaga o'tkazilgan urinmani burchak koeffisienti (urinmani absissa o'qining musbat yo'nalishi bilan hosil qilgan burchak tangensi)ga teng bo'ladi. $y=f(x)$ egri chiziqni x_0 nuqtasidan urinma tenglamasi

$$y - f(x_0) = f'(x_0)(x - x_0)$$

va shu nuqtadan o'tkazilgan normal tenglamasi

$$y - f(x_0) = \frac{1}{f'(x_0)}(x - x_0).$$

$M_0(x_0; y_0)$ nuqtada kesishuvchi $y = f_1(x)$ va $y = f_2(x)$ chiziqlar orasidagi burchak deganda shu nuqtadan egri chiziqlarga o'tkazilgan urinmalar orasidagi burchak tushuniladi:

$$tg\varphi = \frac{f_2'(x_0) - f_1'(x_0)}{1 + f_1'(x_0)f_2'(x_0)} \qquad (7.14)$$

Egri chiziqlarga o'tkazilgan urinmalarning tenglamalari yozilsin va egri chiziqlar hamda urinmalar yasalsin.

7.121. $y = \frac{8}{4 + x^2}$ lokonga (zulfga) $x = 2$ nuqtada.

7.122. $y = \sin x$ sinusoidaga $x = \pi$ nuqtada.

7.123. $y = \sin x$ egri chiziq Ox o'q bilan qanday burchak orasida kesishadi?

7.124. $2y = x^2$ va $2y = 8 - x^2$ egri chiziqlar qanday burchak ostida keshishadi?

7.125. 1) $y = x^2$; 2) $y^2 = x^3$ egri chiziqlarga $x = 1$ nuqtada o'tkazilgan urinma osti, normal osti, urinma va normalning uzunliklari topilsin.

7.126. $y^2 = 2px$ parabolaning urinma nuqta abssissasining ikkilanganiga, normal ostiesa p gatengekani isbotqilinsin.

7.127. Agar $y = x^2 + bx + c$ parabola $x = 2$ nuqtada $y = x$ to'g'ri chiziqqa urinsa, parabola tenglamasidagi b va c aniqlansin. .

7.128. $y = x^2 - 4x + 5$ parabola uchidan unga Oy o'q bilan kesishgan nuqtasigacha bo'lgan masofa topilsin.

7.129. $y = 0{,}5$ to'g'ri chiziq $y = \cos x$ egri chiziqni qanday burchak ostida kesadi?

7.130. $y = x^2 + 4x$ parabolaga qaysi nuqtada o'tkazilgan urinma Ox o'qqa parallel bo'ladi?

7.131. $y = x^2 - 2x + 5$ parabolaga o'tkazilgan urinma, birinchi koordinatalar burchagining bissektrissasiga perpendikulyar bo'lishi uchun, urinma parabolaning qaysi nuqtasida o'tkazilishi kerak?

7.132. $y=\frac{2}{1+x^2}$ egri chiziqqa $x=1$ nuqtada o'tkazilgan urinma osti, normal osti, urinma va normal uzunliklari topilsin.

§ 7.4 Hosilaning tatbiqlari

Hosilani mexanik ma'nosi. Agar material nuqta $S=S(t)$ qonuniyat bilan harakatlanayotgan bo'lsa, (S -yo'l; t -vaqt), u holda $S'(t)$-material nuqta tezligini ifodalaydi. Tezlikdan olingan ikkinchi hosila $[S'(t)]=S''(t)$ material nuqtaning tezlanishini ifodalaydi.

Rollteoremasi. Farazqilaylik, $y=f(x)$ funksiya quyidagi shartlarni qanoatlantirsin:

1) $[a,b]$ oraliqda uzluksiz;

2) (a,b) intervalda differensiallanuvchi;

3) oraliq oxirlarida funksiya bir xil qiymatlarni qabul qiladi, ya'ni $f(a)=f(b)$; u holda shunday $\xi\in(a,b)$ nuqta topiladiki, $f'(\xi)=0$ bo'ladi.

Lagranj teoremasi. Faraz qilaylik, $y=f(x)$ funksiya quyidagi shartlarni qanoatlantirsin:

1) $[a,b]$ oraliqda uzluksiz; 2) (a;b) intervalda differensiallanuvchi;

U holda shunday $\xi\in(a,b)$ nuqta topiladiki, $f'(\xi)=\frac{f(b)-f(a)}{b-a}$ tenglik bajariladi.

Lopital qoidasi. $f(x)$ va $g(x)$ funksiyalar x_0 nuqtaning atrofida differensiallanuvchi bo'lib, $g'(x)\neq 0$ bo'lsin. Agar $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0} g(x)=0$, yoki $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0} g(x)=\infty$ bo'lib, $\lim\limits_{x\to x_0}\frac{f'(x)}{g'(x)}$ mavjud bo'lsa, u holda $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}=\lim\limits_{x\to x_0}\frac{f'(x)}{g'(x)}$ tenglik o'rinli bo'ladi. Lopital qoidasi $\frac{0}{0};\frac{\infty}{\infty}$ shaklidagi aniqmasliklarning limitini hisoblashda qo'llaniladi. $0\cdot\infty$ va $\infty-\infty$ shaklidagi aniqmasliklar, algebraic o'zgartirishlar yordamida $\frac{0}{0}$ *va* $\frac{\infty}{\infty}$ shakliga keltiriladi. 1^{∞}, ∞^0, 0^0

shaklidagi aniqmasliklar esa avval logarifmlash orqali $0\cdot\infty$ shaklidagi aniqmaslikka keltirilib, so'ngra $\frac{0}{0}$ *yoki* $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltiriladi.

Berilgan masalalarni yeching.

7.133. Zenit snaryad boshlang'ich *am/sek* tezlik bilan vertikal yo'nalishda otilgan. *t* sekunddan so'ng snaryar qanday *x* balandlikda bo'ladi? Snaryadning harakat tezligi va tezlanishi aniqlansin. Necha sekunddan so'ng snaryad eng yuqori balandlikka ko'tariladi va yerdan qanday masofada bo'ladi?

7.134. Jism $x=\frac{t^3}{3}-2t^2+3t$ qonunga asosan Ox to'g'ri chiziq bo'yicha harakat qiladi. harakat tezligi va tezlanishi aniqlansin. Qaysi paytlarda jism harakat yo'nalishini o'zgartiradi?

7.135. Moddiy nuqta $x=a\cos\omega t$ qonun bo'yicha tebranma harakat qiladi. $x=\pm a$ va $x=0$ nuqtalardagi tezlik va tezlanish aniqlansin. $\frac{d^2x}{dt^2}$ tezlanish hamda nuqtaning uzoqlashishi *x* ushbu $\frac{d^2x}{dt^2}=-\omega^2x$ "differensial" tenglama bilan bog'langani ko'rsatilsin.

7.136. $f(x)=x^2-4x+3$ funksiya ildizlari orasida uning hosilasining ham ildizi bor ekani tekshirilsin. Bu grafik usulda tushuntirilsin.

7.137. Roll teoremasini $f(x)=\sqrt[3]{x^2}$ funksiyaga [-1; 1] segmentda tatbiq qilish mumkinmi?

7.138. $y=|\sin x|$ egri chiziqning $[-\frac{\pi}{2};\frac{\pi}{2}]$ segmentdagi $\breve{AB}$ yoyi yasalsin. Nima uchun bu yoyda AB vatarga parallel urinma yo'q? Roll teoremasining qaysi sharti bu yerda bajarilmaydi?

7.139. [1, 4] segmentda $f(x)=\sqrt{x}$ funksiya uchun Lagranj formulasi yozilsin va *c* topilsin.

7.140.[-1, 2] segmentda $\frac{4}{x}$ va $1-\sqrt[3]{x^2}$ funksiyalarga Lagranj teoremasini tatbiq qilish mumkin emasligi ko'rsatilsin. Grafik usulda tushuntirilsin.

7.141. $\left[0,\frac{2\pi}{3}\right]$ segmentda $y=|\cos x|$ egri chiziqning $\breve{AB}$ yoyi yasalsin. Nima uchun bu yoyda AB vatarga parallel urinma yo'q? Lagranj teoremasining qaysi sharti bunda bajarilmaydi?

7.142 $f(x)=x^3$ funksiya uchun Lagranjning $f(b)-f(a)=f(b-a)f'(c)$ formulasi yozilsin va c topilsin.

7.143. Quyidagi funksiyalar uchun Lagranj formulasi yozilsin va c topilsin.

1) [0, 1] segmentda $f(x)=arctgx$;

2) [0, 1] segmentda $f(x)=\arcsin x$;

3) [1, 2] segmentda $f(x)=\ln x$.

7.144. Quyidagi funksiyalar uchun Koshi formulasi yozilsin va c topilsin:

1) [0, $\frac{\pi}{2}$] segmentda $\sin x$ va $\cos x$;

2) [1, 4] segmentda x^2 va $\sqrt{x}$.

Quyidagi funksiyalarning limitlarini hisoblang:

7.145. $\lim\limits_{x\to 1}\frac{x-\sqrt{x}}{\sqrt{x}-1}$

7.146. $\lim\limits_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$

7.147. $\lim\limits_{x\to 0}\frac{tgx}{\sqrt{1-tgx}-1}$

7.148. $\lim\limits_{x\to 4}\frac{\sqrt{x}-2}{\sqrt{3+x}-\sqrt{x-3}}$

7.149. $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

7.150. $\lim\limits_{x\to 0}\frac{x}{\sqrt{3+x}-\sqrt{3-x}}$

7.151. $\lim\limits_{x\to 8}\frac{\sqrt[3]{x}-2}{\sqrt{x}-2\sqrt{2}}$

7.152. $\lim\limits_{x\to -27}\frac{x+27}{\sqrt[3]{x}+3}$

7.153. $\lim\limits_{x\to 5}\frac{5-x}{3-\sqrt{2x-1}}$

7.154. $\lim\limits_{x\to 0}\frac{1-\sqrt{1-x^2}}{x^2}$

7.155. $\lim\limits_{x\to 0}\frac{e^{3x}-1}{x}$

7. 156. $\lim\limits_{x\to 3}\frac{e^x-e^3}{x-3}$

7.157. $\lim\limits_{x\to a}\frac{e^{ax}-e^{a^2}}{x-a}$

7.158. $\lim\limits_{x\to b}\frac{a^x-a^b}{x-b}$

7.159. $\lim\limits_{x\to 0}\frac{(1+x)^{\frac{1}{3}}-1}{2x}$

7.160. $\lim\limits_{x\to 0}\frac{\ln(a+x)-\ln a}{x}$

7.161. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$

7.162. $\lim_{x\to\infty} x(a^{\frac{1}{x}}-1)$

7.163. $\lim_{x\to 0}\frac{e^{mx}-1}{nx}$

7.164. $\lim_{x\to 0}\frac{\ln(1+mx)}{nx}$

§ 7.5 Funktsiyaning ekstremumi va uni to'la tekshirish

Agar x_0 nuqtaning qandaydir ε atrofida, istalgan musbat $h<\varepsilon$ uchun $f(x_0-h)<f(x_0)<f(x_0+h)$ bo'lsa, $f(x)$ funksiya x_0 nuqtada o'suvchi deyiladi.

Agar [a,b] segmentdagi istalgan x1 va x2 uchun $x_1<x_2$ bo'lganda $f(x_1)<f(x_2)$ $[f(x_1)>f(x_2)]$ bo'lsa, $f(x)$ funksiya shu segmentda o'suvchi (kamayuvchi) deyiladi.

$y=f(x)$ **funksiyaning nuqatda yoki segmentda o'suvchi yoki kamayuvchi bo'lishining yetarli alomati:**

Agar biror oraliqda $y'>0$ bo'lsa, funksiya shu oraliqda o'suvchi bo'ladi; agar $y'<0$ bo'lsa, kamayuvchi bo'ladi.

Ekstremumning zaruriy shartlari. $y=f(x)$ funksiya faqat $y'=0$ bo'lgan nuqtalarda yoki bu hosila mavjud bo'lmagan yoki cheksiz bo'lgan nuqtalardagina ekstremumga ega bo'lishi mumkin. Bu nuqtalar kritik (statsionar) nuqtalar deb ataladi.

Ekstremumning yetarli shartlari. Agar $y=f(x)$ funksiya x_0 nuqtada uzluksiz bo'lsa va o'sha nuqta ixtiyoriy atrofida (x_0 nuqtaning o'zidan boshqa nuqtalarda) chekli hosilaga ega bo'lsa, va x_0 nuqtadan o'tishda y'hosila o'z ishorasini (+) dan (–) ga o'zgartirsa, u holda

$f(x_0)=y_{max}$ bo'ladi;

y' o'z ishorasini (–) dan (+) ga o'zgartirsa, u holda

$f(x_0)=y_{min}$ bo'ladi;

y' o'z ishorasini o'zgartirmasa, u holda funksiya ekstremumga ega bo'lmaydi.

Demak, funksiyaning ekstremumlarini topish uchun :

1) y' ni topib, uni nolga aylantiruvchi yoki u mavjud bo'lmagan kritik nuqtalarni topish kerak;

2) har bir kritik nuqtaning chap va o'ng tomonlarida y' ning ishorasini, masalan, ushbu

x		x_1		x_2		x_3		x_4	
y'	-	0	+	yo'q	-	0	-	$-\infty$	-
y	kamayadi	$\cup$ min	o'sadi	$\wedge$ max	Kamayadi	bukilish	kamayadi	bukilish	kamayadi

ko'rinishdagi jadval tuzib, aniqlash kerak.

So'ngra y_{max} va y_{min} larni topib, egri chiziqni (funksiya grafigini) yasash mumkin.

Ekstremum mavjudligining yetarli shartlari (tekshirishni ikkinchi tartibli hosila yordamida topish usuli). Agar biror $x = x_0$ nuqtada;

1) $y' = 0$ va $y'' < 0$ bo'lsa, u holda $f(x_0) = y_{max}$ bo'ladi;

2) $y' = 0$ va $y'' > 0$ bo'lsa, u holda $f(x_0) = y_{min}$ bo'ladi;

3) $y' = 0$ va $y'' = 0$ bo'lsa, u holda masala yechilmasdan qoladi va uni yechish uchun birinchi usulga murojaat qilish kerak.

Funksiyaning differensiali. Agar $y = f(x)$ funksiya x nuqtada differensiallanuvchi bo'lsa, ya'ni o'sha nuqtada chekli y' hosilaga ega bo'lsa, u holda

$$\Delta y = y'\Delta x + a\Delta x$$

funksiya orttirmasi Δy ning Δx ga nisbatan chiziqli bo'lgan bosh qismi $y'\Delta x$ ga funksiyaning differensiali deyiladi va dy bilan belgilanadi.

$$dy = y'\Delta x$$

(7.17) formulada $y = x$ deb $dx = x'\Delta x = 1 \cdot \Delta x = \Delta x$ ga ega bo'lamiz, shuning uchun ham

$$dy = y'dx \qquad (7.18)$$

Differensiallash qoidalari.

1.Agar $c = const$ bo'lsa, $d(c) = 0$ bo'ladi.

2. Agar $c = const$ bo'lsa, $d(c \cdot f(x)) = c \cdot d(f(x))$ bo'ladi.

3. $d(f(x) \pm g(x)) = d(f(x)) \pm d(g(x))$

4. $d(f(x)g(x)) = d(f(x))g(x) + f(x)d(g(x))$

5. Agar $g(x) \neq 0$ bo'lsa,

$d\left(\frac{f(x)}{g(x)}\right) = \frac{d(f(x))g(x) - f(x)d(g(x))}{g^2(x)}$ bo'ladi.

Quyidagi funksiyalarning ekstremumlari topilsin va grafiklari yasalsin:

7.165.*a*) $y=4x-x^2$ *в*) $y=\frac{1}{3}x^3-\frac{5}{2}x^2+6x$

7.166. $y=x^2+2x-3$

7.167. $y=\frac{x^3}{3}+x^2$

7.168. $y=x^3+6x^2+9x$

7.169. $y=\frac{x^2}{x-2}$

7.170. $y=x^3+\frac{x^4}{4}$

7.171. $y=\frac{x^4}{4}-2x^2$

7.172. $y=2x-3\sqrt[3]{x^2}$

7.173. $y=\frac{(x-1)^2}{x^2+1}$

7.174. $y=xe^{-\frac{x^2}{2}}$

7.175. $y=x^4-2x^2+5$ funksiya berilgan [-2;3] kesmada eng katta va eng kichik qiymatlari topilsin

Takrorlash uchun savollar

1. Funksiyaning nuqtadagi hosilasi qanday ta'riflanadi?

2. Hosilaning geometrik ma'nosi nimadan iborat?

3. Hosilaning mexanik ma'nosi nimadan iborat?

4. Differensiallanuvchi funksiyaning uzluksizligi haqida nima deyish mumkin?

5. Qachon funksiya oraliqda differensiallanuvchi deyiladi?

6. O'zgarmas sonning hosilasi nimaga teng?

7. Funksiyalar algebraik yig'indisini hosilasi qanday hisoblanadi?

8. Funksiyalar ko'paytmasini hosilasi qanday hisoblanadi?

9. Funksiyalar nisbatining hosilasi qanday hisoblanadi?

10. Hosila olishda o'zgarmas ko'paytuvchini nima qilish mumkin?

11. Murakkab funksiyaning hosilasi qanday topiladi?

12. Teskari funksiyaning hosilasi qanday topiladi?

FUNKSIYANING HOSILASIGGA DOIR NAZORAT TESTLARI

1. Funksiya hosilasi ta'rifi, uning mexanik va geometrik ma'nosi

1. $y=f(x)$ funksiyaning Δx argument orttirmasiga mos keladigan Δf funksiya orttirmasi qayerda to'g'ri ifodalangan ?

A) $\Delta f=f(x)-f(\Delta x)$. B) $\Delta f=f(x+\Delta x)-f(\Delta x)$. C) $\Delta f=f(x+\Delta x)-f(x)$.
D) $\Delta f=f(x+\Delta x)-f((x-\Delta x)$. E) $\Delta f=f(x)\Delta x$.

2. $y=1$ o'zgarmas funksiyaning Δy orttirmasi qayerda to'g'ri ko'rsatilgan ?

A) Δx. B) $-\Delta x$. C) 1. D) -1. E) 0 .

3. $y=x^3$ funksiyaning Δy orttirmasi qayerda to'g'ri ko'rsatilgan ? .

A) $3x^2\Delta x+(\Delta x)^3$. B) $3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3$.
C) $3x^2\Delta x+3x(\Delta x)^2$. D) $3x(x+3\Delta x)\Delta x$. E) $3x(x+\Delta x)(\Delta x)^2+(\Delta x)^3$.

4. $y=x^3$ funksiya uchun $\Delta y/\Delta x$ orttirmalar nisbatini toping .

A) $3x^2+(\Delta x)^2$. B) $3x(x+3\Delta x)$.
C) $3x^2+3x\Delta x$. D) $3x^2+3x\Delta x+(\Delta x)^2$. E) $3x\Delta x(x+\Delta x)+(\Delta x)^3$.

5. $y=f(x)$ funksiya hosilasini ta'rif bo'yicha hisoblashda quyidagi amallardan qaysi biri bajarilmaydi ?

A) x argumentga Δx orttirma beriladi.
B) funksiya orttirmasi Δf hisoblanadi.
C) orttirmalar nisbati $\Delta f/\Delta x$ hisoblanadi.
D) $\Delta f/\Delta x$ nisbatning $\Delta x\to 0$ bo'lgandagi limiti hisoblanadi.
E) ko'rsatilgan amallarning barchasi bajariladi.

6. $y=f(x)$ funksiya hosilasining ta'rifi qayerda to'g'ri ko'rsatilgan ?

A) $f'(x)=\lim\limits_{\Delta x\to\infty}\frac{\Delta f}{\Delta x}$. B) $f'(x)=\lim\limits_{\Delta x\to 0}\frac{\Delta x}{\Delta f}$.

C) $f'(x)=\lim\limits_{\Delta x\to 0}\frac{\Delta f}{\Delta x}$. D) $f'(x)=\lim\limits_{\Delta x\to\infty}\frac{\Delta x}{\Delta f}$. E) $f'(x)=\lim\limits_{\Delta f\to 0}\frac{\Delta x}{\Delta f}$.

7. Hosilaning mexanik ma'nosi qayerda to'g'ri ko'rsatilgan ?

A) harakatda bosib o'tilgan masofa.

B) harakatda sarflangan vaqt.

C) harakatda oniy tezlik.

D) harakatda to'xtash holati.

E) harakatni boshlash holati.

8. Hosilaning geometrik ma'nosi qayerla to'g'ri ko'rsatilgan ?

A) chiziqqa o'tkazilgan kesuvchining burchak koeffitsenti.

B) chiziqqa o'tkazilgan urinmaning burchak koeffitsenti.

C) chiziqqa o'tkazilgan normalning burchak koeffitsenti.

D) chiziqqa o'tkazilgan urinma va OX o'q orasidagi burchak tangensi.

E) chiziqqa o'tkazilgan urinma va OY o'q orasidagi burchak tangensi.

9. Quyidagilardan qaysi biri $y=f(x)$ funksiya grafigiga (x_0,y_0) nuqtada o'tkazilgan urinma tenglamasini ifodalaydi ?

A) $y-y_0=f'(x_0)(x-x_0)$. B) $y-y_0=f'(x_0)x$. C) $y-y_0=f'(x_0)x_0$.

D) $x-x_0=f'(x_0)(y-y_0)$. E) $y-y_0=\frac{1}{f'(x_0)}(x-x_0)$.

10. Quyidagilardan qaysi biri $y=f(x)$ funksiya grafigiga (x_0,y_0) nuqtada o'tkazilgan urinma tenglamasini ifodalamaydi ?

A) $y-y_0=f'(x_0)(x-x_0)$. B) $\frac{y-y_0}{x-x_0}=f'(x_0)$.

C) $y=y_0+f'(x_0)(x-x_0)$. D) $x-x_0=f'(x_0)(y-y_0)$.

E) $\frac{x-x_0}{y-y_0}=\frac{1}{f'(x_0)}$.

2. Funksiyani differensiallash qoidalari. Hosilalar jadvali

1. Differensiallash qoidasi qayerda xato ko'rsatilgan ?

A) $(Cu)'=Cu'$ (C-const.). B) $(u\pm v)'=u'\pm v'$.

C) $(u\cdot v)'=u'v+uv'$. D) $\left(\frac{u}{v}\right)'=\frac{u'v+uv'}{v^2}$.

E) $(f(u))'=f'(u)u'$.

2. Diffrentsiallanuvchi u va v funksiyalar u/v nisbatining hosilasini hisoblash formulasi to'g'ri yozilgan javobni ko'rsating.

A) $\frac{u'v+uv'}{v}$. B) $\frac{u'v+uv'}{v^2}$. C) $\frac{u'v-uv'}{v^2}$.

D) $\frac{u'v-uv'}{v}$ E) $\frac{u'v'-uv}{v^2}$.

3. $y=x^2/\sin x$ funksiyaning y' hosilasini hisoblang.

A) $y'=x^2/\cos x$. B) $y'=2x/\sin x$. C) $y'=2x/\cos x$.

D) $y'=x(2\sin x+x\cos x)/\sin^2 x$. E) $y'=x(2\sin x-x\cos x)/\sin^2 x$.

4. Diffrentsiallanuvchi u va v funksiyalar $u\cdot v$ ko'paytmasining hosilasini hisoblash formulasi qayerda to'g'ri yozilgan ?

A) $u'v'$. B) $u'v'+uv$. C) $u'v+uv'$.

D) $u'v-uv'$. E) $u'v'-uv$.

5. $y=x^2\sin x$ funksiyaning y' hosilasini hisoblang.

A) $y'=x^2\cos x$. B) $y'=x(x\sin x-2\cos x)$.

C) $y'=2x\sin x$. D) $y'=x(x\cos x+2\sin x)$.

E) $y'=x(x\cos x-2\sin x)$.

6. Agar $y=f(x)$, $u=u(x)$ differensiallanuvchi funksiyalar bo'lsa, $y=f(u)$ murakkab funksiya hosilasini hisoblash formulasini ko'rsating.

A) $y'=f'(u)$. B) $y'=f(u')$. C) $y'=f'(u)$.

D) $y'=f'(u)u'$. E) $y'=f(u')u'$.

7. $f(x)=\sin x$, $u(x)=\ln x$ funksiyalar bo'yicha tuzilgan $y=f(u)=\sin\ln x$ murakkab funksiya hosilasini hisoblang.

A) $y'=\cos\ln x$. B) $y'=\sin(1/x)$.

C) $y'=(\sin\ln x)/x$. D) $y'=(\cos\ln x)/\ln x$.

E) $y'=(\cos\ln x)/x$.

8. $y=\sin\arcsin x$ $(-1 \leq x \leq 1)$ murakkab funksiya hosilasini hisoblang.

A) $y'=\cos\arcsin x$. B) $y'=1$. C) $y'=\sin\arccos x$.

D) $y'=\cos\arccos x$. E) $y'=x$.

9. $y=\cos(x^2+1)$ funksiyaning y' hosilasini hisoblang.

A) $y'=\sin(x^2+1)$. B) $y'=-\sin(x^2+1)$.

C) $y' = \sin 2x$. D) $y'=2x\sin(x^2+1)$.

E) $y'=-2x\sin(x^2+1)$.

10. $y=e^{x^4}$ funksiyaning hosilasi to'g'ri yozilgan javobni toping.

A) $x^4 \cdot e^{x^4-1}$. B) $4x^3 \cdot e^{x^4}$. C) $4x^3 \cdot e^{x^4} \cdot \lg e$.

D) $4x^3 \cdot e^{x^4-1} \cdot \lg e$. E) e^{x^4}.

3.Differensiallanuvchi funksiyalar haqidagi asosiy teoremalar

1.Roll teoremasida $y=f(x)$ funksiyaga qaysi shart qo'yilmaydi ?

A) Biror $[a,b]$ kesmada aniqlangan.

B) $[a,b]$ kesmada uzluksiz.

C) $[a,b]$ kesma ichida differensiallanuvchi.

D) $[a,b]$ kesma chegaralarida $f(a)=f(b)$.

E) Keltirilgan barcha shartlar qo'yiladi .

2.Roll teoremasining tasdig'ini ko'rsating: Agar $y=f(x)$ funksiya $[a,b]$ kesmada uzluksiz va uning ichki nuqtalarida differensiallanuvchi hamda chegaraviy nuqtalarda $f(a)=f(b)$ bo'lsa, u holda bu kesma ichida kamida bitta shunday $x=c$ nuqta topiladiki, unda ..... bo'ladi.

A) $f'(c)>0$. B) $f'(c)<0$. C) $f'(c)=0$.

D) $f'(c)\neq 0$. E) $f'(c)=f(c)$.

3.$f(x)=x\sin x$ funksiya uchun Roll teoremasi qaysi kesmada o'rinli bo'ladi?

A) $[0, 1]$. B) $[0, \pi/2]$. C) $[0, \pi]$.

D) $[2, \pi]$. E) $[\pi/2, \pi]$.

4.$f(x)=x^2-7x+9$ funksiya uchun Roll teoremasining shartlari bajariladigan $[a,b]$ kesmalarning umumiy ko'rinishini aniqlang.

A) $[1-\alpha, 1+\alpha]$, $\alpha>0$. B) $[2-\alpha, 2+\alpha]$, $\alpha>0$.

C) [3–α, 3+α], $\alpha>0$. D) [4–α, 4+α], $\alpha>0$.

E) bunday kesmalar mavjud emas .

5. $f(x)=x^2-7x+9$, $x\in[1,5]$, funksiya uchun Roll teoremasining tasdig'i o'rinli bo'ladigan c nuqtani toping.

A) c=1.5 . B) c=2 . C) c=2.5 . D) c=3 . E) c=4.5 .

6. $f(x)=\sqrt[3]{8x-x^2}$ funksiya uchun [0,8] kesmada Roll teoremasining tasdig'i o'rinli bo'ladigan c nuqtani toping.

A) c=2 . B) c=3 . C) c=4 . D) c=5 . E) c=7 .

7. Lagranj teoremasida $y=f(x)$ funksiyadan qaysi shart talab etilmaydi ?

A) Biror $[a,b]$ kesmada aniqlangan .

B) $[a,b]$ kesmada uzluksiz .

C) $[a,b]$ kesma ichida differensiallanuvchi .

D) $[a,b]$ kesma chegaralarida $f(a)=f(b)$.

E)Keltirilgan barcha shartlar talab etiladi .

8. Lagranj teoremasining tasdig'ini ko'rsating: Agar $y=f(x)$ funksiya $[a,b]$ kesmada uzluksiz va uning ichki nuqtalarida differensiallanuvchi bo'lsa, u holda bu kesma ichida kamida bitta shunday $x=c$ nuqta topiladiki, unda ..... tenglik o'rinli bo'ladi.

A) $f(b)+f(a)=f'(c)(b+a)$. B) $f(b)-f(a)=f'(c)(b-a)$.

C) $f(b)-f(a)=f'(c)(b+a)$. D) $f(b)+f(a)=f'(c)(b-a)$.

E) $f(b)-f(a)=f'(c)$.

9. $f(x)=x\cos x$ funksiya uchun Lagranj teoremasi o'rinli bo'lmaydigan kesmani aniqlang .

A) $[\pi/2,\pi]$. B) $[0,\pi/2]$. C) $[0,\pi]$. D) $[\pi,2\pi]$.

E) Barcha kesmalarda Lagranj teoremasi o'rinli bo'ladi .

10. $f(x)=x^2-2x+9$ funksiya uchun $[a,b]$ kesmada Lagranj teoremasining tasdig'i o'rinli bo'ladigan c nuqtani toping.

A) $c=a+b/2$.

B) $c=b-a/2$.

C) $c=(a+b)/2$.

D) $c=(b-a)/2$.

E) umumшy holda c nuqtani aniqlab bo'lmaydi.

4.Funksiya differensiali. Yuqori tartibli hosila va differensiallar

1. Agar funksiya orttirmasi $\Delta f=A\Delta x+\alpha(\Delta x)\Delta x$ ($\Delta x\to 0$ => $\alpha(\Delta x)\to 0$) ko'rinishda bo'lsa, uning df differensiali qanday aniqlanadi ?

A) $df=\Delta x$. B) $df=A+\Delta x$. C) $df=A\Delta x$. D) $df=A-\Delta x$. E) $df=\alpha(\Delta x)\Delta x$.

2. Agar $y=f(x)$ funksiya hosilasi $f'(x)$ mavjud va chekli bo'lsa, quyidagi hollardan qaysi birida df differensial mavjud bo'ladi ?

A) $f'(x)>0$. B) $f'(x)<0$. C) $f'(x)=0$. D) $f'(x)\neq 0$.

E) barcha hollarda df differensial mavjud bo'ladi.

3. Differensiallanuvchi $y=f(x)$ funksiyaning $f'(x)$ hosilasi df differensial orqali qanday ifodalanadi ?

A) $f'(x)=df$. B) $f'(x)=\frac{df}{dx}$. C) $f'(x)=\frac{dx}{df}$.

D) $f'(x)=df\cdot dx$. E) $f'(x)=df+dx$.

4. Agar $y=f(x)$ funksiyaning hosilasi $f'(x)$ mavjud va chekli bo'lsa, uning df

differensiali qanday topiladi ?

A) $df=f'(x)+dx$. B) $df=f'(x)-dx$ C) $df=f'(x)/dx$.

D) $df=f'(x)dx$. E) $df=f'(x)$.

5. Differensiallanuvchi $y=f(x)$ funksiya argumentining orttirmasi Δx kichik bo'lganda uning differensiali df va orttirmasi Δf orasida doimo qaysi munosobat o'rinli bo'ladi?

A) $\Delta f=df$. B) $\Delta f>df$. C) $\Delta f<df$. D) $\Delta f\cdot df>0$. E) $\Delta f\approx df$

6. Funksiyani differensiallash qoidasi qayerda noto'g'ri ko'rsatilgan ?

A) $dCf = Cdf$. B) $d(u+v) = du + dv$. C) $d(u-v) = du - dv$.

D) $d(uv) = udv - vdu$. E) $df(u) = f'(u)du$.

7. $y=\cos(3x+4)$ funksiya differensiali dy qayerda to‘g‘ri ko‘rsatilgan ?

A) $dy= \sin(3x+4)dx$.

B) $dy= 4\sin(3x+4)dx$.

C) $dy= -3\sin(3x+4)dx$.

D) $dy= -4\sin(3x+4)dx$.

E) $dy= 3\sin(3x+4)dx$.

8. $y=x\ln x$ funksiyaning dy differensialini toping .

A) $dy= xdx$. B) $dy= \ln xdx$. C) $dy=(1/x)dx$.

D) $dy= (1+\ln x)dx$. E) $dy=(1-\ln x)dx$.

9. Funksiyani differensial yordamida taqribiy hisoblash formulasi qayerda

to‘g‘ri ifodalangan ?

A) $f(x+\Delta x)\approx f(x)df$. B) $f(x+\Delta x)\approx f(x)+df$.

C) $f(x+\Delta x)\approx f(x)/df$. D) $f(x+\Delta x)\approx df / f(x)$.

E) $f(x+\Delta x)\approx f(x)\pm df$.

10. Qaysi funksiyaning n-tartibli hosilasi noto‘g‘ri yozilgan ?

A) $(e^x)^{(n)} = e^x$. B) $(a^x)^{(n)} = a^x(\ln a)^n$. C) $(x^n)^{(n)} = n!$.

D) $\sin^{(n)} x = \sin\left(x + \frac{\pi n}{2}\right)$. E) barcha hosilalar to‘g‘ri yozilgan.

5.Funksiyani I tartibli hosila yordamida tekshirish

1. Differensiallanuvchi $y=f(x)$ funksiyaning kamayish sohasi uning $f'(x)$ hosilasi yordamida qanday munosabatdan topiladi?

A) $f'(x)=0$. B) $f'(x)\neq 0$. C) $f'(x)>0$.

D) $f'(x)<0$. E) $f'(x)<\infty$.

2. $f(x)=x^3-3x$ funksiyaning kamayish oralig‘ini toping.

A) $(-1, 1)$. B) $(1, \infty)$. C) $(-1, \infty)$.

D) $(-\infty, -1)$. E) $(-\infty, 1)$.

3. $y=x\ln x$ funksiyaning kamayish sohasi qayerda to'g'ri ko'rsatilgan ?

A) $(0, e)$. B) $(-\infty, 1/e)$. C) $(0, \infty)$. D) $(0, 1/e)$. E) $(1/e, \infty)$.

4. Differensiallanuvchi $y=f(x)$ funksiyaning o'sish sohasi uning $f'(x)$ hosilasi

yordamida qanday munosabatdan topiladi?

A) $f'(x)=0$. B) $f'(x)\neq 0$. C) $f'(x)>0$. D) $f'(x)<0$. E) $f'(x)<\infty$.

5. $f(x)=xe^x$ funksiyaning o'sish oralig'ini toping.

A) $(-1, 1)$. B) $(1, \infty)$. C) $(-1, \infty)$.

D) $(-\infty, -1)$. E) $(-\infty, 1)$.

6. $y=x\ln x$ funksiya o'sish sohasi qayerda to'g'ri ko'rsatilgan ?

A) $(0, e)$. B) $(-\infty, 1/e)$. C) $(0, \infty)$.

D) $(0, 1/e)$. E) $(1/e, \infty)$.

7. $y=f(x)$ funksiya x_0 nuqta atrofida aniqlangan bo'lib, unda lokal maksimumga ega. Agar argument orttirmasi Δx yetarlicha kichik bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli bo'lmaydi ?

A) $f(x_0+\Delta x)<f(x_0)$.

B) $f(x_0-\Delta x)<f(x_0)$.

C) $\Delta f<0$.

D) $f(x_0+\Delta x)+f(x_0-\Delta x)<2f(x_0)$.

E) barcha tasdiqlar o'rinli bo'ladi.

8. $y=f(x)$ funksiya x_0 nuqta atrofida aniqlangan bo'lib, unda lokal minimumga ega. Agar argument orttirmasi Δx yetarlicha kichik bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli bo'lmaydi ?

A) $f(x_0+\Delta x)>f(x_0)$. B) $f(x_0-\Delta x)>f(x_0)$. C) $\Delta f>0$.

D) $f(x_0+\Delta x)+f(x_0-\Delta x)>2f(x_0)$.

E) barcha tasdiqlar o'rinli bo'ladi.

9. Differensiallanuvchi $y=f(x)$ funksiya x_0 nuqtada lokal ekstremumga ega bo'lsa, quyidagi shartlardan qaysi biri o'rinli bo'ladi ?

A) $f'(x_0)>0$. B) $f'(x_0)<0$. C) $f'(x_0)=0$.

D) $f'(x_0)\neq 0$. E) $f'(x_0)$ mavjud emas.

10. $f(x)=x^3-3x$ funksiyaning kritik nuqtalarini toping.

A) ± 1 . B) 0 va 1 . C) -1 va 0 . D) 2 va 3 . E) ± 2 .

Qiyin matematik masalani yechishni -
mustahkam qal'ani zabt etishga taqqoshlash mumkin.
Vilenkin

VIII-BOB. ANIQMAS INTEGRAL

§ 8.1 Aniqmas integral. Bevosita integrallash. Yoyish usuli yordamida integrallash.

§ 8.2.O'zgaruvchini almashtirish usuli bilan integrallash. Bo'laklab integrallash. Ratsional kasrlarni integrallash

§ 8.3. Ba'zi bir irratsional funksiyalardan tashkil topgan va trigonometrik funksiyalardan tashkil topgan ifodalarni integrallash.

§ 8.1 Aniqmas integral. Bevosita integrallash. Yoyish usuli yordamida integrallash.

1. Agar [a,b] kesmaning istalgan ichki nuqtasida $F(x)$ funksiyaning hosilasi $f(x)$ ga teng bo'lsa, bu $F(x)$ funksiya $f(x)$ uchun boshlang'ich funksiya deyiladi.

$$F'(x)=f(x)\Rightarrow dF(x)=f(x)dx, \quad x\in[a,b].$$

Boshlang'ich funksiyani uning hosilasi $f(x)$ yoki differensiali $f(x)dx$ bo'yicha izlash differensiallashga teskari amaldir, bu amal **integrallash** deyiladi. $f(x)$ funksiya yoki $f(x)dx$ differensialning boshlang'ich funksiyalari to'plami aniqmas integral deyiladi va $\int f(x)dx$ simvol bilan belgilanadi. Shunday qilib, agar $d[F(x)+C]=f(x)dx$ bo'lsa,

$$\int f(x)dx=F(x)+C \tag{8.1}$$

bo'ladi.

Bu yerda $f(x)$ integral ostidagi funksiya; $f(x)dx$ - integral ostidagi ifoda; C – integrallash o'zgarmasi, x-integrallash o'zgaruvchisi, $\int$ -integral belgisi.

2. Aniqmas integralning asosiy xossalari:

a) funksiya differensialining aniqmas integrali shu funksiya bilan ixtiyoriy o'zgarmasning yig'indisiga teng:

$$\int dF(x) = F(x) + C$$

b) Aniqmas integralning differensiali integral ostidagi ifodaga, aniqmas integralning hosilasi esa integral ostidagi funksiyaga teng:

$$d\left(\int f(x)dx\right) = f(x)dx, \quad \left(\int f(x)dx\right)' = f(x) \qquad (8.2)$$

c) Funksiyalar algebraik yig'indisining (ayirmasining) aniqmas integrali bu funksiyalar aniqmas integrallarining yig'indisiga (ayirmasiga) teng:

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx \qquad (8.3)$$

d) o'zgarmas ko'paytuvchi aniqmas integral belgisidan tashqariga chiqarish mumkin $(a = const)$:

$$\int af(x)dx = a\int f(x)dx, \quad (a = const) \qquad (8.4)$$

e) agar $\int f(x)dx = F(x) + C$ bo'lib, $u = \varphi(x)$ uzluksiz hosilaga ega bo'lgan istalgan ma'lum funksiya bo'lsa,

$$\int f(u)du = F(u) + C \qquad (8.5)$$

bo'ladi.

3. Integrallashning asosiy formulalari.

$$\int 0dx = C \qquad (8.6)$$

$$\int dx = x + C \qquad (8.7)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; \quad (n \neq -1) \qquad (8.8)$$

$$\int \frac{dx}{x} = \ln|x| + C \qquad (8.9)$$

$$\int a^x dx = \frac{a^x}{\ln a} + C; \quad a > 0; \quad a \neq 1 \qquad (8.10)$$

$$\int e^x dx = e^x + C \qquad (8.11)$$

$$\int \sin x dx = -\cos x + C \qquad (8.12)$$

$$\int \cos x dx = \sin x + C \qquad (8.13)$$

$$\int \frac{dx}{\cos^2 x} = tgx + C \qquad (8.14)$$

$$\int \frac{dx}{\sin^2 x} = -ctgx + C \qquad (8.15)$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C; \quad -a < x < a; \quad a > 0 \qquad (8.16)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \qquad (8.17)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C \qquad (8.18)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctg\frac{x}{a} + C \qquad (8.19)$$

$$\int \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| + C \qquad (8.20)$$

$$\int \frac{dx}{\cos x} = \ln\left|tg\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C \qquad (8.21)$$

$$\int shxdx = chx + C \qquad (8.22)$$

$$\int \frac{dx}{ch^2 x} = thx + C \qquad (8.23)$$

$$\int \frac{dx}{sh^2 x} = -cthx + C \qquad (8.24)$$

Yoyish yo`li bilan integrallash berilgan integralni sodda integrallarning yigindisiga keltirishdan iboratdir.

4.Bevosita integrallash. Bevosita integrallash jadval integrallaridan bevosita foydalanishga asoslangandir. Bu yerda quyidagi hollar ro'y berishi mumkin:

a) berilgan integral tegishli jadval integrali yordamida topiladi;

b) berilgan integralga (8.3) va (8.4) xossalarni qo'llanilgandan so'ng bir yoki bir necha jadval integraliga keltiriladi;

v) berilgan integral integral osti funksiya ustida elementar shakl almashtirishdan so'ng (8.3) va (8.4) xossalar qo'llanilgandan bir yoki bir nechta jadval integraliga keltiriladi.

Ushbu aniqmas integrallarni topib, to'g'riligini differensiallash yordamida tekshiring.

8.1. $\int\left(3x^4 + \frac{2}{x^3} - \sqrt[4]{x^3}\right)dx$ 8.2. $\int\left(5x^5 + 2\sqrt[3]{x^2} - \frac{8}{x^3}\right)$

8.3. $\int\left(4x^2+3\sqrt{x}-\frac{3}{\sqrt{x}}\right)dx$

8.4. $\int\left(3x^2+4\sqrt{x}-\frac{4}{\sqrt[3]{x}}\right)dx$

8.5. $\int\left(7x^6-3\sqrt[3]{x}+\frac{5}{\sqrt[3]{x^2}}\right)dx$

8.6. $\int\left(1-5\sqrt[4]{x^3}+\frac{3}{\sqrt[6]{x}}\right)dx$

8.7. $\int\left(\sqrt[5]{x^2}+\frac{2x^3}{3}+\frac{1}{x^3}\right)dx$

8.8. $\int\left(1+2\sqrt[4]{x}-\frac{3x^4}{5}\right)dx$

8.9. $\int\left(\sqrt[3]{x}-\frac{2}{7}x^6+\frac{8}{x^4}\right)dx$

8.10. $\int\left(2\sqrt[6]{x}-\frac{5x^6}{7}+\frac{3}{\sqrt[6]{x}}\right)dx$

Aniqmas integrallarni toping.

8.11 $\sqrt{\cos x}\sin xdx$

8.12 $\int(\ln x)^3\frac{dx}{x}$

8.13 $\int\frac{arctgx}{1+x^2}dx$

8.14 $\int\frac{\cos x}{\sqrt[3]{\sin x}}dx$

8.15 $\int e^{-x^2}xdx$

8.16 $\int\frac{x}{2+x^4}dx$

8.17 $\int\sqrt{\ln x}\cdot\frac{dx}{x}$

8.18 $\int\frac{x}{\sqrt{1-2x^2}}dx$

8.19 $\int\frac{x}{2x^4+5}dx$

8.20 $\int\frac{dx}{x\ln x}$

8.21 $\int\frac{\sin x}{\cos^2 x}dx$

8.22 $\int\frac{x^2}{2x^3+3}dx$

8.23 $\int\sqrt{5x^4+3}x^3dx$

8.24 $\int x^2e^{x^3+1}dx$

8.25 $\int\frac{x^3}{\sqrt{8x^4-1}}dx$

8.26 $\int\frac{x}{2x^2+3}dx$

8.27 $\int\arcsin^2 x\frac{dx}{\sqrt{1-x^2}}$

8.28 $\int\frac{\sqrt{arcrgx}}{1+x^2}dx$

8.29 $\int\frac{\ln x+3}{x}dx$

8.30 $\int\sqrt{1+2x^2}xdx$

Integrallarni toping.

8.31 *a*) $\int\frac{dx}{\sqrt{25-9x^2}}$; *в*) $\int\frac{dx}{\sqrt{x+4}+\sqrt{x-1}}$;

c) $\int\frac{(2x^2-3)dx}{x^2(x^2-3)}$ *d*) $\int\frac{dx}{\sqrt[3]{x^2}\cos^2\sqrt[3]{x}}$; *e*) $\int\frac{x^4dx}{x^2+2}$

8.32 *a*) $\int\frac{x^3-5x^2+1}{\sqrt{x}}dx$ *b*) $\int\left(\sqrt{t^3}+\frac{1}{\sqrt[3]{t^2}}\right)dt$

8.33 $\int \sqrt{x^3}\left(5\sqrt[3]{x^2}-1\right)dx$ 8.34. $\int \frac{d\varphi}{\sqrt{16-\varphi^2}}$

8.35 $\int\left(y\sqrt{y}-\frac{1}{\sqrt{y}}+\frac{1}{(y+2)^2}\right)dy$ 8.36. $\int \frac{x-16}{\sqrt{x}+4}dx$

8.37. $\int \frac{dx}{\sqrt{9-4x^2}}$ 8.38. $\int \frac{dx}{\sqrt{x^2+5}}$

8.39. $\int \frac{dx}{16x^2-9}$ 840. $\int \frac{2^{x-1}+3^{x+1}}{6^x}dx$

8.41. $\int 5^{x-1}e^x dx$ 8.42. $\int (3^x-2)(3^{-x}+2)dx$

8.43. $\int \frac{dx}{\sqrt{x+2}-\sqrt{x-1}}$ 8.44. $\int \frac{\sqrt{2-x^2}+\sqrt{2+x^2}}{\sqrt{4-x^4}}dx$

8.45. $\int \frac{\sqrt{x^2+4}-6}{x^2+4}dx$ 8.46. $\int \frac{x^4dx}{x^2+1}$

8.47. $\int \frac{x^3-1}{x+1}dx$ 8.48. $\int \sin^2\frac{x}{2}dx$

8.49. $\int ctg^2 x dx$ 8.50. $\int cth^2 x dx$

8.51. $\int \frac{5-\cos x}{\cos x}dx$ 8.52. $\int \frac{e^{2x}-4}{e^x+2}dx$

8.53. $\int \frac{dx}{\sin^2 x+\cos 2x}$ 8.54. $\int \frac{1-5x^2}{x^2(1-x^2)}dx$

8.55. $\int \frac{8-\sqrt{x}}{\sqrt[3]{x}+2\sqrt[6]{x}+4}dx$ 8.56. $\int e^{-x}(e^x+5)dx$

8.57. $\int (\sqrt{x}-1)(x+\sqrt{x}+1)dx$ 8.58. $\int (3ctgx-1)(3tgx+1)dx$

8.59. $\int \frac{1+2\cos^2 x}{1+\cos 2x}dx$ 8.60. $\int \sqrt{x\sqrt{x\sqrt{x}}}\,dx$

§ 8.2.O'zgaruvchini almashtirish usuli bilan integrallash. Bo'laklab integrallash. Ratsional kasrlarni integrallash

O'zgaruvchini almashtirish (o'rniga qo'yish) usuli bilan integrallashning mohiyati shundan iboratki, $\int f(x)dx$ integralni asosiy integrallash formulalarining birortasi oson integrallanadigan $\int F(u)du$ integralga keltirishdan iboratdir. Faraz qilamiz, $x=\varphi(u)$ o'rganiladigan oraliqda uzluksiz, differensiallanuvchi funksiya bo'lsin, u holda

$$\int f(x)dx = \int f(\varphi(u))\varphi'(u)du = \int F(u)du \qquad (8.25)$$

Yangi o'zgaruvchi u ga nisbatan integral topilgandan so'ng $u=\psi(x)$ o'rniga qo'yish yordamida uni x o'zgaruvchiga keltiriladi.

1-misol. Ushbu

$$\int_0^1 \sqrt{1-x^2}\,dx$$

Integral hisoblansin.

Berigan integralda $x=\sin t$ almashtirishni bajaramiz.Unda

$$\int_0^1 \sqrt{1-x^2}\,dx=\int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\cos t dt=\int_0^{\frac{\pi}{2}} \cos^2 t dt=$$

$$=\int_0^{\frac{\pi}{2}} (\frac{1}{2}+\frac{1}{2}\cos 2t)dt=(\frac{1}{2}t+\frac{1}{4}\sin 2t)\Big|_0^{\frac{\pi}{2}}=\frac{\pi}{4}$$

bo'ladi.

Bo'laklab integrallash. $d(u\vartheta)=ud\vartheta+\vartheta du$ tenglikning ikkala tomonini integrallab, quyidagini hosil qilamiz:

$$\int d(u\vartheta)=\int ud\vartheta+\int \vartheta du;\quad u\vartheta=\int ud\vartheta+\int \vartheta du$$

bu yerdan

$$\int ud\vartheta=u\vartheta-\int \vartheta du \qquad (8.26)$$

2-misol.Ushbu

$$\int x\ln x dx$$

Integral hisoblansin.

Bu intervalda $u(x)=\ln x, dv(x)=x$ deb $du(x)=\frac{1}{x}dx, v(x)=\frac{x^2}{2}$ bo'lishini topamiz. Unda (8.26) formulaga ko'ra:

$$\int x\ln x dx=(\frac{x^2}{2}\ln x)-\int \frac{x^2}{2}\cdot\frac{1}{x}dx=\frac{x^2}{2}\ln x-\frac{1}{2}\int xdx=\frac{x^2}{2}\ln x-\frac{x^2}{4}+c$$ bo'ladi.

Ratsional kasrlarni integrallash. Rastsional kasr deb $P_n(x)/Q_m(x)$ ko'rinishidagi kasrga aytiladi, bu yerda $P_n(x)$ va $Q_m(x)$ mos ravishda n va m darajali ko'phadlar. Agar $n<m$ bo'lsa, ratsional kasr to'g'ri , $n\geq m$ bo'lsa noto'g'ri kasr deyiladi. Har qanday noto'g'ri ratsional kasrni maxrajga suratni bo'lish orqali ko'phad to'g'ri rasional kasr yig'indisi ko'rinishida tasvirlash mumkin. Shuning uchun ratsional kasrlarni integrallash to'g'ri

ratsional kasrlarni integrallashga keltiriladi. To'g'ri ratsional kasrni integrallash uchun uni eng sodda ratsionallar yig'indisi ko'rinishida

$$\frac{P(x)}{Q(x)}=\frac{A_{11}}{(x-\alpha_1)}+\frac{A_{12}}{(x-\alpha_1)^2}+...+\frac{A_{1k}}{(x-\alpha_1)^k}+...+\frac{A_{e1}}{x-\alpha_1}+...+\frac{A_{ek}}{(x-\alpha_1)^k}$$

bu yerda α va A_{11}, A_{12},...A_{ek} -o'zgarmas haqiqiy sonlar; k - butun musbat sonlar

Eng sodda ratsional kasrlardan tashkil topgan integrallarni hisoblash.

1)Birinchi turdagi eng sodda ratsional kasrlar:

$$\int\frac{A}{x-a}dx=A\ln|x-a|+C$$

2)Ikkinchi turdagi eng sodda ratsional kasrlar:

$$\int\frac{A}{(x-a)^n}dx=-\frac{A}{n-1}\frac{1}{(x-a)^{n-1}}+C\ n\in N;n\geq 2$$

3)Uchinchi turdagi eng sodda ratsional kasrlar:

$$\int\frac{Ax+B}{x^2+px+q}dx=\int\frac{At}{t^2+a^2}dt+\int\frac{B-\frac{Ap}{2}}{t^2+a^2}dt;\quad p^2-4q<0;\quad t=x+\frac{p}{2};\quad a^2=q-\left(\frac{p}{2}\right)^2>0$$

4) To'rtinchi turdagi eng sodda ratsional kasrlar:

$$\int\frac{Ax+B}{(x^2+px+q)^n}dx=A\int\frac{tdt}{(t^2+a^2)^n}+\left(B-\frac{Ap}{2}\right)\int\frac{dt}{(t^2+a^2)^n};$$

$$n\in N;\quad n\geq 2;\quad p^2-4q<0;\quad t=x+\frac{p}{2};\quad a^2=q-\left(\frac{p}{2}\right)^2>0$$

5) Beshinchi turdagi eng sodda ratsional kasrlar:

$$\int\frac{tdt}{(t^2+a^2)^n}=\frac{1}{2(n-1)(t^2+a^2)^{n-1}}+C;\quad n\geq 2$$

6) Oltinchi turdagi eng sodda ratsional kasrlar:

$$\int\frac{dt}{(t^2+a^2)^n}=\frac{1}{2a^2(n-1)(t^2+a^2)^{n-1}}+\frac{1}{a^2}\left(1-\frac{1}{2(n-1)}\right)\int\frac{dt}{(t^2+a^2)^{n-1}};\quad n\geq 2$$

3-misol.Ushbu

$$\int\sin^3x\cos^4xdx$$

integral hisoblansin.

Ayrim hollarda $t=\cos x$, $t=\sin x, t=\operatorname{tg}x$ almashtirishlar qulay bo'ladi.

Aytaylik, $R(u,v)$ ratsional funksiya uchun

$$R(-u,v)=-R(u,v)$$

bo'lsin. Bu holda

$$\int R(\sin x,\cos x)\,dx=\int R_2(\sin^2 x,\cos x)\sin x\,dx=$$

$$=\left|\begin{matrix} t=\cos x \\ dt=-\sin x dx \end{matrix}\right|=-\int R_2(1-t^2,t)\,dt$$

bo'ladi.

Aytaylik, $R(u,v)$ ratsional funksiya uchun

$$R(u,-v)=-R(u,v)$$

bo'lsin. Bu holda

$$\int R(\sin x,\cos x)\,dx=\int R_3(\sin x,\cos^2 x)\cos x\,dx=$$

$$=\left|\begin{matrix} t=\sin x \\ dt=\cos x dx \end{matrix}\right|=\int R_3(t,1-t^2)\,dt$$

bo'ladi.

Aytaylik, $R(u,v)$ ratsional funksiya uchun

$$R(-u,-v)=R(u,v)$$

bo'lsin. Bu holda

$$\int R(\sin x,\cos x)\,dx=\int R_2(tgx,\cos^2 x)\,dx=$$

$$=\left|\begin{matrix} t=tgx \\ dx=\dfrac{1}{1+t^2}dt \end{matrix}\right|=\int R_2\left(t,\frac{1}{1+t^2}\right)\frac{1}{1+t^2}\,dt$$

bo'ladi.

Integral ostidagi funksiya uchun $R(-u,v)=-R(u,v)$ bo'ladi.

Shuning uchun $\cos x=t$ deyilsa,unda $-\sin x dx=dt$ bo'lib,

$$\int\sin^3 x\cos^4 x dx=\int(t^2-1)t^4 dt=\frac{t^7}{7}-\frac{t^5}{5}+C=\frac{1}{7}\cos^7 x-\frac{1}{5}\cos^5 x+C$$

bo'ladi.

O'zgaruvchilarni almashtirish usuli yordamida integrallarni toping.

8.61 a) $\int\frac{5x^4-6}{x^5-6x}dx$ в) $\int\frac{arctg^3 x}{1+x^2}dx$ c) $\int\frac{x^2dx}{\sqrt{1+x^3}}$

d) $\int\frac{arctg^3 x}{1+x^2}dx$ e) $\int\frac{\ln(1+\sqrt{x})}{x+\sqrt{x}}dx$

8.62. а) $\int\left(\sin\frac{\pi}{20}x-\cos\frac{x}{20}\right)dx$ в) $\int(11x-7)^7dx$

8.63. $\int\frac{d(arctgx)}{arctg^2x}$

8.64. $\int\frac{dx}{\sin^2x(1+ctgx)^{3/2}}$

8.65. $\int\sqrt{3x+5}dx$

8.66. $\int\frac{dx}{(2x-3)^2}$

8.67. $\int4^{3x+7}dx$

8.68. $\int\pi^{2x-1}dx$

8.69. $\int\frac{3ctg^2x-1}{\cos^2x}dx$

8.70. $\int\frac{dx}{x(9+\ln^2x)}$

8.71. $\int\frac{xdx}{\sqrt{x^2-5}}$

8.72. $\int2^{x^5+1}x^4dx$

8.73. $\int\frac{dx}{\sqrt{1-x^2}\arcsin x}$

8.74. $\int e^x\cos e^xdx$

8.75. $\int\sec^2(1+\ln^2x)d(1+\ln^2x)$

8.76. $\int\frac{\pi^{1/x^2}dx}{x^3}$

8.77. $\int\frac{2x^3+x}{x^4+x^2}dx$

8.78. $\int\frac{\sqrt{\ln z}}{z}dz$

8.79. $\int\frac{e^{3x}dx}{\sqrt{1+e^{6x}}}$

8.80. $\int\frac{5\cos x-ctgx}{\sin^2x}dx$

8.81. $\int x^{-2}e^{-1/x}dx$

8.82. $\int\frac{udu}{\cos^2u^2}$

8.83. $\int\frac{5^{\sqrt{x}}}{\sqrt{x}}dx$

8.84. $\int\frac{dx}{25\cos^2x+9\sin^2x}$

8.85. $\int5^{\sin^2x}\sin2xdx$

8.86. $\int\frac{dx}{\sqrt{1-x^2}\sqrt{\arccos^2x-4}}$

8.87. $\int\frac{\sin xdx}{\cos^2x\sqrt{\cos x}}$

8.88. $\int\frac{dx}{\sqrt[3]{tg^5x\sin^2x}}$

8.89. $\int\frac{dx}{(x^2+1)\sqrt{arctgx+1}}$

8.90. $\int e^{5\sin x-2}\cos xdx$

8.91. $\int\frac{3^xdx}{7+3^{2x}}$

8.92. $\int\frac{dx}{e^{2x}+5}$

8.93. $\int\frac{(x-1)dx}{\sqrt{2x-x^2}}$

8.94. $\int\sqrt[3]{2-3\sin2x}\cos2xdx$

8.95. $\int\frac{e^{5x}dx}{(e^{5x}+11)^2}$

8.96. $\int\frac{x^2dx}{\sin^2x^3}$

8.97. $\int\frac{dx}{x\sqrt{9-\ln^22x}}$

8.98. $\int\frac{t^2dt}{t^6-1}$

8.99. $\int\frac{\sin2xdx}{1+\cos^22x}$

8.100. $\int\frac{xdx}{16+x^4}$

8.101. $\int \frac{xdx}{1-x^4}$

8.102. $\int \frac{dx}{\sqrt{x}(1+x)}$

8.103. $\int \cos^2\left(\sin\frac{x}{2}\right)\cos\frac{x}{2}dx$

8.104. $\int th2xdx$

8.105. $\int xtg(x^2-5)dx$

8.106. $\int \frac{x^2 e^{\sqrt{x^3+1}}}{\sqrt{x^3+1}}dx$

8.107. $\int \left[\cos\left(3x-\frac{\pi}{6}\right)\right]^{-2} dx$

8.108. $\int \frac{xdx}{(x^2+2)\sqrt{x^2+2}}$

8.109. $\int \frac{\sin 2xdx}{\sin^2 x+5}$

8.110 $\int (1-x)^{99} xdx$

8.111. $x^2(1+x^3)^4 dx$

Bo'laklab integrallash usuli yordamida toping.

8.112. *a*) $(x+3)\sin xdx$; *b*) $\int x^2 arctgxdx$; *c*) $\int \arcsin xdx$;
d) $\int (x^2+3x)e^x dx$; e) $\int e^{2x}\sin 3xdx$.

8.113. *a*) $\int x\cos 2xdx$ *в*) $\int \arcsin 2xdx$

8.114. $\int \ln xdx$

8.115. $\int xarctgx^2 dx$

8.116. $\int \sqrt{x}\ln\sqrt{x}dx$

8.117. $\int x\ln(x^2+4)dx$

8.118. $\int x^3 arctgxdx$

8.119. $\int \frac{\ln x}{x^2}dx$

8.120. $\int \frac{x\arcsin x}{\sqrt{1-x^2}}dx$

8.121. $\int \frac{x\cos x}{\sin^3 x}dx$

8.122. $\ln(x^2+1)dx$

8.123. $\int x^2 5^x dx$

8.124. $\int x^{99}(x\cos x+100\sin x)dx$

8.125. $\int x^{3/2}\cos\sqrt{x}\frac{dx}{\sqrt{x}}$

8.126. $\int \frac{\ln^3 x}{\sqrt{x}}dx$

8.127. $\int x^{1/3}\ln^2 xdx$

8.128. $\int \sec^2 x\ln(\sec x)dx$

8.129. $\int xchxdx$

8.130. $\int (3x^2-8x)e^{2x}dx$

8.131. $\int (x^2+3x+5)e^{3x+1}dx$

8.132. $\int (x+2)\cos(3x+7)dx$

8.133. $\int (1+x)^2\sin xdx$

8.134. $\int (3x^2+6x+5)arctgxdx$

8.135. $\int (x-2)\arcsin xdx$

8.136. $\int e^{3x}\cos 2xdx$

8.137. $\int (1-3x-x^2)\ln xdx$

8.138. $\int (x^3-x-1)\ln^2 xdx$

Maxrajida kvadrat uchhad qatnashgan integrallarni toping

8.139. $\int \frac{dx}{15-9x^2-6x}$ 8.140. $\int \frac{dx}{x^2+2x+17}$

8.141. $\int \frac{dx}{\sqrt{x^2-8x+25}}$ 8.142. $\int \frac{dx}{\sqrt{3+2x-x^2}}$

8.143. $\int \frac{dx}{x^2+\sqrt{3}x+1}$ 8.144. $\int \frac{(x-2)dx}{x^2-4x+13}$

8.145. $\int \frac{(4x-5)dx}{x^2-2x+5}$ 8.146. $\int \frac{(3x-5)dx}{\sqrt{2x^2-12x+15}}$

8.147. $\int \frac{e^x dx}{e^{2x}-2e^x+6}$ 8.148. $\int \frac{xdx}{\sqrt{x^2-6x+1}}$

8.149. $\int \frac{(2x-1)dx}{\sqrt{5+4x-x^2}}$ 8.150. $\int \frac{6x+1}{\sqrt{x-x^2}}dx$

8.151. $\int \frac{dx}{x\sqrt{x^2-5x+4}}$ 8.152. $\int \frac{dx}{x^2\sqrt{1+x^2}}$

8.153. $\int \frac{dx}{(x^2-3x+3)^2}$ 8.154. $\int \frac{dx}{(x^2+2x)^2}$

8.155. $\int \frac{dx}{(x^2+1)^3}$ 8.156. $\int \frac{xdx}{(2x^2+3x+5)^2}$

Ratsional kasrni integrallang

8.157. a) $\int \frac{x^2dx}{(x-1)^2(x+1)}$, в) $\int \frac{dx}{(x-1)(x^2-x+1)}$

8.158. a) $\int \frac{dx}{(x-1)(x-3)}$ в) $\int \frac{xdx}{(2+x)(x-3)}$

8.159. $\int \frac{2x^2+3x+6}{(1+x)^2(4-x)}dx$ 8.160 $\int \frac{x^2dx}{(x+5)^2(x+4)^2}$

8.161. $\int \frac{(x^2-5x)dx}{x^2-5x+6}$ 8.162. $\int \frac{dx}{x^3+2x^2+x}$

8.163. $\int \frac{dx}{x(x^2+1)}$ 8.164. $\int \frac{dx}{(x+1)(x^2+4)}$

8.165. $\int \frac{dx}{(x+1)(x^2+x+1)^2}$ 8.166. $\int \frac{(x^3+1)dx}{(x^2-4x+5)^2}$

8.167. $\int \frac{1-2x+5x^2-2x^3}{(1-x)^4(x^2+1)}dx$ 8.168. $\int \frac{x^3-10x+25}{x^4-10x^3+25x^2}dx$

8.169. $\int \frac{x^5dx}{8-x^3}$ 8.170. $\int \frac{xdx}{(1-x^3)^2}$

8.171. $\int \frac{xdx}{(x^2+2x+2)^2}$ 8.171. $\int \frac{dx}{x^7+x^5}$

8.172. $\int\frac{x^2dx}{(2-3x)^7}$

8.173. $\int\frac{dx}{x(1+x)^5}$

8.174. $\int\frac{dx}{(1+x^2)^5}$

8.175. $\int\frac{9x^2-14x+1}{x^3-2x^2-x+2}dx$

8.176. $\int\frac{6dx}{x(x-1)(x^2-5x+6)}$

8.177. $\int\frac{(x^4+1)dx}{x^3-x^2+x-1}$

8.178. $\int\frac{dx}{(x^2+4)^4}$

§ 8.3. Ba'zi bir irratsional va trigonometrik funksiyalardan tashkil topgan ifodalarni integrallash

Ba'zi bir irratsional funksiyalardan tashkil topgan integrallarni integrallash. $\sqrt{x^2-a^2}$, $\sqrt{x^2+a^2}$, $\sqrt{a^2-x^2}$ ko'rinishdagi radikallar qatnashgan integrallarni hisoblashda quyidagi almashtirishlardan (o'rniga qo'yishlardan) foydalaniladi:

1) $\sqrt{a^2-x^2}$ qatnashgan integrallarda: $x=a\sin u$ (yoki $x=a\cos u$);
2) $\sqrt{a^2+x^2}$ qatnashgan integrallarda: $x=a tgu$ (yoki $x=a ctgu$);
3) $\sqrt{x^2-a^2}$ qatnashgan integrallarda: $x=a\sin u$ (yoki $x=a/\cos u$).

Agar $\int R(x,\sqrt[n]{x})dx$ integral berilgan bo'lsa, $t=\sqrt[n]{x}$ almashtirish bajarib integral sodda ko'rinishga olib kelinadi.

Agar integral $\int R\left(x,\sqrt[n]{\frac{ax+b}{cx+d}}\right)dx$ ko'rinishda berilgan bo'lsa, $t=\sqrt[n]{\frac{ax+b}{cx+d}}$

almashtirish bajarilib, integral sodda ko'rinishga olib kelinadi.

4-misol. Ushbu

$$\int\frac{dx}{x+\sqrt{x^2+x+1}}$$

integral hisoblansin.

Bu integralda

$$t=x+\sqrt{x^2+x+1}$$

almashtirishni bajaramiz.Natijada

$$x=\frac{t^2-1}{1+2t}\quad,\quad dx=2\frac{t^2+t+1}{(1+2t)^2}dt$$

bo'lib,

$$\int \frac{dx}{x+\sqrt{x^2+x+1}} = 2\int \frac{t^2+t+1}{(1+2t)^2 t}dt$$

bo'ladi.

Agar

$$\frac{2(t^2+t+1)}{t(1+2t)^2} = \frac{2}{t} - \frac{3}{1+2t} - \frac{3}{(1+2t)^2}$$

bo'lishini e'tiborga olsak,unda

$$\int \frac{dx}{x+\sqrt{x^2+x+1}} = \int\left(\frac{2}{t} - \frac{3}{1+2t} - \frac{3}{(1+2t)^2}\right)dt =$$

$$= 2\ln|t| - \frac{3}{2}\ln|1+2t| + \frac{3}{2(1+2t)} + C =$$

$$= 2\ln\left|x+\sqrt{x^2+x+1}\right| - \frac{3}{2}\ln\left|1+2x+2\sqrt{x^2+x+1}\right| +$$

$$+ \frac{3}{2(1+2x+2\sqrt{x^2+x+1})} + C$$

bo'lishi kelib chiqadi.

5-misol. Ushbu

$$I = \int \frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}}dx$$

integral hisoblansin.

Ravshanki,

$$x^2+3x+2 = (x+1)\cdot(x+2).$$

Shuni e'tiborga olib berilgan integralda

$$t = \frac{1}{x+1}\sqrt{x^2+3x+2}$$

almashtirishni bajaramiz.U holda

$$x = \frac{2-t^2}{t^2-1} \quad , \qquad dx = -\frac{2tdt}{(t^2-1)^2}$$

bo'lib,

$$\int \frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}}dx = \int \frac{-2t^2-4t}{(t-2)\cdot(t-1)\cdot(t+1)^3}dt$$

bo'ladi.

Endi

$$\frac{-2t^2-4t}{(t-2)\cdot(t-1)\cdot(t+1)^3}=\frac{\frac{3}{4}}{t-1}-\frac{\frac{16}{27}}{t-2}-\frac{\frac{17}{108}}{t+1}+\frac{\frac{5}{18}}{(t+1)^2}+\frac{\frac{1}{3}}{(t+1)^3}$$

bo'lishini e'tiborga olib topamiz:

$$I=\int\frac{-2t^2-4t}{(t-2)\cdot(t-1)\cdot(t+1)^3}dt=\frac{3}{4}\int\frac{dt}{t-1}-\frac{16}{27}\int\frac{dt}{t-2}-$$

$$-\frac{17}{108}\int\frac{dt}{t+1}+\frac{5}{18}\int\frac{dt}{(t+1)^2}+\frac{1}{3}\int\frac{dt}{(t+1)^3}=\frac{3}{4}\ln|t-1|-$$

$$-\frac{16}{27}\ln|t-2|-\frac{17}{108}\ln|t+1|-\frac{5}{18}\cdot\frac{1}{t+1}-\frac{1}{6}\cdot\frac{1}{(t+1)^2}+C.$$

Trigonometrik funksiyalardan tashkil topgan ifodalarni integrallash. Bu integrallarni $\int R(\sin x,\cos x)dx$ (bu yerda R – ratsional funksiya). Integrallash $u=tg\frac{x}{2}$ $(-\pi<x<\pi)$ universal trigonometrik almashtirish yordamida ratsional kasrlarni integralalshga keltiriladi.

U holda $\sin x=\frac{2u}{1+u^2}$, $\cos x=\frac{1-u^2}{1+u^2}$, $dx=\frac{2du}{1+u^2}$

va $\int R(\sin x;\cos x)dx=\int R\left(\frac{2u}{1+u^2};\frac{1-u^2}{1+u^2}\right)\frac{2du}{1+u^2}$ bo'ladi.

Trigonometrik funksiyalar qatnashgan ba'zi ifodalarni integrallarini hisoblang.

8.179. *a)* $\int\frac{dx}{3+5\cos x}$ b) $\int\frac{dx}{\cos^3 x}$

8.180. *a)* $\int\frac{\sin x dx}{\sin x-\cos x}$ *b)* $\int\frac{\cos x dx}{1+\cos x}$

8.181. $\int\frac{dx}{1-ctgx}$ 8.182. $\int\frac{\sin x+\cos x}{\sin x-\cos x}dx$

8.183. $\int\frac{dx}{(\sin x-\cos x)^2}$ 8.184. $\int\frac{dx}{1+\cos x+\sin x}$

8.185. $\int\frac{dx}{\sin^2 x-5\sin x\cos x}$ 8.186. $\int\frac{dx}{\cos x(1+\sin x)}$

8.187. $\int\frac{\sin x dx}{6\cos^2 x+\sin^2 x}$ 8.188. $\int\frac{(\sin x+\sin^3 x)dx}{\cos 2x}$

8.189. $\int\frac{dx}{\sin^2 x-2tgx+3\cos^2 x}$ 8.190. $\int\frac{1+tgx}{1-tgx}dx$

8.191. $\int \frac{\sin^3 x+1}{\cos^2 x}dx$

8.192. $\int \frac{dx}{3+5\cos x}$

8.193. $\int \frac{dx}{8-4\sin x+7\cos x}$

8.194. $\int \frac{dx}{4+3tgx}$

8.195. $\int \frac{\cos x dx}{\sin^3 x-\cos^3 x}$

8.196. $\int \frac{1+tgx}{\sin x}dx$

Sinus va kosinuslarni ko'paytmasi va darajasi qatnashgan ba'zi ifodalarni integrallarini toping.

8.197. $\int \cos^3 2x dx$

8.198. $\int \cos^2 5x dx$

8.199. $\int \sin^2 x \cos^2 x dx$

8.200. $\int \sin^2 2x \cos^3 x dx$

8.201. $\int \cos^7 x dx$

8.202. $\int \sin^6 x dx$

8.203. $\int \frac{\sin^3 x}{\cos^4 x}dx$

8.204. $\int \frac{dx}{\sin^2 x\cos^2 x}$

8.205. $\int \frac{\sin^2 2x}{\cos^4 2x}dx$

8.206. $\int \sin^2 x \cos^3 x dx$

8.207. $\int \cos 3x \sin 5x dx$

8.208. $\int \sin\frac{x}{2}\sin\frac{x}{4}dx$

8.209. $\int \sin\left(5x-\frac{\pi}{2}\right)\cos\left(x+\frac{\pi}{4}\right)dx$

8.210. $\int tg^2 x dx$

8.211. $\int \sin x\cdot\sin 2x\cdot\sin 3x dx$

8.212. $\int \frac{\cos^2 x}{\sin^3 x}dx$

8.213. $\int ctg^3 x dx$

8.214. $\int tg^6 3x dx$

8.215. $\int tg^4\varphi\cdot\sec^4\varphi d\varphi$

8.216. $\int \frac{ctg^{3/2} x dx}{\sin^4 x}$

8.217. $\int \frac{\cos 2x dx}{\sin^4 x}$

8.218. $\int \frac{dx}{\sin^5 x\cos x}$

Ba'zi bir irratsional ifodalar qatnashgan integrallarni toping

8.219. a) $\int \frac{\sqrt{x^2-1}}{x}dx$ b) $\int x(2x-1)^{1/3}dx$

8.220. $\int \left(\frac{x}{x+1}\right)^{3/2}dx$

8.221. $\int \frac{x+1+\sqrt{x+2}}{x+3}dx$

8.222. $\int \sqrt{\frac{x-3}{x}}\frac{dx}{x}$

8.223. $\int \sqrt[3]{\frac{1-x}{1+x}}\frac{dx}{(1+x)^2}$

8.224. $\int \frac{\sqrt{x+25}}{x}dx$

8.225. $\int \frac{\sqrt{\left(\frac{x}{1-x}\right)^3}}{x+\sqrt{\frac{x}{1-x}}}dx$

8.226. $\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x^5}-\sqrt[6]{x^7}}dx$

8.227. $\int \sqrt{\frac{x+1}{x-1}}dx$

8.228. $\int \frac{\sqrt[6]{x}dx}{1+\sqrt[3]{x}}$

8.229. $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx$

8.230. $\int \frac{dx}{(\sqrt{x+2}-\sqrt[3]{x+2})\sqrt{x-2}}$

8.231. $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}dx$

8.232. $\int \frac{\sqrt[3]{x}dx}{\sqrt{1+\sqrt[3]{x^2}}}$

8.233. $\int \sqrt{x}(1+2\sqrt[6]{x})^3 dx$

8.234. $\int x^5\sqrt{4+x^2}dx$

8.235. $\int \frac{dx}{\sqrt[3]{1+x^3}}$

8.236. $\int \frac{\sqrt[3]{1+x^2}}{x^2}dx$

8.237. $\int \frac{x^7dx}{\sqrt{1+x^2}}$

8.238. $\int \frac{dx}{x^4\sqrt{x^2-1}}$

8.239. $\int \frac{dx}{x(1+x^3)^{1/4}}$

Eyler almashtirish yordamida integrallarni toping

8.240. $\int \frac{dx}{x\sqrt{x^2+4x+1}}$

8.241. $\int \frac{dx}{x+\sqrt{x^2-x+4}}$

8.242. $\int \frac{xdx}{\sqrt{(6x-8-x^2)^3}}$

8.243. $\int \frac{x^2dx}{\sqrt{x^2+2x+5}}$

8.244. $\int \frac{dx}{x\sqrt{x^2-x-5}}$

8.245. $\int \frac{dx}{x-\sqrt{x^2-1}}$

8.246. $\int \frac{dx}{(x-2)\sqrt{(7x-x^2-10)^3}}$

Takrorlash uchun savollar

1.Aniqmas integral deb nimaga aytiladi?

2.Aniqmas integralni hisoblash usullari haqida ayting.

3.Bo'laklab integrallash usuli formulasini yozing.

4.O'zgarmas sonning integrali nimaga teng?

5.Trigonometrik funksiyali ifodalarni ratsional funksiyaga ketiruvchi universal almashtirmani ko'rsating.

6.I tur eng sodda ratsional funksiyani ayting.

7.Boshlang'ich funksiya ta'rifini ayting.

ANIQMAS INTEGRALGA DOIR NAZORAT TESTLARI

1.Boshlang'ich funksiya va aniqmas integral. Integrallar jadvali

1. Quyidagi shartlarning qaysi birida $F(x)$ berilgan $f(x)$ funksiyaning boshlang'ich funksiyasi deyiladi ?

A) $F(x)=f(x)+C$ (C-const). B) $\lim\limits_{t\to x} F(t)=f(x)$.

C) $F'(x)=f(x)$. D) $F''(x)=f(x)$. E) $F(x)=f'(x)$.

2. Quyidagilardan qaysi biri $f(x)=\ln x$ uchun boshlang'ich funksiya bo'ladi?

A) $\frac{1}{x}$. B) $x\ln x$. C) $x\ln x+x$.

D) $x\ln x-x$. E) $\frac{1}{x}\ln x-x$.

3. Quyidagilardan qaysi birining boshlang'ich funksiyasi $F(x)=x\cos x$ bo'ladi?

A) $x^2\sin x$. B) $-\sin x$. C) $\cos x-x\sin x$.

D) $\sin x+x\cos x$. E) $x(\sin x+\cos x)$.

4. ***Teoremani to'ldiring:*** Agar $F(x)$ biror $f(x)$ funksiya uchun boshlang'ich funksiya bo'lsa, unda ixtiyoriy C o'zgarmas soni uchun ... funksiya ham $f(x)$ uchun boshlang'ich funksiya bo'ladi.

A) $C\cdot F(x)$. B) $C-F(x)$. C) $C+F(x)$.

D) $C/F(x)$. E) $F(x+C)$.

5. Agar $F_1(x)$ va $F_2(x)$ berilgan $f(x)$ funksiya uchun boshlang'ich funksiyalar bo'lsa, unda biror C o'zgarmas soni uchun quyidagi tengliklardan qaysi biri o'rinli bo'ladi?

A) $F_1(x)F_2(x)=C$. B) $F_1(x)/F_2(x)=C$.

C) $F_1(x)+F_2(x)=C$. D) $F_1(x)-F_2(x)=C$. E) $F_1(x)\pm F_2(x)=C$.

6. Agar $F(x)$ biror $f(x)$ funksiya uchun boshlang'ich funksiya bo'lsa, unda ta'rif bo'yicha $\int f(x)dx$ aniqmas integral qanday aniqlanadi ?

A) $C\cdot F(x)$. B) $C-F(x)$.

C) $C+F(x)$. D) $C/F(x)$. E) $F(x+C)$

7. Aniqmas integralning geometrik ma'nosi qayerda to'g'ri va to'liq ko'rsatilgan?

A) Qandaydir to'g'ri chiziq.
B) Qandaydir egri chiziq.
C) Qandaydir chiziqlar sinfi.
D) OX o'qi bo'yicha o'zaro parallel chiziqlar sinfi.
E) OY o'qi bo'yicha o'zaro parallel chiziqlar sinfi.

8. Qayerda aniqmas integralning xossasi xato ko'rsatilgan ?

A) $\left(\int f(x)dx\right)' = f(x)$. B) $d\left(\int f(x)dx\right) = f(x)dx$.
C) $\int dF(x) = F(x) + C$. D) $\int F'(x)dx = F(x) + C$.
E) Barcha xossalar to'g'ri ko'rsatilgan.

9. Aniqmas integral uchun qaysi tenglik bajarilmaydi ?

A) $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$.
B) $\int f(x)g(x)dx = \int f(x)dx \cdot \int g(x)dx$.
C) $\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$.
D) $\int kf(x)dx = k\int f(x)dx \quad (k - const)$.
E) keltirilgan barcha tengliklar bajariladi.

10. Agarda $F(x)$ va $G(x)$ mos ravishda $f(x)$ va $g(x)$ funksiyalar uchun boshlangich funksiyalar , a va b ixtiyoriy o'zgarmas sonlar bo'lsa, $\int [af(x) + bg(x)]dx$ aniqmas integral javobi qayerda to'g'ri ko'rsatilgan?

A) $(a+C)F(x)+(b+C)G(x)$. B) $aF(x)+bG(x)+C$.
C) $aF(x)-bG(x)+C$. D) $F(ax)+G(bx)+C$.
E) $(a+C)F(x)-(b+C)G(x)$.

2.Aniqmas integralni hisoblash usullari

1. Aniqmas integralni hisoblashning qaysi usuli mavjud emas ?

A) ko'paytirish usuli. B) o'zgaruvchini almashtirish usuli.
C) differensial ostiga kiritish usuli. D) yoyish usuli.
E) bo'laklab integrallash usuli.

2. Qaysi tenglik aniqmas integralni yoyish usulida hisoblashni ifodalaydi?

A) $\int f(x)dx = \int udv = uv - \int vdu$. B) $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt$.

C) $\int f(x)dx = \int \sum_{k=1}^{n} a_k f_k(x)dx = \sum_{k=1}^{n} a_k \int f_k(x)dx$.

D) $\int f(ax+b)dx = \frac{1}{a}F(ax+b)+C$. E) $\int f(x)dx = \int f[\varphi(t)]\varphi'(t)dt$.

3. $\int \frac{2x^2-3x+5}{x^2}dx$ integralni yoyish usulida hisoblang.

A) $2-\frac{3}{x}+\frac{5}{x^2}+C$. B) $2x-3\ln|x|+\frac{5}{x^2}+C$.

C) $2x-\frac{3}{x}-\frac{5}{3x^3}+C$. D) $2x-3\ln|x|-\frac{5}{x}+C$. E) $2x+\frac{3}{x^2}-\frac{5}{x}+C$

4. Quyidagi integrallardan qaysi biriga differensial ostiga kiritish usulini qo'llab bo'lmaydi?

A) $\int f(x)f'(x)dx$. B) $\int \frac{f'(x)dx}{f(x)}$.

C) $\int [f(x) \pm f'(x)]dx$. D) $\int \sqrt{f(x)}f'(x)dx$.

E) barcha integrallarga differensial ostiga kiritish usulini qo'llab bo'ladi .

5. Quyidagi integrallardan qaysi biri differensial ostiga kiritish usulida xato hisoblangan?

A) $\int f(x)f'(x)dx = \frac{1}{2}f^2(x)+C$. B) $\int \cos[f(x)]f'(x)dx = \sin[f(x)]+C$.

C) $\int \sin[f(x)]f'(x)dx = \cos[f(x)]+C$. D) $\int \frac{f'(x)dx}{f(x)} = \ln|f(x)|+C$.

E) keltirilgan barcha integrallar to'g'ri hisoblangan.

6. Qaysi tenglik aniqmas integralni o'zgaruvchilarni almashtirish usulida hisoblashni ifodalaydi ?

A) $\int f(x)dx = \int udv = uv - \int vdu$. B) $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt$.

C) $\int f(x)dx = \int \sum_{k=1}^{n} a_k f_k(x)dx = \sum_{k=1}^{n} a_k \int f_k(x)dx$.

D) $\int f(ax+b)dx = \frac{1}{a}F(ax+b)+C$. E) $\int f(x)dx = \int f[\varphi(t)]\varphi'(t)dt$.

7. $\int f(x)dx$ aniqmas integralda $x=\varphi(t)$ almashtirma bajarilganda u qanday ko'rinishga keladi

A) $\int f[\varphi(t)]\varphi(t)dt$. B) $\int f[\varphi'(t)]\varphi'(t)dt$. C) $\int f[\varphi(t)]dt$

D) $\int f[\varphi(t)]\varphi'(t)dt$. E) $\int f[\varphi'(t)]\varphi(t)dt$.

8. $\int\frac{x^4dx}{\sqrt{x^{10}-1}}$ integral qaysi almashtirma orqali jadval integraliga keltiriladi ?

A) $t=x^2$. B) $t=x^3$. C) $t=x^4$. D) $t=x^5$. E) $t=x^6$.

9. $\int\frac{\sin xdx}{\sqrt{\cos x}}$ integral qaysi almashtirma orqali jadval integraliga keltiriladi ?

A) $t=\sin x$. B) $t=\cos x$. C) $t=\sqrt{\cos x}$.

D) $t=\mathrm{tg}x$. E) $t=\mathrm{ctg}x$.

10. $\int\frac{\cos xdx}{\sqrt{\sin x}}$ integral qaysi almashtirma orqali jadval integraliga keltiriladi ?

A) $t=\sin x$. B) $t=\cos x$. C) $t=\sqrt{\sin x}$.

D) $t=\mathrm{tg}x$. E) $t=\mathrm{ctg}x$.

3.Ratsional funksiyalar va ularni integrallash

1. Quyidagi yig'indilardan qaysi biri ko'phadni ifodalaydi ?

A) $\sum_{k=0}^{n}a_k\sin kx$. B) $\sum_{k=0}^{n}a_k\sin^k x$. C) $\sum_{k=0}^{n}a_kx^k$.

D) $\sum_{k=0}^{n}a_kx^{-k}$. E) $\sum_{k=0}^{n}a_kx_k$.

2. $P(x)=(x^2+2x-10)^2(3x+5)$ ko'phadning darajasini aniqlang:

A) 1. B) 2. C) 3. D) 4. E) 5.

3. Agar $P_n(x)$ va $Q_m(x)$ ko'phadlar bo'lsa, unda quyidagi funksiyalardan qaysi biri ratsional funksiya deyiladi ?

A) $P_n(x)+Q_m(x)$. B) $P_n(x)-Q_m(x)$.

C) $P_n(x)/Q_m(x)$. D) $P_n(x)\cdot Q_m(x)$. E) $P_n[Q_m(x)]$.

4. Qaysi shartda $R(x)=P_n(x)/Q_m(x)$ to'g'ri ratsional funksiya deyiladi ?

A) $m\geq n$. B) $m\leq n$. C) $m>n$;D) $m<n$. E) $m\neq n$.

5. Qaysi holda $R(x)=P_n(x)/Q_m(x)$ noto'g'ri ratsional funksiya bo'lmaydi?

A) $m \geq n$. B) $m \leq n$. C) $m=n$. D) $m>n$. E) $m=n-1$.

6. Quyidagilardan qaysi biri to'g'ri ratsional funksiya bo'ladi ?

A) $\frac{x^2+x+1}{x+1}$. B) $\frac{(x+1)^3}{x^2+x+1}$. C) $\frac{x^2+x+1}{x^3+1}$.

D) $\frac{x^2+x+1}{(x+1)^2}$. E) $\frac{x^2+x+1}{x^2+1}$.

7. I tur eng sodda ratsional funksiyani ko'rsating.

A) $\frac{Ax+B}{x-a}$. B) $\frac{Ax+B}{(x-a)^k}$. C) $\frac{A}{(x-a)^k}, k \geq 2$. D) $\frac{A}{x-a}$.

E) $\frac{x+b}{x-a}$.

8. II tur eng sodda ratsional funksiyani ko'rsating.

A) $\frac{Ax+B}{x^2+px+q}$. B) $\frac{Ax+B}{(x-a)^k}$. C) $\frac{A}{x-a}$.

D) $\frac{A}{x^2+px+q}$. E) $\frac{A}{(x-a)^k}, k \geq 2$.

9. III tur eng sodda ratsional funksiyani ko'rsating.

A) $\frac{Ax+B}{x^2+px+q}$. B) $\frac{Ax+B}{(x^2+px+q)^k}, k \geq 2$. C) $\frac{Ax+B}{(x-a)^k}, k \geq 3$

D) $\frac{Ax^2+Bx+C}{x^2+px+q}$. E) $\frac{Ax^2+Bx+C}{(x^2+px+q)^k}$.

10. IV tur eng sodda ratsional funksiyani ko'rsating.

A) $\frac{Ax+B}{x^2+px+q}$. B) $\frac{Ax+B}{(x^2+px+q)^k}, k \geq 2$.

C) $\frac{Ax+B}{(x-a)^k}, k \geq 3$.

D) $\frac{Ax^2+Bx+C}{x^2+px+q}$. E) $\frac{Ax^2+Bx+C}{(x^2+px+q)^k}$.

4.Ayrim irratsional ifodali integrallarni hisoblash

1. Binomial integralning umumiy ko'rinishi qayerda to'g'ri ifodalangan ?

A) $\int x(a+bx)^p dx$. B) $\int x^r(a+bx)^p dx$. C) $\int x^r(a+bx^s)^p dx$.

D) $\int x^r(a+bx^s)dx$. E) $\int x^r(ax^t+bx^s)^p dx$.

2. Qaysi shartda $\int x^r(a+bx^s)^p dx$ binomial integral albatta elementar funksiyalar orqali ifodalanadi ?

A) p–butun son. B) r–butun son. C) s–butun son.

D) $r+s$–butun son. E) keltirilgan barcha hollarda.

3. Qaysi shartda $\int x^r(a+bx^s)^p dx$ binomial integral elementar funksiyalarda integrallanuvchi bo'lmasligi mumkin ?

A) $\frac{r+1}{s}+p$–butun son. B) $\frac{r+1}{s}$ –butun son.

C) s–butun son. D) p–butun son.

E) keltirilgan barcha hollarda integrallanuvchi bo'ladi.

4. $\int x^r(a+bx^s)^p dx$ binomial integralda $(r+1)/s$– butun son, $p=k/m$ bo'lsa , qaysi almashtirma orqali undan ratsional funksiyali integralga o'tiladi ?

A) $a+bx^s=t$. B) $a+bx^s=t^k$. C) $a+bx^s=t^m$.

D) $a+bx=t^m$. E) $ax^{-s}+b=t^m$.

5. $\int x^r(a+bx^s)^p dx$ binomial integralda p– butun son, $r=k/m$ va $s=q/m$ bo'lsa , qaysi almashtirma orqali undan ratsional funksiyali integralga o'tiladi ?

A) $x=t^p$. B) $x=t^m$. C) $x=t^k$. D) $x=t^q$. E) $x=t^{kq}$.

6. $\int x^r(a+bx^s)^p dx$ binomial integralda $p+(r+1)/s$– butun son, $p=k/m$ bo'lsa , qaysi almashtirma orqali undan ratsional funksiyali integralga o'tiladi ?

A) $a+bx^s=t$. B) $a+bx^s=t^k$. C) $a+bx^s=t^m$.

D) $a+bx=t^m$. E) $ax^{-s}+b=t^m$.

7. $\int x^{2/3}(a+bx^{3/4})^2 dx$ binomial integraldan ratsional funksiyali integralga qaysi almashtirma orqali o'tiladi ?

A) $a+bx^{3/4}=t$. B) $a+bx^{3/4}=t^3$. C) $x=t^{12}$.

D) $x=t^3$. E) $x=t^4$.

8. $\int x^{2/3}(a+bx^{5/6})^{1/2} dx$ binomial integraldan ratsional funksiyali integralga qaysi almashtirma orqali o'tiladi ?

A) $a+bx^{5/6}=t$. B) $a+bx^{5/6}=t^2$. C) $ax^{-5/6}+b=t$.

D) $ax^{-5/6}+b=t^2$.   E) $x=t^6$.

9. $\int x^{2/3}(a+bx^{3/4})^{7/9}dx$ binomial integraldan ratsional funksiyali integralga qaysi almashtirma orqali o'tiladi ?

A) $a+bx^{3/4}=t$.   B) $a+bx^{3/4}=t^9$.   C) $ax^{-3/4}+b=t$.

D) $ax^{-3/4}+b=t^9$.   E) $x=t^{12}$.

10. $\int R(x, x^{m/n}, \ldots, x^{r/s})dx$ irratsional funksiyali integral qanday almashtirma orqali ratsional funksiyali integralga keltiriladi ?

A) $x=t^n$.   B) $x=t^s$.   C) $x=t^k, k=EKUK(n,\ldots,s)$.

D) $x=t^k, k=EKUB(n,\ldots,s)$.   E) $x=t^{ns}$.

5.Ayrim trigonometrik ifodali integrallarni hisoblash

1. Trigonometrik funksiyali ifodalarni ratsional funksiyaga ketiruvchi

universal almashtirmani ko'rsating.

A) sin$x=t$.   B) cos$x=t$.   C) tg$x=t$.   D) ctg$x=t$.

E) tg$(x/2)=t$.

2. $t=\mathrm{tg}\frac{x}{2}$ universal almashtirma qatnashgan quyidagi tengliklardan qaysi biri noto'g'ri ?

A) $\sin x=\frac{2t}{1+t^2}$.   B) $\cos x=\frac{1-t^2}{1+t^2}$.   C) $dx=\frac{2dt}{1+t^2}$.

D) $x=2\mathrm{arctg}\,t$.   E) keltirilgan barcha tengliklar to'g'ri.

3. $\int R(\cos x)\sin x dx$ ko'rinishdagi integrallarni hisoblash uchun qaysi almashtirmadan foydalaniladi?

A) cos$x=t$.   B) sin$x=t$.   C) tg$x=t$.

D) ctg$x=t$.   E) tg$2x=t$.

4. Trigonometrik ifodali $\int(1-\cos^4 x)\sin x dx$ integralni hisoblang.

A) $\cos x-\sin^4 x+C$.   B) $\sin x-\frac{\cos^5 x}{5}+C$.

C) $-\cos x+\frac{\cos^5 x}{5}+C$ D) $\sin x-\frac{\sin^5 x}{5}+C$.

E) $-\cos x + \dfrac{\sin^5 x}{5} + C$.

5. $\int R(\sin x)\cos x dx$ ko‘rinishdagi integral qanday almashtirma yordamida hisoblanishi mumkin ?

A) $\cos x = t$. B) $\sin x = t$. C) $\mathrm{tg}\, x = t$.
D) $\mathrm{ctg}\, x = t$. E) $\mathrm{tg}\, 2x = t$.

6. $\int \dfrac{\cos x}{1-\sin x} dx$ integral javobi qayerda to‘g‘ri ko‘rsatilgan ?

A) $\dfrac{\sin x}{1-\cos x} + C$. B) $\dfrac{\sin x}{1-\sin x} + C$. C) $\dfrac{1+\sin x}{1-\sin x} + C$.
D) $-\ln|1-\sin x| + C$. E) $\ln|1-\cos x| + C$.

7. Quyidagi almashtirmalarning qaysi biridan $\int R(\mathrm{tg}\, x) dx$ ko‘rinishdagi integralni ratsional funksiyali integralga keltirishda foydalanib bo‘lmaydi?

A) $t = \mathrm{tg}\, x$. B) $t = \sin x$. C) $t = \mathrm{ctg}\, x$. D) $t = \mathrm{tg}\dfrac{x}{2}$.

E) ko‘rsatilgan barcha almashtirmalardan foydalanib bo‘ladi .

8. $\int \sin^{2m+1} x \cos^n x dx$ integralni hisoblash uchun qaysi almashtirmadan foydalanish qulay ?

A) $\sin x = t$. B) $\cos x = t$. C) $\mathrm{tg}\, x = t$. D) $\mathrm{ctg}\, x = t$. E) $\sin^2 x = t$.

9. $\int \sin^3 x \cos^5 x dx$ integralni hisoblang.

A) $\dfrac{\cos^8 x}{8} - \dfrac{\cos^6 x}{6} + C$. B) $\dfrac{\sin^8 x}{8} - \dfrac{\cos^6 x}{6} + C$. C) $\dfrac{\cos^8 x}{8} - \dfrac{\sin^6 x}{6} + C$.
D) $\dfrac{\sin^8 x}{8} - \dfrac{\sin^6 x}{6} + C$. E) $\dfrac{\sin^8 x}{8} + \dfrac{\sin^6 x}{6} + C$.

10. $\int \sin^m x \cos^{2n+1} x dx$ integralni hisoblash uchun qaysi almashtirmadan foydalanish qulay ?

A) $\sin x = t$. B) $\cos x = t$. C) $\mathrm{tg}\, x = t$. D) $\mathrm{ctg}\, x = t$. E) $\cos^2 x = t$.

Matematika fanlar ichra shoh,
uning sirlaridan bo'lingiz ogoh!
Qori Niyoziy

XI-BOB. ANIQ INTEGRAL

§ 9.1. Aniq integralni hisoblash

1. [a,b] kesmada $f(x)$ funksiya aniqlangan bo'lsin. [a,b] oraliqni $a = x_0 < x_1 < ... < x_n = b$ nuqtalar bilan n ta bo'laklarga ajrataylik. Har bir $[x_{i-1}, x_i]$ kesmadan bittadan $\xi_i(\xi_i \in [x_{i-1}, x_i])$ nuqta olib, $\Delta J = \sum_{i=1}^{n} f(\xi_i)\Delta x_i$ yig'indi tuzamiz, bunda $\Delta x_i = x_i - x_{i-1}$. ΔJ - ko'rinishdagi yig'indi,integral yig'indi deyiladi. Uning $\max \Delta x_i \to 0$ dagi limiti, (u mavjud va chekli bo'lsa) $f(x)$ funksiyaning a dan b gacha aniq integrali deyiladi hamda

$$\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i \qquad (9.1)$$

ko'rinishida yoziladi.

2.Aniq integralning xossalari.

$$1)\ \int_a^b \alpha f(x)dx = \alpha \int_a^b f(x)dx; \quad (\alpha = const); \qquad (9.2)$$

$$2)\ \int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx; \qquad (9.3)$$

$$3)\ \int_a^b f(x)dx = -\int_b^a f(x)dx; \qquad (9.4)$$

$$4)\ \int_a^a f(x)dx = 0; \qquad (9.5)$$

$$5)\ \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx; \qquad (9.6)$$

6) Agar $y = f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo'lsa, u holda $\xi \in (a,b)$ topiladiki,

$$\int_a^b f(x)dx = f(\xi)(b-a); \qquad (9.8)$$

bo'ladi.

8) Agar $y = f(x)$ juft funksiya bo'lsa, u holda

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx; \qquad (9.8)$$

) Agar $y = f(x)$ toq funksiya bo'lsa, u holda

$$\int_{-a}^{a} f(x)dx = 0 \qquad (9.9)$$

3. Aniq integral Nyuton-Leybnits

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) \qquad (9.10)$$

formulasi orqali hisoblanadi.

4. $\int_a^b f(x)dx$ integralni hisoblash uchun $x = \varphi(t)$, $\alpha \le t \le \beta$ almashtirishni qo'llaymiz. Agar $[\alpha;\beta]$ kesmada $x = \varphi(t)$, $\varphi'(t)$, $f(\varphi(t))$ funksiyalar uzluksiz va $\varphi(\alpha) = a$, $\varphi(\beta) = b$ bo'lsa, quyidagi

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt \qquad (9.11)$$

tenglik o'rinli.

5. [a,b] kesmada $u = u(x)$, $\vartheta = \vartheta(x)$ funksiyalar uzluksiz hosilalarga ega bo'lsa, quyidagi **bo'laklab** integrallash formulasi o'rinli bo'ladi:

$$\int_a^b u d\vartheta = u\vartheta\Big|_a^b - \int_a^b \vartheta du \qquad (9.12)$$

1-misol. Ushbu

$$J_n = \int_0^{\frac{\pi}{2}} \sin^n x dx \ (n = 0,1,2,...)$$

integral hisoblansin.

◀ Ravshanki,

$$J_0 = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2} \quad , \quad J_1 = \int_0^{\frac{\pi}{2}} \sin x dx = (-\cos x)\Big|_0^{\frac{\pi}{2}} = 1.$$

$n \geq 2$ bo'lganda berilgan integralni

$$J_n = \int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \sin^{n-1} x\, d(-\cos x)$$

ko'rinishda yozib, unga bo'laklab integrallash formulasini qo'llaymiz. Natijada

$$J_n = (-\sin^{n-1} x \cdot \cos x)\Big|_0^{\frac{\pi}{2}} + (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x dx =$$

$$= (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x(1-\sin^2 x)dx = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x dx - (n-1)\int_0^{\frac{\pi}{2}} \sin^n x dx =$$

$$= (n-1)J_{n-2} - (n-1)J_n$$

bo'lib, undan ushbu

$$J_n = \frac{n-1}{n} J_{n-2}$$

recurrent formula kelib chiqadi.

Bu formula yordamida berilgan integralni $n = 1,2,3,........$ bo'lganda ketma-ket hisoblash mumkin.

Aytaylik, $n = 2m$ - juft son bo'lsin. Unda

$$J_{2m} = \frac{2m-1}{2m} \cdot \frac{2m-3}{2m-2} \cdots\cdots \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot J_0 = \frac{(2m-1)!!}{(2m)!!} \cdot \frac{\pi}{2}$$

bo'ladi.

Aytaylik, $n = 2m+1$ - toq son bo'lsin. Unda

$$J_{2m+1} = \frac{2m}{2m+1} \cdot \frac{2m-2}{2m-1} \cdots\cdots \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3} \cdot J_1 = \frac{(2m)!!}{(2m+1)!!}$$

bo'ladi. ($m!!$ simvol m dan katta bo'lmagan va u bilan bir xil juftlikka ega bo'lgan natural sonlarning ko'paytmasini bildiradi.

Integrallarni hisoblang (Nyuton – Leybnis formulasini qo'llash yo'li bilan)

9.1 $a)\ \int_0^1 \frac{xdx}{\sqrt{1+x^2}}$ $\quad$ $в)\ \int_0^1 \sqrt{1+x}\,dx$ $\quad$ 9.2 $\int_{-2}^{-1} \frac{dx}{(11+5x)^3}$

9.3 $\int_2^{-11} \frac{dx}{\sqrt[5]{(3-x)^4}}$ $\quad$ 9.4 $\int_0^1 \frac{y-1}{\sqrt{y}+1}dy$ $\quad$ 9.5 $\int_0^{T/2} \sin\left(\frac{2\pi t}{T}-\varphi_0\right)dt$

9.6 $\int_0^{16} \frac{dx}{\sqrt{x+9}-\sqrt{x}}$ $\quad$ 9.8 $\int_0^1 (e^x-1)^4 e^x dx$ $\quad$ 9.8 $\int_0^{2a} \frac{3dx}{26-x}$ $(b>a>0)$

9.9 $\int_0^1 \frac{xdx}{(x^2+1)^2}$ $\quad$ 9.10 $\int_1^e \frac{dx}{x\sqrt{1-(\ln x)^2}}$ $\quad$ 9.11 $\int_1 \frac{1+\lg x}{x}dx$

9.12 $\int_1^2 \frac{e^{1/x}dx}{x^2}$ $\quad$ 9.13 $\int_0^{\sqrt[n]{a/2}} \frac{x^{n-1}dx}{\sqrt{a^2-x^{2n}}}$ $\quad$ 9.14 $\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}}$

9.15 $\int_{1/2}^{\sqrt{5}/2} \frac{x^3dx}{\left(\frac{5}{8}-x^4\right)\sqrt{\frac{5}{8}-x^4}}$ $\quad$ 9.16 $\int_0^{a/2} \frac{adx}{(x-a)(x-2a)}$ $\quad$ 9.18 $\int_2^3 \frac{dx}{2x^2+3x-2}$

9.18 $\int_0^1 \frac{dx}{x^2+4x+5}$ $\quad$ 9.19 $\int_1^2 \frac{dx}{x+x^3}$ $\quad$ 9.20 $\int_{-0.5}^1 \frac{dx}{\sqrt{8+2x-x^2}}$

9.21 $\int_{-\pi/2}^{\pi/2} \frac{dx}{1+\cos x}$ $\quad$ 9.22 $\int_0^{\pi/2} \cos^5 x\sin 2x dx$ $\quad$ 9.23 $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x-\cos^3 x}\,dx$

9.24 $\int_0^{\pi/\omega} \sin^2(\omega t+\varphi_0)dt$ $\quad$ 9.25 $\int_{-\pi/2}^{-\pi/4} \frac{\cos^3 xdx}{\sqrt[3]{\sin x}}$ $\quad$ 9.26 $\int_0^{\pi/4} ctg^4\varphi d\varphi$

9.28 $\int_{1/\pi}^{2/\pi} \frac{\sin\frac{1}{x}}{x^2}dx$ $\quad$ 9.28 $\int_{-\pi/2}^{\pi/2} \cos t\sin\left(2t-\frac{\pi}{4}\right)dt$ $\quad$ 9.29 $\int_{\pi/8}^{\pi/6} \frac{dx}{\cos^2 2x}$

9.30 $\int_{-1}^{0} \sqrt{2-3x}\,dx$ $\quad$ 9.31 $\int_1^2 \frac{dx}{4x-3}$ $\quad$ 9.32 $\int_{-1}^1 \left(5+\frac{1}{1+x^2}\right)dx$

9.33 $\int_0^{\sqrt{3}/2} \left(\frac{3}{\sqrt{1-x^2}}+\frac{4}{\sqrt{3-x^2}}\right)dx$ $\quad$ 9.34 $\int_{-2}^{2} e^{3x+4}dx$ $\quad$ 9.35 $\int_{\pi/4}^{\pi/6} \left(\sin 3x-\frac{1}{2}\cos 4x\right)dx$

9.36 $\int_0^2 \frac{2dx}{x^2+4}$ $\quad$ 9.38 $\int_{-3}^{-2} \frac{dx}{9+(x-2)^2}$ $\quad$ 9.38 $\int_5^8 \frac{3dx}{\sqrt{-4x+x^2}}$

9.39 $\int_{-\pi/2}^{\pi/2} \frac{\sin xdx}{1+\cos x}$ $\quad$ 9.40 $\int_0^1 \frac{xdx}{\sqrt{3x^2+8}}$ $\quad$ 9.41 $\int_0^4 \left(1+e^{\frac{x}{4}}\right)dx$

9.42 $\int_1^4 \frac{x+1}{\sqrt{x}}dx$ $\quad$ 9.43 $\int_1^4 \frac{dx}{\sqrt{3x+4}}$ $\quad$ 9.44 $\int_0^1 \left(1-\sqrt[3]{x}\right)dx$

9.45 $\int_1^2 \frac{dx}{3x-2}$ $\quad$ 9.46 $\int_0^1 \frac{dz}{(2z+1)^3}$ $\quad$ 9.48 $\int_0^{\pi} \cos\frac{x}{2}\cos\frac{3x}{2}dx$

9.48 $\int\limits_0^1 \frac{xdx}{\sqrt{1+3x}}$ 9.49 $\int\limits_{\ln 2}^{\ln 3} \frac{dx}{e^x+e^{-x}}$ 9.50 $\int\limits_0^{\pi/2} \frac{d\varphi}{2+\cos\varphi}$

9.51 $\int\limits_0^1 \frac{x^2dx}{\sqrt{4-x^2}}$ 9.52 $\int\limits_0^{a} x^2\sqrt{a^2-x^2}dx$ 9.53 $\int\limits_0^1 \ln(x+1)dx$

9.54 $\int\limits_a^{a\sqrt{3}} \frac{dx}{a^2+x^2}$ 9.55 $\int\limits_0^1 \frac{dx}{\sqrt{x^2+1}}$ 9.56 $\int\limits_1^{\sqrt{e}} \frac{dx}{x\sqrt{1-(\ln x)^2}}$

9.58 $\int\limits_{\pi/2}^{\pi/4} \frac{\cos^3 xdx}{\sqrt[3]{\sin x}}$ 9.58 $\int\limits_0^{\sqrt{2}/2} \frac{dx}{\sqrt{1-x^2}}$ 9.59 $\int\limits_1^e \frac{\sin(\ln x)}{x}dx$

9.60 $\int\limits_{\ln 2}^{\ln 3} \frac{dx}{ch^2x}$ 9.61 $\int\limits_0^1 \frac{z^3}{z^8+1}dz$ 9.62 $\int\limits_{\pi/6}^{\pi/3} \frac{dx}{\cos^2 x\sin^4 x}$

9.63 $\int\limits_2^3 \frac{dx}{\sqrt{5+4x-x^2}}$ 9.64 $\int\limits_1^2 \frac{e^xdx}{e^x-1}$ 9.65 $\int\limits_{\pi/6}^{\pi/3} tg^4xdx$

9.66 $\int\limits_0^2 \frac{2x-1}{2x+1}dx$ 9.68 $\int\limits_{e^2}^{e^3} \frac{dx}{x\ln x}$ 9.68 $\int\limits_{\pi/4}^{\pi/3} \frac{1+tg^2x}{(1+tgx)^2}dx$

Integrallarni hisoblang.

9.69. $\int\limits_4^5 x\sqrt{x^3-16}dx$ 9.80. $\int\limits_1^5 \frac{xdx}{\sqrt{1+3x}}$ 9.81. $\int\limits_1^2 \frac{4x+2}{2x-1}dx$

9.82. $\int\limits_4^9 \frac{\sqrt{x}dx}{\sqrt{x}-1}$ 9.83. $\int\limits_e^{e^2} \frac{2\ln x+1}{x}dx$ 9.84. $\int\limits_0^4 \frac{dx}{\sqrt{2x+1}}$

9.85. $\int\limits_{-1}^1 x^2\sqrt{1-x^2}dx$ 9.86. $\int\limits_{\ln 3}^{\ln 8} \frac{e^xdx}{\sqrt{e^x+1}}$

9.88. a) $\int\limits_0^{2\pi} x\cos xdx$ в) $\int\limits_0^{\ln 2} xe^xdx$ 9.88. $\int\limits_1^e x\ln xdx$ 9.89. $\int\limits_1^e \ln^2 xdx$

9.80. $\int\limits_0^{\sqrt{3}} arctgxdx$ 9.81. $\int\limits_{-1}^1 x^2e^{-x}dx$ 9.82. $\int\limits_0^{\pi} e^x\sin xdx$

9.83. $\int\limits_0^{\pi/3} (x+3)\sin xdx$ 9.84. $\int\limits_{-\pi}^{\pi} x\sin x\cos xdx$ 9.85. $\int\limits_0^{\pi/2} \cos^3 x\sin xdx$

9.86. $\int\limits_{-7}^7 \frac{x^4\sin x}{x^6+2}dx$ 9.88. $\int\limits_0^{\pi/2} \sin^3 xdx$ 9.88. $\int\limits_0^{\ln 2} \sqrt{e^x-1}dx$

9.89. $\int\limits_0^1 \frac{dx}{x^2+4x+5}$ 9.90. $\int\limits_0^{\pi} \frac{dx}{3+2\cos x}$ 9.91. $\int\limits_1^3 \frac{dx}{x\sqrt{x^2+5x+1}}$

9.92 . $\int\limits_0^{1/2} \sqrt{\frac{1+x}{1-x}}dx$ 9.93. $\int\limits_1^2 \frac{dx}{x^2+2x}$ 9.94. $\int\limits_{-1}^{7} \frac{dx}{1+\sqrt[3]{x+1}}$

9.95. $\int\limits_0^{\pi/4} tg^3xdx$ 9.96. $\int\limits_{-3}^3 x^2\sqrt{9-x^2}dx$ 9.98 $\int\limits_4^9 \frac{dx}{\sqrt{x}-1}$ $(x=t^2)$

9.98 $\int_{3}^{8} \frac{xdx}{\sqrt{1+x}}$ 9.99 $\int_{1}^{2} \frac{dx}{x\sqrt{1+x^2}}$ 9.100 $\int_{1}^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2} dx$

9.101 $\int_{0}^{1} \frac{\sqrt{e^x} dx}{\sqrt{e^x+e^{-x}}}$ 9.102 $\int_{3}^{29} \frac{\sqrt[3]{(x-2)^2} dx}{3+\sqrt[3]{(x-2)^2}}$ 9.103 $\int_{3}^{6} \frac{\sqrt{x^2-9}}{x^4} dx$

9.104 $\int_{\sqrt{2}/2}^{1} \frac{\sqrt{4-x^2}}{x^2} dx$ 9.105 $\int_{0}^{a} \frac{dx}{x+\sqrt{a^2-x^2}}$ 9.106 $\int_{0}^{\pi/2} \frac{dx}{2+\cos x}$

O'zgaruvchilarni almashtirish yordamida quyidagi integrallarni shaklini almashriting!

9.212 $a)\int_{0}^{3} x\sqrt{x+1}dx$ $в)\int_{1}^{3} \sqrt{x+1}dx$, $x=2t-1$ 9.213 $\int_{1/2}^{1} \frac{dx}{\sqrt{1-x^2}}$, $x=\sin t$

9.214 $\int_{3/4}^{4/3} \frac{dx}{\sqrt{x^2+1}}$, $x=\sin t$ 9.215 $\int_{0}^{\pi/2} f(x)dx$

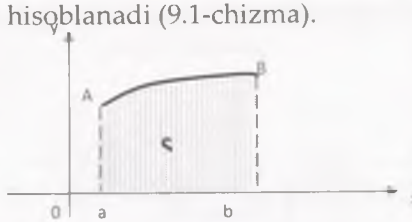
9.216 $\int_{0}^{4} \frac{dx}{1+\sqrt{x}}$, $x=t^2$

§ 9.2. Yassi figuralar yuzlarini hisoblash

1) Uzluksiz $y=f(x)$ $(f(x)\geq 0)$ egri chiziq $x=a$ va $x=b$ to'g'ri chiziqlar hamda Ox o'qining [a,b] kesmasi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

$$S=\int_{a}^{b} f(x)dx \qquad (9.13)$$

formula bilan hisoblanadi (9.1-chizma).



9.1- chizma

Agar [a,b] kesmada $f(x)\leq 0$ va uzluksiz bo'lsa, u holda aABb yuzasi

$$S=\left|\int_{a}^{b} f(x)dx\right| \qquad (9.14)$$

formula bilan topiladi.

2) Uzluksiz $x=\varphi(y)$ $(\varphi(y)\geq 0)$ egri chiziq, $y=c$ va $y=d$ to'g'ri chiziqlar hamda Oy o'qining [c,d] kesmasi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

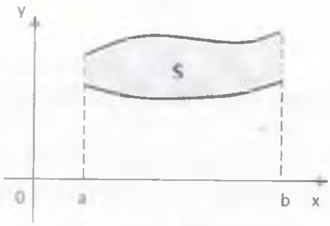
$$S=\int_c^d \varphi(y)dy \tag{9.15}$$

formula bilan hisoblanadi .

3) Uzluksiz $y=f_1(x)$ va $y=f_2(x)$ egri chiziqlar hamda $x=a$, $x=b$ $(a<b)$ to'g'ri chiziqlar bilan chegaralangan sohaning yuzi

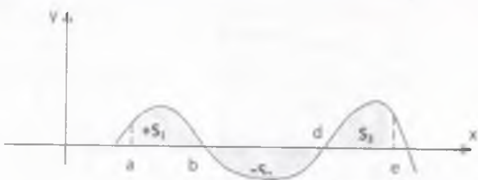
$$S=\int_a^b (f_2(x)-f_1(x))dx, \tag{9.16}$$

formula bilan hisoblanadi (9.2 - chizma).



9.2 – chizma

4) Agar [d,e] kesmada $y=f(x)$ funksiya uzluksiz va chekli sonda o'z ishorasini almashtirsin (9.3 - chizma). Masalan, [a,b], [b;d] musbat va [d,e] kesmada manfiy qiymatlarni qabul qilsin.



9.3 – chizma

U holda

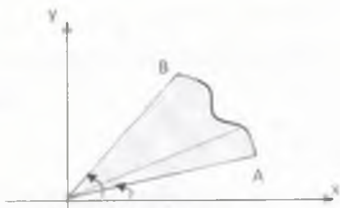
$$S=S_1+S_2+S_3=\int_a^b f(x)dx+\left|\int_b^d f(x)dx\right|+\int_d^c f(x)dx \; .$$

5) Yuqori chegarasi parametrik ko'rinishda berilgan egri chiziq $(x=\varphi(t), y=\psi(t))$ $(\alpha \le t \le \beta)$ va yon tomonlari $a=\varphi(x)$ va $b=\psi(x)$ chiziqlar bilan chegaralangan egri chiziqli trapetsiya yuzasini hisoblash uchun (9.13) formuladan foydalanamiz.

$$S=\int_a^b f(x)dx=\int_a^b ydx=\left|\begin{array}{l} y=\psi(t) \\ x=\varphi(t) \\ dx=\varphi'(t)dt \\ x=a \Rightarrow t=\alpha \\ x=b \Rightarrow t=\beta \end{array}\right|=\int_\alpha^\beta \psi(t)\varphi'(t)dt$$

6)Qutb koordinatalar sistemasida berilgan egri chiziqli OAB sektorning yuzini (9.4-chizma) quyidagi formula yordamida topamiz:

$$S=\lim_{\Delta\varphi\to 0}\sum\frac{1}{2}r^2\Delta\varphi=\frac{1}{2}\int_{\varphi_1}^{\varphi_2} r^2 d\varphi \qquad (9.18)$$



4 – chizma.

Quyidagi chiziqlar bilan chegaralangan figuralarning yuzini toping

9.108 $y=4-x^2,\quad y=0$

9.108 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

9.109 $y^2=2px,\quad x=h$

9.110 $y=3-2x-x^2,\quad y=0$

9.111 $xy=4,\quad x=1,\quad x=4,\quad y=0$

9.112 $y=\ln x,\quad x=e,\quad y=0$

9.113 $y^2=2x+4,\quad x=0$

9.114 $y^2=x^3,\quad y=8,\quad x=0$

9.115 $y^2=(4-x)^3,\quad x=0$

9.116 $4(y^2-x^2)+x^3=0$ egri chiziq ilmog'i

9.118 $y=x^2,\quad y=2-x^2$

9.118 $y=x^2+4x,\quad y=x+4$

9.119 $a^2y^2=x^3(2a-x)$

9.120 $(y-x)^2=x^3,\quad x=1$

9.121 $y^2(2a-x)=(x-a)^3$ atrofidagi ilmog'i

9.122 $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ zanjir chiziq $x=\pm a$ va $y=0$

9.123 $x=a(-\sin t)$, $y=a(1-\cos t)$ sikloidaning bir davri (arkasi) va OX o'qi

9.124 $x=a\cos^3 t$, $y=a\sin^3 t$ astroida

9.125 $r^2=a^2\cos 2\varphi$ limneskata

9.126 $r=a(1-\cos\varphi)$ kardioida

9.128 $y=4x-x^2$ parabola va Oxo'qi bilan chegaralangan.

9.128. $y=e^x$; $y=e^{x/2}$; $y=e^2$

9.129. $y=x^4-2x^2$; $y=0$

9.130. $y=3+2x-x^2$; $y=x+1$

9.131. $y=x^2+3$; $xy=4$; $y=2$; $x=0$

9.132. $y=x^3$; $y=-2x^2+3x$

9.133 . $y=\sqrt{1-x}$; $y=x+1$; $y=0$

9.134. $y=\cos 2x$; $y=0$; $x=0$; $x=\pi/4$

9.135. $y=2-x^4$; $y=x^2$

9.136. $xy=1$; $y=x^2$; $x=3$; $y=0$

9.138. $x=0$; $x=2$; $y=2^x$; $y=2x-x^2$

9.138. $y=\arcsin 2x$; $x=0$; $y=-\pi/2$

9.139. $y=x^2+1$; $x=y^2$; $3x+2y-16=0$; $x=0$

9.140. $y=(x+1)^2$; $y^2=x+1$

9.141. $y=4-x^2$; $y=x^2-2x$; $y=0$ (2-chorak)

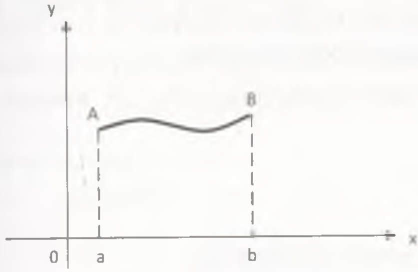
9.142. $y=-1/2x^2+2$; $x+2y-4=0$; $y=0$ (1-chorak)

§ 9.3. Yoy uzunligini hisoblash

1) To'g'ri burchakli Dekart koordinatalari sistemasida berilgan egri chiziq yoyining uzunligi [a;b] kesmada $y=f(x)$ tenglama bilan berilgan. Egri chiziq uchun $f'(x)$ uzluksiz bo'lsa, u holda uning yoyi uzunligi (9.5 - chizma).

$$l=\int_a^b\sqrt{1+(f'(x))^2}\,dx \tag{9.18}$$

formula bilan hisoblanadi.



9.5 – chizma

2) Agar egri chiziq $x = \varphi(y)$ $(c \le y \le d)$ ko'rinishda berilsa, yoyi uzunligi

$$l = \int_c^d \sqrt{1 + (\varphi'(y))^2}\,dy \qquad (9.19)$$

formula bilan hisoblanadi.

3) Agar egri chiziq $x = \varphi(t)$, $y = \psi(t)$ ko'rinishda berilgan bo'lib, $\varphi'(t)$, $\psi'(t)$ uzuliksiz funksiyalar bo'lsa, u holda egri chiziq yoyining uzunligi

$$l = \int_a^\beta \sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\,dt \qquad (9.20)$$

formula bilan hisoblanadi.

4) Qutb koordinatalar sistemasida berilgan silliq egri chiziq $\rho = f(\varphi)$, $\varphi_0 \le \varphi \le \varphi_1$ yoyining uzunligi

$$l = \int_{\varphi_0}^{\varphi_1} \sqrt{(\rho(\varphi))^2 + (\rho'(\varphi))^2}\,d\varphi \qquad (9.21)$$

formula bilan hisoblanadi.

Quyidagi egri chiziqlarning yoy uzunligini toping.

9.143 $a)\ \rho = a(1 + \cos\varphi)$, $a > 0$ $\quad в)\ x^{2/3} + y^{2/3} = a^{2/3}$ egri chiziqning uzunligi

9.144 $a)\ r = a\sqrt{\cos 2\varphi}$ $\quad в)\ x^2 + y^2 = a^2$ egri chiziqning uzunligi

9.145 $y^2 = (x+1)^3$ egri chiziqning $x = 4$ to'g'ri chiziq bilan kesilgan qismining uzunligi

9.146 $x = a(t - \sin t)$, $y = a(1 - \cos t)$ sikloida bir davrining uzunligi

9.148 $x=\frac{t^6}{6}$, $y=2-\frac{t^4}{4}$ egri chiziq koordinata o'qlari bilan nuqtalari orasidagi qismining uzunligi

9.148 $y=\frac{x^2}{2}-1$ egri chiziqning OX o'q kesgan bo'lagining uzunligi.

9.149 $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)=ach\frac{x}{a}$ egri chiziqning $x=\pm a$ to'g'ri chiziqlar orasidagi qismining uzunligi

9.150 $y=\ln x$ ning $x=\frac{3}{4}$ dan $x=\frac{12}{5}$ gacha bo'lgan qismining uzunligi

9.151 $y=\ln(2\cos x)$ egri chiziqning OY va OX o'qlar kesishgan ikki qo'shni nuqtalari orasidagi qismining uzunligi

9.152. $y=2\sqrt{x}$ chiziqni $x=0$ dan $x=1$ bo'lgan oraliqda.

9.153. $y=\ln x$ chiziqni $x=\sqrt{3}$ dan $x=\sqrt{8}$ bo'lgan oraliqda.

9.154. $y=\arcsin e^{-x}$ chiziqni $x=0$ dan $x=1$ bo'lgan oraliqda.

9.155 $y=t-\sin t$, $y=1-\cos t$ chiziqni $t=0$ dan $t=2\pi$ bo'lgan oraliqda.

§ 9.4. Aylanma jism sirtining yuzi ва hajmlarni hisoblash

1) Faraz qilaylik, $f(x)$ va $f'(x)$ funksiyalar [a;b] kesmada uzluksiz bo'lsin, hamda AB chiziq $y=f(x)\geq 0$ funksiyani grafigi bo'lsin. U holda AB egri chiziqning Ox o'q atrofida aylanishidan hosil bo'lgan sirtining yuzi (9.6 - chizma)

$$P=2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx \tag{9.22}$$

formula bilan hisoblanadi.

2) Agar silliq chiziq $x=\varphi(t)$, $y=\psi(t)$, $\alpha\leq t\leq\beta$ parametrik ko'rinishda berilgan bo'lsa, sirt yuzi

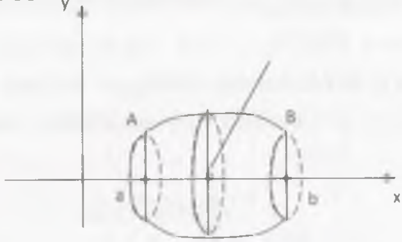
$$P=2\pi\int_\alpha^\beta \psi(x)\sqrt{(\varphi'(t))^2+(\psi'(t))^2}\,dt \tag{9.23}$$

formula bilan hisoblanadi.

3) Silliq egri chiziq qutb koordinatalar sistemasida $\rho = f(\varphi)$, $\varphi_0 \le \varphi \le \varphi_1$ ko'rinishda berilgan bo'lsa, uning qutb o'qi atrofida aylanishidan hosil bo'lgan jism sirtining yuzi

$$P = 2\pi \int_{\varphi_0}^{\varphi_1} \rho(\varphi)\sqrt{\rho^2(\varphi) + (\rho'(\varphi))^2}\, d\varphi \qquad (9.24)$$

formula bilan hisoblanadi.



9.6 – chizma.

9. Hajmlarni hisoblash

1) Agar jismni Ox o'qining x nuqtasida o'tkazilgan perpendikulyar tekisliklar bilan kesishdan hosil bo'lgan kesim yuzi $S(x)$ berilgan bo'lsa, jism hajmi (8.6-chizma)

$$V = \int_a^b S(x)dx$$

formula bilan hisoblanadi.

2) $y = f(x)$ egri chiziq, Ox o'q va $x = a$, $x = b$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Ox o'q atrofida aylanishidan hosil bo'lgan jism hajmi (8.6-chizma)

$$V_x = \pi \int_a^b (f(x))^2 dx = \pi \int_a^b y^2 dx \qquad (9.25)$$

formula bilan hisoblanadi.

3) $x = \varphi(y)$ egri chiziq Oy o'q va $y = c$, $y = d$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Oy o'q atrofida aylanishidan hosil bo'lgan jism hajmi

$$V_y = \pi \int_c^d x^2(y)dy \qquad (9.26)$$

formula bilan hisoblanadi.

4) Umumiy holda $y=f_1(x)$, $y=f_2(x)$ $(0\le f_1(x)\le f_2(x))$ egri chiziqlar bilan chegaralangan sohaning Ox o'q atrofida aylanishidan hosil bo'lgan jism hajmi

$$V_x=\pi\int_a^b\left(f_2^2(x)-f_1^2(x)\right)dx \qquad (9.28)$$

formula bilan hisoblanadi.

5) Agarda $y=f(x)$ $(a\le x\le b)$ egri chiziq parametrik $x=x(t)$, $y=y(t)$, $\alpha\le t\le\beta$ ko'rinishda berilgan bo'lsa, egri chiziqli trapetsiyaning Ox o'q atrofida aylanishidan hosil bo'lgan jismning hajmi

$$V=\pi\int_\alpha^\beta (y(t))^2x'(t)dt \qquad (9.28)$$

formula bilan hisoblanadi.

Quyidagi egri chiziqlarning aylanishidan hosil bo'lgan figurani sirtini toping

9.156 $y=x^3$, $x\in\left[0;\sqrt[4]{1/3}\right]$, OX o'q atrofida

9.158 $9y^2=x(3-x)^2$, $x\in[0;3]$, OX o'q atrofida

9.158 $x^2+y^2=9$, $x\in[-2;1]$, OX o'q atrofida

9.159 $x=\cos^3 t$, $y=\sin^3 t$, $t\in[0;\pi/2]$, OX o'q atrofida

9.160 $x^2+y^2=R^2$, OX o'q atrofida

9.161 $y=x^2/2$ ning $y=1,5$ to'g'ri chiziq bilan kesishgan qism, OY o'q atrofida

9.162 $y=ach\frac{x}{a}$ ning $x=\pm a$ to'g'ri chiziqlar orasidagi qismi, OX o'q atrofida

9.163 $4x^2+y^2=4$, OY o'q atrofida. Ko'rsatma: $P=\pi\int_0^2\sqrt{16-3y^2}dy$ bo'ladi. $y=\frac{4}{\sqrt{3}}\sin t$ almashtirish bilan hisoblanadi.

9.164 $y=\sin x$ egri chiziqning bitta yarim to'lqinni, OX o'q atrofida

9.165 $x=a(t-\sin t)$, $y=a(1-\cos t)$ sikloidaning bir davri, OX o'q atrofida

9.166 $x=t^2,\quad y=\frac{t}{3}(t^2-3)$ egri chiziq ilmog'i, OX o'q atrofida

9.168 $x^2+y^2=a^2,\quad x=b>a$ to'g'ri chiziq atrofida.Ko'rsatma: $2P=2\pi(b+x/ds+2\pi(b-x)ds)$

Quyidagi figuralarni Ox va Oy o'qlari atrofida aylanishidan hosil bo'lgan jism hajmini toping.

9.168 $y=x^3,\quad y=4x$

9.169 $y=\sin x,\quad y=0$ va $0\le x\le\pi$

9.180 $y=4/x,\quad x=1,\quad x=4,\quad y=0$

9.181 $y=1/2x^2-2x,\quad y=0$

9.182 $y=x^2,\quad xy=8,\quad y=0,\quad x=4$

9.183 $x=\sqrt{y-1},\quad x=0,\quad y=5$

9.184 $y=\ln x,\quad y=0,\quad x=e$

9.185 $y=-x^2+4,\quad y=x^2,\quad x=0$

9.186 $y=\sqrt{6x},\quad y=\sqrt{16-x^2},\quad x=0$

9.188 $y=x^2+1,\quad x=y^2+1,\quad y=0,\quad x=0,\quad x=2$

§ 9.5. Xosmas integral

$$\int_{-\infty}^{\infty} f(x)dx=\lim_{a\to-\infty}\int_a^c f(x)dx+\lim_{b\to+\infty}\int_c^b f(x)dx.$$

Agar ko'rsatilgan limitlar mavjud va chekli bo'lsa, xosmas integrallar yaqinlashuvchi, aks holda uzoqlashuvchi deyiladi. Ko'p hollarda berilgan xosmas integralning yaqinlashuvchi yoki uzoqlashuvchi ekanligini bilish va uning qiymatini baholash yetarli bo'ladi.

3) $f(x)$ funksiya $[\,a,b-\varepsilon\,]$ $(\varepsilon>0,\ \ \varepsilon<b-a)$ kesmada aniqlangan va integrallanuvchi bo'lib, b nuqta atrofida chegaralanmagan bo'lsa, $x=b$ nuqta $f(x)$ funksiyaning maxsus nuqtasi deyiladi $\varepsilon\to 0$ da $\int_a^{b-\varepsilon} f(x)dx$ integralning chekli limiti mavjud bo'lsa, bu limit $f(x)$ funksiyaning a dan b gacha xosmas integrali (2 tur xosmas integrali) deyiladi va

$$\int_a^b f(x)dx=\lim_{\varepsilon\to 0}\int_a^{b-\varepsilon} f(x)dx \qquad (9.30)$$

kabi belgilanadi. Bu holda (8.30) xosmas integral yaqinlashuvchi, aks holda uzoqlashuvchi deyiladi.

11. Funksiyaning o'rta qiymati. Agar $f(x)$ funksiya [a,b] kesmada integrallanuvchi bo'lsa, u holda, shunday $c \in [a;b]$ nuqta topiladiki,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

bo'ladi.

Xosmas integralni hisoblang (agar yaqinlashuvchi bo'lsa).

9.188 $a) \int_2^{+\infty} \frac{dx}{x^2-1}$ $b) \int_1^{\infty} \frac{dx}{x^2}$

9.189 a) $\int_0^{\pi/2} \frac{dx}{\cos x}$ b) $\int_1^{\infty} \frac{dx}{x}$

9.180 a) $\int_1^{3} \frac{dx}{\sqrt{4x-x^2-3}}$; b) $\int_1^{\infty} \frac{dx}{\sqrt{x}}$

9.181 $\int_1^{\infty} \frac{dx}{x^4}$ 9.182 $\int_0^{\infty} e^{-x}dx$ 9.183 $\int_0^{\infty} xe^{-x^2}dx$

9.184 $\int_1^{\infty} \frac{dx}{1+x^2}$ 9.185 $\int_1^{\infty} \frac{dx}{x^2\sqrt{x^2-1}}$ 9.186 $\int_1^{\infty} \frac{dx}{x^2+x}$

9.188 $\int_0^{\infty} x^2e^{-x^3}dx$ 9.188 $\int_1^{\infty} \frac{dx}{x\sqrt{x^2-1}}$ 9.189 $\int_1^{\infty} \frac{arctgx}{x^2}dx$

9.190 $\int_1^{\infty} \frac{dx}{(x^2+1)^2}$ 9.191 $a) \int_{-1}^{1} \frac{dx}{x^2}$ $b) \int_2^{6} \frac{dx}{\sqrt[3]{(4-x)^2}}$

9.192 $a) \int_0^1 \frac{dx}{\sqrt{1-x^4}}$ $b) \int_1^2 \frac{dx}{(x-1)^2}$ 9.193 $\int_0^2 \frac{dx}{\sqrt[3]{(x-1)^2}}$

9.194 $\int_1^{\infty} \frac{dx}{(1+x)\sqrt{x}}$ 9.195 $\int_1^{\infty} \frac{x^4dx}{(x^5+1)^4}$ 9.196 $\int_{-\infty}^{-2} \frac{dx}{x\sqrt{x^2-1}}$

9.198 $\int_0^{\infty} \frac{shx}{sh2x}dx$ 9.198 $\int_0^{\infty} \frac{dx}{e^x+\sqrt{e^x}}$ 9.199 $\int_2^{\infty} \frac{xdx}{x^3-1}$

9.200 $\int_1^{\infty} \frac{dx}{(x+1)^4}$ 9.201 $\int_{-\infty}^{-1} \frac{dx}{\sqrt[3]{(2x+1)^2}}$ 9.202 $\int_1^{\infty} \frac{\ln x}{x^2}dx$

9.203 $\int_{-\infty}^{\infty} \frac{dx}{x^2+4x+9}$ 9.204 $\int_0^{\infty} e^{-x}\sin xdx$

9.205 $\int_0^{\infty} arctgxdx$

9.206 $\int\limits_{-\infty}^{\infty} xe^{-x^2}dx$ 9.208 $\int\limits_{-\infty}^{0} e^{-x^2}dx$ 9.208 $\int\limits_{0}^{3} \frac{dx}{\sqrt{9-x^2}}$

9.209 $\int\limits_{1}^{5} \frac{dx}{\sqrt{5-x}}$ 9.210 $\int\limits_{-1}^{0} \frac{dx}{(x+1)^2}$ 9.211 $\int\limits_{1}^{e} \ln x dx$

Xosmas integrallarni hisoblang (yoki uzoqlashuvchi ekanligini ko'rsatib bering).

9.212. $\int\limits_{0}^{1} \frac{dx}{\sqrt{x}}$ 9.213. $\int\limits_{-1}^{2} \frac{dx}{x}$ 9.214. $\int\limits_{0}^{1} \frac{dx}{x^p}$

9.215. $\int\limits_{0}^{3} \frac{dx}{(x-1)^2}$ 9.216. $\int\limits_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$ 9.218. $\int\limits_{1}^{\infty} \frac{dx}{x}$

9.218. $\int\limits_{1}^{\infty} \frac{dx}{x^c}$ 9.219. $\int\limits_{1}^{\infty} \frac{dx}{x^p}$ 9.220. $\int\limits_{-\infty}^{\infty} \frac{dx}{1+x^2}$

9.221. $\int\limits_{-\infty}^{\infty} \frac{dx}{x^2+4x+9}$ 9.222. $\int\limits_{0}^{\infty} \sin x dx$ 9.223. $\int\limits_{0}^{1/2} \frac{dx}{x\ln x}$

9.224. $\int\limits_{0}^{1/2} \frac{dx}{x\ln^2 x}$ 9.225. $\int\limits_{a}^{\infty} \frac{dx}{x\ln^2 x}$ $(a>1)$ 9.226. $\int\limits_{0}^{\pi/2} ctgx dx$

9.228. $\int\limits_{0}^{\infty} e^{-kx}dx$ $(k>0)$ 9.228. $\int\limits_{0}^{\infty} \frac{arctgx}{x^2+1}dx$ 9.229. $\int\limits_{2}^{\infty} \frac{dx}{(x^2-1)^2}$

9.230. $\int\limits_{0}^{\infty} \frac{dx}{x^3+1}$

Quyidagi integralni hisoblang.

9.231. $\int\limits_{3}^{29} \frac{(x-2)^{2/3}}{(x-2)^{2/3}+3}dx$, $x-2=z^3$ 9.232. $\int\limits_{0}^{\ln 2} \sqrt{e^x-1}dx$, $e^x-1=z^2$

9.233. $\int\limits_{0}^{\pi} \frac{dt}{3+2\cos t}$, $tg\frac{t}{2}=z$ 9.234. $\int\limits_{0}^{\pi/2} \frac{dx}{1+a^2\sin^2 t}$, $tgx=t$

Quyidagi funksiyalarning o'rta qiymatlari aniqlansin:

9.235 $y=\sin x$, $[0;\pi]$ intervalda

9.236 $y=tgx$, $\left[0;\frac{\pi}{3}\right]$ intervalda

9.238 $y=\ln x$, $[1;e]$ 9.238 $y=x^2$, $[a;b]$ intervalda

9.239 $y=\frac{1}{x^2+1}$, $[-1;1]$ intervalda

<u>Takrorlash uchun savollar</u>

1.Aniq integral deb nimaga aytiladi?

2.Aniq integralni hisoblash usullari haqida ayting.

3.Nyuton-Leybnits formulasi qanday ko'rinishda bo'ladi?
4.Aniq integralni bo'laklab integrallash formulasini keltirib chiqaring.
5.Tekislikdagi geometrik shakllarning yuzi aniq integral orqali qanday hisoblanadi?
6.Aylanma jismning hajmi aniq integral orqali qanday hisoblanadi?
7.I- tur xosmas integral qanday ta'riflanadi?
8.II-tur xosmas integralning geometrik mazmuni nimadan iborat?

ANIQ INTEGRALGA DOIR NAZORAT TESTLARI

1.Aniq integral va uning xossalari

1. $[a, b]$ kesma bo'yicha $y=f(x)$ funksiya uchun $S_n(f)$ integral yig'indi tuzishda quyidagi amallardan qaysi biri bajarilmaydi ?

A) $[a, b]$ kesma x_i $(i=1, 2, \cdots, n-1)$ va $x_0=a$, $x_n=b$ nuqtalar bilan n bo'lakka ajratiladi.

B) $[x_{i-1}, x_i]$ $(i=1, 2, \cdots, n)$ kesmalardan ixtiyoriy ξ_i nuqtalar tanlanadi.

C) tanlangan ξ_i nuqtalarda $f(x)$ funksiya qiymatlari $f(\xi_i)$ hisoblanadi.

D) $[x_{i-1}, x_i]$ $(i=1, 2, \cdots, n)$ kesmalarning uzunliklari $x_i - x_{i-1} = \Delta x_i$ topiladi.

E) ko'rsatilgan barcha amallar bajariladi .

2. $[a, b]$ kesmada aniqlangan $y=f(x)$ funksiya uchun tuzilgan

$$S_n(f) = \sum_{k=1}^{n} f(\xi_k)\Delta x_k$$

integral yig'indi orqali aniq integral qanday aniqlanadi ?

A) $\int_a^b f(x)dx = S_n(f)$. B) $\int_a^b f(x)dx = \max S_n(f)$.

C) $\int_a^b f(x)dx = \min S_n(f)$. D) $\int_a^b f(x)dx = \lim\limits_{\substack{n\to\infty,\\ \max\Delta x\to 0}} S_n(f)$

E) to'g'ri javob keltirilmagan.

3. $\int_a^b f(x)dx$ aniq integral uchun quyidagi tushunchalardan qaysi biri aniqlanmagan?

A) a - integralning quyi chegarasi .

B) b - integralning yuqori chegarasi .

C) $f(x)$ – integral osti funksiyasi .

D) $f(x)dx$ – integral osti ifodasi .

E) keltirilgan tushunchalarning hammasi aniqlangan.

4. Aniq integralning geometrik ma'nosini ko'rsating.

A) egri chiziqli trapetsiyaga urinma chiziqning burchak koeffitsienti.

B) egri chiziqli trapetsiyaning perimetri.

C) egri chiziqli trapetsiyaning o'rta chizigi uzunligi.

D) egri chiziqli trapetsiyaning yuzi.

E) to'g'ri javob keltirilmagan.

5. Aniq integralning mexanik ma'nosini ko'rsating.

A) o'zgaruvchi kuchning eng katta qiymati.

B) o'zgaruvchi kuchning eng kichik qiymati.

C) o'zgaruvchi kuchning momenti.

D) o'zgaruvchi kuchning bajargan ish.

E) o'zgaruvchi kuchning o'rta qiymati.

6. $[a,b]$ kesmada aniqlangan $y=f(x)$ funksiya qanday shartni qanoatlantirganda $\int_a^b f(x)dx$ aniq integral doimo mavjud bo'ladi ?

A) yuqoridan chegaralangan.

B) quyidan chegaralangan.

C) o'suvchi. D) kamayuvchi. E) uzluksiz.

7. Aniq integralning xossasi qayerda xato ko'rsatilgan ?

A) $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.

B) $\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$.

C) $\int\limits_a^b f(x)g(x)dx=\int\limits_a^b f(x)dx\cdot\int\limits_a^b g(x)dx$.

D) $\int\limits_a^b kf(x)dx=k\int\limits_a^b f(x)dx\ (k-const.)$.

E) barcha xossalar to'g'ri ko'rsatilgan.

8. Aniq integral xossasini ifodalovchi

$$\int\limits_a^b f(x)dx=\int\limits_a^c f(x)dx+\int\limits_c^b f(x)dx$$

tenglik bajarilishi uchun c nuqta qanday shartni qanoatlantirishi kerak ?

A) $c<a$. B) $c>b$, C) $c=a$ yoki $c=b$. D) $a<c<b$.

E) ko'rsatilgan barcha shartlarda tenglik bajariladi.

9. Aniq integral uchun quyidagi tengliklardan qaysi biri o'rinli emas ?

A) $\int\limits_a^a f(x)dx=0$. B) $\int\limits_a^b f(x)dx=\int\limits_b^a f(x)dx$.

C) $\int\limits_a^b kf(x)dx=k\int\limits_a^b f(x)dx$. D) $\int\limits_a^b f(x)dx=\int\limits_a^b f(t)dt$.

E) keltirilgan barcha tengliklar o'rinli.

10. O'rta qiymat haqidagi teoremaning tasdiqini ifodalovchi tenglik qayerda to'g'ri ko'rsatilgan ?

A) $\int\limits_a^b f(x)dx=f(\xi),\ \xi\in[a,b]$. B) $\int\limits_a^b f(x)dx=f(\xi)(b-a),\ \xi\in[a,b]$.

C) $\int\limits_a^b f(x)dx=\dfrac{f(\xi)}{b-a},\ \xi\in[a,b]$. D) $\int\limits_a^b f(x)dx=\dfrac{b-a}{f(\xi)},\ \xi\in[a,b]$.

E) $\int\limits_a^b f(x)dx=\xi[f(b)-f(a)],\ \xi\in[a,b]$.

11. $I=\int\limits_0^{\pi/2}\sin 2x dx$ integral uchun baho qayerda to'g'ri ko'rsatilgan ?

A) $0\le I\le 1$. B) $0\le I\le\pi$. C) $0\le I\le 1$.

D) $0\le I\le\pi/2$. E) $\pi/2\le I\le\pi$.

12. $\int_{-1}^{2}|x-1|dx$ integralning qiymati qaysi oraliqqa tegishli bo'ladi ?

A) (–1, 2). B) (0, 6). C) (1, 3).
D) (–1, 3). E) (0,1).

13. $\int_{a}^{b}dx$ integral qiymati nimaga teng ?

A) $b-a$. B) $a-b$. C) $b+a$. D) b^2-a^2. E) $(a+b)/2$.

14. $\int_{-2}^{5}dx$ integral qiymati nimaga teng ?

A) 3. B) –3. C) 7. D) –7. E) 1.5 .

15. $\int_{-1}^{4}3dx$ integral qiymati nimaga teng ?

A) 5. B) –5. C) –15. D) 15. E) 9.

16. $\int_{-1}^{4}(-4)dx$ integral qiymati nimaga teng ?

A) 5. B) –5. C)–20. D) 20. E) 12.

2.Aniq integralni hisoblash usullari

1. Agar $y=f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo'lsa, unda yuqori chegarasi o'zgaruvchi bo'lgan aniq integral orqali aniqlanadigan

$$\Phi(x)=\int_{a}^{x}f(t)dt, x\in[a,b]$$

funksiya xossasi qayerda to'g'ri ko'rsatilgan ?

A) $\Phi'(x)=f(a)$. B) $\Phi'(x)=f(x)$. C) $\Phi'(x)=0$.
D) $\Phi'(x)=xf(x)$. E) $\Phi'(x)=f(x)/x$.

2. $\int_{a}^{b}f(x)dx$ aniq integralni Nyuton–Leybnits formulasi bilan hisoblashda quyidagi amallardan qaysi biri bajarilmaydi ?

A) $f(x)$ funksiyaning biror $F(x)$ boshlang'ich funksiyasi topiladi.

B) boshlang'ich funksiyaning $F(a)$ va $F(b)$ qiymatlari hisoblanadi.

C) $F(b)+F(a)$ yig'indi hisoblanadi.

D) $F(b)-F(a)$ ayirma hisoblanadi.

E) ko'rsatilgan barcha amallar bajariladi .

3. Agar $y=F(x)$ berilgan $[a,b]$ kesmada $y=f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, unda aniq integral uchun Nyuton–Leybnits formulasi qayerda to'g'ri ifodalangan ?

A) $\int_a^b f(x)dx = F(x)$. B) $\int_a^b f(x)dx = F(a)\cdot F(b)$.

C) $\int_a^b f(x)dx = F(b)/F(a)$. D) $\int_a^b f(x)dx = F(b)-F(a)$.

E) $\int_a^b f(x)dx = F(b)+F(a)$.

4. $\int_0^1 xdx$ aniq integralning qiymatini Nyuton–Leybnits formulasi yordamida toping.

A) 1. B) 0. C) 1/2. D) –1. E) 2/3.

5. $\int_0^{\pi/3} \sin 3x dx$ aniq integralning qiymatini Nyuton–Leybnits formulasidan foydalanib aniqlang.

A) 0. B) 1. C) 2/3. D) 0.75. E) –0.3.

6. $\int_0^{\pi} \sin^2 x dx$ aniq integralning qiymatini Nyuton–Leybnits formulasi yordamida hisoblang.

A) π . B) $\pi/2$. C) $\pi/3$. D) $\pi/4$. E) $\pi/6$.

7. $\int_0^{\pi} \cos^2 x dx$ aniq integral qiymatini Nyuton–Leybnits formulasi yordamida toping.

A) π . B) $\pi/2$. C) $\pi/3$. D) $\pi/4$. E) $\pi/6$.

8. $\int_a^b udv$ aniq integralni bo'laklab integrallash formulasi yordamida hisoblash jarayonida quyidagi amallardan qaysi biri bajarilmaydi ?

A) $u=u(x)$ funksiyaning du differensiali hisoblanadi.

B) $dv=dv(x)$ differensial bo'yicha $v=v(x)$ funksiya topiladi.

C) $u(b)v(b)+u(a)v(a)$ yig'indi hisoblanadi.

D) $\int_a^b vdu$ integral hisoblanadi.

E) ko'rsatilgan barcha amallar bajariladi.

9. Aniq integralni bo'laklab integrallash formulasi qayerda to'g'ri yozilgan?

A) $\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$. B) $\int_a^b udv = uv\Big|_a^b + \int_a^b vdu$.

C) $\int_a^b udv = \int_a^b vdu$. D) $\int_a^b udv = uv\Big|_a^b$. E) $\int_a^b udv = uv - \int vdu$.

10. $\int_0^\pi x\sin xdx$ aniq integral qiymatini bo'laklab integrallash formulasi yordamida toping.

A) π. B) $\pi/2$. C) $\pi/3$. D) $\pi/4$. E) $\pi/6$.

11. $\int_0^1 xe^x dx$ aniq integralni bo'laklab integrallash formulasi yordamida hisoblang.

A) e. B) $e/2$. C) 1. D) 0.5. E) 2.

12. $\int_a^b f(x)dx$ aniq integralni o'zgaruvchilarni almashtirish formulasi orqali hisoblashda quyidagi amallardan qaysi biri bajarilmaydi ?

A) $x=\phi(t)$ almashtirma tanlanadi.

B) $\phi(t)=a$ va $\phi(t)=b$ tenglamalarning yechimlari α va β topiladi.

C) $f[\phi(t)]$ murakkab funksiya tuziladi.

D) almashtirmaning hosilasi $\phi'(t)$ hisoblanadi.

E) ko'rsatilgan barcha amallar bajariladi.

13. $\int_a^b f(x)dx$ aniq integralni $x=\varphi(t)$ almashtirma orqali hisoblash formulasini ko'rsating.

A) $\int_a^b f(x)dx = \int_a^b \varphi(t)dt$. B) $\int_a^b f(x)dx = \int_a^b f[\varphi(t)]dt$.

C) $\int_a^b f(x)dx=\int_\alpha^\beta f(t)\varphi'(t)dt$.   D) $\int_a^b f(x)dx=\int_a^b f[\varphi(t)]\varphi(t)dt$.

E) $\int_a^b f(x)dx=\int_\alpha^\beta f[\varphi(t)]\varphi'(t)dt$.

14. $\int_0^{\sqrt{\pi}} x\sin x^2 dx$ aniq integralni hisoblash uchun qaysi almashtirmadan foydalanish mumkin?

A) $x=t^2$.   B) $x^2=t$.   C) $x=\sin t$.

D) $\sin x=t$.   E) $\sin x^2=t$.

15. $\int_0^{\sqrt{\pi}} x\sin x^2 dx$ aniq integral qiymatini o'zgaruvchilarni almashtirish usulida hisoblang.

A) $\sqrt{\pi}$.   B) $\pi/2$.   C) 0.   D) 1.   E) π.

16. $\int_1^5 \frac{dx}{x\sqrt{3x+1}}$ aniq integral qiymatini hisoblash uchun qaysi almashtirmadan foydalanish mumkin?

A) $t=1/x$. B) $t=\sqrt{3x+1}$. C) $t=1/\sqrt{3x+1}$. D) $t=\sqrt{x}$. E) $t=x\sqrt{3x+1}$.

3.Aniq integralni taqribiy hisoblash formulalari

1.Qaysi holda $\int_a^b f(x)dx$ aniq integralni taqribiy hisoblashga to'g'ri keladi ?

A) integral elementar funksiyalarda ifodalanmaydi.

B) integral ostidagi funksiyani boshlang'ich funksiyasi $F(x)$ noma'lum.

C) integral ostidagi funksiya elementar emas.

D) integral ostidagi $f(x)$ funksiya jadval ko'rinishida berilgan.

E) keltirilgan barcha hollarda .

2. $\int_a^b f(x)dx$ integral qiymatini to'g'ri to'rtburchaklar formulasi yordamida taqribiy hisoblash uchun quyidagi amallardan qaysi biri bajarilmaydi?

A) $[a,b]$ kesma x_k (k=1, 2, ..., n–1) nuqtalar bilan n ta teng Δx_k (k=1, 2, ..., n) bo'laklarka ajratiladi.

B) x_k (k=1, 2, ..., n) bo'linish nyqtalarida integral ostidagi funksiyaning $f(x_k)$ qiymatlari hisoblanadi.

C) asoslari $\Delta x_k=(b-a)/n$ va balandliklari $h_k=f(x_k)$ (k=1, 2, ..., n) bo'lgan to'g'ri to'rtburchaklarning yuzalari $S_k= f(x_k)\Delta x_k$ hisoblanadi.

D) $S_k= f(x_k)\Delta x_k$ (k=1, 2, ..., n) yuzalarning ko'paytmasi topiladi;

E) ko'rsatilgan barcha amallar bajariladi.

3. $[a, b]$ kesma $a=x_0, x_1, x_2,..., x_n=b$ nuqtalar bilan n ta teng bo'lakka ajratilgan. Quyidagi yig'indilardan qaysi biri $\int_a^b f(x)dx$ integralni taqribiy hisoblash uchun qo'llaniladigan to'g'ri to'rtburchaklar formulasini to'g'ri ifodalaydi ?

A) $\int_a^b f(x)dx \approx \frac{b+a}{n}\sum_{k=1}^{n} f(x_k)$. B) $\int_a^b f(x)dx \approx \frac{b-a}{n}\sum_{k=1}^{n} f(x_k)$.

C) $\int_a^b f(x)dx \approx \frac{b+a}{2n}\sum_{k=1}^{n} f(x_k)$. D) $\int_a^b f(x)dx \approx \frac{b-a}{2n}\sum_{k=1}^{n} f(x_k)$.

E) $\int_a^b f(x)dx \approx (b-a)\sum_{k=1}^{n} f(x_k)$.

4. To'g'ri to'rtburchaklar formulasi yordamida [0,1] kesmani n=10 bo'lakka ajratish orqali $S=\int_0^1 (1+x^2)dx$ integralning taqribiy qiymatini toping.

A) $S \approx 1.145$. B) $S \approx 1.275$. C) $S \approx 1.335$. D) $S \approx 1.385$. E) $S \approx 1.405$.

5. $\int_a^b f(x)dx$ integralni taqribiy hisoblash uchun qo'llaniladigan to'g'ri to'rtburchaklar formulasining absolut xatoligi Δ qanday baholanadi ?

A) $\Delta \le \frac{(b-a)^2}{12n} M$, $M = \max_{x\in[a,b]} |f'(x)|$.

B) $\Delta \le \frac{(b-a)^2}{4n} M$, $M = \max_{x\in[a,b]} |f'(x)|$.

C) $\Delta \le \frac{(b-a)^3}{12n^2} M$, $M = \max\limits_{x\in[a,b]} |f'(x)|$.

D) $\Delta \le \frac{(b-a)^2}{n} M$, $M = \max\limits_{x\in[a,b]} |f''(x)|$.

E) $\Delta \le \frac{(b-a)^3}{n^2} M$, $M = \max\limits_{x\in[a,b]} |f'(x)|$.

6. To'g'ri to'rtburchaklar formulasi yordamida [0,1] kesmani n=10 bo'lakka ajratish orqali $S = \int_0^1 (1+x^2)dx$ integralning topilgan taqribiy qiymatining Δ absolut xatoligini baholang.

A) $\Delta \le 0.05$. B) $\Delta \le 0.04$. C) $\Delta \le 0.03$. D) $\Delta \le 0.02$. E) $\Delta \le 0.01$.

7. $\int_a^b f(x)dx$ integral qiymatini trapetsiyalar formulasi yordamida taqribiy hisoblash uchun quyidagi amallardan qaysi biri bajarilmaydi?

A) $[a,b]$ kesma x_k (k=1, 2, ..., n–1) nuqtalar bilan n ta teng Δx_k (k=1, 2, ..., n) bo'laklarga ajratiladi.

B) x_k (k=1, 2, ..., n) bo'linish nuqtalarida integral ostidagi funksiyaning $f(x_k)$ qiymatlari hisoblanadi.

C) asoslari $f(x_{k-1})$ va $f(x_k)$, balandligi $h_k=\Delta x_k=(b-a)/n$ (k=1, 2, ..., n) bo'lgan trapetsiyalarning yuzalari $S_k=\Delta x_k[\,f(x_{k-1})+f(x_k)]/2$ hisoblanadi.

D) S_k(k=1, 2, ..., n) yuzalarning yig'indisi topiladi.

E) ko'rsatilgan barcha amallar bajariladi.

8. $[a, b]$ kesma $a=x_0, x_1, x_2,\ldots, x_n=b$ nuqtalar bilan n ta teng bo'lakka ajratilgan. Quyidagi yig'indilardan qaysi biri $\int_a^b f(x)dx$ integralni taqribiy hisoblash uchun qo'llaniladigan trapetsiyalar formulasini to'g'ri ifodalaydi ?

A) $\int_a^b f(x)dx \approx \frac{b-a}{n}\{2[f(a)+f(b)]+\sum_{k=1}^{n-1} f(x_k)\}$.

B) $\int_a^b f(x)dx \approx \frac{b-a}{n}[\frac{f(a)+f(b)}{2}+2\sum_{k=1}^{n-1} f(x_k)]$.

C) $\int_a^b f(x)dx \approx \frac{b-a}{n}[f(a)+f(b)+2\sum_{k=1}^{n-1} f(x_k)]$.

D) $\int_a^b f(x)dx \approx \frac{b-a}{n}[f(a)+f(b)+\frac{1}{2}\sum_{k=1}^{n-1} f(x_k)]$.

E) $\int_a^b f(x)dx \approx \frac{b-a}{n}[\frac{f(a)+f(b)}{2}+\sum_{k=1}^{n-1} f(x_k)]$.

9. Trapetsiyalar formulasi yordamida [0,1] kesmani n=10 bo'lakka ajratish orqali $S=\int_0^1(1+x^2)dx$ integralning taqribiy qiymatini toping.

A) $S\approx 1.145$. B) $S\approx 1.275$.
C) $S\approx 1.335$. D) $S\approx 1.385$. E) $S\approx 1.405$.

10. $\int_a^b f(x)dx$ integralni taqribiy hisoblash uchun qo'llaniladigan trapetsiyalar formulasining absolut xatoligi Δ qanday baholanadi ?

A) $\Delta\le\frac{(b-a)^2}{12n}M$, $M=\max_{x\in[a,b]}|f(x)|$.

B) $\Delta\le\frac{(b-a)^2}{8n}M$, $M=\max_{x\in[a,b]}|f'(x)|$.

C) $\Delta\le\frac{(b-a)^3}{12n^2}M$, $M=\max_{x\in[a,b]}|f''(x)|$.

D) $\Delta\le\frac{(b-a)^2}{n}M$, $M=\max_{x\in[a,b]}|f''(x)|$.

E) $\Delta\le\frac{(b-a)^3}{n^2}M$, $M=\max_{x\in[a,b]}|f'(x)|$.

11. Trapetsiyalar formulasi yordamida [0,1] kesmani n=10 bo'lakka ajratish orqali $S=\int_0^1(1+x^2)dx$ integralning topilgan taqribiy qiymatining Δ absolut xatoligini baholang.

A) Δ≤0.005. B) Δ≤0.004. C) Δ≤0.003. D) Δ≤0.002.
E) Δ≤0.001.

12. $[a, b]$ kesma $a=x_0, x_1, x_2,\ldots, x_{2n}=b$ nuqtalar bilan $2n$ ta teng bo'lakka ajratilgan. Quyidagi yig'indilardan qaysi biri $\int_a^b f(x)dx$ integralni taqribiy hisoblash uchun qo'llaniladigan parabolalar formulasini to'g'ri ifodalaydi ?

A) $\int_a^b f(x)dx \approx \frac{b-a}{6n}\sum_{k=0}^{n-1}[f(x_{2k})+f(x_{2k+1})+f(x_{2k+2})]$.

B) $\int_a^b f(x)dx \approx \frac{b-a}{6n}\sum_{k=0}^{n-1}[4f(x_{2k})+f(x_{2k+1})+f(x_{2k+2})]$.

C) $\int_a^b f(x)dx \approx \frac{b-a}{6n}\sum_{k=0}^{n-1}[f(x_{2k})+4f(x_{2k+1})+f(x_{2k+2})]$.

D) $\int_a^b f(x)dx \approx \frac{b-a}{6n}\sum_{k=0}^{n-1}[f(x_{2k})+f(x_{2k+1})+4f(x_{2k+2})]$.

E) $\int_a^b f(x)dx \approx \frac{b-a}{6n}\sum_{k=0}^{n-1}[4f(x_{2k})+f(x_{2k+1})+4f(x_{2k+2})]$.

13. $\int_a^b f(x)dx$ integralni taqribiy hisoblash uchun qo'llaniladigan parabolalar formulasining absolut xatoligi Δ qanday baholanadi ?

A) $\Delta \le \frac{(b-a)^2}{12n}M$, $M = \max_{x\in[a,b]}|f(x)|$.

B) $\Delta \le \frac{(b-a)^2}{4n}M$, $M = \max_{x\in[a,b]}|f'(x)|$.

C) $\Delta \le \frac{(b-a)^3}{12n^2}M$, $M = \max_{x\in[a,b]}|f''(x)|$.

D) $\Delta \le \frac{(b-a)^5}{2880n^4}M$, $M = \max_{x\in[a,b]}|f^{(4)}(x)|$.

E) $\Delta \le \frac{(b-a)^3}{n^3}M$, $M = \max_{x\in[a,b]}|f'''(x)|$.

4. Aniq integralning geometrik masalalarga tatbiqlari

1. $y=f(x)$ funksiya grafigi, $x=a$ va $x=b$ vertikal chiziqlar hamda $y=0$ gorizontal to'gri chiziq bilan chegaralangan egri chiziqli trapetsiyaning S yuzini umumiy holda hisoblash formulasi qayerda to'g'ri ko'rsatilgan ?

A) $S=\int_a^b f(x)dx$. B) $S=\left|\int_a^b f(x)dx\right|$. C) $S=\int_a^b |f(x)|dx$.

D) $S=\pm\left|\int_a^b f(x)dx\right|$. E) $S=\int_a^b f^2(x)dx$.

2. $y=x^3$, $x=0$, $x=2$ va $y=0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning S yuzini toping .

A) $S=1$. B) $S=2$. C) $S=3$.

D) $S=4$. E) $S=5$.

3. $y=x^3$, $x=-2$, $x=2$ va $y=0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning S yuzini toping .

A) $S=0$. B) $S=2$. C) $S=4$. D) $S=6$. E) $S=8$.

4. $y=\sin x$, $x=0$, $x=\pi/2$ va $y=0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning S yuzini toping .

A) $S=1$. B) $S=2$. C) $S=3$. D) $S=4$. E) $S=5$.

5. $y=\sin x$, $x=0$, $x=\pi$ va $y=0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning S yuzini toping .

A) $S=1$. B) $S=2$. C) $S=3$. D) $S=4$. E) $S=5$.

6. $y=\cos x$, $x=0$, $x=\pi$ va $y=0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning S yuzini toping .

A) $S=0$. B) $S=1$. C) $S=2$. D) $S=3$. E) $S=4$.

7. Quyidagi $y=f(x)$ funksiya grafigi, $x=0$ va $x=2$ hamda $y=0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning S yuzini toping:

$$f(x)=\begin{cases} x, & 0\le x<1, \\ x^2, & 1\le x\le 2 \end{cases}.$$

A) $S=2$. B) $S=8/3$. C) $S=17/6$.

D) $S=20/7$. E) $S=5$.

8. $y=f(x)$ va $y=g(x)$, $f(x)\le g(x)$, funksiyalarning grafiklari, $x=a$ va $x=b$ vertikal chiziqlar bilan chegaralangan geometrik shaklning S yuzini hisoblash formulasi qayerda to'g'ri ko'rsatilgan ?

A) $S=\int_a^b[f(x)+g(x)]dx$. B) $S=\int_a^b[f(x)-g(x)]dx$.

C) $S=\int_a^b f(x)g(x)dx$. D) $S=\int_a^b[f(x)\pm g(x)]dx$.

E) $S=\int_a^b[g(x)-f(x)]dx$.

9. $y=x^2$ va $y=x^4$ egri ciziqlar bilan chegaralangan shaklning S yuzini toping.

A) $S=4$. B) $S=2$. C) $S=3/10$.

D) $S=2/15$. E) $S=3/8$.

10. $y=f(x)$ va $y=g(x)$, $f(x)\le g(x)$, funksiyalarning grafiklari, $x=a$ va $x=b$ vertikal chiziqlar bilan chegaralangan geometrik shaklning S yuzini hisoblash formulasi qayerda noto'g'ri ko'rsatilgan ?

A) $S=\int_a^b |f(x)-g(x)|dx$. B) $S=\left|\int_a^b [f(x)-g(x)]dx\right|$.

C) $S=\int_a^b |f(x)|dx-\int_a^b |g(x)|dx$. D) $S=\left|\int_a^b g(x)dx-\int_a^b f(x)dx\right|$.

E) barcha formulalar S yuzani to'g'ri ifodalaydi .

11. Qutb koordinatalat sistemasida $r=r(\phi)$ tenglama berilgan L egri chiziq va $\phi=\alpha$, $\phi=\beta$ $(\alpha<\beta)$ radius vektorlar bilan chegaralangan sektor yuzi S aniq integral orqali hisoblanadigan formula qayerda to'gri ko'rsatilgan?

A) $S=\frac{1}{2}\int_\alpha^\beta r^2(\varphi)d\varphi$. B) $S=\frac{1}{2}\int_\alpha^\beta r(\varphi)d\varphi$. C) $S=\frac{1}{2}\int_\alpha^\beta \sqrt{1+r^2(\varphi)}d\varphi$

D) $S=\frac{1}{2}\int_\alpha^\beta \sqrt{1+r(\varphi)}d\varphi$. E) $S=\frac{1}{2}\int_\alpha^\beta |r(\varphi)|d\varphi$.

12. Qutb koordinatalar sistemasida $r=3\phi$ tenglama berilgan Arximed spirali va $\phi=0$, $\phi=2\pi$ radius vektorlar bilan chegaralangan sektorning S yuzini toping.

A) $S=8\pi^3$. B) $S=10\pi^3$. C) $S=12\pi^3$.

D) $S=14\pi^3$. E) $S=16\pi^3$

13. Qutb koordinatalar sistemasida $r=2\sqrt{\cos 2\varphi}$ tenglama berilgan va lemniskata deb ataluvchi egri chiziq hamda $\phi=0$, $\phi=2\pi$ radius vektorlar bilan chegaralangan sektorning S yuzini toping.

A) $S=4\pi^3$. B) $S=4\pi^2$. C) $S=4\pi$.

D) $S=4\sqrt{\pi}$. E) $S=4$.

14. $y=f(x)$, $x \in [a, b]$, funksiya grafigidan iborat yoyning l uzunligini hisoblash formulasini ko'rsating .

A) $l=\int_a^b \sqrt{1+|f(x)|}dx$. B) $l=\int_a^b \sqrt{1+f^2(x)}dx$.

C) $l=\int_a^b \sqrt{1+|f'(x)|}dx$ D) $l=\int_a^b \sqrt{1+[f'(x)]^2}dx$.

E) $l=\int_a^b \sqrt{1+[f''(x)]^2}dx$.

15. $x=\phi(t)$, $y=\psi(t)$, $\alpha \leq t \leq \beta$, parametrik tenglamasi bilan berilgan egri chiziq yoyi l uzunligini hisoblash formulasini ko'rsting .

A) $l=\int_\alpha^\beta [\varphi(t)+\psi(t)]dt$. B) $l=\int_\alpha^\beta [\varphi^2(t)+\psi^2(t)]dt$.

C) $l=\int_\alpha^\beta \sqrt{\varphi^2(t)+\psi^2(t)}dt$.

D) $l=\int_\alpha^\beta \sqrt{[\varphi'(t)]^2+[\psi'(t)]^2}dt$. E) $l=\int_\alpha^\beta \sqrt{[\varphi''(t)]^2+[\psi''(t)]^2}dt$.

16. Parametrik tenglamasi $x=a(t-\sin t)$, $y=a(1-\cos t)$ $(0 \leq t \leq 2\pi)$ bo'lgan egri chiziq yoyining uzunligini toping.

A) $2a$. B) $4a$. C) $6a$. D) $8a$. E) $10a$.

5. Aniq integralning mexanik va fizik masalalarga tatbiqlari

1. Moddiy nuqta to'g'ri chiziq bo'ylab $v=v(t)$ o'zgaruvchan tezlik bilan notekis harakat qilganda $[T_1,T_2]$ vaqt oralig'ida bosib o'tgan s masofani ifodalovchi formulani ko'rsating .

A) $s=v(T_2)-v(T_1)$. B) $s=v'(T_2)-v'(T_1)$.

C) $s=\int_{T_1}^{T_2} v(t)dt$. D) $s=\int_{T_1}^{T_2} v^2(t)dt$. E) $s=\int_{T_1}^{T_2} v'(t)dt$.

2. Moddiy nuqta to'g'ri chiziq bo'ylab $v(t)=2t+5$ o'zgaruvchan tezlik bilan notekis harakat qilganda [4,8] vaqt oralig'ida bosib o'tgan s masofani toping.

A) $s=28$. B) $s=48$. C) $s=68$. D) $s=88$. E) $s=108$.

3. Chiziqli zichligi $\varrho=\varrho(x)$ funksiya bilan berilgan $[a,b]$ kesma ko'rinishdagi bir jinsli bo'lmagan sterjen massasi m qanday topiladi ?

A) $m=\varrho(b-a)$. B) $m=\varrho(b)-\varrho(a)$.

C) $m=\int_a^b \rho^2(x)dx$. D) $m=\int_a^b \rho(x)dx$. E) $m=\int_a^b \sqrt{\rho(x)}dx$.

4. Chiziqli zichligi $\varrho(x)=x^2-x$, $x\in[0,6]$ funksiya bilan berilgan bir jinsli bo'lmagan sterjenning m massasini toping .

A) m=30. B) m=54. C) m=60. D) m=48. E) m=31.

5. $y=f(x)$, $x\in[a,b]$ differensiallanuvchi funksiya bilan berilgan va chiziqli zichlik funksiyasi uzluksiz $\varrho=\varrho(x)$ funksiyadan iborat bo'lgan moddiy L egri chiziqning m massasini hisoblash formulasi qayerda to'g'ri ko'rsatilgan?

A) $m=\int_a^b \rho(x)\sqrt{1+[f'(x)]^2}dx$. B) $m=\int_a^b \rho(x)(1+[f'(x)]^2)dx$.

C) $m=\int_a^b f(x)(1+[\rho'(x)]^2)dx$. D) $m=\int_a^b f(x)\sqrt{1+[\rho'(x)]^2}dx$.

E) $m=\int_a^b f'(x)[1+\rho^2(x)]dx$.

6. $y=ax$, $x\in[0,1]$ funksiya bilan berilgan va chiziqli zichlik funksiyasi $\varrho=2x+1$ bo'lgan moddiy L egri chiziqning m massasini toping.

A) $\sqrt{1+a^2}/2$. B) $\sqrt{1+a^2}$. C) $3\sqrt{1+a^2}/2$.

D) $2\sqrt{1+a^2}$. E) $3\sqrt{1+a^2}$.

7. $y=f(x)$, $x\in[a,b]$ differensiallanuvchi funksiya bilan berilgan va chiziqli zichlik funksiyasi uzluksiz $\varrho=\varrho(x)$ funksiyadan iborat bo'lgan moddiy L egri chiziqning OX koordinata o'qi bo'yicha statik momenti M_x formulasi qayerda to'g'ri ko'rsatilgan?

A) $M_x=\int_a^b \rho(x)\sqrt{1+[f'(x)]^2}dx$. B) $M_x=\int_a^b \rho(x)f(x)\sqrt{1+[f'(x)]^2}dx$.

C) $M_x=\int_a^b f'(x)\rho(x)(1+[\rho'(x)]^2)dx$.

D) $M_x=\int_a^b f(x)\rho(x)\sqrt{1+[\rho'(x)]^2}dx$.

E) $M_x = \int_a^b f(x)\rho(x)(1+[\rho'(x)]^2)dx$.

8. $y=ax$, $x \in [0,1]$ funksiya bilan berilgan va chiziqli zichlik funksiyasi $\varrho=3x+2$ bo'lgan moddiy L egri chiziqning OX o'qi bo'yicha M_x statik momentini toping.

A) $M_x = \sqrt{1+a^2}$. B) $M_x = a\sqrt{1+a^2}$.

C) $M_x = 2a\sqrt{1+a^2}$

D) $M_x = \sqrt{1+a^2}/a$. E) $M_x = \sqrt{1+a^2}/2a$.

9. $y=f(x)$, $x \in [a,b]$ differensiallanuvchi funksiya bilan berilgan va chiziqli zichlik funksiyasi uzluksiz $\varrho=\varrho(x)$ funksiyadan iborat bo'lgan moddiy L egri chiziqning OY koordinata o'qi bo'yicha statik momenti M_y formulasi qayerda to'g'ri ko'rsatilgan?

A) $M_y = \int_a^b \rho(x)\sqrt{1+[f'(x)]^2}dx$. B) $M_y = \int_a^b \rho(x)f(x)\sqrt{1+[f'(x)]^2}dx$.

C) $M_y = \int_a^b x\rho(x)(1+[\rho'(x)]^2)dx$. D) $M_y = \int_a^b x\rho(x)\sqrt{1+[f'(x)]^2}dx$.

E) $M_y = \int_a^b \rho'(x)f(x)\sqrt{1+[f'(x)]^2}dx$.

10. $y=ax$, $x \in [0,1]$ funksiya bilan berilgan va chiziqli zichlik funksiyasi $\varrho=3x+2$ bo'lgan moddiy L egri chiziqning OX o'qi bo'yicha M_y statik momentini toping.

A) $M_y = \sqrt{1+a^2}$. B) $M_y = 2\sqrt{1+a^2}$. C) $M_y = \sqrt{1+a^2}/2$.

D) $M_y = a\sqrt{1+a^2}$. E) $M_y = \sqrt{1+a^2}/a$.

11. $y=f(x)$ egri chiziq, $x=a$ va $x=b$ hamda $y=0$ to'gri chiziqlar bilan chegaralangan moddiy egri chiziqli trapetsiyaning x abssissali nuqtalaridagi zichligi uzluksiz $\varrho=\varrho(x)$ funksiyadan iborat bo'lsa, uning m massasi qaysi formula bilan hisoblanadi?

A) $m = \int_a^b [\rho(x)+f(x)]dx$. B) $m = \int_a^b [\rho(x)-f(x)]dx$.

C) $m = \int_a^b \rho(x)f(x)dx$. D) $m = \int_a^b [\rho(x)/f(x)]dx$.

E) $m = \int_a^b [f(x)/\rho(x)]dx$.

12. $y=x^2$ parabola, $x=0$ va $x=2$ hamda $y=0$ to'gri chiziqlar bilan chegaralangan moddiy egri chiziqli trapetsiyaning x abssissali nuqtalaridagi zichligi $\varrho=0.5x$ funksiyadan iborat bo'lsa, uning m

massasini toping.

A) m=1. B) m=1.5. C) m=2. D) m=2.5. E) m=3.

13.$y=f(x)$ egri chiziq, $x=a$ va $x=b$ hamda $y=0$ to'gri chiziqlar bilan chegaralangan moddiy egri chiziqli trapetsiyaning x abssissali nuqtalaridagi zichligi uzluksiz $\varrho=\varrho(x)$ funksiyadan iborat bo'lsa, uning OX koordinata o'qiga nisbatan M_x statik momenti qaysi formula bilan hisoblanadi?

A) $M_x = \frac{1}{2}\int_a^b [\rho(x) + f^2(x)]dx$. B) $M_x = \frac{1}{2}\int_a^b [\rho(x) - f^2(x)]dx$.

C) $M_x = \frac{1}{2}\int_a^b \rho(x) f^2(x)dx$. D) $M_x = \frac{1}{2}\int_a^b [\rho(x) / f^2(x)]dx$.

E) $M_x = \frac{1}{2}\int_a^b \rho^2(x) f(x)dx$.

14.$y=f(x)$ egri chiziq, $x=a$ va $x=b$ hamda $y=0$ to'gri chiziqlar bilan chegaralangan moddiy egri chiziqli trapetsiyaning x abssissali nuqtalaridagi zichligi uzluksiz $\varrho=\varrho(x)$ funksiyadan iborat bo'lsa, uning OY koordinata o'qiga nisbatan M_y statik momenti qaysi formula bilan hisoblanadi?

A) $M_y = \int_a^b x[\rho(x) + f(x)]dx$. B) $M_y = \int_a^b x[\rho(x) - f(x)]dx$.

C) $M_y = \int_a^b x\rho(x) f(x)dx$. D) $M_y = \int_a^b x[\rho(x) / f(x)]dx$.

E) $M_y = \int_a^b x[f(x) / \rho(x)]dx$.

15. Bir jinsli moddiy L egri chiziq uzluksiz differensiallanuvchi $y=f(x)$, $x \in [a,b]$, funksiya bilan berilgan bo'lsa, uning $M(x_0,y_0)$ og'irlik markazining abssissasi $x_0 = X / \int_a^b \sqrt{1+[f'(x)]^2}$ ko'rinishdagi kasrdan iborat formula bilan hisoblanadi. Bu kasr surati X qayerda to'g'ri ifodalangan?

A) $X = \int_a^b f(x)\sqrt{1+[f'(x)]^2}dx$. B) $X = \int_a^b f(x)\sqrt{1-[f'(x)]^2}dx$.

C) $X = \int_a^b x\sqrt{1+[f'(x)]^2}dx$. D) $X = \int_a^b x\sqrt{1-[f'(x)]^2}dx$.

E) $X = \int_a^b xf(x)\sqrt{1+[f'(x)]^2}dx$.

16. Bir jinsli moddiy L egri chiziq uzluksiz differensiallanuvchi $y=f(x)$, $x \in [a,b]$, funksiya bilan berilgan

bo'lsa, uning $M(x_0,y_0)$ og'irlik markazining ordinatasi $y_0 = Y / \int_a^b \sqrt{1+[f'(x)]^2}$ ko'rinishdagi kasrdan iborat formula bilan hisoblanadi. Bu kasr surati Y qayerda to'g'ri ifodalangan?

A) $Y = \int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$. B) $Y = \int_a^b f(x)\sqrt{1-[f'(x)]^2}\,dx$.

C) $Y = \int_a^b x\sqrt{1+[f'(x)]^2}\,dx$. D) $Y = \int_a^b x\sqrt{1-[f'(x)]^2}\,dx$.

E) $Y = \int_a^b xf(x)\sqrt{1+[f'(x)]^2}\,dx$.

6.Xosmas integrallar

1. Agar $f(x)$ uzluksiz funksiya bo'lsa, qaysi holda $\int_a^b f(x)dx$ I tur xosmas integral bo'lmaydi ?

A) $b=+\infty$. B) $a=-\infty$. C) $a=-\infty$, $b=+\infty$. D) $a=-\infty$ yoki $b=+\infty$.

E)keltirilgan barcha hollarda I tur xosmas integral bo'ladi.

2. Agar $f(x)$ uzluksiz funksiya bo'lsa, quyidagi integrallardan qaysi biri I tur xosmas integral bo'lmaydi ?

A) $\int_a^{+\infty} f(x)dx$. B) $\int_{-\infty}^{b} f(x)dx$. C) $\int_{-\infty}^{+\infty} f(x)dx$. D) $\int_a^{b} f(x)dx$.

E) keltirilgan barcha integrallar I tur xosmas integral bo'ladi.

3. ***Ushbu ta'rifni yakunlang:*** I tur xosmas integral I yaqinlashuvchi deyiladi, agar …

A) uning qiymati musbat bo'lsa.

B) uning qiymati manfiy bo'lsa.

C) uning qiymati nolga teng bo'lsa.

D) uning qiymati chekli bo'lsa.

E) uning qiymati cheksiz bo'lsa.

4. ***Ushbu ta'rifni yakunlang:*** I tur xosmas integral I uzoqlashuvchi deyiladi, agar …

A) uning qiymati musbat bo'lsa.

B) uning qiymati manfiy bo'lsa.

C) uning qiymati nolga teng bo'lsa.

D) uning qiymati chekli bo'lsa.

E) uning qiymati cheksiz bo'lsa.

5. $\int_0^{\infty} \frac{xdx}{1+x^4}$ I tur xosmas integral qiymatini toping.

A) 2/3 . B) $\pi/4$. C) ∞ . D) 3.5 . E) 2π .

6. $\int_1^{\infty} \frac{dx}{x^{\alpha}}$ I tur xosmas integral α parametrning qanday qiymatlarida yaqinlashuvchi bo'ladi ?

A) $\alpha>0$. B) $\alpha<0$. C) $\alpha>1$. D) $\alpha<1$. E) $\alpha\neq0$

7. $\int_1^{\infty} \frac{dx}{x^{\alpha}}$ I tur xosmas integral uzoqlashuvchi bo'ladigan α parametrning barcha qiymatlari qayerda to'g'ri ko'rsatilgan ?

A) $\alpha>0$. B) $\alpha\leq0$. C) $\alpha>1$. D) $\alpha\leq1$. E) $\alpha\neq0$.

8. Quyidagi I tur xosmas integrallardan qaysi biri uzoqlashuvchi ?

A) $\int_0^{\infty} \frac{dx}{x^2+1}$. B) $\int_{-\infty}^{0} \frac{dx}{x^2+1}$. C) $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$.

D) Barcha xosmas integrallar yaqinlashuvchi.

E) Barcha xosmas integrallar uzoqlashuvchi.

9. $y=1/x^3$ $(x\geq1)$, $y=0$ chiziqlar bilan chegaralangan cheksiz egri chiziqli trapetsiyaning S yuzini toping.

A) $S=2$. B) $S=1.5$. C) $S=1$.

D) $S=0.5$. E) $S=+\infty$.

10. $y=e^x$ $(x\leq0)$, $y=0$ chiziqlar bilan chegaralangan cheksiz egri chiziqli trapetsiyaning S yuzini toping.

A) $S=2$. B) $S=1.5$. C) $S=1$. D) $S=0.5$.

E) $S=+\infty$

11. Qaysi shartda $\int_{-\infty}^{b} f(x)dx$ I tur xosmas integral absolut yaqinlashuvchi deyiladi ?

A) $\left|\lim_{a\to-\infty}\int_a^b f(x)dx\right|<\infty$. B) $\lim_{a\to-\infty}\left|\int_a^b f(x)dx\right|>0$.

C) $\lim_{a\to-\infty}\int_a^b |f(x)|dx>0$. D) $\lim_{a\to-\infty}\left|\int_a^b f(x)dx\right|<\infty$.

E) $\lim_{a\to-\infty}\int_a^b |f(x)|dx<\infty$.

12. *Ta'rifni yakunlang:* Chekli $[a,b]$ kesma bo'yicha olingan $\int_a^b f(x)dx$ integral II tur xosmas integral deyiladi, agarda shu kesmada $f(x)$ funksiya … .

A) chegaralangan bo'lsa . B) monoton bo'lsa .
C) chegaralanmagan bo'lsa . D) uzluksiz bo'lsa .
E) davriy bo'lsa.

13. Quyidagi integrallardan qaysi biri II tur xosmas integral bo'ladi ?

A) $\int_0^1 \sqrt{1+x}dx$. B) $\int_0^1 \frac{dx}{\sqrt{1+x}}$. C) $\int_0^1 \sqrt{1-x}dx$. D) $\int_0^1 \frac{dx}{\sqrt{1-x}}$

E) keltirilgan barcha integrallar II tur xosmas integral bo'ladi .

14. $\int_0^4 \frac{dx}{\sqrt{4-x}}$ II tur xosmas integral qiymatini hisoblang .

A) 0.5 . B) 1 . C) 2 . D) 4 . E) ∞ .

15. $\int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2 x}$ II tur xosmas integral qiymatini hisoblang .

A) 1 . B) π . C) π/4 . D) 0.5 . E) ∞ .

16. $\int_0^2 \frac{dx}{x^\alpha}$ II tur xosmas integral uzoqlashuvchi bo'ladigan α parametrning qiymatlari qayerda to'g'ri ko'rsatilgan?

A) α≥0 . B) α≤0 . C) α≤1 . D) α≥1 .
E) α≠0

Musiqa qalb matematikasi bo'lsa,
matematika miya musiqasidir.
N.G. Chernishevskiy

X-BOB. BIR NECHA O'ZGARUVCHILI FUNKTSIYALARNING DIFFERENSIAL HISOBI

§ 10.1.Bir necha argumentli funktsiyalar haqida tushunchalar

1. Agar biror X to'plamning har bir $(x_1,x_2,...x_n)$ haqiqiy sonlarga biror qoida bilan *E* to'plamdagi yagona *z* haqiqiy songa mos qo'yilgan bo'lsa, u holda to'plamda bir necha o'zgaruvchining funksiyasi $z = f(x_1,x_2,...x_n)$ **aniqlangan** deb ataladi. Bu yerda *X* – to'plam funksiyaning **aniqlanish** sohasini; *E* – to'plam funksiyaning **qiymatlar** (o'zgarish) sohasini ifodalaydi. $x_1,x_2,...x_n$ - funksiyaning argumenti; z – funksiya.

1-misol. Quyidagi

$$u = 2x\arcsin\frac{x}{y^2} + 3y^2\arcsin(1-y)$$

funksiyaning aniqlanish sohasi topilsin va chizmada tasvirlansin.

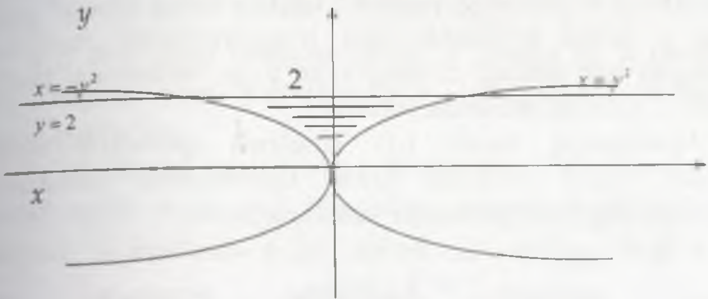
Arksinus funksiyaning aniqlanish sohasidan foydalanamiz.

$$D(u) = \begin{cases} y \neq 0, \\ \left|\frac{x}{y^2}\right| \leq 1, \\ |1-y| \leq 1. \end{cases} \Rightarrow \begin{cases} y \neq 0, \\ -1 \leq \frac{x}{y^2} \leq 1 \\ -1 \leq 1-y \leq 1 \end{cases} \Rightarrow \begin{cases} y \neq 0, \\ -y^2 \leq x \leq y^2 \\ 0 < y \leq 2 \end{cases}$$

yoki

$$D(u)=\{(x,y)\in R^2 : 0<y\le 2,\ -y^2\le x\le y^2\}$$

bo'lib, bu soha 10.1-chizmada tasvirlangan.



10.1-chizma

2-misol. Quyidagi

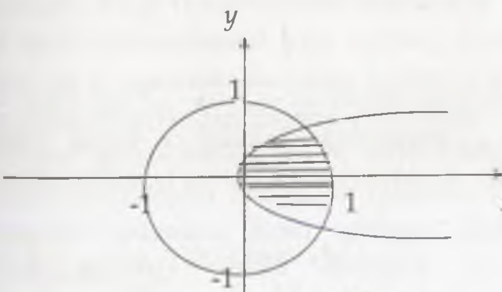
$$u=\frac{\sqrt{4x-y^2}}{\ln(1-x^2-y^2)}$$

funksiyaning aniqlanish sohasi topilsin va chizmada tasvirlansin.

Aniqlanish sohasini topish uchun kasr ratsional funksiya, logarifmik funksiya va kvadrat ildiz ostidagi funksiya xossalaridan foydalanamiz:

$$D(u)=\begin{cases}\ln(1-x^2-y^2)\neq 0,\\ 1-x^2-y^2\ge 0,\\ 4x-y^2\ge 0.\end{cases}\Rightarrow\begin{cases}(x,y)\neq 0,\\ x^2+y^2<1,\\ x\ge\dfrac{y^2}{4}\end{cases}\Rightarrow\left\{(x,y)\in R^2 : (x,y)\neq(0,0),\ x^2+y^2<1,\ x\ge\frac{y^2}{4}\right\}$$

bo'lib, bu soha 2.2-chizmada tasvirlangan.



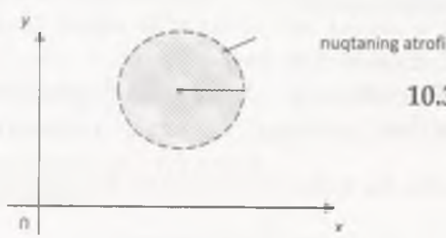
10.2-chizma

2. Agar to'plaminng har bir (x,y) haqiqiy sonlar juftligi biror qoida bilan E to'plamdagi yagona z haqiqiy songa mos qo'yilgan bo'lsa, u holda to'plamda ikki o'zgaruvchining funksiyasi aniqlangan deb ataladi. z ning x va y ga funktsional bog'liq bo'lishi $z=f(x,y)$ ko'rinishida yoziladi.

3.Geometrik tasvir. (.1) tenglama geometrik nuqtai nazardan to'g'ri burchakli dekart koordinatalar sistemasida umumlashtirilganda qandaydir sirtni aniqlaydi.

4.Agar $\lim\limits_{p\to p_0} f(p)=f(p_0)$ bo'lsa, $f(x,y)$ funktsiya P_0 nuqtada uzluksiz deyiladi. Boshqacha aytganda, agar $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}} f(x+\Delta x,\ y+\Delta y)=f(x,y)$ bo'lsa, $f(x,y)$ funktsiya (x,y) nuqtada **uzluksiz** deyiladi.

5. Ikki o'zgaruvchili $z=f(x,y)=f(p)$ funktsiyaning limiti tushunchasini kiritishda Oxy tekisligida nuqtaning atrofini qaraymiz. $P_0(x_0,y_0)$ nuqtaning atrofi deb markazi shu nuqtada bo'lgan doiraning ichki nuqtalar to'plamiga aytiladi. Agar bu doiraning radiusi δ-ga teng bo'lsa, u holda y nuqtaning δ **- atrofi** to'g'risida gapiriladi (10.1-chizma).



10.3-chizma.

$P(x,y)$ nuqtaning δ atrofi $\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$. Agar istalgan ε son uchun $P_0(x_0,y_0)$ nuqtaning shunday δ -atrofi topilsaki, bu atrofning istalgan $P(x,y)$ nuqtasi (P_0 nuqta bundan istisno bo'lishi mumkin) uchun

$$|f(x,y)-A|<\varepsilon \text{ yoki } |f(p)-A|<\varepsilon$$

tengsizlik o'rinli bo'lsa, u holda A son ikki o'zgaruvchi $z = f(x,y) = f(p)$ funksiyaning $P \to P_0(x \to x_0,\ y \to y_0)$ dagi **limiti** deb ataladi.

$M \subset R^m$to`plam berilgan bo`lib, a nuqta M to`plamning limit nuqtasi va$y= f(x)=f(x_1,\ldots,x_m)$ funksiya$M$ to`plamda aniqlangan bo`lsin.

Ta`rif (Koshi ta`rifi) .*Agar* $\forall\varepsilon >0$ *uchun* $\exists\delta = \delta(\varepsilon, a) >0$ *son topilsaki, ushbu* $0<\rho(x,a)< \delta$ *tengsizlikni qanoatlantiruvchi* $\forall x \in M$ *uchun*

$$|f(x) - b| < \varepsilon$$

tengsizlik bajarilsa, unda b son $f(x)$ *funksiyaning a nuqtadagi limiti (yoki* ***karrali limiti*** *) deyiladi va*

$$\lim_{x\to a} f(x) = b$$

yoki

$$\lim_{\substack{x_1 \to a_1 \\ \cdots \\ x_m \to a_m}} f(x_1, x_2, \ldots, x_m) = b$$

kabi belgilanadi.

Ta`rif (Geyne ta`rifi).*Agar M to'plamning nuqtalaridan tuzilgan, a ga intiluvchi har qanday* $\{x^{(n)}\}\left(x^{(n)} \neq a,\quad n = 1,2,\ldots\right)$ *ketma – ketlik olinganda ham mos* $\{f(x^{(n)})\}$ *ketma – ketlik hamma vaqt yagona b (chekli yoki cheksiz) limitga intilsa, b son f(x) funksiyaning a nuqtadagi limiti deb ataladi.*

Agar $a=\infty$ yoki $b=\infty$ bo`lsa, unda ham shu kabi ta`riflarni berish mumkin. Ko`p o`zgaruvchili funksiyalar uchun boshqa formadagi limit tushunchasini ham kiritish mumkin. Masalan, bunda avval bir o`zgaruvchi bo`yicha limitga o`tilib, qolgan $m$-1 ta o`zgaruvchini fiksirlangan (tayin) deb faraz qilinadi. Keyin, qolgan m-1 ta o`zgaruvchining biri bo`yicha limitga o`tilib, $m$-2 ta o`zgaruvchini fiksirlangan deb faraz qilinadi. Bu jarayonni m marta qaytarish natijasida hosil qilingan limitga $f(x_1,\ldots,x_m)$ funksiyaning **takroriy limiti** deyiladi. m o`zgaruvchili funksiyaning jami **m!** ta takroriy limitini qarash mumkin.

Masalan, ikki o`zgaruvchili $f(x_1,x_2)$ funksiya uchun 2 ta $\lim\limits_{x_1\to a_1}\lim\limits_{x_2\to a_2} f(x_1,x_2)$ va $\lim\limits_{x_2\to a_2}\lim\limits_{x_1\to a_1} f(x_1,x_2)$ takroriy limitni ko`rish mumkin.

3 - misol. Ushbu

$$f(x,y)=\begin{cases} x+y\sin\dfrac{1}{x}, & x\neq 0,\\ 0, & x=0\end{cases}$$

funksiyaning (0,0) nuqtadagi takroriy va karrali limitlari hisoblansin.

$$\lim_{x\to 0}\lim_{y\to 0} f(x,y)=\lim_{x\to 0}\lim_{y\to 0}\left(x+y\lim\frac{1}{x}\right)=\lim_{x\to 0} x=0 \quad \text{mavjud.}$$

$$\lim_{y\to 0}\lim_{x\to 0} f(x,y)=\lim_{y\to 0}\left(y\lim_{x\to 0}\sin\frac{1}{x}\right) \qquad \text{mavjud emas, lekin}$$

$\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y)$ mavjud va u nolga teng.

Darhaqiqat,

$$0\le |f(x,y)-0|=\left|x+y\sin\frac{1}{x}\right|\le |x|+|y| \quad (x\neq 0)\Rightarrow \lim_{\substack{x\to 0\\ y\to 0}} f(x,y)=0. ▶$$

4– misol. Ushbu

$$f(x,y)=\frac{x-y}{x+y}$$

funksiyaning $x\to 0,\ y\to 0$ dagi takroriy va karrali limitlarni toping.

$$\lim_{x\to 0}\left(\lim_{y\to 0} f(x,y)\right)=\lim_{x\to 0}\left(\lim_{y\to 0}\frac{x-y}{x+y}\right)=\lim_{x\to 0}\frac{x}{x}=1,$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} f(x,y)\right)=\lim_{y\to 0}\left(\lim_{x\to 0}\frac{x-y}{x+y}\right)=\lim_{y\to 0}\frac{-y}{y}=-1.$$

Karrali limitini topishda Geyne ta`rifidan foydalanamiz: $n\to\infty$ da ikkita (0,0) nuqtaga intiluvchi

$$\{x_n,y_n\}=\left\{\frac{1}{n},\frac{1}{n}\right\},\quad \{x'_n,y'_n\}=\left\{\frac{2}{n},\frac{1}{n}\right\}$$

ketma – ketliklarda funksiya limiti har xil sonlarga intiluvchiligini ko`rsatamiz.

$$f(x_n,y_n)=\frac{\frac{1}{n}-\frac{1}{n}}{\frac{1}{n}+\frac{1}{n}}=0,\quad f(x_n,y_n)\to 0$$

$$f(x_n,y_n)=\frac{\frac{2}{n}-\frac{1}{n}}{\frac{2}{n}+\frac{1}{n}}=\frac{\frac{1}{n}}{\frac{3}{n}},\quad f(x_n,y_n)\to \frac{1}{3}$$

Demak, $n\to\infty$ $\lim\limits_{\substack{x\to0\\ y\to0}} f(x,y)$ mavjud emas.

5– misol. Ushbu

$$\lim_{\substack{x\to+\infty\\ y\to+\infty}}\left(x^2+y^2\right)e^{-(x+y)}$$

limitni hisoblang.

Elementar tengsizliklardan foydalanamiz. (x>0, y>0).

$$0<\left(x^2+y^2\right)e^{-(x+y)}=\frac{x^2}{e^{x+y}}+\frac{y^2}{e^{x+y}}<\frac{x^2}{e^x}+\frac{y^2}{e^y}.$$

$$0\le\lim_{\substack{x\to+\infty\\ y\to+\infty}}\left(x^2+y^2\right)e^{-(x+y)}\le\lim_{\substack{x\to+\infty\\ y\to+\infty}}\left(\frac{x^2}{e^x}+\frac{y^2}{e^y}\right)=0.$$

Demak,

$$\lim_{\substack{x\to+\infty\\ y\to+\infty}}\left(x^2+y^2\right)e^{-(x+y)}=0$$

6 – misol. Ushbu

$$\lim_{\substack{x\to\infty\\ y\to\infty}}\frac{x^2+y^2}{x^4+y^4}$$

limitni hisoblang $(x\neq0;\ \ y\neq0)$.

$$0<\frac{x^2+y^2}{x^4+y^4}=\frac{x^2}{x^4+y^4}+\frac{y^2}{x^4+y^4}\le\frac{x^2}{x^4}+\frac{y^2}{y^4}=\frac{1}{x^2}+\frac{1}{y^2}$$

$$\lim_{\substack{x\to+\infty\\ y\to+\infty}}\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=0 \qquad \text{dan} \qquad \lim_{\substack{x\to+\infty\\ y\to+\infty}}\frac{x^2+y^2}{x^4+y^4}=0$$

Ayrim hollarda $x=a+r\cos\varphi,\quad y=b+r\sin\varphi$ almashtirish

$$\lim_{\substack{x\to 0\\ y\to 0}} f(x,y)$$

karrali limitni topishni yengillashtiradi. Bunda

$$f(x,y)=f(a+r\cos\varphi,\ b+r\sin\varphi)=F(r,\varphi)$$

bo`lib,

$$\lim_{\substack{x\to a\\ y\to b}} f(x,y)=c \Leftrightarrow \lim_{r\to 0}F(r,\varphi)=c$$

7-misol. Ushbu

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt[3]{x^4y^3}}{x^2+y^2}$$

limitni hisoblang.

$x=a+r\cos\varphi,\quad y=b+r\sin\varphi$ almashtirishdan foydalamiz, bunda *a=0; b=0* va $x\to 0$ $y\to 0$ da $r\to 0$;

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt[3]{x^4y^3}}{x^2+y^2}=\lim_{r\to 0}\frac{\sqrt[3]{(r\cos\varphi)^4\cdot(r\sin\varphi)^3}}{r^2\cos^2\varphi+r^2\sin^2\varphi}=$$

$$=\lim_{r\to 0}\frac{r^2\sqrt[3]{r}\cdot\sqrt[3]{\cos^4\varphi\sin^3\varphi}}{r^2}=\lim_{r\to 0}\sqrt[3]{r}\cdot\sqrt[3]{\cos^4\varphi\sin^3\varphi}=0,$$

chunki

$$\sqrt[3]{\cos^4\varphi\sin^3\varphi}$$

chegaralangan.

8-misol. Agar $f(x,y)=\dfrac{x^y}{1+x^y}$ bo`lsa, ushbu takroriy limitlarni hisoblang.

$$\lim_{x\to+\infty}\left(\lim_{y\to+0}f(x,y)\right) \qquad \text{va} \qquad \lim_{y\to+0}\left(\lim_{x\to+\infty}f(x,y)\right).$$

x ni o`zgarmas desak $y>0$ da $x^y$ $y-$ning funksiyasi sifatida uzliksiz bo`ladi, shu sababli

$$\lim_{y\to+0} x^y = 1$$

bo`ladi;

y ning o`zgarmas ($y>0$) qiymatida,x ning barcha $x>0$ qiymatida x^y x ning funksiyasi sifatida uzluksizligidan.

$$\lim_{y\to+\infty} x^y = +\infty$$

bo'ladi

$$\lim_{x\to+\infty}\left(\lim_{y\to+0}\frac{x^y}{1+x^y}\right)=\lim_{x\to+\infty}\frac{1}{1+1}=\lim_{x\to+\infty}\frac{1}{2}=\frac{1}{2},$$

$$\lim_{y\to+0}\left(\lim_{x\to+\infty}\frac{x^y}{1+x^y}\right)=\lim_{y\to+0}\left(\lim_{x\to+\infty}\frac{1}{\frac{1}{x^y}+1}\right)=\lim_{x\to+0}1=1$$

6. Oxy tekislikdagi $f(x,y)=C$ tenglikni qanoatlantiruvchi barcha nuqtalar to'plami $z=f(x,y)$ funksiyaning yuksaklik (sath) chizig'i deyiladi. Fazodagi $f(x,y,z)=C$ tekislikni qanoatlantiruvchi barcha nuqtalar to'plami $u=f(x,y,z)$ funksiyaning yuksaklik sirti (ekvipotentsial sirti) deyiladi, bu yerda C – berilgan o'zgarmas son ($C\in R$).

Quyidagi funksiyalarni aniqlanish sohasini toping.

10.1a) $z=\arcsin\frac{x^2+y^2}{4}+arc\sec(x^2+y^2)$ б). $u=x^2+z^2-y^2$ yuksaklik sirtini toping.

10.2.*a)* $z=\frac{1}{x^4+y^4}$ *в)* $z=\frac{1}{x^2+y^3}$

10.3.*a)* $z=\frac{\sqrt{4-x^2-y^2}}{\ln(x^2+y^2-1)}$ *в)* $z=\sqrt[8]{1-x^2+y}$

10.4. $z=\sqrt[7]{1-x^2+y}$ 10.5. $z=\ln(x+y)$

10.6. $z=\ln(x^2+y^2)$ 10.7. $z=\arcsin x+\arccos y$

$10.8.\ z=\frac{\arcsin(x+y)}{\arccos(x-y)}$

$10.9.\ z=\sqrt{x^2+y^2-1}$ $10.10\ z=1/\sqrt{1-x^2-y^2}$

$10.11\ z=\arcsin(x+y)$

$10.12\ z=\sqrt{\cos(x^2+y^2)}$ $10.13\ z=\ln(-x+y)$

$10.14\ z=y+\sqrt{x}$

$10.15\ u=\sqrt{a^2-x^2-y^2-z^2}$ $10.16\ u=\arcsin\left(z/\sqrt{x^2+y^2}\right)$

$10.17\ u=\sqrt{x^2+y^2-1}$ $10.18\ u=\sqrt{x+y+z}$

$10.19\ z=x^2+y^2$ $10.20\ az=a^2-x^2-y^2$

$10.21\ z=\frac{4}{x^2+y^2}$ $10.22\ z=\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}$

$10.23\ z=\ln(y^2-4x+8)$

$10.24\ z=\frac{1}{R^2-x^2-y^2}$ $10.25\ z=\sqrt{x+y}+\sqrt{x-y}$

$10.26\ z=\frac{1}{\sqrt{x+y}}+\frac{1}{\sqrt{x-y}}$ $10.27\ z=\ln xy$

$10.28\ z=\sqrt{x-\sqrt{y}}$

$10.30\ z=xy\sqrt{\ln\frac{R^2}{x^2+y^2}}+\sqrt{x^2+y^2-R^2}$

$10.31\ z=ctg\,\pi(x+y)$ $10.32\ z=\sqrt{\sin\pi(x^2+y^2)}$

$10.33\ z=\ln x-\ln\sin y$

$10.34\ u=\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}+\frac{1}{\sqrt{z}}$

$10.35\ u=\sqrt{R^2-x^2-y^2-z^2}+\frac{1}{\sqrt{x^2+y^2+z^2-r^2}}\ (R>r)$

$10.36\ z=\frac{1}{\sqrt{xy}}$ $10.37\ z=\sqrt{(1+x)(y-4)}$

$10.38\ z=\sqrt{x^2+y^2-1}+\ln(4-x^2-y^2)$ $10.39\ z=\arcsin(x+y)$

10.40. Uchburchakning perimetri $2p$ berilgan. Uchburchakning ikk tomonini x va y deb, uning yuzi S shu tomonlarining funksiyasi sifatida aniqlansin. x va y ning qabul qilishi mumkin bo'lgan qiymatlariniung sohasi aniqlansin.

10.41 $F(x,y)=\dfrac{x-2y}{2x-y}$; a) $F(3;1)$; b) $F(1;3)$; c) $F(1;2)$; d) $F(2;1)$ e) $F(a;a)$ k) $F(a;-a)$ lar bo'lganda hisoblang.

10.42 $F(x,y)=\sqrt{x^4+y^4}-2xy$; $F(tx,ty)=t^2F(x,y)$ ekani isbot qilinsin.

10.43 Agar $F(x,y)=xy+\dfrac{x}{y}$ bo'lsa, $F\left(\dfrac{1}{2};3\right)$, $(1;-1)$ ni hisoblang.

10.44 Agar $F(x,y)=\dfrac{x^2-y^2}{2xy}$ bo'lsa, $F(y;x)$, $(-x;-y)$, $F\left(\dfrac{1}{x};\dfrac{1}{y}\right)$, $\dfrac{1}{F(x,y)}$ ni hisoblang.

10.45 Agar $F(x,y,z)=\dfrac{x+y+z}{x^2+y^2+z^2}$ bo'lsa, $F(0;2;-3)$, $F\left(x;\dfrac{1}{x};\dfrac{1}{x^2}\right)$ larni hisoblang.

Quyidagi funksiyalarning yuksaklik chiziqlari yasalsin.

10.46 $z=2x+y$ 10.47 $z=x/y$ 10.48 $z=\ln\sqrt{y/x}$

10.49 $z=\sqrt{x}/y$ 10.50 $z=e^{xy}$ 10.51 $z=x+y$

10.52 $z=y/x^2$ 10.53 $z=y-x$ 10.54 $z=x^2-y$

10.55 $z=x^2+y^2$ 10.56 $z=\ln(x^2+y)$ 10.57 $z=\arccos(xy)$

10.58 $z=xy^3$ 10.59 $z=x\ln(x^2+y)$ 10.60 $z=e^{x+y}$

10.61 $z=\sqrt{y-x^2}$ 10.62 $z=\dfrac{1}{x+2y}$ 10.63 $z=\dfrac{y-x^2}{x^2}$

10.64 $z=tg(x+y)$ 10.65 $z=x^2+y^2$

Quyidagi limitlarni toping.

10.66 *a)* $\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{\arcsin(x^2+y^2)}{\ln\left(1-\sqrt{x^2+y^2}\right)}$ *в)* $\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{\sin(xy^2)}{xy}$ 10.67. $\lim\limits_{\substack{x\to 2\\ y\to 0}}\dfrac{tg(xy)}{y}$

10.68. $\lim\limits_{\substack{x\to 0\\ y\to 0}}(x^2+y^2)\sin\dfrac{1}{xy}$ 10.69. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{2-\sqrt{xy+4}}{xy}$

10.70 $\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{a-\sqrt{a^2-xy}}{xy}$ 10.71. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{\sin(x^3+y^3)}{x^2+y^2}$

10.72. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{x^2-y^2}{x^2+y^2}$ 10.73 $\lim\limits_{\substack{x\to 0\\ y\to 0}}\left[xy\sqrt{1+xy}\right]$

10.74. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{x^3y^3}{x^3+y^3}$ 10.75. $\lim\limits_{\substack{x\to 1\\ y\to -1}}\dfrac{\sin(x+y)}{x+y}$

10.76. $\lim\limits_{\substack{x\to 0\\ y\to 0}}[xy\ln(xy)]$ 10.77. $\lim\limits_{\substack{x\to 0\\ y\to 0}}[x+2y]e^{\frac{1}{x}}$

10.78. $\lim\limits_{\substack{x\to 1\\ y\to 1}} \frac{x-y}{x^3-y^2}$

10.79. $\lim\limits_{\substack{x\to 0\\ y\to 0}} (x^2+y^2)\sin\frac{1}{xy}$

10.80. $\lim\limits_{\substack{x\to 0\\ y\to 2}} \frac{\sin xy}{x}$

10.81. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{x}{x+y}$

10.82. $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} \frac{x-y}{x^2+y^2}$

10.83 $\lim\limits_{\substack{x\to\infty\\ y\to k}} \left(1+\frac{y}{x}\right)^k$

10.84. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{x^2-y^2}{x^2+y^2}$

10.85. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$

10.86. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{x^2y^2+1}-1}{x^2+y^2}$

10.87. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x^3+y^3)}{x^2+y^2}$

10.88. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{1-\cos(x^2+y^2)}{(x^2+y^2)x^2y^2}$

10.89. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{e^{-\frac{1}{x^2+y^2}}}{x^4+y^4}$

10.90. $\lim\limits_{\substack{x\to 0\\ y\to 0}} (1+x^2y^2)^{-\frac{1}{x^2+y^2}}$

§ 10.2 Birinchi tartibli xususiy va to`liq hosilalar

7. $z=f(x,y)$ funktsiyada y ni o'zgarmas deb qarab,undan x bo'yicha olingan hosila z ning x bo'yicha **xususiy hosilasi** deyiladi va u $\frac{\partial z}{\partial x}$ yoki $f'_x(x,y)$ ko'rinishida belgilanadi $f'_x(x,y)=\frac{\partial f}{\partial x}=\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$ z ning y bo'yicha xususiy hosilasi ham shunga o'xshash ta'riflanadi va quyidagicha belgilanadi

$$\frac{\partial z}{\partial y}=f'_y(x,y), \text{ ya'ni } f'_y(x,y)=\lim_{\Delta y\to 0}\frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}.$$

8. Agar $z=f(x,y)$ funksiya (x,y) nuqtada uzluksiz xususiy hosilalarga ega bo'lsa, uning to'liq ortirmasi $\Delta z=\frac{\partial z}{\partial x}\Delta x+\frac{\partial z}{\partial y}\Delta y+\varepsilon\cdot\rho$ ko'rinishda yoziladi, bunda $\rho=\sqrt{|\Delta x|^2+|\Delta y|^2}$ nolga intilganda $(\rho\to 0)\varepsilon\to 0$. U holda $\frac{\partial z}{\partial x}\Delta x+\frac{\partial z}{\partial y}\Delta y$ ifoda to'liq ortirrma Δz ning bosh qismi bo'ladi: u funktsiyaning to'liq differensiali deyiladi va dz orqali belgilanadi: $dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$

9 – misol. Ushbu

$$f(x,y)=e^{x+y}$$

funksiyaning (2,2) nuqtada $f_x^{'}$, $f_y^{'}$ xususiy hosilalarini hisoblang.

$$\frac{\partial f(2,2)}{\partial x}=\lim_{\Delta x\to 0}\frac{f(2+\Delta x,2)-f(2,2)}{\Delta x}=\lim_{\Delta x\to 0}\frac{e^{2+\Delta x+2}-e^{2+2}}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}\frac{e^{4+\Delta x}-e^4}{\Delta x}=\lim_{\Delta x\to 0}\frac{e^4\left(e^{\Delta x}-1\right)}{\Delta x}=e^4\lim_{\Delta x\to 0}\frac{e^{\Delta x}-1}{\Delta x}=e^4.$$

Xuddi shunga o`xshash,

$$\frac{\partial f(2,2)}{\partial y}=\lim_{\Delta y\to 0}\frac{f(2,2+\Delta y)-f(2,2)}{\Delta y}=\lim_{\Delta y\to 0}\frac{e^{2+\Delta y+2}-e^{2+2}}{\Delta y}=e^4.$$

Demak,

$$\frac{\partial f(2,2)}{\partial x}=e^4,\qquad \frac{\partial f(2,2)}{\partial y}=e^4.$$

10– misol. Ushbu

$$f(x,y)=\ln\left(x^2+y^2+1\right)+\sin^2 xy$$

funksiyaning xususiy hosilalarini hisoblang.

Hosila olishni qoidalaridan foydalanib topamiz

$$\frac{\partial f}{\partial x}=\frac{2x}{x^2+y^2+1}+2\sin xy\cdot\cos xy\cdot y=\frac{2x}{x^2+y^2+1}+y\sin 2xy,$$

$$\frac{\partial f}{\partial y}=\frac{2y}{x^2+y^2+1}+2\sin xy\cdot\cos xy\cdot x=\frac{2y}{x^2+y^2+1}+x\sin 2xy$$

Quyidagi birnchi tartibli xususiy hosilalarni hisoblang.

10.91. a) $z=x^2e^{y^2}$ в) $z=x^3+3x^2y-y^3$ 10.92. $z=\ln\left(x^2+y^2\right)$

10.93. $z=\frac{y}{x}$ 10.94. $z=arctg\frac{y}{x}$ 10.95. $z=\frac{xy}{x-y}$

10.96. $z=xe^{-yx}$ 10.97. $z=\sqrt{x^2-y^2}$ 10.98. $z=\frac{x}{\sqrt{x^2+y^2}}$

10.99. $z=arctg\frac{y}{x}$ 10.100 $z=x^y$ 10.101. $z=e^{\sin\frac{y}{x}}$

10.102. $z=\ln\sin\frac{x+a}{\sqrt{y}}$ 10.103. $u=(xy)^z$ 10.104. $z=\frac{3u}{\vartheta}+\frac{2\vartheta}{u}$

10.105. $z=\frac{x^3+y^3}{x^2+y^2}$ 10.106 $z=\left(5x^2y-y^3+7\right)^y$ 10.107. $z=x\sqrt{y}+\frac{y}{\sqrt[3]{x}}$

10.108 $z = \frac{1}{arctg\ \frac{x}{y}}$ 10.109 $z = \arcsin\frac{\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}$ 10.110. $z = \ln\frac{\sqrt{x^2+y^2}-x}{\sqrt{x^2+y^2}+x}$

10.111 $z = \ln tg\frac{x}{y}$ 10.112 $z = (1+\log_y x)^3$ 10.113. $z = xye^{\sin xy}$

10.114 $z = (x^2+y^2)\frac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}}$ 10.115. $z = arctg\sqrt{x^y}$ 10.116 $z = 2\sqrt{\frac{1-\sqrt{xy}}{1+\sqrt{xy}}}$

10.117 $u = (\sin x)^{yz}$ 10.118 $u = arctg(x-y)^z$

10.119. $u = \sqrt{az^3 - bt^3}$. $\frac{\partial u}{\partial z}$ va $\frac{\partial u}{\partial t}$ ni $z = b, t = a$ dagi qiymatini toping.

10.120. $z = \frac{x\cos y - y\cos x}{1+\sin x+\sin y}$. $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ ni $x = y = 0$ dagi qiymatini hisoblang.

Quyidagi funksiyani to'liq hosilasini toping.

10.133. $z = e^{xy}(x+y)$ 10.134. $z = \ln(1+e^x+y^2)$

10.135. $z = \frac{\sqrt{x}-\sqrt{y}}{x+y}$ 10.136. $z = \sin\left(\frac{x}{y}\right)$

10.137. $z = \frac{x\arcsin y}{y}$ 10.138. $z = x^y + y^x$

10.139. $z = \ln(x^2+y^2)$ 10.140. $z = \ln tg(y/x)$

10.141 $z = \sin(x^2+y^2)$

10.142. $z = x^y$ 10.143. $u = \ln\left(x+\sqrt{x^2+y^2}\right)$

10.144. $u = e^x(\cos y + x\sin y)$

10.145. $u = e^{x+y}(x\cos y + y\sin x)$ 10.146. $z = arctg\frac{2(x+\sin y)}{4-x\sin y}$

10.147. $u = e^{xyz}$ 10.148. $z = 5x^3y^2$ 10.149. $u = (xy)^z$

10.150. $z = (\sin x)^{\cos y}$ 10.151. $z = \frac{y}{x} - \frac{x}{y}$ 10.152. $z = e^{x^2+y^2}$

10.153. $z = \sin^2 x + \cos^2 y$ 10.154. $z(x,y) = \frac{y}{x^2}$; $df(1;1) - ?$

10.155. $z(x,y,z) = e^{x^2+y^2+z^2}$; $dz = (0;1;2)$

10.156. $z = \ln\left(x+\sqrt{x^2+y^2}\right)$ 10.157. $u = x^{y^2 z}$

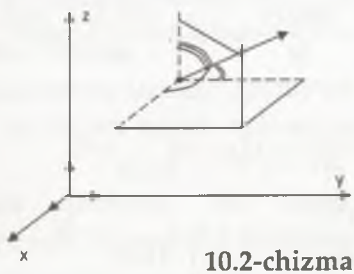
§ 10.3. Yo`nalish bo`yicha olingan hosilalar haqida tushunchalar

8. $u = f(x,y,z)$ tenglama biror sohaning har bir (x,y,z) nuqtasida u ni aniqlab beradi, o'sha soha skalyar u ning maydoni deyiladi. Bu maydonning $P(x,y,z)$ nuqtasini va P nuqtadan

$$\vec{e} = \cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k}$$

birlik vektor yo'nalishida chiquvchi e nurni qaraymiz, bu yerda α,β va γ - shu vektorning koordinata o'qlari bilan tashkil qilgan burchaklari. Bu yerda $|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$ (10.3-chizma).

$P_1(x+\Delta x,\ \ y+\Delta y,\ \ z+\Delta z)$ -bu nurning boshqa biror nuqtasi bo'lsin. Skalyar maydon u funktsiyaning P va P_1 nuqtalardagi qiymatlari ayirmasini bu funksiyaning l yo'nalish bo'yicha orttirmasi deb ataymiz. U holda $\Delta_e u = f(x+\Delta x,\ \ y+\Delta y,\ \ z+\Delta z)$. P va P_1 nuqtalar orasidagi masofani Δl orqali belgilaymiz: $\Delta l = PP_1$. $\lim\limits_{\Delta e \to 0} \frac{\Delta_e u}{\Delta e}$ limit $u = F(x,y,z)$ funksiyaning P nuqtadagi l yo'nalish bo'yicha **hosilasi** deb ataladi. Yo'nalish bo'yicha hosilagacha xususiy hosilalar bilan quyidagicha ifodalanadi.



10.2-chizma

$$u'_e = u'_x \cos\alpha + u'_y \cos\beta + u'_z \cos\gamma\,,$$

bu yerda birlik vektor $\vec{l} = (\cos\alpha, \cos\beta, \cos\gamma)$ chiziq bo'yicha yo'nalgan bo'ladi.

**Ta`rif.** *Agar A nuqta to`gri chiziq bo`ylab A_0 nuqtaga intilganda ushbu*

$$\lim_{A \to A_0} \frac{f(A) - f(A_0)}{\rho(A_0, A)}$$

*limit mavjud bo`lsa, uning qiymatiga f(x,y)=f(A)funksiyaning $A_0=(x_0,y_0)$ nuqtadagi **yo`nalish bo`yicha hosilasi** deyiladi va*

$$\frac{\partial f(A_0)}{\partial l} \text{ yoki } \frac{\partial f(x_0,y_0)}{\partial l} \text{ kabi belgilanadi.}$$

Demak,

$$\frac{\partial f(A_0)}{\partial l} = \lim_{A\to A_0} \frac{f(A)-f(A_0)}{\rho(A_0,A)}$$

11 – misol.

$z=x^2-xy-2y^2$ funksiyaning A(1;2) nuqtada ox o`qining musbat yo`nalish bilan 60^0 burchak tashkil qilgan yo`nalish bo`yicha hosilasini va o`zgarish tezligini toping:

Yo`nalish bo`yicha hosilani (9) formuladan topamiz. Funksiya (1;2) nuqtada differensiallanuvchi.

Agar l chiziq ox o`qning musbat yo`nalishi bo`yicha 60^0 burchak tashkil qilsa, OY bilan 30^0 burchak tashkil qiladi $(\beta=30^0)$

Quyidagini hisoblashimiz kerak

$$\frac{\partial z(1;2)}{\partial l} = \frac{\partial z(1;2)}{\partial x}\cos 60^0 + \frac{\partial z(1,2)}{\partial x}\cos 30^0 .$$

Ravshanki,

$$\frac{\partial z(1;2)}{\partial l} = (2x-y)\Big|_{(1,2)} = 0,$$

$$\frac{\partial z(1;2)}{\partial l} = (-x-4y)\Big|_{(1,2)} = -9$$

Demak,

$$\frac{\partial z}{\partial l} = 0\cdot\frac{1}{2} - 9\cdot\frac{\sqrt{3}}{2} = -\frac{9\sqrt{3}}{2}.$$

$M_0(x_0,y_0,z_0)$ nuqtadan $M_1(x_1,y_1,z_1)$ yo'nalish bo'yicha funksiyani hosilasini toping.

10.121 $u=x^2y+xz^2-2,\quad M_0(1;1;-1),\quad M_1(2;-1;3)$

10.122. $u=xe^y+ye^x-z^2,\quad M_0(3;0;2),\quad M_1(4;1;3)$

10.123. $u = \frac{x}{y} - \frac{y}{x}$, $M_0(1;1)$, $M_1(4;5)$

Skalyar maydonnig M nuqtasining gradiyentini toping.

10.124. $u = \ln(x^2 + y^2 + z^2)$ $M(1,1,-1)$ 10.125. $u = ze^{x^2+y^2+z^2}$ $M(0,0,0)$

10.126. $z = \sqrt{2xy + y^2}$, $M(3;2)$ 10.127. $z = arctg(xy)$, $M(1;1)$

$z = f(x,y)$ **funksiyani M nuqtadagi gradiyenti va uning modulini toping.**

10.128. $z = 7 - x^2 - y^2$ $M(1;2)$ 10.129. $z = (x-y)^2$ $M(0;3)$

10.130. $z = xye^{1+x+y}$ $M(0;-1)$ 10.131. $z = x\ln(x+y)$ $M(-1;2)$

10.132. $z = \sin(x + y^2)$ $M\left(\frac{\pi}{2}; \sqrt{\frac{\pi}{2}}\right)$

§ 10.4 . Taqribiy hisoblash va xatolik bahosi

11. Faraz qilaylik, $u = f(x_1, x_2, ... x_i ... x_n)$ funksiyani $M_0(x_1, x_2, ..., x_n)$ nuqtadagi qiymati ma'lum bo'lsa, $M_0(x_1 + \Delta x_1, x_2 + \Delta x_2, ..., x_n + \Delta x_n)$ nuqtadagi qiymatini hisoblash talab etilsin. Argumentning $|\Delta x_i|$-orttirmasi kichikson bo'ladi, u holda $f(M) = f(M_0) + \Delta u \approx f(x_1, x_2, ..., x_n) + du$, bu yerda $du = \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} \mu(x_0)\Delta x_i$. Xatolik $\rho = \sqrt{\sum_{i=1}^{n} (x_i)^2}$ dan oshib ketmaydi.

12– misol. Ushbu

*$1{,}03^{1{,}98}$*miqdorning taqribiy qiymatini toping.

Taqribiy qiymatini topish uchun

$$f(x,y) = x^y$$

funksiyani qaraymiz. Bu funksiya (1,2) nuqtada differensiallanuvchi va berilgan funksiyani (1,03; 1,98) nuqtada qiymatini topish uchun yuqoridagi formuladan foydalanamiz:

$$\Delta f(1,2) \approx \frac{\partial f(1,2)}{\partial x}\Delta x + \frac{\partial f(1,2)}{\partial y}\Delta y$$

Berilgan miqdorni quyidagicha yozsak

$$1{,}03^{1{,}98} = (1 + 0{,}03)^{2 - 0{,}02}$$

unda $\Delta x = 0{,}03$, $\Delta y = -0{,}02$ deyish mumkin.

Funksiya qiymatini va xususiy hosilalarini (1;2) nuqtada hisoblaymiz:

$f(1,2)=1$,

$$\frac{\partial f(1,2)}{\partial x} = yx^{y-1}\Big|_{(1,2)} = 2\cdot 1^{2-1} = 2,$$

$$\frac{\partial f(1,2)}{\partial y} = x^{y}\ln x\Big|_{(1,2)} = 0.$$

Natijada berilgan miqdorni qiymati quyidagicha hisoblanadi .

$$f(1+0{,}03;2-0{,}02) \approx f(1,2) + \frac{\partial f(1,2)}{\partial x}\Delta x + \frac{\partial f(1,2)}{\partial y}\Delta y \approx$$

$$\approx 1 + 2\cdot 0{,}03 + 0\cdot(-0{,}02) = 1{,}06.$$

Demak,

$$1{,}03^{1{,}98} \approx 1{,}06.$$

13– misol. Ushbu

$$\frac{2{,}02^2}{\sqrt[3]{7{,}98\ \sqrt[4]{1{,}03^3}}}$$

miqdorning taqribiy qiymatini toping.

Taqribiy qiymatini topish uchun

$$f(x,y,z) = \frac{x^2}{\sqrt[3]{y\ \sqrt[4]{z^3}}} = x^2\cdot y^{-\frac{1}{3}}\cdot z^{-\frac{1}{4}}$$

funksiyani qaraymiz.

Bu funksiya (2;8;1) nuqtada differensiallanuvchi va berilgan funksiyani (2,02; 7,98; 1,03) nuqtadagi qiymatini hisoblash uchun yuqoridagi formuladan foydalanamiz:

$$\Delta f(2,8,1) \approx \frac{\partial f(2,8,1)}{\partial x}\Delta \bar{o} + \frac{\partial f(2,8,1)}{\partial y}\Delta y + \frac{\partial f(2,8,1)}{\partial z}\Delta z$$

Berilgan miqdorni quyidagi ko`rinishda yozsak

$$\frac{2{,}02^2}{\sqrt[3]{7{,}98\ \sqrt[4]{1{,}03^3}}} = \frac{(2+0{,}02)^2}{\sqrt[3]{(8-0{,}02)\ \sqrt[4]{(1+0{,}03)^3}}}$$

unda $\Delta x = 0,02$, $\Delta y = -0,02$, $\Delta z = 0,03$ deyish mumkin.

Funksiyaning qiymatini va xususiy hosilalarni (2;8;1) nuqtada hisoblaymiz:

$$f(2,8,1) = \frac{2^2}{\sqrt[3]{8}\,\sqrt[4]{1}} = \frac{4}{2} = 2,$$

$$\frac{\partial f(2,8,1)}{\partial x} = 2x \cdot y^{-\frac{1}{3}} \cdot z^{-\frac{1}{4}}\Big|_{(2,8,1)} = 2 \cdot 2 \cdot 8^{-\frac{1}{3}} \cdot 1^{-\frac{1}{4}} = 2,$$

$$\frac{\partial f(2,8,1)}{\partial y} = -\frac{1}{3}x^2 \cdot y^{-\frac{4}{3}} \cdot z^{-\frac{1}{4}}\Big|_{(2,8,1)} = -\frac{1}{3} \cdot 2^2 \cdot 8^{-\frac{4}{3}} \cdot 1^{-\frac{1}{4}} = -\frac{1}{12},$$

$$\frac{\partial f(2,8,1)}{\partial z} = -\frac{1}{4}x^2 \cdot y^{-\frac{1}{3}} \cdot z^{-\frac{5}{4}}\Big|_{(2,8,1)} = -\frac{1}{4} \cdot 2^2 \cdot 8^{-\frac{1}{3}} \cdot 1^{-\frac{5}{4}} = -\frac{1}{2},$$

Natijada berilgan qiymat quyidagicha hisoblanadi.

$$f(2+0,02,\ 8-0,02,\ 1+0,03) \approx f(2,8,1) +$$
$$+\frac{\partial f(2,8,1)}{\partial x}\Delta x + \frac{\partial f(2,8,1)}{\partial y}\Delta y + \frac{\partial f(2,8,1)}{\partial z}\Delta z =$$
$$= 2 + 2 \cdot 0,02 - \frac{1}{12} \cdot (-0,02) - \frac{1}{4} \cdot 0,03 \approx 2,034.$$

Quyidagi ifodalarni taqribiy hisoblang.

10.158. a) $\sqrt{3,01^2 + 3,98^2}$ b) $(1,02)^{3,01}$.

10.159. $\sqrt{\sin^2 1,55 + 8e^{0.015}}$

10.160. $arctg(1,02/0,95)$ 10.161. $1,02^{4,05}$ 10.162. $\ln(0,09^3 + 0,99^3)$

10.163. $\sqrt[3]{1,02^2 + 0,05^2}$ 10.164. $\sqrt{5e^{0,02} + 2,03^2}$

10.165. $\sqrt{1,04^{1,99} + \ln 1,02}$

§ 10.5. Yuqori tartibli hosilalar ikki o'zgaruvchi funsiyaning ekstremumi

12. G sohada P_0 nuqtaning shunday atrofi mavjud bo'lsaki, bu atrofning P_0 dan farqli barcha nuqtalari uchun $f(P_0) > f(P)$ tengsizlik bajarilsa, ikki o'zgaruvchining $z = f(x,y) = f(P)$ funksiyasi sohaning P_0 nuqtasida **maksimumga ega** deyiladi. G soha P_0 nuqtaning shunday atrofi mavjud bo'lsaki, bu atrofning P_0 dan farqli barcha nuqtalari uchun $f(P_0) < f(P)$ tengsizlik bajarilsa, ikki o'zgaruvchining $z = f(x,y) = f(P)$ funksiyasi G sohaning P_0 nuqtasida **minimumga ega** deyiladi. Maksimum va minimum umumiy nom bilan **eksremumlari** deb ataladi.

13. Agar $P_0(x_0, y_0)$ nuqta $z = f(x,y)$ funktsiyaning ekstremum nuqtasi bo'lsa, u holda bu funksiyaning shu nuqtadagi xususiy hosilalari mavjud bo'lgan taqdirda $f'_x(x_0, y_0) = 0$, $f'_y(x_0, y_0) = 0$ bo'ladi (ekstremum mavjudligining zaruriy sharti).

14. $z = f(x,y)$ funktsiyaning birinchi tartibli xususiy hosiladan olingan xususiy hosila ikkinchi tartibli xususiy hosila deyiladi va uni quyidagicha yoziladi:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = f''_{xx}(x,y)$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = f''_{yy}(x,y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = f''_{xy}(x,y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = f''_{yx}(x,y)$$

Xuddi shuningdek, uch va undan yuqori tartibli xususiy hosilalar ham yuqoridagi kabi aniqlanadi. $f''_{xy}(x,y)$ va $f''_{yx}(x,y)$ xususiy hosilalar $z = f(x,y)$ funktsiyaning aralash hosilalari deyiladi.

15. Agar $P_0(x_0, y_0)$ nuqtada ikkita $z = f(x,y)$ xususiy hosila ham nolga teng bo'lsa, bu nuqtaning tasviri $\Delta = AC - B^2$ $\left(A = z''_{xx},\ B = z''_{xy},\ C = z''_{yy}\right)$ bilan aniqlanadi. $\Delta > 0$ bo'lsa ekstremum mavjud ($A < 0$ maksimum va $A > 0$ bo'lsa minimum) bo'ladi. $\Delta < 0$ bo'lsa ekstremum mavjud bo'lmaydi va $\Delta = 0$ ekstremum mavjud bo'lish ham, bo'lmasligi ham mumkin.

16. Funsiyani ekstremumlar tekshirish uchun quyidagi sxema tavsiya etiladi:

1. Xususiy hosila z'_x va z'_y larni topamiz.

2. $z'_x = 0$ va $z'_y = 0$ tenglamalar sistemasini yechib funktsiyani kritik nuqtalari topiladi.

3. Ikkinchi tartibli xususiy hosilalari olib har bir kritik nuqtaga hisoblanadi va ekstremumning yetarlilik shartidan foydalaniladi.

17. Xususiy hosilani geometrik ma'nosi. $f(x,y,z)=0$ tenglama bilan berilgan sirt va o'rta sirtdan $M(x,y,z)$ nuqta olingan bo'lsin. Bu nuqtada o'tkazilgan normal tenglamalari quyidagicha yoziladi

$$\frac{X-x}{\frac{\partial f}{\partial x}} = \frac{Y-y}{\frac{\partial f}{\partial y}} = \frac{Z-z}{\frac{\partial f}{\partial z}} \qquad (10.1)$$

Urinma tekislik tenglamasi:

$$\frac{\partial f}{\partial x}(X-x)+\frac{\partial f}{\partial y}(Y-y)+\frac{\partial f}{\partial z}(Z-z)=0 \qquad (10.2)$$

dan iborat bo'ladi. (10.1) va (10.2) tenglamalardagi x, y, z - normalning yoki urinma tekislikning o'zgaruvchi koordinatalaridan iborat. $\vec{N}\left(\frac{\partial f}{\partial x};\frac{\partial f}{\partial y};\frac{\partial f}{\partial z}\right)$ vektor sirtining normal vektori deyiladi. Agar sirtda $\frac{\partial f}{\partial x}=0; \frac{\partial f}{\partial y}=0; \frac{\partial f}{\partial z}=0$ bo'lsa, u maxsus nuqta deyiladi. Bunday nuqtada sirtning normali ham urinma tekisligi ham bo'lmaydi.

18. Eng kichik kvadratlar usuli – xatolar nazariyasida tasodifiy xatolarni o'z ichiga olgan o'lchash natijalaridan bir yoki bir necha miqdorni topishda qo'llaniladi. x no'malum miqdorning qiymatini izlab topish uchun n ta mustaqil o'lchash o'tkazilgan, bu o'lchamlardan $y_1, y_2 ... y_n$ qiymatlar, ya'ni $y_i = x + \delta_i$ $(i = 1,2...n)$ qiymatlar topilgan bo'lsin deb faraz qilaylik, bundagi δ_i-tasodifiy xatolar matematik kutilishi nolga $M\delta_i = 0$ va dispersiyasi $D\delta_i = \sigma_i^2$ bo'lgan erkli tasodifiy miqdorlar bo'ladi. Bu usulga x miqdor sifatida shunday x olinadiki, uning uchun

$S(x)=\sum_{i=1}^{n} p_i(y_i-x)^2=\sum_{i=1}^{n}(f(x_i)-y_i)^2$ - kvadratlar yig'indisi eng kichik bo'ladi. Agar $f(x)$ - chiziqli funktsiya bo'lsa, $y=ax+b$, u holda $S=\sum_{i=1}^{n}(ax_i+b-y_i)$. Noma'lum parametrlar a va b quyidagi normal tenglamalar sistemasidan aiqlanadi.

$$\begin{cases}\left(\sum_{i=1}^{n} x_i^2\right)a+\left(\sum_{i=1}^{n} x_i\right)b=\sum_{i=1}^{n} x_i y_i \\ \left(\sum_{i=1}^{n} x_i\right)a+nb=\sum_{i=1}^{n} y_i\end{cases}$$

2. Agar $f(x)$ funktisya kvadrat funksiya ko'rinishida $y=ax^2+bx+c$ bo'lsa, u holda $S=\sum_{i=1}^{n}(ax_i^2+bx_i+c-y_i)^2$, no'malum kattaliklar a, b, c quyidagi normal tenglamalar sistemasidan aniqlanadi.

$$\begin{cases}\left(\sum_{i=1}^{n} x_i^4\right)a+\left(\sum_{i=1}^{n} x_i^3\right)b+\left(\sum_{i=1}^{n} x_i^2\right)c=\sum_{i=1}^{n} x_i^2 y_i \\ \left(\sum_{i=1}^{n} x_i^3\right)a+\left(\sum_{i=1}^{n} x_i^2\right)b+\left(\sum_{i=1}^{n} x_i\right)c=\sum_{i=1}^{n} x_i y_i \\ \left(\sum_{i=1}^{n} x_i^2\right)a+\left(\sum_{i=1}^{n} x_i\right)b+nc=\sum_{i=1}^{n} y_i\end{cases}$$

14-misol. Ushbu

$$z=x^3+y^3-3xy+10$$

funksiyani ekstremumga tekshiring.

Xususiy hosilalarini topamiz:

$$z'_x=3x^2-3y; \qquad z'_y=3y^2-3x.$$

Statsionar nuqtalarini quyidagi sistemadan tapamiz:

$$\begin{cases}3x^2-3y=0, \\ 3y^3-3x=0.\end{cases} \begin{cases}x^2-y=0, \\ y^3-x=0.\end{cases} \begin{cases}y=x^2, \\ x^6-x=0.\end{cases}$$

Bu sistema $x_1=0, y_1=0$ va $x_2=1, y_1=1$ yechimlarga ega bo'ladi. Demak, (0,0) va (1,1) statsionar nuqtalarga ega bo'ladi.

Endi ekstremumga erishishning yetarli shartini tekshirish uchun ikkinchi tartibli xususiy hosilalarni topamiz:

$$\frac{\partial^2 z}{\partial x^2}=6x, \quad \frac{\partial^2 z}{\partial x\partial y}=-3, \quad \frac{\partial^2 z}{\partial y^2}=6y.$$

Yetarli shartni (0,0) nuqtada tekshiramiz. Buning uchun ikkinchi tartibli xususiy hosilalarni bu nuqtada hisoblaymiz:

$$a_{11} = \frac{\partial^2 z(0,0)}{\partial x^2} = 0,\ a_{12} = \frac{\partial^2 z(0,0)}{\partial x \partial y} = -3,\ \frac{\partial^2 z(0,0)}{\partial y^2} = 0.$$

Unda

$$a_{11}a_{22} - a_{12}^2 = 0 - (-3)^2 = -9 < 0$$

bo'lib, 3) shartdan funksiya (0,0) nuqtada ekstremumga ega bo'lmaydi.

Endi yetarli shartni (1,1) nuqtada tekshiramiz:

$$a_{11} = \frac{\partial^2 z(1,1)}{\partial x^2} = 6,\ a_{12} = \frac{\partial^2 z(1,1)}{\partial x \partial y} = -3,\ a_{22} = \frac{\partial^2 z(1,1)}{\partial y^2} = 6.$$

$$a_{11}a_{22} - a_{12}^2 = 6 \cdot 6 - (-3)^2 = 36 - 9 = 27 > 0.$$

Bulardan, $a_{11} > 0$, $a_{11} \cdot a_{22} - a_{12}^2 > 0$ bo'lib, 1) shart o'rinli bo'lmoqda. Demak, funksiya (1,1) nuqtada minimumga ega bo'ladi va u quyidagiga teng

$$\min_{x=1,y=1} z = 1^3 + 1^3 - 3 \cdot 1 \cdot 1 + 10 = 9.$$

Quyidagi funksiyalardan ikkinchi tartibli xususiy hosilalarini oling.

10.166. *a*) $z = y\ln x$ *b*) $u = 4x^3 + 3x^2y + 3xy^2 - y^3$. $\frac{\partial^2 u}{\partial x \partial y}$-ni toping.

10.167. *a*) $z = \ln(1 + x + 2y)$ *b*) $u = xy + \sin(x + y)$. $\frac{\partial^2 u}{\partial x^2}$ - ni toping.

10.168. $u = \ln tg(x + y)$. $\frac{\partial^2 u}{\partial x \partial y}$-ni toping.

10.169. $z = arctg\frac{x + y}{1 - xy}$. $\frac{\partial^2 z}{\partial x \partial y}$ -ni toping.

10.170. $z = x^2 \ln(x + y)$. $\frac{\partial^2 z}{\partial x \partial y}$-ni toping.

10.171. $u = x\sin xy + y\cos xy$. $\frac{\partial^2 u}{\partial x^2}$ - ni toping.

10.172. $u = \sin x(x + \cos y)$. $\frac{\partial^3 z}{\partial x^2 \partial y}$ -ni toping.

10.173. $z = 0{,}5\ln(x^2 + y^2)$. d^2z-ni toping.

10.174. $z = \cos(x + y)$. d^2z-ni toping.

10.175. $z = \cos(ax + e^{y})$. $\frac{\partial^3 z}{\partial x \partial y^2}$-ni toping.

Quyidagi funktsiyalarning ekstremumlarini toping.

10.176.a) $z = e^{\frac{x}{2}}(x + y^2)$ в) $z = x^2 - xy + y^2 + 9x - 6y + 20$

10.177 a) $z = x^y - xy$ в) $z = y\sqrt{x} - y^2 - x + 6y$

10.178. $z = x^3 + 8y^3 - 6xy + 1$

10.179. $z = 2xy - 4x - 2y$

10.180. $z = \sin x + \sin y + \sin(x + y);\ 0 \le x \le \frac{\pi}{2}$ va $0 \le x \le \frac{\pi}{2}$

9.171. $z = e^{\frac{x}{2}}(x + y^2)$

10.181. $z = x^2 + y^2 + xy - 4x - 5y$

10.182. $z = xy(1 - x - y)$

10.184. $z = x^3y^2(2 - x - y)$

10.185. $z = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$

10.186. $z = 2x^3 - xy^2 + 5x^2 + y^2$

10.187. $z = 3x + 6y - x^2 - xy + y^2$

10.188. $z = x^2 + y^2 - 2\ln x - 18\ln y$

10.189. $z = 2 - \sqrt[3]{x^2 + y^2}$

10.190. $z = (2x^2 + y^2)e^{-(x^2+y^2)}$

10.191. $z = \frac{xy}{x^2 + y^2}$

10.192. $z = xy - \ln(x + y)$

10.193. $z = \sqrt{x}\sqrt[4]{y} - x - 2y$

10.194.a) $z = 3x + 6y - x^2 - xy - y^2$ б) $z = x^2 + y^2 - 2x - 4\sqrt{xy} - 2y + 8$

10.195. $z = 2x^3 - xy^2 + 5x^2 + y^2$

10.196. $z = 3x^2 - 2x\sqrt{y} + y - 8x + 8$

10197. $z = (x - 1)^2 + 2y^2$

10.198. $z = (x - 1)^2 - 2y^2$

10.199. $z = x^2 + xy + y^2 - 2x - y$

10.200. $z = x^3y^2(6 - x - y)(x > 0, y > 0)$

10.201. $z = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

10.202. $z = xy\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$

10.203. $z = 1 - (x^2 + y^2)^{2/3}$

10.204. $z = (x^2 + y^2)e^{-(x^2+y^2)}$

Murakkab funksiyalarning hosilalari

10.205. $z = \frac{1}{2}\ln\frac{u}{\vartheta}$ bunda $u = tg^2x, \vartheta = ctg^2x$ bo'lsa, $\frac{dz}{dx}$ ni toping.

10.206. $z = \frac{x^2 - y}{x^2 + y}$ bunda $y = 3x + 1$ bo'lsa, $\frac{dz}{dx}$ ni toping.

10.207. $z = x^2y$ bunda $y = \cos x$ bo'lsa, $\frac{\partial z}{\partial x}$ va $\frac{dz}{dx}$ ni toping.

10.208. $z = x^2/y$ bunda $x = u - 2\vartheta, y = \vartheta + 2u$ bo'lsa, $\frac{\partial z}{\partial u}$ va $\frac{\partial z}{\partial \vartheta}$ ni toping.

10.209. $u = e^{x-2y}$ bunda $x = \sin t, y = t^3$ $\frac{du}{dt} = ?$

10.210. $u = z^2 + y^2 + zy$ bunda $z = \sin t,\ y = e^t$ $\frac{du}{dt} = ?$

10.211. $z = \arcsin(x - y)$ bunda $x = 3t,\ y = 4t^3$ $\frac{dz}{dt} = ?$

10.212. $z = x^2y - y^2x$ bunda $z = u\cos\vartheta,\ y = u\sin\vartheta$ $\frac{\partial z}{\partial u} = ?$ $\frac{\partial z}{\partial \vartheta} = ?$

10.213. $z = x^2 \ln y$ bunda $x = \frac{u}{\vartheta},\ y = 3u - 2\vartheta$ $\frac{\partial z}{\partial u} = ?$ $\frac{\partial z}{\partial \vartheta} = ?$

10.214. $z = arctg(xy)$ bunda $y = e^x$ $\frac{dz}{dx}$ ni toping

Quyidagi tenglamalardan y' ni toping.

10.215. $x^2 + y^2 + \ln(x^2 + a^2) = a^2$

10.216. $(y/x) + \sin(y/x) = a$

10.217. $(xy - \alpha)^2 + (xy - \beta)^2 = r^2$

10.218. $x^3 + 2y^3 - 2xy\sqrt{2xy} + 1 = 0$

10.219. $\ln tg(y/x) - y/x = a$

10.220. $3\sin(\sqrt{x}/y) - 2\cos(\sqrt{x}/y) + 1 = 0$

10.221. $x^3y - y^3x = a^4$

10.222. $x^2y^2 - x^4 - y^4 = a^4$

10.223. $xe^y + ye^x - e^{xy} = 0$

10.224. $(x^2 + y^2)^2 + a^2(x^2 - y^2) = 0$

Quyidagi tenglamalardan $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ lar topilsin.

10.225. $x + y + z = e^x$

10.226. $x^3 + y^3 + z^3 - 3xyz = 0$

10.227. $x^2 + y^2 + z^2 - 6x = 0$

10.228. $z^2 = xy$

10.229. $\cos(ax + by - cz) = k(ax + by - cz)$

10.230. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

10.231. $x^2 - 2y^2 + z^2 - 4x + 2z - 5 = 0$

10.232. $z^3 + 3xyz = a^3$

10.233. $e^z - xyz = 0$

10.234(224). $e^8 + x^2y + z + 5 = 0$

Quyidagi funksiyalarning ekstremumlari berilgan shart asosida topilsin.

10.235. $z = \frac{1}{x} + \frac{1}{y}$, agar $x + y = z$ bo'lsa.

10.236. $z = x - y$, agar $x^2 + y^2 = 1$ bo'lsa.

10.237. $z = xy^2$, agar $x + 2y = 4$ bo'lsa.

10.238. $z = \frac{x - y - 4}{\sqrt{2}}$, agar $x^2 + y^2 = 1$ bo'lsa.

10.239. $z = \sqrt[4]{x}\sqrt[3]{y}$ agar $2x + 5y = 100$ bo'lsa.

Quyidagi sirtlarga o'tkazilgan urinma tekisliklar tenglamasi yozilsin.

10.240. $z = x^2 + 2y^2, (1;1;3)$ nuqtada. 10.241. $xy = z^2, (x_0;y_0;z_0)$ nuqtada.

10.242. $xyz = a^2, (x_0;y_0;z_0)$ nuqtada.

Takrorlash uchun savollar

1.Ko'p o'zgaruvchili funksiya deb nimaga aytiladi?

2.Ko'p o'zgaruvchili funksiya uzluksizligi deb nimaga aytiladi?

3.Karrali limit nima?

4.Takroriy limit nima?

5.Karrali va takroriy limitlar qachon teng bo'ladi?

6.Gradiyent nima?

7.Yo'nalish bo'yicha hosila qanday olinadi?

8.Taqribiy hisoblash formulasini yozing.

BIR NECHA O'ZGARUVCHILI FUNKSIYALARNING DIFFERENSIAL HISOBIGA DOIR TESTLAR

1. Ko'p o'zgaruvchili funksiyalar

1. Tomonlari x va y bo'lgan to'g'ri to'rtburchakka doir qaysi masalaning javobi ikki o'zgaruvchili funksiya bilan ifodalanmaydi ?

A) To'g'ri to'rtburchak yuzasini topish ;

B) To'g'ri to'rtburchak perimetrini topish ;

C) To'g'ri to'rtburchak diagonalini topish ;

D)To'g'ri to'rtburchakning ikkita qarama-qarshi tomonining yig'indisini topish ;

E)Barcha masalalarning javoblari ikki o'zgaruvchili funksiya bilan ifodalanadi.

2. Ikki o'zgaruvchili funksiyani ko'rsating.

A) $f = ax^2 + bx + c$. B) $f = x + \frac{1}{y}$. C) $f = y + \frac{1}{y}$.

D) $f = t^2 + t + 1$. E) $f = \sin(ax^2 + b)$.

3. Ikki o'zgaruvchili $z=f(x,y)$ funksiya aniqlanish sohasidagi har bir $M(x,y)$ nuqtaga nimani mos qo'yadi ?

A) to'g'ri chiziqdagi biror nuqtani .

B) tekislikdagi biror nuqtani .

C) fazodagi biror nuqtani .

D) biror to'plamni . E) biror chiziqni .

4. Ushbu ikki o'zgaruvchili funksiyaning $D\{f\}$ aniqlanish sohasini toping:

$$f(x,y)=\sqrt{9-x^2-y^2}.$$

A) Tomoni $a=3$ va markazi O(0,0) nuqtada joylashgan kvadrat .

B) Tomoni $a=3$ va markazi O(0,0) nuqtada joylashgan muntazam uchburchak .

C) Radiusi R=3 va markazi O(0,0) nuqtada joylashgan doira .

D) Radiusi R=3 va markazi O(0,0) nuqtada joylashgan aylana

E) [–3,3] kesmadan iborat soha .

5. Ushbu ikki o'zgaruvchili funksiyaning qiymatlar sohasini toping:

$$f(x,y)=\sqrt{9-x^2-y^2}.$$

A) [–3,3] . B) (–3,3) . C) [0,3] . D) (0,3) . E) [0,9] .

6. Ushbu ikki o'zgaruvchili funksiyaning $D\{f\}$ aniqlanish sohasini toping:

$$f(x,y)=\sqrt{16+x^2+y^2}.$$

A) Tomoni $a=4$ va markazi O(0,0) nuqtada joylashgan kvadrat ;

B) Tomoni $a=4$ va markazi O(0,0) nuqtada joylashgan muntazam uchburchak;

C) Radiusi R=4 va markazi O(0,0) nuqtada joylashgan doira ;

D) Radiusi R=4 va markazi O(0,0) nuqtada joylashgan aylana ;

E) R^2 tekislik .

7. Ushbu ikki o'zgaruvchili funksiyaning qiymatlar sohasini toping:

$$f(x,y)=\sqrt{16+x^2+y^2}.$$

A) [0,4] . B) (0,4) . C) [4,+∞) .

D) (–∞,4] . E) [0,16] .

8. Ikki o'zgaruvchili $z=f(x,y)$ funksiya grafigi qanday geometrik ob'ektni ifodalaydi?

A) tekislikdagi to'g'ri chiziqni .

B) tekislikdagi egri chiziqni .

C) fazodagi tekislikni .

D) fazodagi sirtni . E) fazodagi jismni .

9. *Ta'rifni to'ldiring:* Ikki o'zgaruvchili $z=f(x,y)$ funksiyaning sath chizig'i deb ··· tenglama bilan aniqlanadigan chiziqqa aytiladi .

A) $f(C,y)=0$. B) $f(x,C)=0$. C) $f(0,y)=C$.

D) $f(x,0)=C$. E) $f(x,y)=C$.

10. Ikki o'zgaruvchili $z=9x^2+9y^2$ funksiyaning sath chiziqlari nimadan iborat?

A) parabola B) giperbola C) ellips D) aylana

E) kesishuvchi ikkita to'g'ri chiziq .

11. Ikki o'zgaruvchili $z=9x^2+16y^2$ funksiyaning sath chiziqlari nimadan iborat?

A) parabola B) giperbola C) ellips D) aylana

E) to'g'ri javob keltirilmagan.

12. Ikki o'zgaruvchili $z=9x^2-16y^2$ funksiyaning sath chiziqlari nimadan iborat?

A) parabola ; B) giperbola; C) ellips; D) aylana;

E) kesishuvchi ikkita to'g'ri chiziq .

13. Ikki o'zgaruvchili $z=9y^2-x$ funksiyaning sath chiziqlari nimadan iborat?

A) parabola B) giperbola C) ellips D) aylana

E) kesishuvchi ikkita to'g'ri chiziq

14. Tomonlari x , y va z bo'lgan to'g'ri burchakli parallelopipedga doir qaysi masalaning javobi uch o'zgaruvchili funksiya bilan ifodalanmaydi ?

A) to‘g‘ri burchakli parallelopiped hajmini topish .

B) to‘g‘ri burchakli parallelopiped yon sirtini topish .

C) to‘g‘ri burchakli parallelopiped to’la sirtini topish .

D) to‘g‘ri burchakli parallelopiped diagonalini topish .

E) Barcha masalalarning javoblari uch o‘zgaruvchili funksiya bilan ifodalanadi .

15. Uch o’zgaruvchili $f(x,y,z)$ funksiya aniqlanish sohasidagi har bir $M(x,y,z)$ nuqtaga nimani mos qo‘yadi ?

A) to’g’ri chiziqdagi biror nuqtani . B) tekislikdagi biror nuqtani

C) fazodagi biror nuqtani .

D) fazodagi biror chiziqni .

E) fazodagi biror sirtni .

16. Uch o‘zgaruvchili $f(x,y,z)$ funksiya grafigi qanday geometrik ob’ektni ifodalaydi?

A) fazodagi chiziqni

B) fazodagi tekislikni

C) fazodagi sirtni

D) fazodagi jismni

E) to‘g’ri javob keltirilmagan .

2. Ko’p o’zgaruvchili funksiyalarning limiti

1. Biror chekli A soni $z=f(x,y)$ funksiyaning $M(x,y)\to M_0(x_0,y_0)$ bo’lgandagi limiti ekanligini tekshirish uchun quyidagi amallardan qaysi biri bajarilmaydi?

A) ixtiyoriy kichik $\varepsilon>0$ son tanlanadi .

B) $M_0(x_0,y_0)$ nuqtaning $r(\varepsilon)$ radiusli atrofi $U_r(x_0,y_0)$ topiladi .

C) ixtiyoriy $M(x,y) \in U_r(x_0,y_0)$ uchun $|f(x,y)-A|<\varepsilon$ shart tekshiriladi .

D) $f(x,y)$ funksiyaning $M_0(x_0,y_0)$ nuqtadagi qiymati hisoblanadi ;

E) keltirilgan barcha amallar bajariladi .

2. $\lim\limits_{\substack{x\to x_0 \\ y\to y_0}} f(x,y)=A$ ekanligini aniqlashda ixtiyoriy kichik $\varepsilon>0$ sonida qaysi munosabat bajarilishi tekshiriladi?

A) $|f(x,y)-A|=\varepsilon$

B) $|f(x,y)-A|\neq\varepsilon$

C) $|f(x,y)-A|>\varepsilon$.

D) $|f(x,y)-A|<\varepsilon$.

E) to'g'ri javob keltirilmagan .

3. $f(x,y)=(x^2-y^2)/(x^2+y^2)$ funksiyaning $M(x,y)\to O(0,0)$ holdagi limitini toping.

A) 0 B) 1 C) $\frac{1}{2}$ D) limit mavjud emas E) 2

4. $f(x,y)=(x^2-y^2)/(x^2+y^2)$ funksiyaning $M(x,y)\to M_0(1,1)$ holdagi limitini toping.

A) 0 B) 1 C) $\frac{1}{2}$ D) limit mavjud emas E) 2

5. $f(x,y)=(x^2-y^2)/(x+y)$ funksiyaning $M(x,y)\to O(0,0)$ holdagi limitini toping.

A) 0 B) 1 C) $\frac{1}{2}$ D) limit mavjud emas E) 2

6. $f(x,y)=(x^2-y^2)/(x-y)$ funksiyaning $M(x,y)\to M_0(1,1)$ holdagi limitini toping.

A) 0 B) 1 C) $\frac{1}{2}$ D) limit mavjud emas E)2

7. $\lim\limits_{\substack{x\to 0 \\ y\to 0}} \cos(x^2+y^2)$ limitni hisoblang.

A) 0 B) 1 C) $\frac{1}{2}$ D) limit mavjud emas E) 2

8. $\lim\limits_{\substack{x\to 0 \\ y\to 0}} \sin(x^2+y^2)$ limitni hisoblang.

A) 0 B) 1 C) $\frac{1}{2}$ D) limit mavjud emas E) 2

9. $\lim\limits_{\substack{x\to 0 \\ y\to \pi/3}} \cos(x^2+y)$ limitni hisoblang.

A) 0 B) 1 C) $\frac{1}{2}$

D) limit mavjud emas . E) 2 .

10. Quyidagilardan qaysi biri karrali limit bo'ladi?

A) $\lim\limits_{\substack{x\to x_0\\ y\to y_0}} f(x,y)$. B) $\lim\limits_{x\to x_0} f(x,y)$. C) $\lim\limits_{y\to y_0} f(x,y)$.

D) $\lim\limits_{x\to x_0}\{\lim\limits_{y\to y_0} f(x,y)\}$. E) $\lim\limits_{x\to x_0}\{\lim\limits_{x\to x_0} f(x,y)\}$.

11. Quyidagilardan qaysi biri takroriy limit bo'ladi?

A) $\lim\limits_{\substack{x\to x_0\\ y\to y_0}} f(x,y)$. B) $\lim\limits_{x\to x_0} f(x,y)$. C) $\lim\limits_{y\to y_0} f(x,y)$.

D) $\lim\limits_{x\to x_0}\{\lim\limits_{y\to y_0} f(x,y)\}$. E) $\lim\limits_{x\to x_0}\{\lim\limits_{x\to x_0} f(x,y)\}$.

12. Takroriy $\lim\limits_{x\to x_0}\{\lim\limits_{y\to y_0} f(x,y)\} = A_1$ va $\lim\limits_{y\to y_0}\{\lim\limits_{x\to x_0} f(x,y)\} = A_2$ limitlar uchun quyidagi munosabatlardan qaysi biri bajarilishi mumkin?

A) $A_1 = A_2$ B) $A_1 \neq A_2$

C) $A_1 < A_2$ D) $A_1 > A_2$.

E) keltirilgan barcha munosabatlar bajarilishi mumkin.

13. $f(x,y)=(x-y)/(x+y)$ funksiya uchun $\lim\limits_{x\to 0}\{\lim\limits_{y\to 0} f(x,y)\}$ takroriy limit qiymatini toping.

A) 0 B) 1 C) –1 D) ∞ E) mavjud emas

14. $f(x,y)=(x-y)/(x+y)$ funksiya uchun $\lim\limits_{y\to 0}\{\lim\limits_{x\to 0} f(x,y)\}$ takroriy limit qiymatini toping.

A) 0 B) 1 C) –1 D) ∞ E) mavjud emas

15. $f(x,y)=(x-y)/(x+y)$ funksiya uchun $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y)$ karrali limitni hisoblang.

A) 0 B) 1 C) –1 D) ∞

E) limit mavjud emas

16. Agar $\lim\limits_{\substack{x\to x_0\\ y\to y_0}} f(x,y) = A$, $\lim\limits_{x\to x_0}\{\lim\limits_{y\to y_0} f(x,y)\} = A_1$ va $\lim\limits_{y\to y_0}\{\lim\limits_{x\to x_0} f(x,y)\} = A_2$ bo'lsa, quyidagi tasdiqlardan qaysi biri doimo o'rinli bo'ladi?

A) A mavjud va chekli bo'lsa, unda A_1 , A_2 mavjud va $A_1 \neq A_2$

B) A_1 , A_2 mavjud va chekli bo'lsa, unda A mavjud .

C) A_1 , A_2 mavjud va $A_1=A_2$ bo'lsa, unda A mavjud .

D)A mavjud va chekli bo'lsa, unda A_1 , A_2 mavjud va $A_1=A_2=A$.

E) A mavjud va chekli bo'lsa, unda A_1 , A_2 mavjud va $A_1=A_2\neq A$.

3. Ko'p o'zgaruvchili funksiyalarning uzluksizligi

1. $z=f(x,y)$ funksiyaning Δf to'la orttirmasi qayerda to'g'ri ko'rsatilgan?

A) $\Delta f=f(x+\Delta x, y+\Delta y)-f(x,y)$ B) $\Delta f=f(x, y+\Delta y)-f(x,y)$

C) $\Delta f=f(x+\Delta x, y)-f(x,y)$ D) $\Delta f=f(x, y+\Delta y)-f(x+\Delta x,y+\Delta y)$

E)$\Delta f=f(x-\Delta x, y+\Delta y)-f(x+\Delta x,y+\Delta y)$.

2. $z=f(x,y)$ funksiyaning x bo'yicha $\Delta_x f$ xususiy orttirmasi qayerda to'g'ri ko'rsatilgan?

A) $\Delta_x f=f(x+\Delta x, y+\Delta y)-f(x,y)$ B) $\Delta_x f=f(x, y+\Delta y)-f(x,y)$.

C) $\Delta_x f=f(x+\Delta x, y)-f(x,y)$ D) $\Delta_x f=f(x+\Delta x, y+\Delta y)-f(x+\Delta x,y)$

E)$\Delta_x f=f(x, y+\Delta y)-f(x+\Delta x,y)$.

3. $z=f(x,y)$ funksiyaning y bo'yicha $\Delta_y f$ xususiy orttirma qayerda to'g'ri ko'rsatilgan?

A) $\Delta_y f=f(x+\Delta x, y+\Delta y)-f(x,y)$ B) $\Delta_y f=f(x, y+\Delta y)-f(x,y)$.

C) $\Delta_y f=f(x+\Delta x, y)-f(x,y)$ D) $\Delta_y f=f(x, y+\Delta y)-f(x+\Delta x,y+\Delta y)$

E)$\Delta_y f=f(x, y+\Delta y)-f(x+\Delta x,y)$.

4. Quyidagilardan qaysi biri $z=f(x,y)$ funksiyaning x bo'yicha $\Delta_x f$ xususiy orttirmasini ifodalamaydi?

A) $\Delta_x f=f(x+\Delta x, y+\Delta y)-f(x,y+\Delta y)$

B) $\Delta_x f=f(x, y)-f(x-\Delta x,y)$.

C) $\Delta_x f=f(x+\Delta x, y)-f(x,y)$.

D) $\Delta_x f=f(x+\Delta x, y+\Delta y)-f(x, y)$

E)$\Delta_x f=f(x, y+\Delta y)-f(x-\Delta x, y+\Delta y)$

5. Quyidagilardan qaysi biri $z=f(x,y)$ funksiyaning y bo'yicha $\Delta_y f$ xususiy orttirmasini ifodalamaydi?

A) $\Delta_y f = f(x+\Delta x, y+\Delta y) - f(x,y)$
B) $\Delta_y f = f(x, y+\Delta y) - f(x,y)$
C) $\Delta_y f = f(x+\Delta x, y+\Delta y) - f(x+\Delta x,y)$
D) $\Delta_y f = f(x, y) - f(x, y-\Delta y)$.
E) $\Delta_y f = f(x+\Delta x, y) - f(x+\Delta x, y-\Delta y)$

6. $f(x,y)=x^2+y^2$ funksiyaning Δf to'la orttirmasini toping.

A) $\Delta f=2(\Delta x^2+\Delta y^2)$ B) $\Delta f=2(x\Delta x+ y\Delta y)+ \Delta x^2+\Delta y^2$
C) $\Delta f=2(x\Delta y+y\Delta x)+ \Delta x^2+\Delta y^2$ D) $\Delta f=\Delta y^2 +2(x+y)\Delta x+\Delta x^2$
E) $\Delta f=\Delta x^2+\Delta y^2+2 (\Delta x+\Delta y)$

7. $z=x^2+y^2+2xy$ funksiyaning x bo'yicha $\Delta_x z$ xususiy orttirmasini toping.

A) $\Delta_x z=2(\Delta x+\Delta y)$
B) $\Delta_x z=2(x\Delta x+ y\Delta y+\Delta x\cdot\Delta y)$.
C) $\Delta_x z=2x\Delta x+2y\Delta y+2(x\Delta y+y\Delta x)$
D) $\Delta_x z=2(x+y)\Delta x+\Delta x^2$.
E) $\Delta_x z=2x+2y+2 (\Delta x+\Delta y)$.

8. $z=x^2+y^2+2xy$ funksiyaning y bo'yicha $\Delta_y z$ xususiy orttirmasini toping.

A) $\Delta_y z=2(\Delta x+\Delta y)$ B) $\Delta_y z=2(x\Delta x+ y\Delta y+\Delta x\cdot\Delta y)$
C) $\Delta_y z=2x\Delta x+2y\Delta y+2(x\Delta y+y\Delta x)$ D) $\Delta_y z=2(x+y)\Delta y+\Delta y^2$
E) $\Delta_y z=2x+2y+2 (\Delta x+\Delta y)$.

9. Qaysi shartda $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada uzluksiz bo'lmaydi ?

A) $\lim\limits_{\substack{x\to x_0\\ y\to y_0}} f(x,y)=f(x_0,y_0)$.

B) $\lim\limits_{\substack{x\to x_0\\ y\to y_0}} [f(x,y)-f(x_0,y_0)]=0$.

C) $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}} f(x_0+\Delta x,y_0+\Delta y)=f(x_0,y_0)$.

D) $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}} \Delta f=0$.

E)keltirilgan barcha shartlarda uzluksiz bo'ladi .

10. α parametrning qanday qiymatida

$$f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x-y}, & x\neq y;\\ \alpha, & x=y.\end{cases}$$

funksiya $M_0(1, 1)$ nuqtada uzluksiz bo'ladi ?

A) 0 B) 1 C) 2 D) 3 E) –1

11. Quyidagi funksiyalardan qaysi biri O(0,0) nuqtada uzlukli?

A) $z=\sin xy$ B) $z=\cos(x+y)$

C) $z=\text{tg}(x-y)$ D) $z=\text{ctg}(x/y)$

E) keltirilgan barcha funksiyalar O(0,0) nuqtada uzluksiz .

12. *Ta'rifni to'diring:* Berilgan $z=f(x,y)$ funksiya biror D sohada uzluksiz deyiladi, agar u bu sohaning ··· nuqtasida uzluksiz bo'lsa.

A) birorta B) ba'zi bir

C) har bir D) bitta E) ma'lum bir

13. Agar $f(x,y)$ va $g(x,y)$ ikki o'zgaruvchili funksiyalar $M_0(x_0,y_0)$ nuqtada uzluksiz bo'lsa, bu nuqtada quyidagi funksiyalardan qaysi biri uzlyksiz bo'lishi shart emas ?

A) $f(x,y)+g(x,y)$ B) $f(x,y)-g(x,y)$

C) $f(x,y)\cdot g(x,y)$ D) $f(x,y)/g(x,y)$.

E) ko'rsatilgan barcha funksiyalar $M_0(x_0,y_0)$ nuqtada doimo uzluksiz bo'ladi .

14. Agar $f(x,y)$ va $g(x,y)$ ikki o'zgaruvchili funksiyalar $M_0(x_0,y_0)$ nuqtada uzluksiz bo'lsa, qaysi holda bu nuqtada $f(x,y)/g(x,y)$ funksiya uzlukli bo'lishi mumkin?

A) $g(x_0,y_0)>0$. B) $g(x_0,y_0)<0$. C) $g(x_0,y_0)=0$. D) $g(x_0,y_0)\neq 0$.

E) keltirilgan barcha hollarda $g(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada uzluksiz bo'ladi.

15. Agar $f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada uzluksiz va C ixtiyoriy noldan farqli o'zgarmas son bo'lsa, quyidagi funksiyalardan qaysi biri $M_0(x_0,y_0)$ nuqtada uzlukli bo'lishi mumkin?

A) $C \cdot f(x,y)$ B) $C+f(x,y)$

C) $C-f(x,y)$ D) $C/f(x,y)$ E) $f(x,y)/C$

16. Quyidagi sohalardan qaysi biri yopiq emas ?

A) $D=\{(x,y): x^2+y^2 \leq 1\}$. B) $D=\{(x,y): \frac{x^2}{a^2}+\frac{y^2}{b^2} \leq 1\}$.

C) $D=\{(x,y): \frac{x}{a}+\frac{y}{b} \leq 1\}$. B) $D=\{(x,y): |x| \leq 1, |y| \leq 1\}$.

E) keltirilgan barcha sohalar yopiq .

4.Ko'p o'zgaruvchili funksiyaning hosila va differensiallari

1. $z=f(x,y)$ funksiyaning x argumenti bo'yicha f'_x xususiy hosilasini ta'rif bo'yicha aniqlashda quyidagi amallardan qaysi biri bajarilmaydi?

A) x argumentga Δx orttirma beriladi .

B)funksiyaning $\Delta_x f=f(x+\Delta x,y)-f(x,y)$ xususiy orttirmasi hisoblanadi .

C) orttirmalar nisbati $\Delta_x f/\Delta x$ aniqlanadi .

D) $\Delta x \to 0$ bo'lganda $\Delta_x f/\Delta x$ nisbat limiti topiladi .

E) Keltirilgan barcha amallar bajariladi .

2. $z=f(x,y)$ funksiyaning x argumenti bo'yicha f'_x xususiy hosilasi qanday aniqlanadi ?

A) $f'_x = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x, y+\Delta y)-f(x,y)}{\Delta x}$.

B) $f'_x = \lim\limits_{\Delta x \to 0} \frac{f(x, y+\Delta x)-f(x,y)}{\Delta x}$.

C) $f'_x = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x, y+\Delta y)-f(x+\Delta x,y)}{\Delta x}$.

D) $f'_x = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x, y)-f(x,y)}{\Delta x}$.

E) $f'_x = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x, y)+f(x,y)}{\Delta x}$.

3. $z=f(x,y)$ funksiyaning y argumenti bo'yicha f'_y xususiy hosilasini ta'rif bo'yicha aniqlashda quyidagi amallardan qaysi biri bajarilmaydi?

A) y argumentga Δy orttirma beriladi .

B) funksiyaning $\Delta_y f=f(x,y+\Delta y)-f(x,y)$ xususiy orttirmasi hisoblanadi .

C) orttirmalar nisbati $\Delta_y f/\Delta y$ aniqlanadi .

D) $\Delta y \to 0$ bo'lganda $\Delta_y f/\Delta y$ nisbat limiti topiladi .

E) Keltirilgan barcha amallar bajariladi .

4. $z=f(x,y)$ funksiyaning y argumenti bo'yicha f'_y xususiy hosilasi qanday aniqlanadi ?

A) $f'_y = \lim\limits_{\Delta y \to 0} \dfrac{f(x+\Delta x, y+\Delta y) - f(x,y)}{\Delta y}$.

B) $f'_y = \lim\limits_{\Delta y \to 0} \dfrac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$.

C) $f'_y = \lim\limits_{\Delta y \to 0} \dfrac{f(x+\Delta x, y+\Delta y) - f(x,y+\Delta y)}{\Delta y}$.

D) $f'_y = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x, y) - f(x,y+\Delta y)}{\Delta x}$.

E) $f'_y = \lim\limits_{\Delta y \to 0} \dfrac{f(x, y+\Delta y) + f(x,y)}{\Delta y}$.

5. $z=x^2+y^3+xy$ funksiyaning x bo'yicha f'_x xususiy hosilasini toping.

A) $f'_x = 2x+3y^2+y$. B) $f'_x = 2x+y^3+x$. C) $f'_x = 2x+y$.

D) $f'_x = 2x+3y^2+x$. E) $f'_x = 2x+xy$.

6. $z=x^2+y^3+xy$ funksiyaning y bo'yicha f'_y xususiy hosilasini toping.

A) $f'_y = 2x+3y^2+y$. B) $f'_y = x^2+3y^2+y$. C) $f'_y = 2x+3y^2$.

D) $f'_y = 2x+3y^2+x$. E) $f'_x = 3y^2+x$.

7. $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada differensiallanuvchi bo'lishi uchun qaysi shart talab etilmaydi ?

A) $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqta va uning biror atrofida aniqlangan .

B) $M_0(x_0,y_0)$ nuqta va uning biror atrofida f'_x, f'_y xususiy hosilalar mavjud

C) $M_0(x_0,y_0)$ nuqta va uning biror atrofida f'_x, f'_y xususiy hosilalar uzluksiz .

D) $M_0(x_0,y_0)$ nuqta va uning biror atrofida xususiy hosilalar $f'_x \neq 0, f'_y \neq 0$.

E) Keltirilgan barcha shartlar talab etiladi.

8. Ikki o'zgaruvchili $z=f(x,y)$ funksiyaning df to'liq differensiali formulasini ko'rsating.

A) $df = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$.

B) $df = \frac{\partial f}{\partial x}dy + \frac{\partial f}{\partial y}dx$.

C) $df = \frac{\partial f}{\partial x}dx \cdot \frac{\partial f}{\partial y}dy$.

D) $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$.

E) $df = \frac{\partial f}{\partial x}dx - \frac{\partial f}{\partial y}dy$.

9. $z=x^2+y^3+xy$ funksiyaning df to'liq differensialini toping.

A) $df = (x^2 + x)dx + (y^3 + y)dy$.

B) $df = (2x + 3y^2 + x)dx + (2x + 3y^2 + y)dy$.

C) $df = (2x + y)dx + (3y^2 + x)dy$. D) $df = 2xdx + 3y^2dy$.

E) $df = (3y^2 + x)dx + (2x + y)dy$.

10. $z=y^x$ funksiyaning df to'liq differensialini toping.

A) $df = y^x(\ln y dx - \frac{x}{y}dy)$. B) $df = y^x(\ln y dx - \frac{y}{x}dy)$.

C) $df = y^x(\ln y dx + \frac{y}{x}dy)$. D) $df = y^x(\ln y dx + \frac{x}{y}dy)$.

E) $df = y^x(\ln y dy - \frac{x}{y}dx)$.

11. $z=f(x,y)$ funksiyaning to'liq differensiali $df=(2x+5y)dx+(3xy-4y)dy$ bo'lsa, $f'_x(-3,2)$ qiymatini toping.

A) -11 . B) -26 . C) 4 . D) -6 . E) 8 .

12. $z=f(x,y)$ funksiyaning to'liq differensiali $df=(2x+5y)dx+(3xy-4y)dy$ bo'lsa, $f'_y(-3,2)$ qiymatini toping.

A) −11 . B) −26 . C) 4 . D) −6 . E) 8 .

13. Qaysi shartda $z=f(x,y)$ funksiyaning grafigi bo'lgan S sirtning $P_0(x_0,y_0,z_0)$ nuqtasida urinma tekislik mavjud bo'ladi ?

A) $z=f(x,y)$ funksiya M_0 (x_0,y_0) nuqtada uzluksiz .

B) $z=f(x,y)$ funksiya M_0 (x_0,y_0) nuqtada $f'_x(x_0,y_0)$ xususiy hosilaga ega

C) $z=f(x,y)$ funksiya M_0 (x_0,y_0) nuqtada $f'_y(x_0,y_0)$ xususiy hosilaga ega

D) $z=f(x,y)$ funksiya $f'_x(x_0,y_0)$ va $f'_y(x_0,y_0)$ xususiy hosilalarga ega .

E) $z=f(x,y)$ funksiya M_0 (x_0,y_0) nuqtada differensiallanuvchi .

14. $z=f(x,y)$ funksiya orqali aniqlangan S sirtning $P_0(x_0, y_0, f(x_0,y_0))$ nuqtasidagi urinma tekislik tenglamasini yozing.

A) $z+f(x_0,y_0)=f'_x(x_0,y_0)(x+x_0)+f'_y(x_0,y_0)(y+y_0)$.

B) $z-f(x_0,y_0)=f'_x(x_0,y_0)(x+x_0)-f'_y(x_0,y_0)(y+y_0)$.

C) $z-f(x_0,y_0)=f'_x(x_0,y_0)(x-x_0)+f'_y(x_0,y_0)(y-y_0)$.

D) $z-f(x_0,y_0)=f''_x(x_0,y_0)(x-x_0)+f''_y(x_0,y_0)(y-y_0)$.

E) $z+f(x_0,y_0)=f''_x(x_0,y_0)(x+x_0)-f''_y(x_0,y_0)(y+y_0)$.

15. $z=3x^2y+5x-2y+1$ funksiya ifodalaydigan S sirtning M_0 $(3,-1,-9)$ nuqtasiga o'tkazilgan urinma tekislik tenglamasini toping.

A) $2x-3y+z=0$ B) $2x+4y+z+7=0$ C) $13x-25y+z-55=0$.

D) $25x+13y+z-53=0$. E) $3x+y+z+1=0$.

16. $z=f(x,y)$ funksiyada argument orttirmalari Δx va Δy kichik bo'lganda o'rinli bo'ladigan taqribiy tenglik qayerda to'g'ri yozilgan?

A) $f(x+\Delta x, y+\Delta y)\approx f(x,y)+df$.

B) $f(x+\Delta x, y+\Delta y)\approx f(x,y)-df$.

C) $f(x+\Delta x, y+\Delta y)\approx f(x,y)\cdot df$.

D) $f(x+\Delta x, y+\Delta y)\approx f(x,y)/df$.

E) to'g'ri javob keltirilmagan .

5. Ikki o'zgaruvchili murakkab funksiyaning hosilalari va to'la differensiali. To'la hosila. Yo'nalish bo'yicha hosila va gradient

1. $f(u,v)=u^2-v$ murakkab funksiyada $u(x,y)=x+2y$ va $f[u(x,y),v(x,y)]=4xy$ bo'lsa, $v=v(x,y)$ funksiyani toping.

A) $v(x,y)=x\pm y$. B) $v(x,y)=(x+y)^2$. C) $v(x,y)=4xy$.
D) $v(x,y)=(x-y)^2$. E) to'g'ri javob keltirilmagan .

2. $z=f(u,v)$ $[u=u(x,y)$ va $v=v(x,y)]$ murakkab funksiyaning x argument bo'yicha xususiy hosilasining formulasi qayerda to'g'ri ko'rsatilgan?

A) $\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}+\frac{\partial f}{\partial v}$.

B) $\frac{\partial z}{\partial x}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}$.

C) $\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x}$.

D) $\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}+\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}+\frac{\partial v}{\partial x}$.

E) $\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}-\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x}$.

3. Agar $z=u^2+3uv-v^2$ murakkab funksiyada $u=x^3+y^3$ va $v=x^2+y^2$ bo'lsa, uning x argument bo'yicha xususiy hosilasini toping.

A) $\frac{\partial z}{\partial x}=3(2u+v)x^2+2(5u-2v)x$.

B) $\frac{\partial z}{\partial x}=2(2u+v)x+3(5u-2v)x^2$,

C) $\frac{\partial z}{\partial x}=3(2u+3v)x^2+2(3u-2v)x$.

D) $\frac{\partial z}{\partial x}=3(2u+v)x^2-2(5u-2v)x$.

E) to'g'ri javob keltirilmagan .

4. $z=f(u,v)$ $[u=u(x,y)$ va $v=v(x,y)]$ murakkab funksiyaning y argument bo'yicha xususiy hosilasining formulasi qayerda to'g'ri ko'rsatilgan?

A) $\frac{\partial z}{\partial y}=\frac{\partial f}{\partial u}+\frac{\partial f}{\partial v}$.

B) $\frac{\partial z}{\partial y}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}$.

C) $\frac{\partial z}{\partial y}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}-\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y}$.

D) $\frac{\partial z}{\partial y}=\frac{\partial f}{\partial u}+\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}+\frac{\partial v}{\partial y}$

E) $\frac{\partial z}{\partial y}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y}$.

5. Agar $z=u^2+3uv-v^2$ murakkab funksiyada $u=x^3+y^3$ va $v=x^2+y^2$ bo'lsa, uning y argument bo'yicha xususiy hosilasini toping.

A) $\frac{\partial z}{\partial y}=3(2u+v)y^2+2(5u-2v)y$.

B) $\frac{\partial z}{\partial y}=2(2u+v)y+3(5u-2v)y^2$.

C) $\frac{\partial z}{\partial y}=3(2u+3v)y^2+2(3u-2v)y$.

D) $\frac{\partial z}{\partial y}=3(2u+v)y^2-2(5u-2v)y$.

E) to'g'ri javob keltirilmagan .

6. $z=f(x,y)=f(x,\phi(x))$ funksiya to'la hosilasining formulasi qayerda to'g'ri ko'rsatilgan?

A) $\frac{dz}{dx}=\frac{\partial z}{\partial x}+\frac{dy}{dx}$ B) $\frac{dz}{dx}=\frac{\partial z}{\partial x}-\frac{dy}{dx}$ C) $\frac{dz}{dx}=\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\cdot\frac{dy}{dx}$.

D) $\frac{dz}{dx}=\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dx}$ E) $\frac{dz}{dx}=\frac{\partial z}{\partial y}\cdot\frac{dy}{dx}$.

7. $z=x^3y+\sin y$ ($y=\ln x$) funksiyaning to'la hosilasini toping.

A) $\frac{dz}{dx}=x^2(3y+x)+\frac{\cos y}{x}$

B) $\frac{dz}{dx}=x^3+\cos y$.

C) $\frac{dz}{dx}=x^2(3y-\frac{1}{x})+\frac{\cos y}{x}$.

D) $\frac{dz}{dx}=x^2(3y+1)+\frac{\cos y}{x}$ E) $\frac{dz}{dx}=x^2(3y-\frac{1}{x})-\frac{\cos y}{x}$.

8. $z=f(x,y)$ funksiyadan $P(x_0,y_0)\in D\{f\}$ nuqtada l yo'nalish bo'yicha hosilani hisoblash uchun quyidagi amallardan qaysi biri bajarilmaydi?

A) l yo'nalisgning yo'naltiruvchi kosinuslari cosα va cosβ=sinα topiladi .

B) l yo'nalisgning normal vektori topiladi .

C) l yo'nalish bo'yicha $\Delta_l f=f(x_0+\Delta x,y_0+\Delta y)-f(x_0,y_0)$ orttirma hisoblanadi .

D) l to'g'ri chiziqdagi kesma uzunligi $\Delta l=\sqrt{(\Delta x)^2+(\Delta y)^2}$ aniqlanadi .

E) $\Delta_l f\,/\,\Delta l$ nisbatning $\Delta l\to 0$ holdagi limiti topiladi .

9. $z=f(x,y)$ funksiyadan $P(x_0,y_0)\in D\{f\}$ nuqtada OX o'qi bilan α burchak tashkil etuvchi l yo'nalish bo'yicha hosilani hisoblash formulasini ko'rsating.

A) $\frac{\partial f(P)}{\partial l}=\frac{\partial f(P)}{\partial x}\cos\alpha+\frac{\partial f(P)}{\partial y}\cos\alpha$.

B) $\frac{\partial f(P)}{\partial l}=\frac{\partial f(P)}{\partial x}\sin\alpha+\frac{\partial f(P)}{\partial y}\sin\alpha$.

C) $\frac{\partial f(P)}{\partial l}=\frac{\partial f(P)}{\partial x}\cos\alpha-\frac{\partial f(P)}{\partial y}\sin\alpha$.

D) $\frac{\partial f(P)}{\partial l}=\frac{\partial f(P)}{\partial x}\cos\alpha+\frac{\partial f(P)}{\partial y}\sin\alpha$.

E) to'g'ri javob keltirilmagan .

10. $z=f(x,y)$ funksiyadan $P(x_0,y_0)$ nuqtada $a=\{a_1,\ a_2\}$ vektor yo'nalishi bo'yicha olingan hosilani ko'rsating.

A) $\frac{\partial z(P)}{\partial a}=\frac{\partial z(P)}{\partial x}\cdot\frac{a_1}{\sqrt{a_1^2+a_2^2}}$

B) $\frac{\partial z(P)}{\partial a}=\frac{\partial z(P)}{\partial y}\cdot\frac{a_2}{\sqrt{a_1^2+a_2^2}}$.

C) $\frac{\partial z(P)}{\partial a}=\frac{\partial z(P)}{\partial x}\cdot\frac{a_1}{\sqrt{a_1^2+a_2^2}}-\frac{\partial z(P)}{\partial y}\cdot\frac{a_2}{\sqrt{a_1^2+a_2^2}}$.

D) $\frac{\partial z(P)}{\partial a}=\frac{\partial z(P)}{\partial x}\vec{i}+\frac{\partial z(P)}{\partial y}\vec{j}$.

E) $\frac{\partial z(P)}{\partial \bar{a}}=\frac{\partial z(P)}{\partial x}\cdot\frac{a_1}{\sqrt{a_1^2+a_2^2}}+\frac{\partial z(P)}{\partial y}\cdot\frac{a_2}{\sqrt{a_1^2+a_2^2}}$.

11. $z=x^2+y^3$ funksiyaning $M(2,1)$ nuqtadagi $\boldsymbol{a}=\{-3,\ 4\}$ vektor yo'nalishi bo'yicha olingan hosilasini hisoblang.

A) 5.8 B) −4.8 C) 0 D) 1 E) −1

12. $z=x^2+y^3$ funksiyaning $M(2,1)$ nuqtadagi $\boldsymbol{a}=\{3,\ 4\}$ vektor yo'nalishi bo'yicha olingan hosilasini hisoblang.

A) 4.8 . B) −5.8 . C) 0 . D) 1 . E) −1 .

13. $z=f(x,y)$ funksiyaning $P(x_0\ ,y_0)$ nuqtadagi gradienti grad$f(P)$ qaysi formula bilan topiladi ?

A) $\text{grad}f(P)=\frac{\partial f(P)}{\partial x}+\frac{\partial f(P)}{\partial y}$

B) $\text{grad}f(P)=\frac{\partial f(P)}{\partial x}\bar{i}+\frac{\partial f(P)}{\partial y}\bar{j}$.

C) $\text{grad}f(P)=\frac{\partial f(P)}{\partial x}-\frac{\partial f(p)}{\partial y}$.

D) $\text{grad}f(P)=\frac{\partial f(P)}{\partial x}\bar{i}-\frac{\partial f(P)}{\partial y}\bar{j}$.

E) $\text{grad}f(P)=\frac{\partial f(P)}{\partial x}\cdot\frac{\partial f(P)}{\partial x}$.

14. $z=f(x,y)$ funksiyaning $P(x_0\ ,y_0)$ nuqtadagi gradienti grad$f(P)$ to'g'risidagi quyidagi tasdiqlardan qaysi biri noto'g'ri?

A) $z=f(x,y)$ funksiyaning gradienti grad$f(P)$ vektor kattalikdan iborat .

B) $z=f(x,y)$ funksiyaning grad$f(P)$ yo'nalishi bo'yicha hosilasi eng katta qiymatga ega bo'ladi .

C) $z=f(x,y)$ funksiyaning grad$f(P)$ yo'nalishi bo'yicha hosilasining qiymati $|\text{grad}f(P)|$ bo'ladi .

D) $z=f(x,y)$ funksiyaning grad$f(P)$ yo'nalishiga perpendikulyar yo'nalish bo'yicha hosilasi nolga teng

E) $z=f(x,y)$ funksiyaning gradienti ushbu formula bilan topiladi:

$$\text{grad}f(P)=\sqrt{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\ .$$

15. $f(x,y)=x^2+y^2+xy$ funksiyaning $P(1,2)$ nuqtadagi gradientini toping.

A) $\text{grad}f(P)=4\bar{i}+5\bar{j}$. B) $\text{grad}f(P)=5\bar{i}+4\bar{j}$. C) $\text{grad}f(P)=9$

D) $\text{grad}f(P)=-1$. E) $\text{grad}f(P)=9\bar{i}-\bar{j}$.

16. l yo'nalishning OX koordinata o'qi bilan tashkil etgan α burchagi kosinusining qiymati qanday bo'lganda $f(x,y)=x^2+y^2+xy$ funksiyaning $P(2,5)$ nuqtadagi hosilasi eng katta qiymatga ega bo'ladi ?

A) $\cos\alpha=0$. B) $\cos\alpha=1$. C) $\cos\alpha=1/2$. D) $\cos\alpha=2/3$.
E) $\cos\alpha=3/5$.

6. Ko'p o'zgaruvchili funksiyalarning yuqori tartibli hosila va differensiallari

1. $z=f(x,y)$ funksiyaning x bo'yicha II tartibli xususiy hosilasi $\frac{\partial^2 f}{\partial x^2}$ qanday aniqlanadi ?

A) $\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial y}(\frac{\partial f}{\partial x})$ B) $\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial y}(\frac{\partial f}{\partial y})$ C) $\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial x}(\frac{\partial f}{\partial y})$

D) $\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial x}(\frac{\partial f}{\partial x})$ E) to'g'ri javob keltirilmagan .

$z=f(x,y)$ funksiyaning y bo'yicha II tartibli xususiy hosilasi $\frac{\partial^2 f}{\partial y^2}$ qanday aniqlanadi ?

A) $\frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial y}(\frac{\partial f}{\partial x})$ B) $\frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial y}(\frac{\partial f}{\partial y})$ C) $\frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial x}(\frac{\partial f}{\partial y})$

D) $\frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial x}(\frac{\partial f}{\partial x})$ E) to'g'ri javob ko'rsatilmagan .

2. $z=f(x,y)$ funksiyaning II tartibli $f''_{xy}=\frac{\partial^2 f}{\partial x\partial y}$ aralash hosilasi qanday aniqlanadi ?

A) $f''_{xy}=\frac{\partial}{\partial y}(\frac{\partial f}{\partial x})$ B) $f''_{xy}=\frac{\partial}{\partial y}(\frac{\partial f}{\partial y})$ C) $f''_{xy}=\frac{\partial}{\partial x}(\frac{\partial f}{\partial y})$.

D) $f''_{xy}=\frac{\partial}{\partial x}(\frac{\partial f}{\partial x})$ E) to'g'ri javob ko'rsatilmagan .

3. $z = x^3 \cos y$ funksiyaning x bo'yicha II tartibli f''_{xx} xususiy hosilasi hisoblansin.

A) $-6x\sin y$ B) $6x\sin y$ C) $-6x\cos y$ D) $6x\cos y$ E) $6x$

4. $z = x^3 \cos y$ funksiyaning y bo'yicha II tartibli f''_{yy} xususiy hosilasi hisoblansin.

A) $x^3 \sin y$ B) $-3x^2 \sin y$ C) $-x^3 \sin y$

D) $-6x\sin y$ E) $-x^3 \cos y$.

5. $z = x^3 \cos y$ funksiyaning II tartibli f''_{xy} aralash hosilasi hisoblansin.

A) $-3x^2 \sin y$. B) $3x^2 \sin y$. C) $6x\cos y$. D) $-6x\sin y$.

E) $-3x^2 \cos y$.

6. $z = x^3 \cos y$ funksiyaning II tartibli f''_{yx} aralash hosilasi hisoblansin.

A) $-3x^2 \sin y$ B) $3x^2 \sin y$

C) $6x\cos y$ D) $-6x\sin y$ E) $-3x^2 \cos y$

7. Aralash hosilalar haqidagi teoremada qaysi tasdiq ifodalanadi ?

A) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x} \cdot \frac{\partial f}{\partial y}$ B) $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$ C) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

D) $\frac{\partial^2 f}{\partial x \partial y} < \frac{\partial^2 f}{\partial y \partial x}$ E) $\frac{\partial^2 f}{\partial x \partial y} > \frac{\partial^2 f}{\partial y \partial x}$

8. $z=f(x,y)$ funksiyaning III tartibli hosilalari mavjud bo'lsa, ularning soni nechta bo'ladi?

A) 3 B) 6 C) 8 D) 9 E) 12

9. $z=3x^3y^2-2y^3\sin x$ funksiyaning x argument bo'yicha III tartibli xususiy hosilasini toping.

A) $18y^2+2y^3\cos x$ B) $18xy^2+2y^3\sin x$ C) $18y^2-2y^3\cos x$

D) $18xy^2-2y^3\cos x$ E) $18y^2+2xy^3\cos x$

10. $z=3x^3y^2-2y^3\sin x$ funksiyaning y argument bo'yicha III tartibli xususiy hosilasini toping.

A) $6x^3+12y\cos x$ B) $6x^3-12y\cos x$

C) $-12\cos x$ D) $6xy^2-12\cos x$ E) $-12\sin x$

11. $z=3x^3y^2-2y^3\sin x$ funksiyaning III tartibli f'''_{xyx} aralash hosilasini toping.

A) $6x^3+12y\cos x$ B) $6xy-12y\cos x$

C) $24xy-6y^2\cos x$ D) $6xy-12x^2\sin x$ E) $24xy+12y\cos x$.

12. Agar $z=f(x,y)$ funksiyaning IV tartibli differensiali d^4f mavjud bo'lsa, undagi $\frac{\partial^4 f}{\partial x^2 \partial y^2}$ aralash hosila oldidagi koeffitsient qiymati nimaga teng ?

A) 2 B) 4 C) 6 D) 8 E) 10

13. Agar $z=f(x,y)$ funksiyaning IV tartibli differensiali d^4f mavjud bo'lsa, undagi $\frac{\partial^4 f}{\partial x^3 \partial y}$ aralash hosila oldidagi koeffitsient qiymati nimaga teng ?

A) 2 B) 4 C) 6 D) 8 E) 10

14. Uch o'zgaruvchili $w=x^3y^2z+y\sin x+z\cos x$ funksiyaning$x$ argument bo'yicha II tartibli xususiy hosilasining $M(0,-2,3)$ nuqtadagi qiymatini hisoblang.

A) 1 B) 0 C) −1 D) −2 E) −3

15. Uch o'zgaruvchili $w=x^3y^2z+y\sin x+z\cos x$ funksiyaning$y$ argument bo'yicha II tartibli xususiy hosilasining $M(1,-2,3)$ nuqtadagi qiymatini hisoblang.

A) −1 B) 0 C) 3 D) 6 E) 8

7. Ikki o'zgaruvchili funksiyaning ekstremumlari

1. *Ta'rifni to'ldiring:* $z=f(x,y)=f(M)$ funksiya aniqlanish sohasidagi ichki $M_0(x_0,y_0)$ nuqtada lokal maksimumga ega deyiladi, agar shu nuqtaning biror atrofidagi ··· uchun $f(M_0)\geq f(M)$ shart bajarilsa.

A) bitta $M(x,y)$ nuqta B) ayrim $M(x,y)$ nuqtadalar .

C) barcha $M(x,y)$ nuqtadalar D) birorta $M(x,y)$ nuqta

E) To'g'ri javob keltirilmagan .

2. *Ta'rifni to'ldiring:* $z=f(x,y)=f(M)$ funksiya aniqlanish sohasidagi ichki $M_0(x_0,y_0)$ nuqtada lokal minimumga ega

deyiladi, agar shu nuqtaning biror atrofidagi ··· uchun $f(M_0)\leq f(M)$ shart bajarilsa.

A) bitta $M(x,y)$ nuqta B) ayrim $M(x,y)$ nuqtadalar

C) barcha $M(x,y)$ nuqtadalar D) birorta $M(x,y)$ nuqta

E) To'g'ri javob keltirilmagan .

3. Ikki o'zgaruvchili funksiyaning lokal ekstremumi nimadan iborat ?

A) faqat lokal maksimumlardan

B) faqat lokal minimumlardan .

C) lokal maksimum yoki lokal minimumlardan .

D) lokal maksimum va lokal minimumlardan .

E) lokal maksimumlarning eng kattasi va lokal minimumlarning eng kichigidan .

4. Berilgan $z=f(x,y)$ funksiya aniqlanish sohasidagi ichki $M_0(x_0,y_0)$ nuqtada lokal maksimumga ega bo'lishi uchun uning biror atrofida $\Delta f(x_0,y_0)$ to'la orttirma qanday shartni qanoatlahtirishi kerak ?

A) $\Delta f(x_0,y_0)=0$. B) $\Delta f(x_0,y_0)\leq 0$. C) $\Delta f(x_0,y_0)\geq 0$.

D) $\Delta f(x_0,y_0)\neq 0$. E) to'g'ri javob keltirilmagan .

5. Berilgan $z=f(x,y)$ funksiya aniqlanish sohasidagi ichki $M_0(x_0,y_0)$ nuqtada lokal minimumga ega bo'lishi uchun uning biror atrofida $\Delta f(x_0,y_0)$ to'la orttirma qanday shartni qanoatlahtirishi kerak ?

A) $\Delta f(x_0,y_0)=0$. B) $\Delta f(x_0,y_0)\leq 0$. C) $\Delta f(x_0,y_0)\geq 0$.

D) $\Delta f(x_0,y_0)\neq 0$. E) to'g'ri javob keltirilmagan .

6. Berilgan $z=f(x,y)$ funksiya aniqlanish sohasidagi ichki $M_0(x_0,y_0)$ nuqtada lokal ekstremumga ega bo'lmasligi uchun $\Delta f(x_0,y_0)$ to'la orttirma qanday shartni qanoatlahtirishi kerak ?

A) $M_0(x_0,y_0)$ nuqtaning biror atrofida $\Delta f(x_0,y_0)$ ishorasi o'zgarmas .

B) $M_0(x_0,y_0)$ nuqtaning biror atrofida $\Delta f(x_0,y_0)\leq 0$.

C) $M_0(x_0,y_0)$ nuqtaning biror atrofida $\Delta f(x_0,y_0)\geq 0$.

D) $M_0(x_0,y_0)$ nuqtaning har qanday atrofida $\Delta f(x_0,y_0)$ ishorasi o'zgaruvchi .

E) to‘g‘ri javob keltirilmagan .

7. ***Ferma teoremasini yakunlang:*** $z=f(x,y)$ funksiya aniqlanish sohasidagi ichki $M_0(x_0,y_0)$ nuqtada lokal ekstremumga ega va bu nuqtada uning xususiy hosilalari mavjud bo‘lsa, unda ··· nolga teng bo‘ladi .

A) xususiy hosilalardan bittasi

B) xususiy hosilalarning ikkalasi ham

C) xususiy hosilalardan kamida bittasi

D) xususiy hosilalardan birortasi

E) to‘g‘ri javob keltirilmagan .

8. Agar differensiallanuvchi $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada lokal ekstremumga ega bo‘lsa, unda quyidagi tasdiqlardan qaysi biri o‘rinli emas ?

A) $\frac{\partial f(M_0)}{\partial x}=0,\quad \frac{\partial f(M_0)}{\partial y}=0$ B) $\left|\frac{\partial f(M_0)}{\partial x}\right|+\left|\frac{\partial f(M_0)}{\partial y}\right|=0$.

C) $\left[\frac{\partial f(M_0)}{\partial x}\right]^2+\left[\frac{\partial f(M_0)}{\partial y}\right]^2=0$ D) $\frac{\partial f(M_0)}{\partial x}+\frac{\partial f(M_0)}{\partial y}=0$.

E) Barcha tasdiqlar o‘rinli bo‘ladi.

9. Agar differensiallanuvchi $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada lokal ekstremumga ega bo‘lsa, unda quyidagi tasdiqlardan qaysi biri o‘rinli emas ?

A) $\frac{\partial f(x_0,y_0)}{\partial x}=0$ B) $df=\frac{\partial f(x_0,y_0)}{\partial x}dx+\frac{\partial f(x_0,y_0)}{\partial y}dy=0$

C) $\frac{\partial f(x_0,y_0)}{\partial y}=0$ D) $\mathrm{grad}f=\frac{\partial f(x_0,y_0)}{\partial x}\bar{i}+\frac{\partial f(x_0,y_0)}{\partial y}\bar{j}=0$

E) barcha tasdiqlar o‘rinli .

10. Differensiallanuvchi $z=f(x,y)$ funksiyaning kritik nuqtalari qaysi shartdan topiladi ?

A) $\frac{\partial f}{\partial x}=0$ B) $\frac{\partial f}{\partial y}=0$ C) $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}=0$

D) $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}$ E) $\frac{\partial f}{\partial x}\neq\frac{\partial f}{\partial y}$

11. $z=(x-1)^2+(y+2)^2+1$ funksiyaning $M_0(x_0,y_0)$ kritik nuqtasini toping.

A) $M_0(2,1)$ B) $M_0(1,2)$

C) $M_0(1,-2)$ D) $M_0(-1,2)$ E) $M_0(0,0)$

12. Differensiallanuvchi $z=f(x,y)$ funksiyani $M_0(x_0,y_0)$ nuqtada ekstremumga ega bo'lishining zaruriy shartini ko'rsating.

A) $f'_x(x_0,y_0)=0$ B) $f'_y(x_0,y_0)=0$

C) $f'_x(x_0,y_0)=0$, $f'_y(x_0,y_0)=0$ D) $f'_x(x_0,y_0)-f'_y(x_0,y_0)=0$

E) $f'_x(x_0,y_0)+f'_y(x_0,y_0)=0$.

13. Differensiallanuvchi $z=f(x,y)$ funksiya uchun $M_0(x_0,y_0)$ kritik nuqta va $\Delta=f''_{xx}(M_0)\cdot f''_{yy}(M_0)-[f''_{xy}(M_0)]^2$ bo'lsin. Bu holda ekstremum mavjudligining yetarli sharti qayerda to'g'ri ko'rsatilgan ?

A) $\Delta<0$ B) $\Delta>0$ C) $\Delta=0$ D) $\Delta\neq0$ E) $\Delta\geq0$

14. Differensiallanuvchi $z=f(x,y)$ funksiya uchun $M_0(x_0,y_0)$ kritik nuqta va $A=f''_{xx}(M_0)$, $C=f''_{yy}(M_0)$, $B=f''_{xy}(M_0)$, $\Delta=AC-B^2$ bo'lsin. Qaysi shartda bu funksiya $M_0(x_0,y_0)$ kritik nuqtada minimumga ega bo'ladi ?

A) $\Delta<0$, $A<0$ B) $\Delta<0$, $A>0$ C) $\Delta>0$; $A<0$.

D) $\Delta>0$, $A>0$ E) $\Delta\neq0$, $A\neq0$.

15. Differensiallanuvchi $z=f(x,y)$ funksiya uchun $M_0(x_0,y_0)$ kritik nuqta va $A=f''_{xx}(M_0)$, $C=f''_{yy}(M_0)$, $B=f''_{xy}(M_0)$, $\Delta=AC-B^2$ bo'lsin. Qaysi shartda bu funksiya $M_0(x_0,y_0)$ kritik nuqtada maksimumga ega bo'ladi ?

A) $\Delta<0$, $A<0$ B) $\Delta<0$, $A>0$

C) $\Delta>0$; $A<0$ D) $\Delta>0$, $A>0$ E) $\Delta\neq0$, $A\neq0$.

16. Differensiallanuvchi $z=f(x,y)$ funksiya uchun $M_0(x_0,y_0)$ kritik nuqta va $\Delta=f''_{xx}(M_0)\cdot f''_{yy}(M_0)-[f''_{xy}(M_0)]^2$ bo'lsin. Qaysi shartda funksiya bu kritik nuqtada ekstremumga ega bo'lmaydi ?

A) $\Delta<0$ B) $\Delta>0$ C) $\Delta=0$ D) $\Delta\neq0$ E) $\Delta\geq0$

Matematika shunday tilki-
barcha aniq fanlar shu tilda gapiradi.
Lobachevskiy

XI-BOB. DIFFERENSIAL TENGLAMALAR

§ 11.1 Differensial tenglama haqida tushunchalar

Differensial tenglamaga olib keluvchi masala.

Masala : Massasi m bo'lgan jism biror balandlikdan tashlab yuborilgan. Agar jismga og'irlik kuchidan tashqari havoning tezlikka proportsional bo'lgan (proportsionallik koeffitsienti k) qarshilik kuchi ta'sir etsa, bu jismning tushish tezligi v qanday qonun bilan o'zgarishini bilish, ya'ni $v=v(t)$ munosobatni topish talab etiladi.

Yechimi. Nyutonning ikkinchi qonuniga muvofiq

$$m\cdot\frac{dv}{dt}=F,$$

bunda $\frac{dv}{dt}$ harakatdagi jismning tezlanishi, F esa jismga harakat yo'nalishida ta'sir etuvchi kuch bo'lib, u og'irlik kuchi va havoning qarshilik kuchidan tashkil topadi. Demak,

$$m\frac{dv}{dt}=mg-kv$$

Biz noma'lum v funksiya bilan uning $\frac{dv}{dt}$ hosilasi orasidagi bog'lanishni ifodalovchi tenglamani topdik. Uning yechimi

$$v = ce^{\frac{-kt}{m}} + \frac{mg}{k}$$

bo'lishinitekshiribko'rishmumkin.

1. n – tartibli oddiy differensial tenglama deb

$$F\left(x, y, y', y'', \ldots y^{(n)}\right) = 0 \qquad (11.1)$$

ko'rinishdagi tenglamaga aytiladi. Agar noma'lum funksiya bitta o'zgaruvchiga bog'liq bo'lsa, bunday differensial tenglama oddiy differensial tenglama deyiladi. Agar noma'lum funksiya ikki yoki undan ortiq o'zgaruvchilarga bog'liq bo'lsa, bunday differensial tenglama xususiy hosilali differensial tenglama deyiladi.

2.Differensial tenglamaning yechimi yoki integrali deb tenglamaga qo'yganda uni ayniyatga aytiladigan har qanday differensiallanuvchi $y = \varphi(x)$ funksiyaga aytiladi. Shu funksiyani aniqlovchi $y = \varphi(x)$ yoki $f(x, y) = 0$ funksiya differensial tenglamaning integrali deyiladi. Har bir integral xOy tekisligida differensial tenglamaning integral chizig'i deb ataluvchi egri chiziqni aniqlaydi.

1-misol. Birinchi tartibli

$$y' = -\frac{y}{x}$$

tenglama uchun $y = \frac{C}{x}$ funksiyalar umumiy yechim bo'ladi, ularning grafiklari esa integral chiziqlar deb aytiladi. Umumiy yechimda C=1 deb olib $y = \frac{1}{x}$ xususiy yechimni hosil qilamiz.

3. Agar x, y va n ta ixtiyoriy $c_1, c_2, \ldots, c_n$ o'zgarmaslarni o'z ichiga olgan

$$\phi(x, y, c_1, c_2, \ldots, c_n) = 0 \qquad (11.2)$$

tenglamadagi ixtiyoriy o'zgarmaslarga har xil qiymatlar berganda (11.1) tenglama yechimlarining mavjudlik va yagonalik sohasidan o'tuvchi hamma integral chiziqlar va faqat o'sha chiziqlargina hosil bo'lsa, (11.2) tenglama (11.1) differensial tenglamaning o'sha sohadagi umumiy integrali deyiladi. Ixtiyoriy o'zgarmaslarga aniq qiymatlar berib, umumiy integraldan hosil qilingan integral xususiy integral deyiladi.

Umumiy integral (11.2) ni n marta x bo'yicha differensiallab, hosil bo'lgan n ta tenglamadan va (11.2) tenglamadan n ta ixtiyoriy o'zgarmasni yo'qotsak, berilgan (11.1) differensial tenglamaga ega bo'lamiz. Differensial tenglamaning tartibi deb noma'lum funksiyaning bu tenglamaga kiruvchi hosilalarning eng yuqori tartibiga aytiladi.

4. Ushbu $F(x,y,y')=0$ tenglama umumiy ko'rinishdagi birinchi tartibli differensial tenglama deb ataladi. Agar uni y' ga nisbatan yechish mumkin bo'lsa, bu quyidagicha yoziladi,

$$y' = f(x,y) \tag{11.3}$$

Hosilaga nisbatan yozilgan bu shakldan differensiallar ishtirok etgan

$$dy - f(x,y)dx = 0 \text{ yoki } M(x,y)dx + N(x,y)dy = 0 \tag{11.4}$$

shaklga yozish mumkin.

5. $y=\varphi(x,c)$ umumiy yechim $c=c_0$ qiymati uchun olingan $y=\varphi(x,c_0)$ yechim xususiy yechim deyiladi.

$y'=f(x,y)$ tenglamaning $y(x_0)=y_0$ boshlang'ich shartni qanoatlantiruvchi xususiy yechimini topish masalasi Koshi masalasi deyiladi.

6. $M(x)dx + N(y)dy = 0$ ko'rinishdagi tenglama o'zgaruvchilari ajralgan tenglama deyiladi.

2-misol. Tenglamani yeching.

$$(1+x)ydx+(1-y)dy=0$$

Yechim: O'zgaruvchilarni ajratamiz.:

$$\frac{(1+x)ydx}{xy} + \frac{(1-y)xdy}{xy} = 0,$$

$$\frac{(1+x)dx}{x} + \frac{(1-y)dy}{y} = 0,$$

$$\int\left(\frac{1}{x}+1\right)dx+\int\left(\frac{1}{y}-1\right)dy,\ ln|x|+x+ln|y|-y+c,$$

$$ln|xy|+x-y=c.$$

7. $M_1(x)N_1(y)dx+M_2(x)N_2(y)dy=0,\quad (N_1(y)\neq 0\quad va\quad M_2(x)\neq 0)$ (11.5)

ko'rinishdagi tenglama o'zgaruvchilari ajraladigan tenglama deyiladi. (11.5) tenglamani $M_2(x)N_1(y)$ ko'paytmaga bo'lib

$$\frac{M_1(x)}{M_2(x)}dx+\frac{N_2(y)}{N_1(y)}dy=0 \qquad (11.6)$$

tenglamani hosil qilamiz. (11.13) tenglamaning umumiy integrali

$$\int\frac{M_1(x)}{M_2(x)}dx+\int\frac{N_2(y)}{N_1(y)}dy=C$$

dan iborat bo'ladi.

Quyidagi funksiyalar berilgan differensial tenglamalarning umumiy integrallari bo'ladimi?

11.1. *a)* $y=c_1e^x+c_2e^{2x}$; $y''-3y'+2y=0$

в) $x^2-xy+y^2=c^2$; $(x-2y)y'=2x-y$

11.2. *a)* $y^3-cx^3+3xy=0$; $y^3-(xy^2+x^2)y'+2xy=0$

в) $x\sqrt{1+y^2}=cy$; $xy'-y=y^3$

11.3. $y^2-2=ce^{1/x}$; $2x^2yy'+y^2=2$

11.4. $y=c_1\ln x-x^2/4+c_2$; $x(y''+1)+y'=0$

Berilgan *a* kattalikning qanday qiymatlarida funksiya differensial tenglamani yechimi bo'ladi?

11.5. $x=cy^2-y^a$, $y^2-(2xy+3)y'=0$ 11.6. $x=ax^4+c/x^2$, $y'+2y/x=x^3$

Quyidagi berilgan egri chiziqlar oilasining differensial tenglamasini tuzing.

11.7. $y=ce^x$ 11.8. $x^2+cy^2=2y$ 11.9. $cy=\sin cx$

11.10. $y^3=c_1(x+c_2)^2$

11.11.O'rniga qo'yish yo'li bilan $y=cx^3$ funksiya $3y-xy'=0$ tenglamaning yechimi ekanligi tekshirilsin. Ushbu 1) $\left(1;\frac{1}{3}\right)$; 2) $(1;1)$; 3) $\left(1;-\frac{1}{3}\right)$ nuqtalardan o'tuvchi integral chiziqlar yasalsin.

11.12. O'rniga qo'yish yo'li bilan 1) $y''+4y=0$ va 2) $y'''-9y'=0$ differensial tenglamalar mos ravishda 1) $y=c_1\cos 2x+c_2\sin 2x$ va 2) $y=c_1+c_2e^{3x}+c_3e^{-3x}$ umumiy integrallarga ega ekanliklari tekshirilsin.

11.13. $c=0,\pm1;\pm2$ bo'lganda $y=cx^2$ parabolalar yasalsin va shu parabolalar oilasining differensial tenglamasi tuzilsin.

11.14. 1) $x^2+y^2=2cx$ aylanalar, 2) $y=x^2+2cx$ parabolalar oilasining tasviri yasalsin va ularning differensial tenglamalari tuzilsin.

O'zgaruvchilari ajraladigan 1-tartibli differensial tenglamalarning umumiy yechimini toping.

11.15. $(3x-1)dy+y^2dx=0$ 11.16. $3x^2ydx+2\sqrt{4-x^3}dy=0$

11.17. $xy'+2y=2xyy'$ 11.18. $e^{1-2x}(y^2-1)dy-dx=0$

11.19. $x(y'-1)=2y'$ 11.20. $e^{x+y}dx+dy=0$

11.21. $y'=(x+y)^2$

11.22. $(2x+3y-1)dx+(4x+6y-5)dy=0$ |

11.23. a) $y'+1=\sqrt{x+y+1}$ б) $y'=\sqrt{4x+2y-1}$

11.24. $y'e^{-x}=x-1$

Quyidagi differensial tenglamalarda :

1) umumiy integral topilsin; 2) bir necha integral chiziqlar chizilsin; 3) $y_{x=-2}=4$ boshlang'ich shart bo'yicha xususiy integral topilsin.

11.25. $xy'-y=0$ 11.26. $xy'+y=0$

11.27. $yy'+x=0$ 11.28. $y'=y$

Quyidagi tenglamalarning umumiy integrallari topilsin.

11.29. $x^2y'+y=0$ 11.30. $x+xy+y'(y+xy)=0$

11.31. $\varphi^2dr+(r-a)d\varphi=0$ 11.32. $2st^2ds=(1+t^2)dt$

Quyidagi tenglamalarning umumiy integrallari va berilgan boshlang'ich shartlar bo'yicha xususiy integrallari topilsin:

11.33. $2y'\sqrt{x}=y, x=4$ bo'lganda $y=1$.

11.34. $y'=(2y+1)ctgx,\ x=\frac{\pi}{4}$ bo'lganda $y=\frac{1}{2}$.

11.35. $x^2y'+y^2=0,\ x=4$ bo'lganda $y=1$.

11.36. 1) $y'(x^2-4)=2xy$;

2) $y'+ytgx=0$ tenglamalardan har birining 1) $(0;1)$ 2) $\left(0;\frac{1}{2}\right)$ 3) $\left(0;-\frac{1}{2}\right)$ 4) $(0;-1)$ nuqtalardan o'tuvchi integral chiziqlar yasalsin.

Differensial tenglamalarni yeching.

11.37.a) $xy'+y=y^2$ $y(1)=0,5$ b) $2x^2yy'+y^2=2$

11.38. $\sqrt{y^2+1}dx=xydy$ 11.39. $yy'=-2xSecy$

11.40. $5e^x tgydx+(1-e^x)\sec^2 ydy=0$ 11.41. $\ln\cos ydx+xtgydy=0$

Berilgan boshlang'ich shartni qanoatlantiruvchi differensial tenglamani yeching.

11.42. $(1+x^2)y^3dx-(y^2-1)x^3dy=0;\quad y(1)=-1$

11.43. $(\sqrt{xy}+\sqrt{x})y'-y=0;\quad y(1)=1$

11.44. $x^2(2yy'-1)=1;\quad y(1)=0$

11.45. $ydx+ctgxdx=0;\quad y\left(\frac{\pi}{3}\right)=-1$

Berilgan differensial tenglamalarning umumiy yechimini toping.

11.46.a) $yx^2dy-\ln xdx=0$ в) $(xy^2+x)dx+(y-x^2y)dy=0$

11.47. $xyy'=1-x^2$ 11.48. $yy'=\frac{1-2x}{y}$ 11.49. $y'tgx-y=a$

11.50. $y'+\sqrt{\frac{1-y^2}{1-x^2}}=0$

§ 11.2. Bir jinsli va birinchi tartibli chiziqli differensial tenglamalar

Agar $f(x,y)$ funksiyada x va y o'zgaruvchilarni mos ravishda tx va ty ga almashtirilganda (bu yerda t – ixtiyoriy parametr) $f(xt,yt)=t^n f(x,y)$ shart bajarilsa, $f(x,y)$ funksiya n o'lchovli bir jinsli funksiya deb ataladi. $f(x,y)=\sqrt{x^2+y^2}$ funksiya bir o'lchovli bir jinsli funksiyadir, chunki $f(tx,ty)=\sqrt{t^2x^2+t^2y^2}=t\sqrt{x^2+y^2}=tf(x,y)$ .Ushbu $f(x,y)=\frac{x-y}{x+y}$ funksiya nol o'lchovli bir jinsli funksiya, chunki $f(tx,ty)=f(x,y)$ yoki $f(tx,ty)=t^0f(x,y)$. $f(tx,ty)=f(x,y)$ shartga bo'ysunadigan nol o'lchovli bir jinsli funksiyani $f(x,y)=\varphi\left(\frac{y}{x}\right)$ ko'rinishda yozish mumkin. (11.4) ko'rinishdagi tenglama M va N funksiyalar x va y ning bir xil o'lchovli bir jinsli funksiyalari bo'lganda, bir jinsli tenglama deyiladi. Bu tenglamani $\frac{dy}{dx}=\varphi\left(\frac{y}{x}\right)$ ko'rinishga keltirib, $\frac{y}{x}=z$ yoki $y=zx$ almashtirish bilan yechiladi. Haqiqat $y=zx$ almashtirish bajarilsa, $\frac{dy}{dx}=x\frac{dz}{dx}+z;\ x\frac{dz}{dx}+z=f(z)$, yoki ba'zi bir almashtirishlar bajarilgandan so'ng $\frac{dz}{f(z)-z}=\frac{dx}{x}$ ko'rinishdagi tenglamaga kelamiz. Oxirgi tenglamani integrallab $\ln|x|=\int\frac{dz}{f(z)-z}+C$ integralni olamiz.

Differensial tenglama ko'rinishda $y'=h\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)$ bo'lsa, bir jinsli tenglamaga olib kelish uchun $a_1x+b_1y+c_1=0$ yoki $a_2x+b_2y+c_2=0$ to'g'ri chiziqlarni kesishish nuqtasiga koordinata boshi olib kelinadi. Agar to'g'ri chiziqlar kesishmasa, ya'ni $a_1x+b_1y+c_1$ va $a_2x+b_2y+c_2$ bir-biriga proporsional bo'lsa, u holda $z=a_1x+b_1y$ almashtirish bajariladi.

3-misol. Tenglamani yeching.

$$y'=\frac{x+y}{x}$$

Yechim. $f(x,y)=\frac{x+y}{x}$, $f(\lambda x,\lambda y)=\frac{\lambda x+\lambda y}{\lambda x}=\frac{x+y}{x}$.

Berilgan tenglama bir jinsli ekan. Umumiy yechimini

$$y=t\cdot x,\ y'=t+xt'$$

ko'rinishda qidiramiz:

$$t+xt'=\frac{x+tx}{x},\qquad xt'=1+t-t,\qquad \frac{dt}{dx}=\frac{1}{x},$$

$$dt=\frac{dx}{x},\ \int dt=\int\frac{dx}{x},\qquad t=\ln|x|+C,$$

$y=t\,x=(\ln|x|+C)\cdot x.$

8.Birinchi tartibli chiziqli differensial tenglama. Noma'lum funksiya y ham uning hosilasi y' ham faqat birinchi darajada qatnashgan differensial tenglama birinchi tartibli chiziqli differensial tenglama deyiladi. Uning umumiy ko'rinishi quyidagicha bo'ladi.

$$y' + f(x)y = g(x) \qquad (11.7)$$

bu yerda $f(x)$ va $g(x)$ funksiyalar x ning uzluksiz funksiyalar. Agar $g(x)=0$ bo'lsa, (11.7) differensial tenglama bir jinsli, aks holda bir jinsli bo'lmagan differensial tenglama deyiladi. Bu (11.7) tenglamaning yechimi $y=f(x)$ ni $y=u\vartheta$ ko'rinishda izlanadi. Bu yerda $u=u(x)$ va $\vartheta=\vartheta(x)$ bo'ladi, ya'ni

$$y' = u'\vartheta + \vartheta' u \qquad (11.8)$$

(11.8) ni (11.7) ga qo'ysak, $u'\vartheta + \vartheta' u + f(x)u\vartheta = g(x)$ yoki

$$\vartheta\frac{du}{dx} + u\left(\frac{d\vartheta}{dx} + f(x)\vartheta\right) = g(x)$$

u yoki ϑ funksiyalarning birini ixtiyoriy tanlash mumkinligi hisobiga, ϑ funksiyani

$$\frac{d\vartheta}{dx} + f(x)\vartheta = 0 \qquad (11.9)$$

ko'rinishda olish mumkin. U holda

$$\vartheta\frac{du}{dx} = g(x) \qquad (11.10)$$

ham o'rinli bo'ladi. (11.9) tenglamani quyidagi ko'rinishda yozish mumkin

$$\frac{d\vartheta}{\vartheta} = -f(x)dx,$$

bundan $\int\frac{d\vartheta}{\vartheta} = -\int f(x)dx$, $\ln|\vartheta| = -\int f(x)dx$ va $\vartheta = e^{-\int f(x)dx}$

ϑ funksiyani bunday tanlab olganimizda (11.10) tenglama ushbu ko'rinishga ega bo'ladi:

$$e^{-\int f(x)dx}\cdot\frac{du}{dx} = g(x) \qquad (11.11)$$

(11.11) ni integrallab, $u = \int\left[g(x)e^{\int f(x)dx}\right]dx + C$ va nihoyat

$$y = e^{-\int f(x)dx}\left\{\int g(x)e^{\int f(x)dx}dx + C\right\} \qquad (11.12)$$

Oxirgi (11.12) funksiya (11.7) chiziqli differensial tenglamaning umumiy yechimini ifodalaydi.

4-misol. Tenglamani yeching.

$$y' - \frac{2}{x+1}y = (x+1)^3$$

Yechim. Umumiyyechimni$y=u\cdot v$ *ko'rinishdaqidiramiz.*

$y=u\cdot v$ va $y' = u'v + uv'$ hosilaning ifodalarini dastlabki tenglamaga qo'ysak, u

$$u'v + uv' - \frac{2}{x+1}u\cdot v = (x+1)^3 \quad ,$$

$$u'v + u(v' - \frac{2}{x+1}v) = (x+1)^3 \qquad (1)$$

ko'rinishga keladi.

v ni aniqlash uchun

$$v' - \frac{2}{x+1}v = 0$$

tenglamani yechamiz :

$$\frac{dv}{v} = \frac{2dx}{x+1} \quad , \int\frac{dv}{v} = 2\int\frac{dx}{x+1}$$

$$\ln|v| = 2\ln|x+1| \quad , v = (x+1)^2$$

Berilgan v funksiyaning qiymatini (1) tenglamaga qo'yib u funksiyani aniqlaymiz:

$$u'(x+1)^2 = (x+1)^3 \quad , u' = x+1$$

$$du = (x+1)dx \quad , \quad \int du = \int(x+1)dx \quad . \quad u = \frac{(x+1)^2}{2} + C$$

Endi dastlabki tenglamaning umumiy yechimini yozishimiz mumkin:

$$y = (x+1)^2\left(\frac{(x+1)^2}{2} + C\right) \quad .$$

9.Bernulli tenglamasi. $y' + py = Qy^n$ (11.13) chiziqli tenglamaga o'xshash $y = u\vartheta$ almashtirish bilan yoki ixtiyoriy o'zgarmasni variatsiya qilish bilan yechiladi. Bernulli tenglamasi $z = y^{1-n}$ almashtirsh natijasida chiziqli tenglamaga keltiriladi, ya'ni

$$y^{-n}\frac{dy}{dx} + p(x)y^{1-n} = Q(x) \Rightarrow y^{1-n} = z \Rightarrow (1-n)y^{-n}\frac{dy}{dx} = \frac{dz}{dx} \Rightarrow \frac{1}{1-n}\frac{dz}{dx} + p(x)z = Q(x).$$

(11.13) tenglama $n = 0$ da chiziqli tenglamaga keladi, $n = 1$ da esa o'zgaruvchilari ajraladigan tenglamaga keladi.

5-misol. Ushbu

$$y'' = \sin x$$

tenglamaning umumiy yechimini toping.

Yechimi: $y' = \int_0^x \sin dx + C_1 = -\cos x + 1 + C_1$

$$y = -\int_0^x (\cos x - 1)dx + \int_0^x c_1 dx + c_2 \ , \ e = -\sin x + xc_1 + c_2 .$$

Quyidagi bir jinsli differensial tenglamalarning umumiy yechimi topilsin.

11.51. а) $y' = \frac{x^2 + y^2}{2x^2}$ в) $y' = \frac{y^2}{x^2} - 2$

11.52. $y' = \frac{x + y}{x - y}$

11.53. $xdy - ydx = ydy$

11.54. $y' = \frac{2xy}{x^2 - y^2}$

11.55. $y' = \frac{x}{y} + \frac{y}{x}$

11.56. $xy' - y = \sqrt{x^2 + y^2}$

11.57. $y^2 + x^2 y' = xyy'$

11.58. $y' = e^{y/x} + \frac{y}{x}$

11.59. $xy' = y \ln \frac{y}{x}$

11.60. $(3y^2 + 3xy + x^2)dx = (x^2 + 2xy)dy$

11.61. $y^2 + x^2 y' = xyy'$

11.62. $xy' = y - xe^{y/x}$

11.63. $xy \sin \frac{y}{x} + x = y \sin \frac{y}{x}$

11.64. $yy' = 2y - x$

11.65. a) $y^2 dx + (x^2 - xy)dy = 0$ в) $x^2 + y^2 - 2xyy' = 0$

11.66. $\frac{dS}{dt} = \frac{S}{t} - \frac{t}{S}$

11.67.a) $y' = \frac{y}{x} + tg \frac{y}{x}$ в) $y' - \frac{3y}{x} = x$

11.68. $y' = \frac{x + 2y - 3}{2x - 2}$

11.69. $y' = \frac{x + 2y - 3}{4x - y - 3}$

11.70. $y' = \frac{x + 6y - 7}{8x - y - 7}$

Berilgan boshlang'ich shartlarni qanoatlantiruvchi differensial tenglamalarning xususiy yechimini toping

11.71. a) $xy' - y = xtg \frac{y}{x}$, $y(1) = \frac{\pi}{2}$ в) $(xy' - y)arctg \frac{y}{x} = x, \quad y(1) = 0$

11.72. $(y^2 + 3x^2)dy + 2xydx = 0, \quad y(0) = 1$

11.73. a) $y' = \frac{x - 2y + 3}{2x - 4y - 1}$ в) $y' = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}; \ y(1) = -1$

11.74. $(x^2 - 3y^2)dx + 2xydy = 0; \quad y(2) = 1$

Quyidagi chiziqli differensial tenglamalarning yechimi topilsin.

11.75. $y' + 2y = 4x$

11.76 $y' + 2xy = xe^{-x^2}$

11.77. $y' + \frac{1-2x}{x^2}y = 1$

11.78. $(1+x^2)y' - 2xy = (1+x^2)^2$

11.79. $y' + y = \cos x$

11.80. $y' + ay = e^{mx}$

11.81. $2ydx + (y^2 - 6x)dy = 0$

11.82. $y' - y\sin x = \sin x \cos x$

11.83. $y' - 2xy = 3x^2 - 2x^4$

11.84. $y' + y\cos x = e^{-\sin x}$

11.85. $y' + ytgx = \frac{1}{\cos x}$

11.86. $y' - yctgx = 2x - \frac{x^2 \cos x}{\sin x}$

11.87. $ydx - (x + y^2 \sin y)dy = 0$

11.88. $xy' - y = x^2 \cos x$

11.89. $y' + 2xy = xe^{-x^2}$

11.90. $y'\cos x + y = 1 - \sin x$

Quyidagi differensial tenglamalar yechilsin

11.91. $y' + \frac{n}{x}y = \frac{a}{x^n}, \quad y(1) = 0$

11.92. $(1+x^2)y' + y = arctgx$

11.93. $y'\sqrt{1-x^2} + y = \arcsin x, \quad y(0) = 0$

11.94. $y' - \frac{y}{\sin x} = \cos^2 x \ln tg\frac{x}{2}$

11.95. $y' - \frac{y}{x\ln x} = x\ln x, \quad y(e) = e^2/2$

11.96. $y'\sin x - y\cos x = 1, \quad y(\pi/2) = 0$

11.97. $y' + 3ytg3x = \sin 6x, \quad y(0) = 1/3$

11.98. $xy' + 2y = x^2$

11.99. $y' + 2xy = 2xe^{-x^2}$

11.100. $y' + 2xy = 2xe^{-x^2}$

Bernulli tenglamalarini yeching

11.101. a) $y' - 2xy = 2x^3y^2, \quad y(0) = 1$ в) $y' = y^4 \cos x + ytgx$

11.102.a) $y' - \frac{y}{2x} = \frac{x^2}{2y}$ в) $xy^2y' = x^2 + y^3$

11.103. $y' + \frac{2y}{x} = 2x^2y^{4/3}$

11.104. $y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x}$

11.105. $y' + y = e^{x/2}\sqrt{y}$

11.106. $y' + xy = xy^3$

11.107. $y' = \frac{y^2}{x^2} - \frac{y}{x}, \quad y(-1) = 1$

11.108. $3y^2y' + y^3 = (x+1), \quad y(1) = -1$

11.109. $\frac{dy}{dx} + \frac{y}{x} = -xy^2$

11.110. a) $y' = \frac{4}{x}y + x\sqrt{y}$ b) $2xy\frac{dy}{dx} - y^2 + x = 0$

§ 11.3 To'liq differensial tenglama. Tartibini pasaytirish mumkin bo'lgan yuqori tartibli differensial tenglamalar

Agar $M(x,y)dx + N(x,y)dy = 0$ differensial tenglamada

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{11.13}$$

bo'lsa, bu tenglama $du = 0$ ko'rinishga va uning umumiy integrali $u = c$ bo'ladi.

Agar (11.13) shart bajarilmasa, ya'ni $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ bo'lsa, u holda ba'zi bir shartlar bajarilganda shunday $\mu(x,y)$ funksiya topish mumkinki, $\mu M(x,y)dx + \mu N(x,y)dy = du$ bo'ladi. Bu $\mu(x,y)$ funksiya integrallovchi ko'paytuvchi deyiladi.

Quyidagi hollarda integrrallovchi ko'paytuvchini topish oson

1) $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \phi(x)$ bo'lganda $\ln\mu = \int\phi(x)dx$ bo'ladi.

2) $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \phi_1(x)$ bo'lganda $\ln\mu = \int\phi_1(x)dx$ bo'ladi.

10. Tartibini pasaytirish mumkin bo'lgan yuqori tartibli differensial tenglamalar. $y^{(n)} = f(x)$ ko'rinishdagi tenglama o'ng tomonini ketma-ket n marta integrallab yechiladi. Har bir integrallashda bitta ixtiyoriy o'zgarmas hosil bo'ladi, oxirgi natijada n ta ixtiyoriy o'zgarmas ishtirok etadi.

$$y = \int dx \int \ldots \int f(x)dx + c_1 x^{n-1} + c_2 x^{n-2} \ldots + c_n .$$

y oshkor ishtirok etmagan $F(x, y', y'') = 0$ tenglama $y' = p,\ \ y'' = \frac{dp}{dx}$ almashtirish bilan $F\left(x, p, \frac{dp}{dx}\right) = 0$ ko'rinishga keltiriladi.

x oshkor etmagan $F(x, y', y'') = 0$ tenglama $y' = p,\ \ y'' = \frac{dp}{dx} = p\frac{dp}{dy}$ almashtirish bilan $F\left(y, p, p, \frac{dp}{dy}\right) = 0$ ko'rinishga keltiriladi.

6-misol. Ushbu

$$3y'' = y^{-\frac{5}{3}}$$

tenglamaning umumiy integralini toping.

Yechim. p *ni y ning* funksiyasi ekanini bilgan holda $y'=p$ deb olamiz. Bu holda $y''=p'\cdot p$ bo'ladi va biz yordamchi p funksiya uchun birinchi tartibli tenglama hosil qilamiz:

$$3pp'=y^{-\frac{5}{3}}$$

Bu tenglamani integrallaymiz:

$$p^2=C_1 y^{-\frac{2}{3}}, \qquad p=\pm\sqrt{C_1-y^{-\frac{2}{3}}}$$

Ammo $y'=p$, demak, y ni aniqlash uchun

$$\pm\frac{dy}{\sqrt{C_1-y^{-\frac{2}{3}}}}=dx\,, \qquad \frac{y^{\frac{1}{3}}dy}{\pm\sqrt{C_1y^{\frac{2}{3}}-1}}=dx$$

tenglamani hosil qilamiz, bundan

$$x+C_2=\pm\int\frac{y^{\frac{1}{3}}dy}{\sqrt{C_1y^{\frac{2}{3}}-1}}$$

keying integralni hisoblash uchun

$$C_1y^{\frac{2}{3}}-1=t^2$$

almashtirish bajaramiz. Bu holda

$$y^{\frac{1}{3}}=(t^2+1)^{\frac{1}{2}}\cdot\frac{1}{C_1^{\frac{1}{2}}}\,, \qquad dy=3t(t^2+1)^{\frac{1}{2}}\cdot\frac{1}{C_1^{\frac{1}{2}}}\,dt$$

Demak

$$\int\frac{y^{\frac{1}{2}}dy}{\sqrt{C_1y^{\frac{2}{3}}-1}}=\frac{1}{C_1^2}\int\frac{3t(t^2+1)}{t}dt=\frac{3}{C_1^2}=\left(\frac{t^3}{3}+t\right)=$$

$$=\frac{1}{C_1^2}\sqrt{C_1y^{\frac{2}{3}}-1}\cdot(C_1y^{\frac{2}{3}}+2)\,.$$

Oxirgi natijadan

$$x+C_2=\pm\frac{1}{C_1^2}\sqrt{C_1y^{\frac{2}{3}}-1}\cdot(C_1y^{\frac{2}{3}}+2)$$

ekanini topamiz.

Quyidagi to'liq differensialli differensial tenglamalar yechilsin:

11.111. a) $(2x^3 - xy^2)dx + (2x^3 - xy^2)dy = 0$

b) $\left(4 - \frac{y^2}{x^2}\right)dx + \frac{2y}{x}dy = 0$

11.112. $3x^2e^y dx + (x^3e^y - 1)dy = 0$

11.113. $e^{-y}dx + (1 - xe^{-y})dy = 0$

11.114. $2x\cos^2 ydx + (2x - x^2\sin 2y)dy = 0$

11.115. $2x\cos^2 ydx + (2x - x^2\sin 2y)dy = 0$

11.116. $(3x^2 + 6xy^2)dx + (6x^2y + 4x^3)dy = 0$

11.117. $(x + y)dx + (x + 2y)dy = 0$

11.118. $(x^2 + y^2 + 2x)dx + 2xydy = 0$

11.119. $(x^3 - 3xy^2 + 2)dx - (3x^2y - y^2)dy = 0$ 11.120. $xdx + ydy = \frac{xdy - ydx}{x^2 + y^2}$

Quyidagi differensial tenglamalarning integrallovchi ko'paytuvchilarini topilsin va tenglamalar yechilsin:

11.121. a) $(y + \ln x)dx - xdy = 0$ b) $(x^2 - \sin^2 y)dx + x\sin 2ydy = 0$

11.122. $(x^2 - 3y^2)dx + 2xydy = 0$

11.123. $(x\sin y + y\cos y)dx + (x\cos y - y\sin y)dy = 0$

11.124. $\cos xdy + (\sin x + e^y)dx = 0$

11.125. $(x^2 - y)dx + xdy = 0$

11.126. $2xtgydx + (x^2 - 2\sin y)dy = 0$

11.127. $(e^{2x} - y^2)dx + ydy = 0$

11.128. $(1 + 3x^2\sin y)dx - xctgydy = 0$

11.129. $y^2dx + (yx - 1)dy = 0$

11.130. $(\sin x + e^y)dx + \cos xdy = 0$

Quyidagi tenglamalar yechilsin.

11.131.a) $y'' = y'ctgx$ в) $y''' = \frac{6}{x^3}$ boshlang'ich shartlar $x = 1$ bo'lganda $y = 2,\ y' = 1,\ y'' = 1$

11.132. $y'' = 4\cos 2x$ boshlang'ich shartlar $x = 0$ bo'lganda $y = 0,\ y' = 0$

11.133. $y'' = \frac{1}{1 + x^2}$ 11.134. $y'' = \frac{1}{x}$ 11.135. $y'' = \frac{1}{2y^3}$

11.136. $y'' = 1 - y'^2$ 11.137. $xy'' + y' = 0$ 11.138. $yy'' = y'^2$

11.139. $yy'' + y'^2 = 0$ 11.140. $(1 + x^2)y'' + y'^2 + 1 = 0$

11.141. $y'(1 + y'^2) = ay''$

11.142. $x^2 y'' + xy' = 1$ 1.143. $yy'' = y^2 y' + y'^2$

11.144. $yy'' - y'(1 + y') = 0$

11.145. $y'' = -\frac{x}{y}$ 11.146. $x^3 y'' + x^2 y' = 1$

11.147. $y'' + y' tgx = \sin 2x$

11.148. $y'' + 2y(y')^3 = 0$ 11.149. $y'' x \ln x = y'$ 11.150. $y'' tgy = 2(y')^2$

11.151. $y'' = -\frac{x}{y}$ 11.152. $xy'' + y' = 0$ 11.153. $xy'' = y' \ln \frac{y'}{x}$

11.154. $yy'' - y(1 + y') = 0$ 11.155. $y''' = 2(y'' - 1)ctgx$

§ 11.4 O'zgarmas koeffisisyentli ikkinchi tartibli chiziqli differensial tenglamalar

11.O'zgarmas koeffisisyentli ikkinchi tartibli chiziqli differensial tenglamalar. Ushbu

$$y'' + py' + qy = r(x) \tag{11.14}$$

ko'rinishdagi tenglama $(p = const,\ q = const)$ ikkinchi tartibli chiziqli differensial tenglama deyiladi. Bu yerda $r(x)$ berilgan funksiya. Agar $r(x) = 0$ bo'lsa, (11.14) bir jinsli $r(x) \neq 0$ bo'lsa, bir jinsli bo'lmagan tenglama differensial tenglama deyiladi.

Bir jinsli ikkinchi tartibli differensial tenglamani

$$y'' + py' + qy = 0 \tag{11.15}$$

yechimni

$$y = Ce \tag{11.16}$$

ko'rinishda qidiramiz. Agar (11.16) ni (11.15) qo'ysak

$$\lambda^2 + p\lambda + q = 0, \quad (\lambda = const)$$

xarakteristik tenglamani olamiz.

1. Agar xarakteristik tenglama ikkita λ_1 va λ_2 yechimlari mavjud bo'lsa, $(\lambda_1 \neq \lambda_2)$ u holda differensial tenglama (11.16) umumiy yechimi

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} \tag{11.17}$$

2. Agar xarakteristik tenglamani yechimlari $\lambda_1 = \lambda_2$ bo'lsa, u holda $y = c_1 e^{\lambda_1 x} + c_2 x e^{\lambda_2 x}$.

3. Agar xarakteristik tenglamani yechimlari kompleks son, ya'ni $\lambda = \alpha \pm \beta i \left(\alpha = -p/2,\ \ \beta = \sqrt{q - p^2/4}\right)$ bo'lsa,

u holda $y = c_1 e^{\alpha x} \sin \beta x + c_2 e^{\alpha x} \cos \beta x$

Ixtiyoriy o'zgarmasni variatsiyalash usuli. Bir jinsli bo'lmagan chiziqli tenglamani yechish usullari umumiyrog'I bo'lib, Lagranj metodi yoki ixtiyoriy o'zgarmasni variatsiyalash metodi hisoblanadi.

$y'' + py' + qy = 0$ tenglamaning o'zaro bog'liq bo'lmagan ikkita yechimi bo'lsa, u holda $y'' + py' + qy = r(x)$ tenglamaning yechimi, Lagranj metodiga asosan, $y = Ay_1 + By_2$ ko'rinishda izlanadi, bundagi A va B lar x ning funksiyalari bo'lib, ular

$$\left.\begin{array}{l} A'y_1 + B'y_2 = 0 \\ A'y_1' + B'y_2' = r(x) \end{array}\right\}$$

Tenglamlar sistemasini qanoatlantirishi kerak. Bundan

$$A' = -\frac{y_2 r(x)}{\Delta},\quad B' = \frac{y_1 r(x)}{\Delta},\quad \Delta = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

7-misol. Tenglamaning umumiy yechimini toping:

$$y^{(4)} - y = 0$$

Yechim. Xarakteristik tenglamani tuzib ildizlarini aniqlaymiz:

$$k^4 - 1 = 0\ ,\ k_1 = 1,\ k_2 = -1\ ,\ k_3 = i\ ,\ k_4 = -i$$

Endi umumiy yechimni yozishimiz mumkin:

$$y = C_1 e^x + C_2 e^{-x} + C_3 \cos x + C_4 \sin x\ .$$

Quyidagi bir jinsli ikkinchi tartibli *y* differensial tenglamalar yechilsin

11.156. $y'' - 4y' + 3y = 0$

11.157. $y'' - 4y' + 4y = 0$

11.158. $y'' - 4y' + 13y = 0$

11.159. $y'' - 4y' = 0$

11.160. $y'' + 4y = 0$

11.161. $y'' + 4y' = 0$

11.162. $y'' + 3y' + 2y = 0$

11.163. $y'' + 2ay' + a^2 y = 0$

11.164. $y'' + 2y' + 5y = 0$

11.165. $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 0$

11.166. $\frac{d^2x}{dt^2} + \omega^2 x = 0$

11.167. $\frac{d^2S}{dt^2} + a\frac{dS}{dt} - 3x = 0$

11.168. $\ddot{x}_{tt} + 2\dot{x}_t + 3x = 0$

11.169. $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} - 4x = 0$

11.170. $4\frac{d^2S}{d\varphi^2} + S = 0$

11.171. $\frac{d^2S}{dt^2} + 2\frac{dS}{dt} + 2S = 0$

Bir jinsli bo'lmagan differensial tenglamalarni yeching

11.172. $y'' - 4y = x^2 e^{2x}$

11.173. $y'' + 9y = \cos 2x$

11.174. $y'' - 4y' + 4y = \sin 2x + e^{2x}$

11.175. $y'' + 2y' + 2y = e^x \sin x$

11.176. $y'' - 7y' + 12y = -e^{4x}$

11.177. $y'' - 2y' = x^2 - 1$

11.178. $y'' - 2y' + y = 2e^x$

11.179. $y'' - 2y' = e^{2x} + 5$

11.180. $y'' - 4y' + 4y = x^2$

11.181. $y'' - y' + y = x^3 + 6$

Boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimni toping

11.182. $y'' + 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = -6$

11.183. $y'' - 10y' + 25y = 0, \quad y(0) = 0, \quad y'(0) = 1$

11.184. $y'' - 2y' + 10y = 0, \quad y(\pi/6) = 1, \quad y'(\pi/6) = e^{\pi/6}$

11.185. $3y'' + y = 0, \quad y(3\pi/2) = 2, \quad y'(3\pi/2) = 0$

11.186. $y'' + 3y' = 0, \quad y(0) = 1, \quad y'(0) = 2$

11.187. $y'' + 9y = 0, \quad y(0) = 0, \quad y'(\pi/4) = 1$

11.188. $y'' + y = 0, \quad y'(0) = 1, \quad y'(\pi/3) = 0$

11.189. $y'' - 4y' + 3y = e^{5x}, \quad y(0) = 3, \quad y'(0) = 9$

11.190. $y'' + y = \cos 3x, \quad y(\pi/2) = 4, \quad y'(\pi/2) = 1$

11.191. $y'' - 8y' + 16y = e^{4x}, \quad y(0) = 0, \quad y'(0) = 1$

11.192. $2y'' - y' = 1, \quad y(0) = 0, \quad y'(0) = 1$

11.193. $y'' - y = 2shx, \quad y(0) = 0, \quad y'(0) = 1$

11.194. $y'' + 9y = 2\sin x \sin 2x, \quad y(0) = y(\pi/2) = 0$

11.195. $y'' + 4y = \cos 2x, \quad y(0) = y(\pi/4) = 0, \quad y = (1/16)(4x - \pi)\sin 2x$

§ 11.5. O'zgarmas koeffisiyentli chiziqli differensial tenglamalar sistemasi

Bizga ushbu differensial tenglamlar sistemasi

$$\left.\begin{aligned} \frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n, \\ \frac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \frac{dx_n}{dt} &= a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n \end{aligned}\right| \quad (11.18)$$

berilgan bo'lsin, bunda a_{ij} koeffiyentlar o'zgarmas sonlar. Bu yerda *t* argument, $x_1(t)$, $x_2(t),\ldots,x_n(t)$ izlanayotgan funksiyalar, (11.18) sistema o'zgarmas koeffiyentli chiziqli bir jinsli differensial sistemasi deyiladi.

Sistemning xususiy yechimini quyidagi ko'rinishda izlaymiz:

$$x_1 = \alpha_1 e^{kt}, \quad x_2 = \alpha_2 e^{kt}, \ldots, \quad x_n = \alpha_n e^{kt} \qquad (11.19)$$

$\alpha_1 e^{kt}$, $\alpha_2 e^{kt},\ldots$, $\alpha_n e^{kt}$ funskiyalar (11.18) tenglamlar sistemsini qanoatlantiruvchi o'zgarmas $\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_n$ va *k* sonlarni aniqlash talab qilinadi. Ularni (11.18) sistemaga qo'yib, ushbuni hosil qilamiz:

$$\left.\begin{aligned} (a_{11}-k)\alpha_1 + a_{12}\alpha_2 + \ldots + a_{1n}\alpha_n &= 0, \\ a_{21}\alpha_1 + (a_{22}-k)\alpha_2 + \ldots + a_{2n}\alpha_n &= 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}\alpha_1 + a_{n2}\alpha_2 + \ldots + (a_{nn}-k)\alpha_n &= 0 \end{aligned}\right| \qquad (11.20)$$

$\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_n$ va k_n larni (11.20) sistemani qanoatlantiradigan qilib tanlab olamiz. bu sistema $\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_n$ ga nisbatan chiziqli allgebraik tenglamalar sistemasidir. (11.20) sistemaning determinantini tuzamiz:

$$\Delta(k) = \begin{vmatrix} a_{11}-k & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-k & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & (a_{nn}-k) \end{vmatrix} \qquad (11.21)$$

Agar k shunday bo'lsaki, Δ determinant noldan farqli bo'lsa, u holda (11.20) sistema faqat $\alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$ nol yechimga ega bo'ladi, demak, (11.19) formula faqat privial yechimni beradi.

$$x_1(t) = x_2(t) = \ldots = x_n(t) \equiv 0$$

Shunday qilib, (11.19) trivial bo'lmagan yechimlarni biz k ning shunday qiymatlarida hosil qilamizki, bu qiymatlarda (11.21) determinant nolga aylanadi. Biz k ni aniqlash uchun ushbi n-tartibli tenglamaga kelamiz:

$$\begin{vmatrix} a_{11}-k & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-k & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn}-k \end{vmatrix} = 0 \qquad (11.22)$$

Bu tenglama (11.18) sistemaning xarakteristik tenglamasi deyiladi, uning ildizlari xarakteristik tenglamaning ildizlari deyiladi.

Bir necha holni ko'rib chiqamiz.

1. Xarakteristik tenglamaning ildizlari haqiqiy va har xil $k_1, k_2, \ldots, k_n$ bilan xarakteristik tenglamaning ildizlari belgilaymiz. Har bir k_i ildiz uchun (3) sistemani yozamiz va

$$\alpha_1^{(i)}, \alpha_2^{(i)}, \ldots, \alpha_n^{(i)}$$

koeffiyentlarni aniqlaymiz. Bulardan bittasining ixtiyoriy bo'lishini va uni birga teng deb hisoblash mumkinligini ko'rsatish mumkin. Shunday qilib, quyidagilarni hosil qilamiz:

k_1 ildiz uchun (11.18) sistemaning yechimi

$$x_1^{(1)} = \alpha_1^{(1)} e^{k_1 t}, \quad x_2^{(1)} = \alpha_2^{(1)} e^{k_1 t}, \ldots, \quad x_n^{(1)} = \alpha_n^{(1)} e^{k_1 t}$$

k_2 ildiz uchun (11.18) sistemaning yechimi

$$x_1^{(2)} = \alpha_1^{(2)} e^{k_2 t}, \quad x_2^{(2)} = \alpha_2^{(2)} e^{k_2 t}, \ldots, \quad x_n^{(2)} = \alpha_n^{(2)} e^{k_2 t}$$

....

k_n ildiz uchun (11.18) sistemaning yechimi

$$x_1^{(n)} = \alpha_1^{(n)} e^{k_n t}, \quad x_2^{(n)} = \alpha_2^{(n)} e^{k_n t}, \ldots, \quad x_n^{(n)} = \alpha_n^{(n)} e^{k_n t}$$

Bevosita tenglamaga qo'yish yo'li bilan

$$\left.\begin{array}{l} x_1 = C_1\alpha_1^{(1)}e^{k_1x} + C_2\alpha_1^{(2)}e^{k_2x} + \ldots + C_n\alpha_1^{(n)}e^{k_nx}, \\ x_2 = C_1\alpha_2^{(1)}e^{k_1x} + C_2\alpha_2^{(2)}e^{k_2x} + \ldots + C_n\alpha_2^{(n)}e^{k_nx}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n = C_1\alpha_n^{(1)}e^{k_1x} + C_2\alpha_n^{(2)}e^{k_2x} + \ldots + C_n\alpha_n^{(n)}e^{k_nx}, \end{array}\right| \quad (11.23)$$

Funksiyalar sistemasi ham, bunda $C_1, C_2, \ldots, C_n$ ixtiyoriy o'zgarmas miqdorlar, (11.18) differensial tenglmalar sistemasining yechimi bo'lishiga ishonch qilish mumkin. Bu (11.18) sistemaning umumiy yechimidir. O'zgarmas miqdorlarning shunday qiymatlarini topish mumkinki, bu qiymatlarda yechimning berilgan boshlang'ich shartlarni qanoatlantirishini ko'rsatish mumkin.

II. Xarakteristik tenglamaning ildizlari har xil, ammo ular orasida kompleks ildizlari bor. Xarakteristik tenglamaning ildizlari orasida ikkita qo'shmakompleksildibo'lsin:

$$k_1 = \alpha + i\beta, \quad k_2 = \alpha - i\beta$$

Bu ildizga ushbu yechimlar mos bo'ladi:

$$x_j^{(1)} = \alpha_j^{(1)} e^{(\alpha + i\beta)x} \qquad (j = 1, 2, \ldots, n) \quad (11.24)$$

$$x_j^{(2)} = \alpha_j^{(2)} e^{(\alpha - i\beta)x} \qquad (j = 1, 2, \ldots, n) \quad (11.25)$$

$\alpha_j^{(1)}$ va $\alpha_j^{(2)}$ koeffisiyentlar (11.20) tnglamalar sistemasidan aniqlanadi. Shunday qilib, biz ikkita xususiy yechim hosil qilamiz:

$$\left.\begin{array}{l} \overline{x}_j^{(1)} = e^{\alpha x}\left(\lambda_j^{(1)} \cos \beta x + \lambda_j^{(2)} \sin \beta x\right), \\ \overline{x}_j^{(2)} = e^{\alpha x}\left(\lambda_j^{(1)} \sin \beta x + \lambda_j^{(2)} \cos \beta x\right), \end{array}\right\} \quad (11.26)$$

bunda $\lambda_j^{(1)}, \lambda_j^{(2)}, \overline{\lambda}_j^{(1)}, \overline{\lambda}_j^{(2)}$ lar $\alpha_j^{(1)}$ va $\alpha_j^{(2)}$ orqali aniqlanadigan haqiqiy sonlar.

(11.26) funksiyalarning mos kombinasiyalari sistemaning umumiy yechimiga kiradi.

Quyidagi tenglamalar sistemasini umumiy yechimini toping

11.196. Ushbu $\frac{dx_1}{dt} = 2x_1 + 2x_2$, $\frac{dx_2}{dt} = x_1 + 3x_2$ tenglamalar sistemasining umumiy yechimi topilsin.

11.197. Ushbu $\frac{dx_1}{dt} = -7x_1 + x_2$, $\frac{dx_2}{dt} = -2x_1 - 5x_2$ tenglamalar sistemasining umumiy yechimi topilsin.

11.198. $\begin{cases} \frac{dx_1}{dt} = 7x_1 + 3x_2, \\ \frac{dx_2}{dt} = 6x_1 + 4x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.199. $\begin{cases} \frac{dx}{dt} = 6x - 12y - z, \\ \frac{dy}{dt} = x - 3y - z, \\ \frac{dz}{dt} = -4x + 12y + 3z. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.200. $\begin{cases} \frac{dx_1}{dt} = 4x_1 - 3x_2, \\ \frac{dx_2}{dt} = 3x_1 + 4x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.201. $\begin{cases} \frac{dx_1}{dt} = x_1 - x_3, \\ \frac{dx_2}{dt} = x_1, \\ \frac{dx_3}{dt} = x_1 - x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.202. $\begin{cases} \frac{dx_1}{dt} = 5x_1 - x_2, \\ \frac{dx_2}{dt} = x_1 + 3x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.203. $\begin{cases} \frac{dx_1}{dt} = -ax_2, \\ \frac{dx_2}{dt} = -ax_1. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.204. $\begin{cases} \frac{dx_1}{dt} = 8x_2 - x_1, \\ \frac{dx_2}{dt} = x_1 + x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.205. $\begin{cases} \frac{dx_1}{dt} = -x_1 + x_2 + x_3, \\ \frac{dx_2}{dt} = x_1 - x_2 + x_3, \\ \frac{dx_3}{dt} = x_1 + x_2 + x_3. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.206. $\begin{cases} \frac{dx_1}{dt} = x_1 - x_2 + x_3, \\ \frac{dx_2}{dt} = x_1 + x_2 - x_3, \\ \frac{dx_3}{dt} = 2x_1 - x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.207. $\begin{cases} \frac{dx_1}{dt} = 12x_1 - 5x_2, \\ \frac{dx_2}{dt} = 5x_1 + 12x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

11.208. $\begin{cases} \frac{dx_1}{dt} = x_1 - 2x_2, \\ \frac{dx_2}{dt} = x_1 - x_2. \end{cases}$ tenglamalar sistemasining umumiy yechimi topilsin.

§ 11.6. Bessel tenglamasi

O'zgarmas koeffisiyentli quyidagi ko'rinishdagi oddiy diffferensial tenglamaga

$$x^2 y'' + xy' + (x^2 - \lambda^2)y = 0 \quad (\lambda = const) \qquad (11.27)$$

Bessel tenglamasi deyiladi. Bu tenglamani $\xi = mx$ almashtirish yordamida

$$x^2 y''+xy'+\left(m^2x^2 - \lambda^2\right)y = 0$$

tenglamani olamiz.

(11.27) tenglmani yechimini umumlashgan darajali qator ko'rinishda izlaymiz.

$$y = x^r\left(a_0 + a_1 x + a_2 x^2 + \ldots\right) = \sum_{k=0}^{\infty} a_k x^{r+k} \qquad (11.28)$$

(11.28) yechimni (11.27) ga qo'yib x darajasi oldidagi koeffiyentlarni nolga tenglashtirsak, u holda

Faraz qilaylik, $a_0 \neq 0$ bo'lsin, u holda $r_{1,2} = \pm\lambda$ bo'ladi. $r_1 = \lambda$ bo'lsin.

U holda ikkinchi tenglamadan $a_1 = 0$, $\left[(r+k)^2 - \lambda^2\right]a_k = a_{k-2}$ tenglamadan $(k = 3,5,7\ldots)$ $a_3 = a_5 = a_7 = \ldots = a_{2k+1} = 0$. Juft nomerdgi koeffisiyentlar uchun

$$a_2 = \frac{-a_0}{(2\lambda+2)\cdot 2}; \quad a_4 = \frac{-a_2}{(2\lambda+4)\cdot 4} = \frac{a_0}{(\lambda+1)(\lambda+2)\cdot 1\cdot 2\cdot 3\cdot 4},$$

$$a_{2k} = \frac{-a_{2k-2}}{(2\lambda+2)\cdot 2k} = (-1)^{k+1}\frac{a_0}{2\cdot 4\cdot 6\cdot\ldots\cdot 2k(2\lambda+2)(2\lambda+4)\ldots(2\lambda+2k)}$$

Topilgan koeffisiyentlarni (2) yechimga qo'ysak,

$$y_1(x) = a_0 x^{\lambda}\left[1 - \frac{x^2}{2(2\lambda+2)} + \frac{x^4}{2\cdot 4\cdot(2\lambda+2)(2\lambda+4)} - \frac{x^6}{2\cdot 4\cdot 6\cdot(2\lambda+2)(2\lambda+4)(2\lambda+6)} + \ldots\right] =$$

$$= a_0\sum_{k=0}^{\infty}\frac{(-1)^k x^{\lambda+2k}}{n^k k!(\lambda+1)(\lambda+2)\ldots(\lambda+k)}$$

bu yerda koeffisiyentiga a_0 - ixtiyoriy o'zgarmas kattalik.

Agar $r_2 = -\lambda$ bo'lsa, u holda

$$y_2(x) = a_0 x^{-\lambda}\left[1 - \frac{x^2}{2(-2\lambda+2)} + \frac{x^4}{2\cdot 4\cdot(-2\lambda+2)(-2\lambda+4)} - \frac{x^6}{2\cdot 4\cdot 6\cdot(-2\lambda+2)(-2\lambda+4)(-2\lambda+6)} + \ldots\right] =$$

$$= a_0\sum_{k=0}^{\infty}\frac{(-1)^k x^{-\lambda+2k}}{n^k k!(-\lambda+1)(-\lambda+2)\ldots(-\lambda+k)}$$

Agar $y_1(x)$ yechimi $a_0 = \frac{1}{2^{\lambda}\Gamma(\lambda+1)}$, ko'paytirsak, hosil bo'lgan funksiyaga Bessel funksiyasi deyiladi yoki u holda

$y(x)=C_1J_\lambda(x)+C_2J_{-\lambda}(x)$, C_1 va C_2-ixtiyoriy o'zgarmas kattaliklar. $J_\lambda(x)$ - birinchi tartibli λ-tartibli Bessel funksiyasi deyiladi. Bu yerda

$$\Gamma(\lambda)=\int_0^\infty e^{-x}x^{\lambda-1}dx \quad (\lambda>0).$$

Agar $\lambda=n$ bo'lsa, u holda

$$J_n(x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\Gamma(n+k+1)}\cdot\left(\frac{x}{2}\right)^{2k+n}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\Gamma(n+k)!}\cdot\left(\frac{x}{2}\right)^{2k+n}$$

11.209. $\lambda=0$ bo'lganda Bessel funksiyasini toping.

11.210. $x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y=0$ tenglamani yeching.

11.211. $J_1(x)$ ni toping.

11.212. $x^2y''+xy'+\left(x^2-\frac{9}{4}\right)y=0$ tenglamani yeching.

11.213. $x^2y''+xy'+\left(x^2-\frac{4}{9}\right)y=0$ tenglamani yeching.

Takrorlash uchun savollar

1.Differensial tenglama qanday ta'riflanadi?

2.Differensial tenglamaning tartibi deb nimaga aytiladi?

3.Differensial tenglamaning yechimi nima?

4.I tartibli differensial tenglama umumiy holda qanday ko'rinishda yoziladi?

5.Eng sodda I tartibli differensial tenglama qanday ko'rinishda bo'ladi?

6.II tartibli differensial tenglamaning umumiy ko'rinishi qanday yoziladi?

7.II tartibli differensial tenglamaning yechimi nima?

8.II tartibli differensial tenglama uchun boshlang'ich shart qanday kiritiladi?

9.Tartibini pasaytirish usulining mohiyati nimadan iborat?

DIFFERENSIAL TENGLAMALARGA DOIR NAZORAT TESTLARI

1.Differensial tenglamalar va ular bilan bog'liq tushunchalar

1. Differensial tenglama ta'rifini ko'rsating .

A) noma'lum funksiya qatnashgan tenglama .

B) noma'lum funksiyaning turli qiymatlari qatnashgan tenglama

C) noma'lum funksiyaning hosilalari qatnashgan tenglama .

D) noma'lum funksiya va uning hosilalarining x_0 nuqtadagi qiymatlari qatnashgan tenglama .

E) noma'lum funksiya va uning integrallari qatnashgan tenglama .

2. Quyidagilardan qaysi biri differensial tenglama bo'ladi ?

A) $y^2+5y-3\cos x=0$ B) $3x^2+4xy-1=0$

C) $y(x)+2\,y'(x_0)-x=0$ D) $y-2xy'+5=0$

E) $\int ydx+\sin(x+y)=0$.

3. $(\alpha^2-1)y'+\alpha y+5x+9=0$ tenglama α parametrning qanday qiymatlarida differensial tenglama bo'ladi?

A) $\alpha\neq0$. B) $\alpha\neq1$. C) $\alpha\neq-1$. D) $\alpha\neq\pm1$. E) $\alpha\in(-\infty,\infty)$.

4. $(\alpha^2-1)y''+\alpha y'+5xy+7=0$ tenglama α parametrning qanday qiymatlarida differensial tenglama bo'ladi?

A) $\alpha\neq0$. B) $\alpha\neq1$. C) $\alpha\neq-1$. D) $\alpha\neq\pm1$. E) $\alpha\in(-\infty,\infty)$.

5. ***Ta'rifni to'ldiring:*** Differensial tenglamaning tartibi deb unda qatnshuvchi noma'lum funksiya hosilalarning ……… aytiladi

A) eng katta darajasiga B) eng katta tartibiga

C) soniga D) eng katta qiymatiga

E) to'g'ri javob keltirilmagan

6. $(y')^3-(y')^2+\ y''-y+5y^4+x^5=0$ differensial tenglama nechanchi tartibli ?

A) I B) II C) III D) IV E) V

7. $(\alpha^2-1)y''+\alpha y'+5xy+7=0$ differensial tenglama I tartibli bo'ladigan α parametrning barcha qiymatlarini ko'rsating.

A) $\alpha=0$ B) $\alpha=1$ C) $\alpha=-1$ D) $\alpha=\pm1$ E) $\alpha\in(-\infty,\infty)$

8. $(\alpha^2-1)y''+\alpha y'+5xy+7=0$ differensial tenglama II tartibli bo'ladigan α parametrning barcha qiymatlarini ko'rsating.

A) $\alpha\neq0$ B) $\alpha\neq1$ C) $\alpha\neq-1$ D) $\alpha\neq\pm1$ E) $\alpha\in(-\infty,\infty)$

9. n tartibli differensial tenglama eng umumiy holda qanday ko'rinishda bo'ladi ?

A) $F(y,y', ..., y^{(n)})=0$ B) $F(x, y,y', ..., y^{(n-1)})= y^{(n)}$

C) $F(x, y,y', ..., y^{(n-1)}, y^{(n)})=0$ D) $F(y,y', ..., y^{(n)})= x$

E) $F(x, y,y', ..., y^{(n-1)})=0$.

10. Biror $y=\phi(x)$ funksiya $F(x, y,y', ..., y^{(n-1)}, y^{(n)})=0$ differensial tenglamaning yechimi bo'lishi uchun qaysi shart talab etilmaydi?

A) $y=\phi(x)$ funksiya biror chekli yoki cheksiz D oraliqda aniqlangan .

B) $y=\phi(x)$ funksiya D oraliqda n marta differensiallanuvchi .

C) $y=\phi(x)$ funksiya D oraliqda monoton .

D) $y=\phi(x)$ funksiya va uning $y^{(k)}(x)=\phi^{(k)}(x)$ $(k=1,2, ... , n)$ hosilalari differensial tenglamani ayniyatga aylantiradi;

E) keltirilgan barcha shartlar talab etiladi .

11. Differensial tenglamaning yechimi yana nima deb ataladi ?

A) ildiz B) differensial

C) boshlang'ich funksiya D) integral

E) tenglashtiruvchi funksiya

12. Qyuidagi funksiyalardan qaysi biri $y'-2y=0$ differensial tenglamaning yechimi bo'ladi?

A) $y=x^2$ B) $y=\sin2x$ C) $y=\cos2x$ D) $y=e^{2x}$ E) $y=\ln2x$

13. Qyuidagi funksiyalardan qaysi biri $y''+4y=0$ differensial tenglamaning yechimi bo'ladi?

A) $y=x^4$ B) $y=2x^2$ C) $y=\cos2x$ D) $y=e^{2x}$ E) $y=\ln2x$

14. Quyidagi funksiyalardan qaysi biri $y''+4y=0$ differensial tenglamaning yechimi bo'lmaydi?

A) $y=\sin 2x$ B) $y=\cos 2x$ C) $y=\cos 2x+\sin 2x$

D) $y=\cos 2x-\sin 2x$ E) $y=\cos 2x\cdot\sin 2x$

15. Quyidagi funksiyalardan qaysi biri $y''-4y=0$ differensial tenglamaning yechimi bo'ladi?

A) $y=x^2$ B) $y=\sin 2x$ C) $y=\cos 2x$ D) $y=e^{2x}$ E) $y=\ln 2x$

16. λ parametrning qanday qiymatida $y=e^{\lambda x}$ funksiya $y'-4y=0$ differensial tenglamaning yechimi bo'ladi?

A) $\lambda=2$ B) $\lambda=-2$ C) $\lambda=4$ D) $\lambda=-4$ E) $\lambda=\pm 1$

2. I tartibli differensial tenglamalar

1. I tartibli differensial tenglama eng umumiy holda qanday ko'rinishda bo'ladi ?

A) $F(x,y,y')=0$ B) $F(x,y)=y'$ C) $F(x,y')=y$.

D) $F(y,y')=x$ E) $F(x,y,y',y'')=0$

2. I tartibli differensial tenglama uchun boshlang'ich shart qanday ko'inishda bo'ladi ?

A) $y(x_0)=y_0$ B) $y'(x_0)=y_0$ C) $\lim\limits_{x\to x_0} y(x)=y_0$

D) $\max\limits_{0\le x\le x_0} y(x)=y_0$ E) $\min\limits_{0\le x\le x_0} y(x)=y_0$

3. I tartibli differensial tenglama uchun Koshi masalasini ko'rsating .

A) $y'=f(x,y)$, $y'(x_0)=y_0$ B) $y'=f(x,y)$, $y(x_0)=y_0$

C) $y'=f(x_0,y)$ D) $y'=f(x,y_0)$ E) $y'=f(x_0,y_0)$

4. I tartibli tenglama uchun Koshi masalasi Koshi teoremasi shartlarida nechta yechimga ega ?

A) kamida bitta B) ko'pi bilan bitta C) faqat bitta

D) cheksiz ko'p E) yechimga ega emas

5. I tartibli eng sodda differensial tenglama qanday ko'rinishda bo'ladi ?

A) $y'=f(x,y)$ B) $y'=f(y)$

C) $y'=f(x)$ D) $y'=f(y')$ E) $y'=0$

6. I tartibli eng sodda $y'=xe^x$ differensial tenglamani integrallang .

A) $y=xe^x+C$ B) $y=(x-1)e^x+C$ C) $y=(x+1)e^x+C$

D) $y=(x-2)e^x+C$ E) $y=(x+2)e^x+C$

7. I tartibli eng sodda $y'=xe^x$ differensial tenglama umumiy yechimining x=1 nuqtadagi qiymati $y(1)$ uchun qaysi javob to'g'ri ?

A) $y(1)=0$ B) $y(1)>0$ C) $y(1)<0$ D) $y(1)\neq 0$

E) $y(1)=C$, C –ixtiyoriy chekli son .

8. $y'=xe^x$, $y(0)=2$ Koshi masalasini yechimini toping .

A) $y=xe^x+2$ B) $y=(x-1)e^x+3$ C) $y=(x+1)e^x+1$

D) $y=(x-2)e^x+4$. E) $y=(x+2)e^x$

9. I tartibli o'zgaruvchilari ajralgan differensial tenglamani ko'rsating .

A) $y'=f(xy)$ B) $y'=f(x/y)$ C) $y'+P(x)y=Q(x)$

D) $M(x)dx+N(y)dy=0$ E) $M_1(x)\, N_1(y)dx+M_2(x)N_2(y)dy=0$

10. O'zgaruvchilari ajralgan $\frac{dx}{\sqrt{x}}+\frac{dy}{\sqrt{y}}=0$ differensial tenglamaning umumiy integralini toping.

A) $x^2+y^2=C$ B) $\sqrt{x}+\sqrt{y}=C$ C) $\frac{x}{2}+\frac{y}{2}=C$

D) $x^2+4y^2=C$ E) $\frac{\sqrt{x}}{y}+\frac{\sqrt{y}}{x}=C$.

11. I tartibli o'zgaruvchilari ajraladigan differensial tenglamani ko'rsating

A) $y'=f(xy)$ B) $y'=f(x/y)$ C) $y'+P(x)y=Q(x)$

D) $M(x)dx+N(y)dy=0$ E) $M_1(x)\, N_1(y)dx+M_2(x)N_2(y)dy=0$.

12. O'zgaruvchilari ajraladigan $ydx+2xdy=0$ differensial tenglamaning umumiy yechimini toping.

A) $y=Cx$ B) $y=Cx^2$ C) $y=C\sqrt{x}$ D) $y=Cx^2+y^4$ E) $y=C/\sqrt{x}$

13. I tartibli bir jinsli differensial tenglamani ko'rsating .

A) $y'=f(xy)$ B) $y'=f(x/y)$ C) $y'+P(x)y=Q(x)$

D) $M(x)dx+N(y)dy=0$ E) $M_1(x)\, N_1(y)dx+M_2(x)N_2(y)dy=0$.

14.I tartibli bir jinsli differensial tenglamani qanday almashtirma yordamida integrallanadi ?

A) $y=u/x$ B) $y=u+x$ C) $y=u-x$ D) $y=ux$ E) $y=x/u$

15. I tartibli chiziqli differensial tenglama qanday ko'rinishda bo'ladi ?

A) $y'=f(x+y)$ B) $y'=f(x/y)$ C) $y'+P(x)y=Q(x)$

D) $M(x)dx+N(y)dy=0$ E) $M_1(x)\, N_1(y)dx+M_2(x)N_2(y)dy=0$.

16.I tartibli $y'+P(x)y=Q(x)$ chiziqli differensial tenglama qaysi holda bir jinsli deyiladi?

A) $Q(x)>0$ B) $Q(x)<0$ C) $Q(x)=0$ D) $Q(x)\neq 0$ E) $P(x)=0$

3. II tartibli differensial tenglamalar. Tartibni pasaytirish usuli

1.Quyigagi differensial tenglamalardan qaysi biri II tartibli ?

A) $(y')^2+2yy'-x=0$ B) $y'+2y^2-x=0$ C) $y'+2y-x^2=0$

D) $y''+2y-x=0$. E) $(y')^2+2y^2-x^2=0$.

2. $(\alpha^2-1)y''+(\alpha^2+1)y'-y\sin x=0$ differensial tenglama II tartibli bo'ladigan αparametrning barcha qiymatlarini toping.

A) $\alpha\neq 0$ B) $\alpha\neq 1$ C) $\alpha\neq -1$ D) $\alpha\neq\pm 1$ E) $\alpha\neq 0$ va $\alpha\neq 1$

3. II tartibli differensial tenglama uchun Koshi masalasida boshlang'ich shartlar qanday ko'rinishda bo'ladi ?

A) $y(x_0)=y_0\,,\, y(x_1)=y_1$ B) $y(x_0)=y_0\,,\, y'(x_1)=y_1'$

C) $y(x_0)=y_0\,,\, y'(x_0)=y_0'$. D) $y(x_0)=y'(x_0)=y_0$.

E) $y'(x_0)=y_0'\,,\, y''(x_0)=y_0''$.

4. II tartibli differensial tenglama uchun Koshi masalasi qayerda to'g'ti ifodalangan ?

A) $y''=f(x,y,y')\,,\, y(x_0)=y_0\,,\, y(x_1)=y_1$.

B) $y''=f(x,y,y')\,,\, y(x_0)=y_0\,,\, y'(x_1)=y_1'$.

C) $y''=f(x,y,y')\,,\, y(x_0)=y_0\,,\, y'(x_0)=y_0'$.

D) $y''=f(x,y,y')\,,\, y(x_0)=y'(x_0)=y_0$.

E) $y''=f(x,y,y')\,,\, y'(x_0)=y_0'\,,\, y''(x_0)=y_0''$.

5. Koshi teoremasida $y''=f(x,y,y')\,,\, y(x_0)=y_0\,,\, y'(x_0)=y_0'$ masala yechimi mavjud va yagona bo'lishi uchun qanday shart

talab etilmaydi ?

A) $f(x,y,y')$ funksiya $M_0(x_0,y_0,y'_0)$ nuqta atrofida uzluksiz .

B) $f'_x(x,y,y')$ xususiy hosila $M_0(x_0,y_0,y'_0)$ nuqta atrofida uzluksiz

C) $f'_y(x,y,y')$ xususiy hosila $M_0(x_0,y_0,y'_0)$ nuqta atrofida uzluksiz .

D) $f'_{y'}(x,y,y')$ xususiy hosila $M_0(x_0,y_0,y'_0)$ nuqta atrofida uzluksiz .

E) keltirilgan barcha shartlar talab etiladi .

6. Koshi teoremasi shartlarida $y''=f(x,y,y')$, $y(x_0)=y_0$, $y'(x_0)=y'_0$ Koshi masalasining yechimi haqida qaysi tasdiq o'rinli bo'ladi?

A) yechim mavjud

B) yechim mavjud va yagona

C) yechim mavjud va cheksiz ko'p

D) kamida bitta yechim mavjud .

E) ko'pi bilan bitta yechim mavjud .

7. II tartibli differensial tenglamani tartibni pasaytirish usulida integrallash uchun qanday almashtirma bajariladi ?

A) $y=p$. B) $y'=p$. C) $y''=p$. D) $yx=p$. E) $y/x=p$.

8. Quyidagi ko'rinishdagi II tartibli differensial tenglamalardan qaysi birini tartibni pasaytirish usulida integrallab bo'lmaydi ?

A) $y''=f(x,y)$ B) $y''=f(x,y')$ C) $y''=f(y,y')$ D) $y''=f(x)$

E) barcha tenglamalarni tartibni pasaytirish usulida integrallab bo'ladi .

9. Quyidagi II tartibli differensial tenglamalardan qaysi birini tartibni pasaytirish usulida integrallab bo'lmaydi ?

A) $y''=xy$. B) $y''=xy'$. C) $y''=yy'$. D) $y''=x$.

E) barcha tenglamalarni tartibni pasaytirish usulida integrallab bo'ladi .

10.Quyidagi II tartibli differensial tenglamalardan qaysi birini tartibni pasaytirish usulida integrallab bo'lmaydi ?

A) $y''= x+y$. B) $y''= x+y'$. C) $y''= y+y'$. D) $y''= x+1$.

E) barcha tenglamalarni tartibni pasaytirish usulida integrallab bo'ladi .

11. II tartibli eng sodda differensial tenglamani ko'rsating .

A) $y''= f(x,y)$ B) $y''= f(x,y')$

C) $y''= f(y,y')$ D) $y''= f(x)$ E) $y''= f(y')$.

12. $y''=\sin 2x$ differensial tenglamaning umumiy yechimini toping .

A) $y= -0.25\cos 2x+C$ B) $y= -0.25\sin 2x+C$

C) $y= -0.25\cos 2x+C_1x+C_2$ D) $y= -0.25\sin 2x+C_1x+C_2$

E) to'g'ri javob keltirilmagan .

13. $y''=12x+2$, $y(0)=1$, $y'(0)=2$ Koshi masalasining yechimini toping .

A) $y=4x^3+x^2+x+1$ B) $y=4x^3+x^2+2x+1$ C) $y=4x^3+x^2+3x+1$.

D) $y=4x^3+x^2+4x+1$ E) $y=4x^3+x^2-x+1$.

14. $y''=6x+1$, $y(0)=2$, $y'(0)=1$ Koshi masalasi yechimining $x=2$ nuqtadagi $y(2)$ qiymatini nimaga teng?

A) $y(2)=8$ B) $y(2)=10$ C) $y(2)=14$ D) $y(2)=16$ E) $y(2)=18$

4. II tartibli chiziqli o'zgarmas koeffitsientli bir jinsli differensial tenglamalar

1. Quyidagilardan qaysi biri II tartibli chiziqli differentsal tenglama bo'ladi?

A) $(y'')^2 + a_1 y' + a_2 y = f(x)$ B) $y'' + a_1 (y')^2 + a_2 y = f(x)$

C) $y'' + a_1 y' + a_2 y^2 = f(x)$ D) $y'' + a_1 y' + a_2 y = f^2(x)$

E) bu tenglamalar orasida II tartibli chiziqli differentsal tenglama yo'q

2. Qaysi shartda II tartibli chiziqli $y'' + a_1 y' + a_2 y = f(x)$ differentsaial tenglama bir jinsli deyiladi ?

A) $f(x)>0$ B) $f(x)<0$ C) $f(x)=0$ D) $f(x)\neq 0$ E) $f(x)\approx 0$

3. II tartibli chiziqli $y'' + a_1 y' + a_2 y = f(x)$ differentsaial tenglama quyidagi hollardan qaysi birida bir jinsli bo'lmasligi mumkin?

A) $f(x)=0$ B) $\ln[1+f(x)]=0$ C) $e^{f(x)}=1$ D) $\sin f(x)=0$

E) keltirilgan barcha hollarda bir jinsli bo'ladi .

4. Quyidagi II tartibli chiziqli tenglamalardan qaysi biri bir jinsli emas ?

A) $y'' - 3y' = 0$. B) $y'' - 3y = 0$. C) $y'' - 3y' + 3y = 0$.

D) $y'' - 3 = 0$. E) keltirilgan tenglamalarning hammasi bir jinsli .

5. II tartibli chiziqli $y'' + a_1 y' + a_2 y = f(x)$ differentsaial tenglama quyidagi hollardan qaysi birida bir jinslimas bo'ladi ?

A) $f(x)>0$. B) $f(x)<0$. C) $f(x)=0$. D) $f(x)\neq 0$.

E) barcha hollarda bir jinslimas bo'ladi .

6. ***Ta'rifni to'lldiring:*** Agar $y'' + a_1 y' + a_2 y = f(x)$ tenglamada a_1 va a_2 koeffitsientlardan .... o'zgarmas son bo'lsa, u II tartibli o'zgarmas koeffitsientli chiziqli differentsaial tenglama deyiladi.

A) birortasi B) faqat bittasi C) ikkalasi ham

D) kamida bittasi E) ko'pi bilan bittasi

7. Quyidagilardan qaysi biri II tartibli o'zgarmas koeffitsientli chiziqli differensial tenglama bo'lmaydi ?

A) $y'' - 4y' = 0$ B) $y'' - 4y = 0$ C) $y'' - 4x = 0$

D) $y'' - 4x^2 = 0$ E) $y'' - 4y^2 = 0$

8. Agar y_1 va y_2 II tartibli bir jinsli chiziqli differensial tenglamaning ikkita xususiy yechimlari bo'lsa, unda quyidagi tasdiqlardan qaysi biri o'rinli emas?

A) ixtiyoriy C_1 va C_2 o'zgarmas sonlar uchun C_1y_1 va C_2y_2 funksiyalar bu tenglama yechimlari bo'ladi .

B) y_1+y_2 bu tenglama yechini bo'ladi .

C) y_1-y_2 bu tenglama yechini bo'ladi .

D) ixtiyoriy C_1 va C_2 o'zgarmas sonlar uchun $C_1y_1+C_2y_2$ funksiyalar bu tenglama yechimlari bo'ladi .

E) keltirilgan barcha tasdiqlar o'rinlidir .

9. Quyidagi shartlardan qaysu birida II tartibli chiziqli differensial tenglamaning ikkita y_1 va y_2 xususiy yechimlari chiziqli bog'liq bo'lmaydi ?

A) birorta $C\neq0$ o'zgarmas son uchun $y_1=Cy_2$.

B) birorta $C\neq0$ o'zgarmas son uchun $y_2=Cy_1$.

C) qandaydir noldan farqli C_1 va C_2 o'zgarmas sonlar uchun $C_1y_1+C_2y_2=0$.

D) qandaydir noldan farqli C_1 va C_2 o'zgarmas sonlar uchun $C_1y_1-C_2y_2=0$.

E) bu shartlarning barchasida y_1 va y_2 yechimlar chiziqli bog'liq bo'ladi .

10.Quyidagi hollardan qaysi birida II tartibli chiziqli differensial tenglamaning ikkita y_1 va y_2 xususiy yechimlari chiziqli erkli bo'ladi ?

A) $2y_1+y_2=0$ B) $2y_1+y_2=1$ C) $2y_1-y_2=0$ D) $2y_1/y_2=1$ E) $2y_1/y_2=-1$

11. Agar y_1 va $y_2=y_1+C$ ($C\neq0$) bo'lsa, C parametrning qanday qiymatlarida y_1 va y_2 funksiyalar chiziqli bog'liq bo'lmaydi?

A) $C>0$ B) $C<0$ C) ixtiyoriy $C\neq0$ uchun D) $C\in\varnothing$ E) $C=\pm1$.

12. $y_1=\cos^2x$, $y_2=1-\cos2x$, $y_3=1+\cos2x$ funksiyalardan qaysi juftlik chiziqli bog'liq bo'ladi ?

A) y_1 va y_2 B) y_1 va y_3 C) y_2 va y_3

D) uchala jurtlik ham juftlik chiziqli bog'liq emas .

E) uchala jurtlik ham chiziqli bog'liq .

13.Quyidagilardan qaysi biri y_1 va y_2 funksiyalarning Vronskiy aniqlovchisini ifodalamaydi ?

A) $\begin{vmatrix} y_1 & y_1' \\ y_2 & y_2' \end{vmatrix}$. B) $\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$. C) $\begin{vmatrix} y_1' & y_2' \\ -y_1 & -y_2 \end{vmatrix}$. D) $\begin{vmatrix} -y_1' & y_1 \\ -y_2' & y_2 \end{vmatrix}$.

E) keltirlgan barcha aniqlovchilar Vronskiy aniqliochisini ifodalaydi .

14. y_1=cos$x$ va y_2=sin$x$ funksiyalarning Vronskiy aniqlovchisi W(y_1, y_2) qayerda to'g'ri ko'rsatilgan ?

A) W(y_1, y_2)=cos$x$+sin$x$. B) W(y_1, y_2)=cos$x$−sin$x$.
C) W(y_1, y_2)=cos$x$·sin$x$. D) W(y_1, y_2)=1 . E) W(y_1, y_2)=0 .

15. $y_1=e^x$ va $y_2=e^{-x}$ funksiyalarning Vronskiy aniqlovchisi W(y_1, y_2) qayerda to'g'ri ko'rsatilgan ?

A) W(y_1, y_2)= e^{2x} B) W(y_1, y_2)= e^{-2x} C) W(y_1, y_2)= −1
D) W(y_1, y_2)=0 E) W(y_1, y_2)=−2

16. $y_1=e^x$cos$x$ va $y_2=e^x$sin$x$ funksiyalarning Vronskiy aniqlovchisi W(y_1, y_2) qayerda to'g'ri ko'rsatilgan ?

A) W(y_1, y_2)= e^{2x}(cos$x$+sin$x$) B) W(y_1, y_2)= e^{2x}(cos$x$+sin$x$) .
C) W(y_1, y_2)= e^{2x}cos$x$·sin$x$ D) W(y_1, y_2)= e^{2x}
E) W(y_1, y_2)= e^{2x}sin2x .

5. II tartibli chiziqli o'zgarmas koeffitsientli bir jinslimas differensial tenglamalar

1. II tartibli chiziqli $y''+py'+qy=f(x)$ differensial tenglama qaysi shartda bir jinslimas deb ataladi ?

A) $f(x)=0$ B) $f(x)\neq 0$ C) $f(x)>0$ D) $f(x)<0$ E) $f(x)\geq 0$

2. II tartibli chiziqli $y''+py'+qy=f(x)$ differensial tenglama qaysi holda birjinslimas bo'lmaydi ?

A) $f(x)=0$ B) $f(x)\neq 0$
C) $f(x)>0$ D) $f(x)<0$ E) $f(x)\geq 0$

3. II tartibli chiziqli $y''+py'+qy=(\alpha^2-1)f(x)$ differensial tenglama α parametrning qanday qiymatlarida birjinslimas bo'ladi ?

A) $\alpha>0$ B) $\alpha\neq 0$ C) $\alpha<0$ D) $\alpha\neq\pm 1$ E) $\alpha=\pm 1$

4. II tartibli chiziqli $y''+py'+qy=(\alpha^2-1)f(x)$ differensial tenglama α parametrning qanday qiymatlarida birjinslimas bo'lmaydi ?

A) $\alpha>0$ B) $\alpha\neq 0$ C) $\alpha<0$ D) $\alpha\neq\pm 1$ E) $\alpha=\pm 1$

5. Quyidagi II tartibli chiziqli tenglamalardan qaysi biri bir jinslimas bo'ladi?

A) $y''+py'+qy=0$ B) $y''+py'+q=0$
C) $y''+py'=0$ D) $y''+qy=0$

E) keltirilgan barcha differensial tenglamalar bir jinslidir .

6. II tartibli bir jinslimas chiziqli $y''+py'+qy=f(x)$ differensial tenglamaning xususiy yechimi y, unga mos keluvchi bit jinsli tenglamaning umumiy yechimi y^0 bo'lsa, birjinslimas tenglamaninh umumiy yechimi y qanday ko'rinishda bo'ladi ?

A) $y= y+ y^0$ B) $y= y/ y^0$

C) $y= y \cdot y^0$ D) $y=y^0/ y$ E) $y= C_1y+ C_2 y^0$

7. AgarII tartibli bir jinslimas chiziqli $y''+py'+qy=f(x)$ differensial tenglama mos keluvchi bir jinsli tenglamaning chiziqli erkli yechimlari y_1 va y_2 bo'lsa, o'zgarmaslarni variatsiyalash usulida bir jinlimas tenglamaning xususiy yechimi y qanday ko'rinishda izlanadi ?

A) $y=C_1(x)y_1 \cdot C_2(x)y_2$

B) $y=C_1(x)y_1+ C_2(x)y_2$

C) $y=C_1(x)y_1/ C_2(x)y_2$

D) $y=[C_1(x)+C_2(x)](y_1+y_2)$

E) $y=[C_1(x)-C_2(x)](y_1-y_2)$.

8. II tartibli bir jinslimas chiziqli $y''+py'+qy=f(x)$ differensial tenglamaning y xususiy yechimini o'zgarmaslarni variatsiyalash usulida $y=C_1(x)y_1+C_2(x)y_2$ ko'rinishda izlanganda (bunda y_1 va y_2 tegishli bir jinsli tenglamaning chiziqli erkli yechimlari) noma'lum $C_1(x)$ va $C_2(x)$ funksiyalar qaysi sistemadan topiladi?

A) $\begin{cases} C_1'(x)y_1 + C_2'(x)y_2 = f(x) \\ C_1'(x)y_1' + C_2'(x)y_2' = 0 \end{cases}$ B) $\begin{cases} C_1'(x)y_1 + C_2'(x)y_2 = 0 \\ C_1'(x)y_1' + C_2'(x)y_2' = f(x) \end{cases}$

C) $\begin{cases} C_1(x)y_1 + C_2(x)y_2 = 0 \\ C_1(x)y_1' + C_2(x)y_2' = f(x) \end{cases}$ D) $\begin{cases} C_1(x)y_1 + C_2(x)y_2 = f(x) \\ C_1(x)y_1' + C_2(x)y_2' = 0 \end{cases}$

E) to'g'ri javob keltirilmagan.

9. O'zgarmaslarni variatsiyalash usulida $y''-4y'+3y=x\sin 2x$ II tartibli chiziqli differensial tenglamaning xususiy yechimi y qanday ko'rinishda izlanadi ?

A) $y=C_1(x)e^{2x}+C_2(x)e^{-2x}$. B) $y=C_1(x)e^{x}+C_2(x)e^{-2x}$.

C) $y=C_1(x)e^{x}+C_2(x)e^{3x}$. D) $y=C_1(x)\sin 2x+C_2(x)\cos 2x$.

E) $y=C_1(x)e^{x}\sin 2x+C_2(x)e^{3x}\cos 2x$

10. Agar $y''+py'+qy=P_n(x)e^{\alpha x}$ ($P_n(x)$–n-darajali ko'phad) differensial tenglamada α soni $\lambda^2+p\lambda+q=0$ xarakteristik

tenglamaning ildizi bo'lmasa va $Q_n(x)$ n-darajali ko'phadni ifodalasa, unda differensial tenglamaning xususiy yechimi y qanday ko'rinishda izlanadi ?

A) $y^* = Q_n(x)e^{\alpha x}$ B) $y^* = xQ_n(x)e^{\alpha x}$ C) $y^* = Q_n(x) + e^{\alpha x}$
D) $y^* = x^2 Q_n(x)e^{\alpha x}$ E) $y^* = Q_n^2(x)e^{\alpha x}$

6. I tartibli differensial tenglamalar sistemasi

1. Quyidagilardan qaysi biri I tartibli differentsal tenglamalar sistemasining umumiy ko'rinishini ifodalaydi ?

A) $\begin{cases} F_1(x,y_1,y_2,y_1',y_2',y_1'',y_2'')=0 \\ F_2(x,y_1,y_2,y_1',y_2',y_1'',y_2'')=0 \end{cases}$ B) $\begin{cases} F_1(x,y_1,y_2,y_1',y_2')=0 \\ F_2(x,y_1,y_2,y_1',y_2')=0 \end{cases}$

C) $\begin{cases} F_1(x,y_1,y_2,y_1',y_2')=0 \\ y_2'=f_2(x,y_1,y_2) \end{cases}$ D) $\begin{cases} y_1'=f_1(x,y_1,y_2) \\ F_2(x,y_1,y_2,y_1',y_2')=0 \end{cases}$

E) $\begin{cases} y_1'=f_1(x,y_1,y_2) \\ y_2'=f_2(x,y_1,y_2) \end{cases}$

2. Quyidagilardan qaysi biri differentsal tenglamalarning I tartibli normal sistemasini ifodalaydi

A) $\begin{cases} F_1(x,y_1,y_2,y_1',y_2',y_1'',y_2'')=0 \\ F_2(x,y_1,y_2,y_1',y_2',y_1'',y_2'')=0 \end{cases}$

B) $\begin{cases} F_1(x,y_1,y_2,y_1',y_2')=0 \\ F_2(x,y_1,y_2,y_1',y_2')=0 \end{cases}$

C) $\begin{cases} F_1(x,y_1,y_2,y_1',y_2')=0 \\ y_2'=f_2(x,y_1,y_2) \end{cases}$

D) $\begin{cases} y_2'=f_1(x,y_1,y_2) \\ F_2(x,y_1,y_2,y_1',y_2')=0 \end{cases}$

E) $\begin{cases} y_1'=f_1(x,y_1,y_2) \\ y_2'=f_2(x,y_1,y_2) \end{cases}$.

3. Quyidagilardan qaysi biri differensial tenglamalarning normal sistemasi bo'ladi ?

A) $\begin{cases} y_1'=xy_1+y_2' \\ y_2'=x+y_1y_2 \end{cases}$ B) $\begin{cases} y_1'=y_1+y_2' \\ y_2'=y_1+y_1'+y_2 \end{cases}$ C) $\begin{cases} y_1'=y_1+(1-x)y_2' \\ y_2'=x+y_1y_2 \end{cases}$

D) $\begin{cases} y_1'=x-y_1+y_2^2 \\ y_2'=\sin x+y_1y_2 \end{cases}$ E) $\begin{cases} y_1'=xy_1+y_2+y_2' \\ y_2'=y_1'-x+y_1y_2 \end{cases}$

4. $y_1=\phi_1(x)$ va $y_2=\phi_2(x)$ funksiyalar biror I oraliqda

$$\begin{cases} y_1' = f_1(x, y_1, y_2) \\ y_2' = f_2(x, y_1, y_2) \end{cases}$$

normal sistemaning yechimi bo'lishi uchun quyidagi shartlardan qaysi biri talab etilmaydi ?

A) $y_1=\phi_1(x)$ va $y_2=\phi_2(x)$ funksiyalar I oraliqda aniqlangan .

B) $y_1=\phi_1(x)$ va $y_2=\phi_2(x)$ funksiyalar I oraliqda differensiallanuvchi .

C) ixtiyoriy $x\in I$ uchun $(x, \phi_1(x), \phi_2(x))$ nuqta $f_i(x,y_1,y_2)$ $(i=1,2)$ funksiyalarning G aniqlanish sohasiga tegishli .

D) $y_1=\phi_1(x)$ va $y_2=\phi_2(x)$ funksiyalar normal sistemadagi tenglamalarni ayniyatga aylantiradi .

E) keltirilgan barcha shartlar talab etiladi.

5. Differensial tenglamalarning I tartibli

$$\begin{cases} y_1' = f_1(x, y_1, y_2) \\ y_2' = f_2(x, y_1, y_2) \end{cases}$$

normal sistemasi uchun Koshi masalasida boshlang'ich shartlar qanday ko'rinishda bo'ladi ?

A) $y_1'(x_0) = y_{10}$, $y_2'(x_0) = y_{20}$ B) $y_1'(x_0) = y_{10}$, $y_2'(x_1) = y_{21}$

C) $y_1(x_0) = y_{10}$, $y_2(x_0) = y_{20}$ D) $y_1(x_0) = y_{10}$, $y_2(x_1) = y_{21}$

E) $y_1'(x_0) = y_{10}$, $y_2'(x_0) = y_{20}$, $y_1(x_1) = y_{11}$, $y_2(x_1) = y_{21}$

6. Differensial tenglamalarning I tartibli chiziqli normal sistemasi umumiy holda qanday ko'rinishda bo'ladi ?

A) $\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 + f_1(x) \\ y_2' = a_{21}y_1 + a_{22}y_2 + f_2(x) \end{cases}$

B) $\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 + a_{13}y_2' + f_1(x) \\ y_2' = a_{21}y_1 + a_{22}y_2 + a_{23}y_1' + f_2(x) \end{cases}$

C) $\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 + f_1(x) \\ y_2' = a_{21}y_1 + a_{22}y_2 \end{cases}$

D) $\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 \\ y_2' = a_{21}y_1 + a_{22}y_2 + f_2(x) \end{cases}$

E) $\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 \\ y_2' = a_{21}y_1 + a_{22}y_2 \end{cases}$

7. *Ta'rifni to'ldiring:* Differensial tenglamalarning I tartibli chiziqli normal sistemasi

$$\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 + f_1(x) \\ y_2' = a_{21}y_1 + a_{22}y_2 + f_2(x) \end{cases}$$

bir jinsli deb ataladi , agar $f_1(x)$ va $f_2(x)$ funksiyalardan .... .

A) kamida bittasi aynan nolga teng bo'lsa .

B) faqat bittasi aynan nolga teng bo'lsa .

C) ikkalasi ham aynan nolga teng bo'lsa .

D) birortasi aynan nolga teng bo'lsa .

E) ikkalasi ham aynan nolga teng bo'lmasa .

8. Differensial tenglamalarning I tartibli chiziqli normal sistemasi

$$\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 + f_1(x) \\ y_2' = a_{21}y_1 + a_{22}y_2 + f_2(x) \end{cases}$$

qaysi holda bir jinslimas bo'ladi ?

A) $f_1(x)$ va $f_2(x)$ funksiyalardan birinchisi aynan nolga teng emas

B) $f_1(x)$ va $f_2(x)$ funksiyalardan ikkinchisi aynan nolga teng emas

C) $f_1(x)$ va $f_2(x)$ funksiyalardan ikkalasi ham aynan nolga teng emas

D) keltirilgan barcha hollarda sistema bir jinslimas bo'ladi .

E) keltirilgan barcha hollarda sistema bir jinslimas bo'lmaydi

9. Agar $y_1=y_1(x)$ va $y_2=y_2(x)$ differensial tenglamalarning I tartibli bir jinsli chiziqli normal sistemasining yechimlari bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli emas ?

A) $y=y_1+y_2$ funksiya ham bu sistema yechimi bo'ladi .

B) $y=y_1 \cdot y_2$ funksiya ham bu sistema yechimi bo'ladi .

C) $y=y_1-y_2$ funksiya ham bu sistema yechimi bo'ladi .

D) ixtiyoriy C_1 va C_2 o'zgarmas sonlar uchun $y=C_1y_1+C_2y_2$ funksiya ham bu sistema yechimi bo'ladi .

E)keltirilgan barcha tasdiqlar o'rinli bo'ladi .

10. *Ta'rifni to'ldiring:* Differensial tenglamalarning I tartibli bir jinsli chiziqli normal sistemasining $y_1=y_1(x)$ va $y_2=y_2(x)$ yechimlari chiziqli erkli deyiladi, agar $\alpha_1y_1+\alpha_2y_2=0$ tenglik α_1 va α_2 koeffitsientlardan ...... nolga teng bo'lganda bajarilsa .

A) kamida bittasi B) faqat bittasi C) ikkalasi ham

D) birortasi E) to'g'ri javob keltirilmagan

Mushohada yuritmay turib, qancha ko'p o'qisangiz,
shuncha ko'p bilayotganga o'xshayverasiz,
agar o'qiyotganda qanchalik ko'p mushohada yuritsangiz,
shu qadar oz narsa bilishingizni aniqroq his etasiz.
F. Volter

XII-BOB. QATORLAR

§12.1 Sonli qatorlar

1. Agar $u_1+u_2+u_3+...+u_n+...$ qatorning birinchi n ta hadning yig'indisi S_n, n cheksizlikka intilganda $(n\to\infty)$ chekli S limitga intilsa: $\lim\limits_{n\to\infty} S_n = S$, qator yaqinlashuvchi deyiladi. S son yaqinlashuvchi qatorning yig'indisi deyiladi.

Qatorning yaqinlashuvchi bo'lishi uchun n cheksizlikka intilganda $(n\to\infty)$ $u_n$ning nolga intilishi $(u_n\to 0)$ zarurdir (ammo yetarli emas).

2. Hadlari musbat hamda kamayuvchi bo'lgan qatorning yaqinlashishi uchun integral alomat:

Agar $u_n=f(n)$ deb olinsa, bunda $f(n)$ kamayuvchi funksiya va

$$\int_1^{\infty} f(x)dx=\begin{cases} A & bo'lsa, u \quad holda \quad qator \quad yaqinlashdi, \\ \infty & bo'lsa, u \quad holda \quad qator \quad uzoqlashadi \end{cases}$$

1-misol : $\sum\limits_{n=1}^{\infty}\frac{1}{n(n+1)}$ qatorning yig'indisini toping.

Yechimi: $\sum\limits_{n=1}^{\infty}\frac{1}{n(n+1)}=\sum\limits_{n=1}^{\infty}(\frac{1}{n}-\frac{1}{n+1})$

$$S_n=\sum_{i=1}^{n}(\frac{1}{i}-\frac{1}{i+1})=1-\frac{1}{n+1}, \qquad S=\lim_{n\to\infty} S_n=\lim_{n\to\infty}(1-\frac{1}{n+1})=1$$

Qator yig'indisi 1 ga teng, u yaqinlashuvchi ekan.

2-misol : $\sum_{n=1}^{\infty}\frac{1}{(n+1)^{n+1}}$ qatorni tekshiring.

Yechimi: Yordamchi $\sum_{n=1}^{\infty}\frac{2}{2^{n+1}}$ qatorni qaraymiz.

Bu qator geometrik progressiya hadlaridan tuzilgan $(q=1/2)$ qator va u yaqinlashuvchidir.

$$\frac{1}{(n+1)^{n+1}} \le \frac{2}{2^{n+1}}$$

Demak $\sum_{n=1}^{\infty}\frac{1}{(n+1)^{n+1}}$ qator yaqinlashuvchi ekan.

3. Musbat hadli qatorning yaqinlashishi uchun Dalamber alomati:

Agar

$$D=\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=r\begin{cases} <1 & bo'lsa,\ u\ \ holda\ \ qator\ \ yaqinlashadi, \\ >1 & bo'lsa,\ u\ \ holda\ \ qator\ \ uzoqlashadi, \\ =1 & bo'lsa,\ u\ \ holda\ \ masala\ \ yechilmay\ \ qoladi. \end{cases}$$

4. Musbat hadli qatorlarning yaqinlashishi uchun Koshi alomati

$$K=\lim_{n\to\infty}\sqrt[n]{u_n}=\begin{cases} K<1 & qator\ \ yaqinlashuvchi, \\ K>1 & qator\ \ uzoqlashuvchi, \\ K=1 & bu\ \ alomat\ \ bilan\ \ tekshirilmaydi. \end{cases}$$

3-misol : $\sum_{n=1}^{\infty}\left(\frac{n}{4n+3}\right)^n$ qatorni tekshiring.

Yechimi: Koshi alomatiga ko'ra

$$\lim_{n\to\infty}\sqrt[n]{u_n}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{4n+3}\right)^n}=\lim_{n\to\infty}\frac{n}{4n+3}=\frac{1}{4}<1$$

qator yaqinlashuvchi.

5. Musbat hadli qatorlarni taqqoslash

$$u_1+u_2+u_3+...+u_n+... \qquad (12.1)$$

$$v_1+v_2+v_3+...+v_n+... \qquad (12.2)$$

ikkita musbat hadli qator bo'lsin.

1) Agar $u_n \le v_n$ bo'lib, (12.2) qator yaqinlashsa, u holda (12.1) qator ham yaqinlashadi.

2) Agar $u_n \geq v_n$ bo'lib, (12.1) qator uzoqlashsa, u holda (12.2) qator ham uzoqlashadi.

6. Agar ishoralari navbatlashuvchi $u_1 - u_2 + u_3 - u_4 + \ldots$ qatorda $u_1 > u_2 > u_3 > \ldots$ va $\lim\limits_{n\to\infty} u_n = 0$ bo'lsa, qatoryaqinlashuvchibo'ladi.

7. Absolyut yaqinlashish

$$u_1 + u_2 + u_3 + \ldots + u_n + \ldots \qquad (12.3)$$

qator hadlarning absolyut qiymatlaridan tuzilgan

$$|u_1| + |u_2| + |u_3| + \ldots + |u_n| + \ldots \qquad (12.4)$$

qator yaqinlashsa, (12.3) qator ham yaqinlashadi. Bu holda (12.3) qator yaqinlashuvchi bo'lib (12.4) qator uzoqlashuvchi bo'lsa, (12.3) qator absolyut yaqinlashuvchi deyiladi. (12.3) qator shartli (absolyutmas) yaqinlashuvchi deyiladi.

Quyidagi qatorlar uchun yaqinlashishning zaruriy sharti bajariladimi?

12.1. $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\ldots$ 12.2. $\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\ldots$ 12.3. $\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+\ldots$

Qatorlar uchun: 1) qatorni yaqinlashishini isbotlang; 2) qatorning yig'indisi (S) ni toping.

12.4) $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\ldots+\frac{1}{n(n+1)}+\ldots$

12.5) $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\ldots+\frac{1}{(2n-1)(2n+1)}+\ldots$

12.6) $\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\ldots+\frac{1}{(3n-2)(3n+1)}+\ldots$

12.7) $\frac{1}{1\cdot4}+\frac{1}{2\cdot5}+\ldots+\frac{1}{n(n+3)}+\ldots$

12.8) $\frac{1}{1\cdot7}+\frac{1}{3\cdot9}+\ldots+\frac{1}{(2n-1)(2n+5)}+\ldots$

12.9) $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\ldots+\frac{1}{n(n+1)(n+2)}+\ldots$

12.10) $\frac{5}{6}+\frac{13}{36}+\ldots+\frac{3^n+2^n}{6^n}+\ldots$

12.11) $\frac{3}{4}+\frac{5}{36}+\ldots+\frac{2n+1}{n^2(n+1)^2}+\ldots$

12.12) $\frac{1}{9}+\frac{2}{225}+\ldots+\frac{1}{(2n-1)^2(2n+1)^2}+\ldots$

12.13) $arctg\,\frac{1}{2}+arctg\,\frac{1}{8}+\ldots+arctg\,\frac{1}{2n^2}+\ldots$

12.14) $\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\dots$

12.15) $\sum_{n=1}^{\infty}\frac{1}{(n+2)(n+3)}$

12.16). $\sum_{n=1}^{\infty}\frac{6n+1}{(3n-1)^2(3n+2)^2}$

12.17) $\sum_{n=1}^{\infty}\ln\left(\frac{5n-3}{5n+2}\right)$.

Qatorlarning yaqinlashishini solishtirish alomati yordamida yeching

12.18) $\frac{1}{1\cdot 2}+\frac{1}{3\cdot 2^3}+\dots+\frac{1}{(2n-1)\cdot 2^{2n-1}}+\dots$

12.19) $\sin\frac{\pi}{2}+\sin\frac{\pi}{4}+\dots+\sin\frac{\pi}{2^n}+\dots$

12.20) $1+\frac{1+2}{1+2^2}+\dots+\frac{1+n}{1+n^2}+\dots$

12.21) $\frac{1}{2\cdot 5}+\frac{1}{3\cdot 6}+\dots+\frac{1}{(n+1)(n+4)}+\dots$

12.22) $\frac{2}{3}+\frac{3}{8}+\dots+\frac{n+1}{(n+2)n}+\dots$

12.23) $tg\frac{\pi}{4}+tg\frac{\pi}{8}+\dots+tg\frac{\pi}{4n}+\dots$

12.24) $\frac{1}{2}+\frac{1}{5}+\dots+\frac{1}{n^2+1}+\dots$

12.25) $\frac{1}{2}+\frac{1}{5}+\dots+\frac{1}{3n-1}+\dots$

12.26) $\frac{1}{\ln 2}+\frac{1}{\ln 3}+\dots+\frac{1}{\ln(n+1)}+\dots$

12.27) $\sum_{n=1}^{\infty}\frac{1}{n^2-4n+5}$

12.28) $\sum_{n=1}^{\infty}\left(\frac{1+n^2}{1+n^3}\right)^2$

12.29) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+2n}}$

12.30) $\sum_{n=1}^{\infty}\left(\sqrt{n}-\sqrt{n-1}\right)$

12.31) $\sum_{n=1}^{\infty}\sqrt{\frac{1}{n^4+1}}$

12.32. $\sum_{n=1}^{\infty}\frac{2n}{3n^2-5}$

12.33. $\sum_{n=1}^{\infty}\frac{2n+7}{3n^3+11}$

12.34. $\sum_{n=1}^{\infty}\frac{\sqrt{n^2+7}}{n^3+12}$

12.35. $\sum_{n=1}^{\infty}\frac{1}{\ln(n+3)}$

12.36. $\sum_{n=1}^{\infty}\frac{3^n}{n(3^n-4)}$

12.37. $\sum_{n=1}^{\infty}\frac{\ln(n+3)}{n^2}$

12.38. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^3}\ln(n+1)}$

12.39. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^3}\ln(n+1)}$

12.40.

12.41. $\sum_{n=1}^{\infty}\sqrt{n}\left(1-\cos\frac{1}{n^2}\right)$

Qatorlarning yaqinlashishini Dalamber alomati yordamida isbotlang

12.42) $\frac{1}{3!}+\frac{1}{5!}+\dots+\frac{1}{(2n+1)!}+\dots$

12.43) $\frac{1}{2}+\frac{2}{2^2}+\dots+\frac{n}{2^n}+\dots$

12.44) $tg\frac{\pi}{4}+2tg\frac{\pi}{8}+\dots+ntg\frac{\pi}{2^{n+1}}+\dots$

12.45) $\frac{2}{1}+\frac{2\cdot 5}{1\cdot 5}+\dots+\frac{2\cdot 5\cdot\ldots\cdot(3n-1)}{1\cdot 5\cdot\ldots\cdot(4n-3)}+\dots$

12.46) $\frac{1}{3}+\frac{4}{9}+\dots+\frac{n^2}{3^n}+\dots$

12.47) $\frac{1}{3}+\frac{1\cdot 3}{3\cdot 6}+\dots+\frac{1\cdot 3\cdot\ldots\cdot(2n-1)}{3^n\cdot n!}+\dots$

12.48) $\sin\frac{\pi}{2} + 4\sin\frac{\pi}{4} + \ldots + n^2\sin\frac{\pi}{2^n} + \ldots$

12.49) $\frac{1}{2!} + \frac{2}{3!} + \ldots + \frac{n}{(n+1)!} + \ldots$

12.50) $\frac{2}{2} + \frac{2\cdot 3}{2\cdot 4} + \ldots + \frac{(n+1)!}{2^n\cdot n!} + \ldots$

Qatorlarning yaqinlashishini Koshi alomati yordamida isbotlang

12.51) $\frac{1}{\ln 2} + \frac{1}{\ln^2 3} + \ldots + \frac{1}{\ln^n(n+1)} + \ldots$

12.52) $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \ldots + \left(\frac{n}{2n+1}\right)^n + \ldots$

12.53) $\arcsin 1 + \arcsin^2\frac{1}{2} + \ldots + \arcsin^n\frac{1}{n} + \ldots$

12.54) $\frac{2}{3} + \frac{\left(\frac{3}{2}\right)^4}{9} + \ldots + \frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n} + \ldots$

12.55) $\frac{1}{2\ln^2 2} + \frac{1}{3\ln^2 3} + \ldots + \frac{1}{(n+1)\ln^2(n+1)} + \ldots$

12.56) $\frac{1}{2\ln 2} + \frac{1}{3\ln 3} + \ldots + \frac{1}{n\ln n} + \ldots$

12.57) $\left(\frac{1+1}{1+1^2}\right)^2 + \left(\frac{1+2}{1+2^2}\right)^2 + \ldots + \left(\frac{1+n}{1+n^2}\right)^2 + \ldots$

12.58) $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}}\ln\frac{n+1}{n-1}$

Dalamber alomati asosan quyidagi qatorlarning yaqinlashishi tekshirilsin:

12.59. $\sum_{n=1}^{\infty}\frac{2^n}{3^n+7}$

12.60. $\sum_{n=1}^{\infty}\frac{n\cdot 2^n}{5^n+12}$

12.61. $\sum_{n=1}^{\infty}\frac{n!}{3^n(n+1)}$

12.62. $\sum_{n=1}^{\infty}\frac{n^2+1}{n^3-5}$

12.63. $\sum_{n=1}^{\infty}\frac{2^n}{2^n+n}$

12.64. $\sum_{n=1}^{\infty}\frac{n!\,3^n}{n^n}$

12.65. $\sum_{n=1}^{\infty}\frac{n!}{5^n+n^2}$

12.66. $\sum_{n=1}^{\infty}\frac{(n+1)!}{n^n+2^n}$

12.67. $\sum_{n=1}^{\infty}\left(\frac{2n+1}{5n+4}\right)^n$

12.68. $\sum_{n=1}^{\infty}\left(\frac{3n+2}{3n+1}\right)^{-2n^2}$

12.69. $\sum_{n=1}^{\infty}\frac{\ln(n+2)}{3^n}$

12.70. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{n^2+5}\right)^n$

12.71. $\sum_{n=1}^{\infty}\sin\frac{\pi\cdot 3^n}{4^n+e^n}$

12.72. $\sum_{n=1}^{\infty}n\left(1-\cos\frac{1}{3^n}\right)$

12.73. $\frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \ldots$

12.74. $1 + \frac{2}{2!} + \frac{4}{3!} + \frac{8}{4!} + \ldots$

12.75. $1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \ldots$

12.76. $1 + \frac{3}{2\cdot 3} + \frac{3^2}{2^2\cdot 5} + \frac{3^3}{2^3\cdot 7} + \ldots$

12.77. $\frac{1}{2}+\frac{3!}{2\cdot 4}+\frac{5!}{2\cdot 4\cdot 6}+\frac{7!}{2\cdot 4\cdot 6\cdot 8}+\dots$

2.78. $\frac{1}{\sqrt{3}}+\frac{5}{\sqrt{2\cdot 3^3}}+\frac{9}{\sqrt{3\cdot 3^3}}+\frac{13}{\sqrt{4\cdot 3^3}}+\dots$

Integral alomati bilan quyidagi qatorlarning yaqinlashishi tekshirilsin

12.79. $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\dots$

12.80. $1+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{7}}+\frac{1}{\sqrt{10}}+\dots$

12.81. $\frac{1}{2^3}+\frac{2}{3^3}+\frac{3}{4^3}+\dots$

12.82. $\frac{1}{1+1^2}+\frac{1}{1+2^2}+\frac{1}{1+3^2}+\dots$

12.83. $\frac{1}{1+1^2}+\frac{2}{1+2^2}+\frac{3}{1+3^2}+\dots$

12.84. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\dots$

12.85. $\frac{1}{2\ln^2 2}+\frac{1}{3\ln^2 3}+\frac{1}{4\ln^2 4}+\dots$

Qatorlarning yaqinlashuvchi yoki uzoqlahuvchi ekanligini aniqlang.

12.86) $\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{(n+1)\sqrt{n+1}}+\dots$

12.87) $1+\frac{2}{3}+\dots+\frac{n}{2n-1}+\dots$

12.88) $\sqrt{2}+\sqrt{\frac{3}{2}}+\dots+\sqrt{\frac{n+1}{n}}+\dots$

12.89) $1+\frac{4}{1\cdot 2}+\dots+\frac{n^2}{n!}+\dots$

12.90) $2+\frac{5}{8}+\dots+\frac{n^2+1}{n^3}+\dots$

12.91) $\frac{1}{1001}+\frac{2}{2001}+\dots+\frac{n}{1000n+1}+\dots$

12.92) $\frac{1}{1+1^2}+\frac{2}{1+2^2}+\dots+\frac{1}{1+n^2}+\dots$

12.93) $\frac{1}{3}+\frac{3}{3^2}+\dots+\frac{2n-1}{3^n}+\dots$

12.94) $arctg\,1+arctg^2\frac{1}{2}+\dots+arctg^n\frac{1}{n}+\dots$

Quyida keltirilgan qatorlarni qaysi biri absolyut yaqinlashuvchi, yaqinlashuvchi va uzoqlashuvchi

12.95. $1-\frac{1}{3}+\dots+(-1)^{n+1}\frac{1}{2n-1}+\dots$

12.96. $1-\frac{1}{3^3}+\dots+(-1)^{n+1}\frac{1}{(2n-1)^3}+\dots$

12.97. $\frac{1}{2}+\dots+(-1)^{n+1}\frac{1}{(2n-1)^2}+\dots$

12.98. $\frac{\sin\alpha}{1}+\frac{\sin 2\alpha}{4}+\dots+\frac{\sin n\alpha}{n^2}+\dots$

12.99. $\frac{1}{2}-\frac{1}{2}\cdot\frac{1}{2^2}+\dots+(-1)^{n+1}\frac{1}{n}\cdot\frac{1}{2^n}+\dots$

12.100. $2-\frac{3}{2}+\dots+(-1)^{n+1}\frac{n+1}{n}+\dots$

12.101. $-1+\frac{1}{\sqrt{2}}-\dots+(-1)^n\frac{1}{\sqrt{n}}+\dots$

12.102. $\frac{1}{2}-\frac{8}{4}+\dots+(-1)^{n+1}\frac{n^3}{2^n}+\dots$

12.103. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n-\ln n}$

12.104. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^{n^2}}{n!}$

Garmonik qator kamayuvchi progressiya bilan taqqoslab, quyidagilarning yaqinlashishi tekshirilsin

12.105. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\ldots$

12.106. $1+\frac{1}{2\cdot 5}+\frac{1}{3\cdot 5^2}+\frac{1}{4\cdot 5^3}+\ldots$

12.107. $\frac{1}{\ln 2}+\frac{1}{\ln 3}+\frac{1}{\ln 4}+\frac{1}{\ln 5}+\ldots$

12.108. Qatorlarni taqqoslash usuli bilan $\frac{1}{1+x^2}+\frac{1}{1+x^4}+\frac{1}{1+x^6}+\ldots$ qatorning $|x|\le 1$ bo'lganda uzoqlashishi, $|x|>1$ bo'lganda esa yaqinlashishi ko'rsatilsin.

12.109. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots$ qatorning yig'indisi topilsin.

12.110. $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+\ldots$ qatorning yig'indisi topilsin.

Quyidagi qatorlarning yaqinlashishi tekshirilsin:

12.112. $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\ldots$

12.112. $1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+\ldots$

12.113. $\frac{1}{2\ln 2}-\frac{1}{3\ln 3}+\frac{1}{4\ln 4}-\ldots$

12.114. $\frac{\sin\alpha}{1}+\frac{\sin 2\alpha}{2^2}+\frac{\sin 3\alpha}{3^2}+\ldots$

12.115. $1+\frac{1}{3\sqrt{3}}+\frac{1}{5\sqrt{5}}+\ldots$

12.116. $1+\frac{1}{101}+\frac{1}{201}+\frac{1}{301}+\ldots$

12.117. $\frac{1}{1+1^4}+\frac{2}{1+2^4}+\frac{3}{1+3^4}+\ldots$

12.118. $1+\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\ldots$

12.119. $1+\frac{1}{4^2}+\frac{1}{7^2}+\frac{1}{10^2}+\ldots$

12.120. $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}+\ldots$

12.121. $\frac{21}{3}+\frac{41}{9}+\frac{61}{27}+\ldots$

12.122. $\frac{2}{1}+\frac{4}{3!}+\frac{6}{5!}+\ldots$

12.123. $1-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\ldots$

12.124. $1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+\ldots$

12.125. $1-\frac{1}{2a^2}+\frac{1}{3a^4}-\frac{4}{4a^6}+\ldots$

12.126. $\sum_{n=1}^{\infty}\frac{1}{(2n+3)\ln^2(n+1)}$

12.127. $arctg1+4arctg^2\frac{1}{\sqrt{2}}+9arctg^3\frac{1}{\sqrt{3}}+\ldots$

12.127.b) Shartli yaqinlashuvchi $1-\frac{1}{\sqrt[3]{2}}+\frac{1}{\sqrt[3]{3}}-\frac{1}{\sqrt[3]{4}}+\ldots+\frac{(-1)^n}{\sqrt[3]{n}}+\ldots$ qatorni kvadratga oshirilsa yaqinlashuvchi bo'ladimi?

12.128.

$$1-2\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+\frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!}+\ldots\right)^2=1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\ldots+\frac{(-1)^n(2x)^{2n-2}}{(2n-2)!}+\ldots$$

tenglik o'rinli ekanligini isbotlang.

12.129.

$$\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots+\frac{(-1)^n x^{2n-2}}{(2n-2)!}+\ldots\right)^2+\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+\frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!}+\ldots\right)^2=1$$

tenglik o'rinli ekanligini isbotlang.

12.130. Faraz qilaylik, $y=f(x)$ funksiya $[1;\infty)$ musbat va monoton kamayuvchi bo'lsin. U holda $\lim\limits_{n\to\infty}\left[f(1)+\ldots+f(n)-\int\limits_1^n f(x)dx\right]=A$ limit mavjud va $f(1)+\ldots+f(n)=\int\limits_1^n f(x)d+A+\varepsilon_n$ ($\lim\limits_{n\to\infty}\varepsilon_n=0$) ekanligini isbotlang.

12.131. $1+\frac{1}{2}+\ldots+\frac{1}{n}=\ln n+C+\varepsilon_n,\quad \varepsilon_n\to 0$ ekanligini isbotlang. Bu tenglikdan quyidagini keltirib chiqaring. $1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}=\ln\sqrt{4n}+\frac{1}{2}C+\varepsilon_k,\quad \varepsilon_k\to 0$

12.132. $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$ munosabat va (12.130) masalani shartidan foyadalanib quyidagini isbotlang. $1+\frac{1}{5}+\frac{1}{9}+\ldots+\frac{1}{4n-3}=\ln\sqrt[4]{8k}+\frac{\pi}{8}+\frac{C}{4}+\varepsilon_k,\quad \varepsilon_k\to 0$

§12.2. Funksional qatorning tekis yaqinlashishi. Darajali qatorlar.

1. x ning

$$u_1(x)+u_2(x)+\ldots+u_n(x)+\ldots \qquad (12.5)$$

funksional qator yaqinlashadigan qiymatlarining to'plami bu qatorning yaqinlashish sohasi deyiladi. $S(x)=\lim\limits_{n\to\infty}S_n(x)$ funksiya qatorning yig'indisi, $R_n(x)=S(x)-S_n(x)$ ayirma esa qatorning qoldig'i deyiladi.

2. Agar har qanday $\varepsilon>0$ uchun shunday N nomer ko'rsatish mumkin bo'lsaki, $n>N$ bo'lganda $[a,\ b]$ segmentdan olingan istalgan x uchun $|R_n(x)|<\varepsilon$ tengsizlik bajarilsa, (12.5) qator $[a,\ b]$ segmentda tekis yaqinlashuvchi deyiladi.

3. Tekis yaqinlashishning alomati

Agar hadlari musbat va yaqinlashuvchi $c_1 + c_2 + c_3 + ... + c_n + ...$ sonlar qatori mavjud bo'lib, x ning $[a, b]$ dagi barcha qiymatlari uchun $|u_n(x)| \le c_n$ bo'lsa, (12.5) qator $[a, b]$ segmentda absolut va tekis yaqinlashadi. Quyidagi qatorlarning xususiy yig'indisi S_n ni toping.

$$a_0 + a_1x + a_2x^2 + ... + a_nx^n + ... \qquad (12.6)$$

darajali qator berilgan bo'lsin. Agar $|x| < R$ bo'lganda qator yaqinlashuvchi va $|x| > R$ bo'lganda qator uzoqlashuvchi bo'lsa, R son (12.5) qatorning yaqinlashish radiusi deyiladi. R ni, (12.5) qatorning absolyut yaqinlashishini Dalamber alomatiga asosan tekshirib yoki, barcha a_i lar noldan farqli bo'lgan holda, $R = \lim\limits_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$ formula bo'yicha topish mumkin. Jumladan, agar limit ∞ ga teng bo'lsa, (12.5) qator butun Ox o'qda absolyut yaqinlashadi.

Darajali qator o'zining yaqinlashish intervali ($-R$, R) ichiga yotuvchi har qanday [a, b] segmentda absolyutgina emas, balki tekis ham yaqinlashadi.

4-misol: $\sum\limits_{n=0}^{\infty} x^n = 1 + x + x^2 + ... + x^n + ...$

funksional qatorning aniqlanish sohasi D va hadlar yig'indisi S(x) funksiyani toping.

Yechimi: Aniqlanish sohasi D (-1;1) oraliqdagi qiymatlardan iborat, chunki bu qiymatlar uchun berilgan funksional qator cheksiz kamayuvchi geometrik progressiyaga teng. Uning birinchi hadi v va maxraji $q = x$ *ga teng*. Demak,

$$S(x) = \frac{1}{1-q} = \frac{1}{1-x}, \qquad x \in (-1,1)$$

bo'ladi.

5-misol: $\sum\limits_{n=1}^{\infty} \frac{\cos nx}{n^2}$ va $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$ yaqinlashuvchi qator uchun $x \in (-\infty;\infty)$ da $\left|\frac{\cos nx}{n^2}\right| \le \frac{1}{n^2}, n = 1,2,...$ bajariladi.

Demak $\sum\limits_{n=1}^{\infty} \frac{\cos nx}{n^2}$ катор $(-\infty;\infty)$ da kuchaytirilgan qatordir.

6-misol: $\sum_{n=1}^{\infty}\frac{1}{n!}x^n = x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$

qator ning yaqinlashish radiusini toping.

Yechimi

$$R = \lim_{n\to\infty}\left(\frac{1}{n!} : \frac{1}{(n+1)!}\right) = \lim_{n\to\infty}\frac{n+1}{1} = \infty$$

Demak yaqinlashish intervali $(-\infty;\infty)$ bo'ladi.

Funksional qatorlarni yaqinlashish sohasini toping.

12.133. $1+x+...+x^n+...$

12.134. $\ln x+\ln^2 x+...+\ln^n x+...$

12.135. $x+x^4+...+x^{n^2}+...$

12.136. $x+\frac{x^2}{2^2}+...+\frac{x^n}{n^2}+...$

12.137. $x+\frac{x^2}{\sqrt{2}}+...+\frac{x^n}{\sqrt{n}}+...$

12.138. $\frac{1}{1+x}+\frac{1}{1+x^2}+...+\frac{1}{1+x^n}+...$

12.139. $2x+6x^2+...+n(n+1)x^n+...$

12.140. $\frac{x}{2}+\frac{x^2}{2+\sqrt{2}}+...+\frac{x^n}{n+\sqrt{n}}+...$

12.141. $\frac{x}{1+x^2}+\frac{x^2}{1+x^4}+...+\frac{x^n}{1+x^{2n}}+...$

12.142. $\sin\frac{x}{2}+\sin\frac{x}{4}+...+n^2\sin\frac{x}{2^n}+...$

12.143. $x\,tg\frac{x}{2}+x^2tg\frac{x}{4}+...+x^n tg\frac{\pi}{2^n}+...$

12.144. $\sin x+\frac{\sin 2\alpha}{2^2}+...+\frac{\sin nx}{n^2}+...$

12.145. $\frac{\cos x}{e^x}+\frac{\cos 2x}{e^{2x}}+...+\frac{\cos nx}{e^{nx}}+...$

12.146. $e^{-x}+e^{-4x}+...+e^{n^2x}+...$

12.147. $\frac{x}{e^x}+\frac{2^x}{e^{2x}}+...+\frac{nx}{e^{nx}}+...$

12.148. $1+\frac{\sin x}{1!}+...+\frac{\sin nx}{n!}+...$

12.149. $\sum_{n=1}^{\infty}\frac{1}{n^2\left[1+(nx)^2\right]}$

12.150. $\sum_{n=1}^{\infty}\frac{\sin nx}{2^n}$

12.151. $\sum_{n=1}^{\infty}\frac{n}{3^n(n+1)}x^n$

Quyidagi qatorlarning yaqinlashish intervali aniqlansin va qatorlar intervalning chegaralarida ham yaqinlashishi tekshirilsin:

12.152. $1+\frac{x}{3\cdot 2}+\frac{x^2}{3^2\cdot 3}-\frac{x^3}{3^3\cdot 4}+...$

12.153. $1-\frac{x^2}{5\sqrt{2}}+\frac{x^3}{5^2\sqrt{3}}-\frac{x^5}{5^3\sqrt{4}}+...$

12.154. $1+\frac{2x}{3^2\sqrt{2}}+\frac{4x^2}{5^2\sqrt{3^2}}+\frac{8x^3}{7^2\sqrt{3^3}}+...$

12.155. $\sum_{n=1}^{\infty}\frac{x^n}{n!}$

12.156. $\sum_{n=1}^{\infty}\frac{(-x)^{n-1}}{n}$

12.157. $\sum_{n=1}^{\infty}\frac{3^n x^n}{\sqrt{(3n-3)2^n}}$

12.158. 1) $\sum_{n=1}^{\infty} x^{n-1}\cdot n!$; 2) $\sum_{n=1}^{\infty}\frac{n!x^n}{(n+1)^n}$

12.159. $(x+1)+\frac{(x+1)^2}{2\cdot 4}+\frac{(x+1)^3}{3\cdot 4^2}+\frac{(x+1)^4}{4\cdot 4^3}+\dots$

12.160. $\frac{2x-3}{1}-\frac{(2x-3)^2}{3}+\frac{(2x-3)^3}{5}-\dots$

Quyidagi qatorlarning yaqinlashish intervallari aniqlansin va yig'indilari topilsin:

12.161. $1+2x+3x^2+4x^3+\dots$

12.162. $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\dots$

12.163. $1+3x+5x^2+7x^3+\dots$

12.164. $1+\frac{m}{1}x+\frac{m(m-1)}{1\cdot 2}x^2+\frac{m(m-1)(m-2)}{1\cdot 2\cdot 3}x^3+\dots$

Quyidagi qatorlarning yaqinlashish intervali aniqlansin va qatorlarning intervalning chegaralarida ham yaqinlashishlari tekshirilsin:

12.165. $1+\frac{2x}{\sqrt{5\cdot 5}}+\frac{4x^2}{\sqrt{9\cdot 5^2}}+\frac{8x^3}{\sqrt{13\cdot 5^3}}+\dots$

12.166. $1-\frac{x^2}{3\cdot 2\sqrt{2}}+\frac{x^4}{3^2\cdot 3\sqrt{3}}-\frac{x^6}{3^3\cdot 4\sqrt{4}}+\dots$

12.167. $\sum_{n=1}^{\infty}\frac{10^n x^n}{\sqrt{n}}$

12.168. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$

12.169. $\frac{x-1}{1\cdot 2}+\frac{(x-1)^2}{3\cdot 2^2}+\frac{(x-3)^3}{5\cdot 2^3}+\dots$

12.170. $\frac{2x+1}{1}+\frac{(2x+1)^2}{4}+\frac{(2x+1)^3}{7}+\dots$

Quyidagi qatorlarning yaqinlashish intervallari aniqlansin va ularning yig'indilari topilsin:

12.171. $1-3x^2+5x^4-7x^6+\dots$

12.172. $x+\frac{x^2}{2}+\frac{x^3}{3}+\dots$

12.173. $1-4x+7x^2-10x^3+\dots$

Quyidagi qatorlarning yaqinlashish intervallari aniqlansin:

12.174. $\frac{x}{1\cdot 2}+\frac{x^2}{2\cdot 3}+\frac{x^3}{3\cdot 4}+\frac{x^4}{4\cdot 5}+...$

12.175. $\sum_{n=1}^{\infty}\frac{x^n}{2^{n-1}}$

12.176. $\sum_{n=1}^{\infty}\frac{x^n}{n}$

12.177. $\sum_{n=1}^{\infty}n!x^n$

12.178. $\frac{x^3}{8}+\frac{x^6}{8^2\cdot 5}+\frac{x^9}{8^3\cdot 9}+\frac{x^{12}}{8^4\cdot 13}+...$

12.179. $\sum_{n=1}^{\infty}10^n x^n$

Funksional qatorlarni yaqinlashish sohasini toping

12.180. $\frac{x}{2}+\frac{x^2}{2+\sqrt{2}}+...+\frac{x^n}{n+\sqrt{n}}+...$

12.181. $\frac{x}{1+x^2}+\frac{x^2}{1+x^4}+...+\frac{x^n}{1+x^{2n}}+...$

12.182. $\sin\frac{x}{2}+\sin\frac{x}{4}+...+n^2\sin\frac{x}{2^n}+...$

12.183. $x tg\frac{x}{2}+x^2 tg\frac{x}{4}+...+x^n tg\frac{\pi}{2^n}+...$

12.184. $\sin x+\frac{\sin 2\alpha}{2^2}+...+\frac{\sin nx}{n^2}+...$

12.185. $\frac{\cos x}{e^x}+\frac{\cos 2x}{e^{2x}}+...+\frac{\cos nx}{e^{nx}}+...$

12.186. $e^{-x}+e^{-4x}+...+e^{n^2 x}+...$

12.187. $\frac{x}{e^x}+\frac{2^x}{e^{2x}}+...+\frac{nx}{e^{nx}}+...$

12.188. $1+\frac{\sin x}{1!}+...+\frac{\sin nx}{n!}+...$

12.189. $\sum_{n=1}^{\infty}\frac{1}{n^2\left[1+(nx)^2\right]}$

12.190. $\sum_{n=1}^{\infty}\frac{\sin nx}{2^n}$

12.191. $\sum_{n=1}^{\infty}\frac{e^{-n^2}x^2}{n^2}$

12.192. $\frac{x}{2+3}+\frac{x^2}{2^2+3^2}+\frac{x^3}{2^3+3^3}+...$

12.193. $\sum_{n=1}^{\infty}x^n tg\frac{1}{n}$

12.194. $5x+\frac{5^2x^2}{2!}+\frac{5^3x^3}{3!}+\frac{5^4x^4}{4!}+...$

12.195. $\sum_{n=1}^{\infty}(1-)^{n+1}\frac{x^n}{n}$

12.196. $\frac{x}{1\cdot 3}+\frac{x^2}{2\cdot 4}+...+\frac{x^n}{n(n+2)}+...$

12.197. $\sum_{n=1}^{\infty}\frac{x^n}{3^n(n+1)}$

12.198. $\sum_{n=1}^{\infty}\frac{(1-)^n x^{2n-1}}{5^n\sqrt{n+1}}$

12.199. $\sum_{n=1}^{\infty}\frac{x^{n^2}}{2^n}$

12.200. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}x^n$

12.201. $\sum_{n=1}^{\infty}(1-)^{n+1}\frac{x^{2n-1}}{3^{n-1}n\sqrt{n}}$

12.202. $\sum_{n=1}^{\infty}\frac{(-1)^n n!}{3^{n^2}}(x-1)^n$

12.203. $\sum_{n=1}^{\infty}\frac{(3n-2)(x-5)^n}{(n+1)^2 2^{n+1}}$

12.204. $\sum_{n=1}^{\infty}\sin^2\frac{1}{n}(x-1)^n$

12.205. $\sum_{n=1}^{\infty}\frac{3^{-\sqrt{n}}x^n}{\sqrt{n^2+1}}$

§12.3. Teylor va Makloren qatorlari

1. $f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+R_n(x)$ (12.7)

ko'rinishdagi formula Makloren formulasi deyiladi, bunda

$$R_n(x)=\frac{x^n}{n!}f^{(n)}(\theta x);\quad 0\le\theta<1.$$

2. $f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+...+R_n(x)$ (12.8)

ko'rinishdagi formula Teylor formulasi deyiladi, bunda

$$R_n(x)=\frac{(x-a)^n}{n!}f^{(n)}[a+\theta(x-a)].$$

3. Teylor va Makloren qatorlari. (12.7) va (12.8) formulalarda n cheksizlikka intilganda $(n\to\infty)$ R_n nolga intilsa $(R_n(x)\to 0)$, u holda bu formulalardan x ning $\lim_{n\to\infty}R_n(x)=0$ bo'lgandagi qiymatlari uchun $f(x)$ ga yaqinlashuvchi quyidagi

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+... \quad (12.9)$$

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+... \quad (12.10)$$

cheksiz qatorlar hosil bo'ladi.

4. Elementarfunksiyalarning qatorlarga yoyilmalari:

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...,$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-....$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-....$$

bu qator x ning har qanday qiymatlari uchun mos (ko'rsatilgan) funksiyalarga yaqinlashadi.

$(1+x)^m = 1 + \frac{m}{1}x + \frac{m(m-1)}{1\cdot 2}x^2 + \ldots$ - binominal qator bo'lib, $|x|<1$ bo'lganda $(1+x)^m$ binomga yaqinlashadi.

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots$ qator $-1 < x \le 1$ bo'lganda $\ln(1+x)$ ga yaqinlashadi.

$arctgx = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$ qator $|x| \le 1$ bo'lganda $arctgx$ ga yaqinlashadi.

7-misol: $f(x)=\sin x$ funksiyaning Makloren qatorini yozing

Yechimi: Berilgan funksiyaning n ta hosilasini topib, $x=0$ dagi qiymatini hisoblaymiz

(12.8) –tenglikdan quyidagi darajali qator kelib chiqadi.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots(-1)^{n+1}\frac{x^{2n-1}}{(2n-1)!} + \ldots$$

8-misol: $y=\cos x$, $y=e^x$, $y=e^{-x}$, $y=shx$, $y=chx$ funksiyalarning Makloren qatorini yozing.

Yechimi: Oldingi misolda ko'rsatilgan usulga ko'ra

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots(-1)^{n-1}\frac{x^{2n-2}}{(2n-2)!} + \ldots.$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots,$$

$$shx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots,$$

$$chx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$$

12.206. Quyidagi funksiyalar x ning darajalari bo'yicha qatorga yoyilsin va qoldiq hadning forulasi yozilsin va u tekshirilsin: 1) $\cos(x-a)$; 2) $\sin^2 x$; 3) ; 4) $\sin\left(mx + \frac{\pi}{3}\right)$.

12.207. $f(x) = \ln(1+e^{kx})$ funksiyaning qatorga yoyilmasidagi birinchi hadi yozilsin.

12.208. $\left(1+\frac{x}{a}\right)^m$ binom Makloren formulasiga asosan x dagi darajalari bo'yicha qatorga yoyilsin va hosil bo'lgan qator $|x|<a$ bo'lganda yaqinlashuvchi ekanligi aniqlansin.

12.209.Binomial qatorga asosan $|x|<1$ bo'lganda $\frac{1}{(1+x)^3}=1-3x+6x^2-10x^3+...=\sum_{n=1}^{\infty}\frac{n(n+1)}{2}(-x)^{n-1}$ ekanligi ko'rsatilsin.

12.210. Binomial qatorga asosan $|x|<1$ bo'lganda $\frac{1}{\sqrt{1+x^2}}=1-\frac{1}{2}x^2+\frac{1\cdot3}{2^2\cdot2!}x^4-\frac{1\cdot3\cdot5}{2^3\cdot3!}x^6+...$ yoyilmasi hosil qilinsin.

12.212. Quyidagi funksiyalarni x ning darajalari bo'yicha qatorga yoyilsin:

1) $\ln\frac{1+x}{1-x}$; 2) $\ln(2-3x+x^2)$; 3) $\ln(1-x+x^2)$.

12.212. $f(x)=e^{\frac{x}{a}}$ funksiyani $x-a$ ning darajalari bo'yicha qatorga yoyilsin, qoldiq hadning formulasi yozilsin va tekshirilsin.

12.213. $f(x)=x^3-3x$ funksiya $x+1$ ning darajalari bo'yicha qatorga yoyilsin.

12.214. $f(x)=x^4$ funksiya $x+1$ ning darajalari bo'yicha qatorga yoyilsin.

12.215. $f(x)=\frac{1}{x}$ funksiyani $x+2$ ning darajalari bo'yicha qatorga yoyib, hosil bo'lgan qatorning yaqinlashishi Dalamber alomatiga asosan tekshirilsin.

12.216. 1) $f(x)=\cos\frac{x}{2}$ funksiya $x-\frac{\pi}{2}$ ning darajalari bo'yicha; 2) $f(x)=\sin 3x$ funksiya $x+\frac{\pi}{3}$ ning darajalari bo'yicha qatorga yoyilsin.

12.217. $f(x)=\sqrt[3]{x}$ funksiyani $x+1$ ning darajalari bo'yicha qatorga yoyib, hosil bo'lgan qatorning yaqinlashishi Dalamber alomatiga asosan tekshirilsin.

Funksiyaning qatorga yoyilmasidagi birinchi hadi yozilsin:

12.218. $y=e^{-2x}$ 12.219. $y=\sin\frac{x}{2}$

12.220. $y=x^3\cos x$ 12.221. $y=\ln(1+5x)$.

12.222. $y=\ln(5+2x)$. 12.223. $y=\sqrt{1+x^2}$.

12.224. $y=\frac{1}{1+x^4}$. 12.225. $y=\frac{3}{4-x}$.

12.226. $y=x^2e^{-2x}$. 12.227. $y=x\,arctgx$.

12.228. $y=(1+x)\ln(1+x)$. 12.229. $y=\frac{x+\ln(1-x)}{x^2}$.

Teylor qatorining x=0 nuqtadagi 5 ta hadini toping.

12.230. $y=\ln(1+e^x)$ 12.231. $y=e^{\cos x}$

12.232. $y=\cos^8 x$ 12.233. $y=-\ln\cos x$

12.234. $y=(1+x)^x$

Limitlarni Teylor qatoriga yoyish yordamida hisoblang.

12.235. $\lim\limits_{x\to 0}\frac{x+\ln\left(\sqrt{1+x^2}-x\right)}{x^3}$ 12.236. $\lim\limits_{x\to 0}\frac{2(tgx-\sin x)-x^3}{x^5}$

12.237. $\lim\limits_{x\to 0}\frac{\ln(1+x+x^2)+\ln(1-x+x^2)}{x(e^x-1)}$ 12.238. $\lim\limits_{x\to\infty}\left[x-x^2\ln\left(1+\frac{1}{x}\right)\right]$

12.239. $\lim\limits_{x\to 0}\left(\frac{1}{x^2}-ctg^2x\right)$

§12.4 Fur\`e qatori. Fur\`e inegrali

1. Ta'rif: Agar $[a, b]$ segmentda $f(x)$ funksiya

1) soni chekli uzilishlarga ega bo'lib, ularning hammasi 1 – tur uzilishlar bo'lsa;

2) sonli chekli ekstremumlarga ega bo'lsa;

3) (a, b) oraliqning har bir nuqtasida $f(x)=\frac{f(x-0)+f(x+0)}{2}$ bo'lsa, funksiya shu segmentda Dirixle shartlariga bo'ysunadi deyiladi.

2. $[-l, l]$ segmentda Dirixle shartlariga bo'ysunuvchi $f(x)$ funksiya kesmaning har bir nuqtasida quyidagi Fur'e qatori bilan aniqlanishi mukin:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{l}+b_n\sin\frac{n\pi x}{l}\right] \qquad (12.11)$$

bunda

$$a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}dx;\quad b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi x}{l}dx \qquad (12.12)$$

Agar $f(x)=f(-x)$, ya'ni $f(x)$ - juft funksiya bo'lsa, u holda $b_n=0$ va

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{l} \qquad (12.13)$$

Agar $f(x)=f(x)$, ya'ni $f(x)$ - toq funksiya bo'lsa, u holda $a_n=0$ va

$$f(x)=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{l} \qquad (12.14)$$

Agar [-*l*, *l*] segmentda (12.11) qator bilan aniqlangan $f(x)$ funksiyani $f(l)=\frac{f(l-0)+f(l+0)}{2}$ shartning bajarilishini talab etib, uni $2l$ ga teng davr bilan davom etirsak, funksiya o'zining butun davomida ham (12.11) qator bilan aniqlanadi.

3. $f(x)$ funksiya $(-\infty,\infty)$ oraliqda absolyut integrallanuvchi (ya'ni $\int_{-\infty}^{\infty}|f(x)|dx$ yaqinlashadi) bo'lsa va har qanday chekli segmentda Dirixle shartlariga bo'ysunsa, u holda bu funksiya quyidagi Fur'e integrli bilan ifodaladi:

$$f(x)=\frac{1}{\pi}\int_0^{\infty}d\alpha\int_{-\infty}^{\infty}f(t)\cos\alpha(x-t)dt=\int_0^{\infty}[a(\alpha)\cos\alpha x+b(\alpha)\sin\alpha x]d\alpha \qquad (12.15)$$

bunda

$$a(\alpha)=\frac{1}{\pi}\int_{-\infty}^{\infty}f(t)\cos\alpha t dt \text{ va } b(\alpha)=\frac{1}{\pi}\int_{-\infty}^{\infty}f(t)\sin\alpha t dt \qquad (12.16)$$

Davri 2π bo'lgan quyidagi funksiyalar Fur'e qatorlariga yoyilsin:

12.240. $0<x<\pi$ bo'lganda $f(x)=1$ va $f(-x)=-f(x)$. Hosil bo'lgan qator yordami bilan $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...=\frac{\pi}{4}$ ekanligi ko'rsatilsin.

12.241. $0\le x\le\pi$ bo'lganda $f(x)=x$ va $f(-x)=f(x)$. Hosil bo'lgan qator yordami bilan $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+...=\frac{\pi^2}{8}$ ekanligi ko'rsatilsin.

12.242. $-\pi\le x\le\pi$ bo'lganda $f(x)=x^2$. Hosil bo'lgan qator yordami bilan

1) $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...=\frac{\pi^2}{12}$ 2) $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+...=\frac{\pi^2}{6}$

ekanligi ko'rsatilsin.

12.243. $f(x)=\begin{cases}\pi, -\pi < x < 0 & bo'lganda, \\ \pi - x, 0 \le x \le \pi & bo'lganda.\end{cases}$

12.244. $f(x)=1$; $0<x<l$ bo'lganda, $f(-x)=-f(x)$.

12.245. $f(x)=1-x$; $0 \le x \le 1$ bo'lganda, $f(-x)=f(x)$, $l=1$.

12.246. $f(x)=\begin{cases}0, -l < x \le 0 & bo'lganda, \\ x, 0 \le x \le l & bo'lganda.\end{cases}$

12.247. Uzunligi l ga teng sterjenda issiqlik tarqalishi $\frac{1}{a^2}\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ tenglama bilan, bunda $u(x,t)$ - temperatura va quyidagi shartlar bilan aniqlanadi:

1) chegaraviy shartlar: $x=0$ va $x=l$ bo'lganda $u=0$;

2) boshlang'ich shartlar: $t=0$ bo'lganda

$$u=\begin{cases} x, & x<\frac{l}{2} \quad bo'lganda, \\ l-x, & x>\frac{l}{2} \quad bo'lganda.\end{cases}$$

Fur'e metodiga asosan $u(x,t)$ funksiya aniqlansin.

12.248. Uzunligi l, bir uchi $(x=0)$ biriktirilgan ikkinchi $(x=l)$ uchi esa erkin bo'lgan sterjenning bo'ylama tebranishlari $\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$ tenglama bilan, bunda $u(x,t)$ - bo'ylama siljishi va quyidagi shartlar bilan aniqlanadi:

1) chegaraviy shartlar: $x=0$ bo'lganda $u=0$; $x=l$ bo'lganda $\frac{\partial u}{\partial x}=0$;

2) boshlang'ich shartlar: $t=0$ bo'lganda $u=f(x)$, $\frac{\partial u}{\partial t}=0$. $u(x,t)$ funksiya Fur'e metodi bilan aniqlansin.

12.249. Uzunligi l, ikki uchi birkitilgan sterjenning ko'ngdalang tebranishlari $\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2}+\frac{\partial^4 u}{\partial x^4}=0$ tenglama va quyidagi shartlar bilan beriladi:

1) chegaraviy shartlar: $x=0$ va $x=l$ bo'lganda $u=0$ va $\frac{\partial^2 u}{\partial x^2}=0$.

2) boshlang'ich shartlar: $t=0$ bo'lganda $u=f(x)$ va $\frac{\partial u}{\partial t}=0$. Fur'e metodi bilan $u(x,t)$ funksiya aniqlansin.

12.250. va 12.251. misollarda berilgan funksiyalar uchun Fur'e integrali yozilsin.

12.250. $f(x)=\begin{cases}1, & 0<x<1 \quad bo'lganda,\\ 0, & x>1 \quad bo'lganda.\end{cases}$ va $f(-x)=-f(x)$.

12.251. $f(x)=e^{-\beta x}$, $x\geq 0$ bo'lganda $f(-x)=f(x)$.

Makloren qatorlarini yoyish orqali integrallarni hisoblang.

12.252. $\int_0^x e^{-t^2}dt$ 12.253. $\int_0^x \frac{\sqrt[3]{1+t^3}-1}{t^2}dt$

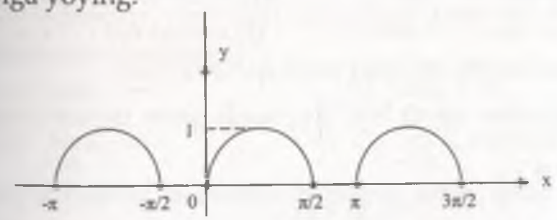
12.254. $\int_0^x \frac{arctgx}{x}dx$ 12.255. $\int_0^x \frac{\arcsin t}{t}dt$

12.256. Darajali qatorga yoyish orqali limitni hisoblang. $\lim\limits_{x\to 0}\frac{\ln(1+x+x^2)+\ln(1-x+x^2)}{x(e^x-1)}$

12.257. Faraz qilaylik, $f(x)=1+x+\frac{x^2}{2!}+...+\frac{x^n}{n!}+...$ o'rinli bo'lsa, $f(x)f(y)=f(x+y)$ ekanligini isbotlang.

12.258. $y=(1+x)^{-p}$ funksiyani darajali qatorga yoyish va $(1+x)^{-p}(1+x)^{-k-1}=(1+x)^{p-k-1}$ ayniyatdan foydalanib $\sum\limits_{S=0}C^m_{p-s}C^q_{q-s}=C^{p-m}_{p+q-1}$ keltirib chiqaring.

12.259 $f(x)=\begin{cases}0, & x\in\left[-\frac{\pi}{2};0\right]\\ \sin 2x, & x\in\left[0;\frac{\pi}{2}\right]\end{cases}$ funksiyani davri $T=\pi$ bo'lgan Fur'e qatoriga yoying.



12.1 rasm

12.260. $f(x)=\begin{cases}x+1, & x\in[-1;0]\\ 0, & x\in[0;3]\end{cases}$ funksiyani davri $T=\pi$ bo'lgan Fur'e qatoriga yoying.

12.261. $f(x)=\begin{cases} x, & x\in\left[0;\frac{\pi}{2}\right] \\ \frac{\pi}{2}, & x\in\left[\frac{\pi}{2};\pi\right] \end{cases}$ funksiyani a) kosinus va b) sinuslar bo'yicha Fur'e qatoriga yoying.

Takrorlash uchun savollar

1.Sonli qator deyilganda nima tushuniladi?

2.Sonli qator hadlari nima?

3.Qachon Sonli qator yaqinlashuvchi deyiladi?

4.Sonli qator yaqinlashuvining zaruriy sharti nimadan iborat?

5.Qanday sonli qator musbat hadli deyiladi?

6.Majoranta qator nima?

7.Qachon sonli qator ishorasi navbatlanuvchi deyiladi?

8.Sonli qator qachon absolyut yaqinlashuvchi deyiladi?

9.Funksional qator ta'rifi qanday ifodalaniladi?

10.Darajali qator qanday ta'riflaniladi?

QATORLARGA DOIR NAZORAT TESTLARI

1. Sonli qatorlar va ularning yaqinlashuvi

1. Ushbu ifodalardan qaysi biri sonli qator bo'ladi ?

A) $u_1\cdot u_2\cdot u_3\cdot \cdots \cdot u_n\cdot \cdots$. B) $u_1 : u_2 : u_3 : \cdots : u_n : \cdots$.

C) $u_1+ u_2+ u_3+ \cdots + u_n+ \cdots$. D) $u_1+ u_2+ u_3+ \cdots + u_n$.

E) keltitilgan barcha ifodalar sonli qator bo'ladi .

2. Quyidagilardan qaysi biri $\sum_{k=1}^{\infty} u_k$ sonli qator uchun xususiy yig'indi bo'lmaydi ?

A) u_1 B) $u_1 + u_2$

C) $u_1 + u_2 + u_3 + u_3$ D) $u_1 + u_2 + u_3 + \cdots + u_n$.

E) keltirilgan barcha ifodalar sonli qator uchun xususiy yig'indi bo'ladi.

3. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ sonli qator uchun S_{10} xususiy yig'indining qiymatini toping.

A) $\frac{1}{10}$ B) $\frac{1}{110}$ C) $\frac{10}{11}$ D) 1 E) $\frac{11}{12}$

4. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\cdots$ sonli qatorning u_n umumiy hadini ko'rsating.

A) $u_n=\frac{n+1}{n}$ B) $u_n=\frac{n-1}{n}$ C) $u_n=\frac{n}{n+1}$

D) $u_n=\frac{n}{n-1}$ E) $u_n=\frac{n-1}{n+1}$

5. Umumiy hadi $u_n=1+\frac{1}{n}$ bo'lgan sonli qatorni toping .

A) $1+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$. B) $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$. C) $\frac{2}{1}+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\cdots$.

D) $1-\frac{1}{2}+\frac{2}{3}-\frac{3}{4}+\cdots$. E) $\frac{2}{1}-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\cdots$.

6. n-xususiy yig'indisi S_n bo'lgan sonli qator qaysi shartda yaqinlashuvchi deyiladi ?

A) $\lim_{n\to\infty} S_n=\infty$. B) $\lim_{n\to\infty} S_n=-\infty$. C) $\lim_{n\to\infty} S_n=C, |C|<\infty$.

D) $\lim_{n\to\infty} S_n=\pm\infty$. E) $\lim_{n\to\infty} S_n$ mavjud emas .

7. n-xususiy yig'indisi S_n bo'lgan sonli qator qaysi shartda uzoqlashuvchi bo'lmaydi?

A) $\lim_{n\to\infty} S_n=-\infty$. B) $\lim_{n\to\infty} S_n=+\infty$.

C) $\lim_{n\to\infty} S_n=\pm\infty$. D) $\lim_{n\to\infty} S_n$ mavjud emas .

E) ko'rsatilgan barcha hollarda sonli qator uzoqlashuvchi bo'ladi .

8. n-xususiy yig'indisi S_n bo'lgan yaqinlashuvchi sonli qatorning S yig'indisi qanday aniqlanadi ?

A) $S=\lim_{n\to\infty}\frac{S_n}{S_{n-1}}$ B) $S=\lim_{n\to\infty}\frac{S_{n-1}}{S_n}$ C) $S=\lim_{n\to\infty}\frac{S_n}{n}$

D) $S=\lim_{n\to\infty} nS_n$ E) $S=\lim_{n\to\infty} S_n$

9. $1+\frac{1}{3}+\frac{1}{3^2}+...+\frac{1}{3^{n-1}}+......$sonli qator yig'indisini toping .

A)2 B)1.5 C) ∞ D) 1.33 E) 1.

10. $1-\frac{1}{4}+\frac{1}{4^2}-\frac{1}{4^3}+......+(-1)^{n+1}\frac{1}{4^n}+........$ sonli qator yig'indisini toping .

A)1 B)0 C) 0.8 D) 1.5 E) $1\frac{1}{3}$.

11. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ sonli qatorning S yig'indisini toping .

A) S=2 B) $S=\frac{1}{2}$ C) $S=\frac{1}{4}$ D) S =1 E) $S=\frac{2}{3}$

12. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}+\cdots$ sonli qator yig'indisini toping.

A) ∞ B) $\pi/2$ .C) e D) 1.5 E) 0.75

13. $\frac{1}{a}+\frac{1}{a^2}+....+\frac{1}{a^n}+....$ sonli qator qaysi shartda yaqinlashuvchi bo'ladi ?

A) $|a|\neq 0$ B) $|a|=1$ C) $|a|<1$ D) $|a|>1$ E) $|a|\neq 1$

14. Agar $\sum_{k=1}^{\infty}u_k$ va $\sum_{k=1}^{\infty}v_k$ yaqinlashuvchi sonli qatorlar bo'lsa, quyidagi sonli qatorlardan qaysi biri yaqinlashuvchi bo'lmaydi ?

A) $\sum_{k=1}^{\infty}(u_k+v_k)$

B) $\sum_{k=1}^{\infty}(u_k-v_k)$

C) $\sum_{k=1}^{\infty}Cu_k$ $(C-const.)$

D) $\sum_{k=1}^{\infty}(C+v_k)$ $(C-const.)$ E) $\sum_{k=1}^{\infty}Cv_k$ $(C-const.)$.

15. $\sum_{k=1}^{\infty}u_k$ sonli qator yaqinlashuvchi bo'lishining zaruriy sharti nimadan iborat ?

A) $\lim_{n\to\infty}u_n\neq 0$ B) $\lim_{n\to\infty}u_n>0$ C) $\lim_{n\to\infty}u_n<0$ D) $\lim_{n\to\infty}u_n=0$

E) $\lim_{n\to\infty}u_n$ mavjud emas .

16. Quyidagilarning qaysi biri uchun sonli qator yaqinlashuvining zaruriy sharti bajarilmaydi ?

A) $\sum_{k=1}^{\infty}\frac{\sin k}{k}$ B) $\sum_{k=1}^{\infty}\frac{\cos k}{k}$ C) $\sum_{k=1}^{\infty}\sin\frac{1}{k}$ D) $\sum_{k=1}^{\infty}\cos\frac{1}{k}$

E) barcha qatorlar uchun qator yaqinlashuvining zaruriy sharti bajariladi .

2. Musbat hadli sonli qatorlarning yaqinlashish alomatlari

1. ***Ta'rifni to'ldiring:*** $\sum_{n=1}^{\infty}u_n$ sonli qator musbat hadli deyiladi, agar uning ... .

A) barcha S_n xususiy yig'indilari musbat bo'lsa
B) birinchi hadi musbat bo'lsa
C) ayrim hadlari musbat bo'lsa .
D) barcha hadlari musbat bo'lsa .
E) yig'indisi musbat bo'lsa .

2. Quyidagi qatorlardan qaysi biri musbat hadli bo'ladi ?

A) $\sum_{k=1}^{\infty}\frac{\sin k}{k}$ B) $\sum_{k=1}^{\infty}\frac{\cos k}{k}$ C) $\sum_{k=1}^{\infty}\frac{e^k}{k}$ B) $\sum_{k=1}^{\infty}\frac{1}{k}\ln\frac{1}{k}$

E) keltirilgan barcha qatorlar musbat hadli emas .

3. Quyidagi qatorlardan qaysi biri musbat hadli emas ?

A) $\sum_{k=1}^{\infty}\frac{1}{k}\sin\frac{1}{k}$ B) $\sum_{k=1}^{\infty}\frac{1}{k}\cos\frac{1}{k}$ C) $\sum_{k=1}^{\infty}\frac{1}{k}e^{1/k}$ B) $\sum_{k=1}^{\infty}\frac{1}{k}\ln\frac{1}{k}$.

E) keltirilgan barcha qatorlar musbat hadli .

4. Musbat hadli $\sum_{n=1}^{\infty}a_n$ sonli qator yaqinlashuvchi bo'lishligining zaruriy shartini ko'rsating.

A) $\lim_{n\to\infty}a_n=0$ B) $\lim_{n\to\infty}a_n=+\infty$ C) $\lim_{n\to\infty}a_n=1$

D) $\lim_{n\to\infty}a_n\neq 0$ E) $\lim_{n\to\infty}a_n>0$.

5. Taqqoslash alomati shartini ko'rsating : Agar $\sum_{n=1}^{\infty}u_n$ va $\sum_{n=1}^{\infty}v_n$ musbat hadli sonli qatorlar hamda $\sum_{n=1}^{\infty}v_n$ yaqinlashuvchi bo'lsa,

unda ··· shart bajarilganda $\sum_{n=1}^{\infty} u_n$ sonli qator ham yaqinlashuvchi bo'ladi .

A) $u_n \approx v_n$ B) $u_n > v_n$ C) $u_n \geq v_n$ D) $u_n \leq v_n$ E) $u_n \neq v_n$

6. Limitik taqqoslash alomatida musbat hadli $\sum_{n=1}^{\infty} u_n$ va $\sum_{n=1}^{\infty} v_n$ sonli qatorlarni yaqinlashuvchi ekanligini tekshirish uchun quyidagi limitlarning qaysi biridan foydalaniladi ?

A) $\lim\limits_{n\to\infty} u_n v_n$ B) $\lim\limits_{n\to\infty}(u_n + v_n)$ C) $\lim\limits_{n\to\infty} \frac{u_n}{v_n}$

D) $\lim\limits_{n\to\infty}(u_n - v_n)$ E) $\lim\limits_{n\to\infty}(u_n \pm v_n)$

7. Limitik taqqoslash alomati shartini to'ldiring: Agar $\sum_{k=1}^{\infty} u_k$ (1) va $\sum_{k=1}^{\infty} v_k$ (2) musbat hadli sonli qatorlar va ... limit qiymati chekli hamda musbat bo'lsa , unda (1) va (2) sonli qatorlar bir paytda yoki yaqinlashuvchi, yoki uzoqlashuvchi bo'ladi.

A) $\lim\limits_{n\to\infty} u_n v_n$ B) $\lim\limits_{n\to\infty}(u_n + v_n)$ C) $\lim\limits_{n\to\infty} \frac{u_n}{v_n}$.

D) $\lim\limits_{n\to\infty}(u_n - v_n)$ E) $\lim\limits_{n\to\infty}(u_n \pm v_n)$.

8. Musbat hadli $\sum_{n=1}^{\infty} u_n$ sonli qatorni Dalamber alomati orqali tekshirish uchun qaysi limit hisoblanadi ?

A) $\lim\limits_{n\to\infty} u_{n+1} u_n$. B) $\lim\limits_{n\to\infty}(u_{n+1} + u_n)$.

C) .

D) $\lim\limits_{n\to\infty} \frac{u_{n+1}}{u_n}$. E) $\lim\limits_{n\to\infty} \sqrt[n]{u_n}$.

9. Musbat hadli $\sum_{n=1}^{\infty} u_n$ sonli qatorni Dalamber alomati orqali tekshirishda

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = d$$

bo'lsa, quyidagi tasdiqlardan qaysi biri noto'g'ri ?

A) $d<1$ bo'lsa qator yaqinlashuvchi

B) $d>1$ bo'lsa qator uzoqlashuvchi

C) d=1 bo'lsa qator yoki yaqinlashuvchi, yoki uzoqlashuvchi

D) $d=\infty$ bo'lsa qator uzoqlashuvchi

E) barcha tasdiqlar to'g'ri .

10. Quyidagi musbat hadli sonli qatorlardan qaysi birining yaqinlashuvini Dalamber alomati orqali aniqlab bo'ladi ?

A) $\sum_{n=1}^{\infty}\frac{n}{1+n^2}$ B) $\sum_{n=1}^{\infty}\frac{1}{1+2n}$ C) $\sum_{n=1}^{\infty}\frac{1}{n}$ D) $\sum_{n=1}^{\infty}\frac{n}{3^n}$ E) $\sum_{n=1}^{\infty}\frac{n}{(1+n)^2}$

11. Umumiy hadi $u_n=(3n+1)/2^n$ bo'lgan sonli qator Dalamber alomati orqali tekshirilganda $d=\lim\limits_{n\to\infty}(u_{n+1}/u_n)$ qiymati va qator yaqinlashuvi haqidagi tasdiq qaysi javobda to'g'ri ko'rsatilgan ?

A) d=1 va qator uzoqlashuvchi

B) d=0 vaqator yaqinlashuvchi .

C) d=0.5 va qator yaqinlashuvchi

D) $d=\infty$ va qator uzoqlashuvchi .

E) d=1vaqator yaqinlashuvchi .

12. Musbat hadli $\sum_{n=1}^{\infty}u_n$ sonli qatorni Koshi alomati orqali tekshirish uchun qaysi limit hisoblanadi ?

A) $\lim\limits_{n\to\infty}\sqrt{u_n}$ B) $\lim\limits_{n\to\infty}\sqrt[n]{u_n}$ C) $\lim\limits_{n\to\infty}\sqrt[n]{u_n u_{n+1}}$

D) $\lim\limits_{n\to\infty}\sqrt[n]{\frac{u_{n+1}}{u_n}}$. E) $\lim\limits_{n\to\infty}\sqrt[n]{\frac{u_n}{u_{n+1}}}$.

13. Musbat hadli $\sum_{n=1}^{\infty}u_n$ sonli qatorni Koshi alomati orqali tekshirishda

$\lim\limits_{n\to\infty}\sqrt[n]{u_n}=k$

bo'lsa, quyidagi tasdiqlardan qaysi biri noto'g'ri ?

A) $k<1$ bo'lsa qator yaqinlashuvchi

B) $k>1$ bo'lsa qator uzoqlashuvchi

C) k=1 bo'lsa qator yaqinlashuvchi ham, uzoqlashuvchi ham bo'lishi mumkin .

D) $k=\infty$ bo'lsa qator uzoqlashuvchi

E) barcha tasdiqlar to'g'ri .

14. Quyidagi qatorlardan qaysi birining yaqinlashuvini Koshi alomati yordamida aniqlab bo'ladi ?

A) $\sum_{n=1}^{\infty}\frac{n}{(n+1)^2}$ B) $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ C) $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

D) $\sum_{n=1}^{\infty}\frac{n^4}{n^5+5}$ E) $\sum_{n=1}^{\infty}\frac{1}{n}$

15. Umumiy hadi $u_n=[(2n+1)/n]^n$bo`lgan sonli qator Koshi alomati yordamida tekshirilganda $k=\lim_{n\to\infty}\sqrt[n]{u_n}$ limit qiymati va qator yaqinlashuvi haqidagi tasdiq qaysi javobda to'g'ri ko'rsatilgan ?

A) k=0 va qator yaqinlashuvchi

B) k=1 va qator uzoqlashuvchi .

C) k=2 va qator uzoqlashuvchi

D) k =0.5 va qator yaqinlashuvchi .

E) k =∞ va qator uzoqlashuvchi .

16. Umumiy hadi $u_n=[(2n+3)/(4n+5)]^n$bo'lgan sonli qator Koshi alomati yordamida tekshirilganda $k=\lim_{n\to\infty}\sqrt[n]{u_n}$ limit qiymati va qator yaqinlashuvi haqidagi tasdiq qaysi javobda to'g'ri ko'rsatilgan ?

A) k=0 va qator yaqinlashuvchi

B) k=1 va qator uzoqlashuvchi

C) k=2 va qator uzoqlashuvchi

D) k =0.5 va qator yaqinlashuvchi .

E) k =∞ va qator uzoqlashuvchi .

3. Ishorasi navbatlanuvchi va o'zgaruvchi sonli qatorlar

17. Qaysi shartda $\sum_{n=1}^{\infty}u_n$ sonli qator ishorasi navbatlanuvchi bo'ladi ?

A) $u_n+u_{n+1}<0$ (n=1,2,3, ···) B) $u_n-u_{n+1}<0$ (n=1,2,3, ···)

C) $u_n\cdot u_{n+1}>0$ (n=1,2,3, ···) D) $u_n\cdot u_{n+1}<0$ (n=1,2,3, ···)

E) to'g'ri javob keltirilmagan

18. Quyidagi qatorlarning qaysi biri ishorasi navbatlanuvchi bo'ladi ?

A) $\sum_{k=1}^{\infty}(-1)^k \sin k$ B) $\sum_{k=1}^{\infty}(-1)^k \cos k$

C) $\sum_{k=1}^{\infty}(-1)^k e^{-k}$ D) $\sum_{k=1}^{\infty}(-1)^k \operatorname{tg} k$.

E) keltirilgan barcha qatorlar ishorasi navbatlanuvchidir .

19. Leybnits alomatida ishorasi navbatlanuvchi $\sum_{n=1}^{\infty} u_n$ sonli qator yaqinlashuvchi bo'lishi uchun uning hadlariga quyidagi shartlardan qaysi biri qo'yiladi ?

A) $\lim_{n\to\infty} u_n = 0$ B) $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = d < 1$ C) $\lim_{n\to\infty} \sqrt[n]{u_n} = k < 1$.

D) $\lim_{n\to\infty} u_n \neq 0$ E) $\lim_{n\to\infty} (u_n - u_{n+1}) = 0$

20. Leybnits alomatida ishorasi navbatlanuvchi $\sum_{n=1}^{\infty}(-1)^n u_n \quad (u_n > 0, n = 1,2,3,...)$ sonli qator yaqinlashuvchi bo'lishi uchun uning hadlariga quyidagi shartlardan qaysi biri qo'yiladi ?

A) $u_{n+1} > u_n$ B) $u_{n+1} \geq u_n$ C) $u_{n+1} < u_n$

D) $u_{n+1} \leq u_n$ E) $u_{n+1} \neq u_n$

21. Agar Leybnits alomatiga asosan ishorasi navbatlanuvchi $\sum_{n=1}^{\infty}(-1)^{n-1} u_n$ sonli qator yaqinlashuvchi bo'lsa, uning yig'indisi S uchun quyidagi tasdiqlardan qaysi biri o'rinli bo'ladi ?

A) $S=u_1$ B) $S>u_1$ C) $S\geq u_1$ D) $S\leq u_1$ E) $S\neq u_1$

22. Quyidagi ishorasi navbatlanuvchi sonli qatorlardan qaysi biri yaqinlashuvchi emas ?

A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$ B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

C) $\sum_{n=1}^{\infty} (-1)^n \cos\frac{1}{n}$ D) $\sum_{n=1}^{\infty} (-1)^n \sin\frac{1}{n}$.

E) keltirilgan barcha qatorlar yaqinlashuvchi .

23. Quyidagilardan qaysi biri ishorasi o'zgaruvchi qator bo'lmaydi ?

A) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n^2}}$ B) $\sum_{n=1}^{\infty}\frac{\sin n}{n^3}$ C) $\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{1}{n}$ D) $\sum_{n=1}^{\infty}\frac{1}{n}\cos n$

E) keltirilgan barcha qatorlar ishorasi o'zgaruvchi bo'ladi

24.Qaysi shartda ishorasi o'zgaruvchi $\sum_{n=1}^{\infty}u_n$ sonli qator yaqinlashuvchi bo'lmaydi

A) Agar $\lim_{n\to\infty}\sum_{k=1}^{n}u_k$ limit mavjud va chekli bo'lsa .

B) Agar biror n_0 natural sonda $\sum_{k=n_0}^{\infty}u_k$ qator yaqinlashuvchi bo'lsa

C) Agar biror n_0 natural sonda $\sum_{k=n_0}^{\infty}|u_k|$ qator yaqinlashuvchi bo'lsa .

D) Agar $\sum_{k=1}^{\infty}|u_k|$ qator yaqinlashuvchi bo'lsa .

E) Keltirilgan barcha shartlarda qator yaqinlashuvchi bo'ladi

25. Qaysi holda ishorasi o'zgaruvchi $\sum_{n=1}^{\infty}u_n$ sonli qator absolut yaqinlashuvchi bo'lmasligi mumkin ?

A) $\lim_{n\to\infty}|u_n|=0$ B) $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|=d<1$ C) $\lim_{n\to\infty}\sqrt[n]{|u_n|}=k<1$

D) $\lim_{n\to\infty}(|u_1|+|u_2|+|u_3|+\cdots+|u_n|)<\infty$.

E) ko'rsatilgan barcha hollarda sonli qator absolut yaqinlashuvchi bo'ladi .

26. Quyidagi qatorlardan qaysi biri absolut yaqinlashuvchi bo'ladi ?

A) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n^2}}$ B) $\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$ C) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ D) $\sum_{n=1}^{\infty}(-1)^n\cos\frac{1}{n}$

E) keltirilgan barcha qatorlar absolut yaqinlashuvchi bo'ladi

27. Quyidagi qatorlardan qaysi biri absolut yaqinlashuvchi emas ?

A) $\sum_{n=1}^{\infty}\frac{\cos n}{n^3}$ B) $\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$ C) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n\sqrt{n}}$ D) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$.

E) keltirilgan barcha qatorlar absolut yaqinlashuvchi bo'ladi

28. Agar ishorasi o'zgaruvchi $\sum_{n=1}^{\infty} u_n$ sonli qator absolut yaqinlashuvchi bo'lsa, unda quyidagi tasdiqlardan qaysi biri o'rinli emas ?

A) Ixtiyoriy C o'zgarmas son uchun $\sum_{n=1}^{\infty} Cu_n$ sonli qator absolut yaqinlashuvchi bo'ladi .

B) Ixtiyoriy o'zgarmas m natural son uchun $\sum_{n=m}^{\infty} u_n$ sonli qator absolut yaqinlashuvchi bo'ladi .

C) $\sum_{n=1}^{\infty} u_n$ sonli qatorga $\sum_{n=1}^{\infty} v_n$ yig'indini qo'shsak, absolut yaqinlashuvchi sonli qator hosil bo'ladi .

D) $\sum_{n=1}^{\infty} u_n$ sonli qator hadlarining o'rnini ixtiyoriy tarzda o'zgartirsak, absolut yaqinlashuvchi sonli qator hosil bo'ladi .

E) keltirilgan barcha tasdiqlar o'rinli bo'ladi.

29. Ishorasi o'zgaruvchi $\sum_{n=1}^{\infty} u_n$(1) sonli qator bo'yicha musbat hadli $\sum_{n=1}^{\infty} |u_n|$ (2) qator tuzilgan. Qachon (1) qator shartli yaqinlashuvchi deb ataladi ?

A) Agar (1) qator yaqinlashuvchi bo'lsa .

B) Agar (2) qator yaqinlashuvchi bo'lsa .

C)Agar (1) qator yaqinlashuvchi, (2) qator esa uzoqlashuvchi bo'lsa .

D) Agar (1) va (2) qatorlarning ikkalasi ham yaqinlashuvchi bo'lsa .

E) Agar (1) va (2) qatorlarning birortasi yaqinlashuvchi bo'lsa .

30.Quyidagi qatorlar orasidan shartli yaqinlshuvchi qatorni ko'rsating.

A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$. B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$. C) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^3}{3^n}$.

D) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$. E) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n}}$.

31.Shartli yaqinlashuvchi $\sum_{n=1}^{\infty}u_n$ sonli qator uchun Riman teoremasida quyidagi tasdiqlardan qaysi biri ifodalanmaydi?

A)qator hadlarining o'rnini almashtirsak qator yig'indisi o'zgaradi .

B)qator hadlarining o'rnini almashtirish orqali uning yig'indisini ixtiyoriy songa tenglashtirish mumkin .

C) qator hadlarining o'rnini almashtirish orqali uni uzoqlashuvchi qatorga aylantirish mumkin .

D) qator hadlarini o'zgarmas C soniga ko'paytirsak, uning yig'indisi ham C marta o'zgaradi .

E)keltirilgan barcha tasdiqlar Riman teoremasida ifodalanadi .

4. Funksional qatorlar

1. Quyidagi ifodalardan qaysi biri funksional qator bo'lmaydi ?

A) $u_0(x)+u_1(x)+u_2(x)+\cdots+u_n(x)+\cdots$.

B) $u_0(x)+u_2(x)+u_4(x)+\cdots+u_{2n}(x)+\cdots$.

C) $u_0(x)+u_3(x)+u_6(x)+\cdots+u_{3n}(x)$.

D) $u_k(x)+u_{k+1}(x)+u_{k+2}(x)+\cdots+u_{k+n}(x)+\cdots$.

E) keltirilgan barcha ifodalar funksional qatorni ifodalaydi .

2. Quyidagilardan qaysi biri funksional qator emas ?

A) $\sum_{n=0}^{\infty}\frac{\cos^2 nx}{(n+1)^2}$ B) $\sum_{n=0}^{\infty}\frac{\sin^2 nx}{(n+1)^2}$ C) $\sum_{n=0}^{\infty}\frac{\cos nx+\sin nx}{(n+1)^2}$

D) $\sum_{n=0}^{\infty}\frac{\cos^2 nx+\sin^2 nx}{(n+1)^2}$ E) barcha qatorlar funksional qatordir.

3. Qaysi hollarda $\sum_{n=0}^{\infty}u_n(x)$ funksional qator $x=x_0$ nuqtada yaqinlashuvchi bo'ladi ?

I. musbat hadli $\sum_{n=0}^{\infty} u_n(x_0)$ sonli qator yaqinlashuvchi bo'lsa .

II. ishorasi o'zgaruvchi $\sum_{n=0}^{\infty} u_n(x_0)$ sonli qator absolut yaqinlashuvchi bo'lsa

III. ishorasi o'zgaruvchi $\sum_{n=0}^{\infty} u_n(x_0)$ sonli qator shartli yaqinlashuvchi bo'lsa

A) I va II hollarda B) I va III hollarda

C) II va III hollarda D) I, II va III hollarda

E) faqat I holda

4. Quyidagi x_0 nuqtalardan qaysi birida $\sum_{n=1}^{\infty} 2^{n+1} x^n$ funksional qator yaqinlashuvchi bo'ladi ?

A) $x_0 = -1$ B) $x_0 = \frac{1}{2}$ C) $x_0 = \frac{1}{4}$ D) $x_0 = 1$ E) $x_0 = -\frac{1}{2}$

5. Funksional qatorning yaqinlashish sohasida uning qoldig'i $r_n(x)$ qaysi shartni qanoatlantiradi ?

A) $\lim_{n\to\infty} r_n(x) > 0$. B) $\lim_{n\to\infty} r_n(x) < 0$. C) $\lim_{n\to\infty} r_n(x) = 0$.

D) $\lim_{n\to\infty} r_n(x) \neq 0$. E) $\lim_{n\to\infty} r_n(x) = \pm\infty$.

6. $\sum_{k=0}^{\infty} \frac{\sin^k x}{k^2+1}$ funksional qatorning yaqinlashish sohasini toping.

A) $(-\infty, \infty)$. B) $\pi n < x < \pi(n+1), n = 0,1,2,\cdots$.

C) $x \neq \pi n, n = 0,\pm1,\pm2,\cdots$. D) $x \neq \frac{\pi}{2} + \pi n, n = 0,\pm1,\pm2,\cdots$.

E) $x \neq \frac{\pi}{2} n, n = 0,\pm1,\pm2,\cdots$.

7. $\sum_{k=0}^{\infty} \frac{\cos x^k}{(k+1)^2}$ funksional qatorning yaqinlashish sohasini aniqlang.

A) $(-\infty, \infty)$. B) $\pi n < x < \pi(n+1), n = 0,1,2,\cdots$.

C) $x \neq \pi n, n = 0,\pm1,\pm2,\cdots$. D) $x \neq \frac{\pi}{2} + \pi n, n = 0,\pm1,\pm2,\cdots$. E) $x>0$

8. $\sum_{k=0}^{\infty} x^k$ funksional qatorning yaqinlashish sohasini toping.

A) $(-\infty, \infty)$ B) $(0, \infty)$ C) $(-2, 2)$ D) $(-\infty, 0)$ E) $(-1, 1)$

9. $\sum_{k=0}^{\infty} 2^{-k} x^k$ funksional qatorning yaqinlashish sohasini toping.

A) $(-\infty, \infty)$ B) $(0, \infty)$

C) $(-2, 2)$ D) $(-\infty, 0)$ E) $(-1/2, 1/2)$

10. $\sum_{k=0}^{\infty} \cos^k x$ funksional qatorning $x \in [\pi/6, \pi/2]$ bo'lgandagi yig'indisini toping.

A) $\frac{1}{1-\sin x}$ B) $\frac{1}{1+\sin x}$

C) $\frac{1}{1-\cos x}$ D) $\frac{1}{1+\cos x}$ E) $\frac{1}{\cos x}$

11. $\sum_{k=0}^{\infty} (-1)^k \cos^k x$ funksional qatorning $[\pi/6, \pi/2]$ kesmadagi yig'indisini toping.

A) $\frac{1}{1-\sin x}$ B) $\frac{1}{1+\sin x}$

C) $\frac{1}{1-\cos x}$ D) $\frac{1}{1+\cos x}$ E) $\frac{1}{\cos x}$

12. $\sum_{n=0}^{\infty} x^n$ funksional qatorning yaqinlashish sohasidagi yig'indisini aniqlang.

A) $\frac{1}{1+x}$ B) $\frac{x^{n+1}-x}{x-1}$ C) $\frac{1}{1-x}$ D) $\frac{x}{1-x}$ E) $\frac{x}{1+x}$

13. $\sum_{n=1}^{\infty} x^n$ funksional qatorning yaqinlashish sohasidagi yig'indisini aniqlang.

A) $\frac{1}{1+x}$ B) $\frac{x^{n+1}-x}{x-1}$ C) $\frac{1}{1-x}$ D) $\frac{x}{1-x}$ E) $\frac{x}{1+x}$

14. $\sum_{n=0}^{\infty} (-1)^n x^n$ funksional qatorning yaqinlashish sohasidagi yig'indisini aniqlang.

A) $\frac{1}{1+x}$ B) $\frac{x^{n+1}-x}{x-1}$ C) $\frac{1}{1-x}$ D) $\frac{x}{1-x}$ E) $\frac{x}{1+x}$

15. $\sum_{n=1}^{\infty}(-1)^{n+1}x^n$ funksional qatorning yaqinlashish sohasidagi yig'indisini aniqlang.

A) $\frac{1}{1+x}$ B) $\frac{x^{n+1}-x}{x-1}$ C) $\frac{1}{1-x}$ D) $\frac{x}{1-x}$ E) $\frac{x}{1+x}$

16. Weyershtrass alomatida funksional qator biror $[a,b]$ kesmada tekis yaqinlashuvchi bo'lishi uchun uning $u_n(x)$ (n=0,1,2, 3, ...) hadlariga qanday shart qo'yiladi?

A) $|u_n(x)|\leq c_n$ B) $\lim_{x\to a} u_n(x)\leq c_n$

C) $|u_n(x)|=c_n$ D) $|u_n(x)|\geq c_n$ E) $\lim_{x\to b} u_n(x)\leq c_n$

5. Darajali qatorlar

1. Quyidagilardan qaysi biri darajali qator emas ?

A) $\sum_{k=0}^{n}a_kx^k$ B) $\sum_{k=0}^{\infty}a_kx^{2k}$ C) $\sum_{k=0}^{\infty}a_kx^{2k+1}$ D) $\sum_{k=0}^{\infty}a_kx^{k^3}$

E) keltirilgan qatorlarning barchasi darajali qator bo'ladi .

2. Quyidagilardan qaysi biri darajali qator bo'ladi ?

A) $\sum_{k=0}^{\infty}a_kx^{2k}$. B) $\sum_{k=0}^{\infty}a_kx^{k/2}$. C) $\sum_{k=0}^{\infty}a_kx^{\sqrt{k}}$. D) $\sum_{k=0}^{\infty}a_kx^{-k}$.

E) keltirilgan qatorlarning barchasi darajali qator bo'ladi .

3. $\sum_{k=0}^{\infty}\frac{x^k}{k+1}$ darajali qator qaysi x_0 nuqtada yaqinlashuvchi ?

A) $x_0=2$. B) $x_0=1$. C) $x_0=-2$. D) $x_0=-1$.

E) ko'rsatilgan barcha nuqtalarda qator yaqinlashuvchi .

4. $\sum_{k=0}^{\infty}x^{2k}$ darajali qatorning yaqinlashish sohasidagi yig'indisini toping.

A) $\frac{1}{1+x}$ B) $\frac{1}{1-x}$ C) $\frac{1}{1-x^2}$ D) $\frac{1}{1+x^2}$ E) $\frac{x^2}{1-x^2}$

5. $\sum_{k=1}^{\infty}x^{2k}$ darajali qatorning yaqinlashish sohasidagi yig'indisini toping.

A) $\frac{1}{1+x}$ B) $\frac{1}{1-x}$ C) $\frac{1}{1-x^2}$ D) $\frac{1}{1+x^2}$ E) $\frac{x^2}{1-x^2}$

6. Agar R soni $\sum_{k=0}^{\infty} a_k x^k$ darajali qatorning yaqinlashish radiusi bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli emas ?

A) $0 \le x < R$ sohada qator yaqinlashuvchi .

B) $-R < x \le 0$ sohada qator yaqinlashuvchi .

C) $-R < x < R$ sohada qator yaqinlashuvchi .

D) $|x| > R$ sohada qator uzoqlashuvchi .

E) keltirilgan barcha tasdiqlar o'rinli .

7. Agar R soni $\sum_{k=0}^{\infty} a_k x^k$ darajali qatorning yaqinlashish radiusi bo'lsa, quyidagi tasdiqlardan qaysi biri har doim ham o'rinli emas ?

A) $x > R$ sohada qator uzoqlashuvchi .

B) $x < -R$ sohada qator uzoqlashuvchi .

C) $|x| > R$ sohada qator uzoqlashuvchi .

D) $x = \pm R$ nuqtalarda qator uzoqlashuvchi .

E) keltirilgan barcha tasdiqlar doimo o'rinli bo'ladi .

8. $\sum_{k=0}^{\infty} a_k x^k$ darajali qatorning yaqinlashish radiusi R uchun Dalamber formulasini ko'rsating .

A) $R = \lim\limits_{n\to\infty} |a_n|$ B) $R = \lim\limits_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$ C) $R = \lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

D) $R = \lim\limits_{n\to\infty} \sqrt[n]{|a_n|}$ E) $R = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}}$

9. $\sum_{k=0}^{\infty} a_k x^k$ darajali qatorning yaqinlashish radiusi R uchun Koshi formulasini ko'rsating .

A) $R = \lim\limits_{n\to\infty} |a_n|$ B) $R = \lim\limits_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$ C) $R = \lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

D) $R = \lim\limits_{n\to\infty} \sqrt[n]{|a_n|}$ E) $R = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}}$.

10. $\sum_{k=0}^{\infty} 3^k x^k$ darajali qatorning R yaqinlashish radiusini toping .

A) $R=3$ B) $R=2$ C) $R=1$ D) $R=0$

E) to'g'ri javob keltirilmagan .

11. $\sum\limits_{k=0}^{\infty} 3^{-k} x^k$ darajali qatorning R yaqinlashish radiusini toping .

A) $R=3$ B) $R=2$ C) $R=1/3$ D) $R=1/2$ E) $R=0$

12. $\sum\limits_{k=0}^{\infty} \frac{x^k}{k+1}$ darajali qatorning R yaqinlashish radiusini toping

A) $R=3$ B) $R=2$ C) $R=1$ D) $R=0$

E) to'g'ri javob keltirilmagan .

13. $\sum\limits_{k=0}^{\infty} \frac{x^k}{k!}$ darajali qatorning R yaqinlashish radiusini toping

A) $R=3$ B) $R=2$ C) $R=1$ D) $R=0$ E) $R=\infty$

14. $\sum\limits_{k=0}^{\infty} 3^k x^k$ darajali qator yig'indisi qaysi $[a,b]$ kesmada uzluksiz funksiya bo'ladi?

A) $[a,b]=[0,1/3]$ B) $[a,b]=[-1/3, 0]$

C) $[a,b]=[-1/3, 1/3]$ D) $[a,b]\subset(-1/3, 1/3)$

E) $[a,b]\supset(-1/3, 1/3)$.

15. Agar $\sum\limits_{n=0}^{\infty} a_n x^n$ darajali qatorning yaqinlashish radiusi R bo'lsa, $\sum\limits_{n=0}^{\infty} a_n (x-c)^n$ darajali qatorning yaqinlashish oralig'i qanday topiladi?

A) $(R-c, R+c)$ B) $(c-R, c+R)$ C) $(-R-c, R+c)$

D) $(-R-c, R-c)$ E) $(R-c, c-R)$

16. $\sum\limits_{n=0}^{\infty} \frac{(x-2)^n}{2^n(n+1)}$ darajali qatorning yaqinlashish oralig'inii toping.

A) $(-2 , 2)$ B) $(0 , 4)$ C) $(-4 , 0)$ D) $(0 , 2)$ E) $(-4, 4)$.

6. Teylor va Makloren qatorlari

1. $y=f(x)$ funksiyaning Teylor qatori qayerda to'g'ri ifodalangan ?

A) $f(0)+\frac{f'(0)}{1!}(x-a)+\frac{f''(0)}{2!}(x-a)^2+\dots+\frac{f^{(n)}(0)}{n!}(x-a)^n+\cdots$.

B) $f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots+\frac{f^{(n)}(a)}{n!}(x-a)^n+\cdots$.

C) $f(a)+\frac{f'(a)}{1!}x+\frac{f''(a)}{2!}x^2+\dots+\frac{f^{(n)}(a)}{n!}x^n+\cdots$.

D) $f(a)+\frac{f(a)}{1!}(x-a)+\frac{f(2a)}{2!}(x-a)^2+\dots+\frac{f(na)}{n!}(x-a)^n+\cdots$.

E) $f(a)+\frac{f(a)}{1!}x+\frac{f(2a)}{2!}x^2+\dots+\frac{f(na)}{n!}x^n+\cdots$.

2. Agar $R_n(x)$ berilgan $f(x)$ funksiya Teylor qatorining qoldiq hadi bo'lsa, bu qatorning yig'indisi (a,b) oraliqda $f(x)$ funksiyaga teng bo'lishi uchun qaysi shart zarur va yetarli ?

A) (a,b) oraliqda $R_n(x)$ yuqoridan chegaralangan .

B) (a,b) oraliqda $R_n(x)$ quyidan chegaralangan .

C) (a,b) oraliqda $\lim\limits_{n\to\infty} R_n(x)=0$.

D) (a,b) oraliqda $R_n(x)$ monoton o'suvchi .

E) (a,b) oraliqda $R_n(x)$ monoton kamayuvchi .

3. Berilgan $f(x)$ funksiyaning $x-x_0$ darajalari bo'yicha Teylor qatorining $R_n(x)$

qoldiq hadi Lagranj ko'rinishida qanday ifodalanadi ?

A) $R_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-x_0)^{n+1}, c\in(x_0,x)$.

B) $R_n(x)=\frac{f^{(n+1)}(x_0)}{(n+1)!}(c-x_0)^{n+1}, c\in(x_0,x)$.

C) $R_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-c)^{n+1}, c\in(x_0,x)$.

D) $R_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}, c\in(x_0,x)$.

E) $R_n(x)=\dfrac{(x-c)^{n+1}}{(n+1)!}, c\in(x_0,x)$

4. Berilgan $f(x)$ funksiyaning $x-x_0$ darajalari bo'yicha Teylor qatorining yigindisi (a,b) oraliqda shu funksiyaning o'ziga teng bo'lishi uchun qanday shart yetarli bo'ladi ?

A) (a,b) oraliqda biror chekli M soni uchun $|f(x)|\le M$.

B) (a,b) oraliqda biror chekli M soni uchun $|f(x)|\ge M$.

C) (a,b) oraliqda biror chekli M soni va barcha $n=0,1,2, \cdots$ uchun $|f^{(n)}(x)|\le M$

D) (a,b) oraliqda biror chekli M soni va barcha $n=0,1,2, \cdots$ uchun $|f^{(n)}(x)|\ge M$.

E) to'g'ri javob keltirilmagan .

5. Berilgan $f(x)$ funksiyaning Makloren qatori qayerda to'g'ri ko'rsatilgan ?

A) $f(0)+\dfrac{f'(0)}{1!}(x-a)+\dfrac{f''(0)}{2!}(x-a)^2+\cdots+\dfrac{f^{(n)}(0)}{n!}(x-a)^n+\cdots$.

B) $f(a)+\dfrac{f'(a)}{1!}(x-a)+\dfrac{f''(a)}{2!}(x-a)^2+\cdots+\dfrac{f^{(n)}(a)}{n!}(x-a)^n+\cdots$.

C) $f(a)+\dfrac{f'(a)}{1!}x+\dfrac{f''(a)}{2!}x^2+\cdots+\dfrac{f^{(n)}(a)}{n!}x^n+\cdots$.

D) $f(a)+\dfrac{f(a)}{1!}x+\dfrac{f(2a)}{2!}x^2+\cdots+\dfrac{f(na)}{n!}x^n+\cdots$.

E) $f(0)+\dfrac{f'(0)}{1!}x+\dfrac{f''(0)}{2!}x^2+\cdots+\dfrac{f^{(n)}(0)}{n!}x^n+\cdots$.

6. $f(x)=\sin x$ funksiyaning Makloren qatoriga yoyilmasini ko'rsating.

A) $1-x+\dfrac{x^2}{2!}-\dfrac{x^3}{3!}+\cdots+(-1)^n\dfrac{x^n}{n!}+\cdots$.

B) $x+\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+\cdots+\dfrac{x^{2n+1}}{(2n+1)!}+\cdots$.

C) $1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots+\dfrac{x^n}{n!}+\cdots$.

D) $x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\cdots+\dfrac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$.

E) $1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\cdots+\dfrac{(-1)^n x^{2n}}{(2n)!}+\cdots$.

7. $f(x)=\cos x$ funksiyaning Makloren qatorini ko'rsating.

A) $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots+(-1)^n\frac{x^n}{n!}+\cdots$.

B) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots+\frac{x^{2n+1}}{(2n+1)!}+\cdots$.

C) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots+\frac{x^{2n}}{(2n)!}+\cdots$.

D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$.

E) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots$.

8. $f(x)=e^x$ funksiyaning Makloren qatorini toping.

A) $1+x+x^2+\cdots+x^n+\cdots$.

B) $1+\frac{x}{1}+\frac{x^2}{2}+\cdots+\frac{x^n}{n}+\cdots$.

C) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$.

D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$.

E) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots$.

9. $f(x)=e^{-x}$ funksiyaning Makloren qatorini toping.

A) $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots+(-1)^n\frac{x^n}{n!}+\cdots$.

B) $1+\frac{x}{1}+\frac{x^2}{2}+\cdots+\frac{x^n}{n}+\cdots$.

C) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$.

D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$.

E) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots$.

10. Quyidagi darajali qatorlardan qaysi biri $(-1,1)$ oraliqda $f(x)=1/(1-x)$ funksiyaning Makloren qatorini ifodalaydi ?

A) $1+x+x^2+\cdots+x^n+\cdots$.

B) $1+\frac{x}{1}+\frac{x^2}{2}+\dots+\frac{x^n}{n}+\dots$.

C) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots+\frac{x^n}{n!}+\dots$.

D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\dots$;

E) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\dots+\frac{(-1)^n x^{2n}}{(2n)!}+\dots$.

11.Quyidagi darajali qatorlardan qaysi biri $(-1,1)$ oraliqda $f(x)=1/(1+x)$ funksiyaning Makloren qatori bo'ladi ?

A) $1+x+x^2+x^3+\dots+x^n+\dots$.

B) $1-\frac{x}{1}+\frac{x^2}{2}-\frac{x^3}{3}+\dots+(-1)^n\frac{x^n}{n}+\dots$.

C) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots+\frac{x^n}{n!}+\dots$.

D) $1-x+x^2-x^3+\dots+(-1)^n x^n+\dots$

E) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\dots$;

12. $(-1,1)$ oraliqda $f(x)=\ln(1+x)$ funksiyaning Makloren qatorini toping .

A) $1+x+x^2+\dots+x^n+\dots$.

B) $1-\frac{x}{1}+\frac{x^2}{2}-\dots+(-1)^n\frac{x^n}{n}+\dots$.

C) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots+\frac{x^n}{n!}+\dots$.

D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\dots$.

E) $x-\frac{x^2}{2}+\frac{x^3}{3}-\dots+\frac{(-1)^{n+1}x^n}{n}+\dots$.

13. $f(x)=\text{arctg}x$ funksiyaning Makloren qatori qayerda ifodalangan ?

A) $1+x+x^2+\dots+x^n+\dots$.

B) $1-\frac{x}{1}+\frac{x^2}{2}-\dots+(-1)^n\frac{x^n}{n}+\dots$.

C) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots+\frac{x^n}{n!}+\dots$.

D) $x-\frac{x^3}{3}+\frac{x^5}{5}-\dots+\frac{(-1)^n x^{2n+1}}{2n+1}+\dots$.

E) $x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+\frac{(-1)^{n+1}x^n}{n}+\cdots$.

14. $f(x)=\sqrt{x+4}$ funksiya Makloren qatorining 3-hadini toping .

A) $\frac{x^3}{6}$ B) $-\frac{x^2}{64}$ C) $\frac{x}{24}$ D) $-\frac{x^2}{32}$ E) $\frac{x^2}{2!}$

15. $f(x)=(1+x)^{\alpha}$ funksiya uchun binomial qatorning yaqinlashish oralig'ini ko'rsating .

A) (0, 1) B) (–1, 1) C) (–1, 0) D) (–∞, ∞) E) (–α, α)

16. $f(x)=e^{3x}$ funksiya uchun Makloren qatorida x^3 daraja oldidagi koeffitsent qiymati nimaga teng ?

A) 3 . B) 3.5 . C) 4 . D) 4.5 . E) 5 .

7. Darajali qatorlarning ayrim tatbiqlari

1. Darajali qator yordamida quyidagi masalalardan qaysi birini yechib bo'lmaydi ?

A) ildizlarning taqribiy qiymatini hisoblash .

B) funksiyaning taqribiy qiymatini hisoblash .

C) noelementar boshlang'ich funksiyani topish .

D) funksiyaning davriyligini aniqlash .

E) keltirilgan barcha masalalar darajali qator yordamida yechiladi .

2. $\sqrt[3]{9}$ ildiz taqribiy qiymatini tegishli binomial qatorning dastlabki uchta hadini olish orqali toping .

A) $\frac{643}{312}$. B) $\frac{435}{211}$. C) $\frac{377}{183}$. D) $\frac{599}{288}$. E) $\frac{243}{115}$.

3. $\sqrt[3]{7}$ ildiz taqribiy qiymatini tegishli binomial qatorning dastlabki uchta hadini qo'shish orqali toping .

A) $\frac{603}{312}$. B) $\frac{551}{288}$. C) $\frac{379}{203}$. D) $\frac{549}{277}$. E) $\frac{247}{135}$.

4. sin1 taqribiy qiymatini $y=\sin x$ funksiya uchun Makloren qatorining dastlabki uchta hadini olish orqali toping.

A) 11/12 . B) 101/120 . C) 111/121 . D) 110/121 . E) 107/120 .

5. cos1 taqribiy qiymatini y=cos$x$ funksiya uchun Makloren qatorining dastlabki uchta hadini olish orqali toping.

A) 1/2 . B) 11/21 . C) 13/24 . D) 9/20 . E) 17/35 .

6. e sonining taqribiy qiymatini $y=e^x$ funksiya uchun Makloren qatorining dastlabki to'rtta hadini olish orqali toping.

A) 8/3 . B) 11/4 . C) 14/5 . D) 17/7 . E) 25/9 .

7. $\pi/4$=arctg1tenglik va y=arctg$x$ funksiya uchun Makloren qatorining dastlabki uchta hadini olish orqali π sonini taqribiy qiymatini toping.

A) 52/15 . B) 55/17 . C) 257/75 .

D) 334/105 . E) 397/120 .

8. $\lim\limits_{x\to 0}\dfrac{x-\ln(1+x)}{\cos x-1}$ qiymatini darajali qator yordamida hisoblang.

A) 0 . B) 1 . C) –1 . D) ∞ . E) 1/3 .

9. $\lim\limits_{x\to 0}\dfrac{x-\text{arctg}x}{x^2\sin x}$ qiymatini darajali qator yordamida hisoblang.

A) 0 . B) 1/2 . C) –1/2 . D) ∞ . E) 1/3 .

10. $\lim\limits_{x\to 0}\dfrac{\sin x-\text{arctg}x}{x(1-\cos x)}$ qiymatini darajali qator yordamida hisoblang.

A) 0 . B) 1/3 . C) –1/3 . D) ∞ . E) 1/2 .

11. Ushbu elementar bo'lmagan $F(x)=\int\limits_0^x\dfrac{1-\cos t}{t}dt$ boshlang'ich funksiyani darajali qator ko'rinishida ifodalang .

A) $F(x)=\sum\limits_{k=1}^{\infty}\dfrac{x^{2k}}{(2k)!}$. B) $F(x)=\sum\limits_{k=1}^{\infty}(-1)^{k+1}\dfrac{x^{2k}}{2k(2k)!}$.

C) $F(x)=\sum\limits_{k=1}^{\infty}(-1)^{k+1}\dfrac{x^{2k}}{(2k)!}$. D) $F(x)=\sum\limits_{k=1}^{\infty}(-1)^{k+1}\dfrac{x^{2k}}{2k+1}$.

E) $F(x)=\sum\limits_{k=1}^{\infty}\dfrac{(2k+1)x^{2k}}{(2k)!}$.

12. $\int\limits_0^x\dfrac{e^t-1}{t}dt$ integralni darajali qator ko'rinishida ifodalang.

A) $\sum\limits_{k=1}^{\infty}\dfrac{x^k}{k^2}$. B) $\sum\limits_{k=1}^{\infty}(-1)^k\dfrac{x^k}{k}$. C) $\sum\limits_{k=1}^{\infty}\dfrac{x^k}{k\cdot k!}$.

D) $\sum_{k=1}^{\infty}(-1)^k \frac{x^k}{k \cdot k!}$. E) $\sum_{k=1}^{\infty}\frac{x^k}{k!}$.

13. $\int_0^1 \frac{e^x - 1}{x}dx$ integral qiymatini sonli qator ko'rinishda ifodalang.

A) $\sum_{k=1}^{\infty}\frac{1}{k^2}$. B) $\sum_{k=1}^{\infty}(-1)^k\frac{1}{k}$. C) $\sum_{k=1}^{\infty}\frac{1}{k \cdot k!}$.

D) $\sum_{k=1}^{\infty}(-1)^k\frac{1}{k \cdot k!}$. E) $\sum_{k=1}^{\infty}\frac{1}{k!}$.

14. $\int_0^1 \frac{\sin x}{x}dx$ aniq integralning taqribiy qiymatini integral ostidagi funksiya uchun Makloren qatorining dastlabki uchta hadini olish orqali toping.

A)1493/1500. B)1579/1600. C) 1671/1700.

D)1703/1800. E) 1837/1900.

15. $\int_0^1 \frac{\ln(x+1)}{x}dx$ aniq integralning taqribiy qiymatini integral ostidagi funksiya uchun Makloren qatorining dastlabki uchta hadini olish orqali toping.

A)21/26 . B)31/36 . C) 41/46 . D)51/56 . E) 61/66

8. Furye qatorlari

1. Trigonometrik qator qayerda to'g'ri ifodalangan?

A) $\frac{a_0}{2}+\sum_{k=1}^{\infty}[a_k\cos(k+x)+b_k\sin(k+x)]$.

B) $\frac{a_0}{2}+\sum_{k=1}^{\infty}\left[a_k\cos\frac{x}{k}+b_k\sin\frac{x}{k}\right]$.

C) $\frac{a_0}{2}+\sum_{k=1}^{\infty}[a_k\cos(x-k)+b_k\sin(x-k)]$.

D) $\frac{a_0}{2}+\sum_{k=1}^{\infty}[a_k\cos kx+b_k\sin kx]$.

E) $\frac{a_0}{2}+\sum_{k=1}^{\infty}[a_k\cos^k x+b_k\sin^k x]$.

2. $\int_{-\pi}^{\pi}\sin mx\cos nx dx=0$ tenglik m va n butun sonlarning qaysi qiymatlarida o'rinli bo'lmaydi?

A) $m>n$. B) $m<n$. C) $m=n$. D) $m\neq n$.

E) ko'rsatilgan barcha hollarda tenglik o'rinli bo'ladi .

3. $\int_{-\pi}^{\pi} \cos mx \cos nx dx = 0$ tenglik m va n butun sonlarning qaysi qiymatlarida o'rinli bo'lmaydi?

A) $m > n$. B) $m < n$. C) $m = n$. D) $m \neq n$.

E) ko'rsatilgan barcha hollarda tenglik o'rinli bo'ladi .

4. $\int_{-\pi}^{\pi} \sin mx \sin nx dx = 0$ tenglik m va n butun sonlarning qaysi qiymatlarida o'rinli bo'lmaydi?

A) $m > n$. B) $m < n$. C) $m = n$. D) $m \neq n$.

E) ko'rsatilgan barcha hollarda tenglik o'rinli bo'ladi .

5. Davri 2π bo'lgan $f(x)$ funksiyaning Furye qatoridagi ozod had a_0 qaysi formula bilan aniqlanadi?

A) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$. B) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin xdx$. C) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos xdx$.

D) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)dx$. E) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |f(x)|dx$.

6. $f(x)=x+1$, $-\pi \le x \le \pi$, funksiyaning Furye qatorining ozod hadi a_0 qaysi formula bilan hisoblanadi?

A) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+x)^2 dx$. B) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+x)\sin xdx$.

C) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+x)\cos xdx$.

D) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+x)dx$. E) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |(1+x)|dx$.

7. $f(x)=x-1$, $-\pi \le x \le \pi$, funksiyaning Furye qatoridagi ozod had a_0 qiymati nimaga teng?

A) 2π . B) -2π . C) 2 . D) -2 . E) 0 .

8. Davri 2π bo'lgan $f(x)$ funksiya Furye qatorining $\cos kx$ oldidagi a_k koeffitsient qaysi formula bilan aniqlanadi?

A) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos^k xdx$.

B) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(x+k)xdx$.

C) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos kxdx$.

D) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x\cos kx)dx$.

E) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos x^k dx$.

9. $f(x)=x+1$, $-\pi\le x\le\pi$, funksiya Furye qatorining $\cos kx$ oldidagi a_k koeffitsient qaysi formula bilan hisoblanadi?

A) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)\cos^k xdx$. B) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)^k \cos xdx$.

C) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)\cos kxdx$. D) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)\cos x^k dx$;

E) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi}|1+x|\cos kxdx$.

10. $f(x)=x+1$, $-\pi\le x\le\pi$, funksiya Furye qatorining $\cos kx$ oldidagi a_k $(k\ne 0)$ koeffitsienti qiymati nimaga teng?

A) $a_k = \frac{1}{k}$ B) $a_k = \frac{(-1)^k}{k}$ C) $a_k = \frac{(-1)^k}{k^2}$ D) $a_k = (-1)^k$ E) $a_k = 0$.

11. Davri 2π bo'lgan $f(x)$ funksiy Furye qatorining $\sin kx$ oldidagi b_k koeffitsient qaysi formula bilan aniqlanadi?

A) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin^k xdx$ B) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x\sin kx)dx$

C) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi}|f(x)\sin kx|dx$ D) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin kxdx$

E) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi}|f(x)|\sin kxdx$

12. $f(x)=x+1$, $-\pi\le x\le\pi$, funksiya Furye qatorining $\sin kx$ oldidagi b_k koeffitsient qaysi formula bilan hisoblanadi?

A) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)\sin^k xdx$

B) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)^k \sin xdx$

C) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)\sin kxdx$

D) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+x)\sin x^k dx$

E) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi}|1+x|\sin kxdx$

13. $f(x)=x+1$, $-\pi\leq x\leq\pi$, funksiya Furye qatorining $\sin kx$ oldidagi b_k ($k\neq0$) koeffitsienti qiymati nimaga teng?

A) $b_k=0$

B) $b_k=\frac{(-1)^k}{k}$

C) $b_k=\frac{(-1)^k}{k^2}$.

D) $b_k=\frac{2}{k}(-1)^{k+1}$

E) $b_k=\frac{2}{k^2}(-1)^k$

14. Dirixle teoremasining tasdig'ini ko'rsating: Agar $f(x)$ davri 2π va $[-\pi,\pi]$ kesmada bo'lakli - monoton hamda chegaralangan funksiya bo'lsa, uning Furye qatori barcha x nuqtalarda yaqinlashuvchi va uning yig'indisini ifodalovchi $S(x)$ funksiya . . . .

I. barcha x nuqtalarda $f(x)$ qiymatga ega bo'ladi .

II. $f(x)$ funksiya uzluksiz bo'lgan x_0 nuqtada $f(x_0)$ qiymatga ega bo'ladi .

III. $f(x)$ funksiya uzlukli bo'lgan x_0 nuqtada $[f(x_0-0)+f(x_0+0)]/2$ qiymatga ega .

A) I . B) II . C) III . D) II va III .

E) to'g'ri javob keltirilmagan .

15. Dirixle teoremasida $f(x)$ funksiya uchun quyidagi shartlardan qaysi biri talab etilmaydi?

A) $f(x)$ funksiya 2π davrli .

B) $f(x)$ funksiya differensiallanuvchi .

C) $f(x)$ funksiya $[-\pi,\pi]$ kesmada bo'lakli - monoton .

D) $f(x)$ funksiya $[-\pi,\pi]$ kesmada chegaralangan .

E) keltirilgan barcha shartlar talab etiladi .

16. 2π davrli toq $f(x)$ funksiyaning Furye qatori qanday ko'rinishda bo'ladi?

A) $f(x)=\sum_{n=1}^{\infty}b_n(\cos nx-\sin nx)$.

B) $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$.

C) $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}b_n(\cos nx+\sin nx)$.

D) $f(x)=\sum_{n=1}^{\infty}b_n\sin nx$.

E) to'g'ri javob keltirilmagan .

Insondagi eng asl qobiliyatlardan biri-
matematik fikrlay olishidir.
Bernard Shou

XIII-BOB. IKKI O'LCHOVLI, UCH O'LCHOVLI VA EGRI CHIZIQLI INTEGRALLAR

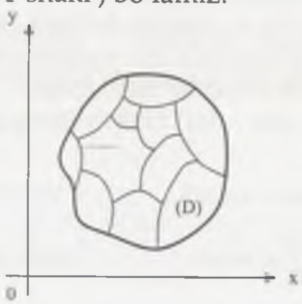
§ 13.1. Ikki o'lchovli integrallar va ularni hisoblash

xOy tekislikda *l* yopiq chiziq bilan chegaralangan *D* soha va unda uzluksiz $z = f(x,y)$ funksiya bilan aniqlangan sirt berilgan bo'lsin.

D sohani ixtiyoriy chiziqlar bilan *n* ta:

$\Delta S_1, \Delta S_2, ..., \Delta S_n$

bo'laklarga (13. 1-shakl) bo'lamiz.



13. 1-shakl

Ularning yuzalarini mos ravishda (belgilashni o'zgartirmasdan) $\Delta S_1, \Delta S_2, ..., \Delta S_n$ deb belgilaymiz. Har bir bo'lakchada ixtiyoriy (xoh bo'lakchaning ichida , xoh chegarada bo'lsin) *n* ta

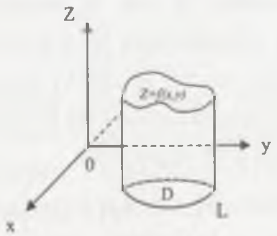
$P_1, P_2, ..., P_n$ nuqtalarni tanlaymiz. Tanlangan nuqtalarning har birida funksiyaning $f(P_1), f(P_2), ..., f(P_n)$ qiymatlarini hisoblaymiz va $f(P_i) \cdot \Delta S_i$ ko'rinishidagi ko'paytmalardan yig'indi tuzamiz:

$$V_n = f(P_1) \cdot \Delta S_1 + f(P_2) \cdot \Delta S_2 + ... + f(P_n) \cdot \Delta S_n = \sum_{i=1}^{n} f(P_i) \cdot \Delta S_i \qquad (13.1)$$

Bu yig'indi D sohada $f(x,y)$ funksiya uchun integral yig'indi deyiladi.

Agar D sohada $f(x,y) \geq 0$ bo'lsa, u holda $f(P_i) \cdot \Delta S_i$ ko'paytmani geometrik ma'noda asosi ΔS_i, balandligi $f(P_i)$ bo'lgan kichik silindrchaning hajmi deb qarash mumkin.

V_n yig'indi, elementar (kichik) silindrchalar hajmlari yig'indisidan iborat bo'lib, yuqoridan tenglamasi $z = f(x,y)$ ($f(x,y) \geq 0$) bo'lgan sirtning qismi, pastdan $z = 0$ tekislikdagi D soha va yasovchisi Oz o'qqa parallel, yo'naltiruvchisi D-sohaning chegarasi l chiziqdan iborat bo'lgan silindrik sirt bilan chegaralangan jismning (12.2 -shakl) hajmidan iborat bo'ladi.



13.2 -shakl

Ta'rif (1). Integral yig'indining $n \to \infty$ da, ΔS_i bo'lakchaning eng katta diametri nolga intilgandagi limitiga (agar bu limit mavjud bo'lsa) D sohada $f(x,y)$ funksiyadan olingan ikki o'lchovli integral deyiladi va

$$\iint_{(D)} f(P) dS \text{ yoki } \iint_{(D)} f(x,y) dxdy$$

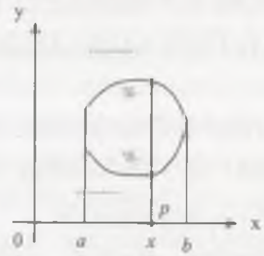
ko'rinishida belgilanadi, ya'ni

$$V = \lim_{diam \Delta S_i \to 0} \sum_{i=1}^{n} f(P_i) \cdot \Delta S_i = \iint_{(D)} f(x,y) dxdy \qquad (13.2)$$

Bu yerda D-integrallash sohasi deyiladi.

Ikki o'lchovli (1) integralni hisoblash masalasini qaraymiz.

D soha $y=f_1(x)$ va $y=f_2(x)$, $x=a$ va $x=b$ chiziqlar bilan chegaralangan egri chiziqli trapetsiya bo'lsin (13.3-shakl).



13.3-shakl

Bunda $f_1(x)$ va $f_2(x)$ funksiyalar $[a;b]$ kesmada uzluksiz bo'lib, $f_1(x)\le f_2(x)$, $a<b$ deb qabul qilamiz. D soha shundayki,uning ichki nuqtalaridan o'tadigan va koordinata o'qlaridan biriga, masalan Oy o'qiga parallel har qanday to'g'ri chiziq, uning chegarasini M_1 va M_2 nuqtalarda kesib o'tadi. Bunday soha Oy o'qi yo'nalishida to'g'ri soha deyiladi. [Ox o'qi bo'yicha to'g'ri soha ham xuddi shunday aniqlanadi].

$f(x,y)$ funksiya D sohada uzluksiz bo'lsin.

Ikki karrali integral deb ataluvchi

$$J_D=\int_a^b\left(\int_{f_1(x)}^{f_2(x)} f(x,y)dy\right)dx$$

ifodani qaraymiz. Bu integralni hisoblash uchun avvalo x ni o'zgarmas miqdor deb qarab, ichki integral hisoblanadi. Natija x ga bog'liq bo'lgan uzluksiz funksiya bo'ladi,

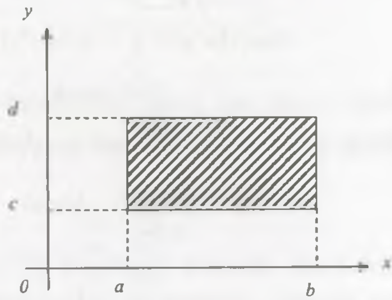
$$\Phi(x)=\int_{f_1(x)}^{f_2(x)} f(x,y)dy$$

Bu funksiyani $[a,\ b]$ kesmada x o'zgaruvchi bo'yicha integrallasak,

$$J_D=\int_a^b \Phi(x)dx$$

Natija qandaydir o'zgarmas son bo'ladi.

Xususiy holda to'g'ri soha to'g'ri to'rtburchak ko'rinishida bo'lishi ham mumkin. Bu holda *D* soha $x=a, x=b, y=c, y=d$ to'g'ri chiziqlar bilan chegaralangan bo'ladi (13.4-shakl). Ham *Ox*, ham *Oy* o'qlar yo'nalishida to'g'ri bo'lgan soha qisqacha to'g'ri soha, yoki standart soha deb ataladi.



13.4-shakl

Biz yuqorida, 2-shaklda ifodalangan jismning hajmini ikki o'lchovli

$$V=\iint_D f(x,y)dxdy \quad (13.3)$$

integral yordamida hisoblagan edik. Endi shu hajmni parallel kesimlar yuzalari yordamida hisoblaymiz.

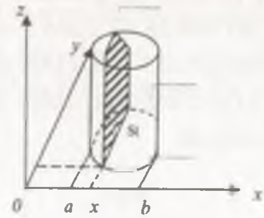
Qaralayotgan jismni $x=const(a<x<b)$ tekislik bilan kesamiz. Kesimda hosil bo'lgan $S(x)$ yuzani hisoblaymiz (13.5-shakl). Bu $z=f(x,y)$ $(x=const)$, $z=0$, $y=f_1(x)$, $y=f_2(x)$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzasidir.

Bu yuza

$$S(x)=\int_{f_1(x)}^{f_2(x)} f(x,y)dy \quad (13.4)$$

Injtegral bilan ifodalanadi. Parallel kesimlar yuzalarini bilgan holda jismning hajmini oson hisoblash mumkin.

$$V=\int_a^b S(x)dx$$



13.5- shakl

yoki $S(x)$ yuza uchun (6) ifodani qo'ysak,

$$V=\int_a^b\left(\int_{f_1(x)}^{f_2(x)}f(x,y)dy\right)dx \tag{13.5}$$

formulani hosil qilamiz (13.3) va (13.5) formulalarda chap tomonlar teng, demak, ularning o'ng tomonlari ham teng bo'ladi.

$$V=\iint_D f(x,y)dxdy=\int_a^b\left(\int_{f_1(x)}^{f_2(x)}f(x,y)dy\right)dx \tag{13.6}$$

Tekis figuralarning yuzi S ham ikki o'lchovli integral ko'rinishida berilishi mumkin. Bunda $f(x,y)=1$ deb olinadi.

$$S=\iint_D dxdy \tag{13.7}$$

Bu yerda integrallash sohasi to'g'ri soha bo'lsa, u holda:

$$S=\int_a^b\left(\int_{f_1(x)}^{f_2(x)}f(x,y)dy\right)dx=\int_a^b[f_2(x)-f_1(x)]dx \tag{13.8}$$

Ikki o'lchovli integralni hisoblashda o'zgaruvchilarni almashtirish ba'zan qulaylik beradi.

Faraz qilaylik,

$$\left.\begin{matrix}u=u(x,y)\\ v=v(x,y)\end{matrix}\right\} \tag{13.9}$$

Funksiyalar berilgan, bu funksiya xOy tekislikning biror D sohasida aniqlangan, uzluksiz xususiy hosilaga ega.

$$\left.\begin{matrix}x=x(u,v)\\ y=y(u,v)\end{matrix}\right\} \tag{13.10}$$

bo'lsin, u holda D sohaning har bir $u(x,y)$ nuqtasiga biror qiymat aniqlovchi (u,v) sonlar jufti mos keladi, ya'ni quyidagi formula o'rinli bo'ladi

$$\iint_D f(x,y)dxdy = \iint_{D_1} f[x(u,v), y(u,v)][J(u,v)]dudv \qquad (13.11)$$

yoki

$$\iint_D f(x,y)dxdy = \iint_{D_1} F(u,v)|J|dudv \qquad (13.12)$$

Bu yerda J determinant $x(u,v)$ va $y(u,v)$ funksiyalarning funksional determinanti yoki yakobian deb yuritilib, uning qiymati

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \qquad (13.11)$$

determinant bilan hisoblanadi.

Formula D soha bo'yicha olingan ikki o'lchovli integralni hisoblashni D_1 soha bo'yicha olingan ikki o'lchovli integralni hisoblashga olib kelishga imkon beradi.

Xususiy holda qutb koordinatalar sistemasida (13.10)-formula quyidagi ko'rinishiga ega bo'ladi.

$$\left.\begin{array}{l} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{array}\right\} \qquad (13.12)$$

Bu almashtirishda yakobian quyidagicha hisoblanadi

$$J(\rho,\varphi) = \frac{\partial x}{\partial \rho}\cdot\frac{\partial y}{\partial \varphi} - \frac{\partial x}{\partial \varphi}\cdot\frac{\partial y}{\partial \rho} = cos\rho cos\varphi + \rho\sin\varphi\sin\varphi = \rho(cos^2\varphi + \sin^2\varphi) = \rho$$

U vaqtda (12.11) formula quyidagi ko'rinishni oladi.

$$\iint_D f(x,y)dxdy = \iint_{D_1} f(\rho\cos\varphi, \rho\sin\varphi)\rho d\rho d\varphi \qquad (13.13)$$

Agar D soha qutb burchaklari φ_1 va φ_2 $(\varphi_1 < \varphi_2)$bo'lgan nurlar va $\rho = \rho_1(\varphi)$ $\rho = \rho_2(\varphi)$ $(\rho_1 < \rho_2)$ egri chiziqlar bilan chegaralangan bo'lsa, bu sohaga mos keluvchi qutb koordinatalari

$$D_1\{\varphi_1 \le \varphi \le \varphi_2; \quad \rho_1(\varphi) \le \rho \le \rho_2(\varphi)\}$$

sohada o'zgaradi va (13.13) formula quyidagi ko'rinishni oladi

$$\iint_D f(x,y)dxdy = \int_{\varphi_1}^{\varphi_2} d\varphi \int_{\rho_1(\varphi)}^{\rho_2(\varphi)} f(\rho\cos\varphi, \rho\sin\varphi)dy\rho d\rho$$

Ikki o'lchovli integrallarni hisoblang

$\iint_{(S)} f(x,y)dxdy$ integralning integrallanish chegralari aniqlansin. Agarda integrallanish sohasi S : $y^2-x^2=1$ giperbola va $x=2$ va $x=-2$ ikkita to'g'ri chiziqlar bilan chegaralangan soha.

13.2. $\iint_D xydxdy, \quad (0\le x\le 1, 0\le y\le 1)$

13.3 . $\iint_D e^{x+y}dxdy, \quad (0\le x\le 1, 0\le y\le 2)$

13.4. $\iint_D \sin(x+y)dxdy, \quad \left(0\le x\le \pi, 0\le y\le \frac{\pi}{2}\right)$

13.5 $x=0$, $y=3$ to'g'ri chiziqlar va $y=x^2$ parabola bilan chegaralangan D sohaning yuzini toping.

13.6. $x=1$, $x=2$, $y=0$ to'g'ri chiziqlar $y=e^x$ egri chiziq bilan chegaralangan D sohaning yuzasi hisoblansin.

Ikki karrali integrallarni hisoblang

13.7 $\int_0^1 dx \int_{x^2}^{\sqrt{x}} \frac{x}{y}dy$ b) $\int_0^1 dx \int_{x^2}^{\sqrt{x}} xydy$ ikki karrali integrallarni hisoblang.

13.8. $\int_0^1 dx \int_{x^2}^{\sqrt{x}} xydy$ ikki karrali integrallarni hisoblang.

13.9. $\int_2^4 dx \int_1^2 xydy$ **13.10.** $\int_3^5 dx \int_0^2 (x+y)dy$ **13.11.** $\int_0^1 dy \int_0^1 e^{x-y}dy$

13.12. $\int_{\pi/2}^0 dy \int_0^{\pi/2} \cos(x+y)dy$

$\iint_D (x+y)dxdy$ **ikki o'lchovli integral hisoblansin. Bu yerda** D **soha uchburchak bo'lib, quyidagi to'g'ri chiziqlar bilan chegaralangan.**

13.13. $x=0$, $y=0$, $x+y=3$ **13.14.** $x=a$, $y=0$, $y=x$

13.15. $\iint_D xdydx$ integralni $xy=6$, $x+y-7=0$ chiziqlar bilan chegaralangan D sohada hisoblang.

13.16. $\iint\limits_D xy^2 dxdy$ integral $x^2+y^2=4,\ \ x+y-2=0$ chiziqlar bilan chegaralangan D sohada hisoblang.

13.17. $\iint\limits_D x^2 ydxdy$ integral $xy=1,\ \ y-x=0$ va $x=2$ chiziqlar bilan chegaralangan D sohada hisoblang

13.18. $J=\int_0^4 dy \int_0^y \frac{y^3}{x^2+y^2}dx$ ikki karrali integralni hisoblang.

13.19. $\int_0^{2\pi} dy \int_0^{1+\sin y} \frac{x}{3}dx$ ikki karrali integral hisoblansin.

13.20. $\int_0^{\frac{1}{2}} dx \int_0^{\sqrt{1-4x^2}} (x+y)dy$ ikki karrali integral hisoblansin.

Karrali integralda integrallash tartibi o'zgartirilsin

13.21. $\int_2^4 dy \int_y^4 f(x,y)dx$ 13.22. $\int_1^2 dy \int_{4y}^{y} zdx$

Ikki o'lchovli integralda integrallanish chegaralari aniqlansin

13.23. $\iint\limits_D f(x,y)dxdy$ D soha $x^2+y^2=4x,\ \ x^2+y^2=8x$ aylanalar va $y=x,\ \ y=2x$ to'g'ri chiziqlar bilan chegralangan

13.24. $\iint\limits_D f(x,y)dxdy$ D soha $x=1,\ \ y=x,\ \ y=0$ to'g'ri chiziqlar bilan chegralangan sohaning yuzasi hisoblansin.

13.25. Markazi koordinata boshida va radiusi ρ bo`lgan doiraning yuzasi topilsin.

13.26 $z=0,\ \ y+z=2$ tekisliklar va $y=x^2$ silindr bilan chegaralangan jismning hajmi topilsin.

13.27 $x=1,\ \ y=0$ to'g'ri chiziqlar va $y=x^3$ parabola bilan chegaralangan sohaning yuzasi hisoblansin.

13.28. $x=0,\ \ x=2,\ \ y=0$ to'g'ri chiziqlar va $y=e^x$ egri chiziq bilan chegaralangan sohaning yuzasi hisoblansin.

13.29. $x=0,\ \ y=1,\ \ y=3$ to'g'ri chiziqlar va $y=e^x$ egri chiziq bilan chegaralangan sohaning yuzasi hisoblansin.

13.30. $y=1,\ \ y=-x$ to'g'ri chiziqlar va $x^2+y^2=-2y$ aylanalar bilan chegaralangan sohaning yuzasi hisoblansin.

13.31. $y^2 = x + 2$ parabola va $x = 2$ to`g'ri chiziq bilan chegaralangan sohaning yuzasi hisoblansin.

13.32. $y = x,\ \ y = 5x$ va $x = 1$ to'g'ri chiziqlar bilan chegaralangan sohaning yuzasi topilsin.

13.33. $y = \sqrt{x},\ \ y = 2\sqrt{x}$ parabolalar va $x = 4$ to`g'ri chiziqlar va bilan chegaralangan sohaning yuzasi hisoblansin.

13.34 $\iint\limits_D \sqrt{1-(x^2+y^2)}dxdy$ ikki o'lchovli integralda o'zgaruvchi almashtirilsin. Bu yerda D soha $x^2 + y^2 = 1$ aylana bilan chegaralangan.

13.35. $\iint\limits_D \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}dxdy$ ikki o'lchovli integral qutb koordinatalar sistemasida hisoblansin. D soha $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips bilan chegralangan.

13.36. $\iint\limits_D \sqrt{4-\frac{x^2}{a^2}-\frac{y^2}{b^2}}dxdy$ ikki o'lchovli integral hisoblansin. D soha: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ va $\frac{x^2}{4a^2}+\frac{y^2}{4b^2}=1$ chiziqlar bilan chegralangan.

Quyidagi integrallar qutb koordinatalar sistemasiga o`xshash yo`li bilan hisoblansin

13.37. $\int\limits_0^c \int\limits_0^{\sqrt{c-x^2}} \sqrt{c^2-x^2y^2}\,dxdy$ **13.38.** $\int\limits_0^a \int\limits_0^{\sqrt{c^2-y^2}} (x^2+y^2)dxdy$

13.39. $y = 0,\ \ y = x$ to'g'ri chiziqlar va $x^2 + y^2 = 2x$ aylana bilan chegaralangan D sohaning yuzasi hisoblansin.

13.40. Pastdan va yon tomondan koordinata tekisliklari va $x + y - 3 = 0$ tekislik bilan yuqiridan $z = 4x^2 + 2y^2 + 1$ elliptik paraboloidning mos qismi bilan chegaralangan jismning hajmini hisoblang.

13.41. $x = \frac{y^2}{b^2}+\frac{z^2}{c^2}$ elliptik paraboloidning $x = k\ (k > 0)$ tekislik bilan kesilgan qismining hajmini toping.

13.42. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning hajmini toping.

13.43. $x=0,\ y=0,\ z=0,\ x=y,\ y=4$ tekisliklar va $z=x^2+y^2+1$ paraboloid bilan chegaralangan jismning hajmi topilsin.

13.44. $x=0,\ y=0,\ z=0$ va $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ tekisliklar bilan chegaralangan jismning hajmi topilsin.

13.45. $x=0,\ y=0,\ z=0$ va $2x+3y-12=0$ tekisliklar bilan chegaralangan jismning hajmi topilsin.

13.46. $x^2+y^2-az=0$ paraboloid, $(x^2+y^2)^2=a^2(x^2-y^2)$ silindr va $z=0$ tekislik bilan chegaralangan jismning hajmi topilsin.

13.47. $z=0,\ x+y=1,\ x+y+z=3$ va $2x+3y-12=0$ tekisliklar bilan chegaralangan jismning hajmi topilsin.

13.48. $x^2+y^2=R^2$ silindr bilan kesilgan $x^2+y^2+z^2=a^2$ sfera sirtining yuzasi topilsin.

13.49. $z^2=x^2+y^2$ konusning $x^2+y^2=2x$ silindr bilan kesilganda hosil bo`lgan sirtning yuzasi topilsin.

13.50. $x^2+y^2=Rx$ silindr bilan $x^2+y^2+z^2=R^2$ sfera kesilganda hosil bo`lgan sirtning yuzasi topilsin.

13.51. $x^2+y^2=a^2$, $y^2+z^2=a^2$ silindrlarning umumiy qismi bo`lgan sirtning yuzasi topilsin.

13.52. $2z=x^2+y^2$ paraboloidning $x^2+y^2=1$ silindr bilan kesilgan qismining yuzasi topilsin.

13.53. Bir jinsli doiraning radiusi R gat eng bo'lsa, uning urinmaga nisbatan statik momentini toping.

13.54. Radiusi R ga teng bo`lgan doira markaziga va diametriga nisbatan inertsiya momenti topilsin. (Doira xOy tekisligida bo'lib, $\mu=1$ bo'lsin).

13.55. $x^2+y^2=a^2$ doira ustki yarmi og'irlik markazining koordinatalari aniqlansin.

13.56. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning a) Oy o`qiga nisbatan, b) koordinata boshiga nisbatan inertsiya momentlari topilsin.

§ 13.2. Massasi tekis taqsimlangan yuzaning (zichligi $\mu = 1$ bo'lganda) og'irlik markazi va inertsiya momentlari

Massasi tekis taqsimlangan S yuzaning og'irlik markazi koordinatalari:

$$x_c = \frac{\iint xdxdy}{S}, \quad y_c = \frac{\iint ydxdy}{S} \tag{13.14}$$

S yuzaning inertsiya momentlari:

$$I_x = \iint\limits_{(S)} y^2 dxdy, \quad I_y = \iint\limits_{(S)} x^2 dxdy, \quad I_0 = \iint\limits_{(S)} r^2 dxdy \tag{13.15}$$

Quyidagi chiziqlar bilan chegaralangan yuzaning og'irlik markazi topilsin:

13.57. $y = 0$ va $y = \sin x$ sinusoidaning bitta yarim to'lqini.

13.58. $y = x^2, \quad x = 4, \quad y = 0$

13.59. $y^2 = ax$ va $y = x$.

13.60. $x^2 + y^2 = a^2$ va $y = 0$.

13.61. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ astroida va Ox o'q bilan chegaralangan yuzaning og'irlik markazi.

Quyidagilarning og'irlik markazlari aniqlansin :

13.62. $y^2 = ax, \quad x = a, \quad y = 0$ lar bilan chegaralangan parabola yarim segmentining ($y > 0$ bo'lganda).

13.63. Ox o'q bilan kesilgan $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ yarim ellisning.

13.64. $y = a + \frac{x^2}{a}, \quad y = 2x$ va $x = 0$ chiziqlar bilan chegaralangan yuzning Oy o'qqa nisbatan inertsiya momenti aniqlansin.

13.65. Uchlari $A(1;1)$, $B(2;1)$, $C(3;3)$ nuqtalarda bo'lgan uchburchak yuzining Ox o'qqa nisbatan inertsiya momenti aniqlansin.

§ 13.3. Uch o'lchovli integral va uning tadbiqi

Agar (V) soha $a \le x \le b, \quad y_1(x) \le y \le y_2(x), \quad z_1(x_1, y) \le z \le z_2(x_1, y)$ tengsizliklar bilan aniqlangan bo'lsa, u holda

$$\iiint\limits_{(V)} F(x,y,z)dxdydz = \int\limits_a^b dx \int\limits_{y_1(x)}^{y_2(x)} dy \int\limits_{z_1(x,y)}^{z_2(x,y)} F(x,y,z)dz$$

$F(x,y,z)=1$ bo'lganda V ning hajmi hosil bo'ladi. Hajmi V ga teng bo'lgan bir jinsli jismning og'irlik markazi koordinatalari quyidagi formulalar bilan topiladi:

$$x_c = \frac{1}{V}\iiint\limits_{(V)} xdxdydz\,, \quad y_c = \frac{1}{V}\iiint\limits_{(V)} ydxdydz \text{ va hokazo.}$$

13.66. $az = x^2 + y^2$, $2az = a^2 - x^2 - y^2$ sirtlar bilan chegaralangan jismning hajmi aniqlansin.

13.67. $x^2 + y^2 - z^2 = 0$, $x^2 + y^2 + z^2 = a^2$ sirtlar bilan chegaralangan jismning konus ichidagi qismining hajmi aniqlansin.

13.68. $x^2 + y^2 - z^2 = 0$ konus sirti $x^2 + y^2 + z^2 = 2az$ sharning hajmini 3:1 nisbatda bo'lishi ko'rsatilsin.

13.69. Yoqlari $x + y + z = a$, $x = 0$, $y = 0$, $z = 0$ tekisliklar bilan tashkil etilgan piramidaning har bir nuqtasidagi zichlik shu applikatasi z ga teng. Piramidaning massasi aniqlansin.

Quyidagi sirtlar bilan chegaralangan bir jinsli jismning og'irlik markazi aniqlansin:

13.70. $x + y + z = a$, $x = 0$, $y = 0$, $z = 0$ 13.71. $az = a^2 - x^2 - y^2$, $z = 0$

Quyidagi uch o'lchovli integralni hisoblang.

13.72. $\int\limits_0^1 dx \int\limits_0^2 dy \int\limits_0^3 dz$ 13.73. $\int\limits_0^a dx \int\limits_0^b dy \int\limits_0^c (x+y+z)dz$

13.74. $\int\limits_0^a dx \int\limits_0^x dy \int\limits_0^x xyzdz$ 13.75. $\int\limits_0^a dx \int\limits_0^x dy \int\limits_0^{xy} x^3y^3zdz$ 13.76. $\int\limits_0^{e-1} dx \int\limits_0^{e-x-1} dy \int\limits_0^{x+y+e} \frac{\ln(z-x-y)}{(x-e)(x+y-e)}dz$

§ 13.4. Birinchi tur egri chiziqli integrallar

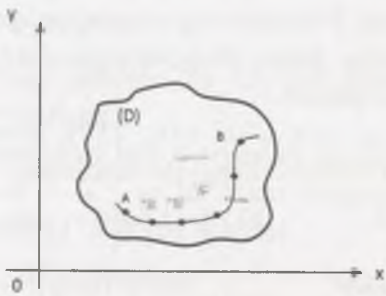
xOy tekislikning biror D sohasida tenglamasi $y = \varphi(x)$ $(a \le x \le b)$ bo'lgan L silliq chiziq to'laligicha joylashgan (yotgan) bo'lsin. $f(x,y)$ funksiya shu L chiziqdagi AB yoyining har bir nuqtasida aniqlangan va uzluksiz bo'lsin.

AB yoyni $A = A_0$, A_1, $A_2, \ldots, A_n = B$ nuqtalar yordamida uzunliklari $\Delta l_1, \Delta l_2, \ldots, \Delta l_n$ bo'lgan n ta ixtiyoriy bo'laklarga bo'lamiz.

Bu bo'laklarning har birida ixtiyoriy ravishda $M_1(x_1,y_1)$, $M_2(x_2,y_2),...,M_n(x_n,y_n)\left(M_i(x_i,y_i)\in\Delta l_i, i=\overline{1,n}\right)$ nuqtalarni tanlab olamiz. Tanlab olingan nuqtalarning har birida $f(x,y)$ funksiyaning qiymatlarini hisoblab

$$f(x_1,y_1)\cdot\Delta l_1+f(x_2,y_2)\cdot\Delta l_2+...+f(x_n,y_n)\cdot\Delta l_n=\sum_{i=1}^{n}f(x_i,y_i)\cdot\Delta l_i \qquad (13.16)$$

yig'indini tuzamiz. Hosil bo'lgan bu yig'indi L chiziqning AB yoyida $f(x,y)$ funksiyaning integral yig'indisi deyiladi. Bu yerda shuni ta'kidlash kerakki, har qanday berilgan $f(x,y)$ funksiya va AB yoy uchun bu yoyni har xil ko'rinishda n ta bo'lakka bo'lib, ularda bittadan $M_i\in\Delta l_i$ nuqtalarni tanlab olish mumkin. Bu yerda shuni ta'kidlash kerakki, har qanday berilgan $f(x,y)$ funksiya va AB yoy uchun bu yoyni har xil ko'rinishda n ta bo'lakka bo'lib, ularda bittadan $M_i\in\Delta l_i$ nuqtalarni tanlab olish mumkin.



13.6- shakl

Natijada (13.16) ko'rinishdagi to'g'ri integral yig'indilarni tuza olamiz. n ni cheksiz orttirib,

$$\max_{n\to\infty}\Delta l_i\to 0 \qquad (*)$$

bo'lganda (13.16) integral yig'indining limitini (agar mavjud bo'lsa) hisoblash mumkin.

Ta'rif: (*) shartlar asosida (13.16) ko'rinishdagi integral yig'indilarning limiti mavjud bo'lib, u yagona bo'lsa, bu limit

$f(x,y)$ funkiyadan AB yoy bo'yicha olingan I tur egri chiziqli integral deyiladi va

$$\int_{AB} f(x,y)dl = \lim_{\substack{\max \Delta l_i \to 0 \\ n\to\infty}} \sum_{i=1}^{n} f(x_i, y_i)\cdot \Delta l_i \quad (13.17)$$

ko'rinishda yoziladi. Bu yerda dl-yoy differensiali deyiladi va $dl = \sqrt{(dx)^2 + (dy)^2}$ formula yordamida hisoblanadi.

I tur egri chiziqli integralni hisoblashda quyidagi hollar bo'lishi mumkin.

1. Agar L chiziqdagi AB yoyning tenglamasi $y=\varphi(x)$ $(a \le x \le b)$ ko'rinishda bo'lsa, u holda (13.17) I tur egri chiziqli integral

$$\int_{AB} f(x,y)dl = \int_{AB} f[x;\varphi(x)]\sqrt{1+[\varphi(x)]^2}\,dx \quad (13.18)$$

formula yordamida hisoblanadi.

2. Agar L chiziqning tenglamasi $x=\psi(y)$ $(c \le y \le d)$ ko'rinishda bo'lsa, u holda (13.17) integral

$$\int_{AB} f(x,y)dl = \int_{AB} f[\psi(y);y]\sqrt{[\psi(y)]^2+1}\,dy \quad (13.18')$$

formula yordamida hisoblanadi.

3.Agar L egri chiziq parametrik ko'rinishdagi $x=\varphi(t)$, $y=\psi(t)$ $(t_1 \le t \le t_2)$ tenglamalar bilan berilgan bo'lsa, u holda (13.17) egri chiziqli integral

$$\int_{AB} f(x,y)dl = \int_{t_1}^{t_2} f[\varphi(t),\psi(t)]\sqrt{[\varphi(t)]^2+[\psi(t)]^2}\,dt \quad (13.19)$$

formula yordamida hisoblanadi.

4. Agar L egri chiziq fazoda $x=x(t)$, $y=y(t)$ va $z=z(t)$ $(t_1 \le t \le t_2)$ tenglama bilan berilgan bo'lsa, u holda (13.17) integral

$$\int_{AB} f(x,y)dl = \int_{t_1}^{t_2} f[x(t),y(t),z(t)]\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt \quad (13.20)$$

formula yordamida hisoblang.

13.77. $\int_L (x-y)dl$ egri chiziqli integralni L-to'g'ri chiziqning A(0;0) va B(4;3) nuqtalari orasidagi kesmada hisoblang.

13.78. $\int_L (x^2+y^2)dl$ egri chiziqli integralni to'g'ri chiziqning A(a;a) va B(b;b) nuqtalari orasidagi kesmada hisoblang $(b>a)$.

13.79. $\int\limits_L y^2 dl$ egri chiziqli integralni $x=\ln y$ funksiyaning $y=1$ va $y=2$ nuqtalari orasidagi yoyda hisoblang.

13.80. $A(2;\ 2)$ va $B(2;\ 0)$ nuqtalar berilgan. 1) OA to'g'ri chiziq; 2) $y=\frac{x^2}{2}$ parabolaning $\breve{OA}$ yoyi; 3) OBA siniq chiziq bo'yicha $\int\limits_{(C)}(x+y)dx$ hisoblansin.

Quyidagi egri chiziqli integrallarni hisoblang.

13.81. $\int\limits_L \frac{ds}{x-y}$, bu yerda L: $y=\frac{1}{2}x-2$ to'g'ri chiziqning nuqtalari $A(0;\ -2)$ va $B(4;\ 0)$ orasidagi kesma.

13.82. $\int\limits_L xyds$, bu yerda L: uchlari $A(0;\ 0)$, $B(4;\ 0)$, $C(4;\ 2)$ va $D(0;\ 2)$ bo'lgan to'rtburchak.

13.83. $\int\limits_L ydl$, bu yerda L: $y^2=2px$ parabolaning $A(0;\ 0)$ va B(2p;2p) nuqtalari orasidagi L yoyni hisobalng.

13.84. $\int\limits_L (x^2+y^2)^n ds$, bu yerda L: $x=a\cos t,\quad y=a\sin t$ aylana.

13.85. $\int\limits_L xyds$, bu yerda L: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning choragi.

§13.5. Ikkinchi tur egri chiziqli integrallar

$P(x,y)$ va $Q(x,y)$ funksiyalar $y=\varphi(x)$ $(a\le x\le b)$ tenglama orqali ifodalangan L egri chiziqning AB yoyi nuqtalarida aniqlangan va uzluksiz bo'lsin. AB yoyni $A=A_0,A_1,A_2,...,A_{n-1},A_n=B$ nuqtalar yordamida ixtiyoriy ravishda uzunliklari $\Delta l_1,\Delta l_2,...,\Delta l_n$ bo'lgan bo'laklarga bo'lamiz.

Bu bo'laklarda $M_i(x_i;y_i)$ $(i=\overline{1,n})$ nuqtalarni ixtiyoriy tanlab

$$\sum_{i=1}^{n}\left[P(x_i;y_i)\cdot\Delta x_i+Q(x_i;y_i)\cdot\Delta y_i\right] \qquad (**)$$

ko'rinishdagi yig'indini hosil qilamiz. Bu yig'indi $P(x,y)$ va $Q(x,y)$ funksiyalar uchun koordinatalar bo'yicha integral yig'indi deyilib, unda $\Delta x_i,\Delta y_i$ miqdorlar Δl_i yoy bo'lakchasining mos ravishda Ox va Oy o'qlaridagi proyeksiyalarini (akslarini) bildiradi.

Ta'rif: (**) integral yig'indining $\max_{n\to\infty} \Delta x_i \to 0$ va $\max_{n\to\infty} \Delta y_i \to 0$ bo'lganda limiti mavjud bo'lsa va u $M_i(x_i;y_i)$ $(i=\overline{1,n})$ nuqtalarning tanlab olinishiga bog'liq bo'lmasa, u holda bu limit $P(x,y)dx+Q(x,y)dy$ ifodadan AB yoy yo'nalishi bo'yicha olingan II tur (yoki koordinatalar bo'yicha) egri chiziqli integral deyiladi va

$$\int_{AB} P(x,y)dx + Q(x,y)dy$$

ko'rinishda yoziladi.

Ta'rifga asosan

$$\int_{AB} P(x,y)dx+Q(x,y)dy = \lim_{\substack{n\to\infty \\ \max\Delta x\to 0 \\ \max\Delta y\to 0}} \sum_{i=1}^{n}\left[P(x_i;y_i)\cdot\Delta x_i + Q(x_i;y_i)\cdot\Delta y_i\right] \qquad (13.21)$$

formula II tur egri chiziqli integralni hisoblash formulasi bo'ladi.

II tur egri chiziqli integralni hisoblashda quyidagi hollar bo'lishi mumkin.

1. AB yoy $y=\varphi(x)$ $(a\le x\le b)$ tenglama bilan berilgan bo'lsa, u holda II tur egri chiziqli integral

$$\int_{AB} P(x,y)dx + Q(x,y)dy = \int_a^b \{P[x,\varphi(x)]+Q[x,\varphi(x)]\varphi'(x)\}dx \qquad (13.22)$$

formula bo'yicha hisoblanadi.

1'. Agar $x=\psi(y)$ $(c\le y\le d)$ bo'lsa, u holda

$$\int_{L} P(x,y)dx + Q(x,y)dy = \int_c^d \{P[\psi(y),y]\cdot\psi'(y)+Q[\psi(y),y]\}dy \qquad (13.22')$$

formula bo'yicha hisoblanadi.

2. Agar L egri chiziq parametrik ko'rinishdagi $x=\varphi(t)$, $y=\psi(t)$ $(t_1\le t\le t_2)$ tenglamalar bilan berilgan bo'lsa, u holda II tur egri chiziqli integral

$$\int_{L} P(x,y)dx + Q(x,y)dy = \int_{t_1}^{t_2} \{P[\varphi(t),\psi(t)]\cdot\varphi'(t)+Q[\varphi(t),\psi(t)]\cdot\psi'(t)\}dt \qquad (13.23)$$

formula bo'yicha hisoblanadi.

3. Agar L egri chiziq fazoda $x=x(t),\quad y=y(t),\quad z=z(t)\quad (t_1\le t\le t_2)$ formulalar bilan berilgan bo'lsa, u holda II tur egri chiziqli integral

$$\int_L P(x,y,z)dx+Q(x,y,z)dy+R(x,y,z)dz=$$

$$=\int_{t_1}^{t_2}\{P[x(t),y(t),z(t)]\cdot x'(t)+Q[x(t),y(t),z(t)]\cdot y'(t)+R[x(t),y(t),z(t)]\cdot z'(t)\}dt$$

(13.24)

formula bo'yicha hisoblanadi.

13.86. $\int_L xdy$ egri chiziqli integral $\frac{x}{a}+\frac{y}{b}=1$ to'g'ri chiziqning absissalar o'qi bilan kesishgan nuqtasidan ordinatalar o'qi bilan kesishgan nuqtasigacha bo'lgan kesmada hisoblansin.

13.87. $\int_L xydx+(y-x)dy$ egri chiziqli integral

1) $y=x$, 2) $y=x^2$, 3) $y=x^3$,

4) $y^2=x$ egri chiziqlarning $A(0;0)$, $B(1;1)$ nuqtalar orasidagi burchagida hisoblansin.

13.88. $y=x^2,\quad dy=2xdx,\quad 0\le x\le 1$, (13.22) formulaga ko'ra

$$\int_L xydy+(y-x)dy=\int_0^1\left[x\cdot x^2+\left(x^2-x\right)\cdot 2x\right]dx=\int_0^1\left(x^3+2x^3-2x^2\right)dx=$$

$$=\int_0^1 3x^3dx-\int_0^1 2x^2dx=3\cdot\frac{x^4}{4}\Big|_0^1-2\cdot\frac{x^3}{3}\Big|_0^1=\frac{3}{4}-\frac{2}{3}=\frac{1}{12}$$

13.89. $y=x^3,\quad dy=3x^2dx,\quad 0\le x\le 1$, (13.22) formuladan foydalansak, u holda

$$\int_L xydy+(y-x)dy=\int_0^1\left[x\cdot x^3+\left(x^3-x\right)\cdot 3x^2\right]dx=\int_0^1\left(x^4+3x^5-3x^3\right)dx=$$

$$=\frac{x^5}{5}\Big|_0^1+3\cdot\frac{x^6}{6}\Big|_0^1-3\cdot\frac{x^4}{4}\Big|_0^1=\frac{1}{5}+\frac{1}{2}-\frac{3}{4}=-\frac{1}{20}$$

13.90. $y^2=x,\quad dx=2ydy,\quad 0\le y\le 1$, (13.22′) formulaga asosan

$$\int_L xydy+(y-x)dy=\int_0^1\left[y^2\cdot y2y+\left(y-y^2\right)\right]dy=2\int_0^1 y^4dy+\int_0^1 ydy-\int_0^1 y^2dy=$$

$$=2\cdot\frac{y^5}{5}\Big|_0^1+\frac{y^2}{2}\Big|_0^1-\frac{y^3}{3}\Big|_0^1=\frac{2}{5}+\frac{1}{2}-\frac{1}{3}=\frac{17}{30}$$

13.91. Tenglamalari $x=\cos^3 t,\quad y=\sin^3 t$ bo'lgan astroidaning $A(-1;0)$ va $B(0;1)$ nuqtalar orasidagi yoyida $\int\limits_L 3\sqrt[3]{x}dx-\int\limits_L 3\sqrt[3]{y}dy$ egri chiziqli integral hisoblansin.

13.92. $\int\limits_L xdy$ egri chiziqli integral $\frac{x}{2}+\frac{y}{3}=1$ to'g'ri chiziq va koordinata o'qlarining kesishidan hosil bo'lgan uchburchakning konturida hisoblansin (yo'nalish soat strelkasi yo'nalishiga qarama-qarshi olinsin). Javob: 3

13.93. $\int\limits_L (x^2-y^2)dy$ egri chiziqli integral $y=x^2$ parabolaning (0; 0) va (2; 4) nuqtalari orasida joylashgan yoyda hisoblansin. J: $-\frac{56}{15}$

13.94. $\int\limits_L ydx+xdy$ egri chiziqli integral $x=R\cos t,\quad y=R\sin t$ tenglamalar bilan berilgan aylananing $t_1=0$ va $t_2=\frac{\pi}{2}$ qiymatlarga mos keluvchi nuqtalari orasidagi bo'lagida hisoblansin. J: 0

13.95. Parametrik tenglamalari $x=R\cos t,\quad y=R\sin t,\quad z=\frac{at}{2\pi}$ bo'lgan vint chizig'i $z=0$ va $z=a$ tekisliklar bilan kesishgan nuqtalari orasidagi yoyda $\int\limits_L yzdx+zxdy+xydz$ egri chiziqli integral hisoblansin. J: 0

13.96. $\int\limits_{AB} 2y\sin 2xdx-\cos 2xdy$ egri chiziqli integral ixtiyoriy chiziqning $A\left(\frac{\pi}{4};2\right)$ va $B\left(\frac{\pi}{6};1\right)$nuqtalari orasida hisoblansin. J: -0.5

13.97. $y=x^2$ va $y=9$ chiziqlar bilan chegaralangan figuraning konturida (chegarasida) $\oint\limits_C 2x(y-1)+x^2dy$ egri chiziqli integral hisoblansin. J: 0

13.98. Grin formulasidan foydalanib $\oint\limits_C (y+x)^2dx+x^2dy$ egri chiziqli integral uchlari

O(0; 0), A(a; a) va B(0; a) nuqtalarda joylashgan OAB uchburchakning perimetrida (konturida) hisoblansin. J: $-\frac{a^3}{3}$

§ 13.6. Grin formulasi. Sirt bo'yicha olingan integrallar. Ostrogradskiy va Stoks formulalari

1. Egri chiziqli integralning mexanik ma'nosi $\int\limits_{AB}(Pdx+Qdy+Rdz)$ ko'rinishidagi integral, birlik massaning $F(P;Q;R)$ kuch hosil qilgan maydonda $\breve{AB}$ yoy bo'ylab harakat qilishidagi ishini aniqlaydi.

2. To'liq differensial bo'lgan hol. Agar biror (V) sohada $Pdx+Qdy+Rdz=du$ bo'lsa, u holda $\int\limits_{AB}(Pdx+Qdy+Rdz)=u_B-u_A$ bo'ladi, ya'ni u $u(x,y,z)$ funksiyaning B va A nuqtalardagi qiymatlarining ayirmasiga teng bo'lib, (V) sohada olingan integrallash yo'li AB ga bog'liq emas.

$$\oint\limits_{(C)}(Pdx+Qdy)=\iint\limits_{(S)}\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy$$

$Pdx+Qdy$ funksiyadan (C) yopiq kontur bo'yicha (soat strelkasiga qarshi yo'nalishda) olingan egri chiziqli integralni shu kontur bilan almashtiradi.

3. Grin formulasi

$$\oint\limits_{(C)}(Pdx+Qdy)=\iint\limits_{(S)}\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy$$

$Pdx+Qdy$ funksiyadan (C) yopiq kontur bo'yicha (soat strelkasiga qarshi yo'nalishda) olingan egri chiziqli integralni shu kontur bilan chegaralangan (S) soha bo'yicha olingan ikki o'lchovli integralga almashtiradi.

4. (C) kontur bilan chegralangan yuz:

$$S=\frac{1}{2}\oint\limits_{(C)}(xdy-ydx).$$

5. Ostrogratskiy formulasi quyidagicha yoziladi:

$$\iint\limits_{(S)}(P\cos\alpha+Q\cos\beta+R\cos\gamma)ds=\iiint\limits_{(V)}\left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right)dxdydz\,,$$

bunda α,β va γ - yopiq S sirt tashqi normalining burchaklaridan, V esa shu sirt bilan chegaralangan hajmdan iboratdir. Birinchi integralni $\pm\iint\limits_{(S_z)}\left[P\frac{\partial F}{\partial x}+Q\frac{\partial F}{\partial y}+R\frac{\partial F}{\partial z}\right]\frac{\partial x\partial y}{\frac{\partial F}{\partial z}}$ ko'rinishda yozish mumkin, bunda $F(x,y,z)=0$ - sirtning tenglamasi, S_z esa S ning $z=0$ tekislikdagi proyeksiyasidir.

6. Stoks formulasi quyidagicha yoziladi:

$$\oint_{(C)}(Pdx+Qdy+Rdz)=\iint_{(S)}\begin{vmatrix}\cos\alpha & \cos\beta & \cos\gamma \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R\end{vmatrix}ds$$

bunda β va $\gamma - S$ sirtga o'tkazilgan normal burchaklaridan iborat bo'lib, u sirtning shunday tomoniga yo'naltirilganki, undan C konturni aylanish soat strelkasining yurishiga qarshi ko'rinadi.

13.99. $A(2;\ 2)$ va $B(2;\ 0)$ nuqtalar berilgan. 1) OA to'g'ri chiziq ; 2) $y=\frac{x^2}{2}$ parabolaning OA yoyi ; 3) OBA siniq chiziq bo'yicha $\int_{(C)}(x+y)dx$ hisoblansin.

13.100. $A(4;\ 2)$ va $B(2;\ 0)$ nuqtalar berilgan. 1) OA to'g'ri chiziq ; 2) OBA siniq chiziq bo'yicha $\int_{(C)}[(x+y)dx-xdy]$ hisoblansin.

13.101.13.100- masala $\int_{(C)}(ydx+xdy)$ integral uchun yechilsin. Nima uchun bu yerda integralning qiymati integrallash yo'liga bog'liq emas?

13.102. $A(a;\ 0;\ 0)$, $B(a;\ a;\ 0)$ va $C(a;\ a;\ a)$ nuqtalar berilgan. OC to'g'ri chiziq va $OABC$ siniq chiziq bo'yicha $\int(ydx+zdy+xdz)$ integral hisoblansin.

13.103. Koordinatalari $P=x-y,\ Q=x$ bo'lganda $F\{P,Q\}$ kuch maydon hosil qiladi. Tomonlari $x=\pm a$ va $y=\pm a$ dan iborat kvadratning har bir uchida F kuch yasalsin va birlik massa kvadratning konturi bo'yicha harakat qilgandagi ish hisoblansin.

13.104. $F\{P,Q\}$ kuch maydon hosil qiladi, bunday $P=x+y,\ Q=2x,\ x=a\cos t,\ y=a\sin t$ aylananing har bir choragi boshida F kuch yasalsin va o'sha aylana bo'yicha birlik massa harakat qilgandagi ish hisoblansin. Shu masala $P=x+y,\ Q=x$ hol uchun ham yechilsin. Nima uchun bu yerda ish nolga teng?

13.105. $\vec{F}\{y;a\}$ kuch maydon hosil qiladi. m massa $x = a\cos t,\ \ y = b\sin t$ ellipsning birinchi choragi va koordinata yarim o'qlaridan iborat kontur bo'yicha harakat qilganidagi ish hisoblansin.

Quyidagi egri chiziqli integrallarni hisoblang.

13.106. $\int\limits_L \frac{ds}{x-y}$, bu yerda L: $y = \frac{1}{2}x - 2$ to'g'ri chiziqning nuqtalari A(0; -2) va B(4; 0) orasidagi kesma.

13.107. $\int\limits_L xyds$, bu yerda L: uchlari A(0; 0), B(4; 0), C(4; 2) va D(0; 2) bo'lgan to'rtburchak.

13.108. $\int\limits_L yds$, bu yerda L: $y^2 = 2px$ parabolaning $x^2 = 2py$ parabola bilan kesilgan yoyi.

13.109. $\int\limits_L (x^2 + y^2)^n ds$, bu yerda L: $x = a\cos t,\ \ y = a\sin t$ aylana.

13.110. $\int\limits_L xyds$, bu yerda L: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning choragi.

13.111. $\iint\limits_{(S)} [x\cos\alpha + y\cos\beta + z\cos\gamma]ds$ integral, $x + y + z = a$ tekislikning birinchi oktantada yotgan qismining ustki sirti bo'yicha hisoblansin.

13.112. $\iint\limits_{(S)} [x^2\cos(n,i) + y^2\cos(n,j) + z^2\cos(n,k)]ds$ integral $x^2 + y^2 + 2az = a^2$ paraboloidning ikkinchi oktantada yotgan qismining ustki sirti bo'yicha hisoblansin (bunda $x < 0, y > 0, z > 0$).

13.113. $x^2 + y^2 + z^2 = a^2$ sharning sirti bo'yicha olingan $\iint\limits_{(S)} [x\cos(n,i) + y\cos(n,j) + z\cos(n,k)]ds$ integral uchun Ostrogradskiy formulasi yozilsin va tekshirilsin.

13.114. $x^2 + y^2 + 2az = a^2,\ \ x = 0,\ \ y = 0,\ \ z = 0$ sirtlar bilan chegralangan jismning birinchi oktantada yotgan qismining sirti bo'yicha olingan $\iint\limits_{(S)} [x^2\cos(n,i) + y^2\cos(n,j) + r^2\cos(n,k)]ds$ integral uchun Ostrogradskiy formulasi yozilsin va tekshirilsin.

13.115. Ostrogradskiy formulasida $P = x,\ \ Q = y,\ \ R = z$ deb olib, hajm uchun ushbu $V=\frac{1}{3}\iint\limits_{(S)}[x\cos\alpha+y\cos\beta+z\cos\gamma]ds$ formula hosil qilinsin. Bu formulaga asosan $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning hajmi hisoblansin.

Takrorlash uchun savollar

1.Ko'p o'zgaruvchili funksiya deganda nimani tushunasiz?

2. Ko'p o'zgaruvchili funksiya ta'rifini keltiring.

3.I tur egri chiziqli integral deb nimaga aytiladi?

4.O'zgaruvchilarni almashtirish deganda nimani tushunasiz?

5.Ikki o'lchovli integral deb nimaga aytiladi?

KO'P O'ZGARUVCHILI FUNKSIYALARNING INTEGRAL HISOBIGA DOIR NAZORAT TESTLAR

1. Ikki o'lchovli integral va uning xossalari

1. Ikki o'zgaruvchili $z=f(x,y)$ funksiyadan yopiq D soha bo'yicha integral yig'indini tuzishda quyidagilardan qaysi biri bajarilmaydi ?

A) D sohada $z=f(x,y)$ funksiyaning eng katta va eng kichik qiymati aniqlanadi .

B) D soha qandaydir chiziqlar bilan ΔD_i (i=1,2, ..., n) kichik sohachalarga ajratiladi .

C) har bir ΔD_i (i=1,2, ..., n) sohachadan ixtiyoriy bir $M(x_i,y_i)$ nuqta tanlanadi va unda funksiya qiymati $f(x_i,y_i)$ hisoblanadi .

D) funksiyaning hisoblangan $f(x_i,y_i)$, i=1,2, ..., n, qiymatlari ΔD_i sohacha yuzasi ΔS_i ga ko'paytirilib, $f(x_i,y_i)\Delta S_i$ ko'paytmalar yig'indisi topiladi .

E) ko'rsatilgan barcha amallar bajariladi .

2. $V_n=\sum_{i=1}^{n} f(x_i,y_i)\Delta S_i$ integral yig'indi orqali $I=\iint\limits_{D} f(x,y)dxdy$ ikki o'lchovli integral qanday aniqlanadi ?

A) $I=\max V_n$. B) $I=\min V_n$. C) $I=\lim\limits_{n\to\infty} V_n$.

D) $I=\sum_{k=1}^{n} V_k$. E) $I=\lim_{n\to\infty}\sum_{k=1}^{n} V_k$.

3. ***Teoremaning shartini ko'rsating:*** Agar D yopiq sohada $z=f(x,y)$ funksiya ··· bo'lsa , unda $I=\iint_D f(x,y)dxdy$ ikki o'lchovli integral mavjud bo'ladi .

A) monoton B) uzluksiz

B) C) chegaralangan D) davriy .

E) to'g'ri javob keltirilmagan .

4. Ikki o'lchovli integralning geometrik ma'nosi qayerda to'g'ri ko'rsatilgan ?

A) tekis shakl yuzi

B) egri chiziq yoyi uzunligi

C) silindrik jism hajmi

D) aylanma jism hajmi

E) aylanma jism sirti .

5. Agar integrallash sohasi D tomonlari a=4 va b=5 bo'lgan to'g'ri to'rtburchak bo'lsa, ikki karrali $I=\iint_D dxdy$ integral qiymati nimaga teng ?

A) I=8 . B) I=9 . C) I=10 . D) I=18 . E) I=20 .

6. Agar integrallash sohasi D radiusi R=4 bo'lgan doira bo'lsa, ikki karrali $I=\iint_D 2dxdy$ integral qiymati nimaga teng ?

A) $I=8\pi^2$ B) $I=16\pi^2$ C) $I=32\pi^2$ D) $I=20\pi$ E) $I=16\pi$

7. Ikki o'lchovli integralning mexanik ma'nosi qayerda to'g'ri ko'rsatilgan ?

A) notekis harakatda bosib o'tilgan masofa .

B) bir jinsli bo'lmagan sterjen massasi .

C) bir jinsli bo'lmagan plastinka massasi .

D) o'zgaruvchi kuch bajargan ish .

E) bir jinsli bo'lmagan jism massasi .

8. Ikki o'lchovli integral xossasi qayerda xato ko'rsatilgan ?

A) $\iint_D f_1(x,y)\cdot f_2(x,y)dxdy=\iint_D f_1(x,y)dxdy\cdot\iint_D f_2(x,y)dxdy$.

B) $\iint\limits_D (f_1(x,y)+f_2(x,y))dxdy = \iint\limits_D f_1(x,y)dxdy + \iint\limits_D f_2(x,y)dxdy$.

C) $\iint\limits_D (f_1(x,y)-f_2(x,y))dxdy = \iint\limits_D f_1(x,y)dxdy - \iint\limits_D f_2(x,y)dxdy$.

D) $\iint\limits_D Cf(x,y)dxdy = C\iint\limits_D f(x,y)dxdy$, $C - const$.

E) barcha xossalat to'g'ri ifodalangan .

9. Agar $\iint\limits_D f(x,y)dxdy = 5$ bo'lsa, $\iint\limits_D (-3)f(x,y)dxdy$ integral qiymatini toping.

A) –3 B) 15 C) –15 D) 0

E) bu integral qiymatini aniq ko'rsatib bo'lmaydi .

10. Agar $\iint\limits_D f(x,y)dxdy = 5$, $\iint\limits_D g(x,y)dxdy = -2$ bo'lsa, $\iint\limits_D f(x,y)g(x,y)dxdy$ integral qiymati nimaga teng ?

A) –10 B) 10 . C) –2 . D) 5 .

E) bu integral qiymatini aniq ko'rsatib bo'lmaydi .

2. Ikki o'lchovli integralni hisoblash. Ikki karrali integrallar

1. Tekislikdagi D soha OY koordinata o'qi bo'yicha to'g'ri soha bo'lishi uchun quyidagi shartlardan qaysi biri talab etilmaydi ?

A) D soha chap tomondan $x=a$ va o'ng tomondan $x=b$ ($a<b$) vertikal to'g'ri chiziqlar bilan chegaralangan .

B) D soha $[a,b]$ kesmada uzluksiz bo'lgan $y=\phi_1(x)$ va $y=\phi_2(x)$ $[\phi_1(x)\leq\phi_2(x)]$ funksiyalarning grafiklari bilan chegaralangan .

C) D sohaning ichki nuqtasidan o'tuvchi va OY o'qiga parallel har qanday to'g'ri chiziq soha chegarasini faqat ikkita huqtada kesib o'tadi .

D) D sohani ichki nuqtasidan o'tuvchi va OX koordinata o'qiga parallel bo'lgan har qanday L to'g'ri chiziq soha chegarasini faqat ikkita huqtada kesib o'tadi .

E) keltirilgan barcha shartlar talab etiladi .

2. Tekislikdagi *D* soha OX koordinata o'qi bo'yicha to'g'ri soha bo'lishi uchun quyidagi shartlardan qaysi biri talab etilmaydi ?

A) *D* soha quyidan $y=c$ va yuqoridna $y=d$ $(c<d)$ gorizontal to'g'ri chiziqlar bilan chegaralangan .

B) *D* soha $[a,b]$ kesmada uzluksiz bo'lgan $x=\psi_1(y)$ va $x=\psi_2(y)$ $[\psi_1(y)\leq\psi_2(y)]$ funksiyalarning grafiklari bilan chegaralangan .

C) *D* sohaning ichki nuqtasidan o'tuvchi va OY o'qiga parallel har qanday to'g'ri chiziq soha chegarasini faqat ikkita huqtada kesib o'tadi .

D) *D* sohani ichki nuqtasidan o'tuvchi va OX koordinata o'qiga parallel bo'lgan har qanday *L* to'g'ri chiziq soha chegarasini faqat ikkita huqtada kesib o'tadi .

E) keltirilgan barcha shartlar talab etiladi .

3. Quyidagi sohalardan qaysi biri OY o'qi bo'yicha to'g'ri soha bo'lmaydi ?

A) aylana bilan chegaralangan soha

B) ellips bilan chegaralangan soha

C) to'g'ri to'rtburchak bilan chegaralangan soha .

D) ikkita konsentrik aylana bilan chegaralangan soha .

E) keltirilgan barcha sohalar OY o'qi bo'yicha to'g'ri soha bo'ladi .

4. Quyidagi sohalardan qaysi biri OX o'qi bo'yicha to'g'ri soha bo'lmaydi ?

A) aylana bilan chegaralangan soha

B) ellips bilan chegaralangan soha

C) to'g'ri to'rtburchak bilan chegaralangan soha .

D) ikkita konsentrik aylana bilan chegaralangan soha .

E) keltirilgan barcha sohalar OY o'qi bo'yicha to'g'ri soha bo'ladi .

5. Quyidagidan qaysi biri ikki karrali integral bo'lmaydi ?

A) $I=\int_a^b\left(\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy\right)dx$ B) $I=\int_c^d\left(\int_{\psi_1(y)}^{\psi_2(y)} f(x,y)dx\right)dy$

C) $\int\limits_a^b \left(\int\limits_c^d f(x,y)dy \right) dx$ D) $\int\limits_c^d \left(\int\limits_a^b f(x,y)dx \right) dy$

E) keltirilgan integrallarning hammasi ikki karrali integral bo'ladi .

6. $\int\limits_0^1 \left(\int\limits_0^x (x+y)dy \right) dx$ ikki karrali integral qiymatini toping.

A) 0 B) 0.5 C) 1 D) −1 E) −0.5

7. $\int\limits_0^1 \left(\int\limits_0^y (x-y)dx \right) dy$ ikki karrali integral qiymatini toping.

A) 0 B) 1 C) 1/6 D) −1 E) −1/6

8. $\int\limits_0^1 \left(\int\limits_1^2 (x+y)dy \right) dx$ ikki karrali integral qiymatini toping.

A) 2 . B) 1.5 . C) 1 . D) 0.5 . E) 0 .

9. $\int\limits_0^1 \left(\int\limits_1^2 (x-y)dx \right) dy$ ikki karrali integral qiymatini toping.

A) 2 . B) 1.5 . C) 1 . D) 0.5 . E) 0 .

10. OY koordinata o'qi bo'yicha to'g'ri D soha uchun $\iint\limits_D f(x,y)dxdy$ ikki o'lchovli integralni hisoblash formulasi qayerda to'g'ri ko'rsatilgan ?

A) $\iint\limits_D f(x,y)dxdy = \int\limits_c^d \left(\int\limits_{\psi_1(y)}^{\psi_2(y)} f(x,y)dx \right) dy$.

B) $\iint\limits_D f(x,y)dxdy = \int\limits_a^b \left(\int\limits_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy \right) dx$.

C) $\iint\limits_D f(x,y)dxdy = \int\limits_{\psi_1(y)}^{\psi_2(y)} \left(\int\limits_a^b f(x,y)dx \right) dy$.

D) $\iint\limits_D f(x,y)dxdy = \int\limits_{\varphi_1(x)}^{\varphi_2(x)} \left(\int\limits_c^d f(x,y)dy \right) dx$.

E) $\iint\limits_D f(x,y)dxdy = \int\limits_{\varphi_1(x)}^{\varphi_2(x)} \left(\int\limits_{\psi_1(y)}^{\psi_2(y)} f(x,y)dx \right) dy$.

3. Ikki o'lchovli integralning amaliy tatbiqlari

1. $z=f(x,y)>0$ funksiya bilan aniqlangan σ sirtning XOY koordinata tekisligidagi proyeksiyasi chegarasi L

2. chiziqdan iborat D yopiq soha bo'lsin. Ynda σ sirt bilan chegaralangan va yasovchisi L bo'lgan to'gri silindrik jism hajmi Vqaysi formula bilan hisoblanadi ?

A) $V = \iint\limits_D f''_{xy}(x,y)dxdy$

B) $V = \iint\limits_D f(x,y)dxdy$

C) $V = \pi\iint\limits_D f^2(x,y)dxdy$

D) $V = \pi\iint\limits_D [f''_{xy}(x,y)]^2 dxdy$

E) to'g'ri javob keltirilmagan .

3. Jism $z=f(x,y)$ va $z=g(x,y)$ $[f(x,y)\leq g(x,y)]$ funksiyalar bilan aniqlangan $\sigma(f)$ va $\sigma(g)$ sirtlar bilan chegaralangan, $\sigma(f)$ va $\sigma(g)$ sirtlarning XOY koordinata tekisligidagi proyeksiyalari ustma-ust tushib, biror yopiq D sohadan iborat bo'lsa, jism hajmi V qaysi formula formula bilan hisoblanadi?

A) $V = 2\pi\iint\limits_D [g^2(x,y) - f^2(x,y)]dxdy$.

B) $V = 2\pi\iint\limits_D [g(x,y) - f(x,y)]dxdy$.

C) $V = 2\pi\iint\limits_D [g(x,y) + f(x,y)]dxdy$.

D) $V = \iint\limits_D [g(x,y) - f(x,y)]dxdy$.

E) $V = \iint\limits_D [g(x,y) + f(x,y)]dxdy$.

4. XOY koordinata tekisligida yotuvchi yopiq D soha ko'rinishidagi yassi geometrik shakl yuzasi S qaysi formula bilan hisoblanadi?

A) $S = \iint\limits_D xdxdy$. B) $S = \iint\limits_D ydxdy$. C) $S = \iint\limits_D xydxdy$.

D) $S = \iint\limits_D \sqrt{x^2 + y^2}dxdy$. E) $S = \iint\limits_D dxdy$.

5. $y=\phi(x)$, $y=\psi(x)$ $[\phi(x)\leq\psi(x)]$ va $x=a$, $x=b$ chiziqlar bilan chegaralangan D yopiq sohaning S yuzasi hisoblash formulasi qayerda to'g'ri ifodalangan?

A) $S = \int\limits_a^b \{ \int\limits_{\varphi(x)}^{\psi(x)} dx\}dy$ B) $S = \int\limits_a^b \{ \int\limits_{\varphi(x)}^{\psi(x)} dy\}dx$ C) $S = \int\limits_{\varphi(x)}^{\psi(x)} \{\int\limits_a^b dx\}dy$.

D) $S=\int\limits_{\varphi(x)}^{\psi(x)}\{\int\limits_{a}^{b}dy\}dx$ E) to'g'ri javob keltirilmagan .

6. Quyidagi ikki karrali integrallardan qaysi biri $y=\phi(x)$, $y=\psi(x)$ $[\phi(x)\leq\psi(x)]$ va $x=a$, $x=b$ chiziqlar bilan chegaralangan D yopiq sohaning yuzasini ifodalamaydi?

A) $\int\limits_{a}^{b}\{\int\limits_{\varphi(x)}^{\psi(x)}dy\}dx$ B) $-\int\limits_{a}^{b}\{\int\limits_{\psi(x)}^{\varphi(x)}dy\}dx$ C) $-\int\limits_{b}^{a}\{\int\limits_{\varphi(x)}^{\psi(x)}dy\}dx$

D) $\int\limits_{b}^{a}\{\int\limits_{\psi(x)}^{\varphi(x)}dy\}dx$

E) keltirilgan barcha integrallar D yopiq sohaning yuzasini ifodalaydi .

7. $x=\phi(y)$, $x=\psi(y)$ $[\phi(y)\leq\psi(y)]$ va $y=a$, $y=b$ chiziqlar bilan chegaralangan D yopiq sohaning S yuzasi hisoblash formulasi qayerda to'g'ri ifodalangan?

A) $S=\int\limits_{a}^{b}\{\int\limits_{\varphi(y)}^{\psi(y)}dx\}dy$ B) $S=\int\limits_{a}^{b}\{\int\limits_{\varphi(y)}^{\psi(y)}dy\}dx$ C) $S=\int\limits_{\varphi(y)}^{\psi(y)}\{\int\limits_{a}^{b}dx\}dy$

D) $S=\int\limits_{\varphi(y)}^{\psi(y)}\{\int\limits_{a}^{b}dy\}dx$ E) to'g'ri javob keltirilmagan .

8. Quyidagi ikki karrali integrallardan qaysi biri $x=\phi(y)$, $x=\psi(y)$ $[\phi(y)\leq\psi(y)]$ va $y=a$, $y=b$ chiziqlar bilan chegaralangan D yopiq sohaning S yuzasi hisoblash formulasi qayerda to'g'ri ifodalangan?

A) $\int\limits_{a}^{b}\{\int\limits_{\varphi(y)}^{\psi(y)}dx\}dy$ B) $-\int\limits_{a}^{b}\{\int\limits_{\varphi(y)}^{\psi(y)}dx\}dy$

C) $-\int\limits_{b}^{a}\{\int\limits_{\varphi(y)}^{\psi(y)}dx\}dy$ D) $\int\limits_{b}^{a}\{\int\limits_{\psi(y)}^{\varphi(y)}dx\}dy$.

E) keltirilgan barcha integrallar D yopiq sohaning yuzasini ifodalaydi .

9. $y=x^3$, $y=x$ va $x=0$, $x=1$ chiziqlar bilan chegaralangan D yopiq sohaning S yuzasini toping.

A) $S=1/2$. B) $S=1/3$. C) $S=1/4$. D) $S=1/5$. E) $S=1/6$

10. $x=y^4$, $x=y^2$ va $y=0$, $y=1$ chiziqlar bilan chegaralangan D yopiq sohaning S yuzasini toping.

A) S=1/3 B) S=4/15 C) S=1/5 D) S=2/15 E) S=1/6

11. $z=f(x,y)$ funksiya bilan aniqlangan fazodagi σ sirtning XOY koordinata tekisligidagi proyeksiyasi D yopiq sohadan iborat bo'lsa, σ sirtning S yuzasi hisoblanadigan formula qayerda to'g'ri ifodalangan?

A) $S = \iint\limits_D \sqrt{1 + f^2(x,y)}dxdy$

B) $S = \iint\limits_D \sqrt{1 + [f'_x(x,y)]^2}dxdy$.

C) $S = \iint\limits_D \sqrt{1 + [f'_y(x,y)]^2}dxdy$.

D) $S = \iint\limits_D \sqrt{1 + [f'_x(x,y)]^2 + [f'_y(x,y)]^2}dxdy$.

E) $S = \iint\limits_D \sqrt{1 + [f''_{xy}(x,y)]^2}dxdy$.

4. Uch o'lchov integral va uning xossalari

1. Uch o'zgaruvchili $w=f(x,y,z)$ funksiyadan fazodagi yopiq V soha bo'yicha integral yig'indini tuzishda quyidagilardan qaysi biri bajarilmaydi ?

A) V soha ixtiyoriy bir usulda ΔV_i (i=1,2, ..., n) kichik sohachalarga ajratiladi .

B) har bir ΔV_i (i=1,2, ..., n) sohachadan ixtiyoriy bir $M(x_i,y_i,z_i)$ nuqta tanlanadi va unda funksiya qiymati $f(x_i,y_i,z_i)$ hisoblanadi .

C) funksiyaning hisoblangan $f(x_i,y_i,z_i)$, i=1,2, ..., n, qiymatlari ΔV_i sohacha hajmi Δv_i ga ko'paytirilib, bu ko'paytmalar yig'indisi S_n topiladi .

D) S_n integral yig'indining absolut qiymati baholanadi .

E) ko'rsatilgan barcha amallar bajariladi .

2. $S_n = \sum\limits_{i=1}^{n} f(x_i, y_i, z_i)\Delta v_i$ integral yig'indi orqali $I = \iiint\limits_V f(x,y,z)dxdydz$ uch o'lchovli integral qanday aniqlanadi ?

A) $I= \max V_n$ B) $I= \min V_n$ C) $I = \lim\limits_{n\to\infty} V_n$ D) $I= V_n$

E) to'g'ri javob keltirilmagan .

3. ***Teoremaning shartini ko'rsating:*** Agar fazodagi V yopiq sohada $w=f(x,y,z)$ funksiya ··· bo'lsa , unda $I=\iiint\limits_V f(x,y,z)dxdydz$ uch o'lchovli integral mavjud bo'ladi .

A) monoton B) uzluksiz

C) chegaralangan D) davriy

E) to'g'ri javob keltirilmagan .

4. Uch o'lchovli integralning geometrik ma'nosi qayerda to'g'ri ko'rsatilgan ?

A) tekis shakl yuzasi B) egri chiziq yoyi uzunligi

C) silindrik jism sirti D) fazodagi jism hajmi

E) aylanma jism yon sirti .

5. Integrallash sohasi $V=\{(x, y, z): x^2+y^2+z^2\le 9\}$ yopiq shar bo'lgan uch karrali $\iiint\limits_V dxdydz$ integral qiymati nimaga teng?

A) 27π B) 30π C) 33π D) 36π E) 39π

6. Integrallash sohasi $V=\{(x, y, z): |x|\le 2, |y|\le 3, |z|\le 1\}$ yopiq to'g'ri burchakli parallelopiped bo'lgan uch karrali $\iiint\limits_V dxdydz$ integral qiymati nimaga teng?

A) 6π B) 36 C) 48 D) 24 E) 36π

7. Uch o'lchovli integralning mexanik ma'nosi qayerda to'g'ri ko'rsatilgan ?

A) notekis harakatda bosib o'tilgan masofa

B) bir jinsli bo'lmagan sterjen massasi

C) bir jinsli bo'lmagan plastinka massasi .

D) o'zgaruvchi kuch bajargan ish

E) bir jinsli bo'lmagan jism massasi .

8. Uch o'lchovli integral xossasi qayerda xato ko'rsatilgan ?

*A) $\iiint\limits_V f_1(x,y,z)\cdot f_2(x,y,z)dxdydz=$

$=\iiint\limits_V f_1(x,y,z)dxdydz\cdot\iiint\limits_V f_2(x,y,z)dxdydz$.

B) $\iiint\limits_V [f_1(x,y,z)+f_2(x,y,z)]dxdydz=$

$$=\iiint\limits_V f_1(x,y,z)dxdydz+\iiint\limits_V f_2(x,y,z)dxdydz\ .$$

C) $\iiint\limits_V [f_1(x,y,z)-f_2(x,y,z)]dxdydz=$

$$=\iiint\limits_V f_1(x,y,z)dxdydz-\iiint\limits_V f_2(x,y,z)dxdydz\ .$$

D) $\iiint\limits_V Cf(x,y,z)dxdydz=C\iiint\limits_V f(x,y,z)dxdydz \quad (C-\text{const.})$.

E) barcha xossalat to'g'ri ifodalangan .

9. Agar $\iiint\limits_V f(x,y,z)dxdydz=6$ bo'lsa, $\iiint\limits_V (-4)f(x,y,z)dxdydz$ integral qiymatini toping.

A) −6 . B) 12 . C) −24 . D) 0 .

E) bu integral qiymatini aniq ko'rsatib bo'lmaydi .

10. Agar $\iiint\limits_V f(x,y,z)dxdydz=6$, $\iiint\limits_V g(x,y,z)dxdydz=-2$ bo'lsa, $\iiint\limits_V f(x,y,z)g(x,y,z)dxdydz$ integral qiymati nimaga teng ?

A) −12 . B) 12 . C) −4 . D) 4 .

E) bu integral qiymatini aniq ko'rsatib bo'lmaydi .

5. Uch o'lchovli integrallarni hisoblash

1. Fazodagi V soha to'g'ri soha bo'lishi uchun quyidagi shartlardan qaysi biri talab etilmaydi ?

A) V soha S yopiq sirt bilan chegaralangan .

B) V sohaning ixtiyoriy ichki nuqtasidan OZ koordinata o'qiga parallel qilib o'tkazilgan har qanday to'g'ri chiziq bu sohaning S chegarasini faqat ikkita nuqtada kesib o'tadi .

C) V sohaning XOY koordinata tekisligidagi D proyeksiyasi to'g'ri sohadan iborat.

D) V sohaning chegaraviy nuqtalari bir bog'lamli to'plamni tashkil etadi .

E) keltirilgan barcha shartlar talab etiladi .

2. Fazodagi quyidagi sohalardan qaysi biri uch o'lchovli to'g'ri soha bo'lmaydi ?

A) shar B) piramida C) parallelepiped

D) tor E) aylanma ellipsoid .

3. Uch karrali integral qayerda to'g'ri ifodalangan?

A) $\int\limits_{\varphi_1(x)}^{\varphi_2(x)}\{\int\limits_a^b[\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}f(x,y,z)dz]dx\}dy$

B) $\int\limits_a^b\{\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}[\int\limits_{\varphi_1(x)}^{\varphi_2(x)}f(x,y,z)dz]dy\}dx$

C) $\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}\{\int\limits_{\varphi_1(x)}^{\varphi_2(x)}[\int\limits_a^b f(x,y,z)dz]dy\}dx$

D) $\int\limits_a^b\{\int\limits_{\varphi_1(x)}^{\varphi_2(x)}[\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}f(x,y,z)dz]dy\}dx$.

E) $\int\limits_{\varphi_1(x)}^{\varphi_2(x)}\{\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}[\int\limits_a^b f(x,y,z)dz]dy\}dx$.

4. $\int\limits_0^1\{\int\limits_0^1[\int\limits_0^1(x+y+z)dz]dy\}dx$ uch karrali integral qiymatini toping.

A) 0 . B) 0.5 . C) 1.0 . D) 1.5 . E) –0.5 .

5. $\int\limits_0^1\{\int\limits_0^x[\int\limits_0^{xy}(x+z)dz]dy\}dx$ uch karrali integral qiymatini toping.

A) 0 B) 5/18 C) 13/36 D) 19/144 E) 23/180

6. $\int\limits_0^1\{\int\limits_0^x[\int\limits_0^{xy}(x+y+z)dz]dy\}dx$ uch karrali integral qiymatini toping.

A) 0 B) 5/16 C) 7/36 D) –3/26 E) –11/46

7. Fazodagi V to'g'ri soha quyidan va yuqoridan $z=\psi_1(x,y)$ va $z=\psi_2(x,y)$ sirtlar bilan, uning XOY koordinata tekisligidagi proyeksiyasini ifodalovchi yassi D yopiq soha esa $y=\phi_1(x)$, $y=\phi_2(x)$ va $x=a$, $x=b$ chiziqlar bilan chegaralangan bo'lsin. Bu holda uch o'lchovli $I=\iiint\limits_V f(x,y,z)dxdydz$ integralni uch o'lchovli integral orqali hisoblash formulasi qayerda to'g'ri ko'rsatilgan ?

A) $I=\int\limits_{\varphi_1(x)}^{\varphi_2(x)}\{\int\limits_a^b[\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}f(x,y,z)dz]dx\}dy$.

B) $I=\int\limits_a^b\{\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)}[\int\limits_{\varphi_1(x)}^{\varphi_2(x)}f(x,y,z)dz]dy\}dx$.

C) $I = \int\limits_{\psi_1(x,y)}^{\psi_2(x,y)} \{ \int\limits_{\varphi_1(x)}^{\varphi_2(x)} [\int\limits_a^b f(x,y,z)dz]dy\}dx$.

D) $I = \int\limits_a^b \{ \int\limits_{\varphi_1(x)}^{\varphi_2(x)} [\int\limits_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z)dz]dy\}dx$.

E) $I = \int\limits_{\varphi_1(x)}^{\varphi_2(x)} \{ \int\limits_{\psi_1(x,y)}^{\psi_2(x,y)} [\int\limits_a^b f(x,y,z)dz]dy\}dx$.

8. Fazodagi V to'g'ri soha quyidan va yuqoridan $z=0$ va $z=xy$ sirtlar bilan, uning XOY koordinata tekisligidagi proyeksiyasini ifodalovchi yassi D yopiq soha esa $y=0$, $y=x$ va $x=0$, $x=1$ chiziqlar bilan chegaralangan bo'lsin. Uch o'lchovli $I = \iiint\limits_V (x+z)dxdydz$ integral qiymatini toping.

A) 0 B) 5/36 C) 23/180 D) 21/216 E) 1

9. Fazodagi V to'g'ri soha quyidan va yuqoridan $z=0$ va $z=xy$ sirtlar bilan, uning XOY koordinata tekisligidagi proyeksiyasini ifodalovchi yassi D yopiq soha esa $y=0$, $y=x$ va $x=0$, $x=1$ chiziqlar bilan chegaralangan bo'lsin. Uch o'lchovli $I = \iiint\limits_V (x+y+z)dxdydz$ integral qiymatini toping.

A) 7/36 B) 5/46 C) 3/56 D) 1 E) 0

10. Fazodagi V to'g'ri soha quyidan va yuqoridan $z=0$ va $z=xy$ sirtlar bilan, uning XOY koordinata tekisligidagi proyeksiyasini ifodalovchi yassi D yopiq soha esa $y=0$, $y=x$ va $x=0$, $x=1$ chiziqlar bilan chegaralangan bo'lsin. Uch o'lchovli $I = \iiint\limits_V xyzdxdydz$ integral qiymatini toping.

A) 1/16 B) 1/32 C) 1/48 D) 1/ 64 E) 1/80

6. Uch o'lchovli integralning amaliy tatbiqlari

1. Fazoda to'g'ri sohani tashkil etuvchi T jismning V halmi uch o'lchovli integral orqali qaysi formula bilan hisoblanadi?

A) $V = \iiint\limits_T xyzdxdydz$ B) $V = \iiint\limits_T (x+y+z)dxdydz$ C) $V = \iiint\limits_T dxdydz$

D) $V = \iiint\limits_T \sqrt{x^2+y^2+z^2}dxdydz$ E) $V = \iiint\limits_T (x^2+y^2+z^2)dxdydz$

2. $z=x^2+y^2$, $z=a^2(x^2+y^2)$, $y=x$, $y=x^2$ sirtlar bilan chegara-langan jismning V hajmini toping.

A) $V=a^3$ B) $V=3(a^3-1)/35$ C) $V=a^2$

D) $V=3(a^2-1)/35$ E) $8a\sqrt{2a}$.

3. $z=x+y$, $z=axy$, $y+x=1$, $y=0$, $x=0$ sirtlar bilan chegaralangan jismning V hajmini toping.

A) $V=(1-a)/8$. B) $V=(2-a)/12$. C) $V=(4-a)/16$.

D) $V=(6-a)/20$. E) $V=(8-a)/24$.

4. Fazoda to'g'ri sohani tashkil etuvchi va har bir $M(x,y,z)$ nuqtasidagi zichligi $\rho=\rho(x,y,z)$ funksiya bilan aniqlanadigan bir jinsli bo'lmagan T jismning m massasiqaysi formula bilan topiladi ?

A) $m=\iiint_T xyz\rho(x,y,z)dxdydz$

B) $m=\iiint_T (x+y+z)\rho(x,y,z)dxdydz$

C) $m=\iiint_T (x^2+y^2+z^2)\rho(x,y,z)dxdydz$

D) $m=\iiint_T \rho(x,y,z)dxdydz$

E) $m=\iiint_T \sqrt{x^2+y^2+z^2}\rho(x,y,z)dxdydz$

5. Har bir $M(x,y,z)$ nuqtasidagi zichligi $\rho(x,y,z)=a(x+y+z)$ funksiya bilan aniqlanadigan $0\le x\le 1$, $0\le y\le 1$, $0\le z\le 1$ kubning m massasini toping.

A) $a/2$. B) $a/3$. C) $3a/2$. D) $2a/3$. E) $3a^2$.

6. Har bir $M(x,y,z)$ nuqtasidagi zichligi $\rho(x,y,z)=x+y+z$ funksiya bilan aniqlanadigan $0\le x\le a$, $0\le y\le a$, $0\le z\le a$ kubning m massasini toping.

A) $3a/2$. B) $3a^2/2$. C) $3a^3/2$. D) $3a^4/2$. E) $3a^5/2$.

7. Har bir $M(x,y,z)$ nuqtasidagi zichligi $\rho(x,y,z)=x+y+z$ funksiya bilan aniqlanadigan $0\le x\le a$, $0\le y\le b$, $0\le z\le c$ parallelepipedning m massasini toping.

A) $abc/2$ B) $(a+b+c)/2$ C) $abc(a+b+c)/2$

D) $abc/2(a+b+c)$ E) $(a+b+c)/2abc$

8. Zichligi $\rho=\rho(x,y,z)$ funksiya bilan aniqlanadigan bir jinsli bo'lmagan T jismning XOY koordinata tekisligiga nisbatan S_{xy} statik momenti qaysi uch o'lchovli integral orqali ifodalanadi?

A) $\iiint\limits_T y\rho(x,y,z)dxdydz$

B) $\iiint\limits_T x\rho(x,y,z)dxdydz$

C) $\iiint\limits_T z\rho(x,y,z)dxdydz$

D) $\iiint\limits_T xy\rho(x,y,z)dxdydz$

E) to'g'ri javob keltirilmagan .

9. Zichligi $\rho=\rho(x,y,z)$ funksiya bilan aniqlanadigan bir jinsli bo'lmagan T jismning XOZ koordinata tekisligiga nisbatan S_{xz} statik momenti qaysi uch o'lchovli integral orqali ifodalanadi?

A) $\iiint\limits_T y\rho(x,y,z)dxdydz$

B) $\iiint\limits_T x\rho(x,y,z)dxdydz$

C) $\iiint\limits_T z\rho(x,y,z)dxdydz$

D) $\iiint\limits_T xz\rho(x,y,z)dxdydz$

E) to'g'ri javob keltirilmagan

10. Zichligi $\rho=\rho(x,y,z)$ funksiya bilan aniqlanadigan bir jinsli bo'lmagan T jismning YOZ koordinata tekisligiga nisbatan S_{yz} statik momenti qaysi uch o'lchovli integral orqali ifodalanadi?

A) $\iiint\limits_T y\rho(x,y,z)dxdydz$

B) $\iiint\limits_T x\rho(x,y,z)dxdydz$

C) $\iiint\limits_T z\rho(x,y,z)dxdydz$

D) $\iiint\limits_T yz\rho(x,y,z)dxdydz$

E) to'g'ri javob keltirilmagan .

7. I tur egri chiziqli integrallar

1. AB egri chiziq bo'yicha $f(x,y)$ funksiya uchun I tur integral yig'indini tuzishda quyidagi amallardan qaysi biri bajarilmaydi?

A) AB egri chiziq n ta $M_{i-1}M_i(i=1,2,3, \dots, n)$ kichik yoychalarga ajratiladi .

B) har bir $M_{i-1}M_i(i=1,2,3, \dots, n)$ kichik yoychalardan ixtiyoriy bir $P_i(x_i,y_i)$ nuqta tanlanadi .

C) tanlangan $P_i(x_i,y_i)$ nuqtalarda funksiya qiymatlari $f(x_i,y_i)$ (i=1,2,3, ... , n) hisoblanadi .

D) hisoblangan $f(x_i,y_i)$ (i=1,2,3, ... , n) qiymatlar orasidan ularning eng kattasi va eng kichigi aniqlanadi .

E) $f(x_i,y_i)$ qiymatlar $M_{i-1}M_i$ yoychalarning Δl_i(i=1,2,3, ... , n) uzunliklariga ko'paytirilib, bu ko'paytmalar yig'indisi hosil etiladi .

2. $S_n(f)=\sum_{i=1}^{n} f(x_i,y_i)\Delta l_i$ integral yig'indi orqali I tur egri chiziqli integral $I=\int_{AB} f(x,y)dl$ qanday aniqlanadi?

A) $I=\max S_n(f)$. B) $I=\min S_n(f)$. C) $I=[\max S_n(f)+\min S_n(f)]/2$.

D) $I=\int S_n(f)dl$. E) $I=\lim_{\substack{n\to\infty,\\ \max\Delta l_i\to 0}} S_n(f)$.

3. Quyidagi masalalardan qaysi birlari I tur egri chiziqli integral yordamida yechilishi mumkin?

I. moddiy egri chiziqning massasini topish .

II. egri chiziq yoyining uzunligini hisoblash .

III. moddiy egri chiziqning og'irlik markazini topish .

A) I va III . B) II va III .

C) I va II . D) I, II va III . E) II .

4. I tur egri chiziqli integralning geometrik ma'nosi nimadan iborat ?

A) *AB* egri chiziqning o'rta nuqtasi . B) *AB* egri chiziqning urinmasi .

C) *AB* egri chiziqning uzunligi . D) *AB* egri chiziqning normali .

E) *AB* egri chiziqning OX koordinata o'qiga proeksiyasi .

5. AB egri chiziqning l uzunligi qaysi formula bilan topiladi?

A) $l=\int_{AB} dl$ B) $l=\int_{AB}(x^2+y^2)dl$ C) $l=\int_{AB}\sqrt{x^2+y^2}\,dl$.

D) $l=\int_{AB} x\,dl$ E) $l=\int_{AB} y\,dl$

6. $x=a(t-\sin t)$, $y=a(1-\cos t)$ $(0\le t\le\pi)$ sikloida yoyining l uzunligini toping.

A) $2a$ B) $4a$ C) $6a$ D) $8a$

E) to'g'ri javob keltirilmagan .

7. I tur egri chiziqli integralning mexanik ma'nosi nimadan iborat ?

A) AB egri chiziqning og'irlik markazi

B) AB egri chiziqning statik momenti

C) AB egri chiziqning inertsiya momenti

D) AB egri chiziqning massasi

E) to'g'ri javob keltirilmagan .

8. Chiziqli zichlik funksiyasi $\rho=\rho(x,y)$ bo'lgan moddiy L chiziqning m massasi qaysi formula bilan topiladi?

A) $m=\int\limits_L \frac{dl}{\rho(x,y)}$ B) $m=\int\limits_L \rho(x,y)dl$ C) $m=\int\limits_L \rho^2(x,y)dl$

D) $m=\int\limits_L \sqrt{\rho(x,y)}dl$ E) $m=\int\limits_L \frac{dl}{\rho^2(x,y)}$

9. Chiziqli zichlik funksiyasi $\rho(x,y)=|y|$ bo'lgan $y^2=12x$ $(0\le x\le 3)$ parabola yoyining m massasini toping.

A) $12(2\sqrt{2}-1)$ B) $16(2\sqrt{2}-1)$ C) $20(2\sqrt{2}-1)$

D) $24(2\sqrt{2}-1)$ E) $28(2\sqrt{2}-1)$

10. Chiziqli zichlik funksiyasi $\rho=\rho(x,y)$ va massasi m bo'lgan moddiy AB chiziq og'irlik markazining abssissasi x_0 qaysi formula bilan aniqlanadi ?

A) $x_0=m\int\limits_{AB} x\rho(x,y)dl$ B) $x_0=m\int\limits_{AB} y\rho(x,y)dl$

C) $x_0=\frac{1}{m}\int\limits_{AB} x\rho(x,y)dl$ D) $x_0=\frac{1}{m}\int\limits_{AB} y\rho(x,y)dl$

E) $x_0=\frac{1}{m}\int\limits_{AB} xy\rho(x,y)dl$

8. II tur egri chiziqli integrallar

1. AB egri chiziq bo'yicha $u(x,y)$ va $v(x,y)$ funksiyalar uchun II tur integral yig'indini tuzishda quyidagi amallardan qaysi biri bajarilmaydi?

A) AB egri chiziq n ta $M_{i-1}M_i(i=1,2,3, \ldots, n)$ kichik yoychalarga ajratiladi .

B) har bir $M_{i-1}M_i(i=1,2,3, \ldots, n)$ kichik yoychalardan ixtiyoriy bir $P_i(x_i,y_i)$ nuqta tanlanadi .

C) tanlangan $P_i(x_i,y_i)$ nuqtalarda $u(x,y)$ va $v(x,y)$ funksiyalarning qiymatlari $u(x_i,y_i)$ va $v(x_i,y_i)$ (i=1,2,3, ... , n) hisoblanadi .

D) $u(x_i,y_i)\Delta x_i+v(x_i,y_i)\Delta y_i$ (i=1,2,3, ... , n) ifodalarning yig'indisi hosil etiladi

E) keltirilgan barcha amallar bajariladi .

2. $S_n(u,v)=\sum_{i=1}^{n}[u(x_i,y_i)\Delta x_i+v(x_i,y_i)\Delta y_i]$ integral yig'indi orqali II tur egri chiziqli integral $I=\int_{AB} u(x,y)dx+v(x,y)dy$ qanday aniqlanadi?

A) $I=\max S_n(u,v)$ B) $I=\min S_n(u,v)$

C) $I=[\max S_n(u,v)+\min S_n(u,v)]/2$. D) $I=\int S_n(u,v)dl$.

E) $I=\lim\limits_{\substack{n\to\infty,\\ \max(\Delta x_i,\Delta y_i)\to 0}} S_n(u,v)$.

3. Quyidagi masalalardan qaysi birlari II tur egri chiziqli integral yordamida yechilishi mumkin?

I. yopiq egri chiziq bilan chegaralangan to'g'ri sohaning yuzasini topish .

II. o'zgaruvchi kuch moddiy nuqtani egri chiziq bo'yicha harakatlantirganda bajarilgan ishni hisoblash .

III. moddiy egri chiziqning og'irlik markazini topish .

A) I va III B) II va III C) I va II D) I, II va III E) III

4. II tur egri chiziqli integral yordamida quyidagi geometrik masalalardan qaysi biri yechiladi?

A) *AB* egri chiziqning o'rta nuqtasini topish .

B) *AB* egri chiziqqa o'tkazilgan urinmani topish .

C) *AB* yopiq egri chiziqning uzunligini topish .

D) *AB* yopiq egri chiziq bilan chegaralangan soha yuzasini topish .

E) *AB* yopiq egri chiziqning normalini topish .

5. L yopiq egri chiziq bilan chegaralangan to'g'ri sohaning yuzasi S qaysi formula bilan topiladi?

A) $S-\frac{1}{2}\oint_L ydx-xdy$ B) $S=\frac{1}{2}\oint_L xdy-ydx$ C) $S=\frac{1}{2}\oint_L xdx-ydy$

D) $S=\frac{1}{2}\oint_L ydy-xdx$ E) $S=\frac{1}{2}\oint_L ydx+xdy$

6. $x=a\cos^3t$, $y=a\sin^3t$ $(0\leq t\leq 2\pi)$ astroida bilan chegaralangan sohaning S yuzasini toping.

A) $S=3\pi a^2/8$ B) $S=2\pi a^2/5$ C) $S=\pi a^2/3$.

D) $S=4\pi a^2/7$ E) $S=5\pi a^2/2$.

7. II tur egri chiziqli integral yordamida quyidagi masalalardan qaysi birini yechish mumkin ?

A) AB moddiy egri chiziqning og'irlik markazini topish .

B) AB moddiyegri chiziqning massasini topish .

C) AB egri chiziqning inertsiya momentini topish .

D) AB egri chiziq bo'yicha o'zgaruvchi kuchning bajargan ishni topish .

E) AB egri chiziqning statik momentini topish .

8. $F(x,y)= u(x,y)\mathbf{i}+v(x,y)\mathbf{j}$ o'zgaruvchi kuch moddiy nuqtani L egri chiziq bo'yicha harakatlantirganda bajarilgan A ish qiymati qaysi formula bilan hisoblanadi?

A) $A=\int\limits_L u(x,y)dx - v(x,y)dy$ B) $A=\int\limits_L u(x,y)dx + v(x,y)dy$

C) $A=\int\limits_L v(x,y)dx - u(x,y)dy$ D) $A=\int\limits_L v(x,y)dx + u(x,y)dy$

E) $A=\int\limits_L [u(x,y)+v(x,y)]dx + [u(x,y)-v(x,y)]dy$.

9. $F(x,y)=xy\mathbf{i}+(x+y)\mathbf{j}$ o'zgaruvchi kuch moddiy nuqtani $y=x$ $(0\leq x\leq 1)$ to'gri chiziq kesmasi bo'yicha harakatlantirganda bajarilgan A ishni hisoblang .

A) $A=4/3$ B) $A=7/5$ C) $A=11/8$

D) $A=17/12$ E) $A=21/16$

10. $F(x,y)=xy\mathbf{i}+(x+y)\mathbf{j}$ o'zgaruvchi kuch moddiy nuqtani $y=x^2$ $(0\leq x\leq 1)$ parabola yoyi bo'yicha harakatlantirganda bajarilgan A ishni hisoblang .

A) $A=4/3$ B) $A=7/5$ C) $A=11/8$

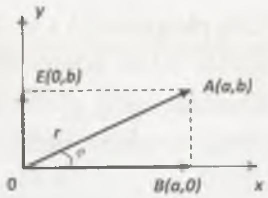
D) $A=17/12$ E) $A=21/16$.

XIV-BOB. KOMPLEKS SONLAR VA KOMPLEKS ARGUMENTLI FUNKSIYALAR

§14.1. Kompleks sonlar ustida amallar

Ta'rif: $z=a+bi$ ko'rinishidagi son kompleks son deyiladi. Bu yerda a=Rez;b=ImZ $i^2=-1$ yoki $\sqrt{-1}=i$. $\bar{z}=a-ib$ -berilgan kompleks sonning qo'shmasi deyiladi. E- kompleks sonlar to'plami bo'lsin. $z = x + i\,y$ kompleks son shu to'plam ixtiyoriy elementi bo'lishi mumkin. $z_1=a_1+ib_1$, $z_2=a_2+ib_2$ kompleks sonlar berilgan bo'lsa, ular ustida quyidagi amallar bajarish mumkin:

1) Qo'shish: $z_1+z_2=(a_1+b_1)+i(b_1+b_2)$
2) Ayirish: $z_1-z_2=(a_1-b_1)+i(b_1-b_2)$
3) Ko'paytirish: $z_1z_2=(a_1b_2-b_1b_2)+i(a_1b_2+a_2b_1)$
4) Bo'lish: $\frac{z_1}{z_2}=\frac{z_1}{z_2}\cdot\frac{z_2}{z_2}=\frac{a_1+ib_1}{a_2+ib_2}\cdot\frac{a_2-ib_2}{a_2-ib_2}=\frac{(a_1a_2+b_1b_2)+i(a_2b_1-a_1b_2)}{a_2^2+b_2^2}$



14.1-chizma. Kompleks sonning geometrik tasviri

Kompleks sonning trigonometrik shakli. $z=a+bi$ kompleks son $\overline{OA}$ vektor bilan tasvirlangan bo'lsin (14.1-chizma). $\overline{OA}$ vektorning OA uzunligini r deb, bu vektor bilan OX o'qning musbat yo'nalishi orasidagi burchakni φ deb belgilasak $a = r\cos\varphi$; $b = r\sin\varphi$.

Bu qiymatlarni $z = a + bi$ ga qo'yib, r ni qavsdan tashqariga chiqarsak: $z = r(\cos\varphi + i\sin\varphi)$ ni $z = a + bi$ kompleks sonning trigonometrik shakli deyiladi, bu yerda $r = |z|$, $r = \sqrt{a^2 + b^2}$, $\arg z = \varphi = arctg\frac{b}{a}$.

Trigonometrik shakldagi kompleks sonlarni ko'paytirish va bo'lish. Ushbu :

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1),\ z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$$

kompleks sonlarni ko'paytirib, quyidagini hosil qilamiz:

$$z_1 z_2 = r_1 r_2[\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\sin\varphi_2 + i(\sin\varphi_1\cos\varphi_2 + \cos\varphi_1\sin\varphi_2)]$$

yoki $z_1 z_2 = r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$.

Umuman, matematik induksiya metodi bilan, n ta

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1), z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2), \ldots\ldots, z_n = r_n(\cos\varphi_n + i\sin\varphi_n)$$

kompleks sonlar uchun :

$$z_1 z_2 \ldots z_n = r_1 r_2 \ldots r_n[\cos(\varphi_1 + \varphi_2 + \ldots + \varphi_n) + i\sin(\varphi_1 + \varphi_2 + \ldots + \varphi_n)]$$

ekanligini ko'rsatih mumkin. Shunday qilib, bu hol uchun

$$z_1 z_2 \ldots z_n = r_1 r_2 \ldots r_n[\cos(\varphi_1 + \varphi_2 + \ldots + \varphi_n) + i\sin(\varphi_1 + \varphi_2 + \ldots + \varphi_n)]$$

tenglikdan

$$z^n = r^n(\cos n\varphi + i\sin n\varphi)$$

yoki

$$[r(\cos n\varphi + i\sin n\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi)$$

kelib chiqadi.

Kompleks sondan ildiz chiqarish. $a + bi$ kompleks sonning kvadrat ildizi deb, kvadrati $a + bi$ songa teng bo'lgan $x + yi$ kompleks songa aytiladi.Shunday qilib, $a + bi = (x + yi)^2$.

$a + bi$ kompleks sonning kvadrat ildizi $\sqrt{a + bi}$ ko'rinishida belgilanadi.

Demak, x va y ning $x = \pm\sqrt{\frac{a + \sqrt{a^2 + b^2}}{2}}$ va $y = \pm\sqrt{\frac{-a + \sqrt{a^2 + b^2}}{2}}$ qiymatlarini $a + bi = x + yi$ ga qo'yib, b>0 va b<0 ga mos quyidagi ikki formulani hosil qilamiz:

$$\sqrt{a + bi} = \pm\left(\sqrt{\frac{a + \sqrt{a^2 + b^2}}{2}} + i\sqrt{\frac{-a + \sqrt{a^2 + b^2}}{2}}\right)$$ b>0 uchun,

$\sqrt{a+bi}=\pm\left(\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}}-i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\right)$ $b<0$ uchun.

Ushbu $z=r(\cos\varphi+i\sin\varphi)$ kompleks sonning n – darajali ildizini

$$\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n}+i\sin\frac{\varphi+2k\pi}{n}\right) \qquad (14.1)$$

ko'rinishida yozish mumkin.

Formulada k istalgan butun sonni ifodalaydi. Lekin k ga $0, 1, 2, 3, \ldots, n-1$ qiymatlarni berish kifoya.

Quyidagilarni toping: 1) z_1+z_2; 2) z_1-z_2; 3) $\bar{z}_1$; 4) $z_1\cdot z_2$; 5) $z_1:z_2$; 6) z_2^3, 7) z_1+z_2 va z_1-z_2 chizmada tasvirlang.

14.1. $z_1=-2,\quad z_2=2i$ 14.2. $z_1=2i,\quad z_2=-2i$

14.3. $z_1=-2i,\quad z_2=1+i$ 14.4. $z_1=2i,\quad z_2=-2i$

14.5. $z_1=\sqrt{15}-i,\quad z_2=1-i\sqrt{15}$ 14.6. $z_1=-3+4i,\quad z_2=3-4i$

14.7. $z_1=6+11i,\quad z_2=7+3i$ $\frac{75}{58}+\frac{59}{58}i$; $154+414i$.

14.8. $z_1=3-i,\quad z_2=4+5i$

14.9 Quyidagi kompleks sonlarni tasvirlovchi vektorlar yasalsin:

a) $3+i$; b) $-4+5i$; v) $7-4i$; g) $-2-6i$;

d) 1; e) -1; j) $i\sqrt{2}$; z) $1+i\sqrt{3}$;

i) $2-\sqrt{3}+4i$; k) 8; l) $1-2i\sqrt{2}$;

m) -7; n) $-5i$.

14.10. Quyidagi ildizlarni hisoblang:

a) $\sqrt{2i}$; b) $\sqrt{-8i}$; v) $\sqrt{3-4i}$; g) $\sqrt{-15+8i}$;

d) $\sqrt{-11+60i}$; e) $\sqrt{2-3i}$; j) $\sqrt{1-i\sqrt{3}}$; z) $\sqrt{4+i}+\sqrt{4-i}$

i) $\sqrt[4]{-7+24i}$; k) $\sqrt[4]{-4}$; l) $\sqrt[4]{-1}$; m) $\sqrt[4]{2+i\sqrt{12}}$.

n) $\sqrt{15+8i}$ ni hisoblang.

o) $x^2-(5-i)x+(8-i)=0$ kvadrat tenglamani yeching.

14.11. $\sqrt{7+3i}$ ni hisoblang.

14.12. Ushbu kvadrat tenglamalarni yeching.

a) $x^2-(3+2i)x+(1+3i)=0$; b) $(7+i)x^2-(7+i)x+2=0$;

v) $(1+i)x^2-(7+2i)x+(8-i)=0$.

14.13. Ushbu kompleks sonlarni trigonometrik shaklga keltiring:

a) 3; b) -2; v) $\frac{1}{2}-i\frac{\sqrt{3}}{2}$; g) $2i$;

d) $\frac{\sqrt{2}}{2}(1-i)$; e) $-1+i\sqrt{3}$; j) $3+i$; z) $4-i$;

i) $-2+i$; k) $-1-2i$;

Quyidagi ildizlarining barcha qiymatlarini toping. Ildiz ostida turgan sonlarni ko'rsatkichli ko'rinishda tasvirlang.

14.14. $\sqrt{1+i}$ 14.15. $\sqrt{3+4i}$ 14.16. $\sqrt{-1+i}$

14.17. $\sqrt[3]{\sqrt{15}-i}$ 14.18. $\sqrt[3]{-3+4i}$ 14.19. $\sqrt[3]{3-4i}$

14.20. $\sqrt[4]{-1}$ 14.21. $\sqrt[5]{1-i}$ 14.22. $\sqrt[3]{-1-i}$

14.23. $\sqrt[8]{-2+2i}$

14.24. Quyidagi ildizlarni hisoblang.

a) $\sqrt[3]{-1+i}$; b) $\sqrt[4]{-1}$; v) $\sqrt[4]{32}$; g) $\sqrt[3]{i}$; d) $\sqrt[3]{-i}$; e) $\sqrt[6]{2}$.

Quyidagi ifodalarni sinus va kosinusning darajalari orqali ifodalang.

14.25. $\sin 3x$ 14.26. $\cos 3x$ 14.27. $\sin 4x$

14.28. $\cos 4x$ 14.29. $\sin 5x$

Quyidagi ifodalarni sinus va kosinusning karrali yoylari orqali ifodalang.

14.30. $\sin^2 x$ 14.31. $\sin^3 x$ 14.32. $\sin^4 x$

14.33. $\cos^4 x$ 14.34. $\cos^5 x$ 14.35. $\cos^6 x$

14.36. Chiziq tenglamasini kompleks ko'rinishda yozing.

a) $3x+y=1$ b) $(x-1)y=2$ v) $2(x-1)^2+3(y+2)^2=6$

14.37. $z=2+2i$ sonni trigonometrik shaklda ifodalang.

14.38. $z=3\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)$ sonning moduli va argumentini toping.

14.39. Ushbu $z_1=r_1(\cos\varphi_1+i\sin\varphi_1)$ kompleks sonni $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$ kompleks soniga bo'ling.

14.40. $z_1=3(\cos 30^0+i\sin 30^0)$, $z_2=2(\cos 60^0+i\sin 60^0)$ berilgan. $\frac{z_1}{z_2}$ ni hisoblang.

14.41. $\sin 2\varphi$ va $\cos 2\varphi$ ni $\sin\varphi$ va $\cos\varphi$ orqali ifodalang.

14.42. $\left[2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]^{n}$ ni hisoblang.

14.43. $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{20}$ ni hisoblang.

14.44. Quyidagi kompleks sonlarni trigonometrik shaklga keltirib, so'ngra ko'rsatilgan amallarni bajaring:

a) $(-5+5i\sqrt{3})(1-i)$; b) $\frac{-5+5i\sqrt{3}}{(1-i)^2}$; v) $(1+i)^{25}$; g) $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$;

d) $\frac{(-1+i\sqrt{3})^5}{(1-i)^{20}}+\frac{(-1-i\sqrt{3})^5}{(1+i)^{20}}$.

14.45. Ushbu $(\cos 15^0+i\sin 15^0)(\cos 30^0+i\sin 30^0)=\cos 45^0+i\sin 45^0$ ayniyatdan foydalanib, $\sin 15^0$ va $\cos 15^0$ ning qiymatlarini toping.

14.46. $\frac{(1-i\sqrt{3})(\cos\alpha+i\sin\alpha)}{2(1-i)(\cos\varphi-i\sin\varphi)}$ ni hisoblang.

14.47. Quyidagi ayniyatlarni isbotlang (n – butun son):

a) $(1+i)^n=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right)$, b) $(\sqrt{3}-i)^n=2^n\left(\cos\frac{n\pi}{6}-i\sin\frac{n\pi}{6}\right)$.

14.48. $(1+\cos\varphi+i\sin\varphi)^n$ ni bajaring.

14.49. $(\cos\varphi-i\sin\varphi)^n=\cos n\varphi-i\sin n\varphi$ ayniyatni isbotlang.

14.50. $\left(\frac{1+itg\varphi}{1-itg\varphi}\right)^n=\frac{1+itgn\varphi}{1-itgn\varphi}$ ayniyatni isbotlang.

14.51. $|\alpha^{-n}|=|\alpha|^{-n}$ tenglikni isbotlang.

14.52. Quyidagi modullarni hisoblang :

a) $\left|\left(\frac{1}{2}-\frac{1}{3}i\right)(3+2i)\right|$; b) $\left|\frac{\sqrt{2}-i}{\sqrt{3}+i}\right|$; v) $\left|\left(8+\frac{1}{\sqrt{5}}i\right)^5\right|$;

g) $\left|\frac{\left(2+\frac{1}{2}i\right)^4\cdot(\sqrt{2}-2i)^3\cdot(1-\sqrt{3}i)}{(2+i\sqrt{2})^2\cdot(\sqrt{3}+i)\cdot\left(\frac{1}{2}-2i\right)^4}\right|$; d) $\left|\left(\sqrt{5}+\frac{1}{\sqrt{3}}i\right)^{-2}\right|$;

e) $\left|\sqrt[3]{(2+7i)^2+(1-i)^{-4}\cdot(5+3i)}\right|$; j) $\left|\left(\frac{\sqrt[3]{(3-4i)^3}\cdot\sqrt{(1+2i)(3-i)}}{\sqrt{(4+5i)^3}(1+i)\sqrt{2+i}}\right)^2\right|$.

§ 14.2. Komleks o'zgaruvchili funksiysiyalar va ularni differensiallash

Hosila mavjudligining zaruriy va yetarli shartlari. Koshi – Riman shartlari. Biror G sohada bir qiymatli $w = f(z)$ funksiya berilgan bo'lsin. Agar $z = x + iy$ deb olsak, u holda bu funksiyani $w = f(z) = u + iv$ shaklida ifodalash mumkin. Biz yuqorida ko'rdikki, $f(z) = \bar{z} = x - iy$ funksiya tekislikning hech bir nuqtasida differensiallanuvchi emas. Lekin uning haqiqiy qismi $u = x$ va mavhum qismining koeffitsenti $v = -y$ hamma yerda ixtiyoriy tartibli uzluksiz xususiy hosilalarga ega. Bu yerdan $f(z)$ kompleks funksiyaning differensiallanuvchi bo'lishi uchun u va v funksiyalar bir-biri bilan qandaydir bo'glangan bo'lishi kerak degan xulosa kelib chiqadi. Biz quyida ushbu bog'lanishni oshkor qilamiz. Faraz qilaylik $w = f(z)$ funksiya biror $z = z_0 = x_0 + iy_0$ nuqtada differensiallanuvchi bo'lsin. U holda bu nuqtada

$$\lim_{\Delta z \to 0} \frac{\Delta w}{\Delta z} = \lim_{\Delta x \to 0, \Delta y \to 0} \frac{\Delta u + i\Delta y}{\Delta x + i\Delta y} = f'(z_0)$$

limit mavjud bo'ladi. U holda intilish yoliga bog'liq holda tanlangan quyidagi xususiy limitlar ham mavjud va ularning qiymatlari ham $f'(z_0)$ ga teng bo'lishi lozim:

1. $\Delta y = 0, \Delta x \to 0$ bo'lganda

$$\lim_{\Delta x \to 0} \frac{\Delta u + i\Delta v}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} + i \lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} = \frac{\partial u(x_0, y_0)}{\partial x} + i\frac{\partial v(x_0, y_0)}{\partial x} = f'(z_0); \qquad (14.2)$$

2. $\Delta x = 0, \Delta y \to 0$ bo'lganda esa

$$\lim_{\Delta y \to 0} \frac{\Delta u + i\Delta v}{i\Delta v} = \frac{1}{i} \lim_{\Delta y \to 0} \frac{\Delta u}{\Delta y} + \lim_{\Delta y \to 0} \frac{\Delta v}{\Delta y} = \frac{1}{i}\frac{\partial u(x_0, y_0)}{\partial y} + \frac{\partial v(x_0, y_0)}{\partial y} = f'(z_0). \qquad (14.3)$$

(14.2) va (14.3) dan

$$\frac{\partial u(x_0, y_0)}{\partial x} + i\frac{\partial v(x_0, y_0)}{\partial x} = \frac{1}{i}\frac{\partial u(x_0, y_0)}{\partial y} + \frac{\partial v(x_0, y_0)}{\partial y}$$

tenglikni olamiz. Bu yerda kompleks sonlarning tenglik ta'rifini qo'llasak

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \qquad (14.4)$$

munosabatlarning o'rinli ekanligi kelib chiqadi. Shunday qilib, agar $w = f(z)$ funksiya biror $z = z_0$ nuqtada differensiallanuvchi bo'lsa, u holda shu nuqtada (14.4) munosabatlarning bajarilishi zarurdir. Odatda (14.4) shartlar Koshi –Riman yoki Dalamber – Eyler shartlari deyiladi. (14.4) shartlar

$$w = f(z) = u + iv$$

funksiyaning $z = z_0$ nuqtada differensiallanuvchi bo'lishi uchun yetarli bo'la olmaydi

Bunda $u(x, y)$ va $v(x, y)$ mos ravishda kompleks funksiyaning haqiqiy va mavhum qismlaridir.

Agar tekislikning biror qismini qaraydigan bo'lsak, bu qismini tekislikning qolgan qismidan chiziq bilan ajratish kerak. Tekislik qismini ajratuvchi chiziq tenglamalari $x = \varphi(t)$, $y = \psi(t)$ bo'lsin. Bu chiziq ochiq va yopiq bo'lishi mumkin.

Agar chiziqning bosh va oxirgi nuqtalari bitta nuqtada bo'lsa, bunday chiziq yopiq deyiladi. Biz qaraydigan chiziqlarning karrali nuqtalari yo'q, ya'ni t parametrning ikkita qiymatiga (chiziqning bosh va oxirgi nuqtasidan boshqa) ikkita nuqta mos keladi, boshqacha aytganda, chiziq o'z-o'zini kesmaydi va o'z-o'ziga urinmaydi.

Tekislikning ushbu: 1) agar birorta nuqta to'plamga tegishli bo'lsa, bu nuqtaning atrofi ham shu to'plamga tegishli; 2) to'plamning ikkita ixtiyoriy nuqtasini birlashtiruvchi har qanday uzluksiz egri chiziqning nuqtalari to'plamga tegishli degan shartlarga bo'ysunuvchi nuqtalar to'plami soha deb ataladi.

14.2 –chizma.

Biz yuqorida ta'riflangan chiziqlar bilan chegaralangan sohalar bilangina ish ko'ramiz. Soha chegralangan chiziq sohaning konturi yoki chegarasi deyiladi. Sohaning ichki nuqtasi deganda nuqtaning o'zi va atrofi shu sohaga tegishli bo'lgan nuqta tushuniladi. Soha konturi bilan birga qaralsa, yopiq soha deyiladi. Konturi bitta chiziq bo'lgan soha bir bog'lamli, konturi bir necha yopiq chiziqlar bo'lgan soha ko'p bog'lamli deyiladi. Umuman, agar sohaning hamma nuqtalarini markazi koordinatalar boshida bo'lgan ixtiyoriy katta radiusli aylana ichiga joylashtirish mumkin bo'lsa, bu holda soha chegaralangan deyiladi. Aks holda soha chegaralanmagan deyiladi.

Kontur bo'ylab harakatlanganda soha chap tomonda qolsa, yo'nalish musbat, aks holda manfiy deyiladi (14.2-chizma).

Misol. $|z|<1$ yoki $x^2+y^2<1$ markazi koordinatalar boshida, radiusi 1 ga teng bo'lgan aylana bilan chegaralangan ochiq soha. $|z|=1$ yoki $x^2+y^2=1$ shu sohaning konturi.

z o'zgaruvchi xOy tekislikdagi B sohaning nuqtasi bo'lsin. Agar $f(z)$ funksiya B sohaning har bir buqtasida aniq chekli qiymatga ega bo'lsa, u holda B soha $w=f(z)$ funksiya xOy tekislikdagi (yoki z tekislikdagi) nuqtalarni uOv teskislikdagi (yoki w tekislikdagi) nuqtalarga o'takazadi. Xususan, funskiyaning aniqlash sohasi B bo'lsa, bu funksiya B sohani B sohaga o'zkazadi yoki akslantiradi deyiladi.

Kompleks o'zgaruvchili funksiyaning uzluksizligi. B soha $w=f(z)$ funksiyaning mavjudlik sohasi bo'lib, z_0 nuqta B sohaga tegishli bo'lsin.

Oldindan berilgan har qanday kichik musbat ε son uchun shunday musbat $\eta(\varepsilon)=\eta$ sonni toppish mumkin bo'saki, bunda $|z-z_0|<\eta(\varepsilon)$ bo'lganda, $|f(z)-A|<\varepsilon$ tengsizlik bajarilsa, $f(z)$ funksiya o'zgarmas A g intiladi deyiladi va $\lim\limits_{z\to z_0} f(z)=A$ ko'rinishida yoziladi. Xususan, agar $A=f(z_0)$ bo'lsa, $f(z)$ funsksiya z_0 nuqtada uzluksiz deyiladi. Demak, oldindan berilgan har qanday kichik musbat ε soni uchun musbat $\eta(\varepsilon)=\eta$ sonni topish mumkin bo'lsaki, bunda $|z-z_0|<\eta(\varepsilon)$ bo'lganda

$|f(z)-f(z_0)|<\varepsilon$ tengsizlik bajarilsa, $f(z)$ funksiya z_0 nuqtada uzluksiz deyiladi.

Hosila. B sohada aniqlangan va bir qiymatli $f(z)$ funksiya berilgan bo'lsin; $f(z)$ funksiyaning z nuqtadagi hosilasi

$$f'(z)=\lim_{\Delta z\to z}\frac{f(z+\Delta z)-f(z)}{\Delta z} \qquad (14.4)$$

bo'ladi. Elementar funksiyalardan hosilalar jadvali:

$$\left(z^n\right)'=nz^{n-1},\ \left(e^z\right)'=e^z,$$

$$\left(\cos z\right)'=-\sin z\ \ \left(\sin z\right)'=\cos z$$

$$\left(\ln z\right)'=\frac{1}{z}\ \left(\arcsin z\right)'=\frac{1}{\sqrt{1-z^2}}\ \left(\arccos z\right)'=\frac{1}{\sqrt{1-z^2}}\ \left(arctgz\right)'=-\frac{1}{\sqrt{1+z^2}}$$

Agar $w=f(z)=u(x,y)+iv(x,y)$ funksiya $z=x+iy$ nuqtada differensiallanuvchi bo'lsa, u holda bu nuqtada xususiy hosilalar $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$ mavjud bo'ladi. Binobarin,

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\quad \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x} \qquad (14.5)$$

Koshi-Riman shartlari (differensialanuvchi bo'lishining zaruriy va yetarli shartlari) o'rinli bo'ladi.

Misol.

$w=z^2=(x+iy)^2=x^2-y^2+2xyi:\quad u(x;y)=x^2-y^2,\quad v(x;y)=2xy:$

$$\begin{bmatrix}\frac{\partial u}{\partial x}=(x^2-y^2)'_x=2x, & \frac{\partial v}{\partial y}=(2xy)'_y=2x\end{bmatrix}\Rightarrow 2x=2x \qquad (14.5)$$
$$\begin{bmatrix}\frac{\partial v}{\partial x}=(2xy)'_x=2y, & \frac{\partial u}{\partial y}=(x^2-y^2)'_y=-2y\end{bmatrix}\Rightarrow 2y=-(-2y).$$

shartlar bajariladi, demak $w=z^2$ funksiya differensiallanuvchi.

B sohaning har bir nuqtasida hosilaga ega bo'lgan $w=f(z)$ funksiya shu sohada analitik deyiladi.

Bu tengliklarning bajarilishi funksiyaning hosilaga ega bo'lishligining zaruriy va yetarli shartlaridir. (14.5) Koshi-Riman yoki Dalamber-Eyler shartlari deyilib, bu funksiyaning analitiklik shartlaridir.

Berilgan misollarni yeching

143.53. $\left|\sqrt[n]{\alpha}\right|=\sqrt[n]{|\alpha|}$ tenglikni isbotlang.

14.54. $w=z^2+z$ funksiya berilgan. Funksiyaning 1) $z=1+i$;
2) $z=2-i$; 3) $z=i$;
4) $z=-1$ nuqyalaridagi qiymatini toping.

14.55. $f(z)=x^2+iy^2$ ($z=x+yi$) funksiya berilgan. Quyidagilarni toping. 1) $f(1+2i)$; 2) $f(2-3i)$; 3) $f(0)$; 4) $f(-i)$.

14.56. $w=|z|$ z 14.57. $w=z^2$ z

14.58. $\ln(\sqrt{3}+i)$ ni toping.

14.59. $\cos(i/2)$ ni $\varepsilon=0{,}0001$ aniqlikda toping.

14.60. $w=e^z$ funksiyani 1) $z=\pi i/2$ 2) $z=\pi(1-i)$ 3) $z=1+(\pi/2+2\pi k)i$, ($k\in Z$) buqtadagi qiymatin topilsin.

14.61. $f(z)=1/(x-yi)$, $z=x+yi$. $f(1+i)$, $f(i)$, $f(3-2i)$ larni toping.

14.62. $w=2z^3$ 14.63. $\ln(1-i)$ ni toping.

14.64. $\sin i\cdot ch1=i\cos i\cdot sh1$ ni isbotlang.

14.65. $\cos z=2$ tenglamani yeching.

14.66. $\arcsin i$ ni toping.

14.67. $\sin i$ ni hisoblang ($\varepsilon=0{,}001$).

14.68. $\sin(\pi/6+i)$ ni hisoblang ($\varepsilon=0{,}001$).

14.69. $f(z)=e^{z^2}$ funksiya berilgan. 1) $z=i$; 2) $z=1+\pi i/2$

14.70. Quyidagi tenglamalar bilan xOy tekisligida qanday chiziqlar ifodalangan.

a) $z=3+t^2+it$ b) $z=2t-4+i(3-4t^2)$ v) $z=i+3e^{it}$

14.71. Limitlarni toping.

a) $\lim\limits_{n\to\infty}\left(\frac{2n}{n+2}-i\frac{n^2-2}{n^2+3}\right)$; b) $\lim\limits_{n\to\infty}\sqrt{\frac{2n+3i}{8n-7i}}$;

c) $\lim\limits_{n\to\infty} n\sin\frac{2i}{n}$ d) $\lim\limits_{z\to 3i}\frac{z^2+9}{z-3i}$ e) $\lim\limits_{z\to -i}\frac{z^2+4}{z+i}$

f) $\lim\limits_{z\to 2+i}\frac{z^2-4z+5}{z^2-2iz-5}$ g) $\lim\limits_{z\to 0}\frac{1-\cos z}{z^2}$

14.72. Funksiyaning uzluksizligini tekshiring.

a) $w=z\,\mathrm{Re}\,z-2z^2$ b) $w=\frac{2z-1}{z^2-6z+10}$ v) $w=\frac{2-3z}{1-|z|}$

Agar $z=re^{i\varphi}$ bo'lsa, $Arg f(z)$ ni toping.

14.73. $f(z)=z^2$ 14.74. $f(z)=z^3$. 14.75. $f(z)=\sqrt[3]{z+1}$

14.76. $f(z)=\sqrt{z-8}$ 14.77. $f(z)=\sqrt{z^2-4}$ 14.78. $f(z)=\sqrt{\frac{z-2}{z+1}}$

Ko'rsatilgan nuqtadagi funksiyaning qiymatini hisoblang

14.79. $\cos(1+i)$ 14.80. $ch\,i$ 14.81. $sh(-2+i)$

14.82. $Ln(-1)$ 14.83. $\ln i$ 14.84. $Ln\frac{1+i}{\sqrt{2}}$

Funksiyani Koshi –Riman shartlariga $f'(z)$ ni toping

14.85. $f(z)=e^{1/z}$ 14.86. $f(z)=shz$

14.87. $f(z)=z^n, \quad n\in N$ 14.88. $f(z)=\cos z$

14.89. $f(z)=\ln(z^2)$ 14.90. $f(z)=\sin\frac{z}{3}$

14.91. Quyidagi o'zgaruvchilarning geometrik o'rni aniqlansin: a) $|z|\le\sqrt{3}$; b) $|z-2|=2$; v) $|z-2|=|z+2|$; g) $|z-C|+|z+C|\le 2a$; d) $\alpha<\arg z<\beta,\quad \alpha<\operatorname{Re} z<b$.

14.92. Quyidagi funksiyalarning analitikligi tekshirilsin:

$$e^{\bar z};\quad \sin z;\quad \cos z;\quad z^{2};\quad e^{-z^{2}}.$$

14.93. Funksiya hosilasining ta'rifidan foydalanib, quyidagi tengliklarning to'g'riligi isbotlansin:

$$(e^z)'=e^z;\quad (\sin z)'=\cos z;\quad (\cos z)'=-\sin z.$$

14.94. Mavhum qismi $v=\frac{y}{x^2+y^2}$, va o'zi $w(2)=0$ shartni qanoatlantiruvchi funskiya topilsin.

14.95. Haqiqiy qismi $u=x^2-y^2+xy$ va o'zi $w(0)=0$ shartni qanoatlantiruvchi funksiya topilsin.

14.96. $w=z^2+2i$ funksiya vositasida akslantirishda $z=i$ nuqtadagi cho'zilish koeffisiyenti va burilish burchagi aniqlansin.

14.97. $x=2+3\cos t,\quad y=-1+3\sin t\quad (0\le t\le 2\pi)$ chiziq tenglamasini kompleks ko'rinishda yozing.

14.98. $2x^2+3y^2=12$ chiziq tenglamasini kompleks ko'rinisda yozing.

14.99. a) $z=5e^{it}+2e^{-it}$ b) $z=2t-1+i(1+2t-4t^2)$ tenglama xOy tekisligidan qanday chiziqni ifodalaydi?

14.100. Kompleks z tekisligida berilgan chiziqlarning xOy tekisligidagi tenglamasini toping. a) $|z+4-i\cdot 2|=3$; b) $\operatorname{Re}\frac{z-2}{z+i}=2$; v) $\arg(z-1)=\frac{\pi}{4}$

14.101.Limitni hioblang.

a) $\lim\limits_{n\to\infty}\left(\frac{2n-3}{n+1}+i\frac{3n^2-2n+1}{1+n^2}\right)$; b) $\lim\limits_{z\to 2-3i}\frac{\operatorname{Re}(z^2)}{|z|^2}$; v) $\lim\limits_{z\to -1+2i}\frac{z^2+2z+5}{z+1-2i}$.

14.102. Quyidagi funksiyalarning uzluksizligini tekshiring.

a) $w=z^2\operatorname{Re}\bar{z}-i\operatorname{Im}(z^2)$ b) $w=\frac{z^2-3z+1}{|z-2i|-3}$

14.103. w funksiyaning analitik bo'lishini tekshiring.

a) $w=(z-1)\operatorname{Re}(z+1)$ b) $w=e^{i2z+1}$

14.104. $u=x^2e^{y}$ garmonik funksiya bo'la oladimi?

14.105. Quyidagi funksiyalarning analitikligini tekshiring.

a) $\omega=z\bar{z}-z\operatorname{Im}z$ b) $\omega=x^2-2iy$ v) $\omega=2z^2-3iz$

Quyidagi limitlar hisoblansin

14.106. a) $\lim\limits_{n\to\infty}\sqrt{\frac{2n+3i}{8n-7i}}$ b) $\lim\limits_{n\to\infty} n\sin\frac{2i}{n}$

v) $\lim\limits_{n\to\infty}\frac{1-\cos z}{z^2}$ g) $\lim\limits_{n\to\infty}\frac{z^2+4}{z+i}$

14.107. $f(z)=(x^2-y^2)+2xyi$ differensillanuvchi funksiya bo'ladimi?

14.108. $f(z)=e^x\cos y+ie^x\sin y$ differensillanuvchi funksiya bo'ladimi?

14.109. Differensillanuvchi $f(z)$ funksiyaning haqiqiy qismi $u(x,y)=x^2-y^2-x$, $x=x+yi$ berilgan bo'lsa, $f(z)$ ni toping.

14.110. Differensillanuvchi $f(z)$ funksiyaning mavhum qismi $v(x,y)=x+y$ bo'lsa, $f(z)$ ni toping.

14.111. $f(z)=(x^2+y^2)-2xyi$ differensillanuvchi funksiya bo'ladimi?

14.112 $f(z)=(x^3-3xy^2)+i(3x^2y-y^3)$ funksiyaning hosilasini toping.

14.113 $f(z)=\sin x\operatorname{ch}y+i\cos x\operatorname{sh}y$ funksiyaning hosilasini toping.

14.11 $f(z)=a\bar{z}$ $(\bar{z}=x-yi)$ a ning qanday qiymatida differensiallanuvchi bo'ladi?

§ 14.3. Kompleks o'zgaruvchili funksiyani integrallash

xOy tekislikda (z tekislikda) tenglamalari $x=\varphi_1(t)$; $y=\varphi_2(t)$ bo'lgan yo'naltirilgan silliq yoki bo'laklari silliq egri chiziq (C) berilgan. Bu egri chiziqda f(z) uzluksiz funksiya aniqlangan bo'lsin. Egri chiziqni M_0~A, M_1, M_2,...,M_{n-1}, M_n~B, nuqtalar bilan n ta bo'lakka (bo'laklar o'zaro teng bo'lishi shart emas) bo'lamiz. $M_k(z_k)$ deb $M_{k-1}M_k$ bo'lakning ixtiyoriy ζ_k nuqtasini olib, quyidagi yig'indini tuzaylik:

$$\sum_{k=1}^{n} f(\zeta_k)(z_k - z_{k-1}).$$

Bo'laklar sonini cheksiz orttira borib, max $|z_k - z_{k-1}| \to 0$ shart bajarilgandagi yig'indining limitiga f(z) funksiyadan (C) chiziq bo'yicha olingan egri chiziqli integral deyiladi va quyidagicha belgilanadi:

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(\zeta_k)(z_k - z_{k-1}) = \int_{(C)} f(z)dz.$$

Agarda $z_k = x_k + iy_k$; $\zeta_k = \xi_k + i\eta_k$ desak,

$$f(z_k)=u(x_k,y_k)+iv(x_k,y_k),\quad f(\zeta_k)=u(\xi_k,\eta_k)+iv(\xi_k,\eta_k)$$

va yig'indini haqiqiy o'zgaruvchilar orqali ifodalash mumkin:

$$\sum_{k=1}^{n} f(\zeta_k)f(z_k - z_{k-1}) = \sum_{k=1}^{n}[u(\xi_k,\eta_k)+iv(\xi_k,\eta_k)][(x_k+iy_k)-(x_{k-1}+iy_{k-1})]=$$

$$=\sum_{k=1}^{n}[u(\xi_k,\eta_k)(x_k - x_{k-1}) + u(\xi_k,\eta_k)(y_k - y_{k-1})]$$

Agar C chiziq $x=x(t)$, $y=y(t)$ parametrik tenglamalar yordamida yoki $z=z(t)=x(t)+iy(t)$ formada berilgan bo'lsa, u holda $z(t_1)=a$, $z(t_2)=b$ (a,b lar C chiziqning oxirlari), u holda

$\int_C f(z)dz = \int_{t_1}^{t_2} f(z(t))z'(t)dt$ bo'ladi. Faraz qilaylik, $w=f(z)$ bir bog'-lamli D sohada analitik funksiya va C yopiq bo'lgan chiziq (z_1-boshi z_2 -oxirgi nuqtalari) bo'lsa, u holda

$\int_C f(z)dz = \int_{t_1}^{t_2} f(z(t))z'(t)dt = F(z_2) - F(z_1)$, bu yerda $F(z)$ funksiya $f(z)$ ning boshlang'ich funksiyasi $F'(z) = f(z)$

Yopiq B sohada analitik f(z) funksiya berilgan bo'lsin. Sohaning konturini C deylik. Funksiyaning konturdagi qiymatlari bo'yicha kontur ichidagi qiymatlarini aniqlash mumkin. Bu $f(z) = \frac{1}{2\pi i}\int_C \frac{f(\zeta)}{\zeta - z}d\zeta$ ko'rinishdagi Koshi formulasiga asosan topiladi. Koshi integrali yopiq kontur bo'yicha olinsa,

$$\oint_C \frac{f(z)dz}{(z-\zeta)^{n+1}} = \frac{2\pi i}{n!} f^{(n)}(\zeta), \quad (n = 0,1,2...)$$

bu yerda ζ -C chiziqning ichki nuqtasi bo'lib, konturi musbat yo'nalishda aylanib o'tadi. $w = f(z)$ analitik funksiya uchun $f^{(n)}(\zeta) = \frac{n!}{2\pi i}\oint_C \frac{f(z)dz}{(z-\zeta)^{n+1}}$ $(n = 0,1,2...)$ bu yerda $f^{(0)}(\zeta) = f(\zeta)$. C – bir bog'lamli soha.

Misol. 1) $\int_{(C)} \frac{(z^2-1)^n}{z-x}dz$ integral hisoblansin. Koshi formulasi va Koshi teoremasiga asosan, x nuqta c kontur ichida bo'lsa, integral $2\pi i(x^2-1)^n$ ga, x nuqta konturdan tashqarida bo'lsa 0 ga teng.

2) Koshi formulasiga asosan $\int_{(C)} \frac{dz}{1+z^2}$ integralning a) C kontur $|z-i|=1$ aylana, b) C kontur $|z+i|=1$ aylana, c) C kontur $|z|=2$ aylana bo'lgandagi qiymati hisoblansin. Bu integralni $\frac{1}{2i}\left[\int_{(C)} \frac{dz}{z-i} - \int_{(C)} \frac{dz}{z+i}\right]$ ko'rinishda yozish mumkin. Qavs ichidagi ikkala integralning ham suratidagi funksiyalar o'zgarmas son 1 ga teng, shuning uchun $|z-i|=1$ aylana bo'yicha integrallaganda Koshi formulasiga asosan birinchi integral $2\pi i$ ga, ikkinchi integral esa 0 ga teng.

$|z+i|=1$ aylana bo'yicha integrallaganda aksincha, birinchi integral 0 ga, ikkinchisi esa $2\pi i$ ga teng. $|z|=2$ aylana bo'yicha integrallaganda $z=i$ va $z=-i$ nuqtalar aylana ichida bo'lib, birinchi integral suratdagi funskiyaning i nuqtadagi qiymatining

$2\pi i$ ga ko'paytmasini, ikkinchi integral esa o'sha funksiyaning $-i$ nuqtadagi qiymatining yana $2\pi i$ ga ko'paytmasiga teng bo'lib, natija ikkalasining yig'indisi bo'lishi kerak edi. Integral ostidagi ifodaning suratidagi funksiya o'zgarmas 1 ga teng bo'lgani uchun ikkala integral 0 ga teng.

Berilgan integrallarni hisoblang

14.115. $\int_{AB} z^2 dz$, AB $z_A = 1$ *va* $z_B = i$ nuqtalarni tutashtiruvchi to'g'ri chiziq.

14.116. $\int_C z^{10} dz$, C: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$-ellips.

14.117. $\int_C \frac{dz}{z^2}$, C: $(x-4)^2 + (y-3)^2 = 1$ - aylana.

14.118. $\int_C \frac{dz}{z}$, C: $z = e^{it}$ - aylana.

14.119. $\int_{AB} f(z) dz$ integralni $f(z) = x^2 + y^2 i$, $AB - A(1+i)$ va $B(2+3i)$ nuqtalarni tutashtiruvchi to'g'ri chiziq bo'lgan hol uchun hisoblang.

14.120. $\int_1^{1+i} z dz$ integralni hisobalng.

14.121. $\int_\gamma \bar{z} dz$, γ -yopiq soha bo'lib parametrik tenglamasi $x = \cos t$, $y = \sin t$.

14.122. $\int_\gamma \frac{dz}{z-4}$, γ-ellips $x = 3\cos t$, $y = 2\sin t$

14.123. $\int_\gamma \frac{dz}{z-(1+i)}$, γ-aylana $|z-(i+1)| = 1$

14.124. $w = f(z)$ analitik funksiya z tekislikdagi B sohani w tekislikdagi B_1 sohaga akslantiradi. $B_1 = \iint_{(B)} |f'(z)|^2 dxdy$ ekanligi isbotlansin. Bu formulaga asosan $w = z^2$ funksiya vositasida $1 \le |z| \le 2$; $\frac{\pi}{4} \le \arg z \le \frac{\pi}{4}$ soha aksining yuzi topilsin.

14.125. Integralning ta'rifiga asosan $\int_{|z|=r} |dz|$ va $\int_{|z|=r} dz$ integrallar hisoblansin.

14.126. Ushbu $\int\limits_{(L)} \frac{dz}{z^2+4}$ integral Koshi formulasiga asosan hisoblansin, bunda L: a) $-2i$ va $+2i$ nuqtalarni o'z ichiga olgan kontur; b) $-2i$ nuqtani o'z ichiga olgan kontur; v) $+2i$ nuqtani o'z ichiga olgan kontur.

14.127. Ushbu $\int |z| z dz$ integral yarim halqa konturi bo'yicha hisoblansin.

14.128. $\int\limits_C \left(iz^2 - 2ze^{z^2}\right)dz$, $C: z_1 = e^{-i\pi/4}$ va $z = i$ nuqtalarni tutashtiruvchi ixtiyoriy chiziq.

14.129. $\oint \frac{2z - e^z}{z^2(z-3)} dz$, $C: |z+1| = 2$-aylana.

14.130. $\int\limits_C \frac{2z-1-i}{(z-1)(z-i)} dz$ $|z| = 2$-aylana.

14.131. $\int\limits_C f(z)dz$, agar $f(z) = y + xi$ C – uchi $z_0 = 0$, $z_A = i$, $z_B = 1 + i$ bo'lgan siniq chiziq.

Takrorlash uchun savollar

1.Kompleks son ta'rifini ayting

2.Kompleks sonlar ustida qanday amallar bajarish mumkin?

3.Kompleks sonning trigonometric ko'rinishini yozing.

4.i-nima?

5.Kompleks sondan qanday ildiz chiqariladi?

6.Kompleks sonning geometric tasviri deganda nimani tushinasiz?

7.Kompleks son haqiqiy qismi, mavhum qismi va u qanday belgilanadi?.

KOMPLEKS O'ZGARUVCHILI FUNKSIYALAR NAZARIYASI ELEMENTLARIGA DOIR NAZORAT TESTLAR

1. Kompleks sonlar va ular ustida amallar .

1. Mavhum birlik i qanday aniqlanadi ?

A) $i = \sqrt{-1}$ B) $i = \sqrt[3]{-1}$ C) $i = \sqrt{1}$ D) $i = \sqrt[3]{1}$.

E) to'g'ri javob keltirilmagan .

2. Mavhum birlik i qaysi tenglik bilan aniqlanadi ?

A) $i=-1$ B) $i^2=-1$ C) $i^3=-1$ D) $i^4=-1$ E) $i^2=1$

3. Mavhum birlik i uchun qaysi tenglik o'rinli emas?

A) $i^{4n}=1$ B) $i^{4n+1}=i$ C) $i^{4n+2}=-1$ D) $i^{4n+3}=-i$

E) barcha tengliklar to'g'ri .

4. Mavhum birlik i uchun qaysi tenglik o'rinli ?

A) $i^{4n}=1$ B) $i^{4n}=i$ C) $i^{4n}=-1$

D) $i^{4n}=-i$ E) $i^{4n}=(-1)^n$

5. Agar n-ixtiyoriy natural son bo'lsa, unda mavhum birlik i uchun qaysi tenglik o'rinli ?

A) $i^{2n}=1$ B) $i^{2n}=i$ C) $i^{2n}=-1$

D) $i^{2n}=-i$ E) $i^{2n}=(-1)^n$

6. ***Ta'rifni to'ldiring:***$z=x+iy$ ko'rinishdagi ifoda kompleks son deb ataladi. Bunda i–mavhum birlik ($i^2=-1$) va x,y – ··· sonlarni ifodalaydi.

A) ratsional B) irratsional C) haqiqiy

D) natural E) butun

7. $z=x+iy$ kompleks son uchun Rez qanday aniqlanadi?

A) Rez=$\sqrt{x^2+y^2}$ B) Rez=$x+y$ C) Rez=x D) Rez=y

E) to'g'ri javob keltirilmagan

8. $z=-3+4i$ kompleks son uchun Rez nimaga teng?

A) 5 . B) 1 . C) 3 . D) 4 . E) −3 .

9. $z=x+iy$ kompleks son uchun Imz qanday aniqlanadi?

A) Imz=$\sqrt{x^2+y^2}$ B) Imz=$x+y$ C) Imz=x

D) Imz=y E) to'g'ri javob keltirilmagan

10. $z=-3+4i$ kompleks son uchun Imz nimaga teng?

A) 5 . B) 1 . C) −4 . D) 4 . E) −3 .

11. $z=x+iy$ kompleks sonning moduli $|z|$ qanday aniqlanadi?

A) $|z|=x+y$ B) $|z|=x^2+y^2$. C) $|z|=\sqrt{x^2+y^2}$.

D) $|z|=|x+y|$ E) $|z|=\sqrt{x^2-y^2}$.

12. $z=3-4i$ kompleks sonning moduli $|z|$ nimaga teng?

A) −1 . B) 1 . C) 7 . D) 5 . E) 25 .

13. $z=x+iy$ kompleks songa qo'shma kompleks son $\bar{z}$ qanday aniqlanadi?

A) $-x+iy$ B) $x-iy$ C) $-x-iy$

D) $y+ix$ E) $y-ix$

14. O'zaro qo'shma kompleks sonlar uchun quyidagi tengliklardan qaysi biri o'rinli emas ?

A) $z+\bar{z}=2\operatorname{Re}z$ B) $z-\bar{z}=2\operatorname{Im}z$ C) $z\cdot\bar{z}=|z|^2$.

D) $z\cdot\bar{z}=|\bar{z}|^2$ E) $|z|=|\bar{z}|$

15. $z_1=x_1+y_1i$ va $z_2=x_2+y_2i$ kompleks sonlar qaysi shartda teng deyiladi?

A) $x_1=x_2$ B) $y_1=y_2$ C) $x_1=x_2$, $y_1\neq y_2$

D) $x_1=x_2$, $y_1=y_2$ E) $x_1\neq x_2$, $y_1=y_2$.

16. Quyidagi tengliklarning qaysi birida $z_1=x_1+y_1i$ va $z_2=x_2+y_2i$ kompleks sonlar teng bo'lmasligi mumkin?

A) $(x_1-x_2)^2+(y_1-y_2)^2=0$ B) $|x_1-x_2|+|y_1-y_2|=0$.

C) $(x_1-x_2)(y_1-y_2)=0$ D) $x_1-x_2=0$, $y_1-y_2=0$.

E) barcha hollarda $z_1=z_2$ bo'ladi.

2. Kompleks o'zgaruvchili funksiyalar

1. $w=z^2-z$ funksiyaning $z=1+2i$ nuqtadagi qiymatini hisoblang.

A) $4-2i$ B) $-4-2i$ C) $-4+2i$.

D) $4+2i$ E) $2-4i$

2. $w=z^2-z$ funksiyaning $z=1-2i$ nuqtadagi qiymatini hisoblang.

A) $4-2i$ B) $-4-2i$ C) $-4+2i$

D) $4+2i$ E) $2-4i$

3. $z=x+iy$ kompleks o'zgaruvchili funksiya $f(z)=x^2-2(x+y)i$ kabi aniqlangan. Bu funksiyaning $z=2-3i$ nuqtadagi qiymatini hisoblang.

A) $4-10i$ B) $4-2i$ C) $4+2i$ D) $4+10i$ E) $4-4i$

4. $w=z^2-z$ $(z=x+iy)$ funksiya uchun $u(x,y)=\operatorname{Re}w$ nimaga teng?

A) x^2-2xy B) x^2+y^2-x C) x^2-y^2+x

D) x^2-y^2-x E) x^2+y^2+x

5. $w=z^2-z$ $(z=x+iy)$ funksiya uchun $v(x,y)=\text{Im}w$ nimaga teng?

A) x^2-2xy B) $2xy-x$ C) $2xy+x$.

D) $2xy-y$ E) $2xy+y$

6. $w=f(z)$ $(z=x+iy)$ funksiya uchun $\text{Re}f(z)=x^2-y^2$, $\text{Im}f(z)=-2xy$ bo'lsa, $w=f(z)$ funksiyani toping .

A) $w=z^2$. B) $w=z\cdot\bar{z}$. C) $w=\bar{z}^2$.

D) $w=z^2+z$. E) $w=z^2+\bar{z}$.

7. $w=f(z)$ $(z=x+iy)$ funksiya uchun $\text{Re}f(z)=x^2-y^2$, $\text{Im}f(z)=2xy$ bo'lsa, $w=f(z)$ funksiyani toping .

A) $w=z^2$. B) $w=z\cdot\bar{z}$. C) $w=\bar{z}^2$.

D) $w=z^2+z$. E) $w=z^2+\bar{z}$.

8. Agar $f(z)=z^2-z$ va $z_0=1+2i$ bo'lsa, $\lim\limits_{z\to z_0} f(z)$ limit qiymatini toping .

A) $4-2i$. B) $-4-2i$. C) $-4+2i$. D) $4+2i$. E) $2-4i$.

9. Agar $f(z)=z^2-z$ va $z_0=1-2i$ bo'lsa, $\lim\limits_{z\to z_0} f(z)$ limit qiymatini toping .

A) $4-2i$. B) $-4-2i$. C) $-4+2i$. D) $4+2i$. E) $2-4i$.

10. Agar $\lim\limits_{z\to z_0} f(z)$ va $\lim\limits_{z\to z_0} g(z)$ limitlar mavjud, quyidagi limitlardan qaysi biri mavjud bo'lmasligi mumkin?

A) $\lim\limits_{z\to z_0}[f(z)+g(z)]$ B) $\lim\limits_{z\to z_0}[f(z)-g(z)]$

C) $\lim\limits_{z\to z_0}[f(z)\cdot g(z)]$ D) $\lim\limits_{z\to z_0}[f(z)/g(z)]$

E) barcha limitlar mavjud bo'ladi .

11. Agar $\lim\limits_{z\to 1+i} f(z)=2-3i$ bo'lsa, $\lim\limits_{z\to 1+i} zf(z)$ limit qiymati nimaga teng ?

A) $3-2i$ B) $4+i$ C) $5-i$

D) $-1+2i$ E) aniqlab bo'lmaydi .

12. Agar $\lim\limits_{z\to i} f(z)=2-3i$ bo'lsa, $\text{iim}\limits_{z\to i}[f(z)/z]$ qiymati nimaga teng ?

A) $3-2i$ B) $2+3i$ C) $-3-2i$

D) $-1+3i$ .E) aniqlab bo'lmaydi .

13. Agar $\lim\limits_{z\to i} f(z)=2-3i$ bo'lsa, $\lim\limits_{z\to i}[z^2 f(z)]$ qiymati nimaga teng ?

A) $3-2i$. B) $-2+3i$. C) $-3-2i$. D) $2-3i$.

E) aniqlab bo'lmaydi .

14. $w=e^z$ ko'rsatgichli funksiya qaysi darajali qator bilan aniqlanadi?

A) $1-\frac{z}{1!}+\frac{z^2}{2!}-\frac{z^3}{3!}+\cdots+(-1)^n\frac{z^n}{n!}+\cdots$.

B) $1+\frac{z}{1!}+\frac{z^2}{2!}+\frac{z^3}{3!}+\cdots+\frac{z^n}{n!}+\cdots$.

C) $1-\frac{z^2}{2!}+\frac{z^4}{4!}-\frac{z^6}{6!}+\cdots+(-1)^n\frac{z^{2n}}{(2n)!}+\cdots$.

D) $z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}+\cdots+(-1)^n\frac{z^{2n-1}}{(2n-1)!}+\cdots$.

E) $z+\frac{z^3}{3!}+\frac{z^5}{5!}+\cdots+\frac{z^{2n+1}}{(2n+1)!}+\cdots$.

15. $w=e^{-z}$ ko'rsatgichli funksiya qaysi darajali qator bilan aniqlanadi?

A) $1-\frac{z}{1!}+\frac{z^2}{2!}-\frac{z^3}{3!}+\cdots+(-1)^n\frac{z^n}{n!}+\cdots$.

B) $1+\frac{z}{1!}+\frac{z^2}{2!}+\frac{z^3}{3!}+\cdots+\frac{z^n}{n!}+\cdots$.

C) $1-\frac{z^2}{2!}+\frac{z^4}{4!}-\frac{z^6}{6!}+\cdots+(-1)^n\frac{z^{2n}}{(2n)!}+\cdots$.

D) $z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}+\cdots+(-1)^n\frac{z^{2n-1}}{(2n-1)!}+\cdots$.

E) $z+\frac{z^3}{3!}+\frac{z^5}{5!}+\cdots+\frac{z^{2n+1}}{(2n+1)!}+\cdots$.

16. $w=\cos z$ trigonometrik funksiya qaysi darajali qator bilan aniqlanadi?

A) $1-\frac{z}{1!}+\frac{z^2}{2!}-\frac{z^3}{3!}+\cdots+(-1)^n\frac{z^n}{n!}+\cdots$.

B) $1+\frac{z}{1!}+\frac{z^2}{2!}+\frac{z^3}{3!}+\cdots+\frac{z^n}{n!}+\cdots$.

C) $1-\frac{z^2}{2!}+\frac{z^4}{4!}-\frac{z^6}{6!}+\cdots+(-1)^n\frac{z^{2n}}{(2n)!}+\cdots$.

D) $z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}+\cdots+(-1)^n\frac{z^{2n-1}}{(2n-1)!}+\cdots$.

E) $z+\frac{z^3}{3!}+\frac{z^5}{5!}+\cdots+\frac{z^{2n+1}}{(2n+1)!}+\cdots$.

3. Kompleks o'zgaruvchili funksiyalarning hosilasi

1. $w=f(z)$ kompleks funksiyaning hosilasini ta'rif bo'yicha topishda quyidagi amallardan qaysi biri bajarilmaydi?

A) z argumentga Δz orttirma beriladi .

B) funksiyaning $\Delta f=f(z+\Delta z)-f(z)$ orttirmasi hisoblanadi .

C) $\Delta f/\Delta z$ nisbat topiladi . D) $\Delta f/\Delta z$ nisbatning $\Delta z\to 0$ holdagi limiti topiladi .

E) keltirilgan barcha amallar bajariladi .

2. $f(z)=u(x,y)+iv(x,y)$ funktsiyaning differensiallanuvchi bo'lishi uchun qaysi shartlar zarur va yetarli bo'ladi?

I. $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}$ II. $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ III. $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial x}$ IV. $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

A) I va II B) I va IV C) II va III

D) II va IV E) III va IV

3. Quyidagi tengliklardan qaysi biri Koshi-Riman shartlalrini ifodalaydi?

A) $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$. B) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$.

C) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$. D) $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$.

E) keltirilgan barcha tengliklar Koshi-Riman shartlarini ifodalaydi .

4. $f(z)=(x^2-3y^2+2)+(6x+8y-7)i$ kompleks funksiya qaysi $z_0=x_0+iy_0$ nuqtada differensiallanuvchi bo'ladi?

A) $z_0=-4-i$. B) $z_0=-4+i$. C) $z_0=4-i$. D) $z_0=4+i$.

E) bu funksiya birorta ham nuqtada differensiallanuvchi bo'lmaydi .

5. $f(z)=(5x-3y+2)+(6x+8y-7)i$ kompleks funksiya qaysi $z_0=x_0+iy_0$ nuqtada differensiallanuvchi bo'ladi?

A) $z_0=-4-i$. B) $z_0=-4+i$. C) $z_0=4-i$. D) $z_0=4+i$.

E) bu funksiya birorta ham nuqtada differensiallanuvchi bo'lmaydi .

6. $f(z)=(x^2-y^2)+2xyi$ kompleks funksiya qaysi $z_0=x_0+iy_0$ nuqtada differensiallanuvchi bo'lmaydi?

A) $z_0=-4-i$ B) $z_0=-4+i$ C) $z_0=4-i$.

D) bu funksiya barcha nuqtalarda differensiallanuvchi bo'ladi .

E) bu funksiya birorta ham nuqtada differensiallanuvchi bo'lmaydi .

7. $f(z)=(x^2+y^2)+axyi$ kompleks funksiya a parametrning qanday qiymatlarida differensiallanuvchi bo'ladi?

A) $a=\pm2$. B) $a=-2$. C) $a=2$. D) $a\neq\pm2$. E) $a\in\varnothing$.

8. $f(z)=(x^2-y^2)+axyi$ kompleks funksiya a parametrning qanday qiymatlarida differensiallanuvchi bo'ladi?

A) $a=\pm2$. B) $a=-2$. C) $a=2$. D) $a\neq\pm2$. E) $a\in\varnothing$.

9. $f(z)=y+axi$ kompleks funksiya a parametrning qanday qiymatlarida differensiallanuvchi bo'ladi?

A) $a=\pm1$. B) $a=-1$. C) $a=1$. D) $a\neq\pm1$. E) $a\in\varnothing$.

10. $f(z)=a\bar{z},\ \bar{z}=x-iy$, kompleks funksiya a parametrning qanday qiymatlarida differensiallanuvchi bo'ladi?

A) $a=0$. B) $a>0$. C) $a<0$. D) $a\neq0$. E) $a\in\varnothing$.

11. $f(z)=u(x,y)+iv(x,y)$ funksiyaning hosilasi uchun quyidagi formulalardan qaysi biri o'rinli emas?

A) $f'(z)=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}$ B) $f'(z)=\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}$

C) $f'(z)=\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial y}$ D) $f'(z)=\frac{\partial v}{\partial y}+i\frac{\partial v}{\partial x}$

E) keltirilgan barcha formulalar o'rinli.

12. $f(z)=(x^2-y^2-x)+(2xy-y+1)i$ kompleks funksiyaning $f'(z)$ hosilasini toping.

A) $f'(z)=(2x-1)+2yi$ B) $f'(z)=(2x-1)-2yi$

C) $f'(z)=-2y+(2x-1)i$ D) $f'(z)=2y-(2x-1)i$

E) $f'(z)=2y+(2x-1)i$.

13. Qaysi kompleks funksiyaning hosilasi noto'g'ri ifodalangan?

A) $(z^n)'=nz^{n-1}$ B) $(e^z)'=e^z$ C) $(\cos z)'=-\sin z$

D) $(\sin z)'=\cos z$ E) barcha hosilalar to'g'ri ko'rsatilgan .

14. Qaysi kompleks funksiyaning hosilasi noto'g'ri ifodalangan?

A) $(\mathrm{Ln}z)'=1/z$ B) $(\mathrm{sh}z)'=\mathrm{ch}z$ C) $(\mathrm{ch}z)'=-\mathrm{sh}z$

D) $(\mathrm{arctg}z)'=1/(1+z^2)$ E) barcha hosilalar to'g'ri ko'rsatilgan

15. Agar $f(z)$ va $g(z)$ funksiyalar z nuqtada differensiallanuvchi bo'lsa, unda bu nuqtada quyidagi funksiyalardan qaysi biri differensiallanuvchi bo'lmasligi mumkin?

A) $f(z)\cdot g(z)$ B) $f(z)/g(z)$ C) $f(z)+g(z)$ D) $f(z)-g(z)$

E) ko'rsatilgan barcha funksiyalar differensiallanuvchi bo'ladi

16. Kompleks funksiyalarni differensiallash qoidasi qayerda noto'g'ri ifodalangan ?

A) $[f(z)+g(z)]'=f'(z)+g'(z)$

B) $[f(z)-g(z)]'=f'(z)-g'(z)$

C) $[f(z)g(z)]'=f'(z)g(z)+f(z)g'(z)$

D) $[Cf(z)]'=Cf'(z)$ (C – const.) .

E) $\left[\dfrac{f(z)}{g(z)}\right]' = \dfrac{f'(z)g(z)+f(z)g'(z)}{g^2(z)}$

4. Kompleks o'zgaruvchili funksiyalarning integrali

1. ***Ta'rifni yakunlang:*** $z=z(t)$, $\alpha\le t\le\beta$, tenglama bilan aniqlanadigan L chiziq silliq deyiladi, agarda $z=z(t)$ funksiya ...... bo'lsa.

A) chegaralangan . B) monoton . C) uzluksiz .

D) differensiallanuvchi . E) integrallanuvchi .

2. Silliq $L=AB$ chiziq va unda aniqlangan $w=f(z)$ kompleks funksiya bo'yicha integral yig'indini tuzishda quyidagi amallardan qaysi biri bajarilmaydi?

A) $L=AB$ chiziq $A=z_0$, z_1, z_2, ..., z_{n-1}, $z_n=B$ nuqtalar bilan ixtiyoriy n ta (z_k, z_{k+1}) $(k=0,1, \dots , n-1)$ oraliqlarga bo'laklanadi .

B) Har bir (z_k, z_{k+1}) $(k=0,1, \dots , n-1)$ oraliqning uzunligi $\Delta z_k=z_{k+1}-z_k$ aniqlanadi .

C) Har bir (z_k, z_{k+1}) $(k=0,1, \dots , n-1)$ oraliqda $w=f(z)$ kompleks funksiyaning orttirmasi $\Delta f_k=f(z_{k+1})-f(z_k)$ topiladi .

D) Har bir (z_k, z_{k+1}) $(k=0,1, \dots , n-1)$ oraliqdan ixtiyoriy bir ξ_k nuqta tanlanadi .

E)Tanlangan ξ_k (k=0,1, ... , n−1) nuqtalarda $w=f(z)$ kompleks funksiyaning qiymatlari $f(\xi_k)$ hisoblanadi .

3. $w=f(z)$ funksiya aniqlangan $L=AB$ chiziq $A=z_0$, z_1, z_2, ..., z_{n-1}, $z_n=B$ nuqtalar bilan ixtiyoriy n ta (z_k, z_{k+1}) (k=0,1, ... , n−1) oraliqlarga bo'laklangan va ularning uzunliklari Δz_kkabi belgilangan . Quyidagilardan qaysi biri L chiziq bo'yicha $w=f(z)$ funksiya uchun integral yig'indi bo'la olmaydi?

A) $\sum_{k=0}^{n-1} f(z_k)\Delta z_k$.

B) $\sum_{k=0}^{n-1} f(z_{k+1})\Delta z_k$.

C) $\sum_{k=0}^{n-1} f(\frac{z_{k+1}-z_k}{2})\Delta z_k$.

D) $\sum_{k=0}^{n-1} f(\frac{z_{k+1}+z_k}{2})\Delta z_k$.

E) barcha javoblar integral yig'indini ifodalaydi .

4. $w=f(z)$ kompleks funksiyadan L chiziq bo'yicha olingan integralni ta'rifidagi $\int_L f(z)dz = \lim \sum_{k=0}^{n-1} f(\xi_k)\Delta z_k$ tenglikda limitga qanday o'tiladi?

A) $n\to\infty$. B) $\Delta z_n\to 0$. C) $n\to\infty$, $\Delta z_n\to 0$.
D) $n\to\infty$, $\max\Delta z_k\to 0$. E) $n\to\infty$, $\min\Delta z_k\to 0$.

5. Teorema shartini ko'rsating: L silliq chiziq bo'yicha $\int_L f(z)dz$ integral mavjud bo'ladi, agar $f(z)$ funksiya ... bo'lsa.

A) chegaralangan . B) monoton . C) uzluksiz .
D) quyidan chegaralangan E) yuqoridan chegaralangan .

6. Agar $f(z)=u(x,y)+iv(x,y)$ bo'lsa, unda $\int_L f(z)dz = \int_L [u(x,y)+iv(x,y)]d(x+iy)$ integral qiymati egri chiziqli integral orqali qaysi formula bilan hisoblanadi?

*A) $\int_L f(z)dz = \int_L udx - vdy + i\int_L udy + vdx$

B) $\int_L f(z)dz = \int_L vdx + udy + i\int_L vdy + udx$

C) $\int_L f(z)dz = \int_L vdy - udx - i\int_L vdy + udx$

D) $\int\limits_L f(z)dz = \int\limits_L udx + vdy - i\int\limits_L vdx - udy$.

E) to'g'ri javob keltirilmagan .

7. Nyuton-Leybnits formulasidan foydalanib $\int\limits_{AB} zdz$ integral qiymatini toping. Bunda AB integrallash yo'li A=2i va B=3+i nuqtalarni tutshtiruvchi silliq chiziqni ifodalaydi.

A) 2+i . B) 1+2i . C) 6+3i . D) 3+6i . E) 0 .

8. Nyuton-Leybnits formulasidan foydalanib $\int\limits_{AB} z^2dz$ integral qiymatini toping. Bunda AB integrallash yo'li A=3i va B=3 nuqtalarni tutshtiruvchi to'g'ri chiziqni ifodalaydi.

A) 9+9i . B) 9–9i . C) –9+9i . D) –9–9i . E) 0 .

9. Integrallash yo'li tenglamasi $z(t)=2t+(4t-3)i$, $0\le t\le 1$, bo'lgan L chiziqdan iborat $\int\limits_L zdz$ integralni hisoblang.

A) 2(3–i) . B) 3(2–i) . C) 2(3+i) .

D) 3(2+i) . E) 0 .

10. Quyidagi tengliklarning qaysi biri kompleks funksiyadan olingan integralning chiziqlilik xossasini ifodalamaydi?

A) $\int\limits_L [f(z)+g(z)]dz = \int\limits_L f(z)dz + \int\limits_L g(z)dz$.

B) $\int\limits_L [f(z)-g(z)]dz = \int\limits_L f(z)dz - \int\limits_L g(z)dz$.

C) $\int\limits_L [Af(z)+Bg(z)]dz = A\int\limits_L f(z)dz + B\int\limits_L g(z)dz \quad (A,B - const.)$.

D) $\int\limits_L [Af(z)-Bg(z)]dz = A\int\limits_L f(z)dz - B\int\limits_L g(z)dz \quad (A,B - const.)$.

E) barcha tengliklar integralning chiziqlilik xossasini ifodalaydi .

11. Agar $\int\limits_L f(z)dz = 3-2i$, $\int\limits_L g(z)dz = 2+3i$ bo'lsa,

$\int\limits_L [f(z)+g(z)]dz$ integral qiymati nimaga teng bo'ladi?

A) 5+5i B) 5–5i C) 5+I

D) 5–i E) aniqlab bo'lmaydi .

12. Agar $\int\limits_L f(z)dz = 3-2i$, $\int\limits_L g(z)dz = 2+3i$ bo'lsa, $\int\limits_L [f(z)-g(z)]dz$ integral qiymati nimaga teng bo'ladi?

A) 1+5i . B) 1–5i . C) 5+i . D) 5–i .

E) aniqlab bo'lmaydi .

13. Agar $\int\limits_L f(z)dz = 3-2i$, $\int\limits_L g(z)dz = 2+3i$ bo'lsa, $\int\limits_L [3f(z)+2g(z)]dz$ integral qiymati nimaga teng bo'ladi?

A) 13+5i . B) 13–5i . C) 13i .
D) 13 . E) aniqlab bo'lmaydi .

14. Kompleks funksiyadan olingan integral xossalarini ifodalovchi quyidagi tengliklardan qaysi biri noto'g'ri?

A) $\int\limits_L Cf(z)dz = C\int\limits_L f(z)dz \quad (C-const.)$

B) $\int\limits_{L_1+L_2} f(z)dz = \int\limits_{L_1} f(z)dz + \int\limits_{L_2} f(z)dz$

C) $\int\limits_{AB} f(z)dz = -\int\limits_{BA} f(z)dz$

D) $\int\limits_G f(z)g(z)dz = \int\limits_G f(z)dz \cdot \int\limits_G g(z)dz$

E) $\int\limits_L [f(z) \pm g(z)]dz = \int\limits_L f(z)dz \pm \int\limits_L g(z)dz$

15. Agar $\int\limits_L f(z)dz = 3-2i$ bo'lsa, $\int\limits_L (4+i)f(z)dz$ integral qiymatini toping.

A) 14+5i B) 14–5i C) 5+14i
D) 5–14i E) 0

16. Agar $\int\limits_L f(z)dz = 3-2i$, $\int\limits_L g(z)dz = 2+3i$ bo'lsa, $\int\limits_L f(z)g(z)dz$ integral qiymatini toping.

A) 1+5i B) 1–5i C) 12+5i
D) 12–5i E) aniqlab bo'lmaydi .

5. Kompleks funksiyalar uchunTeylor va Loran qatorlari. Kompleks funksiyaning maxsus nuqta va chegirmalari

1. Biror ochiq $|z-z_0|<R$ doirada analitik bo'lgan $f(z)$ funksiyaning $z-z_0$ darajalari bo'yicha Teylor qatori qayerda to'g'ri ifodalangan ?

A) $f(z)=\sum\limits_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}(z-z_0)^k$ B) $f(z)=\sum\limits_{k=0}^{n}\frac{f^{(k)}(0)}{k!}(z-z_0)^k$.

C) $f(z)=\sum\limits_{k=0}^{\infty}\frac{f^{(k)}(z_0)}{k!}(z-z_0)^k$ D) $f(z)=\sum\limits_{k=0}^{n}\frac{f^{(k)}(z_0)}{k!}(z-z_0)^k$.

E) $f(z)=\sum_{k=-\infty}^{\infty}c_k(z-z_0)^k$, $c_k=\frac{1}{2\pi i}\int_L\frac{f(z)}{(z-z_0)^{k+1}}dz$.

2. Biror ochiq $|z|<R$ doirada analitik bo'lgan $f(z)$ funksiyaning Makloren qatori qayerda to'g'ri ifodalangan ?

A) $f(z)=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}z^k$ B) $f(z)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}z^k$.

C) $f(z)=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}(z-z_0)^k$ D) $f(z)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}(z-z_0)^k$.

E) $f(z)=\sum_{k=-\infty}^{\infty}c_k z^k$, $c_k=\frac{1}{2\pi i}\int_L\frac{f(z)}{z^{k+1}}dz$.

3. $f(z)=e^z$ funksiyaning Makloren qatorini ko'rsating .

A) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}$ B) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}$ C) $\sum_{n=0}^{\infty}\frac{z^n}{n!}$.

D) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{n+1}}{n+1}$ E) $\sum_{n=0}^{\infty}\frac{z^{2n}}{(2n)!}$

4. $f(z)=\sin z$ funksiyaning Makloren qatorini ko'rsating .

A) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}$ B) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}$ C) $\sum_{n=0}^{\infty}\frac{z^n}{n!}$.

D) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{n+1}}{n+1}$ E) $\sum_{n=0}^{\infty}\frac{z^{2n}}{(2n)!}$

5. $f(z)=\cos z$ funksiyaning Makloren qatorini ko'rsating .

A) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}$ B) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}$ C) $\sum_{n=0}^{\infty}\frac{z^n}{n!}$

D) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{n+1}}{n+1}$ E) $\sum_{n=0}^{\infty}\frac{z^{2n}}{(2n)!}$

6. $f(z)=\text{ch}z$ giperbolik kosinus funksiyaning Makloren qatorini ko'rsating .

A) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}$ B) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}$ C) $\sum_{n=0}^{\infty}\frac{z^n}{n!}$

D) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{n+1}}{n+1}$ E) $\sum_{n=0}^{\infty}\frac{z^{2n}}{(2n)!}$

7. $f(z)=\text{Ln}z$ ko'p qiymatli funksiyaning $z=0$ nuqta atrofidagi bir qiymatli tarmog'ining Makloren qatorini ko'rsating .

A) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}$ B) $\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}$ C) $\sum_{n=0}^{\infty}\frac{z^n}{n!}$

D) $\sum\limits_{n=0}^{\infty}(-1)^n\frac{z^{n+1}}{n+1}$ E) $\sum\limits_{n=0}^{\infty}\frac{z^{2n}}{(2n)!}$

8. $f(z)=z^2e^z$ funksiya Makloren qatorining z^4 daraja oldidagi koeffitsientini toping.

A) 0 . B) 1 . C) 1/2 . D) 1/6 . E) 1/24 .

9. $f(z)=e^{2z}$ funksiya Makloren qatorining z^4 daraja oldidagi koeffitsientini toping.

A) 2/3 . B) 3/4 . C) 1/2 . D) 1/6 . E) 1/24 .

10. Biror ochiq $r<|z-z_0|<R$ halqada analitik bo'lgan $f(z)$ funksiya va bu halqada yotgan L kontur uchun $c_k=\frac{1}{2\pi i}\int_L\frac{f(z)}{(z-z_0)^{k+1}}dz$ bo'lsin. Bu holda $f(z)$ funksiyaning $z-z_0$ darajalari bo'yicha Loran qatori qayerda to'g'ri ifodalangan ?

A) $f(z)=\sum\limits_{k=0}^{\infty}c_k(z-z_0)^k$

B) $f(z)=\sum\limits_{k=0}^{n}c_k(z-z_0)^k$

C) $f(z)=\sum\limits_{k=0}^{\infty}c_{-k}(z-z_0)^{-k}$

D) $f(z)=\sum\limits_{k=0}^{n}c_{-k}(z-z_0)^{-k}$

E) $f(z)=\sum\limits_{k=-\infty}^{\infty}c_k(z-z_0)^k$

11. $f(z)=1/(1-z)$funksiyaning z darajalari bo'yicha Loran qatorini toping.

A) $\sum\limits_{k=0}^{\infty}z^k$ B) $\sum\limits_{k=0}^{\infty}z^{-k}$ C) $\sum\limits_{k=-\infty}^{\infty}z^k$

D) $\sum\limits_{k=0}^{n}z^k$ E) $\sum\limits_{k=-n}^{0}z^k$.

12. $f(z)=1/(z-z^2)$funksiyaning z darajalari bo'yicha Loran qatorini toping.

A) $\sum\limits_{k=0}^{\infty}z^k$ B) $\sum\limits_{k=0}^{\infty}z^{-k}$ C) $\sum\limits_{k=-1}^{\infty}z^k$

D) $\sum\limits_{k=1}^{\infty}z^k$ E) $\sum\limits_{k=-1}^{\infty}z^{-k}$

13. $f(z)=z^2/(1-z)$funksiyaning z darajalari bo'yicha Loran qatorini toping.

A) $\sum_{k=2}^{\infty} z^k$ B) $\sum_{k=2}^{\infty} z^{-k}$ C) $\sum_{k=-2}^{\infty} z^k$

D) $\sum_{k=-2}^{\infty} z^{-k}$. E) $\sum_{k=-\infty}^{\infty} z^k$.

14. ***Tasdiqni yakunlang:***$f(z)$ funksiyaning $z-z_0$ darajalari bo'yicha Loran qatori uchun ... shart bajarilsa, $z=z_0$ tuzatib bo'ladigan maxsus nuqta bo'ladi .

A) $c_{-k}=0$ $(k=m+1, m+2, m+3, ...)$

B) $c_k=0$ $(k=m+1, m+2, m+3, ...)$.

C) $c_{-k}=0$ $(k=1,2, 3, ...)$

D) $c_k=0$ $(k=0,1,2, ...)$

E) $c_k\neq 0$ $(k=0,\pm 1, \pm 2, ...)$.

15. ***Tasdiqni yakunlang:***$f(z)$ funksiyaning $z-z_0$ darajalari bo'yicha Loran qatori uchun ... shart bajarilsa, $z=z_0 m$-tartibli qutb bo'ladi .

A) $c_{-k}=0$ $(k=m+1, m+2, m+3, ...)$.

B) $c_k=0$ $(k=m+1, m+2, m+3, ...)$.

C) $c_{-k}=0$ $(k=1,2, ..., m)$.

D) $c_k=0$ $(k=0,1,2, ..., m)$.

E) $c_k\neq 0$ $(k=0,\pm 1, \pm 2, ...)$.

16. $f(z)$ funksiyaning $z-z_0$ darajalari bo'yicha Loran qatorida qaysi shart bajarilsa, $z=z_0$ muhim maxsus nuqta bo'ladi ?

A) $c_{-k}\neq 0$ $(k=m+1, m+2, m+3, ...)$.

B) $c_k\neq 0$ $(k=\pm(m+1), \pm(m+2), ...)$.

C) $c_{-k}\neq 0$ $(k=1,2, 3, ...)$.

D) $c_k\neq 0$ $(k=0,\pm 1, \pm 2, ...)$.

E) barcha shartlarda .

Matematika fanlar ichra shoh,
uning sirlaridan bo'lingiz ogoh!
Qori Niyoziy

XV-BOB. OPERATSION HISOB

§ 15.1. Boshlang'ich funksiya va tasvir. Laplas almashtirishi

Haqiqiy sonlr o'qida aniqlangan $f(t)$ funksiya quyidagi shartlarni qanoatlantirsin:

a) $f(t)$ funksiya istalgan chekli intervalda uzluksiz yoki I – tur uzulish nuqtalariga ega:

b) $f(t)$ funksiya argumentning manfiy qiymatlarida aynan nolga teng, ya'ni $t<0$ bo'lganda $f(t)\equiv 0$

c) shunday $M>0, S_0>0$ sonlar mavjudki, t argumentning barcha qiymatlari uchun $f(t)$ funksiyaning moduli Me^{S_0t} funksiyadan oshmaydi, ya'ni barcha t lar uchun $|f(t)|\le M^{S_0t}$ tengsizlik bajariladi. Yuqoridagi a), b), c) shartlarni qanoatlantiruvchi har qanday $f(t)$ funksiya boshlang'ich funksiya deyiladi.

Endi $f(t)$ boshlang'ich funksiyani e^{-pt} (p-kompleks sonlar) funksiyaga ko'paytiraylik va undan quyidagi

$$\int_0^{\infty} e^{-pt} f(t)dt, \qquad (\operatorname{Re} p>0)$$

xosmas integralni ko'rib o'taylik. Bu xosmas integral, $f(t)$ boshlang'ich funksiya bo'lganligi uchun, doimo yaqinlashuvchi va p argumentning funksiyasi bo'ladi.

Quyidagi $$g(p)=\int_0^{\infty} e^{-pt} f(t)dt, \quad \operatorname{Re} p>0 \qquad (15.1)$$

munosabat bilan aniqlangan $g(p)$ funksiya $f(t)$ funksiyaning tasviri yoki Laplas almashtirishi deyiladi va

$$g(p) = L[f]$$

kabi belgilanadi.

Shuni ta'kidlab o'tamizki har qanday $f(t)$ boshlang'ich funksiyaning tasviri (15.1) munosabat orqali yagona aniqlanganligi kabi, har qanday $g(p)$ tasvirning ham yagona $f(t)$ boshlang'ich funksiyasi mavjud. Berilgan tasvirdan uning boshlang'ich funksiyasiga o'tish

$$f(t) = L^{-1}[g]$$

tenglik orqali ifodalanadi. Agar $f(t)$ funksiya a), b), c) shartlarning birortasini qanoatlantirmasa, u boshlang'ich funksiya bo'la olmaydi va (15.1) xosmas integral uzoqlashadi.

Masalan, $f(t) = ctgt$ boshlang'ich funksiya bo'la olmaydi, chunki u tasvirning a) shartini qanoatlantirmaydi, ya'ni $t = k\pi$, $k = 0, \pm1, \pm2, \pm3, \ldots$ nuqtalarda II tur uzulishga ega. Shuning uchun $f(t) = ctgt$ funksiyaning tasviri mavjud emas. Bir qator boshlang'ich funksiyalarning tasvirlarini hisoblashni ko'rib chiqaylik.

I. Chiziqlilik xossasi.

a) boshlang'ich funksiya bo'lmish $f(t)$ va ixtiyoriy olingan c o'zgarmas miqdor uchun

$$L[cf(t)] = cL[f(t)] \qquad (15.2)$$

tenglik o'rinlidir.

b) chekli sondagi boshlang'ich funksiyalar yig'indisining tasviri mos tasvirlar yig'indisiga teng, ya'ni

$$L[f_1 \pm f_2 \pm \ldots \pm f_n] = L[f_1] \pm L[f_2] \pm \ldots \pm L[f_3] \qquad (15.3)$$

munosabat o'rinli.

II. O'xshashlik teoremasi. Agar $a > 0$ va $L[f(t)] = g(p)$ bo'lsa, u holda $L[f(at)] = \frac{1}{a} g(\frac{p}{a})$ munosabat o'rinli bo'ladi.

III. Kechikish teoremasi. Agar s ixtiyoriy musbat son va $L[f(t)] = g(p)$ bo'lsa, u holda

$$L[f(t-s)] = e^{-sp} g(p)$$

tenglik o'rinli bo'ladi.

IV. Siljish teoremasi. Agar $L[f] = g(p)$ bo'lsa, u holda

$$L[e^{-\alpha t} \cdot f(t)] = g(p+\alpha)$$

munosabat o'rinli bo'ladi.

V.Kompozitsiyalash teoremasi.

Ta'rif. Berilgan f_1 *va* f_2 funksiyalarning kompozitsiyasi deb

$$f(t) = \int_0^t f_1(s) \cdot f_2(t-s)ds = \int_0^t f_2(s) f_1(t-s)ds$$

tenglik bilan aniqlanuvchi $f(t)$ funksiyaga aytiladi. Odatda ikkita funksiyaning kompozitsiyasi

$$f(t) = f_1(t) * f_2(t)$$

kabi belgilanadi.

15.1-jadval

Ba'zi funksiyalarning tasvirlari

Original	Tasvir	Original	Tasvir
1	$\frac{1}{p}$	$sh\beta t$	$\frac{\beta}{p^2-\beta^2}$
$e^{\alpha t}$	$\frac{1}{p-\alpha}$	$ch\beta t$	$\frac{p}{p^2-\beta^2}$
$\sin\beta t$	$\frac{\beta}{p^2+\beta^2}$	t^n	$\frac{n!}{p^{n+1}}$
$\cos\beta t$	$\frac{p}{p^2+\beta^2}$	$t^n e^{\alpha t}$	$\frac{n!}{(p-\alpha)^{n+1}}$
$e^{\alpha t}\cos\beta t$	$\frac{p-\alpha}{(p-\alpha)^2+\beta^2}$	$e^{\alpha t}\sin\beta t$	$\frac{\beta}{(p-\alpha)^2+\beta^2}$
$t\cos\beta t$	$\frac{p^2-\beta^2}{(p^2+\beta^2)^2}$	$t\sin\beta t$	$\frac{2p\beta}{(p^2+\beta^2)^2}$

Quyidagi funksiyalarni orginali mavjudmi?

15.1. $f(t) = \begin{cases} 0, & agar \quad t<0 \\ 4, & agar \quad t>0 \end{cases}$

15.2. $f(t) = \begin{cases} 0, & agar \quad t<0 \\ tgt, & agar \quad t>0 \end{cases}$

Laplas almashtirishining xossalaridan foydalanib quyidagi funksiyalarning tasvirlarini toping.

15.3. $f(t)=\begin{cases}1, & t\ge 0\\ 0, & t<0\end{cases}$ 15.4. $f(t)=13$

15.5. $f(t)=\begin{cases}e^{\alpha t}, & t\ge 0\\ 0, & t<0\end{cases}$ 15.6. $f(t)=a^t$

15.7. $f(t)=e^{3t}\sin\pi t$ 15.8. $f(t)=\cos\omega t,\ \ t\ge 0$

15.9 $f(t)=t^n$ 15.10 $f(t)=\sin\beta t$ 15.11 $f(t)=74t$

Laplas almashtirishining xossalaridan foydalanib quyidagi funksiyalarning tasvirlari topilsin.

15.12. $f(t)=(t+1)^2$ 15.13. $f(t)=(t-1)^4$ 15.15. $f(t)=te^t$

15.15. $f(t)=e^{-3t}\sin 4t$ 15.16. $f(t)=t^4e^{3t}$ 15.17. $f(t)=t^6e^{-5t}$

15.18 $f(t)=e^{-2t}\cos 4t$

15.19. $e^{-\alpha t}\sin\omega t$ *va* $e^{-\alpha t}\cos\omega t$

15.20. $f(t)=e^{-2t}\cos 13t$

15.21. $f(t)=e^{3t}\sin t$

15.22. $f(t)=2e^{-it}-3+5\cos t$

15.23. $f(t)=\sin 2t\cos 5t$

15.24. $f(t)=e^{-3t}\sin\pi t$

15.25. $f(t)=sh\beta t\cos\beta t$

15.26. $f(t)=\begin{cases}t, & agar\ \ 0<t<2\pi\\ \sin 2t, & agar\ \ t\ge 2\pi\end{cases}$

15.27. $f(t)=\sin^2 t$

15.28. $f(t)=e^t\cos^2 t$

15.29. $f(t)=shat\cos bt$

15.30 $f(t)=chat\sin bt$

15.31. $f(t)=chat\cos bt$

15.32. $f(t)=tchbt$

15.33. $f(t)=cos^3t$

Laplas almashtirishining xossalaridan foydalanib quyidagi funksiyalarning tasvirlari topilsin.

15.34. $f(t)=e^{t+2}\cos 5t$

15.35. $f_1(t)=t,\ \ f_2(t)=e^t$ funksiyalarning kompozitsyasi topilsin.

15.36 $f(t)=e^{t}(\cos 2t-\sin t)$

Quyidagi funksiyalarning tasvirlari mavjudmi?

15.37. $f(t)=\begin{cases}0, & agar \quad t<0\\ e^{(3+i)t}, & agar \quad t>0\end{cases}$ 15.38. $f(t)=\begin{cases}0, & agar \quad t<0\\ \sin(2-i)t, & agar \quad t\ge 0\end{cases}$

15.39. $f(t)=\begin{cases}0, & agar \quad t<0\\ 2^{t^2}, & agar \quad t\ge 0\end{cases}$

§ 15.2. Hosilaning tasviri va tasvirning hosilasi

Faraz qilaylik $f(t)$ funksiya va uning $n-$ tartibligacha bo'lgan $f', f'', ..., f^{(n)}$ hosilalari boshlang'ich funksiyalar bo'lsin.

Hosilaning tasviri. Agar $f(t)$ funksiyaning tasviri $g(p)$ bo'lsa, u holda $f'(t)$ hosilaning tasviri

$L[f']=pg(p)-f(0)$ munosabat bilan aniqlanadi va bu yerda $f(0)=\lim\limits_{t\to+0} f(t)$ deb tushuniladi.

Xuddi shunday usulda $f''(t), f'''(t), ..., f^{(n)}(t)$ yuqori tartibli hosilalarning ham tasvirlarini $f(t)$ funksiyaning tasviri orqali ifodalash mumkin. Agar $L[f]=g(p)$ bo'lsa u holda

$$L[f']=pg(p)-f(0)$$

$$L[f'']=p^2g(p)-pf(0)-f'(0)$$

$$L[f''']=p^3g(p)-p^2f(0)-pf'(0)-f''(0)$$

....

....

$$L[f^{(n)}]=p^n g(p)-p^{n-1}f(0)-...-pf^{(n-2)}(0)-f^{(n-1)}(0)=p^n g(p)-\sum_{k=0}^{n-1} f^{(k)}(0)p^{n-k-1}$$

Tasvirning hosilasi:

$g(p)$ tasvir p parametrning funksiyasi bo'lganligi uchun undan p ga nisbatan hosilalar olish masalasini ko'rib o'taylik.

Tasvirning ta'rifiga asosan $g(p)=\int\limits_0^\infty e^{-pt}f(t)dt$ va shuning uchun

$$g'(p)=\int\limits_0^\infty (e^{-pt})'_p\cdot f(t)dt=\int\limits_0^\infty e^{-pt}(-t)f(t)dt=\int\limits_0^\infty e^{-pt}[-tf(t)]dt$$

olingan natijadan $g'(p)=[-tf(t)]$ ekanligini ko'ramiz.

Demak tasvirning hosilasi $-tf(t)$ boshlang'ich funksiyaning tasviriga teng ekan. Shuningdek, tasvirning yuqori tartibli hosilalari uchun

$$g''(p) = L\left[t^2 f(t)\right]$$

$$g'''(p) = L\left[-t^3 f(t)\right]$$

....

$$g^{(n)}(p) = L\left[(-1)^n t^n f(t)\right]$$

tengliklarni keltirib chiqarish mumkin.

Tasvirning hosilalari uchun topilgan munosabatlardan foydalanib quyidagi

$$L\left[t^n e^{\alpha t}\right] = \frac{n!}{(p-\alpha)^{n-1}}$$

tenglikni keltirib chqaraylik. Darhaqiqat $g_1(p) = L\left[e^{\alpha t}\right] = \frac{1}{p-\alpha}$ tenglikni nazarda tutib, uning uning hosilalarini hisoblaymiz:

$$g'_1(p) = -\frac{1}{(p-\alpha)^2}, \qquad g''_1(p) = \frac{1\cdot 2}{(p-\alpha)^3}$$

$$g'''_1(p) = -\frac{1\cdot 2\cdot 3}{(p-\alpha)^4}, \qquad g_1^{(n)}(p) = (-1)^n \frac{n!}{(p-\alpha)^{n+1}}$$

Lekin biz $g^{(n)}(p) = L\left[(-1)^n t^n f(t)\right]$ ekanligini yuqorida hosil qilgan edik. Demak $f(t) = e^{\alpha t}$ bo'lsa $g_1^{(n)}(p) = L\left[(-1)^n t^n e^{\alpha t}\right]$ bo'ladi. Natijada

$$(-1)^n L\left[t^n e^{\alpha t}\right] = (-1)^n \frac{n!}{(p-\alpha)^{n+1}} \text{ yoki } \quad L\left[t^n e^{\alpha t}\right] = \frac{n!}{(p-\alpha)^{n-1}}$$

tenglikka erishamiz. Bu yerda $\alpha = 0$ deb, ushbu

$$L\left[t^n\right] = \frac{n!}{p^{n+1}}$$

muhim formulaga yana bir bora kelamiz.

15.40. Agar $y(0)=y'(0)=0$ va $y(t)=\overline{y}(p)$ bo'lsa, $y(t)=y''(t)-y'(t)-y(t)$ funksyaning tasvirni toping.

15.41. Agar $y(0)=0,\ y'(0)=1,\ y'(0)=2,\ y(t)=\overline{y}(p)$ bo'lsa, $y(t)=y'''(t)-y''(t)+2y'(t)-2y(t)$ ning tasvirini toping

14.42. Agar $y(0)=0,\ y'(0)=1,\ y'(0)=2,\ y(t)=\overline{y}(p)$ bo'lsa, $y(t)=y'''(t)-y''(t)+2y'(t)-2y(t)$ ning tasvirini toping.

15.43. Agar $y(0)=-1,\ y'(0)=2,\ y'(0)=-3,$ va $y(t)=\overline{y}(p)$ bo'lsa, $F(t)=y'''(t)-3y''(t)+2y'(t)-4y(t)+1$ ning tasvirini toping.

15.44. $\int_0^t (e^{-3t}ch2t + e^{4t}\sin 2t)dt$ tasvirni toping.

15.45. $\int_0^t (t^7 - 5t^4 - 2t^2 + 3)e^{2t}dt$ tasvirni yeching.

15.46. Agar $y(0)=0,\quad y(t)=\overline{y}(p)$ bo'lsa, u holda $F(t)=y'(t)-\int_0^t y(\tau)d\tau$ ning tasvirini toping.

15.47. Agar $y(0)=1,\quad y'(0)=2$ bo'lsa, $y''(t)-4y'(t)+3y(t)$ ning tasvirini toping.

15.48.Agar $y(0)=-3,\quad y'(0)=7,\quad y''(0)=1$ bo'lsa, $y'''(t)+6y''(t)+y'(t)-2y(t)+3$ ning tasvirini toping.

15.49. $\int_0^t (\sin t + 3e^2)dt$ tasvirni yeching.

15.50. $\int_0^t t^4e^{-2t}dt$ tasvirni yeching.

15.51. Davri $T=2\pi$ bo'lgan funksiyani originalini toping.

$$f(t)=\begin{cases}1-2t/\pi, & agar \quad 0<t<\pi\\ 2t/\pi-3, & agar \quad \pi<t<2\pi\end{cases}$$

§ 15.3. Tasvirga ko'ra originalni tiklash

Tasvirga ko'ra originalni tiklash uchun oddiy hollarda (asosan elementar funksiyalarda) 15.1-jadvaldan foydalaniladi. Boshqa hollarda yoyish birinchi va ikkinchi teoremalaridan foydalaniladi. Bu teorema kasr –ratsional funksiyalarni tasviridan $F(p)=u(p)/v(p)$ originalga o'tishga imkon yaratadi.

$$F(p)=\sum_{j=1}^{r}\sum_{S=1}^{k_j}\frac{A_{j,S}}{(p-p_j)^{k_j-S+1}} \qquad (15.4)$$

koeffiyentlar $A_{j,S}$ quyudagi formula bilan aniqlanadi.

$$A_{j,S}=\frac{1}{(S-1)!}\lim_{p\to p_j}\left\{\frac{d^{S-1}}{dp^{S-1}}\left[(p-p_j)^{k_j}F(p)\right]\right\}$$

Agar $v(p)$ ko'p handing ildizlari oddiy bo'lsa,

$$v(p)=(p-p_1)(p-p_2)...(p-p_n) \quad (p_j\neq p_k)$$

u holda

$$F(p)=\sum_{j=1}^{n}\frac{A_j}{p-p_j}, \text{bu yerda } A_j=\frac{u(p_j)}{v'(p_j)}$$

Tasvir funksiyasini (1/p) bo'yicha dajani qatorga yoyib hisoblash mumkin

$$F(p)=\frac{a_0}{p}+\frac{a_1}{p^2}+\ldots+\frac{a_n}{p^{n+1}}+\ldots$$

$F(p)$ qator $|p|>R$ da yaqinlashuvchi $(R=\lim\limits_{n\to\infty}(a_{n+1}/a_n\neq\infty))$, u holda

$$f(t)=a_0+a_1\frac{t}{1!}+a_2\frac{t^2}{2!}+\ldots+a_n\frac{t^n}{n!}+\ldots$$

qator ham t ning barcha qiymatlarini yaqinlashuvchi bo'ladi.

Quyidagi $F(p)$ tasvirlarning originallarini toping.

15.52. $F(p)=\frac{p}{p^2-2p+5}$

15.53. $F(p)=\frac{2p+1}{p^2+5p+10}$

15.54. $F(p)=e^{-p}\frac{p}{p^2-9}+\frac{1}{p^2+5}$

15.55. $F(p)=\frac{1}{p^3-8}$

15.56. $F(p)=\frac{1}{p(p^2+4)}$

15.57. $F(p)=\frac{1}{p^2(p^2+4)}$

Quyidagi tasvirlarning originallari topilsin

15.58. $\frac{2p+1}{p^2+5p+10}$

15.59. $F(p)=\frac{1}{(p-3)^5}$

15.60. $F(p)=\frac{1}{(p-1)^3(p+2)^2}$

15.61. $F(p)=\frac{1}{(p-1)(p^2-4)}$

15.62. $F(p)=\frac{p+3}{p(p^2-4p+3)}$

15.63. $F(p)=\frac{1}{p(p^4-5p^2+4)}$

15.64. $F(p)=\frac{1}{p(1+p^4)}$

15.65. $F(p)=\frac{1}{p^2-1}+\frac{3p-2}{(p-1)^2+3}$

15.66. $F(p)=\frac{p-3}{2p^2-6p+1}$

15.67. $F(p)=\frac{e^{-2p}}{p^2+4p+3}$

15.68. $F(p)=\frac{p}{(p^2+1)(p^2+4)}$

Quyidagi tasvirlarning originallari tiklansin

15.69. $e^{-p}\frac{p}{p^2-9}+\frac{1}{p^2+5}$

15.70. $F(p)=\frac{p^2}{p^4+13p^2+36}$

15.71. $F(p)=\frac{p^3e^{-2p}}{(p^2+9)^2}$

15.72. $F(p)=\frac{p+1}{p(p-1)(p-2)(p-3)}$

15.73. $F(p)=\frac{4-p-p^2}{p^3-p^2}$

15.74. $F(p)=\frac{1}{p^4-6p^3+11p^2-6p}$

15.75. $F(p)=\frac{1}{(p-1)(p^3+1)}$

§ 15.4. Operatsion hisobni oddiy differensial tenglamalarni yechishga qo'llash

Faraz qilaylik, $x(0)=x_0;\ x'(0)=x_1 ... x^{(n-1)}(0)=x_{n-1}$ boshlang'ich shartlarni qanoatlantiruvchi o'zgarmas koeffisiyentli n-tartibli oddiy differensial tenglama

$$a_n x^{(n)}(t)+a_{n-1}x^{(n-1)}(t)+...+a_1 x^1(t)+a_0 x(t)=f(t), \qquad (15.5)$$

berilgan bo'lsin. Hamda $Y(t) \leftarrow X(p)$ va $f(t) \Leftarrow F(p)$ bo'lsin. Laplas almashtirish natijasida $A(p)Y(p)=F(p)+B(p)$, bu yerda $A(p), B(p)$ -ko'p hadlar. Bu tenglamani yechib, $Y(p)=\frac{F(p)+B(p)}{A(p)}$ ni olamiz.

Endi umumiy holda ikkinchi tartibli chiziqli o'zgarmas koeffisentli bir jinsli bo'lmagan differensial tenglamaning

$$y''+ay'+by=\varphi(t) \qquad (15.6)$$

$$y(0)=y_0 \quad va \quad y'(0)=y'_0$$

boshlang'ich shartlarni qanoatlantiruvchi yechimning tasvirini topaylik. Buning uchun (15.6) tenglamaning har ikkala tomonidan Laplas almashtirishi olamiz

$$L[y'']+aL[y']+bL[y]=L[\varphi]$$

yoki

$$p^2\bar{y}(p)-py(o)-y'(0)+a(p\bar{y}(p)-y(0))+b\bar{y}(p)=L[\varphi]$$

yordamchi chiziqli tenglamaga kelamiz. Bundan

$$\bar{y}(p)\left[p^2+ap+b\right]=L[\varphi]+py(0)+y'(0)+ay(0)$$

tenglamani hosil qilamiz va uni yechib

$$\bar{y}(p)=\frac{L[\varphi]+(p+a)y(0)+y'(0)}{p^2+ap+b} \qquad (15.7)$$

tasvirni topamiz. Agar differensial tenglamaning o'ng tomoni $\varphi(t)$ va yechimning boshlang'ich y_0, y'_0 qiymatlari berilgan bo'lsa (15.7) tenglamadan $y(t)$ yechimni yuqoridagi jadval yordamida tiklash mumkin.

15.76. $y''+3y'+2y=0$ differensial tenglamaning $y(0)=0,\ y'(0)=1$ boshlang'ich shartni qanoatlantiruvchi yechimi topilsin.

Yechish: Tasvirini $\bar{y}(p)=L[y]$ orqali belgilasak, y' va y'' funksiyalarning tasvirlari quyidagicha aniqlanadi:

$$L[y']=p\bar{y}(p)-y(0),\quad L[y'']=p^2\bar{y}(p)-py(0)-y'(0)$$

Berilgan differensial tenglamaning har ikkala tomonidan Laplas almashtirishi olib, yordamchi algebraik tenglamaga kelamiz;

$$L[y''+3y'+2y]=L[0]=0,\quad L[y'']+3L[y']+2L[y]=0 \text{yoki}$$

$$p^2\bar{y}(p)-py(0)-y'(0)+3(p\bar{y}(p)-y(0))+2\bar{y}(p)=0$$

misolning shartiga ko'ra $y(0)=0,$ va $y'(0)=1$ ekanligini hisobga olsak, $\bar{y}(p)$ tasvirga nisbatan $p^2\bar{y}(p)+3p\bar{y}(P)+2\bar{y}(p)-1=0$ yoki $\bar{y}(p)[p^2+3p+2]=1$ tenglamaga ega bo'lamiz. Bu tenglamadan izlanayotgan yechimning tasviri

$$\bar{y}(p)=\frac{1}{p^2+3p+2}=\frac{1}{p+1}-\frac{1}{p+2}$$

topiladi va jadvaldagi 3– formuladan uning o'zi $y(t)=e^{-t}-e^{-2t}$ ekanligi aniqlanadi.

15.77. $y'-2y=0,\quad y(0)=1$ 15.78. $y'+y=e^t,\quad y(0)=0$

15.79. $y''-9y=0,\quad y(0)=y'(0)=0$

$y'+cy=\varphi(t)$ **differensial tenglamaning** $y(0)=y_0$ **shartni qanoatlantiruvchi yechimi topilsin:**

15.80. a) $c=2;\ y_0=0;\ \varphi(t)=3$ в) $c=1;\ y_0=1;\ \varphi(t)=5t$

15.81. $c=-4;\ y_0=1;\ \varphi(t)=e^{3t}$ 15.82. $c=0;\ y_0=2;\ \varphi(t)=\sin t$

15.83. $c=0;\ y_0=2;\ \varphi(t)=te^{-t}$

$y''+ay'+by=\varphi(t)$ **differensial tenglamaning** $y(0)=y_0,\ y'(0)=y'_0$ **shartlarni qanoatlantiruvchi yechimi topilsin, agar**

15.84. $a=-2;\ b=-3;\ y_0=0;\ y'_0=0;\ \varphi(t)=e^{3t}$

15.85. $a=3;\ b=3;\ y_0=1;\ y'_0=2;\ \varphi(t)=0$

15.86. $a=-7;\ b=10;\ y_0=1;\ y'_0=1;\ \varphi(t)=5$

15.87. $a=0;\ b=-3;\ y_0=0;\ y'_0=0;\ \varphi(t)=e^{2t}-2$

15.88. $a=-2;\ b=1;\ y_0=1;\ y'_0=0;\ \varphi(t)=\sin t$

15.89. $a=4;\ b=0;\ y_0=0;\ y'_0=0;\ \varphi(t)=\sin 3t$

15.90. $a=-2c_1;\ b=c_1^2+c_2^2\ \ \varphi(t)=0$

15.91. $a=0;\ b=s^2;\ \ \varphi(t)=c\cdot\cos\omega t$

15.92. Yuqori tartibli $y^{(IV)}-y''+y=\sin t$ differensial tenglamaning $y(0)=y'(0)=y''(0)=y'''(0)=0$ shartlarni qanoatlantiruvchi yechimi topilsin.

15.93. $y''+y'-2y=e^t$, $y(0)=-1$, $y'(0)=0$

15.94. $y'''-6y''+11y'-6y=0$, $y(0)=0$, $y'(0)=1, y''(0)=0$

Quyidagi birinchi tartibli, chiziqli, o'zgarmas koiffisentli differensial tenglamalar sistemasining berilgan boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin:

15.95. $\begin{cases} \dfrac{dx}{dt}=x+2y, & x(0)=0 \\ \dfrac{dy}{dt}=2x+y+1, & y(0)=5 \end{cases}$

Yechish. Tasvirga o'tib oddiy tenglamalar sistemasini tuzamiz

$$\begin{cases} p\bar{x}(p)=\bar{x}(p)+2\bar{y}(p) \\ p\bar{y}(p)-5=2\bar{x}(p)+\bar{y}(p)+\dfrac{1}{p} \end{cases}$$

sistemani $\bar{x}$ va $\bar{y}$ larga nisbatan yechamiz.

$$\bar{x}(p)=\frac{10p+2}{p(p+1)(p-3)}, \quad \bar{y}(p)=\frac{5p^2-4p-1}{p(p+1)(p-3)}$$

Original topish uchun §15.2da keltirilgan yoyish usulidan foydalanamiz. U holda

$$x=-\frac{2}{3}-2e^{-t}+\frac{8}{3}e^{3t}; \quad y=\frac{1}{3}+2e^{-t}+\frac{8}{3}e^{3t}.$$

15.96 $\begin{cases} \dfrac{dx}{dt}=2y, & x(0)=2 \\ \dfrac{dy}{dt}=2x, & y(0)=2 \end{cases}$

15.97. $\begin{cases} \dfrac{dx}{dt}=3x+4y, \\ \dfrac{dy}{dt}=4x-3y, & x(0)=y(0)=1 \end{cases}$

15.98 $\begin{cases} \dfrac{dx}{dt}+4x-y=0, & x(0)=1 \\ \dfrac{dy}{dt}-2x-y=0, & y(0)=2 \end{cases}$

15.99. $\begin{cases} \dfrac{dx}{dt}+x+y=t, & x(0)=0 \\ \dfrac{dy}{dt}+x+y=0, & y(0)=0 \end{cases}$

Takrorlash uchun savollar

3. $f(t)$ funksiyaning tasviri yoki Laplas almashtirishi deb nimaga aytiladi?
4. Chiziqlilik xossasini ayting.
5. Kompozitsiya nima?
6. Hosilaning tasviri haqida gapiring.
7. Tasvirning hosilasi nima?

OPERATSION HISOB ELEMENTLARIGA DOIR NAZORAT TESTLARI

Laplas almashtirishi va uning chiziqlilik xossasi. Ayrim funksiyalarning tasvirlari

1. Quyidagi shartlardan qaysi biri $f(t)$ original uchun talab etilmaydi?

A) $t<0$ bo'lganda $f(t)\equiv 0$. B) $f(t)$ monoton funksiya

C) ixtiyoriy $[a,b]$ chekli kesmada $f(t)$ funksiya bo'lakli-uzluksiz

D) biror musbat M va s o'zgarmas sonlar uchun $|f(t)|<Me^{st}$.

E) keltirilgan barcha shartlar talab etiladi .

2. Quyidagi $f(t)$ $(t\geq 0)$ funksiyalardan qaysi biri original bo'la olmaydi?

A) $f(t)=\sin t^3$ B) $f(t)=\cos t^3$ C) $f(t)=e^{t^3}$ D) $f(t)=t^3$

E) keltirilgan barcha funksiyalar original bo'ladi .

3. Quyidagi $f(t)$ $(t\geq 0)$ funksiyalardan qaysi biri original bo'la olmaydi?

A) $f(t)=\sin^3 t$ B) $f(t)=\cos^3 t$ C) $f(t)=(e^t)^3$ D) $f(t)=t^3$

E) keltirilgan barcha funksiyalar original bo'ladi .

4. $f(t)$ originalning tasviri $F(p)$ $(p=x+iy)$ qaysi formula bilan aniqlanadi?

A) $F(p)=\int\limits_0^\infty f(t)\sin pt\,dt$ B) $F(p)=\int\limits_0^\infty f(t)\cos pt\,dt$

C) $F(p)=\int\limits_0^\infty e^{pt}f(t)dt$ D) $F(p)=\int\limits_0^\infty e^{-pt}f(t)dt$

E) $F(p)=\int\limits_0^\infty e^{ipt}f(t)dt$.

Agar original $|f(t)|<Me^{st}$ shartni qanoatlantirsa, uning tasviri $F(p)$ o'zgaruvchining qanday qiymatlarida aniqlangan bo'ladi?

A) $\mathrm{Re}p>s$ .B) $\mathrm{Re}p<s$. C) $\mathrm{Im}p>s$. D) $\mathrm{Im}p<s$. E) $|p|>s$.

5. Agar $L\{f\}=L\{g\}$ bo'lsa, quyidagi tasdiqlardn qaysi biri to'g'ri ?

A) ixtiyoriy C o'zgarmas son uchun $f(t)\equiv Cg(t)$.

B) ixtiyoriy C o'zgarmas son uchun $f(t)\equiv g(t)+C$.

C) ixtiyoriy C o'zgarmas son uchun $f(t)\equiv g(Ct)$.

D) ixtiyoriy C o'zgarmas son uchun $f(t) \equiv g(t+C)$.

E) keltirilgan barcha tasdiqlar to'g'ri emas.

6. Agar $L\{f\}=L\{g\}$ bo'lsa, quyidagi tasdiqlardn qaysi biri noto'g'ri ?

A) $f(t) \equiv g(t)$. B) $\mathrm{Re}f(t) \equiv \mathrm{Re}g(t)$. C) $\mathrm{Im}f(t) \equiv \mathrm{Im}g(t)$.
D) $|f(t)| \equiv |g(t)|$. E) keltirilgan barcha tasdiqlar to'g'ri .

7. $\sigma_0(t)$ Hevisayd funksiyasining tasviri nimaga teng?

A) $1/(p-1)$. B) $1/(p+1)$. C) $1/(p^2-1)$. D) $1/(p^2+1)$.
E) $1/p$.

8. $\delta(t)$ Dirak funksiyasining tasviri nimaga teng?

A) $1/(p-1)$. B) $1/(p+1)$. C) $1/(p^2-1)$. D) $1/(p^2+1)$.
E) 1 .

9. $f(t)=\sin t$ $(t \geq 0)$ trigonometrik funksiyaning tasviri nimaga teng?

A) $1/(p-1)$ B) $1/(p+1)$ C) $1/(p^2-1)$
D) $1/(p^2+1)$ E) $1/p$

10. $f(t)=\cos t$ $(t \geq 0)$ trigonometrik funksiyaning tasviri nimaga teng?

A) $p/(p-1)$ B) $p/(p+1)$ C) $p/(p^2-1)$
D) $p/(p^2+1)$ E) $1/p$

11. $f(t)=\mathrm{ch}t$ $(t \geq 0)$ giperbolik funksiyaning tasviri nimaga teng?

A) $p/(p-1)$ B) $p/(p+1)$ C) $p/(p^2-1)$
D) $p/(p^2+1)$ E) $1/p$

12. $f(t)=\mathrm{sh}t$ $(t \geq 0)$ giperbolik funksiyaning tasviri nimaga teng?

A) $1/(p-1)$ B) $1/(p+1)$ C) $1/(p^2-1)$
D) $1/(p^2+1)$ E) $1/p$

13. $f(t)=e^t$ $(t \geq 0)$ ko'rsatgichli funksiyaning tasviri nimaga teng?

A) $1/(p-1)$ B) $1/(p+1)$ C) $1/(p^2-1)$
D) $1/(p^2+1)$ E) $1/p$

14. $f(t)=e^{-t}$ $(t \geq 0)$ ko'rsatgichli funksiyaning tasviri nimaga teng?

A) $1/(p-1)$ B) $1/(p+1)$ C) $1/(p^2-1)$
D) $1/(p^2+1)$ E) $1/p$

15. $f(t)=e^{\alpha t}$ ($t\geq0$) ko'rsatgichli funksiyaning tasviri nimaga teng?

A) $1/(p-\alpha)$ B) $1/(p+\alpha)$ C) $1/(p^2-\alpha^2)$
D) $1/(p^2+\alpha^2)$ E) α/p

2. Laplas almashtirishining asosiy xossalalri. Tasvirlar jadvali

1. Laplas almashtirishi uchun o'xshashlik xossasini ko'rsating.

A) $L\{e^{-at}f(t)\}=F(p+a)$ B) $L\{f(t-\theta)\}=e^{-\theta p}F(p)$.

C) $L\{f(\beta t)\}=\frac{1}{\beta}F(\frac{p}{\beta})$ D) $L\{f(t)/t\}=\int_p^\infty F(z)dz$.

E) $L\{\int_0^t f(u)g(t-u)du\}=F(p)\cdot G(p)$.

2. Agar $f(t)$ originalning tasviri $F(p)=4p/(p+1)$ bo'lsa, $f(2t)$ original tasviri nimaga teng bo'ladi?

A) $2p/(p+1)$ B) $8p/(p+1)$ C) $2p/(p+2)$
D) $8p/(p+2)$ E) $p/(2p+1)$

3. Laplas almashtirishi uchun siljish xossasini ko'rsating.

A) $L\{e^{-at}f(t)\}=F(p+a)$ B) $L\{f(t-\theta)\}=e^{-\theta p}F(p)$

C) $L\{f(\beta t)\}=\frac{1}{\beta}F(\frac{p}{\beta})$ D) $L\{f(t)/t\}=\int_p^\infty F(z)dz$

E) $L\{\int_0^t f(u)g(t-u)du\}=F(p)\cdot G(p)$

4. Agar $f(t)$ originalning tasviri $F(p)=p/(p+1)$ bo'lsa, $e^t f(t)$ original tasviri nimaga teng bo'ladi?

A) $(p-1)/(p+1)$ B) $(p+1)/(p-1)$ C) $p/(p+1)$
D) $p/(p-1)$ E) $(p-1)/p$

5. Laplas almashtirishi uchun kechikish xossasini ko'rsating.

A) $L\{e^{-at}f(t)\}=F(p+a)$ B) $L\{f(t-\theta)\}=e^{-\theta p}F(p)$;

C) $L\{f(\beta t)\}=\frac{1}{\beta}F(\frac{p}{\beta})$ D) $L\{f(t)/t\}=\int_p^\infty F(z)dz$

E) $L\{\int_0^t f(u)g(t-u)du\}=F(p)\cdot G(p)$.

6. Agar $f(t)$ originalning tasviri $F(p)=p/(p+1)$ bo'lsa, $f(t+2)$ original tasviri nimaga teng bo'ladi?

A) $e^{2p}p/(p+1)$. B) $e^{2-p}p/(p-1)$. C) $e^{-2p}p/(p+1)$.

D) $e^{2+p}p/(p+1)$. E) $e^{p-2}p/(p+1)$.

7. Laplas almashtirishi uchun kompozitsiya xossasini ko'rsating.

A) $L\{e^{-at}f(t)\}=F(p+a)$. B) $L\{f(t-\theta)\}=e^{-\theta p}F(p)$.

C) $L\{f(\beta t)\}=\frac{1}{\beta}F(\frac{p}{\beta})$. D) $L\{f(t)/t\}=\int_p^\infty F(z)dz$.

E) $L\{\int_0^t f(u)g(t-u)du\}=F(p)\cdot G(p)$.

8. Originalni differensiallash xossasi qayerda to'g'ri ko'rsatilgan ?

A) $L\{f^{(n)}(t)\}=p^n L\{f(t)\}+\sum_{k=0}^{n-1} f^{(k)}(0)p^{n-1-k}$

B) $L\{f^{(n)}(t)\}=p^n L\{f(t)\}+\sum_{k=0}^{n-1}(-1)^k f^{(k)}(0)p^{n-1-k}$

C) $L\{f^{(n)}(t)\}=p^n L\{f(t)\}-\sum_{k=0}^{n-1} f^{(k)}(0)p^{n-1-k}$

D) $L\{f^{(n)}(t)\}=p^n L\{f(t)\}+\sum_{k=0}^{n-1}(-1)^{n-1-k} f^{(k)}(0)p^{n-1-k}$

E) $L\{f^{(n)}(t)\}=(-1)^n p^n L\{f(t)\}+\sum_{k=0}^{n-1} f^{(k)}(0)p^{n-1-k}$

9. Agar $f(t)$ originalning tasviri $F(p)$ va $f(0)=a$ bo'lsa, $f'(t)$ hosilaning tasviri $G(p)$ qanday topiladi?

A) $G(p)=F(p)+ap$ B) $G(p)=F(p)-ap$ C) $G(p)=pF(p)+a$

D) $G(p)=pF(p)-a$ E) $G(p)=apF(p)$

10. Agar $f(t)$ originalning tasviri $F(p)=(p^2-1)/(p^3+p)$ va $f(0)=-1$bo'lsa, $f'(t)$ hosilaning tasviri $G(p)$ nimaga teng?

A) $G(p)=-2p/(p^2+1)$ B) $G(p)=-2/(p^2+1)$

C) $G(p)=(p-2)/(p^2+1)$ D) $G(p)=-2+p/(p^2+1)$

E) $G(p)=-2+1/(p^2+1)$

11. Agar $L\{f(t)\}=F(p)$ bo'lsa, $L\{\int_0^t f(u)du\}$ nimaga teng bo'ladi?

A) $pF(p)$ B) $F(1/p)$ C)$F(p)/p$

D) $1/F(p)$ E) $p/F(p)$.

12. Agar $f(t)$ originalning tasviri $F(p)=(p^3-p)/(p^2+1)$ bo'lsa, $g(t)=\int_0^t f(u)du$ funksiyaning $G(p)$ tasvirini toping .

A) $G(p)=(3p^2-1)/(p^2+1)$ B) $G(p)=(p^2-1)/2$
C) $G(p)=(p^4-p^2)/(p^2+1)$ D) $G(p)=(p^2-1)/(p^2+1)$
E) to'gri javob keltirilmagan .

13. Agar $L\{f(t)\}=F(p)$ bo'lsa, tasvirning $F^{(n)}(p)$ hosilasiga qaysi original mos keladi?

A) $t^n f(t)$ B) $(-t)^n f(t)$ C) $f(t) / t^n$
D) $f(t) / (-t)^n$ E) $f(t^n)$

14. Agar $f(t)$ originalning tasviri $F(p)$ bo'lsa, $tf(t)$ originalning tasviri nimaga teng bo'ladi?

A) $-pF(p)$ B) $-F(p)/p$ C) $-F'(p)$
D) $-\int_p^\infty F(u)du$ E) $F(1/p)$.

15. Agar $f(t)$ originalning tasviri $F(p)=\cos 2p$ bo'lsa, $tf(t)$ originalning tasviri nimaga teng bo'ladi?

A) $p\cos 2p$ B) $(\cos 2p)/p$ C) $2\sin 2p$
D) $(\sin 2p)/2$ E) $\cos 2p^2$

16. Agar $f(t)=(t-1)/(t^2+1)$ originalning tasviri $F(p)$ bo'lsa, $F'(p)$ tasvirga mos keladigan $g(t)$ originalni toping.

A) $g(t)=(t-1)/(t^3+t)$. B) $g(t)=(t^2-t)/(t^2+1)$. C) $g(t)=(t^2+t)/(t^2+1)$. D) $g(t)=(t-t^2)/(t^2+1)$. E) to'g'ri javob keltirilmagan .

3.Laplasning teskari almashtirishi. Operatsion hisob yordamida differensial tenglamalarni yechish

1. Agar $F(p)=1/(p^2+4)$ bo'lsa, Laplasning teskari $L^{-1}\{F\}$ almashtirishi nimaga teng ?

A) $0.5\text{sh}2t$ B) $0.5\sin 2t$ C) $0.5\text{ch}2t$
D) $0.5\cos 2t$ E) $0.5e^{2t}$.

2. Agar $F(p)=p/(p^2+4)$ bo'lsa, Laplasning teskari $L^{-1}\{F\}$ almashtirishi nimaga teng ?

A) $\text{sh}2t$ B) $\sin 2t$ C) $\text{ch}2t$
D) $\cos 2t$ E) e^{2t} .

3. Agar $F(p)=1/(p^2-4)$ bo'lsa, Laplasning teskari $L^{-1}\{F\}$ almashtirishi nimaga teng ?

A) $0.5\text{sh}2t$ B) $0.5\sin 2t$ C) $0.5\text{ch}2t$

D) $0.5\cos 2t$ E) $0.5e^{2t}$.

4. Agar $F(p)=p/(p^2-4)$ bo'lsa, Laplasning teskari $L^{-1}\{F\}$ almashtirishi nimaga teng ?

A) $\mathrm{sh}2t$ B) $\sin 2t$ C) $\mathrm{ch}2t$ D) $\cos 2t$ E) e^{2t}

5. Quyidagi tengliklardan qaysi biri Laplas teskari almashtirishining chiziqlilik xossasini ifodalamaydi ?

A) Har qanday o'zgarmas C soni uchun $L^{-1}\{CF\}=CL^{-1}\{F\}$.

B) Agar $L^{-1}\{F\}$ va $L^{-1}\{G\}$ mavjud bo'lsa, unda $L^{-1}\{F+G\}=L^{-1}\{F\}+L^{-1}\{G\}$.

C) Agar $L^{-1}\{F\}$ va $L^{-1}\{G\}$ mavjud bo'lsa, unda $L^{-1}\{F-G\}=L^{-1}\{F\}-L^{-1}\{G\}$.

D) Agar $L^{-1}\{F\}$ va $L^{-1}\{G\}$ mavjud bo'lsa, unda $L^{-1}\{F\cdot G\}=L^{-1}\{F\}\cdot L^{-1}\{G\}$.

E) Agar $L^{-1}\{F\}$ va $L^{-1}\{G\}$ mavjud bo'lsa, unda ixtiyoriy a va b sonlari uchun $L^{-1}\{aF+bG\}=aL^{-1}\{F\}+bL^{-1}\{G\}$.

6. Agar $F(p)=[(p+4)(p+3)]^{-1}$ bo'lsa, Laplasning teskari $L^{-1}\{F\}$ almashtirishi nimaga teng ?

A) $e^{-3t}-e^{4t}$ B) $e^{3t}-e^{-4t}$ C) $e^{3t}-e^{4t}$

D) $e^{-3t}-e^{-4t}$ E) $e^{3t}+e^{4t}$.

7. Agar $F(p)=2/(p^2+6p+8)$ bo'lsa, Laplasning teskari $L^{-1}\{F\}$ almashtirishi nimaga teng ?

A) $e^{-2t}-e^{4t}$ B) $e^{2t}-e^{-4t}$ C) $e^{2t}-e^{4t}$ D) $e^{-2t}-e^{-4t}$ E) $e^{2t}+e^{4t}$

8. Agar $F(p)=(3+p)/(p^2+9)$ bo'lsa, Laplasning teskari $L^{-1}\{F\}$ almashtirishi nimaga teng ?

A) $\sin 3t+\cos 3t$ B) $\sin 3t-\cos 3t$ C) $\mathrm{sh}3t-\mathrm{ch}3t$

D) $\mathrm{sh}3t+\mathrm{ch}3t$ E) $\sin 3t-\mathrm{ch}3t$

9. Agar $F(p)$ tasvirning Loran qatori $F(p)=\sum_{k=1}^{\infty}\frac{c_k}{p^k}$ bo'lsa, unda $L^{-1}\{F\}=?$

A) $L^{-1}\{F\}=\sum_{k=1}^{\infty}\frac{c_k}{t^k}$ B) $L^{-1}\{F\}=\sum_{k=1}^{\infty}c_k t^k$ C) $L^{-1}\{F\}=\sum_{k=1}^{\infty}c_k\frac{t^k}{k!}$.

D) $L^{-1}\{F\}=\sum_{k=1}^{\infty}c_k\frac{t^{k+1}}{(k+1)!}$ E) $L^{-1}\{F\}=\sum_{k=1}^{\infty}c_k\frac{t^{k-1}}{(k-1)!}$

10. Agar tasvir $F(p)=\sin\frac{1}{p}$ bo'lsa, $L^{-1}\{F\}$ original nimaga teng ?

A) $\sum_{k=1}^{\infty}\frac{(-t^2)^{k-1}}{(2k+1)!(2k+2)!}$ B) $\sum_{k=1}^{\infty}\frac{(-t^2)^{k-1}}{(2k-1)!(2k+2)!}$

C) $\sum_{k=1}^{\infty}\frac{(-t^2)^{k-1}}{(2k+1)!(2k-2)!}$ D) $\sum_{k=1}^{\infty}\frac{(-t^2)^{k-1}}{(2k-1)!(2k-2)!}$

E) barcha javoblar noto'gri .

11.Agar $Q(p)$ va $H(p)$ ko'phadlar umumiy ildizlarga ega bo'lmasa va $F(p)=Q(p)/H(p)$ ratsional funksiya $p_1, p_2, \ldots, p_n$ oddiy qutblarga ega bo'lsa, unda $f(t)= L^{-1}\{F\}$ original qaysi yig'indi bilan aniqlanadi ?

A) $\sum_{k=1}^{n}\frac{Q(p_k)}{H(p_k)}e^{p_k t}$ B) $\sum_{k=1}^{n}\frac{Q(p_k)}{H'(p_k)}e^{p_k t}$ C) $\sum_{k=1}^{n}\frac{Q'(p_k)}{H(p_k)}e^{p_k t}$

D) $\sum_{k=1}^{n}\frac{Q'(p_k)}{H'(p_k)}e^{p_k t}$ E) $\sum_{k=1}^{n}\frac{Q(p_k)}{H''(p_k)}e^{p_k t}$

12. $F(p)=(p^2+3)/(p^3+2p^2-8p)$ bo'lsa, $f(t)= L^{-1}\{F\}$ original nimaga teng?

A) $f(t)=(13-21e^{-4t}+67e^{2t})/56$

B) $f(t)=-(21-19e^{4t}+98e^{-2t})/56$

C) $f(t)=-(25-17e^{-4t}+32e^{-2t})/56$

D) $f(t)=(19-27e^{4t}+42e^{2t})/56$

E) $f(t)=(14e^{t}-23e^{2t}+29e^{3t})/56$

13. $ay''(t)+by'(t)+cy(t)=f(t),\quad y(0)=y_0,\quad y'(0)=y_0'$

Koshi masalasini operatsion hisob yordamida yechishda quyidagi amallardan qaysi biri qatnashmaydi?

A) $L\{y\}=Y(p)$, $L\{f\}=F(p)$ tasvirlar qaraladi .

B) boshlang'ich shartlardan foydalanib $L\{y'\}$ va $L\{y''\}$ topiladi

C) tenglamaning o'ng tomonidan foydalanib $L\{f'\}$ topiladi .

D) $Y(p)$ tasvirga nisbatan operator tenglama tuziladi va yechiladi .

E) Laplasning teskari almashtirishi $L^{-1}\{Y\}$ aniqlanadi .

14. $y''(t)-2y'(t)+5y(t)=e^{2t},\quad y(0)=2,\quad y'(0)=-1$

Koshi masalasi $y(t)$ yechimining $Y(p)$ tasvirini toping.

A) $Y(p)=\frac{p^2-3p+1}{(p+2)(p^2-2p+5)}$ B) $Y(p)=\frac{4p^2-5p+9}{(p-2)(p^2-2p+5)}$

C) $Y(p)=\frac{2p^2-9p+11}{(p-2)(p^2-2p+5)}$ D) $Y(p)=\frac{4p^2+3p-10}{(p+2)(p^2-2p+5)}$

E) to'g'ri javob keltirilmagan .

Yulduzlar hukmiga oid ilm-
matematik fanlarning eng yaxshi mahsulidir.
Abu Rayhon Beruniy

XVI-BOB. MATEMATIK FIZIKANING BA'ZI TENGLAMALARI

§ 16.1. Koshi masalasini Dalamber usuli yordamida yechish

Tor tebranish tenglamasi matematik fizikaning oddiy tenglamasi hisoblanadi. Uning ko'rinishi quyidagicha

$$\frac{\partial^2 u(x,t)}{\partial t^2} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2}, \qquad (16.1)$$

bu yerda $a^2 = T/\rho$, T – tor nuqtalaridagi taranglik kuchi, ρ-tor materiali zichligi, $u(x,t)$ - tor nuqtalarining t vaqtda muvozanat vaziyati atrofidagi ko'chishi. (16.1) quyidagi boshlang'ich shartlarni qanoatlantiradi:

$$u(x,t)\big|_{t=0} = \varphi(x), \quad \frac{\partial u(x,t)}{\partial t}\bigg|_{t=0} = \psi(x) \qquad (16.2)$$

harakat tenglamasi (16.1) ning (16.2) boshlang'ich shartlarni qanoatlantiruvchi $(|x| < \infty, \ t > 0)$ yechimini topish Koshi masalasi deyiladi.

Bu masalani yechishda Dalamber usuli qo'llanilishi mumkin. Buning uchun (16.1) tenglamaning umumiy yechimi quyidagicha izlanadi:

$$u(x,t) = f_1(x + at) + f_2(x - at).$$

Bu yerda f_1, f_2 -ixtiyoriy ikki marta differensiallanuvchi funksiya bo'lib, (16.2) shartlardan topiladi va umumiy yechim quyidagi ko'rinishga ega:

$$u(x,t)=\frac{1}{2}[\varphi(x+at)+\varphi(x-at)]+\frac{1}{2a}\int_{x-at}^{x+at}\psi(S)dS \qquad (16.3)$$

Masalalarda berilgan shartlarni qanoatlantiruvchi Koshi masalasi yechimi $u(x;t)(-\infty<x<\infty)$ ni toping.

16.1 $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$ differensial tenglamaning $u(x,t)|_{t=0}=\cos 2x$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\sin x$

shartlarni qanoatlantiruvchi yechimini toping.

$$u(x,t)=\frac{1}{2}(\cos 2(x+at)+\cos 2(x-at))+\frac{1}{2a}\int_{x-at}^{x+at}\sin S dS=\cos 2x\cos 2at+\frac{1}{2}\sin x\sin at.$$

16.2. $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$ tenglamaning $u(x,t)|_{t=0}=0$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=x$

shartlarni qanoatlantiruvchi yechimini toping.

16.3 $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$ tenglamaning $u(x,t)|_{t=0}=\sin x$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\cos x$ masalalarda berilgan shartlarni qanoatlantiruvchi Koshi masalasi yechimini $u(x,t)$ $(-\infty<x<\infty,\ t>0)$ shartlarni qanoatlantiruvchi yechimini toping.

16.4. Quyidagi $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$ tenglama bilan ifodalanuvchi tor tebranish tenglamasining $u(x,t)|_{t=0}=\cos x$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=2$ boshlang'ich shartlarni qanoatlantiruvchi $t=\frac{\pi}{3}$ vaqtdaga yechimini aniqlang.

16.5. $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$ tenglamaning $u(x,0)=x(1+x^2)^{-1}$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\sin x$ shartlarni qanoatlantiruvchi yechimini toping.

16.6. $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$ tenglamaning yechimini $u(x,t)|_{t=0}=x$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=-x$ shartlar asosida toping.

16.7. $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$ tenglamaning yechimini $u(x,t)|_{t=0}=0$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\cos x$ shartlar asosida toping.

16.8. $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$ tenglamaning $t=\pi$ vaqtdagi $u(x,t)|_{t=0}=\sin x$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\cos x$ shartlarni qanoatlantiruvchi yechimini toping.

16.9. $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$tenglamaning $u(x,t)|_{t=0}=0$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=x$ shartlarni qanoatlantiruvchi yechimini toping.

16.10 $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$, $(-\infty<x<\infty)$tenglamaning $u(x,t)|_{t=0}=\sin x$, $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=1$ shartlarni qanoatlantiruvchi $t=\frac{\pi}{2a}$ vaqtdagi yechimini toping.

§ 16.2. Tor tebranish tenglamasini Fur'e usuli yordamida yechish

Tor tebranish tenglamasining

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}\quad a^2=\frac{T}{\rho} \tag{16.4}$$

$$u(0,t)=0;\quad u(e,t)=0\quad (t\geq 0)$$

chegaraviy va

$$u(x,t)|_{t=0}=\varphi(x),\quad \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\psi(x)\ (0\leq x\leq t)$$

boshlang'ich shartlarni qanoatlantiruvchi yechimini topish talab etiladi. Bu (16.4) tenglamani yechishga Fur'e usulini qo'llash mumkin. U holda $u(x,t)$ yechim qator yordamida ifoda qilinadi:

$$u(x,t)=\sum_{k=1}^{\infty}\left(a_k\cos\frac{k\pi at}{\ell}+b_k\sin\frac{k\pi at}{\ell}\right)\sin\frac{k\pi x}{\ell}, \tag{16.5}$$

bu yerda

$$a_k=\frac{2}{\ell}\int_0^{\ell}\varphi(x)\sin\frac{k\pi x}{\ell}dx;\quad b_k=\frac{2}{k\pi a}\int_0^{\ell}\psi(x)\sin\frac{k\pi x}{\ell}dx$$

Agar $A_k=\sqrt{a_k^2+b_k^2}$, $\sin\varphi_k=\frac{a_k}{A_k}$, $\cos\varphi_k=\frac{b_k}{A_k}$ belgilash kiritsak, u holda yechim

$$u(x,t)=\sum_{k=1}^{\infty}A_k\sin\frac{k\pi x}{\ell}\sin\left(\frac{k\pi at}{\ell}+\varphi_k\right)$$

ko'rinishda bo'ladi. Bu qatorning har bir hadi turg'un to'lqinni ifoda qiladi va uning amplitudasi $A_k \sin(k\pi x/\ell)$, chastotasi - $\omega_k = \frac{k\pi a}{\ell}$ va fazasi - φ_k bo`ladi.

Quyidagi masalalarni yeching.

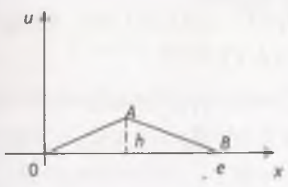
16.11. Uzunligi l bo'lgan ikki tomoni mahkamlangan ($x=0$ va $x=l$) tor nuqtalarining ko'chishi

$$u(x,t)\big|_{t=0} = \varphi(x) = \begin{cases} \frac{x}{5}, & agar \quad 0 < x < \frac{\ell}{2} \\ -\frac{1}{5}(x-\ell), & agar \quad \frac{\ell}{2} < x < \ell \end{cases},$$

chegaraviy $u(0,t)=0;\ \ u(l,t)=0\ \ (t \ge 0)$va boshlang'ich $\frac{\partial u(x,t)}{\partial t}\big|_{t=0} = \psi(x) = 0\ \frac{\partial u(x,t)}{\partial t}\big|_{t=0} = \psi(x) = 0\ (0 \le x \le l)$shartlarni qanoatlantiruvchi yechimini toping.

16.12. Ikki uchi mahkamlangan ($x=0$ va $x=l$) tor boshlang'ich vaqtdagi holati parabola ko'rinishida $u = (4h/l^2)\cdot x(l-x)$ Torning abtsissa o'qi bo'yicha ko'chishini toping (boshlang'ich tezlik hisobga olinmasin).

16.13 Torning boshlang'ich holatdagi ko'rinishi OAB siniq chiziq ko'rinishda bo'lsa, ixtiyoriy t vaqtdagi $u(x,t)$ tor nuqtalari ko'chishini toping (16.1- chizma).



16.1- chizma

16.14. Ikki tomoni mahkamlangan tor nuqtalarining boshlang'ich ko'chishi nolga va tezligi

$$\frac{\partial u}{\partial t} = u_t' = \begin{cases} v_0(const), & agar \quad |x - l/2| < h/2 \\ 0, & agar \quad |x - l/2| > h/2 \end{cases}$$

bo'lsa, t vaqtda torning holatini aniqlang.

16.16. Torning ikkala tomoni mahkamlangan ($x=0$ va $x=2$). Boshlang'ichholatda $u=(2x-x^2)$. Ixtiyoriy$t$vaqtgatornuqtalarning $u(x,t)$ko'chishinitoping.

§ 16.3. Issiqlik tarqalish tenglamasi

Bitta Ox o'qi bo'yicha issiqlik tarqalish tenglamasining

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}, \quad (t>0, \quad -\infty<x<\infty)\ , \qquad (16.6)$$

$u(x,t)\big|_{t=0}=f(x)$ shartni qanoatlantiruvchi yechimi Fur'e usuli yordamida

$$u(x,t)=\frac{1}{2a\sqrt{\pi t}}\int_{-\infty}^{\infty} f(\xi)e^{-(\xi-x)^2/(4a^2t)}d\xi$$

ko'rinishida topiladi.

Quyidagi masalalarni yeching.

16.17. Differensial tenglamaning $u\big|_{t=0}=f(x)=u_0$ va $u\big|_{x=0}=0$ shartlarni qanoatlantiruvchi yechimini toping.

16.18. Differensial tenglamaning $u\big|_{t=0}=f(x)=\begin{cases} x, & agar \ \ 0<x<e/2 \\ e-x, & agar \ \ e/2\le x<e \end{cases}$

va $u\big|_{x=0}=u\big|_{x=e}=0$ shartlarni qanoatlantiruvchi yechimini toping.

16.19. $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ tenglamaning quyidagi boshlang'ich shartlarni qanoatlantiruvchi yechimini toping.

$$u(x,t)\big|_{t=0}=f(x)=\begin{cases} 1-\frac{1}{\ell}, & 0\le x\le \ell; \\ 1+\frac{x}{\ell}, & -\ell\le x\le 0; \\ 0 & , x\ge \ell \text{ и } x\le -\ell \end{cases}$$

16.20. Quyidagi $u_t'=a^2u_{xx}$ tenglamaning

$$u(x,t)\big|_{t=0}=f(x)=\begin{cases} u_0, & agar \ \ x_1<x<x_2 \\ 0, & agar \ \ x<x_1 \ \ yoki \ \ x>x_2 \end{cases}$$

shartni qanoatlantiruvchi yechimi topilsin

16.21. Yarim cheksiz sterjenning chapdagi oxiri issiqlik o'tkazmaydigan bo'lsa va issiqlik tarqalishining boshlang'ich tezligi quyidagi $u|_{t=0} = f(x) = \begin{cases} 0, & x < 0; \\ u_0, & 0 < x < \ell; \\ 0, & \ell < x. \end{cases}$ formula bilan berilgan bo'lsa, issiqlik tarqalish tenglamasining yechimini toping.

16.22. Yupqa bir jinsli, uzunligi ℓ ga teng bo'lgan sterjen berilgan. U tashqi muhitdan ajratilgan bo'lib, boshlang'ich harorati $f(x) = \frac{cx(\ell - x)}{\ell^2}$ ga teng. Sterjenning oxirlarida nolga teng bo'lgan harorat saqlanadi. Sterjenning ixtiyoriy $t>0$ vaqtdagi harorati aniqlansin.

16.23. Umumiy OZ o'qqa ega bo'lgan ikki silindr orasidagi fazoda issiqlikning statsionar tarqalishi qonunini toping. Silindrlar sirtlarida harorat o'zgarmas.

Takrorlash uchun savollar

1.Matematik fizika tenglamasi ta'rifini ayting.

2.Koshi masalasi haqida nimalarni bilasiz?

3.Dalamber formulasini yazing.

4.Boshlang'ich shartlar nima?

5.Chegaraviy shartlar nima?

6.Umumiy holda $u=u(x,y)$ funksiya uchun II tartibli chiziqli matematik fizika tenglamasi qanday ko'rinishda bo'ladi ?

MATEMATIK FIZIKA TENGLAMALARIGA DOIR TESTLAR

1. Matematik fizika tenglamalari va ularning turlari.

17.Matematik fizika tenglamasi ta'rifini ko'rsating .

A) ko'p o'zgaruvchili noma'lum funksiya qatnashgan tenglama .

B) ko'p o'zgaruvchili noma'lum funksiyaning berilgan nuqtalardagi qiymatlari qatnashgan tenglama .

C) ko'p o'zgaruvchili noma'lum funksiyaning hosilalari qatnashgan tenglama .

D) ko'p o'zgaruvchili noma'lum funksiya hosilalarining berilgan nuqtalardagi qiymatlari qatnashgan tenglama .

E) ko'p o'zgaruvchili noma'lum funksiyaning integrallari qatnashgan tenglama .

18.Ikki o'zgaruvchili noma'lum $u=u(x,y)$ funksiya uchun quyidagi tenglamalar- dan qaysi biri matematik fizika tenglamasi bo'ladi?

A) $u'_x(x,0)+u'_x(0,y)=f(x,y)$.

B) $u'_y(x,0)+u'_y(0,y)=f(x,y)$.

C) $u'_x(x,0)+u'_y(0,y)=f(x,y)$.

D) keltirilgan tenglamalarning birortasi ham matematik fizika tenglamasi bo'lmaydi .

E)barcha keltirilgan tenglamalar matematik fizika tenglamasi bo'ladi .

19.Ikki o'zgaruvchili noma'lum $u=u(x,y)$ funksiya uchun quyidagi tenglamalar- dan qaysi biri matematik fizika tenglamasi bo'ladi?

A) $u(x,y)+u^2(x,y)=f(x,y)$.

B) $u'_x(x,y)+u'_y(x,y)=f(x,y)$.

C) $u(x,0)+u(0,y)=f(x,y)$.

D) keltirilgan tenglamalarning birortasi ham matematik fizika tenglamasi bo'lmaydi .

E) barcha keltirilgan tenglamalar matematik fizika tenglamasi bo'ladi .

20.Matematik fizika tenglamasining tartibi qanday aniqlanadi?

A) noma'lum funksiyaning argumentlari soni bo'yicha .

B) noma'lum funksiyaning darajalari bo'yicha .

C) noma'lum funksiya hosilalarining tartibi bo'yicha .

D) noma'lum funksiya hosilalarining darajasi bo'yicha .

E) noma'lum funksiya hosilalarining soni bo'yicha.

21.Umumiy holda $u=u(x,y)$ funksiya uchun I tartibli chiziqli matematik fizika tenglamasi qanday ko'rinishda bo'ladi ?

A) $A\frac{\partial u}{\partial x}+B\frac{\partial u}{\partial y}+Cu=f(x,y)$

B) $A\frac{\partial u}{\partial x}+2B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial u}{\partial y}+Du=f(x,y)$.

C) $A\frac{\partial u}{\partial x}\cdot B\frac{\partial u}{\partial y}+Cu=f(x,y)$

D) $A\frac{\partial u}{\partial x}\cdot B\frac{\partial u}{\partial y}\cdot Cu=f(x,y)$.

E) to'gri javob keltirilmagan .

22.Quyidagilardan qaysi biri $u=u(x,y)$ funksiya uchun I tartibli chiziqli matematik fizika tenglamasi bo'ladi ?

A) $\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial u}{\partial x}+u=f(x,y)$. B) $\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial u}{\partial y}+u=f(x,y)$.

C) $\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial y}+u=f(x,y)$. D) $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+u=f(x,y)$.

E) keltirlgan tenglamalardan birortasi ham I tartibli chiziqli matematik fizika tenglamasi bo'lmaydi .

23.Quyidagilardan qaysi biri I tartibli chiziqli matematik fizika tenglamasi bo'ladi ?

A) $\frac{\partial u}{\partial x}+u=f(x,y)$

B) $\frac{\partial u}{\partial y}+u=f(x,y)$.

C) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+u=f(x,y)$.

D) keltirlgan tenglamalardan barchasi I tartibli chiziqli matematik fizika tenglamasi bo'ladi .

E) keltirlgan tenglamalardan birortasi ham I tartibli chiziqli matematik fizika tenglamasi bo'lmaydi .

24.Quyidagilardan qaysi biri I tartibli chiziqli matematik fizika tenglamasi bo'ladi ?

A) $\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}+u=f(x,y)$

B) $\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}+u=f(x,y)$

C) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+u=f(x,y)$.

D) keltirlgan tenglamalardan barchasi I tartibli chiziqli matematik fizika tenglamasi bo'ladi .

E) keltirlgan tenglamalardan birortasi ham I tartibli chiziqli matematik fizika tenglamasi bo'lmaydi .

25. Umumiy holda u=u(x,y) funksiya uchun II tartibli chiziqli matematik fizika tenglamasi qanday ko'rinishda bo'ladi ?

A) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial y^2}=f(x,y)$ B) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}=f(x,y)$.

C) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial y^2}+C\frac{\partial u}{\partial x}+D\frac{\partial u}{\partial y}+Eu=f(x,y)$.

D) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$.

E) $A\left(\frac{\partial u}{\partial x}\right)^2+B\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}+C\left(\frac{\partial u}{\partial y}\right)^2+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$.

26. Quyidagilardan qaysi biri u=u(x,y) funksiya uchun II tartibli chiziqli matematik fizika tenglamasi bo'lmaydi ?

A) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+Du=f(x,y)$.

B) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial y^2}+C\frac{\partial u}{\partial x}+D\frac{\partial u}{\partial y}+Eu=f(x,y)$.

C) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$.

D) $A\left(\frac{\partial u}{\partial x}\right)^2+B\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}+C\left(\frac{\partial u}{\partial y}\right)^2+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$.

E) Barcha tenglamalar II tartibli chiziqli matematik fizika tenglamasi bo'ladi.

27. Quyidagilardan qaysi biri u=u(x,y) funksiya uchun II tartibli chiziqli matematik fizika tenglamasi bo'ladi ?

A) $A\frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial^2 u}{\partial y^2}+B\frac{\partial^2 u}{\partial x\partial y}+Cu=f(x,y)$.

B) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial y^2}+C\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}+Du=f(x,y)$

C) $A\frac{\partial^2 u}{\partial x\partial y}+Bu=f(x,y)$.

D) $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D(\frac{\partial u}{\partial x})^2+E(\frac{\partial u}{\partial y})^2+Fu=f(x,y)$.

E) Barcha tenglamalar II tartibli chiziqli matematik fizika tenglamasi bo'ladi.

28. Umumiy holdagi II tartibli chiziqli matematik fizika tenglamasi

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$$

qaysi shartda bir jinsli bo'ladi?

A) $f(x,y)>0$. B) $f(x,y)<0$. C) $f(x,y)=0$. D) $f(x,y)\neq 0$.
E) $|f(x,y)|>0$.

29. Matematik fizikada II tartibli chiziqli tenglamaning qaysi turi aniqlanmagan?

A) sferik. B) elliptik. C) giperbolik. D) parabolik.

E) barcha keltirilgan turdagi tenglamalar aniqlangan.

30. Umumiy holdagi II tartibli chiziqli matematik fizika tenglamasi

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$$

qaysi shartda elliptik turga kiradi?

A) $B^2-4AC>0$. B) $B^2-4AC<0$. C) $B^2-4AC=0$.
D) $B^2-4AC\neq 0$. E) $|B^2-4AC|>0$.

31. Umumiy holdagi II tartibli chiziqli matematik fizika tenglamasi

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$$

qaysi shartda giperbolik turga kiradi?

A) $B^2-4AC>0$. B) $B^2-4AC<0$. C) $B^2-4AC=0$.
D) $B^2-4AC\neq 0$. E) to'g'ri javob keltirilmagan.

32. Umumiy holdagi II tartibli chiziqli matematik fizika tenglamasi

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+Fu=f(x,y)$$

qaysi shartda parabolik turga kiradi?

A) $B^2-4AC>0$. B) $B^2-4AC<0$. C) $B^2-4AC=0$.

D) $B^2-4AC\neq 0$. E) to'g'ri javob keltirilmagan.

2.Tor tebranishi tenglamasi va uning uchun chegaraviy masalani Furye usulida yechish

1. Quyidagilardan qaysi biri tor tebranishi tenglamasini ifodalaydi?

A) $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$ B) $(\frac{\partial u}{\partial t})^2=a^2\frac{\partial^2 u}{\partial x^2}$ C) $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$

D) $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial u}{\partial x}$ E) $\frac{\partial^2 u}{\partial t^2}=a^2(\frac{\partial u}{\partial x})^2$

2. Tor tebranishi tenglamasi $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$ matematik fizikaning qaysi turdagi tenglamasi bo'ladi?

A) elliptik. B) giperbolik. C) parabolik.

D) $a<1$ bo'lganda elliptik, $a=1$ bo'lganda parabolik va $a>1$ bo'lganda giperbolik.

E) $a<1$ bo'lganda giperbolik, $a=1$ bo'lganda parabolik va $a>1$ bo'lganda elliptik.

3. Uzunligi l bo'lgan va uchlari $x=0$, $x=l$ nuqtalarda mahkamlab qo'yilgan tor tebranishini ifodalovchi $u=u(x,t)$ funksiya uchun quyidagi tengliklardan qaysi biri chegaraviy shartni ifodalaydi?

A) $u(x,0)=f(x)$. B) $u(0,t)=0$. C) $\frac{\partial u(x,0)}{\partial t}=\varphi(x)$

D) keltirilgan barcha tengliklar chegaraviy shartni ifodalaydi.

E) keltirilgan birorta ham tenglik chegaraviy shartni ifodalamaydi.

4. Uzunligi l bo'lgan va uchlari $x=0$, $x=l$ nuqtalarda mahkamlab qo'yilgan tor tebranishini ifodalovchi $u=u(x,t)$

funksiya uchun quyidagi tengliklardan qaysi biri chegaraviy shartni ifodalaydi?

A) $u(l, t)=0$ B) $u(x,0)=f(x)$ C) $\frac{\partial u(x,0)}{\partial t}=\varphi(x)$

D)keltirilgan barcha tengliklar chegaraviy shartni ifodalaydi.

E)keltirilgan birorta ham tenglik chegaraviy shartni ifodalamaydi.

5. Uzunligi l bo'lgan va uchlari $x=0$, $x=l$ nuqtalarda mahkamlab qo'yilgan tor tebranishini ifodalovchi $u=u(x,t)$ funksiya uchun quyidagi tengliklardan qaysi biri boshlang'ich shartni ifodalaydi?

A) $u(l, t)=0$. B) $u(x,0)=f(x)$. C) $u(0, t)=0$.

D)keltirilgan barcha tengliklar boshlang'ich shartni ifodalaydi.

E)keltirilgan birorta ham tenglik boshlang'ich shartni ifodalamaydi.

6. Uzunligi l bo'lgan va uchlari $x=0$, $x=l$ nuqtalarda mahkamlab qo'yilgan tor tebranishini ifodalovchi $u=u(x,t)$ funksiya uchun quyidagi tengliklardan qaysi biri boshlang'ich shartni ifodalaydi?

A) $u(l, t)=0$. B) $u(0,t)=0$. C) $\frac{\partial u(x,0)}{\partial t}=\varphi(x)$.

D) keltirilgan barcha tengliklar boshlang'ich shartni ifodalaydi.

E) keltirilgan birorta ham tenglik boshlang'ich shartni ifodalamaydi.

7. Uzunligi l bo'lgan tor tebranishini ifodalovchi $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$ tenglamaning $u_n=u_n(x,t)$ xususiy yechimlari Furye usulida qanday ko'rinishda izlanadi?

A) $u_n(x,t)=X_n(x)+T_n(t)$. B) $u_n(x,t)=X_n(x)-T_n(t)$.

C) $u_n(x,t)=X_n(x)\cdot T_n(t)$. D) $u_n(x,t)=X_n(x)/T_n(t)$

E) $u_n(x,t)=X_n(x)\pm T_n(t)$.

8. Chegaraviy masalani yechishning Furye usuli boshqacha qanday nomlanadi?

A) o'zgaruvchilarni qo'shish.
B) o'zgaruvchilarni ayirish.
C) o'zgaruvchilarni bo'lish.
D) o'zgaruvchilarni ajratish.
E) o'zgaruvchilarni ko'paytirish.

9. Uzunligi l bo'lgan tor tebranishini ifodalovchi $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ tenglamaning $u_n(x,t)=X_n(x)\cdot T_n(t)$ ko'rinishdagi xususiy yechimi Furye usulida topilganda $X_n=X_n(x)$ funksiya qanday ko'rinishda bo'ladi?

A) $X_n(x) = A \cdot tg\frac{n\pi}{l}x$ B) $X_n(x) = A \cdot ctg\frac{n\pi}{l}x$

C) $X_n(x) = A \cdot \cos\frac{n\pi}{l}x$ D) $X_n(x) = A \cdot \sin\frac{n\pi}{l}x$

E) $X_n(x) = A \cdot \ln\frac{n\pi}{l}x$

10. Uzunligi l bo'lgan tor tebranishini ifodalovchi $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ tenglamaning $u(0, t)=0$, $u(l, t)=0$ chegaraviy shartlarni qanoatlantiruvchi $u_n(x,t)=X_n(x)\cdot T_n(t)$ ko'rinishdagi xususiy yechimi Furye usulida topilganda $T_n=T_n(t)$ funksiya qanday ko'rinishda bo'ladi?

A) $T_n(t) = B \cdot \cos\frac{an\pi}{l}t + C \cdot \sin\frac{an\pi}{l}t$

B) $T_n(t) = B\cos\frac{an\pi}{l}t \cdot C\sin\frac{an\pi}{l}t$

C) $T_n(t) = B \cdot tg\frac{an\pi}{l}t + C \cdot ctg\frac{an\pi}{l}t$

D) $T_n(t) = Btg\frac{an\pi}{l}t \cdot Cctg\frac{an\pi}{l}t$

E) $T_n(t) = B \cdot \sin\frac{an\pi}{l}t + C \cdot tg\frac{an\pi}{l}t$

11. Uzunligi l bo'lgan tor tebranishini ifodalovchi $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ tenglamaning $u(0, t)=0$, $u(l, t)=0$ chegaraviy shartlarni qanoatlantiruvchi $u=u(x,t)$ umumiy yechimi Furye usulida

qanday ko'rinishda izlanadi?

A) $u(x,t)=\sum_{n=1}^{\infty}[B_n tg\frac{an\pi}{l}t+C_n ctg\frac{an\pi}{l}t]\sin\frac{n\pi}{l}x$.

B) $u(x,t)=\sum_{n=1}^{\infty}[B_n tg\frac{an\pi}{l}t+C_n ctg\frac{an\pi}{l}t]\cos\frac{n\pi}{l}x$.

C) $u(x,t)=\sum_{n=1}^{\infty}[B_n\sin\frac{an\pi}{l}t+C_n\cos\frac{an\pi}{l}t]\sin\frac{n\pi}{l}x$.

D) $u(x,t)=\sum_{n=1}^{\infty}[B_n\sin\frac{an\pi}{l}t+C_n\cos\frac{an\pi}{l}t]\cos\frac{n\pi}{l}x$.

E) $u(x,t)=\sum_{n=1}^{\infty}[B_n\sin\frac{an\pi}{l}t+C_n\cos\frac{an\pi}{l}t]tg\frac{n\pi}{l}x$.

12. Uzunligi l bo'lgan tor tebranishini ifodalovchi $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$ tenglamaning $u(0, t)=0$, $u(l, t)=0$ chegaraviy shartlarni qanoatlantiruvchi

$$u(x,t)=\sum_{n=1}^{\infty}[B_n\sin\frac{an\pi}{l}t+C_n\cos\frac{an\pi}{l}t]\sin\frac{n\pi}{l}x$$

umumiy yechimidagi B_n koeffitsiyentlar Furye usulida $u(x,0)=f(x)$ boshlang'ich shart orqali qanday aniqlanadi?

A) $B_n=\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi}{l}xdx$.

B) $B_n=\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi}{l}xdx$.

C) $B_n=\frac{2}{l}\int_0^l f(x)tg\frac{n\pi}{l}xdx$.

D) $B_n=\frac{2}{l}\int_0^l f(x)ctg\frac{n\pi}{l}xdx$.

E) $B_n=\frac{2}{l}\int_0^l f(x)[\sin\frac{n\pi}{l}+\cos\frac{n\pi}{l}]xdx$.

13. Uzunligi l bo'lgan tor tebranishini ifodalovchi $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$ tenglamaning $u(0, t)=0$, $u(l, t)=0$ chegaraviy shartlarni qanoatlantiruvchi

$$u(x,t)=\sum_{n=1}^{\infty}[B_n\sin\frac{an\pi}{l}t+C_n\cos\frac{an\pi}{l}t]\sin\frac{n\pi}{l}x$$

umumiy yechimidagi C_n koeffitsiyentlar Furye usulida $\frac{\partial u(x,0)}{\partial t}=\varphi(x)$ boshlang'ich shart orqali qanday aniqlanadi?

A) $C_n=\frac{2}{an\pi}\int_0^l\varphi(x)tg\frac{n\pi}{l}xdx$.

B) $C_n=\frac{2}{an\pi}\int_0^l\varphi(x)ctg\frac{n\pi}{l}xdx$.

C) $C_n = \frac{2}{an\pi}\int_0^l \varphi(x)\cos\frac{n\pi}{l}xdx$. D) $C_n = \frac{2}{an\pi}\int_0^l \varphi(x)\sin\frac{n\pi}{l}xdx$.

E) $C_n = \frac{2}{an\pi}\int_0^l \varphi(x)[\sin\frac{n\pi}{l} + \cos\frac{n\pi}{l}]xdx$.

3. Tor tebranishi uchun Koshi masalasi va unu Dalamber usulida yechish

1. Quyidagilardan qaysi biri tor tebranishi tenglamasini ifodalaydi?

A) $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$ B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ C) $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$.

D) $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial u}{\partial x}$ E) $(\frac{\partial u}{\partial t})^2 = a^2\frac{\partial^2 u}{\partial x^2}$

2. Tor tebranishi uchun Koshi masalasida quyidagi shartlardan qaysi biri qaraladi?

A) $u(0,t)=h(t)$ B) $u(l,t)=\phi(t)$ C) $u'_t(x,0)=\psi(x)$.

D) $u'_x(0,t)=f(t)$. E) $u'_x(l,t)=g(t)$.

3. Tor tebranishi uchun Koshi masalasida quyidagi shartlardan qaysi biri qaraladi?

A) $u(x,0)=\phi(x)$ B) $u(l,t)=\psi(t)$ C) $u(0,t)=h(t)$

D) $u'_x(0,t)=f(t)$ E) $u'_x(l,t)=g(t)$.

4. Tor tebranishi uchun Koshi masalasida qanday shartlar qaraladi?

A) faqat chegaraviy shartlar

B) faqat boshlang'ich shartlar

C) ham chegaraviy, ham boshlang'ich shartlar

D) limitik shartlar

E) keltirilgan barcha hollar qaralishi mumkin

5. Tor tebranishi uchun Koshi masalasi nechta yechimga ega bo'ladi?

A) kamida bitta B) ko'pi bilan bitta C)cheksiz ko'p

D) faqat bitta E) barcha hollar bo'lishi mumkin .

6. Tor tebranishi uchun Koshi masalasini Dalamber usulida yechishda noma'lum $u=u(x,t)$ funksiya qanday ko'rinishda izlanadi?

A) $u(x,t)=f(ax+t)+g(ax-t)$. B) $u(x,t)=f(x+at)\cdot g(x-at)$.

C) $u(x,t)=f(x+at)/g(x-at)$. D) $u(x,t)=f(x+at)+g(x-at)$.

E) $u(x,t)=f(ax+t)\cdot g(ax-t)$.

7. Tor tebranishi uchun Koshi masalasini Dalamber usulida yechishda boshlang'ich shartlar $u(x,0)=\phi(x)$, $\frac{\partial u(x,0)}{\partial t}=\psi(x)$ ko'rinishda bo'lsa, $u(x,t)=f(x+at)+g(x-at)$ yechimdagi $f(x)$ va $g(x)$ funksiyalar qaysi tenglamalar sistemasidan topiladi?

A) $\begin{cases} f(x)+g(x)=\psi(x) \\ af'(x)-ag'(x)=\varphi(x) \end{cases}$ B) $\begin{cases} f(x)-g(x)=\varphi(x) \\ af'(x)+ag'(x)=\psi(x) \end{cases}$.

C) $\begin{cases} f(x)+g(x)=\varphi(x) \\ af'(x)-ag'(x)=\psi(x) \end{cases}$ D) $\begin{cases} f(x)-g(x)=\psi(x) \\ af'(x)+ag'(x)=\varphi(x) \end{cases}$.

E) $\begin{cases} f(x)+g(x)=\psi(x) \\ af'(x)+ag'(x)=\varphi(x) \end{cases}$

8. Tor tebranishi uchun Koshi masalasini Dalamber usulida yechishda boshlang'ich shartlar $u(x,0)=\phi(x)$, $\frac{\partial u(x,0)}{\partial t}=\psi(x)$ ko'rinishda bo'lsa, $u(x,t)=f(x+at)+g(x-at)$ yechimdagi $f(x)$ funksiya qaysi formula bilan aniqlanadi?

A) $f(x)=\frac{1}{2}[\psi(x)+\frac{1}{a}\int_{x_0}^{x}\varphi(s)ds+C]$ B) $f(x)=\frac{1}{2}[\psi(x)-\frac{1}{a}\int_{x_0}^{x}\varphi(s)ds+C]$.

C) $f(x)=\frac{1}{2}[\varphi(x)-\frac{1}{a}\int_{x_0}^{x}\psi(s)ds+C]$. D) $f(x)=\frac{1}{2}[\varphi(x)+\frac{1}{a}\int_{x_0}^{x}\psi(s)ds+C]$

E) $f(x)=-\frac{1}{2}[\psi(x)+\frac{1}{a}\int_{x_0}^{x}\varphi(s)ds+C]$.

9. Tor tebranishi uchun Koshi masalasini Dalamber usulida yechishda boshlang'ich shartlar $u(x,0)=\phi(x)$, $\frac{\partial u(x,0)}{\partial t}=\psi(x)$ ko'rinishda bo'lsa, $u(x,t)=f(x+at)+g(x-at)$ yechimdagi $g(x)$ funksiya qaysi formula bilan aniqlanadi?

A) $g(x)=\frac{1}{2}[\psi(x)+\frac{1}{a}\int_{x_0}^{x}\varphi(s)ds+C]$

B) $g(x)=\frac{1}{2}[\psi(x)-\frac{1}{a}\int_{x_0}^{x}\varphi(s)ds+C]$

C) $g(x)=\frac{1}{2}[\varphi(x)-\frac{1}{a}\int\limits_{x_0}^{x}\psi(s)ds+C]$

D) $g(x)=\frac{1}{2}[\varphi(x)+\frac{1}{a}\int\limits_{x_0}^{x}\psi(s)ds+C]$.

E) $g(x)=\frac{1}{2}[\varphi(x)-\frac{1}{a}\int\limits_{x_0}^{x}\psi(s)ds-C]$.

10. Tor tebranishi uchun Koshi masalasida boshlang'ich shartlar

$$u(x,0)=\phi(x) \quad , \qquad \frac{\partial u(x,0)}{\partial t}=\psi(x)$$

ko'rinishda bo'lsa, $u(x,t)$ yechim uchun Dalamber formulasini ko'rsating.

A) $u(x,t)=\frac{1}{2}[\varphi(x+at)+\varphi(x-at)]+\frac{1}{2a}\int\limits_{x-at}^{x+at}\psi(s)ds$.

B) $u(x,t)=\frac{1}{2}[\varphi(ax+t)+\varphi(ax-t)]+\frac{1}{2a}\int\limits_{ax-t}^{ax+t}\psi(s)ds$.

C) $u(x,t)=\frac{1}{2}[\varphi(x+at)-\varphi(x-at)]-\frac{1}{2a}\int\limits_{x-at}^{x+at}\psi(s)ds$.

D) $u(x,t)=\frac{1}{2}[\varphi(ax+t)-\varphi(ax-t)]-\frac{1}{2a}\int\limits_{ax-t}^{ax+t}\psi(s)ds$.

E) $u(x,t)=\frac{1}{2}[\psi(ax+t)+\psi(ax-t)]-\frac{1}{2a}\int\limits_{ax-t}^{ax+t}\varphi(s)ds$.

11. Tor tebranishi uchun $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $u(x,0)=x$, $u_t'(x,0)=1$

Koshi masalasining yechimini Dalamber formulasi yordamida toping.

A) $u(x,t)=x(1+t)+(x+1)t^2$. B) $u(x,t)=x(1+t)-(x+1)t^2$.

C) $u(x,t)=x(t-1)+(x-1)t^2$ D) $u(x,t)=x+t$. E) $u(x,t)=x+t+xt$

12. Tor tebranishi uchun $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$, $u(x,0)=x$, $u_t'(x,0)=2x$

Koshi masalasining yechimini Dalamber formulasi yordamida toping.

A) $u(x,t)=x(1+2t)+(x+1)t^2$

B) $u(x,t)=x(1+2t)-(x+1)t^2$

C) $u(x,t)=x(2t-1)+(x-1)t^2$

D) $u(x,t)=x+xt$

E) $u(x,t)=x+t+xt$.

4. Issiqlik tarrqalishi tenglamasi va uning uchun Koshi masalasini Furye usulida yechish

1. Quyidagilardan qaysi biri sterjenda issiqlik tarqalish tenglamasini ifodalaydi?

A) $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$ B) $(\frac{\partial u}{\partial t})^2=a^2\frac{\partial^2 u}{\partial x^2}$ C) $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$.

D) $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial u}{\partial x}$ E) $\frac{\partial^2 u}{\partial t^2}=a^2(\frac{\partial u}{\partial x})^2$

2. Sterjenda issiqlik tarqalishi uchun birinchi chegaraviy masalada quyidagi shartlardan qaysi biri talab etilmaydi?

A) $u(x,0)=\phi(x)$. B) $u(0,t)=\psi(t)$.

C) $u(l,t)=\omega(t)$. D) $u'_t(x,0)=\phi(x)$.

E) keltirilgan barcha shartlar talab etiladi.

3. Sterjenda issiqlik tarqalishi uchun birinchi chegaraviy masala nechta yechmga ega ?

A) kamida bitta. B) faqat bitta. C) cheksiz ko'p. D) chekli sonda.

E) sanoqli sonda.

4. Sterjenda issiqlik tarqalishi uchun Koshi masalasida qaysi shart qaraladi?

A) $x=0$ nuqta uchun $u(0,t)=\psi(t)$ chegaraviy shart.

B) $x=l$ nuqta uchun $u(l,t)=\omega(t)$chegaraviy shart.

C) $t=0$ uchun $u(x,0)=\phi(x)$ boshlang'ichch shart.

D) $t=0$ uchun $u'_t(x,0)=f(x)$boshlang'ichch shart.

E) keltirilgan barcha shartlar qaraladi.

5. Sterjenda issiqlik tarqalishi uchun Koshi masalasida quyidagi shartlardan qaysi biri qaraladi?

A) $u(x,0)=\phi(x)$ B) $u(l,t)=\psi(t)$ C) $u(0,t)=h(t)$

D) $u'_x(0,t)=f(t)$ E) $u'_x(l,t)=g(t)$

6. Sterjenda issiqlik tarqalishi tenglamasining xususiy yechimlari Furye usulida qanday ko'rinishda izlanadi?

A) $u(x,t)=X(x)+T(t)$ B) $u(x,t)=X(x)-T(t)$
C) $u(x,t)=X(x)\cdot T(t)$ D) $u(x,t)=X(x)/T(t)$
E) $u_n(x,t)=X(x)\pm T(t)$.

7. Sterjenda issiqlik tarqalishi tenglamasining $u(x,t)=X(x)\cdot T(t)$ ko'rinishdagi xususiy yechimi Furye usulida topilganda $X=X(x)$ funksiya qanday ko'rinishda bo'ladi?

A) $X(x)=A\cos(\lambda+x)+B\sin(\lambda+x)$.
B) $X(x)=A\cos(\lambda-x)+B\sin(\lambda-x)$.
C) $X(x)=A\cos(x/\lambda)+B\sin(x/\lambda)$.
D) $X(x)=A\cos\lambda x+B\sin\lambda x$.
E) $X(x)=A\cos(\lambda/x)+B\sin(\lambda/x)$.

8. Sterjenda issiqlik tarqalishi $u'_t=a^2u''_{xx}$ tenglamasining $u(x,t)=X(x)\cdot T(t)$ ko'rinishdagi xususiy yechimi Furye usulida topilganda $T=T(t)$ funksiya qanday ko'rinishda bo'ladi?

A) $T(t)=Ce^{a^2\lambda^2t}$ B) $T(t)=Ce^{-a^2\lambda^2t}$ C) $T(t)=C\cos a^2\lambda^2t$.
D) $T(t)=C\sin a^2\lambda^2t$. E) $T(t)=C\cos a^2\lambda^2t+D\sin a^2\lambda^2t$.

9. Sterjenda issiqlik tarqalishi $u'_t=a^2u''_{xx}$ tenglamasining Furye usulida topilgan $u(x,t)=X(x)\cdot T(t)$ ko'rinishdagi xususiy yechimi qanday ko'rinishda bo'ladi?

A) $u(x,t)=e^{-a^2\lambda^2t}[A(\lambda)\cos(\lambda+x)+B(\lambda)\sin(\lambda+x)]$.
B) $u(x,t)=e^{-a^2\lambda^2t}[A(\lambda)\cos(\lambda-x)+B(\lambda)\sin(\lambda-x)]$.
C) $u(x,t)=e^{-a^2\lambda^2t}[A(\lambda)\cos\lambda x+B(\lambda)\sin\lambda x]$.
D) $u(x,t)=e^{-a^2\lambda^2t}[A(\lambda)\cos(x/\lambda)+B(\lambda)\sin(x/\lambda)]$.
E) $u(x,t)=e^{-a^2\lambda^2t}[A(\lambda)\cos(\lambda/x)+B(\lambda)\sin(\lambda/x)]$.

10. Sterjenda issiqlik tarqalishi $u'_t=a^2u''_{xx}$ tenglamasining Furye usulida topilgan yechimi qanday ko'rinishda bo'ladi?

A) $u(x,t)=\int_0^\infty e^{-a^2\lambda^2t}[A(\lambda)\cos(\lambda+x)+B(\lambda)\sin(\lambda+x)]d\lambda$.

B) $u(x,t)=\int_0^\infty e^{-a^2\lambda^2t}[A(\lambda)\cos(\lambda-x)+B(\lambda)\sin(\lambda-x)]d\lambda$.

C) $u(x,t)=\int_0^\infty e^{-a^2\lambda^2t}[A(\lambda)\cos\lambda x+B(\lambda)\sin\lambda x]d\lambda$.

D) $u(x,t)=\int_0^{\infty} e^{-a^2\lambda^2 t}[A(\lambda)\cos(x/\lambda)+B(\lambda)\sin(x/\lambda)]d\lambda$.

E) $u(x,t)=\int_0^{\infty} e^{-a^2\lambda^2 t}[A(\lambda)\cos(\lambda/x)+B(\lambda)\sin(\lambda/x)]d\lambda$.

11.Sterjenda issiqlik tarqalishi uchun Koshi masalasining Furye usulida topilgan

$$u(x,t)=\int_0^{\infty} e^{-a^2\lambda^2 t}[A(\lambda)\cos\lambda x+B(\lambda)\sin\lambda x]d\lambda$$

yechimidagi $A(\lambda)$ funksiyalar $u(x,0)=\phi(x)$ boshlang'ich shart orqali qanday aniqlanadi?

A) $A(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)\sin\lambda\alpha\,d\alpha$. B) $A(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)[\sin\lambda\alpha+\cos\lambda\alpha]d\alpha$.

C) $A(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)\cos\lambda\alpha\,d\alpha$. D) $A(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)[\sin\lambda\alpha-\cos\lambda\alpha]d\alpha$.

E) $A(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)\sin\lambda\alpha\cos\lambda\alpha\,d\alpha$.

12.Sterjenda issiqlik tarqalishi uchun Koshi masalasining Furye usulida topilgan

$$u(x,t)=\int_0^{\infty} e^{-a^2\lambda^2 t}[A(\lambda)\cos\lambda x+B(\lambda)\sin\lambda x]d\lambda$$

yechimidagi $B(\lambda)$ funksiyalar $u(x,0)=\phi(x)$ boshlang'ich shart orqali qanday aniqlanadi?

A) $B(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)\sin\lambda\alpha\,d\alpha$. B) $B(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)[\sin\lambda\alpha+\cos\lambda\alpha]d\alpha$.

C) $B(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)\cos\lambda\alpha\,d\alpha$. D) $B(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)[\sin\lambda\alpha-\cos\lambda\alpha]d\alpha$.

E) $B(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(\alpha)\sin\lambda\alpha\cos\lambda\alpha\,d\alpha$.

13.Sterjenda issiqlik tarqalishi uchun $u_t'=a^2u_{xx}''$, $u(x,0)=\phi(x)$ Koshi masalasining Furye usulida topilgan yechimi qayerda to'g'ri ifodalangan?

A) $u(x,t)=\frac{1}{\pi}\int_0^{+\infty} e^{-a^2\lambda^2 t}[\int_{-\infty}^{+\infty}\varphi(\alpha)\sin(\alpha-x)\lambda d\alpha]d\lambda$.

B) $u(x,t)=\frac{1}{\pi}\int_0^{+\infty} e^{-a^2\lambda^2 t}[\int_{-\infty}^{+\infty}\varphi(\alpha)\sin(\alpha+x)\lambda d\alpha]d\lambda$.

C) $u(x,t)=\frac{1}{\pi}\int\limits_{0}^{+\infty} e^{-a^2\lambda^2 t}[\int\limits_{-\infty}^{+\infty}\varphi(\alpha)\cos(\alpha-x)\lambda d\alpha]d\lambda$.

D) $u(x,t)=\frac{1}{\pi}\int\limits_{0}^{+\infty} e^{-a^2\lambda^2 t}[\int\limits_{-\infty}^{+\infty}\varphi(\alpha)\cos(\alpha+x)\lambda d\alpha]d\lambda$.

E) $u(x,t)=\frac{1}{\pi}\int\limits_{0}^{+\infty} e^{-a^2\lambda^2 t}\{\int\limits_{-\infty}^{+\infty}\varphi(\alpha)[\cos(\alpha-x)\lambda+\sin(\alpha-x)\lambda]d\alpha\}d\lambda$.

14. Sterjenda issiqlik tarqalishi uchun Koshi masalasining Furye usulida topilgan yechimi qaysi integrali orqali ifodalanadi?

A) Koshi integrali B) Furye integrali

C) Puasson integrali D) Dirixle integrali

E) Laplas integrali.

15. Sterjenda issiqlik tarqalishi uchun $u'_t=a^2u''_{xx}$, $u(x,0)=\phi(x)$ Koshi masalasining Furye usulida topilgan yechimi Puasson integrali orqali qanday ifodalalanadi?

A)

B) $u(x,t)=\frac{1}{2a\sqrt{\pi t}}\int\limits_{-\infty}^{+\infty}[\varphi(\alpha)+e^{-\frac{(\alpha-x)^2}{4a^2t}}]d\alpha$.

C) $u(x,t)=\frac{1}{2a\sqrt{\pi t}}\int\limits_{-\infty}^{+\infty}[\varphi(\alpha)-e^{-\frac{(\alpha-x)^2}{4a^2t}}]d\alpha$.

D) $u(x,t)=\frac{1}{2a\sqrt{\pi t}}\int\limits_{-\infty}^{+\infty}[\varphi(\alpha)\pm e^{-\frac{(\alpha-x)^2}{4a^2t}}]d\alpha$.

E) $u(x,t)=\frac{1}{2a\sqrt{\pi t}}\int\limits_{-\infty}^{+\infty}\varphi(\alpha)e^{\frac{(\alpha-x)^2}{4a^2t}}d\alpha$.

16. Sterjenda issiqlik tarqalishi uchun $u'_t=a^2u''_{xx}$, $u(x,0)=\phi(x)$ Koshi masalasining Furye usulida topilgan yechimi Puasson integrali orqali

$$u(x,t)=\frac{1}{2a\sqrt{\pi t}}\int\limits_{-\infty}^{+\infty}\varphi(\alpha)e^{-Z(\alpha,x,t)}d\alpha$$

ko'rinishda ifodalanadi. Bunda $Z(\alpha,x,t)$ qaysi funksiyadan iborat bo'ladi?

A) $\frac{\alpha^2+x^2}{4a^2t}$. B) $\frac{\alpha^2-x^2}{4a^2t}$. C) $\frac{(\alpha+x)^2}{4a^2t}$. D) $\frac{(\alpha-x)^2}{4a^2t}$.

E) to'g'ri javob keltirilmagan.

5. Laplas tenglamasi va uning uchun chegaraviy masalalar

1. Ikki o'zgaruvchili noma'lum $u=u(x,y)$ funksiya uchun Laplas tenglamasi qayerda to'g'ri ifodalangan?

A) $(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2=0$. B) $(\frac{\partial u}{\partial x})^2-(\frac{\partial u}{\partial y})^2=0$. C) $\frac{\partial^2 u}{\partial^2 x}+\frac{\partial^2 u}{\partial^2 y}=0$

D) $\frac{\partial^2 u}{\partial^2 x}-\frac{\partial^2 u}{\partial^2 y}=0$. E) $(\frac{\partial u}{\partial x}\pm\frac{\partial u}{\partial y})^2=0$.

2. Tekislikda Laplas operatori Δ qanday ko'rinishda bo'ladi?

A) $\Delta=(\frac{\partial}{\partial x})^2+(\frac{\partial}{\partial y})^2$ B) $\Delta=(\frac{\partial}{\partial x})^2-(\frac{\partial}{\partial y})^2$. C) $\Delta=\frac{\partial^2}{\partial^2 x}+\frac{\partial^2}{\partial^2 y}$

D) $\Delta=\frac{\partial^2}{\partial^2 x}-\frac{\partial^2}{\partial^2 y}$. E) $\Delta=(\frac{\partial}{\partial x}\pm\frac{\partial}{\partial y})^2$.

3. Laplas operatori Δ orqali $u=u(x,y)$ funksiya uchun Laplas tenglamasi qanday yoziladi?

A) $\Delta u=0$. B) $\Delta^2 u=0$. C) $\Delta u^2=0$.

D) $(\Delta u)^2=0$. E) $\Delta^2 u+\Delta u^2=0$.

4. $\Delta u=0$ Laplas tenglamasi uchun Dirixle masalasida quyidagi shartlardan qaysi biri talab etilmaydi?

A) *D* biror *C* kontur bilan chegaralangan yopiq soha.

B) $u=u(x,y)$ funksiya *D* sohaning ichki nuqtalarida Laplas tenglamasini qanoatlantiradi.

C) $u=u(x,y)$ funksiya *C* konturdagi barcha $M=M(x,y)$ nuqtalarda $u(M)=f(M)$ chegaraviy shartni qanoatlantiradi.

D) $u|_{M\in C}=f(M)$ chegaraviy shartda berilgan $f(M)$ funksiya *C* konturda uzluksiz. E) keltirilgan barcha shartlar talab etiladi .

5. Dirixle masalasi nechta yechimga ega?

A) cheksiz ko'p B) chekli sonda

C) sanoqli sonda D) kamida bitta E) bitta

6. *Tasdiqni to'ldiring:* Laplas tenglamasi uchun Dirixle masalasi doirada qaralganda x va y Dekart koordinatalaridan ...... koordinatalarga o'tiladi.

A) silindrik B) qutb C) sferik

D)egri chiziqli. E) fazoviy.

7. x va y Dekart koordinatalaridan r va ϕ qutb koordinatalarga o'tishda qutb radiusi r qaysi formula bilan aniqlanadi?

A) $r=\sqrt{|x+y|}$. B) $r=\sqrt{|x-y|}$. C) $r=\sqrt{x^2+y^2}$.

D) $r=\sqrt{|x^2-y^2|}$. E) $r=\sqrt{x^4+y^4}$.

8. x va y Dekart koordinatalaridan r va ϕ qutb koordinatalarga o'tishda qutb burchagi ϕ uchun quyidagi formulalardan qaysi biri noto'g'ri?

A) $\varphi=arctg\frac{y}{x}\,(x>0)$. B) $\varphi=\pi-arctg\frac{y}{x}\,(x<0)$.

C) $\varphi=\frac{\pi}{2}\,(x=0,y>0)$. D) $\varphi=-\frac{\pi}{2}\,(x=0,y<0)$.

E) keltirilgan barcha formulalar to'g'ri.

9. x va y Dekart koordinatalaridan r va ϕ qutb koordinatalarga o'tishda qutb burchagi ϕ uchun quyidagi formulalardan qaysi biri noto'g'ri?

A) $\varphi=arctg\frac{y}{x}\,(x>0)$. B) $\varphi=\pi+arctg\frac{y}{x}\,(x<0)$.

C) $\varphi=\frac{\pi}{2}\,(x=0,y>0)$. D) $\varphi=-\frac{\pi}{2}\,(x=0,y<0)$.

E) keltirilgan barcha formulalar to'g'ri.

10. Qutb koordinatalarida berilgan $v=v(r,\phi)$ funksiya uchun Laplas tenglamasi qanday ifodalanadi?

A) $\frac{\partial^2 v}{\partial^2 r}+\frac{\partial^2 v}{\partial^2 \varphi}=0$. B) $\frac{\partial^2 v}{\partial^2 r}-\frac{\partial^2 v}{\partial^2 \varphi}=0$.

C) $\frac{\partial^2 v}{\partial^2 r}+\frac{1}{r}\cdot\frac{\partial^2 v}{\partial r\partial\varphi}+\frac{\partial^2 v}{\partial^2 \varphi}=0$. D) $\frac{\partial^2 v}{\partial^2 r}+\frac{\partial^2 v}{\partial r\partial\varphi}+\frac{1}{r^2}\frac{\partial^2 v}{\partial^2 \varphi}=0$.

E) $\frac{\partial^2 v}{\partial^2 r}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v}{\partial^2 \varphi}=0$.

11.Qutb koordinatalarida ifodalangan Laplas tenglamasining yechimi $v=v(r,\phi)$ Furye usulida qanday ko'rinishda izlanadi?

A) $v(r,\phi)=P(r)+\Phi(\phi)$ B) $v(r,\phi)=P(r)-\Phi(\phi)$

C) $v(r,\phi)=P(r)\cdot\Phi(\phi)$ D) $v(r,\phi)=P(r)/\Phi(\phi)$

E) $v(r,\phi)=\Phi(\phi)/P(r)$

12.Qutb koordinatalarida ifodalangan Laplas tenglamasining yechimi Furye usulida $v(r,\phi)=P(r)\cdot\Phi(\phi)$ ko'rinishda izlanganda $P(r)$ funksiya qaysi differensial tenglamadan topiladi?

A) $P''(r)+kP'(r)-r^2P(r)=0, k=0,1,2,\cdots$.

B) $r^2P''(r)-P'(r)+kP(r)=0, k=0,1,2,\cdots$.

C) $r^2P''(r)+rP'(r)-k^2P(r)=0, k=0,1,2,\cdots$

D) $P''(r)+k^2P'(r)-r^2P(r)=0, k=0,1,2,\cdots$.

E) $P''(r)+r^2P'(r)-k^2P(r)=0, k=0,1,2,\cdots$.

13. Qutb koordinatalarida ifodalangan Laplas tenglamasining yechimi Furye usulida $v(r,\phi)=P(r)\cdot\Phi(\phi)$ ko'rinishda izlanganda $\Phi(\phi)$ funksiya qaysi differensial tenglamadan topiladi?

A) $k^2\Phi''(\varphi)+\Phi(\varphi)=0, k=0,1,2,\cdots$.

B) $\Phi''(\varphi)+k^2\Phi(\varphi)=0, k=0,1,2,\cdots$.

C) $\Phi''(\varphi)-k^2\Phi'(\varphi)=0, k=0,1,2,\cdots$.

D) $\Phi''(\varphi)+k^2\Phi'(\varphi)=0, k=0,1,2,\cdots$.

E) $\Phi''(\varphi)+r^2\Phi'(\varphi)-k\Phi(\varphi)=0, k=0,1,2,\cdots$.

14. Qutb koordinatalarida ifodalangan Laplas tenglamasining yechimi Furye usulida $v(r,\phi)=P(r)\cdot\Phi(\phi)$ ko'rinishda izlanganda $\Phi(\phi)$ funksiya $\Phi''(\varphi)+k^2\Phi(\varphi)=0, k=0,1,2,\cdots$, differensial tenglamaning yechimi kabi aniqlanadi. $k\neq0$ holda $\Phi(\phi)$ funksiya ko'rinishi qayerda to'g'ri ko'rsatilgan?

A) $\Phi(\varphi)=A_k\cos^k\varphi+B_k\sin^k\varphi$

B) $\Phi(\varphi)=A_k\cos\varphi^k+B_k\sin\varphi^k$

C) $\Phi(\varphi) = A_k \cos(\varphi / k) + B_k \sin(\varphi / k)$

D) $\Phi(\varphi) = A_k \cos k\varphi + B_k \sin k\varphi$.

E) $\Phi(\varphi) = A_k \cos(k \pm \varphi) + B_k \sin(k \pm \varphi)$.

15. Qutb koordinatalarida ifodalangan Laplas tenglamasining yechimi Furye usulida $v(r,\phi) = P(r)\cdot\Phi(\phi)$ ko'rinishda izlanganda $P(r)$ funksiya

$$r^2 P''(r) + rP'(r) - k^2 P(r) = 0, k = 0,1,2,\cdots$$

differensial tenglamaning yechimi kabi aniqlanadi. $k \neq 0$ holda $P(r)$ funksiya ko'rinishi qayerda to'g'ri ko'rsatilgan?

A) $P(r) = C_k (r^k + r^{-k})$.

B) $P(r) = C_k k^r + D_k k^{-r}$.

C) $P(r) = C_k r^{2k} + D_k r^{-2k}$.

D) $P(r) = C_k k^{2r} + D_k k^{-2r}$.

* E) $P(r) = C_k r^k + D_k r^{-k}$.

16. Qutb koordinatalarida ifodalangan Laplas tenglamasining Furye usulida topilgan $v_k(r,\phi)$ (k=0,1,2,...) xususiy yechimlari qanday ko'rinishda bo'ladi?

A) $v_k = (A_k \cos^k \varphi + B_k \sin^k \varphi) r^k \ (k = 1,2,\cdots)$, $v_0 = A_0 / 2$.

B) $v_k = (A_k \cos\varphi^k + B_k \sin\varphi^k) r^k \ (k = 1,2,\cdots)$, $v_0 = A_0 / 2$.

C) $v_k = (A_k \cos k\varphi + B_k \sin k\varphi) r^k \ (k = 1,2,\cdots)$, $v_0 = A_0 / 2$.

D) $v_k = [A_k \cos(\varphi / k) + B_k \sin(\varphi / k)] r^k \ (k = 1,2,\cdots)$, $v_0 = A_0 / 2$.

E) $v_k = [A_k \cos(\varphi \pm k) + B_k \sin(\varphi \pm k)] r^k \ (k = 1,2,\cdots)$, $v_0 = A_0 / 2$.

Kim matematikani bilmasa, haqiqatni bilmaydi.
Kim uni tushunmasa zulmatta yashaydi.
Rene Dekart

XVII-BOB. EHTIMOLLAR NAZARIYASI ASOSLARI

§ 17.1. Ehtimolning ta'riflari

Ehtimol termini hodisaning amalga oshish, ro'y berish imkoniyatining obyektiv o'lchovini ifodalaydi.

Biror tajriba natijasida sondagi $e_1, e_2, ..., e_n$ elementar hodisalardan birortasi ro'y berishi mumkin bo'lsin, ya'ni $U = \{e_1, e_2, ..., e_n\}$ bo'lsin. Bu elementar hodisalarga quyidagi shartlarni qo'yamiz:

1) hodisalar juft-jufti bilan birgalikda emas, boshqacha qilib aytganda, istalgan ikkita e_i va e_j $(i \neq j)$ hodisa uchun ulardan biri ro'y bersa, ikkinchisi albatta ro'y bermaydi.

2) $e_1, e_2, ..., e_n$ hodisalar yagona mumkin bo'lgan hodisalar, ya'ni ularning birortasi albatta ro'y berishi lozim.

3) $e_1, e_2, ..., e_n$ hodisalar teng imkoniyatli. Bu shart $e_1, e_2, ..., e_n$ hodisalardan birortasining boshqalaridan ko'proq ro'y berishiga yordam beradigan hech qanday obyektiv sabablar yo'qligini anglatadi.

Aytaylik, A hodisa berilgan bo'lib, u $e_i(i=\overline{1,n})$ elementar hodisalardan ba'zilari ro'y bergandagina ro'y bersin. Bunday holda biz $e_i(i=\overline{1,n})$ elementar hodisalardan ro'y berishi A hodisaning ro'y berishiga ham olib keladiganlarini A hodisa qulaylik tug'diradigan hodisalar deb ataymiz.

Aytaylik, qaralayotgan n ta $e_1, e_2, ..., e_n$ elementar hodisadan m tasi A hodisaning ro'y berishiga qulaylik tugdirsin, ya'ni $A=(e_{k_1}, e_{k_2}, ..., e_{k_m})$ bo'lsin.

1. Ehtimolning klassik ta'rifi. A hodisaning ehtimoli deb A hodisaning ro'y berishiga qulaylik tug'diruvchi hodisalar sonining teng imkoniyatli barcha elementar hodisalar soniga nisbatiga aytiladi va quyidagicha belgilanadi:

$$P(A)=\frac{m}{n}=\frac{A \quad ga\ kirgan\ elementar\ hodisalar\ soni}{barcha\ elementar\ hodisalar\ soni}$$

Ehtimolning xossalari.

a) muqarrar hodisaning ehtimoli birga teng:

$$P(U)=1.$$

b) mumkin bo'lmagan hodisaning ehtimoli nolga teng:

$$P(V)=0,$$

c) istalgan A hodisaning ehtimoli quyidagi qo'sh tengsizlikni qanoatlantiradi:

$$0 \le P(A) \le 1.$$

Klassik ta'rifdan foydalanib masalalar yechishda kombinatorika elementlari muhim rol o'ynaydi, shuni e'tiborga olib kombinatorikaga doir ba'zi tushunchalar bilan tanishib o'taylik.

Har qanday narsalardan tuzilgan va bir – biridan yo shu narsalarning tartibi bilan, yoki shi narsalarning o'zlari bilan farq qiluvchi turli gruppalar birlashmalar deb ataladi.

Birlashmalar uch xil bo'lishi mumkin: o'rinlashtirish, o'rin almashtirish va gruppalash. Ularning har birini ko'rib chiqaylik.

1) n elementini m tadan (bunda $m \le n$) o'rinlashtirish deb shunday birlashmalarga aytiladiki, ularning har birida berilgan n

ta elementdan olingan m ta element bo'lib, ular bir – biridan yo elementlari bilan, yoki elementlarning tartibi bilan farq qiladi. n ta elementdan m tadan barcha o'rinlashtirishlar soni A_n^m kabi belgilanib, u

$$A_n^m = n(n-1)...[n-(m-1)]$$

formula bilan hisoblanadi.

2) Agar o'rinlashtirishlar n ta elementdan n tadan olingan bo'lsa (ya'ni faqat elementlarining tartibi bilan farq qilsa), bunday o'rinlashtirishlar o'rin almashtirishlar deb ataladi, u holda yuqoridagi formulaga ko'ra n ta elementdan barcha o'rin almashtirishlar soni quyidagicha bo'ladi:

$$P_n = n(n-1)...1 = n!$$

3) Agar n ta elementdan m tadan tuzish mumkin bo'lgan barcha o'rinlashtirishlardan bir – biridan eng kamida bir element bilan farq qiladiganlarini tanlab olsak, u holda gruppalar (kombinatsiyalar) deb atalgan birlashmalarni hosil qilamiz.

n ta elementdan m tadan barcha gruppalashlar (kombinatsiyalar) soni

$$C_n^m = \frac{n!}{m!(n-m)!} = \frac{n(n-1)...[n-(m-1)]}{m!}$$

formula yordamida topiladi.

Yuqoridagi 3 ta formula uchun $A_n^m = C_n^m \cdot P_n$ munosabat o'rinlidir.

2. Ehtimolning geometrik ta'rifi. Biror Q soha berilgan bo'lib, bu soha Q_1 sohani o'z ichiga olsin, Q sohaga tavakkaliga tashlangan nuqtaning Q_1 sohaga tushish ehtimolini topish talab qilinadi. B yerda barcha elementlar hodisalar to'plami Q ning barcha nuqtalaridan iborat. Binobarin, bu holda klassik ta'rifdan foydalana olmaymiz. Tanlangan nuqta Q ga albatta tushsin va uning biror Q_1 qismiga tushish ehtimoli shu Q_1 qismining o'lchoviga (uzunligiga, yuziga, hajmiga) proparsional bo'lib, Q_1 ning formasiga va Q_1 qism Q ning qayerda joylashganligiga bog'liq bo'lmasin. Bu shartlarda qaralayotgan hodisaning ehtimoli

$$P = \frac{mesQ_1}{mesQ}$$

formula yordamida aniqlanadi. Bu formula yordamida aniqlangan p funksiya ehtimolning barcha xossalarini qanoatlantirishini ko'rish qiyin emas.

3. Ehtimolning statistik ta'rifi. Ehtimollar nazariyasining ko'pgina tatbiqlarida ehtimolning statik ta'rifi deb ataluvchi ta'rifdan foydalaniladi. Har birida biror hodisaning ro'y berishi yoki ro'y bermasligi kuzatiladigan tajribalarni sharoitni o'zgartirmagan holda cheksiz ko'p marta tajrorlash mumkin bo'lsin deb faraz qilaylik. Masalan, o'yin soqqasini yoki tangani tahslash, nishonga o'q uzish va shunga o'xshash tajribalarni cheksiz ko'p marta takrorlash mumkin.

Aytaylik, tajribalar soni n yetatlicha katta bo'lganda bizni qiziqtirayotgan *A* hodisa *m* marta ro'y bergan bo'lsin.

$W(A)=\frac{m}{n}$ nisbat *A* hodisaning nisbiy chastotasi deb ataladi.

Ko'p kuzatishlar shuni ko'rsatadiki, agar bir xil shart-sharoitda tajribalar o'tkazilib, ularning har birida sinovlar soni yetarlicha katta bo'lsa, u holda nisbiy chastota turg'unlik xossasiga ega bo'ladi.

Quyidagi masalalarni yeching.

17.1.Gruppada 10 ta fan o'qitiladi. Agar har kuni 4 xil dars o'tilsa, bir kunlik darsni necha xil usul bilan taqsimlash mumkin?

17.2. 8 ta stulga 8 kishini necha xil usul bilan o'tkazish mumkin?

17.3. $C_n^m = C_n^{n-m}$ tenglik o'rinli ekanini isbotlang.

17.4. Ikkita tanga bir vaqtda tashlangan $m\,(m=0,1,2)$ marta gerbli tomon tushish ehtimolini toping.

17.5. Yoqlariga 1, 2, 3, 4, 5, 6 raqamlar yozilgan ikkita soqqa bir vaqtda tashlanadi. Ikkala soqqada tushgan ochkolar yig'indisi 8 ga teng bo'lish ehtimolini toping.

17.6. Ikkita soqqa tashlangan. Tushgan ochkolar yig'indisi beshga, ko'paytmasi to'rtga teng bo'lish ehtimolini toping.

17.7. Tanga ikki marta tashlangan. Hech bo'lmaganda bir marta "gerbli" tomon tushishi ehtimolini toping.

17.8. Yashikda 15 ta detal bo'lib, ulardan 10 tasi bo'yalgan. Yig'uvchi tavakkaliga 3 ta detal oladi. Olingan detallarning bo'yalgan bo'lishi ehtimolini toping.

17.9. Abonent telefon nomerini terayotib nomerning oxirgi uchta raqamini eslay olmadi va bu raqamlarni turli ekanligini bilgani holda ularni tavakkaliga terdi. Kerakli raqamlar terilganligi ehtimolini toping.

17.10.Sexda 6 erkak va 4 ayol ishlaydi. Table nomerlari bo'yicha tavakkaliga 7 kishi ajratilgan. Ajratilganlar orasida 9 ayol bo'lishi ehtimolini toping.

17.11. Radiusi R bo'lgan doiraga radiusi r bo'lgan kichik doira joylashtirilgan. Katta doiraga tasodifan tashlangan nuqtaning kichik doiraga tushish ehtimolini toping. Nuqtaning doiraga tushish ehtimoli doira yuziga proporsional bo'lib, uning joylashishiga bog'liq emas deb faraz qilinadi.

17.12. tekislik bir – biridan $2a$ masofada joylashgan to'gri chiziqlar bilan bo'lingan. Tekislikka radiusi $r<a$ bo'lgan tanga tavakkaliga tashlangan. Tanga to'g'ri chiziqlarning birortasini ham kesmasligi ehtimolini toping.

17.13. Ikki do'st kunduzgi soat 12 bilan 13 orasida tayin bir joyda uchrashishga va oldin kelgan kishi do'stini 1/4 soat kutib, kelmagandan kelmagandan keyin ketishga kelishib olishdi. Agar har bir kishi o'zining kelish momentini tavakkaliga (soat 12 bilan 13 orasida) tanlasa, ularning uchrashish ehtimolini toping.

17.14.Ox o'qining uzunligi L bo'lgan OA kesmasiga ikkita

$B(x)$ va $C(y)$ nuqta tavakkaliga qo'yilgan. Hosil bo'lgan uchta

$B(x)$ kesmadan uchburchak yasash mumkin bo'lishi ehtimolini toping.

17.15. Radiusi R bo'lgan doira ichiga tavakkaliga nuqta tahslangan. Tahslangan nuqta doiraga ichki chizilgan: a) kvadrat ichiga; b) muntazam uchburchak ichiga tushish ehtimolini toping. Nuqtaning doira bo'lagiga tushish ehtimoli bu bo'lakning yuziga proporsional bo'lib, uning doiraa nisbatan joylashishiga bog'liq emas deb faraz qilinadi.

§ 17. 2. Ehtimollarni qo'shish va ko'paytirish teoremalari. Shartli ehtimollar. Hodisalarning bog'liqsizligi

Berilgan hodisaga qulaylik tug'diruvchi hollarni bevosita hisoblash ancha bo'lishi mumkin. Shuning uchun hodisaning ehtimolini hisoblashda uni boshqa soddaroq hodisalar kombinatsiyasi ko'rinishida ifodalash qulayroqdir. Biroq bunda boshqa hodisalarning kombinatsiyasi ko'rinishida ifodalashda hodisaning ehtimoli bo'ysunadigan qoidalarni bilish kerak. quyida ular bilan tanishib o'tamiz.

1. Birgalikda bo'lmagan hodisalar ehtimollarini qo'shish teoremasi. Ikkita birgalikda bo'lmagan A va B hodisadan istalgan birirning ro'y berish ehtimoli bu hodisalar ehtimolarining yig'indisiga teng:

$$P(A+B)=P(A)+P(B).$$

Umuman har ikkitasi birgalikda bo'lmagan bir nechta $A_1, A_2, ..., A_n$ hodisalardan istalgan birining ro'y berish ehtimoli bu ehtimollarining yig'indisiga teng:

$$P(A_1+A_2+...+A_n)=P(A_1)+P(A_2)+...+P(A_n).$$

Natija. Agar $A_1, A_2, ..., A_n$ hodisalardan faqat bittasi albatta ro'y beradigan va ular birgalikda bo'lmagan hodisalar bo'lsa, u holda

$$P(A_1)+P(A_2)+...+P(A_n)=1.$$

Xususiy holda, agar A va $\bar{A}$ hodisalar o'zaro qarama – qarshi hodisalarni ifodalasa, u holda

$$P(A)+P(\bar{A})=1.$$

Bu teoremaga keyinroq misol ko'armiz.

2. Shartli ehtimollar. Hodisalarning bog'liqsizligi.

Hodisalarning ehtimolini aniqlash asosida biror S shartlar komleksi yotishini aytgan edik. Agar $P(A)$ ehtimolni hisoblash S shartlar kompleksidan boshqa hech qanday shartlar talab qilinmasa bunday ehtimol, shartsiz ehtimol deyiladi. Ko'p hollarda A hodisaning ehtimolini biror B hodisa ($P(B)>0$ deb faraz qilinadi) ro'y bergan shartda hisoblashga to'g'ri keladi. Bunday ehtimol shartli ehtimol deyiladi va $P(A/B)$ kabi belgilanadi. Agar ikkita A va B hodisadan birning ehtimoli ikkinchisining ro'y berishi yoki ro'y bermasligi natijasida o'zgarmasa, u holda bu hodisalar o'zaro bog'liqsiz hodisalar deyiladi, aksholda bu hodisalar o'zaro bog'liq hodisalar deyiladi.

Masalan, oq va qora sharlar solingan yashikdan olingan birinchi shar unga qayta solinsa, ikkinchi martaolingan sharning oq bo'lish ehtimoli birirnchi olingan sharning oq yoki qora bo'lishiga bog'liq emas. Shuning uchun birnchi va ikkinchi shar olish natijalari o'zaro bog'liqsiz bo'ladi.

Aksincha, agar birinchi olingan shar yashikka qayta solimasa, u holda ikkinchi marta shar olinishidagi natija birinchi marta shar olish natijasiga bog'liq ravishda o'zgaradi, chunki birinchi marta shar olinishi natijasida yashikdagi sharlarning sostavi o'zgaradi. Bu yerda biz bog'liq hodisalar misoliga egamiz.

Shartli ehtimollar uchun qabul qilingan belgilashlardan foydalanib, A va B hodisalarning o'zaro bog'liqsiz bo'lishi shartini

$$P(A/B)=P(A) \text{yoki} \quad P(A/B)=P(B)$$

ko'rinishda yozish mumkin.

3. Hodisalar ehtimollarini ko'paytirish teoremasi. Ikkita bog'liq hodisaning birgalikda ro'y berish ehtimoli ulardan birinchisining ehtimolini ikkinchisining birinchisi ro'y bergan shart ostidagi shrtli ehtimoliga ko'paytirilganiga teng va aksincha, ya'ni

$$P(A\cdot B)=P(A)\cdot P(B/A),$$
$$P(A\cdot B)=P(B)\cdot P(A/B).$$

Xususiy holda, agar A va B hodisalar o'zaro bog'liq bo'lmasa, ularning birgalikda ro'y berish ehtimoli bu hodisalar ehtimolining ko'paytmasiga teng:

$$P(A\cdot B)=P(A)\cdot P(B).$$

4.Birgalikda bo'lgan hodisalar ehtimollarini qo'shish teoremasi. Ikkita birgalikda bo'lgan *A* va *B* hodisadan hech bo'lmaganda birining ro'y berish ehtimoli bu hodisalar ehtimollari yig'indisidan ularning birgalikda ro'y berish ehtimolining ayrilganiga teng:

$$P(A+B)=P(A)+P(B)-P(A\cdot B).$$

Agar *A* va *B* hodisalar o'zaro bog'liq bo'lmasa, u holda ushbu formula o'rinli bo'ladi:

$$P(A+B)=P(A)+P(B)-P(A)\cdot P(B)$$

Masala-1 Sexda ikkita brigade bir xil maxsulot ishlab chiqarmoqda. Kun davomida I brigada (II brigada) *n* dona (*t dona*) mahsulot tayyorladi va ularning *m donasi* (*s donasi*) oliy navli deb topildi. Har bir brigada tayyorlagan mahsulotlar ichidan tasodifiy ravishda bittadan mahsulot tanlab olindi. Quyidagi tasodifiy hodisalarning ehtimolliklarini toping:

A={I brigadadan tanlangan mahsulot oliy navli},
B={ II brigadadan tanlangan mahsulot oliy navli emas},
C={Ikkala brigadadan tanlangan mahsulotlar oliy navli},
D={Tanlangan mahsulotlarning faqat bittasi oliy navli},
E={ Tanlangan mahsulotlarning kamida bittasi oliy navli },
$n=30$, $m=22$, $t=50$, $s=44$

<u>Yechish:</u> Ehtimollikning klassik ta'rifi bo'yicha

$$P(A)=\frac{m}{n}=\frac{22}{30}=0{,}73$$

Qarama-qarshi hodisalarning ehtimolliklari formulasiga asosan

$$P(B)=1-P(\overline{B})=1-\frac{s}{t}=1-\frac{44}{50}=\frac{6}{50}=0{,}12.$$

Hodisalar ko'paytmasi ta'rifiga asosan $C = A\overline{B}$ va A , $\overline{B}$ bog'liqmas hodisalar bo'lgani uchun ehtimolliklarni ko'paytirish teoremasi bo'yicha

$$P(C)=P(A\overline{B})=P(A)P(\overline{B})=\frac{22}{30}\cdot\frac{44}{50}\approx 0{,}73\cdot 0{,}88\approx 0{,}64$$

D hodisa ehtimolligini topish uchun

D_1 = {Faqat I brigadadan tanlangan mahsulot oliy navli }

D_2 = {Faqat II brigadadan tanlangan mahsulot oliy navli }

Tasodifiy hodisalarni kiritamiz.

Hodisalarni qo'shish ta'rifiga asosan $D = D_1 + D_2$ deb yozish mumkin. Hodisalar ko'paytmasi ta'rifiga asosan $D_1 = AB$, $D_2 = \overline{A}\overline{B}$ deb yozish mumkin. D_1 va D_2 birgalikda bo'lmagan hodisalar bo'lgani uchun, ehtimolliklarni qo'shish va ko'paytirish teoremalariga asosan ushbu natijani olamiz :

$P(D) = P(D_1 + D_2) = P(D_1) + P(D_2) = P(AB) + P(\overline{A}\overline{B}) =$

$P(A)P(B)+P(\overline{A})P(\overline{B})=$

$$\frac{22}{30}\cdot\frac{6}{50}+\left(1-\frac{22}{30}\right)\cdot\frac{44}{50}=0{,}73\cdot 0{,}12+0{,}27\cdot 0{,}88=0{,}0876+0{,}2376=0{,}3252$$

Endi E hodisaga qarama-qarshi $\overline{E}$ hodisani qaraymiz.

$\overline{E}$ = {ikkala mahsulot ham oliy navli emas} = $\overline{A}\cdot B$ bo'lgani uchun

$$P(\overline{E}) = P(\overline{A}\cdot B) = P(\overline{A})\cdot P(B)=0{,}27\cdot 0{,}12=0{,}0324$$

Buholda

$$P(E) = 1- P(\overline{E}) = 1- 0{,}0324 = 0{,}9676 \approx 0{,}97 .$$

Masala-2. Omborda uch partiya mahsulot saqlanmoqda. Bu partiyalardagi mahsulotlar soni mos ravishda n_1, n_2, n_3 bo'lib ular p_1, p_2, p_3 ehtimollik bilan sifatli bo'lishi mumkin. Ombordagi mahsulotlar ichidan bitta mahsulot tasodifiy ravishda tanlab olindi.

a) Tanlangan mahsulotni sifatli bo'lish ehtimolligini hisoblang.

б) Agar tanlangan mahsulot sifatli bo'lsa, uni *i*-partiyaga tegishli bo'lish ehtimolligini toping.

$n_1 = 10$, $n_2 = 15$, $n_3 = 20$, $p_1 = 0{,}9$, $p_2 = 0{,}8$, $p_3 = 0{,}7$ $i = \text{II}$

Yechish. *a*) Mahsulotni tanlab olishda ushbu uchta natija bo'lishi mumkin.

E_1 = { Tanlangan mahsulot I partiyadan}

E_2 = { Tanlangan mahsulot II partiyadan }

E_3 = { Tanlangan mahsulot III partiyadan }

Masala shartiga asosan va ehtimollikning klassik ta'rifi bo'yicha

$$P(E_1) = \frac{n_1}{n_1+n_2+n_3} = \frac{10}{10+15+20} = \frac{10}{45} = \frac{2}{9}$$

$$P(E_2) = \frac{n_2}{n_1+n_2+n_3} = \frac{15}{45} = \frac{3}{9}$$

$$P(E_3) = \frac{n_3}{n_1+n_2+n_3} = \frac{20}{45} = \frac{4}{9}$$

Bunda ushbu A = { Tanlangan mahsulot sifatli} tasodifiy hodisaning P(A) ehtimolligi qiziqtiradi. Masala shartiga ko'ra A hodisaning shartli ehtimolliklari quyidagicha :

$P(A/E_1) = p_1 = 0{,}9$, $P(A/E_2) = p_2 = 0{,}8$, $P(A/E_3) = p_3 = 0{,}7$

Bu holda to'liq ehtimollik formulasiga ko'ra

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) =$$
$$\frac{2}{9}\cdot 0{,}9 + \frac{3}{9}\cdot 0{,}8 + \frac{4}{9}\cdot 0{,}7 = \frac{1{,}8+2{,}4+2{,}8}{9} = \frac{7}{9} \approx 0{,}78$$

b) Bu yerda $P(E_2/A)$ shartli ehtimollikni hisoblash talab etiladi. Bu ehtimollikni Bayes formulasi bo'yicha topamiz:

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(A)} = \frac{\frac{3}{9}\cdot 0{,}8}{\frac{7}{9}} = \frac{2{,}4}{7} \approx 0{,}34$$

Bu natijani shartsiz ehtimolligi $P(E_2) = 3/9 \approx 0{,}33$ edi. Demak kuzatuv natijasi bo'yicha E_2 ehtimolligi 0,01 ga oshdi.

Berilgan masalalarni yeching.

17.16. Pul – buyum loteriyasida 1000 ta biletli har bir seriyaga 120 ta pul yutug'i va 80 ta buyum yutug'i to'g'ri keladi. Bitta loteriyasi bor kishiga pul yutug'i yoki buyum yutug'i, umuman yutuq chiqish ehtimolini toping.

17.17. Merganning bitta o'q uzishda 10 ochko urish ehtimoli 0,15 ga, 9 ochko urish ehtimoli 0,35 ga, 8 yoki undan kam ochko urish ehtimoli 0,5 ga teng. Merganning bitta o'q uzishda kamida 9 ochko urish ehtimolini toping.

17.18. 10 ta detalli partiyada 8 ta standart detal bor. Tavakkaliga olingan ikkita detaldan kamida bittasi standart bo'lish ehtimolini toping.

17.19. Uchta yashikning har birida 10 tadan detal bor. Birnchi yashikda 9 ta, ikkinchi yashikda 8 ta, uchinchi yashikda 7 ta standart detal bor. Har bir yashikdan tavakkaliga bittadan detal olinadi. Olingan uchala detal standart bo'lish ehtimolini toping.

17.20. Yashikda 10 oq va 5 ta qora shar bor. Yashikdan ikki marta tavakkaliga bittadan shar olinadi. Olingan sharlar yashikka solinmaydi. Agar birinchi olingan shar qora bo'lsa (A hodisa), ikkinchi olingan shar oq bo'lish (B hodisa) ehtimolini toping.

17.21. Yashikda 6 ta shar bor, ulardan uchtasi qizil rangda. Yashikdan tavakkaliga 2 ta shar olidi. Ikkala sharning ham qizil rangda bo'lish ehtimolini toping.

17.22. Ikkita mergan bittadan o'q uzishdi. Birinchi merganningnishonga tekkazish ehtimoli 0,7 ga, ikkinchisiniki esa 0,6 ga teng. Merganlardan aqalli bittasining nishonga tekkazish ehtimolini toping.

17.23. Student imtihonga programmadagi 25 ta savoldan 20 tasini bilib keldi. Imtihon oluvchi studentga 3 ta savol berdi. Studentning uchala savolni ham bilish ehtimolini toping.

17.24. Student imtihon biletlaridan ba'zilarini bilmaydi. Student uchun qaysi holda u bilmaydigan biletni olish ehtimoli kichik bo'ladi: birinchi bo'lib olgandami yoki eng oxirida olgandami?

17.25. Uchta o'yin soqqasi tashlandi. Kamida bitta soqqada 6 ochko tushish ehtimolini toping.

§ 17.3. To'la ehtimol formulasi. Bayes formulasi

Bog'liq bo'lmagan tajribalar ketma – ketligi. Bernulli formulasi.

Murakkab hodisalarning ehtimolarini hisoblashda ko'pincha bu hodisalarga qo'shish va ko'paytirish teoremalarini birga tatbiq qilib hosil qilingan formulalardan foydalanishga to'g'ri keladi. Quyidagi ana shunday muhim formulalarning ba'zilari bilan tanishib o'tamiz.

1. To'la ehtimol formulasi. Faraz qilaylik, Ahodisa n ta juft – jufti bilan birgalikda bo'lmagan $H_1;H_2,...,H_n$ hodisalar (gipotezalar)ning bittasi va faqat bittasi bilangina ro'y berishi mumkin bo'lsin, boshqacha qilib aytganda:

$$A = AH_1 + AH_2 + ... + AH_n \ (A \text{ – murakkab hodisa}).$$

Bu yerda $AH_i \cap AH_j = V(i \neq j)$, u holda birgalikda bo'lmagan hodisalar ehtimollarini qo'shish teoremasiga asosan:

$$P(A) = P(AH_1) + P(AH_2) + ... + P(AH_n).$$

Ko'paytirish teoremasiga ko'ra $P(AH_i) = P(H_i) \cdot P(A/H_i)$ ekanligini e'tiborga olsak, u holda

$$P(A) = P(H_1)P(A/H_1) + P(H_2)P(A/H_2) + ... + P(H_n)P(A/H_n)$$

yoki

$$P(A) = \sum_{i=1}^{n} P(H_i)P(A/H_i).$$

Bu tenglik to'la ehtimol formulasi deyiladi. To'la ehtimol formulasidan foydalanib, Bayes formulasi yoki gipotezalar ehtimollari formulasi deb ataluvchi muhim formulani hosil qilish mumkin.

2. Bayes formulasi. Birgalikda bo'lmagan $H_1;H_2,...,H_n$ hodisalarning (gipotezalarning) to'la gruppasi berilgan bo'lib, tajribani o'tkazishga qadar ularning har birining $P(H_i), i=\overline{1,n}$ ehtimollari tayin qiymatiga ega bo'lsin. Tajriba natijasida A hodisa ro'y berdi degan shart ostida $H_i\ (i=\overline{1,n})$ gipotezalarning ehtimollari tajribadan so'ng qanday bo'ladi?

H_i va A hodisalarning ko'paytmasi uchun ushbu

$$P(AH_i)=P(A)\cdot P(A/H_i)=P(H_i)P(A/H_i)$$

formulaning o'rinliligidan

$$P(H_i/A)=\frac{P(H_i)P(A/H_i)}{P(A)}$$

munosabatga ega bo'lamiz, bu yerda to'la ehtimol formulasini qo'llasak, ushbu Bayes formulasi deb ataluvchi formulani hosil qilamiz:

$$P(H_i/A)=\frac{P(H_i)P(A/H_i)}{\sum_{k=1}^{n}P(H_k)P(A/H_k)}.$$

Bu formulalar yordamida yechiladigan masalalarni ko'raylik.

3- masala. Omborga 360 ta mahsulot keltirildi. Bulardan 300 tasi 1 – korxonada tayyorlangan bo'lib, ularning 250 tasi yaroqli mahsulot; 40 tasi 2-korxonada tayyorlangan bo'lib, ularning 30 tasi yaroqli hamda 20 tasi 3-korxonada tayyorlangan bo'lib, ulardan 10 tasi yaroqli. Tavakkaliga olingan mahsulotning yaroqli bo'lish ehtimolini toping.

Tavakkaliga olingan mahsulot uchun quyidagi gipotezalar o'rinli bo'ladi:

H_1 gipoteza – mahsulotning 1 – korxonada tayyorlangan bo'lishi;

H_2 gipoteza – mahsulotning 2 – korxonada tayyorlangan bo'lishi;

H_3 gipoteza – mahsulotning 3 – korxonada tayyorlangan bo'lishi.

Ularning ehtimollari quyidagicha bo'ladi:

$$P(H_1)=\frac{5}{6};\quad P(H_2)=\frac{1}{9};\quad P(H_3)=\frac{1}{18}.$$

Agar olingan mahsulotning yaroqli bo'lishini A hodisa deb belgilasak, u holda bu hodisaning turli gipotezashartlari ostidagi ehtimollari quyidagicha bo'ladi:

$$P(A/H_1)=\frac{5}{6};\quad P(A/H_2)=\frac{3}{4};\quad P(A/H_3)=\frac{1}{2}.$$

Yuqorida topilganlarni to'la ehtimol formulasiga qo'yib, izlanayotgan hodisa ehtimolini topamiz:

$$P(A)=P(H_1)P(A/H_1)+P(H_2)P(A/H_2)+P(H_3)P(A/H_3)=\frac{5}{6}\cdot\frac{5}{6}+\frac{1}{9}\cdot\frac{3}{4}+\frac{1}{18}\cdot\frac{1}{2}=\frac{29}{36}.$$

4-masala. Ikki mergan nishonga bittadan o'q uzdi. Birinchi merganning o'qi nishonga 0,8 ehtimol bilan, ikkinchi merganniki esa 0,4 ehtimol bilan tegadi. O'q uzilgandan so'ng nishonga bitta o'q tekkanligi (A hodisa) ma'lum bo'ldi, bu o'qni birinchi mergan uzgan bo'lishi ehtimolini toping.

Tajriba o'tkazishdan oldin quyidagi gipotezalarni qo'yamiz:

H_1 - birinchi mergan otgan o'q ham, ikkinchi mergan otgan o'q ham nishonga tegmaydi;

H_2 - ikkala merganning otgan o'qi ham nishonga tegadi;

H_3 - birinchi merganing otgan o'qi nishonga tegadi, ikkinchisiniki esa tegmaydi;

H_4 - birinchi merganing otgan o'qi nishonga tegmaydi, ikkinchisiniki esa tegadi.

Gipotezalardan bittasi va faqat bittasi tajriba natijasida albatta ro'y beradi, ya'ni H_1, H_2, H_3, H_4 lar bog'liq bo'lmagan hodisalarning to'liq gruppasini tashkil etadi.

Bu gipotezalarning tajribadan oldingi ehtimollari:

$$P(H_1)=0{,}2\cdot 0{,}6=0{,}12,$$
$$P(H_2)=0{,}8\cdot 0{,}4=0{,}32,$$
$$P(H_1)=0{,}8\cdot 0{,}6=0{,}48,$$
$$P(H_1)=0{,}2\cdot 0{,}4=0{,}08.$$

Bu gipotezalarda kuzatilayotgan A hodisaning shartli ehtimollari quyidagilarga teng:

$$P(A/H_1)=0,\quad P(A/H_2)=0,\quad P(A/H_3)=1,\quad P(A/H_4)=1.$$

Tajribadan keyin (A hodisa ro'y berganidan keyin) H_1,

H_2 gipotezalar ro'y bermasligi ma'lum bo'ladi.

H_3 va H_4 gipotezalarning tajribadan keyingi ehtimollari Bayes formulasiga ko'ra quyidagicha:

$$P(H_3 / A) = \frac{0{,}48 \cdot 1}{0{,}48 \cdot 1 + 0{,}08 \cdot 1} = \frac{6}{7};$$

$$P(H_4 / A) = \frac{0{,}08 \cdot 1}{0{,}48 \cdot 1 + 0{,}08 \cdot 1} = \frac{1}{7}.$$

Demak, nishonga tekkan o'qning birinchi merganga tegishli bo'lish ehtimoli $\frac{6}{7}$ ekan.

3.Agar biror *A* hodisaning ro'y berish yoki ro'y bermasligini kizatish uchun bir nechta tajribalar o'tkazilayotgan bo'lib, ularning har biriga *A* hodisaning ro'y berish yoki ro'y bermaslik ehtimoli qolgan tajribalarning natijalariga bog'liq bo'lmasa (bog'liq bo'lsa), u holda bu tajribalar *A* hodisaga nisbatan bog'liq bo'lmagan (bog'liq bo'lgan) tajribalar ketma-ketligini tashkil etadi deyiladi.

Masalan, yashikda *s* ta oq va *l* ta qora shar bo'lsin. Shu yashikdan bir necha marta bittadan shar olish tajribalari ketma-ketligini ko'raylik. Bunda har bir tajribadan so'ng olingan shar yashikka qaytarib solinsa (qaytarib solinmasa), bu tajribalar ketma-ketligi bir-biriga bog'liq bo'lmaydi (bir-biriga bog'liq bo'ladi).

Bog'liq bo'lmagan *n* ta tajriba o'tkazilayotgan bo'lib, har bir tajribada kuzatilayotgan *A* hodisaning ro'y berish ehtimoli *p* va ro'y bermaslik ehtimoli $q = 1 - p$ bo'lsin. Bu holda kuzatilayotgan *A* hodisaning *n* ta tajribada *k* marta ro'y berish ehtimoli $P_n(k)$ quyidagi formula yordamida topiladi:

$$P_n(k) = C_n^k q^{n-k}, \quad (0 \le k \le n)$$

bu yerda

$$C_n^k = \frac{n(n-1)(n-2)...(n-k+1)}{k!} = \frac{n!}{k!(n-k)!}.$$

Bu formula *Bernulli formulasi* deyiladi.

5-masala. Chigitning unuvchanligi 80 % bo'lsa, ekilgan 4 ta chigitdan:

a)uchtasining unib chiqish;

b) hech bo'lmaganda ikkitasining unib chiqish ehtimolini toping.

a) Shartga ko'ra $n=4; k=3; \quad p=0{,}8; \quad q=0{,}2$.

Bernulli formulasiga ko'ra:

$$P_4(3)=C_4^3\cdot 0{,}8^3\cdot 0{,}2=0{,}4096.$$

b) A hodisa ekilgan 4 ta chigitdan 2 tasi yoki 3 tasi, yoki 4 tasi unib chiqishini, ya'ni hech bo'maganda ikkitasining unib chiqishini bildirsin. Ehtimollarni qo'shish teoremasiga ko'ra:

$P_4(A)=P_4$ (yoki 2, yoki 3, yoki 4) $=P_4(2)+P_4(3)+P_4(4)$.

$P_4(3)$ ehtimol (a) punktda hisoblangan;

$$P_4(2)=C_4^2\cdot 0{,}8^2\cdot 0{,}2^2=0{,}1536;$$

$$P_4(4)=C_4^4\cdot 0{,}8^4\cdot 0{,}2^0=0{,}4096.$$

Demak, $P(A)=0{,}9728$.

6-masala. Bitta detalning yaroqsiz bo'lish ehtimoli $p=0{,}05$ bo'lsin.ixtiyoriy olingan 10 000 detal ichida yaroqsiz detallarning soni 50 tadan ko'p bo'lmaslik ehtimolini toping.

μ - yaroqsiz detallar soni bo'lsin.

$$\mu: 0, 1, 2, \dots, 50.$$

$$P_{10000}\{0\le\mu\le 50\}=\dots?$$

$$P_{10000}\{0\le\mu\le 50\}=\sum_{k=0}^{50}P_n(k)=\sum_{k=0}^{50}C_{10000}^k(0{,}05)^k(0{,}95)^{10000-k}$$

(bundan keyingi hisoblash ancha qiyin, shuning uchun ularni keltirmaymiz).

7-masala. Texnik kontrol bo'limi 24 ta detaldan iborat partiyani tekshirmoqda. Detalning standart bo'lish ehtimoli 0,6 ga teng. Standart deb tan olinadigan detallarning eng katta ehtimolli sonini toping.

Shartga ko'ra $n=24; \quad p=0{,}6; \quad q=0{,}4$. Standart deb tan olingan detallarning eng katta ehtimolli sonini quyidagi qo'sh tengsizlikdan topamiz:

$$np-q\le k_0<np+p.$$

$np-q=24\cdot 0{,}6-0{,}4=14$ butun son bo'lgani uchun eng katta ehtimolli son ikkita: $k_0=14$ va $k_0+1=15$.

Quyidagi masalalarni yeching.

17.26. Sportchilar gruppasida 20 chang'ichi, 6 velosipedchi va 4yuguruvchi bor. Saralash normasini bajarish ehtimoli chang'ichi uchun 0,9 ga, velosipedchi uchun 0,8 ga, yuguruvchi uchun 0,75 ga teng. Tavakkaliga ajratilgan sportchining normani bajara olish ehtimolini toping.

17.27. Birinchi yashikda 10 ta detal bo'lib, ulardan 15 tasi standart, ikkinchi yashikda 30 detal bo'lib, ulardan 24 tasi standart, uchinchi yashikda 10 ta detal bo'lib, ulardan 6 tasi standart. Tavakkaliga tanlangan yashikdan tavakkaliga olingan detalning standart bo'lish ehtimolini toping.

17.28. 1 – masala shartida, agar tavakkaliga olingan mahsulot yaroqli ekanligi ma'lum bo'lsa, uni 1 – korxonada tayyorlangan bo'lish ehtimolini toping.

17.29. Ichida 2 ta shar bo'lgan idishga bitta oq shar solinib, shundan keyin idishdan tavakkaliga bitta shar olingan. Sharlarning dastlabki tartibi (rangi bo'yicha) haqida mumkin bo'lgan barcha gipotezalar teng imkoniyatli bo'lsa, u holda olingan sharning oq rangda bo'lish ehtimolini toping.

17.30. Benzokolonka joylashgan shossedan yuk mashinalari sonining o'sha shossedan o'tadigan yengil mashinalar soniga nisbatan 3:2 kabi. Yuk mashinasining benzin olish ehtimoli 0,1 ga teng, yengilmashina uchun bu ehtimol 0,2 ga teng. Benzokolonka yoniga benzin olish uchun mashina kelibto'xtadi. Uning yuk mashina bo'lish ehtimolini toping.

17.31. Chigitning unuvchanligi 70 % bo'lsa, ekilgan 5 ta chigitdan: a) uchtasining; b) ko'pi bilan uchtasining; c) kamida uchtasining unib chiqish ehtimolini toping.

17.32. Ikki teng kuchli raqib shaxmat o'ynamoqda. Qaysi ehtimol kattaroq:

a) raqiblardan birining ikki partiyadan bittasini yutish ehtimolimi yoki to'rt partiyadan ikkitasini yutish ehtimolimi?

b) to'rt partiyadan kamida ikkitasini yutish ehtimolimi yoki besh partiyadan kamida uchtasini yutish ehtimolimi? Durang natijalar e'tiborga olinmaydi.

17.33. Ikki mergan bir vaqtda nishonga o'q uzmoqda. Bitta o'qni uzishda nishonga tekkizish ehtimoli birinchi mergan uchun 0,8 ga, ikkinchi mergan uchun 0,6 ga teng. Agar bir yo'la 15 marta o'q uziladigan bo'lsa, ikkala merganning ham nishonga tekkizishlarning eng katta ehtimolli sonini toping.

17.34. Agar 49 ta bog'liq bo'lmagan tajribada hodisa ro'y berishning eng katta ehtimolli soni 30 ga teng bo'lsa, tajribalarning har birida hodisaning ro'y berish ehtimoli p ni toping.

§ 17.4. Asimptotik formulalar

Yuqoridagi keltirilgan misollardan ko'rinadiki, tajribalar soni n yetarlicha katta bo'lganda $P_n(k)=C_n^k p^k q^{n-k}$ ehtimolni hisoblash katta qiyinchiliklarga olib keladi. Bunday hollarda hisoblashni osonlashtiruvchi formulalarga, hatto ular izlanayotgan ehtimolning taqribiy qiymatini bersa ham ehtiyoj tug'iladi. Bunday formulalar asimptotik formulalar deb ataladi. Ushbu paragrafda shunday formulalar bilan tanishamiz.

1. Muavr – Laplasning teoremasi. Har birida hodisaning ro'y berish ehtimoli $p(0<p<1)$ ga teng bo'lgan n ta bog'liqsiz tajribada hodisaning k marta ro'y berish ehtimoli (n yetarlicha katta bo'lganda) taqriban

$$P_n(k)\approx\frac{1}{\sqrt{npq}}\varphi(x) \qquad (17.1)$$

ga teng. Bu yerda

$$\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}},\quad x=\frac{k-np}{\sqrt{npq}}.$$

$\varphi(x)$ funksiyaning qiymatlar jadvali 1 – ilovada keltirilgan ($\varphi(-x)=\varphi(x)$ - juft funksiya). (17.1) formula Muavr –Laplasning local formulasidir.

2.Puasson formulasi. Agar tajribalar soni yetarlicha katta bo'lib, har bir tajribada hodisaning ro'y berish ehtimoli p juda kichik bo'lsa, u holda

$$P_n(k)\approx\frac{\lambda^k}{k!}e^{-\lambda}, \qquad (17.2)$$

bu yerda $\lambda = np$. (17.2) – Puasson formulasidir.

3.Muavr – Laplasning integral formulasi. Har birida hodisaning ro'y berish ehtimoli $p(0<p<1)$ ga teng bo'lgan n ta bog'liqsiz tajribada hodisaning kamida k_1 marta ro'y berish ehtimoli (n yetarlicha katt bo'lganda) taqriban

$$P_n(k_1\le\mu\le k_2)\approx\Phi(x'')-\Phi(x') \qquad (17.3)$$

ga teng. Bu yerda

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x e^{-\frac{x^2}{2}}dx$$

Laplas finksiyasi bo'lib, bunda

$$x'=\frac{k_1-np}{\sqrt{npq}},\quad x''=\frac{k_2-np}{\sqrt{npq}}.$$

x ning ($0\le x\le 5$) musbat qiymatlari uchun Laplas funksiyaning qiymatlari jadvali 2 – ilovada keltirilgan. $x>5$ qiymatlar uchun $\Phi(x)=0{,}5$ deb olinadi ($\Phi(x)=-\Phi(x)$, ya'ni $\Phi(x)$ -toq funksiya).

Quyidagi masalalarning yechilishi bilan tanishaylik.

8-masala. Korxonada ishlab chiqarilgan detalning yaroqsiz bo'lish ehtimoli 0,005 ga teng. 10 000 ta detaldan iborat partiyadagi yaroqsiz detallar sonining 40 ta bo'lish ehtimolini toping.

Masalaning shartiga ko'ra $n=10000;\ k=40;\ p=0{,}005;\ q=0{,}995$.

$n=10000$ yetarlicha katta son bo'lgani uchun Muavr – Laplasning local formulasidan foydalanamiz:

$$P_n(k)\approx\frac{1}{\sqrt{npq}}\varphi(x),\ \text{bu yerda}\ x=\frac{k-np}{\sqrt{npq}}.$$

x ning qiymatini topamiz:

$$x=\frac{k-np}{\sqrt{npq}}=\frac{40-10000\cdot 0{,}005}{\sqrt{10000\cdot 0{,}005\cdot 0{,}995}}\approx-\frac{10}{7{,}05}=-1{,}42.$$

Jadvaldan $\varphi(-1{,}42)=\varphi(1{,}42)=0{,}1456$ ni topamiz.

Demak, izlanayotgan ehtimol:

$$P_{10000}(40)\approx\frac{1}{7{,}05}\cdot 0{,}1456\approx 0{,}0206.$$

9-masala. Darslik 200 000 nusxada bosib chiqarilgan. Darslikning brak bo'lish ehtimoli ehtimoli 0,00005 ga teng. Butun tirajda rosa brak bo'lish kitob ehtimolini toping.

Shartga ko'ra $n=200000,\quad p=0{,}00005,\quad k=5$. n son katta va p ehtimol kichik, shu sababli Puasson formulasidan foydalanamiz:

$$P_n(k)\cong\frac{\lambda^k}{k!}e^{-\lambda},$$

bu yerda $\lambda=np,\;\lambda$ ning qiymatini topamiz:

$$\lambda=200000\cdot 0{,}00005=10.$$

Deamk, izlanayotgan ehtimol:

$$P_{200000}(5)=\frac{10^5}{5!}e^{-10}=\frac{10^5}{120}\cdot 0{,}000045\approx 0{,}0375.$$

10-masala. Tavakkaliga olingan pillaning yaroqsiz chiqish ehtimoli 0,2 ga teng. Tasodifan olingan 400 ta pilladan 70 tadan 130 tagacha yaroqsiz bo'lish ehtimolini toping.

Masala shartga ko'ra:

$$p=0{,}2;\quad q=0{,}8;\quad n=400;\quad k_1=70;\quad k_2=130.$$

Muavr – Laplasning integral formulasidan foydalanamiz:

$$P_n(k_1\le\mu\le k_2)=\Phi(x'')-\Phi(x').$$

x' va x'' larning qiymatilarini topamiz:

$$x'=\frac{k_1-np}{\sqrt{npq}}=\frac{70-400\cdot 0{,}2}{\sqrt{400\cdot 0{,}2\cdot 0{,}8}}=-\frac{10}{8}=-1{,}25,$$

$$x''=\frac{k_2-np}{\sqrt{npq}}=\frac{130-400\cdot 0{,}2}{\sqrt{400-0{,}2\cdot 0{,}8}}=\frac{55}{8}=6{,}25.$$

Jadvaldan topamiz:

$$\Phi(-1,25) = -\Phi(1,25) = -0,39435,$$

$\Phi(6,25) = 0,5$, chunki $x > 5$ da $\Phi(x) = 0,5$.

Demak, izlanayotgan ehtimol:

$$P_{400}(70 \le \mu \le 130) = \Phi(6,25) + \Phi(1,25) = 0,5 + 0,39435 = 0,89435.$$

Quyidagi masalalarni yeching

17.35. Har bir tajribada *A* hodisaning ro'y berish ehtimoli 0,2 ga teng bo'lsa, uning 400 ta tajribadan 80 tasida ro'y berish ehtimolini toping.

17.36. O'g'il bola tug'ilish ehtimoli 0,51 ga teng. Tug'ilgan 100 chaqaloqning 50 tasi o'g'il bola bo'lish ehtimolini toping.

17.37. Har birida *A* hodisaning ro'y berish ehtimoli $p = 0,5$ ga teng bo'lgan 10 000 ta tajriba o'tkaziladi. Shuncha tajribada *A* hodisa ro'y berishining eng katta ehtimolli sonining ehtimolini toping.

17.38. Ishchi ayol 800 ta urchuqqa xizmat ko'rsatadi. Δt vaqt oralig'ida har bir urchuqda yigirilayotgan ipning uzilish ehtimolli sonini va bu sonning ehtimolini toping.

17.39. Bir soat davomida istalgan abonentning kommutatorga telefon qilish ehtimoli 0,01 ga teng. Telefon stnsiyasi 300 abonentga xizmat qiladi. Bir soat davomida 4 ta abonentning telefon qilish ehtimolini toping.

17.40. Har bir otilgan o'qning nishonga tegish ehtimoli 0,001 ga teng. Agar 5000 ta o'q otilgan bo'lsa, kamida 2 ta o'qning nishonga tegish ehtimolini toping.

17.41. Fakultet studentlarining imtihon komissiyasidan "4" va "5" baholar bilan o'tish ehtimoli 0,9 ga teng. Tavakkaliga olingan 400 studentdan 34 tadan 55 tagacha hech bo'lmaganda bitta fandan "4" dan past baho olish ehtimolini toping.

17.42. Hodisaning 2100 ta bog'liq bo'lmagan tajribalarning har birida ro'y berish ehtimoli 0,7 ga teng. Hodisaning: a) kamida

1470 marta va ko'pi bilan 1500 marta; b) kamida 1470 marta; v) ko'pi bilan 1469 marta ro'y berish ehtimolini toping.

17.43. O'zaro bog'liq bo'lmagan 625 ta tajribaning har birida A hodisaning ro'y berish ehtimoli 0,8 ga teng. Hodisaning ro'y berish nisbiy chastotasining uning ehtimolidan chetlashishi absolyut qiymati bo'yicha 0,04 dan katta bo'lmaslik ehtimolini toping.

17.44.O'zaro bog'liq bo'lmagan tajribalarning har birida A hodisaning ro'y berish ehtimoli 0,5 ga teng. Hodisa ro'y berish nisbiy chastotasining uning ehtimolidan chetlashishi absolyut qiymati bo'yicha 0,02 dan ortiq bo'lmasligining 0,7698 ehtimol bilan kutish mumkin bo'lishi uchun nechta tajriba o'tkazish kerak?

17.45. O'yin soqqasini ushbu $\left|\frac{m}{n}-\frac{1}{6}\right| \le 0{.}01$ tengsizlikning ehtimoli qarama – qarshi tengsizlikning ehtimolidan kichik bo'lmasligi uchun nechta marta tashlash lozim (bu yerda m o'yin soqqasini n marta tashlashda besh ochko chiqish soni)?

17.46. Texnik kontrol bo'limi 900 ta detalning standartga muvofiqligini tekshiring. Detalning standartga muvofiq bo'lish ehtimoli 0,9 ga teng. Shunday ε musbat son topingki, detalning standart bo'lish ehtimoli nisbiy chastotasining uning ehtimoli 0,9 dan chetlashishining absolyut qiymati ε dan katta bo'lmasligini 0,9544 ehtimol bilan kutish mumkin bo'lsin.

17.47. Texnik kontrol bo'limi 475 ta buyumning yaroqligini tekshiradi. Buyumning brak bo'lish ehtimoli 0,05 ga teng. Tekshirilgan buyumlar orasida braklari soni m ning yotadigan chegaralarini 0,9426 ehtimol bilan toping.

§ 17.5 Diskret tasodifiy miqdor va uning taqsimot qonuni.

Diskret tasodifiy miqdorlarning sonli xarakteristikalari. Uzluksiz tasodifiy miqdorlar va ularning sonli xarakteristikalari.

O'zining turli qiymatlarini tasodifga bog'liq ravishda qabul qiladigan o'zgaruvchi miqdorlar *tasodifiy miqdorlar* deyiladi va X, Y, Z kabi bosh harflar bilan belgilanadi. Tasodifiy miqdor qabul qiladigan qiymatlar uning *mumkin bo'lgan qiymatlari* deb ataladi.

Masalan, o'yin soqqasi tashlanganda chiqqan ochko (X), tasodifiy tanlangan sonning kasr qismi (Y)-tasodifiy miqdorlar bo'ladi. Tasodifiy miqdorlar diskret va uzluksiz bo'lishi mumkin.

Mumkin bo'lgan qiymatlari chekli yoki sanoqli to'plamni tashkil etuvchi tasodifiy miqdorlar *diskret* deb ataladi.

AgardaX diskret tasodifiy miqdor bo'lsa, uning mumkin bo'lgan qiymatlarini $x_1, x_2,\ldots, x_n,\ldots$ kabi belgilab chiqish mumkin. Yuqorida ko'rsatilgan misoldagi X-diskret tasodifiy miqdor bo'lib, uning mumkin bo'lgan qiymatlari $x_1=1,\ x_2=2,\ldots,\ x_6=6$ chekli to'plamni tashkil etadi.

X diskret tasodifiy miqdorni to'la aniqlash uchun faqat uning mumkin bo'lgan $x_1, x_2,\ldots, x_n,\ldots$ qiymatlarini bilish kifoya bo'lmasdan, shu qiymatlarning

$$P\{X = x_1\} = p_1\ ,\quad P\{X = x_2\} = p_2\ ,\ldots,\ P\{X = x_n\} = p_n\ ,\ldots$$

ehtimolliklarini ham bilish zarurdir. Bu holdaX diskret tasodifiy miqdorni ushbu

X	x_1	x_2	...	x_n	...
P	p_1	p_2	...	p_n	...

(1)

jadval ko'rinishida aniqlash mumkin. Bu yerda

$$p_1 + p_2 +\ldots+ p_n +\ldots = 1 \qquad (2)$$

shart bajarilishi kerak. Bu shart (1) jadvalning birinchi satrida X tasodifiy miqdorning barcha mumkin bo'lgan qiymatlari keltirilganligini ifodalaydi.

T A ' R I F : (2) shartni qanoatlantiruvchi (1) jadval X diskret tasodifiy miqdorning *taqsimot qonuni* deyiladi.

Masalan, simmetrik tanga ikki marta tashlanganda uning gerb tomoni bilan tushishlar sonini X deb olaylik. Bu holda X tasodifiy miqdor bo'ladi. Uning mumkin bo'lgan qiymatlari $x_1 = 0$, $x_2 = 1$, $x_3 = 2$ chekli to'plam bo'lgani uchun X diskret tasodifiy miqdor bo'ladi. Bu qiymatlarning ehtimolliklarini klassik ta'rif yordamida topish mumkin. Bunda barcha natijalar soni $n = 4$ va x_1, x_2 va x_3 uchun qulaylik tug'diruvchi natijalar soni $m_1 = 1$, $m_2 = 2$, $m_3 = 1$ bo'lgani uchun

$$P\{X = 0\} = \frac{1}{4} = 0,25 \quad , P\{X = 1\} = \frac{2}{4} = 0,5 \quad , P\{X = 2\} = \frac{1}{4} = 0,25$$

Demak, ko'rilayotgan X tasodifiy miqdorning taqsimot qonuni

X	0	1	2
P	0,25	0,5	0,25

(3)

ko'rinishda bo'lib, (2) shart

$$p_1 + p_2 + p_3 = 0,25 + 0,5 + 0,25 = 1$$ bajariladi.

Diskret X tasodifiy miqdor to'g'risidagi barcha ma'lumotni

$$F(x) = P\{X < x\}, \quad x \in (-\infty, \infty) \qquad (4)$$

funktsiya orqali ham berish mumkin.

$y = F(x)$ funktsiya X tasodifiy miqdorning *taqsimot funktsiyasi* deyiladi.

Ehtimollikning xossalaridan foydalanib, $F(x)$ taqsimot funktsiyasining quyidagi uchtaasosiy xossasini isbotlash mumkin:

I. Ixtiyoriy $x \in (-\infty, \infty)$ uchun $0 \le F(x) \le 1$;

II. $y = F(x)$ kamaymovchi funktsiya, ya'ni $\forall x_1 < x_2$ uchun $F(x_1) \le F(x_2)$ bo'ladi.

III. $F(-\infty) = \lim\limits_{x \to -\infty} F(x) = 0$, $F(\infty) = \lim\limits_{x \to \infty} F(x) = 1$.

Yuqorida ko'rib o'tilgan X diskret tasodifiy miqdorning taqsimot funktsiyasini topamiz:

a) $x \le 0 \Rightarrow F(x) = P\{X < x\} = P\{X < 0\} = P(\varnothing) = 0$;

b) $0 < x \le 1 \Rightarrow F(x) = P\{X < x\} = P\{X = 0\} = 0{,}25$;

c) $1 < x \le 2 \Rightarrow F(x) = P\{X < x\} = P\{X = 0\} + P\{X = 1\} = 0{,}25 + 0{,}5 = 0{,}75$;

d) $x > 2 \Rightarrow F(x) = P\{X < x\} = P\{X = 0\} + P\{X = 1\} + P\{X = 2\} = P(\Omega) = 1$.

Bu misoldan diskret tasodifiy miqdorning taqsimot funktsiyasi uzlukli, pog'onasimon (zinapoyasimon) bo'lishi kelib chiqadi. Bu funktsiyaning uzilish nuqtalari X diskret tasodifiy miqdorning mumkin bo'lgan qiymatlarini, shu nuqtalardagi sakrashlari esa bu qiymatlarning ehtimolliklarini ifodalaydi.

Mumkin bo'lgan qiymatlari biror chekli yoki cheksiz (a,b) oraliqni to'la qoplaydigan X tasodifiy miqdor *uzluksiz* deyiladi.

Masalan, tasodifiy tanlangan sonning kasr qismini ifodalovchi Y tasodifiy miqdor uzluksiz bo'lib, uning mumkin bo'lgan qiymatlari $[0,1)$ yarim oraliqni qoplaydi.

Uzluksiz X tasodifiy miqdorni (1) taqsimot qonuni orqali aniqlab bo'maydi, chunki uning mumkin bo'lgan qiymatlari sanoqsiz bo'lib, ularni natural sonlar bilan belgilab chiqib bo'maydi. Bundan tashqari $\forall x \in (a,b)$ mumkin bo'lgan qiymatning ehtimolligi $p_x = P\{X = x\} = 0$ bo'ladi. Ammo uzluksiz X tasodifiy miqdorning $F(x)$ taqsimot funktsiyasini (4) munosabat bilan aniqlash mumkin va bu funktsiya uzluksiz tasodifiy miqdor to'g'risida to'liq ma'lumotni o'z ichiga oladi. Bu holda uzluksiz tasodifiy miqdorning barcha mumkin bo'lgan qiymatlari to'plamida taqsimot funktsiyasi $F(x)$ uzluksiz bo'lishini ko'rsatish mumkin va shu sababli uning hosilasi to'g'risida so'z yuritish mumkin.

Agarda F(x) taqsimot funktsiya differentsillanuvchi bo'lsa, uning hosilasi $F'(x) = f(x)$ X tasodifiy miqdorning *zichlik funktsiyasi* deyiladi.

Shuni ta'kidlab o'tish kerakki, f(x) zichlik funktsiyasi faqat uzluksiz tasodifiy miqdorlar uchun aniqlangandir.

Har qanday f (x) zichlik funktsiyasi quyidagi ikkita asosiy xossaga ega :

$$\text{I. } f(x) \geq 0, \ x \in (-\infty, \infty) \ ; \qquad \text{II. } \int_{-\infty}^{\infty} f(x)dx = 1$$

Masalan, tasodifiy ravishda tanlangan sonning kasr qismini ifodalovchi uzluksiz tasodifiy miqdorning zichlik funktsiyasi

$$f(x) = \begin{cases} 1, x \in [0,1) \\ 0, x \bar{\in} [0,1) \end{cases} \qquad (5)$$

bo'lishini ko'rsatish mumkin va bu funktsiya yo'qoridagi ikkita shartni qanoatlantiradi.

Tasodifiy miqdor X o'zining taqsimot qonuni yoki taqsimot funktsiyasi yoki zichlik funktsiyasi bilan berilganda u to'liq aniqlangan bo'ladi. Ba'zi hollarda X tasodifiy miqdor to'g'risida bunday to'liq ma'lumot kerak bo'lmasdan, uning ma'lum bir xususiyatlarini ifodalovchi sonli xarakteristikalarini bilish kifoyadir. Masalan, X biror tarmoq xodimlarining ish haqini ifodalovchi tasodifiy miqdor bo'lsa, uning har bir xodim uchun qiymatlarini bilish shart bo'masdan, barcha xodimlar bo'yicha o'rta qiymatini bilish yetarlidir.

Ehtimolliklar nazariyasida tasodifiy miqdorni ifodalovchi juda ko'p sonli xarakteristikalar bo'lib, ularning eng asosiylari matematik kutilish M(X) , dispersiya D(X) va o'rta kvadratik chetlanish σ(X) bo'lib hisoblanadi.

X tasodifiy miqdorning *matematik kutilishi* deb, uning qiymatlarini vaznlashtirilgan o'rta miqdorini ifodalovchi va diskret holda

$$M(X) = x_1p_1 + x_2p_2 + \ldots + x_np_n + \ldots \ , \qquad (6)$$

uzluksiz holda esa

$$M(X)=\int_{-\infty}^{\infty} xf(x)dx, \qquad (7)$$

formulalar bilan hisoblanadigan songa aytiladi.

Masalan , (3) taqsimot qonunli diskret tasodifiy miqdor uchun $M(X)=0\cdot 0,25+1\cdot 0,5+2\cdot 0,25=1$, (5) zichlik funktsiyali uzluksiz tasodifiy miqdor uchun

$$M(X)=\int_0^1 x\cdot 1dx=\frac{x^2}{2}\Big|_0^1=0,5$$

Matematik kutilish quyidagi xossalarga ega:

1. Har qanday o'zgarmas C soni uchun M(C)=C.
2. Har qanday o'zgarmas C ko'paytuvchi uchun M(CX)=C M(X);
3. Matematik kutilishlari mavjud bo'lgan X va Y tasodifiy miqdorlar uchun $M(X\pm Y)=M(X)\pm M(Y)$ tenglik o'rinli bo'ladi.

X tasodifiy miqdorning *dispersiyasi* deb, uning qiymatlarini matematik kutilmasi atrofida tarqoqligini ifodalovchi va diskret holda

$$D(X)=(x_1-m)^2p_1+(x_2-m)^2p_2+\ldots+(x_n-m)^2p_n+\ldots, \qquad (8)$$

uzluksiz holda esa

$$D(X)=\int_{-\infty}^{\infty}(x-m)^2 f(x)dx, \qquad m=M(X), \qquad (9)$$

formula bilan aniqlanuvchi songaaytiladi.

Yuqorida ko'rib o'tilgan (3) X diskret tasodifiy miqdor uchun m=M(X)=1 va

$$D(X)=(0-1)^2\cdot 0,25+(1-1)^2\cdot 0,5+(2-1)^2\cdot 0,25=0,5,$$

(5) uzluksiz tasodifiy miqdor uchun esa m=M(X)=0,5 va

$$D(X)=\int_0^1 (x-0,5)^2dx=\frac{(x-0,5)^3}{3}\Big|_0^1=\frac{1}{12}$$

Dispersiya quyidagi xossalarga ega:

1. Har qanday X tasodifiy miqdor uchun D(X)≥0.
2. Har qanday C o'uzgarmas son uchun D(C)=0.

3. Har qanday o'zgarmas C ko'paytuvchi uchun

$$D(CX)=C^2 D(X).$$

4. Har qanday o'zgarmas C son uchun $D(X \pm C)= D(X)$.

Agarda X biror narsani massasini ifodalab, uning o'lchov birligi kilogramm (kg) bo'lsa, dispersiya o'lchovi kg^2 bo'lib, ma'nosiz bo'ladi. Shu sababli bunday paytlarda$\sigma(X)=\sqrt{D(X)}$ formula bilan aniqlanadigan va <u>*o'rta kvadratik chetlanish*</u> deb ataladigan ko'rsatgichdan foydalaniladi.

Tasodifiy miqdor ehtimollari taqsimotining integral funksiyasi:

Ta'rif: Har bir x qiymat uchun X tasodifiy miqdorning x dan kichik qiymat qabul qilish ehtimoliga taqsimotning integral funksiyasi deyiladi.

$$P(X < x) = F(x)$$

Agar $F(x)$ - integral funksiya uzluksiz differensiallanuvchi bo'lsa, x tasodifiy miqdor uzluksiz deyiladi.

Integral funksiyasi ba'zan taqsimot funksiyasi deb ham nomlanadi.

Integral funksiya xossalari.

1-xossasi: Integral funksiya qiymatlari $[0;1]$ oraliqda joylashgandir.

$$0 \le F(x) \le 1$$

2-xossasi: Integral funksiya kamaymaydigan funksiyadir, ya'ni $x_1 < x_2$ bo'lganda $F(x_1) \le F(x_2)$ bo'ladi.

3-xossasi: X tasodifiy miqdorning (a,b) oraliqda yotgan qiymatlarni qabul qilish ehtimoli: $P(a < x < b) = F(b) - F(a)$ ga teng.

Xulosa: Agar X tasodifiy miqdorning barcha mumkin bo'lgan qiymatlari (a,b) oraliqga tegishli bo'lsa, u holda

$$F(x) = \begin{cases} 0, agar\ x \le a, bo'lsa \\ 1, agar\ x > b, bo'lsa \end{cases}$$

Uzluksiz tasodifiy miqdor ehtimollari taqsimotining differensial funksiyasi.

Integral funksiyadan olingan birinchi tartibli hosilaga ehtimollar taqsimotining differesial funksiyasi deyiladi. $f(x)=F'(x)$

Ba'zan differensial funksiyasini zichlik funksiyasi deb ham atashadi.

X uzluksiz tasodifiy miqdorning (a,b) oraliqqa tegishli qiymatlarni qabul qilish ehtimoli.

$$P(a<x<b)=\int_a^b f(x)dx \quad \text{ga teng.}$$

Integral funksiya differensial funksiya orqali quyidagi formula bilan ifodalanadi.

$$F(x)=\int_{-\infty}^{x} f(x)dx$$

Differensial funksiya xossalari.

1-xossasi: Differensial funksiya manfiy emas:

$$f(x)\geq 0$$

2-xossasi: Quyidagi xosmas integral birga teng:

$$\int_{-\infty}^{\infty} f(x)dx=1$$

Agar tasodifiy miqdor (a,b) oraliqqa tegishli qiymat qabul qilsa, u holda:

$$\int_a^b f(x)dx=1 \text{bo'ladi.}$$

Uzluksiz tasodifiy miqdorning matematik kutilishi dispersiyasi va o'rtacha kvadratik chetlanishi quyidagi formulalar orqali ifodalanadi.

1) $M(x)=\int_{-\infty}^{\infty} xf(x)dx$, yoki $M(x)=\int_a^b xf(x)dx$

2) $D(x)=\int_{-\infty}^{\infty}(x-M(x))^2 f(x)dx$, yoki $D(x)=\int_a^b x^2 f(x)dx-M^2(x)$

3) $\sigma(x)=\sqrt{D(x)}$

Masalalar yechish uchun namunalar

1) Masala. Agar X-tasodifiy miqdor (0:2) oraliqda quyidagi differensial funksiya bilan berilgan bo'lsa,

$$f(x)=\begin{cases} 1/4, agar, x\in(0;1] bo'lsa, \\ 3/4, agar, x\in(1;2) bo'lsa \end{cases}$$

uning integral funksiyasi topilsin.

Yechish: $F(x)=\int_{-\infty}^{x} f(x)dx=\int_0^1 \frac{dx}{4}+\int_1^x \frac{3}{4}dx=\frac{3}{4}x-\frac{1}{2}$ dan $F(x)=$

$$\begin{cases} 0, agar x\le 0, bo'lsa, \\ \frac{x}{4}, agar 0<x\le 1, bo'lsa, \\ \frac{3}{4}x-\frac{1}{2}, agar 1<x<2, bo'lsa, \\ 1, agar, x>2, bo'lsa, \end{cases}$$

ga ega bo'lamiz.

2) X-tasodifiy miqdorning integral funksiyasi berilgan

bo'lsa, $F(x)=\begin{cases} 0, agar x<0, bo'lsa, \\ x, agar 0\le x\le 1, bo'lsa, \\ 1, agar x>1, bo'lsa, \end{cases}$

uning differensial funksiyasi va o'rtacha kvadratik chetlanishi topilsin.

Yechish: $f(x)=F'(x)=\begin{cases} 0, agar, x<0, bo'lsa, \\ 1, agar, 0\le x\le 1, bo'lsa, \\ 0, agar, x>1, bo'lsa \end{cases}$

$$M(x)=\int_0^1 xdx=\frac{x^2}{2}\Big|_0^1=\frac{1}{2},\quad D(x)=\int_0^1 x^2dx-\frac{1}{4}=\frac{1}{3}-\frac{1}{4}=\frac{1}{12},\quad \sigma(x)=\sqrt{D(x)}=\sqrt{\frac{1}{12}}=\frac{\sqrt{3}}{6}$$

Quyidagi masalalarni yeching.

17.58. 2 ta o'yin kubigi bir marta tashlanganda chiqadigan ochkolar ko'paytmasining matematik kutilishi topilsin.

17.59. Avvalgi masala shartlarida yig'indining matematik kutilishi topilsin.

17.60. Qutida 5 ta shar bo'lib, ulardan 2 tasi oq va 3 tasi esa qora rangda. Tavakkaliga qutidan ikkita shar olindi. X-tasodifiy miqdor oq shar chiqish soni taqsimotining matematik kutilishi topilsin.

17.61. Avvalgi masala shartlarida X tasodifiy miqdorining dispersiyasi hisoblansin.

17.62.Diskret 2 ta erkli tasodifiy miqdorlarning taqsimot qonunlari berilgan.

X	1	2
p	0,2	0,8

Y	0,5	1
p	0,3	0,7

XY-ko'paytmasining matematik kutilishini toping.

17.63. X -tasodifiy miqdorning taqsimot qonuni berilgan.

X	2	4	8
p	0,1	0,5	0,4

Uning o'rtacha kvadratik chetlanishi topilsin.

17.64. 10 ta detaldan iborat to'plamda 3 ta nostandart detal bor. Tavakkaliga 2 ta detal olingan. X-diskret tasodifiy miqdor, olingan 2 ta detal orasidagi nostandart detallar sonining matematik kutiliiihini toping.

17.65. Geolog safardan qayta turib, tog'dan 6 ta namuna olib keldi. Shulardan 4 tasida izlanayotgan metal qotishmasi bor. Tavakkaliga 3 ta namuna tanlandi. X-diskret tasodifiy miqdor olingan namunalar orasida izlanayotgan metal qorishmasi borligi soni taqsimotining o'rtacha kvadratik chetlanishi topilsin.

17.66. X-diskret tasodifiy miqdor faqat 2 ta mumkin bo'lgan x_1 va x_2 qiymatlarga ega, shu bilan birga $x_1 < x_2$, X tasodifiy miqdorning x_1 qiymatni qabul qilish ehtimoli 0,2 ga teng. $M(X)=2{,}6$ ni o'rtacha kvadratik chetlanish $\sigma(X)=0{,}8$ ni bilgan holda, X ning taqsimot qonuni topilsin.

17.67. X diskret tasodifiy miqdor faqat 3 ta mumkin bo'lgan $x_1 = 3, x_1, x_2$ qiymatlarga ega shu bilan birga $x_1 < x_2 < x_3$ X ning x_3 va x_2 qiymatlarini qabul qilish ehtimoli mos ravishda 0,5 va 0,4 ga teng. X miqdorning matematik kutilishi $M(X)=2{,}4$ va dispersiyasi $D(X)=0{,}44$ ni bilgan holda uning taqsimot qonunini toping.

17.68. Sinov paytida biror detalning buzilish ehtimoli 0,3 ga teng. Sinov paytida kuzatilgan 20 ta detaldan ishdan chiqqan detallar sonining matematik kutilishi topilsin.

17.69. Har bir tajribada biror qurilmadagi elementning ishdan chiqish ehtimoli 0,1 ga teng. X diskret tasodifiy miqdor-elementning 5 ta erkli tajribada ishdan chiqish sonining dispersiyasi topilsin.

17.70. Agar x va y tasodifiy miqdorlar erkli bo'lib, $D(X)=12$ va $D(Y)=7$ ga teng ekanligi ma'lum bo'lsa, $Z=5X-4Y$ tasodifiy miqdorning dispersiyasini toping.

17.71. 3 ta erkli sinovlar o'tkazilayotgan bo'lib, ularning har birida A hodisasi 0,4 ehtimol bilan ro'y bersin. X tasodifiy miqdor-A hodisaning ro'y berish sonining matematik kutilishi topilsin.

17.72. 14-masala shartlarida X tasodifiy miqdor dispersiyasi topilsin.

17.73. Agar X va Y tasodifiy miqdorlarning matematik kutilishi ma'lum bo'lsa, $M(X)=5;$ $M(Y)=3;$ $Z=4X-2Y$ tasodifiy miqdorning matematik kutilishini toping.

17.74. Agar 3 ta erkli sinovda A hodisaning ro'y berish ehtimoli bir xil va $M(X)=0,6$ bo'lsa, bu sinovlarda A hodisaning ro'y berishlari sonidan iborat X diskret tasodifiy miqdorning dispersiyasi topilsin.

17.75. X diskret tasodifiy miqdor faqat ikkita mumkin bo'lgan x_1 va x_2 qiymatga ega bo'lib, $x_1 < x_2$ bo'lsin. Agar x ning x_1 qiymatni qabul qilish ehtimoli 0,8 ga, matematik kutilishi 2,6 ga, dispersiyasi esa 0,64 ga teng bo'lsa, x ning x_1 qiymatni qabul qilish ehtimoli topilsin.

17.76. X tasodifiy miqdor faqat 3 ta mumkin bo'lgan qiymatlar $x_1 < x_2 < x_3$ qabul qiladi. X ning x_1 va x_3 qiymatlarni qabul qilish ehtimollari mos ravishda 0,2 va 0,5 ga teng. Matematik kutilishi esa 3,2 va dispersiyasi 0,76 ga teng bo'lsa, X ning taqsimot qonuni topilsin.

17.77. Xtasodifiymiqdorintegralfunksiyasibilanberilgan.

$$F(x)=\begin{cases}1-\frac{8}{x^3}, agar, x \geq 2, bo'lsa\\ 0, agar, x < 2, bo'lsa.\end{cases}$$

X ning sonli xarakteristikalari, ya'ni matematik kutilishi, dispersiyasi va o'rtacha kvadratik chetlanishi topilsin.

17.78.X tasodifiy miqdor (0:1) oraliqda differensial funksiyasi $f(x)=x$, bu oraliqdan tashqarida esa $f(x)=0$ bo'lsa, uning sonli xarakteristikalari hisoblansin.

17.79. X tasodifiy miqdorining (3:5) oraliqda $f(x)=\frac{1}{2}$ differensial funksiya bilan berilgan bu oraliqdan tashqarida esa $f(x)=0$ ga teng. X ning integral funksiyasi topilsin.

17.80. X ning integral funksiyasi berilgan bo'lsa, $F(x)=\begin{cases} 1, & agar \quad x \geq a, bo'lsa \\ \frac{1}{2}+\frac{1}{\pi} arcSin\frac{x}{a}, & azap, -a < x < a, bo'lsa \\ 0, & agar, x \leq -a, bo'lsa \end{cases}$ X ning $\left(-\frac{a}{2};\frac{a}{2}\right)$ oraliqqa tushish ehtimoli topilsin.

17.81. X ning zichlik funksiyasi $f(x)=ae^{-x}$ $[0;\infty)$ oraliqda berilgan bo'lsa, a-koeffitsient topilsin.

17.82. X ning differentsial funksiyasi $f(x)=kxe^{-x}$ $x\in[0;\infty)$ oraliqda berilgan bo'lsa, k-koeffitsient topilsin.

17.83. Tekis taqsimlangan tasodifiy miqdor X ning integral funksiyasi berilgan.

$$F(x)=\begin{cases} 0, & agar, x<0, bo'lsa, \\ x, agar & 0\leq x\leq 1 \quad bo'lsa, \\ 1, & agar, x>1, bo'lsa \end{cases}$$

X ning differensial funksiyasi topilsin.

17.84. X ning taqsimot funksiyasi berilgan.

$$F(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}dt$$

Uning zichlik funksiyasi topilsin.

17.85. Agar zichlik funksiyasi $f(x)=\frac{a}{1+x^2}$ $x\in(-\infty;\infty)$ bo'lsa,

1) a-koeffitsient topilsin.
2) integral funksiyasi topilsin.

17.86. Avvalgi 9-masala shartlarida. X tasodifiy miqdorning (-1;1) oraliqqa tegishli qiymat qabul qilish ehtimoli topilsin.

17.87. $f(x)=ae^{-x^2}$ X tasodifiy miqdorning $(-\infty;\infty)$ oraligida zichlik funksiyasi bo'lishi uchun a-qanday bo'lishi kerak?

17.88. X tasodifiy miqdorning taqsimot funksiyasi.

$$F(x)=\begin{cases} 0, & agar, x\le -1, bo'lsa, \\ \frac{1}{\pi}\cdot arc\sin x, agar, & -1\le x\le 1, \quad bo'lsa, \\ 1, agar & x\ge 1, bo'lsa \end{cases}$$

berilgan. Uning matematik kutilishi va dispersiyasi qanday?

17.89. Agar X (10:20) oraliqda tekis taqsimlangan tasodifiy miqdor bo'lsa, uning matematik kutilishi dispersiyasi topilsin.

17.90. (1:7) oraliqda tekis taqsimlangan X tasodifiy miqdorning o'rtacha kvadratik chetlanishi topilsin.

17.91. (a;b) oraliqda tekis taqsimlansin X tasodifiy miqdorning matematik kutilishini toping.

17.92. 15-masala shartlarida X tasodifiy miqdorning dispersiyasini toping.

17.93. X tasodifiy miqdor (0:2) oraliqda $f(x)=\frac{x}{2}$ differensial funksiyasi bilan berilgan, bu oraliqdan tashqarida esa $f(x)=0$ ga teng. X ning dispersiyasi topilsin.

17.94. X tasodifiy miqdorning $(1;\infty)$ oralikda integral funksiyasi $F(x)=1-\frac{1}{x^3}$ ga teng bu oraliqdan tashqarida esa $F(x)=0$, ga teng bo'lsa, uning matematik kutilishini toping.

17.95. X tasodifiy miqdor (2:4) oraliqda $f(x)=\frac{3x^2}{4}-\frac{9}{2x}x+\frac{14}{2}$ differensial funksiya bilan berilgan. Bu oraliqdan tashqarida esa $f(x)=0$ ga teng. X miqdorning matematik kutilishi topilsin.

17.96. X tasodifiy miqdor differensial funksiyasi berilgan.

$$f(x)=\begin{cases} 0, agar & x\le \frac{-\pi}{2}, bo'lsa, \\ \cos x & ,агар \frac{-\pi}{2} < x\le 0, bo'lsa, \\ 0, agar & x>0, bo'lsa \end{cases}$$

Uning integral funksiyasi topilsin.

Takrorlash uchun savollar

1.Ehtimollar nazariyasida hodisa qanday ta'riflanadi?

2.Ehtimolning klassik ta'rifini ayting.

3.O'rinlashtirishlar formulasini yozing.

4.O'rin almashtirishlar qanday topiladi?

5.Birgalikda bo'lmagan hodisalar ehtimollarini qo'shish teoremasini ayting.

6.Muqarrar hodisalar odatda qanday belgilanadi?

7.Qachon ikkita matritsa teng deyiladi?

8."Ikkita o'yin soqqasi tashlanganda ochkolar ko'paytmasi ...."

tasdiq qanday davom ettirilganda tasodifiy hodisaga ega bo'lamiz?

EHTIMOLLIKLAR NAZARIYASI VA MATEMATIK STATISTIKAGA DOIR NAZORAT TESTLARI

1. Hodisalar va ular ustida amallar

1. Ehtimollar nazariyasida hodisa qanday ta'riflanadi?

A) Har qanday tajribaning ixtiyoriy natijalari hodisa deyiladi.

B) Har qanday kuzatuvning ixtiyoriy natijalari hodisa deyiladi.

C) Har qanday tasdiqlar hodisa deyiladi.

D) Har qanday fikrlar hodisa deyiladi.

E) Hodisa tushunchasi ta'rifsiz qabul etiladi.

2. Ehtimollar nazariyasida qanday hodisalar sinfi qaralmaydi?

A) Mumkin bo'lmagan hodisalar.

B) Muqarrar hodisalar.

C) Tasodifiy hodisalar. D) Noma'lum hodisalar.

E) Barcha ko'rsatilgan hodisalar sinflari qaraladi.

3. Muqarrar hodisalar odatda qanday belgilanadi?

A) Q. B) Ω . C) $\varnothing$. D) M . E) G .

4. Quyidagilardan qaysi biri muqarrar hodisa bo'ladi?

A) Tashlangan tanga gerb tomoni bilan tushdi .

B) O'yin soqqasi tashlanganda 7 dan kichikochko chiqdi.

C) Tasodifiy ravishda tanlangan natural son juft bo'ldi.

D) Sotib olingan lotereyaga yutuq chiqmadi.

E) Tekshirilgan mahsulot sifatli bo'lib chiqdi.

5. "Ikkita o'yin soqqasi tashlanganda ochkolar ko'paytmasi ...." tasdiq qanday davom ettirilganda muqarrar hodisaga ega bo'lamiz?

A) 36 dan katta bo'ldi. B) 36 ga teng bo'ldi.

C) 36 dan kichik bo'ldi. D)36 dan katta bo'lmadi.

E) 36 dan kichik bo'lmadi.

6. Qutida 10 ta shar bo'lib, ulardan 7 tasi oq va qolganlari qora ranglidir. Shu qutidan tasodifiy ravishda 4 ta shar olindi. Quyidagilardan qaysi biri muqarrar hodisa bo'ladi?

A) tanlangan sharlardan faqat bittasi oq rangli.

B) tanlangan sharlardan ko'pi bilan bittasi oq rangli.

C) tanlangan sharlardan kamida bittasi oq rangli.

D) tanlangan sharlardan birortasi ham oq rangli emas.

E) Keltirilgan hodisalar orasida muqarrar hodisa mavjud emas.

7. Qutida n ta oq va m ta qora rangli shar bor. "Qutidan tasodifiy ravishda tanlangan k ta shar orasida kamida bittasi oq rangli" hodisa muqarrar bo'ladigan k parametrning eng kichik qiymati nimaga teng?

A) $k=m-1$ B) $k=m$ C) $k=m+1$. D) $k=n-1$. E) $k=n$

8. Quyidagilardan qaysi biri muqarrar hodisa bo'ladi?

A) Tasodifiy ravishda tanlangan natural son 5 ga karrali.

B) Matndan tasodifiy ravishda olingan harf unli.

C) Ikkita o'yin soqqasi tashlanganda ochkolar yig'indisi 2 dan kichik.

D) Talaba testdan muvaffaqiyatli o'tdi.

E) Tanlangan mahsulot yoki sifatli yoki sifatsiz.

9. Mumkin bo'lmagan hodisalar odatda qanday belgilanadi?

A) Q. B) Ω . C) Ø . D) M . E) G .

10. Quyidagilardan qaysi biri mumkin bo'lmagan hodisa bo'ladi?

A) Tashlangan tanga raqamli tomoni bilan tushdi .

B) O'yin soqqasi tashlanganda 1 dan kichikochko chiqdi.

C) Tasodifiy ravishda tanlangan natural son toq bo'ldi.

D) Sotib olingan lotereyaga yutuq chiqdi.

E) Tekshirilgan mahsulot sifatli bo'lib chiqdi.

11."Ikkita o'yin soqqasi tashlanganda ochkolar ko'paytmasi ...." tasdiq qanday davom ettirilganda mumkin bo'lmagan hodisaga ega bo'lamiz?

A) 36 dan katta bo'ldi. B) 36 ga teng bo'ldi.

C) 36 dan kichik bo'ldi. D)36 dan katta bo'lmadi.

E) 36 dan kichik bo'lmadi.

12.Qutida 10 ta shar bo'lib, ulardan 7 tasi oq va qolganlari qora ranglidir. Shu qutidan tasodifiy ravishda 4 ta shar olindi. Quyidagilardan qaysi biri mumkin bo'lmagan hodisa bo'ladi?

A) tanlangan sharlardan faqat bittasi oq rangli.

B) tanlangan sharlardan ko'pi bilan bittasi oq rangli.

C) tanlangan sharlardan kamida bittasi oq rangli.

D) tanlangan sharlardan birortasi ham oq rangli emas.

E) Keltirilgan hodisalar orasida mumkin bo'lmagan hodisa mavjud emas.

13.Quyidagilardan qaysi biri mumkin bo'lmagan hodisa bo'ladi?

A) Tasodifiy ravishda tanlangan natural son 5 ga karrali.

B) Matndan tasodifiy ravishda olingan harf unli.

C) Ikkita o'yin soqqasi tashlanganda ochkolar yig'indisi 2 dan kichik.

D) Talaba testdan muvaffaqiyatli o'tdi.

E) Tanlangan mahsulot yoki sifatli yoki sifatsiz.

14. Quyidagi belgilardan qaysi biri tasodifiy hodisani ifodalashi mumkin?

A) Λ. B) Ω . C) Ø . D) b . E) A .

15. Quyidagilardan qaysi tasodifiy hodisa bo'ladi?

A) Tashlangan tanga raqam yoki gerb tomoni bilan tushdi.

B) O'yin soqqasi tashlanganda 1 dan kichikochko chiqdi.

C) Tasodifiy ravishda tanlangan natural son toq bo'ldi.

D) Keltirilgan barcha hodisalar tasodifiydir.

E) Keltirilgan barcha hodisalar tasodifiy emas.

16. "Ikkita o'yin soqqasi tashlanganda ochkolar ko'paytmasi ...." tasdiq qanday davom ettirilganda tasodifiy hodisaga ega bo'lamiz?

A) 36 dan katta bo'ldi. B) 36 ga teng bo'ldi.

C) 36 dan kichik yoki unga teng bo'ldi. D) 36 dan katta bo'lmadi.

E) 37 dan kichik bo'ldi.

2. Ehtimol va uni hisoblash usullari

1. Qaysi shartda A va B birgalikda bo'lmagan hodisalar deyiladi?

A) A+B=Ω B) A·B=Ω C) A+B=Ø D) A·B=Ø E) A=B

2. O'yin soqqasi tashlanganda quyidagi hodisalarning qaysi juftligi birgalikda bo'lmagan hodisalar bo'ladi:

A={soqqada 4 yoki 5 ochko chiqdi}, B={soqqada toq ochko chiqdi},

C={soqqada tub ochko chiqdi}, D={soqqada 3 ga karrali ochko chiqdi}.

A) A va B . B) A va C . C) A va D . D) B va C . E) C va D .

3. O'yin dastsasi 36 ta qartadan iborat bo'lib, undan tasodifiy ravishda ikkita qarta tanlab olindi. Ushbu hodisalarning qaysi juftligi birgalikda bo'lmagan hodisalar bo'ladi:

A={tanlangan ikkala qarta g'isht turga ega} ,

B={tanlangan ikkala qartadan kamida bittasi g'isht turga ega },
C={ tanlangan ikkala qartadan faqat bittasi g'isht turga ega },
D={ tanlangan ikkala qartadan bittasi tuz}.

A) A va B B) A va C C) A va D
D) B va C E) C va D

4. Biror hodisalar to'plami Θ hodisalar algebrasi bo'lishi uchun qaysi shart tyalab etilmaydi?

A) $A\in\Theta,\ B\in\Theta \Rightarrow A+B\in\Theta$. B) $A\in\Theta,\ B\in\Theta \Rightarrow A\cdot B\in\Theta$.
C) $1\in\Theta \Rightarrow 0\in\Theta$. D) $A\in\Theta \Rightarrow \overline{A}\in\Theta$.
E) barcha shartlar talab etiladi.

5. Agar Θ hodisalar algebrasi bo'lsa quyidagi tasdiqlardan qaysi biri o'rinli bo'lmaydi?

A) $A\in\Theta,\ B\in\Theta \Rightarrow \overline{A}+B\in\Theta$. B) $A\in\Theta,\ B\in\Theta \Rightarrow \quad \in\Theta$
C) $A\in\Theta,\ B\in\Theta \Rightarrow A+\overline{B}\in\Theta$. D) $A\in\Theta,\ B\in\Theta \Rightarrow \overline{A+B}\in\Theta$
E) barcha shartlar talab etiladi.

6. Agar Θ hodisalar algebrasi bo'lsa quyidagi tasdiqlardan qaysi biri o'rinli bo'lmaydi?

A) $A\in\Theta,\ B\in\Theta \Rightarrow \overline{A}\cdot B\in\Theta$. B) $A\in\Theta,\ B\in\Theta \Rightarrow \overline{A}\cdot\overline{B}\in\Theta$.
C) $A\in\Theta,\ B\in\Theta \Rightarrow A\cdot\overline{B}\in\Theta$. D) $A\in\Theta,\ B\in\Theta \Rightarrow \overline{A\cdot B}\in\Theta$
E) barcha shartlar talab etiladi.

7. Berilgan A hodisadan hosil qilingan Θ hodisalar algebrasi qayerda to'g'ri ko'rsatilgan?

A) $\Theta=\{A,\overline{A}\}$. B) $\Theta=\{\Omega,A,\overline{A}\}$. C) $\Theta=\{\varnothing,A,\overline{A}\}$.
D) $\Theta=\{\Omega,\varnothing,A,\overline{A}\}$. E) $\Theta=\{\Omega,\varnothing,A,\overline{A},1,0\}$.

8. Hodisalar algebrasida ehtimol tushunchasi nechta aksioma orqali kiritiladi?

A) 5 B) 4 C) 3 D) 2
E) to'g'ri javob keltirilmagan.

9. Ehtimol xossasi qayerda noto'g'ri ko'rsatilgan?

A) Har qanday muqarrar hodisa Ω ehtimolligi $P(\Omega)=1$.
B) Har qanday mumkin bo'lmagan hodisa $\varnothing$ ehtimolligi $P(\varnothing)=0$.
C) Har qanday A hodisaning ehtimolligi $P(A)\geq 0$.

D) Har qanday A hodisaning ehtimolligi $P(A) \leq 1$.

E) Barcha xossalar to'g'ri ko'rsatilgan .

10. Ehtimol xossasi qayerda to'g'ri ko'rsatilgan?

A) Har qanday muqarrar hodisa Ω ehtimolligi $P(\Omega)=0$.

B) Har qanday mumkin bo'lmagan hodisa $\emptyset$ ehtimolligi $P(\emptyset)=1$.

C) Har qanday A hodisaning ehtimolligi $P(A) \leq 0$.

D) Har qanday A hodisaning ehtimolligi $P(A) \geq 1$.

E) Barcha xossalar noto'g'ri ko'rsatilgan .

11. Ehtimolni hisoblashning qaysi usuli mavjud emas?

A) Ehtimolni klassik ta'rif usulida hisoblash .

B) Ehtimolni algebraik ta'rif usulida hisoblash .

C) Ehtimolni geometrik ta'rif usulida hisoblash .

D) Ehtimolni statistik ta'rif usulida hisoblash .

E) Keltirilgan barcha usullar mavjud .

12. Ehtimolning klassik ta'rifida A hodisa ro'y berishi mumkin bo'lgan $\mathfrak{Z}$ kuzatuv yoki tajriba natijalariga quyidagi shartlardan qaysi biri talab etilmaydi?

A) Barcha natijalar soni chekli

B) Barcha natijalar to'liq guruhni tashkil etadi .

C) Barcha natijalar birgalikda emas

D) Barcha natijalar teng imkoniyatli .

E) Keltirilgan barcha shartlar talab etiladi .

13. Ehtimolning klassik ta'rifida A hodisa ro'y berishi mumkin bo'lgan $\mathfrak{Z}$ kuzatuv yoki tajribaning $E_1, E_2, E_3, ..., E_n$ natijalariga quyidagi shartlardan qaysi biri talab etilmaydi?

A) $E_1+ E_2+ E_3+ ...+ E_n=\Omega$ (Ω-muqarrar hodisa) .

B) $E_i \cdot E_k=\emptyset$ ($i \neq k$, $\emptyset$-mumkin bo'lmagan hodisa) .

C) $P(E_1)=P(E_2)=P(E_3)= ...=P(E_n)$.

D) $E_1, E_2, E_3, ..., E_n$ natijalarning har birida A hodisa yoki albatta ro'y beradi yoki ro'y bermaydi .

E) Keltirilgan barcha shartlar talab etiladi .

14. Ehtimolning klassik ta'rifida n-barcha natijalar soni, m(A)-A hodisaga qulaylik yaratuvchi natijalar soni bo'lsa, P(A) ehtimol qaysi formula bilan topiladi?

A) $P(A)=m(A)\cdot n$ B) $P(A)=m(A)/n$

C) $P(A)=m(A)+n$. D) $P(A)=m(A)-n$.

E) $P(A)=n/m(A)$.

15. Ehtimolning klassik ta'rifida barcha natijalar soni n=20, A hodisaga qulaylik yaratuvchi natijalar soni m(A)=8 bo'lsa, P(A) ehtimol qiymati nimaga teng bo'ladi?

A) 0.8 . B) 0.6 . C) 0.4 . D) 0.2 . E) 2.5

16. Idishda 12 ta oq va qora sharlar bor. Idishdan tavakkaliga bitta oq shar olish ehtimoli 1/3 ga teng bo'lsa, idishda nechta qora shar bo'lgan?

A) 4. B) 6. C) 8. D) 10.

E) Qora sharlar sonini aniq ko'rsatib bo'lmaydi .

3. Ehtimollarni qo'shish va ko'paytirish teoremalari

1. Qaysi shartda A va B hodisalar birgalikdamas deyiladi?

A) $AB=\varnothing$ B) $AB=\Omega$ C) $A+B=\varnothing$

D) $A+B=\Omega$ E) $AB=A+B$.

2. Agar A va B hodisalar birgalikdamas bo'lsa, quyidagi tengliklardan qaysi biri o'rinli bo'lmaydi?

A) $AB=\varnothing$. B) $A+B=\Omega$. C) $AB=BA$. D) $A+B=B+A$

E) Barcha tengliklar o'rinli bo'ladi oladi .

3. A va B hodisalar birgalikdamas bo'lsa, P(A+B) ehtimol uchun qaysi tenglik o'rinli bo'lmaydi?

A) $P(A+B)=P(A)+P(B)$.

B) $P(A+B)=P(A)+P(B)-P(AB)$.

C) $P(A+B)=P(A|B)+P(B|A)$.

D) $P(A+B)=P(A)+P(B)+P(AB)$.

E) $P(A+B)=P(B+A)$.

4. 36 ta qartadan iborat dastadan tasodifiy ravishda bitta qarta tanlandi.

A={ Tanlangan qarta valet} ,

B={Tanlangan qarta tuz}

hodisalar bo'yicha P(A+B) ehtimolni toping.

A) 1/36 . B) 1/18 . C) 2/9 . D) 1/6 . E) 5/12 .

5. A va B birgalikdamas hodisalar uchun P(A)=0.4 va P(A+B)=0.7 bo'lsa, P(B) ehtimol qiymatini toping.

A) 0.4 . B) 0.6 . C) 0.3 . D) 0.55 . E) –0.3 .

6. Qutida n ta oq, m ta qizil va r ta ko'k rangli shar bor. Shu qutidan tasodifiy ravishda tanlangan shar oq yoki qizil rangli bo'lish ehtimolini toping.

A) n/(n+n+r) . B) m/(n+n+r) . C) r/(n+n+r) .

D) (n+m)/(n+n+r) . E) (n+r)/(n+n+r) .

7. Qarama-qarshi A va $\bar{A}$ hodisalarning ehtimollari uchun qaysi tenglik o'rinli?

A) $P(A)/P(\bar{A})=1$.

B) $P(A)-P(\bar{A})=1$.

C) $P(A)\cdot P(\bar{A})=1$.

D) $P(A)+P(\bar{A})=1$.

E) tog'ri javob keltirilmagan.

8. Agar $P(A)$=0.3 bo'lsa, $P(\bar{A})$ qiymati nimaga teng?

A) 0.3 . B) 0.7 . C) 0.33 . D) 0.09 . E) aniqlab bo'lmaydi .

9. Mahsulotni sifatli bo'lish ehtimoli 0.9 bo'lsa, uni sifatsiz bo'lish ehtimoli nimaga teng?

A) 0.1 . B) 0.09 . C) 0.99 . D) 0.19 .

E) to'g'ri javob keltirilmagan .

10. A va B hodisalar birgalikda bo'lsa, quyidagi tengliklardan qaysi biri o'rinli bo'la olmaydi?

A) AB=∅ .

B) A+B=Ω .

C) AB=BA .

D) A+B=B+A .

E) Barcha tengliklar o'rinli bo'ladi .

11.Agar A va B birgalikda bo'lgan hodisalar bo'lsa, P(A+B) ehtimol qaysi formula bilan hisoblanadi?

A) P(A+B)=P(A)+P(B) .

B) P(A+B)=P(A)+P(B)−P(A)P(B) .

C) P(A+B)=P(A)+P(A)+P(A)P(B) .

D) P(A+B)=P(A)+P(B)−P(AB) .

E) P(A+B)=P(A)+P(B)+P(AB).

12.Umumiy holda P(A+B) ehtimol qaysi formula bilan topiladi ?

A) P(A+B)=P(A)+P(B) . B) P(A+B)=P(A)+P(B)+P(AB) .

C) P(A+B)=P(A)+P(B)−P(AB) . D) P(A+B)=P(A)+P(B)±P(AB) .

E) P(A+B)=P(A)+P(B)−P(A)P(B) .

13.P(A|B) shartli ehtimol mazmuni qayerda to'g'ri ko'rsatilgan?

A) A hodisa ro'y bergan holda B hodisaning ehtimoli .

B) A hodisa ro'y bermagan holda B hodisaning ehtimoli .

C) B hodisa ro'y bergan holda A hodisaning ehtimoli .

D) B hodisa ro'y bermagan holda A hodisaning ehtimoli .

E) A va B hodisalarni bir paytda ro'y berish ehtimoli .

14.Qutida 10 ta oq va 6 ta qora shar bor. Shu qutidan tasodifiy ravishda ketma-ket ikkita shar tanlab olindi.

B={I tanlangan shar oq rangli}, A={II tanlangan shar qora rangli}

Hodisalar uchun P(A|B) shartli ehtimol qiymati nimaga teng?

A) 3/8 . B) 5/8 . C) 3/5 . D) 2/5 . E) 1/3 .

15. Qutida 10 ta oq va 6 ta qora shar bor. Shu qutidan tasodifiy ravishda ketma-ket ikkita shar tanlab olindi.

B={I tanlangan shar oq rangli},

A={II tanlangan shar oq rangli}

hodisalar uchun P(A|B) shartli ehtimol qiymati nimaga teng?

A) 3/8 . B) 5/8 . C) 3/5 . D) 2/5 . E) 1/3 .

16. Qutida 10 ta oq va 6 ta qora shar bor. Shu qutidan tasodifiy ravishda ketma-ket ikkita shar tanlab olindi.

B={I tanlangan shar qora rangli}, A={II tanlangan shar qora rangli}

hodisalar uchun P(A|B) shartli ehtimol qiymati nimaga teng?

A) 3/8 . B) 5/8 . C) 3/5 . D) 2/5 . E) 1/3 .

4. To'liq ehtimol va Bayes formulalari

1. A hodisa ehtimolini to'liq ehtimol formulasi yordamida hisoblashda tajriba natijalari $E_1, E_2, E_3, \ldots, E_n$ uchun qaysi shart talab etilmaydi?

A) $E_1, E_2, E_3, \ldots, E_n$ natijalar to'liq guruhni tashkil etadi .

B) $E_1, E_2, E_3, \ldots, E_n$ birgalikda bo'lmagan natijalar .

C) $E_1, E_2, E_3, \ldots, E_n$ teng imkoniyatli natijalar .

D) $P(A|E_1), P(A|E_2), P(A|E_3), \ldots, P(A|E_n)$ shartli ehtimollar berilgan .

E) Barcha shartlar talab etiladi.

2. To'liq ehtimol formulasini qo'llash uchun qaysi ehtimollarni bilish talab etiladi?

A) $E_1, E_2, E_3, \ldots, E_n$ natijalarning shartsiz ehtimollarini .

B) $E_1, E_2, E_3, \ldots, E_n$ natijalarning shartli ehtimollarini .

C) $E_1, E_2, E_3, \ldots, E_n$ natijalar yig'indisining ehtimolini .

D) $E_1, E_2, E_3, \ldots, E_n$ natijalar ko'paytmasining ehtimolini .

E) A tasodifiy hodisa ehtimolini.

3. To'liq ehtimol formulasini qo'llash uchun qaysi ehtimollarni bilish talab etiladi?

A) A tasodifiy hodisaning shartsiz ehtimolini .

B) A tasodifiy hodisa ehtimolini .

C) A tasodifiy hodisaga qarama-qarshi hodisaning ehtimolini .

D) $P(E_1|A), P(E_2|A), P(E_3|A), \ldots, P(E_n|A)$ shartli ehtimollarni .

E) $P(A|E_1)$, $P(A|E_2)$, $P(A|E_3)$, ..., $P(A|E_n)$ shartli ehtimollarni.

4. To'liq ehtimol formulasini keltirib chiqarishda qaysi tenglikdan foydalanilmaydi?

A) $A=A\Omega$.

B) $\Omega=E_1+E_2+E_3+\ldots+E_n$.

C) $P(E_iA)=P(E_i)P(A|E_i)$.

D) $A(E_1+E_2+E_3+\ldots+E_n)= AE_1+AE_2+AE_3+\ldots+AE_n$.

E) $P(AE_i)=P(A)P(E_i|A)$.

5. Agarda A hodisa $E_1,E_2,E_3,\ldots,E_n$ natijalarga ega tajribada ro'y berishi mumkin bo'lsa, unda P(A) uchun to'liq ehtimol formulasi qayerda to'g'ri ifodalangan?

A) $P(A)=P(E_1)+P(E_2)+\ldots+P(E_n)$.

B) $P(A)=P(A|E_1)+P(A|E_2)+\ldots+P(A|E_n)$.

C) $P(A)=P(E_1|A)+P(E_2|A)+\ldots+P(E_n|A)$.

D) $P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+\ldots+P(E_n)P(A|E_n)$.

E) $P(A)=P(E_1)P(E_1|A)+P(E_2)P(E_2|A)+\ldots+P(E_n)P(E_n|A)$.

6. Talaba «Oliy matematika», «Umumiy fizika» va «Ingliz tili» fanlari bo'yicha test savollariga mos ravishda 0.8, 0.7 va 0.9 ehtimol bilan to'g'ri javob bera oladi. Yakuniy testga «Oliy matematika» , «Umumiy fizika» va «Ingliz tili» fanlaridan mos ravishda 40%, 35% va 25% savollar kiritligan. Yakuniy testdan tasodifiy ravishda tanlab olingan savolga talaba to'g'ri javob berish ehtimolini hisoblang.

A) 0.33 . B) 0.79 . C) 0.47 .

D) 0.56 . E) 0.65 .

7. Mahsulotlar partiyasi uch smena davomida ishlab chiqarilgan mahsulotlardan iborat. Bunda I ,II va III smenalarda ishlab chiqarilgan mahsulotlar soni mos ravishda 100, 80 va 70 ta bo'lib, ular 0.9, 0.8 va 0.6 ehtimol bilan sifatli bo'lishi mumkin. Shu partiyadan tasodifiy ravishda tanlangan mahsulotni sifatli bo'lish ehtimolini nimaga teng?

A) 0.638 . B) 0.692 . C) 0.735 .

D) 0.784 . E) 0.836 .

8. "Oliy matematika" fani bo'yicha yakuniy imtihonda "Algebra", "Analitik geometriya" va "Matematik analiz" bo'limlar bo'yicha mos ravishda 60, 40 va 100 ta savollar mavjud va ularga talaba mos ravishda 0.8, 0.7 va 0.6 ehtimol bilan to'g'ri javob bera oladi. Yakuniy imtihonda talaba unga tasodifiy ravishda berilgan savolga to'g'ri javob berish ehtimolini hisoblang.

A) 0.87 . B) 0.83 . C) 0.75 . D) 0.71 . E) 0.69 .

9. Bayes formulasini qo'llash uchun qaysi ma'lumot talab etilmaydi?

A) $P(E_1), P(E_2), P(E_3), \ldots, P(E_n)$ ehtimollar .

B) $P(A|E_1), P(A|E_2), P(A|E_3), \ldots, P(A|E_n)$ shartli ehtimollar .

C) Tasodifiy A hodisaning P(A) ehtimoli .

D) Keltirilgan barcha ma'lumotlar talab etiladi .

E) Keltirilgan barcha ma'lumotlar talab etilmaydi .

10. Bayes formulasi yordamida nima hisoblanadi?

A) Tasodifiy A hodisaning P(A) ehtimoli .

B) Tajribadagi E_i natijalarning $P(E_i)$ ehtimollari .

C) $P(E_i|A)$ shartli ehtimollar .

D) $P(A|E_i)$ shartli ehtimollar .

E) $P(AE_i)$ shartlsiz ehtimollar .

11. Bayes formulasini keltirib chiqarishda qaysi tenglikdan foydalanilmaydi?

A) $AE_i = E_iA$. B) $P(AE_i)=P(E_iA)$. C) $P(AE_i)=P(A)P(E_i|A)$.

D) $P(E_iA)=P(E_i)P(A|E_i)$. E) Barcha tengliklardan foydalaniladi.

12. Quyidagi tengliklardan qaysi biri Bayes formulasini ifodalaydi?

A) $P(A)=\sum_{i=1}^{n} P(E_i)P(A|E_i)$. B) $P(E_k|A)=\dfrac{P(E_k)P(A|E_k)}{P(A)}$.

C) $P(AE_k)=P(A)P(E_k|A)$. D) $P(E_kA)=P(E_k)P(A|E_k)$.

E) $P(A)=\dfrac{P(E_k)P(A|E_k)}{P(E_k|A)}$.

13.Ombordagi 40% mahsulot I korxonada, qolgan 60% mahsulot esa II korxonada ishlab chiqarilgan. I korxonada ishlab chiqarilgan mahsulot 0.9, II korxona mahsuloti esa 0.8 ehtimol bilan sifatli bo'lishi mumkin. Shu ombordan tasodifiy ravishda olingan mahsulot sifatli bo'lib chiqdi. Bu mahsulotni II korxonada ishlab chiqarilganligi ehtimolini toping.

A) 57/84 . B) 12/21 . C) 27/84 . D) 9/21 . E) 29/42 .

14.Ombordagi 40% mahsulot I korxonada, qolgan 60% mahsulot esa II korxonada ishlab chiqarilgan. I korxonada ishlab chiqarilgan mahsulot 0.9, II korxona mahsuloti esa 0.8 ehtimol bilan sifatli bo'lishi mumkin. Shu ombordan tasodifiy ravishda olingan mahsulot sifatli bo'lib chiqdi. Bu mahsulotni I korxonada ishlab chiqarilganligi ehtimolini toping.

A) 57/84 . B) 12/21 . C) 27/84 . D) 9/21 . E) 29/42 "Oliy matematika" fani bo'yicha yakuniy imtihonda "Algebra", "Analitik geometriya" va "Matematik analiz" bo'limlar bo'yicha mos ravishda 60, 40 va 100 ta savollar mavjud va ularga talaba mos ravishda 0.8, 0.7 va 0.6 ehtimol bilan to'g'ri javob bera oladi. Yakuniy imtihonda talaba unga tasodifiy ravishda berilgan savolga to'g'ri javob berdi. Bu savol "Algebra" bo'limiga tegishli ekanligi ehtimolini hisoblang.

A) 24/71 . B) 28/71. C) 60/71 .
D) 42/71 . E) 39/71 .

15. "Oliy matematika" fani bo'yicha yakuniy imtihonda "Algebra", "Analitik geometriya" va "Matematik analiz" bo'limlar bo'yicha mos ravishda 60, 40 va 100 ta savollar mavjud va ularga talaba mos ravishda 0.8, 0.7 va 0.6 ehtimol bilan to'g'ri javob bera oladi. Yakuniy imtihonda talaba unga tasodifiy ravishda berilgan savolga to'g'ri javob berdi. Bu savol "Analitik geometriya" bo'limiga tegishli ekanligi ehtimolini hisoblang.

A) 24/71 . B) 28/71. C) 60/71 . D) 42/71 . E) 39/71 .

5. Bernulli formulasi va Muavr-Laplas, Puasson teoremalari

1. Bernulli sxemasi uchun quyidagi shartlardan qaysi biri talab etilmaydi?

A) Barcha sinovlarda A tasodifiy hodisa ehtimolligi P(A)=p o'zgarmas .

B) Har bir sinov natijasi oldingi sinovlar natijalariga bog'liq emas .

C) Har bir sinov natijasi keyingi sinov natijalariga ta'sir etmaydi .

D) Barcha sinovlar soni chekli .

E) Barcha shartlar talab etiladi.

2. Quyidagilardan qaysi biri Bernulli sxemasi orqali ifodalanmaydi?

A) n ta simmetrik tanga tashlanganda ulardan m tasini gerb tomoni bilan tushishi .

B) n ta o'yin soqqasi tashlanganda ulardan m tasida 3 ochko chiqishi .

C) n ta talabaga bir xil test berilganda ulardan m tasini to'g'ri javob berishi .

D) n ta bir xil mahsulot tekshirilganda ulardan m tasini sifatli chiqishi .

E) Keltirilganlarning barchasi Bernulli sxemasi orqali ifodalanadi .

3. Bog'liqmas sinovlar n marta takrorlanganda ehtimoli P(A)=p bo'lgan A hodisa k marta ro'y berishini ifodalovchi $P_n(m)$ ehtimol uchun Bernulli formulasi qayerda to'g'ri ifodalangan (q=1−p).

A) $P_n(m)=C_n^m p^m q^{n-m}$. B) $P_n(m)=C_n^m q^m p^{n-m}$.

C) $P_n(m)=C_n^m p^m q^n$. D) $P_n(m)=C_n^m p^n q^m$.

E) $P_n(m)=C_n^m p^m q^{n+m}$.

4. Bernulli formulasida P(A)=0.6 bo'lgan holda $P_4(3)$ ehtimol qiymati nimaga teng bo'ladi?

A) 0.3456 . B) 0.2304 . C) 0.1536 .
D) 0.1228 . E) 0.1096 .

5. Simmetrik o'yin soqqasi n=3 marta tashlanganda m=2 marta 4 ochko chiqish ehtimolini hisoblang.

A) 5/6 . B) 5/12 . C) 5/18 . D) 5/36 . E) 5/72

6. Simmetrik tanga 10 marta tashlanganda 6 marta raqam tomoni bilan tushish ehtimolini aniqlang.

A) $C_{10}^{6}(0.5)^{6}$. B) $C_{10}^{6}(0.5)^{4}$. C) $C_{10}^{6}(0.5)^{10}$.
D) $C_{10}^{6}(0.5)^{16}$. E) $C_{10}^{4}(0.5)^{6}$.

7. Mahsulot 0.8 ehtimol bilan sifatli bo'lishi mumkin. Tekshirilgan 20 ta mahsulotdan 15 tasi sifatli bo'lish ehtimoli nimaga teng?

A) $C_{20}^{15}(0.8)^{5}(0.2)^{15}$. B) $C_{20}^{5}(0.8)^{5}(0.2)^{15}$.
C) $C_{20}^{15}(0.8)^{15}(0.2)^{20}$. D) $C_{20}^{15}(0.8)^{15}(0.2)^{5}$.
E) $C_{20}^{5}(0.8)^{15}(0.2)^{5}$.

8. Har biri p=0.8 ehtimol bilan sifatli bo'lishi mumkin bo'lgan n=10 ta mahsulot tekshirilganda, ulardan m=7 tasi sifatli bo'lishi ehtimoli $P_{10}(7)$ nimaga teng?

A) $P_{10}(7)= 0.64\cdot0.8^7$. B) $P_{10}(7)= 0.82\cdot0.8^7$.
C) $P_{10}(7)=0.96\cdot0.8^7$. D) $P_{10}(7)= 0.98\cdot0.8^7$.
E) $P_{10}(7)=7/10$.

9. Bernulli sxemasida sinovlar soni n, P(A)=p , $q=1-p$ bo'lsa, A hodisani eng katta ehtimol bilan ro'y berishi soni m_0 qaysi munosabatdan topiladi?

A) $qn-q<m_0<pn+p$. B) $pn-q<m_0<pn+p$. C) $qn-q<m_0<pn-p$.
D) $pn+q<m_0<pn-p$. E) $pn+q<m_0<pn+p$.

10. Har biri p=0,76 ehtimollik bilan sifatli bo'lishi mumkin bo'lgan n=10 ta mahsulot tekshirilganda, eng katta ehtimol bilan nechta mahsulot sifatli bo'lishi mumkin?

A) 5 . B) 6 . C) 7 . D) 8 . E) 9.

11. Bernulli sxemasida sinovlar soni n yetarli katta bo'lganda

$$P_n(m)=C_n^m p^m q^{n-m}\ (q=1-p,\ m=1,2,3\ )$$

ehtimolning aniq qiymatini hisoblashda paydo bo'ladigan qiyinchilik qayerda noto'g'ri ko'rsatilgan?

A) $n!$ juda katta son bo'ladi .

B) $(n-m)!$ juda katta son bo'ladi .

C) q^{n-m}darajali ifoda juda kichik son bo'ladi .

D) p^mdarajali ifoda juda kichik son bo'ladi .

E) barcha qiyinchiliklar to'g'ri ko'rsatilgan .

12. Bernulli sxemasida sinovlar soni n yetarli katta bo'lganda

$$P_n(m)=C_n^m p^m q^{n-m}\ (q=1-p\ )$$

ehtimol uchun Muavr–Laplasning lokal teoremasini ifodalovchi

$$P_n(m)\approx\frac{1}{\sqrt{npq}}\varphi(x),\quad x=\frac{m-np}{\sqrt{npq}},$$

taqribiy formuladagi $\phi(x)$ funksiya qaysi javobda to'g'ri ifodalangan?

A) $\varphi(x)=\frac{1}{\sqrt{2\pi}}\sin(x^2/2)$. B) $\varphi(x)=\frac{1}{\sqrt{2\pi}}\cos(x^2/2)$.

C) $\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$. D) $\varphi(x)=\frac{1}{\sqrt{2\pi}}10^{-x^2/2}$.

E) $\varphi(x)=\frac{1}{\sqrt{2\pi}}\ln(x^2/2)$.

13. Bernulli sxemasida sinovlar soni n yetarli katta bo'lganda

$$P_n(m)=C_n^m p^m q^{n-m}\ (q=1-p\ )$$

ehtimol uchun Muavr–Laplasning lokal teoremasini ifodalovchi

$$P_n(m)\approx\frac{1}{\sqrt{npq}}\varphi(x),\quad \varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

taqribiy formuladagi x qiymati nimaga teng bo'ladi?

A) $x=\frac{m-np}{\sqrt{npq}}$. B) $x=\frac{n-mp}{\sqrt{npq}}$. C) $x=\frac{m+np}{\sqrt{npq}}$.

D) $x=\frac{n+mp}{\sqrt{npq}}$. E) $x=\frac{mp-nq}{\sqrt{npq}}$.

14. n=600 bog'liqmas sinovlarning har birida o'zgarmas p=0.6 ehtimolga ega bo'lgan A tasodifiy hodisani bu sinovlarda m=372 marta ro'y berishini ifodalovchi $P_{600}(372)$ ehtimolning taqribiy qiymatini Muavr-Laplasning lokal teoremasi va $\phi(1)$=0.242 ekanligidan foydalanib toping.

A) 242/5553 . B) 121/4000 . C) 242/4557 .
D) 121/6000 . E) 6/25.

15. Partiyadagi n=2100 ta mahsulotning har biri p=0.7 ehtimol bilan sifatli bo'lishi mumkin. Bu partiyadagi sifatli mahsulotlar soni m=1491 ta bo'lishini ifodalovchi $P_{2100}(1491)$ ehtimolning taqribiy qiymatini Muavr-Laplasning lokal teoremasi va $\phi(1)$=0.242 ekanligidan foydalanib toping.

A) 0.0115 . B) 0.0195 . C) 0.0015 .
D) 0.0095 . E) 0.1195.

16. Bernulli sxemasida sinovlar soni n yetarli katta bo'lganda

$$P_n(m_1,m_2)=\sum_{m=m_1}^{m_2} C_n^m p^m q^{n-m} \quad (q=1-p)$$

ehtimol uchun Muavr–Laplasning integral teoremasini ifodalovchi

$$P_n(m)\approx \Phi(x_2)-\Phi(x_1), \quad x_i=\frac{m_i-np}{\sqrt{npq}},\ i=1,2 \quad ,$$

taqribiy formuladagi $\Phi(x)$ funksiya qaysi javobda to'g'ri ko'rsatilgan?

A) $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x \sin(t^2/2)dt$. B) $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x \cos(t^2/2)dt$.

C) $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x e^{-t^2/2}dt$. D) $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x \ln(t^2/2)dt$.

E) $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x 10^{-t^2/2}dt$.

6.A Asosiy diskkret taqsimotlar va ularning sonli xarakteristikalari

1. Binomial taqsimot ifodalangan javobni toping.

A) $P\{X = k\} = pq^{k-1}$. B) $P\{X = k\} = C_n^k p^k q^{n-k}$.

C) $P\{X = k\} = \dfrac{\lambda^k}{k!}e^{-\lambda}$. D) $P\{X = k\} = \dfrac{C_M^k C_{N-M}^{n-k}}{C_N^n}$.

E) $P\{X = k\} = C_{r+k-1}^{r-1} p^r q^k$.

2. $P\{X = k\} = C_n^k p^k q^{n-k}$ binomial taqsimotdagi p va q orasidagi bog'lanish qayerda to'g'ri ko'rsatilgan?

A) $p^2+q^2=1$. B) $pq=1$. C) $p+q=1$.

D) $p+q+pq=1$. E) $p+q-pq=1$.

3. $P\{X = k\} = C_n^k p^k q^{n-k}$ binomial taqsimotdagi p va q uchun quyidagi tengliklardan qaysi biri o'rinli emas ?

A) $p+q=1$. B) $|p|+q=p+|q|=1$. C) $|p+q|=1$.

D) $|p|+|q|=1$. E) Barcha tengliklar o'rinli .

4. $P_n(k) = C_n^k p^k q^{n-k}$ binomial taqsimotli X tasodifiy miqdorning matematik kutilishi M(X) qaysi formula bilan hisoblanadi?

A) M(X)=nq . B) M(X)=npq . C) M(X)=np .

D) M(X)=n/p . E) M(X)=n/q.

5. $P_n(k) = C_n^k p^k q^{n-k}$ binomial taqsimotli X tasodifiy miqdorning D(X) dispersiyasini hisoblash formulasi qayerda to'g'ri ko'rsatilgan?

A) D(X)=nq . B) D(X)=npq . C) D(X)=np .

D) D(X)=n/p . E) D(X)=n/q.

6. $P_n(k) = C_n^k p^k q^{n-k}$ binomial taqsimotda matematik kutilish M(X)=8 va dispersiya D(X)=4.8 bo'lsa, n parametrning qiymati nimaga teng?

A) 20 . B) 10 . C) 15 . D) 40 . E) 25 .

7. $P_n(k) = C_n^k p^k q^{n-k}$ binomial taqsimotda matematik kutilish M(X)=8 va dispersiyasi D(X)=4.8 bo'lsa, p parametrning qiymati nimaga teng?

A) 0.8 . B) 0.6 . C) 0.4 . D) 0.2 . E) 0.5 .

8. $P(X=k)=C_{15}^{k}\cdot 0.8^{k}\cdot 0.2^{15-k}$ binomial taqsimotga ega X tasodifiy miqdorning M(X) matematik kutilishining qiymati nimaga teng ?

A) 3 . B) 2.4 . C) 18.75 . D) 75 . E) 12 .

9. $P(X=k)=C_{15}^{k}\cdot 0.8^{k}\cdot 0.2^{15-k}$ binomial taqsimotga ega X tasodifiy miqdorning D(X) dispersiyasining qiymati nimaga teng ?

A) 3 . B) 2.4 . C) 18.75 . D) 75 . E) 12 .

10. Parametrlari n=4 va p=0.8 bo'lgan binomial taqsimotga ega X tasodifiy miqdor uchun P{X=2} ehtimol qiymatini hisoblang.

A) 0.2432 . B) 0.3286 . C) 0.1536 .
D) 0.4354 . E) 0.0784 .

11. Puasson taqsimoti ifodalangan javobni toping.

A) $P\{X=k\}=pq^{k-1}$. B) $P\{X=k\}=C_n^k p^k q^{n-k}$.

C) $P\{X=k\}=\frac{\lambda^k}{k!}e^{-\lambda}$. D) $P\{X=k\}=\frac{C_M^k C_{N-M}^{n-k}}{C_N^n}$.

E) $P\{X=k\}=C_{r+k-1}^{r-1}p^r q^k$.

12. $P(k)=\lambda^k e^{-\lambda}/k!$ Puasson taqsimotida λ parametr qaysi shartni qanoatlantiradi ?

A) λ≤0 . B) λ≥0 . C) λ<0 . D) λ>0 . E) λ≠0 .

13. $P(k)=\lambda^k e^{-\lambda}/k!$ Puasson taqsimotga ega X tasodifiy miqdorning M(X) matematik kutilishi qaysi formula bilan topiladi?

A) M(X)=1/λ . B) M(X)= 1/λ² . C) M(X)= λ .
D) M(X)= λ² . E) M(X)= λ+1/λ .

14. $P(k)=\lambda^k e^{-\lambda}/k!$ Puasson taqsimotiga ega X tasodifiy miqdorning D(X) dispersiyasi qaysi formula bilan topiladi?

A) D(X)=1/λ . B) D(X)= 1/λ² . C) D(X)= λ .
D) D(X)= λ² . E) D(X)= λ+1/λ .

15. Puasson taqsimotida matematik kutilish M(X) va dispersiya D(X) orasida qaysi munosabat o'rinli bo'ladi?

A) M(X)<D(X) . B) M(X)>D(X) .
C) M(X)≠D(X) . D) M(X)=D(X) .

E) Keltirilgan barcha munosabatlar o'rinli bo'ladi .

16. $P(k) = 3^k e^{-3} / k!$ Puasson taqsimotga ega X tasodifiy miqdorning M(X) matematik kutilishi nimaga teng?

A) M(X)=1/3 . B) M(X)= 1/9 . C) M(X)= 3 .
D) M(X)= 9 . E) M(X)= 10/3 .

7. Ehtimollarning taqsimot va zichlik funksiyalari. Uzluksiz tasodifiy miqdorlarning sonli xarakteristikalari

1. X tasodifiy miqdorning F(x) taqsimot funksiyasi qaysi formula bilan aniqlanadi?

A) F(x)=P{X>x} . B) F(x)=1+P{X>x}
C) F(x)=1−P{X<x} . D) F(x)=P{X<x} .
E) F(x)=P{X=x} .

2. Diskret X tasodifiy miqdor ushbu taqsimot qonuni orqali berilgan:

x_i	−1	3	8
p_i	0.3	0.5	0.2

Bu tasodifiy miqdorning F(x) taqsimot funksiyasini toping.

A) $F(x)=\begin{cases} 0, & x<-1 \\ 0.3, & -1\le x<3 \\ 0.5, & 3\le x<8 \\ 1, & x\ge 8 \end{cases}$. B) $F(x)=\begin{cases} 0, & x<-1 \\ 0.5, & -1\le x<3 \\ 0.8, & 3\le x<8 \\ 1, & x\ge 8 \end{cases}$.

C) $F(x)=\begin{cases} 0.3, & x<-1 \\ 0.5, & -1\le x<3 \\ 0.8, & 3\le x<8 \\ 1, & x\ge 8 \end{cases}$. D) $F(x)=\begin{cases} 0, & x<-1 \\ 0.3, & -1\le x<3 \\ 0.8, & 3\le x<8 \\ 1, & x\ge 8 \end{cases}$.

E) $F(x)=\begin{cases} 0, & x<-1 \\ 0.3, & -1\le x<3 \\ 0.5, & 3\le x<8 \\ 0.8, & x\ge 8 \end{cases}$.

3. Ixtiyoriy F(x) taqsimot funksiyasi uchun quyidagi tasdiqlardan qaysi biri o'rinli emas?

A) F(x) – kamaymovchi funksiya .
B) Ixtiyoriy x uchun 0≤F(x)≤1 .

C) $F(-\infty)=\lim\limits_{x\to-\infty}F(x)=0$. D) $F(+\infty)=\lim\limits_{x\to+\infty}F(x)=1$.

E) F(x) – uzluksiz funksiya .

4. Ixtiyoriy F(x) taqsimot funksiyasi uchun $F(-\infty)=\lim\limits_{x\to-\infty}F(x)$ qiymati nimaga teng?

A) $-\infty$. B) –1 . C) 0 . D) 1. E) aniqlanmagan .

5. Ixtiyoriy F(x) taqsimot funksiyasi uchun $F(+\infty)=\lim\limits_{x\to+\infty}F(x)$ qiymati nimaga teng?

A) $-\infty$. B) –1 . C) 0 . D) 1. E) aniqlanmagan .

6. X tasodifiy miqdorning taqsimot funksiyasi F(x) orqali P{a≤X<b} ehtimol

qanday topiladi?

A) P{a≤X<b}=F(a)F(b) . B) P{a≤X<b}=F(a)+F(b) .

C) P{a≤X<b}= F(b)–F(a) . D) P{a≤X<b}=F(a)–F(b) .

E) P{a≤X<b}=F(a)+F(b)– F(a)F(b) .

7. X tasodifiy miqdor ushbu F(x) taqsimot funksiyasi bilan berilgan:

$$F(x)=\begin{cases}0, & x<-2\\ (x+2)/7, & -2\le x<5\\ 1, & x\ge 5\end{cases}.$$

P{–3≤X<4} ehtimol qiymati nimaga teng?

A) 2/7 . B) 3/7 . C) 4/7 . D) 5/7 . E) 6/7 .

8. X tasodifiy miqdor ushbu F(x) taqsimot funksiyasi bilan berilgan:

$$F(x)=\begin{cases}0, & x<-2\\ (x+2)/7, & -2\le x<5\\ 1, & x\ge 5\end{cases}.$$

P{–1≤X<3} ehtimol qiymati nimaga teng?

A) 2/7 . B) 3/7 . C) 4/7 . D) 5/7 . E) 6/7 .

9. X tasodifiy miqdor ushbu F(x) taqsimot funksiyasi bilan berilgan:

$$\mathrm{F}(x)=\frac{1}{2}+\frac{1}{\pi}arctgx,\ x\in(-\infty,\infty)\ .$$

P{–1≤X<1} ehtimol qiymati nimaga teng?

A) $2/\pi$.   B) $\pi/4$.   C) 3/4 .   D) 1/2 .   E) 0 .

10.X tasodifiy miqdorning $f(x)$ zichlik funksiyasi uning $F(x)$ taqsimot funksiyasi orqali qanday aniqlanadi?

A) $f(x)=\int F(x)dx$.   B) $f(x)=F(x)+F(-x)$.

C) $f(x)=dF(x)$.   D) $f(x)=F(x)-F(-x)$.   E) $f(x)=F'(x)$.

11.Ixtiyoriy $f(x)$ zichlik funksiyasi uchun quyidagi tasdiqlardan qaysi biri o'rinli emas?

A) Ixtiyoruy x uchun $f(x)\geq 0$.

B) Ixtiyoruy x uchun $f(x)\leq 1$.

C) Agar $f(x)$ zichlik funksiyasi bo'lsa, $F(x)=\int\limits_{-\infty}^{x} f(t)dt$ taqsimot funksiyasi bo'ladi.

D) $\int\limits_{-\infty}^{\infty} f(t)dt=1$.   E) $P\{a<X<b\}=\int\limits_{a}^{b} f(x)dx$.

12. Agar X tasodifiy miqdorning zichlik funksiyasi $f(x)$ va $a<b$ bo'lsa quyidagi tengliklardan qaysi biri o'rinli bo'lmaydi?

A) $P\{a<X<b\}=\int\limits_{a}^{b} f(x)dx$.   B) $P\{a\leq X<b\}=\int\limits_{a}^{b} f(x)dx$.

C) $P\{a<X\leq b\}=\int\limits_{a}^{b} f(x)dx$.   D) $P\{a\leq X\leq b\}=\int\limits_{a}^{b} f(x)dx$.

E) Keltirilgan barcha tengliklar o'rinli .

13. Ixtiyoriy zichlik funksiyasi $f(x)$ uchun $\int\limits_{-\infty}^{\infty} f(x)dx$ integral qiymati nimaga teng bo'ladi?

A) –2 .   B) –1 .   C) 0 .   D) 1 .   E) 2 .

14. X tasodifiy miqdor ushbu $F(x)$ taqsimot funksiyasi bilan berilgan:

$$F(x)=\begin{cases} 0, & x<0 \\ 2\sin 3x, & 0\leq x<\pi/18. \\ 1, & x\geq \pi/18 \end{cases}$$

Shu tasodifiy miqdorning $f(x)$ zichlik funksiyasini toping.

A) $f(x)=\begin{cases} 2\sin 3x, & x\in[0,\pi/18) \\ 0, & x\notin[0,\pi/18) \end{cases}$

B) $f(x)=\begin{cases} 2\cos 3x, & x\in[0,\pi/18) \\ 0, & x\notin[0,\pi/18) \end{cases}$

C) $f(x)=\begin{cases} 6\cos 3x, & x\in[0,\pi/18) \\ 0, & x\notin[0,\pi/18) \end{cases}$

D) $f(x)=\begin{cases} 6\sin 3x, & x\in[0,\pi/18) \\ 0, & x\notin[0,\pi/18) \end{cases}$.

E) $f(x)=\begin{cases} \sin 3x, & x\in[0,\pi/18) \\ 0, & x\notin[0,\pi/18) \end{cases}$.

15. X tasodifiy miqdor $f(x)=\dfrac{1}{\pi(x^2+1)}$, $x\in(-\infty,\infty)$ zichlik funksiyasi bilan berilgan. Bu tasodifiy miqdorning F(x) taqsimot funksiyasini toping .

A) F(x)=$\dfrac{1}{\pi}arctgx$. B) F(x)=$1+\dfrac{1}{\pi}arctgx$. C) F(x)=$1-\dfrac{1}{\pi}arctgx$

D) F(x)=$\dfrac{1}{2}-\dfrac{1}{\pi}arctgx$. E) F(x)=$\dfrac{1}{2}+\dfrac{1}{\pi}arctgx$.

16. X tasodifiy miqdorning zichlik funksiyasi $f(x)$ quyidagi ko'rinishga ega:

$$f(x)=\begin{cases} ax^2, & x\in[-2,3] \\ 0, & x\notin[-2,3] \end{cases}.$$

Bu funksiyadagi a parametr qiymati nimaga teng?

A) 3/35 . B) 6/35 . C) 8/35 .

D) 9/35 . E) 13/35 .

8. Asosiy uzluksiz taqsimotlar va ularning sonli xarakteristikalari

1. Tekis taqsimotning $f(x)$ zichlik funksiyasini ko'rsating.

A) $f(x)=\begin{cases} 1/(b-a), & x\in[a,b] \\ 0, & x\notin[a,b] \end{cases}$.

B) $f(x)=\begin{cases} \lambda e^{-\lambda x}, & x\geq 0 \\ 0, & x<0 \end{cases}$.

C) $f(x)=\dfrac{1}{\pi(1+x^2)}$.

D) $f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\dfrac{(x-a)^2}{2\sigma^2}\right\}$.

E) $f(x)=\frac{2}{\pi}\cdot\frac{1}{e^{-x}+e^{x}}$.

2. $[a,b]$ kesmada tekis taqsimlangan X tasodifiy miqdorning zichlik funksiyasi $f(x)$ qayerda to'g'ri ifodalangan?

A) $f(x)=\begin{cases} b-a, & x\in[a,b] \\ 1, & x\notin[a,b] \end{cases}$.

B) $f(x)=\begin{cases} b-a, & x\in[a,b] \\ 0, & x\notin[a,b] \end{cases}$.

C) $f(x)=\begin{cases} 1/(b-a), & x\in[a,b] \\ 0, & x\notin[a,b] \end{cases}$.

D) $f(x)=\begin{cases} 1/(b-a), & x\in[a,b] \\ 1, & x\notin[a,b] \end{cases}$.

E) $f(x)=\begin{cases} 1/(b-a), & x\in[a,b] \\ 1/(b+a), & x\notin[a,b] \end{cases}$.

3. $[-3,7]$ kesmada tekis taqsimlangan X tasodifiy miqdorning $f(x)$ zichlik funksiyasi $x\in[-3,7]$ bo'lganda qanday qiymatni qabul etadi?

A) 10 . B) 4 . C) 1/10 . D) 1/4 . E) 1 .

4. $[a,b]$ kesmada tekis taqsimlangan X tasodifiy miqdorning F(x) taqsimot funksiyasining $x\in[a,b]$ bo'lgandagi ifodasi qayerda to'g'ri ko'rsatilgan?

A) $F(x)=\frac{b-x}{b-a}$. B) $F(x)=\frac{x+b}{b-a}$.

C) $F(x)=\frac{x-a}{b-a}$. D) $F(x)=\frac{x+a}{b-a}$.

E) $F(x)=\frac{(x-a)(b-x)}{b-a}$.

5. $[-3,7]$ kesmada tekis taqsimlangan X tasodifiy miqdor uchun $P\{-1\le X\le 4\}$ ehtimolni hisoblang.

A) 0.3 . B) 0.7 . C) 0.4 . D) 0.5 .

E) 0.8 .

6. $[-5,10]$ kesmada tekis taqsimlangan X tasodifiy miqdor uchun $P\{c\le X\le d\}=0.6$ bo'lsa, $d-c$ qiymati aniqlang.

A) 3 . B) 7 . C) 9 .
D) 12 . E) aniqlab bo'lmaydi .

7. [a,b] kesmada tekis taqsimlangan X tasodifiy miqdorning F(x) taqsimot funksiyasi uchun quyidagi tasdiqlardan qaysi biri noto'g'ri?

A) $x<a$ bo'lganda F(x)=0 .
B) $x>b$ bo'lganda F(x)=1.
C) $x \in [a,b]$ bo'lganda F(x)=(x–a)/(b–a) .
D) $x \in (-\infty,\infty)$ bo'lganda F(x) uzluksiz funksiya .
E) Keltirilgan barcha tasdiqlar to'g'ri .

8. [–3,7] kesmada tekis taqsimlangan X tasodifiy miqdorning F(x) taqsimot funksiyasi $x \in [-3,7]$ bo'lganda qanday ifodalanadi?

A) F(x)=(7–x)/10 . B) F(x)=(7–x)/4 .
C) F(x)=(x+3)/10 . D) F(x)=(x+3)/4 .
E) F(x)=(10–x)/10 .

9. [a,b] kesmada tekis taqsimlangan X tasodifiy miqdorning M(X) matematik kutilishi qaysi formula bilan hisoblanadi ?

A) M(X)=(b–a)/2 . B) M(X)=(a+b)/2 .
C) M(X)=(b–a)/ (a+b) . D) M(X)=(a+b)/ (b–a) .
E) M(X)=(b^2–a^2)/2 .

10.[–3,7] kesmada tekis taqsimlangan X tasodifiy miqdorning M(X) matematik kutilishi nimaga teng?

A) 5 . B) 0 . C) 2.5 . D) 2 . E) 0.4 .

11. [a,b] kesmada tekis taqsimlangan X tasodifiy miqdorning D(X) dispersiyasi qaysi formula bilan hisoblanadi?

A) D(X)=(b+a)/12 . B) D(X)=(b+a)^2/12 .
C) D(X)=(b–a)/12 . D) D(X)=(b–a)^2/12 .
E) D(X)=(b^2–a^2) /12

12.[–3,7] kesmada tekis taqsimlangan X tasodifiy miqdorning D(X) dispersiyasi nimaga teng?

A) 1/3 . B) 4/3 . C) 5/6 .
D) 25/3 . E) 10/3 .

13. [–3,7] kesmada tekis taqsimlangan X tasodifiy miqdor uchun M(X)+3D(X) yig'indi qiymatini toping.

A) 29 . B) 27 . C) 25 . D) 23 . E) 21 .

14. Normal taqsimotning $f(x)$ zichlik funksiyasini ko'rsating.

A) $f(x)=\begin{cases} 1/(b-a), & x\in[a,b] \\ 0, & x\notin[a,b] \end{cases}$. B) $f(x)=\begin{cases} \lambda e^{-\lambda x}, & x\geq 0 \\ 0, & x<0 \end{cases}$.

C) $f(x)=\dfrac{1}{\pi(1+x^2)}$. D) $f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\dfrac{(x-a)^2}{2\sigma^2}\right\}$

E) $f(x)=\dfrac{2}{\pi}\cdot\dfrac{1}{e^{-x}+e^{x}}$.

15. X tasodifiy miqdor zichlik funksiyasi

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-a)^2}{2\sigma^2}\right\}$$

bo'lgan normal taqsimotga ega. Bu taqsimotdagi a parametr nimani ifodalaydi?

A) a=P{X<0} . B) a=P{X>0} . C) a=M(X) .

D) a=D(X) . E) a=σ(X) .

16. X tasodifiy miqdor zichlik funksiyasi

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-a)^2}{2\sigma^2}\right\}$$

bo'lgan normal taqsimotga ega. Bu taqsimotdagi σ^2 nimani ifodalaydi?

A) σ^2=P{X<0} . B) σ^2=P{X>0} . C) σ^2=M(X) .

D) σ^2=D(X) . E) σ^2=M(X^2) .

9. Statistik taqsimotlar

1. Matematik statistikada X tasodifiy miqdor haqidagi xulosalar nima asosida chiqariladi?

A) X tasodifiy miqdorning taqsimot qonuni asosida .

B) X tasodifiy miqdorning taqsimot funksiyasi asosida .

C) X tasodifiy miqdorning zichlik funksiyasi asosida .

D) X tasodifiy miqdorning sonli xarakteristikalari asosida .

E) X tasodifiy miqdor ustida o'tkazilgan kuzatuv natijalari asosida .

2. Tanlanma qanday ta'riflanadi?

A) O'rganilayotgan X tasodifiy miqdorning mumkin bo'lgan barcha qiymatlar to'plami tanlanma deyiladi.

*B) O'rganilayotgan X tasodifiy miqdorning statistik kuzatuvlarda ro'y bergan qiymatlari to'plami tanlanma deyiladi.

C) O'rganilayotgan X tasodifiy miqdorning $f(x)$ zichlik funksiyasining kuzatilgan qiymatlari to'plami tanlanma deyiladi.

D) O'rganilayotgan X tasodifiy miqdorning $F(x)$ taqsimot funksiyasining kuzatilgan qiymatlari to'plami tanlanma deyiladi.

E) O'rganilayotgan X tasodifiy miqdorning kuzatilgan eng katta va eng kichik qiymatlari to'plami tanlanma deyiladi.

3. Tanlanma hajmi ta'rifi qayerda to'g'ri ifodalangan?

A) Tanlanmadagi musbat qiymatli variantalar soni tanlanma hajmi deyiladi.

B) Tanlanmadagi manfiy qiymatli variantalar soni tanlanma hajmi deyiladi.

C) Tanlanmadagi noldan farqli qiymatli variantalar soni tanlanma hajmi deyiladi.

D) Tanlanmadagi qiymati nolga teng bo'lgan variantalar soni tanlanma hajmi deyiladi.

E) Tanlanmadagi barcha variantalar soni tanlanma hajmi deyiladi.

4. Berilgan 5, 2, 4, 0, –2, –5, 0, 4, –1, 2, 0, 0 tanlanma hajmi nimaga teng?

A) 3 . B) 4 . C) 12 . D) 8 . E) 5 .

5. X tasodifiy miqdor ustidagi 10 ta kuzatuv natijalari 5, 2, 4, 0, 2, 5, 0, 4, 1, 2 tanlanmadan iborat. Bu tanlanmaning variatsion qatorini ko'rsating.

A) 5, 5, 2, 2, 2, 4, 4, 0, 0, 1 . B) 2, 2, 2, 5, 5, 4, 4, 0, 0, 1 .
C) 4, 4, 5, 5, 2, 2, 2, 0, 0, 1 . D) 0, 0, 1, 2, 2, 2, 4, 4, 5, 5 .
E) 5, 5, 2, 2, 2, 0, 0, 1, 4, 4 .

6. Berilgan 5, –3, 2, –1, 0 tanlanmaning variatsion qatorini yozing.

A) 0, –1, 2, –3, 5 . B) 0, 5, 2, –3, –1 .
C) –1, –3, 2, 0, 5 . D) –3, –1, 0, 2, 5 .

E) 0, 1, 2, 3, 5.

7. X belginingx_1, x_2, x_3,..., x_n variantalarining chastotalalri qanday ta'riflanadi?

*A) X belgining x_1, x_2, x_3,..., $x_n$variantalarining tanlanmada necha marta uchrashishini ifodalovchi sonlarga chastotalar deyiladi.

B) X belgining x_1, x_2, x_3,..., $x_n$variantalarining tanlanmadagi umumiy soniga chastotalar deyiladi.

C) X belgining x_1, x_2, x_3,..., $x_n$tanlanmadagi musbat qiymatli variantalarining soniga chastotalar deyiladi.

D) X belgining x_1, x_2, x_3,..., $x_n$tanlanmadagi manfiy qiymatli variantalarining soniga chastotalar deyiladi.

E) X belgining x_1, x_2, x_3,..., $x_n$tanlanmadagi nol qiymatli variantalarining soniga chastotalar deyiladi.

8. Berilgan 5, 2, 4, 0, 2, 5, 0, 4, 1, 2 tanlanmadagi x=2 variantaning chastotasi nimaga teng?

A) 2 . B) 3 .C) 4 . D) 5 . E) 0.3 .

9. Berilgan 5, 2, 4, 0, 2, 5, 0, 4, 1, 2 tanlanmadagi eng katta chastotali variantani aniqlang.

A) 0 . B) 1 . C) 2 . D) 4 . E) 5 .

10. Hajmi n bo'lgan tanlanmadagi variantalarning chastotalarin_1, n_2, n_3,..., $n_m$bo'lsa, quyidagi tengliklardan qaysi biri o'rinli bo'ladi?

A) $\max(n_1, n_2, n_3,..., n_m)=n$. B) $\min(n_1, n_2, n_3,..., n_m)=n$.

C) $n_1+ n_2+ n_3+...+ n_m=n$. D)$n_1\cdot n_2\cdot n_3\cdot...\cdot n_m=n$.

E) To'g'ri javob keltirilmagan .

11. Hajmi n bo'lgan tanlanmadagi variantalarning chastotalarin_1, n_2, n_3,..., $n_m$bo'lsa, $w_k$nisbiy chastotalar qanday aniqlanadi ?

A) $w_k = n_k\cdot n$. B) $w_k = n_k+n$. C) $w_k = n_k/n$.

D) $w_k =n/n_k$. E) $w_k =n-n_k$.

12. Hajmi n bo'lgan tanlanmadagi variantalarning nisbiy chastotalariw_1, w_2, w_3,..., w_m bo'lsa, quyidagi tengliklardan qaysi biri o'rinli bo'ladi?

A) $w_1+ w_2+ w_3+...+ w_m=n$. B) $w_1+ w_2+ w_3+...+ w_m=1$.

C)$w_1+ w_2+ w_3+...+ w_m=0$. D) $w_1\cdot w_2\cdot w_3\cdot...\cdot w_m=1$.

E) $w_1 \cdot w_2 \cdot w_3 \cdot \ldots \cdot w_m = n$ ..

13. Berilgan 5, 2, 4, 0, 2, 5, 0, 4, 1, 2 tanlanmadagi $x=2$ variantaning nisbiy chastotasi nimaga teng?

A) 2 . B) 3 . C) 4 . D) 0.2 . E) 0.3

14. Hajmi n bo'lgan tanlanmadagi variantalarning nisbiy chastotalari w_i ($i=1,2, \ldots, m$) bo'lsa, variantalarning n_i ($i=1,2, \ldots, m$) chastotalari qanday aniqlanadi?

A) $n_i=n/w_i$. B) $n_i=w_i/n$. C) $n_i=n+w_i$. D) $n_i=nw_i$.

E) Nisbiy w_i chastotalar bo'yicha n_i chastotalarni aniqlab bo'lmaydi .

10. Statistik baholar. Tanlanma o'rta qiymat va dispersiya

1. *Ta'rifni yakunlang:* Berilgan $x_1, x_2, \ldots, x_n$ tanlanma bo'yicha noma'lum θ parametr uchun statistuk baho deb ..... .

A) tanlanmadagi kuzatuv natijalarining eng kattasiga aytiladi .

B) tanlanmadagi kuzatuv natijalarining eng kichigiga aytiladi .

C) tanlanmadagi kuzatuv natijalarining o'rta arifmetik qiymatiga aytiladi .

D) tanlanmadagi kuzatuv natijalarining o'rta geometrik qiymatiga aytiladi .

E) tanlanmadagi kuzatuv natijalaridan tuzilgan ma'lum bir funksiyaga aytiladi .

2. Noma'lum θ parametr uchun tuzilgan θ_n^* statistik baho qaysi shartda siljimagan baho deyiladi?

A) $M(\theta_n^*) > \theta$. B) $M(\theta_n^*) = \theta$. C) $M(\theta_n^*) < \theta$.

D) $M(\theta_n^*) \neq \theta$. E) $\lim\limits_{n\to\infty} M(\theta_n^*) = 0$.

3. Noma'lum θ parametr uchun tuzilgan θ_n^* statistik baho qaysi shartda asosli baho deyiladi?

A) $D(\theta_n^*) > \theta$. B) $D(\theta_n^*) = \theta$. C) $D(\theta_n^*) < \theta$.

D) $D(\theta_n^*) \neq \theta$. E) $\lim\limits_{n\to\infty} D(\theta_n^*) = 0$.

4. Noma'lum θ parametr uchun tuzilgan va dispersiyasi $D(\theta_n^*) = f(\theta, n)$ bo'lgan statistik baho qaysi holda asosli baho bo'ladi?

A) $f(\theta,n)=\theta+\frac{1}{n}$. B) $f(\theta,n)=\theta+\frac{n}{n+1}$. C) $f(\theta,n)=\frac{\theta}{n}$.

D) $f(\theta,n)=\frac{n\theta}{n+1}$. E) $f(\theta,n)=\theta^{1/n}$.

5. *Ta'rifni yakunlang:* Agarda noma'lum θ parametr uchun barcha statistik baholar to'plamida θ_n^* statistik baho . . . shartni qanoatlantirsa, u effektiv baho deyiladi.

A) $M(\theta_n^*)=\max$. B) $D(\theta_n^*)=\max$. C) $M(\theta_n^*)=\min$

D) $D(\theta_n^*)=\min$. E) $D(\theta_n^*)=\max,\quad M(\theta_n^*)=\min$.

6. Berilgan $x_1, x_2, \ldots, x_n$ tanlanma bo'yicha $F(x,\theta)$ taqsimot funksiyasidagi noma'lum θ parametr uchun haqiqatga maksimal o'xshashlik usulida statistik baho tuzishda quyidagi amallardan qaysi biri bajarilmaydi?

A) Taqsimot funksiyasi va tanlanma bo'yicha $F(x_1,\theta)$, $F(x_2,\theta)$, . . ., $F(x_n,\theta)$ ifodalar topiladi .

*B) $F(x_1,\theta)$, $F(x_2,\theta)$, . . ., $F(x_n,\theta)$ ifodalarning o'rta arifmetik qiymati aniqlanadi . C) $L(x_1, x_2, \ldots, x_n, \theta) = F(x_1,\theta)\cdot F(x_2,\theta)\cdot \ldots \cdot F(x_n,\theta)$ funksiya tuziladi .

D) $\frac{\partial L(x_1,x_2,\cdots,x_n,\theta)}{\partial\theta}=0$ tenglama yechiladi .

E) Ko'rsatilgan barcha amallar bajariladi .

7. Berilgan $x_1, x_2, \ldots, x_n$ tanlanma bo'yicha $F(x,\theta)$ taqsimot funksiyasidagi noma'lum θ parametr uchun haqiqatga maksimal o'xshashlik funksiyasi qanday aniqlanadi?

*A) $L(x_1, x_2, \ldots, x_n, \theta) = F(x_1,\theta)\cdot F(x_2,\theta)\cdot \ldots \cdot F(x_n,\theta)$.

B) $L(x_1, x_2, \ldots, x_n, \theta) = F(x_1,\theta)+F(x_2,\theta)+ \ldots +F(x_n,\theta)$.

C) $L(x_1, x_2, \ldots, x_n, \theta) = F(x_1,\theta)-F(x_2,\theta)- \ldots -F(x_n,\theta)$.

D) $L(x_1, x_2, \ldots, x_n, \theta) =\max\{F(x_1,\theta), F(x_2,\theta), \ldots, F(x_n,\theta)\}$.

E) $L(x_1, x_2, \ldots, x_n, \theta) =\min\{F(x_1,\theta), F(x_2,\theta), \ldots, F(x_n,\theta)\}$.

8. Berilgan $x_1, x_2, \ldots, x_n$ tanlanma bo'yichanoma'lum θ parametr uchun haqiqatga maksimal o'xshashlik funksiyasi $L(x_1, x_2, \ldots, x_n, \theta)$ topilgan. Bu holda haqiqatga maksimal o'xshashlik tenglamasi qanday aniqlanadi?

A) $\frac{\partial L(x_1,x_2,\cdots,x_n,\theta)}{\partial x_i}=0$. B) $\frac{\partial L(x_1,x_2,\cdots,x_n,\theta)}{\partial x_i}=\theta$.

C) $\frac{\partial L(x_1,x_2,\cdots,x_n,\theta)}{\partial\theta}=0$. D) $\frac{\partial L(x_1,x_2,\cdots,x_n,\theta)}{\partial\theta}=\theta$.

E) $L(x_1, x_2, \ldots, x_n, \theta)=0$.

9. Berilgan $x_1, x_2, \ldots, x_n$ tanlanma bo'yicha $\overline{X}$ tanlanma o'rta qiymat qaysi formula bilan hisoblanadi?

A) $\overline{X}=\sqrt[n]{x_1+x_2+\cdots+x_n}$. B) $\overline{X}=\sqrt[n]{x_1\cdot x_2\cdot \cdots \cdot x_n}$.

C) $\overline{X}=\dfrac{x_1+x_2+\cdots+x_n}{n}$. D) $\overline{X}=\dfrac{x_1\cdot x_2\cdot \cdots \cdot x_n}{n}$.

E) $\overline{X}=\sqrt{\dfrac{x_1+x_2+\cdots+x_n}{n}}$.

10. X tasodufiy miqdor ustidagi kuzatuv natijalari 5,–3, 2, 1, 3, 1,–2, 1, 3, 3 tanlanmani tashkil etgan. $\overline{X}$ tanlanma o'rta qiymat nimaga teng?

A) $\sqrt[10]{14}$. B) $\sqrt[10]{1620}$. C) 0 . D) 1.4 . E) 3 .

11. X tasodufiy miqdor ustidagi n ta kuzatuv natijalari

x_i	x_1	x_2	. . .	x_m
n_i	n_1	n_2	. . .	n_m

statistik taqsimot qonuni bilan berilgan bo'lsa, uning $\overline{X}$ tanlanma o'rta qiymati qaysi formula bilan hisoblanadi ?

A) $\overline{X}=\dfrac{x_1+x_2+\cdots+x_m}{n}$. B) $\overline{X}=\dfrac{x_1+x_2+\cdots+x_m}{m}$.

C) $\overline{X}=\dfrac{x_1+x_2+\cdots+x_m}{n_1+n_2+\cdots+n_m}$. D) $\overline{X}=\dfrac{x_1n_1+x_2n_2+\cdots+x_mn_m}{n_1+n_2+\cdots+n_m}$.

E) $\overline{X}=\dfrac{x_1n_1+x_2n_2+\cdots+x_mn_m}{m}$.

12. X tasodufiy miqdor ustidagi kuzatuv natijalari

x_i	–3	0	2	10
n_i	4	7	8	1

statistik taqsimot qonuni bilan berilgan bo'lsa, uning $\overline{X}$ tanlanma o'rta qiymati

nimaga teng?

A) 0 . B) 2 . C) 9/4 . D) 9/20 . E) 7/10 .

2. Vektorlarning koordinatalari va ular ustida amallar									
1	2	3	4	5	6	7	8	9	10
A	B	C	A	D	C	C	C	C	A
3. Vektorlarning skalyar ko'paytmasi, uning xossalalri va tatbiqlari									
1	2	3	4	5	6	7	8	9	10
B	D	E	E	C	D	D	D	C	C
4. Vektorial ko'paytma, uning xossalalri va tatbiqlari									
1	2	3	4	5	6	7	8	9	10
D	E	C	B	D	A	E	E	C	E
5. Vektorlarning aralash ko'paytmasi , uning xossalari va tatbiqlari									
1	2	3	4	5	6	7	8	9	10
B	B	E	D	E	D	D	D	A	D

4-BOB

1. To'g'ri chiziq va uning tenglamalari									
1	2	3	4	5	6	7	8	9	10
A	C	D	C	B	E	B	D	A	C
2. To'g'ri ciziqlarga doir asosiy masalalar									
1	2	3	4	5	6	7	8	9	10
E	D	C	A	C	B	D	D	B	B
3. II tartibli tenglama va chiziqlar. Aylana va ellips									
1	2	3	4	5	6	7	8	9	10
C	C	D	A	D	B	B	A	C	C
4. Giperbola va parabola									
1	2	3	4	5	6	7	8	9	10
A	A	D	C	D	A	B	A	C	A

5. Tekislik va uning tenglamalari									
1	2	3	4	5	6	7	8	9	10
A	C	E	C	C	E	B	A	C	C
6. Tekislikka doir asosiy masalalar									
1	2	3	4	5	6	7	8	9	10
E	B	C	D	E	A	D	C	C	D
7. Fazodagi to'g'ri chiziq tenglamalari									
1	2	3	4	5	6	7	8	9	10
C	C	B	C	D	B	A	C	C	D

5-BOB

1. To'plamlar va ular ustida amallar									
1	2	3	4	5	6	7	8	9	10
C	D	D	C	A	C	C	A	B	D
2.Chekli va cheksiz to'plamlar									
1	2	3	4	5	6	7	8	9	10
D	C	C	C	B	C	D	D	D	D
3. Qavariq to'plamlar									
1	2	3	4	5	6	7	8	9	10
E	D	C	E	C	C	E	D	E	D

6-BOB

1. Sonli to'plamlar. Sonning absolut qiymati va uning xossalari									
1	2	3	4	5	6	7	8	9	10
C	A	A	D	A	D	B	C	A	A
2. Sonli ketma-ketlik va uning limiti									
1	2	3	4	5	6	7	8	9	10
E	D	B	E	B	C	E	D	D	C
3. Funksiya va u bilan bog'liq tushunchalar									
1	2	3	4	5	6	7	8	9	10
C	D	B	E	D	B	E	B	B	D

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Test javoblari

1-BOB

1.Matritsalar va ular ustida amallar

1	2	3	4	5	6	7	8	9	10
D	B	B	B	E	B	C	E	C	A

2. Determinantlar va ularning xossalari

1	2	3	4	5	6	7	8	9	10
B	B	C	A	A	D	B	E	C	E

3. Teskari matritsa. Matritsa rangi

1	2	3	4	5	6	7	8	9	10
D	D	C	D	B	E	A	E	E	E

2-BOB

1. Chiziqli tenglamalar sistemasi

1	2	3	4	5	6	7	8	9	10
E	B	C	E	B	A	C	D	C	C

2. Chiziqli tenglamalar sistemasini maxsus holda yechish

1	2	3	4	5	6	7	8	9	10
D	B	D	B	B	B	B	E	E	B

3. Chiziqli tenglamalar sistemasini umumiy holda yechish. Bir jinsli tenglamalar sistemasi

1	2	3	4	5	6	7	8	9	10
E	C	A	C	B	B	D	E	D	D

3-BOB

1. Vektorlar va ular ustida amallar

1	2	3	4	5	6	7	8	9	10
B	E	C	D	D	D	D	C	C	E

4. Funksiya limiti va uning xossalari									
1	2	3	4	5	6	7	8	9	10
D	C	C	D	D	A	A	D	C	A
5. Uzluksiz funksiyalar va ularning xossalari									
1	2	3	4	5	6	7	8	9	10
E	A	D	A	D	E	D	B	A	E
6. Funksiyaning uzilish nuqtalari va ularning turlari									
1	2	3	4	5	6	7	8	9	10
C	E	D	D	E	E	E	D	C	D

7-BOB

1. Funksiya hosilasi ta'rifi, uning mexanik va geometrik ma'nosi									
1	2	3	4	5	7	7	8	9	10
C	E	B	D	E	C	C	B	A	D
2. Funksiyani differensiallash qoidalari. Hosilalar jadvali									
1	2	3	4	5	7	7	8	9	10
D	C	E	C	D	D	E	B	E	B
3.Differensiallanuvchi funksiyalar haqidagi asosiy teoremalar									
1	2	3	4	5	7	7	8	9	10
E	C	C	C	D	C	D	B	E	C
4.Funksiya differensiali. Yuqori tartibli hosila va differensiallar									
1	2	3	4	5	7	7	8	9	10
C	E	B	D	E	D	C	D	B	E
5.Funksiyani I tartibli hosila yordamida tekshirish									
1	2	3	4	5	7	7	8	9	10
D	A	D	C	C	E	E	E	C	A

8-BOB

1.Boshlang'ich funksiya va aniqmas integral. Integrallar jadvali									
1	2	3	4	5	6	7	8	9	10
C	D	C	C	D	C	E	E	B	B
2.Aniqmas integralni hisoblash usullari									
1	2	3	4	5	6	7	8	9	10
A	C	D	C	C	E	D	D	B	A
3.Ratsional funksiyalar va ularni integrallash									
1	2	3	4	5	6	7	8	9	10
C	E	C	C	D	C	D	E	A	B
4.Ayrim irratsional ifodali integrallarni hisoblash									
1	2	3	4	5	6	7	8	9	10
C	A	C	C	B	E	C	B	D	C
5.Ayrim trigonometrik ifodali integrallarni hisoblash									
1	2	3	4	5	6	7	8	9	10
E	E	A	C	B	D	B	B	A	A

9-BOB

9.1.Aniq integral va uning xossalari															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	D	E	D	D	E	C	E	B	B	D	B	A	C	D	C
9.2.Aniq integralni hisoblash usullari															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B	C	D	C	C	B	B	C	A	A	C	E	E	B	D	B
9.3.Aniq integralni taqribiy hisoblash formulalari															
1	2	3	4	5	6	7	8	9	10	11	12	13			
E	D	B	D	B	A	E	E	C	C	D	C	D			

9.4. Aniq integralning geometrik masalalarga tatbiqlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	D	E	A	B	C	C	E	D	C	A	C	E	D	D	B

9.5. Aniq integralning mexanik va fizik masalalarga tatbiqlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	D	B	A	D	B	C	D	B	C	C	C	C	C	A

9.6.Xosmas integrallar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	D	D	E	B	C	D	D	D	C	E	C	D	D	E	D

10-BOB

1. Ko'p o'zgaruvchili funksiyalar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	B	C	D	C	E	C	D	E	D	C	B	A	E	E	E

2.Ko'p o'zgaruvchili funksiyalarning limiti

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	D	D	A	A	E	B	A	C	A	D	E	B	C	E	D

3. Ko'p o'zgaruvchili funksiyalarning uzluksizligi

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	C	B	D	A	C	D	D	E	D	D	C	D	C	D	C

4. Ko'p o'zgaruvchili funksiyaning hosila va differensiallari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	D	E	B	C	E	D	D	C	D	C	B	E	D	C	A

5.Ikki o'zgaruvchili murakkab funksiyaning hosilalari va to'la differensiali. To'la hosila. Yo'nalish bo'yicha hosila va gradient

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	C	E	C	D	D	B	D	E	C	A	B	E	A	E

6. Ko'p o'zgaruvchili funksiyalarning yuqori tartibli hosila va differensiallari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	B	A	D	C	A	A	C	C	A	E	C	C	B	E	D

7. Ikki o'zgaruvchili funksiyaning ekstremumlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	D	B	C	A	B	E	E	C	C	C	B	D	C	A

11-BOB

1. Differensial tenglamalar va ular bilan bog'liq tushunchalar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	D	D	E	B	B	D	D	C	C	D	D	C	E	D	C

2. I tartibli differensial tenglamalar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	A	B	C	C	B	E	B	D	B	E	E	B	D	C	C

3. II tartibli differensial tenglamalar. Tartibni pasaytirish usuli.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	D	C	C	B	B	B	A	A	A	D	C	B	C		

4. II tartibli chiziqli o'zgarmas koeffitsientli bir jinsli differensial tenglamalar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	C	D	D	E	C	E	E	E	B	D	B	E	D	E	D

5. II tartibli chiziqli o'zgarmas koeffitsientli bir jinslimas differensial tenglamalar

1	2	3	4	5	6	7	8	9	10
B	A	D	E	B	A	B	B	C	A

6. I tartibli differensial tenglamalar sistemasi

1	2	3	4	5	6	7	8	9	10
B	E	D	E	C	A	C	D	B	C

12-BOB

1. Sonli qarorlar va ularning yaqinlashuvi

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	E	C	C	C	C	E	E	B	C	D	E	C	D	D	A

2. Musbat hadli sonli qatorlarning yaqinlashish alomatlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	C	B	A	D	C	C	D	E	D	C	B	E	C	C	D

3. Ishorasi navbatlanuvchi va o'zgaruvchi sonli qatorlar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
D	C	A	C	D	C	C	E	A	B	D	E	C	E	D	

4. Funksional qatorlar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	D	D	C	C	A	A	E	C	C	D	C	D	A	E	A

5. Darajali qatorlar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	A	D	C	E	E	D	B	E	E	A	C	E	D	B	B

6. Teylor va Makloren qatorlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B	C	A	C	E	D	E	C	A	A	D	E	D	D	B	D

7. Darajali qatorlarning ayrim tatbiqlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
D	D	B	B	C	A	A	C	E	B	B	C	C	D	B	

8. Furye qatorlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	E	C	C	A	D	D	C	C	E	D	C	D	D	B	D

13-BOB

1. Ikki o'lchovli integral va uning xossalari

1	2	3	4	5	6	7	8	9	10
A	C	B	C	E	C	C	A	C	E

2. Ikki o'lchovli integralni hisoblash. Ikki karrali integrallar

1	2	3	4	5	6	7	8	9	10
D	C	D	D	E	B	E	A	C	B

3. Ikki o'lchovli integralning amaliy tatbiqlari

1	2	3	4	5	6	7	8	9	10
B	D	E	B	E	A	E	E	D	D

4. Uch o'lchovli integral va ularning xossalari									
1	2	3	4	5	6	7	8	9	10
D	C	B	D	D	C	E	A	C	E
5. Uch o'lchovli integrallarni hisoblash.									
1	2	3	4	5	6	7	8	9	10
D	D	D	D	E	C	D	C	A	D
6. Uch o'lchovli integralning amaliy tatbiqlari									
1	2	3	4	5	6	7	8	9	10
C	D	E	D	C	D	C	C	A	B
7. I tur egri chiziqli integrallar									
1	2	3	4	5	6	7	8	9	10
D	E	D	C	A	B	D	B	D	C
8. II tur egri chiziqli integrallar									
1	2	3	4	5	6	7	8	9	10
E	E	C	D.	B	A	D	E	A	D

14-BOB

1. Kompleks sonlar va ular ustida amallar															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	B	E	A	E	C	C	E	D	D	C	D	B	B	D	C
2. Kompleks o'zgaruvchili funksiyalar															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	B	C	D	D	C	A	C	B	D	C	C	B	B	A	C
3. Kompleks o'zgaruvchili funksiyalarning hosilasi															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	C	B	D	E	D	E	C	B	A	E	A	E	C	B	E
5. Kompleks o'zgaruvchili funksiyalarning integrali															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	C	D	C	A	C	B	C	E	C	B	D	D	B	E
5. Kompleks funksiyalar uchunTeylor va Loran qatorlari. Kompleks funksiyaning maxsus nuqta va chegirmalari															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	A	C	A	B	E	D	C	A	E	A	C	D	C	A	E

15-BOB

1. Laplas almashtirishi va uning chiziqlilk xossasi. Ayrim funksiyalarning tasvirlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B	C	E	D	A	E	E	E	E	D	D	C	C	A	B	A

2. Laplas almashtirishining asosiy xossalalri. Tasvirlar jadvali

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	A	E	B	A	E	C	C	B	C	D	B	C	C	D

3. Laplasning teskari almashtirishi. Operatsion hisob yordamida differensial tenglamalarni yechish

1	2	3	4	5	6	7	8	9	10	11	12	13	14		
B	D	A	C	D	D	D	A	E	D	B	B	C	C		

16-BOB

1.Matematik fizika tenglamalari va ularning turlari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	D	B	C	A	E	D	C	D	D	C	C	A	B	A	C

2.Tor tebranishi tenglamasi va uning uchun chegaraviy masalani Furye usulida yechish

1	2	3	4	5	6	7	8	9	10	11	12	13			
C	B	B	A	B	C	C	D	D	A	C	B	D			

3.Tor tebranishi uchun Koshi masalasi va unu Dalamber usulida yechish

1	2	3	4	5	6	7	8	9	10	11	12				
C	C	A	B	D	D	C	D	E	A	D	D				

4.Issiqlik tarrqalishi tenglamasi va uning uchun Koshi masalasini Furye usulida yechish

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	D	B	C	A	C	D	B	C	C	C	A	B	D	E	C

5.Laplas tenglamasi va uning uchun chegaraviy masalalar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	A	E	E	B	C	B	E	E	C	C	B	D	E	C

17-BOB

1.Hodisalar va ular ustida amallar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	D	B	B	D	C	C	E	C	B	A	D	C	E	C	B

2.Ehtimol va uni hisoblash usullari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	C	B	C	E	E	D	C	E	E	B	E	E	B	C	C

3.Ehtimollarni qo'shish va ko'paytirish teoremalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	B	C	C	C	D	D	B	A	A	D	C	C	D	C	E

4.To'liq ehtimol va Bayes formulalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	A	E	C	D	B	D	D	C	D	E	B	B	D	A	B

5.Bernulli formulasi va Muavr-Laplas, Puasson teoremalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	C	A	A	E	C	D	C	B	D	D	C	A	D	A	C

6.Asosiy diskkret taqsimotlar va ularning sonli xarakteristikalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B	C	E	C	B	A	C	E	B	C	C	D	C	C	D	C

7.Ehtimollarningtaqsimotvazichlikfunksiyalari.Uzluksiztasodifiymiqdor larningsonlixarakteristikalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	D	E	C	D	C	E	C	D	E	B	E	D	C	E	A

8. Asosiy uzluksiz taqsimotlar va ularning sonli xarakteristikalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	C	C	C	D	C	E	C	B	D	D	D	B	D	C	D

9. Statistik taqsimotlar

1	2	3	4	5	6	7	8	9	10	11	12	13	14
E	B	E	C	D	D	A	B	C	C	C	B	E	D

10.Statistik baholar. Tanlanma o'rta qiymat va dispersiya

1	2	3	4	5	6	7	8	9	10	11	12
E	B	E	C	D	B	A	C	C	D	D	E

MUNDARIJA

8-BOB. ANIQMAS INTEGRAL

9- BOB.ANIQ INTEGRAL

10-BOB. BIR NECHA O'ZGARUVCHILI FUNKSIYALARNING DIFFERENSIAL HISOBI

11-BOB. DIFFERENSIAL TENGLAMALAR

12-BOB. QATORLAR

13-BOB. IKKI O'LCHOVLI, UCH O'LCHOVLI VA EGRI CHIZIQLI INTEGRALLAR

14-BOB. KOMPLEKS SONLAR VA KOMPLEKS ARGUMENTLI FUNKSIYALAR

15-BOB. OPERATSION HISOB

16-BOB. MATEMATIK FIZIKANING BA'ZI TENGLAMALARI

17-BOB. EHTIMOLLAR NAZARIYASI ASOSLARI

Sharipova N.O'.

OLIY MATEMATIKA

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