O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA‘LIM VAZIRLIGI

B.A. XUDAYAROV

MATEMATIKA

1-QISM

**CHIZIQLI ALGEBRA
VA ANALITIK GEOMETRIYA**

O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta‘lim vazirligi tomonidan qishloq va suv xo‘jaligi sohasi ta’lim yo‘nalishlari talabalari uchun darslik sifatida tavsiya etilgan

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Ushbu darslik matematika fanining birinchi qismi – chiziqli algebra va analitik geometriyadan 1-semestrdagi ma’ruzalarga mos keladi.

Darslik qishloq xo‘jaligi oliy o‘quv yurtlari talabalari uchun mo‘ljallangan.

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Учебник соответствует первой части лекционного курса первого семестра по математике разделы линейной алгебры и аналитической геометрии.

Предназначен для студентов сельскохозяйственных вузов.

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Textbook corresponds to the first part of the lecture course of the first semester math sections of linear algebra and analytic geometry.

This textbook is intended for use by students of agricultural universities.

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SO‘ZBOSHI

Komil inson g‘oyasi azal-azaldan xalqimizning ezgu orzusi, uning ma’naviyatining uzviy bir qismi bo‘lib kelgan. U sharq falsafasidan oziqlanib, yanada kengroq ma’no-mazmun kasb etib kelmoqda.

Erkin fuqarolik jamiyatini barkamol, ezgu g‘oyalari, hayotiy e’tiqodi mustahkam bo‘lgan insonlargina bunyod eta oladi. Shuning uchun yangilanayotgan jamiyatimizda sog‘lom avlodni tarbiyalash, erkin fuqaro ma’naviyatini shakllantirish, ma’naviy-ma’rifiy ishlarni yuksak darajaga ko‘tarish orqali barkamol insonlarni voyaga yetkazishga muhim e’tibor berilmoqda. Mamlakatimizda “Kadrlar tayyorlash Milliy dasturi” asosida ta’lim-tarbiya tizimining tubdan isloh etilayotgani ham ana shu ulug‘vor maqsadni amalga oshirish yo‘lidagi muhim qadamlardir.

Hozirgi davr yoshlari ruhiyatida chuqur va mustahkam bilimlarni shakllantirish, milliy istiqlol g‘oyalariga sadoqatni, ona-Vatanga mehr-muhabbatni, bu yo‘ldagi fidoiylikni tarbiyalashni davom ettirish oliy ta’limning asosiy vazifalaridandir.

“Ta’lim to‘g‘risida”gi Qonun va “Kadrlar tayyorlash Milliy dasturi” vazifalarini amalga oshirishda va yuqori malakali mutaxassislar tayyorlashda aniq fanlarga ehtiyoj kuchayib bormoqda, chunki umumkasbiy va ixtisoslik maxsus fanlari ana shu fanlar asosida qurilgan bo‘ladi.

Maskur darslik oliy malakali ta’lim bo‘yicha yangi davlat ta’lim standartlarining irrigatsiya, qishloq xo‘jaligi va texnik yo‘nalishlari uchun matematik ta’limga qo‘yilgan talablarga mos keladi.

Darslik matematika fanining birinchi qismi bo‘yicha chiziqli algebra va analitik geometriyadan 1-semestrdagi ma’ruza matnlarini o‘z ichiga oladi.

Darslik muallifning Toshkent irrigatsiya va qishloq xo‘jaligini mexanizatsiyalash muhandislari institutida ko‘p yillik ma’ruza o‘qish jarayonlarida to‘plangan ma’lumotlari va orttirilgan tajribasi asosida yozildi.

Darslik irrigatsiya, qishloq xo‘jaligi va ba’zi texnik yo‘nalishlar talabalari hamda matematika fani o‘qituvchilari, shuningdek, o‘zining matematik bilimini oshirish uchun mustaqil o‘rganuvchilar uchun foydali bo‘ladi, deb hisoblaymiz.

KIRISH

Matematika – bizni o‘rab turgan olamning fazoviy shakllari va miqdoriy munosabatlari haqidagi fandir. Keltirilgan ta’rifni keng ma’noda tushunish zarur. Fan va texnikaning rivojlanishi fazoviy shakllar va miqdoriy munosabatlar bo‘yicha uzbiy bog‘liqligini matematikada o‘rganish uzluksiz kengayib boradi.

Matematika tabiiy-ilmiy, injener-texnik va iqtisodiy tadqiqotlarda muhim vazifani bajaradi. U ko‘plab bilim tarmoqlarida faqatgina miqdoriy hisob quroli bo‘lib qolmasdan, shuningdek, aniq tadqiqotlar usuli, muammo va tushunchalarni yetarlicha aniqlikda umumlashtirish vositasi bo‘ladi. Zamonaviy matematika va uning rivojlanayotgan mantiqiy hamda hisoblash metodlaridan foydalanmasdan insoniyot faoliyatining turli sohalarida yuksak natijalarga erishib bo‘lmaydi.

Matematika faqat amaliy masalalarni yechishning kuchli vositasi bo‘lib qolmay, balki umimiy madaniyat elementi hamdir. Shu sababli matematik bilimlar zamonaviy mutaxassislar tayyorlash tizimining fundamental asosini tashkil qiladi deb qarash lozim.

Yevropa va markaziy osiyolik olimlarning matematika fani taraqqiyotiga qo‘shgan hissalari. Tarixdan ma’lumki, ilm-fan, madaniyat va san’at rivojlangan jamiyatda taraqqiy etish, yuksalish bo‘lgan. Qolaversa, ilm-fan ma’lum davlatning jahonga chiqishida, tanilishida asosiy omillardan biridir. Bu jarayonda o‘z iqtidori, bilimi va ilmiy, badiiy ijodi bilan ilm xazinasini kashfiyot durlari bilan boyitgan sharqu-garb olimlarining hissasi beqiyosdir. Ular yaratgan asarlar ayni kunda ham olamni anglash, inson va borliq o‘rtasidagi muammolarni hal etish, diniy va dunyoviy bilimlarni boyitishda dasturamal vazifasini o‘tab kelayotir. Bu borada matematika sohasida farb va sharq allomalari ilmiy ishlarini mo‘jizaga qiyoshlash mumkin.

Qadimiy gretsiyalik matematik va mexanik Arximed (eramizdan avvalgi 287-212-yillar); fransuz matematigi va faylasufi P. Dekart (1596-1650); angliyalik fizik va matematik I. N’yuton (1643-

1727); nemis faylasufi, matematik va fizik G. Leybnits (1646-1716); matematik, mexanik va fizik L. Eyler (1707-1783); fransuz matematigi va mexanigi J. Lagranj (1736-1813); nemis matematigi K. Gauss (1777-1855); fransuz matematigi O. Koshi (1789-1857) va boshqa ko‘plab yirik olimlarning ishlanmalarida oliy matematikaning asoslari keltirilganligini ta’kidlash kerak.

Markaziy Osiyoda ham dunyo ilm-fan taraqqiyotiga hissa qo‘shgan buyuk allomalarimizning ilmiy merosi bebahodir. Matematika sohasida buyuk asarlar yaratgan, ulkan dunyoviy ilmni meros qoldirgan ajdodlarimiz haqida kengroq ma’lumotga ega bo‘lish bugungi avlodlar oldidagi ham farz, ham qarzdir.

Abu Abdulloh Muhammad ibn Muso al-Xorazmiy – (taxminan 780-850-yillarda yashagan) – matematika fanining asoschisi, geografiya, tarix va astronomiya kabi fanlarning rivojlanishiga katta hissa qo‘shgan. Hindlarning o‘nli sistemasini birinchi bo‘lib tadqiq qilgan, algebra faniga asos solgan buyuk astronom, qomusiy olim hisoblanadi.

Al-Xorazmiy o‘z umrining aksariyatini Bag‘dodda tashkil etilgan ilmiy dargoh «Baytul-hikmat» (“Donishmandlik uyi”)da olim sifatida faoliyat yuritdi. Uning matematika bo‘yicha yozgan risolalari: «Kitob al-jabr val muqobala», «Hind hisobi haqida qisqacha kitob», «Astronomik jadvallar», «Kitob ul-suratul-arz», «Hind hisobi haqida qisqacha kitob» asari Yevropada hind raqamlari sistemasining tarqalishida muhim rol o‘ynadi. «Kitob al-jabr val muqobala» asarida algebra mustaqil fan sifatida birinchi bo‘lib o‘rganib chiqildi. Bu risola ikki qismdan iborat bo‘lib, birinchi qismida algebraik miqdorlar ustida amallarni bajarish qoidalari, birinchi va ikkinchi darajali tenglamalar ko‘rib chiqilgan. Qoidalar va yechimlar so‘z bilan bayon etilgan. Noma'lum ildiz yoki buyum deb, noma’lumning kvadrati - kvadrat deb atalgan. Kvadrat tenglamalar geometrik usulda yechilgan. Ikkinchi qismda esa turli xo‘jalik – turmush, savdo va yuridik (yer o‘lchash, meros bo‘lish) masalalarga algebraik metodlarni joriy qilish ko‘riladi. Shuningdek, asarda geometrik masalalar bayon etilgan. Unda π va $\sqrt{10}$ sonlarining bir-biriga yaqinligi hamda bundan tashqari, $\frac{22}{7}$, 3,1416 kabi qiymatlari keltirilgan. Bu asar lotin tiliga XII asrda tarjima qilingan va ko‘p vaqtlar davomida Yevropa mamlakatlarida matematika bo‘yicha asosiy qo‘llanma bo‘lib keldi. “Al-jabr” asari matematikaning alohada bo‘limiga aylanib, “algebra” deb ataladigan bo‘ldi. “Al-Xorazmiy” nomi “Algoritmus’ hozirgi hisoblash matematikasining asosiy atamasi “algoritm”ga aylandi.

Abul Abbos Ahmad ibn Muhammad ibn Nosir al-Farg‘oniy 797-865-yillarida yashab ijod qilgan buyuk ajdodimizdir. Sharqda Al-Farg‘oniy, Yevropada Alfraganus (Alfraganus) tahalluslari bilan mashhurdir. Ahmad al-Farg‘oniy Farg‘onaning Qubo, hozirgi Quva shahrida tavallud topgan. U – buyuk astronom, matematik va geograf. Sharqda “Hosib” (matematik) degan laqab bilan shuhrat topgan. Uni “Munajjim al-Rais” deb ham atashgan. Astronomiya, geografiya va matematika sohasidagi asarlari bu fanlar taraqqiyotiga salmoqli hissa qo‘shdi, hamda bir necha asrlar davomida olimlar uchun qo‘llanma bo‘ldi. Ahmad al-Farg‘oniy Yer kurrasining doiraviy uzunligini, diametri va radiusini aniqlagan. Yer meridianlari haqida bilimlarga asos solgan va samodagi yulduzlarga mukammal tasnif bergan. Chuqur matematik tadqiqotlar natijasida samoviy jismlarning balandligi va ulargacha bo‘lgan masofalarni o‘lchash jihozlarini qurish hamda foydalanish ilmining birinchi mukammal nazariyasini yaratgan olim sifatida jahon ilm ahli tomonidan e’tirof etilgan.

Ahmad al-Farg‘oniy tomonidan ixtiro qilgan suv sathini o‘lchash “Nilometr” qurilmasi Nil daryosi sohiliga barpo qilinadi. Bu qurilma suvning ko‘tarilishi va pasayishini kuzatish imkonini bergan. Bu kuzatish asosida dehqonchilikning yil bo‘yi qanday bo‘lganligini qayd etish mumkin bo‘lgan. U Bag‘dod rasadxonasida ko‘pgina kashfiyotlar qildi. Jumladan, 840-yilgi Quyosh tutilishini oldindan bildi va bu haqda ilmiy kuzatishlar olib bordi. Alloma 1022 ta yulduzni o‘lchab, tasvirladi.

Abu Nasr Muhammad ibn Muhammad ibn Uzlug‘ ibn Tarxon Forobiy (870- 950) qomuschi olim, sharq fanining asoschilaridan biri. Forobiy «Hajm va miqdor haqida kitob», «Fazo geometriyasiga kirish haqida qisqacha kitob» va boshqa asarlari bilan mashhurdir. Asarlarida matematikaning asosiy tushunchalarini asoslash va to‘g‘ri bayon etish usullariga katta e’tibor bergan.

Abu Rayhon Muhammad ibn Ahmad Beruniy (973-1048) astronom, matematik va qomuschi olim, Xorazmda tug‘ilgan, asosiy asarlarini arab tilida yozgan. Beruniyning asosiy ishlari astronomiya, matematika, fizika, falsafa, tarix, botanika, geografiya, mineralogiya va h. k. larga bag‘ishlangan. «Kitob at-tafxim» (1029—1034 yillar) asarida matematika, astronomiya va astrologiya asoslari bayon etilgan. «Doiradagi vatarlarni uning ichiga chizilgan siniq chiziqlar yordamida aniqlash haqidagi risola» (1027-yil) nomli asarida geometriya va trigonometriyaning qator teoremalarining isbotlari berilgan. Beruniy tomonidan arifmetika va algebraning asosiy masalalariga ta‘rif beradi; butun va kasr sonlar ustida amallar, chiziqli, kvadrat hamda kub tenglamalarni taqribiy yechish usullarini bayon etadi; doiraga ichki chizilgan muntazam ko‘pburchakning tomonlarini aniqlaydi; ko‘pyoqlar, aylanma jismlar, konus kesimlari, muntazam ko‘pyoqlarga ta’rif beradi va stereometriyaning asosiy tushunchalarini bayon etadi. Matematikaga bag‘ishlangan 22 ta risola yozib qoldirgan.

Abu Ali Husayn ibn Abdulloh ibn Sino (980-1037) faylasuf-tabiatshunos, tabib, matematik, shoir, Buxoroga yaqin Afshona qishlog‘ida tugilgan, Xorazm va Eronda ishlagan. Asosiy asarlari: «Tib qonunlari», «Ashshifo», «Najot», «Ishorat va tanbih», «Donishnoma» va «Urjuz». Bulardan «Ashshifo» va «Donishnoma» asarlarida matematikaga bag‘ishlangan maxsus bo‘limlar bor.

Shoir, faylasuf, astronom va matematik **G‘iyosiddin Abulfath Umar ibn Ibrohim Hayyom** (1048-1131) Nishopurda tug‘ilgan. U birinchi bo‘lib uchinchi darajagacha bo‘lgan tenglamalarni yechish nazariyasini yaratdi va barcha tenglamalarning umumiy sinflarini bayon etdi. Umar Hayyom birinchi marta geometriya bilan algebraning aloqasi to‘g‘risidagi hamda algebraik tenglamalarni geometrik tushuntirish va yechish haqidagi masalani qo‘ydi.

Muhammad Tarag‘ay Ulug‘bek (1394-1449) – buyuk o‘zbek astronomi, matematigi, davlat arbobi va ma’rifatparvari. U Samarqandda madrasa va dunyoda eng yaxshi rasadxona bunyod etdi. O‘z atrofiga mashhur matematik va munajjimlarni to‘plab, ilmiy maktab tashkil etdi. Ulug‘bek tomonidan juda aniq trigonometrik jadvallarni tuzishga imkon beruvchi al-jabr usullari ishlab chiqildi.

Bu usul istalgan aniqlikda hisoblashlarni amalga oshirishga yordam berar edi.

Matematikaning rivojlanishiga rus matematiklari – N.I. Lobazevckiy (1792-1856), M.V.Ostrogradskiy (1801-1861), P.L. Chebishev (1821-1894), A.A. Marchuk (1856-1922), A.M. Lyapunov (1857-1918) va boshqalarning ishlarini ham e'tirof etish mumkin.

O'zbekistonda matematika fanining rivojlanishi. Zamonaviy o'zbek matematika maktabi ham jahon matematika fanining oldingi qatorlarini egallab kelmoqda. Bu borada o'zbek matematiklari A. Sarimsoqov, S.H. Sirojiddinov, T.N. Qori-Niyoziy, T.Jo'rayev, M. Salohiddinov, Sh. Ayupov, Sh. Alimov, A. Sadullayev, A. A'zamov, N. G'anixodjayev, R. G'anixodjayev, M. Oripov, E. Fayziboyev, Y. Soatov, A. Narmonov, M. Mamatov va boshqalarning ilm-fan sohasidagi erishgan yutuqlarini ta'kidlash lozimdir.

I BOB. MATRITSA VA DETERMINANTLAR

1-§. Matritsa va ular ustida amallar

1.1. Matritsa tushunchasi. Matritsa tushunchasi va matematikaning matritsalar algebrasi bo‘limi iqtisodchilar va boshqa soha mutaxassislari uchun muhim ahamiyatga egadir. Iqtisodiy jarayonlar va obyektlarning matematik modeli sodda va ixcham matritsali tenglama shaklida ifoda etiladi.

Ta’rif. *m* ta yo‘l va *n* ta ustundan iborat bo‘lgan ushbu jadvalga

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \text{ yoki } \begin{Vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{Vmatrix} \qquad (1)$$

$m\times n$ - o‘lchamli ***matritsa*** deyiladi.

Masalan,

$$A = \begin{pmatrix} 2 & 1 & \pi \\ 1 & \sqrt{2} & -5 \end{pmatrix}$$

2×3 -o‘lchamli ***matritsa.***

$$B = \begin{pmatrix} e^t & 1 & -1 & \cos t \\ 0 & 4t & -7 & 1-t \end{pmatrix}$$

2×4 -o‘lchamli ***matritsa.***

Matritsalarni $A, B, C, \ldots$ bosh harflar bilan belgilaymiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \text{ yoki } A = \|a_{ij}\| = (a_{ij})$$

$$(i=1,2,\ldots,m, \quad j=1,2,\ldots,n)$$

a_{ij} -lar ***matritsa elementlari*** deyiladi, bu yerda birinchi indeks i – element turgan yo‘l nomerini, ikkinchi indeks j – esa ustun nomerini bildiradi.

Agar ixtiyoriy i va j larda $a_{ij} = b_{ij}$ bo'lsa, $A = (a_{ij})$ va $B = (b_{ij})$ ***matritsalar teng*** deyiladi va A=B kabi yoziladi.

Matritsalar yordamida ba'zi iqtisodiy bog'liqlarni ifodalash mumkin. Masalan, iqtisodiyotning ba'zi tarmoqlari bo'yicha resurslarning taqsimotini jadvalda ifodalaymiz.

Resurslar	Iqtisodiyotning tarmoqlari	
	Qishloq xo'jaligi	Suv xo'jaligi
Suv	7,2	8,1
Mehnat	4,1	3,2
Elektroenergiya	5,2	6,3

Ushbu jadvalni resurslar taqsimotining ixcham matritsasi ko'rinishida ifodalash mumkin:

$$A = \begin{pmatrix} 7{,}2 & 8{,}1 \\ 4{,}1 & 3{,}2 \\ 5{,}2 & 6{,}3 \end{pmatrix}$$

jadvalga ko'ra, $a_{11} = 7{,}2$ – matritsa elementi – qishloq xo'jaligiga qancha suv resursi, $a_{22} = 3{,}2$ – element esa – suv xo'jaligiga qancha mehnat resursi sarflanishini ko'rsatadi.

Agar (1) matritsaning barcha elementlari nolga teng bo'lsa,

$$0 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

u ***nol matritsa*** deyiladi.

1.2. Kvadrat matritsa. Agar matritsaning yo'llari soni uning ustunlari soniga teng bo'lsa, u holda uni ***kvadrat matritsa*** deyiladi. Kvadrat matritsaning yo'llari soni uning tartibini bildiradi. n - tartibli kvadrat matritsa berilgan bo'lsin:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \qquad (2)$$

Kvadrat matritsada $a_{11}, a_{22}, \dots, a_{nn}$ elementlar joylashgan diagonal bosh diagonal, $a_{1n}, a_{2,n-1}, \dots, a_{1n}$ - yordamchi diagonal deyiladi.

Bosh diagonalidan pastdagi (yoki yuqoridagi) hamma elementlar nolga teng bo‘lgan kvadrat matritsa ***uchburchakli matritsa*** deyiladi. Uchburchakli matritsa quyidagi ko‘rinishda bo‘ladi:

$$A_1 = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \text{ yoki}$$

$$B = \begin{pmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix}.$$

A_1 – ***yuqori uchburchakli***, B – ***quyi uchburchakli*** matritsadir.

Agar (2) kvadratik matritsaning bosh diagonal elementlaridan boshqa barcha elementlari nolga teng bo‘lsa,

$$\begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \tag{3}$$

uni ***diagonal matritsa*** deyiladi. Xususiy holda, (3) matritsada

$$a_{11} = a_{22} = \dots = a_{nn} = 1$$

bo‘lsa,

$$E = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

ko‘rinishga keladi, uni ***birlik matritsa*** deb ataladi.

A kvadrat matritsaning yo‘llarini mos ustunlar bilan almashtirilgan hosil bo‘lgan matritsaga ***transponirlangan matritsa*** deyiladi va A^T – bilan belgilanadi:

$$A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix}.$$

Masalan, agar $A = \begin{pmatrix} 3 & 4 \\ 5 & -2 \\ 2 & 0 \end{pmatrix}$ bo‘lsa, u holda

$$A^T = \begin{pmatrix} 3 & 5 & 2 \\ 4 & -2 & 0 \end{pmatrix}$$

bo‘ladi.

Agar A kvadrat matritsa transponirlangan A^T matritsaga teng bo‘lsa, u holda A ***simmetrik matritsa*** deyiladi.

1.3. Matritsalar ustida amallar va ularning xossalari. *Matritsalarni qo‘shish va ayirish*. $m\times n$ – o‘lchovli $A = (a_{ij})$ va $B = (b_{ij})$ matritsalar berilgan bo‘lsin. Ularning ***yig`indisi*** deb, $m\times n$ – o‘lchovli $C = (c_{ij})$ matritsaga aytiladi va elementlari

$$c_{ij} = a_{ij} + b_{ij} \quad (i=1,2,\ldots,n, j = 1,2,\ldots,m) \qquad (4)$$

tenglik yordamida aniqlanadi ($C = A+B$ deb belgilanadi).

1-misol. 2×3 – o‘lchovli $A = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix}$ matritsalar berilgan bo‘lsin. $A + B$ yig‘indini toping.

Yechish. $A + B = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix} =$

$$= \begin{pmatrix} 1+(-1) & 2+6 & -3+3 \\ 4+8 & 0+12 & 2+14 \end{pmatrix} = \begin{pmatrix} 0 & 8 & 0 \\ 12 & 12 & 16 \end{pmatrix}.$$

Ta’rifga ko‘ra, matritsalarni qo‘shishda mos elementlar qo‘shiladi. Qo‘shish amali bir xil tartibli matritsalar uchun aniqlangan.

$m\times n$ – o‘lchovli A va B matritsalar mos elementlari ayirmalaridan tashkil topgan ushbu $m\times n$ – o‘lchovli $D = (d_{ij})$ matritsa A va B ***matritsalar ayirmasi*** deyiladi va $D=A-B$ kabi belgilanadi.

Matritsalarni qo‘shish xossalari:

1^0. $A+0=0+A=A$.

2^0. $A+B = B+A$.

3^0. $(A+B)+C = A+ (B+C)$ (assotsiativ).

Matritsani songa ko'paytirish. $A=(a_{ij})$ (i=1,2,…,m, j = 1,2,…,n) matritsaning har bir elementini λ haqiqiy songa ko'paytirganda hosil bo'lgan matritsaga λ son bilan *A* ***matritsa ko'paytmasi*** deyiladi va λ*A* kabi belgilanadi. Demak,

$$\lambda A=\begin{pmatrix}\lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n}\\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n}\\ \cdots & \cdots & \cdots & \cdots\\ \lambda a_{n1} & \lambda a_{n2} & \cdots & \lambda a_{nn}\end{pmatrix}.$$

2-misol. **A=**$\begin{pmatrix}-3\\ 4\\ 2t\\ \sin(2t)\end{pmatrix}$ matritsani $\sqrt{2}$ ga ko'paytiring.

Yechish. $\sqrt{2}\ \cdot \mathrm{A}=\sqrt{2}\cdot\begin{pmatrix}-3\\ 4\\ 2t\\ \sin(2t)\end{pmatrix}=\begin{pmatrix}-3\sqrt{2}\\ 4\sqrt{2}\\ 2t\sqrt{2}\\ \sqrt{2}\sin(2t)\end{pmatrix}.$

3-misol. **A=**$\begin{pmatrix}2 & e^t\\ \sin t & 4\end{pmatrix}$ matritsani $\cos t$ ga ko'paytiring.

Yechish. $\cos t\cdot \mathrm{A}=\cos t\cdot\begin{pmatrix}2 & e^t\\ \sin t & 4\end{pmatrix}=\begin{pmatrix}2\cos t & e^t\cos t\\ \cos t\sin t & 4\cos t\end{pmatrix}.$

Matritsalarni songa ko'paytirish xossalari:

4⁰. (λμ)*A*= λ(μ*A*), (λ, μ=const).

5⁰. λ(*A*+*B*)=λ*A*+λ*B*.

6⁰. (λ+μ)*A* =λ*A*+μ*A*.

4-misol. Agar

$$A=\begin{pmatrix}2 & -1\\ 1 & 3\\ 5 & 3\end{pmatrix},\ B=\begin{pmatrix}2 & 0\\ 4 & 1\\ 6 & 5\end{pmatrix}$$

bo'lsa, *A*+*B*, 3*A*-4*B* matritsalarni toping.

Yechish. Matritsalarni qo'shish, ayirish va songa ko'paytirish qoidalaridan foydalanib, izlanayotgan matritsalarni topamiz:

$$A+B=\begin{pmatrix}2 & -1\\ 1 & 3\\ 5 & 3\end{pmatrix}+\begin{pmatrix}2 & 0\\ 4 & 1\\ 6 & 5\end{pmatrix}=\begin{pmatrix}2+2 & -1+0\\ 1+4 & 3+1\\ 5+6 & 3+5\end{pmatrix}=\begin{pmatrix}4 & -1\\ 5 & 4\\ 11 & 8\end{pmatrix}.$$

$$3A-4B=3\cdot\begin{pmatrix}2 & -1\\ 1 & 3\\ 5 & 3\end{pmatrix}-4\cdot\begin{pmatrix}2 & 0\\ 4 & 1\\ 6 & 5\end{pmatrix}=\begin{pmatrix}6 & -3\\ 3 & 9\\ 15 & 9\end{pmatrix}-\begin{pmatrix}8 & 0\\ 16 & 4\\ 24 & 20\end{pmatrix}=$$

$$=\begin{pmatrix}6-8 & -3-0\\ 3-16 & 9-4\\ 15-24 & 9-20\end{pmatrix}=\begin{pmatrix}-2 & -3\\ -13 & 5\\ -9 & -11\end{pmatrix}.$$

Matritsalarni ko‘paytirish. $m\times n$ – o‘lchovli $A=(a_{ij})$ (i=1,2,…,m; j=1,2,…,n) matritsani $n\times p$ – o‘lchovli $B=(b_{ij})$ (i=1,2,…,n; j=1,2,…,p) matritsaga ***ko‘paytmasi*** deb, shunday $m\times p$ – o‘lchovli $R=(r_{ij})$ (i=1,2,…,m; j=1,2,…, p) matritsaga aytiladi va uning elementlari

$$r_{ij}=\sum_{k=1}^{n} a_{ik}b_{kj}$$

$$(\, i=1,2,\ldots,m;\ j=1,2,\ldots,p\,) \qquad (5)$$

formula yordamida aniqlanadi.

Ta‘rifdan ko`rinadiki, A matritsani ixtiyoriy B matritsaga ko‘paytirish uchun A matritsaning ustunlari soni B matritsaning yo‘llari soniga teng bo‘lishi talab qilinadi.

5-мisol. Ushbu

$$A=\begin{pmatrix}1 & 0\\ 2 & -1\\ 0 & 3\end{pmatrix},\ B=\begin{pmatrix}2 & -1 & 4\\ 0 & 3 & 1\end{pmatrix}$$

matritsalar ko‘paytmasini toping.

Yechish. A matritsa 3x2 - o‘lchovli, B matritsa 2x3 - o‘lchovli bo‘lgani uchun ularning ko‘paytmasi 3x3 - o‘lchovli

$$A\cdot B=\begin{pmatrix}r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{pmatrix}$$

matritsa bo‘lib, uning elementlari (5) formulaga ko‘ra

$r_{11}=1\cdot 2+0\cdot 0=2,\qquad r_{12}=1\cdot(-1)+0\cdot 3=-1,$

$r_{13}=1\cdot 4+0\cdot 1=4,\qquad r_{21}=2\cdot 2+(-1)\cdot 0=4,$

$r_{22} = 2 \cdot (-1) + (-1) \cdot 3 = -5, r_{23} = 2 \cdot 4 + (-1) \cdot 1 = 7,$
$r_{31} = 0 \cdot 2 + 3 \cdot 0 = 0, \qquad r_{32} = 0 \cdot (-1) + 3 \cdot 3 = 9,$
$r_{33} = 0 \cdot 4 + 3 \cdot 1 = 3$
bo‘ladi. Demak,

$$A \cdot B = \begin{pmatrix} 2 & -1 & 4 \\ 4 & -5 & 7 \\ 0 & 9 & 3 \end{pmatrix}.$$

6-мisol. Ushbu

$$A = \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix}$$

matritsalar berilgan bo‘lsin. *A·B* va *B·A* matritsalarni toping.

Yechish. Berilgan matritsalarning ko‘paytmasini topamiz:

$$A \cdot B = \begin{pmatrix} -1 \cdot 0 + 2 \cdot (-2) & -1 \cdot 3 + 2 \cdot 4 \\ 3 \cdot 0 + 0 \cdot (-2) & 3 \cdot 3 + 0 \cdot 4 \end{pmatrix} = \begin{pmatrix} -4 & 5 \\ 0 & 9 \end{pmatrix},$$

$$B \cdot A = \begin{pmatrix} 0 \cdot (-1) + 3 \cdot 3 & 0 \cdot 2 + 3 \cdot 0 \\ -2 \cdot (-1) + 4 \cdot 3 & -2 \cdot 2 + 4 \cdot 0 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 14 & -4 \end{pmatrix}.$$

7-мisol. Ushbu

$$A = \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix}, B = \begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix}$$

matritsalar uchun *A·B* ≠ *B·A* munosabatni tekshirib ko‘ring.

Yechish. $A \cdot B = \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} \cdot \begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 8 & 0 \end{pmatrix},$

$$B \cdot A = \begin{pmatrix} -2 & 0 \\ 8 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} -14 & 24 \\ -5 & 12 \end{pmatrix}.$$

Demak, $\boldsymbol{A \cdot B \neq B \cdot A}$. Keltirilgan misollardan ko‘rinadiki, ikki matritsa ko‘paytmasi uchun o‘rin almashtirish qoidasi o‘rinli bo‘lmaydi. Ammo, *n*x*n* - o‘lchovli ***A*** matritsa bilan *n*x*n* - o‘lchovli birlik *E* matritsa uchun

$$\boldsymbol{A \cdot E = E \cdot A = A}$$

tenglik o‘rinli bo‘ladi.

8-мisol. Ushbu matritsalar ko‘paytmasini toping.

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \end{pmatrix}.$$

Yechish. *A* matritsa 2x2 - o‘lchovli, *B* matritsa 2x3 - o‘lchovli bo‘lgani uchun ularning ko‘paytmasi 2x3 - o‘lchovli matrita bo‘ladi.

$$A \cdot B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \end{pmatrix} =$$

$$=\begin{pmatrix}1\cdot1+\ 3\cdot2 & 1\cdot1+3\cdot1 & 1\cdot3+3\cdot4\\ 2\cdot1+5\cdot2 & 2\cdot1+5\cdot1 & 2\cdot3+5\cdot\ 4\end{pmatrix}=\begin{pmatrix}7 & 4 & 15\\ 12 & 7 & 26\end{pmatrix}.$$

A, *B* va *C* matritsalar berilgan bo'lsin. U holda

7^0 . $(A\cdot B)\cdot C = A\cdot(B\cdot C)$

8^0 . $(A+B)\cdot C = A\cdot C+B\cdot C$, $A\cdot(B+C) = A\cdot B+A\cdot C$

xossalar o'rinli bo'ladi. 7^0-xossaning o'rinli bo'lishini ko'rsatamiz. *n* - tartibli ixtiyoriy *A*, *B* va *C* matritsalar berilgan bo'lsin, qisqacha ularni quyidagicha belgilaylik

$A=(a_{ij})$, $B=(b_{ij})$, $C=(c_{ij})$, $A\cdot B=U=(u_{ij})$, $B\cdot C=V=(v_{ij})$,

$(A\cdot B)\cdot C=S=(s_{ij})$, $A\cdot(B\cdot C)=T=(t_{ij})$

$(A\cdot B)\cdot C=A\cdot(B\cdot C)$ ekanligini, ya`ni $S=T$ ekanligini isbotlashimiz kerak. U quyidagi tenglikdan kelib chiqadi.

$$u_{il}=\sum_{k=1}^{n}a_{ik}b_{kl},\qquad v_{kj}=\sum_{l=1}^{n}b_{kl}c_{lj}$$

$S=U\cdot C$, $T=A\cdot V$ tengliklarga ko'ra

$$s_{ij}=\sum_{l=1}^{n}u_{il}c_{lj}=\sum_{l=1}^{n}\sum_{k=1}^{n}a_{ik}\,b_{kl}c_{lj}\,,$$

$$t_{ij}=\sum_{l=1}^{n}a_{ik}v_{kj}=\sum_{k=1}^{n}\sum_{l=1}^{n}a_{ik}\,b_{kl}c_{lj}\,,$$

ya'ni, $s_{ij}=t_{ij}$ $(i,j=1,2,...,n)$.

Demak, $(A\cdot B)\cdot C=A\cdot(B\cdot C)$.

9-мisol. Ushbu matritsalar berilgan:

$$A=\begin{pmatrix}2 & 1\\ 3 & -4\end{pmatrix},\ B=\begin{pmatrix}5\\ -2\end{pmatrix},\mathrm{C}=\begin{pmatrix}3 & 0 & -2\end{pmatrix}.$$

Bu matritsalar uchun $(A\cdot B)\cdot C=A\cdot(B\cdot C)$ xossa o'rinli bo'lishini tekshiring.

Yechish. $A\cdot B=\begin{pmatrix}2 & 1\\ 3 & -4\end{pmatrix}\cdot\begin{pmatrix}5\\ -2\end{pmatrix}=\begin{pmatrix}8\\ 23\end{pmatrix},$

$$(A\cdot B)\cdot C=\begin{pmatrix}8\\ 23\end{pmatrix}\cdot\begin{pmatrix}3 & 0 & -2\end{pmatrix}=\begin{pmatrix}24 & 0 & -16\\ 69 & 0 & -46\end{pmatrix},$$

$$B\cdot C=\begin{pmatrix}5\\ -2\end{pmatrix}\cdot\begin{pmatrix}3 & 0 & -2\end{pmatrix}=\begin{pmatrix}15 & 0 & -10\\ -6 & 0 & 4\end{pmatrix},$$

$$A\cdot(B\cdot C)=\begin{pmatrix}2 & 1\\ 3 & -4\end{pmatrix}\cdot\begin{pmatrix}15 & 0 & -10\\ -6 & 0 & 4\end{pmatrix}=\begin{pmatrix}24 & 0 & -16\\ 69 & 0 & -46\end{pmatrix}.$$

Demak, $(A \cdot B) \cdot C = A \cdot (B \cdot C)$.

2-§. Determinantlar

Ta'rif. 2 - tartibli kvadrat matritsaning

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

determinanti deb, ushbu

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad (6)$$

songa aytiladi. Bu yerda a_{11}, a_{12} , a_{21}, a_{22} – determinant elementlaridir. Determinant elementining birinchi indeksida turgan son yo'l raqamini, ikkinchi indeksida turgan son esa ustun raqamini bildiradi. $a_{11}a_{22} - a_{12}a_{21}$ ayirma determinant qiymati deyiladi va detA, yoki $|A|$, yoki Δ bilan belgilanadi.

10-misol. 2-tartibli determinantni hisoblang.

$$\begin{vmatrix} 2 & -4 \\ 3 & 5 \end{vmatrix} = 2 \cdot 5 - (-4) \cdot 3 = 10 + 12 = 22.$$

Ushbu

$$detA = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

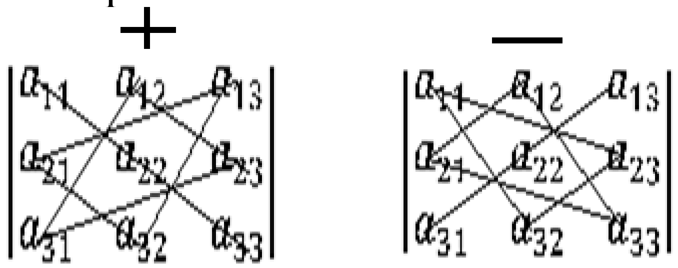
songa ***3-tartibli determinant***,

$$detA = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} +$$
$$+ a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

unung qiymati deyiladi.

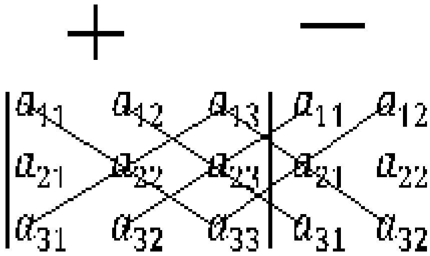
Uchinchi tartibli determinant qiymati oltita hadlar yig'indisidan iborat bo'lib, ulardan uchtasi musbat, qoplgan uchtasi esa manfiy ishoralidir. Bu hadlarni hisoblash quyidagi sxemalar yo'rdamida ifodalansa bo'ladi:

a) Uchburchak qoidasi



yoki

b) Sarryus qoidasi



11-misol. 3-tartibli determinantni hisoblang.

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & 3 & -1 \\ 4 & 5 & 2 \end{vmatrix} =$$

$$= 1 \cdot 3 \cdot 2 + (-2) \cdot (-1) \cdot 4 + 1 \cdot 2 \cdot 5 - -1 \cdot 3 \cdot 4 -$$
$$-1 \cdot (-1) \cdot 5 - (-2) \cdot 2 \cdot 2 = 6 + 8 + 10 - 12 + 5 + 8 = 25.$$

n - tartibli kvadrat matritsaning

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

determinanti deb, ushbu

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \qquad (7)$$

ifodaga aytiladi.

Agar A matritsaning determinanti $detA = 0$ bo‘lsa, u holda A ***xos matritsa*** deyiladi, aks holda $detA \neq 0$ bo‘lsa, u holda A ***xosmas matritsa*** deyiladi.

3-§. Minor va algebraik to‘ldiruvchilar. Determinantlarning xossalari

Uchinchi tartibli

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

determinant berilgan bo‘lsin. Bu determinantning biror a_{ik} $(i,k = 1,2,3)$ elementini olib, shu element turgan yo‘lni hamda ustunni o‘ziramiz. O‘zirilmay qolgan elementlardan ikkiinchi tartibli determinant hosil bo‘ladi. Unga a_{ik} elementning ***minori*** deyiladi va M_{ik} bilan belgilanadi. Masalan, uchinchi tartibli determinantda a_{23} element turgan yo‘l va ustunni o‘chirish

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

natijasida

$$M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

determinant hosil bo‘ladi. Bu berilgan determinant a_{23} elementining minoridir.

12-мisol. $\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$ determinantning minorlarini toping.

Yechish. Determinantning elementlari soni to‘rtta bo‘lgani uchun minorlar soni ham to‘rtta bo‘ladi: $M_{11} = 4, M_{12} = 1,\ M_{21} = -3,\ M_{22} = 2.$

13-мisol. $\begin{vmatrix} 1 & 0 & -2 \\ 3 & 4 & 1 \\ 0 & -3 & 5 \end{vmatrix}$ determinantning M_{11}, M_{12} va M_{32} minorlarini toping.

Yechish. Determinantning elementlari soni to‘qqizta bo‘lgani uchun minorlar soni ham to‘qqizta bo‘ladi. Misol shartiga ko‘ra, $M_{11},\ M_{12}$ va M_{32} minorlarni topamiz:

$$M_{11} = \begin{vmatrix} 4 & 1 \\ -3 & 5 \end{vmatrix}, \quad M_{12} = \begin{vmatrix} 3 & 1 \\ 0 & 5 \end{vmatrix} \text{ va } M_{32} = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix}.$$

Ta’rif. (7) determinant a_{ij} elementining ***algebraik to‘ldiruvchisi*** deb,

$$(-1)^{i+j}M_{ij}$$

miqdorga aytiladi va A_{ij} orqali belgilanadi:

$$A_{ij} = (-1)^{i+j}M_{ij}.$$

Kvadrat matritsa determinantining algebraik to‘ldiruvchilari soni uning elementlari soniga teng bo‘ladi.

Masalan,

$$\begin{vmatrix} 1 & 0 & -2 \\ 3 & 4 & 1 \\ 0 & -3 & 5 \end{vmatrix}$$

determinant elementlariga mos to‘qqizta minorlar mavjud. $a_{32} = -3$ elementining algebraik to‘ldiruvchisini topamiz:

$$A_{32} = (-1)^{3+2} M_{32} = (-1)^5 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = -\big(1 - (-6)\big) = -7.$$

Determinant xossalarini 3-tartibli determinantlarga nisbatan keltiramiz. Uchinchi tartibli

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

determinant berilgan bo‘lsin.

1^0. *Determinant yo‘llarini unga mos ustunlar bilan almashtirilsa, determinant qiymati o‘zgarmaydi*, *ya‘ni*

$$|A^T| = |A|.$$

Isbot. Ushbu xossani 3-tartibli matritsa determinanti ushun tekshirib ko‘ramiz. Uchinchi tartibli transponirlangan matritsa determinantini hisoblaymiz:

$$|A^T| = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} =$$

$$= a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} -$$
$$-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}.$$
$$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$
$$-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}\ .$$

Demak, $|A^T| = |A|$.

2^0. *Determinant ixtiyoriy ikki yo‘lini (ustunini) o‘zaro almashtirsak, determinant qiymati o‘zgarmasdan unung ishorasi qarama-qarshi ishoraga o‘zgaradi*.

Isbot. Birinchi va ikkinchi yo‘llari o‘zaro almashtirilgan 3-tartibli matritsa determinantini $|A_1|$ belgilaymiz va uning qiymatini hisoblaymiz:

$$|A_1| = \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= a_{12}a_{21}a_{33} + a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} -$$

$$-a_{31}a_{12}a_{23} - a_{11}a_{22}a_{33} - a_{21}a_{32}a_{13} =$$
$$= -(-a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} +$$
$$+a_{31}a_{12}a_{23} + a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13}) = -|A|.$$

Demak, $|A_1| = -|A|$.

3^0. *Determinantning ixtiyoriy yo'lida (ustunida) turgan barcha elementlarni k songa ko'paytirilsa, determinant qiymati k marta ortadi.*

$$\begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$
$$= ka_{11}a_{22}a_{33} + ka_{12}a_{23}a_{31} + ka_{13}a_{21}a_{32} -$$
$$-ka_{13}a_{22}a_{31} - ka_{11}a_{23}a_{32} -$$
$$- ka_{12}a_{21}a_{33} = k(a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$
$$-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}) = k \cdot detA$$

4^0. *Determinantning biror yo'li (ustuni)dagi barcha elementlar nolga teng bo'lsa, determinant qiymati nolga teng bo'ladi.*

5^0. *Determinantning ikki yo'li (ustuni) bir xil bo'lsa, uning qiymati nolga teng bo'ladi.*

Isbot. Faraz qilaylik, 3-tartibli determinantning ikkita yo'li elementlari bir xil bo'lsin.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{12}a_{33} + a_{12}a_{13}a_{31} + a_{13}a_{11}a_{32} -$$
$$-a_{13}a_{12}a_{31} - a_{11}a_{13}a_{32} - a_{12}a_{11}a_{33} = 0$$

6^0. *Determinantning ixtiyoriy ikki yo'li (ustuni) o'zaro proporsional bo'lsa, uning qiymati nolga teng bo'ladi.*

Isbot. Faraz qilaylik, 3-tartibli determinantning birinchi va ikkinchi yo'llari o'zaro proporsional bo'lsin:

$$\frac{a_{21}}{a_{11}} = \frac{a_{22}}{a_{12}} = \frac{a_{23}}{a_{13}} = t,$$
$$a_{21} = t \cdot a_{11}, \qquad a_{22} = t \cdot a_{12}, \qquad a_{23} = t \cdot a_{13}.$$

Determinantning ikkinchi yo'l elementlari o'rniga birinchi yo'l elementlari orqali ifodasini qo'yamiz:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ta_{11} & ta_{12} & ta_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= t \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = t \cdot 0 = 0.$$

5-xossaga ko‘ra,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0.$$

7^0. *Agar determinantning biror yo‘li (ustuni)dagi elementlar ikki qo‘shiluvchilar yig‘indisidan iborat bo‘lsa, masalan,*

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + \alpha & a_{22} + \beta & a_{23+\gamma} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

bo‘lsa, u holda

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + \alpha & a_{22} + \beta & a_{23} + \gamma \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ \alpha & \beta & \gamma \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

bo‘ladi.

8^0. *Agar determinantning biror yo‘li (ustuni)ni o‘zgarmas songa ko‘paytirib, boshqa yo‘li (ustuni)ga qo‘shilsa, determinant qiymati o‘zgarmaydi.*

Isbot. 3-tartibli determinantning 1-yo‘lini k-songa ko‘paytirib, 2-yo‘lga qo‘shamiz va 7-xossaga ko‘ra ifodalaymiz:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + ka_{11} & a_{22} + ka_{12} & a_{23} + ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} & ka_{12} & ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + k \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + k \cdot 0 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

9^0. *Determinantning biror yo‘li (ustuni)dagi barcha elementlarning ularga mos algebraik to‘ldiruvchilari bilan ko‘paytmasidan tashkil topgan yig‘indi shu determinant qiymatiga teng bo‘ladi.*

Isbot. 3-tartibli determinantning 1-yo‘l bo‘yicha yoyilmasini algebraik to‘ldiruvchilar orqali ifodalaymiz:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\cdot A_{11} + a_{12}\cdot A_{12} + a_{13}\cdot A_{13} =$$

$$= a_{11}\cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= a_{11}\cdot(a_{22}a_{33} - a_{23}a_{32}) - a_{12}\cdot(a_{21}a_{33} - a_{23}a_{31}) +$$

$$+ a_{13}\cdot(a_{21}a_{32} - a_{22}a_{31}) =$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$

$$-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}.$$

10^0. *Determinantning biror yo‘li (ustuni)dagi barcha elementlari bilan boshqa yo‘l (ustun)da turgan mos elementlarning algebraik to‘ldiruvchilari ko‘paytmalaridan tashkil topgan yig‘indi nolga teng bo‘ladi.*

Isbot. 3-tartibli determinantning 2-yo‘l elementlarini 1-yo‘l elementlarining mos algebraik to‘ldiruvchilari ko‘paytmalaridan tashkil topgan yig‘indini hisoblaymiz:

$$a_{21}\cdot A_{11} + a_{22}\cdot A_{12} + a_{23}\cdot A_{13} =$$

$$= a_{21}\cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{22}\cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{23}\cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= a_{21}\cdot(a_{22}a_{33} - a_{23}a_{32}) - a_{22}\cdot(a_{21}a_{33} - a_{23}a_{31}) +$$

$$+ a_{23}\cdot(a_{21}a_{32} - a_{22}a_{31}) =$$

$$= a_{21}a_{22}a_{33} - a_{21}a_{23}a_{32} - a_{22}a_{21}a_{33} +$$

$$+a_{22}a_{23}a_{31} + a_{23}a_{21}a_{32} - a_{23}a_{21}a_{32} - a_{23}a_{22}a_{31} = 0.$$

Determinantlarning 9–xossasiga ko‘ra, n – tartibli $(n \geq 2)$ determinantning 1-yo‘l bo‘yicha yoyilmasi quyidagicha ifodalanadi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \sum_{j=1}^{n} a_{1j}A_{1j} \qquad (8)$$

n - tartibli determinantning ixtiyoriy yo‘l bo‘yicha yoyilmasini

$$\Delta = \sum_{j=1}^{n} a_{ij}A_{ij}\,, \left(i = \overline{1,n}\right) \qquad (9)$$

yoki ixtiyoriy ustun bo‘yicha yoyilmasini

$$\Delta = \sum_{i=1}^{n} a_{ij} A_{ij}\,, \left(j = \overline{1, n}\right) \qquad (10)$$

formulalar yordamida ifodalash mumkin.

(9) va (10) formulalar ***Laplas formulalari*** deyiladi. Agar *i*=1 bo‘lsa, (9)-formula (8)-formula bilan ustma-ust tushadi.

Diagonal (quyi yoki yuqori) matritsaning determinanti bosh diagonaldagi elementlar ko‘paytmasiga teng bo‘ladi:

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{vmatrix} = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}.$$

Diagonal matritsani 1-ustun bo‘yicha yoyamiz:

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{vmatrix} = a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} & \cdots & a_{1n} \\ 0 & a_{33} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{vmatrix} =$$

$$= a_{11} \cdot a_{22} \cdot \begin{vmatrix} a_{33} & a_{34} & \cdots & a_{1n} \\ 0 & a_{44} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{vmatrix} = \cdots =$$

$$= a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}.$$

14-misol.

$$A = \begin{pmatrix} 1 & 0 & 1 & 2 \\ -3 & 1 & -2 & -4 \\ 2 & 3 & 1 & 0 \\ -4 & -2 & 0 & -7 \end{pmatrix}$$ matritsa determinantini hisoblang.

Yechish. Matritsa determinantining 1-yo‘lidagi a_{11} elementdan boshqa barcha elementlarni xossalardan foydalanib nolga aylantiramiz. Birinchi yo‘lda bitta nol bor, yana ikkita nol hosil qilamiz. Buning uchun, 1-ustun elementlarini -1 ga ko‘paytiramiz va 3-ustunning mos elementlariga qo‘shamiz. Shuningdek, 1-ustun elementlarini -2 ga ko‘paytiramiz va 4-ustunning mos elementlariga qo‘shamiz. Olingan natijalar asosida determinantni yozamiz:

$$\det A=\begin{vmatrix}1&0&1&2\\-3&1&-2&-4\\2&3&1&0\\-4&-2&0&-7\end{vmatrix}=\begin{vmatrix}1&0&0&0\\-3&1&1&2\\2&3&-1&-4\\-4&-2&4&1\end{vmatrix}=$$

{1-yo‘l elementlari bo‘yicha determinantni yoyamiz}

$$=1\cdot(-1)^{1+1}\cdot\begin{vmatrix}1&1&2\\3&-1&-4\\-2&4&1\end{vmatrix}=\begin{vmatrix}1&1&2\\3&-1&-4\\-2&4&1\end{vmatrix}=$$

{Determinantning 1-ustun elementlarini -1 ga ko‘paytiramiz va 2-ustunning mos elementlariga qo‘shamiz. Shuningdek, 1-ustun elementlarini -2 ga ko‘paytiramiz va 3-ustunning mos elementlariga qo‘shamiz. Olingan natijalar asosida determinantni yozamiz va uni 1-yo‘l elementlari bo‘yicha yoyamiz}.

$$=\begin{vmatrix}1&0&0\\3&-4&-10\\-2&6&5\end{vmatrix}=1\cdot(-1)^{1+1}\begin{vmatrix}-4&-10\\6&5\end{vmatrix}=-20+60=40.$$

15-misol. $\Delta=\begin{vmatrix}127&1&2&3\\154&2&3&4\\181&3&4&5\\208&4&5&6\end{vmatrix}$ determinant xossalaridan foydalanib hisoblang.

Yechish. 7^0-xossadan foydalangan holda, deteminantni yig‘indi ko‘rinishda yozamiz:

$$\Delta=\begin{vmatrix}100+27&1&2&3\\100+54&2&3&4\\100+81&3&4&5\\100+108&4&5&6\end{vmatrix}=\begin{vmatrix}100&1&2&3\\100&2&3&4\\100&3&4&5\\100&4&5&6\end{vmatrix}+\begin{vmatrix}27&1&2&3\\54&2&3&4\\81&3&4&5\\108&4&5&6\end{vmatrix}.$$

3^0-xossaga binoan umumiy ko‘paytuvchilarni determinant oldiga chiqaramiz:

$$\Delta = 100 \cdot \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 1 & 4 & 5 & 6 \end{vmatrix} + 27 \cdot \begin{vmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 4 & 5 \\ 4 & 4 & 5 & 6 \end{vmatrix}.$$

Yig'indidagi 2-determinantning ikkita ustun elementlari bir xil. 5^0-xossaga asosan uning qiymati nolga teng:

$$\Delta = 100 \cdot \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 1 & 4 & 5 & 6 \end{vmatrix} + 27 \cdot 0 = 100 \cdot \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 1 & 4 & 5 & 6 \end{vmatrix}$$

Ikkinchi ustun elementlaridan mos ravishda birinchi ustun elementlarini ayiramiz va natijani 2-ustunga yozamiz. Birinchi ustun elementlarini 2 ga ko'paytiramiz va mos ravishda uchunchi ustun elementlaridan ayiramiz. Olingan natijani uchunchi ustunga yozamiz. Natijada determinantning 2 ta ustuni elementlari bir xil bo'ladi. 6^0-xossaga ko'ra, determinant qiymati nolga teng:

$$\Delta = 100 \cdot \begin{vmatrix} 1 & 0 & 0 & 3 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 3 & 6 \end{vmatrix} = 0.$$

4- §. Teskari matritsa

n - tartibli kvadrat matritsa

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

berilgan bo'lsin.

Agar A bilan n-tartibli A^{-1} - kvadrat matritsa ko'paytmasi E - birlik matritsaga teng bo'lsa

$$A \cdot A^{-1} = A^{-1} \cdot A = E$$

u holda A^{-1} matritsa *A* ga ***teskari matritsa*** deyiladi.

Teorema. *A* matritsaga A^{-1} teskari matritsa mavjud bo‘lishi uchun uning xosmas matritsa bo‘lishi zarur va etarlidir.

3 - tartibli kvadrat matritsa

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

berilgan bo‘lsin. *A* matritsaga teskari matritsa A^{-1} quyidagi formula yordamida topiladi:

$$A^{-1} = \frac{1}{detA}\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T =$$

$$= \frac{1}{detA}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{A_{11}}{detA} & \frac{A_{21}}{detA} & \frac{A_{31}}{detA} \\ \frac{A_{12}}{detA} & \frac{A_{22}}{detA} & \frac{A_{32}}{detA} \\ \frac{A_{13}}{detA} & \frac{A_{23}}{detA} & \frac{A_{33}}{detA} \end{pmatrix}.$$

bu yerda A_{ij} $(i, j = 1,2,3)$ berilgan *A* matritsa a_{ij} elementining algebraik to‘ldiruvchisi.

16-misol.

$$A = \begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & 3 \\ 1 & -3 & 1 \end{pmatrix}$$

A matritsaga teskari A^{-1} matritsani toping.

Yechish. det*A*=2-3+0-2+18-0=15. Demak, $detA \neq 0 \implies$ $A^{-1} -$ mavjud. A^{-1} ni topamiz:

$$A^{-1} = \frac{1}{15}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix},$$

bu yerda

$$A_{11}=(-1)^{1+1}\begin{vmatrix}1&3\\-3&1\end{vmatrix}=10,\;\; A_{12}=(-1)^{1+2}\begin{vmatrix}0&3\\1&1\end{vmatrix}=3,$$
$$A_{13}=(-1)^{1+3}\begin{vmatrix}0&1\\1&-3\end{vmatrix}=-1, A_{21}=(-1)^{2+1}\begin{vmatrix}-1&2\\-3&1\end{vmatrix}=-5,$$
$$A_{22}=(-1)^{2+2}\begin{vmatrix}2&2\\1&1\end{vmatrix}=0,\quad A_{23}=(-1)^{2+3}\begin{vmatrix}2&-1\\1&-3\end{vmatrix}=5,$$
$$A_{31}=(-1)^{3+1}\begin{vmatrix}-1&2\\1&3\end{vmatrix}=-5, A_{32}=(-1)^{3+2}\begin{vmatrix}2&2\\0&3\end{vmatrix}=-6,$$
$$A_{33}=(-1)^{3+3}\begin{vmatrix}2&-1\\0&1\end{vmatrix}=2.$$

Demak, berilgan matritsaga teskari matritsa quyidagi ko‘rinishga ega bo‘ladi:

$$A^{-1}=\frac{1}{15}\begin{pmatrix}10&-5&-5\\3&0&-6\\-1&5&2\end{pmatrix}.$$

Tekshirib ko‘ramiz:

$$A^{-1}\cdot A=\frac{1}{15}\begin{pmatrix}10&-5&-5\\3&0&-6\\-1&5&2\end{pmatrix}\cdot\begin{pmatrix}2&-1&2\\0&1&3\\1&-3&1\end{pmatrix}=$$
$$=\frac{1}{15}\begin{pmatrix}15&0&0\\0&15&0\\0&0&15\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=E.$$

Eslatma. Xos matritsaning teskari matritsasi mavjud bo‘lmaydi.

5-§. Matritsa rangi

Biror $m\times n$ - o‘lchamli *A matritsa*

$$A=\begin{pmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\cdots&\cdots&\cdots&\cdots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{pmatrix}$$

berilgan bo‘lsin. *A* matritsaning ixtiyoriy *k* ta yo‘li va *k* ta ustunini olib, ($k\le\min(m,n)$) *k*-tartibli kvadrat matritsa tuzamiz. Bu kvadrat matritsaning determinanti *A* matritsaning ***k-tartibli minori*** deyiladi.

1-ta’rif. *A* matritsaning noldan farqli bo‘lgan eng yuqori (katta) tartibli minoriga uning ***rangi*** deyiladi va ***rankA*** bilan belgilanadi.

Matritsaning rangi uning yo‘llari va ustunlari sonidan katta bo‘lmaydi, ya’ni *rangA* ≤ min(m,n).

1-ta’rifdan quyidagilar kelib chiqadi:

1) agar *rangA=k* bo‘lsa, u holda *A* matritsa minorlari orasida noldan farqli *k*-tartibli kamida bitta minori mavjud bo‘ladi;

2) (*k*+1) va undan yuqori tartibli minorlari (agar ular mavjud bo‘lsa) nolga teng.

17-misol. Ushbu

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ -4 & 1 & 1 \end{pmatrix}$$

matritsaning rangini toping.

Yechish. Berilgan matritsaning 2-tartibli minorlari bir nechta bo‘lib, ulardan biri $\begin{vmatrix} 3 & 1 \\ -4 & 1 \end{vmatrix} = 7$ bo‘ladi. *A* matritsaning 3-tartibli minori

$$\begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ -4 & 1 & 1 \end{vmatrix}$$

determinantlarning xossalariga ko‘ra nolga teng. *rangA*=2.

Eslatma. Diagonal matritsaning rangi bosh diagonaldagi nolga teng bo‘lmagan elementlar soniga teng bo‘ladi.

2-ta’rif. Noldan farqli eng yuqori tartibli minorga ***bazis minor*** deyiladi. Bazis minor bilan kesishgan yo‘llar va ustunlar ***bazis*** deyiladi.

Ta’rifga ko‘ra, bazis minor tartibi matritsa rangiga teng bo‘ladi.

Eslatma. Bazis minorlar soni bir nechta bo‘lishi mumkin.

18-misol. $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 10 \end{pmatrix}$ matritsaning barcha minorlarini toping va ular orasidan basis minorlarni aniqlang.

Yechish. 1) Ta‘rifga ko‘ra, minor – berilgan matritsadan ajratilgan kvadrat qism matritsa determinantidir. Demak, berilgan matritsa ikkita yo‘ldan iborat bo‘lgani uchun uning 3-tartibli minori mavjud bo‘lmaydi. Barcha ikkinchi tartibli minorlarni hisoblaymiz:

$$M_1 = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -2, \; M_2 = \begin{vmatrix} 1 & 5 \\ 2 & 10 \end{vmatrix} = 0, \; M_3 = \begin{vmatrix} 3 & 5 \\ 4 & 10 \end{vmatrix} = 10.$$

M_2 - minor nolga teng va u ta‘rifga ko‘ra bazis bo‘lmaydi. Bazis minorlar sifatida M_1 va M_3minorlarni olamiz.

Ta‘rifga binoan, matritsa rangini aniqlash uchun barcha minorlarini hisoblash yetarlicha murakkablik tug‘diradi. Bu murakkablikni yechish uchun matritsa rangi saqlanadigan almastirishdan foydalanamiz.

Quyidagi almashtirishlar matritsa ustidagi ***elementar almashtirishlar*** deyiladi:

1. Matritsaning nolli ustunlarini (yo‘llarini) tashlab yuborish.

2. Matritsaning yo‘l (ustun) elementlarini noldan farqli songa ko‘paytirish.

3. Matritsaning yo‘llarini (ustunlarini) almashtirish.

4. Matritsaning biror yo‘lidagi (ustunidagi) barcha elementlarni noldan farqli ixtiyoriy o‘zgarmas songa ko‘paytib, boshqa yo‘l (ustun) mos elementlariga qo‘shish.

Teorema. Matritsa ustida bajarilgan elemantar almashtirishlardan so‘ng, uning rangi o‘zgarmaydi.

Ushbu teorema yordamida matritsani zinasimon yoki uchburchakli ko‘rinishga keltirib, uning rangini aniqlash mumkin.

Matritsa elementar almashtirishlar yordamida uchburchakli matritsaga keltiriladi va uning rangini hisoblash katta qiyinchilik tug‘dirmaydi.

19-misol. $A=\begin{pmatrix} 1 & 3 & 0 & 5 \\ 2 & 5 & 2 & 1 \\ 1 & 6 & 1 & 3 \end{pmatrix}$ matritsa rangini toping.

Yechish: Berilgan matritsani elementar almashtirishlar yordamida uchburchakli ko‘rinishga keltiramiz. Yoqorida keltirilgan teoremaga ko‘ra, uning rangi o‘zgarmaydi. Birinchi yo‘l elementlarini -2 ga ko‘paytiramiz va ikkinchi yo‘lning mos elementlariga qo‘shamiz. Xuddi shuningdek, birinchi yo‘l elementlarini -1 ga ko‘paytiramiz va uchunchi yo‘lning mos elementlariga qo‘shamiz:

$A=\begin{pmatrix} 1 & 3 & 0 & 5 \\ 2 & 5 & 2 & 1 \\ 1 & 6 & 1 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 0 & 5 \\ 0 & -1 & 2 & -9 \\ 0 & 3 & 1 & -2 \end{pmatrix} \Rightarrow$ {Keyingi almashtirishlarni bajaramiz: ikkinchi yo‘l elementlarini 3 ga ko‘paytiramiz va uchinchi yo‘lning mos elementlariga qo‘shamiz, natijada quyidagi matritsa hosil bo‘ladi} $\Rightarrow\begin{pmatrix} 1 & 3 & 0 & 5 \\ 0 & -1 & 2 & -9 \\ 0 & 0 & 7 & -29 \end{pmatrix}$. Hosil bo‘lgan matritsa, noldan farqli uchinchi tartibli minorga ega:

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 7 \end{vmatrix} = -7.$$

Demak, $rang\ A = 3$.

6-§. Matritsalarning amaliy masalalarga tatbiqi

Matritsalar yordamida ba’zi iqtisodiy bog‘liqlarni ifodalash mumkinligini birinchi paragrafda ko‘rdik. Endi matritsalar yordamida ba’zi amaliy masalalarni yechishni o‘rganamiz.

1-masala. “Ravot” va “Qahramon” fermer xo‘jaliklarida yetishtirilgan poliz mahsulotlari shahardagi N_1, N_2 va N_3 supermarketlarga har kuni yetkazilib turiladi. Bu fermer xo‘jaliklaridan kundalik poliz mahsulotlarining bir tonnasini N_1 - supermarketga yetkazib berish - 20 ming, N_2 - supermarketga yetkazib berish - 30 ming va N_3 - supermarketga yetkazib berish esa - 50 ming pul birligiga to‘g‘ri keladi. Har bir fermer xo‘jaligining kundalik transport xarajatlarini hisoblang.

Fermer xo‘jaliklari	Supermarketlarga kundalik yetkazilib berilgan poliz mahsulotlari (tonna hisobida)		
	N_1	N_2	N_3
“Ravot”	2	3	1
“Qahramon”	3	1	4

Yechish. A – matritsa bilan har kuni fermer xo‘jaliklaridan supermarketlarga yetkazib berilgan polis mahsulotlari (tonna hisobida), B – matritsa esa fermer xo‘jaligidan bir tonna mahsulotni supermarketga yetkazib berish uchun sarflanadigan transport xarajatlari (narxlari) bo‘lsin:

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix}, \; B = (20 \quad 30 \quad 50).$$

U holda, fermer xo‘jaliklarining poliz mahsulotlarini supermarketlarga yetkasib berish uchun ketgan bir kunlik sarf xarajatlari matritsasi quyidagiga teng bo‘ladi:

$$C = A \cdot B^T = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 20 \\ 30 \\ 50 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 \cdot 20 + 3 \cdot 30 + 1 \cdot 50 \\ 3 \cdot 20 + 1 \cdot 30 + 4 \cdot 50 \end{pmatrix} = \begin{pmatrix} 180 \\ 290 \end{pmatrix}.$$

Demak, “Ravot” fermer xo‘jaligidan polis mahsulotlarini supermarketlarga yetkasib berish uchun kuniga 180 ming, “Qahramon” fermer xo‘jaligidan esa 290 ming shartli pul birligi sarflanadi.

2-masala. Fermer xo‘jaligida 10 tonna kartoshka, 3 tonna piyoz va 6 tonna pomidor yetishtirish rejalashtirilgan. $X = (10 \quad 3 \quad 6)$ – fermer xo‘jaligining rejasi; $S = (1 \quad 1 \quad 3)$ - resurslar narxi (har bir tonna uchun); $P = (0 \quad 3 \quad 7)$ – transport xarajati (har bir tonna uchun).

1) Fermer xo‘jaligi bo‘yicha rejadagi qishloq xo‘jalik mahsulotlarini yetishtirish uchun sarflangan har bir resurslarning miqdorini aniqlang.

2) Mahsulotlar turlari bo‘yicha bir tonna qishloq xo‘jalik mahsulotini yetishtirish uchun sarflangan resurs xarajatlarini aniqlang.

Qishloq xo‘jalik mahsulotlari	1 tonna mahsulotni yetishtirish uchun sarflanadigan resurslar miqdori		
	T_1 suv (ming, litr)	T_2 mahaliy o‘g‘itlar (tonna)	T_3 mineral o‘g‘itlar (tonna)
Kartoshka	2	2	1
Piyoz	3	1	3
Pomidor	4	3	2

1) Rejani bajarish uchun sarflangan jami resurs xarajatlari miqdorini aniqlang.

2) Fermer xo‘jaligi bo‘yicha resurs va transport xarajatlari umumiy yig‘indisini toping.

Yechish. 1) 1 tonna mahsulotni yetishtirish uchun sarflanadigan resurslar miqdorini $A = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 3 \\ 4 & 3 & 2 \end{pmatrix}$- matritsa bilan ifodalaylik. Bu yerda a_{ij} - i-turdagi qishloq xo‘jalik mahsulotining bir tonnasini yetishtirish uchun sarflangan j-turdagi T_j resurs miqdori.

$$T = X \cdot A = (10\ \ 3\ \ 6) \cdot \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 3 \\ 4 & 3 & 2 \end{pmatrix} =$$
$$= (10 \cdot 2 + 3 \cdot 3 + 6 \cdot 4 \quad 10 \cdot 2 + 3 \cdot 1 + 6 \cdot 3$$
$$10 \cdot 1 + 3 \cdot 3 + 6 \cdot 2) = (53\ \ 41\ \ 31).$$

Demak, fermer xo‘jaligi rejadagi qishloq xo‘jalik mahsulotlarini yetishtirishga sarflagah resurslar miqdori quyidagicha: T_1– 53 ming litr; T_2 – 41 tonna; T_3 – 31 tonna.

2) Bir tonna qishloq xo‘jalik mahsulotini yetishtirish maqsadida foydalanilgan resurslar uchun ketgan sarf-xarajatlarni hisoblaymiz:

$$A \cdot S^T = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 3 \\ 4 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 + 2 \cdot 1 + 1 \cdot 3 \\ 3 \cdot 1 + 1 \cdot 1 + 3 \cdot 3 \\ 4 \cdot 1 + 3 \cdot 1 + 2 \cdot 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 13 \\ 13 \end{pmatrix}.$$

Demak, bir tonna qishloq xo‘jalik mahsulotini yetishtirish uchun 1-turdagi mahsulotga - 7 ming, ikkinchi va uchunchi turdagi mahsulotlar uchun -13 ming so‘m sarflanadi.

3) Fermer xo‘jaligining uch turdagi mahsulotlarni yetishtirish uchun resurslarga sarflagan xarajatini aniqlaymiz:

$$X \cdot (A \cdot S^T) = (10 \quad 3 \quad 6) \cdot \begin{pmatrix} 7 \\ 13 \\ 13 \end{pmatrix} =$$
$$=(10 \cdot 7 + 3 \cdot 13 + 6 \cdot 13) = 187.$$

Demak, fermer xo‘jaligining rejadagi kartoshka, piyoz va pomidorni yetishtirish uchun resurslarga sarflagan xarajatlari summasi – 187 ming so‘mni tashkil qiladi.

4) Resurslarni tashish uchun ketgan transport xarajatini hisoblaymiz:

$$T \cdot P = (53 \quad 41 \quad 31) \cdot \begin{pmatrix} 0 \\ 3 \\ 7 \end{pmatrix} = 0 + 123 + 217 = 340.$$

Demak, fermer xo‘jaligining resurslar va transport xarajatlari umumiy yig‘indisi quyidagiga teng:

$$X \cdot (A \cdot S^T) + T \cdot P = 187 + 340 = 527.$$

3-masala. “Ravot” ko‘p tarmoqli fermer xo‘jaligida tashkil etilgan kichik korxona 2 xil qishloq xo‘jalik xom-ashyo mahsulotlaridan uch turdagi konserva mahsulotlari ishlab chiqadi. Qishloq xo’jalik xom-ashyo mahsulotlarining sarf miqdori quyidagi matritsa ko’rinishida berilgan:

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 3 \end{pmatrix}.$$

Bu yerda a_{ij} (i=1,2,3; j=1,2), i - turdagi birlik mahsulotga j – turdagi birlik xom-ashyo sarflanishi. Korxonaning bir kunlik mahsulot ishlab chiqarish rejasi yo‘l matritsa ko‘riishida berilgan:

$$C = (100 \quad 50 \quad 70).$$

Ikki turdagi qishloq xo‘jalik xom-ashyo mahsulotlarining narxi ushbu matritsa ko’rinishida berilgan (bir kilogrami uchun):

$$B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

1) Korxonaning bir kunlik mahsulot ishlab chiqarish rejasi bajarilishi uchun qancha xom-ashyo mahsuloti kerak?

2) Uch turdagi mahsulotlarning har donasi uchun ishlatilgan xom-ashiyoning narxini toping.

3) Korxonaning bir kunlik mahsulot ishlab chiqarish rejasini bajarish uchun sarflagan ikki turdagi xom-ashyoning narxi toping.

Yechish. 1) Bir kunlik mahsulot ishlab chiqarish uchun sarflanadigan $S_1 -$ birinchi va S_2 – ikkinchi turdagi xom-ashiyo miqdorini aniqlaymiz:

$$S_1 = 2 \cdot 100 + 3 \cdot 50 + 1 \cdot 70 = 200 + 150 + 70 = 420 \text{ kg}.$$

$$S_2=1\cdot100+2\cdot50+3\cdot70=100+100+210=410 \text{ kg},$$

yoki, boshqa tartibda, ya’ni matritsa yordamida aniqlaymiz:

$$S=C\cdot A=(100\ 50\ 70)\cdot\begin{pmatrix}2 & 1\\ 3 & 2\\ 1 & 3\end{pmatrix}=(420\ \ 410).$$

2) Uch turdagi tayyorlangan mahsulotning har bir donasiga ishlatilgan xom-ashyo narxini topamiz:

$$R=A\cdot B=\begin{pmatrix}2 & 1\\ 3 & 2\\ 1 & 3\end{pmatrix}\cdot\begin{pmatrix}2\\ 3\end{pmatrix}=\begin{pmatrix}7\\ 12\\ 11\end{pmatrix}.$$

3) Kunlik reja bo‘yicha ishlab chiqarilgan mahsulotlar uchun ishlatilgan 2 turdagi xom-ashyo mahsulotining umumiy narxi:

$$Q=S\cdot B=(420\ 410)\cdot\begin{pmatrix}2\\ 3\end{pmatrix}=840+1230=2070 \text{ ming so‘m}.$$

Kunlik ishlatilgan xom-ashyo mahsulotlarining narxini boshqa tartibda ham hisoblash mumkin:

$$Q=C\cdot R=(100\quad 50\quad 70)\cdot\begin{pmatrix}7\\ 12\\ 11\end{pmatrix}=700+600+770=2070000 \text{ so’m}.$$

Yuqorida yechilgan masalalardan ko‘rinadiki, matritsalarning iqtisodiyotda ahamiyati juda kattadur. Ulardan foydalanish hisobiga iqtisodchilar uchun muhim bo‘lgan ko‘pgina iqtisodiy masalalarni qulay va sodda yechish imkoniyati hosil bo‘ladi.

Mustaqil ishlash uchun misollar

1. Determinantlarni birinchi ustun elementlari bo‘yicha yoyib hisoblang:

$$a)\ \begin{vmatrix}2 & 3 & 4\\ 5 & -2 & 1\\ 1 & 2 & 3\end{vmatrix};\qquad b)\ \begin{vmatrix}a & 1 & a\\ -1 & a & 1\\ a & -1 & a\end{vmatrix}.$$

2. A va B matritsalar berilgan. Quyidagilar topilsin:

1) $2A-3B$; 2) $A\cdot B$; 3) $B\cdot A$; 4) A^{-1}; 5) $A\cdot A^{-1}$.

$$A=\begin{pmatrix}3 & 5 & -6\\ 2 & 4 & 3\\ -3 & 1 & 1\end{pmatrix},\quad B=\begin{pmatrix}2 & 8 & -5\\ -3 & -1 & 0\\ 4 & 5 & -3\end{pmatrix}.$$

3. Matritsalar ko‘paytmasi AB va BA ni hisoblang:

$$1)\ A=\begin{pmatrix}3 & -2\\ 5 & -4\end{pmatrix},\qquad B=\begin{pmatrix}3 & 4\\ 2 & 5\end{pmatrix};$$

$$2)\ A=\begin{pmatrix}5 & 0 & 2 & 3\\ 4 & 1 & 5 & 3\\ 3 & 1 & -1 & 2\end{pmatrix},\ B=\begin{pmatrix}6\\ -2\\ 7\\ 4\end{pmatrix}.$$

4. Determinantlarni hisoblang.

$$a)\ \begin{vmatrix}3 & 1 & 2 & 3\\ 4 & -1 & 2 & 4\\ 1 & -1 & 1 & 1\\ 4 & -1 & 2 & 5\end{vmatrix};\qquad b)\ \begin{vmatrix}2 & -1 & 2 & 0\\ 3 & 4 & 1 & 2\\ 2 & -1 & 0 & 1\\ 1 & 2 & 3 & -2\end{vmatrix};$$

$$c)\ \begin{vmatrix}-1 & -2 & 4 & 1\\ 2 & 3 & 0 & 6\\ 2 & -2 & 1 & 4\\ 3 & 1 & -2 & -1\end{vmatrix};\qquad d)\ \begin{vmatrix}0 & 4 & 1 & 1\\ -4 & 2 & 1 & 3\\ 0 & 1 & 2 & -2\\ 1 & 3 & 4 & -3\end{vmatrix}.$$

5. Determinant xossalaridan foydalanib hisoblang:

$$1)\ \begin{vmatrix}8 & 7 & 2 & 10\\ -8 & 2 & 7 & 10\\ 4 & 4 & 4 & 5\\ 0 & 4 & -3 & 2\end{vmatrix};\ 2)\ \begin{vmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 2 & 2\\ 1 & 1 & -1 & 3\\ 1 & 1 & 1 & -1\end{vmatrix};$$

$$3)\ \begin{vmatrix}2 & -3 & 4 & 1\\ 4 & -2 & 3 & 2\\ a & b & c & d\\ 3 & -1 & 4 & 3\end{vmatrix};\ 4)\ \begin{vmatrix}5 & a & 2 & -1\\ 4 & b & 4 & -3\\ 2 & c & 3 & -2\\ 4 & d & 5 & -4\end{vmatrix}.$$

6. Berilgan tenglamalar dan x ni toping va ildizlarni determinantga qo‘yib tekshiring:

$$1)\ \begin{vmatrix}x^2 & 3 & 2\\ x & -1 & 1\\ 0 & 1 & 1\end{vmatrix}=0;\qquad 2)\ \begin{vmatrix}x^2 & 4 & 0\\ x & 2 & 3\\ 1 & 1 & 0\end{vmatrix}=0.$$

7. Berilgan matritsalarga teskari matritsalarni toping:

1) $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$; 2) $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$;

3) $A = \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$; 4) $A = \begin{pmatrix} 2 & 1 & -1 \\ 5 & 2 & 4 \\ 7 & 3 & 2 \end{pmatrix}$.

8. Matritsali tenglamalarni yeching:

1) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$;

2) $X \cdot \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}$;

3) $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} \cdot X \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}$.

II BOB. CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASI

Iqtisodiy masalalarni (rejalastirish, boshqarish va boshqa masalalarni) yechishda ko‘p noma‘lumli chiziqli algebraik tenglamalar sistemasi qo‘llaniladi. m ta tenglamalardan tuzilgan n ta noma’lumli chiziqli tenglamalar sistemasini quyidagicha yozish mumkin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2, \\ \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m, \end{cases} \qquad (1)$$

bu yerda $a_{11}, a_{12}, \dots, a_{mn}$– sistema koeffitsiyentlari, $x_1, x_2, \dots, x_n$ – noma’lum koeffitsiyentlar, $b_1, b_2, \dots, b_m$ – ozod hadlar.

Chiziqli algebraik tenglamalar sistemasini quyidagi ixchamroq ko‘rinishda ham yozish mumkin:

$$\sum_{j=1}^{n} a_{ij}x_j = b_i, \qquad i = 1,2, \dots, m.$$

Agar $x_1^0, x_2^0, \dots, x_n^0$ sonlarni mos ravishda (1) tenglamalar sistemasidagi $x_1, x_2, \dots, x_n$ - noma’lum koeffitsiyentlar o‘rniga qo‘yilganda sistemaning har bir tenglamasi ayniyatga aylansa, $x_1^0, x_2^0, \dots, x_n^0$ sonlar ***sistemaning yechimi*** deyiladi. (1) chiziqli algebraik tenglamalar sistemasi yechimga ega bo‘lsa, ***sistema birgalikda***, aks holda, ya’ni yechimga ega bo‘lmasa, ***sistema birgalikda emas*** deyiladi. Agar birgalikdagi sistema bitta yechimga ega bo‘lsa, ***sistema aniqlangan***, aks holda, ya’ni ko‘p yechimga ega bo‘lsa, ***sistema aniqlanmagan*** deyiladi.

Tenglamalar sistemasida ozod hadlar nolga teng bo‘lsa, sistema ***bir jinsli*** deyiladi.

1-§. Chiziqli algebraik tenglamalar sistemasini yechishning Kramer usuli

Uch noma’lumli tenglamalar sistemasi berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \quad (2)$$

Noma’lumlar oldidagi koeffitsiyentlardan 3-tartibli determinantni tuzamiz va uni determinant xossalariga ko‘ra 1-ustun bo‘yicha yoyamiz:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot A_{11} + a_{21} \cdot A_{21} + a_{31} \cdot A_{31}. \quad (3)$$

Determinantning birinchi, ikkinchi va uchinchi ustunlarini mos ravishda ozod hadlar almashtirib uchda $\Delta_1,\ \Delta_2, \Delta_3$ determinantlarni hosil qilamiz hamda ularni ham 1-ustun bo‘yicha yoyamiz:

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix} = b_1 \cdot A_{11} + b_2 \cdot A_{21} + b_3 \cdot A_{31}, \quad (4)$$

$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix} = b_1 \cdot A_{12} + b_2 \cdot A_{22} + b_3 \cdot A_{32}, \quad (5)$$

$$\Delta_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} = b_1 \cdot A_{13} + b_2 \cdot A_{23} + b_3 \cdot A_{33}. \quad (6)$$

(2) sistemaning 1-tenglamasini A_{11} algebraik to‘ldiruvchiga, 2-tenglamasini A_{21} ga, 3- tenglamasini A_{31} ga ko‘paytirib hadlab qo‘shamiz:

$$\begin{gathered} (A_{11}a_{11} + A_{21}a_{21} + A_{31}a_{31})x_1 + \\ +(A_{11}a_{12} + A_{21}a_{22} + A_{31}a_{32})x_2 + \\ +(A_{11}a_{13} + A_{21}a_{23} + A_{31}a_{33})x_3 = \\ = A_{11}b_1 + A_{21}b_2 + A_{31}b_3 \end{gathered} \quad (7)$$

Yuqoridagi (3) va (4) munosabatlar hamda determinantlarning xossalariga ko‘ra:

$$a_{11} \cdot A_{11} + a_{21} \cdot A_{21} + a_{31} \cdot A_{31} = \Delta,$$
$$A_{11}a_{12} + A_{21}a_{22} + A_{31}a_{32} = 0,$$
$$A_{11}a_{13} + A_{21}a_{23} + A_{31}a_{33} = 0,$$
$$A_{11}b_1 + A_{21}b_2 + A_{31}b_3 = \Delta_1.$$

Natijada (7) tenglama quyidagi ko‘rinishga keladi:

$$\Delta \cdot x_1 = \Delta_1.$$

Xuddi yuqoridagidek, (2) sistemaning 1-tenglamasini A_{12} ga, 2-tenglamasini A_{22} ga va 3-tenglamasini A_{32} ga ko‘paytirib hadlab qo‘shamiz:

$$\Delta \cdot x_2 = \Delta_2.$$

(2) sistemaning 1-tenglamasini A_{13} ga, 2-tenglamasini A_{23} ga va 3-tenglamasini A_{33} ga ko‘paytirib hadlab qo‘shamiz:

$$\Delta \cdot x_3 = \Delta_3.$$

Natijada (2) sistemaga teng kuchli bo‘lgan

$$\begin{cases} \Delta \cdot x_1 = \Delta_1 \\ \Delta \cdot x_2 = \Delta_2 \\ \Delta \cdot x_3 = \Delta_3 \end{cases} \tag{8}$$

sistemani hosil qilamiz.

(8) sistemaning yechimi unda qatnashgan determinantlarga boq‘liqdir.

1^0. $\Delta \neq 0$ bo‘lsin. U holda, (8) sistemadan noma‘lumlarning

$$x_1 = \frac{\Delta_1}{\Delta}, \; x_2 = \frac{\Delta_2}{\Delta}, \; x_3 = \frac{\Delta_3}{\Delta} \tag{9}$$

qiymatini topamiz. (x_1, x_2, x_3) qiymatlar (1) sistemaning yagona yechimi bo’ladi. (9) formulaga ***Kramer formulasi***[1] deyiladi.

2^0. $\Delta = 0$ bo‘lib, Δ_1, Δ_2 va Δ_3 lardan hech bo‘lmaganda bittasi noldan farqli bo‘lsin. Bu holda (2) sistema yechimga ega bo‘lmaydi.

3^0. $\Delta = 0$ bo‘lib, $\Delta_1 = \Delta_2 = \Delta_3 = 0$ bo’lsin. Bu holda (2) sistema yoki cheksiz ko‘p yechimga ega bo‘ladi yoki bitta ham yechimga ega bo‘lmaydi.

1-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} x_1 - x_3 = 2, \\ 2x_1 - x_2 + 3x_3 = -1, \\ 3x_1 + 2x_2 - 2x_3 = 5. \end{cases}$$

[1] Gabriel Kramer (1704-1752) –fransuz, chiziqli algebraning asoschilaridan biri.

Yechish. Noma'lumlar oldidagi koeffitsyentlardan 3-tartibli determinantni tuzamiz va uning qiymatini topamiz:

$$\Delta=\begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{vmatrix}=2+0-4-3-6-0=-11\neq 0.$$

$\Delta\neq 0$ bo'lgani uchun berilgan tenglamalar sistemasi yagona yechimga ega bo'ladi. Kramer formulasiga ko'ra sistemaning yechimini topamiz.

Δ_1, Δ_2 va Δ_3 larning qiymatini aniqlaymiz:

$$\Delta_1=\begin{vmatrix} 2 & 0 & -1 \\ -1 & -1 & 3 \\ 5 & 2 & -2 \end{vmatrix}=4+0+2-5-12-0=-11,$$

$$\Delta_2=\begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 5 & -2 \end{vmatrix}=2+18-10-3-15+8=0,$$

$$\Delta_3=\begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & -1 \\ 3 & 2 & 5 \end{vmatrix}=-5+0+8+6+2-0=11.$$

Kramer formulasiga ko'ra,

$$x_1=\frac{\Delta_1}{\Delta}=\frac{-11}{-11}=1,\quad x_2=\frac{\Delta_2}{\Delta}=\frac{0}{-11}=0,$$

$$x_3=\frac{\Delta_3}{\Delta}=\frac{11}{-11}=-1.$$

2-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} 2x_1+x_2+x_3=1, \\ x_1-2x_2+3x_3=2, \\ 4x_1+2x_2+2x_3=-2. \end{cases}$$

Yechish. $\Delta=\begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & 3 \\ 4 & 2 & 2 \end{vmatrix}=-8+12+2+8-$

$$-12-2=0.$$

Δ_1, Δ_2 va Δ_3 larning qiymatini aniqlaymiz:

$$\Delta_1=\begin{vmatrix} 1 & 0 & 1 \\ 2 & -2 & 3 \\ -2 & 2 & 2 \end{vmatrix}=-4+0+4-4-6-0=-10,$$

$$\Delta_2=\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 4 & -2 & 2 \end{vmatrix}=8+12+2+8+12-2=40,$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & 2 \\ 4 & 2 & -2 \end{vmatrix} = 8+2+8+8-8+2 = 20.$$

$\Delta = 0$ bo‘lgani uchun 2-xossaga ko‘ra sistema yechimga ega emas.

2-§. Chiziqli algebraik tenglamalar sistemasini yechishning Gauss[2] usuli. Kroneker-Kapelli teoremasi

Uch noma’lumli tenglamalar sistemasi berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

Noma‘lumlar oldidagi koeffisiyentlardan *A* matritsani tuzamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

A matritsaning uchinchi ustunidan so‘ng ozod hadlardan iborat to‘rtinchi ustunni vertikal chiziq bilan ajratgan holda yozamiz va hosil bo‘lgan kengaytirilgan matritsani $\overline{A}$ bilan belgilaymiz:

$$\overline{A} = \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right)$$

Kengaytirilgan $\overline{A}$ matritsada diagonal elementlaridan pastda joylashgan a_{21}, a_{31} va a_{32} elementlar o‘rnida nol hosil qilishimiz kerak. Birinchi yo‘l elementlarini a_{21} ga va ikkinchi yo‘l elementlarini a_{11} ga ko‘paytirib, mos ravishda ayiramiz hamda hosil bo‘lgan natijani ikkinchi yo‘lga yozamiz. Xuddi shuningdek, birinchi yo‘l elementlarini a_{31} ga va uchinchi yo‘l elementlarini a_{11} ga ko‘paytirib, mos ravishda ayiramiz hamda hosil bo‘lgan natijani uchinchi yo‘lga yozamiz:

$$\left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right) \sim \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & a'_{32} & a'_{33} & b'_3 \end{array}\right)$$

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[2] Gauss Iogann Karl Fridrix (Johann Carl Friedrich, 1777-1855) – nemis matematigi.

bu yerda $a'_{22} = a_{12} \cdot a_{21} - a_{22} \cdot a_{11}$, $a'_{32} = a_{12} \cdot a_{31} - a_{32} \cdot a_{11}$, $b'_2 = b_1 \cdot a_{21} - b_2 \cdot a_{11}$ va h.k.

Hosil bo‘lgan matritsaning ikkinchi yo‘l elementlarini $\acute{a}_{32}$ ga va uchinchi yo‘l elementlarini $\acute{a}_{32}$ ga ko‘paytirib, mos ravishda ayiramiz hamda hosil bo‘lgan natijani uchinchi yo‘lga yozamiz:

$$\left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & a'_{32} & a'_{33} & b'_3 \end{array}\right) \sim \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & \acute{a}_{22} & \acute{a}_{23} & \acute{b}_2 \\ 0 & 0 & \ddot{a}_{33} & \ddot{b}_3 \end{array}\right)$$

bu yerda $\ddot{a}_{33}$=$\acute{a}_{23} \cdot \acute{a}_{32} - \acute{a}_{33} \cdot \acute{a}_{22}$, $\ddot{b}_3$=$\acute{b}_2 \cdot \acute{a}_{32} - \acute{b}_3 \cdot \acute{a}_{22}$.

Hosil bo‘lgan matritsani noma’lumlar orqali ifodalaymiz:

$$\left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & \acute{a}_{22} & \acute{a}_{23} & \acute{b}_2 \\ 0 & 0 & \ddot{a}_{33} & \ddot{b}_3 \end{array}\right) \sim \begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ \acute{a}_{22}x_2 + \acute{a}_{33}x_3 = \acute{b}_2 \\ \ddot{a}_{33}x_3 = \ddot{b}_3 \end{cases}$$

Sistemaning uchinchi tenglamasidan noma’lum koeffitsiyent x_3 ning qiymatini topamiz:

$$x_3 = \frac{\ddot{b}_3}{\ddot{a}_{33}}.$$

x_3 ning qiymatini sistemaning ikkinchi tenglamasiga qoyamiz va noma’lum koeffitsiyent x_2 ning qiymatini aniqlaymiz. Shuningdek, 1-tenglamadan x_1 ning qiymati aniqlanadi.

3-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} x_1 - x_3 = 2, \\ 2x_1 - x_2 + 3x_3 = -1, \\ 3x_1 + 2x_2 - 2x_3 = 5. \end{cases}$$

Yechish. Tenglamalar sistemasini Gauss usulida yechamiz. Ushbu kengaytirilgan matritsani tuzamiz:

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 2 & -1 & 3 & -1 \\ 3 & 2 & -2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -5 & 5 \\ 0 & -2 & -1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -5 & 5 \\ 0 & 0 & 11 & -11 \end{array}\right) \sim \begin{cases} x_1 - x_3 = 2, \\ x_2 - 5x_3 = 5, \\ 11x_3 = -11. \end{cases}$$

Sistemaning uchinchi tenglamasidan noma’lum koeffitsiyent x_3 ning qiymatini topamiz:

$$11x_3 = -11 \;\Rightarrow\; x_3 = \frac{-11}{11} \;\Rightarrow\; x_3 = -1.$$

x_3 ning qiymatini sistemaning ikkinchi tenglamasiga qoyamiz va noma'lum koeffitsiyent x_2 ning qiymatini aniqlaymiz:

$$x_2 - 5 \cdot (-1) = 5 \;\Longrightarrow\; x_2 + 5 = 5 \;\Longrightarrow\; x_2 = 0.$$

Birinchi tenglamadan x_1 ning qiymatini aniqlaymiz:

$$x_1 - x_3 = 2 \;\Longrightarrow\; x_1 - (-1) = 2 \;\Longrightarrow\; x_1 = 1.$$

Demak, sistemaning yechimi $\{1;\ 0;\ -1\}$**.**

Teorema (Kroneker[3]-Kapelli[4]). Chiziqli tenglamalar sistemasi birgalikda bo'lishi uchun kengaytirilgan va asosiy matrisalari ranglari teng ($rang(\overline{A}) = rang(A)$) bo'lishi zarur va yetarli.

Usbu teoremadan quyidagi natijalar kelib chiqadi:

1. Agar $rang(\overline{A}) \neq rang(A)$ bo'lsa, sistema birgalikda emas.

2. Agar $rang(\overline{A}) = rang(A) = r$ bo'lsa, sistema birgalikda bo'ladi. $r = n$ bo'lsa, sistema yagona yechimga bo'ladi, $r < n$ bo'lsa, sistema cheksiz ko'p yechimga egadir. Bu yerda n – sistemada qatnashayotgan noma'lumlar soni.

4-misol. Sistemani yeching.

$$\begin{cases} x_1 + 3x_2 - 2x_3 + 4x_4 = 1, \\ -3x_1 + 2x_2 + 6x_3 - 5x_4 = 2, \\ -x_1 + 8x_2 + 2x_3 + 3x_4 = -2. \end{cases}$$

Yechish. Sistemani birgalikda yoki birgalikda emasligini tekshiramiz:

$$\overline{A} = \left(\begin{array}{cccc:c} 1 & 3 & -2 & 4 & 1 \\ -3 & 2 & 6 & -5 & 2 \\ -1 & 8 & 2 & 3 & -2 \end{array}\right) \square \left(\begin{array}{cccc:c} 1 & 3 & -2 & 4 & 1 \\ 0 & 11 & 0 & 7 & 5 \\ 0 & 11 & 0 & 7 & -1 \end{array}\right) \square$$

$$\square \left(\begin{array}{cccc:c} 1 & 3 & -2 & 4 & 1 \\ & & 0 & 7 & 5 \\ 0 & 0 & 0 & 0 & -6 \end{array}\right) \Rightarrow \quad rang(A)=2,\ rang(\overline{A}) = 3.$$

$rang(\overline{A}) \neq rang(A) \quad \Rightarrow$ sistema birgalikda emas.

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[3] Kroneker Leopol'd (Kronecker Leopold, 1823-1891) – nemis matematigi.
[4] Kapelli Al'fredo (Capelli Alfredo, 1855-1910) – italiyalik matematik.

5-misol. Sistemani yeching.

$$\begin{cases} 2x_1 - x_2 + x_3 = 0, \\ \quad x_1 + x_3 = -1, \\ \quad 3x_2 + x_3 = -2. \end{cases}$$

Yechish.

$$\overline{A} = \left(\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 3 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & -1 & -1 & 2 \\ & & & \\ 0 & 3 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & -1 & -1 & 2 \\ & & & \\ 0 & 2 & 0 & 0 \end{array}\right).$$

$$\begin{vmatrix} 0 & -1 & -1 \\ 1 & -1 & 0 \\ 0 & 2 & 0 \end{vmatrix} = -2 \neq 0 \quad \Rightarrow \quad rang(A) = rang(\overline{A}) = 3.$$

Sistema birgalikda, matritsaning rangi noma'lumlar soniga teng. Demak, sistema yagona yechimga ega.

Almashtirishlar yordamida hosil qilingan oxirgi matritsadan foydalanib, berilgan sistemaga teng kuchli bo‘lgan sistemani yozamiz:

$$\begin{cases} -x_2 - x_3 = 2, \\ \quad x_1 - x_2 = 1, \\ \quad 2x_2 = 0. \end{cases} \Rightarrow \quad \begin{array}{l} x_3 = -2, \\ x_1 = 1, \\ x_2 = 0. \end{array}$$

Sistemaning yechimi: $x_1 = 1, \; x_2 = 0, \; x_3 = -2.$

6-misol. Chiziqli tenglamalar sistemasining umumiy va bitta xususiy yechimini toping: $\begin{cases} 3x_1 - 2x_2 - 5x_3 + x_4 = 3 \\ 2x_1 - 3x_2 + x_3 + 5x_4 = -3 \\ x_1 + 2x_2 - 4x_4 = -3 \end{cases}.$

Yechish. 1) Sistemaning kengaytirilgan matritsasini yozamiz:

$$\left(\begin{array}{cccc|c} 3 & -2 & -5 & 1 & 3 \\ 2 & -3 & 1 & 5 & -3 \\ 1 & 2 & 0 & -4 & -3 \end{array}\right).$$

2) Matritsani uchburchakli ko‘rinishga keltiramiz. Hisoblashlarda qulaylik bo‘lishishi uchun matritsaning birinchi va uchunchi yo‘llarini o‘zaro almashtiramiz. Matritsaning 1-ustunida a_{11} elementdan

boshqa elementlar o‘rnida almashtirishlar yordamida nollar hosil qilamiz:

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & -4 & -3 \\ 2 & -3 & 1 & 5 & -3 \\ 3 & -2 & -5 & 1 & 3 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & -4 & -3 \\ 0 & -7 & 1 & 13 & 3 \\ 0 & -8 & -5 & 13 & 12 \end{array}\right) \Rightarrow$$

3) Hisoblashlarda qulaylik bo‘lishishi uchun matritsaning ikkinchi va uchinchi ustunlarini o‘zaro almashtiramiz. Har bir ustun yuqorisiga noma’lum o‘zgaruvchilarni yozamiz. 2-yo‘l elementlarini 5 ga ko‘paytiramiz va 3-yo‘lning mos elementlariga qo‘shamiz:

$$\Rightarrow \left(\begin{array}{cccc|c} x_1 & x_3 & x_2 & x_4 & \\ 1 & 0 & 2 & -4 & -3 \\ 0 & 1 & -7 & 13 & 3 \\ 0 & -5 & -8 & 13 & 12 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} x_1 & x_3 & x_2 & x_4 & \\ 1 & 0 & 2 & -4 & -3 \\ 0 & 1 & -7 & 13 & 3 \\ 0 & 0 & -43 & 78 & 27 \end{array}\right).$$

4) Bazis minor sifatida ushbu noldan farqli determinantni olamiz:

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & -7 \\ 0 & 0 & -43 \end{vmatrix},$$

unga tegishli bo‘lgan noma‘lumlar: x_1, x_2, x_3, – bog‘liqli o‘zgaruvchilar, bundan, x_4 – erkli o‘zgaruvchidir.

$rang(A) = rang(\overline{A}) = 3$ bo‘lgani uchun sistema yechimga ega, ammo matritsaning rangi noma’lumlar sonidan kichik. Shu sababli, sistema aniqlanmagan, ya’ni sistema cheksiz ko‘p yechimga egadir.

5) Bog‘liqli o‘zgaruvchilarni x_4 - erkli o‘zgaruvchi orqali ifodalaymiz:

$$-43x_2 + 78x_4 = 27, \text{ bundan } x_2 = \frac{78x_4 - 27}{43};$$

$$x_3 - 7x_2 + 13x_4 = 3, \text{ bundan } x_3 = \frac{-13x_4 - 60}{43};$$

$$x_1 + 2x_2 - 4x_4 = -3, \text{ bundan } x_1 = \frac{16x_4 - 75}{43}.$$

6) $x_4 = 2$ uchun bitta xususiy yechimni topamiz, u holda

$$x_1 = \frac{16 \cdot 2 - 75}{43} = -1; \; x_2 = \frac{78 \cdot 2 - 27}{43} = 3; \; x_3 = \frac{-13 \cdot 2 - 60}{43} = -2.$$

7) Olingan yechimlarni tekshiramiz:

$$3\cdot(-1)-2\cdot3-5\cdot(-2)+1\cdot2=3$$
$$2\cdot(-1)-3\cdot3+1\cdot(-2)+5\cdot2=-3.$$
$$1\cdot(-1)+2\cdot3-4\cdot2=-3$$

Demak, sistemaning umumiy yechimi:

$$x_1=\frac{16x_4-75}{43};\ x_2=\frac{78x_4-27}{43};\ x_3=\frac{-13x_4-60}{43};\quad x_4\in R..$$

Agar $x_4=2$ bo‘lsa, bog‘liqli noma’lumlarning qiymati:

$$x_1=-1;\ x_2=3;\ x_3=-2$$

bo‘ladi. Bu qiymatlar sistemaning xususiy yechimidir.

7-misol. Sistemani yeching.

$$\begin{cases} 2x_1+x_3-x_4=0, \\ 3x_1-x_2-2x_4=1, \\ x_2+x_3-x_4=2, \\ 5x_1-x_2+x_3-3x_4=1. \end{cases}$$

Yechish. Sistemani birgalikda yoki birgalikda emasligini tekshiramiz:

$$\overline{A}=\left(\begin{array}{cccc:c} 2 & 0 & 1 & -1 & 0 \\ 3 & -1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -1 & 2 \\ 5 & -1 & 1 & -3 & 1 \end{array}\right)\sim\left(\begin{array}{cccc:c} 2 & 0 & 1 & -1 & 0 \\ 3 & -1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

To‘rtinchi yo‘ldagi nollar birinchi va ikkinchi yo‘llarni qo‘shish hamda to‘rtinchi yo‘ldan ayirish natijasida hosil bo‘ldi. Birinchi ustunda nollar hosil qilish uchun birinchi yo‘lni 3 ga, ikkinchi yo‘lni 2 ga ko‘paytiramiz va ikkinchi yo‘ldan ayiramiz:

$$\overline{A}=\left(\begin{array}{cccc:c} 6 & 0 & 3 & -3 & 0 \\ 6 & -2 & 0 & -4 & 2 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)\sim\left(\begin{array}{cccc:c} 6 & 0 & 3 & -3 & 0 \\ 0 & -2 & -3 & -1 & 2 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

To‘rtinchi yo‘lni tashlab yuboramiz va uchinchi yo‘l elementlarini 2 ga ko‘paytiramiz. Ikkinchi yo‘l elementlarini uchinchi yo‘lning mos elementlariga qo‘shamiz:

$$\left(\begin{array}{cccc:c} 2 & 0 & 1 & -1 & 0 \\ 0 & -2 & -3 & -1 & 2 \\ 0 & 1 & 1 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc:c} 2 & 0 & 1 & -1 & 0 \\ & & & & \\ 0 & 2 & 2 & -2 & 4 \end{array}\right) \sim \left(\begin{array}{cccc:c} 2 & 0 & 1 & -1 & 0 \\ & & & & 2 \\ 0 & 0 & -1 & -3 & 6 \end{array}\right).$$

$$\begin{vmatrix} 2 & 0 & 1 \\ 0 & -2 & -3 \\ 0 & 0 & -1 \end{vmatrix} = 4 \neq 0 \quad \Rightarrow \quad rang(A) = rang(\overline{A}) = 3 \quad \Rightarrow$$

sistema yechimga ega, ammo matritsaning rangi noma'lumlar sonidan kichik bo'lgani uchun sistema aniqlanmagan, ya'ni sistema cheksiz ko'p yechimga egadir.

Oxirgi matritsadan foydalanib, sistemani yozamiz:

$$\begin{cases} 2x_1 + x_3 - x_4 = 0, \\ -2x_2 - 3x_3 - x_4 = 2, \\ -x_3 - 3x_4 = 6. \end{cases}$$

Ma'lumki, x_4- erkli o'zgaruvchi, sistemani quyidagi ko'rinichga keltiramiz:

$$\begin{cases} 2x_1 + x_3 = x_4, \\ -2x_2 - 3x_3 = x_4 + 2, \\ -x_3 = 3x_4 + 6. \end{cases} \Rightarrow \begin{cases} 2x_1 = 6 + 3x_4 + x_4, \\ -2x_2 = -18 - 9x_4 + x_4 + 2, \\ x_3 = -3x_4 - 6. \end{cases} \Rightarrow$$

$$\begin{cases} x_1 = 3 + 2x_4, \\ x_2 = 8 + 4x_4, \\ x_3 = 6 - 3x_4, \\ x_4 \in R. \end{cases}$$

3-§. Chiziqli algebraik tenglamalar sistemasini teskari matritsa yordamida yechish

Uch noma'lumli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \tag{10}$$

Noma'lumlar oldidagi koeffitsiyentlardan *A* matritsani, noma'-lumlardan tashkil topgan *X* – ustun matritsani va ozod hadlardan *B* – ustun matritsani tuzamiz:

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix};\ B=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

u holda (10) tenglamalar sistemasini matritsali tenglama, ya'ni

$$AX=B \qquad (11)$$

ko'rinishda ifodalash mumkin.

Agar *A* matritsa xosmas matritsa bo'lsa, u holda (11) tenglama quyidagicha yechiladi. (11) tenglamaning o'ng va chap qismini *A* matritsaga teskarisi matritsa A^{-1} ni ko`paytiramiz:

$$A^{-1}(AX)=A^{-1}B \text{ yoki } (A^{-1}A)X=A^{-1}B,$$

$A^{-1}A=E$ va $EX=X$ bo'lgani uchun tenglamaning

$$X=A^{-1}B \qquad (12)$$

ko'rinishidagi yechimiga ega bo'lamiz.

8-misol. Ushbu tenglamalar sistemasini matritsalar yordamida yeching.

$$\begin{cases} x_1-x_3=2, \\ 2x_1-x_2+3x_3=-1, \\ 3x_1+2x_2-2x_3=5. \end{cases}$$

Yechish.

$$A=\begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{pmatrix};\ X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},B=\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

A matritsa determinantini hisoblaymiz:

$$detA=\begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{vmatrix}=2+0-4-3-6-0=-11\neq 0.$$

Demak, $detA\neq 0 \implies A^{-1}$ – mavjud. A^{-1} ni topamiz:

$$A^{-1} = \frac{1}{15}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix},$$

bu yerda

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 3 \\ 2 & -2 \end{vmatrix} = -4,$$
$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = 13,$$
$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 7,$$
$$A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & -1 \\ 2 & -2 \end{vmatrix} = -2,$$
$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = 1,$$
$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} = -2,$$
$$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & -1 \\ -1 & 3 \end{vmatrix} = -1,$$
$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = -5,$$
$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1.$$

Demak, berilgan matritsaga teskari matritsa quyidagi ko‘rinishga bo‘ladi:

$$A^{-1} = \frac{1}{-11}\begin{pmatrix} -4 & -2 & -1 \\ 13 & 1 & -5 \\ 7 & -2 & -1 \end{pmatrix}.$$

$$A^{-1} = \begin{pmatrix} \frac{4}{11} & \frac{2}{11} & \frac{1}{11} \\ -\frac{13}{11} & -\frac{1}{11} & \frac{5}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{1}{11} \end{pmatrix}$$

(12) tenglikdan sistemaning yechimini topamiz:

$$X = \begin{pmatrix} \frac{4}{11} & \frac{2}{11} & \frac{1}{11} \\ -\frac{13}{11} & -\frac{1}{11} & \frac{5}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{1}{11} \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{8}{11} - \frac{2}{11} + \frac{5}{11} \\ -\frac{26}{11} + \frac{1}{11} + \frac{25}{11} \\ -\frac{14}{11} - \frac{2}{11} + \frac{5}{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

Demak, tenglamalar sistemasining yechimi:

$$x_1 = 1;\ x_2 = 0;\ x_3 = -1.$$

4-§. Bir jinsli chiziqli algebraik tenglamalar sistemasi

Ozod hadlari nolga teng bo‘lgan chiziqli algebraik tenglamalar sistemasini qaraylik.

Ta’rif. Agar har bir tenglamada ozod hadlar nolga teng bo‘lsa, birinchi darajali ***tenglamalar sistemasi bir jinsli*** deyiladi.

Ushbu bir jinsli chiziqli algebraik tenglamalar sistemasini berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0. \end{cases} \tag{13}$$

Ma’lumki, $x_1 = 0;\ x_2 = 0;\ x_3 = 0$ sonlar (13) sistemaning har bir tenglamasini qanoatlantiradi. Bu yechim (13) sistemaning ***trivial yechimi*** deyiladi.

Agar (13) sistemaning asosiy determinanti $detA \neq 0$ bo‘lsa, sistema *trivial* yechimga ega bo‘ladi. (13) sistemada ozod hadlar nolga teng bo‘lgani uchun $\Delta_1 = \Delta_2 = \Delta_3 = 0$ bo‘ladi. Kramer formulasiga ko‘ra, $x_1 = 0;\ x_2 = 0;\ x_3 = 0$.

Demak, (13) sistema noldan farqli yechimga ega bo‘lishi uchun $detA = 0$ bo‘lishi zarur ekan.

Ikkita tenglamadan iborat sistema berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0. \end{cases} \tag{14}$$

1-holat. Noma’lumlar oldidagi koeffitsiyentlar proporsional emas, ya’ni quyidagi uchta determinantlarning kamida bittasi nolga teng bo‘lmaydi:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix},\ \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix},\ \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix}. \tag{15}$$

U holda yechimni simmetrik ko‘rinishda yozish mumkin:

$$x_1 = k\begin{vmatrix} a_{12} & a_{13} \\ a_{21} & a_{23} \end{vmatrix},\ x_2 = -k\begin{vmatrix} a_{11} & a_{31} \\ a_{21} & a_{31} \end{vmatrix},$$

$$x_3 = k\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \tag{16}$$

bu yerda k – ixtiyoriy o‘zgarmas son.

2-holat. Noma'lumlar oldidagi koeffitsiyentlar o'zaro proporsional bo'lsin, ya'ni (15) determinantlarning hammasi nolga bo'lsin. Sistema bitta tenglamaga keltiriladi.

9-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} 2x_1 - x_2 + 3x_3 = 0, \\ x_1 + 2x_2 - 5x_3 = 0. \end{cases}$$

Yechish.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5,$$

$$\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ 2 & -5 \end{vmatrix} = -1,$$

$$\begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ -5 & 1 \end{vmatrix} = 13.$$

(16) formulaga asosan: $x_1 = -k$; $x_2 = -13k$; $x_3 = 5k$.

10-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} 3x_1 + x_2 - 2x_3 = 0, \\ 6x_1 + 2x_2 - 4x_3 = 0. \end{cases}$$

Yechish. Noma'lumlar oldidagi koeffitsiyentlar o'zaro proporsionaldir. U holda, (15) determinantlarning hammasi nolga teng bo'ladi. Sistemani bitta tenflama bilan ifodalash mumkin. Tenglamadagi ixtiyoriy ikkita noma'lumga ixtiyoriy qiymatlar beriladi va uchinchi noma'lum aniqlanadi.

11-misol.

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ 3x_1 + x_2 + 2x_3 = 0 \\ 2x_1 + 3x_2 + x_3 = 0 \end{cases}$$

Yechish. Sistemaning asosiy determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 1 + 8 + 27 - 6 - 6 - 6 = 18 \neq 0.$$

Demak, sistema $x_1 = 0$; $x_2 = 0$; $x_3 = 0$ trivial yechimga egadir.

12-misol.

$$\begin{cases} -2x_1 - 2x_2 + x_3 = 0, \\ x_1 + x_2 + 4x_3 = 0, \\ 3x_1 + 3x_2 - x_3 = 0. \end{cases}$$

Yechish. Sistemaning asosiy determinantini hisoblaymiz:

$$\Delta=\begin{vmatrix} -2 & -2 & 1 \\ 1 & 1 & 4 \\ 3 & 3 & -1 \end{vmatrix} = 2-24+3-3+24-2=0.$$

Demak, sistema noldan farqli yechimga egadir. Sistemaning ixtiyoriy ikkita tenglamasini olib, (16) formulaga asosan, sistemaning yechimni topamiz. Bu yerda k – ixtiyoriy o‘zgarmas son.

$$x_1 = k\begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} = -9k,\ x_2 = -k\begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} = 9k,$$

$$x_3 = k\begin{vmatrix} -2 & -2 \\ 1 & 1 \end{vmatrix} = 0.$$

Sistemaning echimi: $\{-9k;\ 9k;\ 0\}$.

5-§. Chiziqli algebraik tenglamalar sistemasining tatbiqlari

Chiziqli algebraik tenglamalar sistemasi yordamida iqtisodiyot tarmoqlariga tegishli masalalarning yechimlarini topish mumkin. Chiziqli tenglamalar sistemasining amaliy masalalarga tatbiqini quyidagi masalalarda ko‘rib chiqamiz.

1-masala. Zavodda 3 xil turdagi temir-buyum mahsulotlari ishlab chiqariladi. Mahsulotlar uchun 3 turdagi S_1, S_2 va S_3 xom-ashyo ishlatiladi. Bitta mahsulot uchun har bir xom-ashyodan ishlatish me’yori va bir oylik xom-ashyo ishlatish hajmi 1-jadvalda berilgan. Zavodning har bir mahsulot bo‘yicha bir oylik ishlab chiqarish hajmini toping.

1-jadval

Xom-ashyo turlari	Bitta mahsulot ishlab chiqarish uchun xom-ashyo ishlatilish me’yori (shartli birlikda)			Bir oylik xom-ashyo islatilishi (shartli birlikda)
	darvoza	deraza panjarasi	zinapoya to‘siqlari	
S_1	2	0	3	69
S_2	1	2	1	60
S_3	5	0	4	120

Yechish. Masalani chiziqli algebraik tenglamalar sistemasi yordamida yechamiz.

Faraz qilaylik, zavod bir oyda x dona darvoza, y dona deraza panjarasi, z dona zinapoya to‘siqlari ishlab chiqarsin. U holda, har bir turdagi mahsulot uchun xom-ashyo sarflanishiga mos holda, quyidagi sistemani hosil qilamiz:

$$\begin{cases} 2x + 3z = 69, \\ x + 2y + z = 60, \\ 5x + 4z = 120. \end{cases}$$

Bu sistemani turli usullar bilan yechish mumkin. Biz Kramer usulidan foydalanamiz. Buning uchun asosiy determinantni tuzamiz va hisoblaymiz:

$$\Delta = \begin{vmatrix} 2 & 0 & 3 \\ 1 & 2 & 1 \\ 5 & 0 & 4 \end{vmatrix} = -14.$$

Asosiy determinant noldan farqli, demak, sistema birgalikda va yagona yechimga ega. Yordamchi determinantlarni tuzamiz va hisoblaymiz:

$$\Delta x = \begin{vmatrix} 69 & 0 & 3 \\ 60 & 2 & 1 \\ 120 & 0 & 4 \end{vmatrix} = -168,$$

$$\Delta y = -231, \;\; \Delta z = -210.$$

Kramer formulasiga asosan, masalaning matematik nuqtai-nazardan yechimini topamiz:

$$x = \frac{-168}{-14} = 12, \;\; y = \frac{-231}{-14} = 16.5, \qquad z = \frac{-210}{-14} = 15.$$

Masala yechimi butun bo‘lishini hisobga olsak, sistemaning noma’lumlari qiymatidan quyidagi xulosaga kelamiz, ya’ni zavod bir oyda 12 ta darvoza, 16 ta deraza va 15 ta zinapoya to‘siqlarini ishlab chiqaradi.

2-masala. Ma’lum bir sondagi o‘ramli materialdan fabrikada A-ko‘rinishda 360 ta, B – ko‘rinishdagi – 300 ta va C – ko‘rinishdagi 675 ta mahsulot tikiladi. 3- xil usuldagi bichishdan foydalanish mumkin. Har bir material o‘ramidan bichish usullari bo‘yicha mahsulotlar tayyorlash miqdori 2-jadvalda berilgan. Reja bajarilish shartini matematik shaklda yozing.

Yechish. x, y va z bilan mos ravichda birinchi, ikkinchi va uchinchi bichish usullari bo‘yicha ishlatilgan material o‘ramlari bo‘lsin. U holda, 1-bichish usulida x ta o‘ramda $3x$ ta, 2-bichish usulida – $2y$ ta, 3-bichish usulida – z ta A turdagi mahsulotlar rejasini bajarish uchun quyidagi tenglama o‘rinli bo‘lishi kerak: $3x + 2y + z = 360$. Xuddi shu yo‘l bilan $x + 6y + 2z = 300,\ 4x + y + 5z = 675$ tenglamalarni hosil qilamiz. Ularni ushbu sistema ko‘rinishida ifodalaymiz:

$$\begin{cases} 3x + 2y + z = 360, \\ x + 6y + 2z = 300, \\ 4x + y + 5z = 675. \end{cases}$$

2-jadval

Mahsulot turlari	Bichish shakllari		
	1	2	3
A	3	2	1
B	1	6	2
C	4	1	5

Sistemani Gauss usulida yechamiz:

$$\left(\begin{array}{ccc|c} 3 & 2 & 1 & 360 \\ 1 & 6 & 2 & 300 \\ 4 & 1 & 5 & 675 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 6 & 2 & 300 \\ 3 & 2 & 1 & 360 \\ 4 & 1 & 5 & 675 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 6 & 2 & 300 \\ 0 & -16 & -5 & -550 \\ 0 & -7 & 2 & 15 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 6 & 2 & 300 \\ 0 & 16 & 5 & 550 \\ 0 & -14 & 4 & 30 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 6 & 2 & 300 \\ 0 & 16 & 5 & 550 \\ 0 & 2 & 9 & 570 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 6 & 2 & 300 \\ 0 & 2 & 9 & 570 \\ 0 & 16 & 5 & 540 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 6 & 2 & 300 \\ 0 & 2 & 9 & 570 \\ 0 & 0 & -67 & -4020 \end{array}\right) \sim \begin{cases} x + 6y + 2z = 300, \\ 2y + 9z = 570, \\ -67z = -4020. \end{cases}$$

Bu tenglamalar sistemasi yuqorida masala shartiga binoan tuzilgan tenglamalar sistemasiga teng kuchlidir. Hosil bo‘lgan sistemadan $x = 90, y = 15, z = 60$ qiymatlarni aniqlaymiz.

Yuqorida ko‘rilgan masalalardan ma’lumki, chiziqli algebraik tenglamalar sistemasining iqtisodiy masalalarni yechishda o‘rni kattadir.

Mustaqil yechish uchun misollar

1. Tenglamalar sistemasini yeching:

1) $\begin{cases} 2x-3y+z-2=0, \\ x+5y-4z+5=0, \\ 4x+y-3z+4=0. \end{cases}$ 2) $\begin{cases} 7x+2y+3z=15, \\ 5x-3y+2z=15, \\ 10x-11y+5z=36. \end{cases}$

3) $\begin{cases} x+2y+3z=4, \\ 2x+y-z=3, \\ 3x+3y+2z=10. \end{cases}$ 4) $\begin{cases} 2x-y+z=2, \\ 3x+2y+2z=-2, \\ x-2y+z=1. \end{cases}$

2. Sistemalarni Gauss usuli bilan yeching:

1) $\begin{cases} x-y+3z=-4, \\ 2x+3y-2z=5, \\ 3x+5y+z=4. \end{cases}$ 2) $\begin{cases} x-2y-3z=8, \\ 3x+y+z=3, \\ 4x+3y-2z=-1. \end{cases}$

3) $\begin{cases} x-2y+3z=6, \\ 2x+3y-4z=16, \\ 3x-2y-5z=12. \end{cases}$ 4) $\begin{cases} 2x+y-3z=3, \\ 3x+4y-5z=9, \\ 2y+7z=11. \end{cases}$

3. Tenglamalar sistemasini matritsalar yordamida yeching:

1) $\begin{cases} x+2y+z=4, \\ 3x-5y+3z=1, \\ 2x+7y-z=8. \end{cases}$ 2) $\begin{cases} 2x-4y+9z=28, \\ 7x+3y-6z=-1, \\ 7x+9y-9z=5. \end{cases}$

3) $\begin{cases} 2x+y+2z=6, \\ x-3y-z=-5, \\ 5x-2y+z=-1. \end{cases}$ 4) $\begin{cases} x-2y+3z=5, \\ 2x+3y-z=-4, \\ 3x+y-2z=-1. \end{cases}$

5) $\begin{cases} 2x+y+2z=6, \\ x-3y-z=-5, \\ 5x-2y+z=-1. \end{cases}$ 6) $\begin{cases} x-2y+3z=5, \\ 2x+3y-z=-4, \\ 3x+y-2z=-1. \end{cases}$

4. Bir jinsli chiziqli algeraik tenglamalar sistemasini yeching:

1) $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+3y-4z=0. \end{cases}$ 2) $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+y-z=0. \end{cases}$

3) $\begin{cases} x+2y-4z=0, \\ 2x-y-3z=0, \\ x+3y+z=0. \end{cases}$ 4) $\begin{cases} 5x-3y+2z=0, \\ 2x+4y-3z=0, \\ 3x-7y+5z=0. \end{cases}$

5) $\begin{cases} 3x-y+2z=0, \\ 2x+3y-5z=0, \\ x+y+z=0. \end{cases}$ 6) $\begin{cases} 2x-y+3z=0, \\ x+2y-5z=0, \\ 3x+y-2z=0. \end{cases}$

III BOB. TEKISLIKDA ANALITIK GEOMETRIYANING SODDA MASALALARI

1-§. Ikki nuqta orasidagi masofa

Tekislikda $A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalar berilgan bo'lsin. A va B nuqtalardan *Ox* o'qiga perpendikulyar tushiramiz (1-rasm). Perpendikulyarning *Ox* o'qi bilan kesishgan nuqtalarini mos ravishda A_1 va B_1 bilan belgilaymiz. Ma'lumki,

$$OA_1 = x_1, \;\; OB_1 = x_2, \; AA_1 = y_1, \; BB_1 = y_2 \tag{1}$$

A nuqtadan *Ox* o'qiga perellel to'g'ri chiziq o'tkazib, uning BB_1 bilan kesishgan nuqtasini *C* bilan belgilaymiz. U holda

$$AC = A_1B_1, \; CB_1 = A_1A_1 \tag{2}$$

bo'ladi. Agar $A_1B_1 = OB_1 - OA_1$, $BC = BB_1 - CB_1$ ekanini e'tiborga olsak, (1) va (2) munosabatlardan

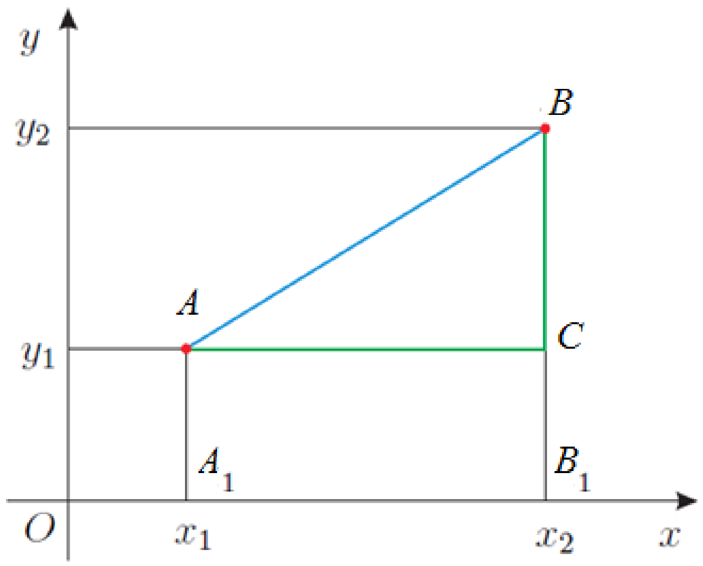
$$AC = x_2 - x_1, \; BC = y_2 - y_1 \tag{3}$$

kelib chiqadi.

ΔACB – to'g'ri burchakli uchburchak. Pifagor teoremasiga binoan $AB^2 = AC^2 + BC^2$ bo'ladi. (3) munosabatga ko'ra

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4}$$

bo'lishini topamiz. (4) formula tekislikda ikki nuqta orasidagi masofani topish formulasidir.



1-rasm.

1-misol. $A(1; -2)$, $B(5; 1)$ nuqtalar orasidagi masofani toping.

Yechish. (4) formulaga ko‘ra, A va B nuqtalar orasidagi masofa:

$$AB = \sqrt{(5-1)^2 + (1-(-2))^2} = \sqrt{16+9} = 5.$$

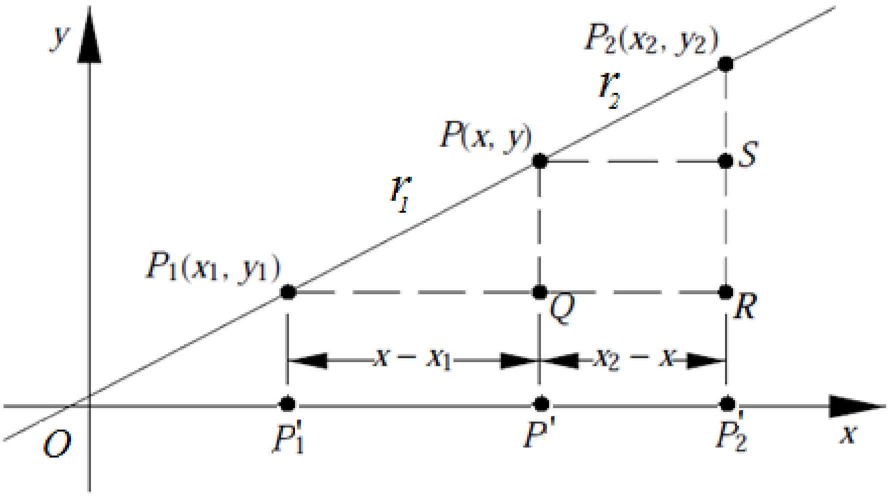
2-§. Kesmani berilgan nisbatda bo‘lish

Tekislikda $P_1(x_1, y_1)$ va $P_2(x_2, y_2)$ nuqtalarni tutashtiruvchi kesmani qaraylik. Bu kesmada shunday $P(x, y)$ nuqtani topish kerakki, P_1P kesmaning PP_2 kesmaga nisbati $\lambda = \frac{r_1}{r_2}$ songa teng bo‘lsin:

$$\frac{P_1P}{PP_2} = \frac{r_1}{r_2} = \lambda \qquad (5)$$

P_1, P, P_2 nuqtalardan Ox o‘qiga perpendikulyar tushiramiz (2-rasm). Unda $OP_1' = x_1$, $OP' = x, OP_2' = x_2$ bo‘ladi. P_1 va P nuqtalardan Ox o‘qiga parellel chiziqlar o‘tkazamiz. Ularning PP' va P_2P_2' bilan kesishgan nuqtalarini Q va S orqali belgilaylik. Ma’lumki,

$$\begin{gathered} P_1Q = P_1'P' = OP' - OP_1' = x - x_1, \\ PS = QR = P'P_2' = OP_2' - OP' = x_2 - x, \\ PQ = SR = PP' - QP' = PP' - P_1P_1' = y - y_1, \\ P_2S = P_2P_2' - SP_2' = P_2P_2' - PP' = y_2 - y. \end{gathered} \qquad (6)$$



2-rasm.

ΔP_1QP va ΔPSP_2 uchburchaklar o‘xshashligidan

$$\frac{P_1Q}{PS} = \frac{P_1P}{PP_2}, \qquad \frac{PQ}{P_2S} = \frac{P_1P}{PP_2}$$

bo‘lishini topamiz.

(5) va (6) tengliklarga asosan,

$$\frac{x-x_1}{x_2-x}=\frac{r_1}{r_2}, \qquad \frac{y-y_1}{y_2-y}=\frac{r_1}{r_2}$$

kelib chiqadi.

Demak,

$$\frac{x-x_1}{x_2-x}=\frac{r_1}{r_2} \Rightarrow x=\frac{r_2x_1+r_1x_2}{r_1+r_2}, \tag{7}$$

$$\frac{y-y_1}{y_2-y}=\frac{r_1}{r_2} \Rightarrow y=\frac{r_2y_1+r_1y_2}{r_1+r_2}. \tag{8}$$

Tengliklarning o‘ng tomonidagi kasrlarning surat va maxrajini r_2 ga bo‘lamiz. $\lambda=\frac{r_1}{r_2}$ belgilash kiritamiz, u holda *C* nuqtaning koordinatalari:

$$x=\frac{x_1+\lambda x_2}{1+\lambda}, \tag{9}$$

$$y=\frac{y_1+\lambda y_2}{1+\lambda}. \tag{10}$$

Xususiy holda, *C* nuqta *AB* kesmaning o‘rtasi bo‘lsa, $\lambda=1$ bo‘lib, *C* nuqtaning koordinatalari

$$x=\frac{x_1+x_2}{2},\ y=\frac{y_1+y_2}{2} \tag{11}$$

bo‘ladi.

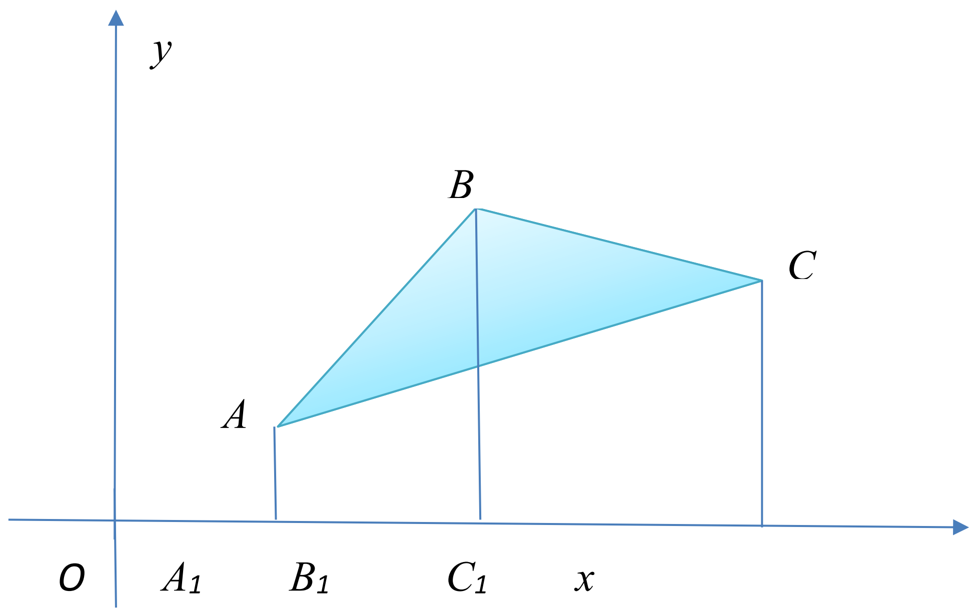
2-misol. Tekislikda *A*(2; 1), *B*(3; -2) nuqtalar berilgan. *AB* kesmani $\lambda=\frac{1}{2}$ nisbatda bo‘luvchi *C*(*x, y*) nuqtaning koordinatalarini toping.

$$x=\frac{2+\frac{1}{2}\cdot 3}{1+\frac{1}{2}}=\frac{\frac{4+3}{2}}{\frac{3}{2}}=\frac{7}{3},$$

$$y=\frac{1+\frac{1}{2}\cdot(-2)}{1+\frac{1}{2}}=\frac{0}{\frac{3}{2}}=0.$$

3-§. Uchburchak va ko‘pburchak yuzi

Tekislikda $A(x_1,y_1)$, $B(x_2,y_2)$ va $C(x_3,y_3)$ nuqtalar berilgan bo‘lsin. Bu nuqtalarni kesmalar bilan tutashtiramiz va *ABC* uchburchak hosil qilamiz (3-rasm).



3-rasm.

A, B, C nuqtalardan Ox o‘qiga perpendikulyar tushiramiz va ularni mos ravishda A_1, B_1, C_1 bilan belgilaymiz.

Bunda $OA_1 = x_1$, $OB_1 = x_2$, $OC_1 = x_3$, $AA_1 = y_1, BB_1 = y_2$, $CC_1 = y_3$ bo‘lib,

$$A_1B_1 = OB_1 - OA_1 = x_2 - x_1,$$
$$B_1C_1 = OC_1 - OB_1 = x_3 - x_2,$$
$$A_1C_1 = OC_1 - OA_1 = x_3 - x_1$$

bo‘ladi.

3-rasmga ko‘ra, AA_1B_1B, BB_1C_1C va AA_1C_1C trapetsiyalar yuzalari uchun ushbu

$$S_{AA_1B_1B} = \frac{AA_1 + BB_1}{2} \cdot A_1B_1 = \frac{y_1 + y_2}{2}(x_2 - x_1)$$
$$S_{BB_1C_1C} = \frac{BB_1 + CC_1}{2} \cdot B_1C_1 = \frac{y_2 + y_3}{2}(x_3 - x_2) \quad (12)$$
$$S_{AAC_1C} = \frac{AA_1 + CC_1}{2} \cdot A_1C_1 = \frac{y_3 + y_1}{2}(x_2 - x_1)$$

formulalar o‘rinlidir. 3-rasmdan ma’lumki, ABC uchburchakning yuzi

$$S_{\Delta ABC} = S_{AA_1B_1B} + S_{BB_1C_1C} - S_{AAC_1C}$$

bo‘ladi.

(12) tengliklardan foydalanib,

$$S_{\Delta ABC} = \frac{1}{2}[(y_1 + y_2) \cdot (x_2 - x_1) + (y_3 + y_2) \cdot (x_3 - x_2) -$$
$$-(y_3 + y_1) \cdot (x_3 - x_1)]$$ bo‘lishini topamiz.

Oxirgi formulani determinantlar orqali ifodalaymiz:

$$S_{\Delta ABC} = \pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}\right] \quad (13)$$

yoki

$$S_{\Delta ABC} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad (13')$$

(13) formula berilgan uchburchak yuzini topish formulasidir.

3-misol. Uchlari *A*(2; 0), *B*(-1; -3), *C*(1; 4) nuqtalarda bo‘lgan uchburchak yuzini toping.

Yechish. (13) formulaga ko‘ra,

$$S_{\Delta ABC} = \pm\frac{1}{2}\left[\begin{vmatrix} 2 & 0 \\ -1 & -3 \end{vmatrix} + \begin{vmatrix} -1 & -3 \\ 1 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix}\right] =$$
$$= \pm\frac{1}{2}(-6 - 0 - 4 + 3 + 0 - 8) = \pm\frac{1}{2}(-15) = 7{,}5 \text{ kv birlik.}$$

Tekislikda $A_1(x_1, y_1)$, $A_2(x_2, y_2)$, $A_3(x_3, y_3)$, … , $A_n(x_n, y_n)$ nuqtalar berilgan bo‘lsin. Bu nuqtalarni siniq chiziqlar bilan tutashtirishdan hosil bo‘lgan ko‘pburchak yuzi quyidagi formula bilan aniqlanadi:

$$S_{ko‘pbur.} = \pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \cdots + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right]. \quad (14)$$

4-misol. Uchlari *A*(1; 0), *B*(2; 3), *C*(-1; 5), *D*(-2; 1) va *E*(0; -3) nuqtalarda bo‘lgan beshburchak yuzini toping.

Yechish. (14) formulaga ko‘ra,

$$S = \pm\frac{1}{2}\left[\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix} + \begin{vmatrix} -1 & 5 \\ -2 & 1 \end{vmatrix} +\right.$$
$$\left.+\begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 0 & -3 \\ 1 & 0 \end{vmatrix}\right] =$$
$$= \pm\frac{1}{2}(3 - 0 + 10 + 3 - 1 + 10 + 6 - 0 + 0 + 3) = 17$$

kv. birlik.

Mustaqil ishlash uchun misollar

1. Quyidagi *A* va *B* nuqtalar orasidagi masofani aniqlang:

1) $A(5;\ -3)$ va $B(-3;\ 1)$; 2) $A(4;\ 2)$ va $B(7;\ -2)$;

3) $A(0;\ 3)$ va $B(-2;\ 3)$; 4) $A(k;\ l)$ va $B(k+q;\ 0)$.

2. Ordinatalar o‘qida $A(-5;\ 1)$ va $B(3;\ 2)$ nuqtalardan barobar uzoqlashgan nuqtani toping.

3. Abssissalar o‘qida shunday nuqtani topingki, undan (5; 12) nuqtagacha bo‘lgan masofa 13 ga teng bo‘lsin.

4. Nuqta to‘g‘ri chiziqli harakat qilib, $M(5;\ 5)$ va $N(1;\ 3)$ nuqtalardan o‘tadi. Ox o‘qini kesib o‘tgan nuqtasini toping.

5. $A(-2;\ 1)$ va $B(3;\ 6)$ nuqtalar berilgan. AB kesmani $AN: NB = -3: 2$ nisbatda bo‘luvchi $N(x;y)$ nuqtani toping.

6. Uchlari $A(2;\ 0)$, $B(5;\ 3)$ va $C(2;\ 6)$ nuqtalarda bo‘lgan uchburchakning yuzini toping.

7. $A(x;\ 4)$ va $B(-6;\ y)$ nuqtalar orasidagi AB masofa $N(-1;\ 1)$ nuqtada teng ikkiga bo‘lingan. A va B nuqtalarni aniqlang.

8. Uchlari $A(-2;\ 0)$, $B(0;\ -1)$, $C(2;\ 0)$, $D(3;\ 2)$ va $E(-1;\ 3)$ nuqtalarda bo‘lgan beshburchakning yuzini hisoblang.

IV BOB. TEKISLIKDA TO‘G‘RI CHIZIQ TENGLAMALARI

1-§. To‘g‘ri chiziqning umumiy tenglamasi

Tekislikda Dekart koordinatalar sistemasi va $P(a_1, b_1)$, $Q(a_2, b_2)$ nuqtalar berilgan bo‘lsin. Bu nuqtalardan baravar uzoqlikda joylashgan tekislikdagi $\{N(x, y)\}$ nuqtalar to‘plamini qaraylik (1-rasm). Shartga ko‘ra, $PN = QN$ bo‘ladi. Ikki nuqta orasidagi masofani topish formulasiga asosan

$$PN = \sqrt{(x - a_1)^2 + (y - b_1)^2},$$

$$QN = \sqrt{(x - a_2)^2 + (y - b_2)^2}$$

bo‘ladi. PN va QN masofalar teng:

$$\sqrt{(x - a_1)^2 + (y - b_1)^2} = \sqrt{(x - a_2)^2 + (y - b_2)^2}.$$

Tenglikning har ikki tomonini kvadratga ko‘taramiz:

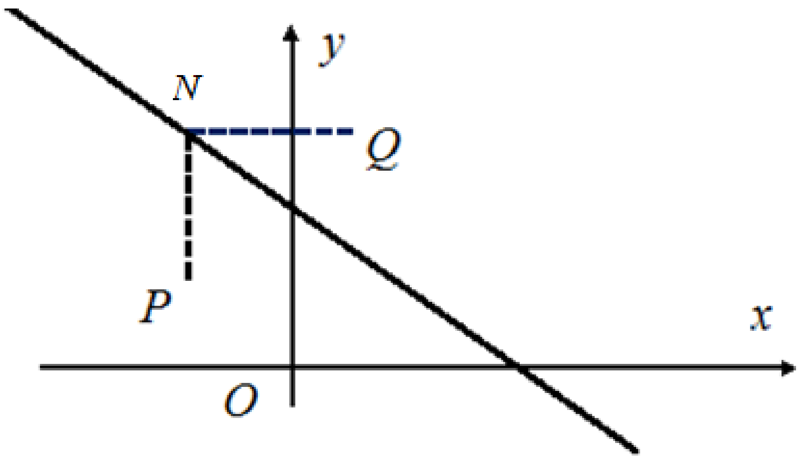
$$(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2,$$
$$x^2 - 2a_1x + a_1^2 + y^2 - 2b_1y + b_1^2 =$$
$$= x^2 - 2a_2x + a_2^2 + y^2 - 2b_2y + b_2^2,$$
$$2(a_2 - a_1)x + 2(b_2 - b_1)y + a_1^2 + b_1^2 - a_2^2 - b_2^2 = 0.$$

Agar $A = 2(a_2 - a_1)$, $B = 2(b_2 - b_1)$ va $C = a_1^2 + b_1^2 \ - a_2^2 - b_2^2$ deb belgilash kiritilsa, u holda

$$Ax + By + C = 0 \tag{1}$$

tenglamani hosil qilamiz. Bu ***to‘g‘ri chiziqning umumiy tenglamasi*** deyiladi.

A, B, C sonlar (1) tenglamaning koeffitsiyentlari bo‘lib, ular to‘g‘ri chiziqning tekislikdagi vaziyatini aniqlaydi.



1-rasm.

(1) tenglamaning ba’zi xususiy hollarini qaraymiz:

1°. $A \neq 0, B \neq 0, C = 0$ bo‘lsin. (1) tenglama

$$Ax + By = 0 \tag{2}$$

ko‘rinishini oladi. Bunday to‘g‘ri chiziqlar koordinata boshidan o‘tadi.

2°. $A = 0, B \neq 0, C \neq 0$ bo‘lsin. U holda (1) tenglama

$$By + C = 0 \tag{3}$$

ko‘rinishni oladi. (3) tenglama bilan belgilangan to‘g‘ri chiziq Ox o‘qiga parallel bo‘ladi.

3°. $A \neq 0, B = 0, C \neq 0$ bo‘lsin. U holda (1) tenglama

$$Ax + C = 0 \tag{4}$$

ko‘rinishni oladi. (4) tenglama bilan belgilangan to‘g‘ri chiziq Oy o‘qiga parallel bo‘ladi.

4°. $A \neq 0, B = C = 0$. Bu holda (1) tenglama

$$Ax = 0, \text{ ya'ni } x = 0 \tag{5}$$

ko‘rinishni oladi. Bu to‘g‘ri chiziq ordinata o‘qini ifodalaydi.

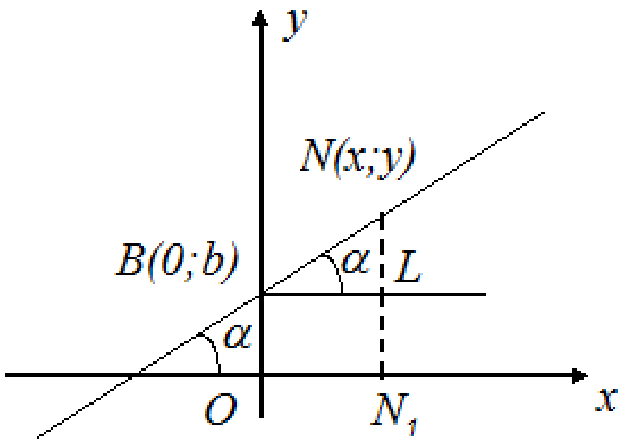
5°. $A = 0, B \neq 0, C = 0$. Bu holda (1) tenglama

$$By = 0, \text{ ya'ni } y = 0 \tag{6}$$

ko‘rinishni oladi. Bu to‘g‘ri chiziq abssissa o‘qini ifodalaydi.

2-§. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi

Aytaylik, tekislikda to‘g‘ri chiziq koordinata o‘qlariga parallel bo‘lmasin. Bu to‘g‘ri chiziq Oy o‘qida $B(0; b)$ nuqtadan o‘tib, Ox o‘qining musbat yo‘nalishi bilan α burchak tashkil etsin (2-rasm).



2-rasm.

Faraz qilaylik, $N(x;y)$ - to‘g‘ri chiziqdagi $B(0;b)$ nuqta bilan ustma-ust tushmaydigan ixtiyoriy nuqta bo‘lsin. N nuqtadan Ox o‘qiga perpendikulyar tushiramiz. B nuqtadan Ox o‘qiga parallel to‘g‘ri chiziq o‘tkazamiz va uning NN_1 bilan kesishgan nuqtasini L orqali belgilaymiz. 2-rasmdan ma’lumki,

$$ON_1 = x, OB = b, ON_1 = BL, OB = N_1L$$

$$NN_1 = y, LN = NN_1 - N_1L = y - b.$$

ΔBLN uchburchakdan

$$\text{tg}\alpha = \frac{LN}{BL}, \quad \text{tg}\alpha = \frac{y-b}{x}$$

$$y = \text{tg}\alpha \cdot x + b.$$

Tenglamada $k = \text{tg}\alpha$ belgilash kiritamiz. U holda,

$$y = kx + b, \qquad (7)$$

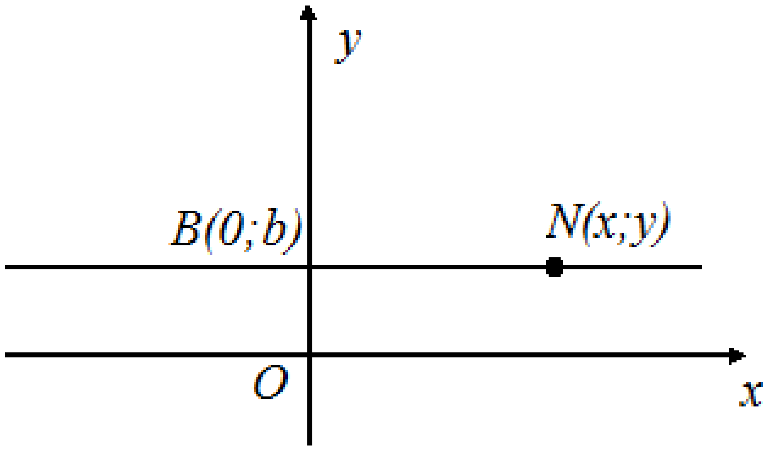
bu yerda k- to‘g‘ri chiziqning burchak koeffitsiyenti deyiladi; b - to‘g‘ri chiziqning Oy o‘qidan ajratgan kesmasi.

(7) – tenglama ***to‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi*** deyiladi.

Agar to‘g‘ri chiziq Ox o‘qiga parallel bo‘lsa, yoki o‘q bilan ustma-ust tushsa, u holda $\alpha = 0$ bo‘ladi (3-rasm).

Faraz qilaylik, to‘g‘ri chiziq Ox o‘qiga parallel va ordinata o‘qini $B(0;b)$ nuqtadan kesib o‘tsin. U holda bu to‘g‘ri chiziqdagi ixtiyoriy N nuqtaning ordinatasini B nuqta ordinatasiga teng bo‘ladi:

$$y = b. \qquad (8)$$



3-rasm.

(8) tenglamada $b = 0$ bo‘lsa, to‘g‘ri chiziq Ox o‘qi bilan ustma-ust tushadi.

Agar $b = 0$ bo‘lsa, (7) tenglama quyidagi ko‘rinishda bo‘ladi:

$$y = kx \tag{9}$$

Bu holatda to‘g‘ri chiziq koordinata boshidan o‘tadi.

1-misol. $B(0;\ -3)$ nuqtadan o‘tib, Ox o‘qi bilan $\alpha = 135^\circ$ burchak tashkil etuvchi to‘g‘ri chiziq tenglamasini toping.

Yechish. To‘g‘ri chiziqning burchak koeffitsiyentini topamiz:

$$k = \mathrm{tg}135^\circ = -1.$$

Shartga ko‘ra, $b = -3$. (7) formulaga ko‘ra

$$y = (-1) \cdot x - 3,$$

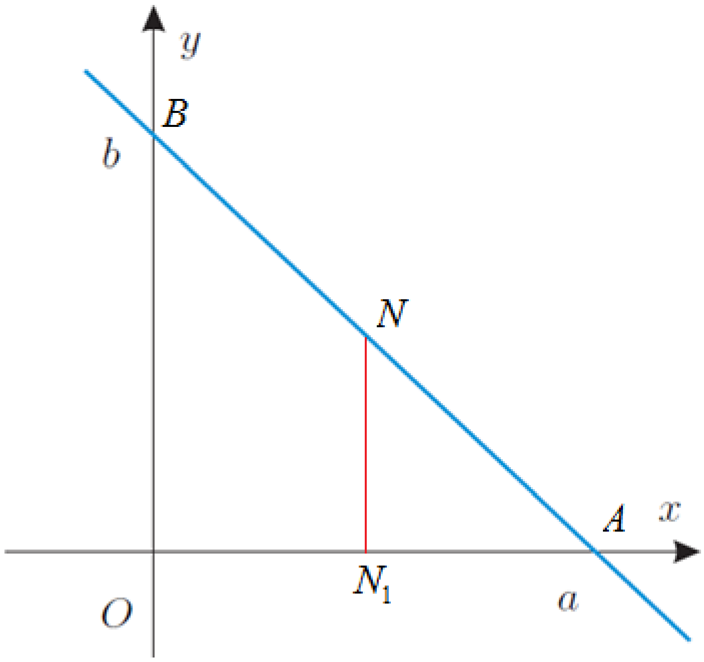
yoki $y = -x - 3$.

2-misol. Koordinata boshidan va (4; -3) nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing.

Yechish. To‘g‘ri chiziq koordinata boshidan o‘tganligi uchun uning burchak koeffitsiyentli tenglamasi $y = kx$ ko‘rinishda bo‘ladi. Tenglamadan k ning qiymatini aniqlaymiz: $-3 = k \cdot 4 \ \Rightarrow \ k = -\frac{3}{4}$. To‘g‘ri chiziq tenglamasi: $y = -\frac{3}{4}x$.

3-§. To‘g‘ri chiziqning kesmalar bo‘yicha tenglamasi

Koordinata o‘qlaridagi ma’lum bir uzunlikdagi kesmalar ajratuvchi to‘g‘ri chiziq tenglamasini keltirib chiqaramiz. Faraz qilaylik, to‘g‘ri chiziq Ox o‘qidan $OA = a, Oy$ o‘qidan $OB = b$ (4-rasm) kesma ajratsin.



4-rasm.

To‘g‘ri chiziq Oy o‘qidan b birlik kesma ajratganligi uchun uning burchak koeffitsiyentli tenglamasi $y = kx + b$ ko‘rinishda bo‘ladi. Ma’lumki, to‘g‘ri chiziq $A(a;0)$ nuqtadan o‘tadi. A nuqta koordinatalarini to‘g‘ri chiziq tenglamasiga qo‘yamiz va to‘g‘ri chiziqning burchak koeffitsiyentini aniqlaymiz:

$$0 = k \cdot a + b \implies k = -\frac{b}{a}.$$

Burchak koeffitsiyent qiymatini $y = kx + b$ tenglamaga qo‘yamiz: $y = -\frac{b}{a}x + b.$

Tenglikning o‘ng va char qismini b ga qisqartiramiz:

$$\frac{y}{b} = -\frac{1}{a}x + 1 \iff \frac{y}{b} + \frac{x}{a} = 1.$$

Natijada

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad (10)$$

tenglama hosil bo‘ladi. Bu tenglama ***to‘g‘ri chiziqning kesmalar bo‘yicha tenglamasidir***.

To‘g‘ri chiziqning kesmalar bo‘yicha tenglamasini uchburchaklarning o‘xshashligi xossasidan foydalanib ham keltirib chiqarish mumkin. Buning uchun to‘g‘ri chiziqda bitta ixtiyoriy $N(x;y)$ nuqtani tanlaymiz (4-rasm). N nuqtadan Ox o‘qiga perpendikulyar o‘tkazamiz va uning Ox o‘qi bilan kesishish nuqtasini N_1 bilan belgilaylik.

Natijada, ikkita ΔOAB va ΔNN_1A o‘xshash uchburchaklar hosil bo‘ladi. Bu uchburchaklarning o‘xshashligidan

$$\frac{NN_1}{OB} = \frac{N_1A}{OA} \tag{11}$$

tenglik o‘rinli bo‘ladi.

Ma’lumki, $OA = a, OB = b, ON_1 = x, NN_1 = y;\ N_1A = OA - ON_1 = a - x$. Ushbu qiymatlarni (11) formulaga qo‘yamiz:

$$\frac{y}{b} = \frac{a - x}{a} \iff \frac{x}{a} + \frac{y}{b} = 1. \tag{12}$$

Agar to‘g‘ri chiziq $Ax + By + C = 0\ (A \neq 0, B \neq 0, C \neq 0)$ umumiy tenglama bilan berilgan bo‘lsa, uning koordinata o‘qlaridan ajratgan kesmalar uzunligini quyidagicha aniqlaymiz:

$$Ax + By + C = 0 \implies Ax + By = -C$$

$$\frac{A}{-C}x + \frac{B}{-C}y = 1 \implies \frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1.$$

$a = -\frac{C}{A},\ b = -\frac{C}{B}$ belgilash kiritamiz:

$$\frac{x}{a} + \frac{y}{b} = 1.$$

3-misol. $2x + 3y - 6 = 0$ to‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalar uzunliklarini toping.

Yechish. Berilgan tenglamani quyidagi ko‘rinishga keltiramiz:

$$2x + 3y = 6.$$

Hosil bo‘lgan tenglamani 6 ga bo‘lamiz:

$$\frac{2x}{6} + \frac{3y}{6} = 1 \implies \frac{x}{3} + \frac{y}{2} = 1.$$

Demak, $a = 3, b = 2$.

4-§. Ikki to‘g‘ri chiziq orasidagi burchak. Ikki to‘g‘ri chiziqning parallellik va perpendikulyarlik shartlari

L_1 va L_2 to‘g‘ri chiziqlar mos ravishda burchak koeffitsiyentli tenglamalari bilan berilgan bo‘lsin (5-rasm):

$$y = k_1x + b_1 \text{ va } y = k_2x + b_2.$$

Bu to‘g‘ri chiziqlar orasidagi φ burchak tangensini topamiz. Faraz qilamiz, L_1 va L_2 to‘g‘ri chiziqlar o‘zaro perpendikulyar bo‘lmasin, aks holda $\mathrm{tg}\varphi$ mavjud bo‘lmaydi.

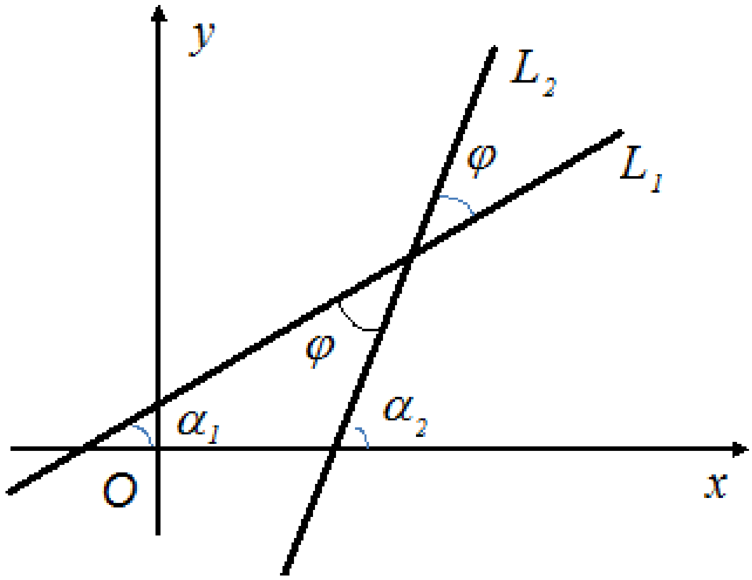
5-rasmdan ko‘rinadiki, $\alpha_2 = \alpha_1 + \varphi$ yoki $\varphi = \alpha_2 - \alpha_1$.

Natijada,

$$\mathrm{tg}\varphi = \mathrm{tg}(\alpha_2 - \alpha_1) = \frac{\mathrm{tg}\alpha_2 - \mathrm{tg}\alpha_1}{1 + \mathrm{tg}\alpha_1 \cdot \mathrm{tg}\alpha_2}.$$

$k_1 = \mathrm{tg}\alpha_1, k_2 = \mathrm{tg}\alpha_2$ bo‘lgani uchun:

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 k_2}. \qquad (13)$$



5-rasm.

Shunday qilib, o‘zaro perpendikulyar bo‘lmagan kesishuvchi L_1 va L_2 to‘g‘ri chiziqlar orasidagi φ burchakning tangensi (13) formula yordamida aniqlanadi. φ burchak L_1 to‘g‘ri chiziqdan L_2 to‘g‘ri chiziqqa soat strelkasi yo‘nalishiga qarama-qarshi yo‘nalish bo‘yicha hisoblanadi.

Agar to‘g‘ri chiziqlar parallel yoki ustma-ust tushsa, u holda $\alpha_1 = \alpha_2$ va $\mathrm{tg}\alpha_1 = \mathrm{tg}\alpha_2$ bo‘ladi, ya’ni

$$k_1 = k_2 \qquad (14)$$

(14) formula ikki to‘g‘ri chiziqning parallellik shartini ifodalaydi.

Agar L_1 va L_2 to‘g‘ri chiziqlar perpendikulyar bo‘lsa, (13) formula ma’noga ega bo‘lmaydi. Biroq, bu holda to‘g‘ri chiziqlar orasidagi burchak kotangensini aniqlaymiz:

$$\mathrm{ctg}\varphi = \mathrm{ctg}(\alpha_2 - \alpha_1) = \frac{1 + \mathrm{tg}\alpha_1 \cdot \mathrm{tg}\alpha_2}{\mathrm{tg}\alpha_2 - \mathrm{tg}\alpha_1} = \frac{1 + k_1 \cdot k_2}{k_2 - k_1}.$$

To‘g‘ri chiziqlar perpendikulyar bo‘lgani uchun

$$\mathrm{ctg}\varphi = \mathrm{ctg}\frac{\pi}{2} = 0.$$

Natijada

$$\frac{1+k_1k_2}{k_2-k_1}=0$$

yoki

$$1+k_1k_2=0\ (k_1k_2=-1). \tag{15}$$

Shunday qilib, (15) formula ikki to‘g‘ri chiziqning perpendikulyarlik shartini ifodalaydi.

4-misol. $x+2y-5=0, 3x-4y+7=0$ to‘g‘ri chiziqlar orasidagi burchakni toping.

Yechish. To‘g‘ri chiziq tenglamalarini burchak koeffitsiyentli tenglamalarga keltiramiz:

$$y=-\frac{1}{2}x+\frac{5}{2}, y=\frac{3}{4}x+\frac{7}{4}.$$

To‘g‘ri chiziqlarning burchak koeffitsiyentlari mos ravishda

$$k_1=-\frac{1}{2}, k_2=\frac{3}{4}.$$

(13) formulaga ko‘ra φ burchak tangensini topamiz:

$$\mathrm{tg}\varphi=\frac{\frac{3}{4}-\left(-\frac{1}{2}\right)}{1+\left(-\frac{1}{2}\right)\cdot\frac{3}{4}}=\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}=\frac{\frac{5}{4}}{\frac{5}{8}}=2.$$

5-misol. *M*(-2; 7) nuqtadan o‘tuvchi va $y=3x-5$ to‘g‘ri chiziqqa *a*) parallel, *b*) perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini yozing.

Yechish. *a*) Berilgan to‘g‘ri chiziqning burchak koeffitsiyenti $k=3$. To‘g‘ri chiziqlarning parallellik shartiga $k=k_1=3$. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasini yozamiz: $y=3x+b$ To‘g‘ri chiziq *M* nuqtadan o‘tganligi sababli, nuqta koordinatalarini tenglamaga qo‘yamiz: $7=3\cdot(-2)+b \Rightarrow b=13$. Natijada, berilgan nuqtadan o‘tuvchi va berilgan to‘g‘ri chiziqqa parallel to‘g‘ri chiziq tenglamasini tuzamiz: $y=3x+13$.

b) To‘g‘ri chiziqlarning perpendikulyarlik shartiga $k_1=-\frac{1}{k}=-\frac{1}{3}$. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasini yozamiz:

$y = k_1 x + b \iff y = -\frac{1}{3}x + b$. Oxirgi tenglamadan b ning qiymatini aniqlaymiz: $7 = -\frac{1}{3}\cdot(-2) + b \implies b = \frac{19}{3}$. Natijada, berilgan nuqtadan o‘tuvchi va berilgan to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziq tenglamasini tuzamiz: $y = -\frac{1}{3}x + \frac{19}{3}$.

5-§. To‘g‘ri chiziqlar dastasi tenglamasi. Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi

Tekislikda $M_0(x_0, y_0)$ nuqta va $y = kx + b$ to‘g‘ri chiziq berilgan bo‘lsin.

Ta’rif. Tekislikda M_0 nuqtadan o‘tuvchi to‘g‘ri chiziqlar to‘plamiga ***to‘g‘ri chiziqlar dastasi*** deyiladi. M_0 nuqta dasta markazi deyiladi.

Faraz qilaylik, to‘g‘ri chiziq M_0 nuqtadan o‘tsin, u holda

$y_0 = kx_0 + b$ bo‘ladi. Bundan $b = y_0 - kx_0$. b ning qiymatini berilgan to‘g‘ri chiziq tenglamasiga qo‘yamiz, natijada

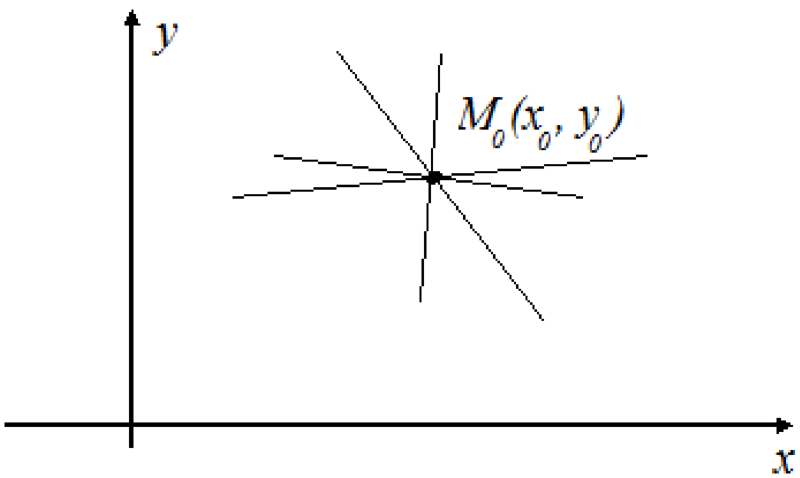
$$y = kx + y_0 - kx_0$$

yoki

$$y - y_0 = k(x - x_0) \qquad (16)$$

tenglama hosil bo‘ladi.

(16) tenglamadagi burchak koeffitsiyent k ga turli qiymatlar berib, M_0 nuqtadan o‘tuvchi cheksiz ko‘p to‘g‘ri chiziqlarni hosil qilishimiz mumkin (6-rasm). Shu sababli (16) tenglama to‘g‘ri chiziqlar dastasi tenglamasi deyiladi.



6-rasm.

M_0 nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasidan bitta to‘g‘ri chiziq tanlab olamiz va bu to‘g‘ri berilgan $M_1(x_1; y_1)$ nuqtadan o‘tsin. U holda,

$$y_1 - y_0 = k(x_1 - x_0)$$

yoki

$$k = \frac{y_1 - y_0}{x_1 - x_0}.$$

k ning qiymatini (16) formulaga qo‘yamiz:

$$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$

yoki

$$\frac{y - y_0}{y_1 - y_0} = \frac{x - x_0}{x_1 - x_0}. \qquad (17)$$

(17) formula berilgan ikki $M_0(x_0; y_0)$ va $M_1(x_1; y_1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasidir.

(17) formulani o‘xshash uchburchaklarning xossalari asosida ham keltirib chiqarish mumkin. Faraz qilaylik, tekislikda $M_0(x_0; y_0)$, $M_1(x_1; y_1)$ nuqtalar va ulardan o‘tuvchi to‘g‘ri chiziq berilgan bo‘lsin (7-rasm). To‘g‘ri chiziqda ixtiyoriy $P(x; y)$ nuqtani tanlaymiz. M_0, P va M_1 nuqtalardan Ox o‘qiga perpendikulyar tushiramiz. Perpendikulyarlarning Ox o‘qidagi asosini mos ravishda $N_0, N,\ N_1$ nuqtalar bilan belgilaymiz. Shuningdek, M_0 nuqtadan Ox o‘qiga parallel chiziq o‘tkazamiz. Natijada, grafikda ΔM_0QP va ΔM_0RM_1 uchburchaklar hosil bo‘ldi. Ma’lumki,

$$ON_0 = x_0,\ ON = x,\ ON_1 = x_1,\ M_0N_0 = QN = RN_1,$$
$$M_0N_0 = y_0,\ \ M_1N_1 = y_1,\ \ PN = y,$$
$$M_0Q = N_0N = ON - ON_0 = x - x_0,$$
$$M_0R = N_0N_1 = ON_1 - ON_0 = x_1 - x_0,$$
$$PQ = PN - QN = PN - M_0N_0 = y - y_0,$$
$$M_1R = M_1N_1 - RN_1 = M_1N_1 - M_0N_0 = y_1 - y_0.$$

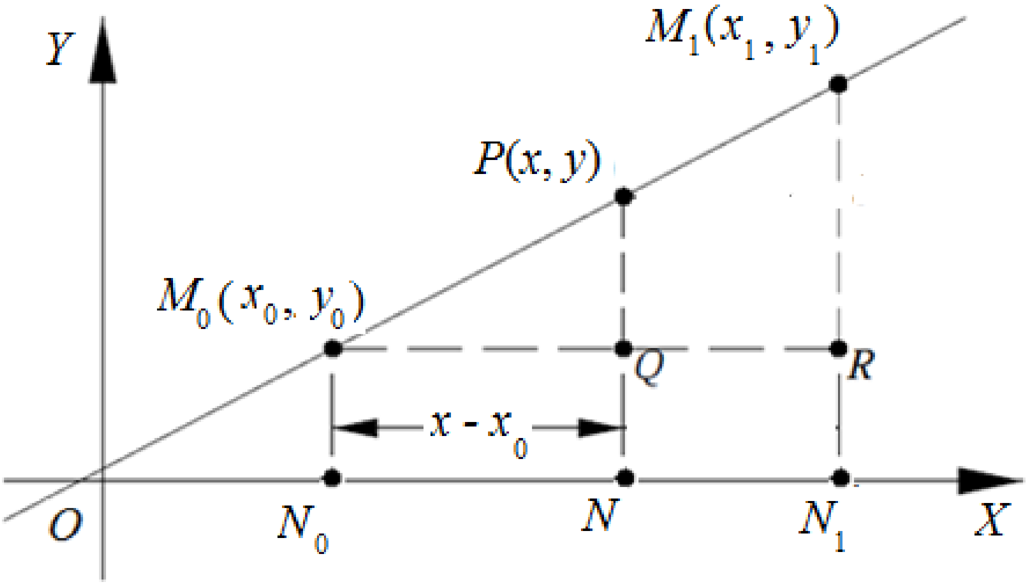
ΔM_0QP va ΔM_0RM_1 uchburchaklarning o‘xshashligidan

$$\frac{M_0Q}{M_0R} = \frac{PQ}{M_1R}$$

tenglik o‘rinlidir. Bu tenglikka yuqoridagi belgilashlarni qo‘yamiz:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0}.$$

Natijada (17) formula hosil bo‘ldi.



7-rasm.

6-misol. Berilgan $M_0(-3; 4), M_1(2; 1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing.

Yechish. A nuqtaning koordinatalarini $x_0 = -3, y_0 = 4$, B nuqtaning koordinatalarini esa $x_1 = 2, y_1 = 1$ deb belgilab, (17) formulaga qo‘yamiz:

$$\frac{x - (-3)}{2 - (-3)} = \frac{y - 4}{1 - 4}$$

yoki

$$\frac{x + 3}{5} = \frac{y - 4}{-3}$$

Hosil bo‘lgan tenglamani to‘g‘ri chiziqning umumiy tenglamasiga keltiramiz:

$$-3(x + 3) = 5(y - 4).$$

Qavslarni ochib chiqib, tenglamaning o‘ng tomonidagi hadni chap tomonga o‘tkazamiz:

$$-3x - 9 - 5y + 20 = 0,$$
$$-3 - 5y + 11 = 0 \text{ yoki } 3x + 5y - 11 = 0.$$

Hosil bo‘lgan $3x + 5y - 11 = 0$ tenglama berilgan M_0 va M_1 nuqtalardan o‘tuvchi to‘g‘ri chiziqning umumiy tenglamasidir.

7-misol. $M_1(-1; 1), M_2(7; 1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing.

Yechish. $y_1 = y_2 = 1$ bo‘lgani uchun to‘g‘ri chiziq Ox o‘qiga parallel bo‘ladi. Izlanayotgan to‘g‘ri chiziq tenglamasi $y = 1$ ko‘rinishda bo‘ladi.

8-misol. (3; -2) nuqtadan o‘tib, $2x - 5y + 3 = 0$ to‘g‘ri chiziq-qa perpendikulyar bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

Yechish. To‘g‘ri chiziqning umumiy $2x - 5y + 3 = 0$ teng-lamasidan uning burchak koeffitsiyenti $k_1 = \frac{2}{5}$ ni aniqlaymiz. (16) tenglamaga binoan, $y - (-2) = k(x - 3)$. Burchak koeffitsiyent k ning qiymatini to‘g‘ri chiziqlarning perpendikulyarlik shartidan aniqlaymiz. Unga ko‘ra

$$k = k_2 = -\frac{1}{k_1} \Rightarrow k = -\frac{1}{\frac{2}{5}} = -\frac{5}{2}.$$

$$y - (-2) = -\frac{5}{2}(x - 3) \Leftrightarrow 2(y + 2) = -5(x - 3);$$

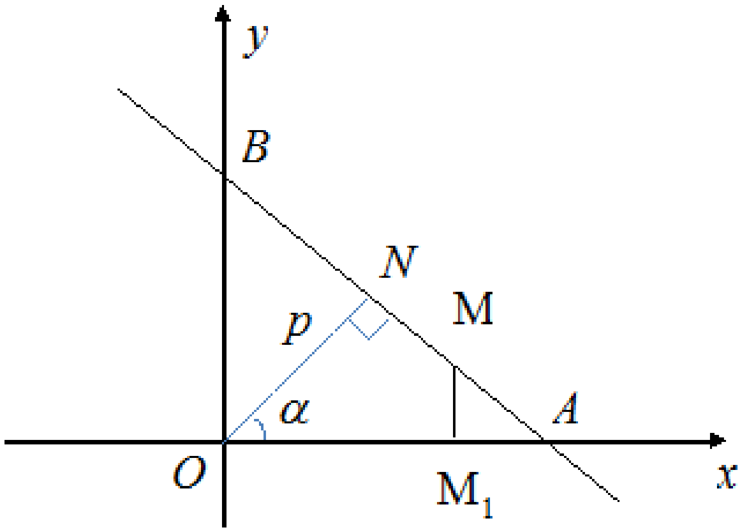
$$2y + 4 + 5x - 15 = 0 \Leftrightarrow 2y + 5x - 11 = 0.$$

6-§. To‘g‘ri chiziqning normal tenglamasi

Tekislikda biror to‘g‘ri chiziq berilgan bo‘lsin. Koordinata boshidan bu to‘g‘ri chiziqqa tushirilgan perpendikulyarning uzunligi p ga, shu perpendikulyar bilan Ox o‘qining musbat yo‘nalishi orasidagi burchak $\alpha \left(\alpha \neq 0,\ \alpha = {}^{\pi}/_{2}\right)$ bo‘lsin (8-rasm).

Demak, $ON = p, \angle AON = \alpha$. To‘g‘ri chiziqda $M = M(x, y)$ nuqtani olib, bu nuqtadan Ox o‘qiga perpendikulyar tushiramiz. Perpendikulyarning asosi M_1 bo‘lsin. Unda

$$OM_1 = x,\ MM_1 = y \qquad (18)$$



8-rasm.

bo‘ladi. AON hamda BON to‘g‘ri burchakli uchburchaklarda $\angle AON = \alpha, \angle BON = 90° - \alpha$.

ΔAON dan:

$$\cos\alpha = \frac{ON}{OA} \Longrightarrow OA = \frac{ON}{\cos\alpha} \Longrightarrow OA = \frac{p}{\cos\alpha}, \quad (19)$$

ΔBON dan:

$$\cos(90° - \alpha) = \frac{ON}{OB} \Longrightarrow \sin\alpha = \frac{ON}{OB} \Longrightarrow OB = \frac{p}{\sin\alpha}. \quad (20)$$

Ma’lumki,

$$M_1A = OA - OM_1 = \frac{p}{\cos\alpha} - x. \quad (21)$$

AOB va AM_1M uchburchaklarning o‘xshashligidan $\frac{M_1M}{OB} = \frac{M_1A}{OA}$ kelib chiqadi.

(18), (19), (20), (21) munosabatlarni e’tiborga olsak, tenglik

$$\frac{y}{\frac{p}{\sin\alpha}} = \frac{\frac{p}{\cos\alpha} - x}{\frac{p}{\cos\alpha}}$$

ko‘rinishga keladi. Bu tenglikdan

$$x\cos\alpha + y\sin\alpha - p = 0 \quad (22)$$

bo‘lishini topamiz. (22) tenglamani to‘g‘ri chiziqning normal tenglamasi deyiladi.

To‘g‘ri chiziqning $Ax + By + C = 0$ umumiy tenglamasini normal ko‘rinishdagi tenglamaga keltirish mumkin. Umumiy tenglamani hozircha noma’lum $\mu(\mu \neq 0)$ songa ko‘paytiramiz:

$$\mu Ax + \mu By + \mu C = 0 \quad (23)$$

Agar (23) tenglamani to‘g‘ri chiziqning normal tenglamasi deb aytadigan bo‘lsak, unda $\mu A = \cos\alpha\,, \mu B = \sin\alpha\,, \mu C = -p$ bo‘ladi. Bu tenglamalardan topamiz:

$$(\mu A)^2 + (\mu B)^2 = \cos^2\alpha + \sin^2\alpha = 1,$$

$$\mu = \pm\frac{1}{\sqrt{A^2+B^2}}, \qquad (24)$$

μ – normallovchi ko‘paytuvchi.

Demak,

$$\mu = \pm\frac{1}{\sqrt{A^2+B^2}}, \cos\alpha = \pm\frac{A}{\sqrt{A^2+B^2}},$$

$$\sin\alpha = \pm\frac{B}{\sqrt{A^2+B^2}}, -p = \pm\frac{C}{\sqrt{A^2+B^2}}.$$

Natijada berilgan $Ax + By + C = 0$ tenglama

$$\frac{A}{\pm\sqrt{A^2+B^2}}x + \frac{B}{\pm\sqrt{A^2+B^2}}y + \frac{C}{\pm\sqrt{A^2+B^2}} = 0$$

normal tenglamaga keladi. μ – normallovchi ko‘paytuvchining ishorasi ozod had C ning ishorasiga qarama-qarshi bo‘ladi.

9-misol. $3x + 4y - 7 = 0$ to‘g‘ri chiziq umumiy tenglamasini normal tenglamaga keltiring.

Yechish. (24) formuladan foydalanib topamiz:

$$\mu = \frac{1}{\sqrt{3^2+4^2}} = \frac{1}{5}.$$

Tenglamani μ ning qiymatiga ko‘paytiramiz:

$$\frac{1}{5}\cdot(3x + 4y - 7) = 0$$

Natijada $\frac{3}{5}x + \frac{4}{5}y - \frac{7}{5} = 0$ to‘g‘ri chiziqning normal tenglamasini hosil qilamiz.

10-misol. To‘g‘ri chiziqdan koordinata boshiga tushirilgan perpendikulyarning uzunligi 3 ga teng. Perpendikulyar *Ox* o‘qining musbat yo‘nalishi bilan 45^0 burchak tashkil etadi. To‘g‘ri chiziqning normal tenglamasini yozing.

Yechish. Misol shartiga ko‘ra $p = 3, \alpha = 45^0$. Bu qiymatlarni (22) tenglamaga qoyamiz:

$$x\cos 45^0 + y\sin 45^0 - 3 = 0,$$

$$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 3 = 0.$$

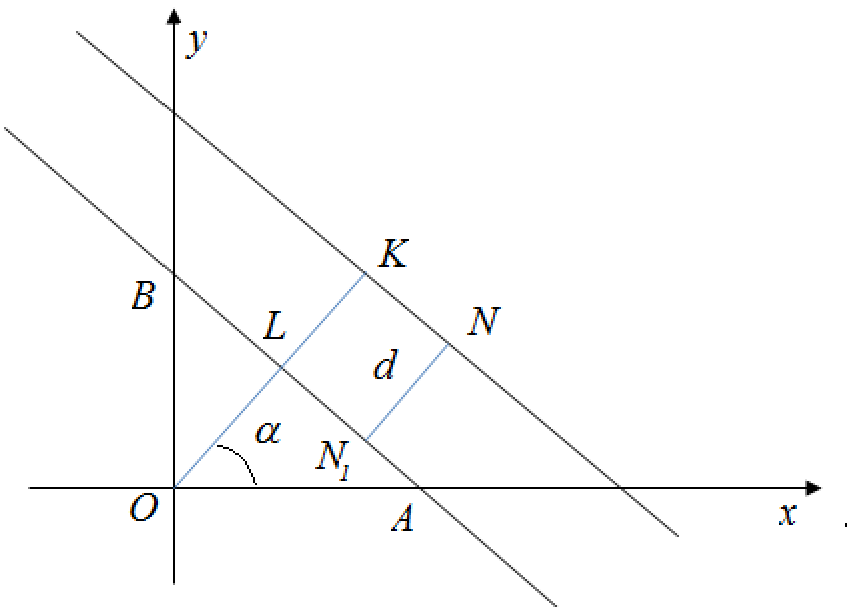
7-§. Nuqtadan to‘g‘ri chiziqgacha masofa

Tekislikda $Ax + By + C = 0$ to‘g‘ri chiziq va bu to‘g‘ri chiziqda yotmagan $N(x_0; y_0)$ nuqta berilgan bo‘lsin. N nuqtadan to‘g‘ri chiziqqa perpendikulyar tushiramiz va uning asosini N_1 nuqta bilan belgilaymiz. N nuqtadan $Ax + By + C = 0$ to‘g‘ri chiziqqacha bo‘lgan masofa NN_1 perpendikulyar uzunligiga teng (9-rasm).

Berilgan to‘g‘ri chiziq tenglamasini normal tenglamaga keltiramiz:

$$x\cos\alpha + y\sin\alpha - p = 0,$$

bu yerda $p = OL$ – koordinata boshidan to‘g‘ri chiziqqa tushirilgan perpendikulyar uzunligi.



9-rasm.

Berilgan $N(x_0; y_0)$ nuqta orqali berilgan to‘g‘ri chiziqqa parallel to‘g‘ri chiziq o‘tkazamiz. Uning normal tenglamasi ushbu

$$x\cos\alpha + y\sin\alpha - \rho = 0 \tag{25}$$

ko‘rinishda bo‘ladi. To‘g‘ri chiziq N nuqta orqali o‘tganligi uchun uning koordinatalari (25) tenglamani qanoatlantiradi

$$x_0\cos\alpha + y_0\sin\alpha - \rho = 0. \tag{26}$$

Ma’lumki, $d = NN_1 = LK, OK = OL + LK, OL = p, OK = \rho, d = OK - OL = p - \rho$.

(26) tenglikdan $\rho = x_0 \cos\alpha + y_0 \sin\alpha$.

Natijada nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofani topish formulasi hosil bo‘ladi:

$$d = |p - \rho| = |p - x_0 \cos\alpha - y_0 \sin\alpha|,$$

yoki

$$d = |x_0 \cos\alpha + y_0 \sin\alpha - p|. \tag{27}$$

Ma’lumki,

$$\cos\alpha = \pm\frac{A}{\sqrt{A^2 + B^2}}, \sin\alpha = \pm\frac{B}{\sqrt{A^2 + B^2}}, -p = \pm\frac{C}{\sqrt{A^2 + B^2}}.$$

$\cos\alpha, \sin\alpha, -p$ ning qiymatlarini (27) formulaga qo‘yamiz:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}. \tag{28}$$

11-Misol. $N(4; 5)$ nuqtadan $2x - 3y - 8 = 0$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

Yechish. (28) formulaga ko‘ra,

$$d = \frac{|2 \cdot 4 - 3 \cdot 5 - 8|}{\sqrt{2^2 + (-3)^2}} = \frac{15}{\sqrt{13}}.$$

Demak, izlanayotgan masofa $\frac{15}{\sqrt{13}}$ ga teng.

8-§. To‘g‘ri chiziqning amaliy masalalarga tatbiqi

To‘g‘ri chiziqning iqtisodiyotga tatbiqi. Iqtisodiyotda asosiy masalada talab *D* (demand) va taklif *S* (supply) mahsulot narxi *P* (price) ga bog‘liqligini o‘rganish masalasidir.

Tadqiqot asosida quyidagi amaliy xulosaga kelamiz: narx qancha past bo‘lsa, mahsulotga talab ko‘p bo‘ladi, ya’ni aholining sotib olish imkoniyati oshadi.

D ning *P* ga bog‘liqligi ko‘r hollarda chiziqli kamayuvchi funksiya sifatida ifodalanadi:

$$D = -aP + c, \ a > 0, \ c > 0 \tag{29}$$

Xuddi shu vaqtda, mahsulot narxi *P* ning oshishi bilan unga bo‘lgan S takliflar ham oshadi. Shu sababli, taklif o‘suvchi chiziqli funksiya bilan ifodalanadi:

$$S = bP + d,\ b > 0,\ d > 0 \tag{30}$$

(29) va (30) munosabatlarda a, b, c, d parametrlar tashqi omillarni (jamiyatning turmush tarzi, siyosiy muhit va shunga o‘xshash holatlarni) ifodalaydi.

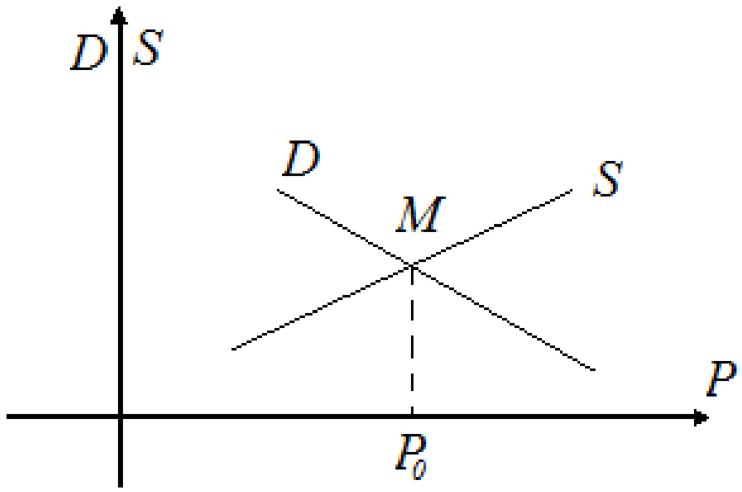
Yuqoridagi formulalarda P, D, S – o‘zgaruvchilar musbat bo‘lganligi uchun funksiya grafigi faqat I-chorakda ma’noga ega bo‘ladi (10-rasm).

Iqtisodiyot uchun muvozanat sharti katta ahamiyatga ega, ya’ni talab va taklif orasidagi tenglikdir.

Bu shart

$$D(P) = S(P)$$

tenglama bilan aniqlanadi. Natijada D va S to‘g‘ri chiziqlarning kesishish nuqtasi M nuqta aniqlanadi. M nuqta muvozanat nuqtasi deb ataladi. Muvozanat ro‘y beradigan P_0 – narxga muvozanat narxi deyiladi.



10-rasm.

Aholining turmush tarzi yaxshilanishi bilan ((29) formulaga asosan c – parametrning o‘sishiga mos keladi) muvozanat nuqtasi M o‘ng tomonga siljiydi, ya’ni D to‘g‘ri chiziq yuqoriga ko‘tariladi. Shu jumladan, S taklif chizig‘i o‘zgarmagan holda mahsulot narxi oshadi.

1-masala. Surxondaryo viloyatining Sariosiyo tumanidagi bog‘dorchilik xo‘jaligi yetishtirilgan xirmoni respublikamizning barcha viloyatlaridagi savdo markazlariga yetkazib beradi. Xo‘jalikda 2 xil turdagi transport vositalari mavjud bo‘lib, ularning yuk tashish xarajatlari quyidagi burchak koeffitsiyentli to‘g‘ri chiziqlar tenglamalari bilan

$$y = 20x + 50 \text{ va } y = 10x + 100$$

ifodalansin. Bunda y – transport xarajati, x –yuk tashish masofasi (bunda 1 – 100 km, 2 – 200 km va h.k.). Xo‘jalik Toshkent shahriga xurmo yetkazib berish uchun qaysi transport vositasidan foydalanishi tejamliroq bo‘ladi?

Yechish. Yuk tashish xarajat funksiyalarini o‘zaro tenglashtiramiz:

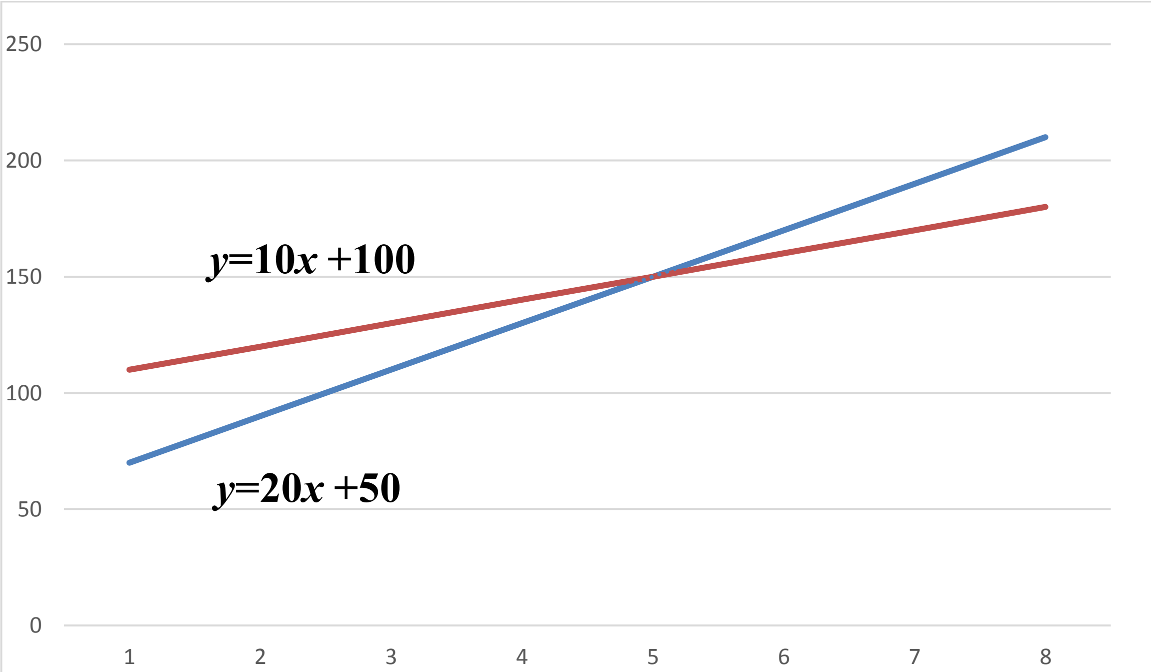
$$20x + 50 = 10x + 100,$$
$$10x = 50,$$
$$x = 5.$$

$x = 5$ qiymatni xarajat funksiyalariga qo‘yamiz va y – transport xarajatini topamiz:

$$y = 20 \cdot 5 + 50 = 150.$$
$$y = 10 \cdot 5 + 100 = 150.$$

$x = 5$, ya’ni $5 \cdot 100 = 500$ km ga ikki transport uchun yuk tashish xarajatlari teng. $x < 5$, ya’ni $5 \cdot 100 = 500$ km gacha 1-transport vositasidan, $x > 5$, ya’ni 500 km dan ortiq masofaga 2-transport vositasidan foydalanish xo‘jalik uchun tejamli bo‘ladi (11-rasm). Sariosiyo tumanidan Toshkent shahrigacha 2-transport vositasi yordamida yuk tashish xo‘jalik uchun tejamlidir.

2-masala. Ko‘p tarmoqli xo‘jalikka tegishli mebel fabrikasida tayyorlangan stullar 64 ming so‘mdan sotilishi rejalashtirilgan. Fabrikada 8 ta stulni tayyorlash uchun 635 ming so‘m, 13 ta stulni tayyorlash uchun esa 750 ming so‘m xarajat qilinishi kerak. Agar xarajatlar funksiyasi chiziqli bo‘lsa, zarar ko‘rmaslik nuqtasini, ya’ni fabrika nechta stul ishlab chiqarishlardan so‘ng, foyda olishini aniqlang.



11-rasm.

Yechish. $M_1(8;\ \ 635)$, $M_2(13;\ \ 750)$ nuqtalardan o‘tuvchi xarajatlar funksiyasi tenglamasini keltirib chiqaramiz va grafigini yasaymiz:

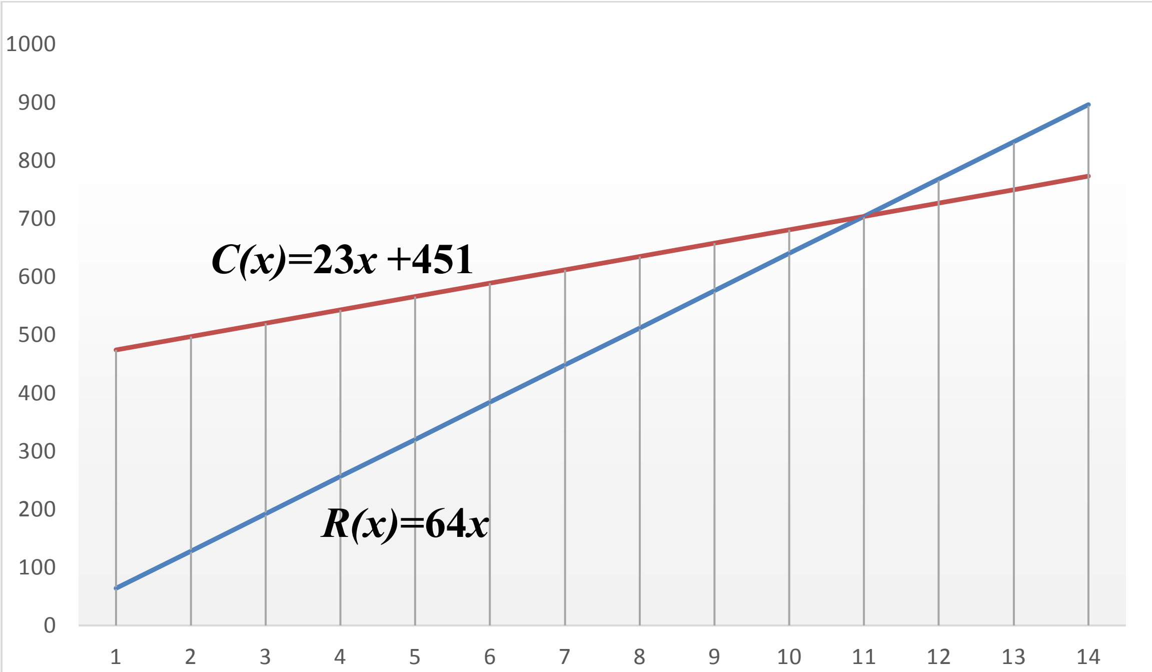
$$\frac{C(x)-635}{750-635}=\frac{x-8}{13-8} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{C(x)-635}{23}=\frac{x-8}{1} \quad \Rightarrow \quad C(x)=23x+451.$$

Shartga ko‘ra, $R(x)$=64x foyda olish funksiyasidir. Zarar ko‘rmaslik nuqtasini aniqlash uchun harajatlar va foyda olish funksiyalari grafiklarining kesishish nurtasi abssitsasini topamiz (12-rasm):

$$23x+451=64x \quad \Rightarrow \quad 41x=451 \quad \Rightarrow \quad x=11.$$

Demak, ishlab chiqarilgan stullar soni 11 tadan oshishi bilan fabrika foyda ko‘ra boshlaydi.



12-rasm.

3-masala. Fermer xo‘jaligi poliz mahsulotlarini yig‘ishtirishda qo‘shimcha ishchi kuchidan foydalanadi va 1 gektarga 100 ming so‘m xarajat qiladi. 5 gektarga esa xarajat 300 ming so‘m bo‘lsin. Agar xarajat funksiyasi chiziqli (to‘g‘ri chiziq) bo‘lsa, 4 gektardagi poliz mahsulotlarini yig‘ishtirishga ketadigan xarajatni toping.

Yechish. Masala shartiga binoan, $A(1,\ 100)$ va $B(5;\ 300)$ belgilashlar kiritamiz. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasiga asosan,

$$\frac{x-1}{5-1}=\frac{y-100}{300-100} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{x-1}{4}=\frac{y-100}{200} \quad \Rightarrow \quad y=50x+50.$$

Oxirgi tenglamadan $x=4$ da y ning qiymatini topamiz:

$$y=50\cdot 4+50 \quad \Rightarrow \quad y=250.$$

Demak, 4 gektardagi poliz mahsulotlarini yig‘ishtirishga 250 ming so‘m xarajat qilinadi.

Mustaqil yechish uchun misollar

1. *A*(-2; 3) nuqtadan o‘tuvchi to‘g‘ri chiziq *Ox* o‘qi bilan 135° li burchak tashkil etadi. Bu to‘g‘ri chiziq tenglamasini toping.

2. Koordinatalar boshidan va (-2; -3) nuqtadan o‘tuvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing.

3. A(0; 3) nuqtadan o‘tuvchi va koordinatalar burchagidan yuzi 3 kv birlikka teng uchburchak kesuvchi to‘g‘ri chiziq tenglamasini yozing.

4. Quyidagi to‘g‘ri chiziqlar orasidagi burchakni aniqlang:

1) $\begin{cases} y = 2x - 3, \\ y = \frac{1}{2}x + 1; \end{cases}$ 2) $\begin{cases} y=3x, \\ y=-2x+5; \end{cases}$ 3) $\begin{cases} y=4x-7, \\ y=-\frac{1}{4}x+2; \end{cases}$

4) $\begin{cases} 5x-y+7=0, \\ 2x-3y+1=0; \end{cases}$ 5) $\begin{cases} 3x+2y=0, \\ 6x+4y+9=0; \end{cases}$

5. *A*(-2; 5) nuqta va $2x-y=0$ to‘g‘ri chiziqni yasang. *A* nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini yozing va o‘sha dastadan berilgan to‘g‘ri chiziqqa: 1) parallel; 2) perpendikulyar bo‘lgan to‘g‘ri chiziqlarni tanlab oling.

6. $(5;\ -3)$ nuqtadan o‘tib, $4x+3y+15=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

7. Berilgan nuqtadan berilgan to‘g‘ri chiziqqacha bo‘lgan masofani hisoblang:

1) $P_1(4; -2)$, $8x-15y-11=0$;
2) $P_2(-3; 2)$, $4x - 7y + 26 = 0$;
3) $P_3(8; 5)$, $3x - 4y - 15 = 0$.

8. $4x - 3y = 0$ to‘g‘ri chiziqdan 4 birlik uzoqlikdagi nuqtalar geometrik o‘rnining tenglamalarini yozing.

9. $3x - 2y + 1 = 0$ va $x + 3y - 7 = 0$ to‘g‘ri chiziqlarning kesishish nuqtasidan ularning birinchisiga perpendikulyar to‘g‘ri chiziq o‘tkazilgan. Hosil qilingan to‘g‘ri chiziqdan koordinatalar boshigacha bo‘lgan masofa qancha?

10. (2;7) nuqtadan shunday to‘gri chiziq o‘tkazilganki, u koordinata o‘qlari bilan yuzi 64 kv birlikka teng bo‘lgan uchburchak tashkil qiladi. Bu chiziqning tenglamasini tuzing.

11. Uchlari A(-4; 2), B(2; -5) va C(5; 0) nuqtalarda bo‘lgan uchburchak medianalarining kesishgan nuqtasini va balandliklarining kesishgan nuqtasini toping.

12. A(-2; -2), B(-3; 1), C(7; 7) va D(3; 1) nuqtalar trapetsiyaning uchlari ekanligini tekshiring. Bu trapetsiyaning o‘rta chizig‘i va diagonallarining tenglamalarini tuzing.

V BOB. IKKINCHI TARTIBLI EGRI CHIZIQLAR

Tekislikda ikkinchi tartibli egri chiziq tenglamasi 2-tartibli algebraik tenglama bilan ifodalanadi. 2-tartibli egri chiziqlar (aylana, ellips, giperbola, parabola) tenglamalari va ularning elementlarini o‘rganamiz.

1-§. Aylana

Tekislikda Dekart koordinatalar sistemasini olaylik. Shu tekislikda biror $C(a, b)$ nuqta berilgan bo‘lsin.

Ta’rif. Berilgan nuqtadan bir xil R masofada joylashgan tekislikdagi barcha nuqtalar to‘plamiga ***aylana*** deyiladi. Bunda $C(a, b)$ nuqta aylana markazi, R – aylana radiusi deyiladi. $M(x, y)$ aylanadagi ixtiyoriy nuqta bo‘lsin (1-rasm).

Aylana ta’rifiga ko‘ra

$$|CM| = R$$

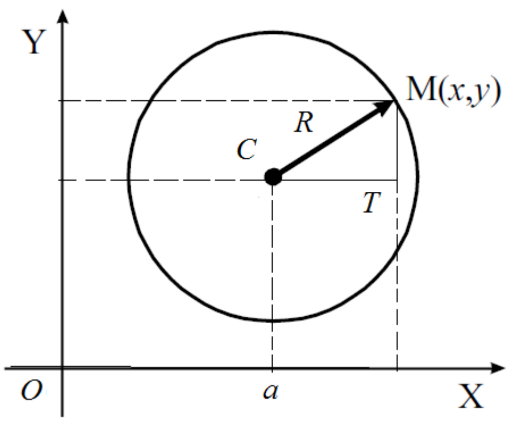
bo‘ladi.

ΔCTM to‘g‘ri burchakli uchburchak bo‘lgani uchun, Pifagor teoremasiga ko‘ra

$$|CT|^2 + |TM|^2 = |CM|^2,$$

bu yerda $|CT| = |x - a|,\ |TM| = |y - b|$. Natijada markazi $C(a, b)$ nuqtada va radiusi R ga teng bo‘lgan aylana tenglamasi hosil bo‘ladi:

$$(x - a)^2 + (y - b)^2 = R^2 \qquad (1)$$

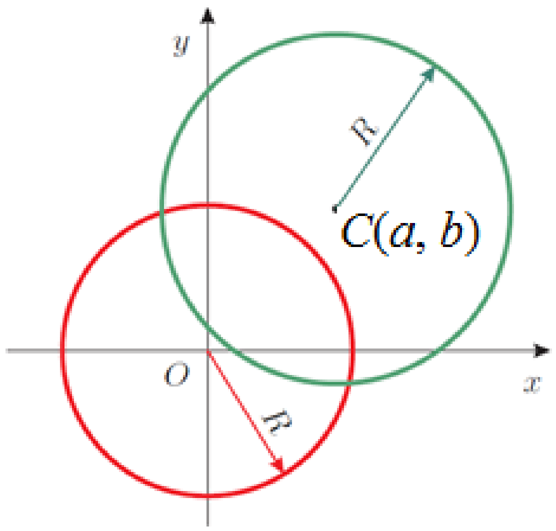


1-rasm.

Xususan, aylana markazi koordinata boshida bo‘lsa, uning kanonik tenglamasi

$$x^2 + y^2 = R^2 \qquad (2)$$

ko‘rinishga ega bo‘ladi (2-rasm).



2-rasm.

1-misol. Markazi (2; 1) nuqtada, radiusi 3 ga teng bo‘lgan aylana tenglamasini yozing.

Yechish. Aylana markazi koordinatalari $a = 2, b = 1$ va radiusi $R = 3$ bo‘ladi. (1) formuladan $(x-2)^2 + (y-1)^2 = 9$ izlanayotgan aylana tenglamasi yoziladi.

(1) tenglamada qavslarni ochish natijasida aylananing quyidagi ko‘rinishdagi umumiy tenglamasini hosil qilamiz:

$$x^2 + y^2 + mx + ny + p = 0 \qquad (3)$$

bu yerda $m = -2a, n = -2b, p = a^2 + b^2 - R^2$.

(3) tenglamani (1) tenglama ko‘rinishga qayta keltirish uchun (3) tenglamaning chap qismida to‘la kvadratlarni ajratamiz:

$$\left(x + \frac{m}{2}\right)^2 + \left(y + \frac{n}{2}\right)^2 = \frac{m^2}{4} + \frac{n^2}{4} - p \qquad (4)$$

2-misol. $x^2 + y^2 - 4x + 6y - 3 = 0$ aylana markazi va radiusini toping.

Yechish. (3) formulaga ko‘ra, $m = -4, n = 6, p = -3$. Bu qiymatlarni (4) formulaga qo‘yamiz:

$$(x-2)^2 + (y+3)^2 = 4 + 9 + 3,$$
$$(x-2)^2 + (y+3)^2 = 16.$$

Demak, aylana markazi $C(2;\ -3)$ nuqtada, radiusi $R = 4$ ga teng bo'ladi.

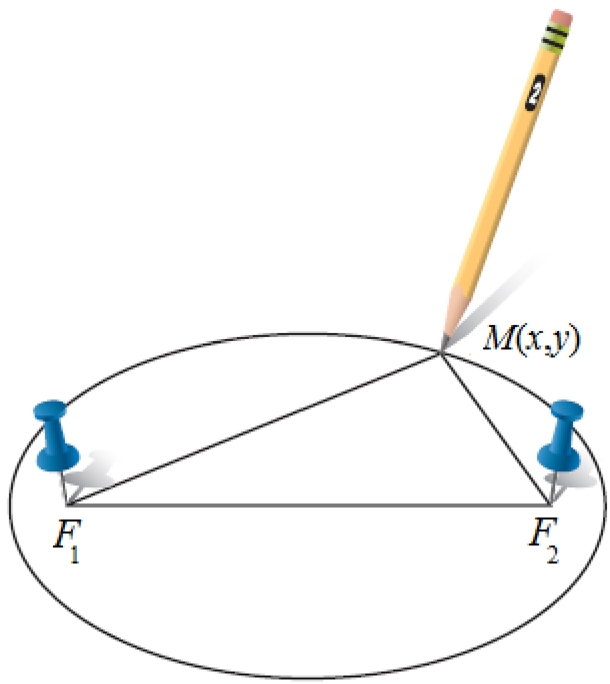
2-§. Ellips

Tekislikda F_1 va F_2 nuqtalar berilgan bo'lib, ular orasidagi masofa $2c$ $(c > 0)$ ga teng bo'lsin. Faraz qilamiz, $2a$ – musbat o'zgarmas son bo'lsin.

Ta'rif. Berilgan qo'zg'almas F_1 va F_2 nuqtalargacha bo'lgan masofalarning yig'indisi o'zgarmas $2a$ songa teng bo'lgan nuqtalarning geometrik o'rniga ***ellips*** deyiladi. F_1 va F_2 nuqtalar ellips fokuslari deyiladi.

Bu ta'rifdan ellipsning kanonik tenglamasini keltirib chiqaramiz. Tekislikda Dekart koordinatalar sistemasini olamiz. F_1 va F_2 nuqtalarni Ox o'qida koordinata boshiga nisbatan simmetrik joylashtiramiz. U holda fokus nuqtalar quyidagicha koordinatalarga ega bo'ladi: $F_1(-c;0), F_2(c;0)$.

F_1, F_2 ellips fokus nuqtalarlarini qo'zg'almas holda mahkamlab, qalam va ip yopdamida 3-rasmda ko'rsatilganidek, ellips grafigini yasash mumkin.



3-rasm.

Oxy koordinatalar sistemasida tasvirlangan ellipsda ixtiyoriy $M(x;y)$ nuqta olamiz. Ta’rifga ko‘ra,

$$|F_1M| + |F_2M| = 2a \tag{5}$$

Bu yerda $2a > 2c$, ya’ni $a > c$.

Ikki nuqta orasidagi masofa formulasiga ko‘ra:

$$|F_1M| = \sqrt{(x-c)^2 + y^2},\ |F_2M| = \sqrt{(x+c)^2 + y^2}. \tag{6}$$

(6) tengliklarni (5) tenglamaga qo‘yamiz:

$$\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a.$$

Ikkinchi kvadrat ildizni tenglikning o‘ng tomoniga o‘tkazamiz va tenglikning har ikki tomonini kvadratga oshiramiz:

$$x^2 + 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} +$$
$$+x^2 - 2cx + c^2 + y^2.$$

Bu tenglikdan: $a\sqrt{(x-c)^2 + y^2} = a^2 - cx$. Hosil bo‘lgan tenglikni kvadratga oshiramiz:

$$a^2[(x-c)^2 + y^2] = (a^2 - cx)^2.$$

Natijada

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2) \tag{7}$$

tenglama hosil bo‘ladi. Ma’lumki, $a > c, a^2 - c^2$ ayirma musbat bo‘lgani uchun $b^2 = a^2 - c^2$ belgilash kiritamiz, u holda (7) tenglama

$$b^2x^2 + a^2y^2 = a^2b^2 \tag{8}$$

ko‘rinishga keladi. $a \neq 0, b \neq 0$ bo‘lgani uchun (8) tenglamaning har ikki tomonini a^2b^2 ga bo‘lamiz:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \tag{9}$$

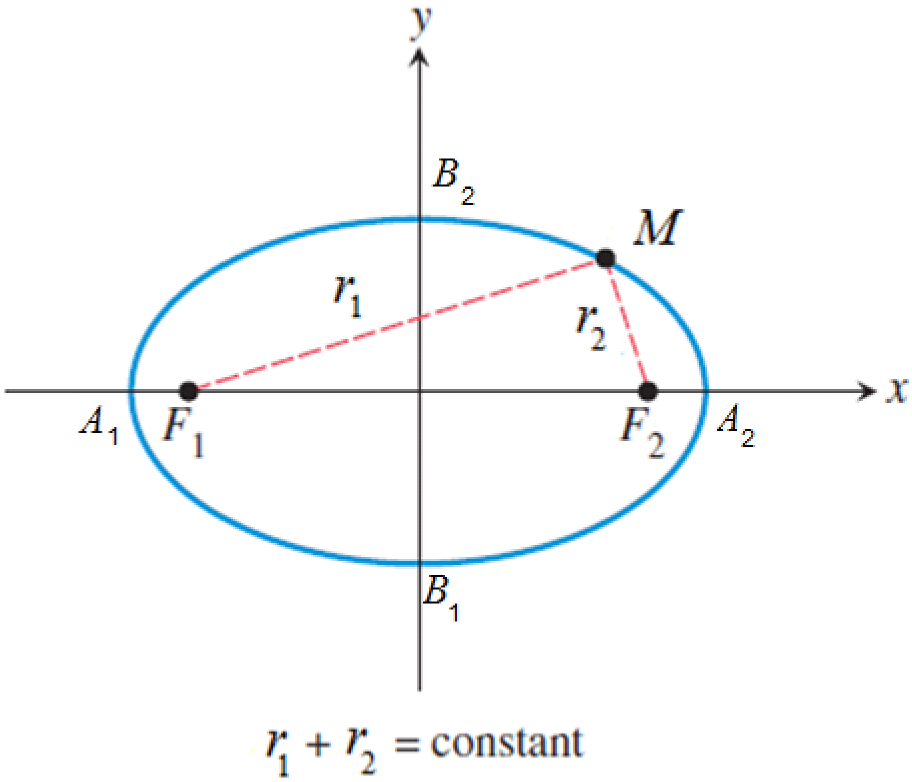
(9) tenglama ellipsning kanonik tenglamasi deyiladi. Uning grafigi 4-rasmda keltirilgan.

Ellipsning kanonik tenglamasiga ko‘ra grafigini tasvirlaymiz va a, b – parametrlarning geometrik ma’nosini aniqlaymiz. (9) tenglamaga ko‘ra $((-y)^2 = y^2, (-x)^2 = x^2)$, ellips koordinata o‘qlariga nisbatan simmetrik. Tenglamaga $x = 0$ va $y = 0$ qiymatlarni navbat bilan qo‘yamiz va ellipsning koordinata o‘qlari bilan kesishish nuqtalarini aniqlaymiz:

$$A_1(-a;0), \qquad A_2(a;0), \qquad B_1(0;\,-b), \qquad B_2(0;b).$$

Topilgan nuqtalar ellipsning uchlari deyiladi. A_1A_2 va B_1B_2 kesmalar mos ravishda ellipsning katta va kichik yarim o‘qlari deyiladi.

Shunday qilib, a – ellipsning katta yarim o‘qi uzunligi, b esa kichik yarim o‘qi uzunligidir ($a > b$).



4-rasm.

Ellipsning fokuslaridan $M(x, y)$ nuqtagacha bo‘lgan masofalar uchun quyidagicha belgilash kiritamiz (3-rasm):

$$|F_1M| = r_1,\ |F_2M| = r_2 \tag{10}$$

U holda (5) tenglama

$$r_1 + r_2 = 2a \tag{11}$$

ko‘rinishda bo‘ladi.

Ma’lumki,

$$r_1 = |F_1M| = \sqrt{(x+c)^2 + y^2},$$
$$r_2 = |F_2M| = \sqrt{(x-c)^2 + y^2}.$$

Oxirgi ikki tenglikni kvadratga oshiramiz va birinchisidan ikkinchisini ayiramiz:

$$r_1^2 - r_2^2 = (x+c)^2 + y^2 - (x-c)^2 - y^2.$$

Qavslarni ochib chiqamiz va quyidagi tenglama hosil bo‘ladi:

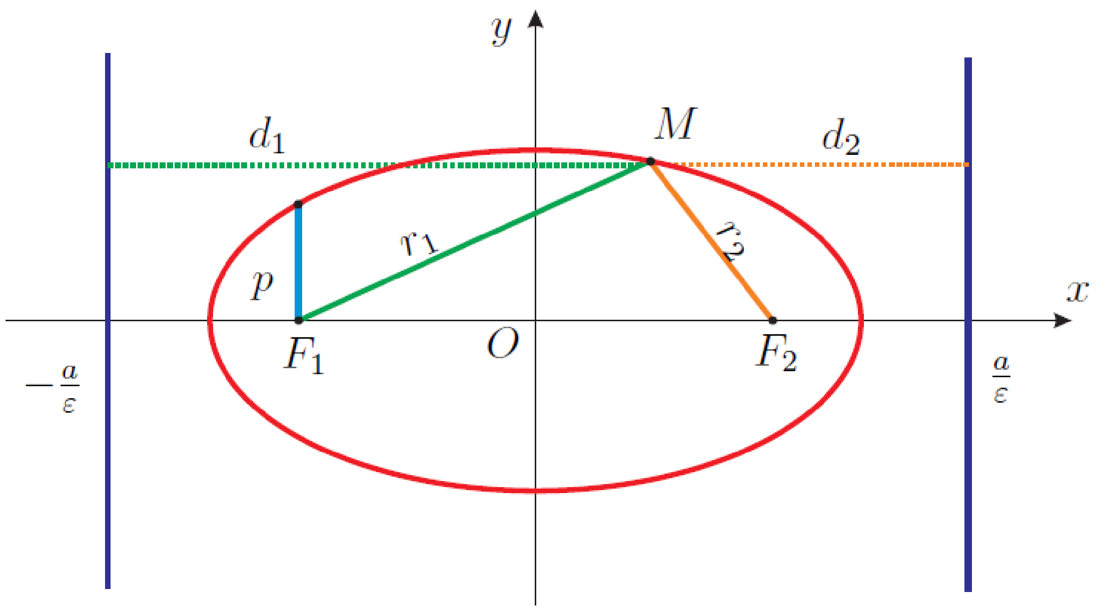
$$r_1^2 - r_2^2 = 4cx. \tag{12}$$

Hosil bo‘lgan tenglamani

$$(r_1 - r_2)(r_1 + r_2) = 4cx$$
ko'rinishda yozamiz. (11) formulaga asosan,
$$(r_1 - r_2) \cdot 2a = 4cx.$$
Bu tenglamani $2a$ ga qisqartiramiz:
$$r_1 - r_2 = 2\frac{c}{a}x. \tag{13}$$
(11) va (13) tenglamalardan r_1 va r_2 topiladi:
$$r_1 = a + \frac{c}{a}x$$
$$r_2 = a - \frac{c}{a}x$$
Natijada, ellipsdagi $M(x, y)$ nuqtadan fokuslargacha bo'lgan r_1 va r_2 masofalarni aniqlash formulasini topamiz:
$$r_{1,2} = a \pm \varepsilon x, \tag{14}$$
bu yerda $\varepsilon = \frac{c}{a}$.

$\varepsilon = \frac{c}{a}$ – miqdor ***ellipsning ekssentrisiteti*** deyiladi. Bu miqdor ellipsning shaklini ifodalaydi, ya'ni ε qiymati qancha katta bo'lsa, ellipsning shakli katta o'q bo'ylab ko'proq cho'zilgan bo'ladi.

Kichik o'qqa parallel va undan $\frac{a}{\varepsilon}$ masofada joylashgan to'g'ri chiziqlar ***ellipsning direktrisalari*** deyiladi (5-rasm). Ellipsning o'ng va chap direktrisa tenglamalari $x = \pm\frac{a}{\varepsilon}$ ko'rinishida bo'ladi. Ellips ekssentrisiteti $\varepsilon < 1$ bo'lgani uchun $\frac{a}{\varepsilon} > a$ bo'ladi.

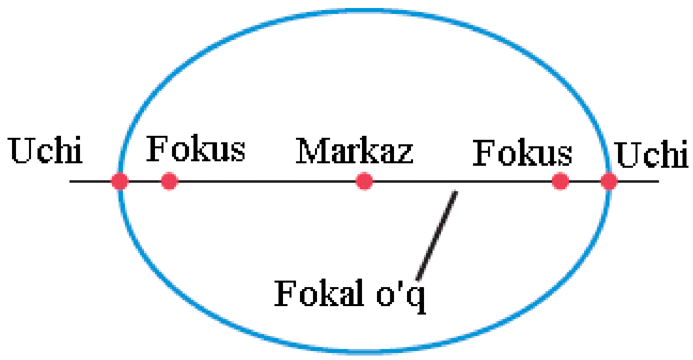


5-rasm.

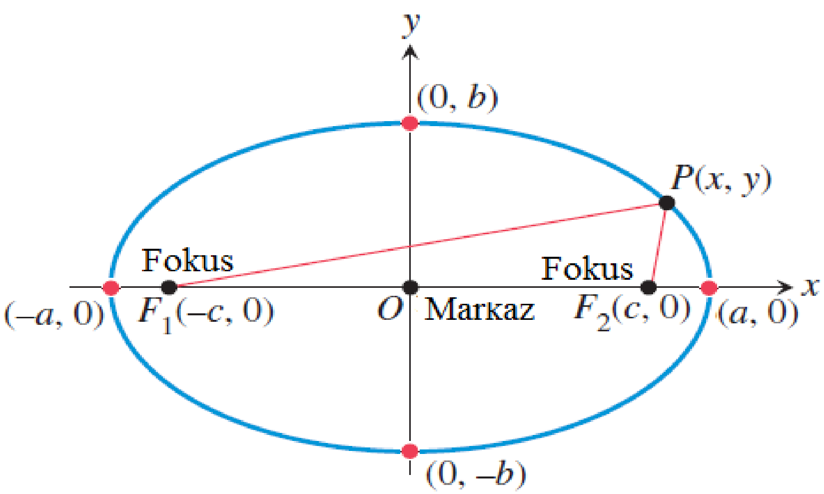
Ellipsning fokal parametri $p = \frac{b^2}{a}$ tenglik bilan aniqlanadi. Bu parametr ellipsning fokusi orqali o‘tgan va kichik o‘qqa parallel bo‘lgan vatar yarmiga tengdir.

Quyidagilar *ellips elementlarini* tashkil etadi:

- O nuqta – ellips markazi (6, 7 - rasmlar);
- $A_1(-a, 0)$; $A_2(a, 0)$; $B_1(0, -b)$; $B_2(0, b)$ nuqtalar – ellips uchlari (4-rasm);
- $F_1(-c; 0), F_2(c; 0)$ – ellips fokuslari;
- $2c$ – fokuslar orasidagi masofa, bu yerda $c = \sqrt{a^2 - b^2}$;
- $A_1A_2 = 2a$ va $B_1B_2 = 2b$ – ellipsning katta va kichik o‘qlari;
- a va b – ellipsning katta va kichik yarim o‘qlari uzunliklari;
- $\varepsilon = \frac{c}{a}, (0 \leq \varepsilon \leq 1)$ – ellipsning ekssentrisiteti;
- $x = \pm\frac{a}{\varepsilon}$ – ellipsning direktrisalari tenglamalari.

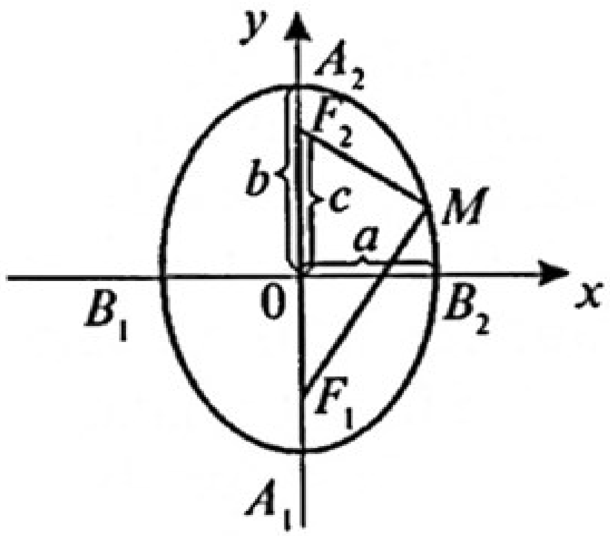


6-rasm.



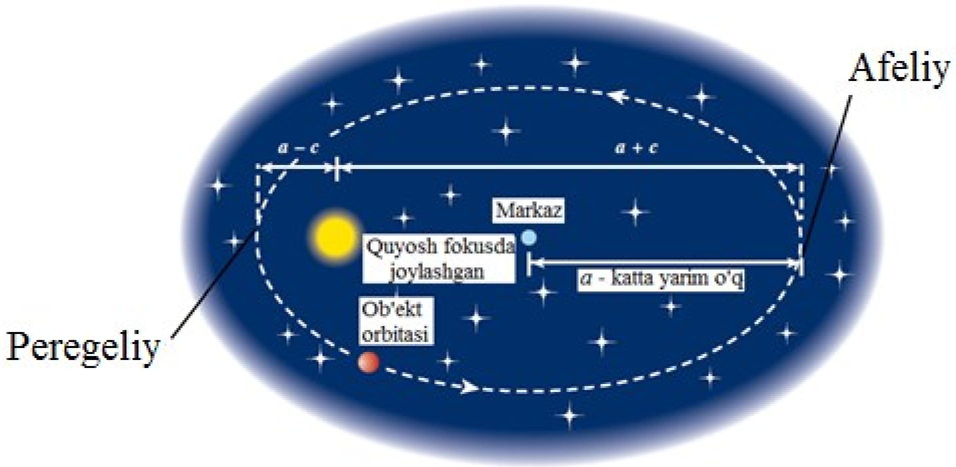
7-rasm.

Agar $a < b$ bo‘lsa, ellips fokuslari Oy o‘qida bo‘lib,
$c = \sqrt{b^2 - a^2}, \frac{c}{b} = \varepsilon, r = b \pm \varepsilon y$ bo‘ladi (8-rasm).



8-rasm.

Osmon obyektlarining elliptik orbitasi. Nemis astronomi I.Keplerning 1609-yilda chop etilgan planetalar harakati haqidagi birinchi qonuniga ko‘ra, planetalar orbitasi fokuslaridan birida Quyosh joylashgan ellipsni tashkil etadi. Asteroidlar, kometalar va boshqa osmon jismlari Quyosh atrofida elliptik traektoriya bo‘yicha harakatlanadi (9-rasm). 9-rasmda massasi Quyosh massasidan kichik bo‘lgan planetaning elliptik orbitasi keltirilgan. Fokuslaridan birida Quyosh joylashgan. Elliptik trayektoriyadagi Quyoshga eng yaqin nuqta – “peregeliy”, eng uzoq nuqta – “afeliy” deyiladi. “Peregeliy” va “afeliy” orasidagi masofa – ellipsning katta o‘qidir.



9-rasm. Osmon obyektlarining Quyosh atrofidagi elliptik orbitasi.

3-misol. $x^2 + 2y^2 = 2$ ellipsning ekssentrisiteti va direktrisasini toping.

Yechish. Tenglamani 2 ga bo‘lib, kanonik ko‘rinishga keltiramiz:

$$\frac{x^2}{2} + \frac{y^2}{1} = 1,$$

$$a^2 = 2, \qquad b^2 = 1. \ c^2 = a^2 - b^2 = 2 - 1 = 1.$$

$$\varepsilon = \frac{c}{a} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Direktrisa formulasiga ko‘ra

$$x = \pm\frac{a}{\varepsilon}, \qquad x = \pm\frac{\sqrt{2}}{\frac{\sqrt{2}}{2}} = \pm 2.$$

4-misol. Ekssentrisiteti $\varepsilon = \frac{2}{3}$ ga teng bo‘lgan ellipsning focuslaridan biri (6;0) nuqtada bo‘lsa, uning kanonik tenglamasini tuzing.

Yechish. Shartga ko’ra, $c = 6$. $\varepsilon = \frac{c}{a} = \frac{2}{3} \Rightarrow c = \frac{2}{3}a$.

$$6 = \frac{2}{3}a \Rightarrow a = 9.$$

$c^2 = a^2 - b^2$ tenglikdan *b* ning qiymatini topamiz:

$$6^2 = 9^2 - b^2 \Rightarrow b^2 = 45.$$

Ellipsning kanonik tenglamasini yozamiz:

$$\frac{x^2}{81} + \frac{y^2}{45} = 1.$$

5-misol. Katta o‘qi kichik o‘qidan uch marta katta bo‘lgan ellipsning ekssentrisitetini toping.

Yechish. Shartga ko‘ra, $a = 3b. c = \sqrt{a^2 - b^2}$ tenglikdan $c = \sqrt{(3b)^2 - b^2} = 2\sqrt{2}b. \varepsilon = \frac{c}{a} \implies \varepsilon = \frac{2\sqrt{2}b}{3b} \implies \varepsilon = \frac{2\sqrt{2}}{3}$.

6-misol. $\frac{x^2}{9} + \frac{y^2}{25} = 1$ ellipsning fokuslari, ekssentrisiteti va direktrisasini toping.

Yechish. Tenglamadan ma’lumki, $a < b$, ya’ni $a = 3, \ b = 5$. U holda ellips fokuslari Oy o‘qida bo‘lib, $c = \sqrt{b^2 - a^2}$; $c = \sqrt{5^2 - 3^2} \implies c = 4, F_1(0; -4), F_2(0; 4). \varepsilon = \frac{c}{b} = \frac{4}{5}$ bo‘ladi. Direktrisa formulasiga ko‘ra

$$y = \pm\frac{b}{\varepsilon}, \qquad y = \pm\frac{5}{\frac{4}{5}} = \pm\frac{25}{4}.$$

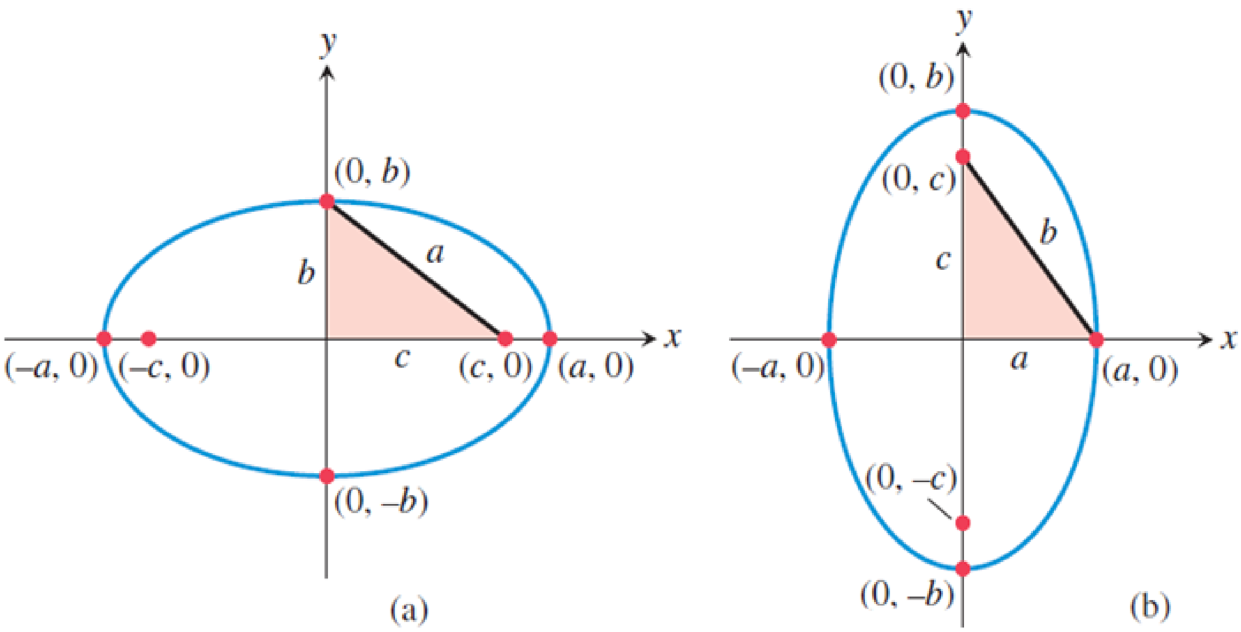
Ellipsning markazi, fokuslarining koordinata o‘qlarida joylashishi va uning elementlarini 1, 2-jadvallarda keltiramiz.

1-jadval

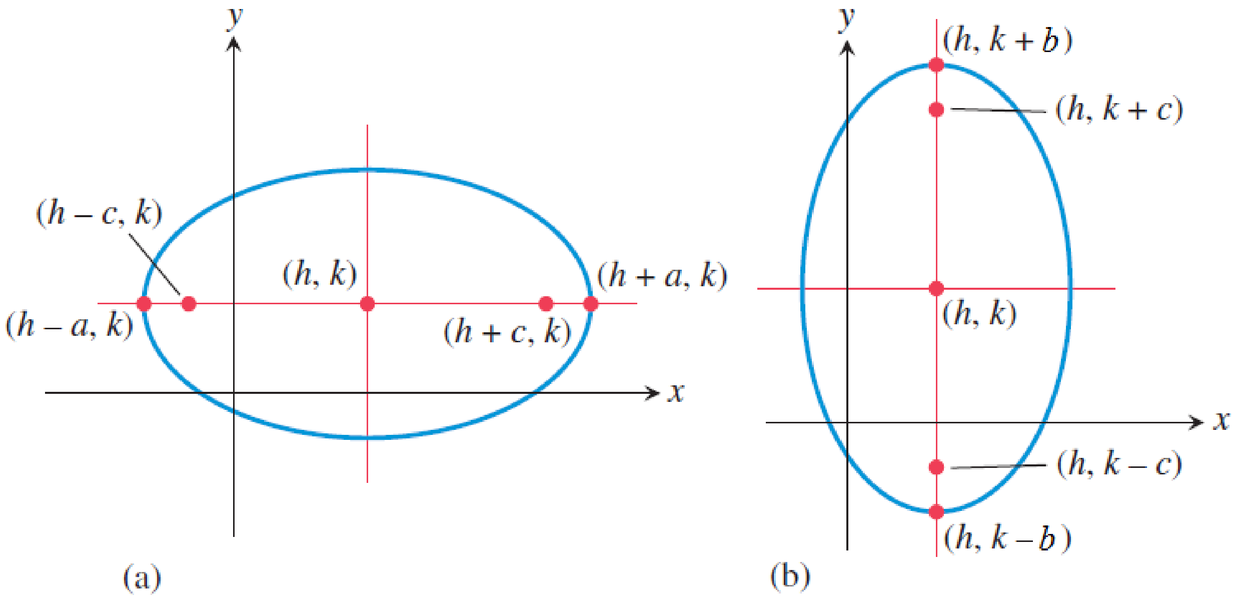
Markazi O(0, 0) nuqtada bo‘lgan ellips		
Standart tenglamasi	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ $a > b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ $a < b$
Fokuslari	$F(\pm c, 0)$	$F(0, \pm c)$
Fokus koordinatasi	$c = \sqrt{a^2 - b^2}$	$c = \sqrt{b^2 - a^2}$
Ekssentrisiteti	$\varepsilon = \frac{c}{a}$	$\varepsilon = \frac{c}{b}$
Katta yarim o‘q uzunligi	a	b
Kichik yarim o‘q uzunligi	b	a
Direktrisasi	$x = \pm\frac{a}{\varepsilon}$	$y = \pm\frac{b}{\varepsilon}$
Fokal o‘qi	Ox	Oy
	10-rasm, (a)	10-rasm, (b)

2-jadval

Markazi O(h, k) nuqtada bo‘lgan ellips		
Tenglama ko‘rinishi	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,$ $a>b$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,$ $a<b$
Fokuslari	$F(h\pm c,k)$	$F(h,k\pm c)$
Fokus koordinatasi	$c=\sqrt{a^2-b^2}$	$c=\sqrt{b^2-a^2}$
Ekssentrisiteti	$\varepsilon=\frac{c}{a}$	$\varepsilon=\frac{c}{b}$
Katta yarim o‘q uzunligi	a	b
Kichik yarim o‘q uzunligi	b	a
Direktrisasi	$x=\pm\frac{a}{\varepsilon}+h$	$y=\pm\frac{b}{\varepsilon}+k$
Fokal o‘qi	$y=k$	$x=h$
Fokal o‘qdagi uchlari	$(h-a,k)$; $(h+a,k)$	$(h,k+b)$; $(h,k-b)$
	11-rasm, (a)	11-rasm, (b)



10-rasm.



11-rasm.

7-misol. (-2, -1) va (8, -1) nuqtalar ellipsning katta o‘qi uchlarini tashkil etadi. Uning kichik o‘qi uzunligi 8 ga teng. Ellipsning kanonik tenglamasini tuzing.

Yechish. Misol shartiga ko‘ra, ellipsning kanonik tenglamasi

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,$$

$(a > b)$ ko‘rinishga bo‘ladi. Ellips markazi (h, k) nuqta koordinatalarini aniqlaymiz:

$$h=\frac{8+(-2)}{2}=3, \qquad k=\frac{-1+(-1)}{2}=-1.$$

Ellipsning katta va kichik yarim o‘qlari uzunliklari:

$$a=\frac{8-(-2)}{2}=5, \qquad b=\frac{8}{2}=4.$$

Aniqlangan qiymatlarni kanonik tenglamaga qo‘yamiz:

$$\frac{(x-3)^2}{5^2}+\frac{(y-(-1))^2}{4^2}=1,$$

$$\frac{(x-3)^2}{25}+\frac{(y+1)^2}{16}=1.$$

8-misol. Ellipsning markazi, uchlari va fokusi koordinatalarini aniqlang:

$$\frac{(x+2)^2}{9}+\frac{(y-5)^2}{49}=1.$$

Yechish. Ellipsning kichik va katta yarim o‘qlari uzunliklari: $a=3,\ b=7.a<b.$ U holda,

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

kanonik tenglamaga asosan, ellips markazi koordinatalari: $h=-2, k=5$. Fokal o‘qdagi uchlari, $(h,k+b)$; $(h,k-b)$ nuqta koordinatalarini aniqlaymiz:

$$(h,k+b)=(-2,5+7)=(-2,12) \text{ va}$$
$$(h,k-b)=(-2,5-7)=(-2,-2).$$

Fokus koordinatasi: $c=\sqrt{b^2-a^2}=\sqrt{49-9}=\sqrt{40}$.

Fokuslari: $F(h,k\pm c)\ \Rightarrow\ F\left(-2,5\pm\sqrt{40}\right)$.

Demak, ellipsning fokuslari $F_1(-2,11.32)$ va $F_2(-2,-1.32)$.

3-§. Giperbola

Tekislikda F_1 va F_2 nuqtalar berilgan bo‘lib, ular orasidagi masofa $2c(c>0)$ ga teng bo‘lsin.

Ta’rif. Berilgan (F_1 va F_2) nuqtalargacha bo‘lgan masofalar ayirmasiining absolyut qiymati o‘zgarmas $2a$ songa teng bo‘lgan nuqtalarning geometrik o‘rniga giperbola deyiladi. F_1 va F_2 nuqtalar giperbola fokuslari deyiladi.

Giperbolaning kanonik tenglamasini keltirib chiqaramiz. F_1 va F_2 nuqtalarni Ox o‘qi bo‘ylab koordinata boshiga nisbatan simmetrik holda c masofada joylashtiramiz. U holda fokus nuqtalar quyidagi koordinatalarga ega bo‘ladi: $F_1(-c;0), F_2(c;0)$.

Oxy koordinatalar sistemasida tasvirlangan giperboladagi ixtiyoriy $M(x,y)$ nuqtani olamiz. Ta’rifga asosan:

$$|F_1M|-|F_2M|=\pm 2a \tag{15}$$

bo‘ladi. Ikki nuqta orasidagi masofa formulasiga ko‘ra:

$$\sqrt{(x+c)^2+y^2}=\pm 2a+\sqrt{(x-c)^2+y^2}.$$

Tenglikninghar ikki tomonini kvadratga oshiramiz va uni soddalashtiramiz:

$$(x+c)^2+y^2=4a^2\pm 4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2,$$

$$cx - a^2 = \pm a\sqrt{(x-c)^2 + y^2},$$
$$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2).$$

$c > a$ bo‘lgani uchun $c^2 - a^2 > 0$ bo‘ladi. U holda $b^2 = = c^2 - a^2$ almashtirish yordamida
$$b^2x^2 - a^2y^2 = a^2b^2$$
tenglama hosil bo‘ladi. Tenglamaning har ikki tomonini a^2b^2 ga bo‘lamiz:
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \qquad (16)$$

(16) tenglama giperbolaning kanonik tenglamasi deyiladi. Giperbolaning koordinata o‘qlari bilan kesishish nuqtasini topamiz. (16) tenglamaga $x = 0$ qiymatni qo‘yamiz:
$$-\frac{y^2}{b^2} = 1.$$

Tenglamaning chap tomoni manfiy yoki 0 ga teng, o‘ng tomoni esa 0 dan katta, tenglama ma’noga ega bo‘lmaydi. Bu natija giperbolaning ordinata o‘qini kesib o‘tmasligini bildiradi. Endi (16) tenglamaga $y = 0$ qiymatni qo‘yamiz:
$$\frac{x^2}{a^2} = 1.$$

Bu tenglik giperbola abssissa o‘qini $A_1(-a; 0)$ va $A_2(a; 0)$ nuqtalarda kesishini bildiradi. A_1 va A_2 nuqtalar giperbola uchlari, A_1A_2 kesma esa uning haqiqiy o‘qi deyiladi.

Oldingi paragrafdagi belgilashlar asosida va giperbola ta’rifiga binoan:
$$r_1 - r_2 = \pm 2a. \qquad (17)$$
2-§ dagi kabi
$$r_1^2 - r_2^2 = 4cx \qquad (18)$$
tenglikni topamiz va uni
$$(r_1 - r_2)(r_1 + r_2) = 4cx$$
ko‘rinishda ifodalaymiz. (3) tenglamaga asosan
$$r_1 + r_2 = \pm 2\frac{c}{a}x \qquad (19)$$
tenglikka kelamiz. (17) va (19) tenglamalardan r_1 va r_2 topiladi:
$$r_{1,2} = |\varepsilon x \pm a|, \qquad (20)$$

bu yerda $\varepsilon = \frac{c}{a} > 1$ – giperbolaning ekssentrisiteti deyiladi. Ekssentrisitet giperbola o‘qlari nisbati bilan aniqlanadi va uning shaklini ifodalaydi. ε qiymati qancha katta bo‘lsa, giperbola asosidagi tomonlari $2a$ va $2b$ ga teng to‘g‘ri to‘rtburchak mavhum o‘q bo‘ylab kattalashadi.

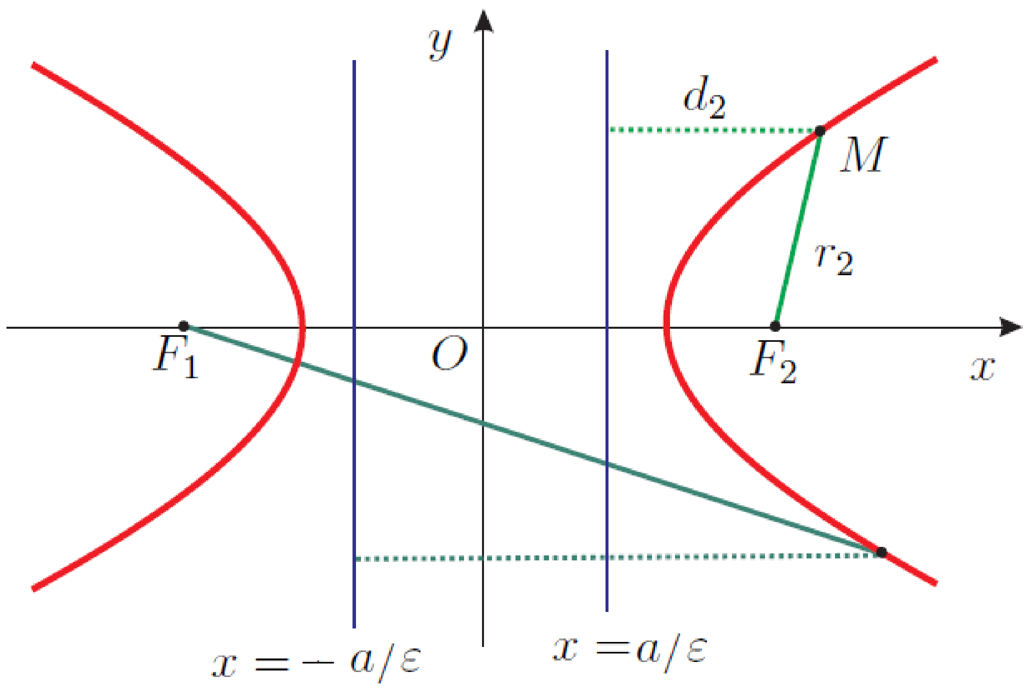
$x = \pm\frac{a}{\varepsilon}$ to‘g‘ri chiziqlar giperbolaning direktrisalari deyiladi (12-rasm). Giperbola ekssentrisiteti $\varepsilon > 1$ bo‘lgani uchun $\frac{a}{\varepsilon} < a$ bo‘ladi.

Giperbola asimptotalari to‘g‘ri chiziqlar bo‘lib, bu to‘g‘ri chiziqlar x ning cheksiz kattalashib borishi bilan giperbolaga borgan sari yaqinlashib boradi. $k = \pm\mathrm{tg}\alpha = \pm\frac{b}{a}$ bo‘lishini hisobga olsak, giperbola asimptotalari tenglamalari

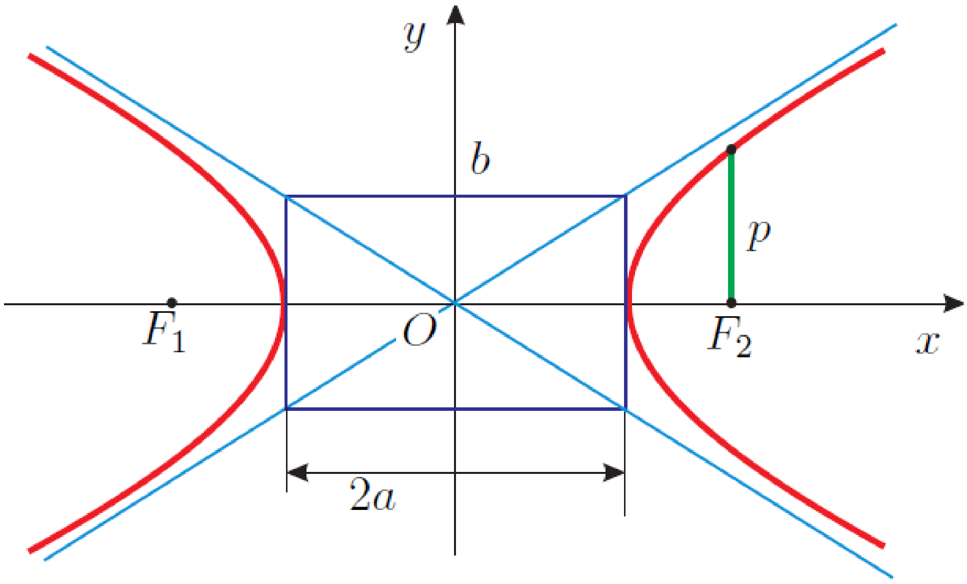
$$y = \pm\frac{b}{a}x \tag{21}$$

ko‘rinishda bo‘ladi.

Giperbola fokal parametri $p = \frac{b^2}{a}$ ga teng (13-rasm).



12-rasm.



13-rasm.

Quyidagilar *giperbola elementlarini* tashkil etadi:

- O nuqta – giperbola markazi;
- $A_1(-a;0)$ va $A_2(a;0)$ nuqtalar – giperbola uchlari;
- $F_1(-c;0),\ F_2(c;0)$ – giperbola fokuslari;
- $2c$ – fokuslar orasidagi masofa, bu yerda $c=\sqrt{a^2+b^2}$;
- $A_1A_2=2a$ – giperbolaning haqiqiy o‘qi, $B_1B_2=2b$ giperbolaning mavhum o‘qi;
- $a>0$ – giperbolaning haqiqiy yarim o‘qi, $b>0$ giperbolaning mavhum yarim o‘qi uzunliklari;
- $\varepsilon=\frac{c}{a}$ – giperbola ekssentrisiteti ($\varepsilon>1$);
- $x=\pm\frac{a}{\varepsilon}$ – giperbola direktrisa tenglamasi;
- $y=\pm\frac{b}{a}x$ – giperbola asimptota tenglamasi.

Agar $a=b$ bo‘lsa, giperbola teng tomonli deyiladi, uning tenglamasi $x^2-y^2=a^2$ ko‘rinishda bo‘ladi.

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ va $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ giperbolalar qo‘shma giperbolalar deyiladi.

Giperbolaning kanonik tenglamasiga ko‘ra grafigini yasash algoritmi:

1. Koordinata boshidan abssissa o‘qining ikki tomoni bo‘ylab haqiqiy yarim o‘q a ning uzunligiga teng kesmalar ajratamiz.

2. Koordinata boshidan ordinata o‘qining ikki tomoni bo‘ylab mavhum yarim o‘q b ning uzunligiga teng kesmalar ajratamiz.

3. Markazi Dekart koordinatalar sistemasi boshida bo‘lgan, tomonlari $2a$ va $2b$ ga teng bo‘lib, koordinata o‘qlariga parallel bo‘lgan to‘g‘ri to‘rtburchak yasaladi.

4. To‘g‘ri to‘rtburchak diagonallari yasaladi va to‘g‘ri to‘rtburchak tashqarisiga ham davom ettiriladi. Bu chiziqlar giperbola asimptotalaridir.

5. Giperbola uchlarini $(-a;0)$ va $(a;0)$ nuqtalarda belgilaymiz. Bu nuqtalardan asimptotalargacha yaqinlashib boradigan va uni kesib o‘tmaydigan shox chiziqlarni yasaymiz.

9-misol. $\frac{x^2}{25}-\frac{y^2}{16}=1$ giperbolaning ekssentrisiteti, direktrisa va asimptotalarini toping.

Yechish. Berilgan tenglamada $a=5, b=4$. Bundan $c=\sqrt{a^2+b^2}=\sqrt{25+16}=\sqrt{41}$. $\varepsilon=\frac{c}{a}=\frac{\sqrt{41}}{5}>1$.

Direktrisa tenglamasi $x=\pm\frac{a}{\varepsilon}$. Bundan $x=\pm\frac{5}{\frac{\sqrt{41}}{5}}=\pm\frac{25}{\sqrt{41}}$ ni topamiz.

$y=\pm\frac{b}{a}x$ formulaga ko‘ra, $y=\pm\frac{4}{5}x$ giperbola asimptota tenglamalarini aniqlaymiz.

10-misol. $\frac{x^2}{16}-\frac{y^2}{9}=1$ giperbola grafigini yasang. Uning ekssentrisiteti, fokusi va direktrisasini toping.

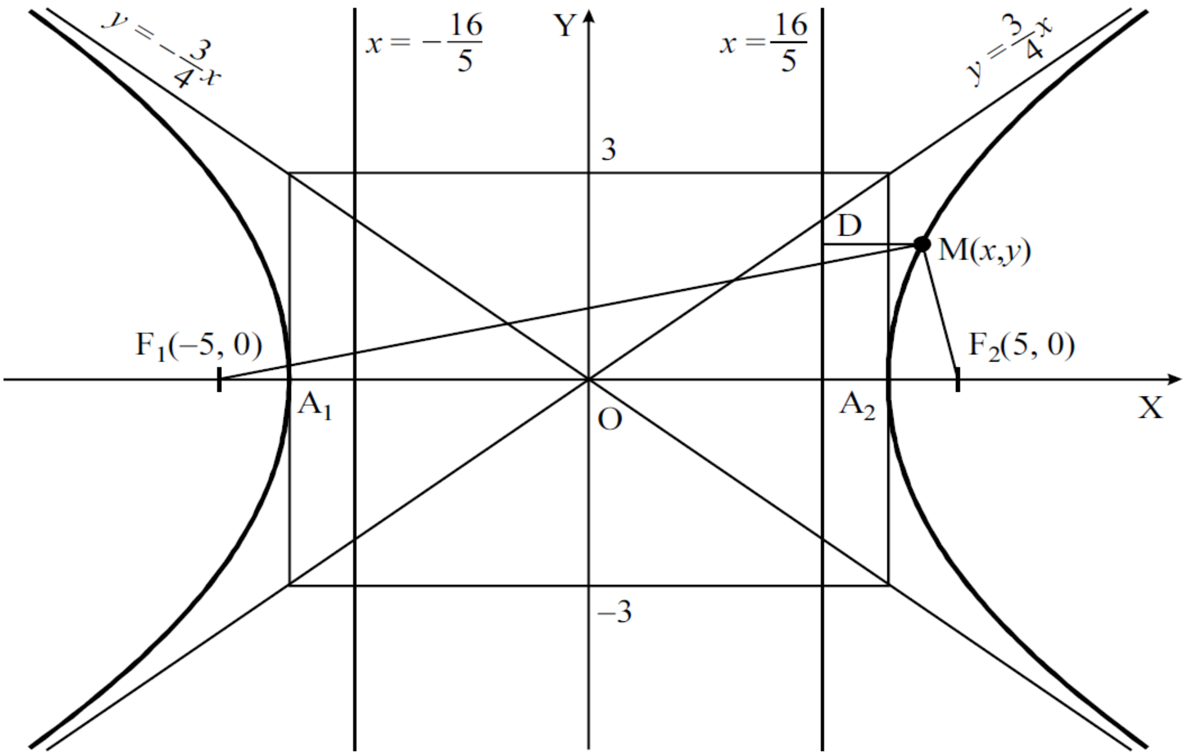
Yechish.

$$c^2=a^2+b^2 \;\Rightarrow\; c=5 \;\Rightarrow\; F_1(-5,0), F_2(5,0).$$

$\varepsilon=\frac{c}{a}=\frac{5}{4},\ x=\pm\frac{a}{\varepsilon}=\pm\frac{16}{5}$ - direktrisa tenglamasi.

$$\frac{|MF_2|}{|MD|}=\varepsilon,\quad \varepsilon=\frac{5}{4};\ \left|\overrightarrow{MF_1}\right|-\left|\overrightarrow{MF_2}\right|=8,\quad |A_1A_2|=8.$$

$y=\pm\frac{3}{4}x$ – giperbola asimptotasi (14-rasm).



14-rasm.

11-misol. Haqiqiy o‘qi 6 ga teng va (9; -4) nuqtadan o‘tuvchi giperbolaning kanonik tenglamasini yozing,

Yechish. Shartga ko‘ra, $2a = 6 \implies a = 3$.

Berilgan (9; -4) nuqta giperbola egri chizig‘iga tegishli bo‘lganligi uchun, nuqta koordinatalarini giperbola kanonik tenglamasiga qo‘yamiz:

$$\frac{9^2}{a^2} - \frac{(-4)^2}{b^2} = 1 \implies \frac{9^2}{3^2} - \frac{(-4)^2}{b^2} = 1 \implies b^2 = 2.$$

a va b ning qiymatlarini kanonik tenglamaga qo‘yamiz:

$$\frac{x^2}{9} - \frac{y^2}{2} = 1.$$

Natijada izlanayotgan kanonik tenglama hosil bo‘ldi.

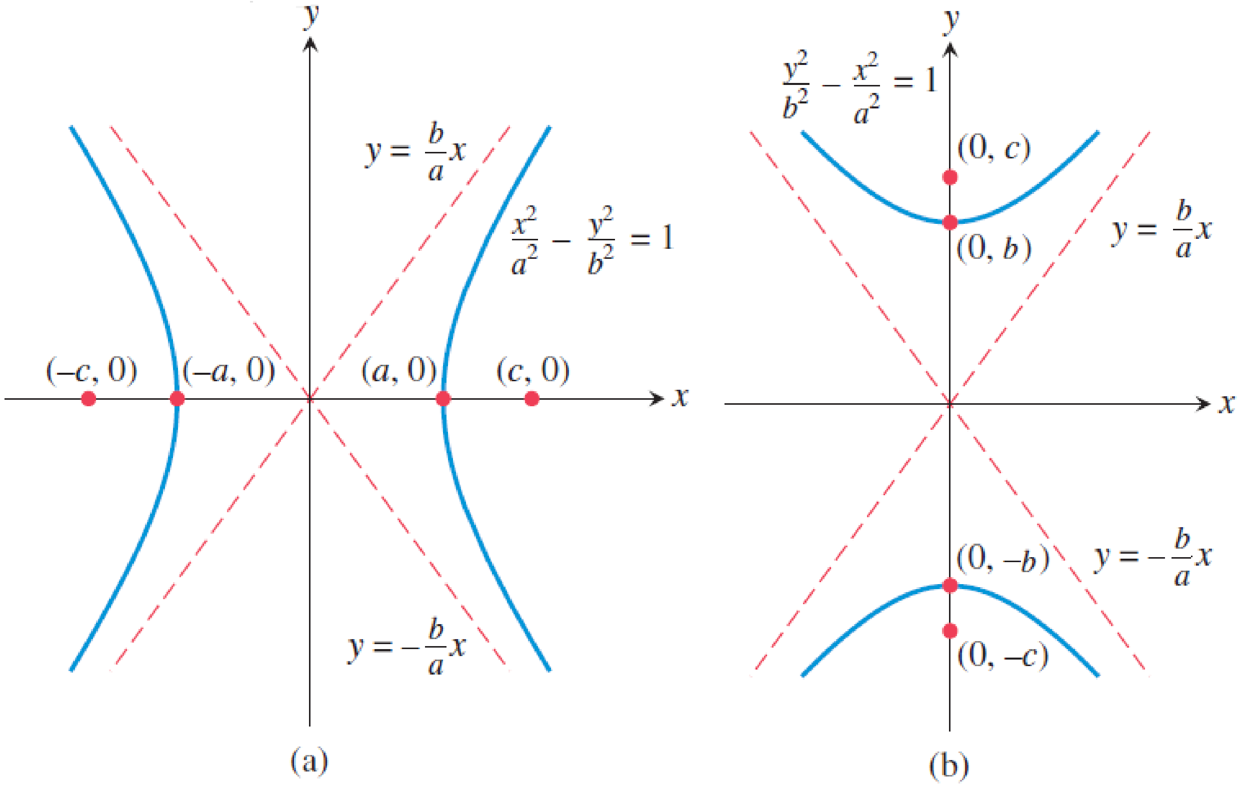
Giperbolaning markazi, fokuslarining koordinata o‘qlarida joylashishi va uning elementlarini 3,4-jadvallarda keltiramiz.

3-jadval

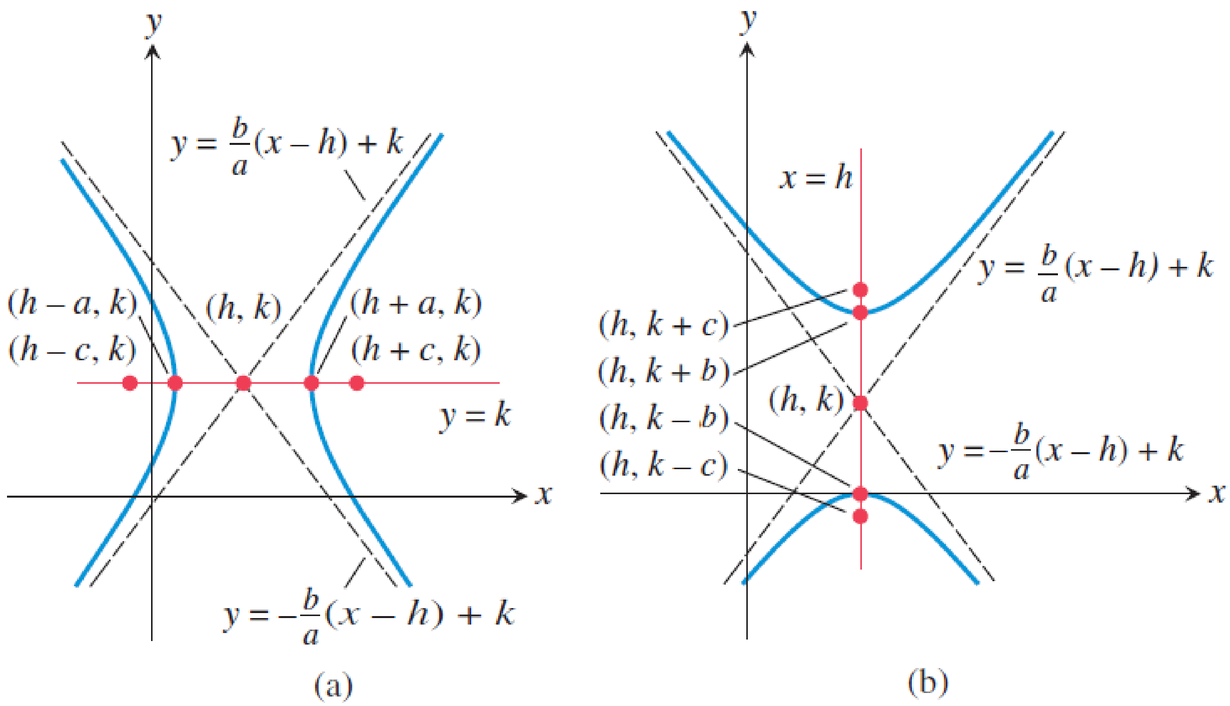
Markazi O(0, 0) nuqtada bo‘lgan giperbola		
Standart tenglamasi	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$
Fokuslari	$F(\pm c,0)$	$F(0,\pm c)$
Fokus koordinatasi	$c=\sqrt{a^2+b^2}$	$c=\sqrt{a^2+b^2}$
Ekssentrisiteti	$\varepsilon=\frac{c}{a}$	$\varepsilon=\frac{c}{b}$
Haqiqiy yarim o‘q uzunligi	a	b
Mavhum yarim o‘q uzunligi	b	a
Direktrisasi	$x=\pm\frac{a}{\varepsilon}$	$y=\pm\frac{b}{\varepsilon}$
Fokal o‘qi	Ox	Oy
Asimptota tenglamasi	$y=\pm\frac{b}{a}x$	$y=\pm\frac{b}{a}x$
	15-rasm, (*a*)	15-rasm, (*b*)

4-jadval

Markazi O(*h*, *k*) nuqtada bo‘lgan giperbola		
Tenglama ko‘rinishi	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1,$	$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$
Fokuslari	$F(h\pm c,k)$	$F(h,k\pm c)$
Fokus koordinatasi	$c=\sqrt{a^2+b^2}$	$c=\sqrt{b^2+a^2}$
Ekssentrisiteti	$\varepsilon=\frac{c}{a}$	$\varepsilon=\frac{c}{b}$
Haqiqiy yarim o‘q uzunligi	a	b
Mavhum yarim o‘q uzunligi	b	a
Direktrisasi	$x=\pm\frac{a}{\varepsilon}+h$	$y=\pm\frac{b}{\varepsilon}+k$
Fokal o‘qi	$y=k$	$x=h$
Asimptota tenglamasi	$y=\pm\frac{b}{a}(x-h)+k$	$y=\pm\frac{b}{a}(x-h)+k$
	16-rasm, (*a*)	16-rasm, (*b*)



15-rasm.



16-rasm.

12-misol. (-2, -1) va (8, -1) nuqtalar giperbolaning haqiqiy o‘qi uchlarini tashkil etadi. Uning mavhum o‘qi uzunligi 8 ga teng. Giperbolaning kanonik tenglamasini tuzing.

Yechish. Misol shartiga ko‘ra, giperbolaning kanonik tenglamasi

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1,$$

ko‘rinishga bo‘ladi. Giperbola markazi (h, k) nuqta koordinatalarini aniqlaymiz:

$$h=\frac{8+(-2)}{2}=3, \qquad k=\frac{-1+(-1)}{2}=-1.$$

Giperbolaning haqiqiy va mavhum yarim o‘qlari uzunliklarini:

$$a=\frac{8-(-2)}{2}=5, \qquad b=\frac{8}{2}=4.$$

Aniqlangan qiymatlarni kanonik tenglamaga qo‘yamiz:

$$\frac{(x-3)^2}{5^2}-\frac{(y-(-1))^2}{4^2}=1,$$

$$\frac{(x-3)^2}{25}-\frac{(y+1)^2}{16}=1.$$

13-misol. Giperbolaning markazi, uchlari va fokusi koordinatalarini aniqlang:

$$\frac{(x+2)^2}{9}-\frac{(y-5)^2}{49}=1.$$

Yechish. Giperbolaning haqiqiy va mavhum yarim o‘qlari uzunliklari: $a=3,\ b=7.$ U holda,

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

kanonik tenglamaga asosan, giperbola markazi koordinatalari: $h=-2, k=5.$ Fokal o‘qdagi uchlari, $(h-a,k)$; $(h+a,k)$ nuqta koordinatalarini aniqlaymiz:

$$(h-a,k)=(-2-3,5)=(-5,5) \text{ va}$$

$$(h+a,k)=(-2+3, \quad 5)=(1, \quad 5).$$

Fokus koordinatasi: $c=\sqrt{a^2+b^2}=\sqrt{9+49}=\sqrt{58}.$

Fokuslari: $F(h\pm c,k) \implies F\left(-2\pm\sqrt{40},5\right).$

Demak, ellipsning fokuslari $F_1(-9.62,5)$ va $F_2(5.62,\ 5).$

4-§. Parabola

Tekislikda Dekart koordinatalar sistemasini olaylik. Shu tekislikda Oy o‘qiga parallel to‘g‘ri chiziq va bu to‘g‘ri chiziqqa tegishli bo‘lmagan $F(p/_2\,; 0)$ nuqta berilgan bo‘lsin.

Ta’rif. Berilgan nuqta va to‘g‘ri chiziqdan bir xil masofada joylashgan tekislikdagi barcha nuqtalar to‘plami parabola deyiladi.

Parabola kanonik tenglamasini keltirib chiqaramiz. $F(\frac{p}{2}; 0)$ nuqtani va $x = -\frac{p}{2}$ to‘g‘ri chiziqni koordinatalar sistemasida yasaymiz $(p > 0)$. $M(x, y)$ - paraboladagi ixtiyoriy nuqta bo‘lsin (17-rasm).

Ta’rifga ko‘ra,

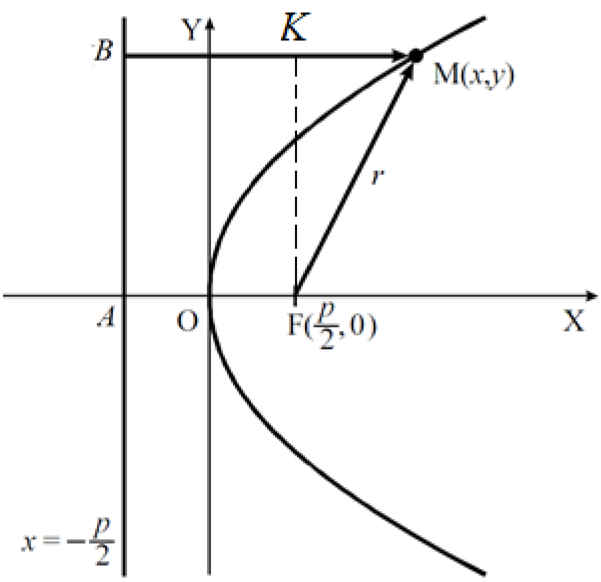
$$|FM| = |BM|. \tag{22}$$

Ikki nuqta orasidagi masofa formulasiga binoan

$$|BM| = \left|x - \left(-\frac{p}{2}\right)\right|, \qquad |FM| = \sqrt{\left(x - \frac{p}{2}\right)^2 + (y - 0)^2},$$

bu qiymatlarni (22) tenglikka qo‘yamiz:

$$\left|x + \frac{p}{2}\right| = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}.$$



17-rasm.

Hosil bo‘lgan tenglamaning ikki tomonini kvadratga oshiramiz va qavslarni ochib chiqamiz:

$$x^2 + px + \frac{p^2}{4} = x^2 - px + \frac{p^2}{4} + y^2.$$

Soddalashtirishlardan so‘ng, tenglama

$$y^2 = 2px \qquad (23)$$

ko‘rinishga keladi. (23) tenglama parabolaning kanonik tenglamasi deyiladi.

Fokusdan paraboladagi ixtiyoriy $M(x, y)$ nuqtagacha bo‘lgan r masofani topamiz. 17-rasmga ko‘ra, ΔFKM – to‘g‘ri burchakli uchburchak. Pifagor teoremasiga asosan

$$FM^2 = FK^2 + KM^2.$$

$FM=r$, $FK=y$, $KM = X - OF = x - \frac{p}{2}$.

$$r^2 = y^2 + \left(x - \frac{p}{2}\right)^2$$

$$r^2 = 2px + x^2 - px + \frac{p^2}{4}$$

$$r^2 = px + x^2 + \frac{p^2}{4} = \left(x + \frac{p}{2}\right)^2$$

$$r = x + \frac{p}{2}.$$

Fokusdan paraboladagi ixtiyoriy $M(x, y)$ nuqtagacha bo‘lgan r masofa

$$r = \frac{p}{2} + x \qquad (24)$$

formula yordamida aniqlanadi.

Quyidagilar *parabola elementlarini* tashkil etadi:

- O nuqta parabola uchi;
- $F\left(\frac{p}{2}; 0\right)$ – parabola fokusi;
- $x = -\frac{p}{2}$ – parabola direktrisasi tenglamasi;
- $\varepsilon = 1$ – parabola ekssentrisiteti;

• p fokal parametr (fokusdan direktrisagacha bo'lgan masofa yoki Ox o'qiga perpendikulyar ravishda fokusdan o'tgan vatar uzunligining yarmi).

Agar parabola Oy o'qiga nisbatan simmetrik bo'lsa, uning kanonik tenlamasi

$$x^2 = 2py \tag{25}$$

ko'rinishda bo'ladi. Uning fokusi Oy o'qidagi $F\left(0;\frac{p}{2}\right)$ nuqtada direktrisasi $y = -\frac{p}{2}$, fokusdan paraboladagi ixtiyoriy $M(x, y)$ nuqtagacha bo'lgan r masofa $r = y + \frac{p}{2}$ formula bilan aniqlanadi.

14-misol. Uchi koordinada boshida va Ox o'qqa nisbatan simmetrik bo'lgan parabola, A(9;6) nuqtadan o'tadi. Uning kanonik tenglamasini yozing.

Yechish. Misol shartiga ko'ra, parabola Ox o'qiga nisbatan simmetrik va uchi koordinata boshida, uning kanonik tenlamasi

$$y^2 = 2px$$

ko'rinishda bo'ladi. A(9;6) nuqta parabolaga tegishli bo'lgani uchun nuqta koordinatalarini parabola tenglamasiga qo'yamiz:

$$6^2 = 2p \cdot 9 \implies p = 2.$$

Parabolaning izlanayotgan kanonik tenglamasini yozamiz:

$$y^2 = 4x\,.$$

15-misol. $y^2 = 16x$ parabolaning fokusi koordinatalari va direktrisa tenglamasini toping.

Yechish. $2px = 16x \implies p = 8.$

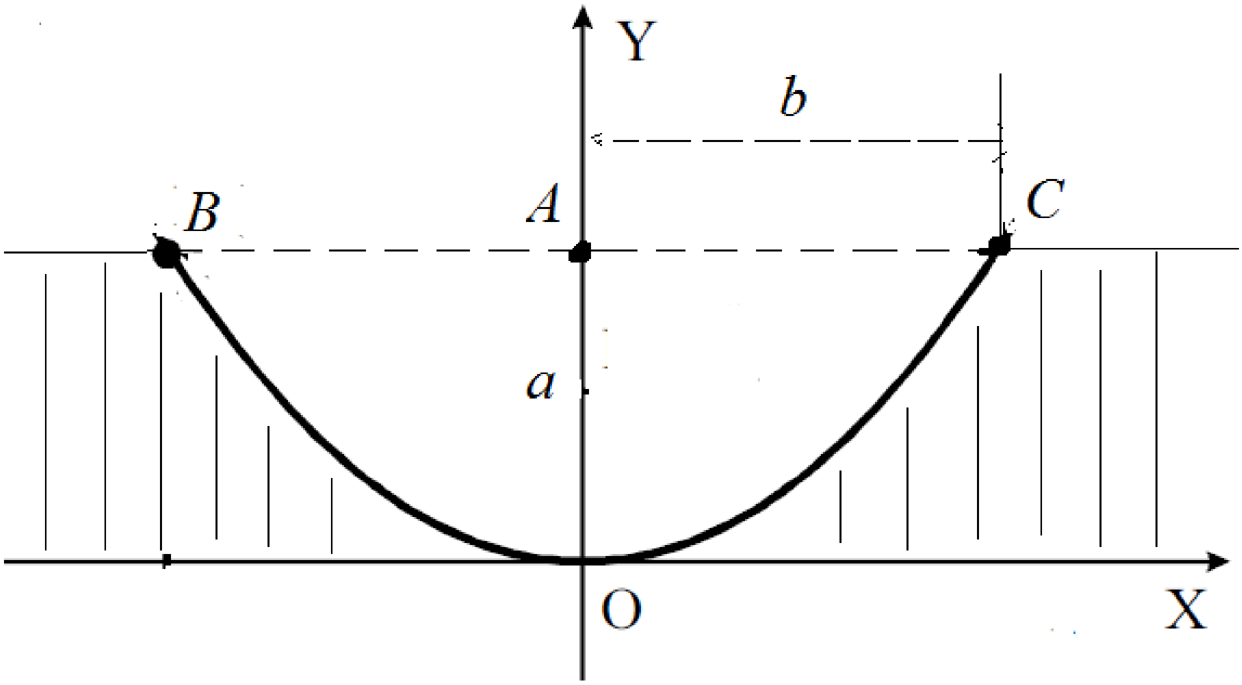
$$F\left(\frac{p}{2};0\right) \implies F\left(\frac{8}{2};0\right) \implies F(4;0).$$

Parabola direktrisasi tenglamasi:

$$x = -\frac{p}{2} \Rightarrow x = -\frac{8}{2} \Rightarrow x = -4.$$

16-misol. Kanalning ko‘ndalang kesimi parabola shakliga ega (18-rasm).

Agar $OA = a,\ BC = 2b$ bo‘lsa, rasmdagi ko‘rsatilgan o‘qlarga nisbatan uning tenglamasini yozing.



18-rasm.

Yechish. Kanal ko‘ndalang kesim chizig‘i Oy o‘qiga nisbatan simmetrik paraboladir. Uning tenglamasi $x^2 = 2py$ ko‘rinishda bo‘ladi. C nuqtaning koordinatalari $(b;\ a)$ dan iborat. C nuqta koordinatalarini parabola tenglamasiga qo‘yamiz:

$$b^2 = 2pa.$$

Bu tenglikdan p parametrni aniqlaymiz:

$$p = \frac{b^2}{2a}$$

p – parametrni parabola tenglamasiga qo'yamiz:

$$x^2 = 2 \cdot \frac{b^2}{2a} y,$$
$$x^2 = \frac{b^2}{a} y.$$

Hosil bo'lgan tenglama parabola shaklidagi kanal ko'ndalang kesim tenglamasidir.

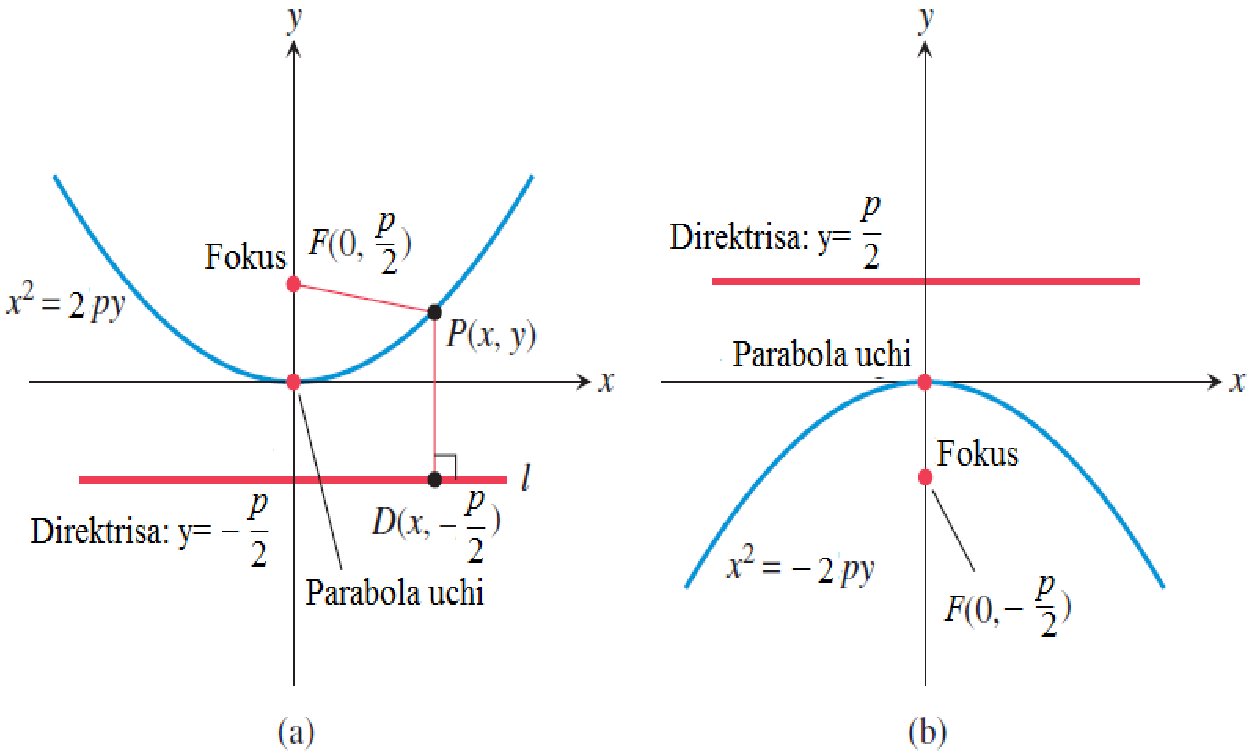
Parabolaning kanonik tenglamalari va ularning mos elementlarini 5,6-jadvallarda keltiramiz.

5-jadval

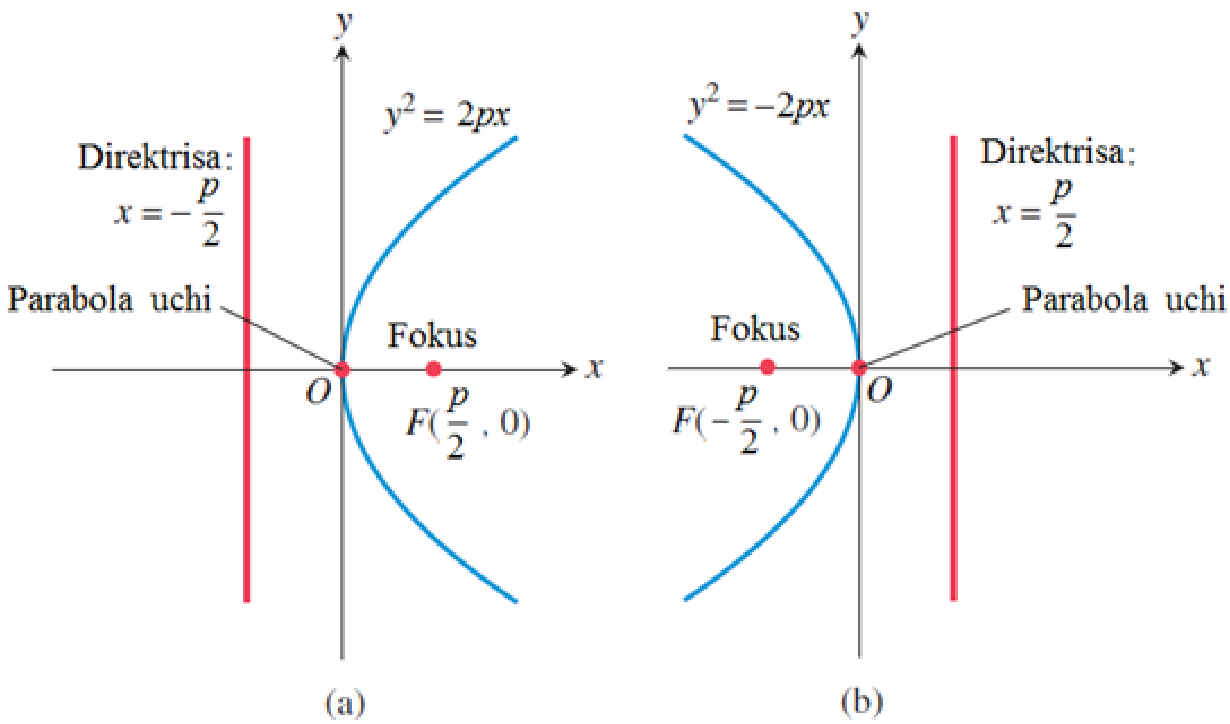
Uchi O(0, 0) nuqtada bo'lgan parabola				
Standart tenglamasi	$x^2 = 2py$	$x^2 = -2py$	$y^2 = 2px$	$y^2 = -2px$
Fokusi	$F(0, \frac{p}{2})$	$F(0, -\frac{p}{2})$	$F(\frac{p}{2}, 0)$	$F(-\frac{p}{2}, 0)$
Direktrisasi	$y = -\frac{p}{2}$	$y = \frac{p}{2}$	$x = -\frac{p}{2}$	$x = \frac{p}{2}$
Simmetriya o'qi	Oy	Oy	Ox	Ox
	19-rasm, (*a*)	19-rasm, (*b*)	20-rasm, (*a*)	20-rasm, (*b*)

6-jadval

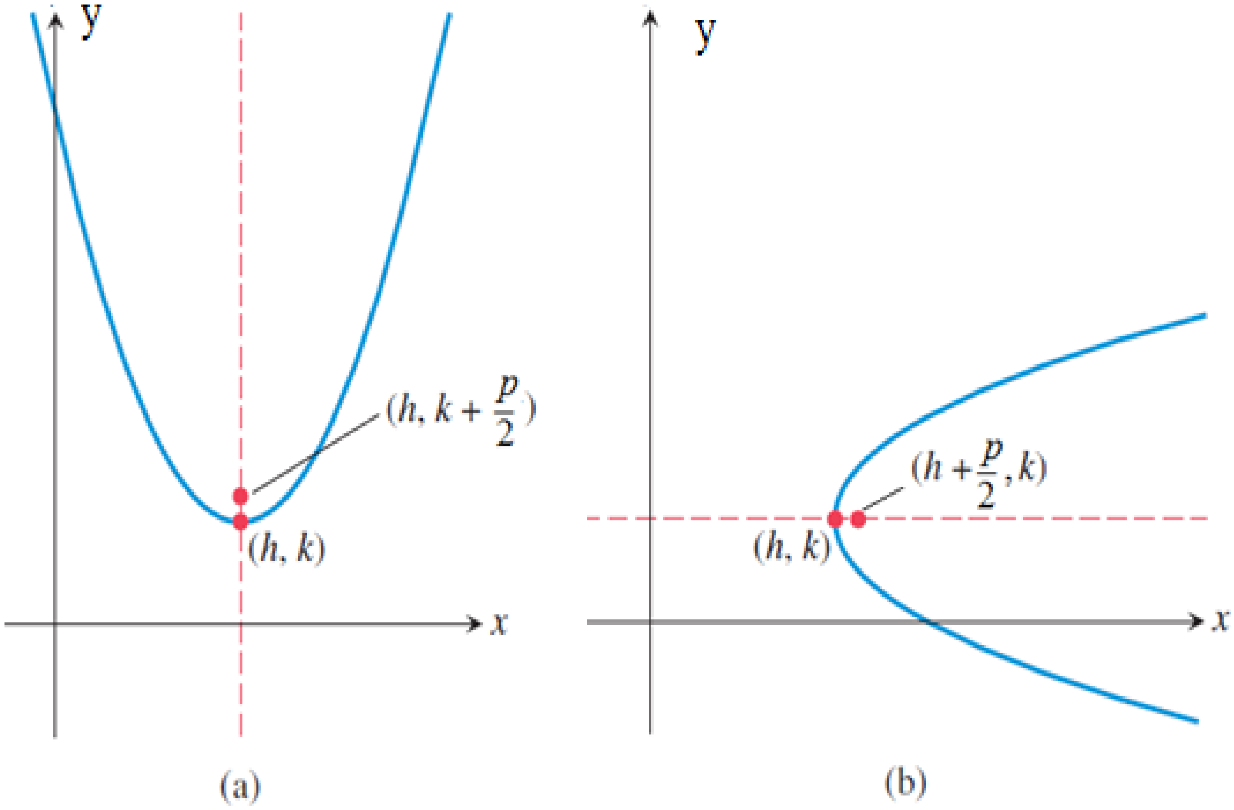
Uchi O(*h*, *k*) nuqtada bo'lgan parabola		
Tenglama ko'rinishi	$(x-h)^2 = 2p(y-k)$	$(y-k)^2 = 2p(x-h)$
Fokusi	$F(h, k + \frac{p}{2})$	$F(h + \frac{p}{2}, k)$
Direktrisasi	$y = k - \frac{p}{2}$	$x = h - \frac{p}{2}$
Simmetriya o'qi	$x = h$	$y = k$
	21-rasm, (*a*)	21-rasm, (*b*)



19-rasm.



20-rasm.



21-rasm.

17-misol. Uchi (3, 4) va fokusi (5, 4) nuqtada bo‘lgan parabolaning kanonik tenglamasini tuzing.

Yechish. Parabola uchi (3, 4) va fokusi (5, 4) nuqtada bo‘lganligi uchun, unung simmetriya o‘qi $y = 4$ to‘g‘ri chiziq bo‘ladi. Parabola kanonik tenglamasi:

$$(y-k)^2 = 2p(x-h).$$

Parabola uchi (3, 4) nuqtada, u holda $h = 3$ va $k = 4$. $F(5,\ 4)$ fokus koordinatasidan p parametrni aniqlaymiz:

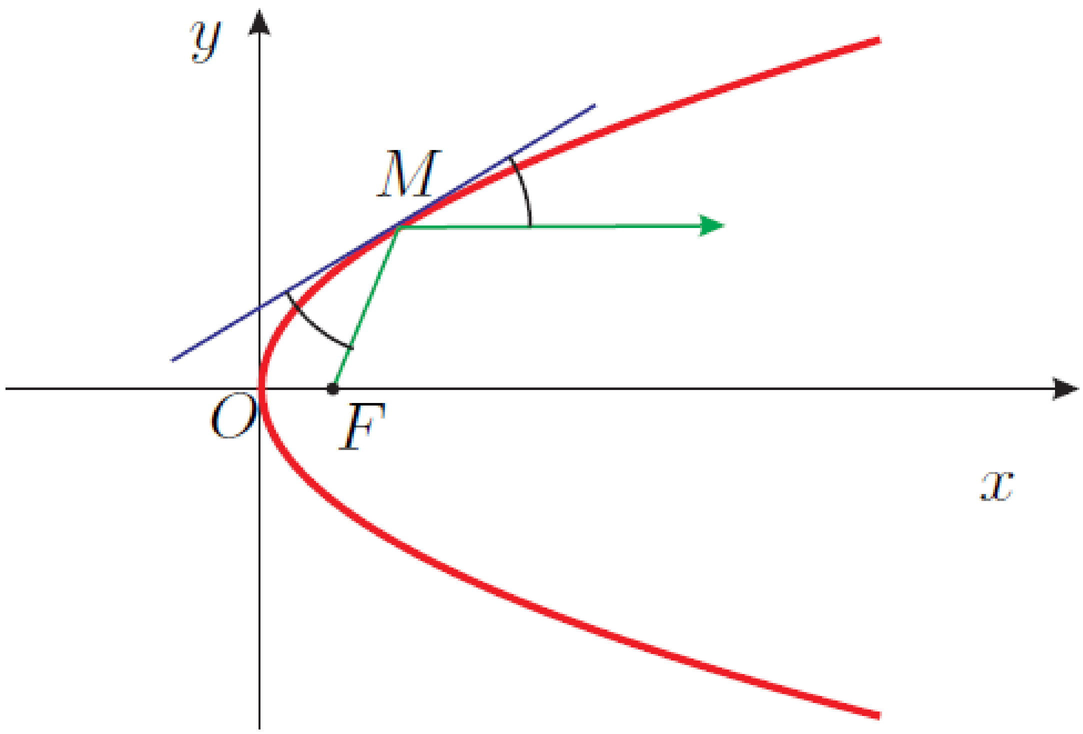
$$F\left(h+\frac{p}{2}, k\right),$$

$$h+\frac{p}{2} = 5 \quad \Longrightarrow \quad 3+\frac{p}{2} = 5 \quad \Longrightarrow \quad p = 4.$$

Demak, parabolaning kanonik tenglamasi:

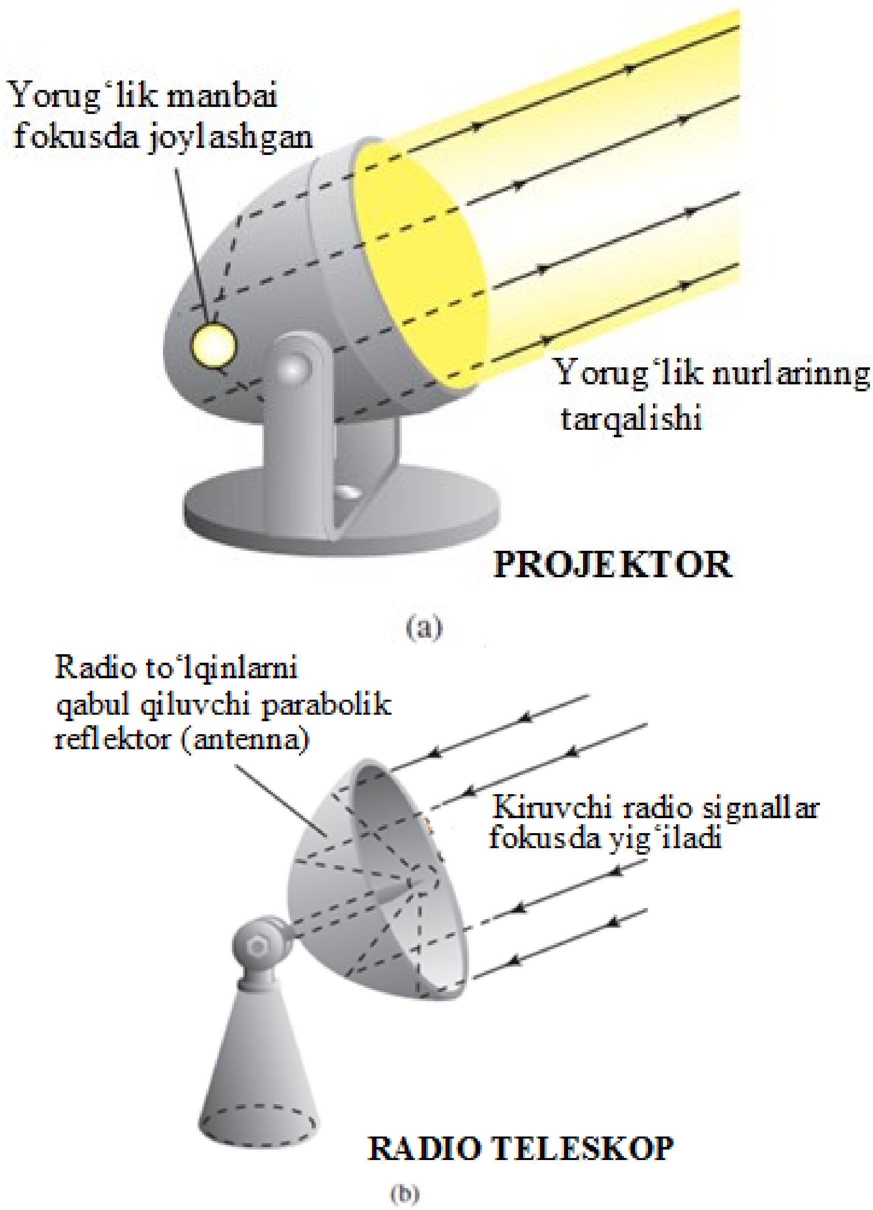
$$(y-4)^2 = 8(x-3).$$

Parabolaning optik xossasi. Parabolaning ixtiyoriy nuqtasiga o‘tkazilgan urinma fokal radius vektor va parabola simmetriya o‘qi bilan bir xil burchak tashkil etadi (22-rasm).



22-rasm.

Shu optik xossadan yorug‘lik, tovush, radiyo va boshqa elektromagnit to‘lqinlarni uzatish va qabul qilish uchun turli sohalarda qo‘llaniladi. Masalan, parabolik ko‘rinishdagi projektorning fokusida yorug‘lik manbai joylashtirilsa, fokusdan chiqgan nurlar paraboloid sirtiga urinadi va 23.a-rasmdagidek, yorug‘lik nurlari parabola simmetriya o‘qiga parallel akslanadi. Bu xossadan shuningdek, fonarlar, avtomobil faralari, projektorlar, mikroto‘lqinli relelar va sputnik antennalarda ham foydalaniladi (24-rasm). Radiyoteleskop va televizon sputnik antennalar yordamida parabolik reflektorning simmetriya o‘qiga parallel holda kiruvchi signallar reflektor fokusiga yo‘naltiriladi va signallar kuchaytiriladi (23.b, 24-rasmlar).



23-rasm.

24-rasm.

5-§. Ikkinchi tartibli egri chiziqlarning umumiy tenglamasi

Ikkinchi tartibli chiziq yoki egri chiziqning umumiy algebraik tenglamasi

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0 \qquad (26)$$

berilgan bo‘lsin. Bu yerda A, B, C, D, E, F – haqiqiy sonlar. Yuqorida o‘rgangan ikkinchi tartibli egri chiziqlar aylana, ellips, giperbola va parabolaning kanonik tenglamalari (26) tenglamaning xususiy holidir. Agar (26) tenglamada $B = 0$ teng bo‘lib, ikkinchi tartibli egri chiziqlarning simmetriya o‘qlari koordinata o‘qlariga parallel bo‘lsa, (26) tenglama koeffitsientlari asosida tekislikda ikkinchi tartibli egri chiziqlarning turini aniqlash mumkin:

1. Agar $A = C$ bo‘lsa, aylana.
2. Agar $A \cdot C > 0$ va $A \neq C$ bo‘lsa, ellips.
3. Agar $A \cdot C < 0$ bo‘lsa, giperbola.
4. Agar $A \cdot C = 0, A^2 + \mathrm{C}^2 \neq 0$ bo‘lsa, parabola.

Masalan:

1. $2x^2 + 5y^2 - 3x + 7y - 5 = 0$ – ellips tenglamasi, chunki $A = 2, C = 5, A \cdot C = 2 \cdot 5 = 10 > 0$.
2. $8x - 7y - 2x^2 - 2y^2 + 4 = 0$ –aylana, chunki $A = -2$, $C = -2$.
3. $x^2 - 6y^2 + 10x - 14y + 20 = 0$ – giperbola tenglamasi, chunki $A = 1, C = -6, A \cdot C = 1 \cdot (-6) = -6 < 0$.
4. $4x^2 - 12y + 16x - 7 = 0$ – parabola tenglamasi, chunki $A = 4, C = 0, A \cdot C = 0, A \neq 0, A^2 + C^2 = 4^2 + 0^2 \neq 0$.

Ikkinchi tartibli egri chiziqlarning umumiy tenglamasini kanonik ko‘rinishga keltirish masalasini o‘rganamiz. Bu masala koordinata o‘qlarini parallel ko‘chirish va koordinata o‘qlarini ma’lum α burchakka burish yordamida hal etiladi.

1. **Koordinata o‘qlarini parallel ko‘chirish**. Koordinata boshini O nuqtadan O_1 nuqtaga parallel ko‘chiramiz. Faraz qilamiz, Oxy koordinata sistemasida $P(x, y)$ nuqta berilgan bo‘lsin. Koordinata o‘qlarini parallel ko‘chirish natijasida P nuqta $O_1x'y'$ koordinata sistemasida x' va y' koordinatalarga ega bo‘ladi (25-rasm). Eski va yangi koordinatalar sistemasida $P(x, y)$ va $P(x', y')$

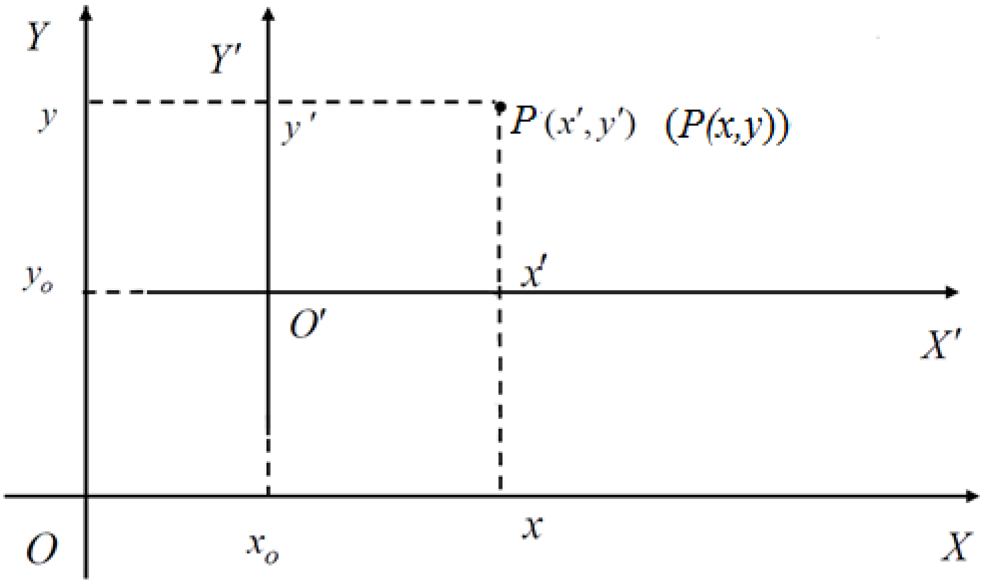
nuqta koordinatalari orasidagi bog‘liqlik quyidagi formulalar bilan ifodalanadi:

$$\begin{cases} x = x' + x_0, \\ y = y' + y_0, \end{cases} \tag{27}$$

yoki

$$\begin{cases} x' = x - x_0, \\ y' = y - y_0. \end{cases} \tag{28}$$

(27) va (28) formulalar koordinata o‘qlarini parallel ko‘chirish formulalari deyiladi.



25-rasm.

2. **Koordinata o‘qlarini burish.** Oxy Dekart koordinatalar sistemasi va $P(x, y)$ nuqta berilgan bo‘lsin. Koordinata o‘qlarini soat strelkasiga qarama-qarshi yo‘nalishda α burchakka buramiz. P nuqtaning koordinatalari $Ox'y'$ koordinatalar sistemasida x' va y' ga teng (33-rasm).

ΔSPD uchburchakda $\angle SPD = \alpha,\ OD = x',\ PD = y'$.

Ravshanki,

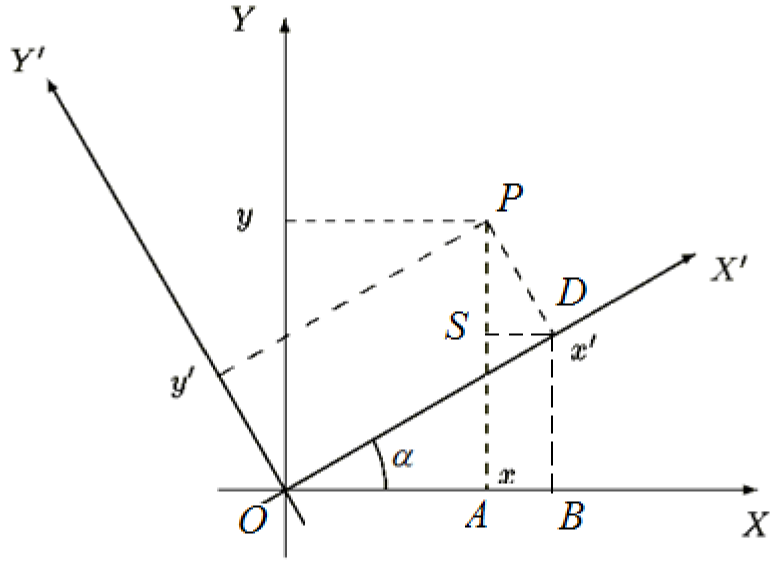
$$x = OA = OB - AB = OB - SD,$$
$$y = PA = AS + SP = DB + SP.$$

Ma’lumki,

$$OB = x' \cos\alpha, \qquad SD = y' \sin\alpha,$$
$$SP = y' \cos\alpha, \qquad DB = x' \sin\alpha.$$

Natijada

$$\begin{cases} x = x' \cos\alpha - y' \sin\alpha, \\ y = x' \sin\alpha + y' \cos\alpha. \end{cases} \tag{29}$$



26-rasm.

Bu formulalar ixtiyoriy P nuqtaning $(x;\ y)$ eski koordinatalarini koordinata o‘qlarini α burchakka burish natijasida hosil bo‘lgan yangi (x', y') koordinatalar orqali ifodasidir. P nuqtaning yangi $(\ x', y'\ )$ koordinatalarini eski $(x;\ y)$ koordinatalar bilan ifodalash uchun (29) formulada eski va yangi koordinatalar o‘rnini va α ni esa $(-\alpha)$ ga almashtirish kerak. Natijada

$$\begin{cases} x' = x\cos\alpha + y\sin\alpha \\ y' = -x\sin\alpha + y\cos\alpha \end{cases} \qquad (30)$$

formula hosil bo‘ladi. (30) formula koordinata o‘qlarini burish formulasi deyiladi.

18-misol. $2x^2 - 8x + 2y^2 + 4y - 62 = 0$ egri chiziq tupini aniqlang va tenglamani kanonik ko‘rinishga keltiring.

Yechish. x^2 va y^2 oldidagi koeffitsiyentlar teng, ya’ni $A = 2, C = 2, A = C$. Shu sababli berilgan tenglama aylana tenglamasidir.

Tenglamani kanonik ko‘rinishdagi tenglamaga keltirish uchun unda qatnashayotgan har bir o‘zgaruvchiga nisbatan to‘la kvadratni ajratamiz.

Maktab matematikasidan ma’lumki, ushbu ko‘rinishdagi

$$m^2 + 2mn + n^2 = (m+n)^2 \text{ va}$$

$$m^2 - 2mn + n^2 = (m-n)^2$$

ifodalar to‘la kvadrat deyiladi.

Berilgan tenglamadan x qatnashgan hadlarni olamiz va to‘la kvadratni ajratamiz:

$$2x^2 - 8x = 2(x^2 - 4x) = 2(x^2 - 2 \cdot x \cdot 2 + 2^2 - 2^2)$$
$$= 2((x-2)^2 - 4) = 2(x-2)^2 - 8.$$

Xuddi shuningdek, y qatnashgan hadlar uchun:

$$2y^2 + 4y = 2(y^2 + 2y) = 2(y^2 + 2 \cdot y \cdot 1 + 1^2 - 1^2)$$
$$= 2((y+1)^2 - 1) = 2(y+1)^2 - 2.$$

To‘la kvadrat qatnashgan ifodalarni berilgan tenglamaga qo‘yamiz:

$$2(x-2)^2 - 8 + 2(y+1)^2 - 2 - 62 = 0$$

Ozod hadlarni tenglikning o‘ng tomoniga o‘tkazamiz va tenglikning har ikki tomonini 2 ga bo‘lamiz:

$$(x-2)^2 + (y+1)^2 = 36.$$

Hosil bo‘lgan tenglama markazi $C(2;\ -1)$ nuqtada radiusi $R = 6$ ga teng bo‘lgan aylana tenglamasidir.

(28) formulaga ko‘ra $x' = x - 2,\ y' = y + 1$ belgilashlar kiritamiz.

Natijada $x'^2 + y'^2 = 36$ aylana tenglamasini hosil qilamiz.

19-misol. $x^2 + 8x + 9y^2 - 36y - 29 = 0$ tenglama bilan aniqlangan ikkinchi tartibli egri chiziq tipini aniqlang va tenglamani kanonik ko‘rinishga keltiring.

Yechish. $A = 1, C = 9, A \cdot C = 1 \cdot 9 > 0.\ A \neq C.$

Berilgan tenglama ellips tenglamasidir. Tenglamani kanonik ko‘rinishga keltirish uchun har bir o‘zgaruvchilar bo‘yicha to‘la kvadratlarni ajratamiz:

$$x^2 + 8x = x^2 + 2 \cdot x \cdot 4 + 4^2 - 4^2 = (x+4)^2 - 16.$$
$$9y^2 - 36y = 9(y^2 - 4y) = 9(y^2 - 2 \cdot y \cdot 2 + 2^2 - 2^2)$$
$$= 9(y-2)^2 - 36.$$

Hosil bo‘lgan ifodalarni tenglamaga qayta qo‘yamiz:

$$(x+4)^2 - 16 + 9(y-2)^2 - 36 - 29 = 0$$

Ozod hadlarni tenglikning o‘ng tomoniga o‘tkazamiz:

$$(x+4)^2 + 9(y-2)^2 = 81.$$

Tenglamani 81 ga bo‘lamiz:

$$\frac{(x+4)^2}{81} + \frac{(y-2)^2}{9} = 1.$$

Ma’lumki, $a = 9, b = 3.$

Tenglamada (3) formulaga ko‘ra

$$x' = x + 4;\ \ y' = y - 2$$

belgilashlar kiritamiz. Natijada ellipsning kanonik tenglamasi hosil bo‘ladi:

$$\frac{x'^2}{9^2}+\frac{y'^2}{3^2}=1.$$

20-misol. $9x^2-18x+16y^2+64y-71=0$ tenglama bilan aniqlangan ikkinchi tartibli egri chiziq tenglamasini kanonik ko‘rinishga keltiring.

Yechish. $A=9, C=16, A\cdot C=9\cdot 16>0.\ A\neq C.$

Berilgan tenglama ellips tenglamasidir. Tenglamani kanonik ko‘rinishga keltirish uchun har bir o‘zgaruvchilar bo‘yicha to‘la kvadratlarni ajratamiz:

$$9(x^2-2x+1)+16(y^2+4y+4)=71+9\cdot 1+16\cdot 4.$$

$$9(x-1)^2+16(y+2)^2=144.$$

Tenglikning o‘ng va chap tomonlarini 144 ga bo‘lamiz:

$$\frac{(x-1)^2}{16}+\frac{(y+2)^2}{9}=1.$$

Ma’lumki, $a=4, b=3$.

Tenglamada (28) formulaga ko‘ra

$$x'=x-1;\ \ y'=y+2$$

belgilashlar kiritamiz. Natijada ellipsning kanonik tenglamasi hosil bo‘ladi:

$$\frac{x'^2}{4^2}+\frac{y'^2}{3^2}=1.$$

21-misol. $x^2+8x-7y-5=0$ tenglama bilan egri chiziq tipini aniqlang va tenglamani kanonik ko‘rinishga keltiring.

Yechish. Tenglamada y^2 qatnashmaydi, demak, $A\cdot C=0$. Bu tenglama parabola tenglamasidir. Tenglamani kanonik ko‘rinishga keltirish uchun x o‘zgaruvchiga nisbatan to‘la kvadratni ajratamiz:

$$x^2+8x=x^2+2\cdot x\cdot 4+4^2-4^2=(x+4)^2-16.$$

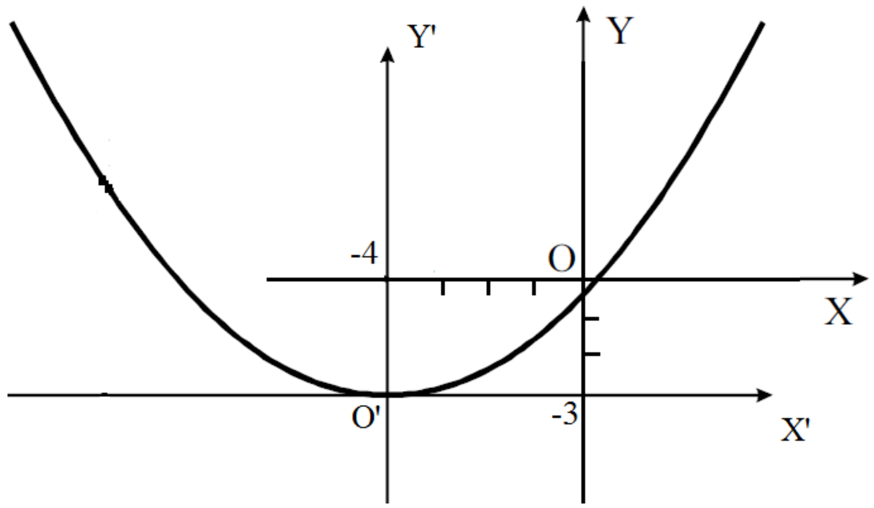
Bu ifodani tenglamaga olib borib qo‘yamiz:

$$(x+4)^2-16-7y-5=0,$$

$$(x+4)^2=7y+21,$$

$$(x+4)^2=7(y+3).$$

Hosil bo‘lgan tenglama $x = -4$ vertikal simmetriya o‘qiga ega bo‘lgan parabola tenglamasidir. Parabola uchi $(-4; -3)$ nuqtada joylashgan (27-rasm).



27-rasm.

(28) formulaga ko‘ra, $x' = x + 4,\ y' = y + 3$ almashtirishlar yordamida parabolaning kanonik tenglamasini hosil qilamiz:

$$x'^2 = 7y'.$$

22-misol. Teng tomonli $x^2 - y^2 = a^2$ giperbola grafigida koordinata o‘qlari $\alpha = 45°$ burchakka burildi. Yangi koordinatalar sistemasida giperbola tenglamasini yozing.

Yechish. Simmetriya o‘qlariga nisbatan teng tomonli

$$x^2 - y^2 = a^2$$

giperbola asimptotalari o‘zaro perpendikulyar. Eski koordinata o‘qlarini soat strelkasi yo‘nalishida $45°$ ga buramiz. (29) formulaga ko‘ra,

$$x = x' \cos\alpha - y' \sin\alpha,$$
$$y = x' \sin\alpha + y' \cos\alpha,$$

bu yerda $\alpha = -45°$ ga teng, u holda

$$x = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' + y'),$$
$$y = -\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(-x' + y').$$

x va y ning koordinatalarini $x^2 - y^2 = a^2$ tenglamaga qo‘yamiz:

$$\frac{1}{2}(x'+y')^2-\frac{1}{2}(-x'+y')^2=a^2$$

Qavslarni ochib, o‘xshash hadlarni soddalashtiramiz:

$$2x'y'=a^2, \text{ yoki } \quad x'y'=+\frac{a^2}{2}$$

Natijada $y'=+\frac{a^2}{2x'}$ tenglama hosil bo‘ladi. Agar $a=\sqrt{2k}$ (k – parametr) belgilash kiritamiz, u holda maktab o‘quvchilariga tanish bo‘lgan $y'=\frac{k}{x'}$ giperbola tenglamasi kelib chiqadi.

3. **Ikkinchi tartibli egri chiziq tenglamasi invariantlari.** Ikkinchi tartibli egri chiziqlar tupini aniqlashning yuqorida ko‘rilgan usuli xususiy hol bo‘lib, endi umumiy holni ko‘ramiz. (26) tenglama koeffitsiyentlaridan ushbu ifodani tuzamiz:

$$I=A+C \; - \text{ birinchi invariant,}$$

$$\delta=\begin{vmatrix} A & B \\ B & C \end{vmatrix} \; - \text{ ikkinchi invariant,}$$

$$\Delta=\begin{vmatrix} A & B & D \\ B & C & E \\ D & E & F \end{vmatrix} \; - \text{ uchinchi invariant.}$$

Δ - determinant **(26)** ***tenglamaning katta diskriminanti*** deyiladi, δ- yuqori tartibli hadlarining ***kichik diskriminanti*** deyiladi. δ va Δ qiymatlariga asosan (26) tenglama bilan quyidagi geometrik tasvirlarni ifodalashini aniqlash mumkin:

	$\Delta \neq 0$	$\Delta = 0$
$\delta > 0$	Ellips (haqiqiy va mavhum)	Nuqta
$\delta < 0$	Giperbola	Kesishuvchi ikkita to‘g‘ri chiziqlar
$\delta = 0$	Parabola	Parallel to‘g‘ri chiziqlar juftligi (haqiqiy yoki mavhum)

Agar $\delta \neq 0$ bo‘lsa, chiziq markazga ega bo‘ladi va u quyidagi tenglamalardan aniqlanadi:

$$Ax+By+D=0, \quad Bx+Cy+E=0 \tag{31}$$

Koordinata boshini $O'(x_0, y_0)$ markazga ko‘chiramiz va (26) tenglamani quyidagi ko‘rinishga keltiramiz:

$$Ax'^2 + 2Bx'y' + Cy'^2 + F_1 = 0, \qquad (32)$$

bu yerda

$$F_1 = Dx_0 + Ey_0 + F = \frac{\Delta}{\delta}. \qquad (33)$$

$O'x'$ va $O'y'$ o‘qlarni qandaydir α burchakka burish natijasida (32) tenglama kanonik ko‘rinishga keladi:

$$A_1X'^2 + C_1Y'^2 + F_1 = 0. \qquad (34)$$

A_1 va C_1 koeffitsiyentlar quyidagi xarakteristik tenglama ildizlaridir:

$$\lambda^2 - I\lambda + \delta = 0. \qquad (35)$$

α burilish burchagi quyidagi formuladan aniqlanadi:

$$tg\alpha = \frac{B}{A_1 - C}. \qquad (36)$$

23-misol. $x^2 + 2xy + y^2 + 2x + y = 0$ tenglama tipini aniqlang.

Yechish. $\delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2 = 1 \cdot 1 - 1^2 = 0.$

Tenglama parabolik tipga tegishlidir.

Katta diskriminant qiymatini aniqlaymiz:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 \end{vmatrix} = -\frac{1}{4}.$$

Demak, berilgan tenglama parabola tenglamasini ifodalaydi.

24-misol. $8x^2 + 24xy + y^2 - 56x + 18y - 55 = 0$ tenglama tipini aniqlang.

Yechish. $\delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2 = 8 \cdot 1 - 12^2 = -136.$

Tenglama giperbolik tipga tegishlidir.

Katta diskriminant qiymatini aniqlaymiz:

$$\Delta = \begin{vmatrix} 8 & 12 & -28 \\ 12 & 1 & 9 \\ -28 & 9 & -55 \end{vmatrix} = 0.$$

Demak, berilgan tenglama kesishuvchi ikkita to‘g‘ri chiziqlarni ifodalaydi.

25-misol. $2x^2 - 4xy + 5y^2 - x + 5y - 4 = 0$ tenglama tipini aniqlang.

Yechish. $\delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2 = 5 \cdot 2 - 2^2 = 6 > 0.$

Tenglama elliptik tipga tegishlidir.

Katta diskriminant qiymatini aniqlaymiz:

$$\Delta = \begin{vmatrix} 2 & -2 & -\frac{1}{2} \\ -2 & 5 & \frac{5}{2} \\ -\frac{1}{2} & \frac{5}{2} & -4 \end{vmatrix} \neq 0.$$

Demak, berilgan tenglama ellips tenglamasini ifodalaydi.

26-misol. $6xy + 8y^2 - 12x - 26y + 11 = 0$ tenglama tipini aniqlang va kanonik ko‘rinishga keltiring.

Yechish. $I = A + C = 0 + 8 = 8.$

$$\delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2 = 0 \cdot 8 - 3^2 = -9.$$

Tenglama giperbolik tipga tegishlidir.

$$\Delta = \begin{vmatrix} 0 & 3 & -6 \\ 3 & 8 & -13 \\ -6 & -13 & 11 \end{vmatrix} = 81 \neq 0.$$

Demak, berilgan tenglama giperbola tenglamasini ifodalaydi.

(35) tenglamaga binoan, xarakteristik tenglamani tuzamiz:

$$\lambda^2 - 8\lambda - 9 = 0.$$

Xarakteristik tenglama ildizlari: $\lambda_1 = 9,\ \lambda_2 = -1.$

(34) tenglamaga asosan,

$$9X'^2 - Y'^2 + \frac{81}{-9} = 0.$$

Bu tenglamani kanonik ko‘rinishga keltiramiz:

$$\frac{X'^2}{1} - \frac{Y'^2}{9} = 1.$$

Umumiy tenglamaning ikkinchi invarianti δ noldan farqli qiymatga ega, demak, ikkinchi tartibli egri chiziq markazga ega bo‘ladi va u (31) tenglamalardan aniqlanadi:

$$3y - 6 = 0,\ \ 3x + 8y - 13 = 0.$$

Bundan $x_0 = -1,\ y_0 = 2.$

Demak, markaz koordinatalari:

$$O'(-1, \quad 2).$$

6-§. Qutb koordinatalari sistemasi. Ikkinchi tartibli egri chiziqlarning qutb koordinatalar sistemasidagi tenglamasi

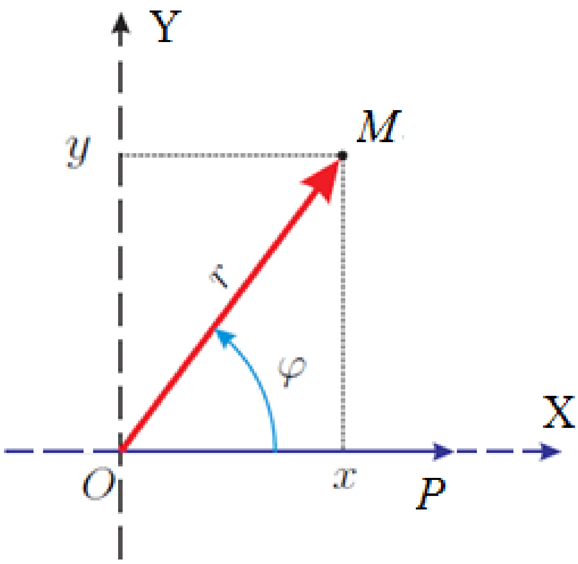
Fizika fanidan ma'lumki, tabiatda ikki xil harakatlar mavjud: ilgarilanma va aylanma. Ilgarilanma harakatni Dekart koordinatalar sistemasida ifodalash mumkin, ammo aylanma harakatni bu sistemada ifodalashda muammolar vujudga keladi.

XVII asrda de Sen-Vensan[5] va Kavaleri[6] bir-biridan bexabar holda qutb koordinatalar sistemasini kiritdi va aylanma harakatni tasvirlash muammosini hal etdilar.

Tekislikda nuqta o'rnini aniqlash uchun Dekart koordinatalar sistemasidan tashqari ko'p hollarda qutb koordinatalar sistemasi ham qo'llaniladi.

Ta'rif. Tekislikda ***qutb koordinatalar sistemasi*** deb, qutb va qutb o'qi bilan aniqlanadigan koordinata sistemasiga aytiladi.

Tekislikda O nuqta qutb va undan chiqqan OP – qutb o'qi berilgan bo'lsin. U holda tekislikda M nuqtaning koordinatalari $(\varphi; r)$ bo'ladi. Bu yerda $\varphi = \angle MOP$ – qutb burchagi, $r = |OM|$ – masofa - radius vektor (28-rasm).



28-rasm.

Agar to'g'ri burchakli Dekart koordinatalar sistemasi boshini qutb, Ox o'qi esa OP qutb o'qi deb qabul qilsak, u holda M

[5]Де Сен-Венсан Грегуар (1584-1667) – бельгиялик математик.
[6]Кавальери Бонавентура (1598-1647) – италиялик математик.

nuqtaning Dekart koordinatalar sistemasidagi (x, y) va qutb koordinatalar sistemasidagi $(\varphi; r)$ koordinatalari orasidagi bog‘liqlik quyidagicha bo‘ladi:

$$\begin{cases} x = r\cos\varphi, \\ y = r\sin\varphi, \end{cases} \tag{37}$$

$$r = \sqrt{x^2 + y^2},\ \mathrm{tg}\varphi = \frac{y}{x}. \tag{38}$$

27-misol. Dekart koordinatalar sistemasida berilgan $M(1; -1)$ nuqtaning qutb koordinatalar sistemasidagi koordinatalarini aniqlang.

Yechish. Shartga ko‘ra, $x = 1, y = -1$.

(38) formulaga asosan: $r = \sqrt{1^2 + (-1)^2} = \sqrt{2},\ \mathrm{tg}\varphi = -1.$ φ ning $\frac{3}{4}\pi$ va $-\frac{\pi}{4}$ qiymatlaridan $\varphi = -\frac{\pi}{4}$ qiymatni olamiz, chunki $\sin\varphi$ bu qiymatda manfiy bo‘lishi kerak.

Demak, qutb koordinatalar sistemasida M nuqtaning koordinatalari $\varphi = -\frac{\pi}{4}$ va $r = \sqrt{2}$ ga teng.

28-misol. Dekart koordinatalar sistemasida berilgan $(x - R)^2 + y^2 = R^2$ aylana tenglamasini qutb koordinatalar sistemasida yozing.

Yechish. Tenglamadagi qavsni ochib chiqamiz:

$$x^2 - 2Rx + R^2 + y^2 - R^2 = 0 \iff x^2 + y^2 - 2Rx = 0.$$

(37) formulaga ko‘ra x va y ning qiymatini hosil bo‘lgan tenglamaga qo‘yamiz:

$$(r\cos\varphi)^2 + (r\sin\varphi)^2 - 2Rr\cos\varphi = 0\,,$$

$$r^2 - 2Rr\cos\varphi = 0.$$

Tenglikni r $(r \neq 0)$ ga bo‘lamiz va natijada aylananing qutb koordinatalar sistemasidagi tenglamasi kelib chiqadi:

$$r = 2R\cos\varphi.$$

Agar ellips, giperbola va parabolaning fokusini qutb va fokal simmetriya o‘qini qutb o‘qi sifatida qabul qilsak, u holda bu egri chiziqlarning qutb koordinatalar sistemasidagi tenglamasi bir xil bo‘ladi:

$$r = \frac{p}{1 - \varepsilon\cos\varphi} \tag{39}$$

bu yerda ε – ekssentrisitet, p – parametr. Ellips va giperbola uchun $p = \frac{b^2}{a}$ ga teng bo‘ladi.

29-misol. Qutb koordinatalar sistemasida ikkinchi tartibli egri chiziq $r=\frac{9}{5-4\cos\varphi}$ tenglama bilan berilgan. Dekart koordinatalar sistemasidagi kanonik tenglamasini yozing.

Yechish. Tenglamaning chap tomonini (39) ko‘rinishga keltirish uchun surat va maxrajini 5 ga bo‘lamiz:

$$r=\frac{\frac{9}{5}}{1-\frac{4}{5}\cos\varphi}.$$

Bu yerda $\varepsilon=\frac{4}{5}<1$. Demak, egri chiziq ellipsdir.

$$p=\frac{b^2}{a}=\frac{9}{5},\qquad \varepsilon=\frac{4}{5}=\frac{c}{a},\qquad b^2=\frac{9}{5}a,\qquad c=\frac{4}{5}a.$$

b^2 va c^2 ning a orqali ifodasini $c^2=a^2-b^2$ tenglikka qo‘yib, a ga nisbatan tenglama hosil qilamiz. Bu tengliklardan $a=5,\ b=3$ qiymatlar aniqlanadi va ularni ellipsning kanonik tenglamasiga qo‘yamiz:

$$\frac{x^2}{5^2}+\frac{y^2}{3^2}=1.$$

Parabola, ellips va giperbolaning fokusini qutb va fokal simmetriya o‘qini qutb o‘qi sifatida qabul qilib, ularning qutb koordinatalar sistemasidagi tenglamasini keltirib chiqaramiz. Bu yerda ellips va giperbola uchun bitta fokusi, masalan chap fokusi olinadi (29-31-rasmlar).

F – fokus, FP – qutb o‘qi bo‘lsin. $FM_0=p$ belgilash kiritaylik va p – fokal parametr deyiladi.

Faraz qilaylik, $M(r,\varphi)$ – egri chiziqda ixtiyoriy nuqta bo‘lsin (29-rasm). Parabola, ellips va giperbola xossalariga ko‘ra,

$$\varepsilon=\frac{FM}{NM}=\frac{r}{NM}. \tag{40}$$

ΔFKM to‘g‘ri burchakli uchburchak bo‘lgani uchun

$$\sin(90^\circ-\varphi)=\frac{KM}{MF}\Longrightarrow KM=r\cos\varphi. \tag{41}$$

$$NM=NK+KM=N_0M_0+r\cos\varphi \tag{42}$$

Ravshanki, $\frac{FM_0}{N_0M_0}=\varepsilon$, bu yerda $FM_0=p$. U holda $N_0M_0=\frac{p}{\varepsilon}$.

$$\text{Natijada } NM=\frac{p}{\varepsilon}+r\cos\varphi. \tag{43}$$

(43) tenglikni (40) ga olib borib qo‘yamiz:

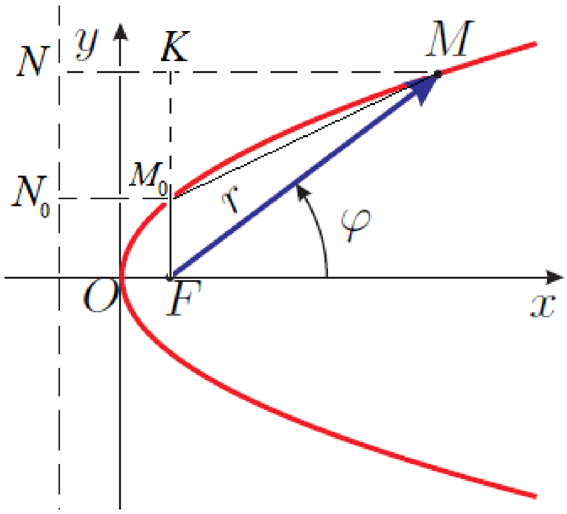
$$r = \varepsilon \cdot NM = p + r\varepsilon \cos\varphi \iff r = \frac{p}{1 - \varepsilon \cos\varphi}.$$

Demak,

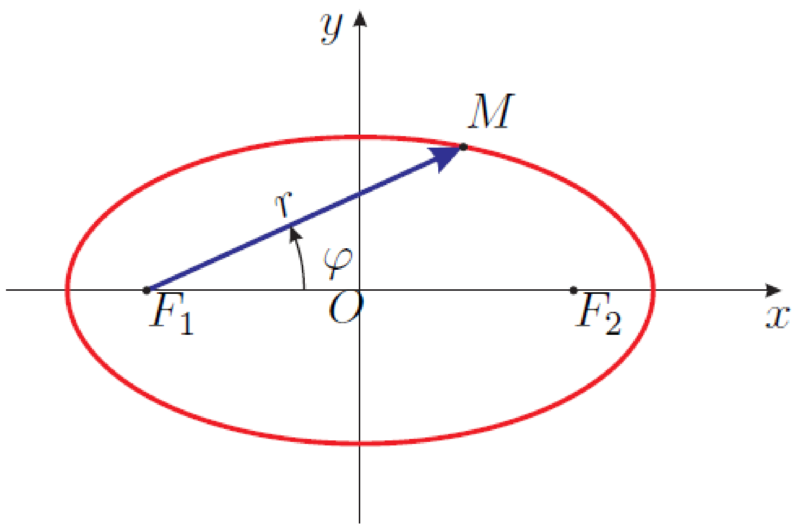
$$r = \frac{p}{1-\varepsilon \cos\varphi} \tag{44}$$

Agar $\varepsilon < 1$ bo‘lsa, ellips; $\varepsilon = 1$ – parabola; $\varepsilon > 1$ – giperbola tenglamasini beradi.

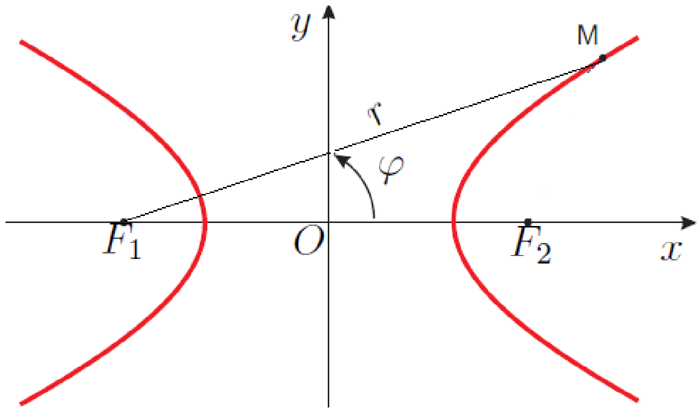
(44) tenglamadagi p parametr parabola uchun o‘zining avvalgi qiymatiga, ya’ni fokusdan direktrisagacha bo‘lgan masofaga teng. Ellips va giperbola uchun p parametrning qiymati a va b yarim o‘qlar bilan ifodalanadi: $p = \frac{b^2}{a}$.



29-rasm.



30-rasm.



31-rasm.

7-§. Ikkinchi tartibli egri chiziqlarga o‘tkazilgan urinmalar tenglamasi

Ikkinchi tartibli egri chiziq (parabola, ellips, giperbola) grafigiga o‘tkazilgan urinma tenglamasini keltirib chiqaramiz.

1. ***Parabolaga o‘tkazilgan urinma tenglamasi.*** Parabola simmetriya o‘qiga parallel bo‘lmagan va parabola bilan bitta umumiy nuqtaga ega to‘g‘ri chiziq parabolaga urinma bo‘ladi.

Faraz qilaylik, (x_0, y_0) nuqta $y^2 = 2px$ parabola va $\frac{x-x_0}{n} = = \frac{y-y_0}{m} = t, m \neq 0$ to‘g‘ri chiziqning kesishish nuqtasi bo‘lsin.

To‘g‘ri chiziq tenglamasini parametrik tenglamaga keltiramiz:

$$x = nt + x_0, \ y = mt + y_0.$$

Ifodalarni parabola kanonik tenglamasiga qo‘yamiz:

$$(y_0 + mt)^2 = 2p(x_0 + nt) \ \Leftrightarrow y_0{}^2 + 2mty_0 + m^2t^2$$
$$= 2px_0 + 2pnt \ \Leftrightarrow m^2t^2 + 2t(my_0 - pn) = 0.$$

Agar $my_0 - pn = 0$ bo‘lsa, hosil bo‘lgan kvadrat tenglama bitta ildizga ega bo‘ladi.

$$my_0 - pn = 0 \ \Leftrightarrow n = \frac{my_0}{p}.$$

Oxirgi tenglikni parabola bilan (x_0, y_0) nuqtada kesishuvchi to‘g‘ri chiziq tenglamasiga qo‘yamiz:

$$\frac{x - x_0}{\frac{my_0}{p}} = \frac{y - y_0}{m} \ \Leftrightarrow p(x - x_0) = y_0(y - y_0) \ \Leftrightarrow$$

$$y_0y - y_0{}^2 = px - px_0.$$

${y_0}^2 = 2px_0$ bo‘lgani uchun tenglikni quyidagi ko‘rinishda yozamiz:

$$y_0 y - 2px_0 = px - px_0.$$

Natijada tenglama

$$yy_0 = p(x + x_0) \tag{45}$$

ko‘rinishga keladi. (45) tenglama ***parabolaning urinma tenglamasi*** deyiladi.

2. **Ellips va giperbolaga o‘tkazilgan urinma tenglamalari.** Faraz qilaylik, (x_0, y_0) nuqta $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellips va $\frac{x-x_0}{n} = \frac{y-y_0}{m}$ to‘g‘ri chiziqning urinma nuqtasi bo‘lsin. To‘g‘ri chiziqning kanonik tenglamasini parametrik tenglamaga keltiramiz:

$$x = x_0 + nt, \qquad y = y_0 + nt.$$

x va y ning t parametr orqali ifodasini ellips tenglamasiga qo‘yamiz va kvadratga ko‘taramiz:

$$\frac{{x_0}^2}{a^2} + \frac{{y_0}^2}{b^2} + 2t\left(\frac{nx_0}{a^2} + \frac{my_0}{b^2}\right) + t^2\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right) = 1,$$

$$t^2\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right) + 2t\left(\frac{x_0 n}{a^2} + \frac{y_0 m}{b^2}\right) = 0.$$

Agar $\frac{x_0 n}{a^2} + \frac{y_0 m}{b^2} = 0$ bo‘lsa, kvadrat tenglama bitta ildizga ega bo‘ladi. Bu shart bajarilishi uchun $n = \frac{y_0}{b^2}, m = -\frac{x_0}{a^2}$ bo‘lishi kerak. n va m ning qiymatlarini to‘g‘ri chiziqning kanonik tenglamasiga qo‘yamiz:

$$\frac{x - x_0}{y_0/b^2} = \frac{y - y_0}{-x_0/a^2} \Leftrightarrow \frac{x_0}{a^2}(x - x_0) + \frac{y_0}{b^2}(y - y_0) = 0,$$

qavslarni ochib chiqamiz va $\frac{{x_0}^2}{a^2} + \frac{{y_0}^2}{b^2} = 1$ tenglikni hisobga olgan holda

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1 \tag{46}$$

tenglamani hosil qilamiz.

(46) tenglama ***ellipsning urinma tenglamasi*** deyiladi.

Xuddi shuningdek,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

giperbolaga $(x_0; y_0)$ nuqtada

$$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1 \qquad (47)$$

urinma tenglamasini keltirib chiqarish mumkin.

(47) tenglama ***giperbolaning urinma tenglamasi*** deyiladi.

Mustaqil ishlash uchun misollar

1. Aylana diametrlaridan birining uchlari $M_1(2;-7)$ va $M_2(-4;-3)$ nuqtalarda yotishi ma'lum. Aylana tenglamasini yozing.

2. Quyidagi aylanalarning radiuslari va markazlarining koordinatalarini aniqlang:

$a)$ $x^2 + y^2 + 2x - 6y + 5 = 0;$ $b)$ $x^2 + y^2 + 4x - 5 = 0;$

$d)$ $x^2 - 5 + y^2 + 4y = 0;$ $e)$ $4x^2 - 5x + 4y^2 - 8 = 0.$

3. Koordinata o'qlariga urinadigan aylana $M(-2;-4)$ nuqtadan o'tadi. Uning tenglamasini yozing.

4. Quyidagi ellipslarning uchlari koordinatalarini, yarim o'qlarini, fokuslarini va ekssentrisitetini toping:

1) $16x^2 + 25y^2 = 400;$ 2) $4x^2 + 9y^2 = 36;$

3) $4x^2 + 6y^2 = 192;$ 4) $9x^2 + 7y^2 = 63.$

5. Ellipsning tenglamasi berilgan: $9x^2 + 25y^2 = 225$. Uning abssissasi 3 bo'lgan nuqtasining radius-vektorlarini aniqlang.

6. Haqiqiy o'qi 6 ga, fokuslari orasidagi masofa 8 ga teng bo'lgan giperbolaning eng sodda tenglamasini tuzing. Qo'shma giperbolaning tenglamasini tuzing.

7. Fokusi $\frac{x^2}{49} + \frac{y^2}{24} = 1$ ellipsning fokusi bilan umumiy bo'lgan va ekssentrisiteti $\varepsilon = 1{,}25$ ga teng giperbolaning tenglamasini tuzing.

8. Quyidagilarga asoslanib, parabolaning tenglamasini tuzing:

1) uchidan fokusigacha bo'lgan masofa 3 ga teng;

2) fokusining koordinatasi – (5;0), direktrisasi – ordinatalar o'qi;

3) $M(1;-4)$ nuqtadan o'tuvchi *Ox* o'qiga simmetrik bo'lgan parabola;

4) fokusi (0;2) da, O*y* o'qiga simmetrik va uchi koordinata boshida bo'lgan parabola;

5) koordinatalar boshidan va $M(6;-2)$ nuqtadan o'tuvchi, *Oy* o'qiga simmetrik bo'lgan parabola.

9. Parabolik ko‘zguning diametri 120 sm ga, botiqligi 15 sm ga teng. Yorug‘lik manbayini parabola uchidan qanday masofada joylashtirilganda qaytgan nur parabola o‘qiga parallel bo‘ladi?

10. Katta yarim o‘qi 2 ga teng, direktrisalari esa $x = \pm\frac{4}{\sqrt{3}}$ to‘g‘ri chiziqlardan iborat ellipsning kanonik tenglamasini yozing.

11. Giperbolaning tenglamasi $9x^2 - 16y^2 = 144$. Uning abssissasi 8 bo‘lgan nuqtasining radius-vektorlarini aniqlang.

12. Koordinata o‘qlariga urinuvchi aylana markazi $2x - y + 3 = 0$ to‘g‘ri chiziqda yotadi. Aylana tenglamasini yozing.

13. $x^2 + y^2 - 4x + 6y - 5 = 0$ aylananing Ox o‘q bilan kesishgan nuqtalariga o‘tkazilgan radiuslari o‘rtasidagi burchaklarni toping.

14. Ellipsning tenglamasi berilgan: $7x^2 + 18y^2 = 126$. Uning absissasi 3 bo‘lgan va ordinatasi musbat bo‘lgan nuqtasining radius-vektorlari orasidagi burchakni toping.

15. Mavhum o‘qi $2\sqrt{2}$ ga teng bo‘lgan giperbola direktrisalarining tenglamalari: $x \pm 2 = 0$. Giperbolaning tenglamasini tuzing.

16. Ko‘prik arkasi tenglamasi $y^2 = 96x$ bo‘lgan parabola ko‘rinishiga ega. Agar balandligi 6 m ga teng bo‘lsa, arka vatarining uzunligini toping.

VI BOB. VEKTORLAR ALGEBRASI

1-§. Fazoda koordinatalar sistemasi

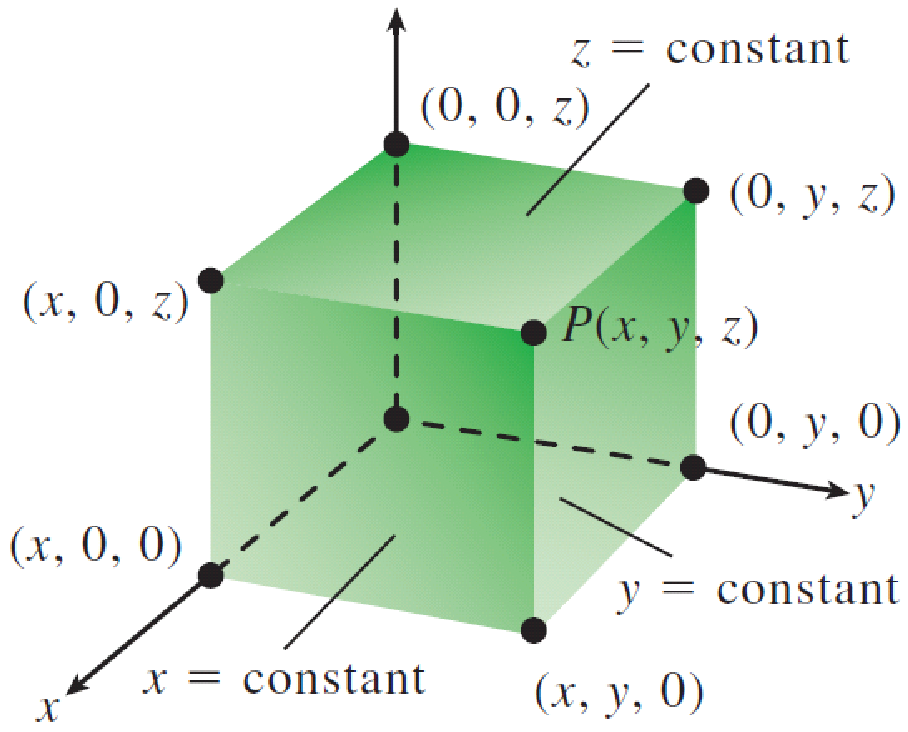
Fazodagi biror jismning o‘rnini aniqlash uchun xuddi tekislikdagi kabi Dekart koordinatalar sistemasini kiritamiz. Fazodagi biror O nuqta va shu nuqta kesishuvchi o‘zaro perpendikulyar bo‘lgan uchta Ox, Oy, Oz to‘g‘ri chiziqlar sistemasi berilgan bo‘lsin. Bu sistemaga fazoda Dekart koordinatalar sistemasi deyiladi va $Oxyz$ kabi belgilanadi. Ox – abssissalar o‘qi, Oy – ordinatalar o‘qi, Oz – applikatalar o‘qi deyiladi. Ox, Oy, Oz koordinatalar o‘qlari kesishgan O nuqta koordinatalar boshi deyiladi.

Jismning joylashgan o‘rnini $P(x, y, z)$ orqali belgilaylik. P nuqtaning x, y, z koordinatalari uning mos ravishda Ox, Oy va Oz o‘qlariga proeksiyalaridir. Koordinatalar o‘qlarida $(x, 0,0)$, $(0, y, 0)$, $(0, 0, z)$ qiymatlarni belgilaymiz. Koordinata o‘qlaridagi shu nuqtalardan o‘zaro perpendikulyar bo‘lgan ikkita koordinata o‘qlariga parallel bo‘lgan to‘g‘ri chiziqlar chiqaramiz. Bu to‘g‘ri chiziqlar $(x, y, 0)$, $(x, 0, z)$ va $(0, y, z)$ nuqtalarda kesishadi. $(x, y, 0)$ nuqtadan Oz o‘qiga, $(x, 0, z)$ nuqtadan Oy o‘qiga va $(0, y, z)$ nuqtadan Ox o‘qiga parallel bo‘lgan to‘g‘ri chiziqlarni yo‘naltiramiz. Yo‘naltirilgan chiziqlar o‘zaro perpendikulyar bo‘lgani uchun bitta nuqtada kesishadi. Kesishish nuqtasi fazoda $P(x, y, z)$ nuqtani ifodalaydi (1-rasm).

Geometrik va fizik miqdorlarning ko‘p qiymatlari qandaydir son orqali ifodalanadi.

Yo‘nalishga ega bo‘lmagan va tanlangan sistemada o‘zining sonli qiymatini to‘liq tasvirlaydigan miqdorga ***skalyar miqdor*** deyiladi. Masalan, zichlik, hajm, harorat va hokazo. Shu sababli, ba’zi hollarda sonlarni skalyar deyiladi. Shunday qilib, skalyar – bu qandaydir sondir.

Boshqa geometrik va fizik miqdorlar son va yo‘nalish bilan ifodalanadi. Masalan, biror nuqtaga qo‘yilgan kuch, moddiy nuqtaning tezligi va hokazo. Bunday miqdorlarga sodda holda to‘g‘ri chiziqdagi yo‘naltirilgan kesmani misol sifatida keltirish mumkin.



1-rasm.

2-§. Vektor tushunchasi

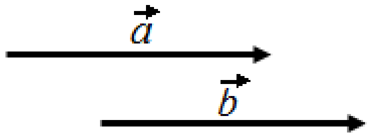
Yo‘naltirilgan kesma ***vektor*** *d*eyiladi va $\vec{a}=\overrightarrow{AB}$ kabi belgilanadi. $\overrightarrow{AB}$ vektorning A nuqtasi uning boshlang‘ich, B esa oxirgi nuqtasi deyiladi. AB kesmaning uzunligi (moduli) ***vektorning uzunligi*** deyilib, $|\vec{a}|=|\overrightarrow{AB}|$ kabi belgilinadi.

Vektor orqali, masalan, tezlik, tezlanish, kuch, momentlar va boshqa yo‘nalishga ega bo‘lgan miqdorlarni ifodalash mumkin.

Boshlang‘ich va oxirgi nuqtalari ustma-ust tushgan vektor ***nol vektor*** deyiladi va $\vec{0}$ vektor bilan belgilanadi.

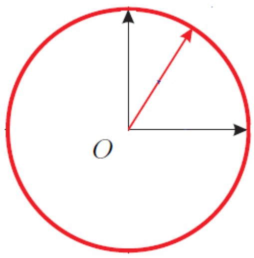
Agar vektor uzunligi birga teng bo‘lsa, uni ***birlik vektor*** yoki ***ort*** deyiladi.

O‘zaro parallel, bir tomonga yo‘nalgan va uzunliklari teng bo‘lgan $\vec{a}$ va $\vec{b}$ vektorlar ***teng vektorlar*** deyiladi va $\vec{a}=\vec{b}$ kabi belgilanadi (1-rasm).

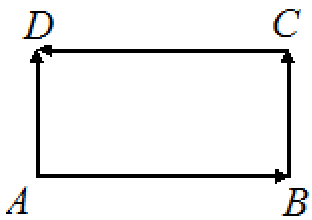


1-rasm.

Shunday qilib, bitta aylananing markazidan chiquvchi turli radius vektorlarning uzunliklari teng, ammo yo‘nalishlari har xil bo‘lganligi uchun ular teng vektorlar emas (2-rasm).



2-rasm.



3-rasm

Shuningdek, 3-rasmdagi $\overrightarrow{AB}$ va $\overrightarrow{CD}$ vektorlar ham o‘zaro teng emas. $\overrightarrow{AD}$ va $\overrightarrow{BC}$ vektorlar teng: $\overrightarrow{AD} = \overrightarrow{BC}$.

Tekislikdagi $A(x_0, y_0)$ va $B(x_1,\ y_1)$, (fazodagi $A(x_0, y_0, z_0)$, $B(x_1,\ y_1, z_1)$ nuqtalar uchun $(x_1 - x_0;\ y_1 - y_0)$ $\langle(x_1 - x_0;\ y_1 - y_0;\ z_1 - z_0)\rangle$ qiymatlar $\overrightarrow{AB}$ ***vektorning koordinatalari*** deyiladi:

$$\overrightarrow{AB} = (x_1 - x_0;\ y_1 - y_0)$$

$$\langle\overrightarrow{AB} = (x_1 - x_0;\ y_1 - y_0;\ z_1 - z_0)\rangle.$$

Vektorning koordinatalari soni uning ***o‘lchovi*** deyiladi. Masalan, $\vec{a}(\alpha;\ \beta)$ – 2-o‘lchovli vektor; $\vec{a}(\alpha;\ \beta;\ \gamma)$ – 3-o‘lchovli vektor; $\vec{a}(\alpha_1;\ \alpha_2;\ \ldots,\ \alpha_n)$ – n-o‘lchovli vektor.

Faraz qilaylik, fermer xo‘jaligi n ta turdagi qishloq xo‘jalik mahsuloti etishtiradi. Jumladan, 1-turdagi mahsulot x_1- tonna, 2-turdagi mahsulot x_2- tonna, va hokazo. Bu holda fermer xo‘jaligining ishlab chiqarish (rejasi) dasturini $(x_1;\ x_2;\ \ldots,\ x_n)$ – n-o‘lchovli vektor sifatida qarash mumkin.

1-misol. A(-2; 4; 1), B(4; 2; 7) nuqtalar berilgan. $\overrightarrow{AB}$ vektor koordinatalarini aniqlang va vektor modulini toping.

Yechish. $\overrightarrow{AB} = (4 - (-2);\ 2 - 4;\ 7 - 1) = (6;\ -2;\ 6)$.

Ta’rifga binoan, vektor moduli (uzunligi) uning boshlang‘ich va oxirgi nuqtalari orasidagi masofaga teng:

$$|\overrightarrow{AB}| = \sqrt{6^2 + (-2)^2 + 6^2} = \sqrt{76} = 2\sqrt{19}.$$

Bitta to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqda yotgan $\vec{a}$ va $\vec{b}$ vektorlar ***kollinear vektorlar*** deyiladi:

$\vec{a} \uparrow\uparrow \vec{b}$ – bir yo‘nalishli vektorlar;

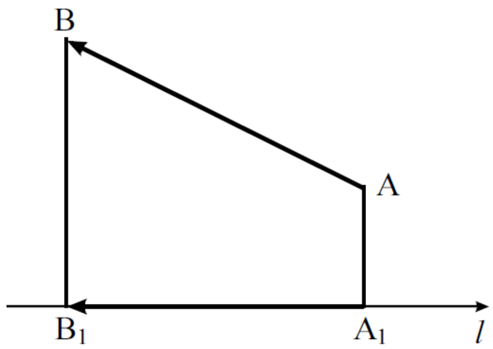
$\vec{a} \downarrow\uparrow \vec{b}$ – qarama-qarshi yo‘nalishli vektorlar;

$\vec{a} \parallel \vec{b}$ – umumiy holda, parallel vektorlar (o‘zaro yo‘nalishi ko‘rsatilmagan hol).

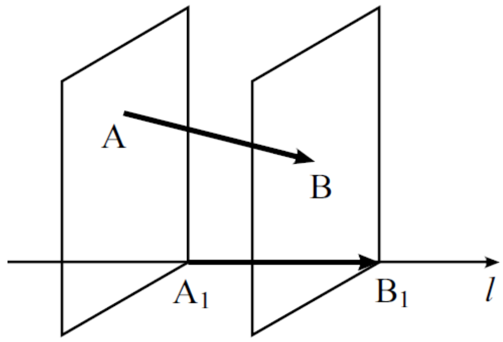
Bitta tekislikka parallel bo‘lgan uchta va undan ortiq vektorlar to‘plami ***komplanar vektorlar*** deyiladi. Jumladan, bitta tekislikda yotgan vektorlar komplanardir.

3-§. Vektorning o‘qdagi proeksiyasi

$\overrightarrow{AB}$ vektor l o‘q berilgan bo‘lsin (5-rasm). A va B nuqtadan l o‘qqa perpendikulyar tushiramiz. Bu nuqtalarning l o‘qdagi proeksiyasini mos ravishda A_1 va B_1 orqali ifodalaymiz.



4-rasm.



5-rasm

A_1B_1 kesmaning uzunligi $\vec{a}$ vektorning l chiziqdagi ***proeksiyasi*** deyiladi va $pr_l\vec{a} = pr_l\overrightarrow{AB}$ kabi belgilanadi. Agar A_1B_1 vektorning yo‘nalishi l ning yo‘nalishi bilan bir xil bo‘lsa, $\vec{a}$ vektorning l o‘qdagi proeksiyasi A_1B_1 ning uzunligiga teng: $pr_l\vec{a} = |A_1B_1|$ (5-rasm), aks holda $pr_l\vec{a} = -|A_1B_1|$ (4-rasm).

$\vec{a}$ vektorni parallel ko‘chiramiz va parallel ko‘chirish natijasida uning boshlang‘ich nuqtasi A_1 nuqta bilan ustma-ust tushsin. U holda $\vec{a}$ vektor l to‘g‘ri chiziq bilan α burchak hosil bo‘lib, bu burchak vektorning l to‘g‘ri chiziqqa nisbatan ***og‘ish burchagi*** deyiladi.

$\vec{a}$ vektorning l o‘qdagi proeksiyasini quyidagicha ifodalash mumkin:

$$pr_l\vec{a} = |\vec{a}| \cdot cos\alpha \qquad (1)$$

$\vec{a}=(a_x,\ a_y,\ a_z)$ vektor koordinatalari bilan berilgan bo‘lsin (6-rasm). Agar α, β, γ lar mos ravishda $\vec{a}$ vektorning Ox, Oy, Oz o‘qlariga nisbatan og‘ish burchaklari bo‘lsa, u holda

$$a_x=|\vec{a}| \cdot cos\alpha, \qquad a_y=|\vec{a}| \cdot cos\beta, \quad a_z=|\vec{a}| \cdot cos\gamma \qquad (2)$$

tengliklar o‘rinlidir. $cos\alpha$, $cos\beta$, $cos\gamma$ - $\vec{a}$ ***vektorning yo‘naltiruvchi kosinuslari*** deyiladi:

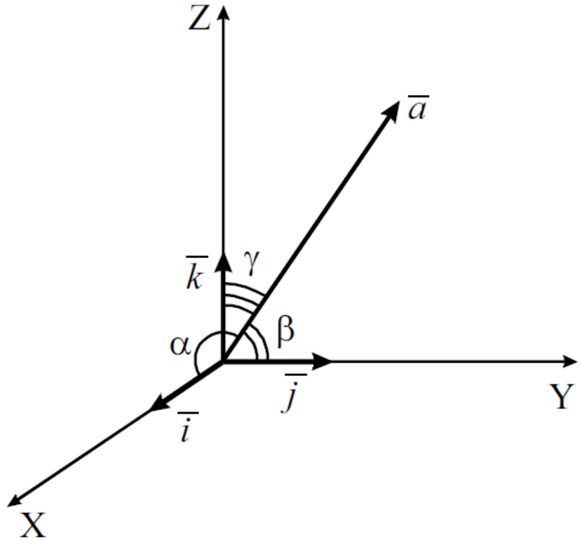
$$cos^2\alpha + cos^2\beta + cos^2\gamma = 1.$$

Proeksiya xossalarini keltiramiz:

1) $pr_{\vec{b}}\vec{a} = |\vec{a}| \cdot \cos\varphi$, bu yerda $\varphi = \widehat{\vec{a}\vec{b}}$;

2) $pr_{\vec{c}}(\lambda\vec{a}+\mu\vec{b}) = \lambda\, pr_{\vec{c}}\vec{a} + \mu\, pr_{\vec{c}}\vec{b}$, λ va μ - ixtiyoriy sonlar;

3) teng vektorlarning proeksiyasi ham teng bo‘ladi.

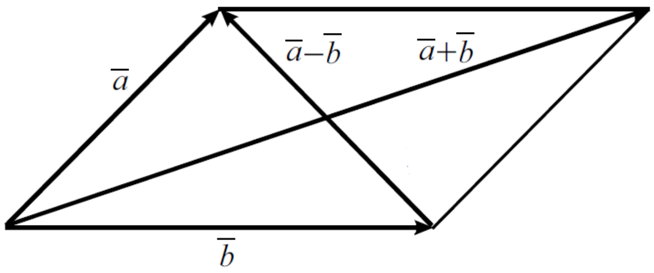


6-rasm.

4-§. Vektorlar ustida arifmetik amallar

n-o‘lchovli $\vec{a}=(\alpha_1;\ \alpha_2;\ \ldots;\alpha_n)$ va $\vec{b}=(\ \beta_1;\ \beta_2;\ \ldots;\ \beta_n)$ ***vektorlar yig‘indisi*** deb, mos koordinatalari yig‘indisidan tashkil topgan $\vec{a}+\vec{b}=(\ \alpha_1+\ \beta_1;\ \alpha_2+\ \beta_2;\ \ldots;\ \alpha_n+\ \beta_n)$ – n-o‘lchovli vektorga aytiladi.

$\vec{a}\,(\alpha_1;\ \alpha_2;\ \alpha_3)$ va $\vec{b}\,(\ \beta_1;\ \beta_2;\ \beta_3)$ vektorlar berilgan bo‘lsin. Vektorlar yig‘indisi va ayirmasini topish uchun “parallelogramm qoidasi” dan foydalanish mumkin (7-rasm). Parallelogramm tomonlari $\vec{a}$ va $\vec{b}$ vektorlardan iborat bo‘lsa, vektorlar yig‘indisi va ayirmasi mos ravishda parallelogramm diagonallarini tashkil etadi.



7-rasm.

Vektorlar yig‘indisini quyidagi masala yordamida tushuntiramiz. Faraz qilaylik, agrofirma tarkibida 2 ta fermer xo‘jaligi mavjud. 1-fermer xo‘jaligi joriy yilda 400 tonna paxta, 210 tonna g‘alla, 70 tonna sabzavot mahsulotlari va 30 tonna poliz mahsulotlari yetishtiradi. 2-fermer xo‘jaligi esa 320 tonna paxta, 230 tonna g‘alla, 80 tonna sabzavot mahsulotlari va 50 tonna poliz mahsulotlari yetishtiradi. 1-fermer xo‘jaligining joriy yildagi qishloq xo‘jaligi mahsulotlari ishlab chiqarish hajmini $\vec{a}$=(400; 210; 70; 30) vektor, 2-fermer xo‘jaligining qishloq xo‘jaligi mahsulotlari ishlab chiqarish hajmi $\vec{b}$=(320; 230; 80; 50) vektor yordamida ifodalash mumkin. U holda agrofirmaning yillik qishloq xo‘jalik mahsulotlari ishlab chiqarish hajmi $\vec{a}$ va $\vec{b}$ vektorlar yig‘indisidan iborat bo‘ladi:

$$\vec{a} + \vec{b}=(720; 440; 150; 80).$$

$(-\alpha_1; -\alpha_2; ...; -\alpha_n)$ vektor $\vec{a}=(\alpha_1; \alpha_2; ...; \alpha_n)$ vektorga qarama-qarshi vektor deyiladi va $-\vec{a}$ kabi belgilanadi (8-rasm). Demak,

$$\vec{a}+(-\vec{a})=0.$$

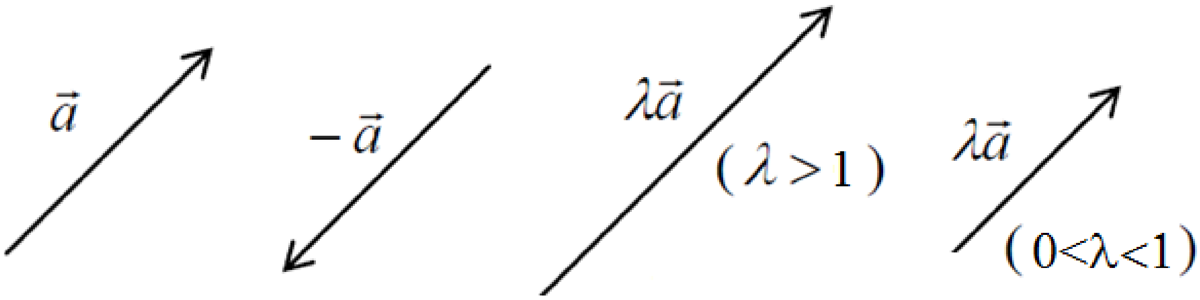
Ikki vektorning ayirmasi qarama-qarshi vektorlar yig‘indisi orqali aniqlanadi:

$$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$$

$\vec{a}=(\alpha_1; \alpha_2; ...; \alpha_n)$ vektorning λ songa ko‘paytmasi quyidagicha aniqlanadi:

$$\lambda\vec{a}=(\lambda\alpha_1; \lambda\alpha_2; \ldots; \lambda\alpha_n). \qquad (3)$$

Agar $\lambda > 0$ bo‘lsa, $\lambda\vec{a}$ vektorning yo‘nalishi $\vec{a}$ vektorning yo‘nalishi bilan ustma-ust tushadi (8-rasm), aks holda, ya’ni $\lambda < 0$ bo‘lsa, $\vec{a}$ vektor yo‘nalishiga qarama-qarshi bo‘ladi.



8-rasm.

Agar yuqoridagi keltirilgan masala bo‘yicha 1-fermer xo‘jaligi ishlab chiqarish hajmini 2 martaga oshirsa, u holda yillik ishlab chiqarishning yangi rejasi vektor ko‘rinishida quyidagicha ifodalanadi:

$$2\vec{a}=(800; 420; 140; 60).$$

2-misol. Agar $\vec{a} = (1;\ -2;\ 3)$ va $\vec{b} = (-5;\ 3;\ -1)$ vektorlar berilgan bo‘lsa, $\vec{a} + 2\vec{b}$ vektor uzunligini toping.

Yechish. $\vec{c} = \vec{a} + 2\vec{b}$ belgilash kiritamiz, $2\vec{b} = (-10;\ 6;\ -2)$. $\vec{c} = (1 - 10;\ -2 + 6;\ 3 - 2) = (-9;\ 4;\ 1)$.

$$|\vec{c}| = \sqrt{(-9)^2 + 4^2 + 1^2} = \sqrt{98} = 7\sqrt{2}.$$

Vektorlar ustida kiritilgan amallarga nisbatan quyidagi xossalar o‘rinli:

1°. $\vec{a} + \vec{b}=\vec{b} + \vec{a}$ (kommutativlik xossasi).

2°. $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$ (assotsiativlik xossasi).

3°. $\vec{a}+0=\vec{a}$.

4°. $\alpha(\vec{a} + \vec{b})= \alpha\vec{a}+\alpha\vec{b}$ $(\alpha+\beta)\,\vec{a}=\alpha\vec{a}+\beta\vec{a}$ (distributivlik xossasi).

5°. $\alpha\cdot(\beta\cdot \vec{a})=\alpha \cdot \beta \cdot \vec{a}$.

Uchta $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ vektorlar uchun quyidagi shartlar bajarilsa, ular ***koordinata bazislari*** deyiladi:

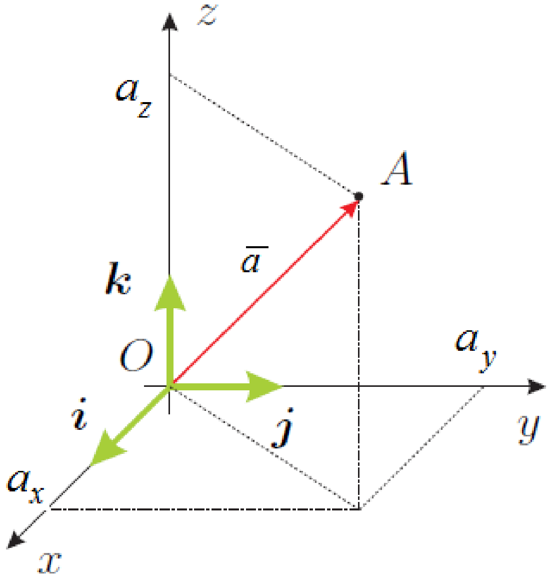
1) $\boldsymbol{i}$ vektor Ox o‘qida, $\boldsymbol{j}$ vektor Oy o‘qida, $\boldsymbol{k}$ vektor Oz o‘qida yotsa;

2) har bir $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ vektorlar o‘z o‘qida musbat tomonga yo‘nalgan bo‘lsa;

3) $|\vec{i}| = 1,\ |\vec{j}| = 1$ va $|\vec{k}| = 1$, ya'ni ular birlik vektorlar bo'lsa.

Ixtiyoriy $\vec{a} = (a_x; a_y; a_z)$ vektorni $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ bazis bo'yicha yoyish mumkin (9-rasm), ya'ni

$$\vec{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}. \qquad (4)$$



9-rasm.

3-misol. $A(3;\ -2;\ 5)$, $B(1;\ 4;\ 8)$ nuqtalar mos ravishda $\overrightarrow{AB}$ vektorning boshlang'ich va oxirgi nuqtalari bo'lsin. $\overrightarrow{AB}$ vektorning $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ bazis bo'yicha yoyilmasini va yo'naltiruvchi kosinuslarini toping.

Yechish. $\overrightarrow{AB}$ vektorning koordinatalarini aniqlaymiz:

$$\overrightarrow{AB} = (1 - 3; 4 - (-2); 8 - 5) = (-2; 6; 3).$$

(4) formulaga ko'ra, $\overrightarrow{AB}$ vektorning $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ bazis bo'yicha yoyilmasi:

$$\overrightarrow{AB} = -2\boldsymbol{i} + 6\boldsymbol{j} + 3\boldsymbol{k}.$$

3-paragrafdagi (2) formuladan foydalanib, yo'naltiruvchi kosinuslar qiymatini topamiz:

$$cos\alpha = \frac{a_x}{|\vec{a}|} = \frac{-2}{\sqrt{4 + 36 + 9}} = -\frac{2}{7};$$

$$cos\beta = \frac{a_y}{|\vec{a}|} = \frac{6}{7}; \quad cos\gamma = \frac{a_z}{|\vec{a}|} = \frac{3}{7}.$$

Ta'rif. Kamida bittasi noldan farqli bo'lgan shunday $\lambda_1, \lambda_2, \ldots, \lambda_k$ sonlar mavjud bo'lib, ular uchun

$$\lambda_1\boldsymbol{a_1} + \lambda_2\boldsymbol{a_2} + \cdots + \lambda_k\boldsymbol{a_k} = 0$$

tenglik o‘rinli bo‘lsa, $\boldsymbol{a_1}, \boldsymbol{a_2}, \dots, \boldsymbol{a_k}$ vektorlar sistemasi ***chiziqli bog‘liq*** deyiladi.

$$b = \lambda_1 \boldsymbol{a_1} + \lambda_2 \boldsymbol{a_2} + \dots + \lambda_k \boldsymbol{a_k}$$

vektor $\boldsymbol{a_1}, \boldsymbol{a_2}, \dots, \boldsymbol{a_k}$ vektorlarning ***chiziqli kombinatsiyasi*** deyiladi.

Vektorlarning chiziqli kombinatsiyasi faqat $\lambda_1 = \lambda_2 = \dots = = \lambda_k = 0$ bo‘lgandagina nolga teng bo‘lsa, u holda $\boldsymbol{a_1}, \boldsymbol{a_2}, \dots, \boldsymbol{a_k}$ vektorlar sistemasi ***chiziqli bog‘liqmas*** (yoki ***chiziqli erkli***) deyiladi.

Teorema. $\boldsymbol{a_1}, \boldsymbol{a_2}, \dots, \boldsymbol{a_k}$ vektorlar sistemasi chiziqli bog‘liq bo‘lishi uchun shu vektorlar sistemasidagi kamida bitta vektor qolgan vektorlarning chiziqli kombinatsiyasiga teng bo‘lishi zarur va yetarlidir.

4-misol. $\vec{a} = (3; -2;\ 5)$ va $\vec{b} = (4;\ 1;\ -2)$ vektorlarning $\lambda_1 = 2$, $\lambda_2 = 3$ koeffitsiyentlar bilan chiziqli kombinatsiyasini toping.

Yechish. Ta’rifga binoan, $\vec{c} = \lambda_1 \vec{a} + \lambda_2 \vec{b}$ vektorni topish mumkin.

1) $\lambda_1 \vec{a} = 2 \cdot (3; -2; 5) = (2 \cdot 3; 2 \cdot (-2); 2 \cdot 5) = (6;\ -4; 10)$.

2) $\lambda_2 \vec{b} = 3 \cdot (4; 1; -2) = (3 \cdot 4; 3 \cdot 1; 3 \cdot (-2)) = (12;\ 3; -6)$.

3) $\lambda_1 \vec{a} + \lambda_2 \vec{b} = (6;\ -4; 10) + (12;\ 3; -6) = (18;\ -1; 4)$.

$$\vec{c} = \lambda_1 \vec{a} + \lambda_2 \vec{b} = (18;\ -1;\ 4).$$

5-misol. $\vec{a} = (3; 1)$ va $\vec{b} = (-1; 4)$ vektorlarning chiziqli bog‘liqmasligini isbotlang.

Yechish. Ta’rifga binoan, $\lambda_1 \vec{a} + \lambda_2 \vec{b} = 0$ vektorli tenglikni hosil qilamiz. Chiziqli kombinatsiyaning λ_1 va λ_2 koeffitsiyentlarini aniqlash uchun vektorlar o‘rniga ularning koordinatalarini qo‘yamiz:

$$\lambda_1 (3; 1) + \lambda_2 (-1; 4) = 0.$$

Tenglikning chap qismida λ_1 va λ_2 koeffitsiyentlarini mos vektorlarga ko‘paytib, ularning yig‘indisini topamiz:

$$(3\lambda_1 - \lambda_2;\ \lambda_1 + 4\lambda_2) = (0; 0).$$

Vektorlar teng bo‘lishi uchun ularning mos koordinatalari teng bo‘lishi kerak. Shu sababli

$$\begin{cases} 3\lambda_1 - \lambda_2 = 0, \\ \lambda_1 + 4\lambda_2 = 0. \end{cases}$$

Sistemadagi ikkinchi tenglamadan λ_1 ning λ_2 orqali ifodasini topamiz $\lambda_1 = -4\lambda_2$ va bu ifodani ikkinchi tenglama qo‘yamiz:

$$3 \cdot (-4\lambda_2) - \lambda_2 = 0,$$
$$-12\lambda_2 - \lambda_2 = 0.$$

$\lambda_2 = 0$ bo‘lganligidan $\lambda_1 = 0$ kelib chiqadi.

Shunday qilib, vektorlarning chiziqli kombinatsiyasi faqat $\lambda_1 = 0$ va $\lambda_2 = 0$ qiymatlarda nolga teng bo’ladi.

Demak, $\vec{a}$ va $\vec{b}$ chiziqli bog‘liqmas vektorlardir.

5-§. Ikki vektorning skalyar ko‘paytmasi

Vektorlarning bu operatsiyasi bizga boshlang‘ish maktabdan ma’lum. Talabalar ham har yili yangi o‘quv yili boshlanishida o‘quv qurollari sotib olishda, bevosita ushbu ko‘paytmadan foydalanadi. Kundalik mahsulotlar xarid qilishimizda ham ushbu vektorlar operatsiyasidan foydalanamiz.

Talaba yangi o‘quv yili boshlanishidan avval, o‘quv jarayoni uchun kerakli o‘quv qurollarini sotib oladi. Masalan, talaba do‘kondan daftar, qalam va ruchka sotib oldi. Ushbu jarayonda talaba ikkita vektordan foydalandi: $\vec{a}$=(7; 2; 3) – miqdor vektori (7 ta daftar; 2 ta qalam; 3 ta ruchka), $\vec{b}$=(2000; 300; 1000) – narx vektori (2000 - daftar narxi; 300 – qalam narxi; 1000 – ruchka narxi). Sotuvchiga borishdan avval, talaba sotib olayotgan o‘quv qurollarining umimiy narxini mustaqil xotirada hisoblaydi:

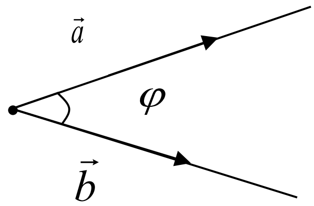
$$7 \cdot 2000 + 2 \cdot 300 + 3 \cdot 1000 = 17600.$$

Xotirada hisoblangan 17600 so‘m vektorlar amali, ya’ni ikki vektorning skalyar ko‘paytmasini tashkil qiladi.

Demak, vektorlarning skalyar ko‘paytmasi, o‘zimiz bilmagan holda, iqtisodiyotda eng ko‘p ishlatiladigan matematik operatsiyalardan biridir.

$\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasi deb, shu vektorlar uzunliklarining ular orasidagi φ burchak kosinusi ko‘paytmasiga aytiladi va quyidagicha belgilanadi:

$$\vec{a} \cdot \vec{b}=|\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi. \qquad (5)$$



(5) formulaga asosan, o‘zaro perpendikulyar bo‘lgan birlik vektorlarning skalyar ko‘paytmasini hisoblaymiz:

$$\boldsymbol{i} \cdot \boldsymbol{i} = |\boldsymbol{i}| \cdot |\boldsymbol{i}| \cdot \cos 0^0 = 1 \cdot 1 \cdot 1 = 1;$$
$$\boldsymbol{j} \cdot \boldsymbol{j} = |\boldsymbol{j}| \cdot |\boldsymbol{j}| \cdot \cos 0^0 = 1; \quad \boldsymbol{k} \cdot \boldsymbol{k} = |\boldsymbol{k}| \cdot |\boldsymbol{k}| \cdot \cos 0^0 = 1;$$
$$\boldsymbol{i} \cdot \boldsymbol{j} = |\boldsymbol{i}| \cdot |\boldsymbol{j}| \cdot \cos 90^0 = 1 \cdot 1 \cdot 0 = 0; \; \boldsymbol{i} \cdot \boldsymbol{k} = 0; \; \boldsymbol{j} \cdot \boldsymbol{k} = 0.$$

Koordinatalari bilan berilgan $\vec{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$ va $\vec{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}$ vektorlarning skalyar ko‘paytmasini topamiz:

$$\vec{a} \cdot \vec{b} = (a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}) \cdot (b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}) =$$
$$= a_x b_x(\boldsymbol{i} \cdot \boldsymbol{i}) + a_x b_y(\boldsymbol{i} \cdot \boldsymbol{j}) + a_x b_z(\boldsymbol{i} \cdot \boldsymbol{k}) +$$
$$+a_y b_x(\boldsymbol{j} \cdot \boldsymbol{i}) + a_y b_y(\boldsymbol{j} \cdot \boldsymbol{j}) + a_y b_z(\boldsymbol{j} \cdot \boldsymbol{k}) +$$
$$+a_z b_x(\boldsymbol{k} \cdot \boldsymbol{i}) + a_z b_y(\boldsymbol{k} \cdot \boldsymbol{j}) + a_z b_z(\boldsymbol{k} \cdot \boldsymbol{k}) =$$
$$= a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z,$$

Demak, $\vec{a}\,(a_x;\; a_y;\; a_z)$, $\vec{b}\,(b_x;\; b_y;\; b_z)$ vektorlarning skalyar ko‘paytmasini ularning koordinatalari orqali quyidagi formula yordamida aniqlash mumkin:

$$\vec{a} \cdot \vec{b} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z. \qquad (6)$$

Tomonlari a, b va c ga teng bo‘lgan uchburchak (10-rasm) uchun kosinuslar teoremasini keltiramiz:

$$a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\varphi = c^2\,. \qquad (7)$$

Agar uchburchak tomonlari uzunliklarini mos vektorlar uzunliklariga $a = |\vec{a}|, b = |\vec{b}|, c = |\vec{a} - \vec{b}|$ teng bo‘lsa, (7) formulaga ko‘ra

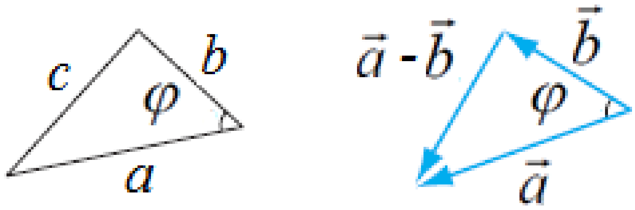
$$|\vec{a}|^2 + |\vec{b}|^2 - 2 \cdot |\vec{a}| \cdot |\vec{b}| \cdot cos\varphi = |\vec{a} - \vec{b}|^2.$$

Tenglikning o‘ng tomonini kvadratga oshiramiz:

$$|\vec{a}|^2 + |\vec{b}|^2 - 2 \cdot |\vec{a}| \cdot |\vec{b}| \cdot cos\varphi = |\vec{a}|^2 + |\vec{b}|^2 - 2 \cdot \vec{a} \cdot \vec{b}.$$

O‘ng va char tomonlardagi o‘xshash hadlarni qisqartiramiz va tenglikni -2 ga bo‘lamiz. Natijada (5) formula hosil bo‘ladi:

$$|\vec{a}| \cdot |\vec{b}| \cdot cos\varphi = \vec{a} \cdot \vec{b}.$$



10-rasm.

(5) va (6) formulalardan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak kosinusi aniqlanadi:

$$cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \cdot \sqrt{b_x^2 + b_y^2 + b_z^2}} \qquad (8)$$

6-misol. Ushbu $\vec{a}$=(1; -3; 4), $\vec{b}$=(2; 1; -1) vektorlarning skalyar ko‘paytmasini toping.

Yechish. (6) formulaga ko‘ra,

$$\vec{a} \cdot \vec{b} = 1 \cdot 2 + (-3) \cdot 1 + 4 \cdot (-1) = -5.$$

7-misol. $\vec{a}$=(6; -4; 3), $\vec{b}$=(3; -2; -4) vektorlar orasidagi burchakni hisoblang.

Yechish. (8) formulaga ko‘ra,

$$cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{6 \cdot 3 + (-4) \cdot (-2) + 3 \cdot (-4)}{\sqrt{36+16+9} \cdot \sqrt{9+4+16}} = \frac{14}{\sqrt{1769}}.$$

Skalyar ko‘paytmani ushbu masalada tushuntiramiz**.** Faraz qilaylik, supermarketda kartoshka 1000 so‘m, pomidor 3000 so‘m va bodring 2000 so‘mdan sotilgan bo‘lsin. U holda bu supermarketda narx vektori $\vec{S}$=(1000; 3000; 2000) ga teng. Agar xaridor $\vec{x}$=(x_1; x_2; x_3) qishloq xo‘jaligi mahsulotlari to‘plamini xarid qilmoqchi bo‘lsa, ya’ni x_1 kg kartoshka, x_2 kg pomidor va x_3 kg bodring, u holda narx vektori $\vec{S}$ va $\vec{x}$ ning skalyar ko‘paytmasi $\vec{S} \cdot \vec{x}$=1000x_1+3000x_2+2000x_3 xarid qilingan mahsulotlar to‘plami uchun sarflangan pul miqdorini ifodalaydi.

Xaridor kartoshka, pomidor va bodring uchun oila byudjetidan Q=50000 so‘m sarflashni rejalashtirgan bo‘lsa,

$$\vec{S} \cdot \vec{x} \leq Q$$

tengsizlikni yoki xaridorning oila byudjeti imkoniyati bo‘yicha faqat

$$1000x_1+3000x_2+2000x_3\leq 50000$$

tengsizlikni qanoatlantiruvchi qishloq xo‘jaligi mahsulotlari to‘plamini xarid qilishi mumkin.

Skalyar ko‘paytmaning xossalari:

1°. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$;

2°. $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$;

3°. $\lambda\vec{a}\cdot\vec{b}=\vec{a}\cdot\lambda\vec{b}=\lambda\cdot(\vec{a}\cdot\vec{b})$;

4°. $\vec{a}\parallel\vec{b}$ bo‘lsa, $\vec{a}\cdot\vec{b}=\pm|\vec{a}|\cdot|\vec{b}|$.

Xususan, $\vec{a}\cdot\vec{a}=|\vec{a}|^2$;

5°. $\vec{a}\cdot\vec{b}=0\Leftrightarrow\vec{a}\perp\vec{b}$ $(\vec{a}\neq 0,\vec{b}\neq 0)$;

6°. $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot Pr_{\vec{a}}\,\vec{b}=|\vec{b}|\cdot Pr_{\vec{b}}\,\vec{a}$.

8-misol. Agar $|\vec{a}|=3,|\vec{b}|=4$ va $\varphi=(\widehat{\vec{a}\vec{b}})=60^0$ bo‘lsa, $3\vec{a}-5\vec{b}$ vektor uzunligini toping.

Yechish. $3\vec{a}-5\vec{b}=\vec{c}$ belgilash kiritamiz. U holda vektorning uzunligi

$$|\vec{c}|=|3\vec{a}-5\vec{b}|=\sqrt{(3\vec{a}-5\vec{b})^2}=\sqrt{9|\vec{a}|^2-30\vec{a}\vec{b}+25|\vec{b}|^2}=$$

$$=\sqrt{81-30\cdot|\vec{a}||\vec{b}|cos\varphi+400}=\sqrt{81-180+400}=\sqrt{301}.$$

9-misol. x ning qanday qiymatida $\vec{a}=(3;\,-2x;7)$ va $\vec{b}=(x;\;4;5)$ vektorlar perpendikulyar bo‘ladi?

Yechish. Vektorlarning skalyar ko‘paytmasini topamiz:

$$\vec{a}\vec{b}=3\cdot x+(-2x)\cdot 4+7\cdot 5=3x-8x+35=-5x+35.$$

Vektorlar perpendikulyar bo‘lsa, u holda ularning skalyar ko‘paytmasi nolga teng bo‘ladi. Shuning uchun

$$-5x+35=0\;\Rightarrow\;x=7.$$

Demak, agar $x=7$ bo‘lsa, u holda $\vec{a}\perp\vec{b}$.

10-misol. Agar $\vec{a}=3\vec{p}+\vec{r},\ \vec{b}=2\vec{p}-\vec{r},\ \ |\vec{p}|=3,|\vec{r}|=4$, $\varphi=(\widehat{\vec{p},\vec{r}})=\frac{\pi}{3}$ bo‘lsa, $\vec{a}\cdot\vec{b}$ ko‘paytmani toping.

Yechish. $\vec{a}$ va $\vec{b}$ vektorlar o‘rniga ularning chiziqli kombinatsiyasini qo‘yamiz:

$$\vec{a}\cdot\vec{b}=(3\vec{p}+\vec{r})\cdot(2\vec{p}-\vec{r})=6\vec{p}\cdot\vec{p}+2\vec{r}\cdot\vec{p}-3\vec{p}\cdot\vec{r}-\vec{r}\cdot\vec{r}.$$

Skalyar ko‘paytmaning xossalariga ko‘ra:

$$\vec{p}\cdot\vec{r}=\vec{r}\cdot\vec{p},\ \vec{p}\cdot\vec{p}=|\vec{p}|^2.$$

U holda,

$$\vec{a}\cdot\vec{b}=6|\vec{p}|^2-\vec{p}\cdot\vec{r}-|\vec{r}|^2=6|\vec{p}|^2-|\vec{p}|\cdot|\vec{r}|cos\varphi-|\vec{r}|^2.$$

$$\vec{a}\cdot\vec{b}=6\cdot 3^2-3\cdot 4\cdot cos\frac{\pi}{3}-4^2=54-12\cdot\frac{1}{2}-16=32.$$

11-misol. Agar $\vec{a}=2\vec{p}+3\vec{r},\ |\vec{p}|=\sqrt{2},|\vec{r}|=3,\ \ \varphi= \ =$ $(\widehat{\vec{p},\vec{r}})=\frac{\pi}{4}$ bo‘lsa, $|\vec{a}|$ ni toping.

Yechish. Ma’lumki, $|\vec{a}|=\sqrt{|\vec{a}|^2}=\sqrt{\vec{a}\cdot\vec{a}}$. Misol shartidan foydalanib $\vec{a}\cdot\vec{a}$ ko‘paytmani topamiz:

$$\vec{a}\cdot\vec{a}=\ (2\vec{p}+3\vec{r})\cdot(2\vec{p}+3\vec{r})=4\vec{p}\cdot\vec{p}+12\vec{r}\cdot\vec{p}+\vec{r}\cdot\vec{r}.$$

Skalyar ko‘paytma xossasiga ko‘ra: $\vec{p}\cdot\vec{p}=|\vec{p}|^2,\ \ \vec{r}\cdot\vec{r}=|\vec{r}|^2.$

U holda

$$\vec{a}\cdot\vec{a}=4|\vec{p}|^2+12|\vec{p}|\cdot|\vec{r}|cos\varphi+|\vec{r}|^2.$$

$$\vec{a}\cdot\vec{a}=4\cdot\left(\sqrt{2}\right)^2+12\cdot\sqrt{2}\cdot 3\cdot cos\frac{\pi}{4}+3^2=$$

$$=4\cdot 2+12\cdot\sqrt{2}\cdot 3\cdot\frac{\sqrt{2}}{2}+3^2=8+36+9=53.$$

$$|\vec{a}|=\sqrt{53}.$$

12-misol. Agar $|\vec{p}|=3,|\vec{r}|=4,\ \varphi=(\widehat{\vec{p},\vec{r}})=\frac{\pi}{3}$ bo‘lsa, ε ning qanday qiymatida $\vec{a}=\vec{p}+\varepsilon\vec{r},\ \vec{b}=\vec{p}-\vec{r}$ vektorlar perpendikulyar bo‘ladi.

Yechish. $\vec{a}\cdot\vec{b}$=0 $\Leftrightarrow$ $\vec{a}\perp\vec{b}$ $(\vec{a}\neq 0,\vec{b}\neq 0)$. $\vec{a}\cdot\vec{b}$ ko‘paytmani topamiz:

$$\vec{a}\cdot\vec{b}=(\vec{p}+\varepsilon\vec{r})\cdot(\vec{p}-\vec{r})=\vec{p}\cdot\vec{p}+(\varepsilon-1)\vec{r}\cdot\vec{p}-\varepsilon\vec{r}\cdot\vec{r}.$$

$$\vec{a}\cdot\vec{b}=|\vec{p}|^2+(\varepsilon-1)|\vec{p}|\cdot|\vec{r}|cos\varphi-\varepsilon|\vec{r}|^2.$$

$$\vec{a}\cdot\vec{b}=3^2+(\varepsilon-1)\cdot 3\cdot 4\cdot cos60^0-\varepsilon\cdot 4^2.$$

$$0=9+(\varepsilon-1)\cdot 6-\varepsilon\cdot 16=-10\varepsilon+3.$$

Demak, $\varepsilon=\frac{3}{10}$ ga teng bo‘lsa, $\vec{a}$ va $\vec{b}$ vektorlar perpendikulyar bo‘ladi.

Skalyar ko‘paytmaning fizik ma’nosi. Biror $\vec{F}$ kuch ta’siri ostida moddiy nuqta to‘g‘ri chiziq bo‘yicha harakat qilsin, bunda kuchning yo‘nalishi harakat yo‘nalishi bilan bir xil bo‘lsin. Moddiy nuqta M_1 nuqtadan M_2 nuqtagacha ko‘chganda $\vec{F}$ kuchning bajargan ishi quyidagi formula (skalyar ko‘paytma) yordamida aniqlanadi:

$$A=\vec{F}\cdot\vec{S}, \qquad (9)$$

bu yerda $\vec{S}=\overrightarrow{M_1M_2}$.

13-misol. Bir nuqtaga $\vec{F_1}=3\vec{i}-4\vec{j}+5\vec{k}, \vec{F_2}=2\vec{i}+\vec{j}-4\vec{k}$ va $\vec{F_3}=-\vec{i}+6\vec{j}+2\vec{k}$ kuchlar qo‘yilgan. Ularning teng ta’sir etuvchi $\vec{F}$ kuch qo‘yilish nuqtasi to‘g‘ri chiziqli harakat qilib, $M_1(4,2,-3)$ nuqtadan $M_2(7,4,1)$ nuqtaga o‘tganda, $\vec{F}$ kuch bajargan ishni hisoblang.

Yechish. Ravshanki, $\vec{F}=\vec{F_1}+\vec{F_2}+\vec{F_3}=(4;3;3)$. $\vec{S}=\overrightarrow{M_1M_2}=(3;2;4)$. U holda $\vec{F}$ kuch yordamida bajarilgan A ish (9) formulaga ko‘ra aniqlanadi:

$$A=\vec{F}\cdot\vec{S}=4\cdot 3+3\cdot 2+3\cdot 4=30\ (J).$$

14-misol. $\vec{F}$ kuch vektorining moduli 6 (kg) ga teng. $\vec{S}$ ko‘chish vektorining uzunligi 7 (m) ga teng. Faraz qilaylik, $\vec{F}$ kuch $\vec{S}$ ko‘chishga nisbatan $\varphi=60^0$ burchak ostida ta’sir etmoqda. $\vec{F}$ kuch bajargan ishni hisoblang.

Yechish. Skalyar ko‘paytma ta’rifiga binoan,

$$A=\vec{F}\cdot\vec{S}=\left|\vec{F}\right|\cdot\left|\vec{S}\right|\cdot cos\varphi=6\cdot 7\cdot cos60^0=42\cdot\frac{1}{2}=21(kGm).$$

6-§. Vektor ko‘paytma

$\vec{a}$ va $\vec{b}$ vektorlarning vektor ko‘paytmasi deb, shunday $\vec{c}$ vektorga aytiladiki, u quyidagi shartlarni qanoatlantiradi:

1) $\vec{c}$ vektorning moduli $\vec{a}$ va $\vec{b}$ vektorlardan yasalgan parallelogramm yuziga teng bo‘ladi, $|\vec{c}|=|\vec{a}|\cdot\left|\vec{b}\right|\cdot sin\varphi$;

2) $\vec{c}\perp\vec{a},\ \vec{c}\perp\vec{b}$ ($\vec{c}$ vektor parallelogramm tekisligiga perpendikulyar);

3) $\vec{a},\vec{b},\vec{c}$ vektorlar o‘ng bog‘lam tashkil etadi.

Vektor ko‘paytma $[\vec{a},\vec{b}]$ yoki $\vec{a}\times\vec{b}$ kabi belgilanadi:

$$[\vec{a},\vec{b}]=\vec{c} \text{ yoki } \vec{a}\times\vec{b}=\vec{c}.$$

Ta’rifga binoan, o‘zaro perpendikulyar bo‘lgan birlik vektorlarning vektor ko‘paytmasini topamiz:

$$|\boldsymbol{i} \times \boldsymbol{i}| = |\boldsymbol{i}| \cdot |\boldsymbol{i}| \cdot \sin 0^0 = 1 \cdot 1 \cdot 0 = 0;$$
$$\boldsymbol{i} \times \boldsymbol{i} = 0; \quad \boldsymbol{j} \times \boldsymbol{j} = 0; \quad \boldsymbol{k} \times \boldsymbol{k} = 0;$$
$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}; \quad \boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}; \quad \boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i}.$$

Koordinatalari bilan berilgan $\vec{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$ va $\vec{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}$ vektorlarning vektor ko‘paytmasini topamiz:

$$\vec{a} \times \vec{b} = (a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}) \times (b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}) =$$
$$= a_xb_x(\boldsymbol{i} \times \boldsymbol{i}) + a_xb_y(\boldsymbol{i} \times \boldsymbol{j}) + a_xb_z(\boldsymbol{i} \times \boldsymbol{k}) +$$
$$+a_yb_x(\boldsymbol{j} \times \boldsymbol{i}) + a_yb_y(\boldsymbol{j} \times \boldsymbol{j}) + a_yb_z(\boldsymbol{j} \times \boldsymbol{k}) +$$
$$+a_zb_x(\boldsymbol{k} \times \boldsymbol{i}) + a_zb_y(\boldsymbol{k} \times \boldsymbol{j}) + a_zb_z(\boldsymbol{k} \times \boldsymbol{k}) =$$
$$= a_xb_x \cdot 0 + a_xb_y \cdot \boldsymbol{k} + a_xb_z(-\boldsymbol{j}) +$$
$$+a_yb_x(-\boldsymbol{k}) + a_yb_y \cdot 0 + a_yb_z \cdot \boldsymbol{i} +$$
$$+a_zb_x \cdot \boldsymbol{j} + a_zb_y(-\boldsymbol{i}) + a_zb_z \cdot 0 =$$
$$= \left(a_xb_y - a_y \cdot b_x\right)\boldsymbol{k} + (a_zb_x - a_xb_z)\boldsymbol{j} + (a_yb_z - a_zb_y)\boldsymbol{i}.$$

Qavslar ichidagi ifodani determinant orqali ifodalaymiz:

$$\vec{a} \times \vec{b} = \boldsymbol{i}\begin{vmatrix} a_y & b_y \\ a_z & b_z \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} a_x & b_x \\ a_z & b_z \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} a_x & b_x \\ a_y & b_y \end{vmatrix}.$$

Determinantning xossalariga ko‘ra, o‘ng tomondagi yig‘indi uning yo‘l bo‘yicha yoyilmasini ifodalaydi.

Demak, $\vec{a}\,(a_x;\ a_y;\ a_z)$ va $\vec{b}\,(b_x;\ b_y;\ b_z)$ vektorlarning vektor ko‘paytmasi quyidagi formula bilan aniqlanadi:

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}. \tag{10}$$

Vektor ko‘paytmaning xossalari:

1°. $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$;

2°. $\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$;

3°. $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$;

4°. $\lambda\vec{a} \times \vec{b} = \vec{a} \times \lambda\vec{b} = \lambda \cdot (\vec{a} \times \vec{b})$;

5°. $\vec{a} \times \vec{a} = 0$;

6°. $\vec{a} \times \vec{b} = 0 \iff \vec{a} \parallel \vec{b}$.

$\vec{a}$ va $\vec{b}$ vektorlardan yasalgan parallelogramm va uchburchak yuzi mos ravishda quyidagi formulalar yordamida aniqlanadi (11-rasm):

$$S_{parallelogramm} = \left|\vec{a} \times \vec{b}\right|, \tag{11}$$
$$S_{uchburchak} = \frac{1}{2}\left|\vec{a} \times \vec{b}\right|. \tag{12}$$

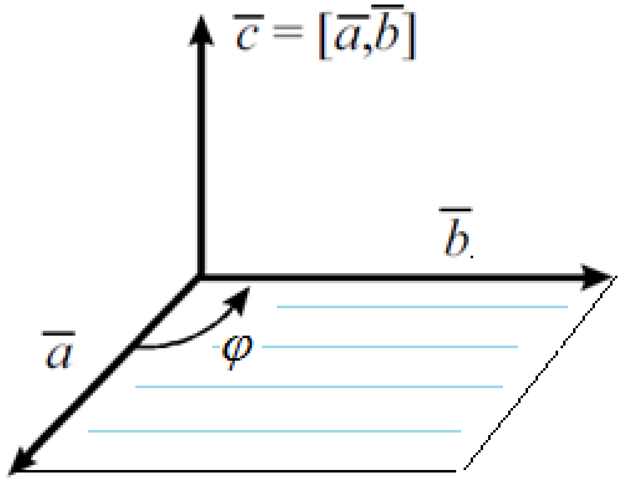
15-misol. $\vec{a}$=(-2; 0; 1), $\vec{b}$=(1; 4; 3) vektorlardan yasalgan parallelogramm yuzini toping.

Yechish. (10) formulaga ko'ra

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 1 \\ 1 & 4 & 3 \end{vmatrix} = \vec{j} - 8\vec{k} - 4\vec{i} + 6\vec{j} = -4\vec{i} + 7\vec{j} - 8\vec{k}.$$

(11) formuladan parallelogramm yuzini topamiz:

$$S_{par} = \left|\vec{a} \times \vec{b}\right| = \sqrt{(-4)^2 + 7^2 + (-8)^2} = \sqrt{129}.$$



11-rasm.

16-misol. Agar $\vec{a} = \vec{p} - 3\vec{r}$, $\vec{b} = 2\vec{p} + 9\vec{r}$, $|\vec{p}| = 2, |\vec{r}| = = 3\sqrt{2}$, $\varphi = (\widehat{\vec{p}, \vec{r}}) = 135^0$ bo'lsa, $\left|\vec{a} \times \vec{b}\right|$ ni hisoblang.

Yechish. $\vec{a} \times \vec{b} = (\vec{p} - 3\vec{r}) \times (2\vec{p} + 9\vec{r}) =$

$$= 2(\vec{p} \times \vec{p}) + 9(\vec{p} \times \vec{r}) - 6(\vec{r} \times \vec{p}) - 27(\vec{r} \times \vec{r}).$$

Vektor ko'paytma xossalariga ko'ra

$$\vec{p} \times \vec{p} = 0, \ \vec{r} \times \vec{p} = -(\vec{p} \times \vec{r}).$$

U holda $\left|\vec{a} \times \vec{b}\right| = |15(\vec{p} \times \vec{r})| = 15|\vec{p}||\vec{r}|\sin 135^0 =$

$$= 15 \cdot 2 \cdot 3\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 90.$$

$$\left|\vec{a} \times \vec{b}\right| = 90.$$

17-misol. Uchlari $A(2; 2; -1)$, $B(1; 3; 0)$, $C(1; 0; -1)$ nuqtalarda bo‘lgan uchburchak yuzini va B uchidan tushirilgan balandlikni toping.

Yechish. Quyidagi vektorlarning koordinatalarini aniqlaymiz:

$$\overrightarrow{AB}=(-1; 1; 1), \quad \overrightarrow{AC}=(-1; -2; 0).$$

Vektor ko‘paytmasini hisoblaymiz:

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ -1 & -2 & 0 \end{vmatrix}=-\vec{j}+2\vec{k}+\vec{k}+2\vec{i}=2\vec{i}-\vec{j}+3\vec{k}.$$

Vektorlar uzunliklarini topamiz:

$$\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\sqrt{2^2+(-1)^2+3^2}=\sqrt{14},$$
$$\left|\overrightarrow{AC}\right|=\sqrt{(-1)^2+(-2)^2+0}=\sqrt{5}.$$

Natijada $S_{\Delta ABC}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{\sqrt{14}}{2}$;

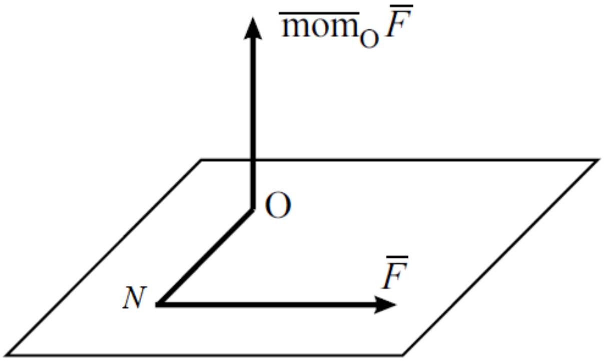
balandlik $h_b=\frac{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|}{\left|\overrightarrow{AC}\right|}=\frac{\sqrt{14}}{\sqrt{5}}=\sqrt{\frac{14}{5}}$.

Vektor ko‘paytmaning tadbiqlarini mexanika va fizikaga doir masalalarda keltiramiz:

1. Mexanika kursida qandaydir qattiq jismning qo‘zg‘almas O nuqtasiga nisbatan $\vec{F}$ kuch momenti deb ataladigan vektor ushbu formuladan aniqlanadi (12-rasm):

$$\overline{mom_0}\overline{F}=\overline{ON}\times\overline{F}. \tag{13}$$

2. Qo‘zg‘almas o‘q atrofida $\overline{\omega}$ burchak tezlik bilan aylanayotgan nuqtaning chiziqli $\overline{V}$ tezligi shu nuqtaning $\overline{R}$ radius vektori va $\overline{\omega}$ burchak tezlik vektorining vektor ko‘paytmasi bilan aniqlanadi: $\overline{V}=\overline{\omega}\times\overline{R}$.



12-rasm.

18-misol. A(0; 2; -1) nuqtaga $\vec{F} = (2;\ 3; 1)$ qo‘yilgan. B(1; 1; 2) nuqtaga nisbatan $\vec{F}$ kuch mometining qiymatini va yonalishini aniqlang.

Yechish. (13) formulaga asosan $\overline{mom}_0\overline{F} = \overline{BA} \times \overline{F}$. $\overline{BA}$ vektor koordinatalarini aniqlaymiz: $\overline{BA} = (-1;\ 1; -3)$. (10) formulaga asosan

$$\overline{mom}_0\overline{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 1 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 10\bar{i} - 5\bar{j} - 5\bar{k},$$

$$\overline{mom}_0\overline{F} = \sqrt{10^2 + (-5)^2 + (-5)^2} = 5\sqrt{6}.$$

$\overline{mom}_0\overline{F}$ vektorning yo‘naltiruvchi kosinuslarini aniqlaymiz:

$$cos\alpha = \frac{10}{5\sqrt{6}} = \frac{2}{\sqrt{6}};\ \ cos\beta = -\frac{1}{\sqrt{6}};\ \ cos\gamma = -\frac{1}{\sqrt{6}}.$$

7-§. Uch vektorning aralash ko‘paytmasi

$\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarning aralash ko‘paytmasi deb, $\vec{a} \cdot (\vec{b} \times \vec{c})$ ifodaga aytiladi.

Agar vektorlar koordinatalari bilan berilgan bo‘lsa, u holda aralash ko‘paytma

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \qquad (14)$$

formula orqali topiladi.

Aralash ko‘paytmaning xossalari

1°. $\vec{a} \cdot (\vec{b} \times \vec{c}) = -\vec{a} \cdot (\vec{c} \times \vec{b})$.

2°. Agar uch vektordan ikkitasi teng yoki parallel bo‘lsa, u holda aralash ko‘paytma nolga teng bo‘ladi.

3°. $(\vec{a}\times\vec{b})\cdot\vec{c}=\vec{a}\cdot(\vec{b}\times\vec{c})$. Aralash ko‘paytmadagi “× ” va “ · ” belgilar o‘rnini almashtirish mumkin. Shu sababli, aralash ko‘paytmani $\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\vec{\boldsymbol{c}}$ ko‘rinishda ham yozish mumkin.

$\vec{a},\vec{b}$ va $\vec{c}$ vektorlardan yasalgan parallelepiped hajmi:

$$V=\left|\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\vec{\boldsymbol{c}}\right|. \tag{15}$$

$\vec{a},\vec{b}$ va $\vec{c}$ vektorlardan yasalgan piramida hajmi:

$$V_{pir}=\pm\frac{1}{6}\,\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\vec{\boldsymbol{c}}\,. \tag{16}$$

$\vec{a},\vec{b}$ va $\vec{c}$ vektorlardan yasalgan prizma hajmi:

$$V_{prizma}=\pm\frac{1}{2}\,\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\vec{\boldsymbol{c}}\,. \tag{17}$$

$\vec{a},\vec{b}$ va $\vec{c}$ vektorlardan yasalgan parallelepiped, piramida, prizmaning $\vec{a}$ va $\vec{b}$ vektorlar tekisligiga tushirilgan h balandlik quyidagi formuladan aniqlanadi:

$$h=\frac{|\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\vec{\boldsymbol{c}}|}{|\vec{\boldsymbol{a}}\times\vec{\boldsymbol{b}}|}. \tag{18}$$

Agar $\vec{a},\vec{b}\ va\ \vec{c}$ vektorlar komplanar bo‘lsa, u holda $\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\vec{\boldsymbol{c}}=0$ bo‘ladi.

19-misol. Uchlari $O(0;0;0)$, $A(5;2;0)$, $B(2;5;0)$ va $C(1;2;4)$ nuqtalarda bo‘lgan piramida hajmini va C uchidan tushirilgan balandlikni toping (13-rasm).

Yechish. $\overrightarrow{OA}=(5;2;0)$, $\overrightarrow{OB}=(2;5;0)$, $\overrightarrow{OC}=(1;2;4)$ vektorlar koordinatalarini aniqlaymiz.

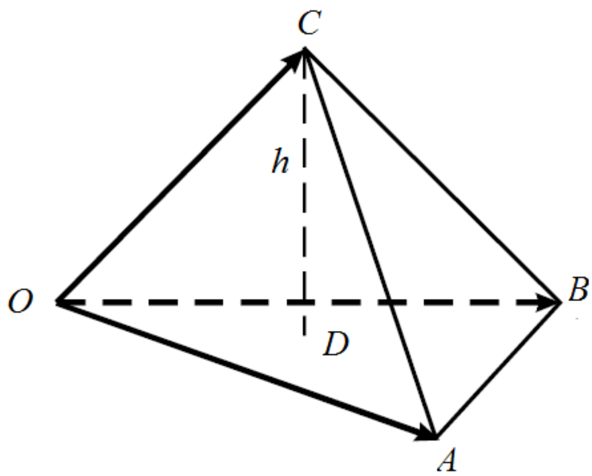
(16) formulaga ko‘ra,

$$V_{pir}=\pm\frac{1}{6}\,\overrightarrow{\boldsymbol{OA}}\,\overrightarrow{\boldsymbol{OB}}\,\overrightarrow{\boldsymbol{OC}}=\pm\frac{1}{6}\begin{vmatrix}5&2&0\\2&5&0\\1&2&4\end{vmatrix}=\pm\frac{1}{6}\,(100-16)=14.$$

$$V_{pir}=14 \text{ kub birlik.}$$

Maktab geometriya fanidan ma’lumki,

$$V_{pir}=\frac{1}{3}\cdot S_{asos}\cdot h\ \Rightarrow\ h=\frac{3\cdot V_{pir}}{S_{\Delta OAB}},$$



13-rasm.

$$S_{\Delta OAB} = \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|,$$

$$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 2 & 0 \\ 2 & 5 & 0 \end{vmatrix} = 21\vec{k}.$$

$$S_{\Delta OAB} = \frac{1}{2} \cdot 21 = \frac{21}{2}.$$

Shunday qilib, $h = \frac{3 \cdot 14}{\frac{21}{2}} = 4.$

Piramidaning C uchidan tushirilgan balandlikni (18) formula yordamida ham aniqlash mumkin.

20-misol. $\vec{a}$=(1; 1; 1), $\vec{b}$=(1; -1; -1) va $\vec{c}$=(1; 1; -1) vektorlar berilgan. 1) $\{\vec{a}, \vec{b}, \vec{c}\}$ vektorlar sistemasi bazis tashkil etishini tekshirib ko‘ring; 2) $\vec{d}$=(4; 0; 2) vektorni shu bazis bo‘yicha yoying.

Yechish. 1) $\{\vec{a}, \vec{b}, \vec{c}\}$ sistema bazis tashkil etishini tekshirib ko‘ramiz. Buning uchun vektorlarning aralash ko‘paytmasini hisoblaymiz:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 4 \neq 0.$$

Bundan ma’lumki, $\{\vec{a}, \vec{b}, \vec{c}\}$ vektorlar komplanar emas. Demak, $\{\vec{a}, \vec{b}, \vec{c}\}$ sistema fazoda bazis tashkil etadi.

2) $\vec{d}$ =(4; 0; 2) vektorni $\{\vec{a}, \vec{b}, \vec{c}\}$ bazis bo‘yicha yoyish uchun $\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$ chiziqli kombinatsiyadan x, y, z koeffitsiyentlarni aniqlash lozim. Chiziqli kombinatsiyani matritsa ko‘rinishda ifodalaymiz:

$$\begin{pmatrix}4\\0\\2\end{pmatrix}=x\begin{pmatrix}1\\1\\1\end{pmatrix}+y\begin{pmatrix}1\\-1\\-1\end{pmatrix}+z\begin{pmatrix}1\\1\\-1\end{pmatrix}.$$

Ma'lumki, vektorlar ustida chiziqli amallar koordinatalari bo'yicha bajariladi, vektorlar teng bo'lishi uchun mos koordinatalari teng bo'lishi kerak. Yuqoridagi vektor tenglikni koordinatalar bilan chiziqli tenglamalar sistemasi ko'rinishida ifodalash mumkin:

$$\begin{cases}x+y+z=4,\\x-y+z=0,\\x-y-z=2.\end{cases}$$

Chiziqli tenglamalar sistemasini biror usul yordamida yechamiz va noma'lum koeffitsiyentlarni aniqlaymiz:

$$x=3,\qquad y=2,\qquad z=-1.$$

Natijada $\vec{d}$ vektorning $\{\vec{a},\vec{b},\vec{c}\}$ bazis bo'yicha yoyilmasi quyidagi ko'rinishda bo'ladi:

$$\vec{d}=3\vec{a}+2\vec{b}-\vec{c}.$$

21-misol. $A(1;\ 2;\ -1)$, $B(0;\ 1;\ 5)$, $C(-1;\ 2;\ 1)$ va $D(2;\ 1;\ 3)$ nuqtalarning bitta tekislikda yotishini tekshiring. $\overrightarrow{BD}$ vektorning $\overrightarrow{BA}$ va $\overrightarrow{BC}$ vektorlar bilan chiziqli ifodasini toping.

Yechish. Agar $\overrightarrow{BA}$, $\overrightarrow{BC}$ va $\overrightarrow{BD}$ vektorlar komplanar bo'lsa, nuqtalarning bitta tekislikda yotishi ma'lum bo'ladi. Vektorlarning koordinatalarini aniqlaymiz:

$$\overrightarrow{BA}=(1;1;-6),\overrightarrow{BC}=(-1;1;-4),\overrightarrow{BD}=(2;0;\ -2).$$

Yuqorida keltirilgan ta'rifga ko'ra, agar $\vec{a},\vec{b}\ va\ \vec{c}$ vektorlar komplanar bo'lsa, u holda $\boldsymbol{\vec{a}\vec{b}\vec{c}}=0$ bo'ladi. $\overrightarrow{BA},\overrightarrow{BC},\overrightarrow{BD}$ vektorlarning aralash ko'paytmasini hisoblaymiz:

$$\boldsymbol{\overrightarrow{BA}\,\overrightarrow{BC}\,\overrightarrow{BD}}=\begin{vmatrix}1&1&-6\\-1&1&-4\\2&0&-2\end{vmatrix}=0\ \Rightarrow$$

$\Rightarrow$ $\overrightarrow{BA},\overrightarrow{BC},\overrightarrow{BD}$ – vektorlar komplanar.

$\overrightarrow{BD} = \lambda_1\overrightarrow{BA} + \lambda_2\overrightarrow{BC}$ chiziqli kombinatsiyadagi λ_1 va λ_2 koeffitsiyentlarni aniqlaymiz. Tenglamadagi vektorlarning mos koordinatalarini tenglashtiramiz:

$$\begin{cases} 2 = \lambda_1 - \lambda_2, \\ 0 = \lambda_1 + \lambda_2, \\ -2 = -6\lambda_1 - 4\lambda_2, \end{cases} \Leftrightarrow \begin{cases} 2 = 2\lambda_1, \\ -2 = -6\lambda_1 - 4\lambda_2, \end{cases} \Rightarrow \begin{matrix} \lambda_1 = 1, \\ \lambda_2 = -1. \end{matrix}$$

Demak, $\overrightarrow{BD} = \overrightarrow{BA} - \overrightarrow{BC}$.

Mustaqil ishlash uchun misollar

1. Oxiri (1;-1;2) nuqtada bo‘lgan $\vec{a} = \{2; -3; -1\}$ vektorning boshlang‘ich nuqtasini toping.

2. $\vec{a} = \{12; -15; -16\}$ vektorning yo‘naltiruvchi kosinuslarini hisoblang.

3. α va β ning qanday qiymatlarida $\vec{a} = -2\boldsymbol{i} + 3\boldsymbol{j} + +\beta\boldsymbol{k}$, $\vec{b} = \alpha\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$ vektorlar kollinear bo‘ladi?

4. $|\vec{a}| = 11$, $|\vec{b}| = 23$ va $|\vec{a} - \vec{b}| = 30$ berilgan. $|\vec{a} + \vec{b}|$ ni hisoblang.

5. Uchburchakni $A(-1; -2; 4)$, $B(-4; -2; 0)$ va $C(3; -2; 1)$ uchlari berilgan. Uning B uchidagi ichki burchagini aniqlang.

6. $\vec{a} = \{2; 1; -1\}$ vektorga kollinear bo‘lgan va $\vec{x} \cdot \vec{a} = 3$ shartni bajaruvchi $\vec{x}$ vektorni toping.

7. Agar $\boldsymbol{f} = \{3; -2; -5\}$ kuchning qo‘yilish nuqtasi to‘g‘ri chiziqli harakatda $A(2; -3; 5)$ dan $B(3; -2; -1)$ holatga ko‘chsa, bu kuch qancha ish bajaradi?

8. $\vec{a}$ va $\vec{b}$ vektorlar $\varphi = \frac{\pi}{6}$ burchak tashkil etadi. Agar $|\vec{a}| = 6$, $|\vec{b}| = 5$ bo‘lsa, $|\vec{a} \times \vec{b}|$ ni hisoblang.

9. $\vec{m}$ va $\vec{n}$ o‘zaro 30° burchak tashkil etuvchi birlik vektorlar bo‘lsa, $\vec{a} = \vec{m} + 2\vec{n}$ va $\vec{b} = \vec{m} + \vec{n}$ vektorlarda yasalgan parallelogramm yuzini toping.

10. $\vec{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$, $\vec{b} = -3\boldsymbol{j} + \boldsymbol{k}$, $\vec{c} = 2\boldsymbol{i} + 5\boldsymbol{k}$ vektorlardan parallelepiped yasang hamda uning hajmini hisoblang.

11. $A(2; -1; -2)$, $B(1; 2; 1)$, $C(2; 3; 0)$ va $D(5; 0; -6)$ nuqtalarning bir tekislikda yotishini ko‘rsating.

VII BOB. FAZODA ANALITIK GEOMETRIYA

1-§. Tekislikning umumiy tenglamasi

Faraz qilaylik, $M_0(x_0, y_0, z_0)$ nuqta $\vec{N}(A, B, C)$ vektorga perpendikulyar bo‘lgan α tekislikka tegishli bo‘lsin (1-rasm). α tekislik nuqtalar to‘plamidan iborat bo‘lgani uchun, undan ixtiyoriy $M(x, y, z)$ nuqtani tanlaymiz. M_0 nuqtadan M nuqtagacha vektor yo‘naltiramiz:

$$\overrightarrow{M_0M} = (x - x_0; y - y_0; z - z_0).$$

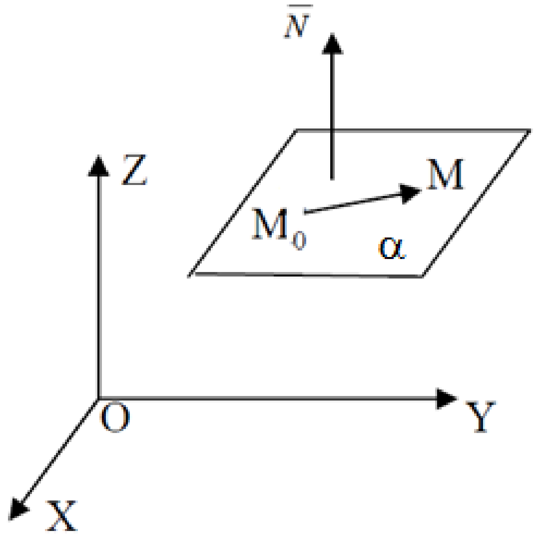
Farazga binoan $\vec{N} \perp \overrightarrow{M_0M}$, u holda skalyar ko‘paytma xossasiga ko‘ra,

$$\vec{N} \cdot \overrightarrow{M_0M} = 0.$$

Ko‘paytmani vektorlar koordinatalari orqali ifodalaymiz:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0. \qquad (1)$$

Hosil bo‘lgan tenglama berilgan nuqta orqali o‘tuvchi va berilgan vektorga perpendikulyar tekislik tenglamasidir.



1-rasm.

(1) tenglamada qavslarni ochib chiqamiz va $D = -Ax_0 - By_0 - Cz_0$ belgilash kiritamiz:

$$Ax + By + Cz + D = 0 \qquad (2)$$

(2) tenglama fazoda ***tekislikning umumiy tenglamasi*** deyiladi. $\vec{N}(A, B, C)$ – tekislikning normal vektori.

(2) tenglama koeffitsiyentlariga binoan tekislikning xususiy hollarini qaraylik.

1°. Tenglamaning barcha koeffitsiyentlari noldan farqli bo‘lsin. Tekislik tenglamasini quyidagi ko‘rinishda yozish mumkin:

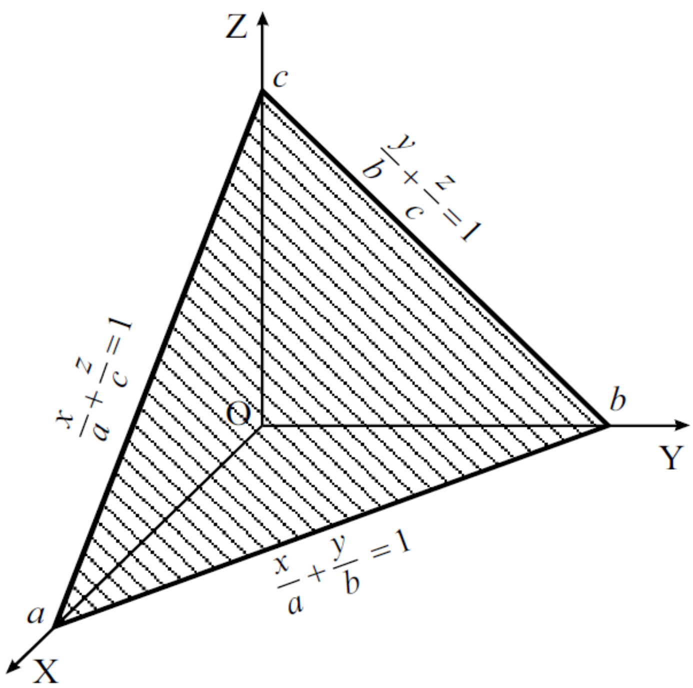
$$\frac{x}{-\frac{D}{A}} + \frac{y}{-\frac{D}{B}} + \frac{z}{-\frac{D}{C}} = 1 \ \text{ yoki } \ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad (3)$$

(3) tenglama ***tekislikning o‘qlardan ajratgan kesmalar bo‘yicha tenglamasi*** deyiladi (2-rasm).

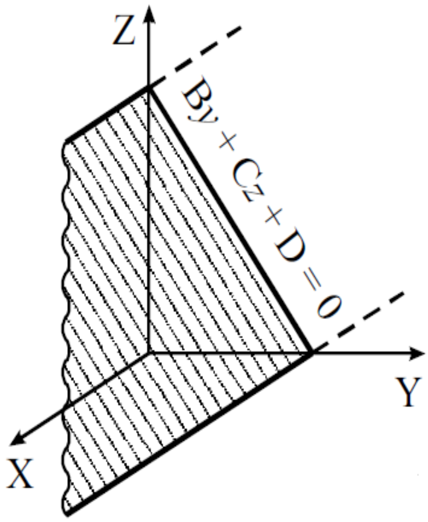
2°. $A = 0 \implies By + Cz + D = 0$. Tekislik Ox o‘qiga parallel bo‘ladi (3-rasm). Xuddi shuningdek, $Ax + Cz + D = 0$ tekislik Oy o‘qiga parallel (4-rasm), $Ax + By + D = 0$ tekislik Oz o‘qiga parallel bo‘ladi (5-rasm).

3°. $D = 0 \implies Ax + By + Cz = 0$. Tekislik koordinata boshidan o‘tadi (6-rasm).

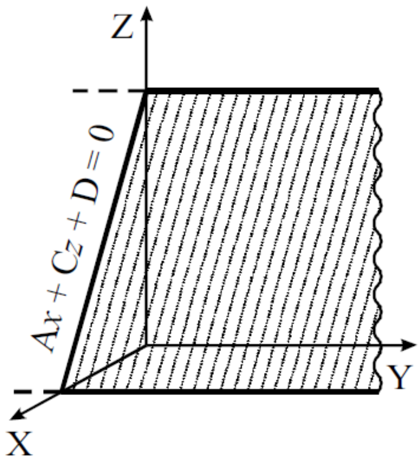
4°. $By + Cz = 0, A = 0, D = 0$ tekislik Ox o‘qi orqali o‘tadi. Xuddi shuningdek, $Ax + Cz = 0$ – tekislik Oy o‘qidan o‘tadi, $Ax + By = 0$ – tekislik Oz o‘qidan o‘tadi (7-9-rasmlar).



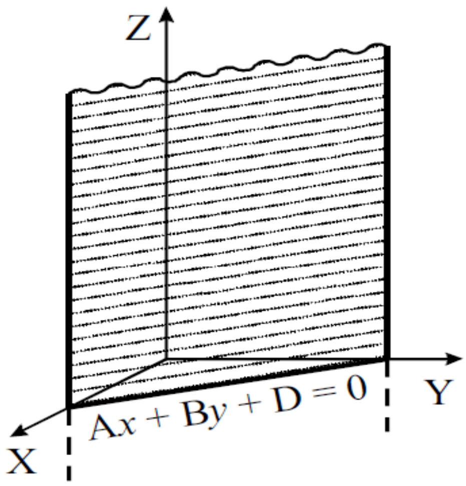
2-rasm.



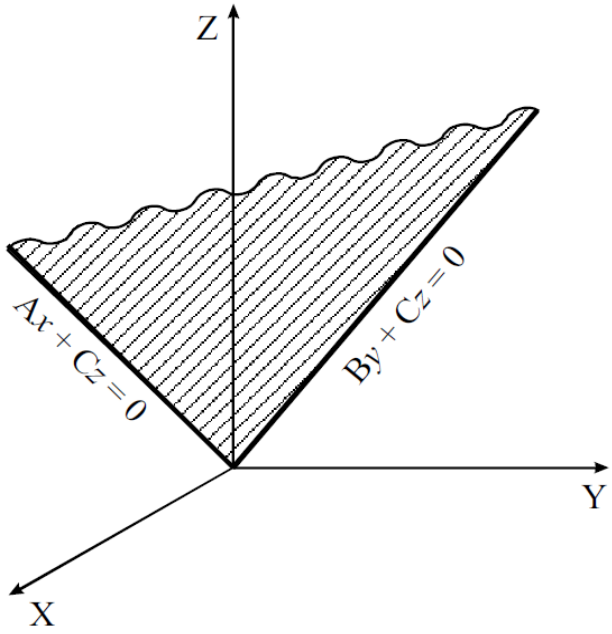
3-rasm.



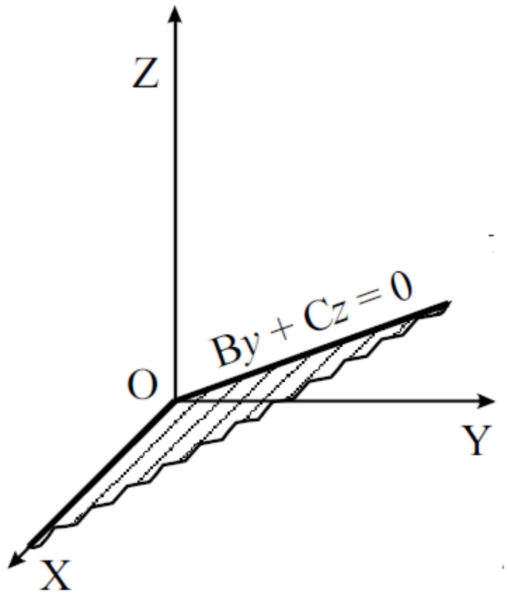
4-rasm.



5-rasm.



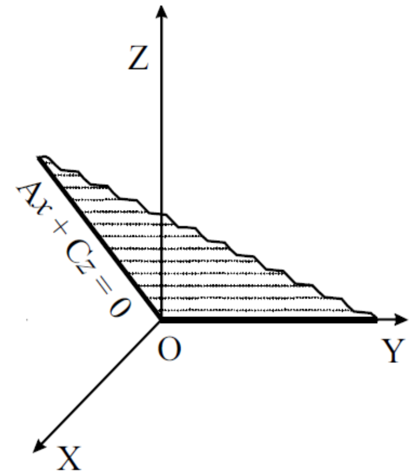
6-rasm.



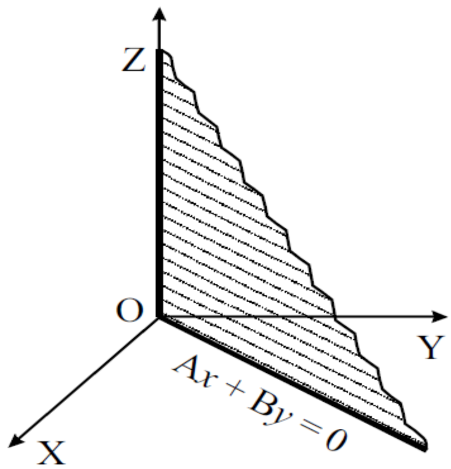
7-rasm.

5°. $Ax + D = 0$ tekislik Oyz tekisligiga parallel boladi (10-rasm). Xuddi shuningdek, $By + D = 0$ tekislik Oxz tekisligiga (11-rasm), $Cz + D = 0$ tekislik Oxy tekisligiga parallel bo‘ladi (12-rasm).

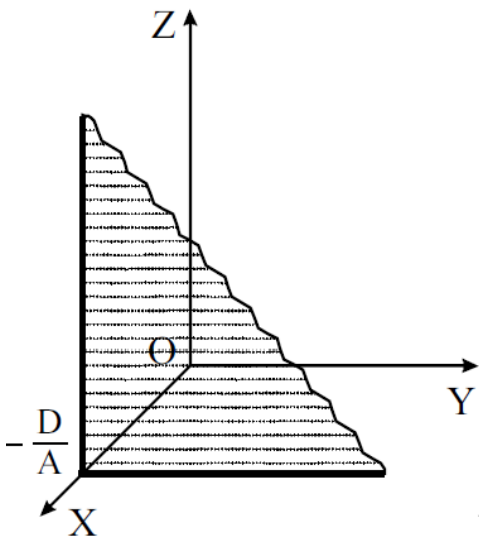
6°. $x = 0,\ y = 0,\ z = 0$ – koordinata tekisliklari tenglamalari.



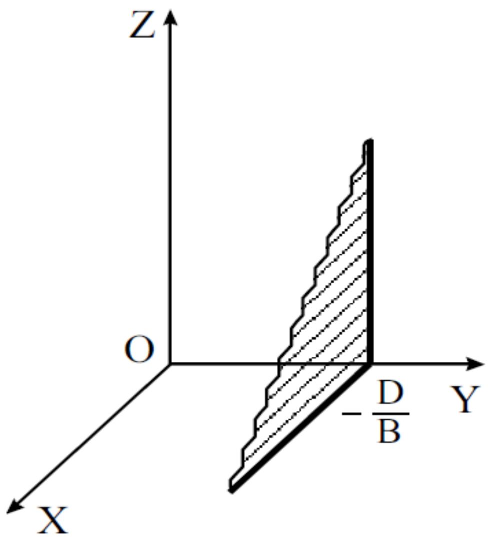
8-rasm.



9-rasm.



10-rasm.



11-rasm.

1-misol. $12x + 15y + 20z - 60 = 0$ tekislikning koordinata o'qlaridan ajratgan kesmasini aniqlang.

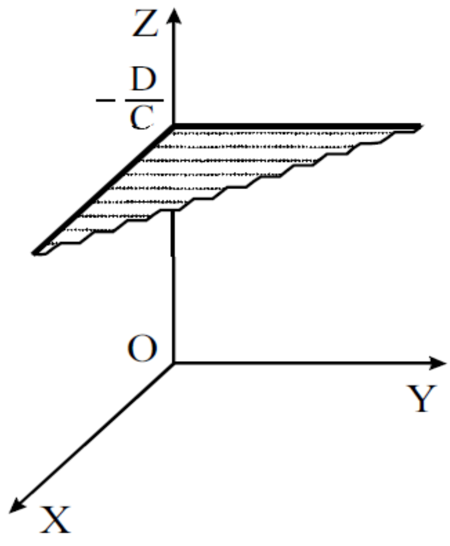
Yechish. Ozod hadni tenglamaning o'ng tomoniga o'tkazamiz (13-rasm):

$$12x + 15y + 20z = 60.$$

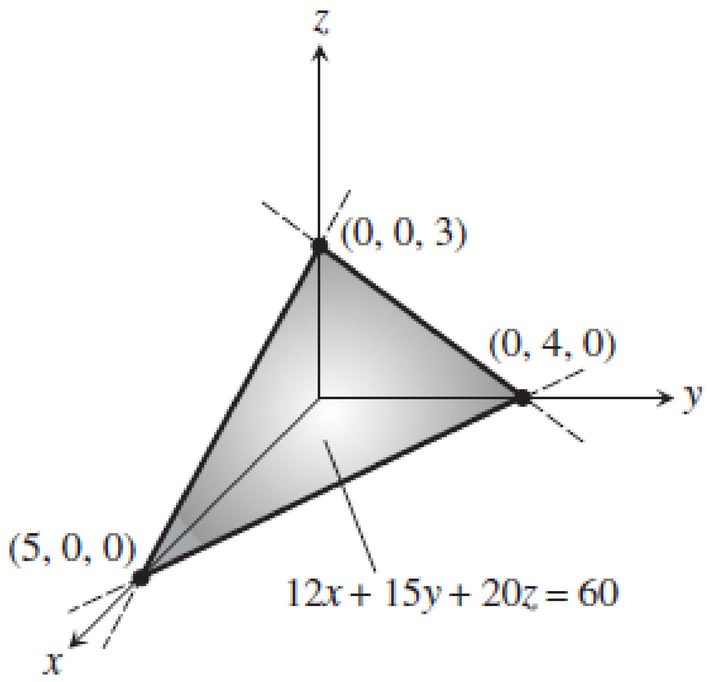
Tenglikning har ikki tomonini 60 ga bo'lamiz:

$$\frac{x}{5} + \frac{y}{4} + \frac{z}{3} = 1.$$

$$a = 5, \qquad b = 4, \qquad c = 3.$$



12-rasm.



13-rasm.

2-misol. Tekislik $M(6;-10;1)$ nuqtadan o‘tib, abssissalar o‘qidan $a=-3$ va applikatalar o‘qidan $c=2$ kesmalar ajratadi. Bu tekislikning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasini tuzing.

Yechish. (3) tenglamaga ko‘ra

$$\frac{x}{-3}+\frac{y}{b}+\frac{z}{2}=1.$$

Tekislik $M(6;-10;1)$ nuqtadan o‘tganligi uchun uning koordinatalari yuqoridagi tenglamani qanoatlantiradi:

$$\frac{6}{-3}+\frac{-10}{b}+\frac{1}{2}=1 \ \Rightarrow \ b=-4.$$

Tekislikning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasi yozamiz:

$$\frac{x}{-3}+\frac{y}{-4}+\frac{z}{2}=1.$$

3-misol. *Oz* o‘qi va $M(3;-4;6)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing.

Yechish. Shartga ko‘ra, tekislik *Oz* o‘qidan o‘tganligi uchun $C=0, D=0$ bo‘lib, tekislik umumiy tenglamasi 4-xossaga binoan $Ax+By=0$ ko‘rinishda bo‘ladi. Bu tenglamaga M nuqtaning koordinatalarini qo‘yamiz:

$$A \cdot 3 + B \cdot (-4) = 0 \;\Rightarrow\; B = \frac{3}{4}A.$$

$$Ax + By = 0 \Rightarrow\; Ax + \frac{3}{4}Ay = 0.$$

Oxirgi tenglikni A ga qisqartiramiz:

$$x + \frac{3}{4}y = 0 \;\;\Rightarrow\;\; 4x + 3y = 0.$$

Demak, misol shartiga ko‘ra, tekislik tenglamasi $4x + +3y = 0$ ko‘rinishda bo‘ladi.

2-§. Nuqtadan tekislikkacha bo‘lgan masofa

Berilgan $M_0(x_0, y_0, z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekislikkacha bo‘lgan masofa nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa formulasi kabi aniqlanadi va quyidagi ko‘rinishda bo‘ladi:

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}. \tag{4}$$

4-misol. $M_0(3; 4; 2)$ nuqtadan $4x + 3y + 12z - 9 = 0$ tekislikkacha bo‘lgan masofani toping.

Yechish. $A = 4,\; B = 3,\; C = 12,\; D = -14.$
$x_0 = 3,\; y_0 = 4,\; z_0 = 2$. Bu qiymatlarni (4) formulaga qo‘yamiz:

$$d = \frac{|4 \cdot 3 + 3 \cdot 4 + 12 \cdot 2 - 9|}{\sqrt{4^2 + 3^2 + 12^2}} = \frac{39}{13} = 3.$$

Demak, berilgan nuqtadan tekislikkacha bo‘lgan masofa $d = 3$ ga teng.

3-§. Berilgan uchta nuqtadan o‘tuvchi tekislik tenglamasi

$M_1(x_1, y_1, z_1),\; M_2(x_2, y_2, z_2),\; M_3(x_3, y_3, z_3)$ nuqtalar berilgan bo‘lib, ular bir to‘g‘ri chiziqqa tegishli bo‘lmasin. Berilgan $M_1,\; M_2$, M_3 nuqtalardan o‘tuvchi tekislik tenglamasini keltirib chiqaramiz. $M(x, y, z)$ – tekislikdagi ixtiyoriy nuqta bo‘lsin. M_1 nuqtadan $M,\; M_2$ va M_3 nuqtalarga vektor yo‘naltiramiz (14-rasm):

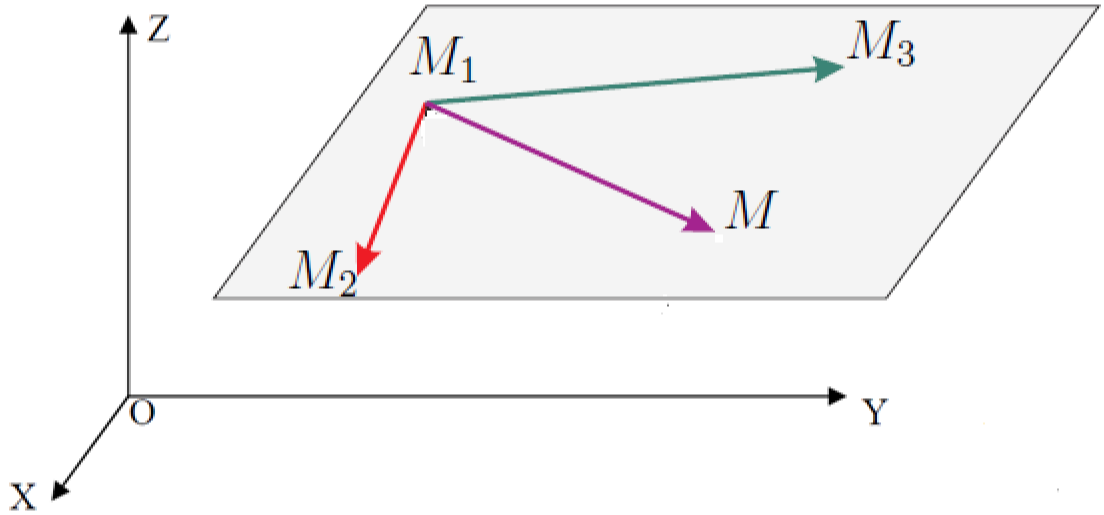
$$\overrightarrow{M_1M} = (x - x_1, \quad y - y_1, \quad z - z_1),$$
$$\overrightarrow{M_2M_1} = (x_2 - x_1, \quad y_2 - y_1, \quad z_2 - z_1),$$
$$\overrightarrow{M_3M_1} = (x_3 - x_1, \quad y_3 - y_1, \quad z_3 - z_1).$$

Uch vektorlarning komplanarlik shartiga asosan,

$$\overrightarrow{M_1M} \cdot \left(\overrightarrow{M_1M_2} \times \overrightarrow{M_1M_3}\right) = 0.$$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0. \qquad (5)$$

(5) formula ***berilgan uchta nuqtadan o'tuvchi tekislik tenglamasi*** bo'ladi.



14-rasm.

5-misol. $M_1(0; 2; 1)$, $M_2(4; -1; 1)$, $M_3(3; 2; 4)$ nuqtalardan o'tuvchi tekislik tenglamasi tuzing.

Yechish. (5) formulaga ko'ra, izlanayotgan tekislik

$$\begin{vmatrix} x - 0 & y - 2 & z - 1 \\ 4 - 0 & -1 - 2 & 1 - 1 \\ 3 - 0 & 2 - 2 & 4 - 1 \end{vmatrix} = 0, \text{ yoki}$$

$$\begin{vmatrix} x & y - 2 & z - 1 \\ 4 & -3 & 0 \\ 3 & 0 & 3 \end{vmatrix} = 0$$

tenglama bilan ifodalanadi. Determinantni hisoblaymiz:

$$-9x + 9(z - 1) - 12(y - 2) = 0,$$

$$-9x + 9z - 9 - 12y + 24 = 0,$$

$$-9x + 9z - 12y + 15 = 0.$$

Oxirgi tenglamani -3 ga bo'lamiz:

$$3x + 4y - 3z - 5 = 0.$$

hosil bo‘lgan tenglama berilgan uchta nuqtadan o‘tuvchi tekislik tenglamasidir.

4-§. Ikki tekislik orasidagi burchak. Tekisliklarning parallellik va perpendikulyarlik shartlari

α_1 va α_2 tekisliklar

$$\alpha_1 \colon A_1x + B_1y + C_1z + D_1 = 0, \qquad \overrightarrow{N_1} = (A_1, \quad B_1, \quad C_1),$$
$$\alpha_2 \colon A_2x + B_2y + C_2z + D_2 = 0, \qquad \overrightarrow{N_2} = (A_2, \quad B_2, \quad C_2)$$

umumiy tenglamasi bilan berilgan bo‘lsin. Tekisliklar orasidagi burchak ularning normal vektorlari orasidagi burchak kabi aniqlanadi:

$$\cos\varphi = \pm\frac{\overrightarrow{N_1}\cdot\overrightarrow{N_2}}{|\overrightarrow{N_1}|\cdot|\overrightarrow{N_2}|} = \pm\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{{A_1}^2+{B_1}^2+{C_1}^2}\cdot\sqrt{{A_2}^2+{B_2}^2+{C_2}^2}}. \qquad (6)$$

Agar tekisliklar parallel bo‘lsa, ularning normal vektorlari kollinear bo‘ladi, ya’ni

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \qquad (7)$$

shart bajarilishi kerak.

Agar $\overrightarrow{N_1} \perp \overrightarrow{N_2}$ bo‘lsa, ya’ni $\overrightarrow{N_1} \cdot \overrightarrow{N_2} = 0$ yoki

$$A_1A_2 + B_1B_2 + C_1C_2 = 0 \qquad (8)$$

shart bajarilsa, α_1 va α_2 tekisliklar o‘zaro perpendikulyar bo‘ladi.

6-misol. $x + y + \sqrt{2}z + 7 = 0$ va $z = 0$ tekisliklar orasidagi burchakni toping.

Yechish. Tekisliklar normal vektori

$$\overrightarrow{N_1}\left(1; 1; \sqrt{2}\right), \qquad \overrightarrow{N_2}(0; 0; 1).$$

Normal vektor koordinatalarini (6) formulaga qo‘yamiz:

$$\cos\varphi = \pm\frac{1\cdot 0 + 1\cdot 0 + \sqrt{2}\cdot 1}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}\cdot\sqrt{0^2 + 0^2 + 1^2}} = \pm\frac{\sqrt{2}}{2}.$$

$$\cos\varphi = \pm\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} \varphi_1 = 45°, \\ \varphi_2 = 135°. \end{cases}$$

Demak, tekisliklar kesishishi natijasida hosil bo‘lgan burchaklar 45° va 135° ga teng.

7-misol. $M(3;2;4)$ nuqtadan o‘tib, $2x-6y-3z+5=0$ tekislikka parallel bo‘lgan tekislik tenglamasini toping.

Yechish. Berilgan tekislikning normal vektori koordinatalari yozamiz: $\overrightarrow{N_1}(2;\ -6;\ -3)$. $\vec{N}(A;\ B;\ C)$ - izlanayotgan tekislikning normal vektori bo‘lsin. Shartga ko‘ra, M nuqtadan o‘tuvchi tekislik berilgan tekislikka parallel, u holda (7) formulaga asosan:

$$\frac{2}{A}=\frac{-6}{B}=\frac{-3}{C} \Rightarrow A=2;\ B=\ -6;\ \ C=-3.$$

Berilgan nuqta orqali o‘tuvchi va berilgan vektorga perpendikulyar

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

tekislik tenglamasiga ko‘ra:

$$2(x-3)-6(y-2)-3(z-4)=0.$$

Qavslarni ochib, tekislik tenglamasini umumiy ko‘rinishga keltiramiz:

$$2x-6y-3z+18=0.$$

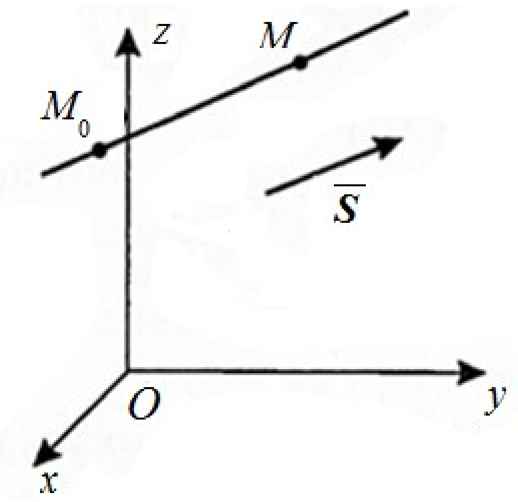
5-§. Fazoda to‘g‘ri chiziq tenglamasi. To‘g‘ri chiziqning kanonik va parametrik tenglamalari

Faraz qilaylik, $M_0(x_0,y_0,z_0)$ nuqta $\vec{S}(n;m;p)$ vektorga parallel bo‘lgan l to‘g‘ri chiziqqa tegishli bo‘lsin. Ma’lumki, to‘g‘ri chiziq nuqtalar to‘plamidan iborat bo‘lgani uchun, unda ixtiyoriy $M(x,y,z)$ nuqtani tanlaymiz. $\overrightarrow{M_0M}$ vektor koordinatalarini aniqlaymiz:

$$\overrightarrow{M_0M}=(x-x_0;y-y_0;z-z_0).$$

Farazga ko‘ra, $\overrightarrow{M_0M} \parallel \vec{S}$, u holda vektorlarning kollinearlik shartiga binoan (15-rasm), ularning koordinatalari proporsionaldir, ya’ni

$$\frac{x-x_0}{n}=\frac{y-y_0}{m}=\frac{z-z_0}{p}.$$



15-rasm.

1-ta'rif. Ushbu ko'rinishdagi

$$\frac{x-x_0}{n}=\frac{y-y_0}{m}=\frac{z-z_0}{p} \qquad (9)$$

tenglama fazoda l ***to'g'ri chiziqning kanonik tenglamasi*** deyiladi, $\vec{S}(n,m,p)$ vektor ***to'g'ri chiziqning yo'naltiruvchi vektori*** deyiladi.

(9) tenglamani t parametrga tenglashtiramiz:

$$\frac{x-x_0}{n}=\frac{y-y_0}{m}=\frac{z-z_0}{p}=t, \qquad (t\epsilon R).$$

t parametr yordamida to'g'ri chiziqning yangi ko'rinishdagi tenglamasini yozamiz.

2-ta'rif. Ushbu ko'rinishdagi

$$\begin{cases} x = nt + x_0 \\ y = mt + y_0 \\ z = pt + z_0 \end{cases} \qquad (10)$$

tenglamalar sistemasi l ***to'g'ri chiziqning parametrik tenglamasi*** deyiladi.

8-misol. $A(3;4;\ -2)$ va $B(7;5;\ -5)$ nuqtalardan o'tuvchi to'g'ri chiziqning kanonik va parametrik tenglamalarini yozing.

Yechish. $\overrightarrow{AB}$ vektor koordinatalarini aniqlaymiz:

$$\overrightarrow{AB}=\langle 7-3; 5-4;\ -5-(-2)\rangle \Rightarrow \overrightarrow{AB}=(4;1;\ -3).$$

Ixtiyoriy to'g'ri chiziq cheksiz ko'p nuqtalar to'plamidan iborat bo'lgani uchun, shu to'g'ri chiziqqa tegishli bo'lgan ixtiyoriy nuqta sifatida $C(x,y,z)$ nuqtani tanlaymiz va $\overrightarrow{AC}$ vektor koordinatalarini topamiz:

$$\overrightarrow{AC} = (x-3; y-4; z+2).$$

Ravshanki, $\overrightarrow{AC}$ va $\overrightarrow{AB}$ vektorlar kollinear, u holda ularning koordinatalari proporsionaldir. Shuning uchun A va B nuqtalar orqali o‘tuvchi to‘g‘ri chiziqning kanonik tenglamasi

$$\frac{x-3}{4} = \frac{y-4}{1} = \frac{z+2}{-3}$$

ko‘rinishda bo‘ladi.

Kanonik tenglamadagi har bir kasrni t parametrga tenglashtiramiz:

$$\begin{cases} \dfrac{x-3}{4} = t, \\ \dfrac{y-4}{1} = t, \\ \dfrac{z+2}{-2} = t. \end{cases}$$

Sistemadagi har bir tenglamani maxrajdagi mos songa ko‘paytiramiz va AB to‘g‘ri chiziqning parametrik tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} x = 4t+3, \\ y = t+4, \\ z = -3t-2. \end{cases}$$

Natijada, AB to‘g‘ri chiziqning kanonik va parametrik tenglamalari topiladi:

$$\frac{x-3}{4} = \frac{y-4}{1} = \frac{z+2}{-3}; \qquad \begin{cases} x = 4t+3, \\ y = t+4, \\ z = -3t-2. \end{cases}$$

9-misol. α: $x - 2y - 2z + 4 = 0$ tekislikdan $M(5; 1; -1)$ nuqtagacha bo‘lgan masofani toping.

Yechish. M nuqta va α tekislik orasidagi masofa M nuqtadan tekislikka tushirilgan perpendikulyar uzunligiga tengdir. Bu perpendikulyar tekislik normal vektori $\vec{N}$ ga parallel bo‘ladi. α tekislikning umumiy tenglamasidan normal vektor koordinatalarini aniqlaymiz:

$$\vec{N} = (1; -2; -2).$$

M nuqtadan $\vec{N}$ vektorga parallel to‘g‘ri chiziq o‘tkazamiz. Uning kanonik va parametrik tenglamalari quyidagicha bo‘ladi:

$$\frac{x-5}{1}=\frac{y-1}{-2}=\frac{z+1}{-2}=t,$$
$$\begin{cases} x=t+5, \\ y=-2t+1, \\ z=-2t-1. \end{cases}$$
α tekislikka perpendikulyar bo‘lgan to‘g‘ri chiziq tekislik bilan $K(x,y,z)$ nuqtada kesishadi. Parametrik tenglamadagi x, y va z ning t parametr orqali ifodasini α tekislik tenglamasiga qo‘yamiz:
$$t+5-2(-2t+1)-2(-2t-1)+4=0,$$
$$t+5+4t-2+4t+2+4=0,$$
$$9t+9=0 \Rightarrow t=-1.$$
t ning qiymatini to‘g‘ri chiziqning parametrik tenglamasiga qo‘yib, $K(x,y,z)$ nuqtaning koordinatalari aniqlanadi:
$$x=-1+5=4,$$
$$y=-2\cdot(-1)+1=3,$$
$$z=(-2)\cdot(-1)-1=1.$$
Demak, K nuqta $(4;\ 3;\ 1)$ koordinataga ega.

U holda M va K nuqtalar orasidagi masofa
$$|MK|=\sqrt{(4-5)^2+(3-1)^2+\big(1-(-1)\big)^2}$$
$$|MK|=3.$$
Shunday qilib, M nuqta va α tekislik orasidagi masofa $d=3$ ga teng.

6-§. Fazoda berilgan ikki nuqta orqali o‘tuvchi to‘g‘ri chiziq tenglamasi

Fazoda $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtalar berilgan bo‘lsin. Bu nuqtalardan o‘tuvchi to‘g‘ri chiziqning kanonik tenglamasini topamiz. $\overrightarrow{M_1M_2}$ vektor koordinatalarini aniqlaymiz:
$$\overrightarrow{M_1M_2}=(x_2-x_1;\ y_2-y_1;\ z_2-z_1).$$
To‘g‘ri chiziq cheksiz ko‘p nuqtalar to‘plamidan iborat bo‘lgani uchun unga tegishli bo‘lgan ixtiyoriy $M(x,y,z)$ nuqtani olamiz va bu nuqtaga M_1 nuqtadan vektor yo‘naltiramiz:
$$\overrightarrow{M_1M}=(x-x_1;\ y-y_1;\ z-z_1).$$

Ravshanki, $\overrightarrow{M_1M}$ va $\overrightarrow{M_1M_2}$ vektorlar kollinear. Ikki vektorning parallellik shartiga asosan

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (11)$$

tengliklar o‘rinli bo‘ladi.

(11) tenglamaga fazoda berilgan ***ikki nuqtadan o‘tuvchi to‘g‘ri chiziqning kanonik tenglamasi*** deyiladi.

10-misol. Fazoda $M_1(3;\ -2;4), M_2(-7;\ -3;6)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning kanonik tenglamasini yozing.

Yechish. M_1 nuqta koordinatalari $x_1=3, y_1=-2, z_1=4$ va M_2 nuqta koordinatalari $x_2=-7, y_2=-3, z=6$ qiymatlarini (11) tenglamaga qo‘yamiz:

$$\frac{x-3}{-7-3}=\frac{y-(-2)}{-3-(-2)}=\frac{z-4}{6-4}$$

natijada

$$\frac{x-3}{-10}=\frac{y+2}{-1}=\frac{z-4}{2}$$

tenglama hosil bo‘ladi. Bu tenglama berilgan M_1 va M_2 nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasidir.

7-§. Fazoda to‘g‘ri chiziqning umumiy tenglamasi

Fazoda har bir to‘g‘ri chiziq orqali cheksiz ko‘p tekisliklar o‘tkazish mumkin. Bu tekisliklardan ixtiyoriy ikkitasi kesishishi natijasida fazoda to‘g‘ri chiziqni hosil qiladi. Ushbu kesishuvchi ixtiyoriy ikki tekislik tenglamalari birgalikda fazoda to‘g‘ri chiziqning umumiy tenglamasini ifodalaydi. Dekart koordinatalar sistemasida o‘zaro parallel bo‘lmagan α_1 va α_2 tekisliklar berilgan bo‘lsin (16-rasm):

$$\alpha_1: A_1x+B_1y+C_1z+D_1=0,$$
$$\alpha_2: A_2x+B_2y+C_2z+D_2=0.$$

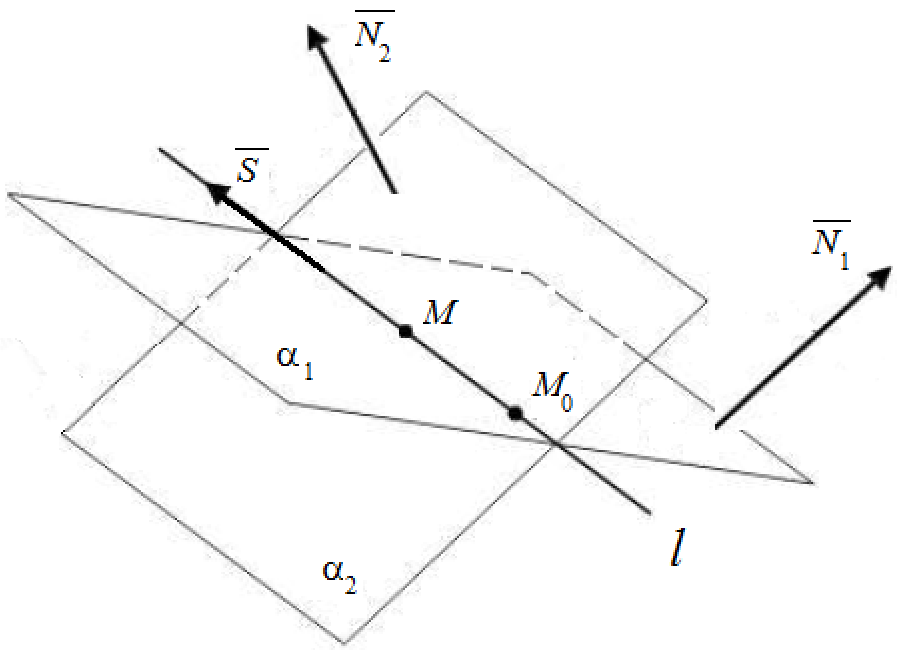
Tekisliklar o‘zaro parallel bo‘lmagani uchun $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$ tengliklar bir vaqtda bajarilmaydi.

Faraz qilaylik, $\frac{A_1}{A_2}\neq\frac{B_1}{B_2}$ bo‘lsin.

Ushbu

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0, \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases} \qquad (12)$$

sistemaga ***fazoda to‘g‘ri chiziqning umumiy tenglamasi*** deyiladi.



16-rasm.

Ma’lumki, bu sistema cheksiz ko‘p yechimga ega. Bu yechimlarni topish uchun noma’lumlardan birining tayinlangan qiymatini olamiz. z ga z_0 qiymat berib, (12) sistemani

$$\begin{cases} A_1x + B_1y = -D_1 - C_1z_0 \\ A_2x + B_2y = -D_2 - C_2z_0 \end{cases} \qquad (13)$$

ko‘rinishda ifodalaymiz. $\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$ munosabatni e’tiborga olib, (13) sistemani Kramer usuli bo‘yicha yechamiz:

$$x = \frac{\begin{vmatrix} -D_1 - C_1z_0 & B_1 \\ -D_2 - C_2z_0 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}, \qquad y = \frac{\begin{vmatrix} A_1 & -D_1 - C_1z_0 \\ A_2 & -D_2 - C_2z_0 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}.$$

x va y ning qiymatlarini z_0 qiymatga mos ravishda x_0 va y_0 orqali belgilaymiz. Shunday qilib, (12) tenglama bilan aniqlangan to‘g‘ri chiziqda $M_0(x_0, y_0, z_0)$ nuqta topildi. To‘g‘ri chiziq cheksiz ko‘p nuqtalar to‘plamidan iborat bo‘lgani uchun, unda ixtiyoriy $M(x, y, z)$ nuqtani tanlaymiz va $\overrightarrow{M_0M}$ vektor koordinatalarini aniqlaymiz:

$$\overrightarrow{M_0M} = (x - x_0, \qquad y - y_0, \qquad z - z_0).$$

To‘g‘ri chiziq ikki tekislikning kesishish chizig‘i bo‘lgani uchun uning yo‘naltiruvchi $\vec{S}$ vektori $\overrightarrow{N_1} \times \overrightarrow{N_2}$ vektorga kollineardir va vektor ko‘paytmani to‘g‘ri chiziqning yo‘naltiruvchi vektori sifatida olinadi:

$$\vec{S} = \overrightarrow{N_1} \times \overrightarrow{N_2}.$$

$$\vec{S} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = \vec{i}\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} + \vec{j}\begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} + \vec{k}\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}.$$

$$\vec{S} = n \cdot \vec{i} + m \cdot \vec{j} + p \cdot \vec{k},$$

bu yerda $n = \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}$, $m = \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}$, $p = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}$.

$\overrightarrow{M_0M}$ va $\vec{S}$ vektorlar parallel bo‘lgani uchun

$$\frac{x - x_0}{n} = \frac{y - y_0}{m} = \frac{z - z_0}{p} \tag{14}$$

tengliklarga ega bo‘lamiz.

Shunday qilib, fazoda to‘g‘ri chiziqning umumiy tenglamasini uning kanonik ko‘rinishdagi tenglamasi orqali ifodaladik.

(12) tenglamadan bir marta y ni, ikkinchi marta x ni yo‘qotib, **to‘g‘ri chiziqning proyeksiyalari bo‘yicha yozilgan tenglamalariga** ega bo‘lamiz:

$$\begin{cases} x = mz + a, \\ y = nz + b. \end{cases} \tag{15}$$

(15) tenglamalarni kanonik ko‘rinishda yozish mumkin:

$$\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-0}{1}.$$

11-misol. To‘g‘ri chiziqning

$$\begin{cases} 2x - 3y + z - 5 = 0, \\ 3x + y - 2z - 4 = 0 \end{cases}$$

umumiy tenglamasini kanonik ko‘rinishda ifodalang.

Yechish. Sistema yechimini topish uchun noma’lumlardan birini ixtiyoriy tanlaymiz. Masalan, $z = 1$. U holda

$$\begin{cases} 2x - 3y = 4, \\ 3x + y = 6. \end{cases}$$

$x = 2,\ y = 0$. To‘g‘ri chiziqdagi $M_0(2; 0; 1)$ nuqtani aniqladik. To‘g‘ri chiziqning yo‘naltiruvchi $\vec{S} = \overrightarrow{N_1} \times \overrightarrow{N_2}$ vektorini topamiz:

$$\vec{S}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & -3 & 1\\ 3 & 1 & -2\end{vmatrix}=6\vec{i}+3\vec{j}+2\vec{k}+9\vec{k}-\vec{i}+4\vec{j}=$$

$$=5\vec{i}+7\vec{j}+11\vec{k}.$$

Demak, to‘g‘ri chiziqning kanonik tenglamasi

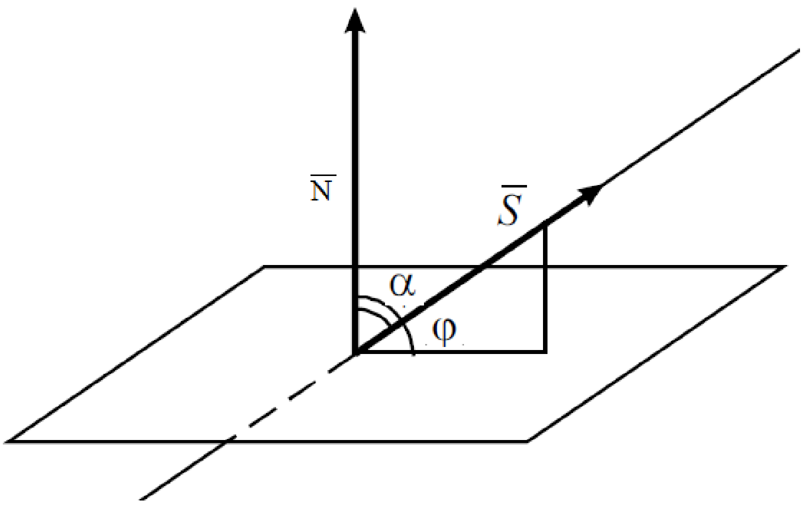
$$\frac{x-2}{5}=\frac{y-0}{7}=\frac{z-1}{11}$$

ko‘rinishda bo‘ladi.

8-§. To‘g‘ri chiziq va tekislik orasidagi burchak. To‘g‘ri chiziq va tekislikning parallellik va perpendikulyarlik shartlari

Fazoda $\frac{x-a}{n}=\frac{y-b}{m}=\frac{z-c}{p}$ to‘g‘ri chiziq va $Ax+By+Cz+D=0$ tekislik berilgan bo‘lsin. $\vec{S}=(n;\ m;\ p)$ - to‘g‘ri chiziqning yo‘naltiruvchi vektori, $\vec{N}=(A;\ B;\ C)$ tekislikning normal vektori.

Faraz qilaylik, to‘g‘ri chiziq tekislikni φ burchak ostida kesib o‘tsin (17-chizma).



17-rasm.

φ to‘g‘ri chiziq va uning tekislikdagi proyeksiyasi orasidagi burchakdir.

$\vec{N}$ va $\vec{S}$ vektorlar orasidagi burchak $\alpha=90°-\varphi$ ga teng. Ikki vektor orasidagi burchak formulasiga ko‘ra

$$\cos\alpha = \frac{\vec{N}\cdot\vec{S}}{|\vec{N}|\cdot|\vec{S}|},$$

$$\cos(90^\circ-\varphi) = \frac{\vec{N}\cdot\vec{S}}{|\vec{N}|\cdot|\vec{S}|} \Rightarrow \sin\varphi = \frac{\vec{N}\cdot\vec{S}}{|\vec{N}|\cdot|\vec{S}|}.$$

Demak, to‘g‘ri chiziq va tekislik orasidagi burchak

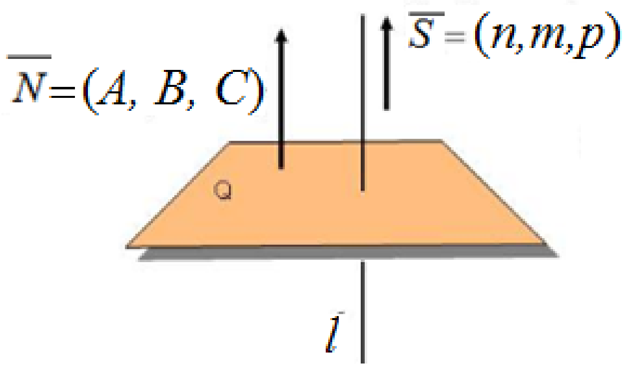
$$\sin\varphi = \frac{|A\cdot n+B\cdot m+C\cdot p|}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{n^2+m^2+p^2}}. \qquad (16)$$

formula bilan aniqlanadi.

Agar to‘g‘ri chiziq tekislikka perpendikulyar bo‘lsa, $\vec{S}\parallel\vec{N}$ bo‘ladi (18-rasm). Ikki vektorning parallellik shartiga binoan,

$$\frac{A}{n} = \frac{B}{m} = \frac{C}{p} \qquad (17)$$

(17) formula ***to‘g‘ri chiziq va tekislikning perpendikulyarlik shartidir***.



18-rasm.

Agar to‘g‘ri chiziq tekislikka parallel bo‘lsa, $\vec{N}\perp\vec{S}$ bo‘ladi (19-rasm).

$$\vec{N}\cdot\vec{S}=0 \Rightarrow A\cdot n+B\cdot m+C\cdot p=0 \qquad (18)$$

(18) formula ***to‘g‘ri chiziq va tekislikning parallellik shartidir***.

12-misol. $\frac{x}{2}=\frac{y-1}{1}=\frac{z+1}{2}$ to‘g‘ri chiziq va $x+2y+$ $+3z-29=0$ tekislik orasidagi burchakni toping.

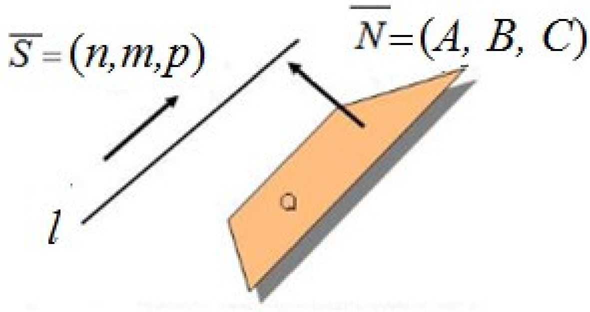
Yechish. Normal va yo‘naltiruvchi vektorlar koordinatalarini yozamiz:

$$\vec{N}(1;2;3), \quad \vec{S}(2;1;2).$$

(16) formulaga asosan

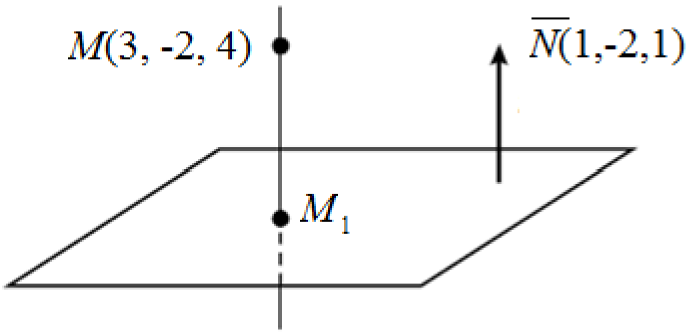
$$\sin\varphi = \frac{|1\cdot 2 + 2\cdot 1 + 3\cdot 2|}{\sqrt{1^2+2^2+3^2}\cdot\sqrt{2^2+1^2+2^2}},$$

$$\sin\varphi = \frac{10}{3\sqrt{14}}.$$



19-rasm.

13-misol. $M(3,-2,4)$ nuqtaning $x - 2y + z - 5 = 0$ tekislikdagi proyeksiyasini toping (20-rasm).



20-rasm.

Yechish. $M(3,-2,4)$ nuqtadan o‘tuvchi va berilgan $x - 2y + z - 5 = 0$ tekislikka perpendikulyar to‘g‘ri chiziq tenglamasini tuzamiz. Tekislikning $\vec{N}(1,-2,1)$ - normal vektori shu to‘g‘ri chiziqning $\vec{S}$ - yo‘naltiruvchi vektori bo‘ladi:

$$\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-4}{1}.$$

M_1 nuqtaning koordinatalarini aniqlash uchun to‘g‘ri chiziqning kanonik tenlamasini parametrik ko‘rinishdagi tenglamaga keltiramiz:

$$\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-4}{1} = t,$$

$$x = t + 3;\ \ y = -2t - 2;\ z = t + 4.$$

t ning qiymatini aniqlash uchun ushbu ifodalarni tekislik tenglamasiga qoyamiz:

$$x - 2y + z - 7 = 0 \Longrightarrow\ t + 3 + 4t + 4 + t + 4 - 5 = 0\ \Longrightarrow t = -1.$$

Bundan, M_1 nuqtaning koordinatalarini aniqlanadi:

$$x = t + 3 = -1 + 3 = 2;\ \ y = -2t - 2 = 2 - 2 = 0;$$
$$z = t + 4 = -1 + 4 = 3.$$

Shuyday qilib, berilgan M nuqtaning tekislikdagi proyeksiyasi aniqlandi: $M_1(2; 0; 3)$.

9-§. Fazoda ikki to‘g‘ri chiziqning parallellik, perpendikulyarlik va bitta tekislikda yotish shartlari

Fazoda l_1 va l_2 to‘g‘ri chiziqlar berilgan bo‘lsin.

$$l_1:\ \frac{x - a_1}{n_1} = \frac{y - b_1}{m_1} = \frac{z - c_1}{p_1},$$

$$\vec{S}_1(n_1, m_1, p_1), \qquad M_1(a_1, b_1, c_1).$$

$$l_2:\ \frac{x - a_2}{n_2} = \frac{y - b_2}{m_2} = \frac{z - c_2}{p_2},$$

$$\vec{S}_2(n_2, m_2, p_2),\ \ M_2(a_2, b_2, c_2).$$

l_1 va l_2 to‘g‘ri chiziqlarning parallellik (yoki ustma-ust tushishi) sharti

$$\frac{n_2}{n_1} = \frac{m_2}{m_1} = \frac{p_2}{p_1} \qquad (19)$$

tengliklarning bajarilishidan, perpendikulyarlik sharti esa

$$n_1 n_2 + m_1 m_2 + p_1 p_2 = 0 \qquad (20)$$

tenglikning bajarilishidan iboratdir.

Ma’lumki, agar $\vec{S}_1 \nparallel \vec{S}_2$ bo‘lib, $\overrightarrow{M_1M_2}$, $\vec{S}_1$ va $\vec{S}_2$ vektorlar komplanar bo‘lsa, ya’ni $\overrightarrow{M_1M_2} \cdot \left(\vec{S}_1 \times \vec{S}_2\right)$=0 o‘rinli bo‘lsa, u holda l_1 va l_2 to‘g‘ri chiziqlar kesishadi.

Demak, l_1 va l_2 to‘g‘ri chiziqlarning bitta tekislikda yotish sharti

$$\begin{vmatrix} a_2-a_1 & b_2-b_1 & c_2-c_1 \\ n_1 & m_1 & p_1 \\ n_2 & m_2 & p_2 \end{vmatrix} = 0 \qquad (21)$$

tenglik bajarilishidan iboratdir.

Agar $\overrightarrow{M_1M_2}\cdot\left(\vec{S}_1\times\vec{S}_2\right)\neq 0$ bo‘lsa, u holda l_1 va l_2 to‘g‘ri chiziqlar ayqash to‘g‘ri chiziqlar bo‘ladi.

14-misol. Berilgan ikki to‘g‘ri chiziqning fazoda o‘zaro joylashishini tushuntiring:

$$l_1: x-1=\frac{y}{2}=z+2,\ \ \vec{S}_1(1,2,1), \qquad M_1(1,0,-2),$$

$$l_2:\ \frac{x+3}{2}=y=z,\ \ \vec{S}_2(2,1,1), \qquad M_2(-3,0,0).$$

Yechish. $\vec{S}_1 \nparallel \vec{S}_2 \ \Longrightarrow\ l_1$ va l_2 parallel emas. $\overrightarrow{M_1M_2}\cdot\left(\vec{S}_1\times\vec{S}_2\right)$ aralash ko‘paytmani hisoblaymiz:

$$\overrightarrow{M_1M_2}\cdot\left(\vec{S}_1\times\vec{S}_2\right)=\begin{vmatrix} -4 & 0 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}=-10.$$

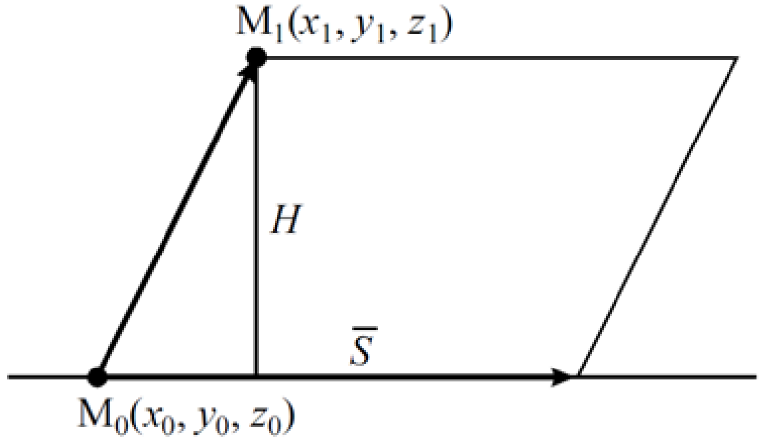
$\overrightarrow{M_1M_2}\cdot\left(\vec{S}_1\times\vec{S}_2\right)\neq 0 \ \Longrightarrow\ l_1$ va l_2 to‘g‘ri chiziqlar ayqash to‘g‘ri chiziqlardir.

10-§. Tekislik va to‘g‘ri chiziqqa doir aralash masalalar

1.$M_1(x_1,x_2,x_3)$ nuqtadan $\frac{x-x_0}{n}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ to‘g‘ri chiziqgacha bo‘lgan masofa (21-rasm).

Ma’lumki, izlanayotgan d masofa $\vec{S}$ va $\overrightarrow{M_0M_1}$ vektorlardan yasalgan parallelogramm balandligiga teng. Shunday qilib,

$$d = H = \frac{\left|\vec{S} \times \overrightarrow{M_0M_1}\right|}{\left|\vec{S}\right|}. \qquad (22)$$



$M_0\ (x_0, y_0, z_0)$,
$\bar{S} = (n, m, p)$.

21-rasm.

2. Ayqash to‘g‘ri chiziqlar orasidagi eng qisqa masofa. Ma’lumki, agar l_1 va l_2 to‘g‘ri chiziqlar ayqash bo‘lsa, u holda ikkita parallel tekisliklar mavjud bo‘lib, ulardan birida l_1 to‘g‘ri chiziq, ikkichisida l_2 to‘g‘ri chiziq yotadi. Shu sabali,

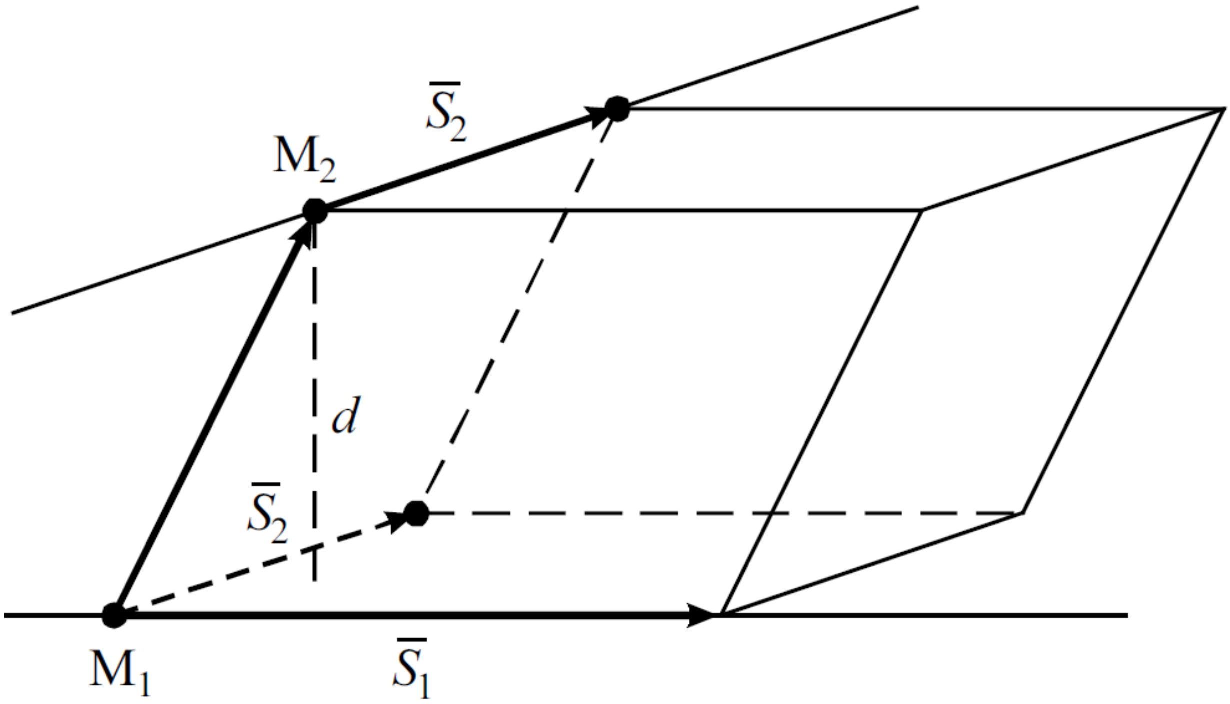
$$l_1: \frac{x - a_1}{n_1} = \frac{y - b_1}{m_1} = \frac{z - c_1}{p_1},$$

bu yerda $\vec{S}_1(n_1, m_1, p_1)$; $M_1(a_1, b_1, c_1)$;

$$l_2: \frac{x - a_2}{n_2} = \frac{y - b_2}{m_2} = \frac{z - c_2}{p_2},$$

bu yerda $\vec{S}_2(n_2, m_2, p_2)$; $M_2(a_2, b_2, c_2)$; ayqash to‘g‘ri chiziqlar orasidagi masofa $\overrightarrow{M_1M_2}$, $\vec{S}_1$ va $\vec{S}_2$ vektorlar asosiga qurilgan parallelepiped balandligiga teng bo‘ladi ($\vec{S}_1$ va $\vec{S}_2$ vektorlar bitta boshlang‘ich nuqtaga keltirilgan) (22-rasm).

$$d = H = \frac{V_{par-d}}{S_{asos}} = \frac{\left|\overrightarrow{M_1M_2} \cdot (\vec{S}_1 \times \vec{S}_2)\right|}{\left|\vec{S}_1 \times \vec{S}_2\right|}. \qquad (23)$$



22-rasm.

3. Berilgan to‘g‘ri chiziqdan o‘tuvchi va berilgan tekislikka perpendikulyar tekislik tenglamasi.

Tekislik va to‘g‘ri chiziq berilgan bo‘lsin:

$$\alpha: Ax + By + Cz + D = 0, \quad \vec{N}(A, B, C),$$

$$l: \ \frac{x-a}{n} = \frac{y-b}{m} = \frac{z-c}{p},$$

bu yerda $\vec{S}(n, m, p)$, $\qquad M_1(a, b, c)$.

Shartga ko‘ra, berilgan l to‘g‘ri chiziq islanayotgan tekislikda yotadi va bu tekislik berilgan α tekislikka perpendikulyardir. U holda, $\vec{N} \parallel \vec{S}$ bo‘ladi.

Ma’lumki, $\vec{S} \parallel \overrightarrow{MM_1}$, bu yerda $M(x, y, z)$ - l to‘g‘ri chiziqdagi ixtiyoriy nuqta. $\vec{S}$ va $\overrightarrow{MM_1}$ vektorlar izlanayotgan tekislikda yotadi. Uch vektorning komplanarlik shartiga asosan,

$$\overrightarrow{MM_1} \cdot (\vec{S} \times \vec{N}) = 0.$$

Aralash ko‘paytmani vektorlarning koordinatalari orqali ifodalaymiz:

$$\begin{vmatrix} x-a & y-b & z-c \\ n & m & p \\ A & B & C \end{vmatrix} = 0.$$

Natijada, berilgan to‘g‘ri chiziqdan o‘tuvchi va berilgan tekislikka perpendikulyar tekislik tenglamasini hosil qilamiz.

15-misol. $\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z-2}{2}$ va $\frac{x}{-1}=\frac{y-2}{3}=\frac{z}{3}$ to‘g‘ri chiziqlar orasidagi eng qisqa masofani toping.

Yechish. Berilgan to‘g‘ri chiziqlarning yo‘naltiruvchi vectorlari:

$$\vec{S}_1(1,-1,2);\ \vec{S}_2(-1,3,3).$$

l_1 to‘g‘ri chiziqdan $M_1(3,1,2)$ nuqtani, l_2 to‘g‘ri chiziqdan $M_2(0,2,0)$ nuqtani olamiz va $\overrightarrow{M_1M_2}$ vektor koordinatalarini aniqlaymiz:

$$\overrightarrow{M_1M_2}(-3,1,-2).$$

$$\overrightarrow{M_1M_2}\cdot\left(\vec{S}_1\times\vec{S}_2\right)=\begin{vmatrix}-3 & 1 & -2\\ 1 & -1 & 2\\ -1 & 3 & 3\end{vmatrix}=18.$$

$$\vec{S}_1\times\vec{S}_2=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ 1 & -1 & 2\\ -1 & 3 & 3\end{vmatrix}=-9\boldsymbol{i}-5\boldsymbol{j}+2\boldsymbol{k}.$$

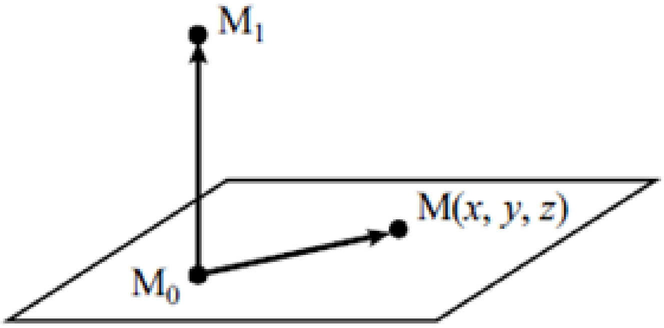
Ayqash to‘g‘ri chiziqlar orasidagi eng qisqa masofani

$$d=H=\frac{V_{par-d}}{S_{asos}}=\frac{\left|\overrightarrow{M_1M_2}\cdot(\vec{S}_1\times\vec{S}_2)\right|}{\left|\vec{S}_1\times\vec{S}_2\right|}$$

formula bilan aniqlaymiz:

$$d=\frac{18}{\sqrt{81+25+4}}=\frac{18}{\sqrt{110}}.$$

16-misol. $M_1(5,3,-4)$ nuqtadan tekislikka perpendikulyar tushirilgan. Perpendikulyarning asosini shu tekislikdagi $M_0(2,4,-1)$ nuqta tashkil etadi (21-rasm). Tekislik tenglamasini tuzing.



23-rasm.

Yechish. Faraz qilaylik, $M(x, y, z)$ - tekislikdagi ixtiyoriy nuqta bo‘lsin. Masala shartiga ko‘ra,

$$\overrightarrow{M_0M_1} \perp \overrightarrow{M_0M} \Rightarrow \overrightarrow{M_0M_1} \cdot \overrightarrow{M_0M} = 0;$$

$$\overrightarrow{M_0M_1} = (3, -1, -3);\ \overrightarrow{M_0M} = (x - 2, y - 4, z + 1);$$

$$\overrightarrow{M_0M_1} \cdot \overrightarrow{M_0M} = 3(x - 2) - (y - 4) - 3(z + 1) = 0 \Longrightarrow$$

$$\Rightarrow 3x - y - 3z - 5 = 0.$$

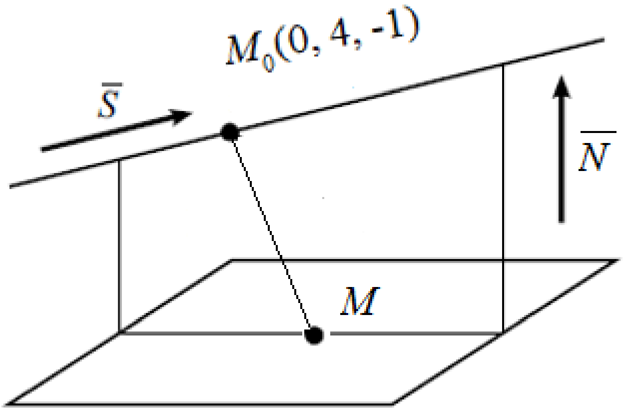
Demak, $3x - y - 3z - 5 = 0$ izlanayotgan tekislik tenglamasidir.

17-misol. $\frac{x}{4} = \frac{y-4}{3} = \frac{z+1}{-2}$ to‘g‘ri chiziqning $x - y + +3z + 8 = 0$ tekislikdagi proyeksiyasini toping (24-rasm).

Yechish. Berilgan tekislik va unga perpendikulyar bo‘lgan hamda berilgan to‘g‘ri chiziq bo‘ylab o‘tgan tekislikning kesishish chizig‘i to‘g‘ri chiziqning tekislikdagi proyeksiyasini ifodalaydi. Perpendikulyar tekislik tenglamasini keltirib chiqaramiz.

Faraz qilaylik, $M(x, y, z)$ tekislikdagi ixtiyoriy nuqta bo‘lsin. To‘g‘ri chiziqning yo‘naltituvchi vektori $\vec{S}(4, 3, -2)$ va tekislikning normal vektori $\vec{N}(1, -1, 3)$ hamda $\overrightarrow{M_0M}(x, y - 4, z + 1)$ vektorlar komplanardir, ya’ni $\overrightarrow{M_0M} \cdot (\vec{S} \times \vec{N}) = 0$. Tenglikni vektorlar koordinatalari bilan ifodalaymiz:

$$\begin{vmatrix} x & y-4 & z+1 \\ 4 & 3 & -2 \\ 1 & -1 & 3 \end{vmatrix} = 0 \Rightarrow x - 2y - z + 7 = 0.$$



24-rasm.

Shunday qilib, ikki tekislikning kesishish chizig‘i, ya’ni to‘g‘ri chiziqning tekislikdagi proyeksiyasi tenglamasini yozamiz:

$$\begin{cases} x - 2y - z + 7 = 0, \\ x - y + 3z + 8 = 0, \end{cases}$$

yoki kanonik ko'rinisda $\frac{x+9}{7} = \frac{y+1}{4} = \frac{z}{-1}$.

11-§. Ikkinchi tartibli sirtlar

Oliy matematikaning ko'p o'zgaruvchili funksiyalar qismini o'rganishda, xususan ularni grafik tasvirlashda, shuningdek, texnika va iqtisodiyot masalalarida ikkinchi tartibli sirtlar haqidagi bilimlarning o'rni muhim ahamiyat kasb etadi.

1. Ellipsoid. Sfera.

1-ta'rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad (24)$$

ko'rinishda bo'lgan sirtga ***ellipsoid*** deyiladi (25-rasm,bu yerda a, b, c – haqiqiy musbat sonlar).

Ellips kabi a, b, c sonlar ***ellipsoid yarim o'qlari*** deyiladi. Yarim o'qlarga nisbatan quyidagi ta'riflarni keltiramiz:

2-ta'rif. Agar $a \neq b, a \neq c, b \neq c$ bo'lsa, ***ellipsoid uch o'qli*** deyiladi.

3-ta'rif. Agar a, b, c yarim o'qlardan ixtiyoriy ikkitasi teng bo'lsa, ellipsoid ***aylanma ellipsoid*** deyiladi.

4-ta'rif. Agar ellipsoidning barcha yarim o'qlari teng bo'lsa: $a = b = c = R,$ u holda ellipsoid R radiusli ***sfera*** deyiladi.

$$x^2 + y^2 + z^2 = R^2 \qquad (25)$$

Ellipsoidning $z = 0$ tekislik bilan kesimini ko'ramiz. Ellipsoid va tekislik kesishish chizig'i tenglamalar sistemasi bilan ifodalanadi:

$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \\ z = 0 \end{cases} \quad \text{yoki} \quad \begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \\ z = 0. \end{cases}$$

Tenglamadan ko'rinadiki, kesishish chizig'i - a va b yarim o'qli ellipsdir.

Ellipsoidning $z = h$ tekislik bilan kesimini ko'ramiz. Kesishish chizig'I quyidagi tenglamalar sistemasi bilan ifodalanadi:

$$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \\ z = h \end{cases} \quad \text{yoki} \quad \begin{cases} \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1, \\ z = h, \end{cases}$$

bu yerda $a_1 = a\sqrt{1-\frac{h^2}{c^2}}, b_1 = b\sqrt{1-\frac{h^2}{c^2}}$. Shunday qilib, agar $0 < h < c$ bo‘lsa, u holda kesim $a_1 < a; b_1 < b$ yarim o‘qli ellips bo‘ladi. Agar $h = c$ bo‘lsa, u holda kesim $(0, 0, c)$ nuqtani tashkil etadi.

18-misol. Markazi $C(a, b, c)$ nuqtada va radiusi R ga teng bo‘lgan sfera tenglamasini yozing.

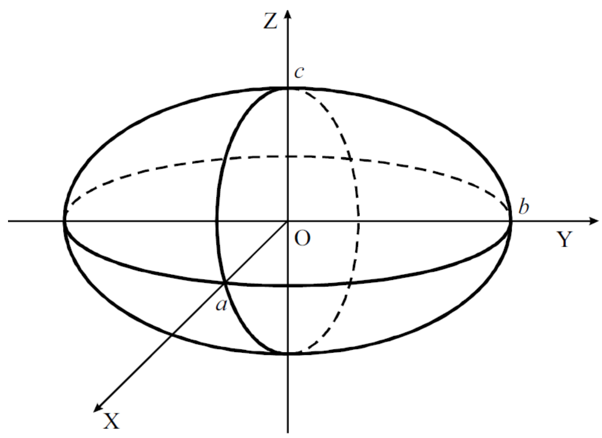
Yechish. $M(x, y, z)$ sferadagi ixtiyoriy nutqa bo‘lsin. Sfera ta’rifiga ko‘ra, $C(a, b, c)$ markazdan ixtiyoriy $M(x, y, z)$ nuqtagacha bo‘lgan masofa R radiusga teng, ya’ni $CM = R$.

Ikki nuqta orasidagi masofa formulasiga ko‘ra

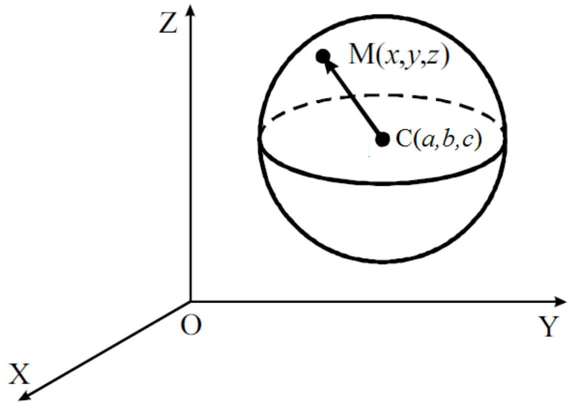
$$CM = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}.$$

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = R$$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2.$$



25-rasm.



26-rasm.

Hosil bo‘lgan tenglama markazi $C(a, b, c)$ nuqtada, radiusi R ga teng bo‘lgan sfera tenglamasidir (26-rasm). Xususiy holda sfera markazi koordinata boshida bo‘lsa, ya’ni $a = b = c = 0$ bo‘lsa, (25) ko‘rinishdagi sferaning sodda tenglamasini hosil qilamiz. (25) tenglama bilan berilgan sferani o‘zaro perpendikulyar uchta yo‘nalish bo‘yicha tekis deformatsiyalash natijasida ellipsoid hosil bo‘ladi.

2. Giperboloidlar.

1-ta’rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \qquad (26)$$

ko‘rinishdagi sirtga ***bir pallali giperboloid*** deyiladi (27-rasm). a, b, c – miqdorlar ***giperboloid yarim o‘qlari*** deyiladi.

2-ta’rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1 \qquad (27)$$

ko‘rinishdagi sirtga ***ikki pallali giperboloid*** deyiladi (28-rasm). Agar $a = b$ bo‘lsa, u holda giperboloid ***aylanma giperboloid*** bo‘ladi.

Bir pallali giperboloidning $\{z = h\}$ tekisliklar bilan kesimi ellipslarni tashkil etadi:

$$\begin{cases} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 + \dfrac{h^2}{c^2}, \\ z = h, -\infty < h < +\infty. \end{cases}$$

$|h|$ ning qiymati kattalashganda ellipsning yarim o‘qlari ham kattalashadi (29-rasm).

Ikki pallali giperboloidning $\{z = h\}$ tekisliklar bilan kesimi $|h| > c$ shart bajarilganda

$$\begin{cases} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{h^2}{c^2} - 1 \\ z = h, \end{cases}$$

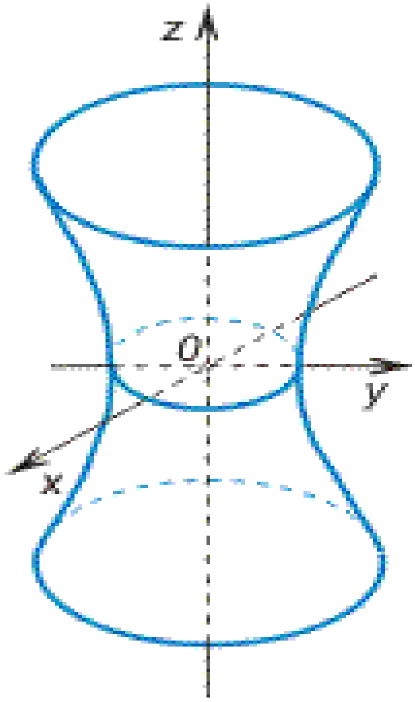
ellipslarni tashkil etadi.

Agar $|h| = c$ bo‘lsa, kesim nuqtaga aylanadi.

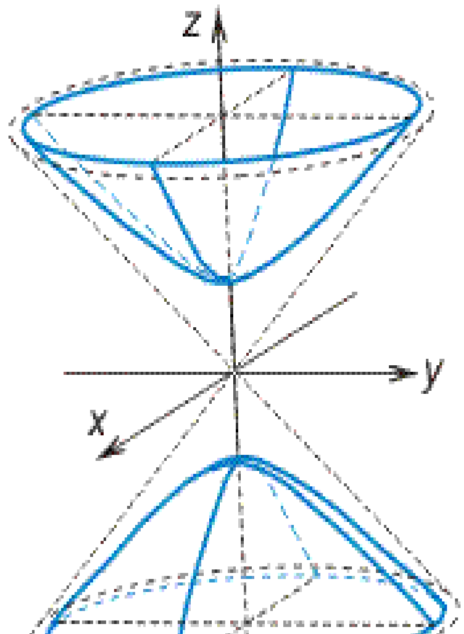
Ikki pallali giperboloidning $x = h$ va $y = h$ tekisliklar bilan kesimida giperbolalar hosil bo‘ladi:

$$\begin{cases} x = h, \\ \dfrac{z^2}{c^2} - \dfrac{y^2}{b^2} = 1 + \dfrac{h^2}{a^2}; \end{cases}$$

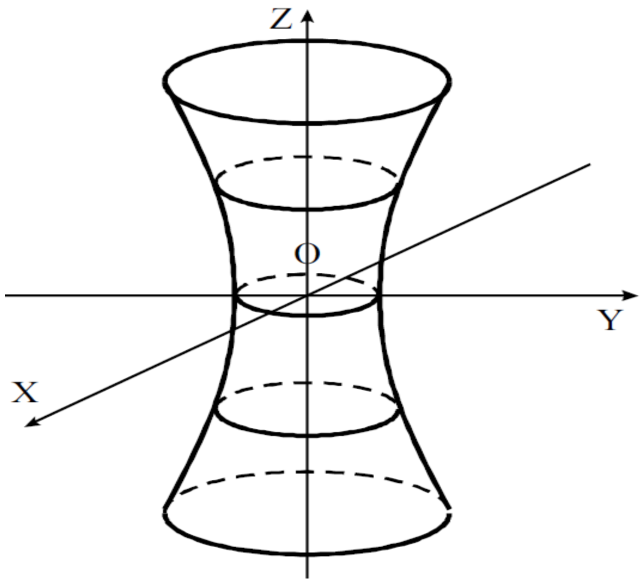
$$\begin{cases} y = h, \\ \dfrac{z^2}{c^2} - \dfrac{y^2}{b^2} = 1 + \dfrac{h^2}{b^2}. \end{cases}$$



27-rasm.



28-rasm.



29-rasm.

3. Paraboloidlar. Oxz tekisligida ushbu

$$x^2 = 2pz, y = 0 \tag{28}$$

tenglama berilgan parabolani qaraylik. Bu parabolani Oz o‘qi atrofida aylantirishdan hosil bo‘lgan sirt ***aylanma paraboloid*** deyiladi.

Aylanma paraboloidning kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 2z \tag{29}$$

ko‘rinishda bo‘ladi.

Aylanma paraboloidni Oy o‘qi bo‘yicha $\frac{a}{b}$ marta tekis deformatsiyalaymiz. Natijada elliptik paraboloid hosil bo‘ladi.

Elliptik paraboloidni $z = h$ $(h > 0)$ tekislik bilan kesish natijasida kesimda ellips hosil bo‘ladi. $x = h$ va $y = h$ tekisliklar bilan kesimi parabolalarni tashkil etadi.

1-ta’rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z, \tag{30}$$

ko‘rinishdagi sirtga (30-rasm) ***elliptik paraboloid*** deyiladi.

2-ta’rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z, \tag{31}$$

ko‘rinishdagi sirtga (31-rasm) ***giperbolik paraboloid*** deyiladi.

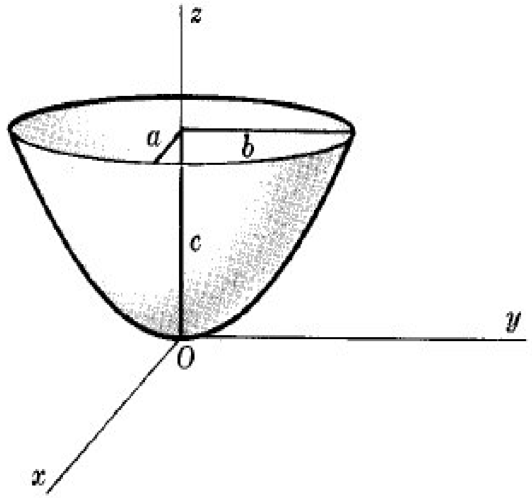
Giperbolik paraboloidning $y = 0$ tekislik bilan kesimida

$$\begin{cases} \frac{x^2}{a^2} = 2z, \\ y = 0, \end{cases} \tag{32}$$

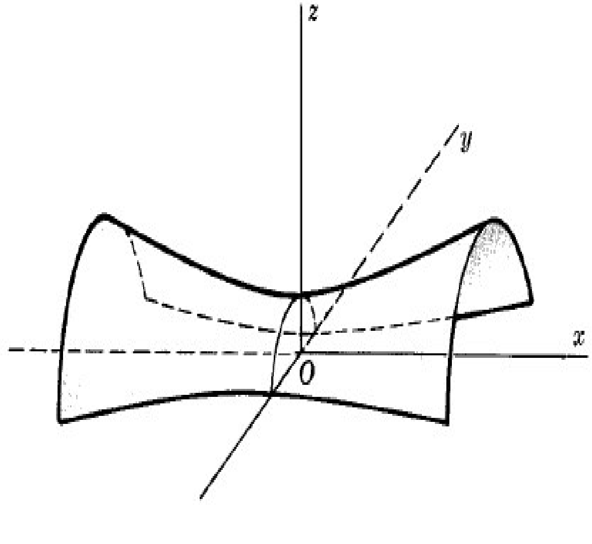
parabola hosil bo‘ladi. Sirtning $x = h$ va $y = h$ tekisliklar bilan kesimida shoxlari pastga va yuqoriga qaragan parabolalar hosil bo‘ladi. Giperbolik paraboloidning $z = h$ tekislik bilan kesimi

$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 2h, \\ z = h, \end{cases} \tag{33}$$

giperbolani tashkil etadi.



30-rasm.



31-rasm.

Oxz tekisligida berilgan $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$ giperbolani Oz o‘qi atrofida aylantirishdan $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ bir pallali giperboloid hosil bo‘ladi.

Bir pallali giperboloidning har bir nuqtasidan ikkita to‘g‘ri chiziq o‘tadi. Bu to‘g‘ri chiziqlar giperboloidning yasovchilari deyiladi.

4. Konus.

Ta’rif. Kanonik tenglamasi

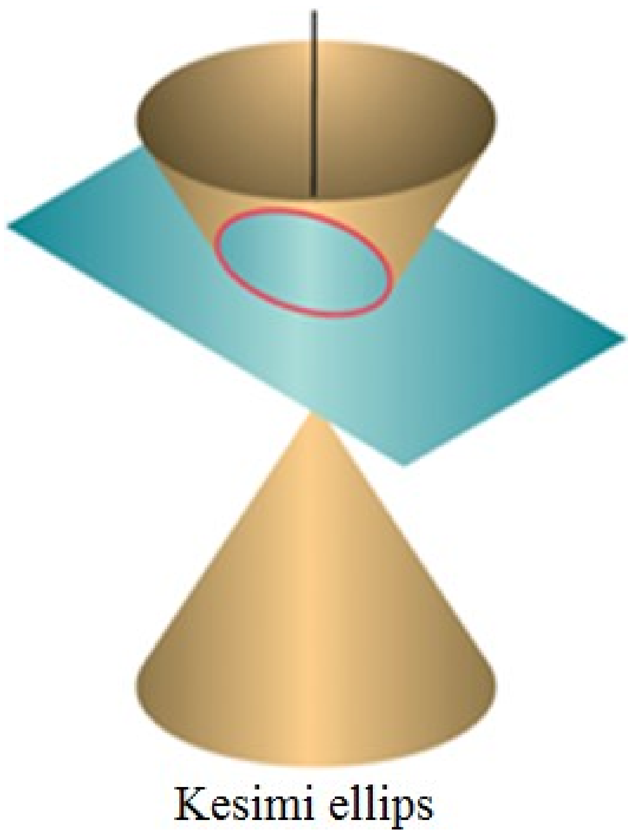
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \tag{34}$$

ko‘rinishdagi sirtga ***konus*** deb ataladi.

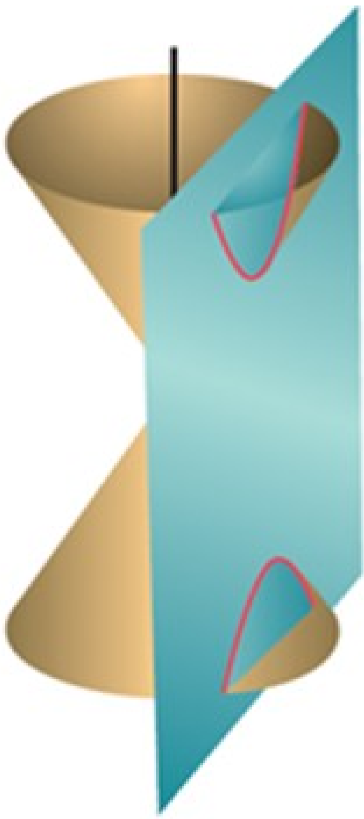
Konusning $z = h$ tekislik bilan kesimida

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{h^2}{c^2}$$

ellips (32-rasm) hosil bo‘ladi. $x = h$ yoki $y = h$ tekisliklar bilan kesimi giperbolalarni (33-rasm) tashkil etadi.



32-rasm.



33-rasm.

12-§. Ikkinchi tartibli sirtlarning umumiy tenglamasi

Biz oldingi paragraflarda kanonik tenglamalari bilan berilgan ikkinchi tartibli sirtlar va ularning tekisliklar bilan kesimlarini o‘rgandik. Bu kanonik tenglamalar

$$Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz + \\ + Gx + Hy + Kz + L = 0 \qquad (35)$$

ko‘rinishdagi tenglamaning xususiy holidir. (35) tenglama ikkinchi tartibli sirtlarning umumiy tenglamasi deyiladi.

(35) tenglamani umumiy holda

$$F(x, y, z) = 0 \qquad (36)$$

ko‘rinishda yozish mumkin.

Agar ikkinchi tartibli sirt tenglamasida o‘zgaruvchilardan ixtiyoriy biri qatnashmasa, bunday sirt silindrik sirtni ifodalaydi.

Masalan, $F(x, y) = 0$ tenglama bilan aniqlangan egri chiziq silindrik sirtning yo‘naltiruvchisi va shu yo‘naltiruvchi egri chiziqni kesib o‘tgan o‘z-o‘ziga parallel to‘g‘ri chiziqlar silindrik sirtning yasovchilari deyiladi. Shunday qilib, silindrik sirtning tenglamasi o‘zining yo‘naltiruvchisi tenglamasi bilan ustma-ust tushar ekan. $F(x, y) = 0$ tenglama bilan aniqlangan silindrlarni ko‘rib chiqamiz.

1-ta’rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (37)$$

ko‘rinishda bo‘lgan sirt ***elliptik silindr*** deyiladi (34-rasm).

2-ta’rif. Kanonik tenglamasi

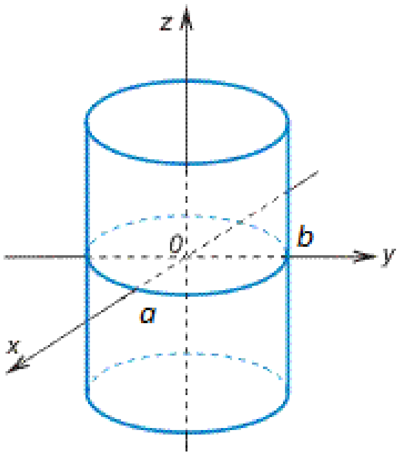
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (38)$$

ko‘rinishda bo‘lgan sirt ***giperbolik silindr*** deyiladi (35-rasm).

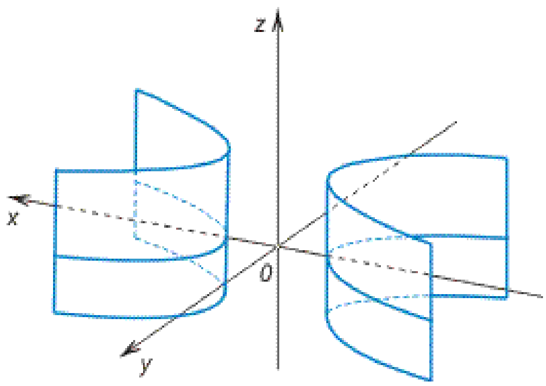
3-ta’rif. Kanonik tenglamasi

$$y^2 = 2px \qquad (39)$$

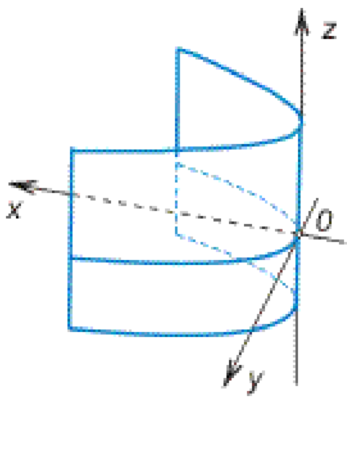
ko‘rinishda bo‘lgan sirt ***parabolik silindr*** deyiladi (36-rasm).



34-rasm.



35-rasm.



36-rasm.

Mustaqil ishlash uchun misollar

1. $2x + 3y + 4z - 48 = 0$ tekislikning koordinata o‘qlari bilan kesishgan nuqtalarini toping.

2. 1) $M_1(7; 2; -3)$ va $M_2(5; 6; -4)$ nuqtalardan o‘tib, Ox o‘qiga parallel; 2) $P_2(2; -1; 1)$ va $P_2(3; 1; 2)$ nuqtalardan o‘tib, Oy o‘qiga parallel; 3) $Q_1(3; -2; 5)$ va $Q_2(2; 3; 1)$ nuqtalardan o‘tib, Oz o‘qiga parallel bo‘lgan tekislik tenglamasini yozing.

3. Koordinatalar boshidan tekislikka tushirilgan perpendikulyarning asosi $M(2; -1; 2)$ nuqtada. Bu tekislik tenglamasini toping.

4. $M(1; -3; 5)$ nuqtadan o‘tib, Oy va Oz o‘qlardan Ox o‘qdagiga ko‘ra ikki marta kesma ajratuvchi tekislik tenglamasini yozing.

5. $M\left(2; 0; -\frac{1}{2}\right)$ nuqtadan $4x - 4y + 2z + 17 = 0$ tekislikgacha bo‘lgan masofani toping.

6. $4x - y + 3z - 6 = 0$ va $x + 5y - z + 10 = 0$ tekisliklarning kesishish chizig‘idan o‘tuvchi va $2x - y + +5z - 5 = 0$tekislikka perpendikulyar bo‘lgan tekislik tenglamasini yozing.

7. $\begin{cases} x - 2y + 3z - 4 = 0, \\ 3x + 2y - 5z - 4 = 0 \end{cases}$ to‘g‘ri chiziq tenglamalarini: 1) proyeksiyalar bo‘yicha; 2) kanonik ko‘rinishda yozing.

8. $M_1(-1; 1; 3)$ nuqtadan o‘tib, 1) $\vec{a}(2; -2; 4)$ vektorga; 2) $\frac{x-1}{2} = \frac{y+2}{4} = \frac{z-1}{0}$ to‘g‘ri chiziqqa; 3) $x = 3t - 1; y = -2t + 3; z = 5t + 2$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziqning parametrik tenglamasini tuzing.

9. Quyidagi to‘g‘ri chiziqlarning parallelligini ko‘rsating:

$$\frac{x+2}{3} = \frac{y-1}{-2} = \frac{z}{1} \quad \text{va} \quad \begin{cases} x + y - z = 0, \\ x - y - 5z - 8 = 0. \end{cases}$$

10. Quyidagi to‘g‘ri chiziqlarning perpendikulyarligini ko‘rsating:

$$\begin{cases} x + y - 3z - 1 = 0, \\ 2x - y - 9z - 2 = 0, \end{cases} \quad \text{va} \quad \begin{cases} 3x + y - 5z + 1 = 0, \\ 2x + 3y - 8z + 3 = 0. \end{cases}$$

11. m va C ning qanday qiymatida $\frac{x-2}{m} = \frac{y+1}{4} = \frac{z-5}{-3}$ to‘g‘ri chiziq $3x - 2y + Cz + 1 = 0$ tekislikka perpendikulyar bo‘ladi?

12. $A(4;-3;1)$ nuqtaning $x+2y-z-3=0$ tekislikdagi proyeksiyasini toping.

13. $P(2;-1;3)$ nuqtaning $x=3t, y=5t-7, z= =2t+2$ to‘g‘ri chiziqdagi proyeksiyasini toping.

14. P(4;1;6) nuqtaga $\begin{cases} x-y-4z+12=0, \\ 2x+y-2z+3=0 \end{cases}$ to‘g‘ri chiziqqa nisbatan simmetrik nuqtani toping.

15. $\frac{x-2}{5}=\frac{y-3}{1}=\frac{z+1}{2}$ to‘g‘ri chiziqdan $x+4y-3z+ +7=0$ tekislikka perpendikulyar tekislik o‘tkazing.

16. $\frac{x}{4}=\frac{y-4}{3}=\frac{z+1}{-2}$ to‘g‘ri chiziqning $x-y+3z+ \ +8=0$ tekislikdagi proyeksiyasini toping.

17. P(7;9;7) nuqtadan $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z}{2}$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

18. $\frac{x-3}{3}=\frac{y+1}{4}=\frac{z}{2}$ va $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ parallel to‘g‘ri chiziqlar orasidagi masofani toping.

19. Markazi $C(1;-1;4)$ nuqtada bo‘lgan sfera $2x+y-3z-3=0$ tekislikka urinadi. Sferaning tenglamasini tuzing.

20. Quyidagi sferalarning markazlari va radiusini toping:
a) $x^2+y^2+z^2-6x+8y+2z+10=0$;
b) $x^2+y^2+z^2+2x-4y-4=0$.

21. ***A*, *B*, *C*** va ***D*** nuqtalarning koordinatalari berilgan bo‘lsin. 1) ***AD*** to‘qri chiziqning kanonik tenglamasini; 2) ***A*, *B*** va ***C*** nuqtalardan o‘tuvchi ***Q*** tekislik tenglamasini; 3) ***D*** nuqtadan o‘tib, ***Q*** tekislikka perpendikulyar bo‘lgan to‘g‘ri chiziqning kanonik tenglamasini; 4) ***D*** nuqtadan ***Q*** tekislikgacha bo‘lgan masofani; 5) ***AD*** to‘qri chiziq bilan ***Q*** tekislik orasidagi burchakni toping.

a) *A*(3, -2, 5), *B* (-2, 4, 3), *C*(1, -1, 6), *D*(2, 0, -1).
b) *A* (1, 2, 4), *B* (3, 0, 1), *C*(0, -1, 1), *D*(2, 1, -1).
c) *A* (3, 0, 4), *B*(6, 3, 0), *C*(0, -9, 1), *D*(1, 2, 10).
d) *A* (2, 7, 3), *B*(4, 5, 6), *C*(2, -3, 0), *D*(5, 1, 12).

ILOVA

VIII BOB. KOMPLEKS SONLAR

Algebraik tenglamalar nazariyasi, shuningdek, elektrotexnika, gidro va aerodinamika hamda qattiq jismlar mexanikasi masalalarining yechimlari haqiqiy va kompleks sonlar bilan ifodalanadi.

Kompleks sonlarning turli shakllari va ular ustidagi asosiy amallarni ko‘rib chiqamiz.

1-§. Kompleks sonlar ustida asosiy amallar

1-ta’rif. $z = a + bi$ ko‘rinishidagi son ***kompleks son*** deyiladi. Bunda $a = ReZ$ – kompleks sonning ***haqiqiy qismi***, $b = ImZ$ – kompleks sonning ***mavhum qismi***, i – ***mavhum birlik*** $(i^2 = -1)$.

z kompleks sonning $a + bi$ ko‘rinishi uning algebraik shakli deyiladi.

Har qanday haqiqiy d sonni

$$d = d + 0 \cdot i$$

ko‘rinishda ifodalash mumkin.

$a + bi$ kompleks sonda $a = 0, b \neq 0$ bo‘lsa, bi ***sof mavhum son*** deyiladi.

2-ta’rif. $z_1 = a_1 + b_1 z$ va $z_2 = a_2 + b_2 z$ kompleks sonlarning mos ravishda haqiqiy va mavhum qismlari teng, ya’ni

$$a_1 = a_2 \text{ va } b_1 = b_2$$

bo‘lsa, z_1 va z_2 ***teng kompleks sonlar*** deyiladi:

$$z_1 = z_2.$$

3-ta’rif. $z_1 = a_1 + b_1 i$ va $z_2 = a_2 + b_2 i$ kompleks sonlar ***yig‘indisi*** deb,

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$$

songa aytiladi.

4-ta’rif. $z_1 = a_1 + b_1 i$ va $z_2 = a_2 + b_2 i$ kompleks sonlar ***ayirmasi*** deb,

$$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)i$$

songa aytiladi.

5-ta'rif. $z_1 = a_1 + b_1 i$ va $z_2 = a_2 + b_2 i$ kompleks sonlar ***ko'paytmasi*** deb,

$$z_1 \cdot z_2 = (a_1 \cdot a_2 - b_1 \cdot b_2) + (a_1 \cdot b_2 + a_2 \cdot b_1)i$$

songa aytiladi.

6-ta'rif. $z = a + bi$ kompleks songa $\overline{z} = a - bi$ son ***qo'shma kompleks son*** deyiladi.

Qo'shma kompleks sonlar yig'indisi va ko'paytmasi haqiqiy son bo'ladi:

$$z + \overline{z} = a + bi + a - bi = (a + a) + \big(b + (-b)\big)i = 2a,$$

$$z \cdot \overline{z} = (a + bi)(a - bi) =$$

$$= \big(a \cdot a - b \cdot (-b)\big) + (a \cdot (-b) + b \cdot a)i = a^2 + b^2.$$

Kompleks sonlarni bo'lish qoidasi: agar $z_2 \neq 0$ bo'lsa, u holda

$$\frac{z_1}{z_2} = \frac{z_1 \cdot \overline{z_2}}{z_2 \cdot \overline{z_2}}.$$

Masalan, $z_1 = a_1 + b_1 i$ va $z_2 = a_2 + b_2 i$ $(z_2 \neq 0)$ kompleks sonlar bo'linmasi

$$\frac{z_1}{z_2} = \frac{a_1 + b_1 i}{a_2 + b_2 i} = \frac{(a_1 + b_1 i)(a_2 - b_2 i)}{(a_2 + b_2 i)(a_2 - b_2 i)} =$$

$$= \frac{a_1 \cdot a_2 + b_1 \cdot b_2}{{a_2}^2 + {b_2}^2} + \frac{a_2 \cdot b_1 - a_1 \cdot b_2}{{a_2}^2 + {b_2}^2} i$$

ko'rinishda bo'ladi.

1-misol. $z_1 = 2 - 7i$ va $z_2 = 5 + 3i$ kompleks sonlar yig'indisi, ayirmasi, ko'paytmasi va bo'linmasini hisoblang.

Yechish.

$$z_1 + z_2 = 2 - 7i + 5 + 3i = 7 - 4i;$$

$$z_1 - z_2 = 2 - 7i - (5 + 3i) = (2 - 5) + (-7 - 3)i = -3 - 10i;$$

$$z_1 \cdot z_2 = (2 - 7i) \cdot (5 + 3i) =$$

$$= (2 \cdot 5 - (-7) \cdot 3) + (2 \cdot 3 + (-7) \cdot 5)i =$$

$$= (10 + 21) + (6 - 35)i = 31 - 29i;$$

$$\frac{z_1}{z_2} = \frac{z_1 \cdot \overline{z_2}}{z_2 \cdot \overline{z_2}} = \frac{2 - 7i}{5 + 3i} = \frac{(2 - 7i)(5 - 3i)}{(5 + 3i)(5 - 3i)} =$$

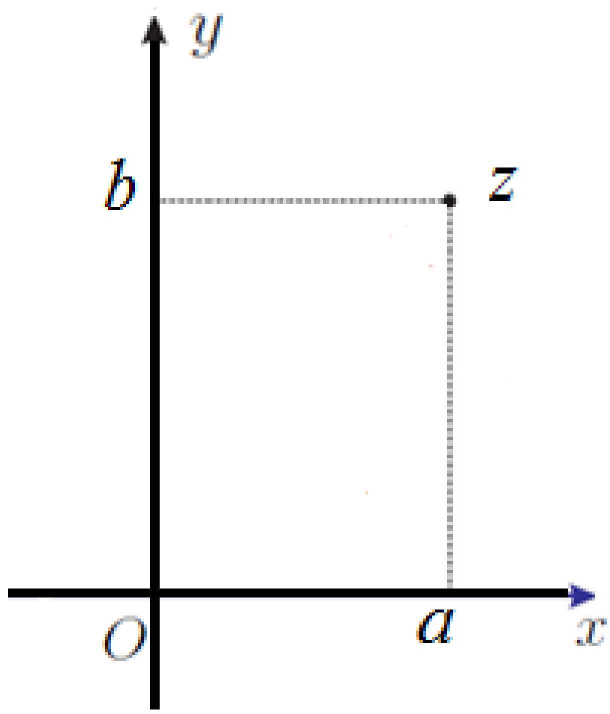
$$= \frac{2 \cdot 5 - (-7) \cdot (-3)}{5^2 + 3^2} + \frac{2 \cdot (-3) + (-7) \cdot 5}{5^2 + 3^2} i =$$

$$= \frac{10 - 21}{34} - \frac{6 + 35}{34} i = -\frac{11}{34} - \frac{41}{34} i.$$

2-§. Kompleks sonning geometrik tasviri

Tekislikda Dekart koordinatalar sistemasi berilgan bo‘lsin. Koordinata tekisligida kompleks son nuqtani ifodalaydi. Tanlangan koordinata sistemasida gorizontal o‘q ***haqiqiy o‘q*** deyiladi va unda kompleks sonning haqiqiy qismi belgilanadi. Vertikal o‘q ***mavhum o‘q*** deyiladi va unda kompleks sonning mavhum qismi belgilanadi.

Har bir $z = a + bi$ kompleks songa koordinata tekisligida (a, b) koordinatali nuqta mos qo‘yiladi (1-rasm).



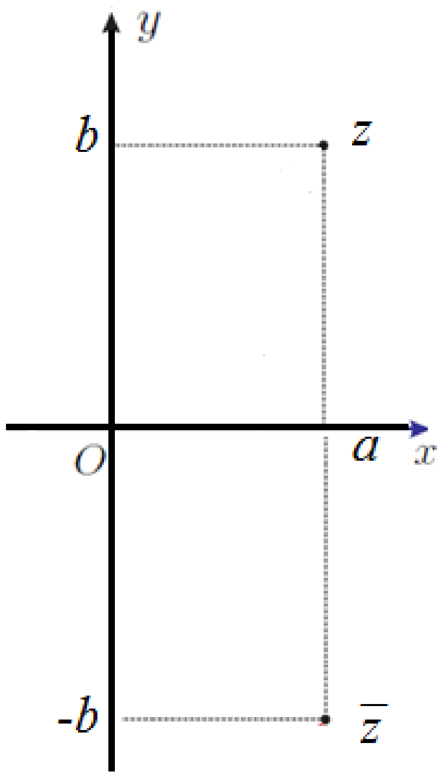
1-rasm.

$z = a + bi$ va $\overline{z} = a - bi$ qo‘shma kompleks sonlar haqiqiy o‘qga nisbatan simmetrik nuqtalarni tasvirlaydi (2-rasm).

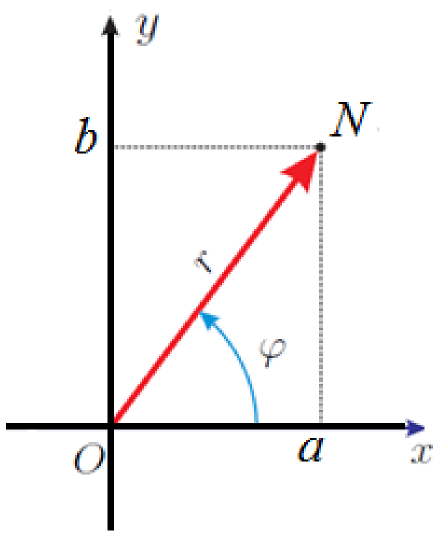
Nuqtalari kompleks sonlarga mos keladigan koordinata tekisligi ***kompleks tekislik*** deyiladi.

3-§. Kompleks sonning moduli va argumenti

$z = a + bi$ kompleks son kompleks tekislikda $N(a, b)$ nuqta bilan tasvirlansin, u holda $\overrightarrow{ON}$ radius vektor ham z kompleks sonni tasvirlashi mumkin (3-rasm).



2-rasm.



3-rasm.

1-ta'rif. $\overrightarrow{ON}$ radius vektorning uzunligi z ***kompleks sonning moduli*** deyiladi va $|z|$ bilan belgilanadi.

Ma'lumki, radius vektorning uzunligi $O(0;\ 0)$ va $N(a, b)$ nuqtalar orasidagi masofaga teng, u holda:

$$|z| = \sqrt{a^2 + b^2}.$$

2-ta'rif. Haqiqiy o'qining musbat yo'nalishi va $\overrightarrow{ON}$ radius vektor orasidagi φ burchakka z ***kompleks sonning argumenti*** deyiladi va $\boldsymbol{\varphi} = \mathbf{Argz}$ bilan belgilanadi.

$$\mathrm{Arg}z = \mathrm{arg}z + 2\pi n, \qquad (n\epsilon Z).$$

bu yerda $\mathrm{arg}z\ -$ $\mathrm{Arg}z$ ning bosh qiymati bo'lib, $-\pi \leq\ \mathrm{arg}z \leq \pi$ shartdan aniqlanadi, ya'ni

$$\mathrm{arg}z = \begin{cases} arctg\dfrac{b}{a}, & agar\ a > 0, \\ \pi + arctg\dfrac{b}{a}, & agar\ a < 0, \quad b \geq 0, \\ -\pi + arctg\dfrac{b}{a}, & agar\ a < 0, \quad b < 0, \\ \dfrac{\pi}{2}, & agar\ a = 0, b > 0, \\ -\dfrac{\pi}{2}, & agar\ a = 0, b < 0. \end{cases}$$

3-rasmga ko'ra, $|z| = r,$ $\mathrm{arg}z = \varphi$, u holda

$$a = r\cos\varphi\,, \qquad b = r\sin\varphi.$$

Bundan $\cos\varphi = \frac{a}{\sqrt{a^2+b^2}}, \sin\varphi = \frac{b}{\sqrt{a^2+b^2}},$ $\mathrm{tg}\varphi = \frac{b}{a}.$

4-§. Kompleks sonning trigonometrik shakli

Kompleks son $z = a + bi$ algebraik shaklda berilgan bo'lsin. Kompleks sondagi a va b o'rniga $a = r\cos\varphi$ va $b = r\sin\varphi$ qiymatni qo'yamiz:

$$z = r\cos\varphi + ir\sin\varphi,$$

$$z = r(\cos\varphi + i\sin\varphi) \qquad (1)$$

bu yerda $r = |z|,$ $\varphi = \mathrm{Arg}z.$

$r(\cos\varphi + i\sin\varphi)$ – kompleks sonning trigonometrik shakli.

2-misol. $z = -1 - i\sqrt{3}$ kompleks sonni trigonometrik shaklda ifodalang.

Yechish. $|z| = r = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = 2,$

$$\text{tg}\varphi = \frac{-\sqrt{3}}{-1} = \sqrt{3},$$

$$\varphi = -\pi + arctg\sqrt{3} = -\pi + \frac{\pi}{3} = -\frac{2}{3}\pi.$$

$$-1 - i\sqrt{3} = 2\left(\cos\left(-\frac{2}{3}\pi\right) + i\sin\left(-\frac{2}{3}\pi\right)\right).$$

Triginometrik shaklda berilgan

$$z_1 = r_1(\cos\varphi + i\sin\varphi) \text{ va } z_2 = r_2(\cos\varphi + i\sin\varphi)$$

kompleks sonlar ko‘paytmasi va bo‘linmasini aniqlaymiz:

$$z_1 \cdot z_2 = r_1(\cos\varphi + i\sin\varphi)\cdot r_2(\cos\varphi + i\sin\psi) =$$
$$= r_1r_2\big(\cos\varphi\cos\psi - \sin\varphi\sin\psi +$$
$$+ i(\cos\varphi\sin\psi + \sin\varphi\cos\psi)\big) =$$
$$= r_1r_2(\cos(\varphi+\psi) + i\sin(\varphi+\psi)).$$

Demak,

$$z_1 \cdot z_2 = r_1r_2(\cos(\varphi+\psi) + i\sin(\varphi+\psi)). \qquad (2)$$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi + i\sin\varphi)}{r_2(\cos\varphi + i\sin\psi)} =$$

$$= \frac{r_1(\cos\varphi + i\sin\varphi)(\cos\psi - i\sin\psi)}{r_2(\cos\psi + i\sin\psi)(\cos\psi - i\sin\psi)} =$$

$$= \frac{r_1\big(\cos\varphi\cos\psi + \sin\varphi\sin\psi + i(\sin\varphi\cos\psi - \cos\varphi\sin\psi)\big)}{r_2(\cos^2\psi + \sin^2\psi)}$$

$$= \frac{r_1}{r_2}[\cos(\varphi-\psi) + i\sin(\varphi-\psi)].$$

Demak,

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\varphi-\psi) + i\sin(\varphi-\psi)]. \qquad (3)$$

1. *Darajaga ko‘tarish*. (2) formuladan foydalanib, trigonometrik shaklda berilgan $z = r(\cos\varphi + i\sin\varphi)$ kompleks sonni darajaga ko‘tarish mumkin.

Faraz qilaylik, n – butun musbat son bo‘lsin.

$$z^n = \underbrace{z\cdot z\cdot \ldots \cdot z}_{n},$$

u holda

$$z^n = \underbrace{r \cdot r \cdot ... \cdot r}_{n}\left[\cos\underbrace{(\varphi + \varphi + \cdots + \varphi)}_{n} + i\sin\underbrace{(\varphi + \varphi + \cdots + \varphi)}_{n}\right].$$

Ifodani soddalashtiramiz:

$$z^n = r^n(\cos n\varphi + i\sin n\varphi). \qquad (4)$$

2. *Ildiz chiqarish***.** Kompleks sonning n – darajali ildizi deb n – darajaga ko‘targanda ildiz ostidagi songa teng bo‘ladigan kompleks songa aytiladi, ya’ni

$$\rho^n(\cos n\psi + i\sin n\psi) = r(\cos\varphi + i\sin\psi)$$

bo‘lsa,

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \rho(\cos\psi + i\sin\psi).$$

Teng kompleks sonlarning ildizlari teng bo‘lishi kerak, argumentlari esa 2π ga karrali songa farq qilishi mumkin bo‘lgani uchun

$$\rho^n = r, \qquad n\psi = \varphi + 2k\pi,$$

$$\rho = \sqrt[n]{r}, \qquad \psi = \frac{\varphi + 2k\pi}{n},$$

bu yerda k – ixtiyoriy butun son. k ga $0, 1, 2, ..., n-1$ qiymatlar berib ildizning n ta har xil qiymatlarini topamiz.

Shunday qilib, kompleks sonning n – darajali ildizi n ta har xil qiymatga ega bo‘ladi:

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right), \quad (5)$$

bu yerda $k = 0, 1, 2, ..., (n-1)$.

(4) va (5) formulalar ***Muavr***[7] ***formulalari*** deyiladi.

3-misol. $z_1 = 2 - 2i$ va $z_2 = \sqrt{3} + i$ sonlar berilgan. Tigonometrik shaklda $z_1 \cdot z_2$ va $\frac{z_1}{z_2}$ ni toping.

Yechish. $z_1 = 2 - 2i$ son uchun

$$r_1 = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}.$$

$$\cos\varphi_1 = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}, \sin\varphi_1 = -\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}, \qquad \Rightarrow \ \varphi_1 = -\frac{\pi}{4}.$$

z_1 kompleks sonni trigonometrik shakli:

$$z_1 = 2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right).$$

Xuddi shuningdek, z_2 kompleks son uchun:

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[7] Abraham de Muavr (Abraham de Moire) (1667-1754) – angliyalik matematik.

$$r_2 = |z_2| = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2;$$

$$\cos\varphi_2 = \frac{\sqrt{3}}{2}, \sin\varphi_2 = \frac{1}{2}, \qquad \Rightarrow \ \varphi_2 = \frac{\pi}{6}.$$

$$z_2 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right).$$

U holda, (2) va (3) formulaga asosan,

$$z_1 \cdot z_2 = 2\sqrt{2} \cdot 2\left(\cos\left(-\frac{\pi}{4} + \frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{4} + \frac{\pi}{6}\right)\right) =$$
$$= 4\sqrt{2}\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right).$$

$$\frac{z_1}{z_2} = \frac{2\sqrt{2}}{2}\left(\cos\left(-\frac{\pi}{4} - \frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{4} - \frac{\pi}{6}\right)\right) =$$
$$= \sqrt{2}\left(\cos\left(-\frac{5\pi}{12}\right) + i\sin\left(-\frac{5\pi}{12}\right)\right).$$

4-misol. $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ kompleks son uchun z^{20} ni toping.

Yechish. Kompleks sonni trigonometrik shaklda yozamiz:

$$|z| = r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1.$$

$$\arg z = \operatorname{arctg}\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \operatorname{arctg}\sqrt{3} = \frac{\pi}{3}.$$

U holda

$$z = 1 \cdot \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}.$$

(4) formulaga ko‘ra,

$$z^{20} = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{20} =$$
$$= \cos\left(20 \cdot \frac{\pi}{3}\right) + i\sin\left(20 \cdot \frac{\pi}{3}\right) = \cos\frac{20\pi}{3} + i\sin\frac{20\pi}{3} =$$

$$= \cos\frac{21\pi - \pi}{3} + i\sin\frac{21\pi - \pi}{3} =$$
$$= \cos\left(7\pi - \frac{\pi}{3}\right) + i\sin\left(7\pi - \frac{\pi}{3}\right) =$$
$$= \cos\left(\pi - \frac{\pi}{3}\right) + i\sin\left(\pi - \frac{\pi}{3}\right) =$$
$$= -\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}.$$
$$z^{20} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i.$$

5-misol. $\sqrt[3]{z} = \sqrt[3]{-1-i}$ kompleks sonning kub ildizlari qiymatlari topilsin.

Yechish. Kompleks sonni trigonometrik shaklda yozamiz:

$$a = -1, \qquad b = -1.$$
$$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}, \qquad \text{tg}\varphi = \frac{-1}{-1} = 1.$$

a va b manfiy bo‘lgani uchun $\varphi = \frac{5\pi}{4}$ ga teng bo‘ladi.

$$\sqrt[3]{-1-i} = \sqrt[3]{\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)} =$$
$$= \sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{4} + 2k\pi}{3} + i\sin\frac{\frac{5\pi}{4} + 2k\pi}{3}\right),$$

$k = 0, 1, 2.$

$$k = 0, \qquad z_0 = \sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{4}}{3} + i\sin\frac{\frac{5\pi}{4}}{3}\right) =$$
$$= \sqrt[6]{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right),$$
$$k = 1, \qquad z_1 = \sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{4} + 2\pi}{3} + i\sin\frac{\frac{5\pi}{4} + 2\pi}{3}\right) =$$
$$= \sqrt[6]{2}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right),$$

$$k=2, \qquad z_2=\sqrt[6]{2}\left(\cos\frac{\frac{5\pi}{4}+4\pi}{3}+i\sin\frac{\frac{5\pi}{4}+4\pi}{3}\right)=$$

$$=\sqrt[6]{2}\left(\cos\frac{21\pi}{12}+i\sin\frac{21\pi}{12}\right).$$

5-§. Ko‘rsatkichi kompleks bo‘lgan ko‘rsatkichli funksiya. Eyler formulalari

Ko‘rsatkichi $z=x+iy$ kompleks bo‘lgan e^z ko‘rsatkichli funksiya quyidagi tenglik bilan aniqlanadi:

$$e^z=\lim_{n\to\infty}\left(1+\frac{z}{n}\right)^n.$$

$$e^z=e^x\left(\cos y+i\sin y\right),$$

yoki

$$e^{x+iy}=e^x\left(\cos y+i\sin y\right) \tag{6}$$

Xususiy holda, agar при $x=0$ bo‘lsa

$$e^{iy}=\cos y+i\sin y \tag{7}$$

bu formula ***Eyler formulasi*** deyiladi.

Ko‘rsatkichli funksiyaning xossalari.

1^0. Agar z_1 va z_2– ikkita komleks son bo‘lsa, unda

$$e^{z_1+z_2}=e^{z_1}e^{z_2}.$$

Isbot. $z_1=x_1+iy_1,\quad z_2=x_2+iy_2$ bo‘lsin. (10) formulaga asosan

$$e^{z_1+z_2}=e^{(x_1+iy_1)+(x_2+iy_2)}=$$

$$=e^{(x_1+x_2)+i(y_1+y_2)}=e^{x_1}e^{x_2}\left[\cos(y_1+y_2)+i\sin(y_1+y_2)\right]. \tag{8}$$

Trigonometrik shakldagi ikkita kompleks sonni ko‘paytirish formulasiga binoan:

$$e^{z_1+z_2} = e^{z_1} e^{z_2}$$

$$e^{z_1} e^{z_2} = e^{x_1+iy_1} e^{x_2+iy_2} =$$

$$= e^{x_1} (\cos y_1 + i \sin y_1) e^{x_2} (\cos y_2 + i \sin y_2) =$$

$$= e^{x_1} e^{x_2} \left[\cos(y_1+y_2)+i\sin(y_1+y_2)\right]. \tag{9}$$

(8) va (9) tengliklardan $e^{z_1+z_2} = e^{z_1} e^{z_2}$ o‘rinli ekanligi kelib chiqadi.

$$2^0. \quad e^{z_1-z_2} = \frac{e^{z_1}}{e^{z_2}} \tag{10}$$

(10) tenglik 1-xossa kabi isbotlanadi.

3^0. Agar m butun son bo‘lsa,

$$\left(e^{z}\right)^{m} = e^{mz}$$

bo‘ladi. Agar m>0 bo‘lsa, bu formula (4) formulaga asosan osongina hosil bo‘ladi; agar m<0 bo‘lsa (4) va (5) formulalarga asosan hosil qilinadi.

4^0. Kompleks ko‘rsatkichli funksiya $2\pi i$ davrga ega, ya'ni $e^{z+2\pi i} = e^{z}$. Xususiy holda, agar $z=0$ bo‘lsa, $e^{2\pi i} = 1$.

Haqiqatan, (9) va (6) formulalarga asosan:

$$e^{z+2\pi i} = e^{z} e^{2\pi i} = e^{z} (\cos 2\pi + i \sin 2\pi) = e^{z} \cdot 1 = e^{z}.$$

Komleks sonning $z = r\left(\cos\phi + i\sin\phi\right)$ trigonometrik shaklini (7) formulaga asosan ***ko‘rsatkichli shaklga*** almashtirish mumkin:

$$z = re^{\phi i}.$$

Ko'rsatkichli shakldagi komleks sonlar uchun ko'paytirish, bo'lish, butun musbat darajaga ko'tarish va butun musbat darajali ildizdan chiqarish quyidagi formulalar bilan bajariladi:

$$r_1 e^{i\phi_1} \cdot r_2 e^{i\phi_2} = r_1 \cdot r_2 e^{i(\phi_1+\phi_2)}; \qquad (11)$$

$$\frac{r_1 e^{i\phi_1}}{r_2 e^{i\phi_2}} = \frac{r_1}{r_2} e^{i(\phi_1-\phi_2)}; \qquad (12)$$

$$\left(re^{i\phi}\right)^n = r^n e^{in\phi}; \qquad (13)$$

$$\sqrt[n]{re^{i\phi}} = \sqrt[n]{r} \cdot e^{\frac{\phi+2\pi k}{n} i} \quad \left(k=0,1,2,...,n-1\right) \qquad (14)$$

(7) Eyler formulasi trigonometrik va ko'rsatkichli funksiyalar orasidagi bog'liqlikni o'rnatadi. Eyler formulasida y ni ϕ va $-\phi$ larga almashtirib, ushbu tengliklarni hosil qilamiz:

$$e^{\phi i} = \cos\phi + i\sin\phi, \qquad e^{-\phi i} = \cos\phi - i\sin\phi.$$

Ularni qo'shish va ayirish yordamida quyidagilarni hosil qilamiz:

$$\cos\phi = \frac{e^{\phi i} + e^{-\phi i}}{2}, \qquad (15)$$

$$\sin\phi = \frac{e^{\phi i} - e^{-\phi i}}{2i}, \qquad (16)$$

Bu ikkita oddiy formulalar ham ***Eyler formulalari*** deyiladi.

6-misol. Toping: 1) $e^{i\pi/4}$; 2) $e^{\pi e^{-i\pi/2}}$; 3) $e^{2+i\pi}$.

Yechish. (7) formulaga asosan:

1) $e^{i\pi/4} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)$;

2) $e^{\pi e^{-i\pi/2}} = e^{\pi\left[\cos(-\pi/2)+i\sin(-\pi/2)\right]} = e^{-\pi i} =$

$= \cos\left(-\pi\right) + i\sin\left(-\pi\right) = -1$;

3) (4) xossaga ko‘ra $e^{2+i\pi} = e^2(\cos\pi + i\sin\pi) = -e^2$.

7-misol. Quyidagilarni toping: 1) $\cos i$; 2) $\cos(1-i)$.

Yechish. (15) formulaga asosan:

1) $\cos i = \dfrac{e^{i^2}+e^{-i^2}}{2} = \dfrac{e^{-1}+e}{2} = \dfrac{1+e^2}{2e}$;

2) $$\cos(1-i) = \frac{e^{i(1-i)}+e^{-i(1-i)}}{2} = \frac{e^{i+1}+e^{-i-1}}{2} =$$

$$= \frac{1}{2}\left[e(\cos 1 + i\sin 1) + e^{-1}(\cos(-1) + i\sin(-1))\right] =$$

$$= \frac{1}{2}\left[e\cos 1 + ei\sin 1 + e^{-1}\cos 1 - e^{-1}i\sin 1\right] =$$

$$= \frac{1}{2}\left[(e+e^{-1})\cos 1 + i(e-e^{-1})\sin 1\right] = \frac{e^2+1}{2e}\cos 1 + i\frac{e^2-1}{2e}\sin 1.$$

8-misol. z kompleks o‘zgaruvchi uchun quyidagi formulalar o‘rinli bo‘lishini ko‘rsating:

1) $\sin^2 z + \cos^2 z = 1$;

2) $\sin 2z = 2\sin z\cos z$;

3) $\cos 2z = \cos^2 z - \sin^2 z$.

Yechish. 1) (15) va (16) formulalarni kvadratga oshiramiz hamda ularni hadma-had qo‘shamiz:

$$\cos^2 z + \sin^2 z = \frac{\left(e^{zi}+e^{-zi}\right)^2}{4} + \frac{\left(e^{zi}-e^{-zi}\right)^2}{-4} =$$

$$= \frac{e^{2zi}+2+e^{-2zi}-e^{2zi}+2-e^{-2zi}}{4} = 1.$$

2) (15) va (16) formulalarning o‘ng va chap qismlarini mos ravishda ko‘paytiramiz

$$2\sin z\cos z = 2\frac{e^{zi}+e^{-zi}}{2}\cdot\frac{e^{zi}-e^{-zi}}{2i} =$$

$$= \frac{e^{2zi}-e^{-2zi}}{2i} = \frac{\cos 2z+i\sin 2z}{2i} - \frac{\cos 2z-i\sin 2z}{2i} = \sin 2z.$$

3) (15) va (16) formulalarning o‘ng va chap qismlarini kvadratga oshiramiz hamda ularni hadma-had ayiramiz:

$$\cos^2 z - \sin^2 z = \frac{\left(e^{zi}+e^{-zi}\right)^2}{4} - \frac{\left(e^{zi}-e^{-zi}\right)^2}{-4} =$$

$$= \frac{e^{2zi}+2+e^{-2zi}+e^{2zi}-2+e^{-2zi}}{4} = \frac{e^{2zi}+e^{-2zi}}{2} = \cos 2z.$$

Mustaqil ishlash uchun misollar

1. Kompleks sonlarni vektorlar bilan tasvirlang, ularning modullari va argumentlarini aniqlang hamda trigonometrik shaklda yozing:

1) $z=3$; 2) $z=-2$; 3) $z=3i$; 4) $z=-2i$;

5) $z=2-2i$; 6) $z=1+i\sqrt{3}$; 7) $z=-\sqrt{3}-i$.

2. $z_1=3+2i$ va $z_2=1+5i$ kompleks sonlar berilgan. $z_1\cdot z_2$; $\frac{z_1}{z_2}$; z_1^3; $\sqrt[3]{z_2}$ larni toping.

3. Quyidagilarni Muavr formulasi bilan hisoblang:

1) $(1+i)^{10}$; 2) $(1-i\sqrt{3})^6$; 3) $(-1+i)^5$;

4) $(1+\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})^4$; 5) $(\sqrt{3}+i)^3$.

4. $z=\sqrt[6]{1}$ ning barcha qiymatlarini toping va radiusi 1 ga teng doira yasab, topilgan qiymatlarini radius-vektorlar bilan tasvirlang.

5. 1) $x^3+8=0$; 2) $x^4+4=0$; 3) $x^3-8=0$; 4) $x^6+64=0$; 5) $x^4-81=0$ ikki hadli tenglamalarni yeching.

O‘TILGAN MAVZULARNING O‘ZLASHTIRILISHINI TEKSHIRISH UCHUN SAVOLLAR

Talabalar darslik bo‘yicha mavzularni va ularga oid masalalar yechish malakasini o‘zlashtirganlaridan so‘ng, o‘zlashtirilgan ta’riflar, formulalar va teoremalar isbotini takrorlash tavsiya etiladi. Ushbu savollar o‘zlashtirilgan materiallarni takrorlashda talabalarga yordam berish maqsadida keltirilgan.

I. Chiziqli algebra elementlari

1. Determinant deb nimaga aytiladi? Uning asosiy xossalarini keltiring.
2. Determinantning minori va algebraik to‘ldiruvchilari deganda nimani tushunasiz?
3. Determinantlarni hisoblash usullarini bilasizmi?
4. Matritsa deganda nimani tushunasiz? Matritsalar ustidagi chiziqli amallar qanday bajariladi? Ularning asosiy xossalarini ayting.
5. Birlik matritsa deb qanday matritsaga aytiladi?
6. Teskari matritsa deb qanday matritsaga aytiladi va u qanday topiladi?
7. Chiziqli tenglamalar sistemasining yechimlari deganda nimani tushunasiz?
8. Tenglamalar sistemasini yechishdagi Kramer formulasi va uni qanday hollarda qo‘llab bo‘ladi?
9. Qanday shart bajarilganda chiziqli tenglamalar sistemasi yagona yechimga ega bo‘ladi?
10. Agar asosiy determinant 0 ga teng bo‘lsa, chiziqli tenglamalar sistemasi haqida nima deyish mumkin?
11. Qanday shart bajarilganda bir jinsli tenglamalar sistemasi noldan farqli yechimga ega bo‘ladi?
12. Chiziqli tenglamalar sistemasini yechishda Gauss usulining ma’nosi nimadan iborat?
13. Tenglamalar sistemasini matritsa usuli bilan yechishni tushuntiring.

II. Tekislikdagi analitik geometriya

1. Chiziqning tenglamasini qanday tuzish mumkin?

2. To‘g‘ri chiziqning burchak koeffitsiyenti deb nimaga aytiladi?

3. To‘g‘ri chiziqning burchak koeffitsiyentli va umumiy tenglamalarini yozing.

4. To‘g‘ri chiziqning kesmalardagi tenglamasi qanday ko‘rinishda bo‘ladi?

5. To‘g‘ri chiziqlar dastasining tenglamasi. Ikki to‘g‘ri chiziq orasidagi burchaklar bissektrisalarining tenglamalarini yozing.

6. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini qanday hosil qilasiz?

7. To‘g‘ri chiziqning normal tenglamasini va umumiy tenglamasini normal ko‘rinishga qanday keltiriladi?

8. Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa qanday aniqlanadi?

9. Ikki to‘g‘ri chiziq orasidagi burchak qanday hisoblanadi?

10. Aylana deb qanday egri chiziqqa aytiladi? Uning tenglamalarini yozing.

11. Ellips deb qanday egri chiziqqa aytiladi? Ellipsning fokuslari va ekssentrisiteti qanday aniqlanadi?

12. Giperbola deb qanday nuqtalarning geometrik o‘rniga aytiladi?

13. Parabola deb qanday nuqtalarning geometrik o‘rniga aytiladi?

14. Ikkinchi tartibli egri chiziqlarning qutb koordinatalaridagi tenglamalarini yozing.

III. Vektorlar algebrasi

1. Vektor va uning moduli deb nimaga aytiladi?

2. Qanday vektorlarga kollinear, komplanar, teng vektorlar deyiladi?

3. Modullari teng bo‘lgan ikki vektor o‘zaro teng bo‘lmasligi mumkinmi? Agar teng bo‘lmasa, farqi nimada?

4. Vektorlar ustida qanday algebraik amallar bajarish mumkin? Nol vektor deb qanday vektorga aytiladi? Vektorlar ustida kiritilgan amallar uchun qanday xossalar o‘rinli?

5. Tekislikda, fazoda basis deb qanday vektorlarga aytiladi?

6. Qanday vektorlarga chiziqli bog‘liq vektorlar deyiladi?

7. Dekart koordinatalar sistemasi qanday tanlanadi?

8. Vektorning komponentalari, uning boshlang‘ich va oxirgi nuqtalarining koordinatalari orqali qanday ifodalanadi?

9. Kesmani berilgan nisbatda bo‘lishni ko‘rsating.

10. Uchburchak og‘irlik markazining koordinatalarini uning uchlarining koordinatalari orqali ifodalang.

11. Nuqtaning va kesmaning o‘qdagi proyeksiyasi deb nimaga aytiladi?

12. Ikki vektorning skalyar ko‘paytmasi deb nimaga aytiladi? Uning xossalari. Proyeksiyalari bilan berilgan ikki vektorning skalyar ko‘paytmasini qanday topasiz?

13. Vektorning uzunligini skalyar ko‘paytma orqali ifodalang.

14. Ikki vektorning vektor ko‘paytmasi deb nimaga aytiladi? Uning xossalari va berilgan vektorlarning proyeksiyalari orqali ifodasi.

15. Uchta vektorning aralash ko‘paytmasi deb nimaga aytiladi? Uning xossalari va geometrik ma’nosini aytib bering.

16. Uchta vektorning komplanarlik shartini ifodalang.

IV. Fazodagi analitik geometriya

1. Qanday parametrlar berilganda fazoda tekislikning o‘rni aniqlangan bo‘ladi?

2. Tekislik tenglamalarini (normal, umumiy, kesmalar bo‘yicha; berilgan bitta nuqtadan, uchta nuqtadan o‘tuvchi) yozing.

3. Ikki tekislik orasidagi burchakni qanday aniqlaysiz? Ikki tekislikning parallellik va perpendikulyarlik shartlarini yozing.

4. Berilgan nuqtadan berilgan tekislikkacha bo‘lgan masofa qanday topiladi?

5. Fazoda ikki tekislik kesishish chizig‘idan o‘tuvchi tekisliklar dastasining tenglamasini yozing. To‘g‘ri chiziqning proyeksiyalar bo‘yicha tenglamalarini yozing.

6. To‘g‘ri chiziqning yo‘naltiruvchi vektori deb qanday vektorga aytiladi? To‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing. Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing.

7. To‘g‘ri chiziq bilan tekislik orasidagi burchak deb qanday burchakka aytiladi va u qanday aniqlanadi? To‘g‘ri chiziq va tekislikning parallellik va perpendikulyarlik shartlarini yozing.

8. To‘g‘ri chiziq bilan tekislikning kesishish nuqtasini qanday topasiz?

9. Ikki to‘g‘ri chiziqning bir tekislikda yotish shartini yozing.

10. Sfera tenglamasini yozing.

11. Yasovchisi *Oz* o‘qiga parallel silindrik sirt tenglamasini yozing.

12. Aylanish sirtini qanday hosil qilasiz? Konus sirtlar tenglamasini yozing.

V. Kompleks sonlar

1. Qanday ifodaga kompleks son deyiladi?

2. Kompleks sonning trigonometrik shaklini yozing. Uning moduli va argumenti deb nimaga aytiladi?

3. Kompleks sonlar ustida qo‘shish, ayirish, ko‘paytirish va ildiz chiqarish amallari qanday bajariladi? Muavr formulasini yozing. Misol keltiring.

4. Haqiqiy sonni trigonometrik shaklda qanday tasvirlash mumkin?

MUSTAQIL YECHISH UCHUN MISOLLAR

Quyidagi determinantlarni hisoblang:

1. $\begin{vmatrix} 5 & 3 \\ 6 & 4 \end{vmatrix}$.

2. $\begin{vmatrix} 8 & 5 \\ 3 & 2 \end{vmatrix}$.

3. $\begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix}$.

4. $\begin{vmatrix} 3 & -2 \\ -4 & 5 \end{vmatrix}$.

5. $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}$.

6. $\begin{vmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{vmatrix}$.

7. $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$.

8. $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$.

9. $\begin{vmatrix} 2 & -3 & 1 \\ 6 & -6 & 2 \\ 2 & -1 & 2 \end{vmatrix}$.

10. $\begin{vmatrix} 0 & x & 0 \\ x & 1 & x \\ 0 & x & 0 \end{vmatrix}$.

11. $\begin{vmatrix} sin\alpha & sin\beta \\ cos\alpha & cos\beta \end{vmatrix}$.

Determinantlarni birinchi ustun elementlari bo‘yicha yoyib hisobla ng:

12. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

13. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$.

14. $\begin{vmatrix} 1 & 2 & -3 \\ 3 & 1 & -1 \\ 1 & 3 & 2 \end{vmatrix}$.

15. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

16. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -b \end{vmatrix}$.

17. $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$.

18. $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$.

19. $\begin{vmatrix} 1 & 2 & 5 \\ 3 & -4 & 7 \\ -3 & 12 & -15\cancel{0} \end{vmatrix}$.

20. $\begin{vmatrix} 12 & 6 & -4 \\ 6 & 4 & 4 \\ 3 & 2 & 8 \end{vmatrix}$.

21. $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$.

22. $\begin{vmatrix} 1+cos\alpha & 1+sin\alpha & 1 \\ 1-sin\alpha & 1+cos\alpha & 1 \\ 1 & 1 & 2 \end{vmatrix}$.

23. 1) $\begin{vmatrix} 2cos^2\frac{\alpha}{2} & sin\alpha & 1 \\ 2cos^2\frac{\beta}{2} & sin\beta & 1 \\ 1 & 0 & 1 \end{vmatrix}$; 2) $\begin{vmatrix} sin\alpha & cos\alpha & 1 \\ sin\beta & cos\beta & 1 \\ sin\gamma & cos\gamma & 1 \end{vmatrix}$.

Determinant xossalaridan foydalanib hisoblang:

24. 1) $\begin{vmatrix} 8 & 7 & 2 & 10 \\ -8 & 2 & 7 & 10 \\ 4 & 4 & 4 & 5 \\ 0 & 4 & -3 & 2 \end{vmatrix}$; 2) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 \end{vmatrix}$;

3) $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}$; 4) $\begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}$;

5) $\begin{vmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 2 & 3 \\ 3 & 1 & 6 & 1 \end{vmatrix}$; 6) $\begin{vmatrix} 2 & 3 & -3 & 4 \\ 2 & 1 & -1 & 2 \\ 6 & 2 & 1 & 0 \\ 2 & 3 & 0 & -5 \end{vmatrix}$;

7) $\begin{vmatrix} 3 & 1 & 2 & 3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix}$; 8) $\begin{vmatrix} 2 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix}$;

9) $\begin{vmatrix} -1 & -2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & -2 & 1 & 4 \\ 3 & 1 & -2 & -1 \end{vmatrix}$; 10) $\begin{vmatrix} 0 & 4 & 1 & 1 \\ -4 & 2 & 1 & 3 \\ 0 & 1 & 2 & -2 \\ 1 & 3 & 4 & -3 \end{vmatrix}$;

11) $\begin{vmatrix} 3 & 5 & 3 & 2 \\ 2 & 4 & 1 & 0 \\ 1 & -2 & 2 & 1 \\ 5 & 1 & -2 & 4 \end{vmatrix}$; 12) $\begin{vmatrix} -4 & 1 & 2 & 0 \\ 2 & -1 & 2 & 3 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & -2 & 3 \end{vmatrix}$;

13) $\begin{vmatrix} 1 & 1 & -2 & 0 \\ 3 & 6 & -2 & 5 \\ 1 & 0 & 6 & 4 \\ 2 & 3 & 5 & -1 \end{vmatrix}$; 14) $\begin{vmatrix} -1 & 1 & -2 & 3 \\ 1 & 2 & 2 & 3 \\ -2 & 3 & 1 & 0 \\ 2 & 3 & -2 & 0 \end{vmatrix}$;

25. Uchlari A(-2; 1), B(2; -2) va C(8; 6) nuqtalarda bo‘lgan uchburchakning yuzini hisoblang.

26. A(1; 3), B(2; 4) va C(3; 5) nuqtalar bir to‘g‘ri chiziqda yotadimi?

27. A va B matritsalar berilgan. Quyidagilar topilsin:

1) $2A-3B$; 2) $A\cdot B$; 3) $B\cdot A$; 4) A^{-1}; 5) $A\cdot A^{-1}$.

27.1. $A=\begin{pmatrix} 3 & 5 & -6 \\ 2 & 4 & 3 \\ -3 & 1 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 8 & -5 \\ -3 & -1 & 0 \\ 4 & 5 & -3 \end{pmatrix}$.

27.2. $A=\begin{pmatrix} 2 & -1 & -3 \\ 8 & -7 & -6 \\ -3 & 4 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 2 & -1 & -2 \\ 3 & -5 & 4 \\ 1 & 2 & 1 \end{pmatrix}$.

27.3. $A=\begin{pmatrix} 1 & 2 & 1 \\ 1 & -2 & 4 \\ 3 & -5 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 7 & 5 & 1 \\ 5 & 3 & -1 \\ 1 & 2 & 3 \end{pmatrix}$.

27.4. $A=\begin{pmatrix} 4 & 1 & -4 \\ 2 & -4 & 6 \\ 1 & 2 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & -1 & 1 \\ 2 & 5 & 0 \\ 1 & -1 & 2 \end{pmatrix}$.

27.5. $A=\begin{pmatrix} 2 & 2 & 5 \\ 3 & 3 & 6 \\ 4 & 3 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{pmatrix}$.

27.6. $A=\begin{pmatrix} 1 & 1 & -1 \\ 2 & -4 & 1 \\ 4 & -3 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 0 & -4 \\ 2 & 5 & -3 \\ 4 & -3 & 2 \end{pmatrix}$.

27.7. $A=\begin{pmatrix} 2 & 6 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 4 & -3 & 2 \\ -4 & 0 & 5 \\ 3 & 2 & -3 \end{pmatrix}$.

27.8. $A=\begin{pmatrix} 3 & -1 & 0 \\ 3 & 5 & 1 \\ 4 & -7 & 5 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 0 & 2 \\ 1 & 8 & 5 \\ 3 & 0 & 2 \end{pmatrix}$.

27.9. $A=\begin{pmatrix} 2 & -1 & -4 \\ 4 & -9 & 3 \\ 2 & -7 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 0 & -4 \\ 5 & -6 & 4 \\ 7 & -4 & 1 \end{pmatrix}$.

27.10. $A=\begin{pmatrix} 1 & 7 & 3 \\ -4 & 9 & 3 \\ 0 & 3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 6 & 5 & 2 \\ 1 & 9 & 2 \\ 4 & 5 & 2 \end{pmatrix}$.

27.11. $A=\begin{pmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 0 & 5 & 2 \end{pmatrix}$.

27.12. $A=\begin{pmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{pmatrix}$.

27.13. $A=\begin{pmatrix} -1 & 0 & 2 \\ 2 & 3 & 2 \\ 3 & 7 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 0 & 1 \\ -3 & 1 & 7 \\ 1 & 3 & 2 \end{pmatrix}$.

27.14. $A=\begin{pmatrix} -3 & 4 & 0 \\ 4 & 5 & 1 \\ -2 & 3 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 7 & -1 \\ 0 & 2 & 6 \\ 2 & -1 & 1 \end{pmatrix}$.

27.15. $A=\begin{pmatrix} 1 & 0 & 3 \\ 3 & 1 & 7 \\ 2 & 1 & 8 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 5 & 4 \\ -3 & 0 & 1 \\ 5 & 6 & -4 \end{pmatrix}$.

27.16. $A=\begin{pmatrix} -3 & 4 & -3 \\ 1 & 2 & 3 \\ 5 & 0 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & -2 & 0 \\ 5 & 4 & 1 \\ 1 & -1 & 2 \end{pmatrix}$.

27.17. $A=\begin{pmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{pmatrix}$.

27.18. $A=\begin{pmatrix} 8 & -1 & -1 \\ 5 & -5 & -1 \\ 10 & 3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 2 & 5 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix}$.

27.19. $A=\begin{pmatrix} 3 & -7 & 2 \\ 1 & -8 & 3 \\ 4 & -2 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 5 & -3 \\ 2 & 4 & 1 \\ 2 & 1 & -5 \end{pmatrix}$.

27.20. $A=\begin{pmatrix} -2 & 3 & 4 \\ 3 & -1 & -4 \\ -1 & 2 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 3 & 1 \\ 0 & 6 & 2 \\ 1 & 9 & 2 \end{pmatrix}$.

27.21. $A=\begin{pmatrix} 5 & 1 & -2 \\ 1 & 3 & -1 \\ 8 & 4 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 5 & 5 \\ 7 & 2 & 1 \\ 1 & 6 & 0 \end{pmatrix}$.

27.22. $A=\begin{pmatrix} 5 & 4 & 2 \\ 1 & 2 & 4 \\ 3 & 0 & 5 \end{pmatrix}$, $B=\begin{pmatrix} 5 & 4 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

27.23. $A=\begin{pmatrix} -3 & 4 & 2 \\ 1 & -5 & 3 \\ 0 & 1 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 4 & 4 \\ 1 & 3 & 2 \\ -4 & 1 & 2 \end{pmatrix}$.

27.24. $A=\begin{pmatrix} 2 & 3 & 2 \\ 1 & 3 & -1 \\ 4 & 1 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 2 & -1 \\ 3 & 1 & 2 \\ 5 & 3 & 0 \end{pmatrix}$.

27.25. $A=\begin{pmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 7 & 0 \\ 5 & 3 & 1 \\ 1 & -6 & 1 \end{pmatrix}$.

27.26. $A=\begin{pmatrix} 1 & -2 & 5 \\ 3 & 0 & 6 \\ 4 & 3 & 4 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{pmatrix}$.

27.27. $A=\begin{pmatrix} 8 & 5 & -1 \\ 1 & 5 & 3 \\ 1 & 1 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 4 & -7 & -6 \\ 3 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

27.28. $A=\begin{pmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 3 & 7 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & -3 & 2 \end{pmatrix}$.

27.29. $A=\begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 6 & 0 \\ 2 & 4 & -6 \\ 1 & -2 & 3 \end{pmatrix}$.

27.30. $A=\begin{pmatrix} 6 & 7 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 2 & 0 & 5 \\ 4 & -1 & -2 \\ 4 & 3 & 7 \end{pmatrix}$

28. Berilgan tenglamalar dan *x* ni toping va ildizlarni determinantga qo‘yib tekshiring:

1) $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0;$ 2) $\begin{vmatrix} x^2 & 4 & 0 \\ x & 2 & 3 \\ 1 & 1 & 0 \end{vmatrix} = 0$

3) $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0;$ 4) $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0.$

Tenglamalar sistemasini determinantlar yordamida yeching:

29. $\begin{cases} 2x + 3y = 1, \\ 3x + 5y = 4. \end{cases}$

30. $\begin{cases} 3x + y = 4, \\ 2x + 4y + 1. \end{cases}$

31. $\begin{cases} x + y = 1, \\ x - y = 2. \end{cases}$

32. $\begin{cases} 3x + 2y = 7, \\ 4x - 5y = 40. \end{cases}$

33. $\begin{cases} ax - 3y = 1, \\ ax + 2y = 6. \end{cases}$

34. $\begin{cases} mx - ny = (m-n)^2, \\ 2x - y = n \ (m \neq 2n). \end{cases}$

35. Ushbu, $3x - ay = 1$, 6x+4y=b tenglamalar sistemasi *a* va *b* ning qaysi qiymatlarida:

1) yagona yechimga ega;

2) yechim mavjud emas;

3) cheksiz ko‘p yechimga ega ekanligini aniqlang.

36. Bir jinsli $13x + 2y = 0$, $5x + ay = 0$ tenglamalar sistemasi *a* ning qanday qiymatida noldan farqli yechimga ega ekanligini aniqlang.

Tenglamalar sistemasini yeching:

37. 1) $\begin{cases} 2x - 3y + z - 2 = 0, \\ x + 5y - 4z + 5 = 0, \\ 4x + y - 3z + 4 = 0. \end{cases}$ 2) $\begin{cases} 7x + 2y + 3z = 15, \\ 5x - 3y + 2z = 15, \\ 10x - 11y + 5z = 36. \end{cases}$

38. 1) $\begin{cases} 2x - 4y + 3z = 1, \\ x - 2y + 4z = 3, \\ 3x - y + 5z = 2. \end{cases}$ 2) $\begin{cases} 2x + y = 5, \\ x + 3z = 16, \\ 5y - z = 10. \end{cases}$

39. 1) $\begin{cases} 2x + y + z = 0, \\ 3x - y + 2z = -3, \\ x + y - 3z = 4. \end{cases}$ 2) $\begin{cases} x + y - 2z = 6, \\ 2x + 3y - 7z = 16, \\ 5x + 2y + z = 16. \end{cases}$

40. 1) $\begin{cases} x + 2y + 3z = 1, \\ 2x - y + 2z = 6, \\ x + y - 3z = 7. \end{cases}$ 2) $\begin{cases} 3x + 2y + z = 5, \\ 2x - y + z = 6, \\ x + 5y = -3. \end{cases}$

41. $\begin{cases} 2x - 5y + 2z = 0, \\ x + 4y - 3z = 0. \end{cases}$

42. $\begin{cases} 3x + 2y + 2z = 0, \\ 5x + 2y + 3z = 0. \end{cases}$

43. $\begin{cases} x - 3y + 5z = 0, \\ 7x - 9y - 11z = 0. \end{cases}$

44. $\begin{cases} x + 2y + 3z = 0, \\ 2x - 3y + 4z = 0, \\ 3x - y + 7z = 0. \end{cases}$

45. $\begin{cases} 3x + 2y - z = 0, \\ 2x - y + 3z = 0, \\ x + 3y - 4z = 0. \end{cases}$

46. $\begin{cases} 3x + 2y - z = 0, \\ 2x - y + 3z = 0, \\ x + y - z = 0. \end{cases}$

47. $\begin{cases} 3x - y + 2z = 0, \\ 2x + 3y - 5z = 0, \\ x + y + z = 0. \end{cases}$

48. $\begin{cases} 2x - y + 3z = 0, \\ x + 2y - 5z = 0, \\ 3x + y - 2z = 0. \end{cases}$

49. $\begin{cases} x + 2y + 3z = 4, \\ 2x + 4y + 6z = 3, \\ 3x + y - z = 1. \end{cases}$

50. $\begin{cases} x + 2y + 3z = 4, \\ 2x + y - z = 3, \\ 3x + 3y + 2z = 7. \end{cases}$

51. $\begin{cases} x + 2y + 3z = 4, \\ 2x + y - z = 3, \\ 3x + 3y + 2z = 10. \end{cases}$

52. $\begin{cases} 2x - y + z = 2, \\ 3x + 2y + 2z = -2, \\ x - 2y + z = 1. \end{cases}$

Sistemalarni Gauss usuli bilan yeching:

53. $\begin{cases} x - y + 3z = -4, \\ 2x + 3y - 2z = 5, \\ 3x + 5y + z = 4. \end{cases}$

54. $\begin{cases} x - 2y - 3z = 8, \\ 3x + y + z = 3, \\ 4x + 3y - 2z = -1. \end{cases}$

55. $\begin{cases} 3x + y + 2z = -4, \\ x - 2y - z = -1, \\ 2x - 3y - 2z = 0. \end{cases}$

56. $\begin{cases} x + y = 4, \\ 2x + 3y + z = 7, \\ 2x + y + 3z = -3. \end{cases}$

57. $\begin{cases} 2x + y + 4z = 20, \\ 2x - y - 3z = 3, \\ 3x + 4y - 5z = -8. \end{cases}$

58. $\begin{cases} x + 2y + 4z = 31, \\ 5x + y + 2z = 29, \\ 3x - y + z = 10. \end{cases}$

59. $\begin{cases} x - 2y + 3z = 6, \\ 2x + 3y - 4z = 16, \\ 3x - 2y - 5z = 12. \end{cases}$

60. $\begin{cases} 2x + y - 3z = 3, \\ 3x + 4y - 5z = 9, \\ 2y + 7z = 11. \end{cases}$

Tenglamalar sistemasini matritsalar yordamida yeching:

64. 1) $\begin{cases} x + 2y + z = 4, \\ 3x - 5y + 3z = 1, \\ 2x + 7y - z = 8. \end{cases}$ 2) $\begin{cases} 2x - 4y + 9z = 28, \\ 7x + 3y - 6z = -1, \\ 7x + 9y - 9z = 5. \end{cases}$

65. $\begin{cases} 2x + y = 5, \\ x + 3z = 16, \\ 5y - z = 10. \end{cases}$

66. $\begin{cases} 7x + 2y + 3z = 15, \\ 5x - 3y + 2z = 15, \\ 10x - 11y + 5z = 36. \end{cases}$

67. $\begin{cases} x + y - z = 36, \\ x + z - y = 13, \\ y + z - x = 7. \end{cases}$

68. $\begin{cases} x + y + z = 36, \\ 2x - 3z = -17, \\ 6x - 5z = 7. \end{cases}$

69. $\begin{cases} 2x + y + 2z = 6, \\ x - 3y - z = -5, \\ 5x - 2y + z = -1. \end{cases}$

70. $\begin{cases} x - 2y + 3z = 5, \\ 2x + 3y - z = -4, \\ 3x + y - 2z = -1. \end{cases}$

71-74- masalalarda ko‘rsatilgan amallarni bajaring:

71. 1) $(2 - 3i)(-4 + 7i)$; 2) $(5 - 6i)(-10 + 8i)$;

3) $(\sqrt{3} + i)(\sqrt{2} - i\sqrt{3})$; 4) $(4 + i\sqrt{5})(4 - i\sqrt{5})$;

5) $(m+i\sqrt{n})(m-i\sqrt{n})$; 6) $(a+bi)(a-bi)$.

72. 1) $\frac{3i}{1-i}$ 2) $\frac{1+2i}{2-i}$; 3) $\frac{5+3i}{2+i}$; 4) $\frac{4-5i}{-2+7i}$.

73. 1) $\frac{1-\sqrt{3}i}{1+\sqrt{3}i}$; 2) $\frac{\sqrt{7}-i}{7-\sqrt{2}i}$; 3) $\frac{\sqrt{s}}{(s-2)i}$; 4) $\frac{m+i\sqrt{n}}{m-i\sqrt{n}}$; 5) $\frac{p+qi}{q-pi}$.

74. 1) $\frac{(3+4i)(-1+3i)}{6-8i}$; 2) $\frac{-4+6i}{(2+i)(3-2i)}$; 3) $\frac{(4-i)(1+2i)}{(-2+i)(1-3i)}$;

4) $\frac{(m+ni)(n+mi)}{n-mi}$.

Quyidagi kompleks sonlarni vektorlar bilan tasvirlang, ularning modullari va argumentlarini aniqlang hamda trigonometrik ko‘rinishda yozing:

75. 1) $z=3$; 2) $z=-2$; 3) $z=3i$; 4) $z=-2i$.

76. 1) $z=2-2i$; 2) $z=1+i\sqrt{3}$; 3) $z=-\sqrt{3}-i$.

77. 1) $z=\sqrt{3}+i$; 2) $z=3+i\sqrt{3}$; 3) $z=-\sqrt{2}+i\sqrt{6}$.

78. 1) $z=-\sqrt{2}+i\sqrt{2}$; 2) $z=\sin\alpha+i(1-\cos\alpha)$.

79. 75-78-masalalarda berilgan sonlarni ko‘rsatkichli $(re^{i\varphi})$ shaklda tasvirlang $(-\pi<\varphi<\pi)$.

80. 1) $x^2+25=0$; 2) $x^2-2x+5=0$; 3) $x^2+4x-13=0$ tenglamalarni yeching va ildizlarni tenglamaga qo‘yib tekshiring.

81. Quyidagi shartlarni qanoatlantiruvchi z nuqtalarning sohalarini yasang.

1) $|z|\le 3$;2) $|z|<2$ va $\frac{\pi}{2}<\varphi<\pi$;

3) $2<|z|<4$ va $-\pi<\varphi<\frac{\pi}{2}$;

4) $|z|>5$; 5) $|z-4|\le 2$; 6) $|z+2i|\ge 4$;

7) $|z-3i|=3$; 8) $|z+1-i|<2$; 9) $|z-i|=|z-1|$.

82. $|z_1 - z_2|$ ifoda z_1 va z_2 nuqtalar orasidagi masofa ekanligini ko‘rsating.

83. $z_0 = -2 + 3i$ nuqta berilgan. $|z - z_0| < 1$ tengsizlikni qanoatlantiruvchi z nuqtalarning sohasini yasang.

84. Quyidagilarni Muavr formulasi bilan hisoblang:

1) $(1+i)^{10}$; 2) $(1 - i\sqrt{3})^6$; 3) $(-1+i)^5$;

4) $(1 + \cos\frac{\pi}{4} + i\sin\frac{\pi}{4})^4$; 5) $(\sqrt{3} + i)^3$; 6) $(\cos 15^\circ + i\sin 15^\circ)^8$; 7) $[\sqrt{2}(\cos 10^\circ + i\sin 10^\circ)]^6$; 8) $[\frac{1}{\sqrt{2}}(\cos 57^\circ + i\sin 57^\circ)]^{10}$.

85. $z = \sqrt[6]{1}$ ning barcha qiymatlarini toping va radiusi 1 ga teng doira yasab, topilgan qiymatlarini radius-vektorlar bilan tasvirlang.

86. 1) $\sqrt[3]{-1}$; 2) $\sqrt[6]{-1}$; 3) $\sqrt[3]{-2+2i}$ larni toping.

87. 1) $\sqrt{i}$; 2) $\sqrt[3]{-1+i}$; 3) $\sqrt[4]{-8 + 8i\sqrt{3}}$ larni toping.

88. 1) $x^3 + \cancel{-}8 = 0$; 2) $x^4 + 4 = 0$; 3) $x^3 - 8 = 0$;

4) $x^6 + 64 = 0$;~~.~~ 5) $x^4 - 81 = 0$ ikki hadli tenglamalarni yeching.

89. Son o‘qida $A(-5), B(+4)$ va $C(-2)$ nuqtalarni yasang va shu o‘qdagi *AB*, *BC* va *AC* kattaliklarni toping. *AB*+*BC*=*AC* ekanligini tekshiring.

90. Berilgan *A* va *B* nuqtalar orasidagi masofani toping:

1) $A(0), B(-1)$; 2) $A(-2), B(8)$; 3) $A(3), B(-1)$;

4) $A(-7), B(-5)$.

91. Quyidagi *A* va *B* nuqtalar orasidagi masofani aniqlang:

1) $A(5;\ -3)$ va $B(-3;\ 1)$;

2) $A(4;\ 2)$ va $B(7;\ -2)$;

3) $A(0;\ 3)$ va $B(-2;\ 3)$;

4) $A(k;\ l)$ va $B(k+q;\ 0)$;

5) $A(-7;\ -9)$ va $B(0;\ 15)$;

6) $A(4;\ -4)$ va $B(6;\ 2)$.

92. Quyidagi nuqtalarning har biri bilan koordinatalar boshi orasidagi masofani toping:

1. A(3; 4); 2. B(5; -12);

3) C(0; 8); 4) D(a; b).

93. Uchlari $A(-4;\ 2), B(0;\ -1)$ va $C(3;\ 3)$ nuqtalarda bo‘lgan uchburchak yasang va uning perimetrini hamda burchaklarini aniqlang.

94. Quyidagi har bir holda ABC uchburchak o‘tkir burchakli, o‘tmas burchakli yoki to‘g‘ri burchakli bo‘lishini aniqlang.

1) A(1; 4), B(5; 8), C(3; 2);

2) A(2; 1), B(-2; 5), C(-1; 3);

3) $A(1;\ -1), B(-2;\ 1)$, C(1; 2);

4) A(-3; 2), B(0; -1), C(-2; 5).

Ko‘rsatma. *Uchburchak katta tomonining kvadrati qolgan ikki tomoni kvadratlarining yig‘indisidan kichik, katta yoki unga teng bo‘lishiga qarab u o‘tkir burchakli, o‘tmas burchakli yoki to‘g‘ri burchakli bo‘ladi.*

95. Ordinatalar o‘qida $A(-5;\ 1)$ va B(3; 2) nuqtalardan barobar uzoqlashgan nuqtani toping.

96. Uchlari A(1; 5), B(2; 2), $C\,(-3;\ -10)$, va $D\,(-3;\ -2)$ nuqtalarda bo‘lgan to‘rtburchak tomonlarining uzunliklarini aniqlang.

97. Ordinatasi 2 bo‘lgan nuqta (10; 3) va (8; 10) nuqtalardan teng uzoqlikda turadi. Bu nuqtaning abssissasini toping.

98. Abssissalar o‘qida shunday nuqtani topingki, undan (5; 12) nuqtagacha bo‘lgan masofa 13 ga teng bo‘lsin.

99. $A(2;\ 2)$, $B(-5;\ 1)$ va $C(3;\ -5)$ nuqtalardan teng uzoqlikda bo‘lgan nuqtani toping.

100. Koordinata o‘qlarida $K(-6;\ 8)$ nuqtadan 10 birlik masofaga uzoqlashgan nuqtalarni toping.

101. $A(-7;\ -3)$ nuqtadan markazi $C(5; -8)$ nuqtada va radiusi 5 ga teng bo‘lgan aylanaga urinmalar o‘tkazilgan. Ularning uzunliklarini toping.

102. $A(1; 2)$, $B(9; 2)$ va $C(2; -5)$ nuqtadan barobar uzoqlashgan D nuqtani toping.

103. Koordinata o‘qlaridan va $C(2; 4)$ nuqtadan barobar uzoqlashgan nuqtani toping.

104. $A(-4; 2)$ nuqtadan o‘tib, abssissalar o‘qining $B(2; 0)$ nuqtasida urinuvchi aylananing markazini toping.

105. Koordinata o‘qlarining har biriga urinuvchi va (2; -1) nuqtadan o‘tuvchi aylananing markazi, radiusini toping.

106. Nuqta to‘g‘ri chiziqli harakat qilib, $M(5; 5)$ va $N(1; 3)$ nuqtalardan o‘tadi. Ox o‘qini kesib o‘tgan nuqtasini toping.

107. $A(3; 5)$ va $B(1;\ -4)$ nuqtalarni yasang. AB kesmani $AN: NB = 2: 3$ nisbatda bo‘luvchi $N(x;y)$ nuqtani toping.

108. $A(-2;\ 1)$ va $B(3; 6)$ nuqtalar berilgan. AB kesmani $AN: NB = -3: 2$ nisbatda bo‘luvchi $N(x;y)$ nuqtani toping.

109. Ox o‘qining $A(x_1)$ va $B(x_2)$ nuqtalariga m_1 va m_2 massalar joylashtirilgan. Bu sistemaning og‘irlik markazini toping.

110. Uzunligi 40 sm va og‘irligi 500 g bo‘lgan bir jinsli sterjenning uchlariga og‘irliklari 100 va 400 g li sharlar osilgan. Shu sistemaning og‘irlik markazini aniqlang.

111. $A(-2;\ 4), B(3;\ -1)$ va $C(2; 3)$ nuqtalarga, mos ravishda, 60, 40 va 100 g massalar qo‘yilgan. Shu sistema massalarining og‘irlik markazini aniqlang.

112. ABC uchburchakning $A(6; 2)$, $B(3; -2)$ uchlari va uning medianalari kesishgan nuqta $M(3; 1)$ berilgan. C uchining koordinatalarini toping.

113. ABC uchburchak berilgan: $A(-1;\ 3), B(2; 1); C(7;\ -3)$. A burchak bissektrisasining uzunligini toping.

114. Uchlari $A(1;\ -1), B(6; 4)$ va $C(2; 6)$ nuqtalarda bo‘lgan uchburchakning og‘irlik markazini toping.

Ko‘rsatma. *Uchburchakning og‘irlik markazi medianalarning kesishgan nuqtasida yotadi.*

115. Uchlari $A(2; 0)$, $B(5; 3)$ va $C(2; 6)$ nuqtalarda bo‘lgan uchburchakning yuzini toping.

116. $A(1; 1)$, $B(-1; 7)$ va $C(0; 4)$ nuqtalarning bir to‘g‘ri chiziqda yotishini ko‘rsating.

117. Uchlari $A(3; 1)$, $B(4; 6)$, $C(6; 3)$ va $D(5; -2)$ nuqtalarda bo‘lgan to‘rtburchakning yuzini hisoblang.

118. (-3; 4) va (2; 2) nuqtalarning teng o‘rtasi bilan (-1; 3) nuqta orasidagi masofani aniqlang.

119. Uchlarining koordinatalari (4; 5), (-2; 3) va (1; 1) bo‘lgan uchburchakning har bir tomoni o‘rtasidagi nuqtalarning koordinatalarini toping.

120. $A(x;\ 4)$ va $B(-6;\ y)$ nuqtalar orasidagi AB masofa N(-1; 1) nuqtada teng ikkiga bo‘lingan. A va B nuqtalarni aniqlang.

121. (-1; 6) va (3; -2) nuqtalar orasidagi kesma to‘rtta teng bo‘lakka bo‘lingan. Bo‘linish nuqtalarining koordinatalarini toping.

122. Parallelogramm burchaklaridan uchtasining uchlari (4; -3), (6; 4) va (-5; -2) nuqtalarda. Uning to‘rtinchi burchagi uchining koordinatalarini toping.

123. Uchlari A(-2; 0), B(0; -1), C(2; 0), D(3; 2) va E(-1; 3) nuqtalarda bo‘lgan beshburchakning yuzini hisoblang.

124. Bir jinsli to‘rtburchakli taxtaning uchlari A(4; 4), B(5; 7), C(10; 10) va D(12; 4) nuqtalarda joylashgan. Taxtaning og‘irlik markazini toping.

125. Markazi koordinatalar boshida bo‘lib, radiusi R gat eng aylananing tenglamasi $x^2 + y^2 = R^2$ bo‘lishini ko‘rsating.

126. Markazi C(3; 4) nuqtada, radiusi $R = 5$ bo‘lgan aylana tenglamasini yozing. $A(-1; 1), B(2; 3), O(0; 0)$ va D(4; 1) nuqtalar shu aylanada yotadimi?

127. A(3; 2) va B(-1; 4) nuqtalardan teng uzoqlikda harakat qiluvchi $M(x;\ y)$ nuqta trayektoriyasining tenglamasini yozing.

C(-1; 1) D(1; -3), E(0; -1) va F(2; 2) nuqtalar o‘sha chiziqda yotadimi?

128. B(0; 4) nuqtaga nisbatan A(0; 12) nuqtadan uch marta uzoqroqda harakat qiluvchi $M(x; y)$ nuqta trayektoriyasining tenglamasini yozing.

129. B(-4; 4) nuqtaga nisbatan A(-1; 1) nuqtadan ikki marta yaqinroqda harakat qiluvchi $M(x;y)$ nuqta trayektoriyasining tenglamasini yozing.

130. Har bir nuqtasidan F(2; 0) va F(-2; 0) nuqtalargacha bo‘lgan masofalarning yig‘indisi $2\sqrt{5}$ ga teng nuqtalarning geometrik o‘rnining tenglamasini yozing va u bo‘yicha chiziq yasang.

131. Ushbu:

1) $y = 2x + 5$; 2) $y = 7 - 2x$; 3) $y = 2x$; 4) $y = 4$;

5) $y = 4 - x^2$; 6) $2x - y + 1 = 0$; 7) $y = \sqrt{9 - x^2}$;

8) $y = \frac{1}{1+x^2}$; 9) $y = \frac{1}{x}$; 10) $y = x^2 - 1$ tenglamalarga mos keladigan chiziqlarni (nuqtalar bo‘yicha) yasang.

132. Tenglamalari $y^2 - x^2 = 5$ va $2x + y - 1 = 0$ bo‘lgan L_1 va L_2 chiziqlarning kesishish nuqtasini toping.

133. Quyidagi chiziqlarning kesishish nuqtalarini toping:

1) $2x - y + 1 = 0$ va $2x + y - 5 = 0$;

2) $x^2 + y^2 = 25$ va $x = 3$;

3) $y = \frac{1}{x+1}$ va $x + y - 1 = 0$;

4) $y = \frac{6}{x}$ va $x^2 + y^2 = 13$.

134. Koordinatalar boshidan va $A\,(-2;\,-3)$ nuqtadan barobar uzoqlashgan nuqtalar geometrik o‘rnining tenglamasini tuzing.

135. Koordinatalar boshidan va A(1; 3) nuqtadan barobar uzoqlashgan nuqtalar geometrik o‘rnining tenglamasini tuzing.

136. A(3; 0) nuqtadan va koordinatalar o‘qidan barobar uzoqlashgan nuqtalar geometrik o‘rnining tenglamasini tuzing.

137. Markazi $C(a; b)$ nuqtada, radiusi r bo‘lgan aylana tenglamasini tuzing.

138. Berilgan $A(1; -2)$ va $B(-1; 2)$ nuqtalargacha masofalarning kvadratlari yig‘indisi 20 ga teng o‘zgarmas kattalik bo‘lgan nuqtalar geometrik o‘rnining tenglamasini tuzing.

139. Berilgan $A(0; 4)$ va $B(-1; 2)$ nuqtalargacha masofalarning kvadratlari yig‘indisi 1 ga teng o‘zgarmas kattalik bo‘lgan nuqtalar geometrik o‘rnining tenglamasini tuzing.

140. Agar M nuqta harakatlanayotgan har bir momentda undan $A(2\sqrt{3}; 2)$ nuqtagacha bo‘lgan masofa $B(\sqrt{3}; -1)$ nuqtagacha masofadan ikki marta ortiq bo‘lsa, M nuqta trayektoriyasining tenglamasini keltirib chiqaring.

141. Oy o‘qdan $b=5$ kesma ajratib, Ox o‘q bilan 1) 45°; 2) 135° burchak tashkil qiluvchi to‘g‘ri chiziqlarni yasang. O‘sha to‘g‘ri chiziqlarning tenglamalarini yozing.

142. Oy o‘qdan $b = -4$ kesma ajratib, Ox o‘q bilan 1) 60°; 2) 120° burchak tashkil qiluvchi to‘g‘ri chiziqlarni yasang. Bu to‘g‘ri chiziqlarning tenglamalarini yozing.

143. Koordinatalar boshidan o‘tib, Ox o‘qi bilan 1) 45°; 2) 60°; 3) 90°; 4) 120°; 5) 135° burchak tashkil qiluvchi to‘g‘ri chiziqlarning tenglamalarini yozing.

144. Koordinatalar boshidan va (-2; -3) nuqtadan o‘tuvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing.

145. $A(-2; 3)$ nuqtadan o‘tuvchi to‘g‘ri chiziq Ox o‘qi bilan 135° li burchak tashkil etadi. Bu to‘g‘ri chiziq tenglamasini toping.

146. 1) $2x - 3y = 6$; 2) $2x + 3y = 0$; 3) $y = -3$;

4) $\frac{x}{4} + \frac{y}{3} = 1$; 5) $x - 4y - 7 = 0$; 6) $y + 5 = 0$;

7) $5x - 2y = 0$; 8) $10x + 5y + 12 = 0$

to‘g‘ri chiziqlarning har qaysisi uchun *k* va *b* parametrlarni aniqlang.

147. Quyidagi tenglamalar bilan ifodalangan to‘g‘ri chiziqlardan qaysilari *Ox* o‘qining musbat yo‘nalishi bilan o‘tkir burchak va qaysilari o‘tmas burchak tashkil qiladi:

1) $y = -x + 5$; 2) $y = \frac{4}{5}x - 1$; 3) $y=x$;

4) $3x - 5y + 1 = 0$; 5) $y = -x$; 6) $3x = 4y$;

7) $x + y + 4 = 0$; 8) $2x + 3y = 0$?

148. Quyidagi to‘g‘ri chiziqlardan qaysilari koordinatalar boshidan o‘tadi? Ikkala koordinata o‘qini kesadi? *Ox* o‘qiga parallel? *Oy* o‘qiga parallel:

1) $y + 3 = 0$; 2) $x + 2y = 0$; 3) $2x - 5 = 0$;

4) $x + y = 2$; 5) $3y + x = 0$; 6) $6y + 7 = 0$;

7) $2x - y - 1 = 0$; 8) $x = y$?

149. Kvadratning uchlaridan biri koordinalar boshi bilan, qarama-qarshi uchi esa *A*(5; -5) nuqta bilan ustma-ust tushadi. Kvadrat tomonlarining tenglamalarini yozing.

150. Kvadrat uchlaridan biri koordinatalar boshida, diagonallari kesishgan nuqta *S*(-1; 1) nuqtada joylashgan. Kvadrat tomonlarining tenglamalarini tuzing.

151. To‘g‘ri to‘rtburchakning ikkita tomoni koordinata o‘qlarida yotadi, uchlaridan biri (-2; -3) koordinataga ega. To‘g‘ri to‘rtburchak tomonlarining tenglamalarini tuzing.

152. 1) $2x - 3y = 6$; 2) $3x - 2y + 4 = 0$; 3) $x + y - 3 = 0$; 4) $x + 2y - 8 = 0$; 5) $x + 8y - 16 = 0$ to‘g‘ri chiziqlarning tenglamalarini o‘qlardan ajratgan kesmalariga nisbatini yozing.

153. O(0;0) va A(-3; 0) nuqtalar berilgan. Bir tomoni OA kesmadan iborat bo‘lgan va diagonallari B(0; 2) nuqtada kesishuvchi parallelogram tomonlarining va diagonallarining tenglamalarini yozing.

154. A(0; 3) nuqtadan o‘tuvchi va koordinatalar burchagidan yuzi 3 kv birlikka teng uchburchak kesuvchi to‘g‘ri chiziq tenglamasini yozing.

155. Abssissalar o‘qining musbat yarmidan 2 birlikka teng, ordinatalar o‘qining manfiy yarmidan 5 birlikka teng bo‘lgan kesma ajratadigan to‘g‘ri chiziqning tenglamasini tuzing ($5x - 2y - 10 = 0$).

156. Rombning diagonallari 8 va 3 birlikka teng. Agar rombning katta diagonalini Ox o‘qi uchun, kichigini Oy o‘qi uchun qabul qilsak, romb tomonlarining tenglamalarini yozing.

($3x+8y-12=0$, $3x-8y+12=0$, $3x+8y+12=0$, $3x-8y-12=0$).

157. Koordinata o‘qlari bilan $2x - 5y + 20 = 0$ to‘g‘ri chiziq orasida joylashgan uchburchak yuzini toping (20 kvadrat birlik).

158. A(1; 2) nuqta orqali o‘tuvchi va koordinatalar o‘qining musbat yarim o‘qlaridan teng kesmalar ajratadigan to‘g‘ri chiziq tenglamasini tuzing.

159. (2; 3) nuqtadan o‘tuvchi va koordinata burchagidan yuzi 12 kv birlikka teng bo‘lgan uchburchak ajratuvchi to‘g‘ri chiziq tenglamasini tuzing $(3x + 2y - 12 = 0)$.

160. Asoslari 10 va 6 bo‘lgan teng yonli trapetsiyaning o‘tkir burchagi 60°, trapetsiyaning katta asosi *Ox* o‘qi, uning simmetriya o‘qini *Oy* o‘qi deb olib, uning tomonlari tenglamasini toping.

$$\left(y = 0;\ \ y = 2\sqrt{3};\ \ y = \sqrt{3}x + 5\sqrt{3};\ \ y = -\sqrt{3}x + 5\sqrt{3}\right).$$

161. Tomoni *a* gat eng bo‘lgan kvadratning diagonallari koordinata o‘qlaridan iborat. Kvadrat tomonlarining tenglamalarini tuzing.

$$\left(y = \pm x \pm \frac{a\sqrt{2}}{2}\right).$$

162. $y = \frac{2}{3}x - 4$ to‘g‘ri chiziq bo‘ylab yo‘nalgan nur *Ox* o‘qiga yetib, qaytadi. Nurning *Ox* o‘qi bilan uchrashgan nuqtasini va qaytgan nur tenglamasini toping.

163. Quyidagi to‘g‘ri chiziqlar orasidagi burchakni aniqlang:

1) $\begin{cases} y = 2x - 3, \\ y = \frac{1}{2}x + 1; \end{cases}$ 2) y=3x,y=-2x+5;

3) y=4x-7,y=-14x+2; 4) $\begin{cases} y = 5x - 3, \\ y = 5x + 8; \end{cases}$

5) $\begin{cases} y = \sqrt{3}x - 5, \\ y = -\sqrt{3}x + 1; \end{cases}$ 6) y=7x-2,y=x-2;

7) 5x-y+7=0,2x-3y+1=0; 8) 2x+y=0,y=3x-4;

9) 3x+2y=0,6x+4y+9=0; 10) 3x-4y=0,8x+6y=11;

11) $\begin{cases} \frac{x}{a} + \frac{y}{b} = 1, \\ \frac{x}{b} + \frac{y}{a} = 1; \end{cases}$ 12) $\begin{cases} y = \frac{2}{3}x + 9, \\ y = 4x + 7; \end{cases}$

13) $\begin{cases} y = \frac{3}{7}x + 5, \\ 3y + 7x - 4 = 0; \end{cases}$ 14) $\begin{cases} 2x - 4y + 5 = 0, \\ 6x - 3y - 7 = 0. \end{cases}$

164. Quyidagi to‘g‘ri chiziqlarning ustma-ust tushishligi, parallelligi yoki kesishishini (bunda kesishish nuqtasini topish kerak bo‘ladi) tekshiring:

1) x-y+3=0, 2x-2y-7=0; 2) 2x-y+4=0, 4x-2y+9=0;

3) x+3y-1=0, 2x+6y-2=0; 4) 5x-y+1=0, 10x-3y+2=0;

5) 3x+2y-4=0, 5x+6y-12=0; 6) 2x-3y=0, 6x-9y=0;

7)$y - 5 = 0, 3y + 15 = 0$; 8) 4x-1=0, 8y+2=0;

9) 2x+3=0, 2x-1=0; 10) 4x-y+1=0, 2x+3y-17=0;

11) 5x+3=0, 10x+7y+2=0.

165. *A*(2; 3) nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini yozing. Shu dastadan *Ox* o‘qi bilan: 1) 45°; 2) 60 ° ; 3) 135°; 4) 0° burchak tashkil etuvchi to‘g‘ri chiziqlarni tanlab oling va yasang.

166. *A*(-2; 5) nuqta va 2x-y=0 to‘g‘ri chiziqni yasang. *A* nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini yozing va o‘sha dastadan berilgan to‘g‘ri chiziqqa: 1) parallel; 2) perpendikulyar bo‘lgan to‘g‘ri chiziqlarni tanlab oling.

167. 2x-5y-10=0 to‘g‘ri chiziqning koordinata o‘qlari bilan kesishgan nuqtalaridan bu to‘g‘ri chiziqqa perpendikulyar chiqarilgan. Ularning tenglamalarini yozing.

168. Berilgan ikki nuqtadan o‘tgan to‘g‘ri chiziqning tenglamasini yozing:

1) A(-1;3) va B4; -2; 2) (-1;4) va (-3;-2) 3) (0; 6) va (3; 6)

4) (0; 0) va (2; 3); 5) $(-1;\ 1)$ va (0; 1).

169. $A(-1;6)$ va $B9;-8$ nuqtalar berilgan. AB kesmaning o‘rtasidan va $2x-3y+5=0$ to‘g‘ri chiziqqa parallel o‘tgan to‘g‘ri chiziq tenglamasini yozing.

170. Uchburchak uchlarining koordinatalari berilgan: $A(-2;3)$, $B(4;-2)$ va $C(1;5)$. Har qaysi uchidan qarama-qarshi tomonga parallel bo‘lib o‘tgan to‘g‘ri chiziqlarning tenglamalarini yozing.

171. Uchlari $A-2;0$, $B(2;6)$ va $C4;2$ nuqtalarda bo‘lgan uchburchakning BD balandligi va BE medianasi o‘tkazilgan. AC tomon, BE medianasi va BD balandlikning tenglamalarini tuzing.

172. To‘rtburchakning uchlari berilgan: $A-4;-2$, $B-3;1$, $C(4;3)$ va $D(5;-3)$. Bu to‘rtburchak tomonlarining o‘rtalari parallelogrammning uchlari ekanligini ko‘rsating.

173. Parallelogrammning $x-y+1=0$ va $2x+3y-6=0$ tomonlarini hamda uning uchlaridan biri $C(7;1)$ni bilgan holda qolgan ikkita tomonning tenglamalarini tuzing.

174. Uchburchakning uchlari $A(0;0)$, $B(1;2)$ va $C(-2;3)$ nuqtalarda. Uning istalgan ikki tomonining o‘rtasidan o‘tgan to‘g‘ri chiziq qolgan tomoniga parallel ekanligini isbot qiling.

175. $(0;2)$ nuqtadan o‘tib, $y-x-5=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

176. $(5;-3)$ nuqtadan o‘tib, $4x+3y+15=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

177. Uchburchakning uchlari $A(2;5)$, $B(2;-3)$ va $C(4;-1)$ nuqtalarda. Bu uchburchak balandliklarining tenglamalarini tuzing.

178. Uchburchakning uchlari A(-2; 3), B(0; 0) va C(3; 5) nuqtalarda. Uning har bir tomonining o‘rtasiga perpendikulyar bo‘lgan to‘g‘ri chiziqlarning tenglamalarini tuzing.

179. Tomonlari $x+y=4$, $3x-y=0$, $x-3y-8=0$ tenglamalar bilan berilgan uchburchak yasang, uning burchaklari va yuzini toping.

180. Uchlari A(-4; 2), B(2; -5) va C(5; 0) nuqtalarda bo‘lgan uchburchak medianalarining kesishgan nuqtasini va balandliklarining kesishgan nuqtasini toping.

181. A(-2; -2), B(-3; 1), C(7; 7) va D(3; 1) nuqtalar trapetsiyaning uchlari ekanligini tekshiring. Bu trapetsiyaning o‘rta chiziqg‘i va diagonallarining tenglamalarini tuzing (BC‖DA;3x-5y+5=0;x-y=0;y-1=0).

182. Uchburchakning A(3; 5), B(6; 1) uchlari va uning medianalarining kesishgan nuqtasi N(4; 0) bo‘yicha uning tomonlari tenglamalarini tuzing (4x+3y-27=0AB; x=3AC; 7x-3y-39=0(BC)).

183. Ikkita: A(-3; 1) va B(3; -7) nuqtalar berilgan. Ordinata o‘qida shunday N nuqta topingki, AN va BN to‘g‘ri chiziqlar bir-biriga perpendikulyar bo‘lsin (N1(0;2) va $N_2(0;\ -8)$).

184. M(4; -3) nuqtadan o‘tib, koordinata o‘qlari bilan, yuzasi 3 kv birlikka teng bo‘lgan uchburchak hosil qiluvchi to‘g‘ri chiziq tenglamasini tuzing.

185. 1) $4x-3y+10=0$; 2) $5x+12y-39=0$;

3) $6x+8y-15=0$; 4) $x-2y+3=0$;

5) $y - x\sqrt{3} = 4$; 6) $xcos10° + ysin10° + 4 = 0$ to‘g‘ri chiziqlarning tenglamalarini normal ko‘rinishga keltiring.

186. A(4;3); B(2;1) va C(1;0) nuqtalardan 3x+4y-10=0 to‘g‘ri chiziqqacha bo‘lgan masofalarni toping va to‘g‘ri chiziqni yasang.

187. Normal uzunligi *p*=2 va uning *Ox* o‘qqa og‘ish burchagi β: 1) 45°; 2) 135°; 4) 315° bo‘lgan to‘g‘ri chiziqlarni yasang.

188. Berilgan nuqtadan berilgan to‘g‘ri chiziqqacha bo‘lgan masofani hisoblang:

1) P14; -2, 8x-15y-11=0;

2) $P_2(2;7)$, $12x + 5y - 7 = 0$;

3) $P_3(-3;5)$, $9x - 12y + 2 = 0$;

4) $P_4(-3;2)$, $4x - 7y + 26 = 0$;

5) $P_5(8;5), 3x - 4y - 15 = 0$.

189. Koordinatalar boshidan $12x - 5y + 39 = 0$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

190. 1) $3x - 4y - 10 = 0$ va $6x - 8y + 15 = 0$; 2) $2x - 3y - 6 = 0$ va $4x - 6y - 25 = 0$ to‘g‘ri chiziqlar o‘zaro parallel ekanligini ko‘rsating va ularning orasidagi masofani aniqlang.

191. $y = kx + 5$ to‘g‘ri chiziq koordinatalar boshidan $d = \sqrt{5}$ masofa uzoqlikda bo‘lsa, *k* ni toping.

192. $4x - 3y = 0$ to‘g‘ri chiziqdan 4 birlik uzoqlikdagi nuqtalar geometrik o‘rnining tenglamalarini yozing.

193. $5x + Ay - 15 = 0$ tenglamadagi *A* koeffitsiyenti qanday bo‘lganida u to‘g‘ri chiziqning ordinatalar o‘qidan kesgan kesma 2 ga teng bo‘ladi?

194. $8x - 15y = 0$ to‘g‘ri chiziqqa parallel bo‘lib, $A(4; -2)$ nuqtadan 4 birlik uzoqlikdagi to‘g‘ri chiziqning tenglamasini yozing.

195. 1) $2x + 3y = 12$ va $3x + 2y = 12$; 2) $3x + 4y = 12$ va $y = 0$; 3) $2x - 9y + 18 = 0$ va $6x + 7y - 21 = 0$ to‘g‘ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalarini yozing.

196. $M(x; y)$ nuqta $y = 4 - 2x$ to‘g‘ri chiziqqa nisbatan $y = 2x - 4$ to‘g‘ri chiziqdan 3 marta uzoqdan harakat qiladi. O‘sha nuqta trayektoriyasining tenglamasini yozing.

197. Uchlari $A\left(\frac{9}{5}; \frac{2}{5}\right)$, $B(0; 4)$ va $C(-3; -2)$ nuqtalarda bo‘lgan uchburchakka ichki chizilgan doira markazining koordinatalarini toping.

198. $N(1; 2)$ nuqtadan o‘tuvchi va $A(3; 3)$, $B(5; 2)$ nuqtalardan bir xil uzoqlikda bo‘lgan to‘g‘ri chiziq tenglamasini toping.

199. Ikkita: $A(2; -3)$ va $B(5; -1)$ nuqtalar berilgan. A nuqtadan 6 birlik va B nuqtadana 4 birlik uzoqlikda bo‘lgan to‘g‘ri chiziq tenglamasini toping.

200. $P(6; -2)$ nuqtadan 4 birlik masofada yotgan va $y = \frac{8}{15}x + 1$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziqning tenglamasini toping.

201. $x + 7y - 6 = 0$ va $5x - 5y + 1 = 0$ to‘g‘ri chiziqlar orasidagi burchak bissektrisalarining tenglamalarini yozing.

202. $A(3; 5)$ va $B(5; 2)$ nuqtalardan teng uzoqlikda bo‘lib, $M(1; 2)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing.

203. $(-1; 2)$ nuqtadan o‘tib, abssissalar o‘qining musbat yo‘nalishi bilan tashkil qilgan burchagining sinusi 0,8 ga teng bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

204. $P(0;1)$ nuqtadan o‘tib, $x-3y+10=0$, $2x+y-8=0$ to‘g‘ri chiziqlarning orasidagi kesmasi P nuqtada teng ikkiga bo‘linuvchi to‘g‘ri chiziqni o‘tkazing.

205. $3x-2y+1=0$ va $x+3y-7=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan ularning birinchisiga perpendikulyar to‘g‘ri chiziq o‘tkazilgan. Hosil qilingan to‘g‘ri chiziqdan koordinatalar boshigacha bo‘lgan masofa qancha?

206. Tenglamasi $4x+3y+1=0$ bo‘lgan to‘g‘ri chiziq berilgan. Bu to‘g‘ri chiziqqa parallel bo‘lgan shunday to‘g‘ri chiziq topilsinki, u bilan berilgan to‘g‘ri chiziq orasidagi masofa 3 birlikka teng bo‘lsin.

207. (2;7) nuqtadan shunday to‘gri chiziq o‘tkazilganki, u koordinata o‘qlari bilan yuzi 64 kv birlikka teng bo‘lgan uchburchak tashkil qiladi. Bu chiziqning tenglamasini tuzing.

208. Tenglamalari $3x-4y+6=0$, $x-2=0$ va $y=2x-1$ bo‘lgan to‘g‘ri chiziqlarning bir nuqtadan o‘tishini isbot qiling.

209. Uchburchakning uchlari berilgan: $A(-8;1)$, $B(1;-2)$ va $C(6;3)$. Uchburchakka tashqi chizilgan aylananing markazi va radiusini toping.

210. Uchburchakning uchlari $A(-2;-1)$, $B(1;3)$ va $C(-2;3)$ nuqtalarda. Uchburchakka ichki chizilgan aylananing markazi va radiusini toping.

211. $2x+5y+3=0$ va $3x-4y-7=0$ to‘g‘ri chiziqlarning kesishgan nuqtasidan shunday to‘g‘ri chiziq o‘tkazingki, uning bilan $y=4x+3$ to‘g‘ri chiziqning orasidagi burchak 45° bo‘lsin.

212. Teng yonli uchburchakda yon tomonlarining tenglamalari $3x - y + 6 = 0$ va $x + 3y - 2 = 0$. Uchburchakning asosi $(1; -2)$ nuqtadan o'tadi. Asosining tenglamasini tuzing.

213. Uchburchakning tomonlaridan ikkitasining tenglamalari $y + 1 = 0$ va $x + 1 = 0$ va medianalari kesishgan nuqta $(-1; 0)$ berilgan. Uchinchi tomon tenglamasini tuzing.

214. Uchburchakning tomonlaridan ikkitasining tenglamalari $3x + 2y + 6 = 0$, $x + y - 3 = 0$ va balandliklarining kesishgan nuqtasi (0;0) berilgan. Uchinchi tomon tenglamasini tuzing.

215. Uchburchakning $A(0; -4)$, $B(3; 0)$ va $C(0; 6)$ uchlari berilgan. C uchidan A burchakning bissektrisasigacha bo'lgan masofani toping.

216. Parallelogrammning AB va BC tomonlari, mos ravishda, $2x - y + 5 = 0$ va $x - y + 4 = 0$ tenglamalar bilan berilgan, diagonallari $M(1; 4)$ nuqtada kesishadi. Uning balandliklarining uzunliklarini toping.

217. Romb ikki tomonining tenglamalari $x + 2y = 4$ va $x + 2y = 10$ hamda diagonallaridan birining tenglamasi y=x+2 ma'lum bo'lsa, romb uchlarining koordinatalarini toping.

218. $2x + y - 6 = 0$ to'g'ri chiziq va unda ordinatalari $y_A = 6$ va $y_B = -2$ bo'lgan ikki A va B nuqta berilgan. AOB uchburchakning AD balandligining tenglamasini yozing, uning uzunligi va DAB burchakni toping.

219. Uchburchakning bitta uchi $A(3; -4)$ va ikkita balandliklarining tenglamalari: $7x - 2y - 1 = 0$ va $2x - 7y - 6 = 0$ ga ko'ra uchburchak tomonlarining tenglamalarini tuzing.

220. Uchburchakning bitta uchi $A(-4;2)$ va ikkita medianalarining tenglamalari: $3x-2y+2=0$ va $3x+5y-12=0$ ga ko‘ra uchburchak tomonlarining tenglamalarini tuzing.

221. Markazi 1) $C(1;2)$ nuqtada va radiusi $R=3$; 2) C(0;-3), $R=5$; 3) $C(-4;3)$, $R=5$ bo‘lgan aylana tenglamasini yozing.

222. $A(-4;6)$ nuqta berilgan. Diametri OA kesmadan iborat aylana tenglamasini yozing.

223. Aylana diametrlaridan birining uchlari $M_1(2;-7)$ va $M_2(-4;-3)$ nuqtalarda yotishi ma‘lum. Aylana tenglamasini yozing.

224. Diametri $12x+5y+60=0$ to‘g‘ri chiziqning koordinata o‘qlari orasidagi kesmasidan iborat bo‘lgan aylananing tenglamasini tuzing.

225. Quyidagi aylanalarning radiuslari va markazlarining koordinatalarini aniqlang:

$a)$ $x^2+y^2+2x-6y+5=0$; $b)$ $x^2+y^2+4x-5=0$;

$d)$ $x^2-5+y^2+4y=0$; $e)$ $4x^2-5x+4y^2-8=0$;

f) $x^2+y^2-8x+6y+21=0$; *g)* $x^2+y^2-10x-6y-15=0$.

226. $x^2+y^2-5x-7y+6=0$ aylananing koordinata o‘qlari bilan kesishgan nuqtalarini aniqlang.

227. Aylana Ox o‘qqa koordinata boshida urinadi va Oy o‘qini (0;10) nuqtada kesib o‘tadi. Aylana tenglamasini yozing.

228. Markazi Ox o‘qda va $A(6;4\sqrt{2})$ hamda $B(0;-2\sqrt{5})$ nuqtalardan o‘tuvchi aylananing tenglamasini yozing.

229. $A(3;-1)$ va $B(-4;-8)$ nuqtalardan o‘tuvchi aylananing radiusi $R=13$ ga teng. Aylana tenglamasini yozing.

230. $A(-3;0)$ va $B(3;6)$ nuqtalar berilgan. Diametri AB kesmadan iborat aylana tenglamasini yozing.

231. 1) $x^2+y^2-6x+4y-23=0$; 2) $x^2+y^2+5x-7y+2{,}5=0$; 3) $x^2+y^2+7y=0$ aylanalarning markazlari va radiuslarini toping. Aylanalarni yasang.

232. Koordinata o‘qlariga urinadigan aylana $M(-2;-4)$ nuqtadan o‘tadi. Uning tenglamasini yozing.

233. Koordinata o‘qlariga urinuvchi va radiusi $\sqrt{5}$ ga teng bo‘lib, IV chorakda yotuvchi aylana tenglamasini yozing.

234. Koordinata o‘qlariga urinuvchi aylana markazi $2x-y+3=0$ to‘g‘ri chiziqda yotadi. Aylana tenglamasini yozing.

235. Berilgan $A(-1;3)$, $B(-2;-4)$ va $C(6;2)$ nuqtalardan o‘tuvchi aylana tenglamasini yozing.

236. Uchburchakning uchlari quyidagi koordinatalarga ega: $A(-2;9), B(-4;9)$ va $C(5;8)$. Uchburchakka tashqi chizilgan aylana tenglamasini yozing.

237. $x^2+y^2=25$ aylananing $2x-y+1=0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmalarini toping.

238. (1;1) nuqtadan o‘tib $7x+y-3=0$ va $x+7y-3=0$ to‘g‘ri chiziqlarga urinuvchi aylana tenglamasini yozing.

239. $A(1;-2), B(0;-1)$ va $C(-3;0)$ nuqtalardan o‘tuvchi aylanaga koordinatalar boshidan o‘tkazilgan urinmalar tenglamalarini yozing.

240. $x^2+y^2-4x+6y-5=0$ aylananing Ox o‘q bilan kesishgan nuqtalariga o‘tkazilgan radiuslari o‘rtasidagi burchaklarni toping.

241. Quyidagi ellipslarning uchlari koordinatalarini, yarim o‘qlarini, fokuslarini va ekssentrisitetini toping:

1) $16x^2 + 25y^2 = 400$; 2) $4x^2 + 9y^2 = 36$;

3) $16x^2 + 9y^2 = 144$; 4) $25x^2 + 9y^2 = 900$;

5) $4x^2 + 6y^2 = 192$; 6) $9x^2 + 7y^2 = 63$.

242. $\frac{x^2}{9} + \frac{y^2}{16} = 1$ ellipsga nisbatan $A(-3; 0)$, $B(0; -5)$, $C(2; 3)$, $D\left(-\frac{3}{4}; \sqrt{15}\right)$, $E(2; -2)$ va $P(3; \frac{4}{3})$ nuqtalar qanday joylashgan?

243. Ellipsning katta yarim o‘qi $a = 4$ va ellipsda yotuvchi $M(-2; \frac{3\sqrt{3}}{2})$ nuqta ma‘lum. Ellipsning eng sodda tenglamasini tuzing va M nuqtadan ellips fokuslarigacha bo‘lgan masofani toping.

244. Kichik yarim o‘qi 24 ga teng bo‘lgan va fokuslardan biri $(-5; 0)$ koordinatalarga ega bo‘lgan ellipsning eng sodda tenglamasini tuzing.

245. Yarim o‘qlarining yig‘indisi 36 ga, Oy o‘qida yotuvchi fokuslari orasidagi masofa 48 ga teng bo‘lgan ellipsning eng sodda tenglamasini tuzing.

246. Agar ellipsning fokuslaridan biri (6;0) nuqtada bo‘lsa va ekssentrisiteti $\varepsilon = \frac{2}{3}$ ga teng bo‘lsa, uning eng sodda tenglamasini tuzing.

247. Ellipsning fokuslari orasidagi masofa katta va kichik o‘qlarining uchlari orasidagi masofaga teng. Ellipsning ekssentrisitetini toping.

248. $9x^2 + 25y^2 = 225$ ellipsda shunday $M(x; y)$ nuqta topingki, undan o‘ng fokusgacha bo‘lgan masofa chap fokusgacha bo‘lgan masofadan 4 marta katta bo‘lsin.

249. $x^2 + y^2 = 36$ aylanadagi barcha nuqtalarning ordinatalari 3 barobar marta qisqartirishdan hosil bo‘lgan yangi egri chiziq tenglamasini yozing.

250. $M(x; y)$ nuqta $x = -4$ to‘g‘ri chiziqqa nisbatan F(-1;0) nuqtaga ikki barobar yaqinroqda harakat qiladi. Uning trayektoriyasini aniqlang.

251. $x^2 + 4y^2 = 16$ ellipsni yasang, uning fokuslari va ekssentrisitetini toping.

252. Quyidagi berilgan parametrlarga nisbatan ellipsning eng sodda tenglamasini yozing:

1) yarim o‘qlari 4 va 2 ga teng;

2) fokuslari orasidagi masofa 6 va katta yarim o‘qi 5 ga teng;

3) katta yarim o‘qi 10 va ekssentrisiteti $\varepsilon = 0{,}8$ ga teng;

4) kichik yarim o‘qi 3 va ekssentrisiteti $\varepsilon = \frac{\sqrt{2}}{2}$ ga teng;

5) yarim o‘qlari yig‘indisi 8 va fokuslar orasidagi masofa 8 ga teng.

253. $25x^2 + 169y^2 = 4225$ ellipsning o‘qlari uzunliklari, fokuslarining koordinatalari va ekssentrisitetini toping.

254. Ellips fokuslarining biridan katta o‘qining uchlarigacha bo‘lgan masofalar 7 va 1 ga teng. Uning eng sodda tenglamasini tuzing.

255. Ellipsdagi ikki nuqtaning koordinatalari (1;4) va (-6;1). Bu ellipsning tenglamasini tuzing.

256. Katta o‘qi kichik o‘qidan uch marta katta bo‘lgan ellipsning ekssentrisitetini toping.

257. Ellipsning tenglamasi berilgan: $9x^2 + 25y^2 = 225$. Uning abssissasi 3 bo‘lgan nuqtasining radius-vektorlarini aniqlang.

258. Ellipsning tenglamasi berilgan: $7x^2 + 18y^2 = 126$. Uning absissasi 3 bo‘lgan va ordinatasi musbat bo‘lgan nuqtasining radius-vektorlari orasidagi burchakni toping.

259. $5x^2 + 9y^2 = 180$ ellipsda shunday nuqtani topingki, nudan o‘ng fokusgacha bo‘lgan masofa chap fokusgacha bo‘lgan masofadan ikki marta kichik bo‘lsin.

260. $16x^2 + 25y^2 = 400$ ellips va markazi ellips kichik o‘qining yuqori uchida bo‘lib, uning fokusidan o‘tuvchi aylana berilgan. Ellips va aylananing kesishish nuqtalarini toping.

261. $8x^2 + 10y^2 = 160$ ellipsga to‘g‘ri to‘rtburchak shunday ichki chizilganki, uning ikkita qarama-qarshi tomoni ellipsning fokuslaridan o‘tadi. Bu to‘rtburchak yuzini toping.

262. $16x^2 - 9y^2 - 144 = 0$ giperbolaning o‘qlari, uchlari, ekssentrisiteti va asimptotalarining tenglamalarini tuzing.

263. Quyidagi giperbolalarning uchlari koordinatalari, o‘qlari, fokuslari va ekssentrisitetini toping:

1) $4x^2 - 5y^2 - 100 = 0$; 2) $9x^2 - 4y^2 - 144 = 0$;

3) $16x^2 - 9y^2 + 144 = 0$; 4) $9x^2 - 7y^2 - 252 = 0$.

264. Haqiqiy abssissalar o‘qida yotadigan va $M_1(3; -2)$, $M_2(-6; 2\sqrt{10})$ nuqtalardan o‘tadigan giperbolaning ekssentrisiteti va fokuslarining koordinatalarini toping.

265. Haqiqiy o‘qi 6 ga, fokuslari orasidagi masofa 8 ga teng bo‘lgan giperbolaning eng sodda tenglamasini tuzing. Qo‘shma giperbolaning tenglamasini tuzing.

266. Giperbolaning yarim o‘qlari yig‘indisi 17 ga, ekssentrisiteti $\varepsilon = \frac{13}{12}$ ga teng. Giperbolaning eng sodda tenglamasini tuzing va fokuslarining koordinatalarini toping.

267. $7x^2 - 5y^2 = 35$ giperbola fokuslaridan o‘tuvchi va *Ox* o‘q bilan 60° li burchak tashkil etuvchi to‘g‘ri chiziq tenglamalarini tuzing.

268. $M(-5; 2)$ nuqta orqali $9x^2 - 4y^2 = 36$ giperbola asimptotalariga parallel bo‘lgan to‘g‘ri chiziqlar o‘tkazing.

269. Giperbolaning ekssentrisiteti $\sqrt{3}$ ga teng, fokuslari (6;0) va (-6;0) nuqtalarda joylashgan. Giperbola va uning asimptotalari tenglamalarini yozing.

270. $x^2 - 3y^2 = 27$ giperbola asimptotalari orasidagi o‘tkir burchak va ekssentrisitetini toping.

271. $M(6; \frac{3}{2}\sqrt{5})$ nuqtadan o‘tuvchi, koordinata o‘qlariga nisbatan simmetrik bo‘lgan giperbolaning haqiqiy yarim o‘qi $a = 4$ ga teng. Giperbolaning chap fokusidan asimptotalariga tushirilgan perpendikulyarning tenglamalarini yozing.

272. M nuqta $9x^2 - 16y^2 = 144$ giperbolaning fokuslari orasidagi masofani $F_2M : MF_1 = 2 : 3$ nisbatda bo‘ladi. F_2 – giperbolaning chap fokusi va M nuqtadan *Ox* o‘qi bilan 135° li burchak tashkil etuvchi to‘g‘ri chiziq o‘tkazilgan. Shu to‘g‘ri chiziqning giperbola asimptotalari bilan kesishgan nuqtalarini toping.

273. $x = -2$ to‘g‘ri chiziqqa nisbatan $F(-8; 0)$ nuqtadan ikki barobar uzoqlikda harakat qiluvchi M nuqtaning trayektoriyasini aniqlang.

274. Quyidagi berilgan parametrlarga ko‘ra giperbolaning eng sodda tenglamasini yozing:

1) uchlarining orasidagi masofa 8 ga, fokuslari orasidagi masofa 10 ga teng;

2) haqiqiy yarim o‘qi 5 ga teng va uchlari markaz bilan fokus oralig‘ini teng ikkiga bo‘ladi;

3) haqiqiy o‘q 6 ga teng va giperbola (9;-4) nuqtadan o‘tadi;

4) giperbola $P(-5;2)$ va $Q(2\sqrt{5};\sqrt{2})$ nuqtalardan o‘tadi.

275. Fokuslari $F_1(10;0)$, $F_2(-10;0)$ bo‘lgan va $M(12;3\sqrt{5})$ nuqtadan o‘tuvchi giperbolaning tenglamasini tuzing.

276. Fokusi $\frac{x^2}{49}+\frac{y^2}{24}=1$ ellipsning fokusi bilan umumiy bo‘lgan va ekssentrisiteti $\varepsilon=1{,}25$ ga teng giperbolaning tenglamasini tuzing.

277. Uchlari $\frac{x^2}{169}+\frac{y^2}{144}=1$ ellipsning fokuslarida, fokuslari esa uning uchlarida bo‘lgan giperbolaning tenglamasini tuzing.

278. $\frac{x^2}{49}-\frac{y^2}{25}=1$ giperbolaning fokuslarini va asimptotalarini yasang.

279. $\frac{x^2}{9}-\frac{y^2}{16}=1$ giperbola berilgan. Talab qilinadi:

1) fokuslarining koordinatalarini hisoblash;

2) ekssentrisitetini hisoblash;

3) asimptotalari va direktrisalarining tenglamalarini yozish;

4) qo‘shma giperbola tenglamalarini yozish va uning ekssentirisitetini hisoblash.

280. Mavhum o‘qi $2\sqrt{2}$ ga teng bo‘lgan giperbola direktrisalarining tenglamalari: $x\pm2=0$. Giperbolaning tenglamasini tuzing.

281. Quyidagi parabolalarning fokusi koordintalarini toping va direktrisasi tenglamasini yozing:

1) $y^2 = 8x$; 2) $y^2 = -12x$;

3) $x^2 = 10y$; 4) $x^2 = -16y$.

282. Quyidagilarga asoslanib, parabolaning tenglamasini tuzing:

1) uchidan fokusigacha bo‘lgan masofa 3 ga teng;

2) fokusining koordinatasi – (5;0), direktrisasi – ordinatalar o‘qi;

3) $M(1;-4)$ nuqtadan o‘tuvchi *Ox* o‘qiga simmetrik bo‘lgan parabola;

4) fokusi (0;2) da, O*y* o‘qiga simmetrik va uchi koordinata boshida bo‘lgan parabola;

5) koordinatalar boshidan va $M(6;-2)$ nuqtadan o‘tuvchi, *Oy* o‘qiga simmetrik bo‘lgan parabola.

283. $y^2 = 8x$ parabolada fokal radius-vektori 20 ga teng bo‘lgan nuqtani toping.

284. $y^2 = 4{,}5x$ parabolada direktrisadan $d = 9{,}125$ masofada bo‘lgan M(x;y) nuqta berilgan. Shu nuqtadan parabola uchigacha bo‘lgan masofani toping.

285. $y^2 = 48x$ parabolaning fokusi orqali o‘tuvchi va $y = \sqrt{3}x + 1$ to‘g‘ri chiziqqa parallel qilib to‘g‘ri chiziq o‘tkazilgan. Hosil bo‘lgan vatarning uzunligini toping.

286. $y^2 = 6x$ parabolaning: 1) $3x + y - 6 = 0$; 2) $2x - y + 5 = 0$; 3) $y - 6 = 0$ to‘g‘ri chiziqlar bilan kesishish nuqtasini toping.

287. $y^2 = 6x$ parabolaning $\frac{x^2}{100} + \frac{y^2}{64} = 1$ ellips bilan kesishish nuqtalarini toping.

288. $y^2 = 36x$ parabola hamda $(x+12)^2 + y^2 = 400$ aylananing umumiy vatari tenglamasini tuzing va uzunligini aniqlang.

289. Uchlari koordinatalar boshida, fokuslari $F_1(-6;0)$ va $F_2(0;-6)$ nuqtalarda bo‘lgan ikkita parabolaning umumiy vatari uzunligini toping.

290. Uchi (7;2) nuqtada, fokusi (7;5) nuqtada bo‘lgan parabola tenglamasini tuzing.

291. Ox o‘qqa nisbatan simmetrik bo‘lgan parabolaning uchi (3;0) nuqtada. U ordinatalar o‘qidan uzunligi 24 ga teng bo‘lgan vatar ajratiladi. Parabola tenglamasini tuzing, uning fokusi va direktrisasini toping.

292. Fokusi $\left(\frac{9}{2};-1\right)$ nuqtada bo‘lib, direktrisasi $2x-3=0$ to‘g‘ri chiziqdan iborat bo‘lgan parabola tenglamasini tuzing.

293. Parabola uchi (-3;4) nuqtada, direktrisasi $2y-9=0$ to‘g‘ri chiziqdan iborat bo‘lgan parabola tenglamasini tuzing.

294. Ko‘prik arki tenglamasi $y^2 = 96x$ bo‘lgan parabola ko‘rinishiga ega. Agar balandligi 6 m ga teng bo‘lsa, ark vatarining uzunligini toping.

295. Parabolik ko‘zguning diametri 120 sm ga, botiqligi 15 sm ga teng. Yorug‘lik manbayini parabola uchidan qanday masofada joylashtirilganda qaytgan nur parabola o‘qiga parallel bo‘ladi?

296. Fontandan otilib chiqayotgan suv oqimining parametri $p=2$ ga teng bo‘lgan parabola shakliga ega. Agar suv ko‘pi bilan 4 m ga ko‘tarila olishi mumkin bo‘lsa, suv chiqayotgan joyidan qancha nariga tushishini aniqlang.

297. O‘tkir burchak ostida gorizontal otilgan jism parabola yoyi chizib, boshlang‘ich holatdan 32 m masofaga borib tushdi. Agar jism ko‘tatrilgan eng yuqori balandlik 12 m bo‘lsa, parabolik trayektoriyaning parametrini aniqlang.

298. $F(0;2)$ nuqtadan va y=4 to‘g‘ri chiziqdan bir xil uzoqlashgan nuqtalar geometrik o‘rnining tenglamasini tuzing. Bu egri chiziqning koordinata o‘qlari bilan kesishgan nuqtalarini toping va uni yasang.

299. 1) $y^2 = 4x$; 2) $y^2 = -4x$; 3) $x^2 = 4y$;

4) $x^2 = -4y$ tenglamalar bilan berilgan parabolalar hamda ularning fokuslari, direktrisalarini yasang va direktrisalarining tenglamalarini tuzing.

300. 1) (0;0) va (1;-3) nuqtadan o‘tuvchi *Ox* o‘qqa nisbatan simmetrik; 2) (0;0) va (2;-4) nuqtalardan o‘tuvchi *Oy* o‘qqa nisbatan simmetrik bo‘lgan parabolaning tenglamasini tuzing.

301. Uchi koordinatalar boshida va fokusininng koordinatalari: 1) $F(0;4)$; 2) $F(0;-3)$; 3)$F(6;0)$; 4) $F(-2{,}5;0)$ bo‘lgan parabolaning tenglamasini tuzing.

302. Parabola *Ox* o‘qqa nisbatan simmetrik, uning uchi koordinatalar boshida, fokusidan uchigacha bo‘lgan masofa 12 ga teng. Parabolaning tenglamasini tuzing.

303. Parabolaning tenglamasi $y^2 = 24x$ va undagi nuqtaning radius-vektori 14 ga teng. Bu nuqtaning parabola boshidan uzoqligini toping.

304. Parabolaning tenglamasi $y^2 = 6x$. Uning shunday vatarini topingki, u (4;3) nuqtada teng ikkiga bo‘linsin.

305. $y = x$ to‘g‘ri chiziq bilan $x^2 + y^2 + 6x = 0$ aylananing kesishgan nuqtalaridan o‘tuvchi va *Ox* o‘qqa nisbatan simmetrik bo‘lgan parabolaning va direktrisalarining tenglamalarini yozing. To‘g‘ri chiziq, parabola, aylanani yasang.

306. $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellips va uning direktrisalarini yasang. Ellipsning $x=-3$ abssissasidan o‘ng fokusgacha va o‘ng direktrisagacha bo‘lgan masofalarni toping.

307. $\frac{x^2}{16}-\frac{y^2}{9}=1$ giperbola va uning direktrisalarini yasang, giperbolaning x=5 abssissasidan chap direktrisasigacha bo‘lgan masofalarni toping.

308. Katta yarim o‘qi 2 ga teng, direktrisalari esa $x=\pm\frac{4}{\sqrt{3}}$ to‘g‘ri chiziqlardan iborat ellipsning kanonik tenglamasini yozing.

309. Asimptotalari $y=\pm x$, direktrisalari esa $x=\pm\sqrt{6}$ bo‘lgan giperbolaning tenglamasini yozing.

310. $x^2+4y^2=16$ ellips, uning $y=\frac{x}{2}$ diametri va unga qo‘shma diametrini yasang. Yasalgan yarim diametrlarning a_1 va b_1 uzunliklarini toping.

311. Ellipsning tenglamasi $5x^2+9y^2=45$. Uning abssissasi 2 va ordinatasi musbat bo‘lgan nuqtasidan o‘tgan urinmaning tenglamasini tuzing.

312. Ellipsning tenglamasi $3x^2+4y^2=13$. Uning shunday nuqtasidan urinma o‘tkazingki, u urinma $y=3x+4$ to‘g‘ri chiziqqa parallel bo‘lsin.

313. Ellipsning tenglamasi $4x^2+7y^2=56$. Uning shunday nuqtasidan urinma o‘tkazingki, u $x-2y-5=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lsin.

314. Ellipsning tenglamasi $4x^2 + 75 = 100$. Uning $(4;\frac{6}{5})$ nuqtasida urinma va normal o‘tkazilgan. Ularning tenglamalarini tuzing.

315. Giperbolaning tenglamasi $x^2 - 4y^2 = 1$. Unga qo‘shma bo‘lgan giperbolaning tenglamasini tuzing va uning ekssentrisitetini toping.

316. Giperbolaning tenglamasi $9x^2 - 16y^2 = 144$. Uning abssissasi 8 bo‘lgan nuqtasining radius-vektorlarini aniqlang.

317. Giperbolaning tenglamasi $16x^2 - 25y^2 = 100$. Uning asimptotalari va direktrisalarining tenglamalarini tuzing.

318. Giperbolaning tenglamasi $4x^2 - y^2 = 15$. Uning shunday nuqtasidan urinma o‘tkazingki, u $8x - y - 3 = 0$ to‘g‘ri chiziqqa parallel bo‘lsin.

319. Giperbolaning tenglamasi $2x^2 - 3y^2 = 5$. Unga (1;3) nuqtadan o‘tkazilgan urinmaning tenglamasini tuzing.

320. Giperbolaning tenglamasi $x^2 - y^2 = 4$. Uning shunday nuqtasidan urinma o‘tkazingki, u $2x + 5y + 1 = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘lsin.

321. Giperbolaning asimptotalari $y = \pm\frac{2}{3}x$. Tenglamasi $x - 4 = 0$ to‘g‘ri chiziq unga urinma bo‘lmoqda. Giperbolaning tenglamasini tuzing.

322. Giperbolaning tenglamasi $8x^2 - 5y^2 - 4 = 0$, uning diametrlaridan birining tenglamasi $y = 2x$. Bunga qo‘shma bo‘lgan diametrning tenglamasini tuzing.

323. Giperbolaning tenglamasi $4x^2 - 6y^2 = 24$. Uning ikki qo‘shma diametrlari orasidagi burchak $45°$. Ikkala diametrning tenglamasini tuzing.

324. $4x^2 - 9y^2 = 36$ giperbolaning $x + 2y = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan urinmalarining tenglamalarini yozing.

325. Parabolaning tenglamasi $y^2 = 2x$. Uning (2;-2) nuqtasidan o‘tkazilgan urinma va normalning tenglamalarini tuzing.

326. Parabolaning tenglamasi $y^2 = 4x$. Uning abssissasi 9 va ordinatasi musbat bo‘lgan nuqtasidan urinma va normal o‘tkazilgan. Ularning tenglamalarini yozing.

327. Parabolaning tenglamasi $y^2 = 8x$. Uning shunday nuqtasidan urinma o‘tkazingki, u $2x + 2y + 5 = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘lsin. Uning tenglamasini tuzing.

328. Parabolaning tenglamasi $y^2 = 6x$. Uning tekisligida (4;3) nuqta berilgan. Parabolaning bu nuqtadan o‘tgan diametrini toping.

329. Parabolaning tenglamasi $y^2 = 6x$. Uning shunday vatarini topingki, u (4;3) nuqtada teng ikkiga bo‘linsin.

330. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ giperbola asimptotalarining direktrisalari bilan kesishgan nuqtalarini toping.

331. $x^2 + 4y^2 = 16$ ellips, uning y=x diametri hamda unga qo‘sh-ma diametrini yasang. Shu diametrlar orasidagi burchakni toping.

332. $\frac{x^2}{9} + \frac{y^2}{7} = 1$ ellipsning koordinata burchaklarining bissektrisalari bilan kesishish nuqtalarini toping.

333. $\frac{x^2}{16} + \frac{y^2}{4} = 1$ ellips va $x^2 + y^2 - 8y = 0$ aylananing umumiy vatari tenglamasini tuzing.

334. Ekssentrisiteti $\varepsilon = 1{,}2$ ga teng degan shartda $\frac{x^2}{64} + \frac{y^2}{28} = 1$ eillips bilan umumiy fokuslarga ega bo‘lgan giperbolaning tenglamasini tuzing.

335. Ko‘prik arki tenglamasi $y^2 = 96x$ bo‘lgan parabola ko‘rinishga ega. Ark balandligi 6 m ga teng bo‘lsa, ark vatarining uzunligini toping.

336. Parabolik ko‘zguning diametri 120 sm ga, botiqligi 15 sm ga teng. Yorug‘lik manbayini parabola uchidan qanday masofada joylashtirganda qaytgan nur parabola o‘qiga parallel bo‘ladi?

337. Qutb koordinatalarida quyidagi nuqtalarni yasang:

$A\left(3;\frac{\pi}{6}\right)$; $B\left(1;\frac{5\pi}{3}\right)$; $C\left(5;\frac{7\pi}{6}\right)$; $D\left(0{,}5;\frac{\pi}{2}\right)$; $E\left(2{,}5;\frac{2\pi}{2}\right)$;

$F(6;\pi)$; $M\left(3;\frac{\pi}{3}\right)$; $N\left(\sqrt{3};-\frac{\pi}{6}\right)$; $K\left(-2;\frac{\pi}{4}\right)$.

338. Dekart koordinatalar sistemasida $M_1(0;2)$, $M_2(-1;0)$, $M_3\left(-\sqrt{3};1\right)$, $M_4(-1;-1)$, $M_5(1;\sqrt{3})$ nuqtalar berilgan. Ularning qutb koordinatalarini toping.

339. Qutb koordinatalari ushbu: a) p=1; b) p=5; d) p=a; e) $\varphi = \frac{\pi}{6}$; $f)\ \varphi = \frac{\pi}{3}$; $g)\ \varphi = \frac{\pi}{2}$; $h)\ \varphi = const$ tenglamalardan birini qanoatlantiradigan nuqtalar qanday joylashgan?

340. Ushbu nuqtalarga $\left(1;\frac{\pi}{4}\right)$; $\left(3;\frac{2\pi}{3}\right)$; $\left(\frac{2}{3};-\frac{\pi}{6}\right)$; $M(\rho;\varphi)$: qutbga nisbatan; b) qutb o‘qiga nisbatan simmetrik bo‘lgan nuqtalarning qutb koordianatalarini toping.

341. Qutb burchaklari 0°, 15°, 30°, 45°, 60°, 75°, 90° ga teng va ularga mos radius-vektorlari $\rho = asin2\varphi$ tenglama bilan hisoblanadigan nuqtalarni yasang. Olingan nuqtalarni silliq egri chiziq bilan birlashtiring.

342. Quyidagi chiziqlarni yasang:

1) r=2+2cos φ; 2) r=a φ (Arxmed spirali);

3) r=a(1-cos φ) (kardoida); 4) $r^2 = a^2 cos\varphi$ (lemniskata);

5) $r = \frac{a}{\varphi}$ (giperbolik spiral); 6) r=a(1+2cos φ) (Paskal chig‘anog‘i).

343. 1) $A\left(2; \frac{\pi}{26}\right)$ va $B\left(1; \frac{5\pi}{12}\right)$; 2) $C\left(4; \frac{\pi}{5}\right)$ va $F\left(4; \frac{\pi}{4}\right)$;

3) $D\left(6; \frac{6\pi}{5}\right)$ va $E\left(3; \frac{11\pi}{18}\right)$ nuqtalar orasidagi masofani toping.

344. Uchburchak uchlari berilgan: $A\left(5; \frac{\pi}{2}\right)$, $B\left(8; \frac{5\pi}{4}\right)$, $C\left(3; \frac{7\pi}{6}\right)$. Bu uchburchak teng tomonli ekanligini ko‘rsating.

345. Qutb o‘qida $A\left(4\sqrt{2}; \frac{\pi}{4}\right)$ nuqtadan 5 birlik uzoqlikda yotgan nuqtani toping.

346. Bitta uchi qutbda, qolganlari $\left(4; \frac{\pi}{9}\right)$ va $\left(1; \frac{5\pi}{18}\right)$ nuqtalarda bo‘lgan uchburchakning yuzini toping.

347. Uchlari $A\left(9; \frac{\pi}{10}\right)$, $B\left(12; \frac{4\pi}{5}\right)$, $C\left(10; \frac{3\pi}{5}\right)$ nuqtalarda bo‘lgan uchburchakning yuzini toping.

348. 1) Qutb o‘qiga perpendikulyar bo‘lib, undan *a* kesma ajratuvchi to‘g‘ri chiziq; 2) $A(\alpha; a)$ nuqtadan o‘tuvchi qutb o‘qiga parallel bo‘lgan to‘g‘ri chiziqning qutb koordinatalaridagi tenglamalarini yozing.

349. Markazi $C(0; a)$ nuqtada va radiusi *a* ga teng ayalananing qutb koordinatalaridagi tenglamasini yozing.

350. Ushbu 1) $x^2 - y^2 = a$; 2) $x^2 + y^2 = a^2$; 3) $xcos\varphi + ysin\varphi - p = 0$; 4) $y = x$; 5) $x^2 + y^2 = ax$; 6) $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ chiziqlarning tenglamalarini qutb koordinatalardagi tenglamalari bilan almashtiring.

Ko‘rsatma. $x = p\cos\varphi,\ \ y = p\sin\varphi$ larni berilgan tenglamalarga qo‘yib soddalashtiring.

351. $\vec{a} = \{6; 3; -2\}$ vektorning modulini hisoblang.

352. Vektorning $X = 4,\ \ Y = -12$ koordinatalari berilgan. Agar $|\vec{a}| = 13$ bo‘lsa, uning uchinchi Z koordinatasini toping.

353. Agar $\vec{a} = \{3; -1; 4\}$ vektorning boshlang‘ich nuqtasi $M(1; 2; -3)$ bo‘lsa, uning oxirgi N nuqtasini toping.

354. Vektorning moduli $|\vec{a}| = 2$ va burchaklari $\alpha = 45°,\ \ \beta = 60°,\ \ \gamma = 60°$. Uning koordinata o‘qlaridagi proyeksiyalarini hisoblang.

355. $\vec{a} = \left\{\frac{3}{13}; \frac{4}{13}; \frac{12}{13}\right\}$ vektorning yo‘naltiruvchi kosinuslarini hisoblang.

356. Vektor ikkita koordinata o‘qlari bilan quyidagi burchaklarni tashkil qilishi mumkinmi: 1) $\alpha = 30°,\ \ \beta = 45°$; 2) $\beta = 60°,\ \ \gamma = 60°$;

3) $\alpha = 150°,\ \ \gamma = 30°$?

357. $\vec{a}$ vektor $Ox,\ \ Oy$ o‘qlar bilan $\alpha = 60°,\ \ \beta = 120°$ burchak tashkil qiladi. Agar $|\vec{a}| = 2$ bo‘lsa, uning koordinatalarini hisoblang.

358. Koordinata o‘qlari bilan bir xil burchaklar tashkil qiluvchi va moduli 3 bo‘lgan radius-vektorning M nuqtasi koordinatalarini toping.

359. $A(3; -1; 2)$ va $B(-1; 2; 1)$ nuqtalar berilgan. $\overrightarrow{AB}$ va $\overrightarrow{BA}$ vektorlarning koordinatalarini toping.

360. Oxiri (1;-1;2) nuqtada bo‘lgan $\vec{a} = \{2; -3; -1\}$ vektorning boshlang‘ich nuqtasini toping.

361. $\vec{a} = \{12; -15; -16\}$ vektorning yo‘naltiruvchi kosinuslarini hisoblang.

362. Vektor koordinata o‘qlari bilan

1) $\alpha = 45°$, $\beta = 60°$, $\gamma = 120°$; 2) $\alpha = 45°$, $\beta = 135°$, $\gamma = 60°$;

3) $\alpha = 90°$, $\beta = 150°$, $\gamma = 60°$ burchaklarni tashkil qilishi mumkinmi?

363. Vektor Ox va Oz o‘qlari bilan $\alpha = 120°$, $\gamma = 45°$ burchak tashkil qilsa, u Oy o‘qi bilan qanday burchak tashkil qiladi?

364. Berilgan $\vec{a}$ va $\vec{b}$ vektorlarga ko‘ra ushbu 1) $\vec{a} + \vec{b}$; 2) $\vec{a} - \vec{b}$; 3) $\vec{b} - \vec{a}$; 4) $-\vec{a} - \vec{b}$; vektorlarni yasang.

365. $|\vec{a}| = 13$, $|\vec{b}| = 19$ va $|\vec{a} + \vec{b}| = 24$ berilgan. $|\vec{a} - \vec{b}|$ ni hisoblang.

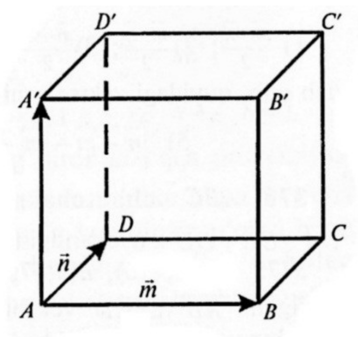
366. $|\vec{a}| = 5$, $|\vec{b}| = 8$ va ular o‘zaro $\varphi = 60°$ li burchak ostida kesishadi. $|\vec{a} + \vec{b}|$ va $|\vec{a} - \vec{b}|$ larni aniqlang.

367. $ABCD\,A'B'C'D'$ parallelepipedda uning qirralari bilan ustma-ust tushuvchi $\overrightarrow{AB} = \vec{m}$, $\overrightarrow{AD} = \vec{n}$, $\overrightarrow{AA_1} = \vec{p}$ vektorlar berilgan.

Quyidagi har bir vektorlarni yasang:

1) $\vec{m} + \vec{n} + \vec{p}$; 2) $\vec{m} + \vec{n} + \frac{1}{2}\vec{p}$;

3) $\frac{1}{2}\vec{m} + \frac{1}{2}\vec{n} + \vec{p}$; 4) $\vec{m} + \vec{n} - \vec{p}$;



5) $-\vec{m} - \vec{n} + \frac{1}{2}\vec{p}$.

368. O‘zaro perpendikulyar yo‘nalishdagi uchta M, N va P kuchlar bir nuqtaga qo‘yilgan. Agar $|\vec{M}| = 2kg$, $|\vec{N}| = 10kg$ va $|\vec{P}| = 11kg$ bo‘lsa, ularning teng ta‘sir etuvchisi R ning qiymatini toping.

369. Ikkita $\vec{a} = \{3; -2; 6\}$ va $\vec{b} = \{-2; 1; 0\}$ vektorlar berilgan. Quyidagi: 1) $\vec{a} + \vec{b}$; 2) $\vec{a} - \vec{b}$; 3) $2\vec{a}$; 4) $-\frac{1}{2}\vec{b}$; 5) $2\vec{a} + 3\vec{b}$; 6) $\frac{1}{3}\vec{a} - \vec{b}$ vektorlarning har birini koordinata o‘qlaridagi proyeksiyasini toping.

370. $\vec{a} = \{2; -1; 3\}$ va $\vec{b} = \{-6; 3 - 9\}$ vektorlarning kollinearligini tekshiring. Ularning uzunliklarini va uzunliklar orasidagi farqni hamda ularning yo‘nalishlarini aniqlang.

371. α va β ning qanday qiymatlarida $\vec{a} = -2\boldsymbol{i} + 3\boldsymbol{j} + \beta\boldsymbol{k}$, $\vec{b} = \alpha\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$ vektorlar kollinear bo‘ladi?

372. $|\vec{a}| = 11$, $|\vec{b}| = 23$ va $|\vec{a} - \vec{b}| = 30$ berilgan. $|\vec{a} + \vec{b}|$ ni hisoblang.

373. $|\vec{a}| = 5$ va $|\vec{b}| = 12$ berilgan. Agar $\vec{a}, \vec{b}$ vektorlar o‘zaro perpendikulyar bo‘lsa, $|\vec{a} + \vec{b}|$ va $|\vec{a} - \vec{b}|$larni aniqlang.

374. $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro 120° burchak tashkil qiladi. $|\vec{a}| = 3$ va $|\vec{b}| = 5$ bo‘lsa, $|\vec{a} + \vec{b}|$, $|\vec{a} - \vec{b}|$ larni aniqlang.

375. ABC uchburchakda vektorlar $\overrightarrow{AB} = \vec{m}$, $\overrightarrow{AC} = \vec{n}$. Quyidagi vektorlarning har birini yasang:

1) $\frac{\vec{m}+\vec{n}}{2}$; 2) $\frac{\vec{m}-\vec{n}}{2}$; 3) $\frac{\vec{n}-\vec{m}}{2}$; 4) $-\frac{\vec{m}+\vec{n}}{2}$. Masshtab birligini $\frac{1}{2}|\vec{n}|$ deb olib, quyidagi vektorni yasang:

5) $|\vec{n}| - \vec{m} + |\vec{m}| - \vec{n}$; 6) $|\vec{n}| - \vec{m} - |\vec{m}| - \vec{n}$.

376. ABC uchburchakning og‘irlik markazi O nuqtada. $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = 0$ ekanligini isbot qiling.

377. $A(-1; 5; -10), B(5; -7; 8), C(2; 2; -7)$ va $D(5; -4; 2)$ nuqtalar berilgan. $\overrightarrow{AB}$ va $\overrightarrow{CD}$ vektorlarning kollinearligini tekshiring, ularning uzunliklari orasidagi farq va yo‘nalishlarni ko‘rsating.

378. $\vec{a}=\{2;-4;4\}$ va $\vec{b}=\{-3;2;6\}$ vektorlar orasidagi burchaknin hisoblang.

379. Uchburchakning $A(3;2;-3)$, $B(5;1;-1)$ va $C(1;2;-1)$ uchlari berilgan. Uning A uchidagi tashqi burchagini aniqlang.

380. $\vec{a}=\{4;-2;-4\}$, $\vec{b}=\{6;-3;2\}$ vektorlar berilgan. Hisoblang:

1) $\vec{a}\cdot\vec{b}$; 2) $\sqrt{\vec{a}^2}$; 3) $\sqrt{\vec{b}^2}$; 4) $(2\vec{a}-3\vec{b})(\vec{a}+2\vec{b})$;

5) $(\vec{a}+\vec{b})^2$; 6) $(\vec{a}-\vec{b})^2$.

381. α ning qanday qiymatida $\vec{a}=\alpha\boldsymbol{i}-3\boldsymbol{j}+2\boldsymbol{k}$ va $\vec{b}=\boldsymbol{i}-2\boldsymbol{j}+\alpha\boldsymbol{k}$ vektorlar o‘zaro perpendikulyar bo‘ladi?

382. $\vec{a}=\{6;-8;-7{,}5\}$ vektorga kollinear bo‘lgan $\vec{x}$ vektor Oz o‘qi bilan o‘tkir burchak tashkil etadi. Agar $|\vec{x}|=50$ bo‘lsa, uning koordinatalarini toping.

383. $\vec{a}=3\boldsymbol{i}+2\boldsymbol{j}+2\boldsymbol{k}$ va $\vec{b}=18\boldsymbol{i}-22\boldsymbol{j}-5\boldsymbol{k}$ vektorlar perpendikulyar bo‘lgan $\vec{x}$ vektor Oy o‘qi bilan o‘tmas burchak tashkil etadi. $|\vec{x}|=14$ bo‘lsa, uning koordinatalarini toping.

384. Uchta $\vec{a}=2\boldsymbol{i}+\boldsymbol{j}+3\boldsymbol{k}$, $\vec{b}=\boldsymbol{i}-3\boldsymbol{j}-2\boldsymbol{k}$ va $\vec{c}=3\boldsymbol{i}+2\boldsymbol{j}-4\boldsymbol{k}$ vektorlar berilgan. $\vec{x}\cdot\vec{a}=-5$, $\vec{x}\cdot\vec{b}=-11$, $\vec{x}\cdot\vec{c}=20$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorni toping.

385. $\vec{S}=\{4;-3;2\}$ vektorning koordinata o‘qlari bilan bir xil o‘tkir burchak tashkil etuvchi o‘qdagi proyeksiyasini toping.

386. l o‘qi koordinata o‘qlari: Ox bilan $\alpha=45°$, Oz bilan $\gamma=60°$ va Oy bilan o‘tkir β burchak tashkil etadi. $\vec{S}=\{\sqrt{2};-3;-5\}$ vektorning l o‘qdagi proyeksiyasini toping.

387. Agar $f = \{3; -2; -5\}$ kuchning qo‘yilish nuqtasi to‘g‘ri chiziqli harakatda $A(2; -3; 5)$ dan $B(3; -2; -1)$ holatga ko‘chsa, bu kuch qancha ish bajaradi?

388. Uchburchakni $A(-1; -2; 4)$, $B(-4; -2; 0)$ va $C(3; -2; 1)$ uchlari berilgan. Uning B uchidagi ichki burchagini aniqlang.

389. Uchlari $A(1; 2; 1)$, $B(3; -1; 7)$, $C(7; 4; -2)$ bilan berilgan uchburchakning ichki burchaklarini hisoblab, uning teng tomonli ekanligini ko‘rsating.

390. $\vec{a} = \{2; 1; -1\}$ vektorga kollinear bo‘lgan va $\vec{x} \cdot \vec{a} = 3$ shartni bajaruvchi $\vec{x}$ vektorni toping.

391. Ikkita $A(1; -4; -2)$, $B(2; 5; -2)$ nuqta berilgan. l o‘qi koordinata o‘qlari Ox, Oy bilan $\alpha = 60°$, $\beta = 120°$, Oz o‘qi bilan γ o‘tmas burchak tashkil etadi. $\overrightarrow{AB}$ ning l o‘qqa proyeksiyasini toping.

392. Bir nuqtaga uchta $M = \{3; -4; -2\}$, $N = \{2; 3; -5\}$, $P = \{-3; -2; 4\}$ kuchlar qo‘yilgan. Ularning teng ta‘sir etuvchi kuchining qo‘yilish nuqtasi to‘g‘ri chiziqli harakat qilib, $M_1(5; 3; -7)$ dan $M_2(4; -1; -4)$ holatga o‘tganda, bu kuch qancha ish bajarishini hisoblang.

393. Agar 1) $\vec{a} = \{2; 3; 0\}$, $b = \{0; 3; 1\}$; 2) $\vec{a} = \{-2; 3; 0\}$, $\vec{b} = \{-2; 0; 4\}$ bo‘lsa, $\vec{c} = \vec{a} \times \vec{b}$ vektorni aniqlang va yasang. Har bir hol uchun berilgan vektorlarda yasalgan parallelogramm yuzini hisoblang.

394. Uchlari 1) $A(7; 3; 4)$, $B(1; 0; 6)$ va $C(4; 5; -2)$; 2) $A(1; 1; 1)$, $B(2; 3; 4)$ va $C(4; 3; 2)$ nuqtalarda bo‘lgan uchburchakning yuzini hiosblang.

395. $\vec{a} = 2\boldsymbol{j} + \boldsymbol{k}$ va $\vec{b} = \boldsymbol{i} + 2\boldsymbol{k}$ vektorlardan parallelogramm yasang hamda uning yuzi va balandligini aniqlang.

396. Uchburchakning $A(1; -1; 2)$, $B(5; -6; 2)$ va $C(1; 6; -1)$ uchlari berilgan. Uning B uchidan AC tomonga tushurilgan balandligini toping.

397. $\vec{a} = \{2; -2; 1\}$ va $\vec{b} = \{2; 3; 6\}$ vektorlar tashkil qilgan burchak sinusini hisoblang.

398. $\vec{a} = \{4; -2; 3\}$ va $\vec{b} = \{0; 1; 3\}$ vektorlarga perpendikulyar bo‘lgan $\vec{m}$ vektor Oy o‘qi bilan o‘tmas burchak tashkil qiladi. Agar $|\vec{m}| = 26$ bo‘lsa, uning koordinatalarini toping.

399. Oz o‘qiga va $\vec{a} = \{8; -15; 3\}$ vektorga perpendikulyar bo‘lgan $\vec{m}$ vektor Ox o‘qi bilan o‘tkir burchak tashkil etadi. Agar $|\vec{m}| = 51$ bo‘lsa, uning koordinatalarini toping.

400. $\vec{a} = \{2; -3; 1\}$ va $\vec{b} = \{1; -2; 3\}$ vektorlarga perpendikulyar bo‘lib, $\vec{x} \cdot (\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}) = 10$ shartni qanoatlantiruvchi $\vec{x}$ vektorni toping.

401. Agar 1) $\vec{a} = 3\boldsymbol{k}$, $\vec{b} = 2\boldsymbol{k}$; 2) $\vec{a} = \boldsymbol{i} + \boldsymbol{j}$; $\vec{b} = \boldsymbol{i} - \boldsymbol{j}$ bo‘lsa, $\vec{c} = \vec{a} \times \vec{b}$ vektorni aniqlang va yasang. Har bir hol uchun berilgan vektorlarda yasalgan parallelogramm yuzini hisoblang.

402. $\vec{a}$ va $\vec{b}$ vektorlar $\varphi = \frac{\pi}{6}$ burchak tashkil etadi. Agar $|\vec{a}| = 6$, $|\vec{b}| = 5$ bo‘lsa, $|\vec{a} \times \vec{b}|$ni hisoblang.

403. $\vec{a} = \{0; 3; -2\}$, $\vec{b} = \{3; -2; 0\}$ va $\vec{c} = \vec{a} \times \vec{b}$ vektorlarni yasang. $\vec{c}$ vektorning moduli hamda $\vec{a}$ va $\vec{b}$ vektorlarda yasalgan uchburchak yuzini hisoblang.

404. Uchlari $A(1;-1;2)$, $B(5;-6;2)$ va $C(1;3;-1)$ bo‘lgan uchburchakning $h=\left|\overrightarrow{BD}\right|$ balandligini toping.

405. $|\vec{a}|=\left|\vec{b}\right|=5$, $\left(\vec{a},\vec{b}\right)=\frac{\pi}{4}\cdot\vec{a}-2\vec{b}$ va $3\vec{a}+2\vec{b}$ vektorlarda yasalgan uchburchak yuzini hisoblang.

406. $\vec{m}$ va $\vec{n}$ o‘zaro $30°$ burchak tashkil etuvchi birlik vektorlar bo‘lsa, $\vec{a}=\vec{m}+2\vec{n}$ va $\vec{b}=\vec{m}+\vec{n}$ vektorlarda yasalgan parallelogramm yuzini toping.

407. O‘ng bog‘lam tashkil etuvchi $\vec{a},\vec{b}$ va $\vec{c}$ vektorlar o‘zaro perpendikulyar. Agar $|\vec{a}|=4$, $\left|\vec{b}\right|=2$, $|\vec{c}|=3$ bo‘lsa, $\vec{a}\vec{b}\vec{c}$ ni hisoblang.

408. Uchta $\vec{a}=\{1;-1;3\}$, $\vec{b}=\{-2;2;1\}$ va $\vec{c}=\{3;-2;5\}$ vektorlar berilgan. $\vec{a}\vec{b}\vec{c}$ ni hisoblang.

409. Quyidagi vektorlarning komplanarligini ko‘rsating:

1) $\vec{a}=\{2;3;-1\}$, $\vec{b}=\{1;-1;3\}$ va $\vec{c}=\{1;9;-1\}$;

2) $\vec{a}=\{3;-2;1\}$, $\vec{b}=\{2;1;2\}$ va $\vec{c}=\{3;-1;-2\}$;

3) $\vec{a}=\{2;-1;2\}$, $\vec{b}=\{1;2;-3\}$ va $\vec{c}=\{3;-4;7\}$.

410. To‘rtta $A(1;2;-1)$, $B(0;1;5)$, $C(-1;2;1)$ va $D(2;1;3)$ nuqtalarning bir tekislikda yotishini ko‘rsating.

411. Uchlari $A(2;-3;5)$, $B(0;2;1)$, $C(-2;-2;3)$ va $D(3;2;4)$ nuqtalarda bo‘lgan tetraedrning hajmini hisoblang.

412. Tetraedrning $A(1;1;1)$, $B(2;0;2)$, $C(2;2;2)$ va $D(3;4;-3)$ uchlari berilgan. Uning D nuqtasidan tushirilgan balandligining uzunligini toping.

413. Tetraedrning hajmi $V = 5$, uning uchta uchlari $A(2; 1; -1)$, $B(3; 0; 1)$, $C(2; -1; 3)$ nuqtalarda. Agar to‘rtinchi D uchi Oy o‘qida yotsa, uning koordinatalarini toping.

414. $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga perpendikulyar, $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak 30°. Agar $|\vec{a}| = 6$, $|\vec{b}| = 3$, $|\vec{c}| = 3$ bo‘lsa, $\vec{a}\vec{b}\vec{c}$ ni hisoblang.

415. $\vec{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$, $\vec{b} = -3\boldsymbol{j} + \boldsymbol{k}$, $\vec{c} = 2\boldsymbol{i} + 5\boldsymbol{k}$ vektorlardan parallelepiped yasang hamda uning hajmini hisoblang. Berilgan $(\vec{a}\vec{b}\vec{c})$ vektorlar qaysi bog‘lamni tashkil etadi?

416. Uchlari $O(0; 0; 0)$, $A(5; 2; 0)$, $B(2; 5; 0)$ va $C(1; 2; 4)$ nuqtalarda bo‘lgan piramida yasang hamda uning hajmini, ABC yoqning yuzini va shu yoqqa tushirilgan balandlikni hisoblang.

417. $A(2; -1; -2)$, $B(1; 2; 1)$, $C(2; 3; 0)$ va $D(5; 0; -6)$ nuqtalarning bir tekislikda yotishini ko‘rsating.

418. $\vec{a} = \boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$, $\vec{b} = \boldsymbol{i} - 2\boldsymbol{j}$ va $\vec{c} = 3\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ vektorlani yasang va ular o‘zaro komplanar ekanligini ko‘rsating. Bu vektorlar orasidagi chiziqli bog‘lanishni toping.

421. 1) Ox va $M_1(4; -1; 2)$ nuqtadan; 2) Oy va $M_2(1; 4; -3)$ nuqtadan; 3) Oz va $M_3(3; -4; 6)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing.

422. 1) $M_1(7; 2; -3)$ va $M_2(5; 6; -4)$ nuqtalardan o‘tib, Ox o‘qiga parallel; 2) $P_2(2; -1; 1)$ va $P_2(3; 1; 2)$ nuqtalardan o‘tib, Oy o‘qiga parallel; 3) $Q_1(3; -2; 5)$ va $Q_2(2; 3; 1)$ nuqtalardan o‘tib, Oz o‘qiga parallel bo‘lgan tekislik tenglamasini yozing.

423. $2x-3y-4z-24=0$ tekislikning koordinata o‘qlari bilan kesishgan nuqtalarini toping.

424. $5x-6y+3z+120=0$ tekislikning Oxy koordinata burchagi bo‘yicha ajratgan uchburchakning yuzini toping.

425. $2x-3y+6z-12=0$ tekislik va koordinata tekisliklari bilan chegaralangan piramidaning hajmini toping.

426. Tekislik $M(6;-10;1)$ nuqtadan o‘tib, abssissalar o‘qidan $a=-3$ va applikatalar o‘qidan $c=2$ kesmalar ajratadi. Bu tekislikning „kesmalardagi“ tenglamasini tuzing.

427. $M_1(4;3;2)$ nuqtadan o‘tib, koordinata o‘qlari bilan noldan farqli, bir xil uzunlikdagi kesmalarni ajratuvchi tekislik tenglamasini tuzing.

428. Oz o‘qidan $c=-5$ kesma ajratuvchi va $\vec{n}(-2;1;3)$ vektorga perpendikulyar bo‘lgan tekislik tenglamasini tuzing.

429. Oz o‘qi va $M(1;-2;1)$ nuqtadan o‘tuvchi tekislik tenglamasini tuzing.

430. $M(2;3;-4)$ nuqtadan o‘tib, Oyz tekislikka parallel bo‘lgan tekislik tenglamasini tuzing.

431. Tekislik $P(3;8;-4)$ nuqtadan o‘tib, abssissalar o‘qidan $a=-3$, applikatalar o‘qidan $c=2$ kesma ajratadi. Bu tekislik tenglamasini tuzing.

432. $x+2y-3z+2=0$ tekislik va koordinata tekisliklari bilan chegaralangan piramidaning hajmini toping.

433. Koordinatalar boshidan tekislikka tushirilgan perpendikulyarning asosi $M(2;-1;2)$ nuqtada. Bu tekislik tenglamasini toping.

434. Oz o‘qqa parallel hamda $M_1(2;2;0)$ va $M_2(4;0;0)$ nuqtalardan o‘tuvchi tekislik tenglamasini yozing.

435. $M(1;-3;5)$ nuqtadan o‘tib, Oy va Oz o‘qlardan Ox o‘qdagiga ko‘ra ikki marta kesma ajratuvchi tekislik tenglamasini yozing.

436. Quyidagi

a) $4x-5y+3z-1=0$ va $x-4y-z+9=0$;
b) $3x-y+2z+15=0$ va $5x+9y-3z-1=0$;
d) $6x+2y-4z+17=0$ va $9x+3y-6z-4=0$;
e) $x+y-1=0$ va $2x-y+\sqrt{3}z+1=0$

tekisliklar orasidagi burchakni toping.

437. Berilgan $M(x;y;z)$ nuqtadan $Ax+By+Cz+D=0$ tekislikgacha bo‘lgan masofani toping: 1) $M_1(-2;-4;3)$, $2x-y+2z+3=0$; 2) $M_2(2;-1;-1)$, $16x-12y+15z-4=0$;3) $M_3(1;2;-3)$, $5x-3y+z+4=0$; 4) $M_4(3;-6;7)$, $4x-3z-1=0$; 5) $M_5(9;2;-2)$, $12y-5z+5=0$.

438. $P(-1;1;-2)$ nuqtadan $M_1(1;-1;1)$, $M_2(-2;1;3)$ va $M_3(4;-5;-2)$ nuqtalardan o‘tuvchi tekislikgacha bo‘lgan masofani toping.

439. Ikkita $x+2y-2z+2=0$, $3x+6y-6z-4=0$ parallel tekisliklar orasidagi masofani toping.

440. 7x-6y+6z+7=0 tekislikdan 2 birlik uzoqlikda bo‘lgan parallel tekislik tenglamasini toping.

441. $M_0(2;-3;-7)$ nuqtadan o‘tib, $2x-6y-3z+5=0$ tekislikka parallel bo‘lgan tekislik tenglamasini toping.

442. $M_0(-2; 7; 3)$ nuqtadan o‘tib, $x - 4y + 5z + 1 = 0$ tekislikka parallel bo‘lgan tekislik tenglamasini toping.

443. $M_0(2; -3; 1)$ nuqtadan o‘tib, $\vec{a}\{-3; 2 - 1\}$ va $\vec{b}\{1; 2; 3\}$ vektorga parallel bo‘lgan tekislik tenglamasini toping.

444. $M_0(2; 2; -2)$ nuqtadan o‘tib, 3x-2y-z+1=0 va x-y-z=0 tekisliklarning kesishish chizig‘iga perpendikulyar bo‘lgan tekislik tenglamasini toping.

445. $M_1(2; -1; 3)$, $M_2(3; 1; 2)$ nuqtalardan o‘tib, $3x - y - 4z = 0$ tekislikka perpendikulyar bol‘gan tekislik tenglamasini toping.

446. $M_1(3; -1; 2)$, $M_2(4; -1; -1)$ va $M_3(2; 0; 2)$ nuqtalardan o‘tuvchi tekislik tenglamasini topng.

447. $A(0; 0; 2)$, $B(3; 0; 5)$, $C(1; 1; 0)$, va $D(4; 1; 2)$ – tetraedr uchlarining koordinatalari. Uning yoqlarining tenglamalarini toping.

448. $x - y + \sqrt{2}z - 5 = 0$ va (yz) tekisliklar orasidagi burchakni toping.

449. $P(3; -6; 2)$ nuqta – koordinatalar boshidan tekislikka tushirilgan perpendikulyarning asosi. Shu tekislik tenglamasini toping.

450. a) $M_1(3; 1; -1)$ nuqtadan 22x+4y-20z-45=0 tekislikgacha;

b) $M_2(4; 3; -2)$ nuqtadan $3x - y + 5z + 1 = 0$ tekislikgacha;

d) $M_3\left(2; 0; -\frac{1}{2}\right)$ nuqtadan $4x - 4y + 2z + 17 = 0$ tekislikgacha bo‘lgan masofani toping.

451. Piramidaning $S(0; 6; 4)$, $A(3; 5; 3)$, $B(-2; 11; -5)$, $C(1; -1; 4)$ uchlari berilgan. Uning (h_s) balandligini hisoblang.

452. $A(1;3;-2)$ va $B(7;-4;4)$ nuqtalar berilgan. AB kesmaga perpendikulyar bo‘lgan va B nuqtadan o‘tuvchi tekislik tenglamasini toping.

453. 1) $(2;-5;3)$ nuqtadan o‘tib, (xz) tekisligiga parallel bo‘lgan;

2) z o‘qi va $(-3;1;-2)$ nuqtadan o‘tuvchi;

3) $(4;0;-2)$ va $(5;1;7)$ nuqtalardan o‘tib, Ox o‘qiga parallel bo‘lgan tekislik tenglamasini toping.

454. $4x-y+3z-6=0$ va $x+5y-z+10=0$ tekisliklarning kesishish chizig‘idan o‘tuvchi va $2x-y+5z-5=0$ tekislikka perpendikulyar bo‘lgan tekislik tenglamasini yozing.

455. $M(-1;1;-3)$ nuqtadan o‘tib, $\vec{S}\{1;-3;4\}$ vektorga parallel bo‘lgan to‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing.

456. $M_1(2;-1;-1)$ va $M_2(3;3;-1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing.

457. $M(-1;-2;2)$ nuqtadan o‘tib, Ox o‘qiga parallel bo‘lgan to‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing.

458. $M(1;-5;3)$ nuqtadan o‘tib, koordinata o‘qlari bilan $\alpha=\frac{\pi}{4},\ \beta=\frac{\pi}{3},\ \gamma=\frac{2\pi}{3}$ burchaklar tashkil qiluvchi to‘g‘ri chiziqning kanonik tenglamalarini yozing.

459. Uchburchakning $A(-5;7;1)$, $B(2;4;-1)$, $C(-1;3;5)$ uchlari berilgan. B uchidan AC tomonga tushirilgan mediananing kanonik tenglamasini toping.

460. Uchburchakning $A(1;-1;3)$, $B(3;-4;9)$, $C(-5;11;7)$ uchlari berilgan. A uchidan tushirilgan bissektrisasining kanonik tenglamasini toping.

461. 1) $\begin{cases} x-3y+2=0, \\ 2y-z+1=0. \end{cases}$ 2) $\begin{cases} 2x-3y-3z=0, \\ x-2y+z+3=0. \end{cases}$

3) $\begin{cases} x-2y+3z+1=0, \\ 2x+y-4z-8=0. \end{cases}$

to‘g‘ri chiziq tenglamalarini kanonik ko‘rinishga keltiring.

462. $M_0(1;-3;5)$ nuqtadan o‘tib, $\begin{cases} 3x-y+2z+7=0, \\ x+3y-2z+3=0 \end{cases}$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziqning kanonik tenglamasini toping.

463. $M(2;1;-1)$ nuqtadan o‘tib, $x-y+z+1=0$ tekislikka perpendikulyar bo‘lgan to‘gri chiziqning kanonik tenglamasini toping.

464. $\begin{cases} 3x-4y-2z=0, \\ 2x+y-2z+1=0 \end{cases}$ va $\begin{cases} 4x+y-6z-2=0, \\ y-3z-2=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi burchakni toping.

465. 1) $\frac{x+3}{1}=\frac{y+2}{-1}=\frac{z}{\sqrt{2}}$ va $\frac{x+2}{1}=\frac{y-3}{1}=\frac{z-5}{\sqrt{5}}$;

2) $\begin{cases} x+3z-7=0, \\ y=0 \end{cases}$ va $\begin{cases} x-2z-5=0, \\ y=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi burchakni toping.

466. Quyidagi to‘g‘ri chiziqlarning perpendikulyarligini ko‘rsating:

1) $\frac{x}{1}=\frac{y-1}{-2}=\frac{z}{3}$ va $\begin{cases} 3x+y-5z+1=0, \\ 2x+3y-8z+3=0; \end{cases}$

2) $\begin{cases} 2x+y-4z+2=0, \\ 4x-y-5z+4=0 \end{cases}$ va $\begin{cases} x=1+2t, \\ y=-2+3t, \\ z=1-6t. \end{cases}$

467. $\frac{x+2}{3}=\frac{y-1}{-2}=\frac{z}{1}$ va $\begin{cases} x+y+z=0, \\ x-y-5z-8=0 \end{cases}$ to‘g‘ri chiziqlarning parallelligini ko‘rsating.

468. $\frac{x+1}{2}=\frac{y}{-1}=\frac{z-2}{3}$ to‘g‘ri chiziqqa $A(2;3;1)$ nuqtadan o‘tkazilgan perpendikulyarning tenglamasini yozing.

469. $\begin{cases} x-2y+3z-4=0, \\ 3x+2y-5z-4=0 \end{cases}$ to‘g‘ri chiziq tenglamalarini: 1) proyeksiyalar bo‘yicha; 2) kanonik ko‘rinishda yozing.

470. Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziqning kanonik tenglamasini tuzing: 1) $(1;-2;1)$ va $(3;1;-1)$; 2) $(3;-1;0)$ va $(1;0;3)$; 3) $(0;-2;3)$ va $(3;-2;1)$; 4) $(-1;2;-4)$ va $(0;2;-4)$.

471. $M_1(-1;1;3)$ nuqtadan o‘tib, 1) $\vec{a}\{2;-2;4\}$ vektorga; 2) $\frac{x-1}{2}=\frac{y+2}{4}=\frac{z-1}{0}$ to‘g‘ri chiziqqa; 3) $x=3t-1$; $y=-2t+3$; $z=5t+2$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziqning parametrik tenglamasini tuzing.

472. $M_1(-6;6;-5)$ va $M_2(12;-6;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning koordinata o‘qlari bilan kesishgan nuqtalarini toping.

473. Quyidagi to‘g‘ri chiziqlarning parallelligini ko‘rsating:

1) $\frac{x+2}{3}=\frac{y-1}{-2}=\frac{z}{1}$ va $\begin{cases} x+y-z=0, \\ x-y-5z-8=0; \end{cases}$

2) $\begin{cases} x=2t+5, \\ y=-t+2, \\ z=t-7 \end{cases}$ va $\begin{cases} x+3y+z+2=0, \\ x-y-3z-2=0; \end{cases}$

3) $\begin{cases} x+y-3z+1=0, \\ x-y+z+3=0 \end{cases}$ va $\begin{cases} x+2y-5z-1=0, \\ x-2y+3z-9=0. \end{cases}$

474. Quyidagi to‘g‘ri chiziqlarning perpendikulyarligini ko‘rsating:

1) $\begin{cases} x+y-3z-1=0, \\ 2x-y-9z-2=0 \end{cases}$ va $\begin{cases} 3x+y-5z+1=0, \\ 2x+3y-8z+3=0; \end{cases}$

2) $\begin{cases} x=2t+1, \\ y=3t-2, \\ z=-6t+1 \end{cases}$ va $\begin{cases} 2x+y-4z+2=0, \\ 4x-y-5z+4=0; \end{cases}$

3) $\begin{cases} x+y-3z-1=0, \\ 2x-y-9z-2=0 \end{cases}$ va $\begin{cases} 2x+y+2z+5=0, \\ 2x-2y-z+2=0. \end{cases}$

475. $\frac{x-1}{4}=\frac{y}{12}=\frac{z-1}{-3}$ to‘g‘ri chiziq bilan $6x-3y+2z=0$ tekislik orasidagi burchakni hisoblang.

476. $A(-1;0;-5)$ va $B(1;2;0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq bilan $x-3y+z+5=0$ tekislik orasidagi burchakni toping.

477. $M_0(3;-2;4)$ nuqtadan o‘tib, 5x+3y-7z+1=0 tekislikka perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

478. $x=3t-2,\ y=-4t+1,\ z=4t-5$ to‘g‘ri chiziqning $4x-3y-6z-5=0$ tekislikka parallelligini ko‘rsating.

479. $\begin{cases}5x-3y+2z-5=0,\\ 2x-y-z-1=0\end{cases}$ to‘g‘ri chiziqning $4x-3y+7z-7=0$ tekislikda yotishini isbot qiling.

480. To‘g‘ri chiziq bilan tekislikning kesishish nuqtasini toping:

1) $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z}{6},\ \ 2x+3y+z-1=0;$

2) $\frac{x+3}{3}=\frac{y-2}{-1}=\frac{z+1}{-5},\ \ x-2y+z-15=0;$

3) $\frac{x+2}{-2}=\frac{y-1}{3}=\frac{z-3}{2},\ \ x+2y-2z+6=0.$

481. $M_0(2;-3;-5)$ nuqtadan o‘tib, 6x-3y-5z+2=0 tekislikka perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini yozing.

482. $M_0(1;-1;-1)$ nuqtadan o‘tib, $\frac{x+3}{2}=\frac{y-1}{-3}=\frac{z+2}{4}$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan tekislik tenglamasini yozing.

483. $M_0(1;-2;1)$ nuqtadan o‘tib, $\begin{cases}x-2y+z-3=0,\\ x+y-z+2=0\end{cases}$ to‘g‘ri chiziqqa perpendikulyar bo‘gan tekislik tenglamasini yozing.

484. n ning qanday qiymatida $\frac{x+1}{3}=\frac{y-2}{n}=\frac{z+3}{-2}$ to‘g‘ri chiziq x-3y+6z+7=0 tekislikka parallel bo‘ladi?

485. C ning qanday qiymatida $\begin{cases} 3x - 2y + z + 3 = 0, \\ 4x - 3y + 4z + 1 = 0 \end{cases}$ to‘g‘ri chiziq $2x - y + Cz - 2 = 0$ tekislikka parallel bo‘ladi?

486. A va D ning qanday qiymatida $x = 3 + 4t,\ y = 1 - 4t,\ z = -3 + 7t$ to‘g‘ri chiziq $Ax + 2y - 4z + D = 0$ tekislikda yotadi?

487. A va B ning qanday qiymatida $Ax + By + 3z - 5 = 0$ tekislik $x = 3 + 2t,\ y = 5 - 3t,\ z = -2 - 2t$ to‘g‘ri chiziqqa perpendikulyar bo‘ladi?

488. m va C ning qanday qiymatida $\frac{x-2}{m} = \frac{y+1}{4} = \frac{z-5}{-3}$ to‘g‘ri chiziq $3x - 2y + Cz + 1 = 0$ tekislikka perpendikulyar bo‘ladi?

489. $A(4; -3; 1)$ nuqtaning $x + 2y - z - 3 = 0$ tekislikdagi proyeksiyasini toping.

490. $A(1; 2; 1)$ nuqtaning $\frac{x+2}{3} = \frac{y}{-1} = \frac{z-1}{2}$ to‘g‘ri chiziqdagi proyeksiyasini toping.

491. $P(2; -1; 3)$ nuqtaning $x = 3t,\ y = 5t - 7,\ z = 2t + 2$ to‘g‘ri chiziqdagi proyeksiyasini toping.

492. P(4;1;6) nuqtaga $\begin{cases} x - y - 4z + 12 = 0, \\ 2x + y - 2z + 3 = 0 \end{cases}$ to‘g‘ri chiziqqa nisbatan simmetrik nuqtani toping.

493. P(2;-5;7) nuqtaga $M_1(5; 4; 6)$ va $M_2(-2; -17; -8)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqqa nisbatan simmetrik nuqtani toping.

494. P(5;2;-1) nuqtaning 2x-y+3z+23=0 tekislikdagi proyeksiyasini toping.

495. $M(3; 1; -2)$ nuqtadan va $\frac{x-4}{5} = \frac{y+3}{2} = \frac{z}{1}$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini yozing.

496. $\frac{x-2}{5}=\frac{y-3}{1}=\frac{z+1}{2}$ to‘g‘ri chiziqdan $x+4y-3z+7=0$ tekislikka perpendikulyar tekislik o‘tkazing.

497. $\frac{x}{4}=\frac{y-4}{3}=\frac{z+1}{-2}$ to‘g‘ri chiziqning $x-y+3z+8=0$ tekislikdagi proyeksiyasini toping.

498. $\frac{x-3}{5}=\frac{y+1}{2}=\frac{z-2}{4}$ va $\frac{x-8}{3}=\frac{y-1}{1}=\frac{z-6}{-2}$ to‘g‘ri chiziqlarning kesishuvchi ekanligini ko‘rsating, ular orqali tekislik tenglamasini yozing.

499. $M(-3;2;5)$ nuqtadan $4x+y-3z+13=0$ va $x-2y+z-11=0$ tekisliklarga tushirilgan perpendikulyar orqali tekislik o‘tkazing.

500. $\frac{x}{7}=\frac{y+2}{3}=\frac{z-1}{2}$ va $\frac{x-1}{7}=\frac{y-3}{3}=\frac{z+2}{2}$ parallel to‘g‘ri chiziqlardan o‘tuvchi tekislik tenglamasini yozing.

501. P(7;9;7) nuqtadan $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z}{2}$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

502. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{2}$ va $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ parallel to‘g‘ri chiziqlar orasidagi masofani toping.

503. $\frac{x+1}{1}=\frac{y}{1}=\frac{z-1}{2}$ va $\frac{x}{1}=\frac{y+1}{3}=\frac{z-2}{4}$ to‘g‘ri chiziqlar orasidagi eng qisqa masofani toping.

Quyidagi tenglamalar qanday sirtlarni aniqlaydi (chizmasini chizing):

504. $x^2+z^2=9.$

505. $16y^2-25z^2=400.$

506. $y^2=-6z.$

507. $x^2=z^2.$

508. $xz = 2$.

509. $x^2 + y^2 = 2ax$.

510. $z^2 + 4z - 2x + 6 = 0$.

511. $4x^2 + 9y^2 - 8x + 36y + 4 = 0$.

512. $x^2 + y^2 = 4y$.

513. $z^2 + 2z - 4x + 1 = 0$ tenglama qanday sirtni ifodalaydi?

514. $9y^2 - 16z^2 + 64z - 18y - 199 = 0$ tenglama qanday sirtni ifodalaydi?

515. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ tenglama qanday sirtni ifodalaydi?

516. L chiziq $x^2 + z^2 = R^2$, $y^2 + z^2 = r^2$ $(R > r)$ tenglamalar bilan berilgan. Bu chiziqning xOy tekislikdagi proyeksiyasini toping.

517. $L\begin{cases} 9y^2 - 6xy - 2xz + 24x - 9y - 3z - 63 = 0, \\ 2x - 3y + z - 9 = 0 \end{cases}$ chiziqning xOy tekislikdagi proyeksiyasini toping.

518. O‘qi Oy o‘qqa parallel va $P(1;2;-1)$ nuqtadan o‘tuvchi, radiusi 3 ga teng bo‘lgan doiraviy silindrning tenglamasini tuzing.

519. $x^2 + y^2 = 9$, $x - z = 0$ tenglamalar sistemasi fazoda qanday chiziqni ifodalaydi?

520. $M(-2;3;6)$ nuqtadan o‘tuvchi, markazi koordinatalar boshida bo‘lgan sferaning tenglamasini tuzing.

521. $M(4;2;2)$ nuqtadan o‘tuvchi, markazi $C(1;-1;-1)$ nuqtada bo‘lgan sferaning tenglamasini tuzing.

522. Markazi $C(1;-1;4)$ nuqtada bo‘lgan sfera $2x + y - 3z - 3 = 0$ tekislikka urinadi. Sferaning tenglamasini tuzing.

523. Markazi $C(0;4;0)$ nuqtada bo‘lgan sfera $2x + 6y - 3z - 3 = 0$ tekislikka urinadi. Sfera tenglamasini tuzing.

524. Quyidagi sferalarning markazlari va radiusini toping:

a) $x^2 + y^2 + z^2 - 6x + 8y + 2z + 10 = 0;$

b) $x^2 + y^2 + z^2 + 2x - 4y - 4 = 0;$

d) $x^2 + y^2 + z^2 - 6x + 10 = 0;$

e) $x^2 + y^2 + z^2 - 4x + 12y - 2z + 41 = 0;$

f) $36x^2 + 36y^2 + 36z^2 - 36x + 24y - 72z - 95 = 0;$

g) $x^2 + y^2 + z^2 - 2x + 4y - 4z - 7 = 0.$

525. Biror diametrning uchlari $A(2; 5; -7)$ va $B(6; -1; 3)$ nuqtalarda bo‘lgan sfera tenglamasini tuzing.

526. $M_1(1; -2; -1)$, $M_2(-5; 10; -1)$, $M_3(-8; -2; 2)$ nuqtalardan o‘tuvchi, radiusi R=9 bo‘lgan sfera tenglamasini tuzing.

527. Koordinata sistemasiga nisbatan $4 - z = x^2 + y^2$ ning joylashishi va ko‘rinishini tekshiring.

Quyidagi sirtlarni kesimlar usuli bilan koordinatalar sistemasiga nisbatan joylashishi va ko‘rinishini aniqlang (chizmasini chizing).

528. $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{4} = 1.$

529. $\frac{x^2}{1} + \frac{y^2}{2} - \frac{z^2}{4} = -1.$

530. $\frac{x^2}{4} + \frac{y^2}{1} - \frac{z^2}{2} = 0.$

531. $\frac{x^2}{3} + \frac{y^2}{1} = 2z.$

532. $x^2 + y^2 = 2(z - 1)^2.$

533. $2y^2 + z^2 = 1 - x.$

534. $3x^2 + y^2 - z^2 = 3.$

535. $x^2 - 2y^2 + z^2 = 1.$

536. $x^2 - y^2 = 2z.$

537. Ushbu:

1) $x^2 + y^2 + z^2 = 2az$;

2) $x^2 + y^2 = 2az$;

3) $x^2 + y^2 = 2az$;

4) $x^2 - y^2 = 2az$;

5) $x^2 - y^2 = z^2$;

6) $x^2 = 2az$;

7) $x^2 = 2yz$;

8) $z = 2 + x^2 + y^2$;

9) $(z - a)^2 = xy$;

10) $(z - 2x)^2 + 4(z - 2x)^2 = y^2$ sirtlardan har birining nomini aniqlang va ularni yasang.

538. $2x^2 - y^2 + 2z^2 + 4x + 2y + 8z + 1 = 0$ tenglama qanday sirtni aniqlaydi?

539. $x^2 + y^2 + 4z^2 = 2$ ellipsoid bilan $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z}{\frac{\sqrt{2}}{2}}$ to‘g‘ri chiziqning kesishish nuqtasini toping.

540. Ushbu tenglamalar qanday sirtni aniqlaydi?

1) $2x^2 + y^2 + z^2 - 4x + 4y + 4z + 7 = 0$;

2) $x^2 - 6y^2 + 3z^2 + 8x + 12y + 1 = 0$;

3) $x^2 + y^2 + 2x - 2y - 2z - 2 = 0$.

541. $\frac{x-4}{2} = \frac{y+6}{-3} = \frac{z-2}{2}$ to‘g‘ri chiziq va $\frac{x^2}{16} + \frac{y^2}{12} + \frac{z^2}{4} = 1$ ellipsoid qaysi nuqtalarda kesishadi?

542. $\frac{x-4}{4} = \frac{y+3}{3} = \frac{z-2}{2}$ to‘g‘ri chiziq va $\frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{4} = 1$ bir kovakli giperboloidning kesishish nuqtalarini toping.

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MUNDARIJA

ILOVA

VIII BOB. KOMPLEKS SONLAR

ОГЛАВЛЕНИЕ

CONTENT

CHAPTER VIII. COMPLEX NUMBERS

BAXTIYAR ALIMOVICH XUDAYAROV

MATEMATIKA

1-QISM

Toshkent – «Fan va texnologiya» – 2018

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