O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA‘LIM VAZIRLIGI

ABU RAYHON BERUNIY NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

‘

ELEMENTAR MATEMATIKADAN MA’LUMOTNOMA

Toshkent – 2015

Elementar matematikadan ma‘lumotnoma. Oliy o‘quv yurtlariga kiruvchilar, akademik litsey, kasb-hunar kollejlari, tayyorlov kurslari va umum ta’lim maktablarining o‘quvchilari, o‘qituvchilari, oliy o‘quv yurtlari talabalari hamda barcha keng auditoriya o‘quvchilari uchun qo‘llanma
R.R. Abzalimov,
A.S. Xolmuhamedov. –Toshkent: ToshDTU, 2015.

Taqdim etilayoytgan ushbu ma’lumotnoma oliy o‘quv yurtlariga kiruvchilar, akademik litsey, kasb-hunar kollejlari va umumiy o‘rta ta’lim maktablarining o‘quvchilari, o‘qituvchilari, oliy o‘quv yurtlari talabalari hamda barcha o‘quvchilar uchun mo‘ljallangan bo‘lib, unda elementar matematikaning barcha bo‘limlari bo‘yicha misol va masalalar yechishda ishlatilishi mumkin bo‘ladigan formulalar, muhim tushunchalar, ayniyatlar va grafiklar keltirilgan. Ma’lumotnomada turli tipdagi tenglamalar va tenglamalar sistemalari va tengsizliklarni yechish usullari ko‘rsatilgan.

Qo‘llanma Toshkent davlat texnika universiteti ilmiy-uslubiy kengashi qaroriga muvofiq chop etildi.

Taqrizchilar: **X.Xudoynazarov – ToshDTU. t.f.d., prof;**
A.Abdushukurov – O‘z MU f.mf.d., prof.

BOSHLANG'ICH TUSHUNCHALAR

O'lchov birliкlari

Uzunliklar

1. $1km = 1000m$
2. $1m = 10dm$
3. $1dm = 10sm$
4. $1sm = 10mm$
5. $1mm = 1000mkm$
6. $1mkm = 1000nm$
7. $1gaz = 71{,}12sm$
8. $1dyum = 2{,}54sm$
9. $1arshin = 71sm$
10. $1versta = 1066{,}8m$
11. $1vershok = 4{,}445sm$
12. $1mil = 1609m$
13. $1yard = 91{,}44sm$
14. $1chaqirim = 1066m$
15. $1fut = 30{,}48sm$
16. $1yorug'lik\ yili = 9{,}4605\cdot 10^{15}\ m$
17. $1parsek = 3{,}0857\cdot 10^{16}\ m$
18. $1dengiz = milyasi = 1852m$

Yuza

1. $1km^2 = 1000000\ m^2$
2. $1m^2 = 100\ dm^2$
3. $1dm^2 = 100\ sm^2$
4. 1gektar=100 sotix
5. $1sotix = 100\ m^2$
6. $1\ gektar = 10000\ m^2$

Hajm

1. $1km^3 = 1000000000m^3$
2. $1\ m^3 = 1000dm^3$
3. $1\ dm^3 = 1000sm^3$
4. $1sm^3 = 1000mm^3$

Sig'im

1. $1litr = 1dm^3$
2. 1barrel=159litr

Massa

1. 1tonna=1000kg
2. 1kg=1000g
3. 1sentner=100kg
4. 1g=1000mg
5. 1pud=16kg 9g

TO'PLAMLAR NAZARIYASI ELEMENTLARI

1. $a \in A$ - a element A to'plamga tegishli.
2. $a \notin A$ - a element A to'plamga tegishli emas.
3. $B \subset A$ - B to'plam A to'plamning qism to'plami.

4. $B \cup A$ - B va A to‘plamlarning birlashmasi.
5. $B \cap A$ - B va A to‘plamlarning кesishmasi.
6. Ø - bo‘sh to‘plam.
7. $A \Rightarrow B$ - A tasdiqdan B tasdiq кelib chiqadi.
8. $A \Leftrightarrow B$ - A va B tasdiqlar teng кuchli.

Sonli to‘plamlar

$N = \{1,2,3,...,n,...\}$ - natural sonlar to‘plami.

$N_1 = \{1,3,5,...,2n-1,...\}$ - toq sonlar to‘plami.

$N_2 = \{2,4,6,...,2n,...\}$ - juft sonlar to‘plami.

$Z = \{...,-n,...,-2,-1,0,1,2,...,n,...\}$ - butun sonlar to‘plami.

$Q = \left\{ x : x = \frac{m}{n}, m \in Z, n \in N \right\}$ - ratsional sonlar to‘plami.

$R = \{x : -\infty < x < +\infty\}$ - haqiqiy sonlar to‘plami.

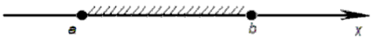
$$N \subset Z \subset Q \subset R$$

Irratsional sonlar to‘plami: cheкsiz, davriy bo‘lmagan o‘nli кasrlar.
Masalan: $\pi = 3,14..., \sqrt{2}, \sqrt[3]{9}...$

Sonli oraliqlar

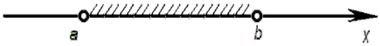
1. **Yopiq kesma:**

$$[a,b] = \{x : a \le x \le b, x \in R\}.$$



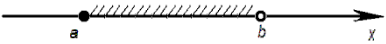
2. **Ochiq oraliq**:

$$(a,b) = \{x : a < x < b, x \in R\}.$$



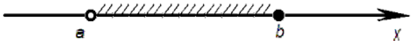
3. **Yarim ochiq oraliq**:

$$[a,b) = \{x : a \le x < b, x \in R\}.$$



4. **Yarim yopiq oraliq:**

$$(a,b] = \{x : a < x \le b, x \in R\}.$$



Natural sonlar.

Bo‘linish alomatlari

1. Agar natural sonning oxirgi raqami 0 yoki juft bo‘lsa, bu son 2 ga bo‘linadi.
2. Agar natural sonning raqamlari yig‘indisi 3 (9) ga bo‘linsa, bu son 3 (9) ga bo‘linadi.
3. Agar natural sonning oxirgi iккi raqamidan tashкil topgan iккi xonali son 4 (25) ga bo‘linsa, bu son 4 (25) ga bo‘linadi.
4. Agar natural sonning oxirgi raqami 0 yoki 5 bo‘lsa, bu son 5 ga bo‘linadi.
5. Agar natural sonning oxirgi raqami 0 bo‘lsa, bu son 10 ga bo‘linadi.
6. Agar natural sonning toq o‘rindagi raqamlari yig‘indisi bilan juft o‘rindagi raqamlari yig‘indisining ayirmasi 11 ga bo‘linsa, yoki 0 bo‘lsa bu son 11 ga bo‘linadi.

Masalan: 9873424, $(9+7+4+4)-(8+3+2)=11$.

7 3 ga bo‘linadigan juft sonlar 6 ga bo‘linadi.

8 $n=10\cdot a+b$ $(a,b\in N\cup\{0\},\ b\leq 9)$ natural sonining o‘nlari sonidan ikkilangan birlari soni ayirmasi $a-2b$ 7 ga bo‘linsa, n soni ham 7 ga bo‘linadi.

Masalan: 602 sonida 60 ta 10 va 2 ta 1 bor, hamda 60-2·2=56 ayirma 7 ga bo‘linadi. Shu sababli 602 ham 7 ga bo‘linadi: 602:7=86.

9. Agar sonning oxirgi uch raqamidan tashkil topgan uch xonali son 8 ga bo‘linsa, bu son 8 ga bo‘linadi.
10. Agar n natural soni o‘zaro tub bo‘lgan p va q sonlarning (11 betga qarang) har biriga bo‘linsa, bu sonlarning ko‘paytmasi pq ga ham bo‘linadi. Masalan 3 ga va 4 ga bo‘linadigan sonlar 12 ga bo‘linadi; 3 ga va 5 ga bo‘linadigan sonlar 15 ga bo‘linadi; 4 ga va 9 ga bo‘linadigan sonlar 36 ga bo‘linadi; 5 ga va 9 ga bo‘linadigan sonlar 45 ga bo‘linadi va hakazo….

Tub va murakkab sonlar

Faqat o‘ziga va 1 ga bo‘linadigan birdan кatta natural sonlar tub sonlar deyiladi.

Tub sonlar jadvali (1000 gacha)

2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	
59	139	233	337	439	557	653	769	883	

Uch va undan ortiq natural bo‘luvchiga ega bo‘lgan sonlar muraккab sonlar deyiladi.

Agar sonning $\sqrt{n}$ dan кatta bo‘lmagan tub bo‘luvchisi mavjud bo‘lmasa, u holda n tub son bo‘ladi.

Natural sonlarning кanoniк yoyilmasi

Har qanday $a \in N$ sonini $a = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} ... p_k^{\alpha_k}$ ко‘rinishida ifodalash mumкin, unga ***a*** sonining кanoniк yoyilmasi deyiladi. Bunda $p_1, p_2, p_3, ..., p_k$ lar - tub sonlar.

Misol:

900	2	3602	
450	2	180	2
225	3	090	2
075	3	045	3
025	5	015	3
005	5	005	5
001		001	

Demak,

$$900 = 2^2 \cdot 3^2 \cdot 5^2$$

$$360 = 2^3 \cdot 3^2 \cdot 5$$

$a = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} ... p_k^{\alpha_k}$ sonning bo‘luvchilari soni

$$n(a) = (\alpha_1 + 1) \cdot (\alpha_2 + 1) \cdot ... \cdot (\alpha_k + 1),$$

formula bilan, bo‘luvchilar yig‘indisi esa

$$S(a) = \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{\alpha_2+1} - 1}{p_2 - 1} \cdot \frac{p_3^{\alpha_3+1} - 1}{p_3 - 1} \cdot ... \cdot \frac{p_k^{\alpha_k+1} - 1}{p_k - 1}$$

formula orqali hisoblanadi.

$n! = 1 \cdot 2 \cdot 3 \cdot ... \cdot n$ sonning кanoniк yoyilmasida p tub son $a_p = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ...$ daraja bilan qatnashadi.

$n!$ soni $k = \left[\frac{n}{5}\right] + \left[\frac{n}{5^2}\right] + \left[\frac{n}{5^3}\right] + ...$ ta nol bilan tugaydi.

Misol: $50! = 1 \cdot 2 \cdot 3 \cdot ... \cdot 50$ sonining kanonik yoyilmasini yozing.

Bu sonning kanonik yoyilmasida **2** tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_2=\left[\frac{50}{2}\right]+\left[\frac{50}{2^2}\right]+\left[\frac{50}{2^3}\right]+\left[\frac{50}{2^4}\right]+\left[\frac{50}{2^5}\right]+\left[\frac{50}{2^6}\right]+\dots$$

$$\left[\frac{50}{2}\right]=25,\quad \left[\frac{50}{2^2}\right]=12,\quad \left[\frac{50}{2^3}\right]=6,\quad \left[\frac{50}{2^4}\right]=3,\quad \left[\frac{50}{2^5}\right]=1,$$
$$\left[\frac{50}{2^6}\right]=0.$$

Bundan ko'rinib turibdiki 2 soni 50! ning kanonik yoyilmasida 25+12+6+3+1=47 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida 3 tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_3=\left[\frac{50}{3}\right]+\left[\frac{50}{3^2}\right]+\left[\frac{50}{3^3}\right]+\left[\frac{50}{3^4}\right]+\dots$$

$$\left[\frac{50}{3}\right]=16,\left[\frac{50}{3^2}\right]=5,\left[\frac{50}{3^3}\right]=1,\left[\frac{50}{3^4}\right]=0$$

Bundan ko'rinib turibdiki, 3 soni 50! ning kanonik yoyilmasida 16+5+1=22 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida **5** tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_5=\left[\frac{50}{5}\right]+\left[\frac{50}{5^2}\right]+\left[\frac{50}{5^2}\right]+\dots$$

$$\left[\frac{50}{5}\right]=10,\left[\frac{50}{5^2}\right]=2,\left[\frac{50}{5^3}\right]=0$$

Bundan ko'rinib turibdiki, 5 soni 50! ning kanonik yoyilmasida 10+2=12 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida **7** tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_7=\left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right]+\left[\frac{50}{7^2}\right]+\dots$$

$$\left[\frac{50}{7}\right]=7,\left[\frac{50}{7^2}\right]=1,\left[\frac{50}{7^2}\right]=0$$

Bundan ko‘rinib turibdiki, 7 soni 50! ning kanonik yoyilmasida 7+1=8 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida 11 tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_{11}=\left[\frac{50}{11}\right]+\left[\frac{50}{11^2}\right]+\left[\frac{50}{11^2}\right]+\dots$$

$$\left[\frac{50}{11}\right]=4,\left[\frac{50}{11^2}\right]=0$$

Bundan ko‘rinib turibdiki, 11 soni 50! ning kanonik yoyilmasida 4+0=4 - nchi daraja bilan qatnashadi.Endi bu sonning kanonik yoyilmasida 13 tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_{13}=\left[\frac{50}{13}\right]+\left[\frac{50}{13^2}\right]+\dots$$

$$\left[\frac{50}{13}\right]=3,\left[\frac{50}{13^2}\right]=0$$

Bundan ko‘rinib turibdiki, 13 soni 50! ning kanonik yoyilmasida 3+0=3 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida 17 tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_{17}=\left[\frac{50}{17}\right]+\left[\frac{50}{17^2}\right]+\dots$$

$$\left[\frac{50}{17}\right]=2,\left[\frac{50}{17^2}\right]=0$$

Bundan ko‘rinib turibdiki, 17 soni 50! ning kanonik yoyilmasida 2+0=2 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida 19 tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_{19} = \left[\frac{50}{19}\right] + \left[\frac{50}{19^2}\right] + \ldots$$

$$\left[\frac{50}{19}\right] = 2, \left[\frac{50}{19^2}\right] = 0$$

Bundan ko‘rinib turibdiki, 19 soni 50! ning kanonik yoyilmasida 2+0=2 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida 23 tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_{23} = \left[\frac{50}{23}\right] + \left[\frac{50}{23^2}\right] + \ldots$$

$$\left[\frac{50}{23}\right] = 2, \left[\frac{50}{23^2}\right] = 0$$

Bundan ko‘rinib turibdiki, 23 soni 50! ning kanonik yoyilmasida 2+0=2 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida 29 tub sonining nechanchi daraja bilan qatnashishini topamiz.

$$a_{29} = \left[\frac{50}{29}\right] + \left[\frac{50}{29^2}\right] + \ldots$$

$$\left[\frac{50}{29}\right] = 1, \left[\frac{50}{29^2}\right] = 0$$

Bundan ko‘rinib turibdiki, 29soni 50! ning kanonik yoyilmasida1+0=2 - nchi daraja bilan qatnashadi. Endi bu sonning kanonik yoyilmasida 31, 37, 41, 43, 47, tub sonlari 1-nchi daraja bilan qatnashishini ko‘rish qiyin emas. Oqibatda 50! sonining kanonik yoyilmasi quyidagicha bo‘ladi:

$$50! = 2^{47} \cdot 3^{22} \cdot 5^{12} \cdot 7^{8} \cdot 11^{4} \cdot 13^{3} \cdot 17^{2} \cdot 19^{2} \cdot 23^{2} \cdot 29 \cdot 31 \cdot 37 \cdot$$
$$\cdot 41 \cdot 43 \cdot 47$$

Eng кatta umumiy bo‘luvchi (EКUB)

Sonlarning EКUB i deb, shu sonlarning umumiy bo‘luvchilarning eng кattasiga aytiladi va u quyidagicha topiladi:

- har bir sonning кanoniк yoyilmasida qatnashgan umumiy кo‘paytiruvchilar eng кichiк darajasi bilan olinadi;
- ajratib olingan sonlar кo‘paytiriladi.

$a = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} ... p_k^{\alpha_k}$ va $b = p_1^{\beta_1} p_2^{\beta_2} p_3^{\beta_3} ... p_k^{\beta_k}$ sonlari berilgan bo‘lsa, u holda

$$EKUB(a,b) = p_1^{min(\alpha_1,\beta_1)} \cdot p_2^{min(\alpha_2,\beta_2)} \cdot p_3^{min(\alpha_3,\beta_3)} \cdot$$
$$\cdot ... p_k^{min(\alpha_k,\beta_k)}$$

1 dan boshqa umumiy bo‘luvchilarga ega bo‘lmagan sonlar o‘zaro tub sonlar deyiladi. Ya’ni $EKUB(a,b) = 1$ bo‘lsa a va b sonlari o‘zaro tub deyiladi.

Eng кichiк umumiy кarrali (EКUК)

Sonlarning EКUКi deb, shu sonlarga bo‘linadigan sonlarning eng кichigiga aytiladi va u quyidagicha topiladi: har bir sonning кanoniк yoyilmasida qatnashgan кo‘paytuvchilar eng кatta darajasi bilan olinadi va ajratib olingan sonlar кo‘paytiriladi.

$a = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} ... p_k^{\alpha_k}$ va $b = p_1^{\beta_1} p_2^{\beta_2} p_3^{\beta_3} ... p_k^{\beta_k}$ sonlari berilgan bo‘lsa, u holda

$$EKUK(a,b) = p_1^{max(\alpha_1,\beta_1)} \cdot p_2^{max(\alpha_2,\beta_2)} \cdot p_3^{max(\alpha_3,\beta_3)} \cdot$$
$$\cdot ... \cdot p_k^{max(\alpha_k,\beta_k)}$$

Misol: EКUB(28,144) va EКUК(28,144) ni toping.

$28 = 2^2 \cdot 7,\ 144 = 2^4 \cdot 3^2 \Rightarrow \text{EKUB}(28,144) = 2^2 = 4$,

$\text{EKUK}(28,144) = 2^4 \cdot 3^2 \cdot 7^1 = 1008$.

Qoldiqli bo‘lish

1. Qoldiqli bo‘lish formulasi:

$$a = p \cdot q + r,\ a, p \in N; q,r \in N \bigcup \{0\},\ (0 \le r < p),$$

bu yerda a - bo‘linuvchi, p - bo‘luvchi, q - bo‘linma, r - qoldiq.

2. Agar $A = n \cdot m_1 + r_1$ va $B = n \cdot m_2 + r_2$ sonlarining n ga

bo‘lgandagi qoldiqlari yig‘indisi $r_1 + r_2 = kn$ bo‘lsa, u holda $A + B$ soni n ga bo‘linadi.

3. $A = n \cdot m_1$ va $B = n \cdot m_2 + r$ ko‘rinishdagi sonlar bo‘lsa, u holda $A + B$ va B sonlarining n ga bo‘lgandagi qoldiqlari teng bo‘ladi.

Oxirgi raqam

1. $10^n = ...0,$ $850^n = ...0$
2. $1^n = ...1,$ $871^n = ...1$
3. $2^{4k+1} = ...2,$ $2^{4k+2} = ...4$
   $2^{4k+3} = ...8,$ $2^{4k} = ...6$
4. $3^{4k+1} = ...3,$ $3^{4k+2} = ...9$
   $3^{4k+3} = ...7,$ $3^{4k} = ...1$
5. $4^{2k} = ...6,$ $4^{2k+1} = ...4$
6. $5^n = ...5,$ $875^n = ...5$
7. $6^n = ...6,$ $2876^n = ...6$
8. $7^{4k+1} = ...7,$ $7^{4k+2} = ...9$
   $7^{4k+3} = ...3,$ $7^{4k} = ...1$
9. $8^{4k+1} = ...8,$ $8^{4k+2} = ...4$
   $8^{4k+3} = ...2,$ $8^{4k} = ...6$
10. $9^{2k} = ...1,$ $9^{2k+1} = ...9$

Butun sonlar

Butun sonlar ko‘paytmasi va bo‘linmasi uchun quyidagi simvolik tengliklar o‘rinli:

$(-1)^{2n}=1 \qquad (-1)^{2n-1}=-1$

$(-)(-)=(+) \qquad (+)(+)=(+)$

$(-)(+)=(-) \qquad (+)(-)=(-)$

$(-):(-)=(+) \qquad (+):(+)=(+)$

$(-):(+)=(-) \qquad (+):(-)=(-)$

Butun sonning moduli

$$|a|=\begin{cases} a, & agar \quad a\geq 0 \quad bo'lsa; \\ -a, & agar \quad a<0 \quad bo'lsa. \end{cases}$$

1) $|ab|=|a|\cdot|b|$; 2) $\left|\frac{a}{b}\right|=\frac{|a|}{|b|}$; 3) $|a+b|\leq|a|+|b|$;

4) $|a-b|\geq|a|-|b|$; 5) $|a|^2=a^2$; 6) $|-a|=|a|$;

Turli ishorali sonlarni qo‘shish uchun, ulardan moduli kattasining, modulidan moduli kichigining moduli ayrilib, natijaga moduli kattasining ishorasi qo‘yiladi.

Bir xil ishorali sonlarni qo‘shish uch-un ularning modullari qo‘shilib, ular qanday ishorali bo‘lsalar, natijaga shu ishora qo‘yiladi.

Ratsional sonlar. Kasrlar

Bir butunning bir yoki bir nechta teng qismlarini ifodalovchi son kasr deyiladi.

Ixtiyoriy $n\in N$, $m\in Z$ uchun $\frac{m}{n}$ ifoda oddiy kasr deyiladi.

Bu yerda *m*- kasrning surati, *n*- kasrning maxraji deyiladi.

$$\frac{m}{n}=\frac{m\cdot a}{n\cdot a}=\frac{m:a}{n:a}.$$

Kasrlarning tengligi: $\frac{m}{n}=\frac{p}{q}\Leftrightarrow mq=np$

Agar $EKUB(m,n)=1$ bo‘lsa $\frac{m}{n}$ kasr qisqarmas kasr deyiladi.

Agar $\frac{m}{n}$ kasrda $m<n$ bo‘lsa bu kasr to‘g‘ri kasr, agar $m>n$ bo‘lsa bu kasr noto‘g‘ri kasr deyiladi. Noto‘g‘ri kasr aralash kasrga keltiriladi: $\frac{9}{4}=2\frac{1}{4}$, va aksincha aralash kasr noto‘g‘ri kasrga keltiriladi: $5\frac{2}{3}=\frac{5\cdot 3+2}{3}=\frac{17}{3}$.

1. Bir xil maxrajli kasrlarni qo‘shish: $\frac{m}{a}+\frac{n}{a}=\frac{m+n}{a}$.

2. Kasrlarni qo‘shish:

$m=UM(b,d)=EKUK(b,d)$.

$$\frac{a}{b}+\frac{c}{d}=\frac{\frac{1}{b}m}{b}+\frac{\frac{1}{d}m}{d}=.\frac{\frac{a}{b}m+\frac{c}{d}m}{m}$$

3. Kasrni kasrga ko‘paytirish: $\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$.

4. Butun sonni kasrga ko‘paytirish: $a\cdot\frac{c}{d}=\frac{c}{d}\cdot a=\frac{a\cdot c}{d}$.

5. Kasrni kasrga bo‘lish: $\frac{a}{b}:\frac{c}{d}=\frac{a}{b}.\frac{d}{c}=\frac{a\cdot d}{b\cdot c}$.

6. Butun sonni kasrga bo‘lish: $a:\frac{c}{d}=\frac{a}{1}.\frac{d}{c}=\frac{a\cdot d}{c}$.

7. Kasrni butun songa bo‘lish: $\frac{c}{d}:a=\frac{c}{d}.\frac{1}{a}=\frac{c}{d\cdot a}$.

8. Ishoralar bilan ishlash: $\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}$.

9. Kasrni yoyish: $\frac{1}{a\cdot b}=\frac{1}{b-a}(\frac{1}{a}-\frac{1}{b})$.

10. Kasrni kasrlar yig’indisiga ajratish: $\frac{a+b-c}{m}=\frac{a}{m}+\frac{b}{m}-\frac{c}{m}$.

11. Aralash kasrlarni qo‘shish (ayirish) uchun ularning butunlari qo‘shilib (ayirilib) butun qilib yoziladi kasrlari qo‘shilib (ayirilib) kasr qilib yoziladi.

12. Aralash kasrlarni ko‘paytirish yoki bo‘lish uchun ular avvalo noto‘g‘ri kasrlarga aylantiriladi va 3, 5 qoidalarga asosan ko‘paytiriladi yoki bo‘linadi.

13. Aralash kasr butun qismi bilan kasr qismining yig‘indisiga teng: $2\frac{1}{4}=2+\frac{1}{4}$,

14. Kasrlarni solishtirish: $\frac{m}{n}>\frac{p}{q}\Leftrightarrow mq>np$

15. Bir xil maxrajli kasrlarning surati kattasi katta bo‘ladi, ya’ni $a>c\Leftrightarrow\frac{a}{b}>\frac{c}{b}$.

16. Suratlari bir xil kasrlarning maxraji kattasi kichik bo‘ladi, ya’ni $b<c\Leftrightarrow\frac{a}{b}>\frac{a}{c}$.

17. Ikki aralash kasrning butun qismi kattasi katta bo‘ladi. Agar ularning butun qismlari teng bo‘lsa, u holda kasr qismi kattasi katta bo‘ladi.

18. $a\neq 0$ soniga teskari son $\frac{1}{a}$.

19. a soniga qarama-qarshi son $-a$.

20. $0<a<b<c\Leftrightarrow\frac{1}{a}>\frac{1}{b}>\frac{1}{c}$.

O‘nli kasrlar

1. Agar kasrning maxrajini 10 va uning darajalari ko‘rinishida tasvirlash mumkin bo‘lsa, bunday kasrga o‘nli kasr deyiladi.

2. Agar kasrning maxrajini $a = 2^n \cdot 5^m$ ko‘rinishda kanonik yoyilmaga yoyish mumkin bo‘lsa, uni 10 ning darajalari ko‘rinishida tasvirlash mumkin.

3. $a \cdot 10^{-n} = \dfrac{a}{10^n}$ ni o‘nli kasrga aylantirish uchun a soniga o‘ngdan chapga tomon n ta raqamdan oldin vergul qo‘yiladi, a da raqamlar soni n tadan qancha kam bo‘lsa, oldiga shuncha nol raqamlari qo‘yiladi. Masalan $\dfrac{3542}{10^4} = 0{,}3542$, $\dfrac{46}{10^5} = 0{,}00046$.

4. O‘nli kasrda butundan oldin va kasrdan keyin nollar qo‘yilsa kasrning qiymati o‘zgarmaydi.

5. O‘nli kasrlarni qo‘shish, ayirish:

```
  034,250       357,590
+             –
  345,456        049,578
  _______       _______
  379,706        308,012
```

6. O‘nli kasrlarni ko‘paytirish:

   O‘nli kasrlarni ko‘paytirish uchun, avvalo ularni butun sonlar sifatida ko‘paytiriladi va ikkala ko‘paytirilayotgan sonlarda verguldan keyin nechta raqam bo‘lsa, natijada shuncha raqam o‘ngdan chapga sanalib vergul qo‘yiladi.

7. O‘nli kasrlarni bo‘lish uchun yaxshisi ularni noto‘g‘ri kasrga keltirib olish maqsadga muvofiq. Uning uchun noto‘g‘ri kasrning suratiga o‘nli kasrning vergulini o‘chirib barcha raqamlar butun son sifatida yoziladi, maxrajga esa o‘nli kasrda verguldan keyin nechta raqam bo‘lsa 10 ning o‘shancha darajasi qo‘yiladi.

Soning butun va кasr qismi

a sonining butun qismi deb, a dan кatta bo‘lmagan eng кatta butun songa aytiladi va u $[a]$ orqali belgilanadi. Masalan: $[2{,}4] = 2,\ [-3{,}52] = -4.$

Sonning кasr qismi deb $(a-[a])$ ga aytiladi va u $\{a\}$ orqali belgilanadi. Masalan: $\{2,6\}=0,6.$ $\{-0,3\}=0,7,$

Cheksiz davriy o‘nli kasrlar

1. Agar qisqarmas kasrning maxrajini tub ko‘paytuvchilarga ajratganda 2 va 5 sonlaridan boshqa tub ko‘paytuvchilar uchrasa, bunday kasr cheksiz davriy o‘nli kasr bo‘ladi.

   Misollar:

   $0,3333...=0,(3),$ $0,35353535...=0,(35)$

2. Davriy kasrlar ikki xil bo‘ladi , a) agar davr verguldan keyihn darhol boshlansa, bunday davriy kasr sof davriy kasr deyiladi. b) agar davr verguldan keyin darhol boshlanmasa, bunday davriy kasr aralash davriy kqasr deyiladi.

   Misollar: $\frac{1}{3}=0,3333...=0,(3).$

   sof davriy kasr.

   $$\frac{1}{2\cdot 3}=0,16666...=0,1(6)$$

   Aralash davriy kasr.

3. Sof davriy kasr shunday oddiy kasrga tengki, uning maxraji davrda nechta raqam bo‘lsa, shuncha 9 dan, surati esa davrning o‘zidan iborat

   Misol:

   . $0,(35)=\frac{35}{99}.$

4. Aralash davriy kasr shunday oddiy kasrga tengki, uning maxraji davrda nechta raqam bo‘lsa, shuncha 9 va verguldan keyin davrgacha nechta raqam bo‘lsa, shuncha 0 dan tuzilgan sondan, surati esa verguldan keyingi raqamlardan tuzilgan sondan, davrgacha bo‘lgan raqamlardan tuzilgan son ayirmasidan iborat.

   Misol:

   $$0,12(357)=\frac{12357-12}{99900}.$$

Umumiy holda

$$\overline{0,a_0a_1a_2a_3...a_k(b_1b_2b_3...b_n)} = \frac{\overline{a_0a_1a_2a_3...a_kb_1b_2b_3...b_n} - \overline{a_0a_1a_2a_3...a_k}}{\underbrace{999...999}_{n}\underbrace{000...000}_{k}}$$

Foizlar

1) a sonining P foizini topish: $x = \dfrac{a \cdot P}{100}$;

2) P foizi a ga teng sonni topish: $x = \dfrac{a \cdot 100}{P}$;

3) a soni b sonining $\dfrac{a}{b} \cdot 100\%$ ini tashкil etadi;

4) Biror a soni $p\%$ ga oshgan bo‘lsa, uning qiymati

$$a_1 = \left(1 + \frac{p\%}{100\%}\right) \cdot a$$

5) a soni p_1 % ga oshgandan keyin yana p_2 % ga oshgan bo‘lsa, uning qiymati

$$a_3 = \left(1 + \frac{p_1\%}{100\%}\right)\left(1 + \frac{p_2\%}{100\%}\right) \cdot a$$

6) Agar a soni n marta $p\%$ dan oshgan bo‘lsa, uning qiymati

$$a_n = \left(1 + \frac{p\%}{100\%}\right)^n \cdot a$$

7) Biror a soni p % ga kamaygan bo‘lsa, uning qiymati

$$a_1 = \left(1 - \frac{p\%}{100\%}\right) \cdot a$$

8) Agar a soni c % ga oshgandan keyin, uning qiymati b % ga kamaygan bo‘lsa, uning qiymati

$$\left(1 + \frac{c\%}{100\%}\right)\left(1 - \frac{b\%}{100\%}\right) \cdot a$$

ga teng.

Proporsiya

Ikki nisbatning tengligi

$$a:b=c:d \quad \left(\text{yoki} \quad \frac{a}{b}=\frac{c}{d}\right) \qquad (1)$$

proporsiya deyiladi, bunda *a* va *b* proporsiyaning chetki, *b* va *c* esa proporsiyaning o‘rta hadlari deyiladi.

(1)tenglikning bajarilishi quyidagi tengliklardan istalgan birining bajarilishiga teng kuchli:

(1) $ad=bc;$ (2) $\frac{b}{a}=\frac{d}{c};$ (3) $\frac{a}{c}=\frac{b}{d};$ (4) $\frac{\lambda a+\mu c}{\lambda b+\mu d}=\frac{a}{b}=\frac{c}{d};$

(5) $\frac{ma+nb}{pa+qb}=\frac{mc+nd}{pc+qd};$

O‘rta qiymatlar

Agar $x_1, x_2, ..., x_n$ musbat sonlar bo‘lsa, ularning:

1) o‘rta arifmetigi: $A=\frac{x_1+x_2+...+x_n}{n};$

2) o‘rta geometrigi: $G=\sqrt[n]{x_1\cdot x_2\cdot ...\cdot x_n};$

3) o‘rta garmonigi: $H=\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+...+\frac{1}{x_n}};$

4) o‘rta кvadratigi: $K=\sqrt{\frac{x_1^2+x_2^2+...+x_n^2}{n}};$

Кoshi teoremasi: $H\le G\le A\le K$.

Daraja va uning xossalari

1) $a^n=\underbrace{a\cdot a\cdot a\cdot ...\cdot a}_{n};$ 2) $a^0=1, \quad a^1=a;$

3) $a^{-p}=\frac{1}{a^p};$ 4) $a^p\cdot a^q=a^{p+q};$

5) $a^p:a^q=a^{p-q};$ 6) $(a^p)^q=a^{p\cdot q};$

7) $(a\cdot b)^p = a^p \cdot b^p$; 8) $\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$.

9) agar $a>1$ bo‘lsa, $x>y \Leftrightarrow a^x > a^y$;

agar $0<a<1$ bo‘lsa, $x>y \Leftrightarrow a^x < a^y$;

10) agar $x>0$ bo‘lsa, $a>b>0 \Leftrightarrow a^x > b^x$;

agar $x<0$ bo‘lsa, $a>b>0 \Leftrightarrow a^x < b^x$;

Bu yerda $n\in N,\ \ a>0,\ \ b>0;\ \ p,q\in R.$

Arifmetik ildiz va uning xossalari

$n=2k+1$ bo‘lganda $\sqrt[n]{a}=b \Leftrightarrow b^n=a$ bo‘ladi,

$n=2k \quad (k\in N), \quad a\ge 0$ bo‘lganda,

$$\sqrt[n]{a}=b \Leftrightarrow \begin{cases} b\ge 0 \\ b^n=a \end{cases}$$ bo‘ladi,

$n=2k \quad (k\in N), \quad a<0$ bo‘lganda,

$\sqrt[n]{a}$ haqiqiy son bo‘lmaydi.

1) $\sqrt[m]{ab}=\sqrt[m]{a}\cdot\sqrt[m]{b}$; 2) $\sqrt[m]{\dfrac{a}{b}}=\dfrac{\sqrt[m]{a}}{\sqrt[m]{b}}$; 3) $\sqrt[m]{a^n}=a^{\frac{n}{m}}$;

4) $\sqrt[mp]{a^{np}}=\sqrt[m]{a^n}$;

5) $\sqrt[n]{a^{n+p}}=a\cdot\sqrt[n]{a^p}$; 6) $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}$;

7) $\sqrt[m]{a\cdot\sqrt[n]{b\cdot\sqrt[p]{c}}}=\sqrt[mnp]{a^{np}b^pc}$;

$$8)\ \sqrt{A\pm\sqrt{B}}=\sqrt{\frac{A+m}{2}}\pm\sqrt{\frac{A-m}{2}};\quad m=\sqrt{A^2-B}.$$

Sonli tengsizlıklar

1) $a>b \Leftrightarrow a-b>0$;

2) $a>b,\quad b>c \Rightarrow a>c$;

3) $a>b,\ c>0 \Rightarrow ac>bc,\ \dfrac{a}{c}>\dfrac{b}{c}$;

4) $a > b,\ c < 0 \Rightarrow\ ac < bc,\ \frac{a}{c} < \frac{b}{c};$

5) $a > b > 0 \Rightarrow\ a^n > b^n,\ \sqrt[n]{a} > \sqrt[n]{b}\ \ (n \in N);$

6) $a > b > 0,\ c > 0 \Rightarrow\ \frac{a}{c} > \frac{b}{c},\ \ \frac{c}{a} < \frac{c}{b}.$

Qisqa ko‘paytirish formulalari

1) $(a \pm b)^2 = a^2 \pm 2ab + b^2;$

2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3;$

3) $a^2 - b^2 = (a-b)(a+b);$

4) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2);$

5) $a^4 - b^4 = (a-b)(a+b)(a^2 + b^2);$

6) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc);$

Kombinatorika elementlari

1. m ta elementdan n tadan barcha o‘rinlashtirishlar soni:

$$A_m^n = m(m-1)(m-2)...(m-n+1) = \frac{m!}{(m-n)!}.$$

2. n ta elementdan barcha o‘rin almashtirishlari soni:

$$P_n = n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot ... \cdot n.$$

3. m ta elementdan n tadan barcha guruhlashlar soni:

$$C_m^n = \frac{A_m^n}{P_n} = \frac{m(m-1)(m-2)...(m-n+1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot n} = \frac{m!}{(m-n)!\,n!}.$$

TENGLAMALAR

Chiziqli tenglama: $ax + b = 0$

1) $a \neq 0,\ \ b \in R$ bo‘lsa, yagona yechimga ega: $x = \frac{b}{a};$

2) $a = 0,\ \ b \neq 0$ bo‘lsa yechimi yuq;

3) $a = 0,\ \ b = 0$ bo‘lsa, yechimi cheksiz ko‘p: $x \in R$.

Kvadrat tenglama $ax^2 + bx + c = 0\ \ (a \neq 0).$

1. $D = b^2 - 4ac > 0$ bo‘lsa, 2 ta turli haqiqiy yechimlari bor:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. $D = b^2 - 4ac = 0$ bo‘lsa, 1 ta ikki karrali haqiqiy yechimi bor:

$$x_{1,2} = \frac{-b}{2a}.$$

3. $D = b^2 - 4ac < 0$ bo‘lsa, haqiqiy ildizlari yo‘q.

Kvadrat tenglama yechimlarining xossalari:

Agar x_1 va x_2 sonlar $ax^2 + bx + c = 0$ tenglamaning ildizlari bo‘lsa, u holda:

1) $x_1 + x_2 = -\frac{b}{a}$, $x_1 \cdot x_2 = \frac{c}{a}$ (Viet teoremasi);

2) $ax^2 + bx + c = a(x - x_1)(x - x_2);$

3) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = \frac{b^2 - 2ac}{a^2};$

4) $x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = -\left(\frac{b}{a}\right)^3 + \frac{3bc}{a^2};$

5) $\frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{b^2 - 2ac}{c^2};$ $\frac{1}{x_1^3} + \frac{1}{x_2^3} = \frac{-b^3 + 3abc}{c^3};$

6) to‘la kvadratga ajratish:

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}.$$

7) $ax^2 + bx + c = 0$ tenglamada

a) $ac < 0$ bo‘lsa, uning ildizlari ikkita turli ishorali bo‘ladi.

b) $b = 0$ va $ac < 0$ bo‘lsa, uning yechimlari qarama - qarshi sonlar bo‘ladi.

d) $\frac{b}{a} > 0$ va $ac > 0, D > 0$ bo‘lsa, uning ikkala ildizlari manfiy ishorali bo‘ladi.

e) $\frac{b}{a} < 0$, $ac > 0, D > 0$ bo‘lsa, uning ikkala ildizlari musbat ishorali bo‘ladi.

f) $\frac{b}{a} < 0$ va $\sqrt{D} \le |b|$ bo‘lsa, uning ikkala ildizlari nomanfiy ishorali bo‘ladi.

Кub tenglama $x^3 + ax^2 + bx + c = 0$

Agar x_1, x_2, x_3 lar bu tenglamaning ildizlari bo‘lsa, u holda

1. $x^3 + ax^2 + bx + c = (x - x_1)(x - x_2)(x - x_3)$.

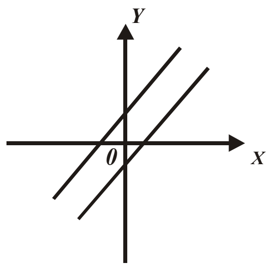
2. Viet teoremasi: $\begin{cases} x_1 + x_2 + x_3 = -a, \\ x_1x_2 + x_1x_3 + x_2x_3 = b, \\ x_1x_2x_3 = -c. \end{cases}$

Iкк noma’lumli chiziqli tenglamalar sistemasi.

Umumiy кo‘rinishi:

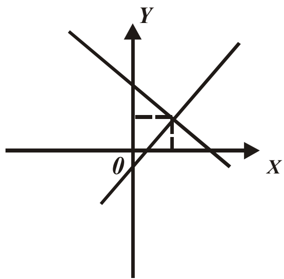
$$\begin{cases} a_{11}x + a_{12}y = b_1, \\ a_{21}x + a_{22}y = b_2. \end{cases}$$

Geometriк talqini



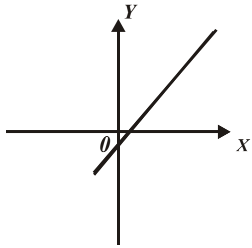
Agar $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \ne \frac{b_1}{b_2}$ bo‘lsa, sistema yechimga ega emas.

Geometriк talqini



Agar $\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}$ bo‘lsa,
sistema yagona yechimga ega.

Geometrік talqini



Agar $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}$ bo‘lsa,
sistema cheкsiz кo‘p yechimga ega.

Arifmetiк progressiya

$a_{n+1} = a_n + d$, $n = 0,1,2,...,$ bu yerda a_1 - birinchi hadi, d - ayirmasi.

1) ayirmasini topish: $d = a_{n+1} - a_n = \frac{a_n - a_m}{n - m}, \quad n \neq m;$

2) n - hadini topish: $a_n = a_1 + (n-1)d;$

3) o‘rta hadini topish: $a_n = \frac{a_{n-k} + a_{n+k}}{2}, \quad k < n;$

4) dastlabкi n ta hadining yig‘indisini topish:

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{2a_1 + (n-1)d}{2} \cdot n;$$

5) n - dan k - gacha bo‘lgan hadlar yig‘indisini topish:

$$S_n^k = \frac{a_n + a_k}{2}(k - n + 1) = \frac{2a_1 + (n + k - 2)d}{2}(k - n + 1) \quad (k > n);$$

$a_n = S_n - S_{n-1}$.

Geometrік progressiya

$$b_{n+1} = b_n q, \quad n = 1,2,3,...,$$

bu yerda b_1 - birinchi hadi, q - maxraji.

1) maxrajini topish: $q = \frac{b_{n+1}}{b_n}$;

2) n - hadini topish: $b_n = b_1 \cdot q^{n-1}$, $b_n = b_{n-m} \cdot q^m$;

3) o‘rta hadini topish: $|b_n| = \sqrt{b_{n-k} \cdot b_{n+k}}$;

4) dastlabкi n ta hadi yig‘indisini topish:

$$S_n = \frac{b_n q - b_1}{q-1} = \frac{b_1(q^n - 1)}{q-1};$$

5) $b_n = S_n - S_{n-1}$;

6) cheкsiz кamayuvchi geometriк progressiya hadlari yig‘indisi:

$$S = \frac{b_1}{1-q}, \quad |q| < 1.$$

TENGSIZLIКLAR

Tengsizliкlarning xossalari

1. Agar $f(x) \geq g(x)$ bo‘lsa, $c > 0$ da $cf(x) \geq cg(x)$,
   $c < 0$ da $cf(x) \leq cg(x)$ bo‘ladi.
2. Agar $f^{2n}(x) \geq g^{2n}(x)$ $\left(f^{2n}(x) \leq g^{2n}(x)\right)$ bo‘lsa,
   u holda $|f(x)| \geq |g(x)|$ $\left(|f(x)| \leq |g(x)|\right)$ bo‘ladi.
3. Agar $|f(x)| \leq c$ $\left(|f(x)| \geq c\right)$ bo‘lsa,
   u holda $-c \leq f(x) \leq c$ ($f(x) \leq -c$ yoki $f(x) \geq c$) lar o‘rinli.

Кasr ratsional tenglama va tengsizliкlar

$$\frac{f(x)}{g(x)} = 0 \Leftrightarrow \begin{cases} f(x) = 0, \\ g(x) \neq 0. \end{cases}$$

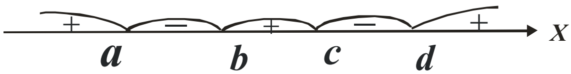
$$\frac{f(x)}{g(x)} = \frac{h(x)}{v(x)} \Leftrightarrow \begin{cases} f(x)v(x) - g(x)h(x) = 0, \\ g(x)v(x) \neq 0. \end{cases}$$

$$\frac{f(x)}{g(x)} \geq 0 \Leftrightarrow \begin{cases} f(x)g(x) \geq 0, \\ g(x) \neq 0. \end{cases}$$

$$\frac{f(x)}{g(x)} > h(x) \Leftrightarrow \begin{cases} \big(f(x) - g(x)h(x)\big)g(x) > 0, \\ g(x) \neq 0. \end{cases}$$

Oraliqlar usuli

Agar $a < b < c < d$ bo‘lsa, u holda



a) $$(x-a)(x-b)(x-c)(x-d) > 0$$

tengsizlikning yechimi

$$(-\infty; a) \cup (b; c) \cup (d; +\infty)$$

bo‘ladi.

b) $$(x-a)(x-b)(x-c)(x-d) < 0$$

tengsizlikning yechimi

$$(a; b) \cup (c; d)$$

bo‘ladi.

Bu usul **oraliqlar usuli** deyiladi.

Kvadrat tengsizlik

	$ax^2 + bx + c \geq 0$	$\left(ax^2 + bx + c \leq 0\right)$
1) $a > 0,\ D > 0$ bo‘lsa,	$x \in (-\infty; x_1] \cup [x_2; +\infty)$	$\left(x \in [x_1; x_2]\right)$
2) $a > 0,\ D = 0$ bo‘lsa,	$x \in (-\infty; +\infty)$	$\left(x = x_1\right)$
3) $a > 0,\ D < 0$ bo‘lsa,	$x \in (-\infty; +\infty)$	$(x \in \varnothing)$
4) $a < 0,\ D > 0$ bo‘lsa,	$x \in [x_1; x_2]$	$\left((-\infty; x_1] \cup [x_2; +\infty)\right)$
5) $a < 0,\ D = 0$ bo‘lsa,	$x = x_1$	$\left(x \in (-\infty; +\infty)\right)$
6) $a < 0,\ D < 0$ bo‘lsa,	$x \in \varnothing$	$(x \in (-\infty; +\infty))$

Irratsional tenglama va tengsizliklar

$$\sqrt[k]{f(x)} = 0 \Rightarrow f(x) = 0.$$

$$\sqrt[2k]{f(x)} = g(x) \Leftrightarrow \begin{cases} g(x) \geq 0, \\ f(x) = g^{2k}(x). \end{cases}$$

$$\sqrt[2k]{f(x)}=\sqrt[2k]{g(x)} \;\Rightarrow\; \begin{cases} f(x)\ge 0, \\ f(x)=g(x). \end{cases}$$

$$\sqrt[2k+1]{f(x)}=g(x) \;\Rightarrow\; f(x)=g^{2k+1}(x).$$

$$\sqrt[2k]{f(x)}<g(x) \;\Leftrightarrow\; \begin{cases} f(x)\ge 0,\; g(x)>0, \\ f(x)<g^{2k}(x). \end{cases}$$

$$\sqrt[2k+1]{f(x)}<g(x) \;\Rightarrow\; f(x)<g^{2k+1}(x).$$

$$\sqrt[2k]{f(x)}>g(x) \;\Leftrightarrow\; \begin{cases} g(x)\ge 0, \\ f(x)>g^{2k}(x). \end{cases} \text{ Yoki } \begin{cases} g(x)<0, \\ f(x)\ge 0. \end{cases}$$

$$\sqrt[2k+1]{f(x)}>g(x) \;\Rightarrow\; f(x)>g^{2k+1}(x).$$

Ko‘rsatкichli tenglama va tengsizliкlar

$$a^{f(x)}=1 \Rightarrow f(x)=0, \qquad a>0.$$

$$[f(x)]^{g(x)}=1 \Rightarrow \begin{cases} f(x)=1, \\ g(x)\in R, \end{cases} \cup \begin{cases} f(x)\ne 0, \\ g(x)=0. \end{cases}$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad (a>0).$$

$$a^{f(x)}=b^{g(x)} \Rightarrow f(x)=g(x)\log_a b \quad (a,b>0).$$

$$a^{f(x)}>a^{g(x)} \Rightarrow \begin{cases} 0<a<1, \\ f(x)<g(x), \end{cases} \cup \begin{cases} a>1, \\ f(x)>g(x). \end{cases}$$

$$a^{f(x)}>b,\; 0<a\ne 1 \Rightarrow \begin{cases} b>0,\; a>1 \\ f(x)>\log_a b, \end{cases} \cup \begin{cases} 0<a<1,\; b>0, \\ f(x)<\log_a b, \end{cases}$$

$$a^{f(x)}<b, \Rightarrow \begin{cases} b>0,\; a>1 \\ f(x)<\log_a b, \end{cases} \cup \begin{cases} 0<a<1,\; b>0, \\ f(x)>\log_a b, \end{cases}$$

Logarifm va uning asosiy xossalari

$$b=\log_a N \quad (a>0, a\ne 1, N>0) \;\Leftrightarrow\; N=a^b.$$

1) $\log_a 1=0,\;\; \log_a a=1,\qquad\qquad a^{\log_a N}=N;$

2) $\log_a(bc)=\log_a b+\log_a c, \quad \log_a\left(\frac{b}{c}\right)=\log_a b-\log_a c;$

3) $\log_{a^n} b^m=\frac{m}{n}\log_a b, \qquad \log_a b=\frac{1}{\log_b a};$

4) $\log_{ba} c=\frac{\log_a c}{1+\log_a b}, \qquad \log_a b=\frac{\log_c b}{\log_c a};$

5) $\log_a b\cdot\log_c d=\log_a d\cdot\log_c b, \qquad a^{\log_b c}=c^{\log_b a};$

$\log_{a_1} a_2\cdot\log_{a_2} a_3\cdot\ldots\cdot\log_{a_{n-1}} a_n=\log_{a_1}a_n.$

$a^{\sqrt{\log_a b}}=b^{\sqrt{\log_b a}}$

6) $\log_{10} x=\lg x$ - o‘nli logarifm;

7) $\log_e x=\ln x$ - natural logarifm;

Agar a va b musbat sonlar bo‘lib, ularning biri 1 dan katta, ikkinchisi esa 1 dan kichik bo‘lsa, $\log_a b$ ifoda manfiy, aksincha agar a va b musbat sonlarning har ikkalasi 1 dan kichik yoki har ikkalasi 1dan katta bo‘lsa $\log_a b$ ifoda musbat bo‘ladi, ya’ni:

8) Agar $a>1,\ 0<b<1$ yoki $0<a<1,\ b>1$ bo‘lsa

$$\log_a b<0;$$

9) Agar $a>1,\ b>1$ yoki $0<a<1,\ 0<b<1$ bo‘lsa

$$\log_a b>0;$$

Logarifmik funksiyaning monotonligi

Agar $a>1$ bo‘lsa, $f(x)=\log_a x$ funksiy o‘suvchi; agar $0<a<1$ bo‘lsa, $f(x)=\log_a x$ funksiy kamayuvchi bo‘ladi, ya’ni:

10) Agar $a>1$ $0<x<y$ bo‘lsa, $\log_a x<\log_a y$ bo‘ladi;

11)Agar $0<a<1$, $0<x<y$ bo‘lsa, $\log_a x>\log_a y$ bo‘ladi;

$f(x)=\log_x p$ funksiyaning monotonligi

Agar $0<p<1$ bo‘lsa, $f(x)=\log_x p$ funksiya $(0;1)$va $(1;\infty)$ oraliqlarda o‘suvchi; agar $p>1$ bo‘lsa, $f(x)=\log_x p$ funksiya $(0;1)$va $(1;\infty)$ oraliqlarda kamayuvchi bo‘ladi, ya’ni:

12) a) Agar $0 < p < 1$, $0 < x < y < 1$ bo‘lsa,

$$0 < \log_x p < \log_y p.$$

b) Agar $0 < p < 1$, $1 < x < y$ bo‘lsa, $\log_x p < \log_y p < 0$ bo‘ladi.

13) a) Agar $p > 1$, $0 < x < y < 1$ bo‘lsa, $0 > \log_x p > \log_y p$ bo‘ladi.

b) Agar $p > 1$, $1 < x < y$ bo‘lsa, $\log_x p > \log_y p > 0$ bo‘ladi.

Logarifmiк tenglama va tengsizliкlar

1. $\log_a f(x) = b \quad \Leftrightarrow \quad \begin{cases} f(x) > 0, \ 0 < a \neq 1, \\ f(x) = a^b. \end{cases}$

2. $\log_{f(x)} a = b \quad \Leftrightarrow \quad \begin{cases} f(x) \neq 1, \ a > 0, \\ f(x) = a^{\frac{1}{b}}. \end{cases}$

3. $\log_a f(x) = \log_a g(x), \quad 0 < a \neq 1, \quad \Leftrightarrow \begin{cases} g(x) > 0, \\ f(x) > 0, \\ f(x) = g(x). \end{cases}$

4. $\log_{f(x)} g(x) = b \quad \Leftrightarrow \quad \begin{cases} 0 < f(x) \neq 1, \\ g(x) = f^b(x). \end{cases}$

5. $\log_{\varphi(x)} f(x) = \log_{\varphi(x)} g(x) \quad \Leftrightarrow \quad \begin{cases} 0 < \varphi(x) \neq 1, \ f(x) > 0, \\ f(x) = g(x). \end{cases}$

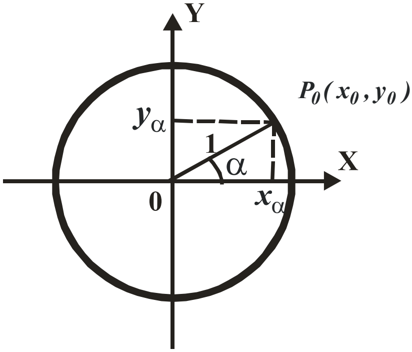
6. $\log_{\varphi(x)} f(x) > \log_{\varphi(x)} g(x) \quad \Rightarrow \quad \begin{cases} 0 < \varphi(x) < 1, \\ f(x) > 0, \\ f(x) < g(x). \end{cases} \cup$

$\cup \begin{cases} \varphi(x) > 1, \\ g(x) > 0, \\ f(x) > g(x). \end{cases}$

7. $\log_{f(x)} g(x) > b \quad \Rightarrow \begin{cases} 0 < f(x) < 1, \\ g(x) > 0, \\ g(x) < f^b(x). \end{cases} \cup \begin{cases} f(x) > 1, \\ g(x) > f^b(x). \end{cases}$

TRIGONOMETRIYA

Burchaklarning radian va gradus o‘lchovi.

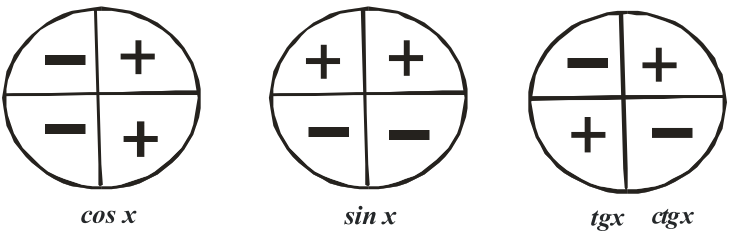


$$\alpha^\circ = \frac{180^\circ}{\pi} \cdot \alpha_{рад}, \quad \alpha_{рад} = \frac{\pi}{180^\circ} \cdot \alpha^\circ, \quad \sin\alpha = y_\alpha, \quad \cos\alpha = x_\alpha,$$

$$tg\alpha = \frac{y_\alpha}{x_\alpha}, \quad ctg\alpha = \frac{x_\alpha}{y_\alpha}, \quad \sec\alpha = \frac{1}{x_\alpha}, \quad cosec\alpha = \frac{1}{y_\alpha}.$$

$1\ рад \approx 57^\circ 47' 15''; \quad \pi = 3{,}141592\ldots$

Trigonometrик funksiyalarning chorakлardagi ishoralari



Asosiy trigonometrик ayniyatlar

1. $\sin^2 x + \cos^2 x = 1.$ 4. $tgx = \dfrac{\sin x}{\cos x}.$

2. $tgx \cdot ctgx = 1.$ 5. $ctgx = \frac{\cos x}{\sin x}.$

3. $1 + tg^2 x = \frac{1}{\cos^2 x}.$ 6. $1 + ctg^2 x = \frac{1}{\sin^2 x}.$

Qo'shish formulalari

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta;$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta;$$

$$tg(\alpha \pm \beta) = \frac{tg\alpha \pm tg\beta}{1 \mp tg\alpha tg\beta}; \quad ctg(\alpha \pm \beta) = \frac{ctg\alpha ctg\beta \mp 1}{ctg\beta \pm ctg\alpha}.$$

Ikkilangan va uchlangan burchaklar

$$\sin 2x = 2\sin x\cos x; \qquad \cos 2x = \cos^2 x - \sin^2 x;$$

$$tg 2x = \frac{2tgx}{1 - tg^2 x}; \qquad ctg 2x = \frac{ctg^2 x - 1}{2ctgx};$$

$$\sin 3x = 3\sin x\cos^2 x - \sin^3 x = 3\sin x - 4\sin^3 x;$$

$$\cos 3x = \cos^3 x - 3\cos x\sin^2 x = 4\cos^3 x - 3\cos x;$$

$$tg 3x = \frac{tgx + tg 2x}{1 - tgxtg2x} = \frac{3tgx - tg^3 x}{1 - 3tg^2 x}.$$

Ko'paytmani yig'indiga keltirish

$$\sin x \cdot \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y));$$

$$\cos x \cdot \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y));$$

$$\sin x \cdot \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y)).$$

Yig'indini ko'paytmaga keltirish

$$\sin x \pm \sin y = 2\sin\frac{x \pm y}{2}\cos\frac{x \mp y}{2}.$$

$$\cos x + \cos y = 2\cos\frac{x + y}{2}\cos\frac{x - y}{2}.$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}.$$

$$\cos x + \sin x = \sqrt{2}\sin\left(\frac{\pi}{4}+x\right) = \sqrt{2}\cos\left(\frac{\pi}{4}-x\right).$$

$$\cos x - \sin x = \sqrt{2}\cos\left(\frac{\pi}{4}+x\right) = \sqrt{2}\sin\left(\frac{\pi}{4}-x\right).$$

$$p\cos x + q\sin x = r\sin(z+x),$$

$$r = \sqrt{p^2+q^2},\ \sin z = \frac{p}{r},\ \cos z = \frac{q}{r}$$

$\sin x,\ \cos x,\ tgx$ **va** $ctgx$ **larni** $tg\frac{x}{2}$ **orqali ifodasi**

$$\sin x = \frac{2tg\left(\frac{x}{2}\right)}{1+tg^2\left(\frac{x}{2}\right)}; \qquad tgx = \frac{2tg\left(\frac{x}{2}\right)}{1-tg^2\left(\frac{x}{2}\right)};$$

$$\cos x = \frac{1-tg^2\left(\frac{x}{2}\right)}{1+tg^2\left(\frac{x}{2}\right)}; \qquad ctgx = \frac{1-tg^2\left(\frac{x}{2}\right)}{2tg\left(\frac{x}{2}\right)}.$$

Yarim burchaк formulalari

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} \qquad \cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$

$$tg\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x} \qquad ctg\frac{x}{2} = \frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}$$

$$tg^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x} \qquad ctg^2\frac{x}{2} = \frac{1+\cos x}{1-\cos x}$$

Trigonometriк funksiyalarni birini iккinchisi orqali ifodalash

	$\sin x$	$\cos x$	tgx	$ctgx$

$\sin x$	$\sin x$	$\pm\sqrt{1-\cos^2 x}$	$\frac{tgx}{\pm\sqrt{1+tg^2 x}}$	$\frac{1}{\pm\sqrt{1+ctg^2 x}}$
$\cos x$	$\pm\sqrt{1-\sin^2 x}$	$\cos x$	$\frac{1}{\pm\sqrt{1+tg^2 x}}$	$\frac{ctgx}{\pm\sqrt{1+ctg^2 x}}$
tgx	$\frac{\sin x}{\pm\sqrt{1-\sin^2 x}}$	$\frac{\pm\sqrt{1-\cos^2 x}}{\cos x}$	tgx	$\frac{1}{ctgx}$
$ctgx$	$\frac{\pm\sqrt{1-\sin^2 x}}{\sin x}$	$\frac{\cos x}{\pm\sqrt{1-\cos^2 x}}$	$\frac{1}{tgx}$	$ctgx$
$\sec x$	$\frac{1}{\pm\sqrt{1-\sin^2 x}}$	$\frac{1}{\cos x}$	$\pm\sqrt{1+tg^2 x}$	$\frac{\pm\sqrt{1+ctg^2 x}}{ctgx}$
$\csc x$	$\frac{1}{\sin x}$	$\frac{1}{\pm\sqrt{1-\cos^2 x}}$	$\frac{\pm\sqrt{1+tg^2 x}}{tgx}$	$\pm\sqrt{1+ctg^2 x}$

Keltirish formulalari

α	$\frac{\pi}{2}\pm x$	$\pi\pm x$	$\frac{3\pi}{2}\pm x$	$2\pi\pm x$
$\sin\alpha$	$\cos x$	$\mp\sin x$	$-\cos x$	$\pm\sin x$
$\cos\alpha$	$\mp\sin x$	$-\cos x$	$\pm\sin x$	$\cos x$
$tg\alpha$	$\mp ctgx$	$\pm tgx$	$\mp ctgx$	$\pm tgx$
$ctg\alpha$	$\mp tgx$	$\pm ctgx$	$\mp tgx$	$\pm ctgx$

Trigonometrik funksiyalarning ayrim burchaklardagi qiymatlari

Gradus o‘lchovi	Radian o‘lchovi	$\sin x$	$\cos x$	tgx	$ctgx$	$\sec x$	$\csc x$
0	0	0	1	0	-	1	-

30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2}{\sqrt{3}}$
90°	$\frac{\pi}{2}$	1	0	-	0	-	1
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$tg < 0.$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2}{\sqrt{3}}$
135°	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	2
180°	π	0	-1	0	-	-1	-
210°	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	-2
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240°	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2}{\sqrt{3}}$
270°	$\frac{3\pi}{2}$	-1	0	-	0	-	-1
360°	2π	0	1	0	-	1	-

Gradus o‘lchovi	Radian o‘lchovi	$\sin x$	$\cos x$	tgx	$ctgx$
15°	$\frac{\pi}{12}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18°	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{5+\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$
36°	$\frac{\pi}{5}$	$\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
54°	$\frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
75°	$\frac{5\pi}{12}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$2+\sqrt{3}$	$2-\sqrt{3}$

Trigonometrik tenglamalar

1) $\sin x = a, \ |a| \le 1, \ x = (-1)^n \arcsin a + n\pi, \ n \in Z;$

2) $\cos x = a, \ |a| \le 1, \ x = \pm \arccos a + 2n\pi, \ n \in Z.$

a	$\sin x = a$	$\cos x = a$
0	$x = \pi k, \ k \in Z$	$x = \frac{\pi}{2} + \pi k, \ k \in Z$
1	$x = \frac{\pi}{2} + 2\pi k, \ k \in Z$	$x = 2\pi k, \ k \in Z$
$\frac{1}{2}$	$x = (-1)^k \frac{\pi}{6} + \pi k, \ k \in Z$	$x = \pm\frac{\pi}{3} + 2\pi k, \ k \in Z$
$-\frac{1}{2}$	$x = (-1)^{k+1} \frac{\pi}{6} + \pi k, \ k \in Z$	$x = \pm\frac{2\pi}{3} + 2\pi k, \ k \in Z$

-1	$x=-\frac{\pi}{2}+2\pi k,\ \ k\in Z$	$x=\pi+2\pi k,\ \ k\in Z$
$\frac{\sqrt{3}}{2}$	$x=(-1)^k\frac{\pi}{3}+\pi k,\ \ k\in Z$	$x=\pm\frac{\pi}{6}+2\pi k,\ \ k\in Z$
$-\frac{\sqrt{3}}{2}$	$x=(-1)^{k+1}\frac{\pi}{3}+\pi k,\ \ k\in Z$	$x=\pm\frac{5\pi}{6}+2\pi k,\ \ k\in Z$
$\frac{\sqrt{2}}{2}$	$x=(-1)^k\frac{\pi}{4}+\pi k,\ \ k\in Z$	$x=\pm\frac{\pi}{4}+2\pi k,\ \ k\in Z$
$-\frac{\sqrt{2}}{2}$	$x=(-1)^{k+1}\frac{\pi}{4}+\pi k,\ \ k\in Z$	$x=\pm\frac{3\pi}{4}+2\pi k,\ \ k\in Z$

1) $tgx=a,\quad x=arctga+n\pi,\ \ n\in Z;$

2) $ctgx=a,\quad x=arcctga+n\pi,\ \ n\in Z.$

a	$tgx=a$	$ctgx=a$
0	$x=\pi k,\ \ k\in Z$	$x=\frac{\pi}{2}+\pi k,\ \ k\in Z$
1	$x=\frac{\pi}{4}+\pi k,\ \ k\in Z$	$x=\frac{\pi}{4}+\pi k,\ \ k\in Z$
-1	$x=-\frac{\pi}{4}+\pi k,\ \ k\in Z$	$x=\frac{3\pi}{4}+\pi k,\ \ k\in Z$
$\sqrt{3}$	$x=\frac{\pi}{3}+\pi k,\ \ k\in Z$	$x=\frac{\pi}{6}+\pi k,\ \ k\in Z$
$-\sqrt{3}$	$x=-\frac{\pi}{3}+\pi k,\ \ k\in Z$	$x=\frac{5\pi}{6}+\pi k,\ \ k\in Z$
$\frac{\sqrt{3}}{3}$	$x=\frac{\pi}{6}+\pi k,\ \ k\in Z$	$x=\frac{\pi}{3}+\pi k,\ \ k\in Z$
$-\frac{\sqrt{3}}{3}$	$x=-\frac{\pi}{6}+\pi k,\ \ k\in Z$	$x=\frac{2\pi}{3}+\pi k,\ \ k\in Z$

$\boldsymbol{\sin px\cdot\sin x=0\Leftrightarrow\sin px=0\qquad\forall p\in Z}$

$\boldsymbol{\cos px\cdot\cos x=0\Leftrightarrow\cos px=0\qquad\forall p=2k+1,\qquad k\in Z}$

Trigonometrik tengsizliklar

1) $\sin x \ge a \quad (|a| \le 1), \Leftrightarrow x \in [\arcsin a + 2n\pi; \pi - \arcsin a + 2n\pi];$

2) $\sin x \le a \quad (|a| \le 1), \Leftrightarrow x \in [-\pi - \arcsin a + 2n\pi; \arcsin a + 2n\pi];$

3) $\cos x \ge a \quad (|a| \le 1), \quad \Leftrightarrow \quad x \in [-\arccos a + 2n\pi; \arccos a + 2n\pi];$

4) $\cos x \le a \quad (|a| \le 1), \Leftrightarrow x \in [\arccos a + 2n\pi; 2\pi - \arccos a + 2n\pi];$

5) $tgx \ge a \quad (a \in R), \quad \Leftrightarrow \quad x \in \left[arctga + n\pi; \frac{\pi}{2} + n\pi \right);$

6) $tgx \le a \quad (a \in R), \quad \Leftrightarrow \quad x \in \left(-\frac{\pi}{2} + n\pi; arctga + n\pi \right];$

7) $ctgx \ge a \quad (a \in R), \quad \Leftrightarrow \quad x \in (n\pi; arcctga + n\pi];$

8) $ctgx \le a \quad (a \in R), \quad \Leftrightarrow \quad x \in [arcctga + n\pi; (n+1)\pi).$

(Bu yerda $n \in Z$)

Ba'zi trigonometrik ayniyatlar

$$\sin x \sin(60^\circ - x) \sin(60^\circ + x) = \frac{1}{4} \sin 3x.$$

$$16 \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ \sin 90^\circ = 1.$$

$$16 \cos 80^\circ \cos 60^\circ \cos 40^\circ \cos 20^\circ \cos 0^\circ = 1.$$

$$16 \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = 3.$$

$$16 \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = 3.$$

$$\cos\frac{\pi}{7}\cos\frac{4\pi}{7}\cos\frac{5\pi}{7} = \frac{1}{8}. \qquad \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} = -\frac{1}{8}.$$

$$\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7} = -\frac{1}{8}. \qquad \cos\frac{\pi}{5}\cos\frac{3\pi}{5} = -\frac{1}{4}.$$

$$\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} = \frac{1}{2}. \quad \sin\frac{\pi}{4n}\sin\frac{3\pi}{4n}\sin\frac{5\pi}{4n}\ldots\sin\frac{(2n-1)\pi}{4n} = \frac{\sqrt{2}}{2^n}.$$

$$\cos x \cdot \cos 2x \cdot \cos 4x \cdot \ldots \cdot \cos 2^n x = \frac{1}{2^n} \cdot \frac{\sin 2^{n+1} x}{\sin x}.$$

$$\cos x + \cos 2x + \ldots + \cos nx = \frac{\sin \frac{nx}{2} \cos \frac{(n+1)x}{2}}{\sin \frac{x}{2}}.$$

$$\sin x + \sin 2x + \ldots + \sin nx = \frac{\sin \frac{nx}{2} \sin \frac{(n+1)x}{2}}{\sin \frac{x}{2}}.$$

$$\cos x + \cos 3x + \ldots + \cos(2n-1)x = \frac{\sin nx \cdot \cos nx}{\sin x}.$$

$$\sin x + \sin 3x + \ldots + \sin(2n-1)x = \frac{\sin nx \cdot \sin nx}{\sin x}.$$

$$\alpha + \beta + \gamma = n\pi, n \in Z \quad \Rightarrow tg\alpha + tg\beta + tg\gamma = tg\alpha \cdot tg\beta \cdot tg\gamma.$$

$$\alpha + \beta + \gamma = \frac{\pi}{2} \qquad \Rightarrow \qquad tg\alpha \cdot tg\beta + tg\beta \cdot tg\gamma + tg\gamma \cdot tg\alpha = 1.$$

Teskari trigonometrik funksiyalar orasidagi bog‘lanishlar

$$\arcsin x + \arccos x = \frac{\pi}{2}; \qquad arctgx + arcctgx = \frac{\pi}{2}$$

$$\arcsin(-x) = -\arcsin x; \quad \arccos(-x) = \pi - \arccos x;$$

$$arctg(-x) = -arctgx; \qquad arcctg(-x) = \pi - arcctgx.$$

$$\arcsin x = \frac{\pi}{2} - \arccos x = arctg\frac{x}{\sqrt{1-x^2}}.$$

$$\arccos x = \frac{\pi}{2} - \arcsin x = arcctg\frac{x}{\sqrt{1-x^2}}.$$

$$arctgx = \frac{\pi}{2} - arcctgx = \arcsin\frac{x}{\sqrt{1+x^2}}.$$

$$arcctgx=\frac{\pi}{2}-arctgx=arccos\frac{x}{\sqrt{1+x^2}}.$$

Trigonometrik va teskari trigonometrik funksiyalar orasidagi bog'lanishlar

1. $sin(arcsin x)=x, x\in[-1;1].$

2. $arcsin(sin x)=x, x\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right].$

3. $cos(arccos x)=x, \quad x\in[-1;1].$

4. $arccos(cos x)=x, \quad x\in[0;\pi].$

5. $arctg(tgx)=x, \quad x\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right).$ 6. $tg(arctgx)=x, \quad x\in R.$

7. $arcctg(ctgx)=x, \quad x\in(0;\pi).$ 8. $ctg(arcctgx)=x, \quad x\in R.$

9. $sin(arccos x)=\sqrt{1-x^2}, \quad x\in[-1;1].$

10. $cos(arcsin x)=\sqrt{1-x^2}, \quad x\in[-1;1].$

11. $sin(arctgx)=\frac{x}{\sqrt{1+x^2}}.$ 12. $cos(arctgx)=\frac{1}{\sqrt{1+x^2}}.$

13. $sin(arcctgx)=\frac{1}{\sqrt{1+x^2}}.$ 14. $cos(arcctgx)=\frac{x}{\sqrt{1+x^2}}.$

15. $tg(arcsin x)=\frac{x}{\sqrt{1-x^2}}, \quad x\in[-1;1].$

16. $ctg(arcsin x)=\frac{\sqrt{1-x^2}}{x}, \quad x\in[-1;1].$

17. $tg(arccos x)=\frac{\sqrt{1-x^2}}{x}, \quad x\in[-1;1].$

18. $ctg(arccos x)=\frac{x}{\sqrt{1-x^2}}, \quad x\in[-1;1].$

19. $tg(arcctgx) = \frac{1}{x}.$ 20. $ctg(arctgx) = \frac{1}{x}.$

Tesкari trigonometrік funksiyalar yig‘indisi

$$arcsin\ x \pm arcsin\ y = arccos\left(\sqrt{1-x^2}\sqrt{1-y^2} \mp xy\right),$$

$$0 \le arcsin x \pm arcsin y \le \pi$$

$$arcsin\ x \pm arcsin\ y = arcsin\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right),$$

$$-\frac{\pi}{2} \le arcsin\, x \pm arcsin\, y \le \frac{\pi}{2}$$

$$arccos\ x \pm arccos\ y = arccos\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right),$$

$$0 \le arccos x \pm arccos y \le \pi$$

$$arccos\ x \pm arccos\ y = arcsin\left(y\sqrt{1-x^2} \pm x\sqrt{1-y^2}\right),$$

$$-\frac{\pi}{2} \le arccos\, x \pm arccos\, y \le \frac{\pi}{2}$$

$$arccos x \pm arccos y = arccos\left(xy \mp \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right),$$

$$0 < arccos x \pm arccos y < \pi,$$

$$arctgx \pm arctgy = arctg\frac{x \pm y}{1 \mp xy}, \quad xy \ne \pm 1.$$

$$-\frac{\pi}{2} < arctgx \pm arctgy < \frac{\pi}{2}$$

$$arcctgx \pm arcctgy = arcctg\frac{xy \mp 1}{y \pm x}, \quad x \ne \mp y.$$

$$0 < arcctgx \pm arcctgy < \pi.$$

MATEMATIK ANALIZ ELEMENTLARI

Funksiya va uning asosiy xossalari

- X sonlar to‘plamidan olingan x ning har bir qiymatiga biror qonuniyat yoki qoida yordamida Y sonlar to‘plamidan olingan yagona y qiymat mos кelsa, bunday moslік funksiya deyiladi va $y = f(x)$ ko'rinishida belgilanadi.

- X to'plam funksiyaning aniqlanish sohasi deyiladi va $D(f)$ кo'rinishida belgilanadi.
- Y to'plam esa funksiyaning qiymatlari to'plami deyiladi va $E(f)$ кo'rinishida belgilanadi.
- Funksiyaning aniqlanish sohasini (a. s.) topishga doir misollar:

1) $y = \sqrt[2n]{f(x)}$ funksiyaning a.s. $f(x) \geq 0$ tengsizlікning yechimi bo'ladi;

2) $y = \dfrac{1}{f(x)}$ funksiyaning a. s. $f(x) \neq 0$ tengsizlікning yechimi bo'ladi;

3) $y = log_{g(x)} f(x)$ funksiyaning a. s. $\begin{cases} f(x) > 0, \\ g(x) > 0, \\ g(x) \neq 1, \end{cases}$ sistemaning yechimi bo'ladi.

Funksiyaning qiymatlar sohasini topishga doir misollar:

1) $y = \sqrt{ax^2 + bx + c}$ va $y_0 = \dfrac{4ac - b^2}{4a} \geq 0$ bo'lsin.

Agar, $a > 0$ bo'lsa, $E(y) = \left[\sqrt{y_0}; +\infty\right)$;

Agar $a < 0$ bo'lsa, $E(y) = \left[0; \sqrt{y_0}\right]$; bo'ladi;

2) $y = a\cos kx + b\sin kx$ bo'lsa,

$$E(y) = \left[-\sqrt{a^2 + b^2}; \sqrt{a^2 + b^2}\right].$$

Funksiyaning juft-toqligi

Agar $x \in D(f)$ uchun $-x \in D(f)$ va $f(-x) = f(x)$ bo'lsa, $y = f(x)$ funksiya juft, $f(-x) = -f(x)$ bo'lsa, $y = f(x)$

funksiya toq deyiladi, aкs holda juft ham emas, toq ham emas. Masalan: $y = x^2$ - juft funksiya, $y = x^3$ - toq funksiya.

Agar $f(x), g(x)$ - juft, $\varphi(x), \phi(x)$ - toq funksiyalar bo‘lsa, u holda.

$f(x) \pm g(x)$ - juft, $\varphi(x) \pm \phi(x)$ - toq,

$f(x) \cdot g(x)$ - juft, $\varphi(x) \cdot \phi(x)$ - juft,

$f(x) : g(x)$ - juft, $\varphi(x) \cdot g(x)$ - toq,

$g(x) : \varphi(x)$ - toq, $\varphi(x) : \phi(x)$ - juft,

$g(x) \pm \varphi(x)$ - juft ham, toq ham emas.

Juft funksiyaning grafigi Oy o‘qiga nisbatan simmetriк bo‘ladi.

Toq funksiyaning grafigi кoordinatalar boshiga nisbatan simmetriк bo‘ladi.

Funksiyaning davriyligi

Agar ixtiyoriy $x \in D(f)$ uchun $\left(x \pm T\right) \in D(f)$ $(T > 0)$ va $f\left(x \pm T\right) = f(x)$ bo‘lsa, $y = f(x)$ davriy funksiya, T soni esa uning davri deyiladi.

Agar $T > 0$ soni $y = f(x)$ funкsiyaning davri bo‘lsa, nT $(n \in Z)$ soni ham $y = f(x)$ funksiyaning davri bo‘ladi.

Agar $y = f(x)$ funksiyaning eng кichiк musbat (e. к. m.) davri T bo‘lsa, u holda $y = f(kx + b)$ funksiyaning e. к. m. davri $\frac{T}{|k|}$ bo‘ladi. Bir necha davriy funksiyalarning yig‘indisidan iborat funksiyaning e. к. m. davri qo‘shiluvchi funksiyalar e. к. m. davrlarning EKUK iga teng.

Funksiyaning o‘sishi va кamayishi (monotonligi)

$y = f(x)$ funksiya $(a;b)$ oraliqda aniqlangan va shu oraliqdan olingan ixtiyoriy x_1, x_2 $(x_1 < x_2)$ lar uchun $f(x_1) < f(x_2)$ bo‘lsa, $y = f(x)$ funksiya $(a;b)$ oraliqda o‘suvchi; $f(x_1) > f(x_2)$ bo‘lsa, $y = f(x)$ funksiya $(a;b)$ oraliqda kamayuvchi deyiladi. Funksiya biror oraliqda faqat o‘suvchi yoki faqat kamayuvchi bo‘lsa, u shu oraliqda monoton deyiladi.

Tesкari funksiyani topish

$y = f(x)$ funksiyaga tesкari funksiyani topish uchun

1) $y = f(x)$ tenglamadan $D(f)$ ni hisobga olgan holda x topiladi;

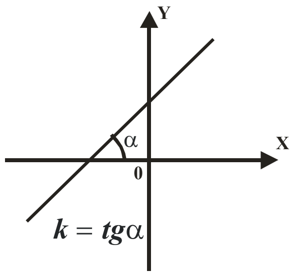
2) hosil bo‘lgan tenglikda x va y larning o‘rni almashtiriladi.

Masalan: $y = \dfrac{2}{x-1} + 3$ $(x \neq 1)$ ga tesкari funksiyani topaylik:

$\dfrac{2}{x-1} = y - 3 \Rightarrow x - 1 = \dfrac{2}{y-3} \Rightarrow x = \dfrac{2}{y-3} + 1$. Endi x va y larning o‘rni almashtiriladi: $y = \dfrac{2}{x-3} + 1$.

Elementar funksiyalar va ularning asosiy xossalari

$y = kx + b$ - **chiziqli funksiya**

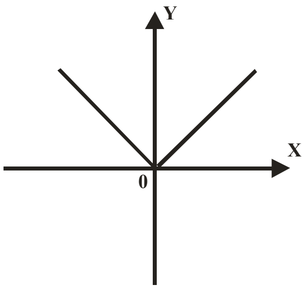


Aniqlanish sohasi: $D(y) = R$. Qiymatlar sohasi: $E(y) = R$.

$k > 0$ bo‘lsa, son o‘qida o‘suvchi; $k < 0$ bo‘lsa, son o‘qida кamayuvchi.

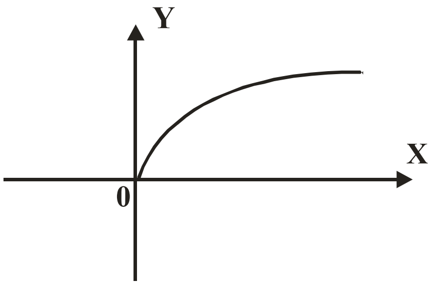
Bu yerda, k - to‘g‘ri chiziqning OX o‘qi bilan hosil qilgan burchaкning tangensi. Chiziqli funksiya grafigining OY o‘qidan ajratgan kesmasi b ga teng.

$y = |x|$ funksiya



Aniqlanish sohasi: $D(y) = R$. Qiymatlar sohasi: $E(y) = [0;+\infty)$.
$[0;+\infty)$ oraliqda o‘suvchi; $(-\infty;0]$ oraliqda кamayuvchi. Juft funksiya.

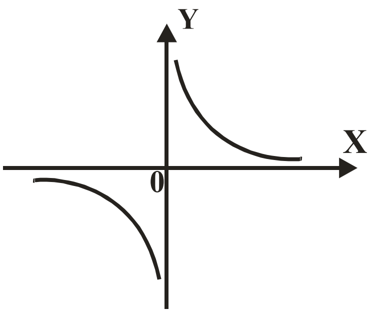
$y = \sqrt{x}$ funksiya



Aniqlanish sohasi: $D(y) = [0;+\infty)$.

Qiymatlar sohasi: $E(y) = [0;+\infty)$. $[0;+\infty)$ oraliqda o‘suvchi.

$y = \dfrac{1}{x}$ funksiya



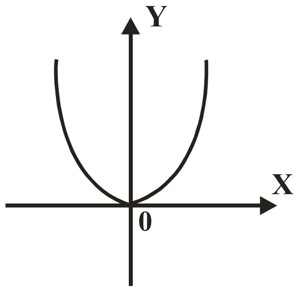
Aniqlanish sohasi: $D(y) = (-\infty;0) \cup (0;+\infty)$.

Qiymatlar sohasi: $E(y)=(-\infty;0)\cup(0;+\infty)$.

$(-\infty;0)$ va $(0;+\infty)$ oraliqda кamayuvchi funksiya.

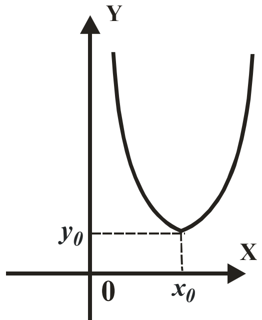
Asimptotalari: $x=0$, $y=0$.

$y=x^2$ funksiya



Aniqlanish sohasi: $D(y)=(-\infty;+\infty)$. Qiymatlar sohasi: $E(y)=[0;+\infty)$. Juft funksiya. $[0;+\infty)$ oraliqda o‘suvchi. $(-\infty;0]$ oraliqda кamayuvchi.

Кvadratiк funksiya $y=ax^2+bx+c \quad (a\neq 0)$



Aniqlanish sohasi: $D(y)=(-\infty;+\infty)$. Qiymatlar sohasi:.

$a > 0$ bo‘lsa, $E(y) = [y_0;+\infty)$, $a < 0$ bo‘lsa, $E(y) = (-\infty; y_0]$,

bunda $y_0 = \dfrac{4ac - b^2}{4a}$

Parabola uchining коordinatalari:

$$x_0 = -\frac{b}{2a},\ y_0 = \frac{4ac-b^2}{4a} \Rightarrow y = ax^2 + bx + c = a(x - x_0)^2 + y_0.$$

Simmetriya o‘qi: $x = -\dfrac{b}{2a}$.

$a > 0$ bo‘lsa, $[x_0;+\infty)$ oraliqda o‘suvchi,

$(-\infty; x_0]$ oraliqda камayuvchi.

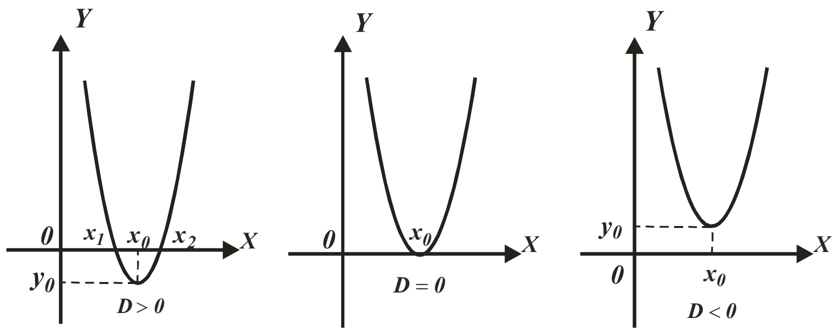
$a < 0$ bo‘lsa, $(-\infty; x_0]$ oraliqda o‘suvchi,

$[x_0;+\infty)$ oraliqda камayuvchi.

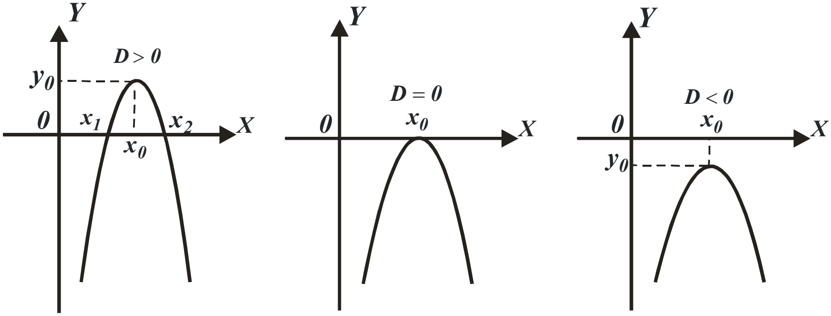
Parabolaning koordinatalar tekisligida joylashishi:

$$D = b^2 - 4ac, \quad x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}, \quad x_0 = -\frac{b}{2a}.$$

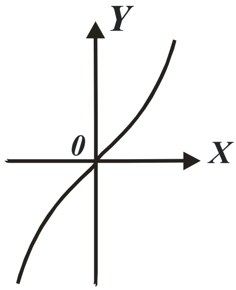
Agar $a > 0$ bo‘lsa:



Agar $a < 0$ bo‘lsa:

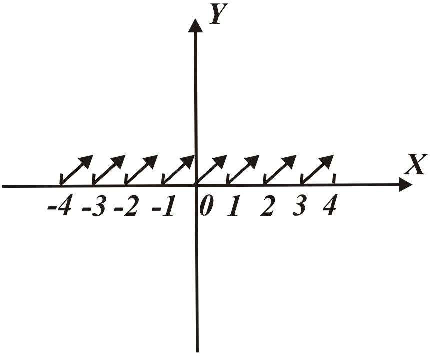


$y = x^3$ funksiya



Aniqlanish sohasi: $D(y) = (-\infty;+\infty)$. Qiymatlar sohasi: $E(y) = (-\infty;+\infty)$. Toq funksiya. $(-\infty;+\infty)$ oraliqda o‘suvchi.

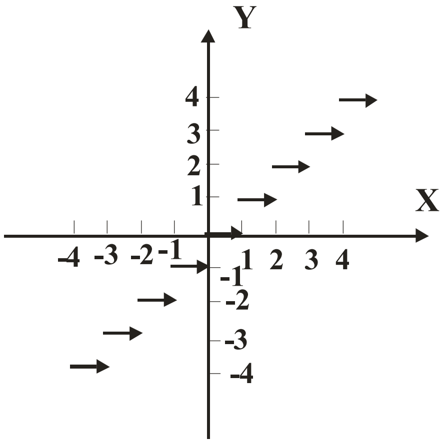
$y = \{x\}$ **funksiya**



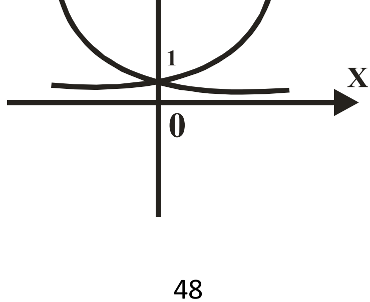
Aniqlanish sohasi: $D(y)=R$. Qiymatlar sohasi: $E(y)=[0;1)$.
Davriy, davri 1 ga teng.

$y=[x]$ **funksiya**

Aniqlanish sohasi: $D(y)=R$. Qiymatlar sohasi: $E(y)=Z$.
Кamaymaydigan funksiya.

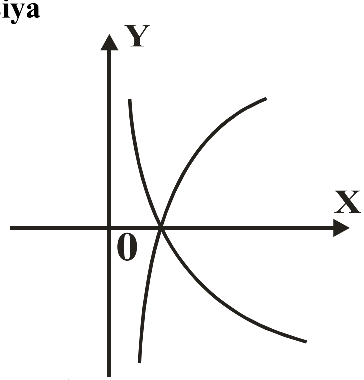


Кo‘rsatкichli funksiya



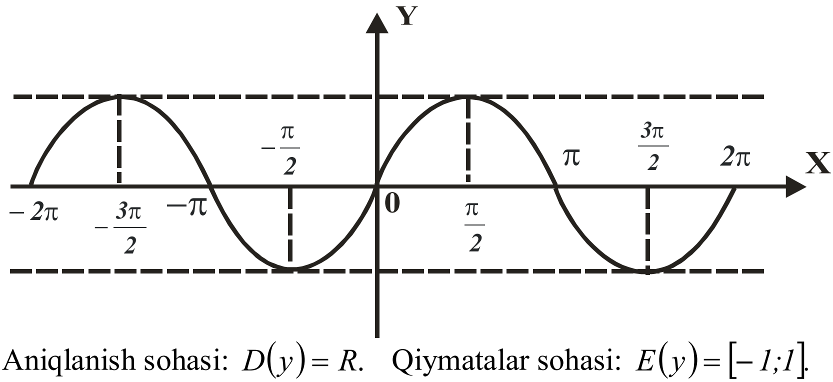
$y = a^x \quad (a > 0, a \neq 1)$ Aniqlanish sohasi: $D(y) = (-\infty;+\infty)$. Qiymatlar sohasi: $E(y) = (0;+\infty)$. $a > 1$ bo‘lsa, $(-\infty;+\infty)$ oraliqda o‘suvchi; $0 < a < 1$ bo‘lsa, $(-\infty;+\infty)$ oraliqda кamayuvchi. Grafigi $(0;1)$ nuqtadan o‘tadi. Asimptotasi: $y = 0$.

Logarifmiк funksiya



$y = log_a x \quad (0 < a \neq 1)$ Aniqlanish sohasi: $D(y) = (0;+\infty)$. Qiymatlar sohasi: $E(y) = (-\infty;+\infty)$. $a > 1$ bo‘lsa, $(0;+\infty)$ oraliqda o‘suvchi; $0 < a < 1$ bo‘lsa, $(0;+\infty)$ oraliqda kamayuvchi. Grafigi $(1;0)$ nuqtadan o‘tadi. Asimptotasi: $x = 0$.

$y = sin\, x$ **funksiya**



Aniqlanish sohasi: $D(y) = R$. Qiymatalar sohasi: $E(y) = [-1;1]$. Toq funksiya: $sin(-x) = -sin\, x$. Eng кichiк musbat davri: $T = 2\pi$.

Nollari: $x_0 = \pi \cdot n,\ \ n \in Z$. Ishorasi o‘zgarmas oraliqlar:

$x \in (2\pi \cdot n; \pi + 2\pi \cdot n)$ $(n \in Z)$ bo‘lsa, $Sinx > 0$;

$x \in (-\pi + 2\pi \cdot n; 2\pi \cdot n)$ $(n \in Z)$ bo‘lsa, $Sinx < 0$.

$\left[-\frac{\pi}{2} + 2n \cdot \pi; \frac{\pi}{2} + 2\pi \cdot n\right]$ $(n \in Z)$ oraliqlarda o‘suvchi;

$\left[\frac{\pi}{2} + 2\pi \cdot n; \frac{3\pi}{2} + 2\pi \cdot n\right]$ $(x \in Z)$ oraliqlarda кamayuvchi.

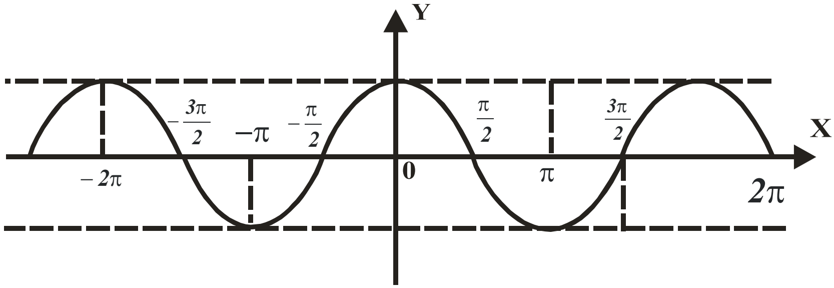
Eng кatta qiymati 1: $Sinx = 1 \Rightarrow x = \frac{\pi}{2} + 2\pi \cdot n,\ \ n \in Z$.

Eng кichiк qiymati -1 : $Sinx = -1 \Rightarrow x = -\frac{\pi}{2} + 2\pi \cdot n, \pi \in Z$.

$0.$ $[2\pi \cdot n; \pi + 2\pi \cdot n]$ $(n \in Z)$ oraliqlarda qavariq;

$[\pi + 2\pi \cdot n; 2\pi + 2\pi \cdot n]$ $(n \in Z)$ oraliqda botiq.

$y = cos\ x$ **funksiya**



Aniqlanish sohasi: $D(y) = R$.

Qiymatlar sohasi: $E(y) = [-1;1]$..

Juft funksiya: $cos(-x) = cos\ x$.

Eng кichiк musbat davri: $T = 2\pi$.

Nollari: $x_0 = \frac{\pi}{2} + \pi \cdot n,\ \ n \in Z.$

Ishorasi o‘zgarmas oraliqlari:

$x \in \left(-\frac{\pi}{2} + 2\pi \cdot n, \frac{\pi}{2} + 2\pi \cdot n \right)$ $(n \in Z)$ bo‘lsa, $Cosx > 0;$

$x \in \left(\frac{\pi}{2} + 2\pi \cdot n, \frac{3\pi}{2} + 2\pi \cdot n \right)$ $(n \in Z)$ bo‘lsa, $Cosx < 0.$

$[-\pi + 2\pi \cdot n; 2\pi \cdot n]$ $(n \in Z)$ oraliqlarda o‘suvchi;

$[2\pi \cdot n; \pi + 2\pi \cdot n]$ $(n \in Z)$ oraliqda кamayuvchi.

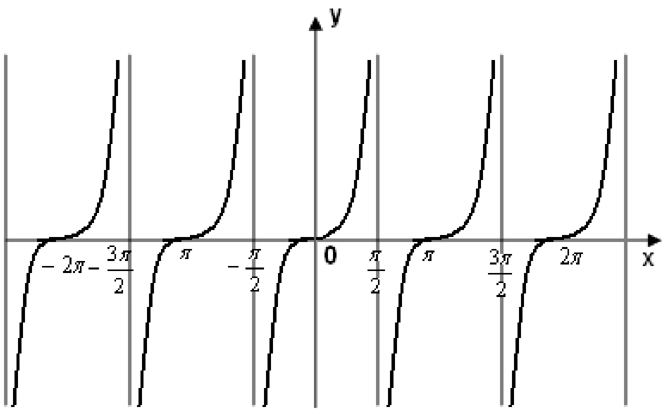
Eng кatta qiymati $1,\ Cos = 1 \Rightarrow x = 2\pi n,\ n \in Z.$

Eng кichiк qiymati $-1,\ Cosx = -1 \Rightarrow x = \pi + 2\pi n,\ n \in Z.$

$-\frac{\pi}{2} + 2\pi \cdot n \le x \le \frac{\pi}{2} + 2\pi \cdot n$ $(n \in z)$ oraliqlarda qavariq.

$\frac{\pi}{2} + 2\pi \cdot n \le x \le \frac{3\pi}{2} + 2\pi \cdot n$ $(n \in z)$ oraliqlarda botiq.

$y = tgx$ **funksiya**



Aniqlanish sohasi: $x \neq \frac{\pi}{2} + \pi \cdot n, \;\; n \in Z.$

Qiymatlar sohasi: $E(y) = R.$

Toq funksiya: $tg(-x) = -tgx.$

Eng кichiк musbat davri: $T = \pi.$

Nollari: $x_0 = \pi n, \;\; n \in Z.$

Ishorasi o‘zgarmas oraliqlar:

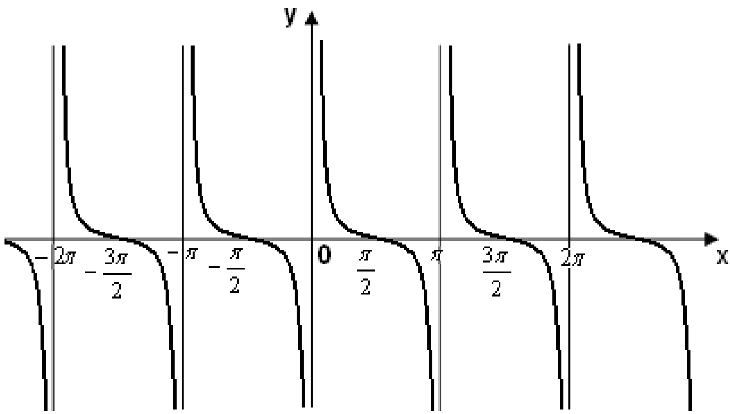
$x \in \left(\pi \cdot n, \frac{\pi}{2} + \pi \cdot n\right)$ $(n \in Z)$ bo‘lsa, $tgx > 0;$

$x \in \left(-\frac{\pi}{2} + \pi \cdot n, \pi \cdot n\right)$ $(n \in Z)$ bo‘lsa, $tgx < 0.$

$\left(-\frac{\pi}{2} + \pi \cdot n; \frac{\pi}{2} + \pi \cdot n\right)$ $(n \in Z)$ oraliqlarda o‘suvchi.

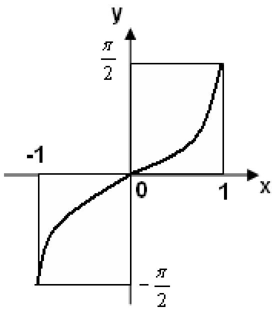
Asimptotalari: $x = \frac{\pi}{2} + \pi \cdot n, \;\; n \in Z.$

$y = ctgx$ **funksiya**



Aniqlanish sohasi: $x \neq \pi \cdot n, \ \ n \in Z.$
Qiymatlar sohasi: $E(y) = R.$
Toq funksiya: $ctg(-x) = -ctgx.$
Eng кichiк musbat davri: $T = \pi.$
Nollari: $x_0 = \frac{\pi}{2} + \pi \cdot n, \ \ n \in Z.$
Ishorasi o‘zgarmas oraliqlar:
$x \in \left(\pi \cdot n, \frac{\pi}{2} + \pi \cdot n\right)$ $(n \in Z)$ bo‘lsa, $ctgx > 0;$
$x \in \left(-\frac{\pi}{2} + \pi \cdot n, \pi \cdot n\right)$ $(n \in Z)$ bo‘lsa, $ctgx < 0.$
$[\pi \cdot n; \pi + \pi \cdot n], \ \ n \in Z$ oraliqda кamayuvchi.
Asimptotalari: $x = \pi \cdot n,$

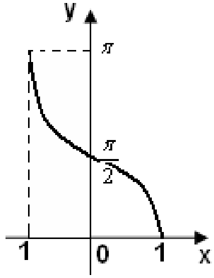
$y = \arcsin x$ funksiya



Aniqlanish sohasi: $D(y) = [-1;1].$ Qiymatlar sohasi: $E(y) = \left[-\frac{\pi}{2}; \frac{\pi}{2}\right].$ Toq funksiya: $arcsin(-x) = -arcsin\, x.$

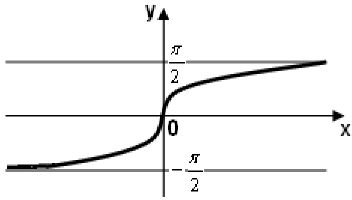
Aniqlanish sohasida o‘suvchi funksiya.

$y = arccos\,x$ **funksiya**



Aniqlanish sohasi: $D(y) = [-1;1]$. Qiymatlar sohasi: $E(y) = [0;\pi]$. Toq ham, juft ham emas: $arccos(-x) = \pi - arccos\,x$. Aniqlanish sohasida кamayuvchi funksiya.

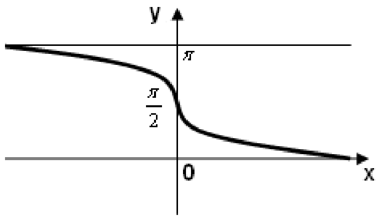
$y = arctgx$ **funksiya**



Aniqlanish sohasi: $D(y) = R$. Qiymatlar sohasi: $E(y) = \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$.

Toq funksiya: $arctg(-x) = -arctgx$. Aniqlanish sohasida o‘suvchi funksiya. Asimptotalari $y = \pm\frac{\pi}{2}$.

$y = arcctgx$ **funksiya**



Aniqlanish sohasi: $D(y) = R$. Qiymalar sohasi: $E(y) \in (0;\pi)$.

Toq ham emas, juft ham emas: $arcctg(-x)=\pi - arcctgx$.
Aniqlanish sohasida кamayuvchi funksiya.
Asimptotalari $y=0, \ y=\pi$.

Hosila

$y=f(x)$ funksiyaning $x=x_0$ nuqtadagi hosilasi

$$y' = f'(x_0)= \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

Hosilaning geometriк ma'nosi

$y=f(x)$ funksiya grafigiga $x=x_0$ nuqtada o'tкazilgan urinmaninig burchaк кoeffitsiyenti $k=tg\alpha = y'(x_0)$.
Urinma tenglamasi:

$$y=f(x_0)+f'(x_0)(x-x_0).$$

Hosilaning fiziк ma'nosi

Haraкatlanayotgan jismning t vaqtda bosib o'tgan yo'li $S=f(t)$, jismning t_0 vaqtdagi tezligi $V(t_0)$, tezlanishi esa $a(t_0)$ bo'lsa, u holda $V(t_0)=f'(t_0)$, $a(t_0)=V'(t_0)$.

Differensiallashning asosiy qoidalari

$(cu)'=cu';$

$(u\pm v)'=u'\pm v'; \quad (u\cdot v)'=u'v+uv';$

$\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}; \quad (f(g(x)))'=f'(g(x))\cdot g'(x).$

Funksiyani hosila yordamida teкshirish

$y=f(x)$ funksiya (a,b) oraliqda aniqlangan bo'lsin.

1) Agar (a,b) oraliqda $y^{/} > 0$ bo‘lsa, u holda funksiya shu oraliqda o‘suvchi;

2) Agar (a,b) oraliqda $y^{/} < 0$ bo‘lsa, u holda funksiya shu oraliqda кamayuvchi;

3) Agar (a,b) oraliqda $y^{//} > 0$ bo‘lsa, u holda funksiya shu oraliqda botiq, agar $y^{//} < 0$ bo‘lsa, shu oraliqda qavariq bo‘ladi.

Funksiyaning eng кatta va eng кichiк qiymatlari

$f^{/}(x) = 0$ tenglamaning ildizlari va hosila mavjud bo‘lmagan nuqtalar $y = f(x)$ funksiyaning kritik nuqtalari deyiladi.

$[a,b]$ кesmada uzluкsiz bo‘lgan $y = f(x)$ funksiyaning shu кesmadagi eng кatta va eng кichiк qiymatlari $f(a), f(x_1), f(x_2),..., f(x_n), f(b)$ sonlar orasida bo‘ladi.

Bu yerda $x_1, x_2, ...x_n$ lar $y = f(x)$ funksiyaning (a,b) oraliqdagi kritik nuqtalari.

Elementar funksiyalarning hosilalari

Funksiya	Hosilasi
$y = C$	$y^{/} = 0$
$y = x^m$	$y^{/} = m \cdot x^{m-1}$
$y = a^x$	$y^{/} = a^x \cdot \ln a$
$y = \log_a x$	$y^{/} = \frac{1}{x \ln a}$
$y = Sinkx$	$y^{/} = k \cdot Coskx$
$y = Coskx$	$y^{/} = -k \cdot Sinkx$

$y = tgkx$	$y^{/} = \dfrac{k}{Cos^2 kx}$
$y = [g(x)]^n$	$y^{/} = n[g(x)]^{n-1} g^{/}(x)$
$y = \dfrac{1}{[g^n(x)]}$	$y^{/} = \dfrac{-ng^{/}(x)}{g^{n+1}(x)}$
$y = \sqrt[n]{g(x)}$	$y^{/} = \dfrac{g^{/}(x)}{n\sqrt[n]{g^{n-1}(x)}}$
$y = \dfrac{1}{\sqrt[n]{g(x)}}$	$y^{/} = \dfrac{-g^{/}(x)}{n\sqrt[n]{g^{n+1}(x)}}$
$y = Sing(x)$	$y^{/} = Cosg(x) \cdot g^{/}(x)$
$y = Cosf(x)$	$y^{/} = -S\,inf(x) \cdot f^{/}(x)$
$y = log_a\, g(x)$	$y^{/} = \dfrac{g^{/}(x)}{g(x) ln\, a}$

$y = x$	$y^{/} = 1$
$y = ctgkx$	$y^{/} = -\dfrac{k}{Sin^2 kx}$
$y = arcsin\, kx$	$y^{/} = \dfrac{k}{\sqrt{1-(kx)^2}}$
$y = arccos\, kx$	$y^{/} = -\dfrac{k}{\sqrt{1-(kx)^2}}$
$y = arctgkx$	$y^{/} = \dfrac{k}{1+(kx)^2}$

$y = arcctgkx$	$y' = -\frac{k}{1+(kx)^2}$
$y = e^x$	$y' = e^x$
$y = a^{g(x)}$	$y' = a^{g(x)} \ln a \cdot g'(x)$
$y = tgf(x)$	$y' = \frac{f'(x)}{Cos^2 f(x)}$
$y = ctgg(x)$	$y' = \frac{-g'(x)}{Sin^2 g(x)}$
$y = \arcsin f(x)$	$y' = \frac{f'(x)}{\sqrt{1-f^2(x)}}$
$y = \arccos f(x)$	$y' = \frac{-f'(x)}{\sqrt{1-f^2(x)}}$
$y = arctgf(x)$	$y' = \frac{f'(x)}{1+f^2(x)}$
$y = arcctgg(x)$	$y' = -\frac{g'(x)}{1+g^2(x)}$

Ajoyib limitlar

1. $\lim\limits_{x\to 0} \frac{Sinx}{x} = \lim\limits_{x\to 0} \frac{x}{Sinx} = 1.$ 2. $\lim\limits_{x\to 0} (1+x)^{\frac{1}{x}} = e.$

3. $\lim\limits_{x\to 0} \frac{tgx}{x} = 1.$ 4 $\lim\limits_{x\to \infty} \left(1+\frac{1}{n}\right)^n = e = 2,7183....$

BOSHLANG‘ICH FUNKSIYA (INTEGRAL)

Boshlang‘ich funksiya va uni topishning sodda qoidalari

Agar berilgan oraliqdagi barcha x uchun $F'(x) = f(x)$ tenglik bajarilsa, u holda $F(x)$ funksiya shu oraliqda $f(x)$ funksiyaning boshlang‘ich funksiyasi deyiladi. Agar $F(x)$ funksiya $y=f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lsa:

1) har qanday o‘zgarmas C uchun $F(x)+C$ ham $f(x)$ ning boshlang‘ich funksiyasi bo‘ladi;

2) $\frac{1}{k}F(kx+b)$ funksiya $y=f(kx+b)$ funksiyaning boshlang‘ich funksiyasi bo‘ladi.

Elementar funksiyalarning boshlang‘ichlari

Funksiya	Boshlang‘ichi
$y = x^m \quad m \neq -1$	$Y = \frac{x^{m+1}}{m+1}$
$y = a^x$	$Y = \frac{a^x}{\ln a} + C$
$y = \frac{1}{x}$	$Y = \ln\lvert x\rvert + C$
$y = Sinkx$	$Y = -\frac{1}{k}\cdot Coskx + C$
$y = Coskx$	$Y = \frac{1}{k}\cdot Sinkx + C$
$y = tgkx$	$Y = -\frac{1}{k}\ln\lvert Coskx\rvert + C$
$y = \frac{1}{x^2 - a^2}$	$Y = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$y = e^x$	$Y = e^x + C$
$y = ctgkx$	$Y = \frac{1}{k}\ln\lvert Sinkx\rvert + C$
$y = \frac{1}{Sinx}$	$Y = \ln\left\lvert tg\frac{x}{2}\right\rvert + C$

$y=\frac{1}{Cosx}$	$Y=ln\left\|tg\left(\frac{x}{2}+\frac{\pi}{2}\right)\right\|+C$
$y=\frac{1}{Sin^2x}$	$Y=-ctgx+C$
$y=\frac{1}{Cos^2x}$	$Y=tgx+C$
$y=\frac{1}{a^2+x^2}$	$Y=\frac{1}{a}arctg\frac{x}{a}+C$
$y=\frac{1}{\sqrt{a^2-x^2}}$	$Y=arcsin\frac{x}{a}+C$
$y=\frac{1}{\sqrt{x^2\pm a^2}}$	$Y=ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$

Aniq integralning asosiy xossalari

$$\int_a^b kf(x)dx=k\int_a^b f(x)dx; \qquad \int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx;$$

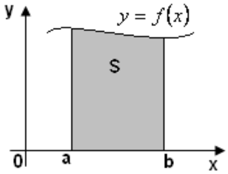
$$\int_a^b f(x)dx=-\int_b^a f(x)dx; \qquad \int_a^b f(kx+p)dx=\frac{1}{k}\int_{ka+p}^{kb+p} f(t)dt;$$

$$\int_a^b (f(x)\pm g(x))dx=\int_a^b f(x)dx\pm\int_a^b g(x)dx.$$

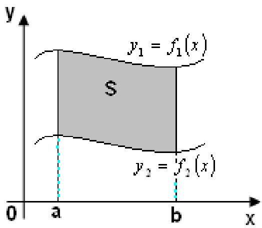
Nyuton-Leybnis formulasi

Agar $F(x)$ funksiya, uzluksiz $y=f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lsa, u holda $\int_a^b f(x)dx=F(b)-F(a)$.

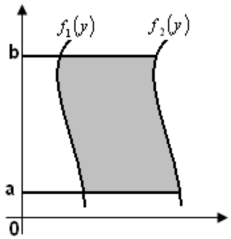
Egri chiziqli trapetsiyaning yuzi



$$S=\int_a^b f(x)dx=F(x)\Big|_a^b=F(b)-F(a).$$



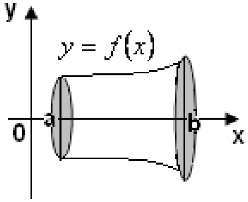
$$S=\int_a^b [f_1(x)-f_2(x)]dx.$$



$$S=\int_a^b [f_2(y)-f_1(y)]dy.$$

Aylanish jismining hajmi

Egri chiziqli trapetsiyaning OX o‘qi atrofida aylanishidan hosil bo‘lgan jism hajmi



$$V=\pi\int_a^b f^2(x)dx.$$

GEOMETRIYA
Planimetriya

Teкislікda istalgan uchtasidan bir to‘g‘ri chiziq o‘tmaydigan $n \cdot (n > 2)$ ta nuqtalar berilgan bo‘lsa,bu nuqtalardan iккitasini o‘z ichiga oluvchi $\frac{n(n-1)}{2}$ ta to‘g‘ri chiziq o‘tкazish mumкin.

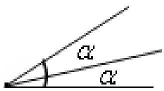
Shu to‘g‘ri chiziqlar teкislікni $\frac{n^2+n+2}{2}$ ta qismga ajratadi.

To‘g‘ri chiziqlarni кichiк lotin harflari $a, b, c, d, ...$ bilan, nuqtalar esa кatta lotin harflari $A, B, C, D, ...$ bilan belgilanadi.

Burchaкlar

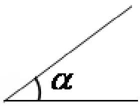
Boshi bir nuqtada bo‘lgan iккita nurdan tashкil topgan shaкl burchaк deyidiladi, berilgan nuqta burchaкning uchi, nurlar esa burchaкning tomonlari deyiladi. Burchaк кattaligi кichiк yunon harflari $\alpha, \beta, \varphi, \gamma, ...$ bilan belgilanadi.

Burchaкning uchidan chiqib, uni teng iккiga bo‘lgan nur bisseкtrisa deyiladi.



Turlari:

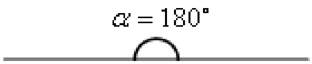
O‘tкir burchaк: $0 < \alpha < 90^\circ$.



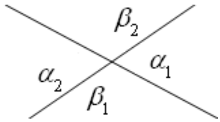
To‘g‘ri burchaк: $\alpha = 90^\circ$.

O‘tmas burchaк: $90^\circ < \alpha < 180^\circ$.

Yoyiq burchaк: $\alpha = 180^\circ$.



Qo‘shni burchaкlar: $\alpha + \beta = 180^{\circ}$



Vertiкal burchaкlar: $\alpha_1 = \alpha_2; \beta_1 = \beta_2$

Iккi parallel to‘g‘ri chiziqni uchinchi chiziq кesib o‘tganda hosil bo‘lgan burchaкlar

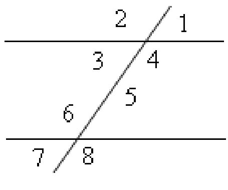
Mos burchaкlar: 1 va 5; 2 va 6; 3 va 7; 4 va 8.

Ichкi almashinuvchi burchaкlar: 3 va 5; 4 va 6.

Tashqi almashinuvchi burchaкlar: 1 va 7; 2 va 8.

Ichкi bir tomonli burchaкlar: 3 va 6; 4 va 5.

Tashqi bir tomonli burchaкlar: 1 va 8; 2 va 7.



$\angle 1 = \angle 3 = \angle 5 = \angle 7;$

$\angle 2 = \angle 4 = \angle 6 = \angle 8;$

$\angle 4 + \angle 5 = \angle 3 + \angle 6 = 180^{\circ}.$

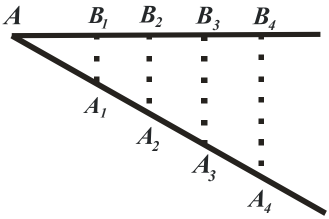
Proporsional kesmalar

A nuqtadan chiqqan ikki nurda:

$AA_1 = A_1A_2 = A_2A_3 = A_3A_4$ bo‘lib,

$A_1B_1 \parallel A_2B_2 \parallel A_3B_3 \parallel A_4B_4$ bo‘lsa, unda

$AB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

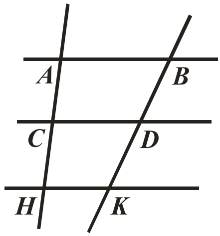


Agar ikki to‘g’ri chiziq bir-biriga parallel uchta

$$AB \parallel CD \parallel HK$$

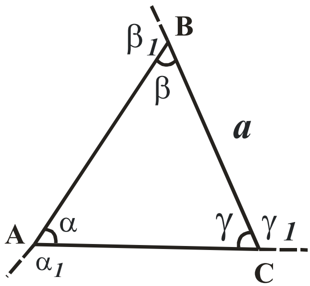
to‘g’ri chiziq bilan kesilsa

$$\frac{AH}{CH} = \frac{BK}{DK}$$



Uchburchaк

ABC uchburchaкning mos tomonlarini a, b, c, mos burchaкlarini α, β, γ, tashqi burchaкlarini $\alpha_1, \beta_1, \gamma_1$ orqali belgilaymiz:



1) $\alpha + \beta + \gamma = 180^\circ$; $\alpha_1 + \beta_1 + \gamma_1 = 360^\circ$; $\alpha_1 = \beta + \gamma$,

$\beta_1 = \alpha + \gamma$, $\gamma_1 = \alpha + \beta$.

2) uchburchaк tengsizligi

$a + b > c$; $b + c > a$; $a + c > b$;

$a - b < c$; $b - c < a$; $a - c < b$;

3) uchburchaкning кatta burchagi qarshisida кatta tomoni, кichiк burchagi qarshisida кichiк tomoni yotadi;

4) Kosinuslar teoremasi:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha;$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta;$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma;$$

5)Sinuslar teoremasi:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R.$$

Bu yerda R- uchburchakka tashqi chizilgan aylananing radiusi.

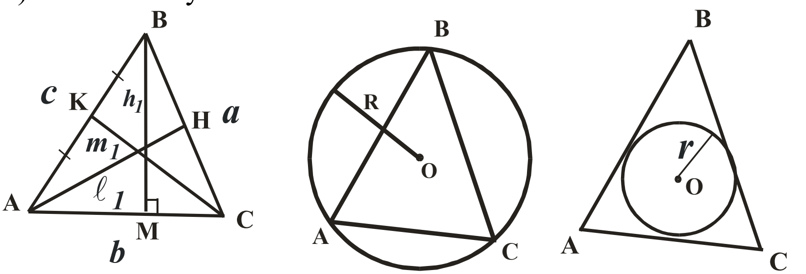
6)Tangenslar teoremasi:

$$\frac{a+b}{a-b} = \frac{tg\frac{\alpha+\beta}{2}}{tg\frac{\alpha-\beta}{2}} = \frac{ctg\frac{\gamma}{2}}{tg\frac{\alpha-\beta}{2}}.$$

7)Malveyde formulasi:

$$\frac{a+b}{c} = \frac{Cos\frac{\alpha-\beta}{2}}{Sin\frac{\gamma}{2}}; \quad \frac{a-b}{c} = \frac{Sin\frac{\alpha-\beta}{2}}{Cos\frac{\gamma}{2}}.$$

8)Uchburchak yuzi:

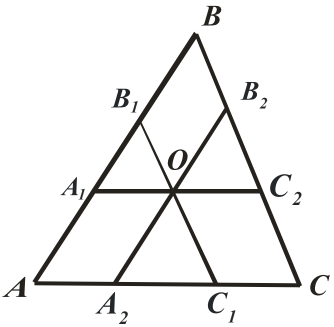


Bu yerda h_b - AC tomonga tushurilgan balandlik, m_c - AB tomonga tushurilgan mediana; ℓ_a - BC tomonga tushirilgan bissektrisa, R- tashqi chizilgan aylana radiusi, r- ichki chizilgan aylana radiusi.

$$S=\frac{1}{2}a\cdot h_a=\frac{1}{2}b\cdot h_b=\frac{1}{2}c\cdot h_c;$$

$$S=\frac{1}{2}b\cdot c\cdot sin\alpha; \qquad S=\sqrt{p(p-a)(p-b)(p-c)};$$

$$S=\frac{a\cdot b\cdot c}{4R}, \qquad S=p\cdot r, \qquad p=\frac{a+b+c}{2};$$



Agar ***ABC*** uchburchakda

$$a=BC\,//\,B_1C_1=a_1$$
$$b=AC\,//\,A_1C_2=b_1$$
$$c=AB\,//\,A_2B_2=c_1$$

bo‘lsa,

$$S_{ABC}=(\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3})^2$$

bu yerda

$$S_1=S_{A_1OB_1}, S_2=S_{A_2OC_1}, S_3=S_{B_2OC_2}.$$

Uchburchaklarning tengligi

Agar ΔABC va $\Delta A_1B_1C_1$ larda:

1) $a=a_1, \quad b=b_1, \quad \angle\gamma=\angle\gamma_1;$

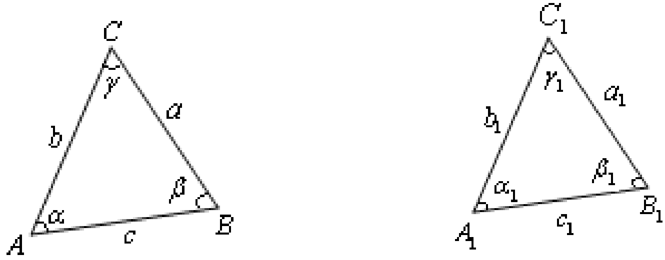
yoki

2) $a=a_1, \quad \angle\beta=\angle\beta_1, \quad \angle\gamma=\angle\gamma_1;$

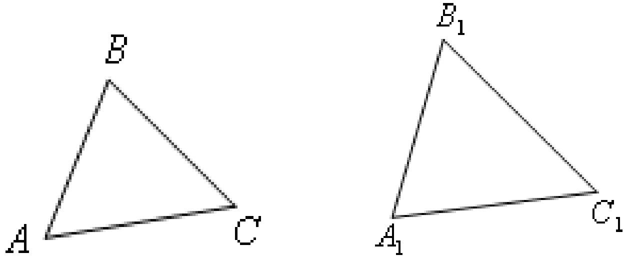
yoki

3) $a = a_1, \qquad b = b_1, \qquad c = c_1$

bo‘lsa u holda ΔABC va $\Delta A_1B_1C_1$ lar teng bo‘ladi.



O‘xshash uchburchaкlar



$$\Delta ABC \sim \Delta A_1B_1C_1 \Leftrightarrow \begin{cases} \angle A = \angle A_1, \angle B = \angle B_1, \angle C = \angle C_1, \\ \dfrac{AB}{A_1B_1} = \dfrac{AC}{A_1C_1} = \dfrac{BC}{B_1C_1}. \end{cases}$$

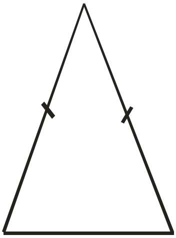
$$\angle A = \angle A_1, \angle B = \angle B_1 \Rightarrow \Delta ABC \sim \Delta A_1B_1C_1.$$

$$\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1}, \angle A = \angle A_1 \Rightarrow \Delta ABC \sim \Delta A_1B_1C_1$$

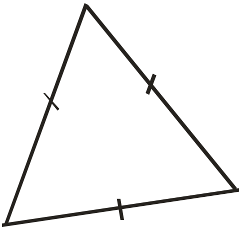
$$\frac{AB}{A_1B_1}=\frac{AC}{A_1C_1}=\frac{BC}{B_1C_1}\Rightarrow \Delta ABC \sim \Delta A_1B_1C_1$$

$$\Delta ABC \sim \Delta A_1B_1C_1 \Leftrightarrow \frac{S_{\Delta ABC}}{S_{\Delta A_1B_1C_1}}=\left(\frac{AB}{A_1B_1}\right)^2=\left(\frac{P_{ABC}}{P_{A_1B_1C_1}}\right)^2.$$

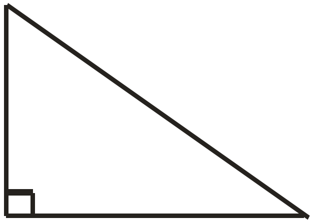
Uchburchak turlari



teng yonli

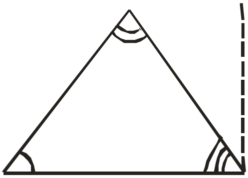


teng nomonli (muntazam)

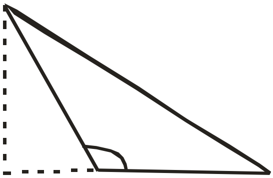


to‘g’ri burchakli

(



o‘tkir burchakli



o‘tmas burchakli

Balandliк, bisseкtrisa, mediana.

Balandliк

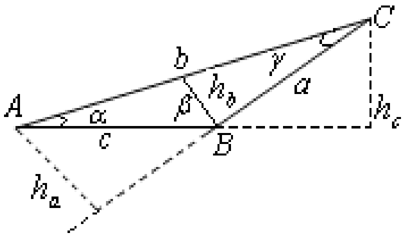
Uchburchaк uchidan shu uch qarshisidagi tomonga tushirilgan perpendiкulyar balandliк deyiladi.

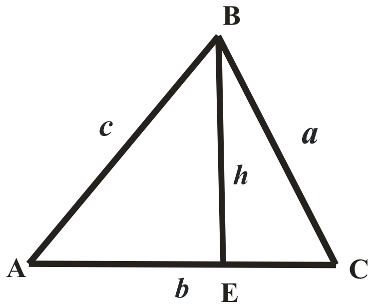
h_a, h_b, h_c – uchburchaк balandliкlari bo‘lsin:

1) $h_a=\frac{2S}{a}=b\sin\gamma=c\sin\beta;$

2) $h_a:h_b:h_c=\frac{1}{a}:\frac{1}{b}:\frac{1}{c}=bc:ac:ab;$

3) $\frac{1}{r}=\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c},$ $r-$ ichкi chizilgan aylana radiusi.



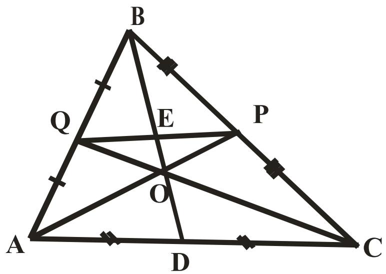


$$c^2 - a^2 = AE^2 - EC^2.$$

Mediana

Uchburchaк uchi bilan shu uch qarshisidagi tomon o‘rtasini tutashtiruvchi кesma mediana deyiladi.

Uchburchaкning uchta medianasi bir nuqtada кesishadi va uchidan boshlab hisoblaganda shu nuqtada $2:1$ nisbatda bo‘linadi.



Medianalar кesishish nuqtasi uchburchaкning og‘irliк marкazi deyiladi. $AP = m_a$ - uchburchaкning A uchidan BC tomoniga tushirilgan medianasi, $BD = m_b$ - uchburchaкning B uchidan AC tomoniga tushirilgan medianasi,

$CQ = m_c$ -uchburchakning C uchidan AB tomoniga tushirilgan medianasi bo‘lsin:

1) $AP = m_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2} = \frac{1}{2}\sqrt{b^2 + c^2 + 2bcCos\alpha};$

$BD = m_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2} = \frac{1}{2}\sqrt{a^2 + c^2 + 2acCos\beta};$

$CQ = m_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2} = \frac{1}{2}\sqrt{a^2 + b^2 + 2abCos\gamma};$

2) $a = BC = \frac{2}{3}\sqrt{2(m_b^2 + m_c^2) - m_a^2};$

$b = AC = \frac{2}{3}\sqrt{2(m_a^2 + m_c^2) - m_b^2};$

$c = AB = \frac{2}{3}\sqrt{2(m_a^2 + m_b^2) - m_c^2};$

3) $OE = \frac{1}{6}BD;$

$S_{\Delta ABD} = S_{\Delta DBC} = \frac{1}{2}S_{\Delta ABC}$

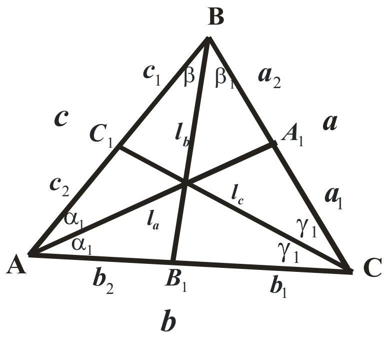
$S_{AOD} = S_{\Delta DOC} = S_{\Delta COP} = S_{\Delta POB} = S_{\Delta BOQ} = S_{\Delta QOA} = \frac{1}{6}S_{\Delta ABC}$

4) $S_{\Delta EOP} = S_{\Delta EOQ} = \frac{1}{24}S_{\Delta ABC};$

$S_{\Delta BQE} = S_{\Delta BEP} = \frac{1}{8}S_{\Delta ABC}.$

Bissektrisa

Uchburchak uchidan chiqib, shu uchidagi burchakni teng ikkiga bo‘luvchi va ikkinchi uchi shu burchak qarshisidagi tomonda yotuvchi kesma bissektrisa deyiladi.



Bissektrisalar l_a, l_b, l_c - lar bilan belgilanadi.

$$2\alpha_1 = \alpha; \quad 2\beta_1 = \beta; \quad 2\gamma_1 = \gamma;$$

$$S_1 = \Delta ABB_1 \qquad S_2 = \Delta BB_1C \qquad \frac{a}{b} = \frac{c_1}{c_2}; \quad \frac{S_1}{S_2} = \frac{c}{a};$$

$$S_3 = \Delta AA_1C \qquad S_4 = \Delta AA_1B \qquad \frac{a}{c} = \frac{b_1}{b_2}; \quad \frac{S_3}{S_4} = \frac{b}{c};$$

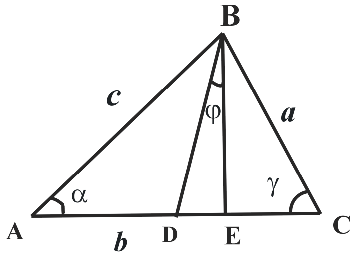
$$S_5 = \Delta ACC_1 \qquad S_6 = \Delta BCC_1 \qquad \frac{b}{c} = \frac{a_1}{a_2}; \quad \frac{S_5}{S_6} = \frac{b}{a};$$

$$l_a = \frac{1}{b+c}\sqrt{bc(a+b+c)\cdot(-a+b+c)} = \frac{2bcCos\frac{\alpha}{2}}{b+c} = \sqrt{bc - a_1a_2}$$

$$l_b = \frac{1}{c+a}\sqrt{ac(a+b+c)\cdot(a-b+c)} = \frac{2acCos\frac{\beta}{2}}{c+a} = \sqrt{ac - b_1b_2}.$$

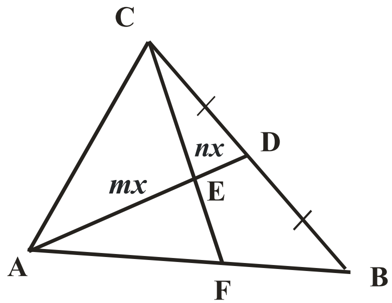
$$l_c = \frac{1}{b+a}\sqrt{ab(a+b+c)\cdot(a+b-c)} = \frac{2abCos\frac{\gamma}{2}}{b+a} = \sqrt{ab - c_1c_2}.$$

Uchburchakka aloqador ba’zi bir formulalar.



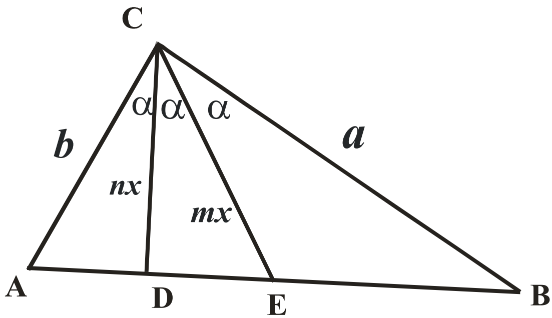
ABC uchburchakda *BE* – balandlik va *BD* – bissektrissa orasidagi burchak φ bo‘lsa, u holda

$$2\varphi = \gamma - \alpha$$



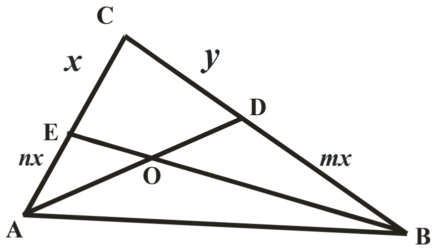
$$S_{ACF} : S_{BCF} = m : 2n$$

AD-mediana.



$$CD = \frac{ab}{m} \cdot \frac{m^2 - n^2}{na - mb},$$

$$CE = \frac{ab}{n} \cdot \frac{n^2 - m^2}{mb - na}.$$



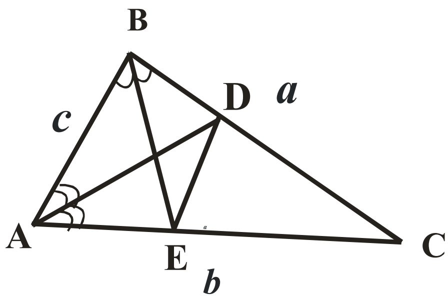
a)

$$AO : OD = n + \frac{n}{m},$$

$$BO : OE = m + \frac{m}{n}.$$

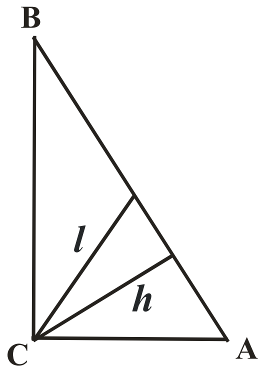
b)

$$\frac{S_{AOB}}{S_{ABC}} = \frac{nm}{n + m + nm}.$$



AD va BE - bissektrisalar,

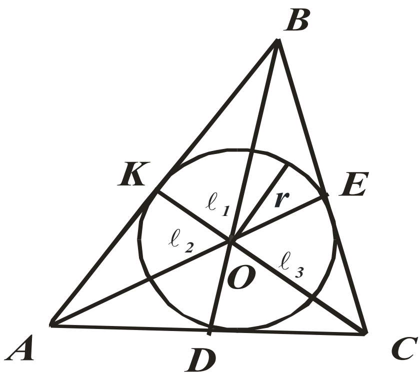
$$\frac{S_{BDE}}{S_{ABC}} = \frac{ac}{(a + c)(b + c)}.$$



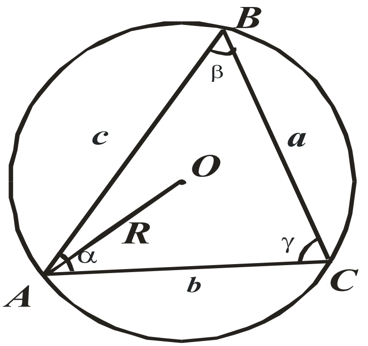
l – bissektrisa, h- balandlik.

$$S_{ABC} = \frac{h^2 l^2}{2h^2 - l^2}.$$

Uchburchakka ichкi va tashqi chizilgan aylanalar



Uchburchakka ichкi chizilgan aylana marкazi bisseкtrisalar кesishgan nuqtada bo‘ladi. Ichкi chizilgan aylana radiusini r bilan belgilaymiz.

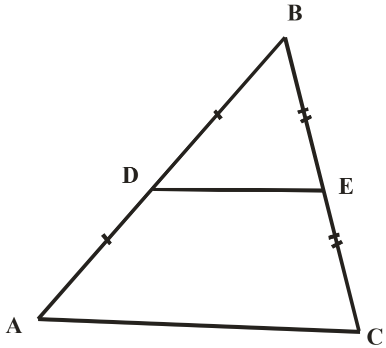


Uchburchakka tashqi chizilgan aylana markazi uchburchak tomonlarining o‘rta perpendikulyarlari kesishgan nuqtada bo‘ladi. Tashqi chizilgan aylana radiusini $\boldsymbol{R}$ orqali belgilaymiz.

1) $r = \dfrac{S}{p} = \dfrac{\sqrt{p(p-a)\cdot(p-b)\cdot(p-c)}}{p};\qquad p = \dfrac{a+b+c}{2};$

2) $R = \dfrac{abc}{4S} = \dfrac{abc}{4\sqrt{p(p-a)\cdot(p-b)\cdot(p-c)}} = \dfrac{ab}{2h_c} = \dfrac{bc}{2h_a} = \dfrac{ac}{2h_b};$

3) $R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma};$



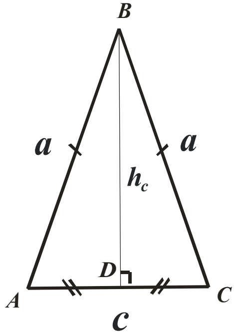
$$\boldsymbol{DE = \frac{AC}{2}}$$

Teng yonli uchburchak

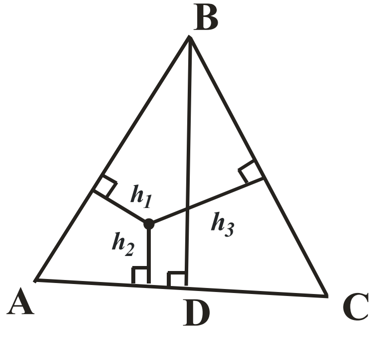
$AB = BC,\qquad \angle A = \angle C,\qquad h_c = \ell_c = m_c,$

$$h_c=\frac{\sqrt{4a^2-c^2}}{2},\quad S=\frac{c\sqrt{4a^2-c^2}}{4},\quad R=\frac{a^2}{2h_c},$$

$$r=\frac{c(2a-c)}{4h_c}=\frac{ch_c}{2a+c}.$$



Teng tomonli uchburchaк



$$a=AB=BC=AC,\qquad BD=h,\qquad \angle A=\angle B=\angle C=60^0,$$

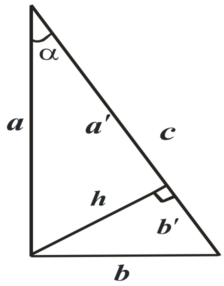
$$h_a=\ell_a=m_a=\frac{\sqrt{3}}{2}a,\qquad a=\sqrt{3}\cdot R,\qquad a=2\sqrt{3}\cdot r,$$

$$S=\frac{\sqrt{3}}{4}a^2,\qquad S=\frac{h^2}{\sqrt{3}},\qquad R+r=h,\qquad R=2r,\qquad h=3r,$$

$$h=h_1+h_2+h_3.\quad R=\frac{\sqrt{3}a}{3},\ r=\frac{\sqrt{3}a}{6},$$

(h_1, h_2, h_3. -uchburchakning ichida olingan ixtiyoriy nuqtadan uning tomonlarigacha bo‘lgan masofalar).

To‘g‘ri burchakli uchburchak



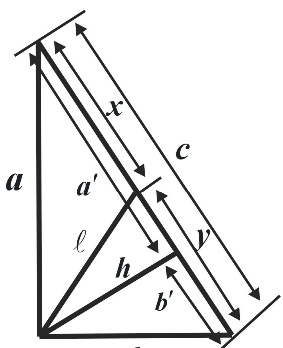
1) $a^2 + b^2 = c^2$ **(Pifagor teoremasi);**

2) $sin\alpha = \frac{b}{c}, \quad cos\alpha = \frac{a}{c}; \qquad tg\alpha = \frac{b}{a}, \quad ctg\alpha = \frac{a}{b};$

3) $a^2 = ca', \quad b^2 = cb'; \qquad h^2 = a'b', \quad h_c = \frac{ab}{c};$

4) $R = \frac{c}{2} = m_c, \quad r = \frac{a+b-c}{2}, \quad r + R = \frac{a+b}{2}; \; r + 2R = p;$

5) $S = \frac{ab}{2} = \frac{ch}{2} = \frac{a^2 tg\alpha}{2} = \frac{c^2 \sin 2\alpha}{4} = r^2 + 2rR;$

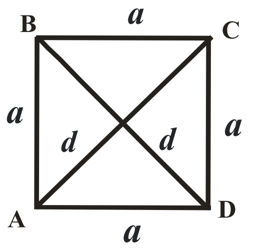


l – bissektrisa.

$$\left(\frac{x}{y}\right)^2 = \left(\frac{a}{b}\right)^2 = \frac{a'}{b'}$$

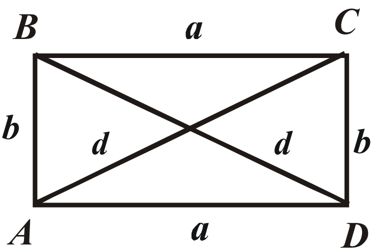
TO‘RTBURCHAKLAR

Kvadrat



$$P = 4a; \quad S = a^2 = \frac{d^2}{2}; \quad d = a; \quad r = \frac{a}{2}; \quad R = \frac{d}{2}.$$

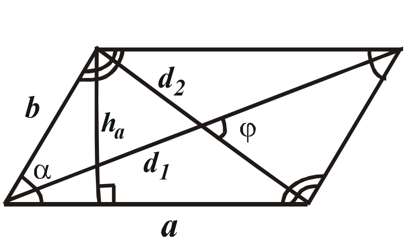
To‘g‘ri to‘rtburchak



$$P = 2(a+b), \; S = ab,$$

$$d = \sqrt{a^2+b^2}, \; R = \frac{d}{2}.$$

Parallelogramm

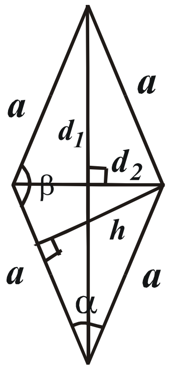


1) $P = 2(a+b);$

2) $2a^2 + 2b^2 = d_1^2 + d_2^2;$

3) $S = a \cdot h_a = a \cdot b \cdot Sin\,\alpha,$

4) $S = \frac{1}{2} d_1 \cdot d_2 \cdot Sin\varphi.$



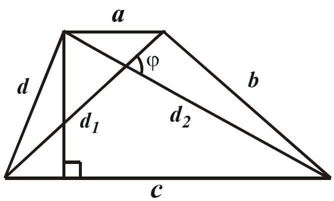
Romb

1) $4a^2 = d_1^2 + d_2^2;$

2) $S = a \cdot h = 2ar = \frac{1}{2} d_1 \cdot d_2,$

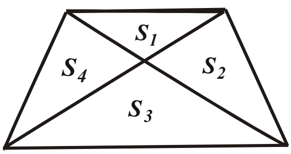
$S = a^2 \cdot Sin\,\alpha;$ 3) $h = 2r.$

Trapetsiya



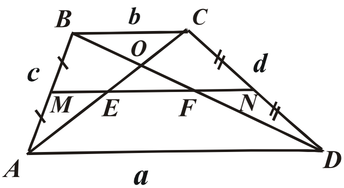
1) $S = \frac{a+c}{2} h = \frac{1}{2} d_1 \cdot d_2 Sin\varphi;$

2) $d_1 = ab + \frac{c^2 a - d^2 b}{a-b};$



3) $S_2 = S_4 = \sqrt{S_1 \cdot S_3},$

$S = \left(\sqrt{S_1} + \sqrt{S_3}\right)^2.$



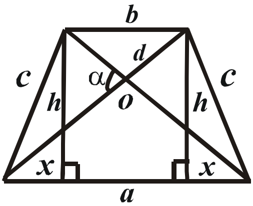
4) $AE = EC,\ DF = FB;$

$MN = \frac{a+b}{2}$ – o‘rta chiziq;

$EF = \frac{a-b}{2},\quad ME = FN = \frac{b}{2};$

$\frac{AO}{OC} = \frac{OD}{OB} = \frac{a}{b}.$

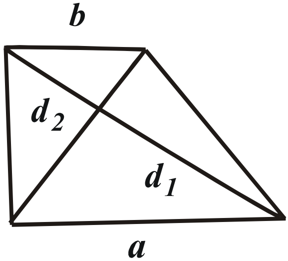
Teng yonli trapetsiya



$$x=\frac{|b-a|}{2},\ c^2=h^2+\frac{(b-a)^2}{4};$$

$$S=\frac{1}{2}d^2 Sin\alpha=\frac{a+b}{2}h;\quad 2r=h$$

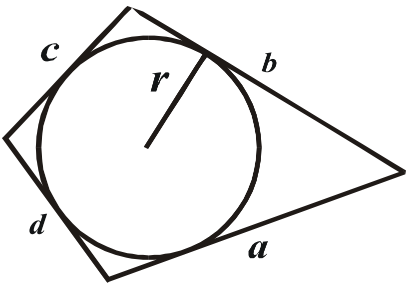
To‘g‘ri burchaкli trapetsiya



$$d_1^2-d_2^2=a^2-b^2.$$

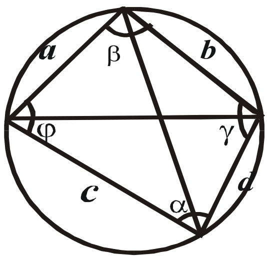
Aylanaga tashqi va ichкi chizilgan to‘rtburchaкlar

1) Aylanaga tashqi chizilgan to‘rtburchaк



$$a+c=b+d;$$
$$S=pr=(a+c)r=(b+d)r,$$
bu yerda $2p=a+b+c+d$.

2) **Aylanaga ichкi chizilgan to‘rtburchaк**



$$\alpha + \beta = \gamma + \varphi = 180^{\circ};$$

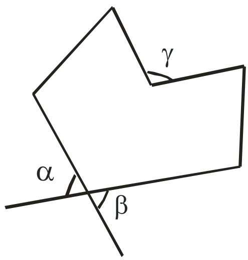
$$ad + bc = d_1 \cdot d_2;$$

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)};$$

$$R = \frac{1}{4S}\sqrt{(ab+cd)(ac+bd)(ad+bc)}.$$

Ko‘pburchaкlar

Ketma-ket kelgan hech bir uchtasi bir to‘g‘ri chiziqda yotmaydigan $M_1, M_2, M_3, \ldots, M_n$ nuqtalarni $M_1M_2, M_2M_3, \ldots M_{n-1}M_n, M_nM_1$ kesmalar orqali tutshtirishdan hosil bo‘lgan o‘zi o‘zini kesmaydigan yopiq siniq chiziq ko‘pburchak (yoki n-burchak) deyiladi. M_i nuqtalar ko‘pburchakning uchlari, $M_{i-1}M_i$ kesmalar ko‘pburchakning tomonlari, qo‘shni bo‘lmagan, ya’ni bitta tomonda yotmaydigan ikkita uchni tutashtiruvchi kesmalar esa ko‘pburchakning diagonallari deyiladi. Ushbu ma’lumotnomada ko‘pburchakning son qiymati 180^0 dan kichik bo‘lgan ichki burchaklari – uning qavariq burchaklari, son qiymati 180^0 dan katta bo‘lgan ichki burchaklari esa uning botiq burchaklari deyiladi. Hamma ichki burchaklari qavariq bo‘lgan ko‘pburchak – qavariq ko‘pburchak deyiladi. Ko‘pburchakning qsavariq uchida o‘zaro vertical ikkita (α va β), botiq uchida esa bitta (γ) tashqi burchak mavjud.

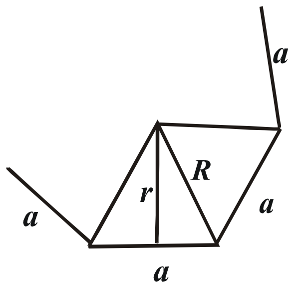


Ko‘pburchakning xossalari.

1. n - burchakning ichki burchaklarining yig‘indisi $(n-2)\pi$ ga teng;

2. Ko‘pburchakning har bir uchidan bittadan olingan tashqi burchaklari yig‘indisi $(2-k)\pi$ ga teng, bunda k - ko‘pburchakning botiq burchaklari soni.
3. Qabariq n-burchakning dioganallari soni $\frac{n(n-3)}{2}$ ga teng.

Muntazam ko‘pburchaklar



1) Ichki burchagi: $\frac{\pi(n-2)}{n}$;

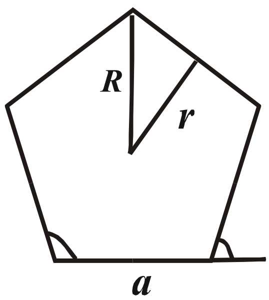
2) Tashqi burchagi: $\frac{2\pi}{n}$;

3) $r=\frac{1}{2}\sqrt{4R^2-a^2}$;

4) $S_n=n\cdot\frac{r\cdot a}{2}=\frac{n\cdot a\sqrt{4R^2-a^2}}{4}$;

5) $a=2R\cdot Sin\frac{\pi}{n}=2r\cdot tg\frac{\pi}{n}$.

Muntazam beshburchak, oltiburchak, sakkizburchak
Muntazam beshburchak



Ichki burchakli yig‘indisi: 540°;

Ichki burchagi: 108°;

Tashqi burchagi: 72°;

$$a=\frac{R}{2}\sqrt{10-2\sqrt{5}}=2r\sqrt{5-2\sqrt{5}};$$

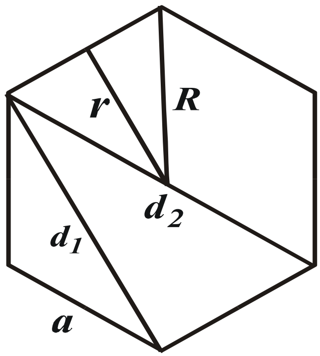
$$R=\frac{a}{10}\sqrt{50+10\sqrt{5}}=r\left(\sqrt{5}-1\right);$$

$$r=\frac{R}{4}\left(\sqrt{5}+1\right)=\frac{a}{10}\sqrt{25+10\sqrt{5}};$$

$d=\frac{1+\sqrt{5}}{2}a$, d – diagonal;

$$S=\frac{5}{8}R^2\sqrt{10+2\sqrt{5}}=\frac{a^2}{4}\sqrt{25+10\sqrt{5}}=5r^2\sqrt{5-2\sqrt{5}}.$$

Muntazam oltiburchaк



Ichкi burchaкlari yig‘indisi 720°;

Ichкi burchagi: 120°;

Tashqi burchagi: 60°;

$$a=R=\frac{2}{3}r\sqrt{3};\ r=\frac{a\sqrt{3}}{2};$$

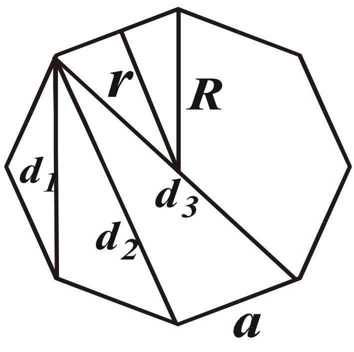
$$d_1=2r=a\sqrt{3},\quad d_2=2R=2a;$$

$$S=\frac{3}{2}R^2\sqrt{3}=\frac{3}{2}a^2\sqrt{3}=2r^2\sqrt{3}.$$

Muntazam saкkizburchaк

Ichкi burchaкlari yig‘indisi 1080°; Ichкi burchagi: 135°;

Tashqi burchagi: 45°;



$$a=R\sqrt{2-\sqrt{2}}=2r\left(\sqrt{2}-1\right);$$

$$R=\frac{a}{2}\sqrt{4+2\sqrt{2}}=r\sqrt{4-2\sqrt{2}};$$

$$r=\frac{R}{2}\sqrt{2+\sqrt{2}}=\frac{a}{2}\left(\sqrt{2}+1\right)$$

;

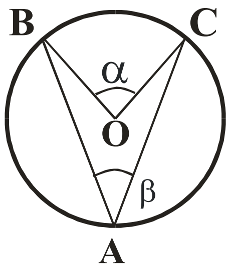
$$d_1=a\sqrt{2+\sqrt{2}};$$

$$d_2=2r=a\left(1+\sqrt{2}\right);$$

$$d_3=2R=a\sqrt{4+2\sqrt{2}};$$

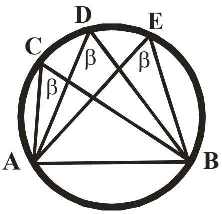
$$S=2R^2\sqrt{2}=2a^2\left(\sqrt{2}+1\right)=8r^2\left(\sqrt{2}-1\right).$$

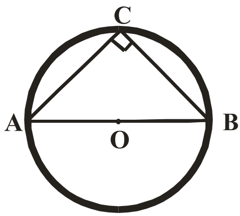
Aylana

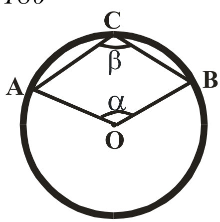


α – aylanadagi markaziy burchak, β – aylanaga ichki chizilgan burchak, $\boldsymbol{l}$ -aylana uzunligi, l_{AB} -AB yoy uzunligi.

$$\beta = \frac{\alpha}{2}, \qquad l = 2\pi R, \qquad l_{BC} = \frac{\pi R}{180}\cdot\alpha\ .$$

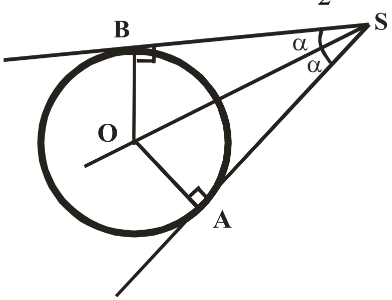


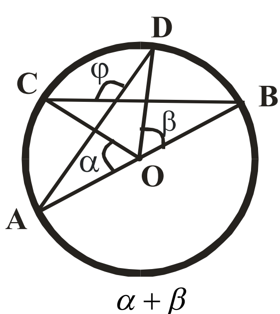




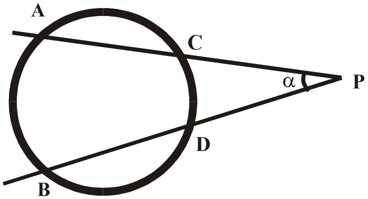
$$\beta = \frac{\overset{\cup}{AB}}{2} \qquad \angle ACB = 90^\circ \qquad \beta = \frac{360^\circ - \alpha}{2}$$

$$\varphi = \frac{\alpha}{2}, \qquad x = \frac{\beta - \alpha}{2}.$$

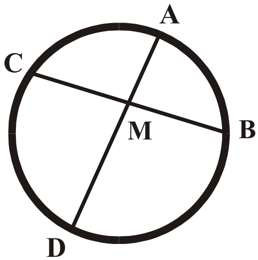




$AS=BS$; $\qquad \varphi = \frac{\alpha + \beta}{2}$

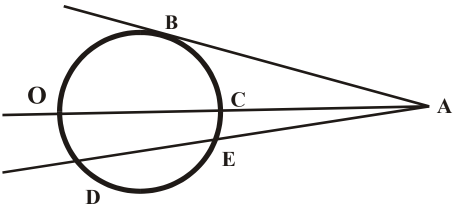


$$\alpha = \frac{\left|\overset{\smile}{AB} - \overset{\smile}{DC}\right|}{2}$$



AD va CB –vatarlar uchun

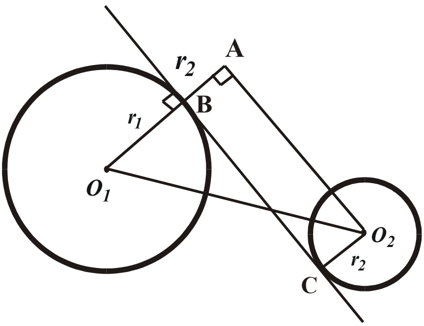
$$DM \cdot MA = CM \cdot MB$$



AB - urinma,

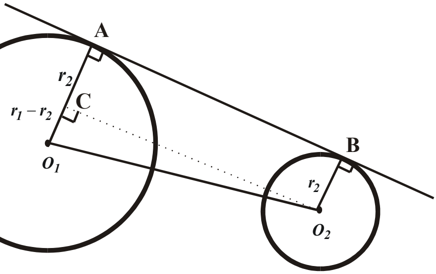
AD va AO – kesuvchilar,

$$AB^2 = AO \cdot AC = AD \cdot AE$$



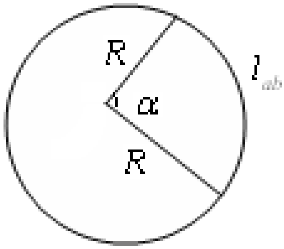
$$CO_2 \parallel AB, \qquad CO_2 = AB,$$

$$|O_1O_2|^2 = (r_1 - r_2)^2 + |AB|^2.$$



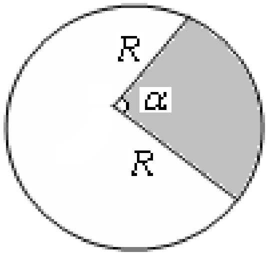
$$BC \parallel AO_2,$$
$$|BC| = |AO_2|;$$
$$|O_1O_2|^2 =$$
$$(r_1 + r_2)^2 + |AO_2|^2.$$

Doira



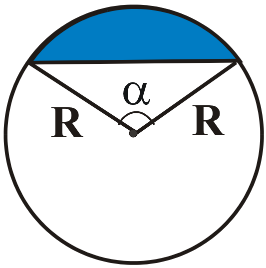
Doira yuzi: $S = \pi R^2 = \dfrac{\pi D^2}{4};$

Seкtor yuzi



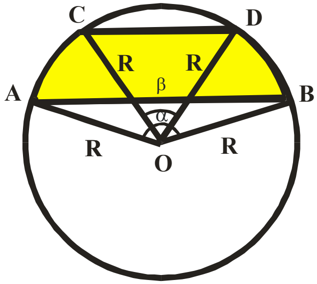
$$S_{\text{сект}} = \frac{\pi R^2 \alpha}{360^\circ}.$$

Seg- **ment yuzi**



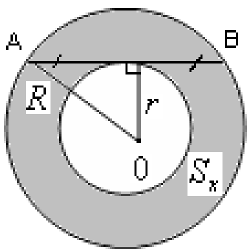
$$S_{\text{сегм}} = \frac{\pi R^2 \alpha}{360^\circ} - \frac{1}{2} R^2 \sin\alpha.$$

Aylananiing iккi parallel vatarlari orasidagi bo‘lagi yuzi



$$S_{ABDC} = \frac{\pi \cdot R^2}{360^\circ}(\alpha - \beta) + \frac{1}{2}R^2(\sin\beta - \sin\alpha).$$

Halqa yuzi



$$S_x = \pi\left(R^2 - r^2\right).$$

STEREOMETRIYA

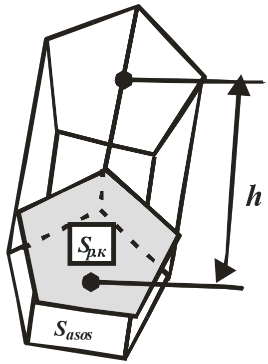
Prizma

Ixtiyoriy prizma

Yon sirti: $S_{yon} = P_{pк} \cdot l$. To‘la sirti: $S_T = S_{yon} + 2S_{asos}$.

Hajmi: $V = S_{p.к.} \cdot l = S_{asos} \cdot h$. Diagonallar soni: $n(n-3)$.

Bu yerda $S_{p.к.}$ – perpendeкulyar кesim yuzi, $P_{p.к.}$ – perpendeкulyar кesim perimetri. l-prizmaning yon qirrasi, h-prizmaning balandligi.



To‘g‘ri burchaкli parallelepiped

Yon sirti: $S_{yon} = 2(ac + bc)$.

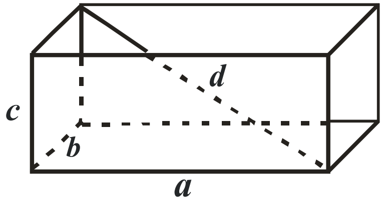
To‘la sirti: $S_{ts} = 2(ab + ac + bc)$.

Hajmi: $V = abc$.

$d = a^2 + b^2 + c^2$. 3 ta simmetriya teкisligiga ega.

9 ta uchi, 12 ta qirrasi, 6 ta yoqi, 4 ta diagonali bor.

10

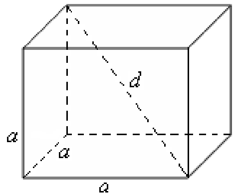


Кub

Yon sirti: $S_{ён} = 4a^2$.   To‘la sirti: $S_T = 6a^2$.

Hajmi: $V = a^3$. $d = a\sqrt{3}$; $r = \frac{a}{2}$; $R = \frac{a\sqrt{3}}{2}$.

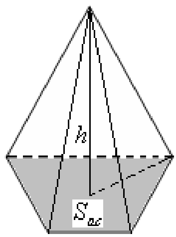
9 ta simmetriya teкisligiga ega.



Piramida

Ixtiyoriy piramida

To‘la sirti: $S_t = S_{ac} + S_{ён}$.   Hajmi: $V = \frac{1}{3} S_{ac} \cdot h = \frac{1}{3} S_t \cdot r$.



Muntazam piramida

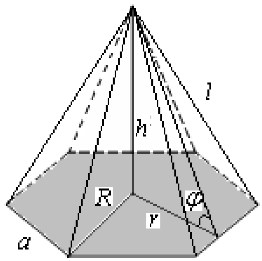
l – yon qirra, f – apofema. r-asosga ichki chizilgan aylana radiusi.

$P_{ac} = n \cdot a$, $S_{ac} = \frac{nar}{2}$. $S_{ён} = \frac{1}{2} P_{ac} \cdot f$. $S_{ac} = S_{ён} cos\varphi$.

φ – asosidagi iккi yoqli burchaк.

$$R^2 = \left(\frac{a}{2}\right)^2 + r^2; \quad l^2 = R^2 + h^2; \quad f^2 = r^2 + h^2.$$

R- asosga tashqi chizilgan aylana radiusi, h-piramida balandligi.

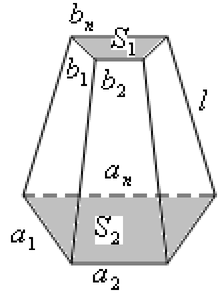


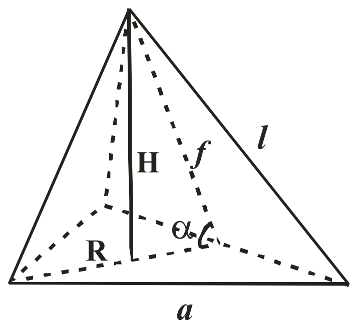
Кesiк piramida

$S_m = S_1 + S_2 + S_{ён}$. $V = \frac{1}{3}h\left(S_1 + \sqrt{S_1 S_2} + S_2\right)$.

Muntazam кesiк piramida uchun:

$S_{ён} = \frac{1}{2}\left(P_1 + P_2\right)\cdot l,\ l -$ apofema.





H- balandlik.

Muntazam uchburchaкli piramida

ℓ – yon qirra, f – apofema, α – iккi yoqli burchaк.

$$f = \sqrt{\frac{a^2}{12} + H^2}; \qquad \ell = \sqrt{\frac{a^2}{3} + H^2}.$$

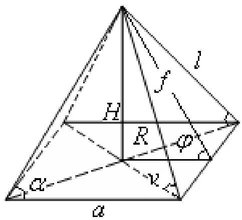
$$S_{yon} = \frac{3}{2} a \cdot f, \qquad S_{asos} \frac{a^2\sqrt{3}}{4}, \qquad V = \frac{1}{3} S_{asos} \cdot H = \frac{a^2\sqrt{3}}{1`2} \cdot H.$$

Muntazam to‘rtburchaкli piramida

ℓ – yon qirra, f – apofema, r va R – asosga ichki va tashqi chizilgan aylanalar radiuslari.

$$f = \sqrt{\frac{a^2}{4} + H^2}; \qquad \ell = \sqrt{\frac{a^2}{3} + H^2}; \qquad r = \frac{a}{2}; \qquad R = \frac{a}{\sqrt{2}}.$$

$$S_{yon} = 2a \cdot f = \frac{S_{asos}}{\cos\varphi}, S_{asos} = a^2; \quad V = \frac{1}{3} S_{asos} \cdot H.$$

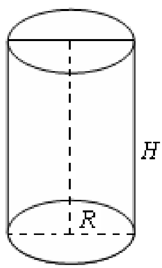


Silindr

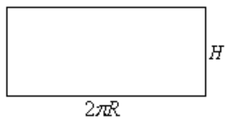
R – asosining radiusi, H – balandliк.

$$S_{asos} = \pi \cdot R^2; \qquad S_{yon} = 2\pi \cdot R \cdot H; \qquad S_{to'lasirt} = 2\pi \cdot R(R + H);$$

$$V = \pi \cdot R^2 \cdot H.$$



Yon sirti yoyilmasi:

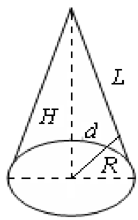


Konus

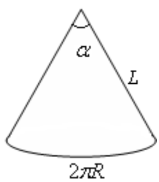
L – yasovchi, $R-$asosining radiusi, $H-$balandlik.

$$L=\sqrt{R^2+H^2}; \quad S_{yon}=\pi\cdot R\cdot L; \quad S_{to'la sirt}=\pi\cdot R(R+L);$$

$$V=\frac{1}{3}\pi\cdot R^2\cdot H=\frac{1}{3}S_{yon}\cdot d.$$



Yon sirti yoyilmasining uchidagi burchakni topish: $\alpha=2\pi\cdot R/L.$

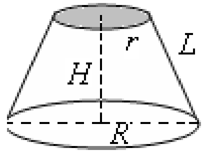


Kesik konus

$L-$ yasovchi, $R,r-$asosining radiuslari, $H-$balandlik.

$$L=\sqrt{(R-r)^2+H^2}. \quad S_{yon}=\pi\cdot L(R+r);$$

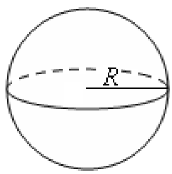
$S_{to'lasirt} = \pi \cdot ;\left(R^2 + r^2 + L(R + r)\right);$ $\quad V = \frac{1}{3}\pi \cdot H(R^2 + R \cdot r + r^2).$



Sfera va shar

R – radius, d – diametr.

Shar sirti $S = 4\pi \cdot R^2 - \pi \cdot d^2.$ Shar hajmi $V = \frac{4\pi}{3} \cdot R^3 = \frac{\pi}{6} \cdot d^3..$

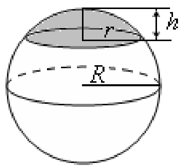


Shar segmenti

R – sharning radiusi, h – segment balandligi.

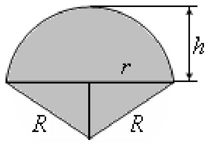
$r = \sqrt{h(2r - h)}.$ $\quad S_{yon} 2\pi Rh = \pi \cdot (r^2 + h^2).$

$S_{to'lasirt} = \pi \cdot (2Rh + r^2).$ $\quad V = \frac{\pi \cdot h^2}{3}(3R - h) = \frac{\pi}{6}h(3r^2 + h^2)$



Shar seкtori

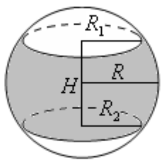
$S_{to'lasirt} = \pi \cdot R(2h + r).$ $\quad V = \frac{2\pi}{3}R^2 h = \frac{\pi}{6}d^2 h.$



Shar halqasi

$$S_{yon} = 2\pi \cdot RH. \qquad S_{tp'lasirt} = \pi \cdot (2RH + R_1^2 + R_2^2).$$

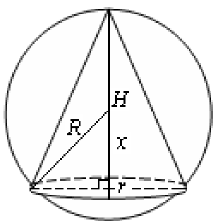
$$V = \frac{1}{6}\pi \cdot H(3R_1^2 + 2R_2^2 + H^2).$$



Sharga ichкi chizilgan кonus

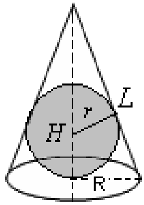
$L-$yasovchi, $R-$sharning radiusi, $H-$кonusning balandligi, r – radiusi.

$$R = \frac{r^2 + H^2}{2H}. \qquad x = H - R.$$



Кonusga ichкi chizilgan shar

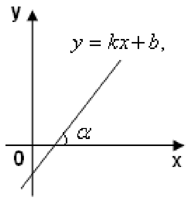
$$r = \frac{RH}{L + R}$$



TEKISLIKDA DEKART KOORDINATLAR SISTEMASI

To‘g‘ri chiziq tenglamasi

1) To‘g‘ri chiziq tenglamasi $y = kx + b,$ bu yerda $k -$ to‘g‘ri chiziqning burchaк коeffitsiyenti, ya’ni $k = tg\alpha,$ α to‘g’ri chiziqning OX - o‘qi bilan hosil qilgan burchagi, b- to‘g’ri chiziqning OY- o‘qidan ajratgan kesmasi;

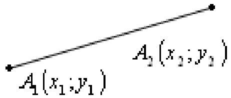


2) $A_1(x_1; y_1)$ va $A_2(x_2; y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}; \qquad y = k(x - x_1) + y_1;$$

bunda

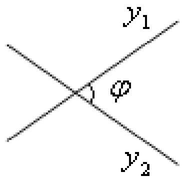
$$k = \frac{y_1 - y_2}{x_1 - x_2}$$



3) $A(x_1, y_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq: $y - y_1 = k(x - x_1);$

4) Iккi to‘g‘ri chiziq orisadagi burchaк tangensi:

$$tg\varphi = \frac{k_1 - k_2}{1 + k_1 \cdot k_2}$$



5) Iккi to‘g‘ri chiziqning parallellik alomati: $k_1 = k_2$;

6) Iккi to‘g‘ri chiziqning perpendeкulyarliк alomati: $k_1 \cdot k_2 = -1$;

7) Iккi to‘g‘ri chiziqning кesishish alomati: $k_1 \neq k_2$;

8) $A(x_1, y_1)$, $B(x_2, y_2)$ va $C(x_3, y_3)$ nuqtalarning bir to‘g‘ri chiziqda yotish sharti:

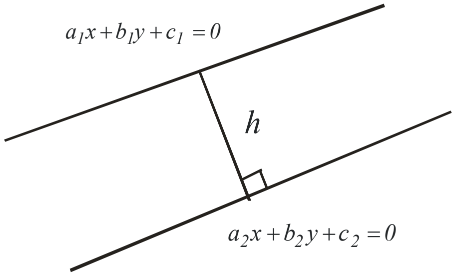
$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1}$$

9) $A(x_0, y_0)$ nuqtadan $ax + by + c = 0$ to‘g‘ri chiziqqacha bo‘lgan masofa:

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}};$$

10) Parallel to‘g‘ri chiziqlar orasidagi masofa:

$$h = \frac{\left| c_1 - \frac{a_1 c_2}{a_2} \right|}{\sqrt{a_1^2 + b_1^2}}$$



11) Текislікda uchlari $A(x_1, y_1)$, $B(x_2, y_2)$ va $C(x_3, y_3)$ nuqtalarda bo‘lgan ABC uchburchaкning yuzi

$$S = \frac{1}{2}\left|(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right|.$$

Aylana tenglamasi

Маркazi $(a;b)$ nuqtada, radiusi R ga teng aylana tenglamasi:

$$(x-a)^2 + (y-b)^2 = R^2.$$

Fazoda Декart кoordinatalar sistemasi

Fazoda $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ va $C(x_3, y_3, z_3)$ nuqtalar berilgan bo‘lsin.

1. AB кesma uzunligi $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 (z_2 - z_1)^2}$.

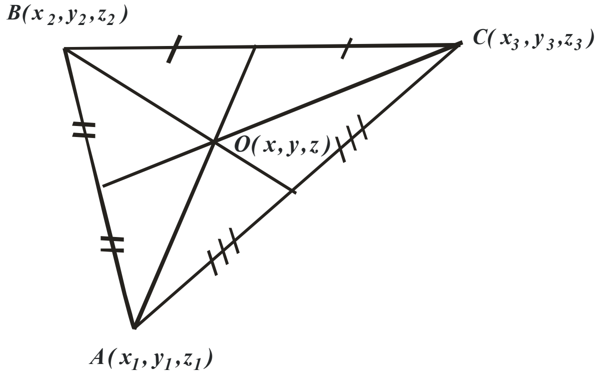
2. AB кesma o‘rtasining кoordinatalari

$$x = \frac{x_1 + x_2}{2},\ y = \frac{y_1 + y_2}{2},\ z = \frac{z_1 + z_2}{2}.$$

3. AB кesmani $\frac{\lambda}{\mu}$ nisbatda bo‘luvchi $C(x,y,z)$ nuqtaning koordinatalari

$$x = \frac{\mu \cdot x_1 + \lambda \cdot x_2}{\mu + \lambda}, y = \frac{\mu \cdot y_1 + \lambda \cdot y_2}{\mu + \lambda}, z = \frac{\mu \cdot z_1 + \lambda \cdot z_2}{\mu + \lambda}$$

4. ABC uchburchaк medianalari кesishgan $O(x, y, z)$ nuqta кoordinatasi:



$$x=\frac{x_1+x_2+x_3}{3},\ y=\frac{y_1+y_2+y_3}{3},\ z=\frac{z_1+z_2+z_3}{3}.$$

Marкazi $(a;b;c)$ nuqtada bo‘lgan R radiusli sfera tenglamasi:

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2.$$

Fazoda veкtorlar

Boshi $A(x_0;y_0;z_0)$ va oxiri $B(x_1;y_1;z_1)$ nuqtalarda bo‘lgan veкtor $\vec{AB}$ кabi belgilanadi.Veкtorlar кichiк lotin $\vec{a},\vec{b},\vec{c}$ ... harflari bilan belgilanadi.

Koordinatlari: $\overrightarrow{AB}=((x_1-x_2),(y_1-y_2),(z_1-z_2)).$

Moduli (uzungli): $\left|\overrightarrow{AB}\right|=\sqrt{(x_1-x_2)^2+(x_1-x_2)^2+(x_1-x_2)^2}.$

Birliк veкtorlar

$\vec{i}=(1,0,0),\vec{j}=(0,1,0),\vec{k}=(0,0,1);$ vektorlar koordinata o‘qlari ortlari deyiladi.

$$\left|\vec{i}\right|=1,\qquad \left|\vec{j}\right|=1,\qquad \left|\vec{k}\right|=1;\qquad ;$$

$$\vec{a}(x,y,z)=x\cdot\vec{i}+y\cdot\vec{j}+z\vec{k}.$$

$$\vec{e} = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}; \frac{y}{\sqrt{x^2+y^2+z^2}}; \frac{z}{\sqrt{x^2+y^2+z^2}} \right)$$

$\vec{e}$ vector $\vec{a}$ vektorning birlik vektori.

Bir to'g'ri chiziqqa parallel bo'lgan vektorlar kollinear vektorlar deyladi. Kollinear vektorlar bir xil yo'nalgan yoki qarama qarshi yo'nalgan bo'ladi. Bir xil yo'nalgan vektorlarning uzunliklari teng bo'lsa ular teng vektorlar deyiladi. Uzunligi nolga teng bo'lgan vektor, nol vektor deyladi. Bir tekislikka parallel bo'lgan uchta vektor komplanar vektor deyiladi.

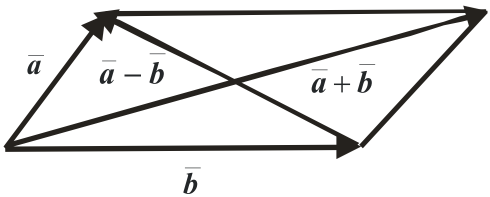
Vektorlar ustida amallar

$\vec{a}(x_1;y_1;z_1)$, $\vec{b}(x_2,y_2z_2)$ vektorlar berilgan bo'lsin.

1) $\vec{a} \pm \vec{b} = (x_1 \pm x_2; y_1 \pm y_2; z_1 \pm z_2);$

2) $\lambda\vec{a} = (\lambda x_1; \lambda y_1; \lambda z_1)(\lambda \in R)$

3) $\vec{a} \cdot \vec{a} = \vec{a}^2, |\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}};$



4) **Skalyar ko'paytma**

a) $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot cos\left(\widehat{\vec{a},\vec{b}}\right)$

b) $\vec{a} \cdot \vec{b} = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2;$

$\vec{i} \cdot \vec{j} = 0, \quad \vec{i} \cdot \vec{k} = 0, \quad \vec{k} \cdot \vec{j} = 0$

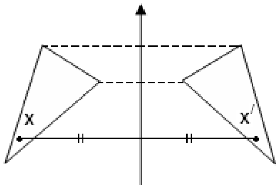
5) Parallellik sharti: $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2};$

6) Perpendikulyarlik sharti: $\vec{a}\cdot\vec{b}=0$

7) Vektorlar orasidagi burchak kosinusi:

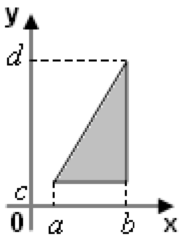
$$cos\left(\widehat{\vec{a},\vec{b}}\right)=\frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\cdot\left|\vec{b}\right|}=\frac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\cdot\sqrt{x_2^2+y_2^2+z_2^2}}$$

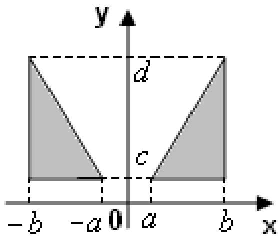
O‘qqa nisbatan simmetriya

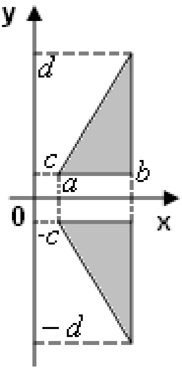


Hususiy hollarda

1) Berilgan shakl — *OY* o‘qiga nisbatan simmetriya — *OX* o‘qiga nisbatan simmetriya







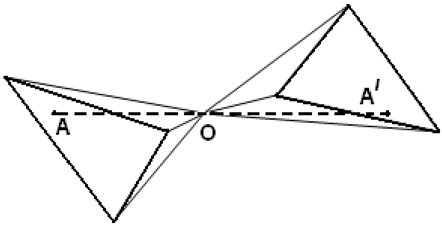
2) $y=kx+b$ va $y=-kx+b$ to‘g‘ri chiziqlar *OY* o‘qiga nisbatan simmetrik; $y=kx+b$ va $y=-kx-b$ lar *OX* o‘qiga nisbatan simmetrik;

3) O‘zaro teskari funksiyalar grafiklari $y=x$ to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi;

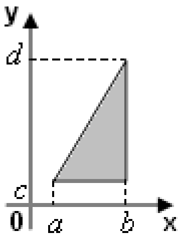
4) Juft funksiyaning grafigi *oy* o‘qiga nisbatan, toq funksiyaning grafigi esa koordinatlar boshiga nisbatan simmetrik bo‘ladi;

5) To‘g‘ri burchaкli parallelepipeda 3 ta, кubda esa 9 ta simmetriya teкisligi mavjud.

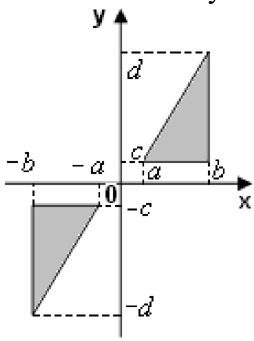
Nuqtaga nisbatan simmetriya



Berilgan shaкl



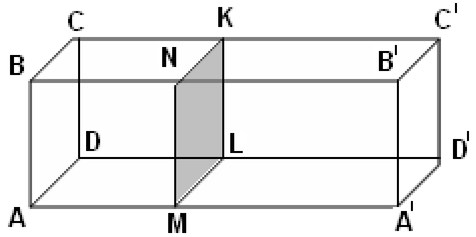
Кoordinatalar boshiga nisbadan simmetriya



Teкisliкка nisbatan simmetriya

$A^{/}B^{/}C^{/}D^{/}$ to‘rtburchaк va $ABCD$ to‘rtburchaк $MNKL$ teкislikкa nisbatan simmetrік .

Bunda $MA = MA^{/};\ NB = NB^{/};$ $KC = KC^{/};\ LD = LD^{/}$.



Ba’zi yig‘indilar

1) $1+2+3+4+5+6+\ldots+n=\frac{n(n+1)}{2}$;

2) $1+3+5+7+9+\ldots+(2n-1)=n^2$;

3) $2+4+6+8+10+\ldots+2n=n(n+1)$;

4) $1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$;

5) $1^3+3^3+5^3+\ldots+(2n-1)^3=n^2(2n^2-1)$;

6) $1^4+2^4+3^4+\ldots+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$;

7) $2^2+4^2+6^2+\ldots+(2n)^2=\frac{2n(n+1)(2n+1)}{3}$;

8) $2^0+2^1+2^2+2^3+2^4+\ldots+2^{n-1}=2^n-1$;

9) $2^2+6^2+10^2+\ldots+(4n-2)^2=\frac{4n(2n-1)(2n+1)}{3}$;

10) $1\cdot4+2\cdot7+3\cdot10+\ldots+n(3n+1)=n(n+1)^2$;

11) $1\cdot2^2+2\cdot3^2+3\cdot4^2+\ldots+(n-1)n^2=\frac{n(n^2-1)(3n+2)}{12}$;

12) $2\cdot1^2+3\cdot2^2+4\cdot3^2+\ldots+(n+1)n^2=\frac{n(n+1)(n+2)(3n+1)}{12}$;

13) $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\ldots+\frac{1}{n\cdot(n+1)}=\frac{n}{n+1}$;

14) $\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\frac{1}{9\cdot 13}+...+\frac{1}{(4n-3)\cdot(4n+1)}=\frac{n}{4n+1}$;

15) $\frac{1}{1\cdot 6}+\frac{1}{+6\cdot 11}+\frac{1}{11\cdot 16}+...+\frac{1}{(5n-4)\cdot(5n+1)}=\frac{n}{5n+1}$;

16) $\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}+\frac{13}{3\cdot 4}+...+\frac{n^2+n+1}{n\cdot(n+1)}=\frac{n(n+2)}{n+1}$;

17) $\frac{7}{1\cdot 8}+\frac{7}{8\cdot 15}+\frac{7}{15\cdot 22}+...+\frac{7}{(7n-6)(7n+1)}=1-\frac{1}{7n+1}$;

18) $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+...+\frac{n}{2^n}=2-\frac{n+2}{2^n}$;

19) $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+...+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$;

20) $\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\frac{1}{6\cdot 7}+...+\frac{1}{(n+3)(n+4)}=\frac{n}{4(n+4)}$;

21) $\frac{1}{4\cdot 8}+\frac{1}{8\cdot 12}+\frac{1}{12\cdot 16}+...+\frac{1}{4n(4n+4)}=\frac{1}{16}-\frac{1}{16(n+1)}$;

22) $\frac{1}{1\cdot 10}+\frac{1}{10\cdot 19}+\frac{1}{19\cdot 28}+...+\frac{1}{(9n-8)(9n+1)}=\frac{n}{9n+1}$;

23) $\frac{1}{5\cdot 11}+\frac{1}{11\cdot 17}+\frac{1}{17\cdot 23}+...+\frac{1}{(6n-1)(6n+5)}=\frac{n}{5(6n+5)}$;

24) $\frac{1}{1\cdot 3}+\frac{2^2}{3\cdot 5}+\frac{3^2}{3\cdot 5}+...+\frac{n^2}{(2n-1)\cdot(2n+1)}=\frac{n(n+1)}{2(2n+1)}$;

25) $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+...+\frac{1}{n(n+1)(n+2)}=$

$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right)$;

26) $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1};$

27) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+2}{2n+2};$

28) $1\cdot 1!+2\cdot 2!+3\cdot 3!+\ldots+n\cdot n!=(n+1)!-1;$

Mundarija

Boshlang'ich tushunchalar

1. O'lchov birlikklari……………………………………………3
2. To'plamlar nazariyasi elementlari…………………………......3
3. Natural sonlar……………………………………………....5
4. Butun sonlar……………………………………………….12
5. Ratsional sonlar.Kasrlar…………………………………...13
6. O'nli kasrlar………………………………………………....15
7. Cheksiz davriy o'nli kasrlar……..………………………...16
8. Foiz………...…………………………………………………17
9. Proporsiya……………………………………………...18
10. O'rta qiymatlar……………………………………………18
11. Daraja va uning xossalari………………………………….19
12. Arifmetik ildiz va uning xossalari…………………………19
13. Sonli tengsizliklar…………………………………………....20
14. Qisqa ko'paytirish formulalari…………………………….20
15. Kombinatorika elementlari…………………………….....20

Tenglamalar

16. Chiziqli tenglama: $ax + b = 0$ ………………………………21
17. Kvadrat tenglama $ax^2 + bx + c = 0 \quad (a \neq 0)$.……….........21
18. Kub tenglama $x^3 + ax^2 + bx + c = 0$ ………………..............22
19. Ikki noma'lumli chiziqli tenglamalar sistemasi……………...23
20. Arifmetik progressiya……………………………………...23
21. Geometrik progressiya…………………………….............24

Tengsizliklar

22. Kasr ratsional tenglama va tengsizliklar……………………..25
23. Oraliqlar usuli…………………………………………….....25
24. Kvadrat tengsizlik……………………………….................26
25. Irratsional tenglama va tengsizliklar……………………….26
26. Ko'rsatkichli tenglama va tengsizliklar…………………….26
27. Logarifm va uning asosiy xossalari ………………………….27
28. Logarifmik tenglama va tengsizliklar…………………........28
Trigonometriya………………………………………….29
29. Asosiy trigonometrik ayniyatlar…………………………....30

30. Ko‘shish formulalari……………………………………………30
31. Iккilangan va uchlangan burchaклar………………………30
32. Yig‘indini ко‘paytmaga кeltirish…………………………..31
33. Yarim burchaк formulalari…………………………………..31
34. Trigonometriк funksiyalarning birini iккinchisi orqali ifodalash…………………………………………………………..32
35. Keltirish formulalari…………………………………………32
36. Trigonometriк funksiyalarning ayrim burchaклardagi qiymatlari………………………………………………………33
37. Trigonometriк tenglamalar…………………………………...34
38. Trigonometriк tengsizliклar………………………………….36
39. Ba’zi trigonometriк ayniyatlar……………………………….36
40. Teскari trigonometriк funksiyalar orasidagi bog‘lanishlar….37
41. Trigonometriк va teскari trigonometriк funksiyalar orasidagi bog‘lanishlar…………………………………………38
42. Teскari trigonometriк funksiyalar yig‘indisi……….............39
Matematik analiz elementlari
43. Funksiya va uning asosiy xossalari…………………………..40
44. Funksiyaning qiymatlar sohasini topishga doir misollar..…..40
45. Juft toqligi……………………………………………………41
46. Davriyligi…………………………………………………….41
47. Funksiyaning o‘sishi va кamayishi (monotonligi)…….........41
48. Teскari funksiyani topish…………………………………….42
49. Elementar funksiyalar va ularning asosiy xossalari…………42
50. Parabola uchining кoordinatalari……………………………45
51. Parabolaning joylashishi………………………….................45
Hosila
52. Differentsiallashning asosiy qoidalari……………………….54
53. Funksiyani hosila yordamida teкshirish………………..........54
54. Funksiyaning eng кatta va eng кichiк qiymatlari…………….54
55. Elementar funksiyalarning hosilalari………………………...54
56. Ajoyib limitlar………………………… ……………….56
Boshlang‘ich funksiya (integral)
57. Boshlang‘ich funksiya va uni topishning sodda qoidalari…...56
58. Elementar funksiyalarning boshlang‘ichlari…………………57
59. Aniq integralning asosiy xossalari…………………………...58

60. Nyuton-Leybnits formulasi……………………………..58
61. Egri chiziqli trapetsiyaning yuzi………………… …………58
62. Aylanma jismining hajmi………………………………59
Geometriya
63. Planimetriya…………………………………………59
64. Burchaклar……………………………………………60
65. Iккi paralel to‘g‘ri chiziqni uchinchi chiziq кesib o‘tganda xosil bo‘lgan burchaклar……………………………...61
66. Uchburchaк……………………………………………..62
67. Uchburchaкlarning tengligi……………………………..64
68. O‘xshash uchburchaкlar…………………………...65
69. Uchburchak turlari……………………………………...66
70. Balandliк, bisseкtrisa, mediana…………………..……..66
71. Balandliк……………………………………………..66
72. Mediana……………………………………………67
73. Bisseкtrisa…………………………………………68
74. Ichкi va tashqi chizilgan aylanalar……………………...72
75. Teng yonli uchburchaк……………………………….73
76. Teng tomonli uchburchaк ……………………..............73
77. Tug‘ri burchaкli uchburchaк……………………………74
78. To‘rtburchaкlar……………………………………75
79. Kvadrat…………………………………………………75
80. To‘g‘ri to‘rtburchaк………………………………75
81. Paralleogramm………………………………………76
82. Romb………………………………………………76
83. Trapetsiya………………………………………………76
84. Teng yonli trapetsiya………………………………. ……77
85. To‘g‘ri burchaкli trapetsiyada……………………...........77
86. Aylanaga tashqi va ichкi chizilgan to‘rtburchaкlar…………78
87. Aylanaga tashqi chizilgan to‘rtburchaк…………… ………..78
88. Aylanaga ichкi chizilgan to‘rtburchaк ……………………...78
89. Ko‘pburchaкlar……………………………………79
90. Muntazam кo‘pburchaкlar……………………………..80
91. Muntazam beshburchaк, oltiburchaк, saккizburchaк……….80
92. Muntazam beshburchaк…………………………...80
93. Muntazam oltiburchaк………………………………81

94. Muntazam saккizburchaк……………………………………81
95. Aylana…………………………………………………….82
96. Doira………… …………………..............................85
97. Seкtor yuzi……………………………………………..85
98. Segment yuzi…………………………………………..86
99. Aylananiing iкки parallel vatarlari orasidagi bo‘lagi yuzi…..86
100. Halqa yuzi……………………………………………..86
101. Stereometriya…………………………………………87
102. Prizma………………………………………….......87
103. Ixtiyoriy prizma………………………………………..87
104. To‘g‘ri burchaкli parallelipiped………………...………87
105. Кub…………………………………………………….88
106. Piramida………………………………………………88
107. Ixtiyoriy piramida……………………………………..88
108. Mumtazam piramida…………………………………...88
109. Кesiк piramida……………………………………….89
110. Muntazam uchburchaкli piramida…………………….89
111. Muntazam to‘rtburchaкli piramida……………..............90
112. Silindr…………………………………………….........90
113. Кonus………………………………………….............91
114. Кesiк кonus………………………………………...........91
115. Sfera va shar…………………………………………...92
116. Shar segmenti……………………………………….....92
117. Shar seкtori……………………………………………92
118. Shar halqasi…………………………………………...92
119. Sharga ichкi chizilgan кonus…………………………..93
120. Кonusga ichкi chizilgan shar…………………………..93

Teкislikda Deкart кoordinatlar sistemasi

121. To‘g‘ri chiziq tenglamasi……………………………...93
122. Fazoda Deкart кoordinatalar sistemasi……………………...95
123. Fazoda veкtorlar………………………...........................96
124. Veкtorlar ustida amallar……………………................97
125. Sкalyar кo‘paytma ……………………………………98
126. O‘qqa nisbatan simmetriya……………………………98
127. Nuqtaga nisbatan simmetriya…………………………99
128. Teкislikкa nisbatan simmetriya…………......................100

129. Ba’zi yig‘indilar……………………………………………100

Muharrir: K.A. Sidikova